STRUCTURAL ENGINEERING

The quality of the materials used in the manufacture of this book is governed by continued postwar shortages.

STRUCTURAL ENGINEERING

By

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PREFACE TO THE SECOND EDITION

The first edition of this book has been used by students and by practical men throughout the United States and in many foreign countries for the past eighteen years.

The author wishes to thank the many engineers, architects and students for the favorable reception accorded the first edition and he hopes that this edition will prove to be of additional service to them.

The work of revision consisted in correcting typographical and arithmetical errors and in revising and rewriting parts to bring the book as a whole in accord with the latest practice in designing.

Chapter XII, on design of highway bridges, has been entirely rewritten and includes new cuts and designs. The part of Chapter XIV on mill building has been revised and the part on high buildings has been entirely rewritten to present the designing of the latest types of high buildings, including riveted and welded construction, using practical formulas based upon slope deflection to obtain stresses due to unsymmetrical loading.

Appendices *A* and *B* have been added.

Appendix *A* gives specifications for railroad bridges.

Appendix *B* is a theoretical and practical presentation of the method of slope deflection as applied in the analysis of rigid frames.

The author wishes to thank Prof. R. E. Kirkham of the Oklahoma Agricultural and Mechanical College for his help in checking the manuscript.

J. E. KIRKHAM.

Stillwater, Okla., December, 1932.

PREFACE TO THE FIRST EDITION

This book is intended as a textbook for college students and as a self-explanatory manual of structural engineering for practical men.

During the author's nineteen years of engineering experience (the greater part of which was spent in actual practice) it fell to his lot to "break in" students from almost all our engineering schools, and he has no apology to offer for the elementary mechanics given in this volume, as he is convinced that no matter how thorough the course in mathematics and theoretical mechanics may be it is quite desirable that students have a short review of the materialistic phase of mechanics at the beginning of the subject of structures to whet their appetites for the work.

As regards the designing given, the author has endeavored to present the usual methods, and this in such a fashion that the average engineering student as well as the practical man can read, understandingly, without having a dictionary, glossary, or a compendium on theoretical mechanics at his elbow.

The author's aim has been to arrange for college use the drawing room exercises throughout the text, so that the work in the classroom and drawing room will go hand in hand. The appreciation of this feature will depend a great deal upon the allotment of hours for the two classes of work. The ratio of two hours' recitation to six in drawing is recommended. "Drawing Room Exercise No. 1," at the end of Chapter II, can be started at the very first drawing room period without wasting any time. Chapters I, II, VII, X, and Chapter XI as far as "Deck Plate Girder Bridges," should be read by the time "Drawing Room Exercise No. 1" is finished, so that "Drawing Room Exercise No. 2" can be taken up. During the time spent on "Drawing Room Exercises No. 2, No. 3, and No. 4" ample time will be found for the necessary advance reading—up to "Through Plate Girder Bridges"—and the reading of Chapters III to VI and also VIII and IX. Beyond this the author refrains from making suggestions as to assignments as he has confidence in the ability of instructors to assign correctly the work to suit conditions.

The practical men of limited theoretical training, and others desiring a review of the subject treated, will find the book well suited to their case if the chapters be taken up in consecutive order.

The designs given in this book are entirely the work of the author and are designed especially for this work, yet he has endeavored to make them as general as possible.

This book, which treats only of simple structures, is the author's first volume on Structural Engineering. A second volume, which will be known as Higher Structures, is in preparation; this will treat of Movable Bridges, Cantilever, Arch, and Suspension Bridges, Secondary Stresses, etc.

The author desires to acknowledge his appreciation of the work of his assistant, Mr. B. S. Myers, in reading and checking proof and preparing and checking drawings, and to thank Prof. F. O. Dufour for valuable suggestions and criticisms.

J. E. KIRKHAM.

Ames, Iowa, Sept. 5, 1914.

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CHAPTER I

PRELIMINARY

1. Structural Engineering.—The part of Civil Engineering pertaining to the designing of steel structures, such as bridges, buildings, towers, etc., is known as Structural Engineering. The work involved consists, principally, in determining the stresses, selecting the material to be used, known as sections, and the contriving and drawing of the details. But, in addition to this work, the structural engineer has, as a rule, the designing of a great deal of incidental construction, such as foundations, concrete floors, roofs, etc.

In order to design structures properly, one must be perfectly familiar with the material used in their construction and have a clear understanding of the manner in which their manufacture and erection are accomplished as well as have a thorough knowledge of the mechanical principles involved throughout. A properly designed structure is one wherein no mechanical principles are seriously violated and which at the same time is economic in material and easily manufactured and erected.

2. Structural Material.—Steel, concrete, stone, and wood are the principal materials used by the structural engineer in the construction of modern structures. However, cast iron, wrought iron, and a few other materials are used in a few cases, as will be designated when these cases present themselves in the text. Concrete and stone will not be treated in this book further than to designate certain allowable pressures.

3. Manufacture of Steel.—In describing in a general way the usual process of making steel, we can say that steel is made from iron ore which is mined and carried to a blast furnace through which it is run together with coke and limestone and thus converted into cast iron. The cast iron is then taken to either an open-hearth furnace or to a Bessemer furnace, usually spoken of as a Bessemer converter, and there converted into steel. When the conversion is complete, the molten steel is run into ingot molds, which are rectangular cast-iron molds, and molded into ingots. These ingots are allowed to cool to some extent. Then the molds are pulled off and the ingots are taken to a rolling mill, which is known as a slab or blooming mill, where they are heated in what is known as a soaking pit until they have the proper temperature for rolling. Then they are rolled to a convenient rectangular cross-section and cut up into convenient-sized pieces known as slabs or billets. These billets or slabs, as the case may be, are then taken to another rolling mill and reheated very much the same as the ingots just described, and then rolled into structural shapes, plates, rails, etc., ready for the market.

Sometimes the cast iron from the blast furnace is cast into rough bars, known as pig iron, which may be stored and converted into steel or molded into castings at any desired time, or it may be shipped to some distant point to be utilized in the same manner. At the most modern plants the molten metal is taken directly from the blast furnace and converted into steel without letting it cool to any great extent.

The steel produced in an open-hearth furnace is known as "open-hearth steel," while the steel produced in a Bessemer converter is known as "Bessemer steel." Open-hearth steel is considered more reliable in every respect than Bessemer, and, consequently, the Bessemer steel is being fast replaced by the open-hearth product. In fact, most engineers at present exclude the use of Bessemer steel in structural work altogether.

4. Grades of Common Structural Steel.—There are three recognized grades of common structural steel: Medium Steel; Soft Steel; Rivet Steel.

Medium Steel is a medium-hard steel which has practically replaced all other grades of steel in structural work. It has an ultimate strength of 60,000 to 70,000 pounds per square inch, and an elastic limit of 30,000 to 35,000 pounds per square inch.

Soft Steel is softer than medium steel. It was the grade of steel first used in structural work, but has been practically replaced by medium steel. It has an ultimate strength of 52,000 to 62,000 pounds per square inch and an elastic limit of 26,000 to 31,000 pounds per square inch.

Rivet Steel is a very soft steel used almost exclusively for making rivets and bolts. It has about the same chemical composition as wrought iron, but has a higher ultimate strength and elastic limit. It has an ultimate strength of 48,000 to 58,000 pounds per square inch and an elastic limit of 24,000 to 29,000 pounds per square inch.

5. Nickel Steel is a steel containing from 2 per cent to 3½ per cent of nickel. It has an ultimate strength of 80,000 to 112,000 pounds per square inch and has a very high elastic limit—something like 60,000 pounds per square inch. This metal has been used recently in some of the large bridges in this country.

6. High Carbon Steel is a hard steel which contains more combined carbon than the ordinary medium steel. It has practically as high ultimate strength and elastic limit as nickel steel. It is used almost exclusively in the form of rods to reinforce concrete.

7. Wood, or timber, as it is usually spoken of, is used in structural work principally for floor and roof covering. The timber mostly used is white oak and yellow pine, both of which have an average ultimate crushing strength of about 7,000 pounds per square inch, and an ultimate tensile strength of about twice that amount.

8. A Casting is made by running molten metal into an impression made in sand by means of a wooden pattern which is shaped to the size of the metal piece desired. The castings used in structural work are made of either cast iron or steel. The castings made of steel are much stronger than those made of cast iron but cost more. The steel used for making castings has a somewhat different chemical composition than that of rolled steel, referred to above. It is produced in the same way, however, the chemical composition being controlled mostly in the mixing of the charge.

9. Structural Steel Shapes.—The I-beam, channel, angle, Z-bar, and T-shape, shown in Fig. 1, are the principal steel shapes used in structural work. The T-shape is not used very extensively except for very special work. Plates, while they are not classified as structural shapes, are really used in a more general way than the shapes.

The tables in the back of this book give the properties, gauges, etc., of the shapes and plates in general use. The use of these tables will be explained as the occasion for their use occurs. Either a Carnegie or Cambria handbook is a convenient book to have—in fact, practically indispensable in structural work—but the student must bear in mind that he is not at liberty to use just any section therein that he happens to pick out, as a great many specials which are not in general use are listed, which will be furnished only when the order for such becomes large enough to warrant the rolling, and, consequently, in the case of ordinary orders containing these special sections, an unusual delay in obtaining the material will likely be experienced.

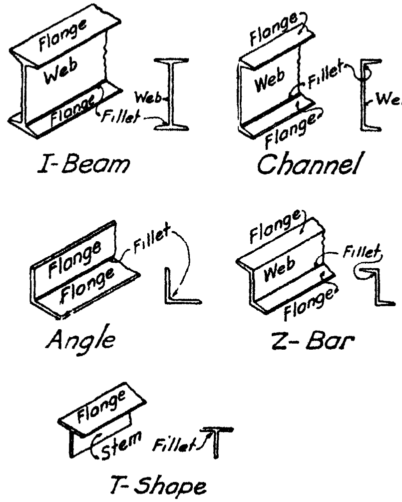


Fig 1

10. Rivets and Riveting.—Rivets are used in structural work to connect the structural shapes and plates together. They are simple bits of metal in themselves, but, indirectly, they are the source of a great deal

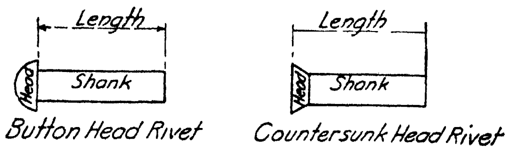


Fig. 2

of trouble. Practically, there are but two kinds: the buttonhead rivet which has a hemispherical head, and the countersunk rivet which has a countersunk head. Each kind is shown in Fig. 2, including the names of their parts. The buttonhead rivet is the kind most used; in fact, the countersunk rivet is never used except where it is absolutely necessary to do so.

As a rule, each structural company manufactures its own rivets. They are made by feeding red-hot rods into a machine which cuts the rods into pieces of proper length and at the same time forms a head on each piece, thus completing the rivets as they are shown in Fig. 2. The way in which this is done can be seen by referring to Fig. 3, where the parts of the machine that directly form the rivets are shown. Let us first consider the plan diagram at (*a*), where *A* represents the "header," *B* a fixed die, *C* a moving die, *D* a forked bar, and *S* a bar known as the gauge. While the parts are in the position shown at (*a*) the heated rod is pushed into the machine, passing through the slot in the bar *D* until it comes in contact with the gauge *S*, as indicated by the dotted outline *e-n-o-k*. Then the die *C* moves toward *B* and cuts the rod off at *o* by shearing with the bar D, and at the same time the piece of the rod thus cut off is caught in the cylindrical grooves in the dies (see section m-m) and held firmly while the gauge S moves up out of the way of the header A which then moves until it comes in contact with the dies B and C, upsetting the part e-n of the piece of the rod into the hemispherical cup in the header A, thus forming the head of the rivet. The parts of the machine are then in the position shown at (b). The header A and then the die C and the gauge S move back to the position shown at (a), while the rivet just formed drops into a cooling basin. Then to obtain the next rivet, the rod is fed into the machine as before. Thus rivets are made at the rate of about one per second.

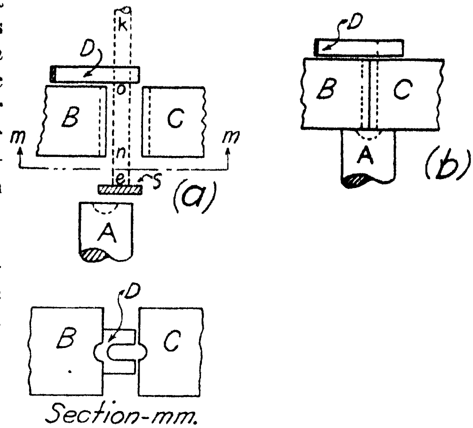


Fig 3

The rivets formed by the machine just described are the buttonhead rivets. However, countersunk rivets can be made on the same machine by replacing the header shown by a header having a plane end (without any cup), and the dies shown by dies which have the cylindrical groove chamfered out on the end next to the header so as to form the countersunk head. In that case the heads are formed in the dies instead of in the header as in the case of the buttonhead rivet.

After the rivet holes have been either punched or drilled into the plates and shapes to be riveted together, and the same have been assembled, each in its proper place, rivets are heated, placed in the holes, and "driven." The driving of a rivet consists principally of forming a head on the plane end, usually the same as the one on the other end formed by the rivet machine just described.

Rivets are driven by machines known as "riveters" except in a few cases where it is necessary to drive them "by hand." The riveter, or rivet-driving machine, has two headers known as tools, one being fixed and the other movable, each of which is very similar to the header used in the rivet-manufacturing machine described above.

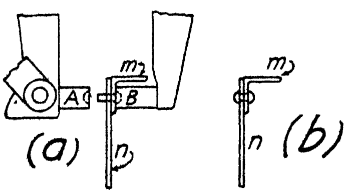


Fig. 4

In order to show the working of a riveter, let m (Fig. 4) represent an angle which is to be riveted to a plate n. The rivet holes are either punched or drilled in the two so as to match. Then the plate and the angle are placed together as shown, and the driving of the rivets proceeds as follows: A rivet is heated and placed in one of the holes, as shown at (a). Then the fixed tool B of the riveter is placed against the head of the rivet and the power is applied (which may be air, steam or water) which moves the tool A toward B whereby the projecting part of the rivet is upset into the cup in the end of *A*, thus forming the head on that side. When the tool *A* moves until it practically touches the plate *n*, the rivet is fully driven, as shown at (*b*). The tool *A* is then brought back to its former position, and the riveter is placed on the next rivet by either moving the pieces being riveted together, or by moving the riveter, and the operation of driving is repeated, and so on.

In case a countersunk rivet is to be driven, the rivet hole is either punched or drilled the same as though a buttonhead rivet were to be used. Then the hole is countersunk, which means that it is reamed out cone-shaped on the side where the countersunk head is to come so that the head will just fit in. The countersinking of the hole is done, usually, with a flat, diamond point drill, as shown at (*a*), Fig. 5. The driving of a countersunk rivet with a riveter is very much the same as driving a buttonhead rivet. For example, suppose a rivet having a countersunk head is to be used to connect the angle *m* to the plate *n* as shown at (*b*), Fig. 5, where the countersunk head is on the same side as the angle. The rivet hole will be countersunk in the angle as shown. The countersunk rivet is heated and placed in the hole from the same side as the angle and driven just the same as was explained above in the case of the buttonhead rivet, but the tool held against the countersunk head would be a plane tool without a cup. If the other tool has a cup in it, the head on that side would be an ordinary buttonhead, as shown at (*c*), Fig 5. If it were necessary to have a countersunk head on each side, the hole would be countersunk in the plate as well as in the angle, as shown at (*d*), Fig. 5. The rivet before driving would have a countersunk head, the same as before, and the driving would take place in the same manner, but each of the tools would have a plane end.

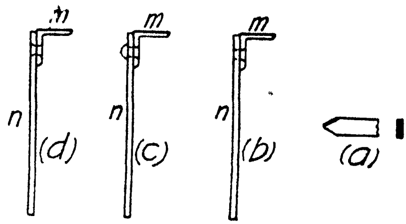


Fig 5

In cases where it is either impossible or impracticable to use the riveter, the rivets are driven by hand. This, in the case of a buttonhead rivet, consists of heating the rivet and placing it in the hole, and while the end of a round bar which contains a cup (known as a dolly bar) is held firmly (by hand) against the head, the projecting end of the rivet is battered down with a hammer and then formed into a finished head by a heading hammer (known as a snap) which is held against the battered end of the rivet and struck by a sledge. A heading hammer, or snap, is really a two-faced hammer with a cup in one end used to form the head of the rivet. In the case of a countersunk rivet, the driving by hand is practically the same as in the case of the buttonhead rivet, but the tools used on the countersunk heads will be plane, that is, without cups.

There are but very few cases where it is necessary to drive rivets by hand in the shop, but practically all of the rivets connecting the individual members in a structure to one another are driven by hand as the structure is being erected. Such rivets are known in structural engineering as "field rivets." Holes left open in the shop for such rivets are known as open holes or field holes.

The thickness of metal connected by a rivet is known as the grip of the rivet. The diameter of the shank of a rivet is known as the diameter of the rivet. It is also known as the size of the rivet.

The part of the rivet projecting beyond the material to be riveted together is just a little longer than that necessary to form a head, for, in driving, some of this is taken up in upsetting the shank of the rivet into the hole which is always about $\frac{1}{16}$ in. larger in diameter than the shank.

The size of the rivets used depends upon the material to be connected. A common rule is that no rivet shall be less in diameter than the thickness of any single piece connected. Rivets vary in size from $\frac{1}{4}$ in. in diameter to $1\frac{1}{4}$ ins. The rivets most often used are the $\frac{7}{8}$- and $\frac{3}{4}$-in. diameter.

11. Method of Procedure in the Designing of Steel Structures.—As a rule, parties desiring structures built specify the location and purpose, and limit to some extent the amount they shall cost. The engineering work really begins with the survey of the site, which, however, does not necessarily require the services of a structural engineer, yet a structural engineer should know how to do such work as it is sometimes required of him; and also such knowledge is often essential in working out the design of a structure. The structural engineering work really begins after the drawings showing the site are made from which the structural engineer works out the preliminary plans of the structure. These preliminary plans show, in a general way, what is desired. They usually consist of what are known as "Stress Sheets" and "General Drawings." A Stress Sheet is usually a single-line drawing of the structure wherein each member is represented by a single line upon which the corresponding stresses and sections are written, and the general dimensions of the structure are given in reference to centers of bearing and to centers of gravity. A General Drawing is usually intended to show the structure as a whole. In many cases it is a general picture of the structure wherein the details are shown in a general way but not fully dimensioned. In the case of very important structures, the general drawings usually include a general picture of the proposed structure showing no specific details at all. Such drawings may be prepared by an engineer or by an architect. The purpose is to present the general appearance of the structure. That such drawings are included does not lessen in the least the necessity of the other general drawings.

After the general drawings have been completed to the satisfaction of all parties concerned, they are used as a guide in making the "Shop Drawings" (sometimes called "Working Drawings") which show the spacing of rivets and all holes, cuts, etc., for each individual member of the structure. After these shop drawings are made and thoroughly checked and are approved by all parties concerned, they are sent to the structural shops where each member of the structure is fabricated accordingly.

All of the drawings referred to above are, as a rule, made on tracing cloth and blueprints are made from these which are furnished to all parties concerned instead of actual drawings. These prints, as a rule, are what are referred to as drawings.

12. A Bridge Company is an organization which designs, manufactures and erects structural work. However, the name is appropriated by various concerns which in many cases are capable of doing only part of this work. A full organization really consists of an operating department and an engineering department. The operating department has the general management of the company, especially the commercial end, while the engineering department attends to everything pertaining to the engineering. It suffices here for us to consider only the engineering department. This department consists of a designing and estimating department, a draughting department, a shop organization, and an erecting department.

The work of the designing and estimating department consists mainly in determining the stresses in structures, the selecting of the required sections, drawing up the stress sheets showing the same, making preliminary estimates of the weight of the material to be used, and computing the cost of the work proposed to be manufactured by the company.

The work of the draughting department consists of the working out of the complete details of structures, using the stresses and sections as specified on the stress sheets (which are furnished by either the designing and estimating department of the company or by outside parties) and making the necessary general and shop drawings. In addition, the draughting department makes out bills of the material required from which the material is ordered from the rolling mills. These bills when completed are known as "Shop Bills" and are sent to the shops along with the shop drawings.

The work of the shop organization consists in fabricating the individual members of the structures in the shops according to the shop drawings furnished by the draughting department.

The work of the erecting department consists in erecting the structures in their final position after the individual members are fully fabricated in the shops.

The shops are divided into departments which are referred to as separate shops. These different shops, or departments, are as follows: Templet Shop, Pattern Shop, Laying-off Shop, Punch Shop, Rivet Shop, Finishing Shop, Forge Shop, Machine Shop, Paint Shop, Foundry, etc., the name of each clearly indicating the nature of the work done therein. Some departments occupy separate buildings, while in some cases several departments are under one roof.

13. Fabrication of Steel Structures.—The draughting department of a bridge company usually orders the material for any structure that the company is to fabricate just as soon as the preliminary work on the shop drawings is far enough along. The steel mills roll this material and ship it to the shops where it is unloaded and placed in the "receiving yards," where it remains until the shops need it. After the shop drawings are completed, blueprints are made of them which are sent to the shops, each department receiving the prints showing the part of the work required of it. The fabrication of riveted work, as a rule, begins in the templet shop where full-sized wooden templets are made for most of the shapes and plates in the structure. These templets are made so that each hole and cut shown on the shop drawings can be located on the actual pieces of metal used. These templets, as a rule, are made of one-inch white pine boards, well seasoned. The templets are either made of one board or of two or more boards connected together, quite often forming a frame. Pasteboard is being used of late, to some extent, in making templets.

The templets are taken to the laying-off shop where they are clamped to the material for which they were made to lay off. Then the workmen mark where each hole is to be in the metal by placing a center punch in each hole in the templet and striking the punch with a hammer, and indicate the cuts required by marking the outline of the templet on the metal. Then the templets are unclamped and thrown to one side and the material thus laid off is taken to the shearing and punch shop where all cuts are made in accordance with the marks, and the rivet holes are punched or drilled as indicated. After this work is completed, the material is taken to the assembling shop where the pieces are assembled so as to form individual members of the structure and bolted together temporarily, just a few bolts being used in each member. Then these members are taken to the rivet shop where the rivet holes are reamed, if such is called for, and all the rivets indicated on the shop drawings are driven. Then the members are taken to the finishing shop where pin holes are bored and all surfaces and joints are finished as called for on the shop drawings. When this work is completed the members are taken to the paint shop where they are cleaned and painted. Then they are taken to the loading yards where they are loaded on cars for shipment to the site. Thus the fabrication is completed.

Usually when plates and shapes are duplicated a great many times, templets are not used. The duplicated pieces, in that case, are run through a multiple punch which is so constructed that the operator can punch the rivet holes by referring directly to the shop drawings.

In addition to the riveted work referred to above, there are usually castings, forged work and machine-shop work included in each structure which are gotten out by the foundry, forge shop and machine shop independently of the other shops, except that the patterns used in molding the castings are made in the pattern shop, which is usually very closely associated with the templet shop. This work, when completed, is loaded on cars and shipped to the site, the same as the riveted work, often going on the same cars.

CHAPTER II

STRUCTURAL DRAUGHTING

14. Preliminary.—It is very essential that all young engineers should be good draughtsmen, as they are usually called upon to do considerable draughting, and good draughting is highly appreciated throughout the engineering profession. Good draughting is an accomplishment; however, one sometimes hears of a "natural born" draughtsman. A newspaper reporter while talking to one of our greatest inventors referred to him as a genius. The inventor retorted that genius and perspiration were synonymous terms. The inventor may have been slightly mistaken in that particular case, but in this case there is no question: anyone can acquire the art of making good drawings, that is, good, neat, plain, practical drawings, nothing of an artistic nature being considered, by simply going about it with a will, being careful, each time trying to do a little better than before.

Structural drawings consist of lines, letters, and figures. The lines should be true and distinct. The letters and figures should be plain, neat, free-hand letters and figures. Each can be practiced independently of the others, but after a fair amount of such practice, good draughtsmanship can be most readily acquired by making exact copies of some good drawings, providing the letters and figures be of the same type as those being practiced by the student, as it is not advisable to shift from one type to another until one type is fairly mastered.

15. The Necessary Equipment for Structural Draughting consists of a drawing board, T-square, two triangles, one decimal scale, one duodecimal scale, one first class, medium size, right line drawing pen, one first class, small ink compass, one large combined ink and pencil compass, one pair of medium size dividers, a slide rule, either a Carnegie or Cambria handbook, one small bottle of black waterproof drawing ink, a writing pen and holder, a scratch pad, drawing pencils, thumb tacks, erasers, drawing paper, and tracing cloth.

There are several very useful things which could be added to the above list, such as logarithmic tables, book of squares, beam compass, etc. There are no objections to a full set of drawing instruments instead of the few instruments mentioned above, but in either case, the instruments should be first class. It is better to have a few good instruments than a full set of poor ones.

16. Free-Hand Letters and Figures.—Some distinct characteristics will always be seen in the free-hand lettering of each individual the same as in the case of ordinary writing, but the type of letters and figures should always conform to some recognized standard. Poor lettering should not be recognized as a characteristic, but as an indication of lack of practice.

Most of the free-hand lettering on drawings is done on tracing cloth, so it is best to practice upon the same. The small remnants of cloth which would otherwise be wasted can often be utilized for that purpose. The student can have the necessary material, which includes a small

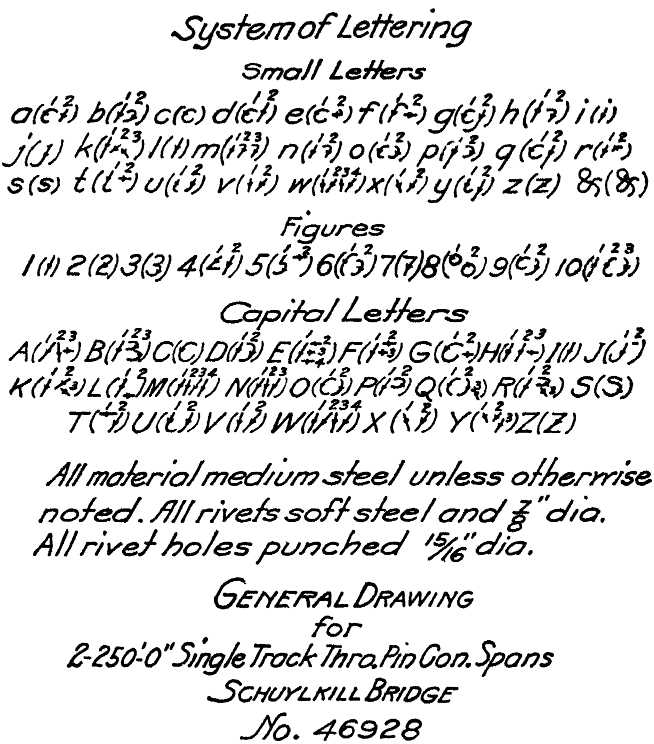


Fig 6

bottle of drawing ink, writing pen, and tracing cloth, at his private room and practice during spare moments as a diversion. Opinions will differ somewhat as to the kind of pen to use in making free-hand letters and figures. The author prefers Gillott's No. 303.

The letters and figures shown in Fig. 6 are of about the type used on structural drawings. Just how each letter and figure is made is fully indicated, the arrows indicating the direction of the strokes, and the small figures above the letters and figures indicating the order in which the strokes are made.

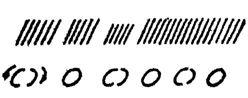


Fig 7

The student can best practice making the letters and figures shown in Fig. 6 by making as nearly as possible exact copies of the work shown there. In addition, at intervals he should practice drawing parallel lines (free hand) having the same slope as the letters, and curves which form the letter O, all of which is outlined in Fig. 7. If there is any letter or figure which gives the student particular trouble, he should keep practicing upon it at intervals until he has mastered it.

17. Size of Drawings.—The usual practice in structural work is to make the drawings 23 x 35 inches inside the border line with a one-half inch margin on all sides. This, however, is not an absolutely fixed size. In some cases it is necessary to make larger drawings, and in other cases it is convenient to make smaller ones. Drawings 18 x 24 inches is a very convenient size in some cases. The border line should be an ordinary plain, single line.

18. Tracings.—It is the usual practice to pencil out all drawings upon ordinary drawing paper, and then make tracings of these pencil drawings upon tracing cloth. The drawing on the tracing cloth should be upon the dull side. Before starting the tracing, the cloth should be rubbed over with talcum powder or with chalk, which should then be brushed off with a cloth or brush. A knife should never be used to make erasures on the cloth. Repeated erasures can be made by using a comparatively soft rubber, as the Ruby Eberhard Faber No. 112. No other than this quality of eraser should be used.

19. General Hints Regarding Shop Drawings.—The shop drawings for a structure are usually the last drawings made, but in order to work up the preliminary drawings satisfactorily, knowledge of shop drawings is indispensable, which makes it necessary that the student take up the making of some simple shop drawings as preliminary work. As stated above, the shop drawings show the complete details of the individual members of structures, that is, all rivets, holes, and cuts are located, and sizes of all shapes and plates are given, and in addition, the kinds and

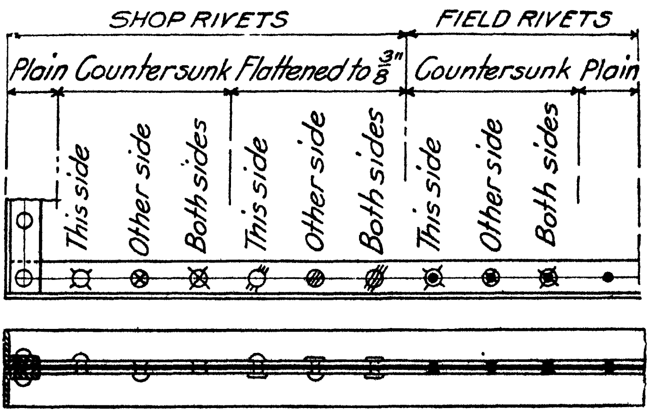


Fig. 8

sizes of rivets and holes and also the nature of the machine work desired are indicated.

In order to indicate the kind of rivets desired, certain conventional signs are used, most of which are shown in Fig. 8. Shop rivets are those driven by riveters while the members are in the shops, and the field rivets are those driven at the site as the structures are being erected. Countersunk rivets are used on the account of clearance, but sometimes sufficient clearance will be obtained by just mashing down the heads of the rivets. In such cases the heads are said to be flattened.

Rivet holes are usually $\frac{1}{16}$ inch larger than the rivets to be driven into them. The sizes of the heads of rivets are given in Fig. 9. The sizes of the heads shown on drawings should correspond with the sizes given here.

In locating rivets, the following rules should be practically followed Never space rivets closer together, that is, center to center, than three

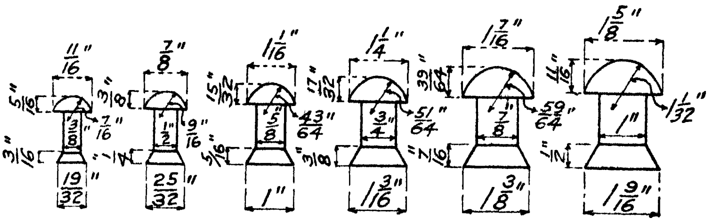


Fig. 9

times the diameter of the rivets, nor farther apart than six inches. Never space rivets closer to the edge or end of a shape or plate than two times their diameter. These rules are not iron-clad, but the deviation from them should be slight. For example: in practice it is customary to space $\frac{7}{8}$-inch rivets $1\frac{1}{2}$ inches from the edge or end of a shape or plate, and $\frac{3}{4}$-inch rivets $1\frac{1}{4}$ inches. It is also customary to limit the minimum distance between $\frac{7}{8}$-inch rivets to 3 inches, and $\frac{3}{4}$-inch rivets to $2\frac{1}{4}$ inches.

The spacing of rivets or holes in reference to one another along one direction is known as the pitch. Rivets or holes passing through the flange of any shape are located in reference to the shape by what is known as the gauge. For example, the distance between any two consecutive rivets or holes along the angles shown in Fig. 10 is the pitch at that point, while the distance *g* from the back of the angles to the line *o-o,* passing through the rivets, is the gauge. The distances marked *e* are

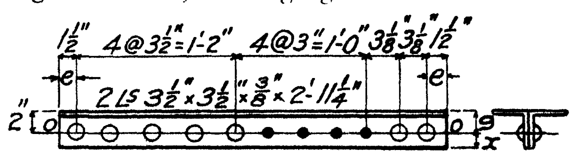


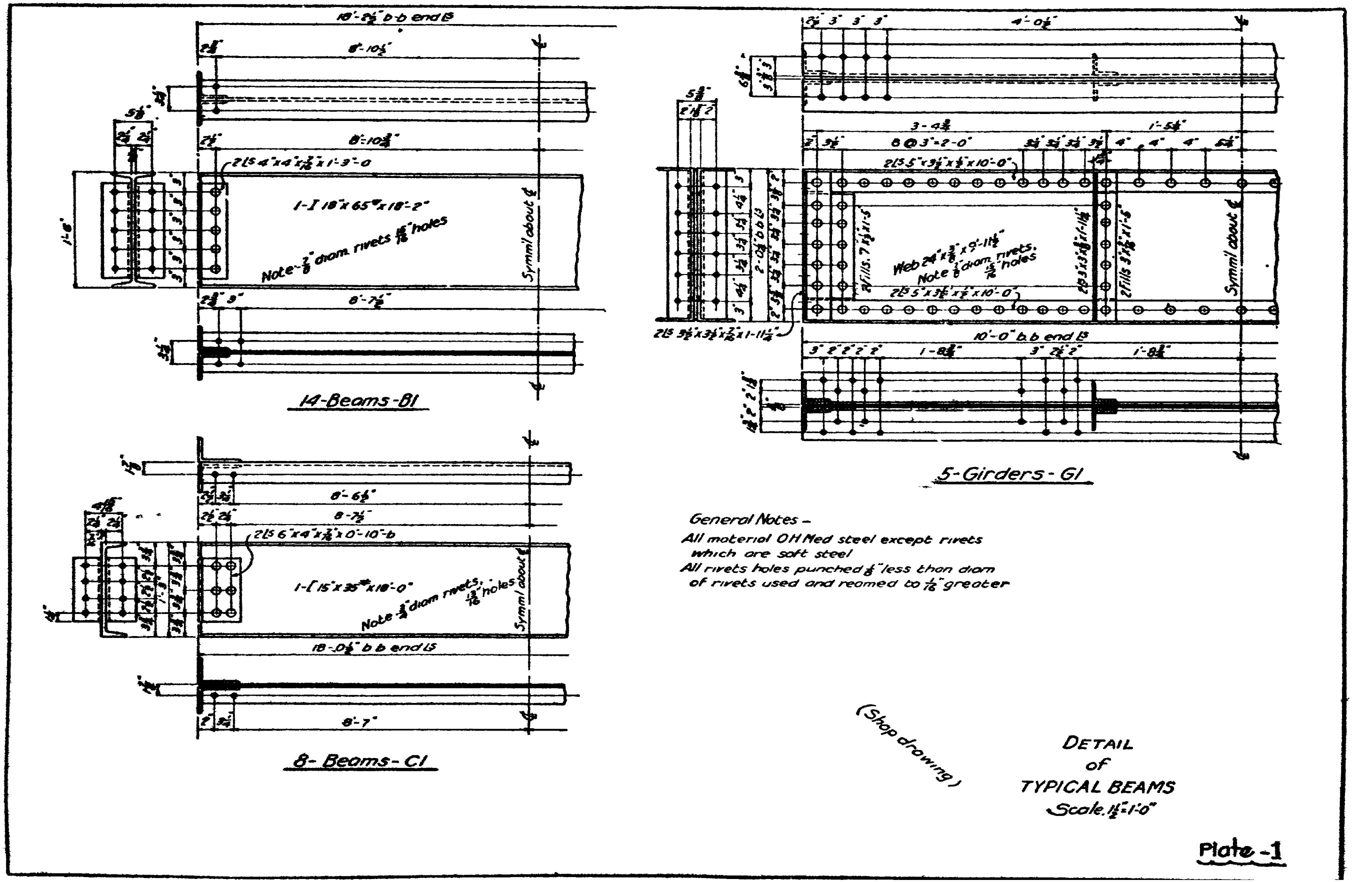
Fig. 10

known as the end distances. In case the flange of a shape has double gauge lines, the pitch is the distance between the rivets measured along the piece and not the distance between the rivets on one gauge line.

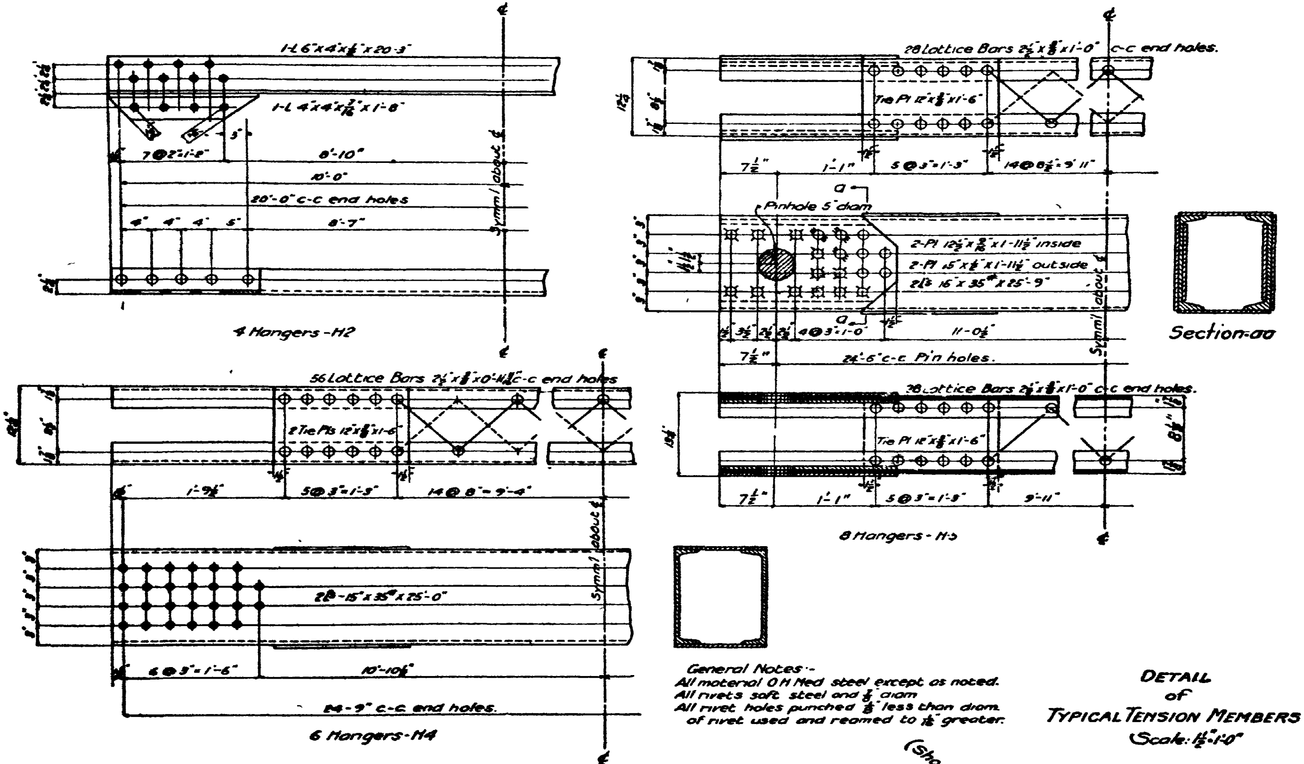
The standard gauges for shapes are given in the tables in the back of this book. The same are to be found in the various "standards" and handbooks gotten out by structural companies.

No rivet or hole should be located in reference to the edge of a flange, as would be done by giving the distance *x* in Fig. 10.

All lines upon which rivets are located are known, in general, as rivet lines, and the lines used in giving the spacing of the rivets, as well as all other distances, are known as dimension lines. All rivet lines (including gauge lines and dimension lines) should be light in comparison

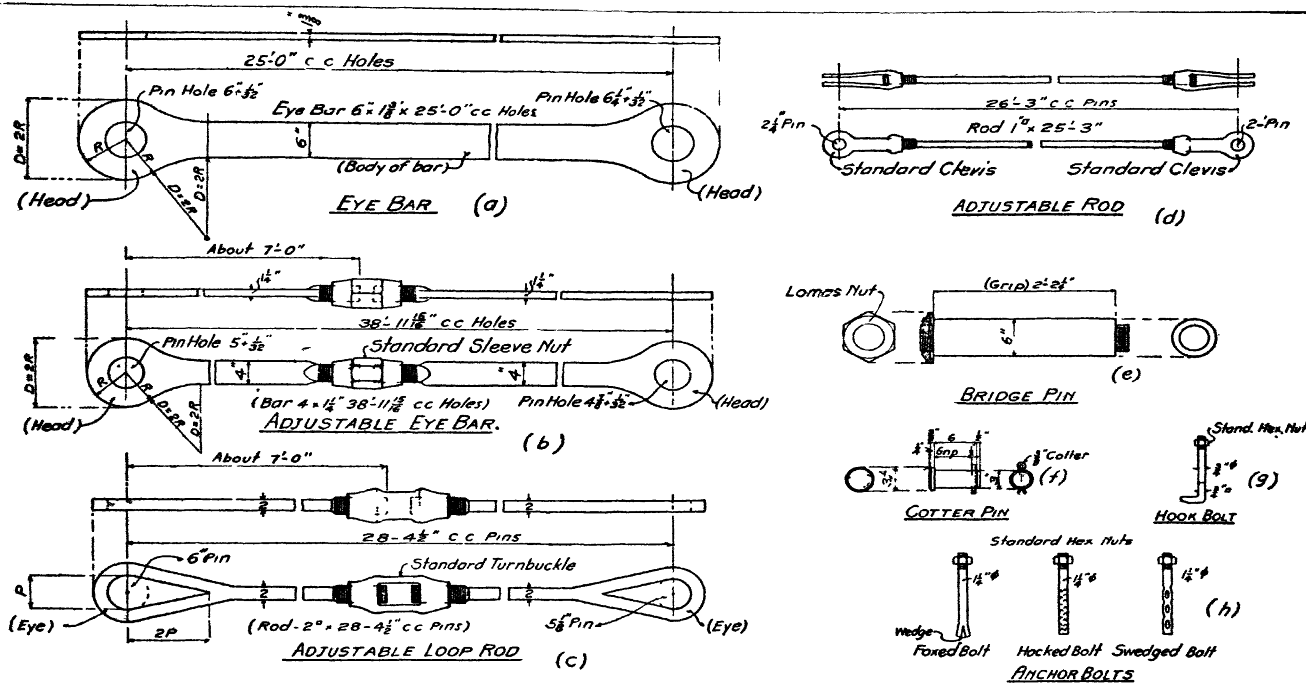


14-Beams-B1
1-I 18"x 65#x 18'-2"
2Ls 4"x4"x7/16"x 1'-3"-0
Note-7/8" diam rivets 15/16" holes
10'-3½" b-b end Ls
Symm'l about ℄
8- Beams-C1
1-[15"x 35#x 18'-0"
2Ls 6"x4"x7/16"x 0'-10"-b
Note-3/4" diam rivets, 13/16" holes
18'-0½" b b end Ls
5-Girders-G1
Web 24"x3/8"x 9'-11½"
Note 3/4" diam rivets, 13/16" holes
2Ls 5"x3½"x½"x 10'-0"
2Fills 7"x3/8"x 1'-5"
2Fills 3"x3/8"x 1'-6"
2Ls 3½"x3½"x7/16"x 1'-11½"
10'-0" b.b end Ls
General Notes -
All material OH Med steel except rivets which are soft steel
All rivets holes punched 1/8" less than diam of rivets used and reamed to 1/16" greater
(Shop drawing)
DETAIL of TYPICAL BEAMS
Scale 1½"=1'-0"
Plate -1



DETAIL
of
TYPICAL TENSION MEMBERS
(Scale: 1½"=1'-0")

Plate-2.



Note The necessary data for drawing the eye bar heads, sleeve nuts, etc can be found in either a Carnegie or Cambria Hand Book

SKETCHES
of
Forged Tension Members, Sleeve Nuts,
Turnbuckles, Clevises, Pins & Bolts.

Plate 3

with the lines used to outline the members. All should be in black ink.

The duo-decimal scale is used in laying out structural drawings. The usual scale for shop drawings is three-quarters inch equals one foot, but in some cases a scale of one-half inch equals one foot is used. In light work it is sometimes advisable to use a larger scale, in which case a scale of one inch equals one foot or one and one-half inches equals one foot is used. It is advisable for the student to use a large scale at the beginning.

In making structural drawings, it is customary to represent feet by putting one dot above the figures specifying the number of feet, and two dots in the case of inches. Thus 4′ is read as four feet, and 4″ is read as four inches. 2′-2″ is read as two feet and two inches; 6′-7½″ is read as six feet and seven and one-half inches, and so on. Square feet and square inches are represented by placing a square in front of the dots. Thus 4□′ is read as four square feet, and 4□″ is read as four square inches.

In specifying the material on structural drawings it is customary to represent the shapes by symbols instead of writing the name. Thus, the I-beam, channel, Z-bar, and angle are represented by I, [, Z, and L, respectively, or in case of plural, as Is, [s, Zs, and Ls, respectively.

All dimensions and sizes given on structural drawings are in feet and inches and fractional parts of inches. The fractional parts are expressed in halves, fourths, eighths, sixteenths, thirty-seconds, and sometimes in sixty-fourths of an inch.

Depths, widths, and thicknesses are always expressed in inches, while length is always expressed in feet and inches.

In the case of I-beams and channels, the number required, depth, weight and length are given thus:

1 – I 12″ x 30# x 20′-4″; 1—[15″ x 33# x 6′-9$\frac{5}{16}$″;

2—Is 15″ x 42# x 21′-8½″; 6—[s 12″ x 35# x 12′-6½″; etc.; etc.

In the case of angles, the number required, width of flanges (known as legs), thickness of metal, and length are given, thus:

1—L 5″ x 3½″ x ½″ x 6′-10½″; 1—L 6″ x 6″ x ¾″ x 9′-7½″;

4—Ls 6″ x 4″ x ¾″ x 12′-8½″; 7—Ls 3″ x 3″ x ⅜″ x 4′-0″; etc.

In case of plates, the number required, width, thickness, and length are given, thus:

1—Plate 36″ x ½″ x 14′-2¼″; 6—Plates 14″ x $\frac{9}{16}$″ x 4′-1″.

In detailing a member, it is the usual practice to show an elevation, top view, and a section through the member, looking downward. However, there should always be as many sections and views taken as is necessary to clearly show the member in detail.

DRAWING ROOM EXERCISE NO. 1

Reproduce Plates 1, 2, and 3 each to a scale of 1½″ = 1′-0″. Each drawing should first be reproduced in pencil upon an 18 x 24-inch sheet of ordinary drawing paper, then traced upon a sheet of tracing cloth of the same size. In making these drawings, the student should study each detail as he goes along. All necessary information concerning the material shown on these drawings will be found in the tables in the back of this book or in a Carnegie or Cambria handbook.

CHAPTER III

FUNDAMENTAL ELEMENTS OF STRUCTURAL MECHANICS

20. Structural Mechanics.—Mechanics, in general, is the science which treats of the effects produced upon bodies by force, while Structural Mechanics, as the name would imply, is simply that part of mechanics that applies directly to structures.

21. Body.—In common usage, the word "body" refers to what is known as an ordinary solid body. But in reality we are unable to think of material (matter) at all other than as bodies. This is most readily realized by attempting to conceive of material in a rarefied state, as in the composition of a gas. We can conceive of it only as being made up of very small bodies, known as particles, molecules, etc. Neither can we conceive of the composition of solid bodies or liquids otherwise if thought of in detail. Such being the case, it is evident that any part of a recognized body, to us, is really a body in itself; and as we are not limited as to subdivision, it is evident that any part of any body may, at will, be treated as an independent body.

22. Force.—In mechanics, the word "force" refers to that which we recognize as the push or pull that bodies exert upon each other, which invariably produces motion or a tendency of motion of the bodies concerned.

Forces exerted by bodies at rest produce only a tendency to motion, and are known as static forces, and the part of mechanics treating of such forces is known as Statics. Forces exerted by moving bodies are known as dynamic forces, and the part of mechanics treating of such forces is known as Dynamics. Most of the mechanics known as Structural Mechanics can be classed under the head of Statics.

Force is known by various names, as pressure, load, stress, reaction, etc. These names are used to indicate certain existing conditions; however, force is the same wherever we find it.

23. Measure of Forces.—Measuring forces is the same as measuring any other thing; it is simply a matter of comparison, that is, we compare forces with some one force which is taken as the basis. We are all familiar with the practice of measuring forces in terms of the common gravity unit, in which case the pull of gravity upon a certain body selected as a standard is taken as the basis of comparison and designated as a pound and known as the unit of weight. It is readily seen that this unit is an arbitrary one, as the pull of gravity upon most any body could have been selected as the unit, although in most cases convenience would have been sacrificed.

In speaking of a force, we say a force of so many pounds, or the intensity of a force is so many pounds. But what we really mean is that the push or pull (as the case may be) exerted upon a certain body by some other body is so many times greater or less than the pull of gravity upon the standard body, which pull, as stated above, is known as a pound.

It is evident that any static force is directly measurable as comparative weight, but dynamic forces are more readily determined by considering the change of motion produced by them. It is known by experiment that the force of gravity will cause the velocity of a body falling freely to increase about 32.2 feet per second for each second of time. This increment of velocity, 32.2 feet, is known as the acceleration of gravity and is usually represented by the letter "g." Then using the force of gravity expressed in the pound unit as our basis of comparison (as in static forces) we can say that the intensity of any dynamic force is to the force of gravity as the acceleration produced by the dynamic force is to the acceleration produced by gravity, regardless of whether the body considered moves horizontally, vertically, or in any other direction. For example, suppose a dynamic force of an unknown intensity F by acting continuously upon a body weighing W pounds increases its velocity a feet per second for each second of time. Then by direct proportion we have

$$F : W :: a : g.$$

Expressing this in words we say: The unknown force (F) in pounds is to the force of gravity (W) in pounds as the acceleration (a) in feet per second caused by the unknown force (F) is to the acceleration (g) caused by gravity. From which we have

$$F=\frac{W}{g}a \qquad \text{(A)},$$

which is one of the fundamental equations of mechanics. The equivalent formula is often written: $F=ma$, where $m=W/g$, and is known as the mass of the body considered. But Formula (A) is the one mostly used in practical work. The mass of a body is the quantity of material it contains; but in a practical sense, the mass of a body is equivalent to what its weight would be if the acceleration of gravity were one foot (considering a foot as unity) per second instead of 32.2 feet,—which it really is. Then by direct proportion, using the force of gravity as our basis as before, we have

$$m : W :: 1 : g, \text{ or } m=\frac{W}{g}$$

which is the expression for mass given above.

The above discussion refers to constant forces, as the forces encountered in structural engineering are mostly constant forces. In case of variable forces it is necessary to know the rate and the limit of variation first, then the intensity can be determined.

Problem 1. A body weighing 200 lbs. is acted upon by a constant force which increases its velocity from 0 to 64.4 ft. in 1 second of time. What is the intensity of the force?

Solution: Substituting in Formula (A), we have

$$F=\frac{200}{32.2}\times 64.4=400 \text{ lbs.}$$

Problem 2. What would be the intensity of the force in Problem 1 if the velocity of the body were increased in 5 seconds from 50 ft. per second to 150 ft. per second?

Solution: Here the increment of velocity or acceleration per second is $(150-50) \div 5 = 20$ ft. Then substituting in Formula (A), we have

$$F = \frac{200}{32.2} \times 20 = 124 \text{ lbs. (about).}$$

Problem 3. If the body in Problem 1 had a velocity of 150 ft. per second, what would be the intensity of a constant force required to stop the body in 3 seconds?

24. Direction of Action of a Force.—Whenever a force is applied to a body it either moves the body or has a tendency to move it. The direction in which the body moves or has a tendency to move is said to be the direction of action of the force causing the motion or tendency.

25. Line of Action of a Force.—Whenever a body receives a force by coming into contact with another body, which is the most common case, there is always a surface upon which the force is exerted. We cannot conceive of this force being transmitted to the surface other than that each infinitesimal area of the surface receives a small amount of the total force transmitted. So it is evident that what we usually consider a force is really made up of an infinite number of forces. But it would be impossible to deal with forces in mechanics when considered in this way, since there would be an infinite number of forces to consider in every case. We avoid this difficulty by assuming the total force to be applied along a line which passes through the surface at the center of mean intensity of the force and in the direction of its action. This line is known as the line of action of the force. The line of action of a force is unlimited in length, and the force may be considered to be applied at any point along the line of action as far as motion or tendency of motion of the body upon which it acts is concerned.

When a force is distributed over quite a large surface it is usually necessary to consider the surface divided up into small portions, say, portions one foot square, or one inch square, as the case may require, and to treat the part of the total force exerted upon each portion as a separate force. This means that the force exerted upon any portion will have a line of action of its own passing through that portion at the center of the mean intensity of the force exerted upon it. In any case we can obtain only an approximation, but the approximation should come within the limits of practicability.

In case of such forces as gravity and magnetism, where each particle of material is assumed to be equally affected, the line of action passes through what is known as the center of gravity of the body. However, if the bodies be large, as is the usual case of structures, it often becomes necessary to consider the body to be divided up into smaller ones and to consider that the force acting upon each of the smaller bodies has a line of action of its own, similar to the case of surfaces.

The plane in which the line of action of a force lies is known as the plane of the force, and the direction of the line of action is known as the direction of the force, and the direction of the action along the line of action is known as the direction of the action of the force. Any number of forces could act upon a body and each force could be in a different plane, but in most of the problems in structural engineering the conditions are such that we can consider them as acting in the same plane.

26. Graphical Indication of Forces.—When considering the effects produced upon a body by forces, for convenience we graphically represent the forces applied without reference to the bodies applying them. This is done by drawing an arrow in the line of action of each force. The arrow point in each case indicates the direction of action, and the point upon the body where the arrow touches indicates the point of application. Thus in Fig. 11, the arrows *P*1, *P*2, *P*3 and *P*4 indicate forces applied upon the body *AB* at the points *a*, *b*, *c*, and *d*, respectively. The arrows would be spoken of as forces *P*1, *P*2, etc. And we would say that the forces *P*1, *P*2, and *P*4 act toward the body, while the force *P*3 acts away from it, this being indicated in each case by the arrow points.

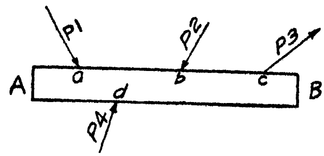


Fig 11

27. Applied Forces and Reactions.—An applied force is any force which we conceive of as being applied to a body from without by some other body, and at the same time as being the initiative of the outward activity. A reaction is the same as an applied force, except we conceive of it as resisting the activity instead of being the cause of it. Both are spoken of as external forces. For example, a body placed at *C* upon the bar *AB* (Fig. 12) by virtue of its own weight will exert a force *P* (pressure) upon the bar at that point. This force *P* will cause the bar to have a tendency to move downward, which is the activity produced. Actual motion is prevented by the supports at *A* and *B* exerting the forces *R*1 and *R*2 acting upward on the bar. We would call *P* an applied force, as we conceive of it as being the initiative of the activity of the bar, while we would call *R*1 and *R*2 reactions, as we conceive of them as resisting the activity, which, in this case, as stated above, is the tendency of motion caused by the force *P*. But regardless of name, all three of these forces are exactly the same in character. This can be seen very readily by imagining the bar to be turned upside down and supported only at *C*, while all three of the forces continue to act the same in reference to the bar. Then the case would be as shown in Fig. 13. This could be actually accomplished by placing a body weighing the same in pounds as the reaction *R*1 where *R*1 is indicated to act and another body weighing the same in pounds as the reaction *R*2 where *R*2 is indicated to act. Then *R*1 and *R*2 would become applied forces, while *P* would become a reaction.

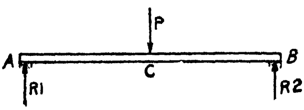


Fig 12

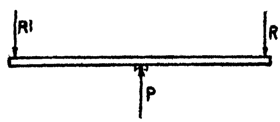


Fig. 13

28. Stress.—Stress is the push or pull which the constituent parts of a body exert upon each other due to the application of external forces. Stresses are known as internal forces, while applied forces and reactions are known as external forces.

As a simple case, suppose a steel rod having a uniform cross-section of *A* square inches is suspended from one end and a weight of *P* pounds is hung on the other. It is known from experience that the pull *P* exerted at the lower end of the rod by the weight is in turn exerted at the upper end. So this pull is, as we say, transmitted through the rod, and we cannot conceive of this transmission taking place other than that the material particles exert a pull upon each other in the direction of the length of the rod. A practical idea of the stress in the rod can be obtained by first imagining the weight P to be supported by a very small wire, so small in cross-section that the wire would be really a row of individual molecules; then we can conceive of the pull P being transmitted along the wire from molecule to molecule, thus producing a stress of P pounds upon each molecule. Next imagine another wire of the same size suspended along the side of the wire just considered, and suppose this second wire now supports half of the weight P: then the stress on the molecules in either wire would be $P/2$; and by adding another wire of the same size the stress on the molecules in each wire would be $P/3$; and by adding another wire it would be $P/4$; and so on. So if there be n number of wires, the stress on each would be P/n. Now imagine the rod made up of n number of such wires and let da be the area of the cross-section of each wire in square inches, and let s be the stress per square inch on each wire; also let p be the stress per square inch on the rod at any cross section. It is obvious that the stress per square inch on the rod at any cross-section will be $p=P/A$. Then, likewise, the stress on each wire being P/n, the stress per square inch in each wire will be

$$s=\frac{\left(\frac{P}{n}\right)}{da}=\frac{P}{nda}.$$

But $$nda=A;$$

therefore, we have

$$s=\frac{P}{A}=p.$$

That is, the stress on the rod per square inch is the same whether we consider the entire cross-section or a portion or a mere molecule in the cross-section. When we say that the stress on a body is so much per square inch, we do not necessarily mean that there is a square inch of material in the body that actually has that stress. We may simply mean that there is some material in the body that has that stress; it may be a square foot or a millionth part of a square inch. The stress per square inch on a body is known in structural mechanics as *Unit Stress.*

If the above rod were stood up vertically—say, upon a floor—and the weight placed upon the top end, the direct stress produced by the weight would be the same in intensity as if suspended as above, but the stress would be produced by a push instead of a pull. When the stress in a body is produced by a direct pull, it is known as *Tensile Stress;* but when it is produced by a direct push, it is known as a *Compressive Stress.*

Whenever a body is subjected to one kind of stress uniformly distributed over its cross-section throughout its length, as was considered in the case of the above rod, the stress is known as *Simple Stress.* The unit stress in that case at any cross-section is always equal to the total stress on the cross-section divided by the area of the cross-section, or, in general, we have $p=P/A$, as given above.

From the above discussion of stress one may be led to think of all bodies as being fibrous in structure. It is true that wood, wrought iron, and some other materials manifest this structure to some extent, but not wholly, while some other materials, as steel and concrete for example, seem to be devoid of it. But in any case we cannot conceive of the arrangement of the ultimate parts of a body other than in some consecutive order, and consequently our demonstration is not faulty, as we only conform to that conception.

Stress, known as shear, cross-bending, and torsion, will be treated later under these specific heads as the occasion for their treatment occurs.

29. Elasticity.—It is an observed fact that whenever a force is applied to a body, the dimensions of the body are changed, and that when the force causing such a change is released, the body has a tendency to regain its original dimensions, and will regain them unless the force be so great as to break or injure the body. This property of regaining their original dimensions which bodies manifest is known as their elasticity. We can conceive of this regaining of dimensions as being due to a stress-resisting force inherent in the material and which we can designate as the force of elasticity, yet the nature of the force is unknown. As an example, suppose a steel rod to be in tension. We can conceive of the material particles exerting a pull upon each other. This we call stress. And we can conceive of this pull being resisted by a force inherent in each particle. This inherent force is the *force of elasticity*, the intensity of which, in accordance with Newton's Law, is necessarily equal to the stress which it resists.

30. Distortion.—In this book the total change of form of a body due to external forces will be known as distortion, while the amount of change per unit of dimension of a body will be known as unit distortion. What really takes place in every case is cubic distortion, but, in determining the distortion of structures, it is usually necessary to consider only the distortion of the length of their parts, and hereafter the word distortion in this book will refer only to the distortion of length unless otherwise stated. Suppose a steel rod, ten feet long, is suspended from one end and a weight is hung on the other end. If the weight causes the rod to stretch one-tenth of an inch in length, the distortion of its length or simple distortion, in that case, is one-tenth of an inch, while the unit distortion, considering an inch as the unit of length, is 1/120 of the distortion, the rod being ten feet long or 120 inches.

31. Elastic Limit.—Whenever a body is distorted the maximum amount that it will sustain and yet return to its original form when permitted to do so, it is said to be distorted to the *elastic limit*. If a body is distorted beyond the elastic limit, it will remain shorter or longer than its original length. Then we say that the body has taken *set*. If a body be in tension, its length would remain longer, and if in compression it would remain shorter.

32. Relation of Stress and Distortion.—Hooke (in 1678) was the first to state that the distortion of a body is directly proportional to the stress. This is known as Hooke's Law, and has been proven by experience to be practically true, provided the distortion does not exceed the elastic limit. Whenever a body is distorted beyond the elastic limit, the distortion increases more rapidly than the stress.

Suppose a steel rod having a length L and a uniform cross-section A is distorted an amount D when subjected to a simple stress P; then, according to Hooke's Law, if the stress were $2P$, the corresponding distortion of its length would be $2D$; if $3P$, it would be $3D$; and so on. While this direct proportion holds good in all cases as long as the stresses are within the elastic limit, yet the actual value of D in any case will depend upon the value of L and A, being directly proportional to L, and inversely proportional to A. This is readily seen, for it is obvious that if the rod were two feet long it would be distorted twice as much when subjected to the same stress as a rod of the same section only one foot long, since each foot of length would be distorted the same in amount in either case.

In regard to the cross-section, it is obvious that if the area is two square inches, the distortion of the rod will be only one-half as much as when its area is one square inch, as the actual stress on the material in the rod in the first case is only one-half as much as it is in the second case.

33. Modulus of Elasticity and Determination of Simple Distortion.—According to Hooke's Law, if we know the distortion of any one piece of material subjected to a known stress, we can determine the distortion of any other piece of the same kind of material subjected to any known stress simply by direct proportion.

Suppose it is found by experiment that a rod of some unknown material, having a length of L inches and a uniform cross-section of A square inches, is distorted D inches in length when subjected to a simple stress of P pounds. What would be the distortion of a rod of the same kind of material having a length of L' inches and a uniform cross-section of A' square inches, if subjected to a simple stress of P' pounds? Let D' be the distortion required. Reducing everything concerned to its lowest terms, which is done for convenience, we have

The unit stress in the first rod $= p = P/A$,
while the unit distortion in inches $= d = D/L$.
The unit stress in the second rod $= p' = P'/A'$,
while the unit distortion in inches $= d' = D'/L'$.

By direct proportion we have

$$\frac{p}{p'} = \frac{d}{d'} \quad \ldots\ldots\ldots\ldots (1).$$

Expressing this in words, we say: The stress (p) per square inch in the first rod is to the stress (p') per square inch in the second rod as the distortion (d) of the first rod per inch of length is to the distortion (d') of the second rod per inch of length; from which we have $d' = dp'/p$, which is the distortion of each inch of the second rod; then, of course, the total distortion of the rod would be L' times this, so we have

$$D' = d'L' = \frac{d}{p}\,p'L' \quad \ldots\ldots\ldots\ldots (2).$$

Now d/p is the ratio of the unit distortion to the unit stress given for the first rod, but according to Hooke's Law, this ratio is the same for any piece of this same kind of material. So if this ratio is determined for any one piece of this material, the elastic property of the material is known, and the distortion of any piece of the material can then be determined if its length, area of cross-section, and stress are known.

The ratio of the unit distortion to the unit stress, as expressed above, would be a very small fraction in any case, so, for convenience, we can invert the expression so as to have a whole number. Then we have $p/d = p'/d' = E$, which is a constant for any piece of the same kind of material composing the two rods. Now substituting $1/E$ for d/p in equation (2), we have $D' = p'L'/E$. E would be known as the *Modulus of Elasticity* of the material composing the two rods. For the modulus of elasticity of any material in general we have

$$E = \frac{p}{d} = \frac{\text{unit stress}}{\text{unit distortion}},$$

which is usually designated as Young's Modulus. It varies with the different kinds of material, but is practically constant for all pieces of the same kind. For the simple distortion of any piece of any kind of material having a uniform cross-section and being subjected to a simple stress, we have the general equation

$$D = \frac{PL}{AE} = \frac{pL}{E} \ldots\ldots (B),$$

where D = distortion of the piece in inches (or feet if L be taken in feet);
P = total stress on the cross-section of the piece in pounds;
A = area of cross-section in square inches;
L = total length of piece in inches or feet;
p = unit stress on the cross-section of the piece;
E = modulus of elasticity of the material composing the piece.

The modulus of elasticity of a material is determined by experiment. The average values of the moduli of elasticity of the principal materials used in structural engineering have been found to be as follows:

29,000,000 for steel;
26,000,000 for wrought iron;
30,000,000 for cast steel;
18,000,000 for cast iron;
720,000 for long-leaf yellow pine;
555,000 for white oak.

In determining the modulus of elasticity, it is laboratory practice to measure the distortion in inches, in which case the length of the test piece must always be reduced to inches; but in using Formula (B), L may be taken either in feet or inches. If taken in feet, the distortion will be given in feet, and if taken in inches, it will be given in inches.

Problem 4. Suppose a steel rod having a length of 10 feet and a uniform cross-section of 2 square inches is suspended from one end and supports a weight of 30,000 pounds hung on the lower end:

(a) What will be the distortion of the rod due to the 30,000 pounds?

Referring to Formula (B), given above, we have in this case $P = 30{,}000$; $p = 30{,}000 \div 2 = 15{,}000$; $E = 29{,}000{,}000$. Substituting these values in the formula, we have for the distortion

$$D = \frac{15{,}000 \times 10}{29{,}000{,}000} = 0.00518 \text{ ft.} = \tfrac{1}{16} \text{ inch (about)}.$$

(b) What will be the distortion of the rod due to its own weight?

In that case the stress varies from zero at the bottom of the rod to a maximum at the top. For convenience, instead of using the numerals given above, let L be the length of the rod and A the cross-section; and let w be the weight of the rod per foot of length. Then the stress on the cross-section at a point b (Fig. 14), x feet from the lower end of the rod, will be $P = wx$, which is really the weight of the rod below b.

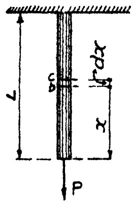


Fig 14

For the distortion of the length of the material in the rod between the cross-section at b and the cross-section at c, which is an infinitesimal distance dx above b, we have

$$d = \frac{wxdx}{AE},$$

which corresponds to Formula (B), given above.

Now as x is a variable, varying from 0 to L, the last expression will give the distortion of any infinitesimal part of the length of the rod anywhere between the ends. Then the total distortion of the rod will be equal to the summation of the distortion of these parts; so, for the total distortion of the rod, we have

$$D = \int_0^L \frac{wxdx}{AE} = \frac{1}{2}\frac{wL^2}{AE} \quad \text{(c)}.$$

Now let wL, which is the total weight of the rod, be represented by P. Then substituting in (c) we have

$$D = \frac{1}{2}\frac{PL}{AE} \quad \text{(d)}.$$

This shows that the distortion of the rod due to its own weight is one-half of what it would be for an equal weight suspended from the lower end. The weight of the rod = 6.8 × 10 = 68 lbs. Substituting in (d) we have for the distortion

$$D = \frac{1}{2} \times \frac{68 \times 10}{2 \times 29{,}000{,}000} = 0.000006 \text{ ft. (about)}.$$

Problem 5. What will be the distortion of length of a solid, circular, cast-iron column having a diameter of 6 ins. and a length of 18 ft., when supporting a load of 282,000 lbs.?

Problem 6. What will be the distortion of an eye-bar 8″ x $1\frac{1}{2}$″ x 39′-0″ (c.c. end holes) when subjected to a stress of 16,000#□″?

34. Motion.—Strictly speaking, a body can have but two distinct kinds of motion, one known as translation and the other as rotation. When a body moves as a whole along an imaginary line, known as its path, its motion is translation, particularly in reference to any relatively fixed position in the immediate vicinity of the body; but if it either moves around or turns about an imaginary line, known as its axis, its motion is rotation in reference to that line.

There is one fact regarding motion which should never be overlooked, that is, motion is absolutely relative, and that the kind of motion is always dependent upon the position of reference. For instance, the motion of a particle in a body rotating about an axis is rotation in reference to that axis; yet at the same time its motion (the particles in the axis excepted) is translation when referred to any relatively fixed position in the immediate vicinity of the particle.

In reference to time, motion is either uniform or variable; in reference to the character of path described it is either rectilinear or curvilinear, being rectilinear when the path is a straight line and curvilinear when a curve.

35. Moment of a Force.—The moment of a force about any point is a measure of its tendency to produce rotation of the body upon which it acts about the point chosen and is equal to the perpendicular distance from the point to the line of action of the force multiplied by the intensity of the force. The point is known as the center of moment, while the perpendicular distance is known as the lever arm of the force, or simply "arm."

In the case shown in Fig. 15, the moments of the forces $P1$, $P2$, and $P3$ about the point O would be expressed as $aP1$, $bP2$, and $cP3$, respectively. As is readily seen, the forces $P1$ and $P2$ would have a tendency to rotate the body A clock-wise about O, while $P3$ would have a tendency to rotate it counter clock-wise. Now, taking one direction as minus and the other direction as plus, say, counter clock-wise minus, and letting M be the algebraic sum of the moments of the three forces about O, we have $M = aP1 + bP2 - cP3$, which is the *equation of moments* of the three forces about O.

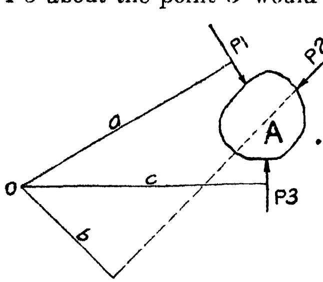


Fig 15

In case of forces not in the same plane, the moments are usually taken about a line, in which case the center of moment of each force is the point on the line nearest the force. The tendency of rotation about the line is what is really measured in that case.

It is obvious that the moment of a force about any point in its line of action is zero, for in that case its arm is zero. If the arm is taken in feet and the force in pounds, the moment will be expressed in what is known as foot pounds, but if the arm is taken in inches and the force in pounds, the moment will be expressed in what is known as inch pounds.

36. A Couple and Moment of Same.—Two equal and opposite parallel forces acting upon a body (other than along the same line of action) form what is known as a couple. Thus in Fig. 16, the force P applied at c upon the body AB and the equal and opposite force P applied at b form a couple.

The moment of a couple is equal to the distance between the forces multiplied by the intensity of one of the forces. For example, the moment of the couple shown in Fig. 16 is $M = Pd$.

The moment of a couple is not changed by changing the center of moments, that is, the moment about one point is the same as about any other point in the plane of the forces. This is readily seen, for taking moments about e or g, we have $M = (es + sg)P = Pd$; taking moments about s,

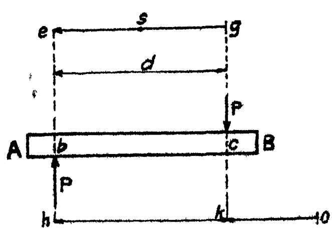


Fig. 16

we have $M = P(se) + P(sg) = P(se + sg) = Pd$; and, again, taking moments about o, we have $M = P(ho) - P(ko) = P(ho - ko) = P(hk) = Pd$. Now, evidently, if the moment of a couple is the same for all points in the plane of the forces, a couple can be considered to be applied anywhere in the plane of its forces, which means that we can consider any couple as being moved bodily to any desired position in the plane of the forces. Of course, the forces must act upon the body concerned.

Any two couples are equivalent when their moments are equal. But the forces and arms in the two cases are not necessarily equal. Rotation or tendency of rotation is always due to the action of a couple. No single force can produce rotation.

37. Concurrent and Non-Concurrent Forces.—When the lines of action of two or more forces acting upon a body meet at a common point, the forces are said to be concurrent, while if their lines of action do not so meet, the forces are said to be non-concurrent.

38. Conspiring Forces.—When the lines of action of two or more forces acting upon a body coincide, the forces are known as conspiring forces.

39. Resultant and Component Forces and Graphical Representation of Same.—A body acted upon by two or more forces may, thereby, have a tendency to move in more than one direction at the same instant; but as a body cannot occupy two positions at the same time, it is evident that motion can take place in but one direction, which is known as the direction of the resultant of the two or more forces. Now, it is evident that a single force acting in this direction with a certain required intensity could produce the same effect as that produced by the two or more forces combined. Such a force would be known as the resultant of the two or more forces, while each of the two or more forces would be known as a component of the resultant force.

For example, if two concurrent forces $P1$ and $P2$ (Fig. 17) be applied simultaneously to a small body resting upon a smooth, horizontal plane at A, we can readily see that the force $P1$ would have a tendency to move the body along its line of action Ac, while at the same time the force $P2$ would have a tendency to move the body along its own line of action Ab. Now we know, from reasons given above, that the body cannot move along both of the lines at the same time, and we can readily see that it would not move along either of the lines, as the two tendencies of motion are not in the same direction; so, undoubtedly, if the two forces moved the body, the motion would take place along some line different from either of the two.

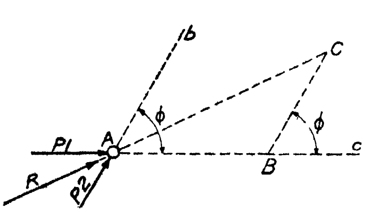


Fig. 17

Suppose the force $P1$, acting alone and continuously for one second, moved the body from A to B, so that at the end of the second the body would be at B. Then suppose the force $P1$ ceased to act at that instant, and suppose the body stopped instantly at that point and the force $P2$ were then applied and in the same manner moved the body from B to C in one second, so that at the end of the next second the body would be at C. Then, the two forces $P1$ and $P2$, each acting separately for one second, would have moved the body from A to C in two seconds, but if they had acted simultaneously upon the body they would undoubtedly have accomplished the same thing in one second, for the simple reason that each force would have been acting for one second, as in the case when acting separately, and with the same intensity, for otherwise they would lose their identity. By this it is not meant that the two forces acting simultaneously would have moved the body along the same paths as when acting separately, but that they would have moved it from A to C along some line in one second. We cannot conceive of these forces doing anything more than they would have to do in moving the body from A to C; then, undoubtedly, they would move it along the straight line AC joining the two positions, as that would be the least required of them in the transaction. Then the line AC would be the direction of the resultant of the forces $P1$ and $P2$.

Now it is evident that the two forces $P1$ and $P2$ acting simultaneously upon the body at A would have the same effect as a single force R (Fig. 17) would have if acting along the line AC with such an intensity as to move the body from A to C in one second. Therefore, the two forces $P1$ and $P2$ could be replaced by this single force R, which is their resultant. Each of the forces $P1$ and $P2$ are, in that case, components of the resultant force R.

Assuming the resistance to be the same everywhere on the plane—which was really the assumption at the start—the distance AB will be directly proportional to the intensity of force $P1$, which moved the body over that distance, and, likewise, the distance BC will be directly proportional to the force $P2$. Then as the sides AB and BC of the triangle ABC (Fig. 17) are directly proportional to the forces $P1$ and $P2$, respectively, it is evident that we can construct a similar triangle such that the sides will represent to convenient scale the intensities of these forces. Such a triangle is shown in Fig. 18, where the side $P1$, parallel to the side AB in Fig. 17, is drawn to represent the intensity of the force $P1$ to scale (so many pounds per inch) and the side $P2$ parallel to the side BC in Fig. 17 is drawn to represent the intensity of the force $P2$ to the same scale as $P1$. Then the side R, which will be parallel to the side AC in Fig. 17, will represent the intensity of the resultant force R to the same scale as $P1$ and $P2$. Now it is evident that any two concurrent forces with their resultant can be represented in magnitude and direction by the sides of a triangle which is known as a *Force Triangle*. It is important to note that the forces act in one direction around the triangle, while their resultant acts in the opposite direction.

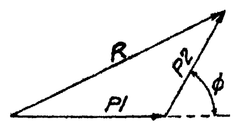


Fig. 18

Suppose, instead of the two forces $P1$ and $P2$ acting upon the small body shown in Fig. 17, that there be four concurrent forces $P1$, $P2$, $P3$, and $P4$, as shown in Fig. 19. The resultant R of the two forces $P1$ and $P2$ is determined by constructing the force triangle ABC in the same manner as was shown above. Then as R is the resultant of the two forces

*P*1 and *P*2, it will replace them, and the remaining forces to be considered are *P*3, *P*4, and *R*. Now constructing the force triangle *ACD* (Fig. 19) for the force *R* and *P*3, we have their resultant *R*1. Next, constructing the force triangle *ADE* for the two remaining forces, *R*1 and *P*4, we have their resultant, *R*2, which is the final resultant, or, in other words, is the resultant force of all the forces *P*1, *P*2, *P*3, and *P*4, and as such can replace them. We now have a polygon *ABCDEA* wherein the forces are seen to act in the same direction around the polygon, while their final resultant *R*2 acts in the opposite direction. Such a polygon is known as a *Force Polygon.* The force polygon *ABCDEA* in this case was constructed by combining force triangles; but it is evident that the same could have been constructed by laying off the forces *P*1, *P*2, *P*3, and *P*4 in consecutive order, thus obtaining the part *ABCDE* of the polygon, and then joining *EA* for completion as shown in Fig. 20.

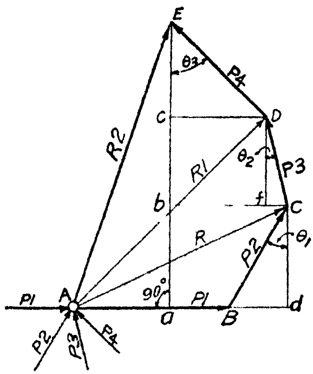


Fig. 19

While it is advisable to begin with some force as *P*1 and lay the forces off in consecutive order around the body as we have done here, yet it is not absolutely necessary to do so, for it will be seen upon inspection that the forces may be taken in any order (except that they must be laid off so that they act in the same direction around the polygon) as shown in Fig. 21, where the line *AE*, which is the final resultant, is the same in length and direction as the final resultant *AE* shown in Fig. 19 and Fig. 20.

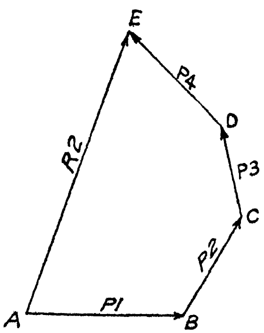


Fig. 20

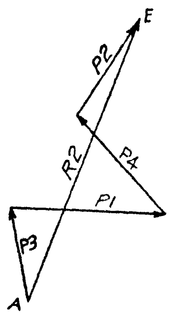


Fig. 21

Now it is evident from the above that any number of concurrent forces with their resultant can be represented in magnitude and direction by the sides of a closed polygon known as a force polygon, wherein the forces act in one direction around the polygon while their resultant acts in the opposite direction, the direction of action in each case being indicated by the arrow points. The force triangle is really a force polygon wherein the component forces are only two in number.

As any number of concurrent forces with their resultant can be represented in magnitude and direction by a closed polygon, it follows that any force may be resolved into any number of components, the only requirements being that the force and its components form a closed polygon. Thus we can construct any number of components as *Aa*, *ab*, *bc, cd,* and *dB* for the force *AB,* as shown in Fig. 22, by simply drawing the components practically at will, the only restrictions being that they form a closed polygon with the force *AB.* Here the force *AB* is considered as a resultant, which is obviously permissible in any case. We would say that the force *AB* was resolved into its component forces *Aa, ab, bc, cd,* and *dB.* Resolving forces into components is known in mechanics as the *Resolution of Forces.* The most convenient components into which a force can be resolved are two components whose directions are at right

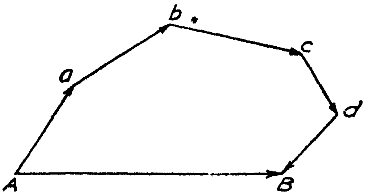


Fig 22

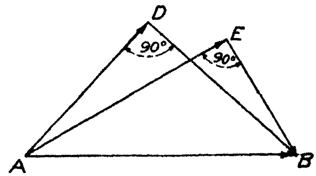


Fig 23

angles to each other, and are known as rectangular components. Thus *AD* and *DB* or *AE* and *EB* (Fig. 23) are rectangular components of the force *AB.* It is obvious that a force can be resolved into any number of pairs of rectangular components—or any number of any other kind.

It will now be shown that the above is as true for non-concurrent forces as it is for concurrent forces. Let *P*1, *P*2, and *P*3 (Fig. 24) represent three non-concurrent forces acting upon the body *AB.* As far as motion or tendency of motion are concerned, any of these forces may be considered as applied at any point along their line of action; however, in making such assumptions in any case, we are compelled to conceive that there always exists the necessary materialistic connection between such points of application and the body concerned. Then, if we

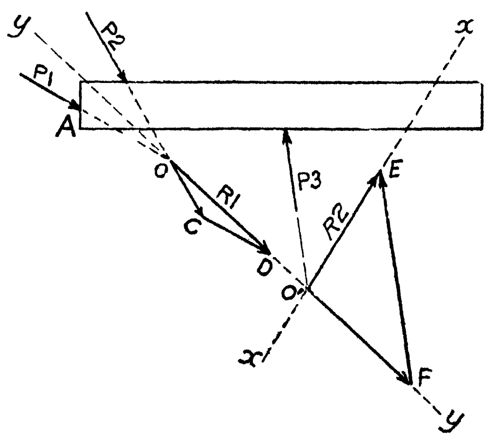


Fig 24

prolong the lines of action of the two forces *P*1 and *P*2 until they intersect at *O,* we can consider them as being two concurrent forces applied at *O,* and by constructing the force triangle *OCD* (where *OC* = *P*2 and *CD* = *P*1 to scale) we have their resultant given in amount and direction by the line *OD.* Now this resultant, which we will designate as *R*1, can be considered as being applied at any point along *yy,* its line of action (the same as any other force). Then prolonging the line of action *yy* of this resultant *R*1 until it intersects the line of action of the third force *P*3 at *O'*, and constructing the force triangle *O'EF*, we have the resultant *R*2, which is the final resultant, or, in other words, the resultant of all three forces, $P1$, $P2$, and $P3$. This final resultant $R2$ can be considered as being applied at any point upon its line of action xx.

From the above it is evident that the resultant of any number of intersecting non-concurrent forces can be represented graphically by first selecting any two forces and prolonging their lines of action until they intersect; then constructing a force triangle and obtaining the resultant of these two forces; then prolonging the line of action of this resultant until it intersects with the line of action of a third force; and then again constructing a force triangle and obtaining the resultant of the first resultant and the third force; and again prolonging the line of action of this second resultant until it intersects the line of action of a fourth force; and so on until all of the forces are combined. The last resultant will be the resultant of all of the forces, or the final resultant.

As an additional example, let $P1$, $P2$, $P3$, $P4$, and $P5$ represent five non-concurrent forces acting upon the body AB, as shown in Fig. 25 (a). Let us first take the two forces $P1$ and $P2$ and prolong their lines of action until they intersect at O. Then by constructing the force triangle ODC we obtain their resultant $R1$. Then by prolonging the line of action of this resultant until it intersects the line of action of a third force $P3$ at O', and constructing the force triangle $O'EF$, we obtain $R2$, the resultant of $R1$ and $P3$. Then prolonging the line of action of $R2$ until it intersects the line of action of a fourth force $P4$ at O'', and constructing the force

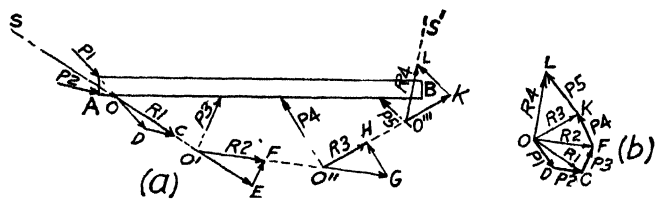


Fig 25

triangle $O''GH$, we obtain $R3$, the resultant of $R2$ and $P4$. Then prolonging the line of action of $R3$ until it intersects the line of action of a fifth force $P5$ at O''', and constructing the force triangle $O'''KL$, we obtain the resultant $R4$, which is the final resultant, or, in other words, the resultant of all the forces $P1 \ldots P5$.

So far our treatment of non-concurrent forces does not seem to have much in common with our treatment of concurrent forces, but let us go a little further and see what we can observe. The force triangles ODC, $O'EF$, $O''GH$, and $O'''KL$ are the same in kind as we found in our treatment of concurrent forces. Now suppose the force triangle ODC in Fig. 25 (a) be moved bodily and parallel to itself to the position ODC in Fig. 25 (b). This would not change the triangle in the least. Next suppose the force triangle $O'EF$ be moved in the same manner to Fig. 25 (b), taking the position OCF, the vertex O' coinciding with O in Fig. 25 (b) and E with C. Next, suppose the force triangles $O''GH$ and $O'''KL$ (in Fig. 25 (a)) to be moved in the same manner to the position in Fig. 25 (b) so that O'' and O''' would fall at O. Then these two triangles would take the positions OFK and OKL, respectively. By thus grouping the force triangles we have constructed the force polygon

ODCFKLO, which is the same kind of a force polygon as was constructed for concurrent forces in Fig. 19, and it can be drawn in the same manner. This shows that any number of non-concurrent forces with their resultant, the same as concurrent forces, forms a closed polygon, known as a force polygon, and hence the resultant of any number of any kind of external forces acting upon any body whatever can be determined simply by constructing a force polygon representing each in intensity and direction, at the same time placing the forces so that they act in the same direction around the polygon.

As a general case, let Fig. 26 represent a body acted upon by four non-concurrent forces $P1 \ldots P4$. We can determine the resultant of these four forces by simply taking any one of them, say, $P4$, and drawing a line as AB equal and parallel to it. Then from B draw BC equal and parallel to $P3$. Then from C draw CD equal and parallel to $P2$. Then from D draw DE equal and parallel to $P1$. Then joining E and A we have EA, which represents the resultant of the four forces in intensity and in direction. This same method of procedure holds for both concurrent and non-concurrent and also for conspiring forces. The conspiring forces would simply be laid off in one straight line. Their resultant would be represented by the distance from the last force laid off to the starting point.

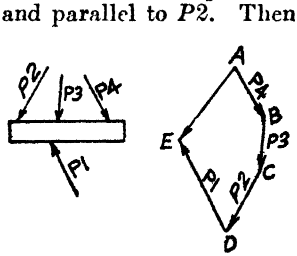


Fig 26

40. Analytical Representation of Resultant and Component Forces.—From the force triangle shown in Fig. 18 (Art. 39) we have

$$R^2 = P1^2 + P2^2 + 2 \cos\phi \, P1 \times P2 \ldots\ldots (1),$$

where ϕ is the angle which the lines of action of the forces $P1$ and $P2$ make with each other. Any one of the forces $P1$, $P2$, R, and the angle ϕ can be determined from the above equation, provided the other three are given. When $\phi = 90°$, the above equation becomes

$$R^2 = P1^2 + P2^2$$

which is the "rectangular equation" that results whenever a force is resolved into rectangular components. As a rule, it is much more convenient to express these rectangular components in terms of the force and angles which they make with the force. Thus, from Fig. 27, we have

$$R\cos\phi = P1; \qquad R = \frac{P1}{\cos\phi} = P1 \times \sec\phi;$$
$$R\cos\theta = P2;$$
$$R\sin\phi = P2;$$
$$R\sin\theta = P1; \qquad R = \frac{P2}{\cos\theta} = P2 \times \sec\theta.$$

The most useful thing to be observed from the above is that the rectangular component of a force along any line is equal to the force multiplied by the cosine of the angle which the force makes with the line.

In the case of conspiring forces, it is evident that the resultant of two or more conspiring forces is equal to their algebraic sum, for we can consider the forces acting in one direction as plus and those acting in the opposite direction as minus. Then by adding all of the plus forces together and all of the minus forces together and subtracting the lesser from the greater sum, we shall thus obtain the intensity and direction of action of their resultant.

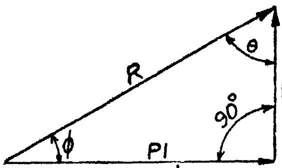


Fig 27

The resultant of three or more forces can be determined analytically most readily by resolving the forces into their horizontal and vertical components, thus obtaining a rectangular equation where the square of the algebraic sum of the horizontal components plus the square of the algebraic sum of the vertical components is equal to the square of the resultant. Thus from Fig. 19 (Art. 39) we have

$$R2^2 = aE^2 + Aa^2.$$

But $$aE = ab + bc + cE = P2\cos\theta_1 + P3\cos\theta_2 + P4\cos\theta_3,$$

which is the algebraic sum of the vertical components of the forces $P1$, $P2$, $P3$, and $P4$;

and $$Aa = AB + Bd - Cf - Dc = P1 + P2\sin\theta_1 - P3\sin\theta_2 - P4\sin\theta_3,$$

which is the algebraic sum of the horizontal components of the same forces, as is readily seen from Fig. 19. θ_1, θ_2, etc., are the angles which the forces make with the vertical line aE. $P1$ has no vertical component, as the cosines of the angles it makes with the vertical is O—the angles being 90°—and, the sine of 90° being 1, it is evident that its horizontal component is equal to $P1$.

41. Proposition.—*The moment of the resultant of two or more forces about any point in the plane of the forces is equal to the algebraic sum of the moments of the two or more forces themselves, about the same point.*

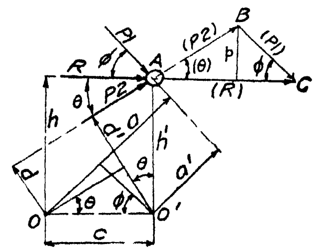


Fig. 28

As the resultant of two or more forces is a force which will produce the same effect (as far as motion or tendency of motion are concerned) as the two or more forces combined, it is practically self-evident that the above proposition is true. However, it is seen to be true from the following demonstration:

Let R (Fig. 28) represent the resultant of two concurrent forces $P1$ and $P2$, acting upon a body at A, and let the intensity of these forces and their resultant be represented by the sides of the force triangle ABC. Select any point O as the center of moments. Then we are to prove that the moment of R about O is equal to the algebraic sum of the moments of the component forces $P1$ and $P2$ about the same point. Now, it makes no difference where O is taken, we can always draw a line through O parallel to the line of action of the resultant R. Then in this case draw the line OO' parallel to R and from A draw h' perpendicular to the line OO'; then the moment of the resultant R about O' is the same as it is about O, since $h' = h$.

For the equation of moments about O', if the above proposition be true, we have $Rh' = a'P1 + d'P2$; but $a' = h'\cos\phi$ and $d' = h'\cos\theta$, and substituting we have $Rh' = (P1\cos\phi + P2\cos\theta)h'$, or $R = P1\cos\phi + P2\cos\theta$.

But from the force triangle ABC we have $R = P1\cos\phi + P2\cos\theta$; then substituting this value for R in the last equation, we have $(P1\cos\phi + P2\cos\theta) = (P1\cos\phi + P2\cos\theta)$ an identity, which proves the proposition when O' is the center of moments. Now, as the moment of R about O is the same as it is about O', it remains for us to prove that the change in the algebraic sum of the moments of $P1$ and $P2$, due to the changing of the center of moments from O' to O is zero. That is, we are to prove that $(a-a')P1 - (d'-d)P2 = 0$, or $(a-a')P1 = (d'-d)P2$. Now from Fig. 28 we have $a - a' = c \sin\phi$ and $d' - d = c \sin\theta$. Then substituting these values, and cancelling c, we have

$$P1\sin\phi = P2\sin\theta$$

which is readily seen to be true from the force triangle ABC, where we have

$$P1\sin\phi = p = P2\sin\theta.$$

So we have thus proven our proposition for the case of two forces. Now as it was shown in Art. 39 that the force polygon representing any number of forces was really made up by combining force triangles, it is obvious that we could show that the moment of the final resultant of any number of forces about any point is equal to the algebraic sum of the moments of the forces about the same point by simply considering the resultants and components in each triangle in consecutive order.

42. Condition of Equilibrium.—We say a body is in equilibrium whenever it is either at rest or in uniform motion. The only state of equilibrium here considered shall be that of rest, as uniform motion is of little interest to us in structural mechanics. As there are but two kinds of motion, translation and rotation, it is evident that a body will be at rest if neither of these motions takes place, and that the problem regarding equilibrium thus reduces to the investigation of the conditions conducive to these two motions.

Our only conception of equilibrium of forces is that of two equal and opposite conspiring forces balancing each other through the body upon which they act. Then, in order that a body be in equilibrium, it is obvious that the forces acting upon it must reduce to that condition. The forces themselves, or resultants, or a combination of the forces and resultants, may form a pair or several pairs of equal and opposite conspiring forces, so that all of the forces acting upon a body will thus be balanced, or, as we say, be in equilibrium, and such being the case, the body upon which they act will be in equilibrium. It makes no difference what combination we choose to consider the forces forming, for so long as we can show that the whole system of forces acting upon a body reduces to equal and opposite conspiring forces, we are assured that the body upon which the system acts is in equilibrium, and of course the converse will be true.

Referring to Fig. 17 (Art. 39), it is obvious that if a force equal in intensity to the resultant of the forces $P1$ and $P2$ be applied to the body at A so that it would act along the line AC in the direction from C to A, it would balance the forces $P1$ and $P2$ and the body would thus be in equilibrium under the action of these three forces. Here it is seen that the resultant of the two forces $P1$ and $P2$ and the balancing force would be two equal and opposite conspiring forces to which the system reduces when the body is in equilibrium.

And, referring to Fig. 19 (Art. 39), it is obvious that if a force equal in intensity to the final resultant $R2$ be applied to the body at A so that it would act along the line EA in the direction from E to A, it would balance all of the forces $P1$, $P2$, $P3$, and $P4$, and thus the body would be in equilibrium under the action of the five forces $P1$, $P2$, $P3$, $P4$, and the balancing force. Here it is seen that the resultant of the forces $P1$, $P2$, $P3$, and $P4$ and the balancing force are two equal and opposite conspiring forces to which the system reduces when the body is in equilibrium.

Further, referring to Fig. 25 (*a*) (Art. 39), it is obvious that if a force equal and opposite to $R4$ be applied along the line $S'O'''$ it would balance all of the forces $P1 \ldots P5$, acting upon the body AB, and thus the body would be in equilibrium. Here again it is seen that the system reduces to equal and opposite conspiring forces when equilibrium exists. So it is in all cases.

It will be observed from the above that any force acting upon a body in equilibrium will always form an equal and opposite conspiring force with the resultant of all the other forces acting upon the body.

Beyond a question, a body will move in translation if the forces acting upon it have a resultant. Then, evidently, a body will be in equilibrium, as far as translation is concerned, whenever the forces acting upon it have no resultant. Then as the forces acting upon a body in equilibrium must have no resultant—in order that no translation may take place—it is obvious that the force polygon representing them, instead of having a final resultant, as $R2$ in Fig. 19, will simply extend on around and close on the starting point without a resultant. This would give us a force polygon wherein all the forces are indicated to act in the same direction around the polygon. From this the following practical statement can be made: A body will be in equilibrium as far as translation is concerned whenever the forces acting upon it (when represented graphically to scale) will form a closed polygon wherein all of the forces act in the same direction around the polygon, or, in other words, the force polygon will close without a resultant, and the converse is just as true, that is, if a body be in equilibrium the polygon will close. This is true regardless of whether the forces be non-concurrent, conspiring, or concurrent forces, for it is evident that a body will have motion of translation if a resultant exists, as the resultant is equivalent to a single force, in which case motion is inevitable; and if no resultant exists, motion of translation will not take place, as there would be no force to produce it. All this is manifestly true regardless of condition.

As the resultant of concurrent or conspiring forces always passes through a common point of action, and the resultant of all the forces except one always forms an opposite and equal conspiring force with the remaining one, in case of equilibrium, and the moments about any point of these two equal and opposite conspiring forces are equal and have opposite signs, it is evident that rotation will not take place if the equilibrium polygon closes. So in the case of concurrent or conspiring forces, if the force polygon closes, we need go no further in our investigations regarding equilibrium, for if no resultant exists, no motion of rotation, as well as no motion of translation, will take place.

But in regard to non-concurrent forces the case is different, for here the forces may reduce to two equal and opposite forces, which indicates that no resultant exists, and thus the force polygon would close, indicating no motion of translation; but at the same time the two forces might not be conspiring forces, in which case, while motion of translation would not take place, motion of rotation would, as the two forces would form a couple which would produce motion of rotation. For example, the forces acting upon the body AB shown at (a) (Fig. 29) could form a closed polygon as shown at (b), which indicates that no resultant exists, and such being the case, no translation would take place; but by mere inspection of the figure at (a) we can see that the forces would form a couple which would rotate the body AB counter clock-wise and hence equilibrium would not exist. So to state the condition of equilibrium fully, we say: *A body is in equilibrium when both the resultant and the algebraic sum of the moments about every point of the external forces acting upon it, equal zero.*

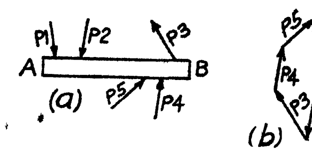


Fig 29

In addition to the above statement, we may add that a body is in equilibrium, as far as motion of translation is concerned, when the algebraic sum of both the horizontal and vertical components of the forces acting upon it is equal to zero, for it was shown in Art. 40 that the sum of the squares of the horizontal and vertical components of any number of forces is equal to the square of their resultant, and, of course, if the components are equal to zero, the resultant will be equal to zero. Further, we can state that the sum of the components of the forces acting upon a body in equilibrium is zero along any line whatsoever, for it is evident that if a component exists along any line, a resultant would exist in some direction, and hence motion of translation would surely take place in that direction.

43. Point of Application of a Force in Relation to Stress and Equilibrium.—In order to determine the nature of the stress produced in any case it is absolutely necessary to know the actual point of application of the force producing the stress. Each force in that case is indicated to act at some particular point as shown in Art. 26, while in reference to equilibrium (as stated in Art. 39), a force may be considered as being applied at any point along its line of action, in which case, of course, we are compelled to conceive of there always being the necessary materialistic connection between the point of application and the body concerned.

In the case of simple stress, whenever a force acts toward a body, the stress produced in the body by the force will always be compression, but if a force acts away from a body, the stress will be tension.

As a simple case, let AB (Fig. 30) represent a steel rod acted upon by two equal and opposite conspiring forces, $P1$ and $P2$. If $P1$ be applied at a and $P2$ at b, as indicated, it is obvious that the rod would be in tension. Now let us suppose the forces interchanged, $P1$ being applied at b and $P2$ at a, then it is obvious that the rod would be in compression. Thus, we see that by changing the point of application of the forces, the stress in the rod would be completely reversed, while the state of equilibrium would not be disturbed.

Then again, suppose $P1$ be applied at a and $P2$ at c, then it is obvious that the stress in the rod between a and c would be the same as when $P2$ is applied at b and $P1$ at a, while the stress in the rod between c and b would be zero. So it is readily seen that any change in the point of application of either of the forces will produce a corresponding change

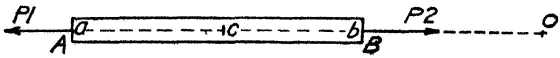


Fig 30

upon the rod in reference to the stress, but it is just as readily seen that any such change will not disturb the state of equilibrium, for it is evident that the forces $P1$ and $P2$ will balance each other so long as their points of application remain in the body anywhere on their common line of action between the points a and b. As for that, either of the forces can even be considered as being applied at any point, as o, off of the body altogether, in which case, of course, we would have to consider the rod as extending to o or being rigidly connected to it in some way.

As another case, let three non-concurrent forces, $P1$, $P2$, and $P3$ (Fig. 31) be applied to the body ZZ at the points a, b, and c, respectively. Now, as far as equilibrium is concerned, we can consider any of these forces as being applied at any point along their line of action, so prolong the lines of action of the two forces $P1$ and $P2$ (see Art. 39) until they meet at A. Then we can consider these two forces as two concurrent forces applied at A, and constructing the force triangle ABC, we have the intensity and direction of their resultant $R1$, which can be considered as a force being applied at any point along its line of action Cy. Then prolong the line of action of the force $P3$ until it meets the line Cy at D. Then we can consider that $R1$ and $P3$ are two concurrent forces applied at D. Then constructing the force triangle DEF, we have their resultant $R2$, which is the final resultant of the three forces $P1$, $P2$, and $P3$.

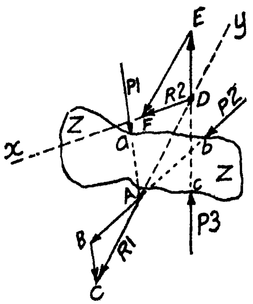


Fig 31

Now it is evident that if a force equal to $R1$ be applied to the body along the line Cy it would produce the same motion as $P1$ and $P2$ if it acted in the same direction as indicated by $R1$, but if it acted in the opposite direction it would balance them. Then again, it is evident that if a force equal to the resultant $R2$ be applied to the body along the line Dx it would produce the same motion of translation as the forces $P1$, $P2$, and $P3$ combined, providing it acted along the line Dx, in the same direction as indicated by $R2$; but if it acted in the opposite direction, it would balance the three forces $P1$, $P2$, and $P3$.

Now in regard to stresses, it is obvious that the stress produced upon the body ZZ by the action of the three forces $P1$, $P2$ and $P3$ will be of a very complicated nature. We can readily see that a force equal to the resultant $R1$, if applied at o or at any other point in or on the body along the line Cy, would not produce the same stress as the two forces themselves produce. And it is more readily seen that a force equal to the resultant $R2$ applied along the line Dx would not produce the same stress in the body as the forces $P1$, $P2$, and $P3$ actually produce. So it is evident that a force representing the resultant of two or more forces, while it will produce the same effect as the forces themselves as far as motion or tendency of motion is concerned, yet will not necessarily produce the same effect in reference to stress. However, in the case of concurrent forces, we assume the stress produced by a force representing a resultant to be the same as that produced by the components, especially when the forces meet at the surface of the body considered. Some cases are very complicated and require very careful consideration, but we can always obtain a reasonable approximation which will come within the limits of practicability, and with this we must be contented.

44. Equilibrium of Couples.—In regard to equilibrium of couples, there is one important fact that should always be borne in mind, and that is, it always requires another couple to balance a couple, and the couples must be equivalent to each other and act in opposite direction to each other. As to the proof that it requires another couple to balance a couple, we have the following:

We know from the definition that a couple can have no single resultant, as the resultant always reduces to zero, and therefore would not balance a single force, and hence any single force or any system of forces reducing to a single resultant could not balance a couple. Then, evidently, the forces balancing a couple must have a resultant equal to zero and an equal and opposite moment, which would undoubtedly constitute another couple.

45. Graphical Determination of the Resultant of Parallel Forces or Those Nearly Parallel.—*First Method:* In the case of intersecting forces, the final resultant can be fully determined in its true position by prolonging in consecutive order the lines of action of both the forces and their successive resultants to intersection, and constructing a force triangle at each intersection, as shown in Art. 39, but it is obvious that in the case of parallel forces or forces nearly parallel the same method will not apply as such forces either do not intersect at all or their intersection is beyond practical limits. However, by resorting to the resolution of forces, the resultant of parallel forces and those nearly parallel can be determined as readily in its true position as the resultant of intersecting forces. As an example, let AB at (a) in Fig. 32 represent a body acted upon by three non-concurrent forces $P1$, $P2$, and $P3$, which are nearly parallel. Select some convenient point on the line of action of one of the forces, say, point O on the line of action of $P1$. Then at this point O resolve $P1$ into any two component forces as c and $c1$ by constructing (at will) a force triangle as abO. This is accomplished by assuming the intensity of one of the components, say c, and laying off aO from O in any convenient direction to represent this intensity, and then from a laying off ab equal and parallel to $P1$, and drawing bO to complete the triangle, we have $c1$, the other component, fully given by this last line bO. Then from O prolong the line of action of the component $c1$ until it intersects the line of action of the force $P2$ at O'. Then at this point resolve $P2$ into two components such that one of them will be equal and opposite to the component $c1$ at O and act along the same line OO'. Then there will be a component force $c1$ at O and an equal and opposite component force $c1$ at O', which will balance each other. The component $c1$ at O' being designated, we obtain the other component of $P2$ which we will call $c2$, by constructing the force triangle deO', which is constructed by laying off dO' equal and parallel to $c1$ and de equal and parallel to $P2$, and drawing eO' for completion. Then $c2$ is given by this last line eO'. Now from O' prolong the line of action of the component force $c2$ until it intersects the line of action of the force $P3$ at O''. Then at this point (O'') resolve $P3$ into two components such that one of them will be equal and opposite

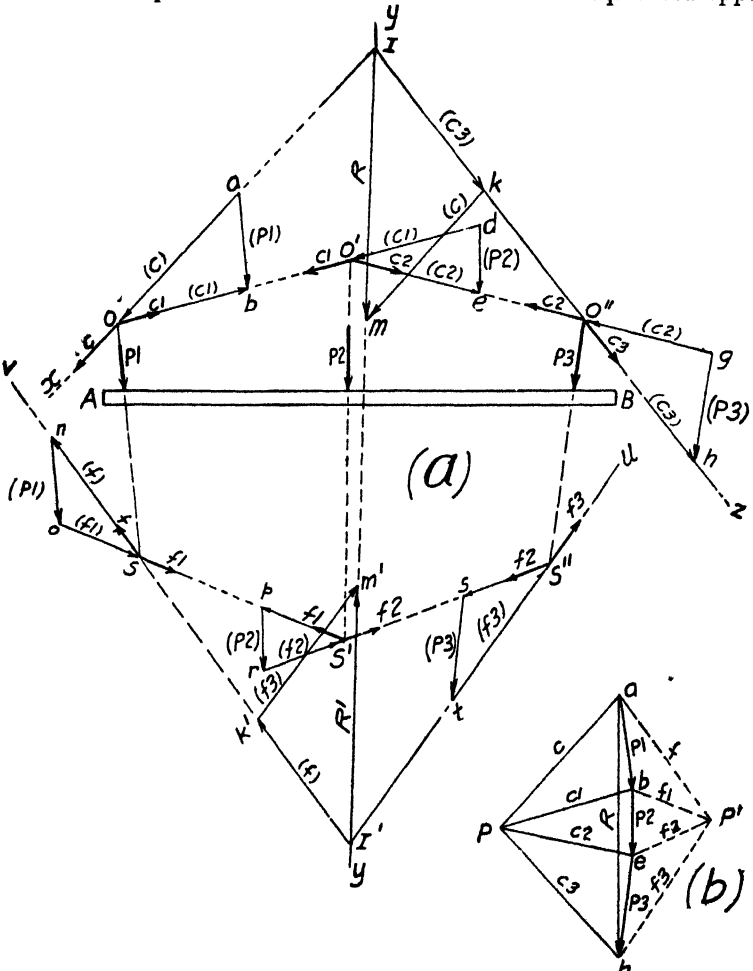


Fig. 32

to the component $c2$ at O' and act along the same line $O'O''$. Then there will be a component force $c2$ at O' and an equal and opposite component force $c2$ at O'' which will balance each other. The one component force $c2$ at O'' being designated, the other component of $P3$, which we will call $c3$, we obtain by constructing another force triangle ghO'', where $c3$ is given by the line hO''.

Now it will be observed that the three forces $P1$, $P2$, and $P3$ at (a)

have each been resolved into two component forces which replace the original force in each case, and that these component forces are all mutually balanced, except c (at O) and $c3$ (at O''), so that the three original forces, $P1$, $P2$, and $P3$, are really replaced by these two component forces, c and $c3$. So, evidently, if we determine the resultant of these two component forces in its true position, we will have the resultant of the three original forces, $P1$, $P2$, and $P3$, fully determined in its true position, as evidently the resultant in the two cases must be identical.

Now as the two component forces c and $c3$ are intersecting forces, their resultant is readily determined in its true position by prolonging their lines of action until they intersect at I and constructing the force triangle Ikm, where their resultant is fully given in its true position by the line Im. This resultant is the same in every respect as the resultant of the original forces $P1$, $P2$, and $P3$, and it may be assumed to be applied at any point along its line of action yy.

Another way of determining the resultant is as follows:

Instead of resolving the forces $P1$, $P2$, and $P3$ into components, as was done above, we can select any point on the line of action of one of the forces, as point S (really the same as we did before) on the line of action of $P1$ (at (a)) and apply two forces f and $f1$ such that the force $P1$ will be balanced by them. This is accomplished by assuming the intensity and direction of one of the forces and then constructing a force triangle to determine the intensity and direction of the other. Thus, beginning at S, lay off Sn (at will) to represent f in intensity and direction. Then from n lay off no equal and parallel to $P1$ and draw oS for completion of the triangle and we have $f1$ represented in intensity and direction by this last line oS. As the three forces at S are in equilibrium, they will act in the same direction around the triangle Sno as indicated, which is in accordance with Art. 42. Next, from S prolong the line of action of the force $f1$ until it intersects the line of action of the force $P2$ at S'. Then at this point apply two forces such that they will balance $P2$ and one of them be equal and opposite to the force $f1$ at S and act along the same line SS'. Then there will be a force $f1$ at S and an equal and opposite force $f1$ at S' which will balance each other. The force $f1$ at S' being known, the other balancing force at S', which we will designate as $f2$, we determine by constructing the force triangle $S'pr$ in the same manner as was explained in the case of triangle snO at S. Then from S' prolong the line of action of the force $f2$ until it intersects the line of action of $P3$ at S''. Then at this point (S'') apply two forces such that they will balance $P3$ and one of them be equal and opposite to $f2$ at S' and act along the same line $S'S''$. Then there will be a force $f2$ at S' and an equal and opposite force $f2$ at S'' which will balance each other. The force $f2$ at S'' being known, the other balancing force, which we will designate as $f3$, at S'', we determine by constructing another force triangle $S''st$, where $f3$ is given by the line $S''t$.

Now it will be observed that each of the three forces $P1$, $P2$, and $P3$ is balanced, that is, held in equilibrium by two forces applied in each case for that purpose, and such being the case, the system will be in equilibrium. But it will be observed that all of the balancing forces are mutually balanced except the force f at S and $f3$ at S''. Then, evidently, these two forces, f and $f3$, hold the three forces $P1$, $P2$, and $P3$ in equilibrium, in which case the resultant of the forces f and $f3$ will be an equal and opposite conspiring force to the resultant of the forces $P1$, $P2$, and $P3$ (see Art. 42). So if we determine the resultant of the two forces f and $f3$ in its true position, we will have a force the same in position and in intensity as the resultant of $P1$, $P2$, and $P3$, but which acts in the opposite direction, so that the resultant of the forces $P1$, $P2$, and $P3$ will thus be relatively determined in its true position. To determine the resultant of the forces f and $f3$, prolong their lines of action until they intersect at I' and construct the force triangle $I'k'm'$ and we have their resultant fully represented by the line $I'm'$.

The only difference in the two methods shown above is due to the components being reversed in direction in the second method so that they are really balancing forces instead of actual components. Otherwise the two methods are identical. Either of the broken lines $xOO'O''z$ or $vSS'S''u$ would be known as an *Equilibrium Polygon,* and the lines xO, OO', $O'O''$, and $O''z$, or vS, SS', $S'S''$, and $S''u$ would be known as their respective segments.

It is evident that the resultant of any number of such forces as $P1$, $P2$ and $P3$ can be determined in its true position in the same manner as shown above for these three forces, but in practice the same thing is accomplished with less work by a somewhat different method of procedure, as will now be shown.

Imagine the triangle abO, at (*a*) Fig. 32, moved bodily and parallel to itself to the position abP at (*b*). Next imagine the triangle deO' at (*a*) moved in the same manner to the position beP, d coinciding with b and O' with P, and likewise imagine the triangle ghO'' moved in the same manner to the position ehP, g coinciding with e and O'' with P. Now we have the three force triangles abO, deO', and ghO'' collected into a single diagram $PabehP$ (at (*b*)) which we will call a *Ray Diagram,* where the line $abeh$ is the load line, and the lines aP, bP, eP, and hP are the rays, and the point P is the pole. It will be observed that these rays, aP, bP, eP, and hP are parallel, respectively, to the segments xO, OO', $O'O''$, and $O''z$ of the equilibrium polygon $xOO'O''z$, at (*a*). So, evidently, if this ray diagram, $PabehP$, had been drawn beforehand, the equilibrium polygon $xOO'O''z$ could have been constructed by beginning at O and drawing xO parallel to the ray aP, then from O drawing OO' parallel to the ray bP, and likewise from O' drawing $O'O''$ parallel to the ray eP, and then from O'' drawing $O''z$ parallel to the ray hP. Then after having drawn the equilibrium polygon $xOO'O''z$ in this manner, the segments xO and zO'' could be prolonged until they intersect at I, and thus the point I would be located, which is one point upon the line of action of the desired resultant. But the things lacking would be the intensity, direction, and direction of action of the resultant. But it will be observed at (*b*) that by drawing the line ah we will have a force polygon $abeha$ which represents the forces $P1$, $P2$, and $P3$, and their resultant, each in intensity, direction, and direction of action. Then the intensity, direction, and direction of action of the desired resultant is given by the line ah. So by drawing through I a line parallel to this line ah we would have the line of action of the desired resultant, and hence the resultant would be fully determined, as its intensity and direction of action, as stated above, as well as its direction, are given by the line *ah* in the force polygon *abcha* at (*b*).

Now, according to Art. 39, in any case the force polygon representing any two or more forces and their resultant can always be drawn. So suppose in the above case the force polygon *abeha* at (*b*) be the very first thing drawn. Then we would have the intensity, direction, and direction of action of the resultant of the three forces *P*1, *P*2, and *P*3 given by the line *ah*, and the only thing lacking would then be the location of the line of action of the resultant, which can be obtained as above by constructing an equilibrium polygon. But we would have so far only the force polygon *abeha* constructed. Now if the point *P* at (*b*) and the point *O* at (*a*) were given, we could draw the rays *aP*, *bP*, etc., and beginning at *O* we could draw the equilibrium polygon $xOO'O''z$, as explained above, and thus obtain the desired line of action *yy*. However, it is not necessary to have the location of either of these particular points given, as the location of *O* was arbitrarily assumed in the first place, and the position taken by the equilibrium polygon $xOO'O''z$ was governed by the arbitrarily constructed triangle *abO*. So, evidently, we might just as well assume the pole *P* and draw the rays of a ray diagram and construct an equilibrium polygon accordingly by starting at any convenient point on the line of action of one of the forces.

As an example, let *AB*, Fig. 33, represent a body acted upon by four forces, *P*1, *P*2, *P*3, and *P*4. First construct the force polygon *abcdea*, at (*b*), by drawing *ab* equal and parallel to *P*1, and *bc* equal and parallel to *P*2, and so on as explained in Art. 39, and we have the resultant of the four forces *P*1 . . . *P*4 given in intensity, direction, and direction of action by the line *ae*. Then to obtain its line of action, take any convenient point *P* as a pole (at (*b*)) and draw the rays *aP*, *bP*, etc., thereby obtaining the ray diagram *PabcdeP*. Then construct a corresponding equilibrium polygon at (*a*) as $xOO'O''O'''z$, which is accomplished in the following manner: First select any convenient point on the line of action of one of the forces, say, point *O* on the line of action of *P*1. Then, from this

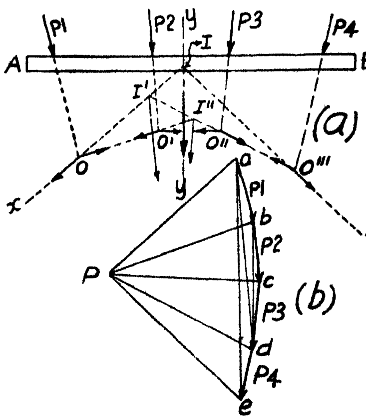


Fig 33

point draw the segments xO and OO' parallel to the rays *aP* and *bP*, respectively. Then from O' draw the segment $O'O''$ parallel to the ray *cP*. Then from O'' draw the segment $O''O'''$ parallel to the ray *dP*. Then from O''' draw the segment zO''' parallel to the ray *eP*. Then prolong the segments xO and zO''' until they intersect at *I*, and through this point *I* draw the line *yy* parallel to the line *ae* in the force polygon *abcdea*, and it will be the desired line of action of the resultant of the four forces, and hence the resultant is fully determined.

If the forces be absolutely parallel, the load line will be a straight line, and the line representing their resultant will coincide with this line, but, however, the work of determining the resultant is the same as shown above. For example, let *AB*, at (*a*) Fig. 34, represent a body acted upon by four parallel forces *P*1 . . . *P*4. We construct the load line *abcde* at (*b*) by laying off the forces in consecutive order as shown. Then assume the pole *P* and construct the ray diagram *PabcdeP* and draw the equilibrium polygon $xOO'O''O'''z$ at (*a*) as before and locate the point *I*, through which draw the line of action *yy* of the resultant parallel to the load line *ae*.

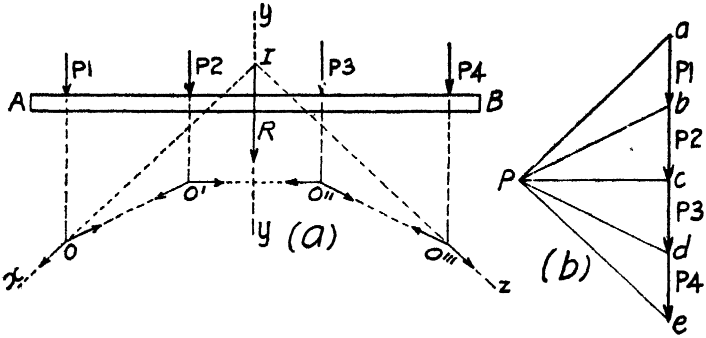


Fig 34

The method of procedure just shown is quite satisfactory in practice, but the student should not acquire the habit of constructing the force polygon and the ray diagram and then the corresponding equilibrium polygon without fully recognizing the exact significance of each step.

In Fig. 32 the force triangles were constructed on the segments of the equilibrium polygons, while in Figs. 33 and 34 the force triangles were constructed on the load lines of the force polygons, but the principle involved is just the same in either case. In the case shown in Figs. 33 and 34 we really resolved each of the forces *P*1, *P*2, *P*3, and *P*4 into two components by constructing, respectively, the triangles *abP*, *bcP*, *cdP*, and *deP*. However, it is easy to overlook this fact when the triangles are constructed by merely drawing the rays *aP*, *bP*, etc. If we consider the forces resolved into components, as is the usual custom, the component in each case will act around the force triangle in the opposite direction to that of the force, while if we reverse the direction of the components, they become balancing forces, in which case they will act in the same direction around the triangle as the force (see Arts. 39 and 42). Take, for example, the case shown in Fig. 33; the two sides *aP* and *bP* of the force triangle *abP*, at (*b*), give directly either the intensity and direction of two components of *P*1 or of two balancing forces. If we assume components, the one represented by the side *bP* will act from *P* to *b*, but if we assume balancing forces, the one represented by the side *aP* will act from *P* to *a*, while the one represented by the side *bP* will act from *b* to *P*. The same is true of all the other force triangles *bcP*, *cdP*, and *deP* shown at (*b*). In constructing an equilibrium polygon it really makes no difference whether we consider the forces resolved into components or balanced by forces applied for that purpose, as the final result will be just the same.

In constructing an equilibrium polygon after the ray diagram is completed, we virtually transfer the two components or the two balancing forces, as the case may be, of each force to a point upon its actual line of action. In order to show what is really involved in the construction, take for an example the case shown in Fig. 33. As the forces here were assumed to be resolved into components, the components will in each case act around the force triangle in the opposite direction to that of the force. Now beginning with $P1$, we can assume its own components, represented by the sides aP and bP of the force triangle abP at (b), as applied at any point O upon the line of action of $P1$. Then as the component represented by the side aP acts from a to P, a line drawn from O in this direction and parallel to aP will represent that component as being applied at O, and we thus obtain the segment xO of the equilibrium polygon. The other component of $P1$, which is represented by the side bP, acts from P to b. Then a line drawn from O in this direction and parallel to bP will represent that component as being applied at O, and we thus obtain the segment OO', which is really the line of action of this other component of $P1$ drawn only to O', where it intersects the force $P2$. We can assume the two components of $P2$, represented by the sides bP and cP of the force triangle bcP at (b), applied at any point upon the line of action of $P2$ the same as in the case of the two components of $P1$. Then, evidently, we can assume them applied at O' as well as at any other point. Now the component of $P2$ represented by the side bP of the force triangle bcP at (b) acts from b to P. Then a line drawn from O' in this direction and parallel to bP will represent that component of $P2$ as being applied at O', and we thus obtain the segment OO' again. The other component of $P2$ represented by the side cP acts from P to c. Then a line drawn in this direction from O' and parallel to cP will represent that component applied at O', and we thus obtain the segment $O'O''$, which is really the line of action of this other component of $P2$ drawn only to O'' where it intersects the force $P3$. The two components of $P3$, represented by the sides cP and dP of the force triangle cdP at (b), can be assumed to be applied at O'', and then the two components of $P4$, represented by the sides dP and eP of the force triangle deP, can be assumed to be applied at O''', and the discussion of the previous cases will apply to each of these cases.

The direction of action of the components of each force can be indicated on the rays of the ray polygon, and on the segments of the equilibrium polygon as in Fig. 33, but usually we omit such indications, because such are usually not necessary to any great extent—however, that is a minor item.

After an equilibrium polygon is drawn for a system of forces as shown in Fig. 33, the resultant of any number of these forces, if taken in consecutive order, can be determined as well as the resultant of all of them. For example, take the three consecutive forces $P1$, $P2$, and $P3$ (Fig. 33). By drawing the line ad (at (b)), we have the force polygon $abcda$, wherein the resultant of the three forces is given in intensity, direction, and direction of action by this line ad. By prolonging the segment $O''O'''$ until it intersects the segment xO at I', we have one point on the line of action of the resultant of these three forces, $P1$, $P2$, and $P3$. Then by drawing a line through I' parallel to the line ad, we obtain the desired line of action of their resultant, and thus we have the resultant of the three forces determined. As another case, the resultant of the two forces $P2$ and $P3$ can be determined by drawing a line bd (at (b)) and prolonging the segments $O''O'''$ and OO' until they intersect at I''.

Second Method: The following graphical method, which may be designated as the *Proportional Triangle Method,* can be used to advantage, quite often, in determining the resultant of parallel forces:

Let $P1$ and $P2$ (Fig. 35) represent any two parallel forces, and let

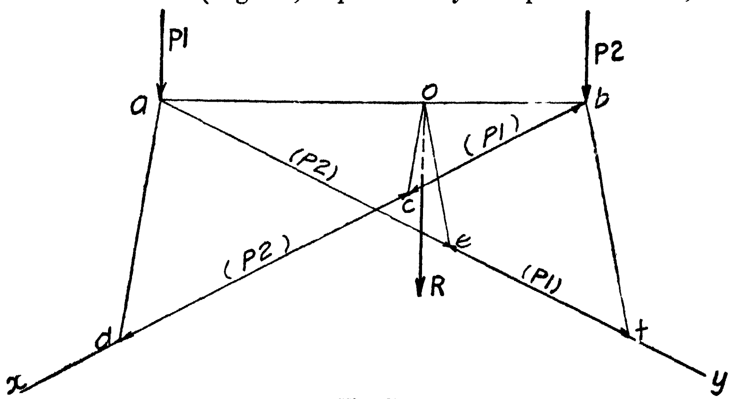


Fig. 35

R represent their resultant. Let a line ab be drawn perpendicular to the two forces and their resultant, intersecting the resultant at o. As there is no question as to the intensity of the resultant being equal to the sum of the forces, that is, $R = P1 + P2$, and the direction being the same as the forces, it remains only to determine the point o through which the line of action of the resultant passes. This can be obtained by the following construction: From b draw a line bx, making any convenient angle with the line ab. Then from b lay off, to any convenient scale, $bc = P1$; and from c lay off, to the same scale, $cd = P2$. Then draw ad and through c draw a line parallel to ad, and where it intersects the line ab will be the required point o through which the resultant passes. The same can be accomplished by drawing the line ay from a and laying off the loads as indicated and then drawing the lines fb and eo.

The above is readily seen to be true, for taking moments about b, according to Art. 41, we have

$$ob \times R = P1 \times ab \text{ or } ob = \frac{P1 \times ab}{P1 + P2}$$

which is the distance of the point o from b; but from similar triangles, bco and bda, we have

$$\frac{ob}{ab} = \frac{cb}{cb + cd} = \frac{P1}{P1 + P2},$$

or

$$ob = \frac{P1 \times ab}{P1 + P2},$$

and thus we have an identity.

In case there are more than two forces, the resultant of any two is determined as above and this resultant is then considered with one of the remaining forces, and so on. For example, let $P1$, $P2$, and $P3$ (Fig. 36)

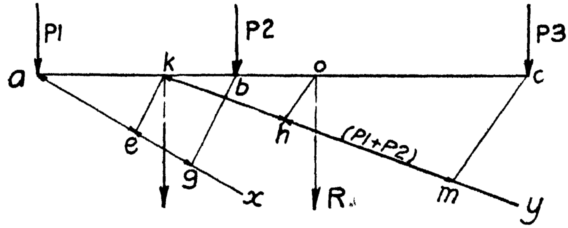


Fig 36

represent three parallel forces. First draw a line as abc perpendicular to the forces. Then from one of the points a, b, or c, say a, draw a line ax, making any convenient angle with the line abc. Then lay off $ae = P2$ and $eg = P1$. Then draw gb and through e draw a line parallel to gb, and the point k where this line intersects the line abc is a point on the line of action of the resultant of the two forces $P1$ and $P2$. Then from this point draw a line ky, making any convenient angle with the line abc. Then lay off $kh = P3$ and $hm = P1 + P2$. Then draw cm and through h draw a line parallel to cm, and the point o where this line intersects the line abc is a point on the line of action of the resultant of the three parallel forces.

It is important to note that the force from which the diagonal line is drawn is laid off last on that line. Thus in Fig. 36, $eg = P1$ and $hm = P1 + P2$.

In cases where the forces are in groups, as locomotive wheel loads, the method is a very convenient one, as the resultant of each group is known from observation and thus the forces really considered in the construction are quickly reduced to only a few.

46. Analytical Determination of the Resultant of Parallel Forces.—It is evident that the resultant of two or more parallel forces must act in the same direction as the forces, and with an intensity equal to their algebraic sum; otherwise it would not produce the same effect as the forces themselves. So the intensity, direction, and direction of action of the resultant of two or more parallel forces are really known directly from the forces themselves, and such being the case, the problem involved in determining their resultant really reduces to the locating of the line of action of the resultant.

Let $P1 \ldots P5$ represent five parallel forces acting upon the body AB (Fig. 37), and let R represent their resultant. Then we have $R = P1 + P2 + P3 + P4 + P5$. According to Art. 41, the moment of this resultant about any point is equal to the algebraic sum of the moments of the five forces about the same points. Then taking moments about some point O, we have

$$aP1 + bP2 + cP3 + dP4 + eP5 = R\bar{x},$$

from which we get

$$\bar{x} = \frac{aP1 + bP2 + cP3 + dP4 + eP5}{R} = \frac{aP1 + bP2 + cP3 + dP4 + eP5}{P1 + P2 + P3 + P4 + P5}.$$

So then we have the line of action of R located in reference to O.

In practice it is usual to take moments about one of the forces,

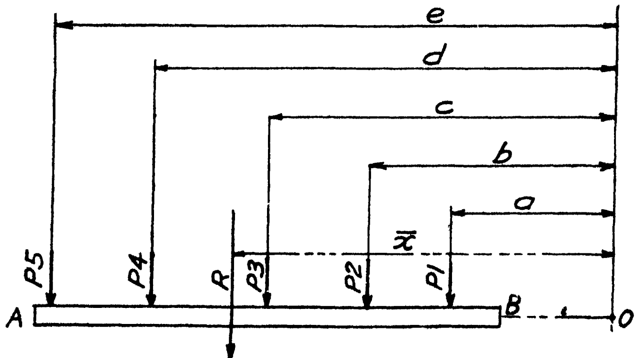


Fig 37

thereby shortening the work by eliminating the moment of that force. Thus taking moments about $P1$ (Fig. 38), we have

$$\bar{x}=\frac{bP2+cP3+dP4+eP5}{P1+P2+P3+P4+P5}.$$

In case the moments about one point are known, the moments about any other point can be determined by either increasing or diminishing the known moments by the sum of the forces multiplied by the common difference of lever arms. For example, the moments about the point z (Fig. 38) are equal to the moments about $P1$ plus the sum of the forces

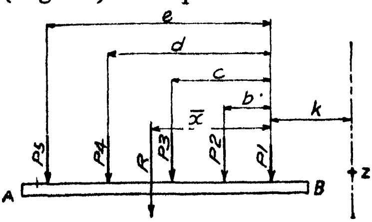


Fig 38

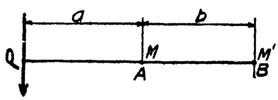


Fig 39

$P1 \ldots P5$ multiplied by k. That is, the moments about z are equal to $bP2+cP3+dP4+eP5+(P1+P2+P3+P4+P5)k$. This is readily seen to be true, for let M be the moment of a force P about A (Fig. 39) and let M' be its moment about B; then we have $M=aP$ and $M'=P(a+b)=aP+bP=M+bP$. It is readily seen that this is as true for a number of forces as it is for one. The product of the sum of the forces and common difference of lever arms would be subtracted from the moments in case the lever arms became shortened by the transferring of the center of moments.

47. Center of Gravity.—The most common case of parallel forces is that of gravity, which acts with equal intensity upon each ultimate part of material contained in a body, regardless of the kind of material. In case of an individual body, the line of action of the resultant of the gravity forces passes through what is known as the center of gravity of the body, but when more than one body is considered, the center of gravity of the bodies considered as a group is the same as the resultant of so many parallel forces, the weight of each body being considered as a force acting through the center of gravity of the respective body.

We can consider symmetrical bodies as being made up of parallel layers of homogeneous material wherein the centers of gravity of the layers coincide each with each, and such being the case, we need treat only one layer, which we may consider as a plane without thickness, from which results the common practice of treating area instead of volume, weight, or mass.

In case the material composing a body be symmetrical about a plane, there is no question but that the center of gravity of the body will be in that plane; and hence, if the material be symmetrical about three or more planes intersecting in a point, their point of intersection will undoubtedly be the center of gravity of the body. So it is evident that the centers of gravity of many bodies and plane figures are known from mere inspection of their state of symmetry. Thus, the center of gravity of a sphere is at its center, a line at its middle point, and we know the center of gravity of a circle, cylinder, cube, an ellipse, etc., from the same deduction. In fact, whatever the method employed in determining the centers of gravity of bodies and plane figures, we assume the center of gravity of certain elements as being predetermined by mere symmetry.

The determination of the center of gravity of loads in groups and the center of gravity of the cross-section of individual members of structures are the principal center of gravity problems occurring in structural engineering.

We can determine the center of gravity of any group of loads in the same manner as the resultants of parallel forces were determined in Arts.

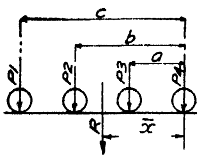


Fig 40

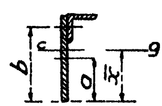


Fig 41

45 and 46. The analytical method outlined in Art. 46 is most used as it is usually more convenient than any other method. As an example, let $P1 \ldots P4$ (Fig. 40) represent four loads. Taking moments about one of the end loads, either $P1$ or $P4$, say $P4$, we have

$$\bar{x}=\frac{cP1+bP2+aP3}{P1+P2+P3+P4}$$

where $\bar{x}$ is the distance the center of gravity of the four loads is from $P4$.

In case of the cross-section of a member, we deal with the area instead of the weight. For example, let the sketch in Fig. 41 represent the cross-section of a plate and an angle which is riveted to the plate. Let A be the area of the plate, and let A' be the area of the angle, and let a and b be the distance from the bottom edge of the plate to the center of gravity of the plate and angle, respectively. Then taking moments about the bottom edge of the plate, we have

$$A'b+Aa=\bar{x}(A'+A) \text{ or } \frac{A'b+Aa}{A'+A}=\bar{x},$$

where $\bar{x}$ is the distance from the bottom edge of the plate to the center of gravity of the total cross-section of the plate and angle combined. The work could be shortened by taking moments about the center of gravity of the plate or angle, thus following out the same scheme as was used in the case of loads (Fig. 40).

The center of gravity of plain areas can be determined quite readily by the aid of calculus, wherein the general formula is

$$\int \frac{yda}{A}=\bar{x}.$$

For example, let Fig. 42 represent a rectangle having a height h and width b. Taking moments about the lower edge of the rectangle, we have

$$\bar{x}=\int \frac{yda}{A}=\int_0^h \frac{bydy}{A}=\frac{bh^2}{2A}=\frac{bh^2}{2bh}=\frac{h}{2}.$$

That is, the center of gravity of the rectangle from its lower edge is equal to one-half of its height, which we really knew from mere inspection.

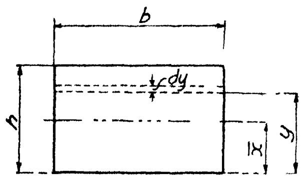


Fig 42

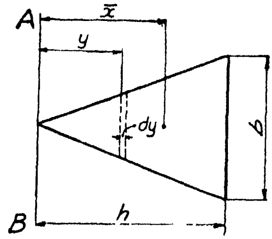


Fig 43

As another example, let Fig. 43 represent a triangle. Taking moments about a line AB parallel to its base, we have

$$x=\int \frac{yda}{A}=\int_0^h \frac{by^2dy}{hA}=\frac{2bh^3}{3bh^2}=\frac{2}{3}h,$$

which is the distance of the center of gravity of the triangle from the axis AB.

Problem 7. Determine the center of gravity in reference to the horizontal axis of the section shown in Fig. 44, which is composed of 3—11″ x ½″ plates and 2—6″ x 6″ x ½″ angles.

Solution: Taking moments of the areas about the center of gravity of the top plate, we have

$$32.5\,\bar{x}=0.5\times 7+1\times 7+2.93\times 11.5,$$

or

$$\bar{x}=\frac{44.19}{32.5}=1.36''$$

Hence the center of gravity of the section is 1.36″ below the center of the top plate, or 0.11″ below the back of the angles. The numerical quantities in this and the preceding problem can be verified by referring to the tables in the back of this book.

Problem 8. Determine the center of gravity in reference to the horizontal axis of the section shown in Fig. 45, which is composed of 2—15″ x 33# channels and one cover plate 22″ x $\frac{1}{2}$″.

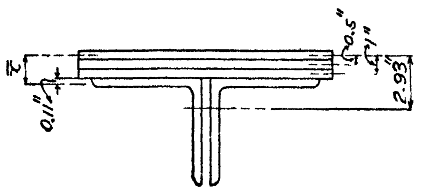


Fig. 44

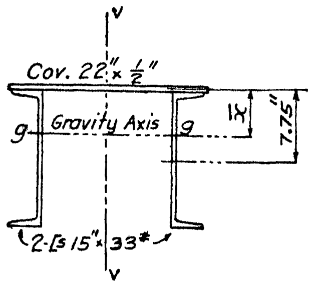


Fig. 45

Solution: Taking moments of the areas about the center of gravity of the cover plate, we have

$$30.8\bar{x} = 7.75 \times 19.8,$$

or

$$\bar{x} = \frac{7.75 \times 19.8}{30.8} = 4.98''.$$

That is, the center of gravity of the entire section is 4.98″ below the center of the cover plate, or 4.73″ from the back of the top flanges of the channels. The center of gravity here lies on the horizontal line marked *gg*, which is known as the gravity line or gravity axis.

It is readily seen that the center of gravity of the above section in reference to the vertical axis would lie on the vertical line *vv* passing through the center of the cover plate; however, the center of gravity of any section in reference to a vertical axis can be obtained by taking moments about some vertical line in the same manner as shown above for the horizontal axis.

48. Inertia.—It is an observed fact that it always requires a force to move a body when at rest, to stop it when in motion, or to change its motion in any respect. This would be true even if friction and all other external resistances were removed. Hence, it is evident that a body in itself offers resistance to every change of motion; otherwise, it would require no force to change the motion of the body if all the external resistances were absent. This property which bodies have in themselves of offering resistance to change of motion is known as *Inertia.* As an example, a body lying upon an absolutely smooth horizontal plane would offer resistance to any change of motion along the plane simply by virtue of its inertia. The actual force exerted would be directly proportional to the mass of the body concerned, and to the change of motion, or in other words to the acceleration produced. Let *W* be the weight of the body concerned, and let *a* be the acceleration produced in feet per second.

Then for the intensity of the force (in pounds) exerted, we have the proportion

$$F : W :: a : g,$$

from which we obtain

$$F = \frac{W}{g} a,$$

which is Formula (A) given in Art. 23.

49. Moment of Inertia and Radius of Gyration.—Let AB (Fig. 46) represent a very thin rectangular plate of homogeneous material. Suppose the plate starts to rotating about a vertical axis YY. The resistance offered by all such strips as cd (which will be due to their inertia) will be directly proportional to their distance out from the axis YY, as their relative increment of velocity or acceleration will be directly proportional to that distance. Then the intensity of the resistance of any strip will be directly proportional to the distance of the strip from the axis and to the mass of the strip. Suppose the plate made up of infinitesimal strips as cd, and let m be the mass of each strip. Then the force resisting the motion of the strip out unit distance from the axis will be (m) 1, while the force out x distance will be mx, and the moment of it about the axis would be $(mx)(x) = mx^2$. Then evidently the moments about the axis YY of all the resisting forces of these strips will be Σmx^2 where m is the mass of each strip, which was assumed to be the same for each and every one, and x the distance out to each strip, no two x's being the same. The expression Σmx^2 is known as the *Moment of Inertia*, which is usually indicated by I, and we have in general

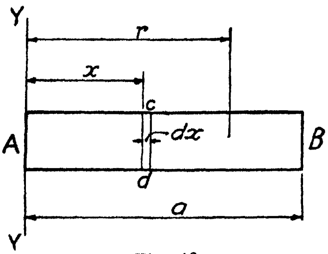


Fig 46

$$I = \Sigma mx^2.$$

In the case of the above plate it is evident that all of its mass could be concentrated at some point out from the axis YY so that its moment of inertia about the axis would be the same as that of the plate. This distance out would be known as the *Radius of Gyration* of the plate in reference to the axis YY. Then we have the general equation

$$I = Mr^2,$$

where M is the total mass of the plate and r the radius of gyration.

The mass of any one of the infinitesimal strips of the above plate is directly proportional to da, the area of the strip; then if we imagine the plate to diminish in thickness until it becomes a plane without thickness, we have for its moment of inertia

$$I = (\Sigma da) r^2 = Ar^2,$$

from which we obtain

$$r = \sqrt{\frac{I}{A}}.$$

Let a be the length of the plate and b its width; then considering the plate as a plane, its moment of inertia about the axis YY is

$$I=\int_0^a bx^2dx=\frac{ba^3}{3}.$$

Knowing its moment of inertia, we can then determine its radius of gyration from the above formula, $I=Ar^2$. Thus substituting, we have

$$\frac{ba^3}{3}=bar^2 \text{ or } r=a\sqrt{\frac{1}{3}}$$

The determination of the moment of inertia and radius of gyration of the cross-section of individual members of structures are the most common problems under this head met with in structural engineering. It is really the case of determining the moment of inertia and the radius of gyration of a plane, or as we may put it, the material cut by a plane. There are a few cases, however, where it is necessary to consider the mass, but in such cases the student will have no trouble if the general formula, $I=Mr^2$, given above, is applied.

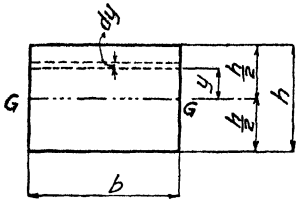


Fig. 47

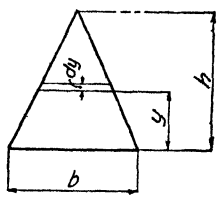


Fig 48

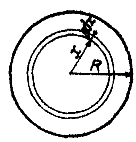


Fig 49

PROBLEMS

(1) Moment of inertia of a rectangle in reference to an axis GG (Fig. 47) through its center of gravity.

Solution: Here we have

$$I=\Sigma day^2=\int_{-\frac{h}{2}}^{+\frac{h}{2}} by^2dy=\frac{bh^3}{24}+\frac{bh^3}{24}=\frac{bh^3}{12}.$$

(2) Moment of inertia of a triangle about its base. (Fig. 48.) Here we have

$$I=\Sigma day^2=\int_0^h b\left(\frac{h-y}{h}y^2\right)dy=\frac{b}{h}\int_0^h(hy^2-y^3)dy=\frac{bh^3}{12}.$$

In case the axis is perpendicular to the plane, we have what is known as the *Polar Moment of Inertia*. For example, for the polar moment of inertia of a circle about its center we have from Fig. 49

$$I=\Sigma dax^2=\int_0^R dax^2=2\pi\int_0^R x^3dx=\frac{\pi R^4}{2}.$$

A more general case of the polar moment of inertia would be that of a rectangle. Here let O (Fig. 50) be the axis and draw XX and YY through O at right angles to each other. Then we have

$$I=\Sigma dap^2=\Sigma da(x^2+y^2)=\Sigma_x^k dax^2+\Sigma_y^t day^2,$$

which is the moment of inertia of the rectangle about the axis XX plus the moment of inertia about the axis YY. This gives the general formula which will apply to any plane figure, and further discussion of the polar moment of inertia is unnecessary. Its principal use is in the designing of shafting. The polar moment of inertia is usually designated by the letter J instead of I, which is known as the *Rectangular Moment of Inertia.*

50. Proposition.—*The rectangular moment of inertia of any plane figure about any axis in the same plane as the figure is equal to its moment of inertia about a parallel axis through its center of gravity, plus its area multiplied by the square of the distance between the two axes.* As proof of this, let $ABCD$ (Fig. 51) represent a rectangle whose area is $A = bh$

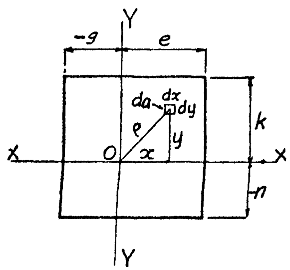


Fig 50

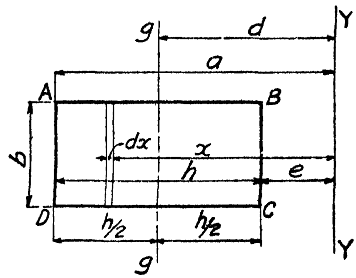


Fig 51

and whose moment of inertia about its gravity axis gg is I'. Then, according to the above, the moment of inertia of the rectangle about any axis as YY is $I = I' + d^2A$. This is shown to be true in the following manner: The moment of inertia of the rectangle about the axis YY is

$$I = \int_e^a bx^2dx = \frac{b}{3}(a^3 - e^3) \quad \dots\dots\dots\dots (1).$$

Now from Fig. 51 we have $a = d + (h/2)$ and $e = d - (h/2)$. Substituting these values of a and e in (1) we have

$$I = \frac{b}{3}\left(d + \frac{h}{2}\right)^3 - \frac{b}{3}\left(d - \frac{h}{2}\right)^3 = \frac{bh^3}{12} + bhd^2.$$

But $bh^3/12$ equals the moment of inertia of the rectangle about its gravity axis gg. Then substituting I' for $bh^3/12$ and A for bh, we have

$$I = I' + Ad^2,$$

which proves the above proposition in the case of a rectangle.

The above proposition is true of any plane figure. This can be shown by following out the same scheme as above. In case of bodies, the only thing different is that we would consider mass instead of area and write the formula

$$I = I' + Md^2$$

instead of

$$I = I' + Ad^2.$$

CHAPTER IV

THEORETICAL TREATMENT OF BEAMS

51. Classification.—Beams are, as a rule, classified according to the number and kind of supports they have. Whenever a beam simply rests upon a support, the support is known as a simple support, but when the beam is held rigidly by a support, the support is known as a fixed support.

Figures 52 to 55 show the most common types of beams. The cantilever beam shown in Fig. 52 has one fixed support, while the fixed beam

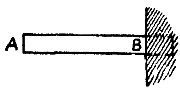


Fig. 52

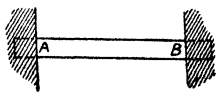


Fig 53

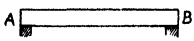


Fig 54

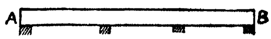


Fig 55

shown in Fig. 53 has two fixed supports. The simple beam shown in Fig. 54, which is the most common type, has two simple supports. The continuous beam shown in Fig. 55 has four simple supports; however, any beam with more than two supports of any kind is a continuous beam.

There are various other types of beams, such as overhanging, inclined, etc., but they are all really modifications of the above types and will be readily recognized as such without any preliminary description.

Beams are usually supported in a horizontal position and support vertical loads which produce vertical reactions. This will be understood to be the case unless otherwise stated.

52. Shearing Stress on Beams.—Let AB (Fig. 56) represent an ordinary rectangular wooden cantilever beam supported in a horizontal position by having the end B built into a wall. Let P represent a load at A which the beam supports in addition to its own weight. Let ab and cd represent the traces of two imaginary planes passing perpendicularly to the longitudinal axis of the beam, mentally separating the very short rectangular block $abcd$ from the other parts of the beam, so that we can think of this block as being a separate body.

By imagining the beam to be cut off instantly along the section ab we readily realize that the part abA of the beam has a tendency to move down vertically and the load P with it. As the material in the block $abcd$ prevents the motion, evidently the part abA of the beam exerts a downward force along the section ab upon the block equal to the combined weight of the part abA of the beam and the load P. Let S be this force.

Now it is evident that the part *cdB* of the beam to the right of the block must exert an equal force *S* (neglecting the weight of the block *abcd*) upward along the section *cd* upon the block in order to prevent the block from being pulled downward by the force *S* acting downward upon it along the section *ab*. It is readily seen that the downward force *S* acting along the section *ab* upon the block and the equal and opposite force *S* acting upward along the section *cd* have a tendency to break the material in the block off vertically. This action is most readily comprehended by imagining the sections *ab* and *cd* to approach each other until the block *abcd* is really a single layer of molecules. Then there would be a force *S* acting downward along *ab* upon the left side of the molecules and an equal and opposite force *S* acting upward along *cd* upon the right side of the molecules, thus tending to break the molecules off crosswise instead of tending to crush or pull them apart as in the case of simple stress, and the stress thus produced would be known as the shearing stress on the beam at that point. *Whenever forces act in this crosswise manner, in any case, the stress directly produced is known as shearing stress.*

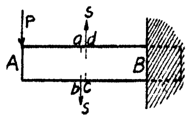


Fig 56

In speaking of the shear on a beam, or on any other body, we refer to it as being x pounds along a certain section of x pounds cut by an imaginary plane; but what we really mean is that a stress of x pounds is produced upon a strip of material infinitesimal in thickness adjoining that section.

It is evident that in the above case the shearing stress on the section *ab* (really the shear on the block *abcd*) of the beam is equal to the load P and to the weight of the part *abA* of the beam. The part of the shear due to the load P is the same for all vertical sections of the beam between A and B, while the part due to the weight of the beam will evidently be directly proportional to the length of the part *abA*. Then, if w be the weight of the beam per foot of length, the shear at any vertical section x feet from the end A will be $S = P + wx$. Here it is seen that the shear on any vertical section of the above beam is equal to the algebraic sum of the forces between the section and the end. If additional forces were applied, they would likewise be included in the summation, provided they were to the left of the section considered. As the forces upon the part of the beam to the right of the block *abcd* must exert an upward force upon the block along the section *cd* equal to the downward force along *ab* exerted by the forces to the left, it is evident that the shear on the block is equal to the algebraic summation of the forces on either side of it. So is the case of all beams, where the forces are applied perpendicularly to the longitudinal axis, and hence we have in general: *The shear on any section of a beam perpendicular to its longitudinal axis is equal to the algebraic summation of the forces on either side of the section, provided all of the forces act perpendicularly to such axis.*

This is readily seen to be true. For, let AB (Fig. 57) represent a beam in equilibrium acted upon by six forces $P1 \ldots P6$, no reference being made as to which are reactions; let *abcd* be an imaginary block cut through the beam, the same as the block *abcd* shown in Fig. 56: It is obvious that the forces $P1$ and $P2$ would cause the part *abA* of the beam to exert a force S upon the block along ab as shown, and it is readily seen that this force S, which is the shear on the block (neglecting the weight of the beam) would be equal to $P1 + P2$, that is, their sum. As the block is in equilibrium, undoubtedly there must be an equal and opposite force S exerted upon the block along cd, as shown, by the part cdB of the beam due to the forces acting upon that part. Then we have $P1+P2=+P6-P5+P4-P3=S$; that is, the shear on the block $abcd$ is equal to the algebraic summation of the forces on either side of it. Likewise, the shear on any section between $P6$ and $P5$ is equal to either $P6$ or $P1+P2-P3+P4-P5$; between $P5$ and $P4$ it is equal to either $P6-P5$ or $P1+P2-P3+P4$; and between $P4$ and $P3$ it is equal to either $P6-P5+P4$ or $P1+P2-P3$. This means that the shear at any cross-section of any beam (as stated above) is equal to the algebraic summation of the forces on either side of the section, provided the forces are perpendicular to the beam.

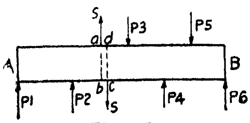


Fig 57

It is common practice to assume this shearing stress to be uniformly distributed over the cross-section of the beam. Then according to this assumption, the shearing stress per square unit on any cross-section of the beam is equal to the shear at the section divided by the area of the cross-section. So if p = the shearing stress in pounds per square inch of cross-section, S = the total shear on the cross-section in pounds, and A = the area of the cross-section in square inches, we have for the shearing stress per square inch the general formula

$$p=\frac{S}{A}.$$

The assumption that the shearing stress is uniformly distributed over the cross-section is not absolutely true, as will be shown later; however, it meets the requirements of practical designing in most cases.

53. Bending Stress on Beams.—Imagine for the time being that the block $abcd$ (Fig. 56) be removed and suppose instead the parts abA and dcB of the beam to be connected by a hinge placed in the center of the cross-section of the beam as shown in Fig. 58. Now it is evident that this hinge could be so constructed that it would prevent the part abA of the beam from moving down vertically. This means that the hinge would resist the shearing force or "shear" which was resisted by the block $abcd$, but it is evident that the part abA would now fall by rotating downward to the left about the hinge. So it is seen that the part abA of the beam has a tendency to rotate about the block $abcd$, and hence will be subjected to stresses therefrom. All stresses produced in this manner in beams, or any other bodies, are known as bending stresses, or as stresses due to cross bending.

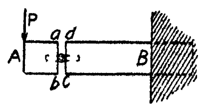


Fig. 58

To determine the bending stress in the block $abcd$ of the above beam (Fig. 56), let us assume all of the block removed except a thin horizontal strip ad at the top of the beam and an equal strip bc at the bottom, as shown in Fig. 59. For the sake of simplicity let us first consider the stress on these strips due to the load P only. The load P would cause the part abA of the beam to exert a downward vertical force s upon each strip (see Fig. 59) which must be resisted by an upward force s exerted upon the part abA in turn by each strip. These two upward forces s upon the part abA and the load P form a couple whose moment is $2sx$ or Px, as $2s = P$. This couple tends to rotate the part abA of the beam counter clock-wise, but actual rotation is prevented by the strips ad and bc, and consequently each will be subjected to a stress. Now as it requires a couple to balance a couple (see Art. 44) the stress produced upon the two strips must form a couple and the stress in strip ad must act to the right, while the stress in the strip bc must act to the left upon the part abA of the beam. Let F be the stress produced on each strip and let h be the vertical distance between their centers of cross-section. Then we have $Fh = Px$, from which we obtain $F = (Px)/h$ as the stress on each strip. So far our problem is very simple.

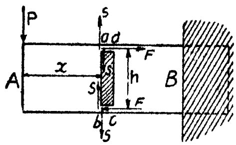


Fig. 59

But suppose we add two other strips, mm and nn, the same in section as the two just considered and which are also symmetrically arranged in reference to the longitudinal axis of the beam as shown in Fig. 60. Now it is evident that the additional strips will help to resist the moment of the couple Px, so that the stress F on the two strips just considered will be reduced. Let f and f' now be the stresses on the strips, and h and h' their lever arms, as indicated in Fig. 60. Then we have $Px = fh + f'h'$. Here it is seen that we have one equation and two unknown quantities, f and f', and it is readily seen that for each additional pair of such strips composing the block added, another unknown force will occur, so our problem is not as simple as it first appeared. However, we get out of our difficulty by resorting to Hooke's Law (see Art. 32). It is seen that the strips at the top of the beam are in tension while those at the bottom are in compression, consequently the strips at the top will be lengthened while those at the bottom will be shortened, and the block $abcd$, which is rectangular when not subjected to bending stress, will really be wedge-shaped as shown in Fig. 61. As the top strips are in tension and the bottom ones in compression, it is evident that there must be a horizontal strip of fibers somewhere between the top and bottom of the beam which has neither tension nor compression; that is, no stress at all. Let og be the location of this strip which is known as the neutral plane or neutral axis. As all the strips above og are in tension and all below are in compression, it is obvious that g is the center of tendency of rotation of the part abA of the beam. Then, evidently, the distortion of the strips both above and below og will vary directly as their distance out from og. Now as the stress on any strip, according to Hooke's Law, will be directly proportional to its distortion, the stress per square inch on any strip will be directly proportional to its distance out from og, and hence the stress on the strips may be represented by the

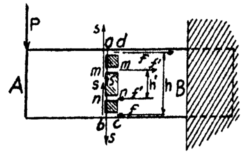


Fig. 60

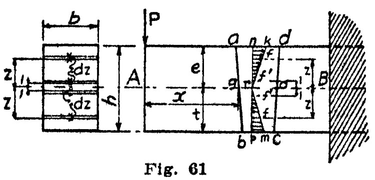


Fig. 61

arrows enclosed in the triangles *nkr* and *mpr* as shown in Fig. 61, where the lines *kr* and *mr* have the same slope with the vertical.

Let f be the stress per square inch on any horizontal strip of fibers y distance out from *og*, and let f' be the stress per square inch on a strip out unit distance from *og*. Then we have the proportion

$$1 : y :: f' : f,$$

from which we get

$$f' = \frac{f}{y}.$$

Now as f/y is the stress on a strip out unit distance from *og*, the stress on a strip out any distance z from *og* would be z times f/y or $(f/y)z$, and the moment of it about g would be z times this or $(f/y)z^2$. Now f was taken as so many pounds per square inch, but as the stress varies continuously from *og* outwardly, there will not be a square inch of material anywhere having the same stress, so that each strip must really be considered as being a horizontal element; that is, a mere plane of fibers having an infinitesimal area of cross-section da. For the sake of conception we may say that da is 1/1,000,000 of a square inch. Then the actual stress or force out z distance from *og* would be 1/1,000,000 of $(f/y)z$ or $(f/y)zda$ $(da = dzb)$ (see section Fig. 61), and the moment of it about g would be z times that, or $(f/y)z^2da$. Now, undoubtedly the summation of these moments $(f/y)z^2da$ about g for all of the strips in the block *abcd*, which we can express as $(f/y)\Sigma z^2da$, must be equal to the moment of the vertical couple Px. So we have

$$Px = (f/y)\Sigma z^2 da.$$

But

$$\Sigma z^2 da = I$$

(see Art. 49), the moment of inertia of the cross-section of the beam. Then we have the equation

$$Px = \frac{f}{y} I.$$

It will be observed very readily that Px is simply the moment of the load P about the section *ab* (*ab* being considered vertical, and practically it is); that is, Px is the measure of the tendency of rotation of the part *abA* of the beam about the section *ab* due to the load P. Then evidently the moment of any other load to the left of the section *ab* about the section would be the measure of the tendency of rotation of the part *abA* of the beam about the section due to any such loads. Then if ΣPx be the algebraic summation of the moments of any number of loads on the left of the section about the section, we have the formula

$$\Sigma Px = \frac{f}{y} I \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1),$$

from which the stress f on any horizontal element of the block *abcd* due to the loads can be computed by substituting for y the distance of the element out from *og*, and f would be known as the bending stress on the element, the block *abcd* being considered infinitesimal in length.

Now it is obvious that the above formula will apply to any section of the above beam if ΣPx be taken as the algebraic summation of the moments of the loads to the left of that section about the section. But it is readily seen that the ΣPx on one side of any section must be equal and opposite to the ΣPx on the other side in order that the beam may not rotate about the section. This is undoubtedly true in the case of any beam whatever. That means that the above formula applies to any section of any beam if ΣPx be taken as the algebraic summation of the moments of the forces (both applied forces and reactions) on either side of the section about the section. This summation is known as the bending moment at the section and is usually represented by the letter M. Then we can write the general equation as

$$M=\frac{fI}{y} \qquad \text{(C)},$$

and by transposing we have

$$f=\frac{My}{I} \qquad \text{(D)},$$

which is the stress on any horizontal element y distance from the neutral axis either above or below it. It will be observed that the neutral axis extends all the way along the beam.

It is seen that y varies from zero to either $+e$ or $-t$ (Fig. 61) and that f will be a maximum when y is a maximum, as M and I are constants for each specific section. So if y be taken as the distance to the farthest element out from g the maximum value of f will be obtained. This is what is usually desired, and for that reason y is usually spoken of as the distance to the extreme fiber and f as the stress on the extreme fiber.

The summation of the bending stresses above the neutral axis is always equal to the summation of the bending stresses below it, but the sums of their moments about the neutral axis are not equal except in the case of symmetrical section. But each force on one side is always paired with an opposite and equal force on the other side, thus forming couples throughout the cross-section.

54. Reaction on Simple Beams.—In order to determine the shearing and bending stress in a beam, it is first necessary to know all of the external forces acting upon it. The applied forces, or loads as they are known, are usually given, but the reactions are usually unknown and hence have to be determined before the stresses in the beam can be determined. We usually resort to the equations of moments to determine the reactions on beams. However, they can be graphically determined, as will be shown later.

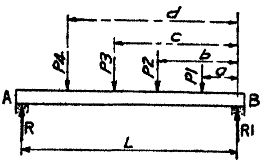


Fig. 62

Let AB (Fig. 62) represent a beam which is supported at A and B and which in turn supports the loads $P1$, $P2$, $P3$, and $P4$. The reaction R at A due to the above loads can be obtained by taking moments about B, and the reaction $R1$ at B can be obtained by taking moments about A. Thus, taking moments about B, we have

$$+RL-aP1-bP2-cP3-dP4=0,$$

from which we obtain

$$R=\frac{aP1+bP2+cP3+dP4}{L}$$

and by taking moments about A we can determine $R1$ in the same manner. It will be observed that by taking moments about the line of action of one of the reactions we eliminate that reaction from the equation of moments, thus reducing the number of unknowns in the equation to one. The sum of the moments being equal to zero is, in accordance with the laws of equilibrium, that the moments of the external forces acting upon a body in equilibrium about every point must be equal to zero (see Art. 42). The above method is quite general, provided the loads are fully known and the reactions are limited to two in number, which is always the case for simple beams.

55. Graphical Representation of Shear on Simple Beams.—The shear on a beam at any section is equal to the algebraic sum of the loads and reaction (as stated in Art. 52), summed up to the section, beginning at either end. For example, the shear at every section between the loads $P3$ and $P2$ (Fig. 62) is equal to either $R-P4-P3$ or $R1-P1-P2$; between A and $P4$ it is equal to either R or $R1-P1-P2-P3-P4$; between B and $P1$ it is equal to either $R1$ or $R-P4-P3-P2-P1$.

This shear on the above beam can be graphically represented in the following manner:

Draw a horizontal line ab (Fig. 63) equal to L, the length of the beam (between centers of end bearings), to some convenient scale. Then draw the lines of action of the forces in their relative positions as shown at 1, 2, 3, and 4. Then beginning at one end, say, at a, draw $ac=R$. Then through c and parallel to ab draw cd and from d lay off $de=P4$. Then from e draw ef and from f lay off $fg=P3$, and so on, thus obtaining the zigzag line $cde\ldots n$. Any ordinate as rr from the line ab to the zigzag line represents the shear on the beam at that point.

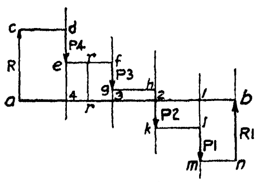


Fig. 63

The value of the shear on any vertical section of a beam will depend upon the weight of the load or loads it supports and upon the position they occupy in reference to the section. Loads are known as *dead load* and *live load.*

The dead load upon a beam is the weight it supports that is fixed in position, while the live load is the weight it supports that is not fixed in position but may move to any position upon it. The dead load of structures usually consists of the weight of the structure itself, while the live load consists of loads which it supports but which at the same time move over it, as a locomotive, train of cars, wagons, etc. The dead load is usually considered as a uniformly distributed load. The live load is sometimes considered as a uniformly distributed load, but more often as concentrated loads.

If the load be a uniformly distributed dead load, each reaction will be $R=wL/2$, where L is the length of the span and w the weight of the dead load per foot of span, and the shear out any distance z from either end will be $S=R-zw=wL/2-zw$. It is seen that when $z=L/2$ the shear is equal to zero, and when $z=0$ the shear is equal to $wL/2$. This shows that the shear due to such a load is a maximum at the ends and zero at the center.

The above equation for shear, $S=wL/2-zw$, is an equation to a straight line, so evidently the shear can be represented graphically by the ordinates to a straight line, as *acb* (Fig. 64), where *AB* represents the length of span and *aA* and *bB* are equal to $wL/2$. The shear, beginning at *A*, decreases until *c*, the center of the span, is reached, where it is zero; then beyond that point, still summing up to the right, the shear will be minus, increasing toward *B*, being equal to $-wL/2$ at *B*. So it is seen that the ordinates to the line *acb* truly represent shear for dead load uniformly distributed. For example, the ordinate *fe* represents the shear at *f*, which is positive, while *kh* represents the negative shear at *k*. The sign of the shear has no practical significance other than algebraic.

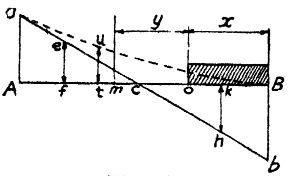


Fig 64

In case of uniform live load moving over a beam, the shear at any section will be a maximum when the load just extends from one of the ends up to the section. For example, suppose a live load of *p* pounds per foot extends from *B* up to some section *O*, a point *x* feet from *B*. Then the shear at *O* is equal to the reaction at *A*, which expressed in terms of the load is $(px)x/2 \div L = px^2/2L$. It is evident that the shear produced at section *O*, as the load moves from *B* to *O*, keeps increasing as the load approaches *O*, as it is equal to the reaction at *A*, which keeps increasing as the load moves from *B* to *O*. Now if the load extends past *O* to any section *m*, the reaction at *A* will be increased a certain amount by the additional load *py*, but in obtaining the shear at *O* we subtract all of the load *py* from the reaction at *A*, and as all of the additional load *py* is not, as we say, transmitted to the end *A*, it is evident that the load extending beyond *O* will diminish the shear at that section. Therefore the above statement is shown to be true, and the equation $S=px^2/2L$ is the general equation for the maximum shear produced on a beam by a uniform live load moving over it. The equation is that of a parabola. Then, evidently, if we construct a parabola as *auB* so that *Aa* is equal to $pL/2$, the shear at any section will be represented by the ordinate to the parabola at that section. For example, the shear at *t* is represented by the ordinate *ut*.

The shear curve *auB* (Fig. 64), as stated above, is an absolute parabola for a uniform live load, but for such loads as the wheels of a locomotive it would be a curve made up of a series of straight lines, however, if the loads are nearly equal in weight and quite uniform in spacing, the curve will approach a parabola, and in most cases it is near enough to be considered one for practical purposes.

56. Graphical Representation of Bending Moments on Simple Beams.—

Case I. When the load is uniformly distributed over the entire length.

Let *AB* (Fig. 65) represent a beam supported at *A* and *B* which in turn supports a uniform load of *w* pounds per lineal foot of length. The bending moment at any section *ss*, *x* feet from *A* due to this load is

$$M = Rx - (wx)\frac{x}{2} = Rx - \frac{wx^2}{2},$$

But

$$R = \frac{wL}{2}$$

so we have

$$M = \frac{wLx}{2} - \frac{wx^2}{2}$$

for the bending moment at any section of the beam. When $x = L/2$, we have

$$M = \frac{wL^2}{4} - \frac{wL^2}{8}$$

from which we obtain

$$M = \frac{wL^2}{8} \quad \text{(E)},$$

which is a formula of great practical value as it expresses the moment at the center of any simple beam uniformly loaded, which is the maximum moment that can occur under such loading, and usually this is what we desire to know.

The above formula, $M = wLx/2 - wx^2/2$, is an equation to a parabola. Then, evidently, the bending moments at the different sections along the beam vary as the ordinates to a parabola. It is seen from the equation that $M = 0$ when $x = 0$, and also when $x = L$; and when $x = L/2$, $M = wL^2/8$. Then if we draw CO equal to $wL^2/8$ to scale, and perpendicular to the beam at its center and pass a parabola through A, O, and B as shown in Fig. 65, any ordinate as rr will represent the bending moment on the beam at that point.

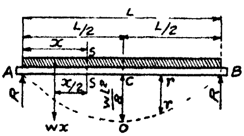


Fig. 65

Case II. When the loads are concentrated at different points along the beam.

Let AB (Fig. 66) represent a beam supported at A and B which in turn supports the loads $P1$ . . . $P5$, as shown. The bending moment anywhere between A and $P5$, due to these loads, is $M = Rx$, which is an equation to a straight line. When $x = 0$, $M = 0$, and when $x = a$, $M = Ra$. Then, if we draw cd equal to Ra, to scale, and join A and d, any ordinate as ss to the line Ad will represent the bending moment on the beam at that point. The bending moment anywhere between $P5$ and $P4$ is $M = R(a + x') - P5x'$, which is also an equation to a straight line. When $x' = 0$, $M = Ra$, so, evidently, one point in the line represented by the equation $M = R(a + x') - P5x'$ is d, as $cd = Ra$. When $x' = b$, $M = R(a + b) - P5b$. So if we draw ef equal to $R(a + b) - P5b$ to scale, using the same scale as in the case of cd, we obtain the line df, the ordinates to which evidently represent the bending moments between $P5$ and $P4$. Now it is evident that the lines fg, gh, hk, and kB can be similarly constructed, the ordinates to

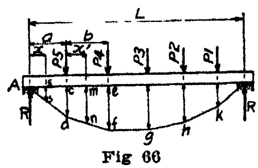


Fig 66

which represent the bending moments between $P4$ and $P3$, $P3$ and $P2$, $P2$ and $P1$, $P1$ and B, respectively. So it is seen that the bending moments on a simple beam due to any number of concentrated loads can be graphically represented by the ordinates to a series of straight lines forming a closed polygon with the ends of the beam. The nearer the loads approach a uniform load, the nearer this polygon approaches a parabola.

It is important to note that the maximum moment always occurs under a load. This is readily seen, for the maximum ordinate to any segment of the above polygon is under a load and hence the maximum of them all will be under a load.

57. Graphical Representation of the Bending Moments and Shears on Cantilever Beams.—

Case I. When the load is uniformly distributed over the entire length.

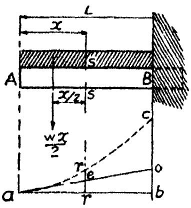


Fig. 67

Let AB (Fig. 67) represent a cantilever beam supporting a uniform load of w pounds per foot of length. The bending moment at any section ss, x feet from A, is $M=(wx)x/2=wx^2/2$, which is an equation to a parabola. Making $x=L$, we have $M=wL^2/2$, which is the bending moment at B. Then if we draw the horizontal line $ab=L$, and the vertical line $cb=wL^2/2$, to scale, and construct the parabola ac, the ordinates between the line ab and the curve ac will graphically represent the bending moments on the beam due to the above load. For example, the moment at the section ss is represented by the ordinate rr.

The shear at any section ss is $S=wx$, which is an equation to a straight line. When $x=0$, $S=0$, and when $x=L$, $S=wL$, which is the shear at B. Then, if we draw bo equal to wL, to scale, and join a and o, the shear at any section in the beam is represented by the ordinate between the line ab and ao immediately under the section. For example, the shear at the section ss is represented by the ordinate er.

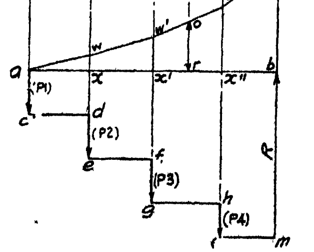


Fig. 68

Case II. When the loads are concentrated at different points along the beam.

Let AB (Fig. 68) represent a cantilever beam supporting the concentrated loads $P1$ . . . $P4$ as shown. As a general example, the bending moment due to the above loads at a section as ss, expressed analytically, is $M=nP1+bP2+tP3$. If we proceed in the same manner as in the case of a simple beam, making the horizontal line $ab=L$, and the verticals wx, $w'x'$, etc., under each load, equal to the bending moment about each, respectively, making bv equal to the moment at B, and drawing the broken line $aww'w''v$, we have the bending moments on the beam graphically represented by the ordinates between the line *ab* and this broken line. For example, the moment at the section *ss* is represented by the ordinate *ro*. The shear at the section *ss* is $S = P1 + P2 + P3$, expressed analytically. Constructing the zigzag line *acd* ... *m*, the same as in the case of simple beams (Art. 55), we have the shear at any section represented by the ordinate between the line *ab* and this zigzag line.

58. Graphical Construction of a Parabola.— The following method of constructing a parabola will be found to be quite convenient. It really consists in passing a parabola through two points, when one of the points is taken at the vertex. Let *A* and *B* (Fig. 69) be any two given points and let *A* be at the vertex of the parabola desired. Draw the line *XX* through *A* as the *X*-axis of the parabola. Then draw *AC* perpendicular

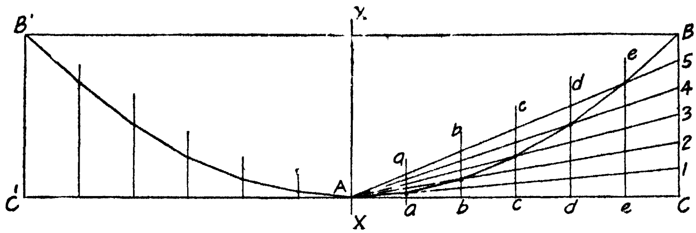


Fig 69

to *XX* and *BC* perpendicular to *AC*. Divide *AC* and *BC* into an equal number of equal parts, say, six, as shown. Then from *A* draw the radial lines 1-*A*, 2-*A*, etc., and where the radial line 1-*A* intersects the vertical *aa* is one point on the curve of the required parabola, and where the next radial line 2-*A* intersects the next vertical *bb* is another point on the curve, and so on. The other side *AB'* of the parabola can be constructed by laying off $AC' = AC$ and dividing it into the same number of equal parts as *AC* and drawing verticals as shown and then projecting the corresponding points over from the curve *AB* just constructed, or by dividing *B'C'* into the same number of equal parts as *AC'* and proceeding as explained for the other side of the curve.

59. Relation of Shear to Bending Moment.—Let *AB* (Fig. 70) represent a simple rectangular wooden beam supported at *A* and *B* and let *P* be a load which the beam supports at mid-span, and let *R* and *R*1 be the reactions (which will be equal) at *A* and *B*, respectively, due to this load *P*. Suppose the reactions applied exactly at the ends of the beams as indicated, which is not at all an imaginary case, and imagine the beam divided up into short rectangular blocks as shown, by imaginary planes passing perpendicularly to the longitudinal axis of the beam. Let Δx be the length of each block. The reaction *R* at *A* applied along the end of the first block, beginning at *A*, tends to move that block upward. If the block does not move undoubtedly the second block prevents it by exerting an equal and downward force *R* where the two blocks join as indicated. As the first block is held entirely by the second block it is obvious that the first block will exert an upward force upon the second block where the two blocks join, and if the second block does not move upward undoubtedly the third block prevents it by exerting a downward force R where the second and third blocks join, and hence it is seen that the blocks to the left of the load resist each other in transmitting the shear, each thereby receiving an upward force R upon one end and an equal downward force upon the other end as is indicated. The same is true of the blocks to the right of the load P except the forces are reversed in order. These vertical forces not only tend to shear the blocks off vertically but tend to rotate them as well, as the two forces on each block form a vertical couple.

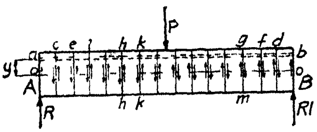


Fig 70

Now beginning at A and considering the blocks to the left of P only, the bending moment at the right end of the first block is $R\Delta x$; at the right end of the second block it is $R(2\Delta x)$; and at the right end of the third block it is $R(3\Delta x)$; and so on, being $R(n\Delta x)$ at the right end of the nth block from A, which is seen at a glance to be nothing more than the summation of the moments of the vertical couples on the blocks beginning at A. This means that the bending moment beginning at the end A increases toward the load P by one $R\Delta x$ at each block. For example, the bending moment at kk is one $R\Delta x$ greater than it is at hh. So, evidently, $R\Delta x$ is the increment of the bending moment to the left of the load P in the case of the above beam. By the same reasoning we have $R1\Delta x$ as the increment of the bending moment to the right of the load P.

It will be observed that all of the blocks to the left of the load P tend to rotate clock-wise while all of the blocks to the right of P tend to rotate in the opposite direction or counter clock-wise, so that the sign of the increments of the bending moment on one side will be different from those on the other side of the load. The bending moment at any point to the left of the load P is equal to the sum of the increments $R\Delta x$ between A and the point and the bending moment at any point to the right of the load is equal to the sum of the increments $R1\Delta x$ between B and the point, although the bending moment at any point is equal to the sum of the increments summed up algebraically from either end of the beam. For example, the bending moment at gm is equal to the sum of the increments to the right of gm or to the sum of the increments to the left of P minus the sum of the increments between P and gm, as the signs are different.

In the case shown in Fig. 70 the increments of the bending moment are all equal, but suppose we consider the case shown in Fig. 71 where R and $R1$ are the reactions at A and B, respectively, due to the three loads $P1$, $P2$, and $P3$. Here it is readily seen that the increment of the bending moment between A and $P1$ due to $P1$, $P2$, and $P3$ is $R\Delta x$; between $P1$ and $P2$ it is $(R-P1)$ Δx; between $P2$ and $P3$ it is $(R-P1-P2)$ Δx; and between $P3$ and B it is $(R-P1$ $-P2-P3)\Delta x$. That is, the increment of the bending moment at any point is equal to the shear at that point multiplied by Δx. It will be observed that this was true in the case shown in Fig. 70, for R is the shear to the left of P and $R1$ the shear to the right of P, which are equal in that case. Now, the vertical couples on any such

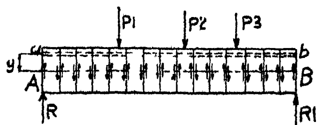


Fig 71

blocks or strips, as here considered, in any beam, could not be anything other than the vertical shear couples on them, so we have, in general,

$$\Delta M = S\Delta x$$

for the increment of the bending moment, where S is the shear on the block in question and Δx its length. Now, Δx could have any practical value but the ultimate increment will be when Δx is an infinitesimal. Then assigning the smallest possible value to Δx making it an infinitesimal dx, we obtain

$$dM = Sdx$$

as the ultimate increment of the bending moment, as it is the smallest increment possible. That is, the differential of the bending moment at any section of any beam is equal to the vertical shear couple on an imaginary vertical block of infinitesimal length. The preceding formula is usually written

$$\frac{dM}{dx} = S \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(F)}.$$

Expressing this in words, we say that the first derivative of the bending moment in reference to x is equal to the shear, where x is the distance from the end of the beam to the section considered.

The above equation (F) is sometimes quite useful, as the determining of the shear from the bending moment is sometimes desirable. The equation, or formula, is very readily applied, as is seen from the following, although its true worth is more readily recognized in more complicated cases.

Considering both the load P and the weight of the beam acting upon the beam shown in Fig. 70, the bending moment at any section x distance from the end A is

$$M = Rx - \frac{wx^2}{2}$$

where w represents the weight of the beam per foot. Differentiating both sides of the equation and dividing through by dx we have

$$\frac{dM}{dx} = R - wx,$$

which, as is readily seen, is the shear at any cross-section x distance from the end A and to the left of the load.

The bending moment at any section x distance from the end of a simple beam having a length L and supporting a uniform load of w pounds per lineal foot is

$$M = \frac{wLx}{2} - \frac{wx^2}{2}$$

(see Art. 56, Case I). Differentiating both sides of this equation, and dividing through by dx, we have

$$\frac{dM}{dx} = \frac{wL}{2} - wx,$$

which, as is readily seen, is the general expression for the shear at any section of the beam. If $x=0$, the shear is equal to $wL/2$, and if $x=L$, it becomes $-(wL/2)$, each of which is recognized as an end shear or reaction. If $x=L/2$ the shear is equal to 0, all of which goes to show how readily Formula (F) can be applied, even for ordinary cases.

60. Increment of the Bending Stress.—Let ab (Fig. 70) represent a very thin horizontal strip through the beam y distance above the neutral axis oo. The end block at A, owing to its tendency to rotate clock-wise, due to the vertical shear couple $R\Delta x$, will exert a force to the right upon all horizontal elements in the second block above the neutral axis and a force to the left upon all such elements in the second block below the neutral axis. Then undoubtedly the first block will exert a force to the right upon the strip ab. Let Δf be this force. The end block at B has the same tendency to rotate as the end block at A but in the opposite direction, and hence the end block at B will exert a force Δf to the left upon the strip ab. Then the strip ab will have a compressive stress of Δf in it from c to d due to the end blocks. The second block from the end A having the same tendency to rotate as the end block will likewise exert a force Δf to the right upon the strip ab and the second block from the end B will exert a force Δf to the left upon the strip ab, and hence the strip ab will have a compressive stress of $2\Delta f$ from e to f due to the first and second blocks from the ends. The third block from the end A will exert a force Δf to the right upon the strip ab and the third block from B will exert a force Δf in the opposite direction upon the strip ab and hence the strip ab will have a compressive stress of $3\Delta f$ from l to g due to the first, second, and third blocks from the ends. Thus it is seen that the stress in the strip ab is increased at each block by one Δf as we pass from either end toward the load P where the stress is a maximum. So, evidently, Δf is the increment of the bending stress in the strip ab. As the strip ab could be any horizontal element out any distance y from the neutral axis, either above or below, it is evident that the bending stress on any horizontal element in the beam will vary by such increments as Δf the same as in the case of the strip ab; of course, their real values will be different owing to their distance out from the neutral axis being different. For the value of Δf at any block to the left of the load P (Fig. 70) we have

$$\Delta f=\frac{(R\Delta x)y}{I},$$

and for its value to the right we have

$$\Delta f=\frac{(R1\Delta x)y}{I},$$

where I is the moment of inertia of the cross-section of the beam in reference to the horizontal gravity axis of the beam; y the distance out from the neutral axis to the strip or element considered; and $R\Delta x$ and $R1\Delta x$ the vertical couple in each respective case. But as the vertical couple on any imaginary vertical block or strip in any beam whatever is simply the shear on the block multiplied by its length, we can write the general expression for the increment of the bending stress at any point in any beam as

$$\Delta f = \frac{(S\Delta x)y}{I} \text{.................................(a).}$$

Then beginning at either end of a beam we can obtain the stress on any horizontal element x distance from the end by summing up the stress increments as expressed by (a). This would give us

$$\Sigma\Delta f = \Sigma S\Delta x \frac{y}{I}.$$

But $\Sigma\Delta f = f$ and $\Sigma S\Delta x = M$, the bending moment (see Art. 59) at a point x distance from the end, when Δx becomes an infinitesimal. Then substituting f for $\Sigma\Delta f$ and M for $\Sigma S\Delta x$ in the last equation we have

$$f = \frac{My}{I},$$

which is Formula (D), Art. 53.

Owing to the shear being constant the increments of the bending stress in the beam shown in Fig. 70 are constant throughout for each horizontal element and are equal to

$$\Delta f = \frac{(R\Delta x)y}{I}$$

But in such cases, as shown in Fig. 71, they will be different owing to the variation of the shear. The increment between A and the load $P1$, in that case, due to the loads $P1$, $P2$, and $P3$ is expressed as

$$\Delta f = \frac{(R\Delta x)y}{I};$$

between $P1$ and $P2$ as

$$\Delta f = \frac{(R - P1)\Delta xy}{I};$$

between $P2$ and $P3$ as

$$\Delta f = \frac{(R - P1 - P2)\Delta xy}{I};$$

and between $P3$ and B as

$$\Delta f = \frac{(R - P1 - P2 - P3)\Delta xy}{I} \text{ or } = \frac{(R1\Delta x)y}{I}.$$

It is seen from the above that the increment of the bending stress varies directly as the shear and hence in most cases the greatest increment will be at the ends of a beam, especially when the weight of the beam is considered.

61. Horizontal Shearing Stress in Beams.—Let AB (Fig. 72) represent a portion of the same beam shown in Fig. 56 (Art. 52), which is here drawn so as to show the imaginary block *abcd* to a large scale. Conceive of the original imaginary block *abcd* sub-divided into smaller imaginary blocks as *egkh*, *gmnk*, etc. We can now conceive of the shearing force exerted upon the block *abcd* along *ab* and *cd* as being distributed to each of the smaller blocks, whereby each is seen to have a vertical couple which we can conceive of as being an increment couple of the vertical couple acting upon the block *abcd* as a whole. Now, evidently, each of the smaller blocks has a tendency to rotate owing to this vertical couple, and if rotation does not take place, evidently it is prevented by the material joining the blocks horizontally. Then the forces resisting the vertical couple on each of the smaller blocks must act through this material, forming a horizontal couple on each block as shown, which must necessarily be equivalent and opposite to the vertical couple that it resists. Now, it is readily seen that these horizontal couples tend to shear the smaller blocks off horizontally the same as the vertical couples tend to shear the same blocks off vertically. If a block be square, it is readily seen that the vertical and horizontal couples on it will not only be equivalent, but will be equal and opposite, which means that each of the forces in the horizontal couple is equal to each of the forces in the vertical couple; that is, the four forces acting upon the block are equal. This means that the horizontal shear on the smaller blocks is equal to the vertical shear on the same. But if the smaller blocks are considered

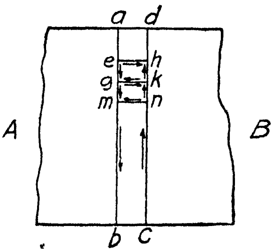


Fig. 72

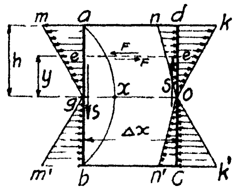


Fig. 73

shorter or longer in one direction than in the other, the unit shear is not changed. The larger sides having the greater area will simply have a greater force acting, but the force per square unit will be just the same, as our mentally changing the block will not alter the unit shear. As far as the two couples on any block remaining equivalent is concerned, it is evident they will, for as the force on one side is increased, the lever arm of the other is increased, so that the moments of the couples are continually equivalent. So it really does not matter what shape our fancy molds the imaginary parts, for it remains evident that the horizontal and vertical shear on any particle in any beam are equal. Then, if we know the horizontal shear on any particle, we know the vertical, as the two are equal, and vice versa. The determining of the total vertical shear at any vertical section of a beam is an easy matter, but just what the intensity of it is on any particular particle is another thing. What we really do is to find the horizontal shear and consider the vertical shear equal to the same. In order to do this, let *abcd*, Fig. 73, represent the imaginary block *abcd* of Fig. 56, as an independent body where the other parts of the beam are replaced by the forces which those parts exert upon the block. Let *go* be the neutral plane. The part of the beam to the left of the block will exert upon it the bending stresses represented by the arrows in triangles *agm* and *bgm'*, and also the vertical shearing force *S* along *ab*, while the part of the beam to the right will exert upon it the forces represented by the arrows in the triangles *dok* and *cok'*, which resist the bending stresses represented by the arrows in the triangles *agm* and *bgm'*, and hence are equal and opposite to them, and also the shearing force S along *dc*, which is equal and opposite to the force S acting along *ab*, and also the bending stress increments represented by the arrows in the triangles *don* and *con'*.

The bending stresses represented by the arrows in triangles *agm* and *bgm'* are transmitted directly through the block and are balanced directly by the forces represented by the arrows in the triangles *dok* and *cok'*, so, evidently, none of these forces causes horizontal shear on the block and can be disregarded. The force S acting along *ab* and the equal and opposite force S acting along *dc* and the bending stress increments represented by the arrows in the triangles *don* and *con'* are the only forces remaining. But as each of the forces S acts vertically, it is evident that the horizontal shear in the block is due entirely to the forces represented by the arrows in the triangles *don* and *con'*; that is, to the increments of the bending stresses on the block, and hence it remains for us to consider these forces only.

Now, evidently, the horizontal shear on any element as *ee* of the block *abcd* is equal to the algebraic summation of these increment forces on either side of *ee*, the same as in the case of any body. Therefore, the horizontal shear at *ee* is equal to the sum of these forces between *d* and *e*, or between *e* and *c*.

The moment of these increment forces is equal to $S\Delta x$, which they resist. Then it is readily seen that the stress at *d*, represented by *nd*, is

$$\Delta f = \frac{(S\Delta x)h}{I},$$

and at *e* it is

$$\Delta f' = \frac{(S\Delta x)y}{I},$$

where I is the moment of inertia of the cross-section of the beam at the block *abcd*. Then the average increment stress between *d* and *e* is

$$f'' = \frac{\Delta f + \Delta f'}{2} = \left(\frac{S\Delta x}{I}\right)\left(\frac{h+y}{2}\right)$$

Multiplying this by $b \times de$, where b = width of beam, we obtain the total horizontal shear along *ee*, which is

$$F = \left(\frac{S\Delta x}{I}\right)\left(\frac{h+y}{2}\right)\left(h-y\right)b,$$

Now this shear is distributed over the horizontal strip through the block at *ee*. If b be the width of the beam, the area of this horizontal strip will be $b\Delta x$. Then the shear per square inch upon it will be

$$S_1 = \left(\frac{S\Delta x}{b\Delta x\, I}\right)\left(\frac{h+y}{2}\right)\left(h-y\right)b,$$

from which we obtain the formula

$$S_1 = \frac{S}{I}\left(\frac{h^2-y^2}{2}\right) \qquad \text{(G)}.$$

From Formula (G) the horizontal shear (and incidentally the vertical) can be obtained at any point in any rectangular beam either above or below the neutral axis.

It is readily seen that Formula (G) is an equation to a parabola, which means that the horizontal shear on any rectangular beam, and likewise the vertical shear, varies on any vertical section as the ordinates to a parabola, as represented by the horizontal ordinates to the curve *axb* shown in Fig. 73. If $y=h$, the shear S_1 is equal to 0. If $y=0$, $S_1=\frac{3}{2}S/A$, which is the maximum, where A = area of cross section of beam. So it is seen that both the vertical and horizontal shear is 0 at the top and likewise at the bottom of any beam, and a maximum at the neutral axis. The same is true of all beams.

In case the width of a beam varies the general equation

$$S_1=\left(\frac{S}{bI}\right)m \qquad\qquad G^1$$

can be used for determining the horizontal shear, where m = the moment about the neutral axis of the part of the cross section above *ee* or below in case *ee* is below the neutral axis and b = width of beam at *ee*.

62. Maximum Stress.—It is seen from the preceding article that every particle in a loaded beam, except the particles at the top and bottom edges, and those in the neutral axis, is subjected to horizontal and vertical shear and to a direct horizontal stress which may be either tension or compression, depending upon the position of the particle in the beam. In the case of a cantilever beam the particles at the top edge are subjected to direct tension only, while all particles between the top edge and neutral axis are subjected to horizontal and vertical shear and also to direct tension, and the particles in the neutral axis are subjected only to horizontal and vertical shear. At the bottom edge of a cantilever beam the particles are subjected to a direct compression only, while the particles between the bottom edge and neutral axis are subjected to horizontal and vertical shear and also to direct compression. In the case of a simple beam we have exactly the reverse of a cantilever beam. As an illustration, let *AB* (Fig. 74) represent a short portion of a simple beam where *oo* represents the neutral axis. The particles at the top edge are subjected to direct compression only, as indicated at *a*, while the particles between the top edge and neutral axis are subjected to horizontal and vertical shear and also to direct compression as indicated at *b*, the particles in the neutral axis are subjected only to horizontal vertical shear as indicated at *c*. At the bottom edge the particles are subjected to direct tension only, as indicated at *e*, while the particles between the bottom edge and the neutral axis are subjected to horizontal and vertical shear and also to direct tension, as indicated at *d*.

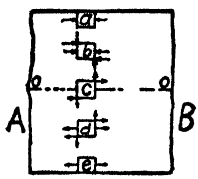


Fig. 74

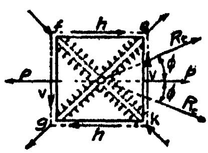


Fig. 75

It is seen from Fig. 74 that the intensity of the maximum stress on any particle at the top or bottom edge of the beam is simply the direct stress on it, but the intensity of the maximum stress on any other particle is not so evident, for here the maximum stress is due to the combined action of the shearing forces and direct stress. Let Fig. 75 represent a small imaginary block taken from a simple beam, corresponding to the block at *d*, Fig. 74. For convenience consider the block to be a cube having sides of unit length, and let *h* represent the horizontal shear and *v* the vertical shear. It is readily seen that the *h* along *fe* and the *v* along *ek* acting against the *v* along *fg* and the *h* along *kg* produce tension upon all such strips through the block as *fk*, while the *h* along *fe* and the *v* along *fg* acting against the *v* along *ek* and the *h* along *kg* will produce compression upon all such strips through the block as *eg*. Now it is evident that the maximum tension and compression as the case may be, due to the shearing forces, will be upon strips perpendicular to the resultant of the shearing forces. As the horizontal and vertical shear are equal it is obvious that their resultant will always be at 45° with the horizontal and vertical and hence the maximum tension or compression stresses due to these forces will be upon strips perpendicular to this direction, that is, 45° from the direction of the shearing forces themselves. It is readily seen that the direct tensile stress indicated as *p* on the block will increase the tension due to the shearing forces on the strip *fk* and decrease the compression on the strip *eg;* while if *p* were a compressive stress just the reverse would be true. While the tensile stress *p* would increase the tension on the strip *fk,* yet this would not be the maximum tension on the block, for evidently the maximum would be on a strip through the block perpendicular to the resultant of the stress *p* and the shearing forces *v* and *h*. This resultant would have some position as R_t, if *p* were tension, and R_c, if *p* were compression, and the maximum tensile stress then would be upon strips through the block perpendicular to R_t; and if *p* were compressive the maximum compressive stress would be upon strips through the block perpendicular to R_c.

If the resultant of the shearing forces on the particles of any beam were graphically combined throughout the beam with the direct stresses on the particles, the balanced resultants obtained would form curves known as the lines of maximum stress. As an example, let *AB* (Fig. 76) represent a simple beam. By combining the forces at *e, d, c,* etc., as indicated at *e′, d′,* and *c′*, the curve *edc* would be obtained and by combining the forces on the particles throughout the entire beam we would obtain some such lines as *t*, *t*1, *t*2, etc., representing the direction of the action of the maximum tensile stresses in the beam. Now, evidently, if any particle in the beam failed in tension the failure would take place perpendicular to these lines of maximum tension.

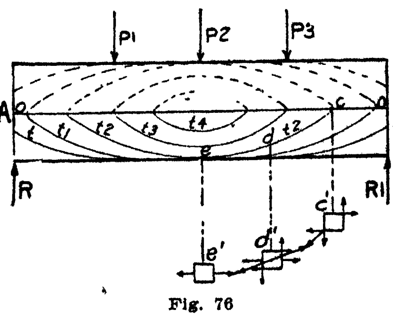


Fig. 76

If the maximum compressive stresses were platted in the same manner they would take some such position as indicated by the dotted lines on the beam, and if any particle failed in compression the failure would evidently take place perpendicular to these lines of maximum compression.

It is readily seen from Fig. 75 that the shear due to the shearing forces v and h along a strip as eg, 45° with the horizontal or vertical, will be zero, for the two shearing forces h and v on each side will just balance each other when resolved along that direction. Then the only shear along that direction will be due to the direct stress p. If we imagine the strip rotated about o it is readily seen that the shearing forces will produce shear along the strip when it slopes other than 45° with the horizontal and the maximum shear on it due to the shearing forces and direct stress combined will occur when the angle of slope has some particular value. But this angle of slope depends upon the relative intensity of the shearing forces and the direct stress. The direction of maximum tension and compression stresses also depends upon the relative intensity of the shearing forces and direct stress. So we will now determine these relations.

Let Fig. 77 represent a material particle assumed rectangular, subjected to horizontal and vertical shear of s pounds per square inch and to a tensile stress of f pounds per square inch. Suppose the particle to be one unit in thickness, and let a be its width and b its length. Then sa will be the total vertical shear and sb the total horizontal shear, and fa the total tensile stress, as indicated in the figure. Let dd be an imaginary diagonal strip through the particle. If the shearing and tensile forces be resolved perpendicularly and parallel to this imaginary strip dd, the components perpendicular to the strip will produce tension on it, while the components along the strip will produce shear.

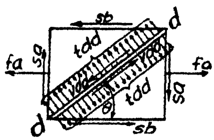


Fig 77

Let t be the tensile stress per square inch perpendicular to the strip dd and let v be the shearing stress per square inch along the strip, and let θ be the angle that the strip dd makes with the horizontal axis of the particle; then, considering the forces on one side of dd only, we have

$$(1)\quad tdd = fa \sin\theta + sb \sin\theta + sa \cos\theta \text{ for the tension,}$$

and

$$(2)\quad vdd = fa \cos\theta + sb \cos\theta - sa \sin\theta \text{ for the shear.}$$

Dividing (1) and (2) by dd and substituting $\sin\theta$ for a/dd and $\cos\theta$ for b/dd, we have

$$(3)\quad t = f \sin^2\theta + 2s \cos\theta\sin\theta = \frac{f}{2}(1-\cos 2\theta) + s \sin 2\theta,$$

and

$$(4)\quad v = f \sin\theta\cos\theta + s (\cos^2\theta - \sin^2\theta) = \frac{f}{2}\sin 2\theta + s \cos 2\theta.$$

It is evident that t will be a maximum when θ has a certain value and v a maximum when θ has a certain other value. Differentiating (3) we have

$$dt = f \sin 2\theta d\theta + 2s \cos 2\theta d\theta.$$

Now, t will be a maximum when

$$\frac{dt}{d\theta}=f\ \sin 2\theta+2s\ \cos 2\theta=0,$$

from which we obtain

$$(5)\quad \tan 2\theta=-\frac{2s}{f}$$

as the value of θ when t is a maximum.

Treating (4) in the same manner we obtain

$$(6)\quad \tan 2\theta=\frac{f}{2s}$$

as the value of θ when v is a maximum. Expressing the value of the tangent in (5) in terms of the sine, we have

$$\frac{\sin 2\theta}{\sqrt{1-\sin^2 2\theta}}=-\frac{2s}{f}.$$

Squaring and reducing, we have

$$(a)\quad \sin 2\theta=\pm\frac{2s}{\sqrt{f^2+4s^2}}.$$

Then expressing the value of the tangent in (5) in terms of the cosine, we have

$$\frac{\sqrt{1-\cos^2 2\theta}}{\cos 2\theta}=-\frac{2s}{f},$$

from which we obtain

$$(b)\quad \cos 2\theta=\pm\frac{f}{\sqrt{f^2+4s^2}}.$$

Referring to equation (5), as $\tan 2\theta=-\sin 2\theta/\cos 2\theta$, a minus quantity, it is evident that $\sin 2\theta$ and $\cos 2\theta$ will have opposite signs, that is, when one is plus the other will be minus. Then substituting the value of $\sin 2\theta$ and $\cos 2\theta$ given in (a) and (b), respectively, in (3), taking one plus and the other minus, we obtain

$$t=\frac{f}{2}\pm\sqrt{s^2+\frac{f^2}{4}}\ \dots\dots\dots\dots\dots\dots\dots\dots\ (\text{H}).$$

By treating (6) in the same manner as (5) was treated above, we obtain

$$\sin 2\theta=\pm\frac{f}{\sqrt{f^2+4s^2}}$$

and

$$\cos 2\theta=\pm\frac{2s}{\sqrt{f^2+4s^2}}.$$

Substituting these values, which will have like sign, in (4), we obtain

$$v=\pm\sqrt{s^2+\frac{f^2}{4}}\ \dots\dots\dots\dots\dots\dots\dots\dots\ (\text{I}).$$

The maximum or minimum tensile or compressive stresses on any material particle subjected to shear and direct stress can be computed from (H). If f be tension, t will be tension when the radical is taken as plus and compression when taken as minus. Thus both tension and compression stresses are obtained. If f be compression, by using the plus sign before the radical we obtain the compressive stress and by using the minus sign we obtain the tension. As the plus sign in (H) gives the maximum stress in all cases it is rarely necessary to use the minus sign.

The maximum shear on any material particle subjected to shear and direct stress can be obtained from (I). It is immaterial whether the plus or minus sign before the radical be used as the two values will be the same.

The direction of the maximum shearing stress can be obtained from (6) and a direction perpendicular to the maximum and minimum tensile and compressive stresses can be obtained from (5).

There are only a few cases where the above formulas need to be applied. They need not be applied in the case of ordinary beams, for here both the maximum tension and compression stresses occur on the outer elements where the shearing stress is zero, and the maximum shear occurs at the neutral axis where the direct stress is zero.

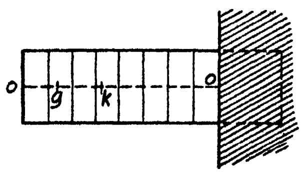


Fig. 78

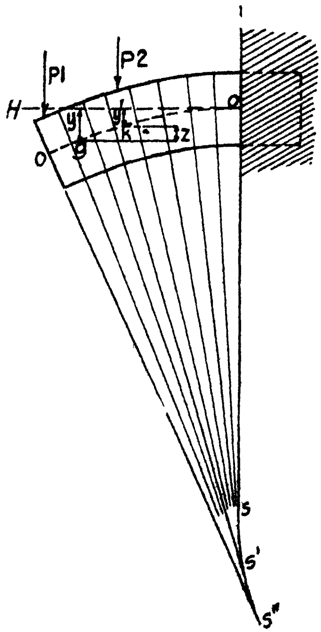


Fig. 79

63. Deflection of Beams.—Beams supporting loads deflect or bend owing to the distortion of their parts which results from the loading. Let Fig. 78 represent an unloaded rectangular wooden cantilever beam which we will assume to be straight and absolutely horizontal. Imagine this beam divided into very short rectangular blocks by imaginary planes passing perpendicularly to its longitudinal axis. Now if loads were applied to this beam, these rectangular blocks would become wedge-shaped as shown in Fig. 79, and the imaginary planes dividing the beam into blocks instead of being parallel would become tangent to some curve as $ss's''$, and the neutral axis *oo* instead of being straight would become a curve which would be known as the "elastic curve" of the beam. The distance from the intersection of any two adjacent imaginary planes to the neutral axis *oo* would be known as the radius of curvature of the elastic curve at the intercepted block. The vertical distance that any point on the neutral axis would move down from its original position would be known as the deflection of the point and of the beam at that point, and the deflection of any one point on the neutral axis in reference to another would be the difference of their deflections. For example, the distance y (Fig. 79) from the horizontal line *Ho* to the point g would be the deflection of point g, and likewise the deflection of the beam at that point, while the distance z would be the deflection of point g in reference to point k.

It is readily seen that the curving of the neutral axis results from the distortion of the blocks. Then it is evident that if the slope of the blocks in reference to one another be determined, an equation of the elastic curve expressing this relation can be derived, from which the deflection of the beam at any point in reference to any other point can be computed. So we shall now proceed to derive such an equation of the elastic curve.

Let Fig. 80 represent any four consecutive distorted blocks of the beam shown in Fig. 79. By drawing the center line of each block we have the broken line curve $BabcA$ which has for its limit the elastic curve SS as the lengths of the blocks decrease; that is, become infinitesimal. It is readily seen that the radius of curvature at any block is perpendicular to its center line at midpoint. So the slope of the center line of any block with the horizontal or X-axis is the same as that of the tangent of the elastic curve at that point. In fact, the two coincide.

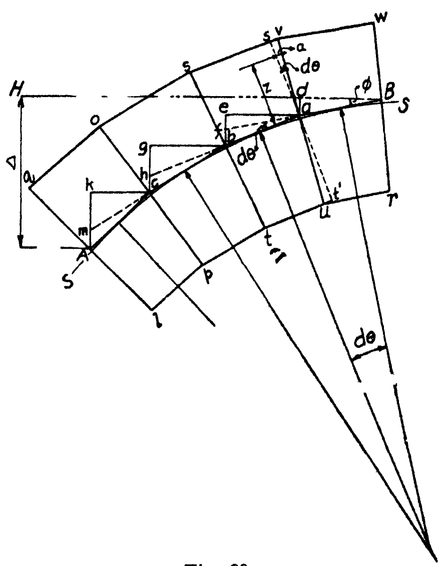


Fig 80

Let HB be a horizontal line. Then ϕ is the angle that the tangent to the elastic curve at the center of the block $vurw$ makes with the horizontal or X-axis. Now, as the length of the block $vurw$ decreases, the center line aB comes nearer and nearer to coinciding with the arc of the elastic curve passing through the block. So if the length of the block be infinitesimal the points a and B will be on the elastic curve and the vertical distance ad will be the deflection of point a in reference to B, and ad would be a first differential of the vertical or y ordinate to the elastic curve at that point. Likewise, the distance eb, gc, and kA will each be a first differential of a y ordinate to the elastic curve when the length of each of the other blocks becomes infinitesimal. Letting Δ represent the deflection of point A in reference to B, we have

$$\Delta = ad + eb + gc + kA,$$

which is simply the summation of the first differentials summed up from B to A. Let $dB = ea = gb = kc = dx$, an infinitesimal, which would really be the case when the lengths of the blocks are infinitesimal. Then by prolonging aB, the center line of block $vurw$, to f we have $ef = ad$. Now as ad and eb are each a first differential of a y ordinate to the elastic curve, fb is a second differential of the same, being the difference between two consecutive first differentials; and similarly hc and mA are also second differentials of y ordinates to the elastic curve. It is readily seen that $gc = ad + fb + hc$ and that $kA = ad + fb + hc + mA$, etc.; that is, each first differential is equal to the summation of the second differentials plus the first differential ad summed up from B to the point considered. Now if the summation be from A to B instead of B to A, as above, we have the case shown in Fig. 81, where ce, bf, ak, and Bn are first differentials, and gb, ha, and mB are second differentials. Taking A as the origin, we have $ak = ce - gb - ha$, and $Bn = ce - gb - ha - mB$; that is, any first differential of a y ordinate to the elastic curve is equal to the summation of the second differentials summed up from the origin to the point considered, plus the first differential ce. (Fig. 81.) The second differentials are minus in the last case, as they are measured down to the curve; while the first differentials are measured up to the curve. For the deflection of point A in reference to B we have $\Delta = ce + bf + ak + Bn$, which is simply the summation of the first differentials summed up from A to B. Now, as stated above, any first differential at any point is equal to the summation of the second differentials summed up from A to the point, plus the first differential ce, while in the case shown in Fig. 80 the first differential at any point is equal to the summation of the second differentials summed up from B to the point, plus the first differential ad. It is readily seen that as far as the deflection of point A in reference to point B is concerned, both of the first differentials ad (Fig. 80) and ce (Fig. 81) are constants, and being such they will appear as constants of integration in the summation of the second differentials. It is first necessary to derive an expression for the second differential of any y ordinate to the elastic curve in terms of known quantities. Referring to Fig. 80, if the length of the two blocks *stuv* and *vurw* be infinitesimal, the angle subtended by their radii of curvature will be an infinitesimal angle $d\theta$, and as one radius is perpendicular to aB, the center line of block *vurw*, and the other one to ab, the center line of block *stuv*, the angle fab will be equal to that subtended by the radii, or $d\theta$. Then, as the angle $d\theta$ (Fig. 80) is infinitesimal, we have

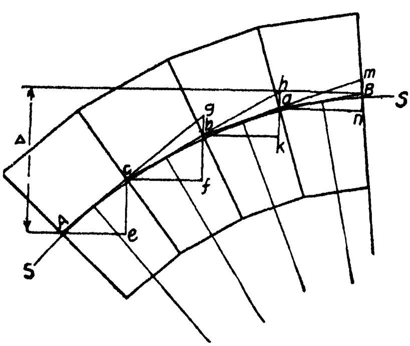


Fig. 81

$$fb = abd\theta \text{ or } d^2y = abd\theta \qquad (1).$$

The permissible deflection of a beam in any case is very small compared to its length (1/1,000), so that no appreciable error will be made if we assume the slope distance $ab = ea = dx$. Then by substituting in (1), we have

$$d^2y \doteq dxd\theta \qquad (2).$$

Now $d\theta$ is the slope that the block *stuv* makes with the block *vurw*, which results entirely from the distortion of the block *stuv*. Then, evidently, the value of $d\theta$ will be directly proportional to the bending moment at the block *stuv* and inversely proportional to the modulus of elasticity of the material composing the block and also inversely proportional to the moment of inertia of its cross-section. Then $d\theta$ can be expressed in terms of these quantities.

Through a (Fig. 80) draw the line $s't'$ parallel to st. Then the angle $s'av = d\theta$. Let α = the longitudinal distortion of an element of the block ($stuv$) distance z above ab. Then we have

$$\alpha = z d\theta.$$

But, according to Art. 33,

$$\alpha = \frac{Mz}{I} \times \frac{ab}{E} = \frac{Mzdx}{IE}$$

(Mz/I = stress on element) (see Formulas B and D, Arts. 33 and 53), where M = the bending moment of the block *stuv* and I = moment of inertia of the cross-section of the block and E the modulus of elasticity of the material composing it. Now, combining these two equations, we have

$$d\theta = \frac{M dx}{IE}.$$

Substituting this value of $d\theta$ in (2) we have

$$d^2 y = \frac{M dx^2}{IE},$$

which is the expression for the second differential of the y ordinate to the elastic curve at block *stuv* in terms of known quantities, but which at the same time is really the expression for the vertical drop of the elastic curve from a to b due wholly to the distortion of block *stuv*. Now, this same expression will hold in the case of any of the blocks, as no special case was taken. Then, evidently, by substituting the proper value of M in the expression and integrating twice, the deflection of point A in reference to B will be obtained. It is evident that the above discussion will apply to any number of blocks as well as to four, so our expression for the second differential of the y ordinates to the elastic curve will apply to the entire beam or any part of it, as A and B can be any two points, and as the bending in the case of any beam can be nothing different from the bending of a cantilever beam here considered, the expression is evidently applicable in the case of any beam whatever, loaded in any manner, and is hence a general differential equation of the elastic curve of any beam or any body whatever, subjected to bending stresses. For convenience, the equation is usually written

$$EI \frac{d^2 y}{dx^2} = M \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(K)}.$$

Referring to Fig. 79, it will be seen that the curve $ss's''$ is an evolute, while the elastic curve oo is an involute. In case of a simple beam there would be two branches to the evolute.

64. Deflection of Cantilever Beams.—

Case I. When the beam supports a single load at its free end.

Let AB (Fig. 82) represent the beam of length L, P the load, and let x and y be the co-ordinates to the elastic curve at any point when the origin is taken at B, and x_1 and y_1 the co-ordinates when the origin is taken at A.

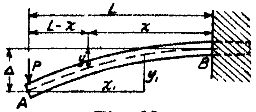


Fig. 82

First, taking B as the origin, we have $M=P(L-x)$ for the bending moment at any point x distance from B. Now substituting this value of M in Formula (K) (Art. 63), we have

$$EI\frac{d^2y}{d^2x}=P(L-x)\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Integrating once, we have

$$EI\frac{dy}{dx}=PLx-P\frac{x^2}{2}+C_1.$$

Now, dy/dx, is the expression for the tangent of the angle that the elastic curve makes with the horizontal or x axis at any point x distance from B. It is readily seen that $dy/dx=0$ at B, the origin, as the elastic curve is horizontal at that point. But $x=0$ also at that point. Then substituting 0 for dy/dx and for x in the above equation, we have $C_1=0$. Now as $C_1=0$, the preceding equation becomes

$$EI\frac{dy}{dx}=PLx-P\frac{x^2}{2}\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

which is the general equation for the slope of the elastic curve at any point between B and A with B as the origin. From this equation, the slope of the beam (= slope of the elastic curve) at any point can be computed by substituting for x its numerical value. For example, the slope of the beam at a point b distance from B is

$$\frac{dy}{dx}=\frac{bPL-P\frac{b^2}{2}}{EI},$$

which is the tangent of the angle that the tangent to the elastic curve at that point makes with the x-axis.

Next integrating (2) we have

$$EIy=PL\frac{x^2}{2}-P\frac{x^3}{6}+C_2.$$

Now at B both y and $x=0$. Then substituting 0 for x and also for y, we have $C_2=0$. Hence, we have

$$y=\frac{P}{EI}\left(L\frac{x^2}{2}-\frac{x^3}{6}\right),$$

which is the algebraic equation of the elastic curve when the origin is at B. From this equation the deflection of the beam at any point can be computed by substituting for x its numerical value. It is readily seen that the deflection will be a maximum when $x=L$. Let Δ = the maximum deflection; then we have

$$y = \Delta = \frac{P}{EI}\left(\frac{L^3}{2} - \frac{L^3}{6}\right) = \frac{PL^3}{3EI}.$$

Now taking A as the origin, we have for the bending moment at any point $x_{,}$ distance from A, $M = Px_{,}$. Then substituting this value of M in Formula (K) (63), we have

$$EI\frac{d^2y_{,}}{dx_{,}^2} = Px_{,}.$$

Integrating once, we have

$$EI\frac{dy_{,}}{dx_{,}} = P\frac{x_{,}^2}{2} + C'.$$

Now, $dy_{,}/dx_{,} = 0$ when $x = L$. Then substituting these values for $dy_{,}/dx_{,}$ and $x_{,}$ respectively, in the preceding equation, we have $C' = -PL^2/2$. Then substituting this value of C' in the equation, we have

$$EI\frac{dy_{,}}{dx_{,}} = \frac{Px_{,}^2}{2} - \frac{PL^2}{2} \quad\text{..............................}\quad (4)$$

for the general equation for the slope of the elastic curve, from which the slope of the curve at any point $x_{,}$ distance from A can be computed by substituting for $x_{,}$ its numerical value.

Next, integrating (4), we have

$$EIy_{,} = \frac{Px_{,}^3}{6} - \frac{PL^2x_{,}}{2} + C''.$$

Now, when $x_{,} = 0$, $y_{,} = 0$. Substituting 0 for $x_{,}$ and also for $y_{,}$ we have $c'' = 0$. Then we have

$$y_{,} = \frac{P}{EI}\left(\frac{x_{,}^3}{6} - \frac{L^2x_{,}}{2}\right) \quad\text{..............................}\quad (5)$$

for the equation of the elastic curve when the origin is at A, from which the vertical ordinate $y_{,}$ at any point $x_{,}$ distance from A can be computed by substituting for $x_{,}$ its numerical value. Then, by subtracting this $y_{,}$ ordinate from the maximum deflection Δ the deflection of the point in question is obtained. It is readily seen from Fig. 82 that the $y_{,}$ ordinate is equal to Δ when $x_{,} = L$. So substituting in (5) we have

$$y_{,} = \Delta = \frac{P}{EI}\left(\frac{L^3}{6} - \frac{L^3}{3}\right) = -\frac{PL^3}{3EI},$$

which is the same as found above except for the sign, which is due to the ordinates being measured in the opposite direction.

Case II. When the beam supports two loads.

Let AB (Fig. 83) represent the beam, P and P' the loads, and let a be the distance that load P is from B, b the distance between the loads and d the distance that the load P' is from A, the end of the beam. Take B as the origin, and let x and y represent the co-ordinates to the elastic curve. When x is less than a, we have for the bending moment at a point x dis-

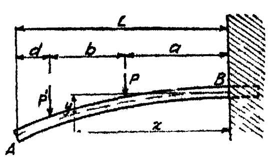


Fig. 83

tance from B

$$M=(a-x)P+(b+a-x)P',$$

therefore,
$$EI\frac{d^2y}{dx^2}=(a-x)P+(b+a-x)P'.$$

Integrating, we have

$$EI\frac{dy}{dx}=Pax-P\frac{x}{2}\ +P'bx+P'ax-\frac{P'x^2}{2}+C'.$$

But $dy/dx = 0$ when $x = 0$; therefore, $C' = 0$, and we have

$$EI\frac{dy}{dx}=Pax-P\frac{x^2}{2}+P'bx+P'ax-\frac{P'x^2}{2}\ldots\ldots\ldots\ldots\ldots\ldots(6).$$

Now integrating (6), we have

$$EIy=Pa\frac{x^2}{2}-\frac{Px^3}{6}+\frac{P'bx^2}{2}+\frac{P'ax^2}{2}-\frac{P'x^3}{6}+C''.$$

But $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$y=\frac{1}{EI}\left(\frac{Pax^2}{2}-\frac{Px^3}{6}+\frac{P'bx^2}{2}+\frac{P'ax^2}{2}-\frac{P'x^3}{6}\right)\ldots\ldots\ldots\ldots(7).$$

Now when x is greater than a and less than $a+b$, we have

$$M=P'(a+b-x)$$

for the bending moment at any point between P and P', x distance from B. Therefore,

$$EI\frac{d^2y}{dx^2}=P'a+P'b-P'x.$$

Integrating, we have

$$EI\frac{dy}{dx}=P'ax+P'bx-\frac{P'x^2}{2}+C_,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8).$$

Now it is readily seen that equations (6) and (8) are equal when $x=a$ in each. So, substituting a for x in each, equating and cancelling, we have

$$C_,=\frac{Pa^2}{2}.$$

Substituting this value of $C_,$ in (8) we have

$$EI\frac{dy}{dx}=P'ax+P'bx-\frac{P'x^2}{2}+\frac{Pa^2}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Integrating, we have

$$EIy=\frac{P'ax^2}{2}+\frac{P'bx^2}{2}-\frac{P'x^3}{6}+\frac{Pa^2x}{2}+C_{,,}\ldots\ldots\ldots\ldots\ldots(10).$$

Now it is readily seen that equations (7) and (10) are likewise equal when $x = a$ in each. So substituting a for x in each, equating and cancelling, we have

$$C_{,,}=-\frac{Pa^3}{6}.$$

Substituting this value of $C_{\prime\prime}$ in (10), we have

$$y=\frac{1}{EI}\left(\frac{P'ax^2}{2}+\frac{P'bx^2}{2}-\frac{P'x^3}{6}+\frac{Pa^2x}{2}-\frac{Pa^3}{6}\right) \quad\ldots\ldots\ldots\ldots(11).$$

Now the slope of the elastic curve at any point between B and the load P can be computed from equation (6) and at any point between the loads from (9), while the deflection at any point between B and the load P can be computed from equation (7) and at any point between the loads from (11) by substituting for x its numerical value in each case.

The maximum deflection will be at A. This can be determined in the following way:

First compute the deflection of the beam at the load P' from equation (11) (substituting $(a+b)$ for x). Then compute the slope, that is, the tangent of the slope angle at that point from (9) and multiply this slope by the distance d, which will give the deflection of the point A in reference to the point at P'. Then add this deflection to the deflection of the beam at P', and we will have the total deflection at A.

Case III. When the beam supports a uniform load.

Let AB (Fig. 84) represent the beam supporting a uniform load of w pounds per foot of length. Take B as the origin, and let x and y represent the co-ordinates to the elastic curve. Then the bending moment at any point x distance from B is

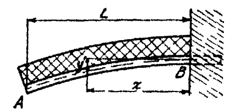


Fig. 84

$$M=w(L-x)\left(\frac{L-x}{2}\right)=\frac{w}{2}(L^2-2Lx+x^2),$$

therefore,

$$EI\frac{d^2y}{dx^2}=\frac{w}{2}(L^2-2Lx+x^2).$$

Integrating once, we have

$$EI\frac{dy}{dx}=\frac{w}{2}\left(L^2x-Lx^2+\frac{x^3}{3}\right)+C'.$$

When $x=0$, $dy/dx=0$; therefore, $C'=0$, and we have

$$EI\frac{dy}{dx}=\frac{w}{2}\left(L^2x-Lx^2+\frac{x^3}{3}\right) \quad\ldots\ldots\ldots\ldots(12).$$

Integrating again, we have

$$EIy=\frac{w}{2}\left(\frac{L^2x^2}{2}-\frac{Lx^3}{3}+\frac{x^4}{12}\right)+C''.$$

Now, when $x=0$, $y=0$; therefore, $C''=0$, and we have

$$y=\frac{w}{2EI}\left(\frac{L^2x^2}{2}-\frac{Lx^3}{3}+\frac{x^4}{12}\right) \quad\ldots\ldots\ldots\ldots(13).$$

From equation (12) the slope at any point of the beam can be computed, and from (13) the deflection at any point can be computed by substituting for x its numerical value in each case. It is evident that the maximum deflection will be at A, the free end. For that point $x=L$.

Then substituting L for x in equation (13), we have

$$y=\Delta=\frac{wL^4}{8EI}$$

for the maximum deflection.

65. Deflection of Simple Beams.—

Case 1. When the beam supports a single load at mid-span.

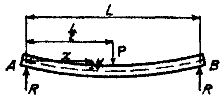


Fig. 85

Let AB (Fig. 85) represent a simple beam of length L supporting a single load P at mid-span. Let R and R' represent the reactions at A and B, respectively, which are equal in this case. Take A as the origin, and let x and y represent the co-ordinates to the elastic curve. For the bending moment at any point to the left of the load, x distance from A, we have

$$M=Rx=\frac{P}{2}x.$$

therefore, we have

$$EI\frac{d^2y}{dx^2}=\frac{P}{2}x.$$

Integrating once, we have

$$EI\frac{dy}{dx}=\frac{Px^2}{4}+C'.$$

Now $dy/dx = 0$ when $x = L/2$. Substituting this value for x, we have

$$C'=-\frac{PL^2}{16}$$

and substituting this value of C' in the above equation, we have

$$EI\frac{dy}{dx}=\frac{Px^2}{4}-\frac{PL^2}{16} \qquad (1).$$

Integrating this equation, we have

$$EIy=\frac{Px^3}{12}-\frac{PL^2x}{16}+C''.$$

Now, $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$y=\frac{P}{EI}\left(\frac{x^3}{12}-\frac{L^2x}{16}\right) \qquad (2),$$

which is the equation of the elastic curve to the left of the load. As the elastic curve is symmetrical about the load, there is no need for deriving the equation for the curve to the right of the load. However, this can be readily accomplished by substituting in Formula (K) (Art. 63), the expression for the bending moment to the right of the load, which is

$$M=\frac{P}{2}x-P\left(x-\frac{L}{2}\right).$$

It is readily seen that the maximum deflection will occur under the load. So letting Δ represent the maximum deflection, and substituting $L/2$ for x in equation (2), we have

$$y = \Delta = -\frac{PL^3}{48EI}.$$

The slope of the elastic curve at any point between A and the load can be computed from equation (1).

Case II. When the beam supports a single load at any point.

Let AB (Fig. 86) represent a simple beam supporting a load P at any point z distance from A. Let R and R' represent the reactions at A and B, respectively, due to this load P. Take A as the origin, and let x and y represent the co-ordinates to the elastic curve. For the bending moment at any point to the left of the load, we have

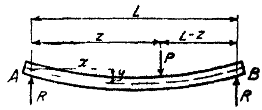


Fig 86

$$M = Rx = \frac{P}{L}(Lx - zx),$$

therefore, we have

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(Lx - zx).$$

Integrating, we have

$$EI\frac{dy}{dx} = \frac{Px^2}{2} - \frac{Pzx^2}{2L} + C' \quad \dots\dots (3).$$

Integrating again, we have

$$EIy = \frac{Px^3}{6} - \frac{Pzx^3}{6L} + C'x + C''.$$

Now, $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$EIy = \frac{Px^3}{6} - \frac{Pzx^3}{6L} + C'x \quad \dots\dots (4).$$

Now for the bending moment at any point to the right of the load, that is, when $x > z$, we have

$$M' = Rx - P(x - z) = -\frac{Pzx}{L} + Pz,$$

therefore, we have

$$EI\frac{d^2y}{dx^2} = Pz - \frac{Pzx}{L}.$$

Integrating, we have

$$EI\frac{dy}{dx} = Pzx - \frac{Pzx^2}{2L} + C_1 \quad \dots\dots (5).$$

Integrating again, we have

$$EIy = \frac{Pzx^2}{2} - \frac{Pzx^3}{6L} + C_1x + C_2 \quad \dots\dots (6).$$

It is readily seen that equations (3) and (5) are equal when $x = z$ in each. Then substituting z for x in each equation, equating and cancelling, we have

$$C' - \frac{Pz^2}{2} = C_1 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7).$$

Now substituting this value of C_1 in (6), we have

$$EIy = \frac{Pzx^2}{2} - \frac{Pzx^3}{6L} + C'x - \frac{Pz^2x}{2} + C_2 \dots\dots\dots\dots\dots\dots (8).$$

It is readily seen that equations (4) and (8) are equal when $x = z$ in each case. Then substituting z for x in each, equating and cancelling, we have

$$C_2 = \frac{Pz^3}{6} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9).$$

Now it is readily seen that y in equation (8) equals zero when $x = L$. Then substituting L for x and $Pz^3/6$ for C_2 in that equation, equating and reducing, we have

$$C' = \frac{Pz^2}{2} - \frac{PzL}{3} - \frac{Pz^3}{6L} \dots\dots\dots\dots\dots\dots\dots\dots\dots (10).$$

Substituting this value of C' in equations (3) and (4), we have, respectively,

$$EI\frac{dy}{dx} = \frac{Px^2}{2} - \frac{Pzx^2}{2L} + \frac{Pz^2}{2} - \frac{PzL}{3} - \frac{Pz^3}{6L} \dots\dots\dots\dots\dots (11),$$

$$EIy = \frac{Px^3}{6} - \frac{Pzx^3}{6L} + \frac{Pz^2x}{2} - \frac{PzxL}{3} - \frac{Pz^3x}{6L} \dots\dots\dots\dots\dots (12).$$

Now, from equation (11) the slope of the elastic curve at any point to the left of the load can be computed, while the deflection at any point to the left of the load can be computed from (12).

From equations (7) and (10) we have

$$C_1 = -\frac{PzL}{3} - \frac{Pz^3}{6L},$$

and from (9) we have

$$C_2 = \frac{Pz^3}{6}.$$

Substituting these values in equations (5) and (6), we have, respectively,

$$EI\frac{dy}{dx} = Pzx - \frac{Pzx^2}{2L} - \frac{PzL}{3} - \frac{Pz^3}{6L} \dots\dots\dots\dots\dots\dots\dots (13),$$

$$EIy = \frac{Pzx^2}{2} - \frac{Pzx^3}{6L} - \frac{PzxL}{3} - \frac{Pz^3x}{6L} + \frac{Pz^3}{6} \dots\dots\dots\dots\dots (14).$$

Now, from equation (13) the slope of the elastic curve at any point to the right of the load can be computed, while the deflection at any point to the right of the load can be computed from (14).

Equations (11) and (12) apply only to the part of the elastic curve to the left of the load, while equations (13) and (14) apply only to the part to the right of the load, and hence we can consider the elastic curve to be composed of two separate curves which are tangent at the load. Now, if the beam supported two loads some distance apart, the elastic curve would be composed of three separate curves; and if it supported three loads, the elastic curve would be composed of four separate curves; and so on. That is to say, if there be n loads, there will be $(n+1)$ separate curves composing the elastic curve. The equations for each of these curves could be derived as readily as the above equations, and then from these equations the slopes and deflections of the elastic curve could be computed. The main labor is the determining of the constants of integration. It will be observed that, at the point of maximum deflection, $dy/dx = 0$, as the tangent to the curve will be horizontal at that point.

Case III. When the beam supports a uniform load.

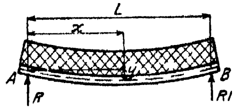


Fig 87

Let AB (Fig. 87) represent a simple beam, length L, supporting a uniform load of w pounds per foot of length. Let R and $R1$ represent the reactions at A and B, respectively, and let x and y represent the co-ordinates of the elastic curve, A being taken as the origin. Then the bending moment at any point x distance from A is

$$M = Rx - \frac{wx^2}{2} = \frac{wLx}{2} - \frac{wx^2}{2},$$

therefore,

$$EI\frac{d^2y}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}.$$

Integrating once, we have

$$EI\frac{dy}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} + C'.$$

Now, $dy/dx = 0$ when $x = L/2$, as the tangent to the elastic curve is horizontal at that point. Now substituting this value of x, we have

$$C' = -\frac{wL^3}{24},$$

and substituting this value of C' in the above equation, we have

$$EI\frac{dy}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} - \frac{wL^3}{24} \quad \text{.......................... (1).}$$

Integrating this equation, we have

$$EIy = \frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3x}{24} + C''.$$

But $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$y=\frac{1}{EI}\left(\frac{wLx^3}{12}-\frac{wx^4}{24}-\frac{wL^3x}{24}\right) \quad \ldots\ldots (2),$$

from which the deflection at any point can be computed.

It is evident that the deflection will be a maximum at midspan; that is, when $x = L/2$. So letting Δ represent the maximum deflection, and substituting $L/2$ for x in (2), we have

$$y=\Delta=-\frac{5wL^4}{384EI}.$$

66. Proposition.—*The bending moment at any point in a beam is equal to the bending moment at any other point plus the shear at the other point multiplied by the distance between the points, plus the algebraic sum of the moments of the forces between the points about the point in question.*

Let AB (Fig. 88) represent a simple beam supporting the loads P, $P1$, and $P2$ as shown. Now, according to the above proposition, the

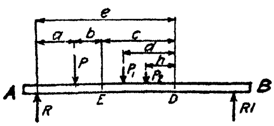


Fig 88

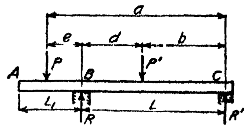


Fig 89

bending moment at D is equal to the bending moment at E, plus the shear at E multiplied by c, plus the sum of the moments of the forces between the points about D. This is readily seen to be true, for, taking moments about E, and letting R represent the reaction at A due to the three loads, we have

$$M_E=R(a+b)-Pb$$

for the bending moment at that point, and taking moments about D, we have

$$M_D=R(a+b+c)-P(c+b)-dP1-hP2$$
$$=R(a+b)-Pb+c(R-P)-dP1-hP2,$$

which is seen to agree with the above proposition, as the part $R(a+b)-Pb$ is the bending moment at E, $R-P$ is the shear at E, which is multiplied by c, the distance between E and D, and $-dP1-hP2$ is the sum of the moments of the forces between E and D, about D. The minus signs in the last are simply the signs of the moments.

The above proposition is readily proven in the case of any beam whatever, but regardless of its simplicity, it is quite useful.

67. Reactions, Shears, and Bending Moments on Overhanging Beams.—Let ABC (Fig. 89) represent an overhanging beam supported at B and C and which in turn supports the loads P and P', as shown. The part AB is known as the cantilever arm, while the part BC is known as the anchor arm. Let R represent the reaction at B due to the two loads, P and P', and let R' represent the reaction at C due to the same. Now, taking moments about C, we have

$$RL-aP-bP'=0.$$

from which we obtain

$$R = \frac{aP + bP'}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Then taking moments about B, we have

$$R'L + eP - dP' = 0,$$

from which we obtain

$$\pm R' = \frac{dP' - eP}{L} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

If eP be greater than dP' it is evident that R' will act downward upon the beam, and hence the beam would pull upward upon the support at C, which would require that the beam be anchored in some manner to the support. In all such cases the reaction is spoken of as being negative. If dP' be greater than eP, of course R' will be positive, the same as for simple beams.

In case of a greater number of loads, the reactions are determined in the same manner as shown above for the two loads. We simply include the moment of each load in the equation of moments. In the case of a uniform load the reactions are determined practically in the same manner, except that we use average lever arms for the uniform load. For example, suppose the beam ABC (Fig. 89) supports a uniform load of w pounds per foot of length, extending from A to C. Let $R_{,}$ represent the reaction at B due to this uniform load, and let $R_{,,}$ represent the reaction at C due to the same. Then taking moments about C, we have

$$R_{,} = \frac{w(L_{,} + L)^2}{2L},$$

and taking moments about B, we have

$$\pm R_{,,} = \frac{1}{L}\left(\frac{wL^2}{2} - \frac{wL_{,}^2}{2}\right).$$

In case the beam overhangs two supports the reactions are determined as above by taking moments about the supports and simply including the moment of each force acting upon the beam in the equation of moments. As an example, let $ABCD$ (Fig. 90) represent a beam overhanging two supports. Let P, P', and P'' represent three loads supported by the beam as shown, and let R represent the reaction at B due to these three loads, and let R' represent the reaction at C due to the same. Now, taking moments about B, we have

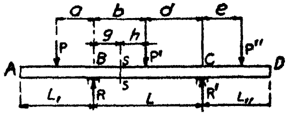


Fig. 90

$$aP - bP' \pm R'L - (e + L)P'' = 0,$$

from which we obtain

$$\pm R' = \frac{bP' + (e + L)P'' - aP}{L}.$$

Then taking moments about C, we have

$$-(L+a)P \pm RL - dP' + eP'' = 0,$$

from which we obtain

$$\pm R = \frac{(L+a)P + dP' - eP''}{L}.$$

Now suppose the beam represented in Fig. 90 supports a uniform load of w pounds per foot of length extending from A to D, and let $R_,$ represent the reaction at B due to this uniform load and let $R_{,,}$ represent the reaction at C due to the same. Then taking moments about B, we have

$$R_{,,} = \frac{w(L+L_{,,})^2}{2L} - \frac{wL_,^2}{2L},$$

and taking moments about C, we have

$$R_, = \frac{w(L+L_,)^2}{2L} - \frac{wL_{,,}^2}{2L}.$$

The determining of the shear at any section of an overhanging beam is just the same as for any other beam; that is, the shear at any section is equal to the algebraic summation of the forces on either side of the section summed up from the end of the beam in each case. For example, the shear at section ss of the beam $ABCD$ (Fig. 90) due to the three loads P, P', and P'', is

$$S = P \pm R \text{ or } P'' \pm R' + P'.$$

The determining of the bending moment at any section of an overhanging beam is just the same as for any other beam; that is, the bending moment at any section is equal to the algebraic summation of the moments of the forces on either side of the section about the section. For example, the bending moment about the section ss of the beam $ABCD$ (Fig. 90) due to the three loads P, P', and P'' is

$$M = (a+g)P \pm Rg \text{ or } (L-g+e)P'' \pm (L-g)R' + hP'.$$

All of the above holds as well for cantilever arms as for anchor arms, but it is more convenient to treat the cantilever arms as independent cantilever beams.

68. Reactions, Shears, and Bending Moments on Beams Fixed at One End and Supported at the Other.—Let AB (Fig. 91) represent such a beam which has the simple support at A and the fixed support at B. As the beam is fixed at B it is evident that any load on the beam will produce tension in the elements above the neutral axis and compression below the neutral axis at that point. But it is equally evident that the reverse is true just to the right of the simple support at A; that is, the top elements of the beam just to the right of A will be in compression and the bottom ones in tension. Then, evidently, there will be some section between A and B where the bending reverses; that is, a section where there is no bending, and hence the bending moment at that point of the

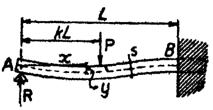


Fig. 91

beam will be equal to zero. Let S be such a point. Then the part SB of the beam, to the right of S, would be simply a cantilever beam, while the part SA of the beam, to the left of S, would be a simple beam, whence the bending or curving of the beam due to loads would be as indicated in Fig. 91. The point S, where the bending moment equals zero, would be known as the point of "contra-flexure," also as the "point of inflection."

As a case of concentrated loads, let R (Fig. 91) represent the reaction of A due to a single load P at any distance kL from A, k being any fraction less than unity. Owing to the unknown forces acting upon the beam to the right of B, this reaction R, at A, cannot be determined in the usual way by taking moments about B. Take A as the origin, and let x and y represent the co-ordinates to the elastic curve. Then, for the bending moment at any point to the left of the load, that is, when x is less than kL, we have

$$M = Rx,$$

therefore,

$$EI\frac{d^2y}{dx^2} = Rx.$$

Integrating once, we have

$$EI\frac{dy}{dx} = \frac{Rx^2}{2} + C' \quad \ldots\ldots (1).$$

Integrating again, we have

$$EIy = \frac{Rx^3}{6} + C'x + C''.$$

But, $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$EIy = \frac{Rx^3}{6} + C'x \quad \ldots\ldots (2).$$

Now for the bending moment at any point to the right of the load, that is, where x is greater than kL, we have

$$M_1 = Rx - Px + PkL,$$

therefore,

$$EI\frac{d^2y}{dx^2} = Rx - Px + PkL.$$

Integrating once, we have

$$EI\frac{dy}{dx} = \frac{Rx^2}{2} - \frac{Px^2}{2} + PkLx + C_1.$$

But, $dy/dx = 0$ when $x = L$. Then substituting L for x, and equating, we have

$$-\frac{RL^2}{2} + \frac{PL^2}{2} - PkL^2 = C_1.$$

Substituting this value of C_1 in the last equation, we have

$$EI\frac{dy}{dx} = \frac{Rx^2}{2} - \frac{Px^2}{2} + PkLx - \frac{RL^2}{2} + \frac{PL^2}{2} - PkL^2 \quad \ldots\ldots (3).$$

Then integrating (3), we have

$$EIy=\frac{Rx^3}{6}-\frac{Px^3}{6}+\frac{PkLx^2}{2}-\frac{RL^2x}{2}+\frac{PL^2x}{2}-PkL^2x+C_2.$$

But here $y = 0$ when $x = L$. Then substituting L for x, equating and cancelling, we have

$$C_2=\frac{RL^3}{3}-\frac{PL^3}{3}+\frac{PkL^3}{2}.$$

Substituting this value of C_2 in the last equation, we have

$$EIy=\frac{Rx^3}{6}-\frac{Px^3}{6}+\frac{PkLx^2}{2}-\frac{RL^2x}{2}+\frac{PL^2x}{2}-PkL^2x$$

$$+\frac{RL^3}{3}-\frac{PL^3}{3}+\frac{PkL^3}{2}\ldots\ldots\ldots(4).$$

It is readily seen that equations (1) and (3) are equal when $x=kL$ in each. Then substituting kL for x in each equation, equating and reducing, we have

$$C'=\frac{Pk^2L^2}{2}-\frac{RL^2}{2}+\frac{PL^2}{2}-PkL^2.$$

Then substituting this value of C' in (2) we have

$$EIy=\frac{Rx^3}{6}+\frac{Pk^2L^2x}{2}-\frac{RL^2x}{2}+\frac{PL^2x}{2}-PkL^2x\ldots\ldots\ldots(5).$$

It is readily seen, also, that equations (4) and (5) are equal when $x = kL$ in each. Then substituting kL for x in each of these equations, equating and reducing, we have

$$R=\frac{P}{2}(k^3-3k+2)\ldots\ldots\ldots(6),$$

from which the reaction at A due to a load at any point on the beam can be computed. For example, suppose the load P is at mid-span. Then $k=\frac{1}{2}$. Substituting this $\frac{1}{2}$ in equation (6) we have

$$R=\frac{P}{2}\left(\frac{1}{8}-\frac{3}{2}+2\right)=\frac{5}{16}P.$$

Again, suppose the load is at the quarter point nearest A. Then $k = \frac{1}{4}$. Substituting this $\frac{1}{4}$ for k in (6) we have

$$R=\frac{P}{2}\left(\frac{1}{64}-\frac{3}{4}+2\right)=\frac{81}{128}P.$$

If there be more than one load on the beam, the reaction at A due to each load would be computed separately and all added together and we would thus obtain the reaction at A due to all the loads. Of course the k for each would be different from the others; that is, no two k's would be the same.

In case the beam supports a uniform load of w pounds per foot of length extending over the full length of the beam, that is, from A to B,

we have

$$M = Rx - \frac{wx^2}{2}$$

for the moment at any point x distance from A, where R represents the reaction at A due to this uniform load. Therefore, we have

$$EI\frac{d^2y}{dx^2} = Rx - \frac{wx^2}{2}.$$

Integrating once, we have

$$EI\frac{dy}{dx} = \frac{Rx^2}{2} - \frac{wx^3}{6} + C'.$$

But $dy/dx = 0$ when $x = L$. So substituting L for x in the preceding equation, we have

$$C' = -\frac{RL^2}{2} + \frac{wL^3}{6}.$$

Then substituting this value of C' in the last equation, we have

$$EI\frac{dy}{dx} = \frac{Rx^2}{2} - \frac{wx^3}{6} - \frac{RL^2}{2} + \frac{wL^3}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7).$$

Integrating this equation, we have

$$EIy = \frac{Rx^3}{6} - \frac{wx^4}{24} - \frac{RL^2x}{2} + \frac{wL^3x}{6} + C''.$$

But here $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$EIy = \frac{Rx^3}{6} - \frac{wx^4}{24} - \frac{RL^2x}{2} + \frac{wL^3x}{6} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8).$$

Now $y = 0$ in (8) when $x = L$. Then substituting L for x (8), we have

$$0 = \frac{RL^3}{6} - \frac{wL^4}{24} - \frac{RL^3}{2} + \frac{wL^4}{6},$$

from which we obtain

$$R = \frac{3}{8}wL.$$

That is, when a beam supported at one end and fixed at the other supports a uniform load, the reaction at the supported end is three-eighths of the total load.

After the reaction at the supported end due to the loads supported is determined, the shear and bending moment at any point in the beam are obtained just the same as in the case of a simple beam, except we deal only with the supported end. The shear at any point is equal to the reaction at the supported end minus all intervening loads, while the bending moment at any point is equal to the reaction at the supported end multiplied by its lever arm, minus the moment of all intervening loads about the point. For example, the shear at any point x distance from the supported end due to a uniform load of w pounds per foot of span is

$$S = \frac{3}{8}wL - wx,$$

while the bending moment is

$$M = \frac{3}{8}wLx - \frac{wx^2}{2}.$$

If $x = L$, we have

$$M = -\frac{1}{8}wL^2,$$

which is the bending moment at the support B or the fixed end.

Deflections can be computed from equations (4), (5), and (8).

69. Shears and Bending Moments on Fixed Beams.—Let AB (Fig. 92) represent a fixed beam. The supports being fixed, any load on the beam, as is seen, will produce tension in the top elements of the beam and compression in the bottom elements of the beam at the supports, while the reverse will be the case out some distance from the supports. Then, evidently, there will be two points of contra-flexure and the bending of the beam due to loads supported will be as indicated in Fig. 92, where S' and S are the points of contra-flexure. It is evident that the parts AS' and SB are cantilevers, while the part $S'S$ is a simple beam. In the case of fixed beams, the point of application of the reactions cannot be definitely fixed, so we deal with the end shears instead of the reactions.

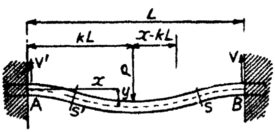


Fig. 92

As a case of concentrated loads let P represent a single load upon the beam at any distance kL from A. Now, when we proceed to determine the shears and bending moments on the beam due to this load P, we quickly realize that we have but little to start with, and it is only through the application of the general differential equation of the elastic curve (K) that we are able to obtain either. Let V' and V represent the end shears at A and B, respectively, due to the load P, and let M' and M'' represent the bending moments at A and B, respectively. Take A as the origin and let x and y represent the co-ordinates to the elastic curve. Then, according to Art. 66, the bending moment at any point to the left of the load any distance x from A is

$$M = M' + V'x.$$

Therefore, we have

$$EI\frac{d^2y}{dx^2} = M' + V'x,$$

and integrating once, we have

$$EI\frac{dy}{dx} = M'x + \frac{V'x^2}{2} + C'.$$

But $dy/dx = 0$ when $x = 0$; therefore, $C' = 0$, and we have

$$EI\frac{dy}{dx} = M'x + \frac{V'x^2}{2} \quad \text{. . . . . (1).}$$

Integrating this equation, we have

$$EIy = \frac{M'x^2}{2} + \frac{V'x^3}{6} + C''.$$

But here $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$EIy = \frac{M'x^2}{2} + \frac{V'x^3}{6} \quad \ldots\ldots\ldots\ldots (2).$$

Now the bending moment at any point to the right of the load x distance from A is

$$M_1 = M' + V'x - Px + PkL.$$

Therefore,

$$EI\frac{d^2y}{dx^2} = M' + V'x - Px + PkL,$$

and integrating this equation, we have

$$EI\frac{dy}{dx} = M'x + \frac{V'x^3}{2} - \frac{Px^2}{2} + PkLx + C_1.$$

Now, $dy/dx = 0$ when $x = L$. So substituting L for x and equating, we have

$$-M'L - \frac{V'L^2}{2} + \frac{PL^2}{2} - PkL^2 = C_1.$$

Then substituting this value of C_1 in the last equation, we have

$$EI\frac{dy}{dx} = M'x + \frac{V'x^2}{2} - \frac{Px^2}{2} + PkLx - M'L - \frac{V'L^2}{2} + \frac{PL^2}{2} - PkL^2 \ldots (3).$$

Integrating (3), we have

$$EIy = \frac{M'x^2}{2} + \frac{V'x^3}{6} - \frac{Px^3}{6} + \frac{PkLx^2}{2} - M'Lx - \frac{V'L^2x}{2} + \frac{PL^2x}{2} - PkL^2x + C_2.$$

But $y = 0$ when $x = L$. So, substituting L for x in the last equation, equating and reducing, we have

$$\frac{M'L^2}{2} + \frac{V'L^3}{3} - \frac{PL^3}{3} + \frac{PkL^3}{2} = C_2.$$

Then substituting this value of C_2 in the last equation, we have

$$EIy = \frac{M'x^2}{2} + \frac{V'x^3}{6} - \frac{Px^3}{6} + \frac{PkLx^2}{2} - M'Lx - \frac{V'L^2x}{2} + \frac{PL^2x}{2}$$
$$- PkL^2x + \frac{M'L^2}{2} + \frac{V'L^3}{3} - \frac{PL^3}{3} + \frac{PkL^3}{2} \quad \ldots\ldots\ldots (4).$$

Now it is evident that equations (1) and (3) are equal when $x = kL$ in each. Then by substituting kL for x in each, equating and reducing, we have

$$M' = \frac{Pk^2L}{2} - \frac{V'L}{2} + \frac{PL}{2} - PkL \quad \ldots\ldots\ldots (5).$$

Then by substituting this value of M' in each of the equations (2) and (4) and then substituting kL for x in each, we have the two equal. Then by equating these equations each to each and reducing, we have

$$V' = P(1 + 2k^3 - 3k^2) \quad \ldots\ldots\ldots (6).$$

By substituting this value of V' in (5), and reducing, we have

$$M' = PL(2k^2 - k^3 - k) \dots\dots (7).$$

Then, having V' and M' determined from (6) and (7) for the point A, the bending moment at any other point in the beam, due to the load P, can be determined according to Art. 66, and the shear at any point can be determined, as it is equal to the summation of the forces summed up from A beginning with V'.

If there be more than one load on the beam, the shear and bending moment at A due to each can be determined separately from (6) and (7). Then adding these shears together and these bending moments together, we obtain the shear and bending moment at A due to all of the loads. Then the bending moment and shear at any point of the beam, due to all of the loads, can be determined. That is, if the shear and bending moment at one support of a fixed beam are known, the shear and bending moment at any other point can be determined, and hence the equations (6) and (7) are all that are absolutely necessary. But from these equations we can readily derive equations for the shear and bending moment at the other support, if such be desired. For example, the shear at B, Fig. 92, is equal to the load P, producing the shear, minus the shear at A. Then subtracting the value of V', given in (6), from P, we have

$$V = P - P(1 + 2k^3 - 3k^2),$$

and reducing, we have

$$V = P(2k^3 - 3k^2) \dots\dots (8).$$

The bending moment at B (Fig. 92) is

$$M'' = M' + V'L - P(L - kL).$$

Then by substituting the value of V' and M' given in (6) and (7), respectively, we have

$$M'' = PL(2k^2 - k^3 - k) + PL(1 + 2k^3 - 3k^2) - P(L - kL),$$

and reducing, we have

$$M'' = PL(k^3 - k^2) \dots\dots (9).$$

As an example of application, suppose the load P (Fig. 92) to be at mid-span. Then $k = 1/2$. Now substituting this 1/2 for k in (6) and (7), we have

$$V' = P\left(1 + \frac{1}{4} - \frac{3}{4}\right) = \frac{P}{2},$$

and

$$M' = PL\left(\frac{1}{2} - \frac{1}{8} - \frac{1}{2}\right) = -\frac{PL}{8},$$

respectively, which is the shear and bending moment at A. For the bending moment under the load we have

$$M = M' + V'kL = -\frac{PL}{8} + \frac{P}{2}\left(\frac{L}{2}\right) = \frac{PL}{8}.$$

In case a uniform load extends over the entire length of a fixed beam, the end shears are each equal to one-half of the total load on the beam. Suppose the beam shown in Fig. 92 supports a uniform load of w pounds per foot of length extending over the entire span. Let V' and V'' represent the end shears at A and B, respectively, due to this uniform load, and let M' and M'' represent the bending moments at A and B, respectively, due to the same.

Taking A as the origin as before, we have for the bending moment at any point x distance from A

$$M = M' + V'x - \frac{wx^2}{2} = M' + \frac{wLx}{2} - \frac{wx^2}{2},$$

therefore,

$$EI\frac{d^2y}{dx^2} = M' + \frac{wLx}{2} - \frac{wx^2}{2}.$$

Integrating this equation, we have

$$EI\frac{dy}{dx} = M'x + \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1.$$

But $dy/dx = 0$ when $x = 0$; therefore, $C_1 = 0$, and we have

$$EI\frac{dy}{dx} = M'x + \frac{wLx^2}{4} - \frac{wx^3}{6} \quad \text{(10)}.$$

Integrating again, we have

$$EIy = \frac{M'x^2}{2} + \frac{wLx^3}{12} - \frac{wx^4}{24} + C_2.$$

But $y = 0$ when $x = 0$; therefore, $C_2 = 0$, and we have

$$EIy = \frac{M'x^2}{2} + \frac{wLx^3}{12} - \frac{wx^4}{24} \quad \text{(11)}.$$

But $y = 0$ also when $x = L$. Then substituting L for x in (11), and reducing, we have

$$M' = -\frac{wL^2}{12} \quad \text{(12)},$$

which is the bending moment at A. Then knowing the bending moment and shear at A, the bending moment and shear at any other point in the beam can be determined. For example, the bending moment at the center is

$$M_C = M' + \frac{V'L}{2} - \frac{wL^2}{8} = -\frac{wL^2}{12} + \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{1}{24}wL^2.$$

To determine the points of contra-flexure, we simply write the equation for the bending moment at any point and equate it to zero. For example, the bending moment at any point x distance from A, when the beam supports a uniform load of w pounds per foot (Fig. 92), is

$$M = M' + V'x - \frac{wx^2}{2} = -\frac{wL^2}{12} + \frac{wLx}{2} - \frac{wx^2}{2}.$$

Then by equating this to zero, we have

$$-\frac{wL^2}{12}+\frac{wLx}{2}-\frac{wx^2}{2}=0,$$

and solving for x, we have

$$x=\frac{L}{2}\pm L\sqrt{\frac{1}{12}}=0.21\,L \text{ or } 0.79\,L \text{ (about)}.$$

That is, one point of contra-flexure is 0.21 of L from A—considered in practice as being $\frac{1}{4}$ of L—and the other 0.79 of L from A—considered in practice as being $\frac{3}{4}$ of L. The points of contra-flexure can be determined in a similar manner in case of any other kind of loading.

Deflections, if desired, can be computed from equations (2), (4), and (11).

70. Bending Moments, Shears, and Reactions on Continuous Beams.—Let ABC (Fig. 93) represent two consecutive spans of a continuous beam of n spans. The bending or curving of the beam in the two

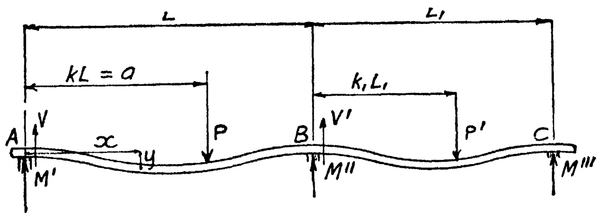


Fig 93

spans is shown to be similar to that of two fixed beams, each span being considered a beam. If the spans were equal in length and symmetrically loaded, this would practically be the case, but otherwise it would not be, for it is readily seen that the loads in one span would tend to produce reverse bending in the adjacent span, which would curve it up instead of down. For example, the loads in span AB could be such that the span BC would be curved upward instead of downward, as it is shown. So it is evident that the tangent to the elastic curve at any support is horizontal only when the spans are equal in length and rigidity, and symmetrically loaded. Hence the slope of the elastic curve at the supports cannot, in general, be utilized in preliminary investigations, as in the case of fixed beams.

In the following treatment it is assumed that the beams have a uniform cross-section and are homogeneous throughout, and are supported upon simple supports of equal elevation. This is an assumption usually made in practice. The equation employed in analyzing continuous beams is known as the *"Three-Moment Equation,"* which we shall now proceed to derive.

Case I. When the beam supports concentrated loads.

Referring to Fig. 93 let L be the length of the span AB and L_1 the length of the span BC. Let P represent a load at any point kL distance from A in span AB and let P' represent a load at any point k_1L_1 distance from B in span BC. Let M', M'', and M''' represent the bending moments at the supports A, B, and C, respectively, due to the loads P and P', and let V and V' represent the shears just to the right of the supports A and B, respectively, due to these same loads.

Considering the span AB, and taking A as the origin, the bending moment (according to Art. 66) at any point to the left of the load P, x distance from A, is

$$M = M' + Vx,$$

therefore,

$$EI\frac{d^2y}{dx^2} = M' + Vx.$$

Integrating once, we have

$$EI\frac{dy}{dx} = M'x + \frac{Vx^2}{2} + C' \qquad \text{(1).}$$

Integrating again, we have

$$EIy = \frac{M'x^2}{2} + \frac{Vx^3}{6} + C'x + C''.$$

But $y = 0$ when $x = 0$; therefore, $C'' = 0$, and we have

$$EIy = \frac{M'x^2}{2} + \frac{Vx^3}{6} + C'x \qquad \text{(2).}$$

Now, the bending moment at any point to the right of the load P, x distance from A, is

$$M = M' + Vx - P(x - kL),$$

therefore,

$$EI\frac{d^2y}{dx^2} = M' + Vx - Px + PkL.$$

Integrating once, we have

$$EI\frac{dy}{dx} = M'x + \frac{Vx^2}{2} - \frac{Px^2}{2} + PkLx + C_, \qquad \text{(3).}$$

Integrating again, we have

$$EIy = \frac{M'x^2}{2} + \frac{Vx^3}{6} - \frac{Px^3}{6} + \frac{PkLx^2}{2} + C_,x + C_{,,} \qquad \text{(4).}$$

It is readily seen that equations (1) and (3) are equal when $x = kL$ in each. Then substituting kL for x in each and equating, we have

$$M'kL + \frac{Vk^2L^2}{2} + C' = M'kL + \frac{Vk^2L^2}{2} - \frac{Pk^2L^2}{2} + Pk^2L^2 + C_,,$$

from which we obtain

$$C' = \frac{Pk^2L^2}{2} + C_, \qquad \text{(5).}$$

Now substituting this value of C' in (2), we have

$$EIy = \frac{M'x^2}{2} + \frac{Vx^3}{6} + \frac{Pk^2L^2x}{2} + C_,x \qquad \text{(6).}$$

Now in equation (4) $y=0$ when $x=L$. Then substituting L for x in that equation, we have

$$0=\frac{M'L^2}{2}+\frac{VL^3}{6}-\frac{PL^3}{6}+\frac{PkL^3}{2}+C_{,}L+C_{,,},$$

from which we obtain

$$C_{,,}=-\frac{M'L^2}{2}-\frac{VL^3}{6}+\frac{PL^3}{6}-\frac{PkL^3}{2}-C_{,}L \quad \text{. . . . . (7).}$$

Substituting this value of $C_{,,}$ in equation (4), we have

$$EIy=\frac{M'x^2}{2}+\frac{Vx^3}{6}-\frac{Px^3}{6}+\frac{PkLx^2}{2}+C_{,}x$$
$$-\frac{M'L^2}{2}-\frac{VL^3}{6}+\frac{PL^3}{6}-\frac{PkL^3}{2}-C_{,}L \quad \text{. . . . . (8).}$$

Equation (6) applies to the part of the elastic curve to the left of the load P, while (8) applies to the part of the curve to the right. Then these two equations are equal when $x = kL$ in each. So substituting kL for x in each, equating and reducing, we have

$$C_{,}=-\frac{Pk^3L^2}{6}-\frac{M'L}{2}-\frac{VL^2}{6}+\frac{PL^2}{6}-\frac{PkL^2}{2} \quad \text{. . . . . (9).}$$

Then substituting this value of $C_{,}$ in (5), we have

$$C'=\frac{Pk^2L^2}{2}-\frac{Pk^3L^2}{6}-\frac{M'L}{2}-\frac{VL^2}{6}+\frac{PL^2}{6}-\frac{PkL^2}{2} \quad \text{. . . . . (10),}$$

and substituting the value of $C_{,}$ given by (9) in (7) and reducing, we have

$$C_{,,}=\frac{Pk^3L^3}{6} \quad \text{. . . . . (11).}$$

So we have thus determined all of the constants of integration so far involved.

Now substituting in (1) the value of C' given in (10), we have

$$EI\frac{dy}{dx}=M'x+\frac{Vx^2}{2}+\frac{Pk^2L^2}{2}-\frac{Pk^3L^2}{6}-\frac{M'L}{2}$$
$$-\frac{VL^2}{6}+\frac{PL^2}{6}-\frac{PkL^2}{2} \quad \text{. . . . . (12),}$$

and substituting the same value of C' in (2), we have

$$EIy=\frac{M'x^2}{2}+\frac{Vx^3}{6}+\frac{Pk^2L^2x}{2}-\frac{Pk^3L^2x}{6}-\frac{M'Lx}{2}$$
$$-\frac{VL^2x}{6}+\frac{PL^2x}{6}-\frac{PkL^2x}{2} \quad \text{. . . . . (13).}$$

Next substituting in (3) and (4) the value of $C_{,}$ and $C_{,,}$, respectively, given in (9) and (11), we have

$$EI\frac{dy}{dx}=M'x+\frac{Vx^2}{2}-\frac{Px^2}{2}+PkLx-\frac{Pk^3L^2}{6}$$
$$-\frac{M'L}{2}-\frac{VL^2}{6}+\frac{PL^2}{6}-\frac{PkL^2}{2}\ldots\ldots\ldots\ldots(14)$$

and

$$EIy=\frac{M'x^2}{2}+\frac{Vx^3}{6}-\frac{Px^3}{6}+\frac{PkLx^2}{2}-\frac{Pk^3L^2x}{6}$$
$$-\frac{M'Lx}{2}-\frac{VL^2x}{6}+\frac{PL^2x}{6}-\frac{PkL^2x}{2}+\frac{Pk^3L^3}{6}\ldots\ldots\ldots(15).$$

Equations (12) and (13) apply to the part of the elastic curve to the left of the load P, while equations (14) and (15) apply to the part of the elastic curve to the right of the load P. Now it is evident that by proceeding in the same manner with span BC, four equations corresponding to (12), (13), (14), and (15) could be derived for the elastic curve in that span. But it is readily seen that the equations would differ from the above equations only in the marking of the *M's, V's, P's, k's,* and *L's*. So for span BC, we can write

$$EI\frac{dy}{dx}=M''x+\frac{V'x^2}{2}+\frac{P'k_,^2L_,^2}{2}-\frac{P'k_,^3L_,^2}{6}$$
$$-\frac{M''L_,}{2}-\frac{V'L_,^2}{6}+\frac{P'L_,^2}{6}-\frac{P'k_,L_,^2}{2}\ldots\ldots\ldots\ldots(16).$$

$$EIy=\frac{M''x^2}{2}+\frac{V'x^3}{6}+\frac{P'k_,^2L_,^2x}{2}-\frac{P'k_,^3L_,^2x}{6}$$
$$-\frac{M''L_,x}{2}-\frac{V'L_,^2x}{6}+\frac{P'L_,^2x}{6}-\frac{P'k_,L_,^2x}{2}\ldots\ldots\ldots\ldots(17)$$

for the part of the elastic curve to the left of the load P', and

$$EI\frac{dy}{dx}=M''x+\frac{V'x^2}{2}-\frac{P'x^2}{2}+P'k_,L_,x-\frac{P'k_,^3L_,^2}{6}$$
$$-\frac{M''L_,}{2}-\frac{V'L_,^2}{6}+\frac{P'L_,^2}{6}-\frac{P'k_,L_,^2}{2}\ldots\ldots\ldots\ldots(18),$$

$$EIy=\frac{M''x^2}{2}+\frac{V'x^3}{6}-\frac{P'x^3}{6}+\frac{P'k_,L_,x^2}{2}-\frac{P'k_,^3L_,^2x}{6}-\frac{M''L_,x}{2}$$
$$-\frac{V'L_,^2x}{6}+\frac{P'L_,^2x}{6}-\frac{P'k_,L_,^2x}{2}+\frac{P'k_,^3L_,^3}{6}\ldots\ldots\ldots(19),$$

for the part of the elastic curve to the right of the load P'.

Now taking moments about B, according to Art. 66, we have

$$M''=M'+VL-P(L-kL),$$

from which we obtain

$$V=\frac{M''-M'}{L}+P-Pk\ldots\ldots\ldots\ldots\ldots\ldots(20).$$

Then substituting this value of V in (14) and reducing, we have

$$EI\frac{dy}{dx}=M'x+\frac{M''x^2}{2L}-\frac{M'x^2}{2L}-\frac{Pkx^2}{2}+PkLx-\frac{Pk^3L^2}{6}$$

$$-\frac{M'L}{2}-\frac{M''L}{6}+\frac{M'L}{6}-\frac{PkL^2}{3}\ldots\ldots\ldots\ldots\ldots\ldots(21).$$

Likewise, taking moments about C, we have

$$M'''=M''+V'L_,-P'(L_,-k_,L_,),$$

from which we obtain

$$V'=\frac{M'''-M''}{L_,}+P'-P'k_,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22).$$

Then substituting this value of V' in (16), we have

$$EI\frac{dy}{dx}=M''x+\frac{M'''x^2}{2L_,}-\frac{M''x^2}{2L_,}+\frac{P'x^2}{2}-\frac{P'k_,x^2}{2}+\frac{P'k_,^2L_,^2}{2}$$

$$-\frac{P'k_,^3L_,^2}{6}+\frac{P'k_,L_,^2}{6}-\frac{M''L_,}{2}-\frac{M'''L_,}{6}+\frac{M''L_,}{6}-\frac{P'k_,L_,^2}{2}\ldots\ldots(23).$$

Equation (21) applies to the part of the elastic curve to the right of the load P in span AB, and equation (23) applies to the part of the elastic curve to the left of the load P' in span BC. Now it is readily seen that these two equations are equal when $x = L$ in (21) and 0 in (23), as the two curves have a common tangent at B. Then substituting L for x in (21), and 0 for x in (23), equating and reducing, we have

$$\frac{M''L}{3}+\frac{M'L}{6}+\frac{PkL^2}{6}-\frac{Pk^3L^2}{6}=-\frac{M''L_,}{3}-\frac{M'''L_,}{6}$$

$$+\frac{P'k_,^2L_,^2}{2}-\frac{P'k_,^3L_,^2}{6}-\frac{P'k_,L_,^2}{3},$$

from which we obtain

$$M'L+2M''(L+L_,)+M'''L_,=-PL^2(k-k^3)-P'L_,^2(2k_,-3k_,^2+k_,^3)\ .\ (\mathrm{L}),$$

which is the *three-moment equation* when the loads are concentrated loads. The more general equation, as usually written, is

$$M'L+2M''(L+L_,)+M'''L_,=-\Sigma PL^2(k-k^3)-\Sigma P'L_,^2(2k_,-3k_,^2+k_,^3).$$

Case II. When the beam supports uniform loads.

Let w be the uniform load per foot on span AB (Fig. 93) and let w' be the uniform load per foot in span BC. Let M', M'', and M''' now represent the bending moments at the supports A, B, and C, respectively, due to the above uniform loads, and let V and V' now represent the shears just to the right of supports A and B, respectively, due to these same loads. Considering span AB, and taking A as the origin, the bending moment at any point x distance from A is

$$M=M'+Vx-\frac{wx^2}{2}.$$

therefore,

$$EI\frac{d^2y}{dx^2} = M' + Vx - \frac{wx^2}{2}.$$

Integrating once, we have

$$EI\frac{dy}{dx} = M'x + \frac{Vx^2}{2} - \frac{wx^3}{6} + C. \quad\ldots\ldots\ldots\ldots(24).$$

Integrating again, we have

$$EIy = \frac{M'x^2}{2} + \frac{Vx^3}{6} - \frac{wx^4}{24} + Cx + C'.$$

But $y = 0$ when $x = 0$; therefore, $C' = 0$, and we have

$$EIy = \frac{M'x^2}{2} + \frac{Vx^3}{6} - \frac{wx^4}{24} + Cx.$$

But also $y = 0$ when $x = L$. Then substituting L for x in the last equation, we have

$$\frac{M'L^2}{2} + \frac{VL^3}{6} - \frac{wL^4}{24} + LC = 0,$$

from which we obtain

$$C = -\frac{M'L}{2} - \frac{VL^2}{6} + \frac{wL^3}{24}.$$

Substituting this value of C in equation (24), we have

$$EI\frac{dy}{dx} = M'x + \frac{Vx^2}{2} - \frac{wx^3}{6} - \frac{M'L}{2} - \frac{VL^2}{6} + \frac{wL^3}{24}. \quad\ldots\ldots\ldots\ldots(25).$$

Similarly, if the origin be taken at the support B, for span BC, we have for the bending moment at any point x distance from B

$$M = M'' + V'x - \frac{w'x^2}{2},$$

therefore,

$$EI\frac{d^2y}{dx^2} = M'' + V'x - \frac{w'x^2}{2}.$$

Integrating once, we have

$$EI\frac{dy}{dx} = M''x + \frac{V'x^2}{2} - \frac{w'x^3}{6} + C_,. \quad\ldots\ldots\ldots\ldots(26).$$

Integrating again, we have

$$EIy = \frac{M''x^2}{2} + \frac{V'x^3}{6} - \frac{w'x^4}{24} + C_,x + C_{,,}.$$

But $x = 0$ when $y = 0$; therefore, $C_{,,} = 0$, and we have

$$EIy = \frac{M''x^2}{2} + \frac{V'x^3}{6} - \frac{w'x^4}{24} + C_,x.$$

But $y = 0$ when $x = L_,$. Then substituting $L_,$ for x in the last equation, we have

$$\frac{M''L_{,}^2}{2}+\frac{V'L_{,}^3}{6}-\frac{w\ L_{,}^4}{24}+C_{,}L_{,}=0,$$

from which we obtain

$$C_{,}=-\frac{M''L_{,}}{2}-\frac{V'L_{,}^2}{6}+\frac{w'L_{,}^3}{24}.$$

Now substituting this value of $C_{,}$ in equation (26), we have

$$EI\frac{dy}{dx}=M''x+\frac{V'x^2}{2}-\frac{w'x^3}{6}-\frac{M''L_{,}}{2}-\frac{V'L_{,}^2}{6}+\frac{w'L_{,}^3}{24}\ldots\ldots(27).$$

It is readily seen that equations (25) and (27) would be equal when $x = L$ in (25) and $x = 0$ in (27), the two elastic curves having a common tangent at B. Then substituting L for x in (25) and 0 for x in (27), equating and reducing, we have

$$12M'L + 8VL^2 - 3wL^3 = -12M''L_{,} - 4V'L_{,}^2 + w'L_{,}^3\ldots\ldots(28).$$

Now (referring to span AB) taking moments about the support B, we have, according to Art. 66,

$$M''=M'+VL-\frac{wL^2}{2}.$$

Transposing and dividing by L, we have

$$V=\frac{M''-M'}{L}+\frac{wL}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(29).$$

Next (referring to span BC) taking moments about C, we have

$$M'''=M''+V'L_{,}-\frac{w'L_{,}^2}{2}.$$

Transposing and dividing by $L_{,}$, we have

$$V'=\frac{M'''-M''}{L}+\frac{w'L_{,}}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(30).$$

Now substituting the values of V and V' as given by (29) and (30), respectively, in equation (28), and reducing, we have

$$M'L + 2M''(L + L_{,}) + M'''L_{,} = -\frac{wL^3}{4}-\frac{w'L_{,}^3}{4}\ldots\ldots\ldots(M),$$

which is the three-moment equation when the loads are uniformly distributed.

The three-moment equation, as is readily seen, is an expression for the bending moments at any three consecutive supports in terms of the

Fig. 94

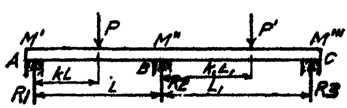


Fig. 95

lengths of the two intervening spans and the loads supported in these spans. The lengths and loads, of course, are known quantities. Now for any continuous beam of n spans, there are always $(n + 1)$ supports, and for each three consecutive supports a three-moment equation can be written. Then, $(n - 1)$ three-moment equations can be written for any continuous beam. For example, let *abcdef* (Fig. 94) represent a continuous beam of five spans. Here we can write four (which is $(n - 1)$) three-moment equations: one for supports *a*, *b*, and *c*; one for supports *b*, *c*, and *d*; one for supports *c*, *d*, and *e*; and one for supports *d*, *e*, and *f*. Now the bending moment at each of the supports would be included in these four three-moment equations, being $(n + 1)$ moments in all. But, as the bending moments at both *a* and *f* are equal to zero, the four unknown moments at *b*, *c*, *d*, and *e* can be determined, as we have four equations and four unknown moments. Then, after these unknown moments at the supports are determined, the reactions at each support can be determined and then the shear and the bending moment at any point of the beam can be determined.

As an example, let *ABC* (Fig. 95) represent a continuous beam having three supports which would be known as a beam continuous over three supports. Let M', M'', and M''' represent the bending moments at the supports *A*, *B*, and *C*, respectively, due to any loads that we choose to consider, and let $R1$, $R2$, and $R3$ represent the reactions at the supports *A*, *B*, and *C*, respectively, due also to any loads that we choose to consider. The intensity of these moments and reactions would, of course, vary with the loads.

Let us first consider two concentrated loads P and P' on the beam as shown in Fig. 95. Now, as there are but three supports, only one three-moment equation can be written. So writing this one, which is really Formula (L), given on page 98, we have

$$M'L + 2M''(L + L_,) + M'''L_, = -PL^2(k - k^3) - P'L_,^2(2k_, - 3k_,^2 + k_,^3).$$

But in this case $M' = 0$ and $M''' = 0$. So the equation becomes

$$2M''(L + L_,) = -PL^2(k - k^3) - P'L_,^2(2k_, - 3k_,^2 + k_,^3),$$

from which we obtain

$$M'' = -\frac{PL^2(k - k^3) - P'L_,^2(2k_, - 3k_,^2 + k_,^3)}{2(L + L_,)} \dots\dots\dots\dots (31).$$

Now considering points *A* and *B*, and taking moments about *B*, according to Art. 66, we have

$$M'' = R1L - P(L - kL).$$

Transposing and reducing, we have

$$R1 = \frac{M''}{L} + P(1 - k).$$

Now, by substituting the value of M'', as given in equation (31), in this last equation, the value of the reaction $R1$ at *A* is obtained. Then, after $R1$ is known, the bending moment at any point in the span *AB* is obtained as readily as in the case of a simple beam, as it is equal to the algebraic sum of the moments about the point considered of the forces to the left of the point, which are now all known.

Now, similarly, considering points B and C, and taking moments about B, we have

$$M'' = R3L_, - P'k_,L_, .$$

Transposing and reducing, we have

$$R3 = \frac{M''}{L_,} + P'k_, .$$

Then by substituting the value of M'', as given in equation (31), in this last equation, the value of the reaction $R3$ at C is obtained. Then after $R3$ is known, the bending moment at any point in the span BC is readily obtained, as it is equal to the algebraic sum of the moments about the point considered of the forces to the right of it, which are now all known. The shear at any point in either span is obtained by simply adding up (algebraically in each case) the forces between the end and the point considered. $R2$, as is readily seen, is equal to the sum of the loads minus the two reactions $R1$ and $R3$. It is also equal to the shear just to the right of B plus the shear just to the left of B. By taking moments about both A and B, equations can be derived from which these shears can be obtained, and consequently $R2$.

Now it is evident that the bending moments, shears, and reactions on the above beam due to any number of such loads could be determined by considering the loads in pairs, as in the above case. But it is more convenient to derive equations expressing the values of these for one single load at any point. So let P (Fig. 95) be the only load on the beam. Then P' and $k_,$ will not occur in the three-moment equation, that is, they will be equal to zero, and hence the three-moment equation becomes

$$2M''(L + L_,) = -PL^2(k - k^3),$$

from which we obtain

$$M'' = -\frac{PL^2(k - k^3)}{2(L + L_,)} \quad \ldots\ldots (33).$$

Now taking moments about B, considering points A and B, we have

$$M'' = LR1 - P(L - kL).$$

Then substituting this value of M'', as given in (33), in this last equation, and reducing, we have

$$R1 = P(1 - k) - \frac{PL(k - k^3)}{2(L + L_,)} \quad \ldots\ldots (34).$$

Then taking moments about B, considering points B and C, we have

$$M'' = L_,R3.$$

Substituting this value of M'', as given in (33), in this last equation, and dividing by $L_,$, we have

$$R3 = -\frac{PL^2(k - k^3)}{2L_,(L + L_,)} \quad \ldots\ldots (35).$$

Now, $R1$ and $R3$ can be determined for any load in the span AB from equations (34) and (35), respectively, and likewise for any load in span BC by interchanging the R's, that is, $R1$ would in that case be at C and $R3$ at A—or just turn the beam end for end. If there are several loads in the two spans, the reactions $R1$ and $R3$ due to each can be determined separately and added algebraically (as $R3$ is always minus), and thus the total reaction at both A and C would be determined. Then the bending moment and shear at any point in the beam is readily determined, as explained above.

Now if the two spans are equal, we have $L = L_1$ in both equations (34) and (35). Then (34) reduces to

$$R1 = \frac{P}{4}(4 - 5k + k^3) \ldots\ldots\ldots\ldots\ldots (36),$$

and (35) reduces to

$$R3 = -\frac{P}{4}(k - k^3) \ldots\ldots\ldots\ldots\ldots (37).$$

Then adding (36) and (37) and subtracting the result from the load P and reducing, we have the formula

$$R2 = \frac{P}{2}(3k - k^3) \ldots\ldots\ldots\ldots\ldots (38).$$

Now the reactions due to a load at any point on any beam continuous over three supports and having equal spans, can be determined from equations (36), (37), and (38). Then the bending moments and shears are readily determined as explained above.

As a practical example, let L and L_1 in Fig. 95, each be equal to 12 feet, and let P be 4 feet from A and let P' be 6 feet from C. Then $k = 4/12 = 1/3$, and $k_1 = 6/12 = 1/2$.

First considering P alone and substituting 1/3 for k in (36) and reducing, we have

$$R1 = \frac{16}{27} P$$

for the value of the reaction at A due to the load P. Next substituting 1/3 for k in (37) and reducing, we have

$$R3 = -\frac{2}{27} P$$

for the reaction at C, which is minus; that is, it pulls down upon the beam instead of pushing up. Now after these reactions are determined, the bending moment and shear at any point in the beam due to the load P alone can be determined as explained above. The bending moments in span BC would be negative, as $R3$ is negative; that is, the top elements in the beam would be in tension while the bottom ones would be in compression.

Now considering P' alone, and substituting 1/2 for k ($=k_1$) in (36) and reducing, we have

$$R1 = \frac{13}{32} P',$$

which is the reaction at C due to load P' in span BC. Next, substituting 1/2 for k in (37) and reducing, we have

$$R3 = -\frac{3}{32}P',$$

which is the reaction at A due to the load P' in span BC. Then the reaction at A due to the two loads is

$$\frac{16}{27}P - \frac{3}{32}P',$$

and the reaction at C due to the same is

$$\frac{13}{32}P' - \frac{13}{54}P.$$

After these reactions are thus determined, the bending moment and shear at any point in the beam are readily determined, as explained above.

Now instead of the concentrated loads just considered, suppose the beam shown in Fig. 95 supported a uniform load of w pounds per foot in span AB and w' pounds per foot in span BC, extending over the entire span in each case. In this case, as the loads are uniformly distributed, we would use the three-moment equation (M), given above, which, as M' and M'' are equal to zero, reduces to

$$2M''(L + L_{,}) = -\frac{wL^3}{4} - \frac{w'L_{,}^3}{4} \quad\ldots\ldots\ldots\ldots (39).$$

Then transposing and dividing through by $2(L + L_{,})$, we have

$$M'' = -\frac{wL^3 + w'L_{,}^3}{8(L + L_{,})} \quad\ldots\ldots\ldots\ldots (40).$$

Now taking moments about B, and considering points A and B, according to Art. 66, we have

$$M'' = LR1 - \frac{wL^2}{2}.$$

Then substituting the value of M'', as given in (40), in this last equation, transposing, and reducing, we have

$$R1 = \frac{wL}{2} - \frac{wL^3 + w'L_{,}^3}{8L(L + L_{,})} \quad\ldots\ldots\ldots\ldots (41).$$

Now taking moments about B, and considering points B and C, we have

$$M'' = L_{,}R3 - \frac{w'L_{,}^2}{2},$$

and substituting the value of M'', as given in (40), in this last equation, transposing and reducing, we have

$$R3 = \frac{w'L_{,}}{2} - \frac{wL^3 + w'L_{,}^3}{8L_{,}(L + L_{,})} \quad\ldots\ldots\ldots\ldots (42).$$

After these reactions are obtained, the bending moment and shear at any point in the beam can be determined very readily as explained above.

Suppose the spans to be equal in length and suppose the loads are equal. Then $L = L_1$ and $w = w'$, and substituting L for L_1 and w for w' in both (41) and (42), and reducing, we have

$$R1 = \frac{3}{8} wL \text{ and } R3 = \frac{3}{8} wL,$$

that is, the reactions at A and C are equal to $\frac{3}{8}$ of the load on one span. Then, evidently, the shear just to the right and also just to the left of B is equal to $\frac{5}{8}(wL)$ or $\frac{5}{8}$ of the load on one span, and hence for the reaction at B we have

$$R2 = \frac{10}{8} wL.$$

Substituting L for L_1 and w for w' in (39) and reducing, we have

$$M'' = -\frac{wL^2}{8},$$

which is the bending moment at B, the central support. This being minus shows that the top elements of the beam are in tension while the bottom ones are in compression. This same moment can be obtained by taking moments about B and considering either span, say, span AB. Then we have

$$M'' = LR1 - \frac{wL^2}{2}.$$

But $R1 = \frac{3}{8}(wL)$, as shown above. So substituting this value for $R1$ in this last equation, and reducing, we have

$$M'' = -\frac{wL^2}{8}.$$

Now taking moments about the center of span AB, we have

$$M = \frac{L}{2} R1 - \left(\frac{wL}{2}\right)\frac{L}{4} = \frac{L}{2} R1 - \frac{wL^2}{8}.$$

Substituting $\frac{3}{8}(wL)$ for $R1$, we have for the bending moment at mid-span

$$M = \frac{3}{16} wL^2 - \frac{wL^2}{8} = \frac{1}{16} wL^2,$$

which is one-half of what it is at B, the central support.

Beams having four or more supports can be analyzed in a similar manner. Of course, there would be more three-moment equations involved —being $(n - 1)$ in each case. However, the same general method as outlined above would hold for all cases.

CHAPTER V

THEORETICAL TREATMENT OF COLUMNS

71. Preliminary.—Let CB (Fig. 96) represent a round steel rod of uniform cross-section A and length L standing vertically upon a smooth, rigid base at B and supporting a load P at C which is symmetrically applied in reference to the longitudinal axis of the rod; that is, the load is applied in the center of gravity of the cross-section of the rod. It is evident that the load P will produce a direct simple compressive stress P at every cross-section of the rod throughout its length, and hence the direct compressive unit stress at every section will be equal to P/A. All bodies having a length much greater than their width, as the above rod, and loaded in the same manner, thereby being in compression, are known in general as columns.

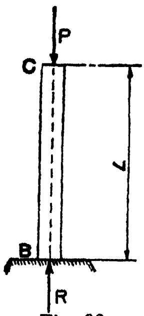


Fig. 96

Now suppose the above rod to be 2 inches in diameter and, say, 4 inches long. Then the unit compressive stress would be equal to

$$\frac{P}{\pi\left(\frac{2}{2}\right)^2}=\frac{P}{\pi},$$

which corresponds to P/A given above. It is evident that this short rod would support a very heavy load without failure, in fact, the compressive stress P/π could be as great as if it were a tensile stress. But suppose the rod to be 20 feet long instead of 4 inches. We know that the rod, if 20 feet long, would not support as heavy a load as it would if only 4 inches long, as the longer rod would be too limber, that is, it would fail by bending transversely; of course, the direct compressive stress would be acting just the same in either case.

If a column were absolutely straight and the load applied absolutely in the longitudinal axis, the bending referred to above would not occur. But such conditions do not exist, as we know from experience. So in the designing of columns the stress due to this bending must be taken into account in addition to the direct compressive stress.

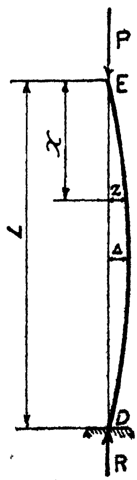


Fig. 97

72. Rankine's Formula.—Let ED (Fig. 97) represent a column which bends as indicated under a load P. It is evident that the elements on the concave side of the column are in compression, owing to the bending, while the elements on the other side are in tension, exactly as in the case of a loaded beam. Now it is readily seen that the maximum stress on the column will occur on the compression or concave side, for here the direct and bending stresses combine, while the tensile stress on the other side reduces the direct stress. This maximum compressive stress will, as is readily observed, occur at the middle of the column, where the deflection is greatest, which is represented as Δ. The bending moment on the column at any point x distance from E due to the load is

$$M = Pz,$$

where z is the deflection. This moment will be a maximum when $z = \Delta$. Then we have

$$M = P\Delta$$

for the maximum bending moment on the column. But from (D), Art. 53, we have $f = My/I$, and substituting $P\Delta$ for M, we have

$$f = \frac{P\Delta y}{I}$$

(where y = the distance from neutral axis of column to the extreme elements) for the maximum compressive stress on the column due to bending. Now if we add this to the direct stress, P/A, we have

$$p = \frac{P}{A} + \frac{\Delta P y}{I} \quad \text{.................................} (1),$$

which is the maximum compression unit stress on the column.

Now—as is proven by experiment—Δ will vary directly as L^2 and inversely as y; that is,

$$\Delta \propto \frac{L^2}{y}.$$

Then if L^2/y be multiplied by some constant c, we have

$$\Delta = c\frac{L^2}{y}.$$

Substituting this value of Δ in (1), we have

$$p = \frac{P}{A} + \frac{PcL^2}{I}.$$

But $I = Ar^2$ (Art. 49). Then substituting this value of I in the last equation and reducing, we have

$$p = \frac{P}{A}\left(1 + c\frac{L^2}{r^2}\right) \quad \text{.................................} (N).$$

This can be written in the form

$$\frac{P}{A} = \frac{p}{1 + c\frac{L^2}{r^2}} \quad \text{.................................} (O),$$

which is known as Rankine's Formula, where

p = maximum compressive unit stress on the column;
P = load on the column;
L = length of column in inches;

A = area of cross-section of the column in square inches;
r = least radius of gyration of the cross-section in inches in reference to the neutral axis of the column;
c = a constant which is determined by experiment, and depends upon the material composing the column and upon end conditions.

The maximum unit stress on any known column due to a given load can be determined from Formula (N) and the allowable direct or average compressive unit stress from (O), providing the proper value of c is known, which is determined from experiment.

The following practical values of c are recommended:

For timber columns $c = 4/3{,}000$;
For cast-iron columns $c = 4/5{,}000$;
For (medium) steel columns $c = 1/11{,}000$.
(American Bridge Co.)

If p be taken as the elastic limit of the material in the column, P/A will be the direct compressive unit stress that the column would be subjected to when the elastic limit of the column was reached, but if p be taken as the ultimate strength of the material, P/A would be the direct compressive unit stress that the column would be subjected to at failure. If p be given either of these values, a factor of safety must be used in determining the "working stress" for any column. If 32,000 lbs. be taken as the value of p at the elastic limit of steel, a factor of safety of 2 should be used, and if 64,000 lbs. be taken as the value of p when the column fails, a factor of 4 should be used. For practical designing, it is more convenient to take p as the allowable or working stress of the material in tension. This for medium steel may be taken as 16,000 lbs., in accordance with the specifications of A. R. E. Ass'n. Now, substituting this value of p and the above value of c in Rankine's Formula (O), we have

$$\frac{P}{A}=\frac{16{,}000}{1+\frac{1}{11{,}000}\left(\frac{L}{r}\right)^2} \quad \ldots\ldots\ldots\ldots \text{(P)}.$$

This Formula (P) may be used for designing any steel column, but a *Straight Line Formula* is preferable, as it is more readily applied and the results obtained are practically as accurate.

73. Straight Line Formula.—The ordinates to the curve *abcd* (Fig. 98) measured from the horizontal line *OH* give the value of P/A corresponding to the different values of L/r, as shown. This curve is obtained by substituting the different values of $(L/r)^2$ in Formula (P). For example, the ordinate *eb* is equal to

$$\frac{P}{A}=\frac{16{,}000}{1+\frac{1}{11{,}000}(30)^2}=14{,}800 \text{ lbs.}$$

It is readily seen from Formula (P) that the ordinate *Oa* is equal to 16,000 lbs., as $L/r = 0$. Now, if some line as *ak* be drawn such that the ordinates to it will be practically the same as to the curve, it is obvious

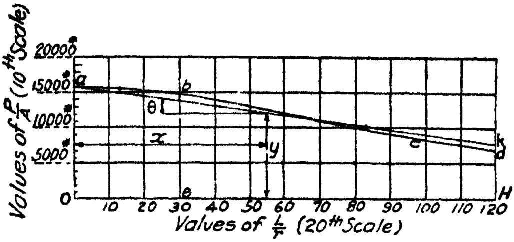


Fig. 98

that the equation to this line can be used as a column formula instead of Formula (P), and the equation to the line would be known as a "Straight Line Formula."

The ordinate Hk to this line is taken here as 7,600 lbs., and the ordinate Oa is 16,000 lbs. Then, as OH is known, the slope of the line ak can be determined, and consequently its equation in reference to 0 can be derived.

Let θ be the angle that the line makes with the horizontal. Then we have any ordinate y, distance x from 0, given by the formula

$$y = 16{,}000 - x\tan\theta \qquad \text{(1).}$$

Now $\tan\theta = (Oa - Hk) \div OH = 0.14$, as Oa and Hk are equal to 16,000 and 7,600 lbs., respectively, and OH, to the same scale, is equal to 60,000 lbs. Now for x, in terms of L/r, we have

$$x = \frac{10{,}000}{20}\left(\frac{L}{r}\right) = 500\frac{L}{r}.$$

Then substituting these values of $\tan\theta$ and x in (1), we have

$$y = \frac{P}{A} = 16{,}000 - 70\frac{L}{r} \qquad \text{(Q),}$$

which is the straight line formula as given in the A. R. E. Ass'n Specifications.

As is obvious, the author just deliberately took the ordinate Hk so that the above formula would result. If the line ak be drawn to conform to mere observation, a slightly different formula would, very likely, be obtained.

It is readily seen that a straight line formula for any other material can be determined in the same manner by simply using the proper c and working stress in each case.

74. Examples in the Application of Column Formulas.—

Example 1. What will be the maximum compressive unit stress produced in a wooden column 10 x 10 ins. in cross-section and 12 ft. long by a load of 20,000 lbs.?

Here $A = 100$, $L = 144$, $r = 2.88$, and $c = 4/3{,}000$. The value of p is desired. Then substituting the above values in Formula (N), we have

$$p = \frac{20{,}000}{100}\left[1 + \frac{4}{3{,}000}\left(\frac{144}{2.88}\right)^2\right] = 868 \text{ lbs.}$$

In this example, the direct unit stress is $20{,}000 \div 100 = 200$ lbs. So the stress due to cross bending is $868 - 200 = 668$ lbs. The working unit stress on timber should be taken at about 1,200 lbs. So the above column will safely support more than 20,000 lbs.

Example 2. What direct unit stress would the above column safely sustain if the working unit stress be taken as 1,200 lbs.? Here P/A is desired. Then using Formula (O), we have

$$\frac{P}{A}=\frac{1,200}{\left[1+\frac{4}{3,000}\left(\frac{144}{2.88}\right)^2\right]}=277 \text{ lbs. per sq. in.}$$

Then the safe load, or P, would be $277 \times 100 = 27,700$ lbs.

What is usually given is the load and the length of the column. In that case the problem is to design a column outright that will safely carry this known load. In all such cases it is practically a matter of first guessing a column which we think will satisfy the conditions and then modifying our guess until the conditions are satisfied, as will be shown in the following problems.

Example 3. Design a timber column 10 ft. long to carry a load of 20,000 lbs. Taking 1,200 lbs. as the working unit stress, then p will be 1,200. Using Formula (N), we have

$$1,200=\frac{20,000}{A}\left[1+\frac{4}{3,000}\left(\frac{120}{r}\right)^2\right].$$

Here it is seen that A and r are unknown. First assume that a column 9 x 9 ins. will do. Then $A = 81$ and $r = 2.6$. Substituting these values in the last equation, we have

$$1,200=\frac{20,000}{81}\left[1+\frac{4}{3,000}\left(\frac{120}{2.6}\right)^2\right]=938 \text{ lbs.},$$

which is too small, or, in other words, the column is too large. So next try an 8 x 8-in. section. Then we have

$$1,200=\frac{20,000}{64}\left[1+\frac{4}{3,000}\left(\frac{120}{2.3}\right)^2\right]=1,430 \text{ lbs.},$$

which is too large, that is, this column is too small. So the correct size is between the two. However, a 9 x 9-in. section would be used, as more than likely nothing between the two could be obtained.

In case of steel columns, the straight line Formula (Q) will be used, although the Formula (P) could be used.

Example 4. Design a steel column 25 ft. long to support a load of 160,000 lbs. Using Formula (Q), we have

$$p=16,000-70\frac{300}{r}.$$

The direct unit stress in accordance with economy should not be less than 9,000 lbs., and it rarely ever exceeds 14,000 lbs. So, guessing in this case, as the column is short, that it will be 13,000 lbs., and dividing the load by that, we have

$$\frac{160,000}{13,000}=12.3 \text{ sq. ins.}$$

From Table 3 it is seen that 2—[s 12″ x 20.5# have about this section. The r for these two channels in reference to the gravity axis perpendicular to their webs will be the same for the two as it is for one channel, which is given in the same table as 4.61. The radius in reference to the gravity axis parallel to the webs can always be made equal to the one given in the table by moving the channels far enough apart. Now substituting the above value of r in the above formula, we have

$$p = 16{,}000 - 70\frac{300}{4.61} = 11{,}450 \text{ lbs.}$$

Then dividing this into the load, we have

$$\frac{160{,}000}{11{,}450} = 14.0 \text{ sq. in. (about)},$$

which is more than the area of the channels assumed. So try 2—[s 12″ x 25# = 14.7□″. Then r, from Table 3, is 4.43, and our formula becomes

$$p = 16{,}000 - 70\frac{300}{4.43} = 11{,}260 \text{ lbs.}$$

Then dividing this into the load, we have

$$\frac{160{,}000}{11{,}260} = 14.2 \text{ sq. ins.},$$

which agrees very closely with the area of the 2—[s 12″ x 25#, which would therefore be used. The channels composing a column as just designed would be latticed together.

The method of applying the column formula in the last example is general, but the form of column is only one of many. It is beyond the limits of practicability to have the radii of gyration tabulated for each form. However, the approximate value of the radius for any form in general use can be obtained from Table 10, where it is given in terms of the height and width of the section, as shown. After the approximate area of the cross-section is obtained by dividing the load by an assumed intensity, as above, the section can be selected and then the corresponding approximate radius of gyration can be obtained from Table 10. In this way the assumed section can be tested, and, if found to be about correct, the exact radius can be computed and substituted in the column formula, and the section modified slightly, if necessary, to satisfy this last requirement. The form of steel columns depends upon the kind of structure and the position they occupy in the structure as well as to the loads they support.

The value of p for the different values of L/r could be taken from such a diagram as shown in Fig. 98 if it be drawn to a sufficiently large scale, or a table giving the same can be computed. The latter is often used in designing offices.

In case a column is fixed at the ends, one-half of the length of the column is taken as L, and if fixed at one end only, three-fourths of the length is taken as L in the column formulas. The reason for so doing is readily seen, as the actual length of the column as far as bending is concerned in the first case is the distance between the points of contraflexure, which is about one-half of the length of the column; and in the second case it is the distance from the free end to the point of contraflexure, which is about three-fourths of the length of the column, the same as in the case of beams. (See Arts. 68 and 69.)

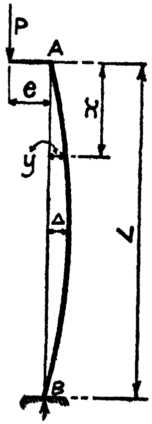


Fig. 99

75. Columns Eccentrically Loaded.—Whenever possible, the load on any column should be applied in the center of gravity of its cross-section. However, in some cases this is not possible. In such cases the bending stresses due to the eccentric loading must be considered.

Let AB (Fig. 99) represent a column, which bends as indicated, due to the eccentrically applied load P. Let e represent the eccentric distance, Δ the maximum deflection, and x and y the co-ordinates to the elastic curve referred to A as origin. Now the bending moment at any point x distance from A is equal to

$$EI\frac{d^2y}{dx^2} = -P(e+y).$$

For convenience let $k = \sqrt{P/EI}$. Then we have

$$\frac{d^2y}{dx^2} = -k^2(e+y).$$

Now multiplying both sides of the equation by $2dy$, we have the form

$$\frac{2dyd(dy)}{dx^2} = -k^2(e+y)2dy.$$

Integrating (dx constant), we have

$$\frac{dy^2}{dx^2} = -k^2(e+y)^2 + C, \quad\ldots\ldots\ldots\ldots (1).$$

Now, as is readily seen, $dy/dx = 0$ when $y = \Delta$. Therefore, $C_{,} = k^2(e+\Delta)^2$. Then substituting this value of $C_{,}$ in (1), we have

$$\frac{dy^2}{dx^2} = -k^2(e+y)^2 + k^2(e+\Delta)^2.$$

Now extracting the square root and transposing, we have

$$dx = \frac{dy}{k\sqrt{(e+\Delta)^2 - (e+y)^2}}.$$

Integrating this equation, we have

$$x = \frac{1}{k}\left(\sin^{-1}\frac{e+y}{e+\Delta}\right) + C_{,,} \quad\ldots\ldots\ldots\ldots (2).$$

Now $y = \Delta$ when $x = L/2$; therefore, $C_{,,} = (L/2 - \pi/2k)$. Then substituting this value in (2), we have

$$x = \frac{1}{k}\left(\sin^{-1}\frac{e+y}{e+\Delta}\right) + \left(\frac{L}{2} - \frac{\pi}{2k}\right) \quad\ldots\ldots\ldots\ldots (3).$$

Now $y = 0$ when $x = 0$. So, substituting 0 in (2) for both y and x, we have

$$C_{,,} = -\frac{1}{k}\left(\sin^{-1}\frac{e}{c+\Delta}\right).$$

Then by equating the two values of $C_{,,}$, we have

$$-\frac{1}{k}\left(\sin^{-1}\frac{e}{e+\Delta}\right) = \left(\frac{L}{2} - \frac{\pi}{2k}\right),$$

from which we obtain

$$\Delta = \frac{e}{\cos\frac{kL}{2}} - e.$$

Then substituting this value of Δ in (3), we have

$$x = \frac{1}{k}\left(\sin^{-1}\frac{(e+y)\cos kL/2}{e}\right) + \left(\frac{L}{2} - \frac{\pi}{2k}\right) \dots\dots\dots\dots\dots (4).$$

Now transposing and reducing, we have

$$\sin\left[\left(kx - \frac{kL}{2}\right) + \frac{\pi}{2}\right] = \frac{e+y}{e}\cos\frac{kL}{2}.$$

But, according to trigonometry,

$$\sin\left[\left(kx - \frac{kL}{2}\right) + \frac{\pi}{2}\right] = \sin\left(kx - \frac{kL}{2}\right)\cos\frac{\pi}{2} + \cos\left(kx - \frac{kL}{2}\right)\sin\frac{\pi}{2}$$

$$= \frac{e+y}{e}\cos\frac{kL}{2}.$$

But, cos $\pi/2 = 0$, and sin $\pi/2 = 1$. Then the last equation reduces to

$$\cos\left(kx - \frac{kL}{2}\right) = \frac{e+y}{e}\cos\frac{kL}{2}.$$

But again, according to trigonometry,

$$\cos\left(kx - \frac{kL}{2}\right) = \cos kx \cos\frac{kL}{2} + \sin kx \sin\frac{kL}{2} = \frac{e+y}{e}\cos\frac{kL}{2}.$$

Dividing through by $\cos kL/2$, we have

$$\cos kx + \sin kx \tan\frac{kL}{2} = \frac{e+y}{e}.$$

But, tan $kL/2 = 1 - \cos kL/\sin kL$. Then substituting this value of tan $kL/2$ in the last equation, and reducing, we have

$$\frac{e[\sin k(L-x) + \sin kx]}{\sin kL} - e = y \dots\dots\dots\dots\dots (5).$$

Now from this last equation, y can be computed for any point along the column, and then the lever arm $(e + y)$ is known and hence the stress due to bending at any point along the column can be determined, as we have

$$P(e+y) = \frac{fI}{d}$$

according to Art. 53. (d is here the distance from the neutral axis to the outermost element in compression.)

Then by adding the unit compressive stress (f) thus obtained to the direct unit stress (P/A), we have the maximum unit stress at the point considered. This, of course, will be a maximum when $y = \Delta$. Now $y = \Delta$ when $x = L/2$. Then, substituting this value of x in (5) and reducing, we have

$$e\left(\sec\frac{kL}{2} - 1\right) = \Delta \qquad (6)$$

This last equation (6) can be used for determining Δ, the maximum deflection. After this is determined, the maximum stress can readily be determined as stated above. For the maximum compressive stress we then have the formula

$$p = \frac{Pd(e+\Delta)}{I} + \frac{P}{A},$$

where p is the maximum compressive unit stress in pounds, I the moment of inertia of the cross-section of the column, A the area of the cross-section in square inches (which is assumed to be constant), and P, e, and Δ are as stated above.

CHAPTER VI

RIVETS, PINS, ROLLERS, AND SHAFTING

RIVETS

76. Kinds of Stress.—Rivets may fail by shearing off transversely, by bearing, that is, by crushing against the metal through which they pass, or by bending, as a beam. So, therefore, we have to consider shearing, bearing, and bending stresses on them.

77. Shearing Stress.—The shear on a rivet at any cross-section is equal to the algebraic sum of the forces on either side of the section—just the same case as a beam. Let A and B (Fig. 100) represent two bars held together by one rivet, as shown. Let P be the tensile stress on each bar. Then the bar B would exert forces along ab upon the rivet, the sum of which (resolved along the bar) would be equal to P, while the bar A would exert forces upon the rivet along cd, the sum of which (resolved along the bar) would be equal to P. The forces along ab, acting against the forces along cd, tend to shear the rivet off transversely. This shear, as is readily seen, varies from O at each end of the shank to a maximum at the cross-section bc. If it were possible to determine the intensity of these forces at all points, the shear at any cross-section along the rivet could be obtained by beginning at either end of the shank and simply summing up the forces to the section considered. However, it is not practical to do so, neither is it necessary, for it is readily seen that the maximum shear occurs just between the two bars at cross-section bc. This shear, as is evident, is equal to P, and as it is the maximum, it is the only shear we need consider. The shear between the pieces connected is what is generally referred to as the shear on rivets, and it is what we shall consider as the shear.

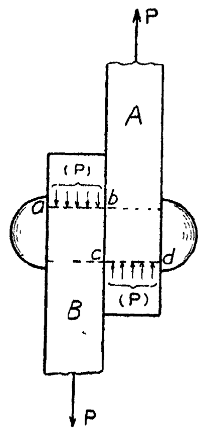


Fig. 100

The unit shearing stress, that is, the stress per square inch on a rivet, is obtained by dividing the shear (in pounds) by the area (in square inches) of the cross-section of the rivet. Thus if S be the shear in pounds at a given section of a rivet having a cross-section of A square inches, and V the unit shearing stress, we have

$$V = \frac{S}{A}.$$

The allowable shear on a given rivet is what we usually desire. This is obtained by multiplying the allowable intensity per square inch by the area of the cross-section of the rivet. We shall take this allowable intensity as 12,000 pounds per square inch for shop rivets and 10,000 pounds for field rivets, as specified by the A. R. E. Ass'n in their specifications for railroad bridges. Then if s be the allowable shearing stress, we have $s = 12{,}000 \times A$ for shop rivets, and $s = 10{,}000 \times A$ for field rivets. Substituting in these formulas we have for the allowable shear on a $\frac{7}{8}''$ shop rivet, $s = 12{,}000 \times 0.6013 = 7{,}216$ lbs., and for a $\frac{7}{8}''$ field rivet, we have $s_1 = 10{,}000 \times 0.6013 = 6{,}013$ lbs. These values for the different size rivets are given in column 3 of Table 11.

A rivet is in single shear when there is (so to speak) one tendency to shear it off, double shear when there are two tendencies, triple shear when there are three, and so on. If two pieces be riveted together, as indicated in the sketch at (*a*) (Fig. 101), the rivets will be in single shear; and if three pieces be riveted together as indicated in the sketch at (*b*), the rivets will be in double shear; and if there were four pieces, the rivets would be in triple shear. If the pieces riveted together are properly designed, rivets in double shear will have twice the allowable

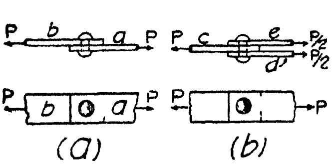


Fig 101

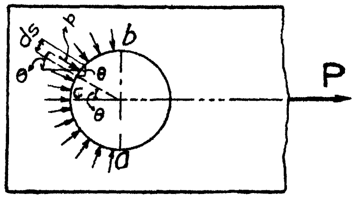


Fig. 102

shearing strength of rivets in single shear, and three times in the case of triple shear, and so on. For example, the shear on the rivet shown at (*a*) (Fig. 101) would be equal to P, while in the case shown at (*b*) it would be equal to $P/2$, providing e and d have equal cross-section.

78. Bearing Stress on Rivets.—Let Fig. 102 represent a bar pulling with a force of P pounds against a rivet which passes through it. The force transmitted to the rivet thereby will be a normal force uniformly distributed from b around to a, as indicated. Let t be the thickness of the bar, D the diameter of the rivet, and let p be the uniform bearing force per square inch. Now, the bearing force on any infinitesimal strip through the plate, as tds, where t is the thickness of the plate, would be $ptds$. Now this force, resolved along the bar, is equal to $ptds \cos\theta$, where θ is the angle the normal force makes with that direction, as indicated. Then if the normal forces on all such strips, from b around to a, be resolved along the bar and summed up, we would have $pt\Sigma ds\cos\theta = P$. But $\Sigma ds\cos\theta = ab = D$, the diameter of the rivet. So we have

$$ptD = P \qquad (1).$$

To obtain the value of P, such that the rivet would not be too highly stressed, we would substitute the working value of p in the above formula, in which case p would be known as the allowable unit-bearing stress on the rivet and P as the allowable bearing of the rivet on the plate of thickness t. For the unit value we will take 24,000 lbs. per square inch for shop rivets and 20,000 lbs. for field rivets, as specified by the A. R. E. Ass'n in their specifications for railroad bridges.

Now suppose the rivet shown in Fig. 102 to be a $\frac{7}{8}''$ shop rivet, and suppose the plate to be $\frac{1}{2}''$ thick. Then substituting in (1), we have

$$24{,}000\times\frac{1}{2}\times\frac{7}{8}=10{,}500 \text{ lbs.} = P,$$

that is, the allowable bearing of a $\frac{7}{8}''$ rivet on a $\frac{1}{2}''$ plate is 10,500 lbs. If the thickness of the plate be $\frac{1}{4}''$ instead of $\frac{1}{2}''$, we would have

$$24{,}000\times\frac{1}{4}\times\frac{7}{8}=5{,}250=P.$$

In this way the allowable bearing for rivets of different diameters on different thicknesses of plates can be computed. These values are given in Table 11.

79. Bending Stress on Rivets.—If rivets are properly driven, bending stresses can be ignored, except in the case of loose fillers as is illustrated in Fig. 103. Here the bars c, d, and e exert double shear on the rivet just the same as the case shown at (b), Fig. 101, and the bearing stresses are just the same, but the idle fillers, f and g, hold the bars apart so that the rivet is really a beam loaded in the center and supported at the ends. Let k be the distance from the center of the bar c to the center of the bar d, and h the distance from the center of the bar c to the center of the bar e. Then the maximum bending moment on the rivet is $(P/2)k$ or $(P/2)h$. Now, according to (D), Art. 53, the maximum bending stress on the rivet per square inch is

Fig. 103

$$f=\frac{My}{I}=\frac{\left(\frac{P}{2}k\right)\frac{D}{2}}{\frac{\pi D^4}{64}}=\frac{16Pk}{\pi D^3},$$

where D is the diameter of the rivet.

The bending stress on a rivet should always be combined with the shearing stress according to Art. 62 so as to obtain the maximum stress.

Details should be so contrived as to avoid bending on rivets. In fact, there is practically no excuse for placing rivets so that they are subjected to bending of any consequence.

80. Examples in Determining Number of Rivets.—Suppose that each of the bars a and b, shown at (a), Fig. 101, be $\frac{1}{4}''$ thick, and suppose the stress P to be 50,000 lbs., how many $\frac{3}{4}''$ shop rivets would be required to transmit this stress?

The allowable single shearing stress on one $\frac{3}{4}''$ shop rivet, from Art. 77, is $12{,}000\times\pi(\frac{3}{4})^2\div4=12{,}000\times0.4418=5{,}301$ lbs. Then the number of rivets required for shear is $50{,}000\div5{,}301 \doteq 9.4$ rivets (use 10).

The allowable bearing stress on one $\frac{3}{4}''$ shop rivet, from (1), Art. 78, is $24{,}000\times\frac{1}{4}\times\frac{3}{4}=4{,}500$ lbs. Then the number required for bearing is $50{,}000\div4{,}500=11$ rivets (about). So 11 rivets is the number required.

Now suppose the bars to be $\frac{3}{8}''$ thick instead of $\frac{1}{4}''$. The allowable shearing stress on each rivet would not change. So 10 rivets would be required for shear the same as before. But the allowable bearing stress is now $24{,}000\times\frac{3}{8}\times\frac{3}{4}=6{,}750$ lbs. Then the number of rivets required for bearing is 50,000 ÷ 6,750 = 7.4 (use 8). So in that case 10 rivets, as required for shear, would be used, as it is the greater number.

As another example, suppose the bar c, shown at (b), Fig. 101, to be ½″ thick, and each of the bars d and e to be ⅜″ thick, and suppose the stress P to be 60,000 lbs.; how many ⅝″ shop rivets would be required to transmit this stress? The allowable double-shearing stress on one ⅝″ shop rivet is 2 × 12,000 × 0.3068 = 7,364 lbs. Then the number of rivets required for shear is 60,000 ÷ 7,364 = 8.15 rivets (use 9). The bearing here, in one direction, is on a ½″ bar, while in the other direction it is on two ⅜″ bars, which is equivalent to one ¾″ bar. So the bearing stress on the ½″ bar (c) will be the greater. The allowable bearing stress on one rivet is 24,000 × ½ × ⅝ = 7,500 lbs. So the number of rivets required for bearing is 60,000 ÷ 7,500 = 8 rivets. Here the number of rivets used would be 9, the number required for shear.

The allowable bearing and shearing intensities for rivets are given in Table 11. However, the student should know how to compute these intensities.

PINS

81. Kinds of Stress.—The same kind of stresses occur on pins as on rivets. However, in the case of pins, the bending stresses are, as a rule, the most serious.

82. Shearing and Bearing Stresses.—The shearing and bearing stresses on pins are determined exactly as on rivets. The allowable unit intensities for shear and bearing are 12,000 and 24,000 lbs., respectively—the same as for shop rivets. The shearing stress very rarely affects the design of a pin, while the same is practically true of the bearing stress. However, the bearing affects the details of the other parts of a structure as each member connected must have the proper thickness of bearing on the pin. As an example, what would be the required thickness of bearing to transmit a stress of 250,000 lbs. to a 6-in. pin? The allowable bearing of a plate 1 in. thick on this pin, according to Art. 78, is 24,000 × 1 × 6 = 144,000 lbs. Then the thickness of the bearing required is 250,000 ÷ 144,000 = 1.76 ins.

83. Bending Stresses on Pins.—The forces applied to pins are assumed to be applied at the center of bearings of the pieces connected. The bending moments on pins can be computed most readily by utilizing the proposition in Art. 66. For example, let AB (Fig. 104) represent a pin acted upon by forces F_a; F_b, etc., applied at sections a, b, c, etc., as shown, and let M_a, M_b, etc., and S_a, S_b, etc., represent the bending moments and shears, respectively, at the sections a, b, c, etc. It is obvious that the bending moments at sections a and g are equal to zero. Then, by starting at either of these sections we can determine the bending moment at each of the other sections, according to Art. 66.

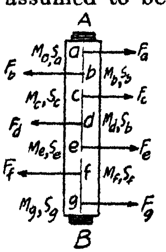


Fig. 104

So, starting from a, the bending moment at b is

$$M_b = M_a(=0) + (ab)S_a = (ab)F_a;$$

at c it is

$$M_c = M_b + (bc)S_b = M_b + bc(F_a - F_b);$$

at d it is

$$M_d = M_c + (cd)S_c = M_c + cd(F_a - F_b + F_c);$$

at e it is

$$M_e = M_d + (de)S_d = M_d + de(F_a - F_b + F_c - F_d);$$

at f it is

$$M_f = M_e + (ef)S_e = M_e + ef(F_a - F_b + F_c - F_d + F_e);$$

at g it is

$$M_g = M_f + (gf)S_f = M_f + fg(F_a - F_b + F_c - F_d + F_e - F_f) = 0.$$

In determining the bending moments in this manner, the maximum is readily observed. Care should be taken in regard to algebraic signs. The shear in some cases will be minus, as is readily seen, and consequently the moments in some cases may be minus.

As a numerical example, let the lever arms be as follows:

$ab = 2$ in., $bc = 1\frac{1}{2}$ in., $cd = 1$ in., $de = 1$ in., $ef = 1\frac{1}{2}$ in., $fg = 2$ in.;

and let

$F_a = 150{,}000$ lbs., $F_b = -200{,}000$ lbs., $F_c = 125{,}000$ lbs., $F_d = -150{,}000$ lbs., $F_e = 125{,}000$ lbs., $F_f = -200{,}000$ lbs., $F_g = 150{,}000$ lbs.;

then

$S_a = 150{,}000$ lbs., $S_b = -50{,}000$ lbs., $S_c = 75{,}000$ lbs., $S_d = -75{,}000$ lbs., $S_e = 50{,}000$ lbs., $S_f = -150{,}000$ lbs.

Now, by substituting these values in the above equations, we have

$$M_b = 2 \times 150{,}000 = 300{,}000 \text{ in. lbs.},$$
$$M_c = 300{,}000 - 75{,}000 = 225{,}000 \text{ in. lbs.},$$
$$M_d = 225{,}000 + 75{,}000 = 300{,}000 \text{ in. lbs.},$$
$$M_e = 300{,}000 - 75{,}000 = 225{,}000 \text{ in. lbs.},$$
$$M_f = 225{,}000 + 75{,}000 = 300{,}000 \text{ in. lbs.},$$
$$M_g = 300{,}000 - 300{,}000 = 0.$$

Here it is seen that the maximum moment on the pin would be 300,000 inch pounds, which happens to occur at three points—b, d, and f.

After the maximum bending moment is thus obtained, the bending stress is computed according to (D), Art. 53, just the same as if the pin were a beam. Thus, in the last example, suppose the pin to be 6″ in diameter. Then we would have

$$f = 300{,}000 \times 3 \div \pi \frac{81}{4} = 14{,}150 \text{ lbs. (about)}.$$

The allowable bending stress intensity for pins, as specified by the A. R. E. Ass'n, is 25,000 lbs. per square inch.

Let D be the diameter of any pin, M, the bending moment, and we have

$$f = M\frac{D}{2} \div \frac{\pi D^4}{64} = \frac{32M}{\pi D^3}.$$

Transposing, we have

$$D = \sqrt[3]{\frac{32M}{f\pi}} \quad \text{(1)}.$$

Now by substituting 25,000 lbs. for f in this last equation, the diameter of pin required for any given bending moment can be determined. The diameters required for given moments are given in Table 12. Here the moments are given in inch pounds, and f taken as 25,000 pounds.

Often the members bearing on a pin act upon it from different directions, as shown in Fig. 105. In such cases the stresses in the members are resolved horizontally and vertically and the bending moments determined in each plane, and the maximum or resultant moment is then obtained by extracting the square root of the sum of the squares of these maximum horizontal and vertical moments. Thus, in the case shown in Fig. 105, $F3$ would be resolved horizontally and vertically. Then the horizontal component would be included with the forces $F1$ and $F2$ in the computations for the horizontal moments, and the vertical component would be included with the force $F4$ in the computations for the vertical moments.

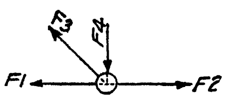


Fig. 105

Let M_h represent the maximum horizontal moment, and let M_v represent the maximum vertical moment; then the maximum or resultant moment would be

$$M = \sqrt{M_h^2 + M_v^2},$$

for which the pin would be designed.

ROLLERS

84. Allowable Pressure.—In modern practice, the allowable pressure on rollers is obtained by the use of an empirical formula, which has the form

$$p = cD \quad \text{(1)},$$

where p = the allowable pressure per lineal inch of roller and D = the diameter of the roller in inches, and c = a constant.

Prof. A. Marston* found that for the elastic limit of soft steel rollers c was about 880. (See paper Trans. A. S. C. E., Vol. 32.) For the allowable pressure, c should be taken as about one-half of this. Then for the allowable pressure per lineal inch on soft steel rollers, we have

$$p = 440\,D \quad \text{(2)}.$$

The formula, for medium steel rollers as specified by the A. R. E. Ass'n, is

$$p = 600\,D \quad \text{(3)}.$$

This is the formula now in general use, as rollers are, as a rule, made of medium steel.

*Dean, Engineering Division, Iowa State College.

The formula is very readily applied. As a practical example, suppose a load of 600,000 pounds be supported upon six 6″ rollers; what will be the required length of each?

Substituting in Formula (3), we have $p = 600 \times 6 = 3{,}600$ lbs. Then the total linear inches required is $600{,}000 \div 3{,}600 = 166.6$ ins.; and as there are six rollers, the length of each will be $166.6 \div 6 = 27.7$ ins.

Rollers should always be as large in diameter as is consistent with accompanying details.

SHAFTING

85. Stress Due to Torsion.—Shafting used to transmit power for operating movable bridges, as well as in all other machinery, is subjected to a twisting about the longitudinal axis known as torsion. The stresses resulting therefrom are known as torsion stresses. This stress is the same as the shearing stress in a beam, except the action on each particle is perpendicular to a radius through the center of rotation, and the stress varies directly as the distance out from this center of rotation. The difference results owing to the stress being caused by a tendency of rotation instead of a tendency of translation, as in the case of a beam.

Let *AB* (Fig. 106) represent a round steel shaft subjected to torsion. Suppose this shaft be cut off at *ab* and spliced by means of inserted steel pins which fit tightly into drilled holes. Now it is readily perceived that the torsion (twisting of the shaft) tends merely to shear each of these pins off transversely and perpendicularly to a radius through the center of rotation, and as far as the torsion is concerned, the pins have only a direct shearing stress on them. Now, evidently, the material particles in every cross-section of the shaft are subjected to exactly the same kind of stress from torsion as the pins, as the action causing the stress is the same.

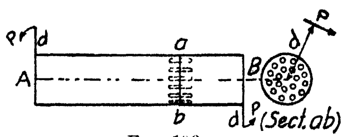


Fig 106

This shearing stress varies directly as the distance out from the center of the shaft. This is readily seen by considering again the pins at section *ab*. Each pin will be distorted a certain amount, and as the tendency of rotation is about the center of the shaft, it is evident that the distortion of the pins will vary directly as their distance out from the center of the shaft, and hence the stress on them will so vary. Then, evidently, if the stress on these pins varies directly as their distance out from the center of the shaft, the stress on the material particles at every cross-section of the shaft will vary directly as their distance out from the center of the shaft.

So we have thus far shown that the stress on the shaft due to torsion is a transverse shearing stress acting perpendicularly to the radii of rotation, and that it varies directly as the distance out from the center of the shaft. Now it remains yet to determine the intensity of the stress. The stress, of course, will depend upon the torsion, which is directly proportional to the moment about the center of the shaft of the force producing the torsion. Let *P* represent the force producing the torsion of the shaft *AB* (Fig. 106) and let *d* be its lever arm. Then the moment of this force, which produces the torsion, is *Pd*. This would be known as the "torque." Now it is evident that this moment *Pd*, or torque, must be balanced at each cross-section of the shaft by the moments of the torsion stresses, at each section, about the center of the shaft.

Let Fig. 107 represent an enlarged cross-section of the shaft AB (Fig. 106). As the torsion stresses vary directly as the distance out from the center of the shaft, the maximum stress will evidently occur at the outermost elements. Let S be this stress per square inch, and let r be the radius of the shaft. Then the stress per square inch out unit distance will be S/r, and the stress per square inch out any distance z will be $(S/r)z$. But as the stress varies continuously outward from the center of the shaft it is evident that there will not be a square inch of material out z distance having a stress of $(S/r)z$, but only an infinitesimal area da of material having that stress. So the actual stress or force out any distance z is $(S/r)zda$, and, of course, the moment of this about the center of the shaft is $(S/r)z^2da$. Now it is evident that the stress is the same on all material equal distances out from the center of the shaft, so da can be taken as a circular strip of width dz and length $2\pi z$. Then we have $da = 2\pi z dz$. Then the moment of the torsion stress out z distance from the center of the shaft is

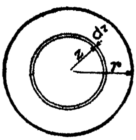


Fig. 107

$$\frac{S}{r}z^2 da = 2\pi\frac{S}{r}z^3dz,$$

and summing up this for the entire cross-section, we have

$$Pd = 2\pi\frac{S}{r}\int_o^r z^3dz \quad \ldots\ldots (1).$$

Integrating, we have $Pd = S\pi r^3/2$, and transposing,

$$S = \frac{2Pd}{\pi r^3} \quad \ldots\ldots (2),$$

from which the maximum torsion stress per square inch on any round solid shaft, as the above, can be computed. S = stress per square inch; Pd = torque; and r = radius of the shaft.

In the case of a hollow shaft, the integration of (1) would be between the limits of the internal and external radii. Thus, let r be the external radius and $r_{,}$ the internal radius of a hollow shaft. Then we would have

$$Pd = 2\pi\frac{S}{r}\int_{r_{,}}^{r} z^3dz.$$

Integrating, we have

$$Pd = \frac{S\pi}{2}\left(r^3 - \frac{r_{,}^4}{r}\right)$$

Transposing, we have

$$S = \frac{2Pd}{\pi\left(r^3 - \frac{r_{,}^4}{r}\right)} \quad \ldots\ldots (3),$$

from which the maximum torsion stress per square inch on any hollow shaft can be computed. S = stress per square inch; r = external radius; $r_,$ = internal radius; and Pd the torque.

In case of a shaft having a square cross-section, da would be taken as any infinitesimal rectangular area. For instance, let Fig. 108 represent the cross-section of a square shaft, the center of which is at O. Then, for the moment of the torsion stress on an infinitesimal area da out any distance z from O, we have $(S/e)z^2da$. Here S is the stress on the outermost element, which is distance e from O. Let Pd be the torque, and we have

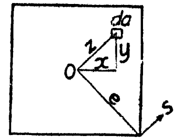


Fig. 108

$$Pd = \frac{S}{e}\Sigma z^2 da.$$

Expressing z in terms of the rectangular co-ordinates x and y, we have

$$Pd = \frac{S}{e}\left(\Sigma x^2 da + \Sigma y^2 da\right).$$

But $(\Sigma x^2 da + \Sigma y^2 da)$ is the polar moment of inertia of the cross-section, which is designated as J in Art. 49. Then we have

$$Pd = \frac{SJ}{e}.$$

Transposing, we have

$$S = \frac{(Pd)e}{J} \quad \ldots\ldots\ldots\ldots (4),$$

from which the maximum torsion stress, not only on square shafting but on any shaft whatever, can be computed. S = stress per square inch; Pd = torque; J = polar moment of inertia; and e = the distance to the outermost element. Formula (4) is a general formula, as it applies to any form of cross-section.

CHAPTER VII

MAXIMUM REACTIONS, SHEARS, AND BENDING MOMENTS ON SIMPLE BEAMS AND TRUSSES, AND STRESSES IN TRUSSES

86. Maximum Reactions on Simple Beams.—The reactions on simple beams in all cases can be determined by taking moments about the supports as explained in Art. 54. In the case of uniform dead load, the two reactions are equal and each is known directly as being equal to half of the dead load supported. In case the dead load is not uniformly distributed, the reactions are obtained by taking moments about the supports, as stated above. In all cases of dead load, the reactions are fixed in intensity, as the loads are fixed in position, but for live load the case is different as the reactions vary with the position of the loads, and the maximum reaction occurs when the loads are in one certain position. If the live load be a uniform load, the reactions at the two supports will be a maximum at the same time and that will be when the load extends over the full length of the beam, in which case each reaction is known directly as being equal to half of the load supported. The live load often consists of wheel loads, as in the case of locomotives, trains, wagons, traction engines, etc., where the loads or wheels are a fixed distance apart. The maximum reaction due to such loading will occur, as is readily seen, when the beam is fully loaded and the heaviest loads are as near as possible to the support considered.

87. Maximum Shear on Simple Beams.—The reactions on simple beams, in all cases, can be determined by taking moments about the

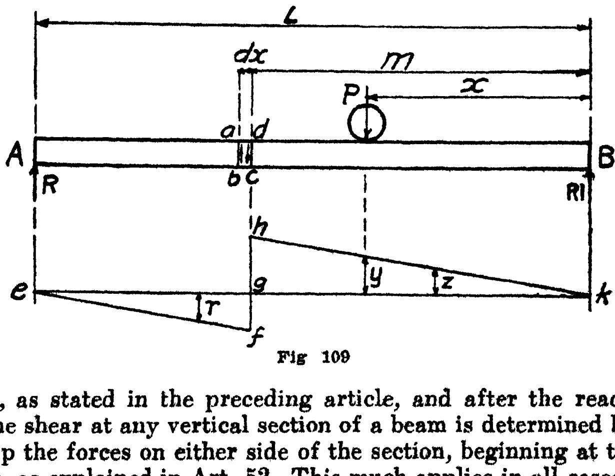


Fig 109

supports, as stated in the preceding article, and after the reactions are known the shear at any vertical section of a beam is determined by simply adding up the forces on either side of the section, beginning at the end of the beam, as explained in Art. 52. This much applies in all cases whether the shear be a maximum or not. The maximum shear on any vertical section of a beam, due to dead load, is readily obtained as such loads are fixed in position and hence the shear obtained by adding up the forces on either side of any section is the maximum for that section, but in the case of live load the maximum shear is not so readily obtained for the shear on every vertical section of a beam changes continually as such loads move over the beam, and hence the maximum shear will occur when the load is in one certain position. As an example of live load, let AB (Fig. 109) represent a simple beam supporting a single moving load P, and let R and $R1$ represent the reactions at A and B, respectively, due to this load when at any point on the beam.

The shear on any short strip through the beam, as $abcd$, due to P, can be expressed as

$$S = \frac{Px}{L} = R,$$

when the load is at any point to the right of the strip. The shearing force exerted upon the strip will then act as indicated. It is readily seen that this shear on the strip varies directly as x and hence will be a maximum when $x = m$. Then if we lay off the vertical line gh equal (by scale) to Pm/L and draw the line hk, any ordinate to this line hk graphically represents the shear on the strip $abcd$ when the load is directly over the ordinate. For example, the ordinate y represents the shear on the strip when the load is in the position shown, and any other ordinate as x represents the shear on the strip when the load is over that ordinate.

When the load is at any point to the left of the strip, the shear on the strip can be expressed as

$$S' = \frac{Px}{L} - P = (R - P),$$

but the shearing forces exerted upon the strip will then act in the opposite direction to those shown. This change in the direction of action of the shearing forces takes place the instant the load passes the strip. We would say that the shear changes signs as the load passes the strip and that the shear on the strip is positive when the load is on one side, and negative when on the other.

It is seen, from the last equation, that S' will be a maximum when R has its least value. This, as is seen, occurs when the load is just to the left of the strip, that is, when $x = m + dx$, but, as dx can be taken as an infinitesimal, we would say when $x = m$. Then if we lay off the vertical line fg equal to $(Pm/L) - P = (R - P)$ and draw the line fe we have the shear on the strip graphically represented for the load P at any point to the left of the strip, as any ordinate r, to the line fe, represents the shear on the strip when the load is over that ordinate. Then we have the variation of the shear on the strip $abcd$ due to the load P, as it moves over the beam from B to A, fully represented by the ordinates to the broken line $efhk$. It is seen from this that the loads on the two sides of any vertical section of a beam really neutralize each other as regards the shear they produce on the section, and hence it appears that the maximum shear would occur when the loads are on one side only and extending on that side from the end of the beam up to the section. This is absolutely true in the case of a uniform live load, as shown in Art. 55, and is practically true for practically all classes of loading, yet it is not absolutely true in some cases of concentrated live loads, for sometimes the maximum shear on a vertical section of a beam, due to such loads, occurs when the loading extends from one end of the beam to a little beyond the section considered. The most common instance of this is that of the typical system of locomotive wheel loads. In such cases the exact position required to produce maximum shear must be determined by trial.

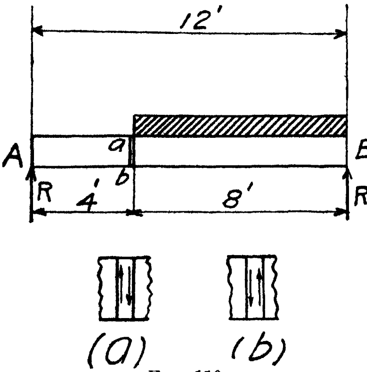


Fig 110

As a general case of uniform load, let it be required to determine the maximum positive and negative shear on a beam 12 ft. long at a point 4 ft. from the end due to a uniform dead load of 100 lbs. per foot and a uniform live load of 1,500 lbs. per foot. Let AB (Fig. 110) represent the beam, and let ba represent the section at which the shear is required.

As the dead load is symmetrically located in reference to the center of the beam, the two reactions due to dead load are equal to each other, and each is equal to $6 \times 100 = 600$ lbs. Then for the dead-load shear at the section ba, adding up from A and taking R ($=600$) as positive, we have

$$600-(4\times 100)=+200 \text{ lbs.}$$

When the uniform live load extends from the end B up to the section, as shown, one of the maximum live-load shears will occur. Taking moments about B when the load is in this position, we have

$$R=\frac{1,500\times 8\times 4}{12}=+4,000 \text{ lbs.}$$

for the live-load reaction at A. Then for the shear due to this load at the section ba, adding up from A, we have $+4,000$ lbs. as there is no intervening live load. This is the maximum positive shear at the section ba due to the live load. It will be readily seen that the dead load and the live load, where the live load extends from B to the section ba, both exert shearing forces at the section which act in the direction shown at (a), hence the two will add, and we have $200+4,000=+4,200$ lbs. for the maximum positive shear at the section ba, due to the dead and live load combined. Now if the live load extends from A to the section ba instead of from B, the reaction at A, due to the live load in that position, would be

$$R=\frac{1,500\times 4\times 10}{12}=5,000 \text{ lbs.}$$

Then for the shear at the section ba due to this load, adding up from A, we have $5,000-6,000=-1,000$ lbs. which is the maximum negative shear at the section due to the live load.

It will be readily seen that the live load when extending from A to the section ba will exert shearing forces at the section which will act as shown at (b), and as this direction is opposite to the direction of action of the shearing forces exerted by the dead load, the shear at the section, due to dead and live load combined, will undoubtedly be equal to the difference of the two shears. So we have $200-1{,}000=-800$ lbs. for the maximum negative shear at the section ba due to dead and live load combined.

Now it is seen that in the above example the maximum positive and negative shear at the section ba, due to dead and live load combined, is simply the algebraic summation of the two simultaneous shears in each case. This will hold in all cases provided the adding up of the forces be in reference to the same end of the beam in each case. As an illustration, let us add up the forces in the above example from the end B instead of from A, as we did above. The reaction at B, due to the dead load, is 600 lbs. Then for the dead-load shear at the section ab, adding up from B, we have, $600-8\times 100=-200$ lbs.

For the live-load reaction at B, when the live load extends from B to the section ba, we have

$$R1=\frac{1{,}500\times 8\times 8}{12}=8{,}000 \text{ lbs.}$$

Then for the shear at the section, adding up from B, we have $8{,}000-12{,}000=-4{,}000$ lbs. Adding this to the dead-load shear, we have $-200-4{,}000=-4{,}200$ lbs., which is the same as we obtained by adding up from A, except the sign is negative instead of positive. If the live load extends from A to the section ba instead of from B to the section, we have

$$R1=\frac{1{,}500\times 4\times 2}{12}=1{,}000 \text{ lbs.}$$

for the reaction at B due to the live load. Then for the shear at the section ba, adding up from B, we have $+1{,}000$ lbs. Now adding this to the dead load, as just determined by adding up the forces from B, we have $-200+1{,}000=+800$ lbs., which is the same as we obtained above by adding up from A, except that the sign is positive instead of negative.

Thus we see that by adding up the forces from one end we obtain positive shear and by adding up the forces from the other end we obtain negative shear. There is no mystery about this at all, as is readily perceived by referring to Fig. 111 where AB represents a simple beam supporting a number of loads, and R and $R1$ represent the reaction at A and B, respectively, due to the loads, and $abcd$ represents a very short strip through the beam. If we take the reactions as positive, then any force acting upward will be positive. By adding up the forces from A to the strip $abcd$ we really obtain the shearing force acting upon the strip along ab and by adding up the forces from B to the strip we really obtain the shearing force acting upon the strip along dc. As these two shearing forces on any vertical section of a beam are always equal and opposite, it is evident that the sum obtained by adding up the forces from one end will be positive, while the equal sum obtained by adding up the forces from the other end will be negative.

There will be no confusion as regards the sign of the shear on any section of a beam in any case, providing the adding up of the forces, that is, the algebraic summation, be started from the same support.

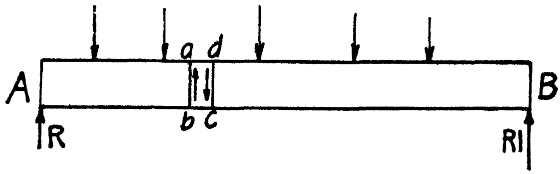


Fig. 111

88. Maximum Bending Moments on Simple Beams.—The bending moment at any cross-section of a beam, in all cases, is equal to the algebraic sum of the moments about the section, of the forces on either side of the section. In case the load be uniformly distributed over the entire length of the beam, the maximum moment will occur at the center of the span, and will be equal to $wL^2/8$ as shown in Art. 56. This is true for all uniformly distributed loads, either live or dead. In the case of fixed concentrated loads, the maximum moment will occur under one of the loads which can be ascertained readily by trial. However, it can be determined otherwise as shown later.

If the live load be wheel loads, it is always necessary to first determine the position of the loads on the beam for maximum moment. As wheel loads roll over a beam it is evident that the moment at every section is continually changing, and it is the absolute maximum of these various moments produced that we desire, although it may last only for an instant during the passage of the load.

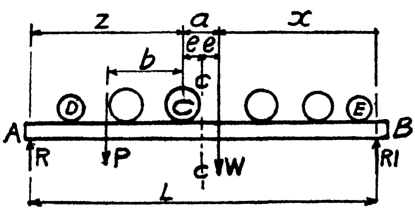


Fig. 112

Let AB (Fig. 112) represent a simple beam supporting a system of wheel loads. According to Art. 56, the maximum moment will occur under a wheel. Let this wheel be the one marked C. Let W be the weight of all the wheels on the beam, and x the distance from the right support to their center of gravity. Further, let P be the weight of the wheels to the left of wheel C, and b the distance of their center of gravity from wheel C, and let z be the distance from the left support to wheel C. Then the bending moment under wheel C can be expressed as

$$M=\frac{(Wx)z}{L}-Pb=(Rz-Pb) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Now from Fig. 112 it is seen that $x=L-a-z$. Substituting this value of x in (1), we have

$$M=\frac{W}{L}(zL-za-z^2)-Pb \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Now any slight movement of the loads either to the right or left will cause a small change in z and M. Let dz and dM represent this increment of z and M, respectively, due to a slight movement of the loads. Adding the increments in equation (2), we have

$$M+dM=\frac{W}{L}\left[(z+dz)L-(z+dz)a-(z+dz)^2\right]-Pb,$$

and reducing we have

$$M+dM=\frac{W}{L}\left[zL-az-z^2\right]+\frac{W}{L}\left[dzL-adz-2zdz\right]-Pb.$$

Now subtracting (2) from this last equation and reducing we obtain

$$dM=Wdz-\frac{W}{L}adz-\frac{W}{L}2zdz \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

which is undoubtedly the expression for the increment of the bending moment under wheel C due to any slight movement of the loads. It is readily seen that if wheel C is to the right of the point of maximum moment any slight movement of the loads to the left will increase the moment M by the amount dM—in other words, dM will be positive—and that all such movements of the loads to the left will increase M the amount dM until wheel C arrives at the point of maximum moment and from there on any slight movement of the loads to the left will decrease M by the amount dM; in other words, dM is negative in that case. Evidently the increment dM is zero just as wheel C arrives at the point of maximum moment as it is positive when the wheel is just to the right of the point of maximum moment and negative when to the left and hence changes signs at the point. So we have from (3), when M is a maximum,

$$dM=Wdz-\frac{W}{L}adz-\frac{W}{L}2zdz=0.$$

Reducing, we obtain

$$z=\frac{L}{2}-\frac{a}{2}$$

which is the value of z when the maximum bending moment occurs under wheel C. This last equation shows that the center of the beam bisects the distance a. From this we see, that *the point of maximum moment and the center of gravity of all the wheels are equidistant from the center of the beam and on opposite sides of the center.* The maximum moment will occur under one of the two wheels adjacent to the center of gravity, practically always under the wheel nearer the center of gravity. So, as a rule, all we have to do to locate a system of wheels on a beam for maximum moment is to find their center of gravity and then place the center of the beam half way between this center of gravity and the wheel nearest to it. For example, take the case shown in Fig. 112. By taking moments about either of the end wheels, either wheel D or E, we can determine the center of gravity of all the wheels which comes nearest to wheel C and hence the center of the beam will come half way between the center of gravity and this wheel. In any case where the center of gravity comes near half way between two wheels, the moment under each of the wheels should be determined in order to ascertain which is the maximum.

Usually the only bending moment used in designing beams is the absolute maximum referred to above, yet there are a few cases where the maximum at other points, other than the point where the absolute maximum occurs, is needed, and which will occur when the wheels are in one certain position in reference to the point considered.

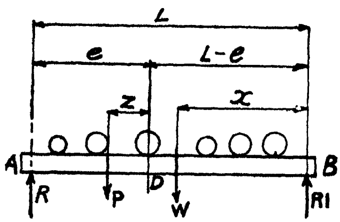


Fig. 113

Let AB (Fig. 113) represent a simple beam supporting a system of wheel loads as shown, and let D be a point distance e from A where the maximum bending moment, due to these wheels as they move over the beam, is desired. Let W represent the weight of all of the wheels on the beam and x the distance of their center of gravity from B. Further, let P represent the weight of the wheels to the left of point D and z the distance of their center of gravity from D.

According to Art. 56, the maximum moment at D will occur when a wheel is at that point. So let a wheel be at the point as shown. Now taking moments about D and considering the forces to the left, we have

$$M = \frac{W}{L} ex - Pz = (Re - Pz) \quad \dots\dots\dots\dots (1)$$

for the bending moment at that point. Now, if the wheels are in the position for maximum moment at D, the increment of the moment that would be caused by the wheels moving to the right or left an infinitesimal distance would be equal to zero. Then assuming we have such a movement, M, x, and z will have corresponding increments which we will designate, respectively, as dM, dx, and dz. Now adding these in equation (1), we have

$$M + dM = \frac{W}{L} ex + \frac{W}{L} edx - Pz - Pdz,$$

and subtracting equation (1), we have

$$dM = \frac{W}{L} edx - Pdz.$$

But, as is readily seen, $dz = dx$, so we have

$$dM = \frac{W}{L} edx - Pdx$$

for the increment of the bending moment, which will be equal to zero when the loads are in the position for maximum moment at D. Then we have

$$dM = \frac{W}{L} edx - Pdx = 0.$$

Reducing, we have

$$\frac{W}{L} = \frac{P}{e} \quad \dots\dots\dots\dots (2).$$

In the last equation we have the total load on the beam divided by the length of the beam equal to the load on the left of the point D divided by the distance from the point to the left support. That is, we have the average unit load on the beam equal to the average unit load to the left of the point. *Then, to obtain the maximum moment at any point in a simple beam, due to wheel loads, the loads must be so placed that there will be a load at the point considered and at the same time the average unit load on the entire beam will be equal to the average unit load to the left of the point.*

It is shown, in Art. 59, that the first derivative of the bending moment, in terms of x, at any section of a beam, is equal to the shear. But at the point of maximum moment the first derivative of the bending moment will be equal to zero. So, it follows that the shear at the point of maximum bending moment will be zero. Then, as the point of maximum bending moment is at the point of zero shear, or where the shear changes signs, which is the same thing, the point of maximum moment can be obtained by adding up the forces from one end of the beam until the point where the shear is zero, or changes signs, is reached, and this point will be the point of maximum bending moment. This method of determining the point of maximum moment is quite convenient in the case of fixed loads, especially where the loads are mixed, uniform, and concentrated.

As an example, let it be required to determine the point of maximum bending moment on the beam shown in Fig. 114, due to the concentrated and uniform load indicated. After determining the reactions shown at A and B, by taking moments about the supports, we can begin at one end of the beam and ascertain the point of zero shear by adding up the forces

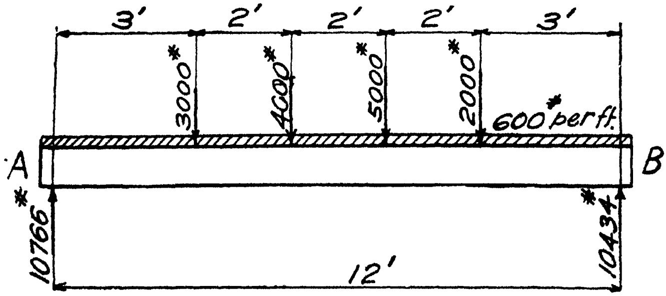


Fig. 114

and noting the sign of the shear as we progress. Thus, beginning at A, we have $10,766 - 1,800 - 3,000 = +5,966$ lbs. for the shear just to the right of the 3,000-pound load, and $10,766 - 3,000 - 7,000 = +766$ lbs. for the shear just to the right of the 4,000-pound load. This last shear shows us that the point of zero shear is just a little to the right of the 4,000-pound load. Let x be the distance. Then we have

$$600\,x = 766,$$

from which we obtain

$$x = \frac{766}{600} = 1.27 \text{ ft.}$$

So the point of maximum moment is 1.27 ft. to the right of the 4,000-pound load. If the 4,000-pound load were something over 766 pounds heavier, it is evident that the point of maximum moment would be under that load. For example, suppose that load weighed 6,000 pounds instead of 4,000, then, all the other loads remaining the same, the reaction at A would be 11,932 lbs. instead of 10,766. Adding up from A we have $11{,}932 - 3{,}000 - 3{,}000 = +5{,}932$ lbs. for the shear just to the left of the second load from A, which is now 6,000 lbs., and $11{,}932 - 3{,}000 - 3{,}000 - 6{,}000 = -68$ lbs. for the shear just to the right of the load. As the shear is positive on one side of the load and negative on the other, the shear undoubtedly passes through zero at the load, and hence the point of maximum moment is at the load.

89. Maximum Reaction on Simple Trusses.— The loads on trusses are transmitted to the joints, either directly or indirectly; directly when applied at the joints, and indirectly when applied between the joints, that is, in the panels. As an example, let the diagram in Fig. 115

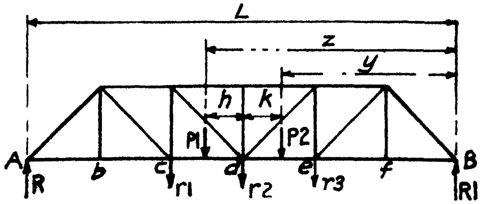


Fig. 115

represent a truss of length L supporting two loads, $P1$ and $P2$, as shown. These loads, as is readily seen, would be transmitted to the joints c, d, and e. Let $r1$, $r2$, and $r3$ represent the amount transmitted to each as indicated. These would be known as concentrations or panel loads. The intensity of $r1$ and $r3$ can be determined by taking moments about d and treating cd and de as simple beams. Thus we have

$$r1 = \frac{hP1}{cd} \text{ and } r3 = \frac{kP2}{de}$$

By taking moments about e we can determine the concentration at d due to $P2$, and by taking moments about c we can determine the concentration at d due to $P1$. Then, by adding these two concentrations together, we obtain the concentration $r2$. Thus we have

$$r2 = \left(\frac{cd - h}{cd}\right)P1 + \left(\frac{de - k}{de}\right)P2.$$

In this manner the concentrations on the joints of any truss, loaded in any manner, can be determined. As the concentrations are really components of the loads, either may be used in determining reactions on trusses. Thus, for the reaction at A (Fig. 115), due to the two loads $P1$ and $P2$, we have

$$R = \frac{zP1 + yP2}{L} \text{ or } R = \frac{(Be)r3 + (Bd)r2 + (Bc)r1.}{L}$$

As a rule, the loads are used instead of the concentrations (owing to convenience), in which case any concentration at the end point due to loads in the end panel must always be subtracted from the reaction found. For instance, if there were loads in the panel Ab, we would take moments about b and determine the concentration at A due to the loads in that panel, and subtract this from the reaction found at A by taking moments of all the loads about B. Other than this one step, the determination of reactions on trusses is exactly the same as for beams.

Dead load (which is usually taken as a uniform load) and uniform live load are always considered as concentrated at the joints. In such cases we deal only with joint loads, or panel loads, which is the same thing. If the panels be of equal length, the panel loads will all be equal, and in such cases the reactions can be determined practically by inspection. For example, let the diagram in Fig. 116 represent a truss of 6

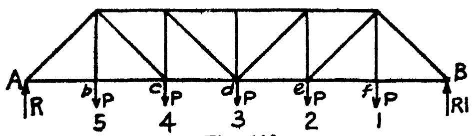


Fig. 116

equal panels supporting a uniform load of w pounds per foot of truss. Let L be the length of the truss and d the length of each panel. If the load extends over the full length of the span there would be five panel loads, represented as P, each equal to wd. In that case we can readily see that each reaction would be equal to $2\frac{1}{2}$ P. This, of course, is mere inspection. In case the panel loads are unequal or some of the joints not loaded, we can obtain the reactions by simply taking moments about the supports just the same as in the case of beams, except as stated above, but if the panels are of equal length it is more convenient to deal with panel lengths instead of feet. For example, to obtain the reaction at A due to a load at f, we would take moments about B and the load at f would be multiplied by d and divided by $6d$, and hence the reaction at A would be $\frac{1}{6}$ of the load. Similarly a load at e would be multiplied by $2d$ and divided by $6d$, and hence the reaction at A, due to this load, would be $\frac{2}{6}$ of the load at e, and likewise the reaction at A, due to a load at d, would be $\frac{3}{6}$ of the load at d, and $\frac{4}{6}$ and $\frac{5}{6}$ for a load at c and b, respectively. From this it is seen that the reactions on trusses of equal panels can be obtained directly by numbering the panels 1, 2, 3, etc., as shown (just below the diagram) in Fig. 116, and multiplying the load on each joint by the number under it, and then adding all the results together and dividing by the number of panels in the truss. If the reaction at the other support be desired, the numbers would simply be reversed in order. If the panel loads are all equal, the work will be shortened, of course, as the numbers under all the joints loaded can be added together and multiplied by one panel load, divided by the number of panels. As an example, suppose the joints f, e, d, and c each support a load P. The reaction at A due to these four loads would be

$$R=\frac{P}{6}(1+2+3+4)=\frac{P}{6}10.$$

If the joint b were loaded also, we would have

$$R=\frac{P}{6}(1+2+3+4+5)=\frac{P}{6}15.$$

If joints d, c, and b alone were loaded, we would have

$$R=\frac{P}{6}(2+4+5)=\frac{P}{6}\ 11.$$

If joint d alone were loaded, we would have

$$R=\frac{P}{6}\ (4)=\frac{P}{6}4,$$

and so on. In general, the maximum reaction on a truss, the same as a beam, will occur when the span is fully loaded and the heaviest loads are near the support considered. Specific cases will be taken up later.

90. Maximum Shear on Simple Trusses.—The determination of the shear on trusses is the same as that of beams, except the loads are considered to be applied to trusses only at the panel points, or joints. Let the diagram in Fig. 117 represent a simple truss supporting a load P at each bottom joint, as indicated, and let R and $R1$ represent the reactions due to these loads. Then the shear in the first panel from the left end is R; in the second it is $R-P$; in the third it is $R-2P$; in the fourth it is $R-3P$; and so on. This case is that of dead load or a uniform live

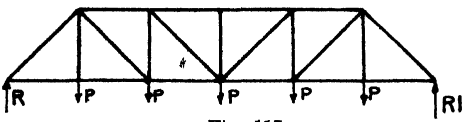


Fig. 117

load extending all the way across the span. In case of dead load, this shear would be a maximum. It is customary to assume that a uniform live load moving over a truss increases by panel loads, in which case the maximum shear in any panel will occur when the joints from one end of the truss up to the panel considered are loaded. By loading from one end we obtain, as we may say, the maximum positive shear, and loading from the other end we obtain the maximum negative shear. For example, one maximum shear would occur in the panel bc of the truss shown in Fig. 118, when the joints e, d, and c alone are loaded, and the other maximum would occur when the joints a and b alone are loaded. Instead of considering the load to move on from the two directions, we usually consider it to move on from only one direction, and by determining the shear in each panel as the load reaches it we obtain all the shears we need, as the positive shears are determined in the panels on one side of the center of the truss while the negative shears are determined in the corresponding panels on the other side of the center.

As the maximum shear in any panel, due to a uniform live load, occurs when the load just reaches that panel, it is readily seen that this shear, in all cases, will be equal to the reaction at the unloaded end of the truss. Thus, the maximum shear in panel ed (Fig. 118) due to a uniform live load moving on from the right, will be equal to the reaction at A due to the load at e; and, similarly, the maximum shear in the panel dc will be equal to the reaction at A due to the loads at e and d; and the maximum shear in panel cb will be equal to the reaction at A due to the loads at e, d, and c; and so on for the other panels. Now if the panels be of equal length, these reactions can be determined by numbering the joints 1, 2, 3, etc., and adding these together for the loaded joints and multiplying the sum by one panel load divided by the total number of panels in the truss, as explained in the last article. As an example, let the diagram in Fig. 118 represent a truss of six equal panels, each of length d (the

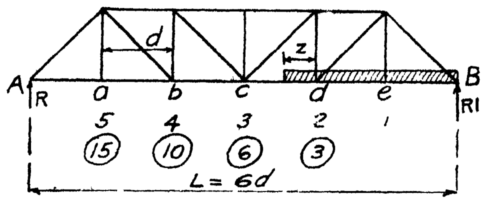


Fig. 118

length of the span then will be $L = 6d$), and let it be required to determine the maximum shear in each panel due to a uniform live load of w pounds per foot of truss moving over it from right to left. Let W ($= wd$) represent the panel load. Then numbering the joints 1, 2, 3, etc., from right to left, as shown, we have

$$\frac{W}{6}(1)$$

for the maximum shear in the panel ed, when joint e alone is loaded,

$$\frac{W}{6}(1+2)$$

for panel dc when joints e and d are loaded,

$$\frac{W}{6}(1+2+3)$$

for panel cb when joints e, d, and c are loaded,

$$\frac{W}{6}(1+2+3+4)$$

for panel ba when joints e, d, c, and b are loaded, and

$$\frac{W}{6}(1+2+3+4+5)$$

for panel aA when joints e, d, c, b, and a are loaded.

From this it is seen that the maximum shear on any truss of equal panels, due to a uniform live load moving over the truss, can be expressed by the general formula,

$$\frac{W}{n}[1+2+3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots+(n-1)],$$

where n is the number of panels in the truss considered.

It will be found quite convenient to write the summation of the numbers in the parentheses under the corresponding panel points as shown in Fig. 118 where the incircled numbers are the respective summations. These summations, as is readily seen, can be made in any case by simply referring directly to the diagram of the truss considered. Then the shear in the panels can be read off of a slide rule directly as,

$$\frac{W}{n}(1), \quad \frac{W}{n}(10),$$
$$\frac{W}{n}(3), \quad \frac{W}{n}(15),$$
$$\frac{W}{n}(6),$$

and so on.

In the case of uniform live load, just considered, we usually assume, as stated above, that the load is applied to the joints of a truss in maximum panel loads. This is not possible, however, as such loads are continuous and must necessarily extend beyond the last loaded joint in order to fully load it. For example, to fully load the joints e and d (Fig. 118) the uniform live load would really have to extend from B to the joint c, as is readily seen. However, as will be shown later, the maximum shear would occur in panel dc when the load extended a short distance z beyond joint d. In that case there would be less than a full panel load at d and a small concentration at c due to the load in the panel dc, and the shear in the panel dc would really be equal to the reaction at A minus the concentration at c. The error resulting from the assumption stated above is small in the case of a uniform live load, as will be seen later, and hence the assumption in that case is permissible, but in the case of wheel loads no such assumption can be made as any slight shifting of the loads may materially change the shear in a panel and consequently it is necessary for us to determine the exact position of such loads for maximum shear.

As a general case, let the diagram in Fig. 119 represent a truss

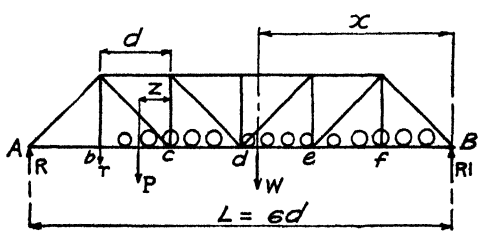


Fig. 119

supporting a system of wheel loads. Now let it be required to determine the position of these wheels for maximum shear in panel bc. Let W be the weight of all the wheels and x the distance from B to their center of gravity, and let P be the weight of the loads in the panel bc, and z the distance from c to their center of gravity. Now if R be the reaction at A due to W, and r the concentration at b due to P, the shear in the panel bc can be expressed as

$$S=R-r \quad \text{.......................................} \quad (1).$$

But $$R=\frac{Wx}{L} \text{ and } r=\frac{Pz}{d}$$

Then substituting these values, we have

$$S=\frac{Wx}{L}-\frac{Pz}{d} \quad \text{.......................................} \quad (2).$$

With P and W constant, any shifting of the wheels to the right or left will affect R and r, not the same, but similarly. The weight of P could be such that any movement to the left would increase the value of R more than r. Then, evidently, any such movements to the left would continue to increase the shear S until another load rolled past c into the panel bc, when the rate of increase of r would be greater than just before the load passed joint c; but R would continue to increase all the while, and, consequently, the shear S would continue to increase, unless the load P were increased so much by the additional load that r would be increased at a greater rate than R for each movement to the left. So it is seen that the shear S will be a maximum when the total load in the panel bc is such that the increment of R is equal to the increment of r. That is, any shifting of the loads with W and P constant (for that instant) would not affect the shear S in panel bc; or, in other words, the increment of the shear would be zero. Now, suppose the loads move an infinitesimal distance to the left; then equation (2) would become

$$S+dS=\frac{Wx}{L}+\frac{Wdx}{L}-\frac{Pz}{d}-\frac{Pdz}{d}\,(dz=dx),$$

and subtracting (2) from this, we have

$$dS=\frac{Wdx}{L}-\frac{Pdx}{d} \quad \text{.......................................} \quad (3),$$

which is the increment of the shear S in panel bc due to the movement. But this increment will be zero when S is a maximum, and then we would have

$$dS=\frac{Wdx}{L}-\frac{Pdx}{d}=0 \quad \text{.......................................} \quad (4).$$

It is readily seen that this occurs when $W/L=P/d$, that is, the shear in panel bc is a maximum when the average unit load on the truss is equal to the average unit load in the panel. This will hold in the case of any truss of the type shown as the case being considered is general. In the case of a truss of n equal panels, each of length d, we have $L=nd$. Substituting this value of L in the last equation, and reducing, we have

$$P=\frac{W}{n} \quad \text{.......................................} \quad (5).$$

That is, the shear in any panel of a truss of n equal panels will be a maximum when the load in the panel is equal to the total load on the truss divided by the number of panels in the truss. This will hold for any class of live load either concentrated or uniform.

It is readily seen that the increment of the shear in panel bc, expressed by equation (3), will pass through zero only as a load passes joint c, so there will be a wheel at joint c when the maximum shear in panel bc occurs. Thus it is in all cases—a wheel will be at the joint on the loaded side of the panel considered when the maximum shear in the panel occurs.

91. Maximum Bending Moments on Simple Trusses.—In the case of bending moments on trusses, the moments are usually taken about the joints only, otherwise the case is the same as that of beams. As a general case of dead load or uniform live load, let the diagram in Fig. 120 represent a truss of six panels supporting a load at each bottom joint as

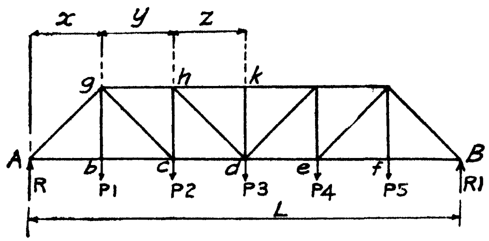


Fig 120

indicated. Let R and $R1$ represent the reaction at A and B, respectively, due to these loads. Then the bending moment at joint b or g is $M' = Rx$; at joint c or h it is $M'' = R(x+y) - P1y$; at joint k or d it is $M''' = R(x+y+z) - P1(y+z) - P2(z)$; and so on.

In the case of uniform live load, the maximum moment at any joint will occur when the load extends the full length of the truss, and hence the moment at all joints, the same as in the case of dead load, is a maximum. In the case of wheel loads it is necessary to determine the exact position of the wheels for the maximum bending moment at each joint. Let the diagram in Fig. 121 represent a truss of six panels supporting a system of wheel loads, as indicated. Now let it be required to determine the position of these wheels for maximum bending moment at joint d.

Let W be the weight of all the wheels on the truss, and x the distance from B to their center of gravity, and let P be the weight of the

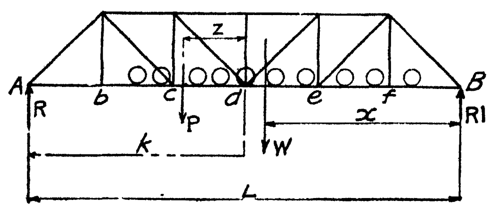


Fig. 121

wheels to the left of joint d and z the distance from d to their center of gravity, and also let k be the distance from A to d and R the reaction at A due to all the wheels when in any position. Then the bending moment at d can be expressed as

$$M = Rk - Pz \quad \text{...} \quad (1).$$

But
$$R=\frac{Wx}{L}$$

so, substituting this value of R, we have

$$M=k\frac{Wx}{L}-Pz \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Now as the loads move to the left, R will be increased, and likewise the moment M, until P becomes so great that the rate of increase of Pz is greater than the rate of increase of $(Wx/L)k$; then, evidently, the moment M at d will be a maximum when its increment is zero, which, as is readily seen, will occur when the increment of $(Wx/L)k$ is equal to the increment of Pz. Now suppose the loads move an infinitesimal distance to the left, then equation (2) becomes

$$M+dM=k\frac{Wx}{L}+k\frac{Wdx}{L}-Pz-Pdx$$

Then subtracting (2) we have

$$dM=k\frac{Wdx}{L}-Pdx \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3),$$

which is the increment of the bending moment at d due to the movement. But this increment will be equal to zero when M is a maximum, and hence we would have

$$dM=k\frac{Wdx}{L}-Pdx=0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4).$$

It is readily seen that this occurs when

$$\frac{W}{L}=\frac{P}{k} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5),$$

that is, the bending moment at joint d is a maximum when the average unit load on the truss is equal to the average unit load on the left of the joint d. Now, as the above is a general case, we have: *The maximum bending at any joint of a truss of the type shown will occur when the average unit load on the truss is equal to the average unit load to the left of the joint considered.* It will be seen that this is the same as found for beams in Art. 88.

92. Stresses in Trusses.—As the loads supported by trusses are applied at the joints, the stress produced in each member is simple stress, and consequently the unit stress in any member is equal to the total stress divided by the area of its cross section. The stresses in the individual members can readily be obtained by resorting to the resolution of forces and to the equation of moments.

As an example, let the diagram at (a) in Fig. 122 represent a truss of six equal panels, supporting a load at each lower joint as indicated. Let d be the length of each panel, h the height of the truss center to center of chords, L the total length of span center to center of end bearings, and let R and $R1$ represent the reactions as indicated. To obtain the stress in the end post $LOU1$ and in the bottom chord $LOL1$, let S and $S1$ be the stress in each, respectively, and suppose the two members cut off along the section mm, and imagine the part of the truss to the left of this section moved bodily to (b) without any stresses or forces being affected in the least. Then we simply have an independent

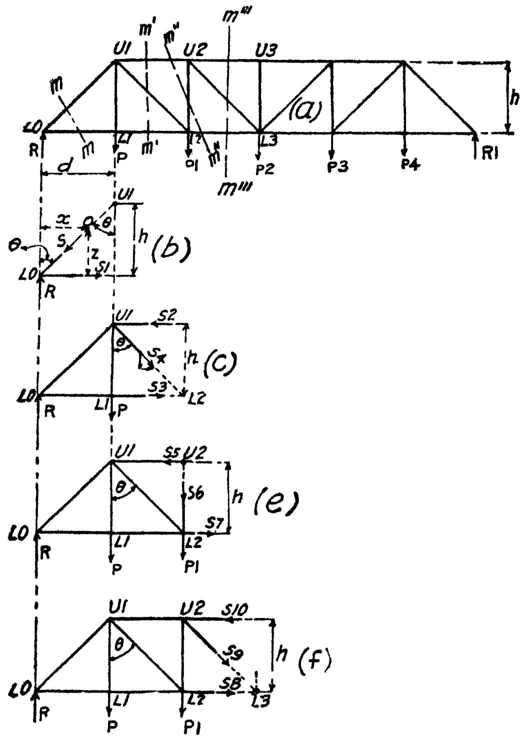


Fig. 122

structure at (b) held in equilibrium by the three external forces R, S, and $S1$—the stresses S and $S1$ being unknown. Now, as these three forces are in equilibrium, the algebraic sum of their components along any direction is equal to zero. So resolving the forces vertically, we have

$$R - S\cos\theta = 0 \quad \ldots\ldots (1),$$

from which we obtain

$$S = R/\cos\theta = R\sec\theta \quad \ldots\ldots (2);$$

and resolving the forces horizontally, we have

$$S1 - S\sin\theta = 0 \quad \ldots\ldots (3),$$

from which we obtain

$$S1 = S\sin\theta \quad \ldots\ldots (4).$$

But, according to (2), $S = R/\cos\theta$. Then, substituting in (4), we have

$$S1 = R\frac{\sin\theta}{\cos\theta} = R\tan\theta \qquad (5).$$

Equation (2) shows that the stress S in $LOU1$ is equal to the shear ($=R$) in the end panel multiplied by the secant of the angle θ which the member makes with the vertical; and equation (5) shows that the stress $S1$ in $LOL1$ is equal to the shear in the end panel multiplied by the tangent of angle θ. It should be observed that R has no horizontal component while $S1$ has no vertical—the cosine of 90° being zero.

Again referring to the figure at (b), by taking moments about any point O on the line of action of S, we have

$$Rx - zS1 = 0,$$

from which we obtain

$$S1 = R\frac{x}{z}.$$

But it is obvious that it is more convenient to take moments about the panel point $U1$, for in that case the lever arms are directly known, and we have

$$Rd - hS1 = 0,$$

from which we obtain

$$S1 = R\frac{d}{h} = R\tan\theta \qquad (6).$$

The stress S could be computed by taking moments about a point in the line of action of the stress $S1$, but it is obvious that S can be computed more readily from equation (2).

It is also obvious that the stress in the hanger $U1L1$ is equal to the load P which it supports directly at $L1$.

To obtain the stress in the chords $U1U2$, $L1L2$, and diagonal $U1L2$, let $S2$, $S3$, and $S4$ be the stress in each, respectively, and suppose the three members cut off along the section $m'm'$, and imagine the part of the truss to the left of this section moved bodily to (c) without any stresses or forces being affected in the least. Then we have an independent structure at (c) held in equilibrium by the external forces R, P, $S2$, $S3$, and $S4$—the stresses $S2$, $S3$, and $S4$ being unknown.

As $S2$ and $S4$ intersect at $U1$, $S3$ can be determined by taking moments about that point. So taking $U1$ as the center of moments we have

$$Rd - hS3 = 0,$$

from which we obtain

$$S3 = R\frac{d}{h} \qquad (7),$$

which is the same as we obtained for the stress in $LOL1$ in equation (6). As $S4$ and $S3$ intersect at $L2$, $S2$ can be determined by taking moments about $L2$. Therefore, taking $L2$ as the center of moments, we have

$$2dR - dP - hS2 = 0,$$

from which we obtain

$$S2 = 2\frac{d}{h}R - \frac{d}{h}P \quad \text{.......................... (8).}$$

$S4$, the stress in the diagonal $U1L2$, can be determined most readily from the algebraic sum of the vertical components of the five forces, for which we have

$$R - P - S4\cos\theta = 0,$$

from which we obtain

$$S4 = \frac{(R-P)}{\cos\theta} = (R-P)\sec\theta \quad \text{.......................... (9).}$$

This shows that the stress $S4$ in the diagonal $U1L2$ is equal to the shear in the panel $L1L2$ multiplied by the secant of angle θ.

To obtain the stress in the vertical post $U2L2$, let $S6$ be the stress, and suppose the three members $U1U2$, $U2L2$, and $L2L3$ be cut off along the section $m''m''$, and imagine the part of the truss to the left of this section moved bodily to (e) without any forces or stresses being affected in the least. Then we have an independent structure at (e) held in equilibrium by six external forces—R, $S7$, $S6$, $S5$, P, and $P1$. The stresses $S7$, $S6$, and $S5$ may all be unknown. As $S7$ and $S5$ have no vertical components (being horizontal members), $S6$ can be determined directly from the algebraic sum of the vertical components of all the forces, for which we have

$$R - 2P - S6 = 0,$$

from which we obtain

$$S6 = R - 2P \quad \text{.......................... (10).}$$

This shows that $S6$ is equal to the shear in the panel $L2L3$.

To obtain the stress in the chords $U2U3$, $L2L3$, and the diagonal $U2L3$, let $S10$, $S8$, and $S9$ be the stress in each, respectively, and suppose the three members cut off along the section $m'''m'''$, and imagine the part of the truss to the left of this section moved bodily to (f) without any forces or stresses being affected in the least. Then we have an independent structure at (f) held in equilibrium by six forces, R, $S10$, $S9$, $S8$, P, and $P1$, the stresses $S10$, $S9$, and $S8$ being unknown. Taking moments about $U2$, we have

$$2dR - dP - hS8 = 0,$$

from which we obtain

$$S8 = \frac{2dR - dP}{h} \quad \text{.......................... (11).}$$

Then taking moments about $L3$, we have

$$3dR - 2dP - dP1 - hS10 = 0,$$

from which we obtain

$$S10 = \frac{d}{h}(3R - 2P - P1) \quad \text{.......................... (12).}$$

Resolving the forces vertically, we have

$$R - P - P1 - S9\cos\theta = 0,$$

from which we obtain

$$S9 = \frac{R - P - P1}{\cos\theta} = \sec\theta(R - P - P1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13).$$

The above loads do not produce any stress in the post $U3L3$. This member could be omitted as far as loads applied at the lower panel points are concerned, but it will support directly any load applied at $U3$.

Stress in the members to the right of $U3L3$ can be determined in the same manner as shown above for the members to the left.

The above shows how readily the stresses in a truss can be determined by considering portions of the truss as independent structures, the portion being selected each time to suit the case considered; the portion may be a single joint or several panels of a structure. This manner of treatment is known as the "Method of Section." It will apply in the case of any truss and may be carried to any desired extent, but after one becomes familiar with a certain type of truss, the analysis is often made without any thought of the method of section, as will be seen later. However, the method is really implied.

Problem 1. Determine the stresses in the members of the truss shown at (*a*), Fig. 120, due to a load of 20,000 lbs., on each lower panel point, assuming the length of each panel to be 25′-0″, and the height of the truss 30′-0″, center to center of chords.

CHAPTER VIII

GRAPHIC STATICS

93. Definition and Limitation of Graphic Statics.—Graphic Statics is that part of Mechanics which treats of the graphical determination of static forces. It results from the fact that forces acting upon a body in equilibrium will form a closed polygon (see Art. 42) wherein all the forces act in the same direction around the polygon. This knowledge applied to geometrical construction constitutes the method.

A force is fully known when its point of application, its intensity, the direction and location of its line of action, and its direction of action (along the line of action) are known.

The determination of the point of application does not come within the scope of Graphic Statics, for it is one of the many things that can be ascertained only from knowledge of actual conditions, and consequently is a presupposed knowledge in graphic problems. The intensity, direction, and location of the line of action and the direction of action can be graphically determined. However, the method is practically limited to the two following cases:

Case I. The *intensity and direction of action of two forces* can be graphically determined provided all of the other forces acting upon the same body are fully known and the lines of action of the two are known. This will hold in the case of one unknown force as well as for two.

Case II. The *intensity, direction of action, and the direction and the location of the line of action of one force* can be graphically determined provided all of the other forces acting upon the same body are fully known.

Concisely expressed, the intensity and direction of action of *two forces* can be found when the lines of action of all the forces are known and the intensity and direction of action of all except those two are known; and the intensity, direction of action, and the direction and location of the line of action of *one force* can be found when all the other forces are known. As a rule, the intensity and direction of action are all that are desired.

94. Preliminary Application.—Let AB, at (a), Fig. 123, represent a body in equilibrium under the action of the five forces $P1$—$P5$. Suppose all of the forces are known except $P4$ and $P5$, which are unknown in intensity and direction of action only, which means their lines of action are known, so that the problem comes under Case I.

To determine their intensity and direction of action, select some convenient scale and lay off AB at (b), equal and parallel to $P1$; and from B lay off BC equal and parallel to $P2$; and from C lay off CD equal and parallel to $P3$. Then, to complete the polygon, $P4$ and $P5$ must close on D and A. So from D draw Dy parallel to $P4$ and from A draw Az parallel to $P5$, intersecting Dy at E. Then the length of the lines DE and AE represents the intensity of $P4$ and $P5$, respectively. Now, as all of the forces will act in the same direction around the polygon. $P4$ will act in the direction from D to E and $P5$ from E to A, as the arrow points indicate. So we have their direction of action, which is toward the body in each case. Then the forces $P4$ and $P5$ are fully known. The two forces $P4$ and $P5$ could close on A and D, as AO and DO, just as well as DE and AE. Either is correct.

It is usually convenient to go around a body laying off the forces in consecutive order as was done at (b), but in some cases it is not possible to do so, and in fact it is not at all necessary. For example, in the above case we could lay off the forces as they are shown at (c) by laying off $P1$ first, then $P3$, then $P2$, and close on K and H with $P4$ and $P5$; or we could lay off either $P2$ or $P3$ first, just so the forces are joined so that they act in the same direction around the polygon, which is the important thing to watch. However, any mistake in that respect is readily detected. For instance, suppose we were to lay off $P1$ as AB (at d) and then lay off $P3$ from A; we could readily see that the forces would not be acting in the same direction around the polygon being constructed, and consequently the polygon would not be correct.

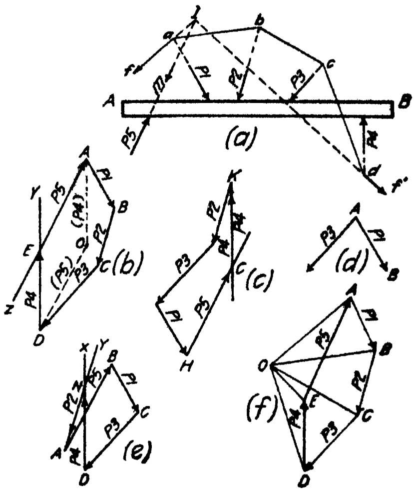


Fig. 123

Now suppose $P2$ and $P4$ at (a) are the two forces unknown in intensity and direction of action instead of $P4$ and $P5$ as considered above. In that case there would be a known force in between the two unknown forces, and consequently it would be impossible to lay off the forces in consecutive order as we pass around the body. However, the case presents no trouble at all, as we can simply lay off the known forces in any order, so long as they are placed so that they all act in the same direction around the polygon. For example, to determine the intensity and direction of action of $P2$ and $P4$, we could lay off $P5$ as AB at (e), then from B lay off $P1$ as BC, and from C lay off $P3$ as CD, then closing on A and D with $P4$ and $P2$ by drawing Dx parallel to $P4$ and Ay parallel to $P2$, intersecting Dx at N. Then DN and NA represent the intensity of $P4$ and $P2$, respectively, and following around the polygon from A to B, C, etc., it is readily seen that $P4$ acts from D to N, and $P2$ from N to A.

It is evident that if only one of the five forces at (a) were unknown in intensity and direction of action, our problem would be quite simple. For example, suppose $P5$ were the only unknown force. We would simply construct a polygon as $ABCDE$, as shown at (b), and join E to A and $P5$ would thus be determined in intensity and direction of action.

As an example, under Case II, suppose all of the forces shown at (a), Fig. 123, are completely known except $P5$, which we will assume is completely unknown. By constructing the force polygon $ABCDEA$ at (f) we have $P5$ given in intensity, direction, and direction of action by the line EA, but the location of its line of action is unknown. To locate its line of action select any point O, at (f), as a pole, and draw the ray diagram as shown, and construct at (a) the equilibrium polygon $fabcdf'$ (as explained in Art. 45). Then prolonging the segments df' and fa until they intersect at I and through I drawing Im parallel to EA, we have the line of action of the resultant of the four forces $P1$, $P2$, $P3$, and $P4$. Now, as all five of the forces are in equilibrium, the resultant just found must balance $P5$, which requires that $P5$ must act along the same line as the resultant. (See Art. 42.) Therefore, the line Im is the line of action of $P5$, and thus $P5$ is fully determined. $P5$ may be applied upon either side of the body as far as equilibrium is concerned.

Referring to the force polygon at (b), Fig. 123, it is evident that if $P4$ and $P5$ were parallel, a straight line joining A and D would represent the direction and intensity of both, but it would be impossible

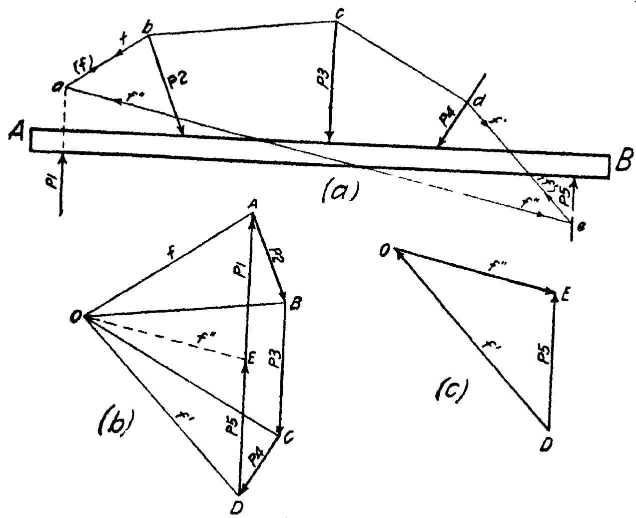


Fig. 124

to determine the intensity of either directly from the polygon, as there would be no break in the line indicating where the two join. However, their intensities are readily determined by the aid of the equilibrium polygon. For example, let AB, at (a), Fig. 124, represent a body held in equilibrium by the five forces $P1 \ldots P5$ as indicated. Let $P1$ and $P5$ be the two unknown parallel forces. By constructing the force polygon $ABCDA$, at (b), we have the combined intensities of the two given by the line DA. Next select any point as O, at (b), as a pole and draw the ray diagram $ABCDOA$. Then, beginning at any point on the line of action of one of the known forces, say at b on the line of action of $P2$, construct the equilibrium polygon $fbcdf'$, as explained in Art. 45. Then the three known forces $P2$, $P3$, and $P4$ are replaced by components, all of which are balanced except f at b and f' at d. As the next step, prolong the segment bf from b until it intersects the line of action of the unknown force $P1$ at a, and likewise prolong the segment df' from d until it intersects the line of action of the other unknown force $P5$ at e. In order to extend the equilibrium polygon farther, it is necessary to resolve $P1$ into two components such that one of them will be equal and opposite to the known component f at b, and $P5$ into two components such that one of them will be equal and opposite to the known component f' at d. This could be readily accomplished if the forces $P1$ and $P5$ were known, but as they are unknown we have only one component in each case to start with; that is, f at a and f' at e.

But as the five forces are in equilibrium, their components will be in equilibrium. Therefore, the unknown component f'' of $P1$ must balance the unknown component of $P5$, and consequently they will act along the same line. So, by drawing the line ea, we have the line of action of the two unknown components, as this is the only line that both could act along. Now at e we have one known force f' and the direction of the two unknown forces $P5$ and f'', each of which can be graphically determined by drawing the force triangle DEO as shown at (c). $P1$ and f'' at a can be graphically determined in the same way. But it is not necessary to construct the triangle at (c) as f' is given in the diagram at (b) by the line OD, and by drawing OE parallel to the line ae we would have the same forces given by the equal triangle ODE at (b), and the forces at a by the triangle OEA, as indicated; and thus $P1$ and $P5$ are fully determined as EA and ED, respectively, in the diagram at (b).

Referring to Fig. 124, it will be observed that the equilibrium polygon closes, which always occurs in the case of any system of forces in equilibrium. The line ea is known as the closing line.

95. Graphical Determination of Reactions and Bending Moments on Simple Beams.—Let AB, at (a), Fig. 125, represent a simple beam supporting the four vertical loads $P1 \ldots P4$ as indicated, and suppose the reactions R and $R1$, due to these loads, are unknown in intensity. First lay off the load line AB as shown at (b). Then select any point O as a pole and draw the ray diagram as shown and construct the equilibrium polygon $A'abcdB'$ at (c). Then from O draw the line OE parallel to the closing line $A'B'$, and, according to the last article, we have R given by the line AE and $R1$ by the line BE, and thus the reactions are determined.

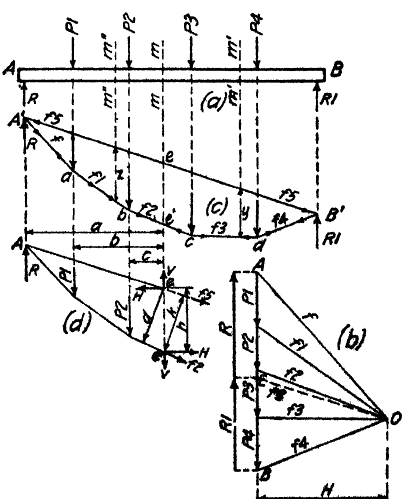


Fig. 125

Conceive of the closing line $A'B'$ at (c) as being a beam and the other part of the equilibrium polygon as a rope suspended from the ends of this beam, and imagine the loads supported upon the rope as indicated. Then the equilibrium polygon really becomes a structure upon which the bending moment at any vertical section will be the same as for the beam, and consequently it may be treated instead of the beam for the purpose of finding these moments.

Then to obtain the bending moment at the section *mm* of the beam, extend this section on down to the equilibrium polygon, cutting it off along the section *ee'*. Then imagine the part of the equilibrium polygon to the left of this section moved bodily to (*d*) without any loads or forces being affected in the least. Then taking moments about *e*, we have

$$aR - bP1 - cP2 = M.$$

Now, this moment is balanced by the moment of $f2$ about *e*, then we have

$$M = d(f2),$$

and taking moments about *e'*, we have

$$M = k(f5).$$

Both $f2$ and $f5$ ($f2$ being the stress in the rope from *b* to *c*, and $f5$ the stress in the beam throughout) are given in the ray diagram, and *d* and *k* can be ascertained by scale. A better way to obtain the moment is to resolve $f2$ and $f5$ into horizontal and vertical components as shown, and then taking moments at either *e* or *e'*, we have

$$M = Hh.$$

H is the same for all segments, as is seen from the ray diagram at (*b*), and is known as the "pole distance." Then, evidently, the bending moment at any vertical section of the beam is equal to this pole distance (*H*) multiplied by the vertical ordinate between the closing line and the broken line of the equilibrium polygon at the same vertical section. For example, the bending moment at the section *m'm'* is equal to Hy, at *m''m''*, Hz, and so on. The ordinates are measured in feet or inches to the same scale as the beam, while *H* is in pounds, so many thousand pounds per inch. In laying out the ray diagram, *H* can always be taken as some convenient number, as 100,000 lbs.

96. Graphical Analysis of Trusses consists in graphically determining the reactions and stresses. The reactions are determined by means of the equilibrium polygon, while the stresses are determined as concurrent forces at the joints by means of force polygons, one polygon for each joint, the joints being considered in the order that the application of the method requires, being usually the consecutive order.

Example 1. Let the diagram at (*a*), Fig. 126, represent a truss supporting the three loads *P*1, *P*2, and *P*3, as indicated, and let *R* and *R*1 represent the reaction at *A* and *B*, respectively, due to the three loads.

Let it be required to determine the reactions on the above truss and the stresses in the members due to the three loads *P*1, *P*2, and *P*3.

To determine the reactions, first construct the ray diagram at (*b*), which is accomplished by laying off the line 1-2 to any convenient scale, equal and parallel to *P*1, and line 2-3 equal and parallel to *P*2, and line 3-4 equal and parallel to *P*3, thus obtaining the load line 1-2-3-4, and then taking any point *O* as a pole and drawing the rays 1-*O*, 2-*O*, etc. Then construct the equilibrium polygon *CghkD*, which is accomplished by taking any point on the line of action of one of the forces, say point g, on the line of action of $P1$, and resolving $P1$ into two components by drawing Cg parallel to the ray $1\text{-}O$ and gh parallel to $2\text{-}O$ and virtually resolving each of the other two loads into two components by drawing hk parallel to $3\text{-}O$ and kD parallel to $4\text{-}O$, and drawing the closing line CD

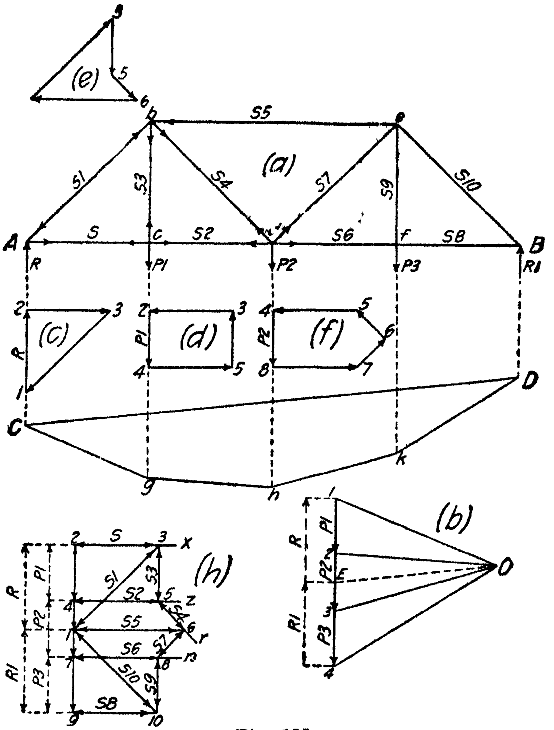


Fig 126

for completion. Then by drawing OE parallel to the closing line CD, we have the reaction R given by the line $1\text{-}E$ and $R1$ by the line $4\text{-}E$. These lines can be scaled and thus the reactions will be fully known as we know their direction of action will be upward, in order to balance the loads.

After the reactions are determined all of the external forces are known and we can proceed to determine the stresses in the truss members.

Any joint of any truss can be considered as being a small body acted upon by the truss members, meeting at the joint, and any load there applied. The details should be such that these will really be concurrent forces in every case and are so considered in graphic analysis.

Then at joint A we have three concurrent forces: the reaction R, the force exerted by the member Ac, and the one exerted by the member Ab. At joint c we have four concurrent forces, the load $P1$ and one for each of the three members meeting at that joint; at joint b we have four concurrent forces, one for each of the members meeting at that joint; at joint d we have five concurrent forces, the load $P2$ and one for each of the members meeting at that joint; and likewise at joints *e*, *B*, and *f* we have, respectively, four, three, and four concurrent forces, as is readily seen.

The intensity of the force exerted by any member on a joint will be equal to the stress in the member exerting the force, and if the force acts toward the joint the member exerting it will evidently be in compression and hence the stress in the member will be compression, while if the force acts away from the joint the member exerting it will be in tension and hence the stress in the member will be tension. So if the force exerted at a joint by a member be known, the stress in the member will be known.

Now it is evident that the graphical determination of the stresses in the truss members is simply a matter of determining the concurrent forces at each joint, which we can do by simply drawing a force polygon for each joint. However, we cannot begin with just any joint, as the application of the method is limited, as stated in Art. 86. It will be seen upon inspection of the diagram at (*a*) that the only joints that can be analyzed at the beginning are joints *A* and *B*. Here we have, in each case, all forces known in direction and only two unknown in intensity and direction of action. So we have simply Case I, Art. 86. Then let us take joint *A* to begin with:

Laying off 1-2 at (*c*), to any convenient scale, equal and parallel to *R*, and drawing 2-3 parallel to the member *Ac* and 1-3 parallel to the member *Ab*, we have the force diagram 1-2-3-1 representing the concurrent forces at joint *A*, where 2-3 represents the force exerted by the member *Ac*, and 1-3 represents the force exerted by the member *Ab*. By scaling 2-3 and 1-3 we obtain the intensity of these forces, respectively. We know that *R* acts upward and hence will act from 1 to 2 in the force polygon. Then as all of the forces must act in the same direction around the polygon, we have the force exerted by *Ac* acting from 2 to 3 (in the polygon) and the one exerted by *Ab* acting from 3 to 1. Now transferring these directions to joint *A* we have the force exerted by the member *Ac* acting away from the joint and the force exerted by the member *Ab* acting toward it, as indicated. Then the stress in *Ac* will be tension and the stress in *Ab* compression, and as the intensity of the first is given by the line 2-3, and the intensity of the second by the line 1-3, in the force polygon, we have the stresses in the two members *Ac* and *Ab* fully determined.

Knowing the force exerted by the member *Ac* at joint *A*, we know the force it exerts at joint *c*, as the two will, undoubtedly, be equal and opposite, and hence the one at *c* will act away from the joint, as indicated, and likewise, knowing the force exerted at joint *A* by the member *Ab*, we know the force it exerts at joint *b*, which acts toward *b*.

Joint *b* cannot be analyzed as yet for there are still three unknown forces acting at that joint, but joint *c* can be, as there are only two unknown forces there. Then laying off 2-3 at (*d*) equal and parallel to the line 2-3 at (*c*) and 2-4 equal and parallel to *P*1 and drawing 4-5 parallel to the member *cd* and 3-5 parallel to *cb*, we have the force polygon 2-3-5-4-2 representing the forces at joint *c*, where the force exerted by the member *cd* is represented by the line 4-5, and the force exerted by the member *cb* is represented by the line 3-5, which is equivalent to saying that the intensity of the stress in *cd* is given by the line 4-5, and the intensity of the stress in *cb* is given by the line 3-5, and by scaling these lines we have the intensity of the stress in each of the two members. As the force exerted at joint *c* by the member *Ac* acts to the left from the joint, it will act from 3 to 2 in the polygon at (*d*), and as the other forces must act in the same direction around the polygon, it is readily seen that the forces exerted by the members *cd* and *cb* both act away from the joint, and, hence, the stress in each of the two members will be tension and thus we have the stresses in the members *cd* and *cb* fully determined.

Now as the force exerted at *c* by the member *cb* is fully determined, the force exerted at *b* by the same member is known and acts away from the joint, as indicated. Then we have only two unknown forces at joint *b* and hence the joint can now be analyzed. Then laying off 1-3 at (*e*), equal and parallel to the line 1-3 at (*c*), and 3-5 equal and parallel to the line 3-5 at (*d*), and drawing 5-6 parallel to the member *bd*, and 1-6 parallel to the member *be*, we have the force polygon 1-3-5-6-1 representing the forces at joint *b*, where 5-6 and 1-6 represent the intensity of the force exerted by the member *bd* and *be*, respectively. Then by scaling these lines we obtain the intensity of the stress in the members *bd* and *be*.

Now the force exerted upon the joint *b* by the member *Ab*, as previously stated, acts toward the joint and the force exerted by *cb* acts away from the joint. Then transferring these directions to the polygon at (*e*), it is readily seen that the forces will act around the polygon, as indicated, and that the force exerted by the member *bd* acts in the polygon from 5 to 6, while the force exerted by the member *be* acts from 6 to 1, and transferring these directions to joint *b*, we have the first acting away from the joint and the second toward it, hence the stress in *bd* is tension while the stress in *be* is compression, and thus we have the stresses in the two members *bd* and *be* fully determined.

The force exerted at *b* by the member *bd* being determined, the force exerted by the same member at *d* is fully known and the same is true of the force exerted at *d* by the member *cd*. Then there are but two unknown forces at *d* and hence the joint can be analyzed by drawing the force polygon shown at (*f*) and thus the stresses in the members *de* and *df* can be determined. Then next the joints *f* and *e* can be analyzed in the same way and thus the stresses in all the members in the truss can be determined, but one continuous diagram as shown at (*h*), wherein the forces at each joint and likewise the stress in each member are represented, can be constructed more readily than the separate diagrams, providing we pass around all the joints in the same direction. Either direction, clock-wise or counter clock-wise, can be taken for the first joint analyzed, but when the direction is once taken we should pass around each of the other joints in the same direction, for otherwise there is apt to be confusion.

For convenience, let *S*, *S*1, *S*2, and so on, represent the stresses in the various members of the truss, as indicated at (*a*).

To construct the continuous diagram at (*h*), let us begin at joint *A*, as before, and pass around the joint counter clock-wise. Beginning with the known force *R* we lay off 1-2 at (*h*) to any convenient scale, equal and parallel to *R*. The next force we come to is the one exerted by the member *Ac*. So from 2 draw a line 2-*x* parallel to the member *Ac*. Then the next force we come to is the one exerted by the member *Ab*. So from 1 draw a line parallel to this member, intersecting the line 2-x at 3, and we have the force polygon 1-2-3-1 representing the forces at joint *A*, where the line 2-3 represents the intensity of the stress *S* in the member *Ac* and the line 1-3 represents the intensity of the stress *S*1 in the member *Ab*. As *R* acts upward the force exerted by *Ac* acts from 2 to 3 (in the polygon), and the force exerted by the member *Ab* acts from 3 to 1,—all in the same direction around the polygon. Transferring these directions to joint *A*, we see that *S* is tension and *S*1 compression, as explained above, and thus we have *S* and *S*1 fully determined. Passing on to joint *c*, we have the force exerted by the member *Ac* represented by the line 2-3 and acting from 3 to 2 as *S* is tension. The next force we come to, passing around the joint counter clock-wise, is *P*1, which we lay off downward as 2-4. Then from 4 draw a line 4-z parallel to the member *cd*, and from 3 draw a line parallel to the member *cb*, and we have the force polygon 3-2-4-5-3, representing the forces at joint *c*, where the line 4-5 represents the intensity of the stress *S*2 in the member *cd*, and the line 5-3 represents the intensity of the stress *S*3 in the member *cb*. As the force exerted by the member *Ac* acts (in the diagram) from 3 to 2, and *P*1 from 2 to 4, the force exerted by the member *cd* will act from 4 to 5 and the force exerted by the member *cb* will act from 5 to 3. Transferring these directions to joint *c*, we have the force exerted by the member *cd* acting away from the joint and the force exerted by the member *cb* acting away from it also. Then *S*2 and *S*3 are both tension and thus we have both fully determined. At joint *b* the force exerted by the member *Ab* is known, as its intensity was determined at joint *A*, and its direction of action at *b* will be in the opposite direction to that of the equal force exerted by the same member at *A*, and hence toward the joint *b*, as indicated, and the force exerted at *b* by the member *cd* is known as its intensity was determined at joint *c*, and its direction of action will be in the opposite direction to that of the equal force exerted by the same member at joint *c*, and hence away from joint *b*. Then only the forces exerted by the members *bd* and *be* remain to be determined at joint *b*. Beginning with the force exerted by the member *Ab* we have its intensity represented by the line 1-3 and it acts from 1 to 3 in reference to joint *b*. Passing on around the joint counter clock-wise, we have next the force exerted by the member *cb*, the intensity of which is represented by the line 5-3, and it acts from 3 to 5. The next force we come to is the one exerted by the member *bd*. So from 5 draw a line 5-r parallel to the member *bd* and close the polygon by drawing a line from 1 parallel to the member *be*, intersecting the line 5-r at 6, and we have the polygon 1-3-5-6-1 representing the forces at joint *b*. As the force exerted by the member *Ab* acts (in the polygon) from 1 to 3 and the one exerted by *cb* acts from 3 to 5, the force exerted by the member *bd* will act from 5 to 6, and the one exerted by the member *be* from 6 to 1. Transferring these directions to joint *b* we have the force exerted by *bd* acting away from the joint and the force exerted by *be* acting toward it. Hence the stress *S*4 in *bd* is tension and the stress *S*5 in *be* is compression, and as the intensity of *S*4 is given by the line 5-6 and that of *S*5 by the line 6-1, we have the stress in the members *bd* and *be* fully determined.

Passing on now to joint *d* we have the forces exerted by the members *bd* and *cd* and also the load *P*1 known, to determine the forces exerted by the members *de* and *df*. Beginning with the force exerted by the member *bd*, and passing around the joint counter clock-wise, we have that force represented by the line 6-5 and it acts from 6 to 5, and the force exerted by the member *cd* represented by the line 5-4 and it acts from 5 to 4. Now, as *P*2 acts downward, from 4 lay off 4-7 downward and equal and parallel to *P*2 and from 7 draw a line 7-*n* parallel to the member *df* and close the polygon by drawing from 6 a line parallel to the member *de* intersecting the line 7-*n* at 8, and we have the polygon 6-5-4-7-8-6 representing the forces at joint *d*. Following around this polygon in the direction of the action of the known forces, that is, from 6 to 5, from 5 to 4, on around to 6, we see that the forces exerted by the member *df* and *de* each act away from joint *d* and hence the stress in each of the members *df* and *de* will be tension, and, as the intensity of the stress *S*6 in the member *df* is given by the line 7-8 and the intensity of the stress *S*7 in the member *de* by the line 8-6, we have the stress in each of the members *df* and *de* fully determined.

Passing on to joint *f* we have the forces exerted by the member *df* and the load *P*3 known, to determine the forces exerted by the members *fB* and *fe*. The force exerted by the member *df* is represented by the line 8-7 and acts from 8 to 7. Then from 7 lay off 7-9 downward and equal and parallel to *P*3 and drawing 9-10 from 9 parallel to the member *fB* and 8-10 from 8 parallel to the member *fe*, we have the force polygon 8-7-9-10-8 representing the forces at joint *f* where the line 9-10 represents the force exerted by the member *fB* and 10-8 represents the force exerted by the member *fe*. Following around the polygon in the direction of the action of the known forces, that is, from 8 to 7, 7 to 9, on around to 8, we have the force exerted by the member *fB* and by the member *fe* both acting away from the joint *f*, and hence the stress *S*8 in *fB* and the stress *S*9 in *fe* are both tension and as the line 9-10 gives the intensity of *S*8, and 10-8 the intensity of *S*9, we have the stresses in the two members *fB* and *fe* fully determined.

Now, passing on to joint *e*, we have here all of the forces known except the one exerted by the member *eB*. Starting with the force exerted by the member *be*, and passing around the joint counter clock-wise, we have the force exerted by the member (*be*) given by the line 1-6 and it acts from 1 to 6; the force exerted by the member *de* given by the line 6-8 and it acts from 6 to 8; and the force exerted by the member *fe* given by the line 8-10 and it acts from 8 to 10. Then a line drawn from 10 parallel to the member *eB* should close the polygon 1-6-8-10-1 representing the forces at joint *e*. If this polygon should not close it shows that the continuous diagram has not been accurately drawn and hence must be wholly redrawn to insure that the intensity of the stresses thus obtained is correct. The intensity of the stress *S*10, in the member *eB*, is given by the line 10-1, and as the force exerted by that member at joint *e* acts from 10 to 1 in the polygon 1-6-8-10-1, *S*10 is seen to be compression and hence is fully determined, and thus the stresses in all of the truss members are found.

For joint *B* we have the force polygon 10-9-1-10, where the line 9-1 represents the reaction *R*1, and 1-10 and 10-9 the stress in *eB* and *fB*, respectively. The continuous diagram at (*h*) would be known as a "stress diagram," in fact all such diagrams representing stresses are known as stress diagrams.

It will be observed that all of the external forces, which consist of the two reactions and the three loads, are laid off on the same line 2-9, which is known as the load line; yet these forces, as they hold the truss in equilibrium, really form a closed polygon, so to speak, where the reaction R is laid off from 1 to 2 and the load $P1$ from 2 down to 4, $P2$ from 4 to 7, $P3$ from 7 to 9, and $R1$ from 9 back to 1—the starting point—all acting, as we may say, in the same direction around the polygon 1-2-4-7-9-1, which would really be a polygon if the forces were not parallel. In constructing any stress diagram the force polygon, known as the load line, formed by the external forces can be laid off first and the remainder of the diagram added to this polygon, as shown in the following example:

Example 2. Let it be required to determine the stresses in the members of the truss shown at (a), Fig. 127, due to the three loads $P1$, $P2$, and $P3$. Let R and $R1$ represent the reaction at A and B, respectively, due to the three loads.

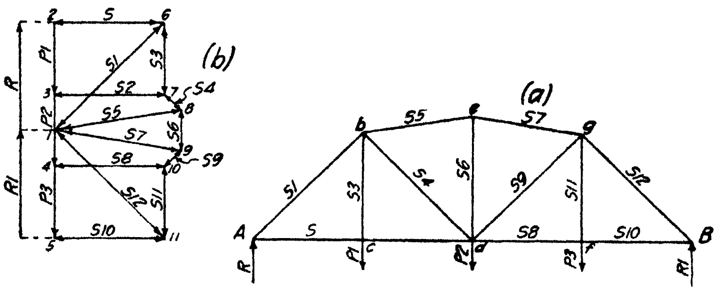


Fig 127

Beginning with R and passing around the truss as a whole counter clock-wise, and taking the forces in consecutive order, we obtain at (b) the force polygon 1-2-3-4-5-1 by laying off 1-2 equal and parallel to R, 2-3 equal and parallel to $P1$, 3-4 equal and parallel to $P2$, 4-5 equal and parallel to $P3$, and 5-1 equal and parallel to $R1$, thus closing the polygon. As is readily seen, the forces act in the order 1-2-3-4-5-1.

To obtain the stresses, we can begin at either A or B, say, B, and passing around that joint counter clock-wise we obtain the polygon 1-11-5-1, representing the forces at that joint, by drawing 1-11 parallel to gB and 5-11 parallel to fB. The line 1-11 gives the stress in gB and 5-11 the stress in fB. As $R1$ acts upward from 5 to 1, the force exerted by gB will act from 1 to 11, and hence toward the joint, and the force exerted by fB will act from 11 to 5, and hence away from the joint. Thus it is seen that the stress $S12$, in the member gB, is compression, while the stress $S10$, in fB, is tension.

Passing on to joint f, we obtain the polygon 4-5-11-10-4 by drawing 11-10 parallel to gf and 10-4 parallel to fd. We really begin with the load $P3$, which acts from 4 to 5. Then the force exerted by fB comes next, which acts from 5 to 11, and next is the force exerted by the member gf which we can lay off from 11 only as a line parallel to gf, and its length is then obtained by drawing a line from 4 parallel to fd, thus closing the polygon. In a similar manner the polygon 10-11-1-9-10 for joint g is obtained, and likewise the polygon 9-1-8-9 for joint e, 3-4-10-9-8-7-3 for joint d, 1-8-7-6-1 for joint b, and 3-7-6-2-3 for joint c. By scaling in each case the line representing the intensity of stress and observing at the same time the direction of action of the force exerted at the joints, the stress in each member is fully determined.

If the direction around the joints in examples 1 and 2 be taken clock-wise instead of counter clock-wise, as above, the stress diagram would simply fall on the left side of the load line instead of the right as shown at (h), Fig. 126, and (b), Fig. 127.

Stress diagrams should be drawn without the aid of any letters or marking of any kind other than the forces on the load line, and the stresses in the members may be indicated in the way shown above; however, after a little practice, even this marking will be found to be superfluous. The student should not acquire the habit of laying off the load line and drawing the stress diagram without fully analyzing each joint of the truss, and if this is done, no marking of the diagram is needed.

Example 3. So far we have considered loads only at the bottom of the trusses, which is often the case for live load, but for dead load, especially, we have loads applied at both the top and bottom of the trusses as shown at (a), Fig. 128. Let it be required to determine the reactions on the truss shown there, and also the stresses in the members, due to the six loads indicated. Let R and $R1$ represent the reaction at A and B, respectively, due to these loads.

The most practical way of determining the reactions is to add the top load at each panel point to the bottom load and treat the sum as a single load in each case. Then the ray diagram is constructed as shown at (b) and an equilibrium polygon as shown at (c) can be drawn. However, the loads can be laid off on a load line in any order and a ray diagram constructed and a corresponding equilibrium polygon drawn. For example, suppose the loads be laid off on the load line in consecutive order, passing around the truss, as a whole, counter clock-wise, as shown at (d). Then taking G as a pole and drawing the ray diagram at (d) and beginning at any convenient point on the line of action of one of the forces, say, point 1, on the line of action of $P1$, resolve $P1$ into two components $f1$ and $f2$, which is accomplished by drawing $f1$ (at point 1) parallel to the ray 1-G and $f2$ parallel to ray 2-G. As $f1$ and $f2$ are components of $P1$, the first will act in the ray diagram from 1 to G and the second from G to 2, and transferring these directions to point 1 we have them acting as shown. By prolonging the line of action of $f2$ from point 1 till it intersects the line of action of $P2$ at point 2, and resolving $P2$ into two components $f2$ and $f3$ by drawing $f2$ parallel to the ray 2-G and $f3$ parallel to the ray 3-G, we have the component at point 2 balancing the component $f2$ at point 1, and drawing the line 2-3 parallel to the ray 3-G, 3-x parallel to the ray 4-G, 3-4 parallel to the ray 5-G, and so on, we obtain the polygon y-1-2-3-x-3-4-5-z, wherein the forces, or components, are all balanced except $f1$ and $f7$. Then prolonging the line of action of $f1$ until it intersects the line of action of $R1$ at K and prolonging the line of action of $f7$ until it intersects the line of action of R at H, and drawing the closing line *HK*, we have the equilibrium polygon *K*-*y*-1-2-3-*x*-3-4-5-*H*-*K* closed, and by drawing *E′*-*G*, at (*d*), parallel to the closing line *HK* we have the reaction *R* given by the line *E′*-1 and the reaction *R*1 by the line *E′*-7. The polygon could be closed just as well by drawing *z*-*K′* and 1-*H′*, and then *H′*-*K′*, which is the closing line in that case, and which is parallel to *H*-*K*.

As another case, suppose the loads are laid off on the load line, as we may, just in any order as shown at (*e*). Then constructing the ray diagram as shown we can draw the equilibrium polygon *V*-6-7-8-9-10-*t*-10-*U*-*V* as shown at (*f*), where *UV* is the closing line. Then drawing

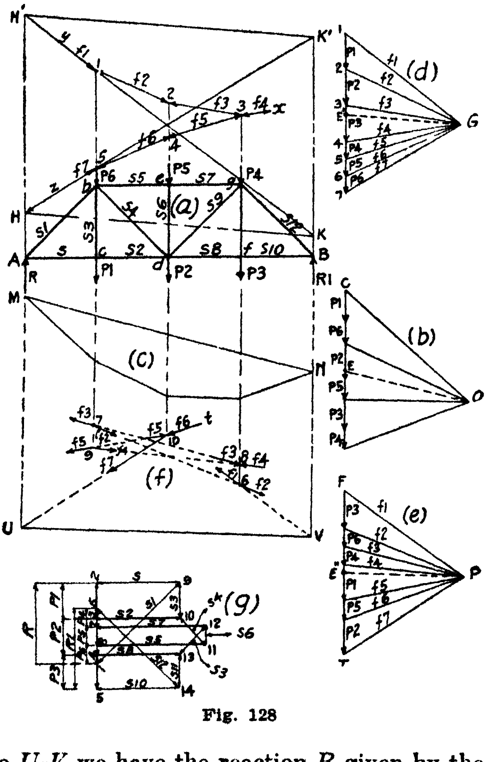


Fig. 128

E″-*P* parallel to *U*-*V* we have the reaction *R* given by the line *E″*-*F* and *R*1 by the line *E″*-*T*.

Complicated diagrams and polygons should be avoided as much as possible. We can accomplish this result only by judicious selection of order and position, for which no fixed rule can be given, and one must be guided by experience and common judgment. Complicated diagrams and polygons are more objectionable on account of inaccuracy, as a rule, than on account of difficult construction. For example, it is obvious that there is more chance of error in constructing the polygon at (*f*) than there is in the case of the one at (*c*).

The most practical way of obtaining the stresses in the truss shown at (*a*) is to lay off the load line as shown at (*g*), which is accomplished by beginning with R and passing around the truss counter clock-wise, taking the forces in consecutive order. Thus we lay off at (g) the line 1-2 upward, equal and parallel to R. Then from 2 lay off 2-3 downward, equal and parallel to $P1$, 3-4 equal and parallel to $P2$, 4-5 equal and parallel to $P3$, and from 5 lay off 5-6 upward, equal and parallel to $R1$, and from 6 lay off 6-7 downward equal and parallel to $P4$, 7-8 equal and parallel to $P5$, and the line 8-1 closing the polygon 1-2-3-4-5-6-7-8-1 should be equal and parallel to $P6$. After the load line is laid off, we can start either from joint A or B, and, passing around each joint counter clock-wise, the stress diagram shown at (g) can be readily constructed.

97. Oblique Reactions.—As a rule, the dead and live load upon structures act vertically, producing vertical reactions, but the pressure due to the wind, known as wind load, produces reactions which are not vertical. However, the method of determining such reactions is prac-

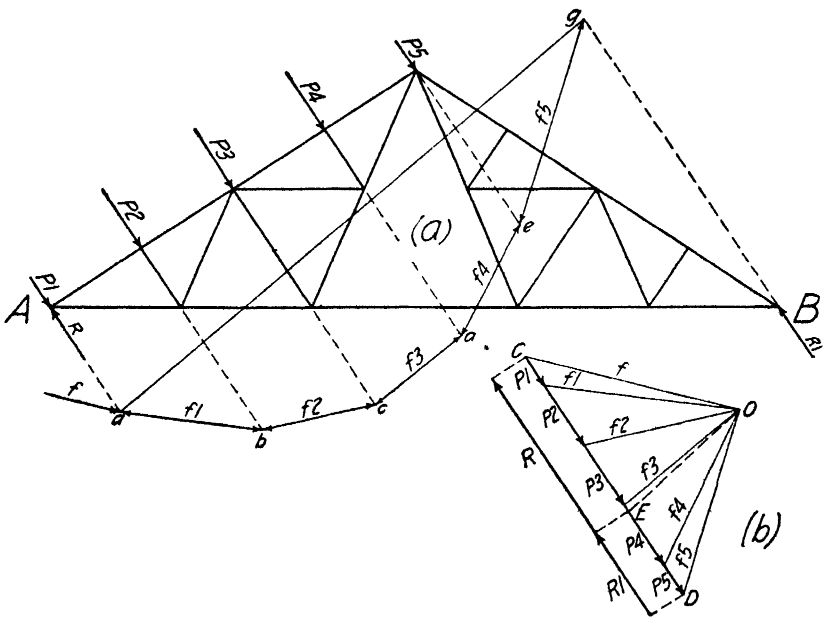


Fig. 129

tically the same as for vertical reactions. As an example, let the diagram at (a), Fig. 129, represent a truss supporting five wind loads, $P1$—$P5$, applied normally to the sloping side of the truss as indicated. In case the truss be equally fixed at its supports A and B, the reactions R and $R1$ will be parallel to the loads as indicated, or in case the loads are not all parallel, the reactions will be parallel to their resultant.

To determine the reactions R and $R1$, first lay off the load line CD at (b) and draw the ray diagram as shown and construct the equilibrium polygon $abcdega$ at (a) the same as for any other forces. Then by drawing OE in the ray diagram parallel to the closing line ag of the equilibrium polygon, we have the reaction R given by the line EC and $R1$ by the line ED.

There is nothing about the equilibrium polygon at (a) out of the ordinary, except at point a. At e the force $P5$ is resolved into two components, and the line of action of the one to the right $(f5)$ is prolonged until it intersects the line of action of $R1$ at g. Now, at a exactly the same method of procedure is followed, but as $P1$ and R have the same line of action, the segment parallel to the ray CO does not appear as its length is zero.

After the reactions are known, the stress in the individual members of the truss can be graphically determined, as explained above, by beginning at either joint A or B.

In case one end of a truss be supported upon rollers, the reaction at that end will be vertical, as the rollers will not resist any horizontal force. In such cases we always know the direction of the one reaction, but, as a rule, the intensity of that one and the intensity and direction of the other remain to be determined.

Let the diagram at (a), Fig. 130, represent the same truss and loads as represented at (a), Fig. 129. First, suppose the truss supported upon

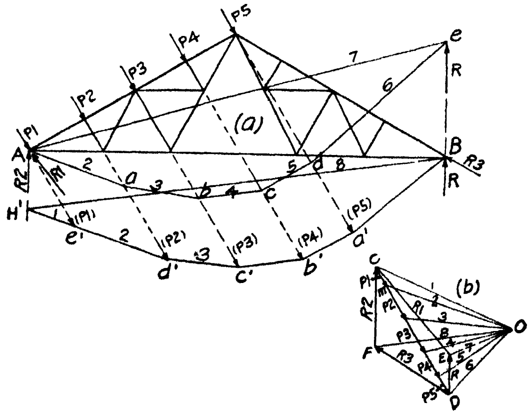


Fig. 130

rollers at B and upon a fixed bearing at A. Then, the end B being upon rollers, the reaction there, indicated by R, will act vertically and, consequently, we know its line of action, while the reaction at A, indicated as $R1$, will evidently act obliquely, as it resists the horizontal component of the wind; but, further than this, $R1$ is unknown.

Lay off the load line CD at (b), Fig. 130, and draw the ray diagram the same as in any other case, and construct the equilibrium polygon $AabcdeA$ at (a) by beginning at A, the point of application of the oblique reaction. Then treating this equilibrium polygon as a truss, we can obtain R by analyzing joint e, as the stress in the member ed is given by ray 6 in the ray diagram at (b). Then by drawing from D, at (b), the line DE parallel to R and closing on O with line OE drawn from O parallel to the member eA (at e), we have the intensity of R given by the line DE. Then, as the forces $P1$—$P5$ and the two reactions R and $R1$ are a system in equilibrium, they will form a closed polygon, *DECD,* wherein the forces act in the same direction around the polygon. So, evidently, the line *EC,* at (*b*), represents the other reaction *R*1 in intensity and direction. However, the analysis of joint *A* will show this, for beginning with *P*1, we have, at (*b*), the force polygon *CmOEC.*

In case the truss were supported upon rollers at *A* and upon a fixed bearing at *B,* we would begin at *B,* as that would be the point of application of the oblique reaction, and construct an equilibrium polygon as *Ba'b'c'd'e'H'* and then analyze the points *H'* and *B* to obtain the reactions *R*2 and *R*3, whence their intensity and direction would be given, at (*b*), by the lines *CF* and *FD,* respectively.

98. Method of Drawing an Equilibrium Polygon through Two and Three Given Points.— As a case of two points, let *P*1, *P*2, and *P*3, Fig. 131, represent three given forces and let *A* and *B* be the two given points. Imagine the line *AB,* joining the two points, as being a beam supporting the three forces. Then the reactions on this beam, due to the three forces, will be parallel to the resultant of the forces. By constructing the force polygon *CabDC,* at (*b*), we have the resultant of the forces given by the line *CD.* Then the reaction at *A* and *B,* represented, respectively, as *R* and *R*1, can be drawn, as shown, parallel to the line *CD.* Taking any point *O,* at (*b*), as a pole and constructing the ray diagram as shown, we can draw the equilibrium polygon *A*-1-2-3-*B'*-*A,* as shown at (*a*). Then drawing *OE* parallel to the closing line *AB'* we have *R* given by the line *EC* and *R*1 by the line *ED.* These reactions will, of course, be constant regardless of the slope of the closing line of the equilibrium polygon used to determine them. So, regardless of the location of the pole in the ray diagram, the line drawn through it and parallel to the closing line of the corresponding equilibrium polygon will pass through *E.* Then evidently the pole of any ray diagram, where the corresponding equilibrium polygon passes through both *A* and *B* will be on a line through *E* parallel to *AB* as *Ex,* and as any equilibrium polygon drawn from *A,* having a closing line parallel to *AB,* will pass through both *A* and *B,* it follows that any point on the line *Ex* can be taken as a pole and the corresponding equilibrium polygon will pass through the two points *A* and *B.* Thus, taking *O*1 on the line *Ex,* at (*b*), as a pole, and constructing the ray diagram, as shown, the equilibrium polygon *A*-4-5-6-*B* can be drawn, and taking *O*2 as a pole and constructing the ray diagram to the left of the load line, as shown, the equilibrium polygon *A*-7-8-9-*B* can be drawn.

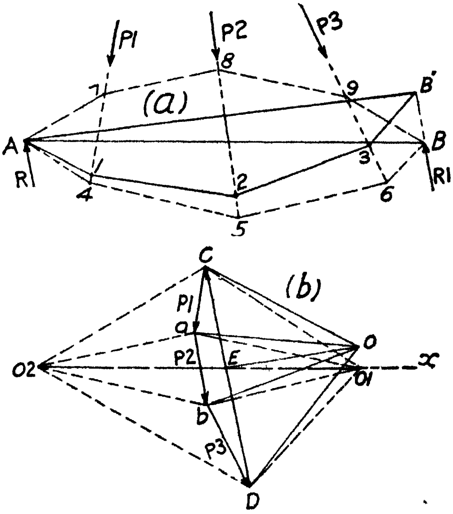


Fig 131

As a case of three points, let *P*1—*P*5, shown at (*a*), Fig. 132, represent five given forces and let *A*, *B*, and *C* be three given points through which an equilibrium polygon is to be drawn.

First construct the ray diagram at (*b*) by laying off the forces in consecutive order, to any convenient scale, thus obtaining the load line *DabGcF*, and taking any point *O* as a pole and drawing the rays.

Imagine the line *AB*, joining the two points *A* and *B*, as being a beam supporting the three forces *P*1, *P*2, and *P*3 (as loads). Then the reactions on beam *AB*, at *A* and *B*, due to the three forces, will be parallel to the resultant of the three forces, which is represented by the line *DG* in the ray diagram. Then by drawing a line, as *yy*, through each of the points *A* and *B*, parallel to *DG*, we have the lines of action of the two reactions on the beam *AB* which are indicated as *R* and *R*1.

We can consider the part *ODG* of the ray diagram as pertaining to beam *AB*. By drawing the equilibrium polygon *A*-1-2-3-*m*-*A*, as shown

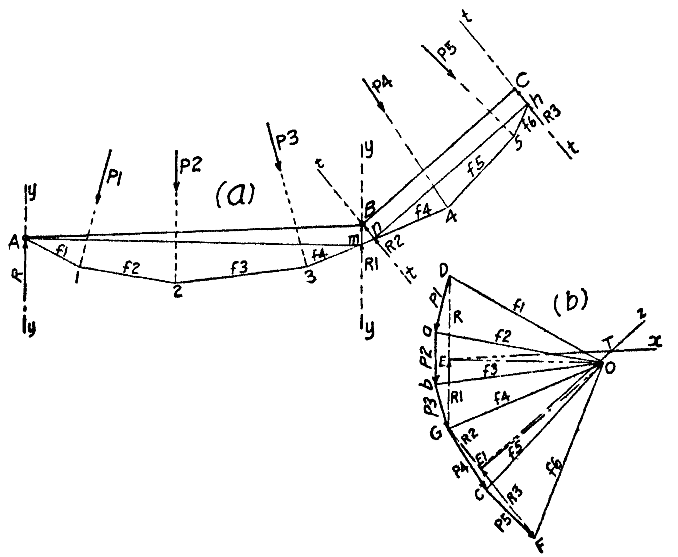


Fig. 132

at (*a*), and then drawing *OE* parallel to the closing line *Am*, and from *E* drawing a line parallel to *AB*, we have the line *Ex*, any point of which could be taken as a pole for a ray diagram, and the corresponding equilibrium polygon would pass through the points *A* and *B* as shown above for the case of two points.

Next, imagine the line *BC* as being a beam supporting the two forces *P*4 and *P*5. Then the reactions on this beam at *B* and *C* would be parallel to *GF* (shown at (*b*)) and by drawing the lines *tt* through *B* and *C* parallel to *GF* we have the lines of action of the reactions on the beam *BC* which are represented as *R*2 and *R*3. The part *OGF* of the ray diagram can be considered as pertaining to the beam *BC*. Then drawing the equilibrium polygon *n*-4-5-*h*-*n* and next the line *OE*1 parallel to the closing line *nh* and from *E*1 drawing a line parallel to *BC* we have the line $E1$-z, any point of which could be taken as a pole for a ray diagram, and the corresponding equilibrium polygon would pass through the two points B and C, provided, of course, that the polygon be started from one of the points. But, as any point on the line E-x can be taken as a pole for a ray diagram, and the corresponding equilibrium polygon would pass through points A and B, provided it be started from one of the points, undoubtedly if the point T, the intersection of E-x and $E1$-z, be taken as a pole of a ray diagram, the corresponding equilibrium polygon will pass through all three points A, B, and C, if started at one of the points.

Problem 1. Determine graphically the bending moment on the beam AB, as shown in Fig. 125, at load $P3$ due to the loads $P1$, $P2$, $P3$, and $P4$, assuming that each of the loads weighs 8,000 lbs. and that all are spaced 4 feet apart along the beam and load $P1$ 4 feet from R, and load $P4$ 5 feet from $R1$.

Problem 2. Determine graphically the stress in the members of the truss shown in Fig. 126 due to the loads indicated. Assuming:

$P1$ = 8,000 lbs., $P2$ = 10,000 lbs., and $P3$ = 12,000 lbs.
Length of span = 4 panels @ 25′ = 100′.
Height of truss = 28′.

Problem 3. Determine graphically the stress in the members of the truss shown in Fig. 127 due to the loads indicated. Assuming:

$P1$ = 20,000 lbs., $P2$ = 30,000 lbs., and $P3$ = 20,000 lbs.
Length of span = 4 panels @ 26′ = 104′.
Height at c and f = 28′.
Height at d = 34′.

Problem 4. Determine graphically the stress in the members of the truss shown in Fig. 128 due to the loads indicated. Assuming:

$P1$ = 22,000 lbs., $P2$ = 24,000 lbs., $P3$ = 25,000 lbs.
$P4$ = 8,000 lbs., $P5$ = 7,000 lbs., $P6$ = 9,000 lbs.
Length of span = 4 panels @ 26′ = 104′.
Height of truss = 31′.

Problem 5. Determine graphically the values of the stresses in the trusses shown in Figs. 132*a*, 132*b*, 132*c*, 132*d* due to the loads indicated in each case.

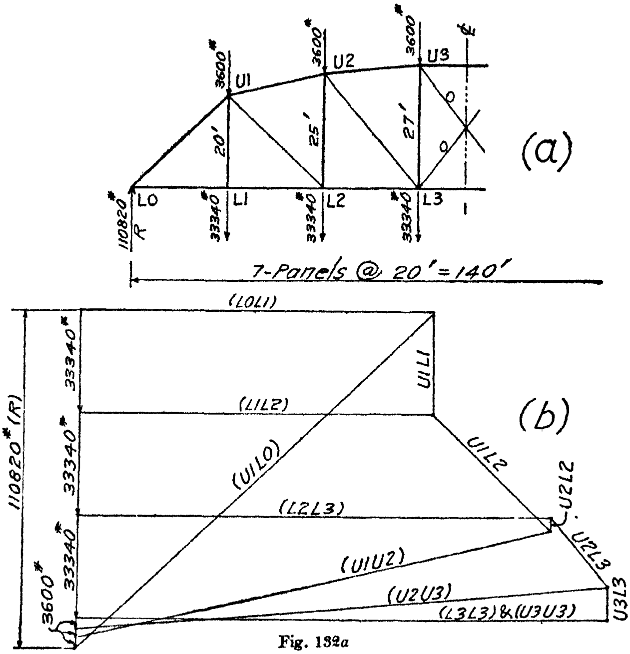


Fig. 132a

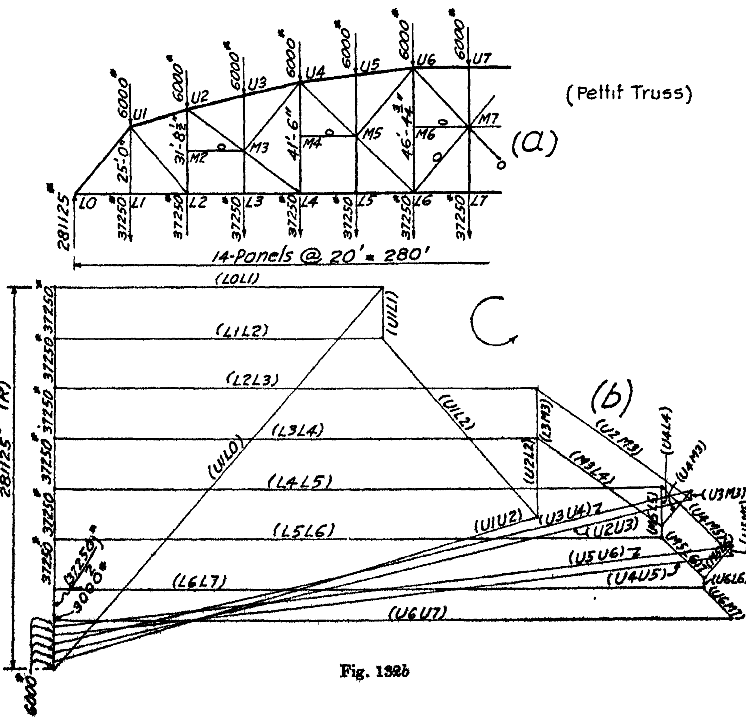


Fig. 132b

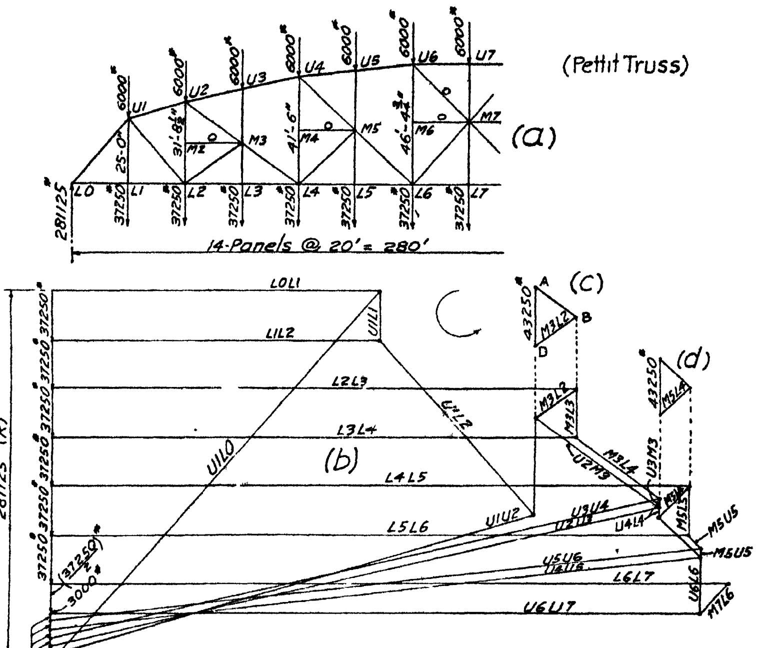


Fig. 132c

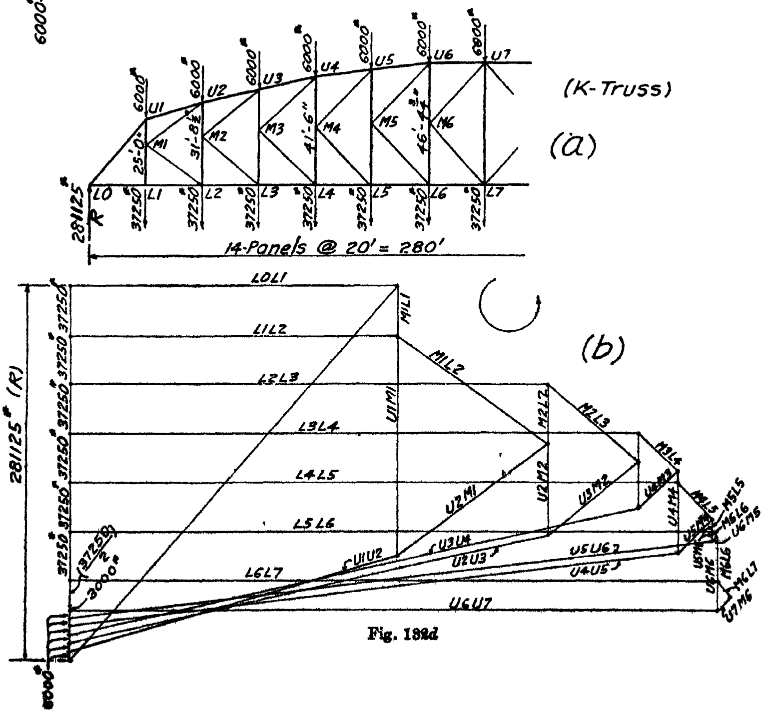


Fig. 132d

CHAPTER IX

INFLUENCE LINES

99. Definition.—An influence line is a line showing the intensity and variation of reactions, shears, moments, and stresses, produced on beams or trusses by a single moving load. Owing to convenience the single moving load is taken as a unit load.

100. Influence Line for Reactions and Shears on a Simple Beam.—Let AB, Fig. 133, represent a simple beam supporting a single moving load P and let R and $R1$ represent the reaction at A and B, respectively, due to this load P when at any point on the beam.

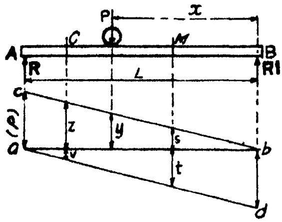


Fig. 133

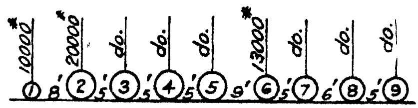
Suppose that the load P moves over the beam from B to A; the shear on all vertical sections of the beam between A and the load P, due to that load, at all times will be equal to R, which varies, of course, with the position of the load. Then for the shear at all points between A and the load, including the point at the load, we have $R = Px/L$, which is an equation to a straight line.

From the equation we have $R = P$ when $x = L$, and $R = 0$ when $x = 0$. Then if we draw the horizontal line ab $(=L)$ and erect the perpendicular ac equal to P (by scale) and draw the line cb, any ordinate as y to the line cb will be equal to the reaction at A and also to the shear on the vertical section of the beam just over the ordinate when the load P is at that point, and from this it is seen that the reaction R and the shear in the beam vary from 0 to P as the load moves from B to A. So then the line cb is the influence line for the reaction at A and also for the shear in the beam when the single moving load moves from B to A. In case the load moved from A to B, the line ad would be the influence line for the reaction at B and for the shear in the beam if bd be laid off equal to P.

As an example of application, suppose we wish to determine the reaction at A and B and the shear on the beam due to a load of 20,000 lbs. at C and a load of 30,000 lbs. at M. For the sake of illustration, suppose P, the single moving load referred to above (which is really a mythical load and is not on the beam at all and is used only to construct the influence line), is equal to 1,000 lbs. Then the ordinate z represents (by scale) what the reaction R would be if the 1,000-lb. load were at C. Then, evidently, the reaction at A, due to the 20,000-lb. load at C, would be 20 times as much or $20z$, and similarly the reaction at A due to the 30,000-lb. load at M will be $30s$, and hence the reaction at A due to both the 20,000- and 30,000-lb. loads is equal to $20z + 30s$, which is also equal to the shear anywhere between A and C, while the reaction at B and also the shear anywhere between B and M is equal to $20v + 30t$ and the shear anywhere between M and C is equal to $20z + 30s - 20{,}000$ or $20v + 30t - 30{,}000$.

The value of any ordinate, as z, s, etc., will always be given to the same scale as that used in laying off the single moving load P. It is always convenient to take this single moving load P as unity, for in that case it does not appear in the computations at all.

Example 1. Determine the maximum reaction and shear on a simple beam 30 ft. long due to the wheel loads as per diagram.



The placing of the wheels in order to obtain the maximum reaction or shear on a simple beam is really a matter of trial, yet in most cases we can determine the position by mere inspection. As in this case, we can readily see that the maximum reaction will occur when wheel 2 is at the end of the beam and wheel 1 off of the beam altogether. Then, to obtain the maximum reaction, draw the line ab (Fig. 134) to represent the length of the beam to some convenient scale, say, $\frac{3}{16}'' = 1'$-$0''$, and space the loads to the same scale, placing wheel 2 at a. Then wheels 2, 3, 4, 5, 6, and 7 will be on the beam as shown in Fig. 134. Next draw the vertical ordinate ac to represent 1 lb. to a convenient scale, say, 1 in. = 1 lb., and then draw the influence line cb. Next draw the vertical lines through the loads as shown and we are then ready to find the reaction at a, which we do in the following manner:

Take a pair of dividers, put one of the points at c and bring the other point to a, then we have the length ac on our dividers; then set one point at e and rest the other point at o $(eo = ca)$; hold the point firmly at o and open the dividers, bringing the point e to f; then we will have the length of ca and ef on our dividers. Then put one point of the dividers at g and rest the other point at o' $(o'g = ca + ef = of)$; holding the point firmly at o' bring the point at g down to h. Then put one point of the dividers at k and rest the other at o'' $(o''k = ac + ef + gh)$; holding the point firmly at o'' bring the point at k down to m, and the distance $o''m$ on the dividers equals the sum of the ordinates ac, ef, gh, and km, each of which is an ordinate under a 20,000-lb. load. Then lay the dividers on a scale divided to 1/10 inches and suppose we find that we have 3.03 inches $(= o''m)$. Then the reaction at a due to the four 20,000-lb. loads is equal to $20{,}000 \times 3.03 = 60{,}600$ lbs. Then with our dividers we get, in the same manner, the distance nt and rp, the sum of which suppose

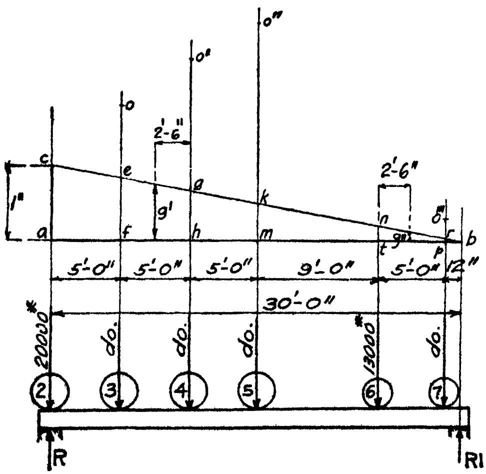


Fig. 134

we find is 0.25 of an inch. Then the reaction at a due to the two 13,000-lb. loads is equal to $13,000 \times 0.25 = 3,250$ lbs. Then the total reaction at a is $60,600 + 3,250 = 63,850$ lbs. In the case of wheel loads, as a rule, time can be saved by using the ordinates at the centers of gravity of the different groups of wheels instead of the ordinates directly under the wheels. For example, the maximum reaction in the above case is equal to

$$80,000 \times g' + 26,000 \times g''.$$

The maximum reaction at b due to the above wheel loads would be the same as at a, and would be determined in the same manner, but of course the loads and the influence line would be reversed, end for end, from the position shown in Fig. 134.

To determine the maximum shear at any intermediate point in the beam, the first thing to do after constructing the influence line is to place some wheel which we think will be at the point in question when the maximum shear occurs. Then determine the end reaction in the same manner as was shown above and subtract the intervening loads, if any, from the reaction, and the difference will be the shear at the point.

Thus, let ab (Fig. 135) represent a beam to a convenient scale, and let it be required to find the maximum shear at point d due to a system of wheel loads as shown. First construct the influence line cb, making ac

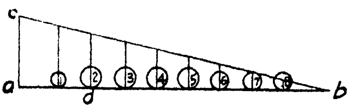


Fig. 135

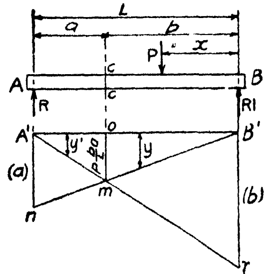


Fig. 136

equal unity. Say we place wheel 2 at d, as shown, and find the reaction at a in the same manner as was shown above. Then subtract wheel 1 from this reaction and the difference will be the shear at d. If there be any question as to this being the maximum shear at d, wheels 1 and 3, and possibly 4, should each in turn be placed at d and the shear computed. In this way we can readily ascertain the wheel to place at d for maximum shear at that point.

101. Influence Line for Bending Moments on Simple Beams.—As a general case, let AB (Fig. 136) represent a simple beam of length L and let cc be any section a distance from A and b distance from B. Imagine a single load P moving over the beam from B to A. The reaction at A due to this load P, at any time, will be $R = Px/L$. When P is to the right of cc, the moment at that section is

$$M_r = Ra = P\frac{x}{L}a,$$

which is an equation to a straight line wherein $M_r = 0$ when $x = 0$, and $M_r = P(b/L)a$ when $x = b$. Then drawing $A'B'$ $(=L)$ as a reference line, and laying off $om = P(b/L)a$, we can draw the line mB', which evidently is the influence line for the bending moment at the section cc for loads on the right of the section, as any ordinate y represents the bending moment at the section cc due to P when P is directly over that ordinate, and the moment for any other load can be obtained by direct proportion. When P is to the left of cc, the moment at that section is

$$M_l = Ra - P(x-b) = P\frac{x}{L}a - P(x-b),$$

which is also an equation to a straight line wherein $M_l = 0$ when $x = L$ and $M_l = P(b/L)a$ when $x = b$. Then, evidently, the line mA' is the influence line for the bending moment at the section cc for loads to the left of the section, as any ordinate y' represents the bending moment at the section cc due to P when P is directly over that ordinate, and the moment for any other load can be obtained by direct proportion.

The two lines $B'm$ and mA' combined would be known as the influence line, which would be referred to as influence line $A'mB'$. By prolonging the line mB' until it intersects the line of action of R at n, we have two similar triangles, $A'B'n$ and $oB'm$. From these similar triangles we have

$$\frac{A'n}{om} = \frac{A'B'}{oB'}$$

or

$$\frac{A'n}{P\frac{b}{L}a} = \frac{L}{b}$$

from which we obtain $A'n = Pa$. But if $P =$ unity, we would have $A'n = a$, and the influence line for unit load could then be constructed by making $A'n = a$, drawing $B'n$, dropping the perpendicular om, and drawing $A'm$. Or the same thing could be accomplished by laying off $B'r = b$, drawing $A'r$, dropping the perpendicular om, and drawing mB'.

Example 1. Let it be required to determine the maximum bending moment on a 40-ft. girder due to the loading given in Example 1 of the preceding article. Wheels 1 to 6 are the heaviest that can be placed on the girder, and consequently will very likely produce the maximum moment—this much being determined by mere inspection. It is first necessary to determine the center of gravity of these wheels in order to place them on the girder so as to satisfy the position for maximum moment. (See Art. 88.) In such problems it is quite convenient to determine the center of gravity by means of proportional triangles. (See Art. 45.) This is accomplished in the following manner:

First lay out the diagram of the loads to a convenient scale (say, $\frac{1}{4}'' = 1'$-$0''$) as shown on line AB in Fig. 137. We know from observation that the center of gravity of wheels 2 to 5 inclusive is at d. Then the center of gravity of wheels 2 to 5 inclusive being at d, the problem reduces to finding the center of gravity of wheels 1, 6, and the 80,000 pounds acting through d. From wheel 1 draw the line ax, making a convenient angle with line AB and lay off ab = 80,000 lbs. and bc = 10,000 lbs. (weight of wheel 1) to any convenient scale. Then join c and d and through b draw a line parallel to cd and the point e where it intersects the line AB will be the center of gravity of all of the wheels from 1 to 5 inclusive. Next, from e draw the line ex' at random, making any convenient angle with AB, and lay off ef = 13,000 lbs. (weight of wheel 6)

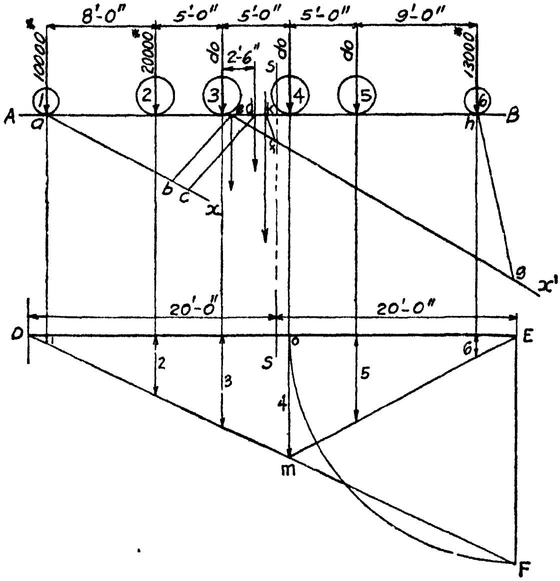


Fig. 137

and fg = 90,000 lbs. (weight of wheels 1 to 5). Then join g and h, and through f draw a line parallel to gh and the point k where it intersects the line AB is the center of gravity of all of the wheels from 1 to 6 inclusive, which was desired.

Now, as this point k is nearest wheel 4, the maximum moment will (very likely) occur under that wheel, and the line SS bisecting the distance from k to wheel 4 will be the center of the span. Then, by laying off 20 ft. on each side of this line SS to the same scale as the load diagram, we have the girder in position represented as DE. By dropping the perpendicular from wheel 4 we have the point o on the beam where the maximum moment occurs. The influence line DmE for the moment at that point is then constructed by laying off $EF = oE$ and drawing FD, om, and mE, as explained above. Then by dropping perpendiculars down from the loads, we have the ordinates 1, 2, 3, ... 6, and by multiplying each by the load above it, and adding these products, we will have the bending moment at o, which is the maximum bending moment on the beam. The sum of the ordinates 2, 3, 4, and 5

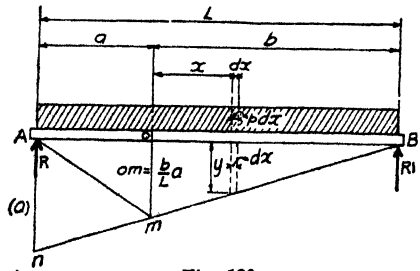


Fig. 138

can be obtained by the use of dividers as explained in the preceding article.

Example 2. As a case of uniform load, let AB (Fig. 138) represent a simple beam supporting a uniform load of p pounds per foot, and let o be any section a distance from A and b distance from B. The influence line AmB for the bending moment at o is constructed in the same manner as explained above by laying off $An = Ao = a$, then drawing nB, om, and Am in consecutive order. From proportional triangles we obtain

$$om = \frac{b}{L} a.$$

We can consider the uniform load as made up of infinitesimal loads each weighing pdx pounds. Then for the bending moment at o due to any such infinitesimal load at x distance to the right of o we have

$$dM = pdxy \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1).$$

But from proportional triangles we have

$$\frac{y}{\left(\frac{b}{L}\right) a} = \frac{b-x}{b},$$

from which we obtain

$$y = \frac{a}{L}(b-x).$$

Now substituting this value of y in (1), we have

$$dM = \frac{p}{L} a(b-x)dx \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2).$$

Then for the bending moment at o due to all such loads to the right, we have

$$M = \frac{p}{L} a \int_0^b (b-x)dx = p\left[\left(\frac{1}{2} \times \frac{b}{L} a\right) b\right].$$

But $\frac{1}{2}(b/L)a \times b$ = area of the triangle omB, since the ordinate $om = (b/L)a$. So we have the bending moment at o due to the uniform load to the right equal to the area of the triangle omB multiplied by the uniform load per linear foot of span. In a similar manner it can be shown that the bending moment at o due to the uniform load to the left of o is equal to the area of the triangle omA multiplied by the uniform load per linear foot. Then letting M represent the total bending moment at o due to the total uniform load on the girder, we have

$$M_t = p\left(\frac{b}{L} a \times \frac{b}{2}\right) + p\left(\frac{b}{L} a \times \frac{a}{2}\right) = p\left(\frac{b}{L} a \frac{(b+a)}{2}\right)$$
$$= \text{area of triangle } AmB \times p \dots\dots (3)$$

that is, the total bending moment at o is equal to the area of the triangle AmB (formed by the influence line and the reference line) multiplied by the uniform load per linear foot of span.

If the section be taken at the center of the span, we have $b = a = L/2$. Substituting in equation (3), we have

$$M = p\left(\frac{L}{4} \times \frac{L}{2}\right) = \frac{pL^2}{8},$$

which is readily recognized as the formula for the bending moment at the center of a beam uniformly loaded.

102. Influence Lines for Shears and Bending Moments on Trusses.—Let the diagram at (*a*), Fig. 139, represent a simple truss and let it be required to construct an influence line for the shear in any panel as *CD*. It is evident that any load as *W* when at *D* or to the right of *D* will produce tension in the diagonal *UD*, while any load as *W*1 when at *C* or to the left of *C* will produce compression in the same diagonal.

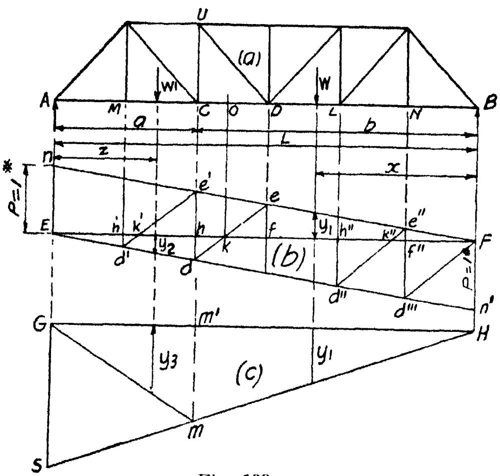


Fig. 139

Then evidently there must be some point between *C* and *D* where, if a load were placed, the stress in the diagonal due to the load would be zero. Then evidently the influence line for the shear in the panel will be of the form *EdeF* shown at (*b*), which is readily constructed for a unit load by laying off *En* and *Fn'* each equal to unity and dropping the perpendiculars *ef* and *hd* under the panel points and drawing *ed*. The shear in the panel producing tension in the diagonal *UD*, due to any load as *W* when at any point *x* distance from *B*, is equal to Wy_1, y_1 being the ordinate under the load as shown, while the shear in the panel producing compression in the diagonal due to any load as *W*1 is equal to $W1y_2$.

It is evident then that to obtain the maximum shear in panel *CD* producing tension in the diagonal *UD*, which we will call positive shear, there should be no loads to the left of *O*, and to obtain the maximum shear producing compression in the same diagonal, which we will call negative shear, there should be no loads to the right of *O*. That is, one kind of shear will be a maximum when the loads extend from *B* to *O*, and the other kind will be a maximum when the loads extend from *A* to *O*.

In case of a uniform live load, the maximum positive shear would be equal to the area of the triangle Fke multiplied by the load per foot, while the maximum negative shear would be equal to the area of the triangle Edk multiplied by the load per foot. In case of uniform dead load, the shear is equal to the difference of the areas of the two triangles Edk and Fke multiplied by the dead load per foot. In the case of concentrated live loads, as wheel loads, to obtain the maximum shear in the panel the loads would be so placed that a load would be at the panel point and the front load as near the point O as possible (not beyond). For example, to obtain the maximum positive shear in panel CD a load would be placed at D, the one that would bring the front load closest to the point O, and to obtain the maximum negative shear the load would be placed at C that would bring the front load as near O as possible. The same result is obtained in this manner (graphically) as would be obtained by satisfying the criterion of Art. 90.

The influence line for the shear in each of the other panels can be drawn on the same preliminary construction at (b) as used for panel CD. For example, the influence lines for panel MC, LN, NB are $Ed'e'F$, $Ed''e''F$, and $Ed'''F$, respectively. The construction of each is accomplished in the same manner as explained above in the case of panel CD.

In the case of trusses, the bending moments are desired only at the panel points. The influence lines for the moments at these points are constructed the same as though the points were on a simple beam, which was treated in the preceding article. For example, the influence line for bending moment at the panel point C of the truss represented at (a) (same for point U) is constructed as shown at (c) by laying off $Gs = a$, the distance of the panel point from A, and drawing sH, mm', and mG in consecutive order, as explained in the last article for simple beams. The influence line for the bending moment at any other panel point would be constructed in the same manner. The moment at any panel point would be obtained in the same way as explained in the preceding article for the case of any point on a simple beam. The exact placing of concentrated loads for maximum moments at the different panel points would be according to Art. 91.

103. Influence Lines for Stresses in Truss Members.—Instead of constructing influence lines for the shears and moments as in the last article and then determining the stresses in the truss members from these, it is more convenient to construct influence lines for the stresses in the members, thereby obtaining the maximum stresses directly from the influence lines without dealing with either shears or moments.

Let the diagram at (a), Fig. 140, represent a truss and let it be required to construct an influence line for the stress in the diagonal ED. First draw the line $A'B'$ at (b) ($=L$) and lay off $A'n$ and $B'n'$ each equal to unity and draw nB' and $A'n'$. Then by drawing the ordinates de and fb and the line db we have the influence line $A'bdB'$ for the shear in the panel CD the same as in the preceding article. The ordinate ed is equal to the positive shear that would be produced in the panel by a unit load at D, and the ordinate fb the negative shear in the panel that would be produced by a unit load at C. Now, as the stress in the diagonal ED varies as the shear in the panel, it is evident that if the ordinates de and bf were laid off to represent the stress in the diagonal that would be produced by a unit load at these same points and an influence line be constructed accordingly, we would have an influence line for the stress in the diagonal.

For example, if $e'd'$, at (c), were equal to the stress produced in the diagonal ED by a unit load at D, and $f'b'$ were equal to the stress produced in the same by a unit load at C, the line $A''b'd'B''$ would be the influence line for the stress in the diagonal. And likewise, if $f't''$, at (c), and $r''e'$ were equal, respectively, to the stress produced in the post EC by a unit load at C and D, the line $A''t''r''B''$ would be the influence line for the stress in the post.

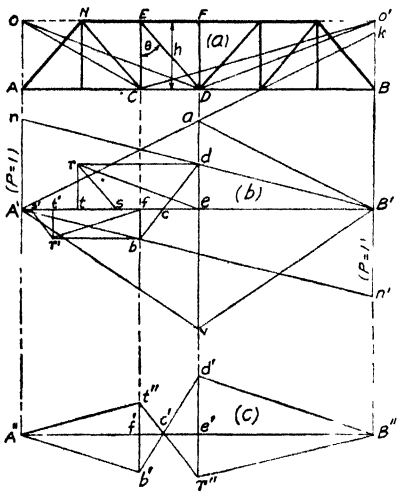


Fig. 140

Laying off $B'k$ at (b), equal to BD and drawing kA', ea, and aB' in consecutive order, we have the influence line for the bending moment at panel point D, as explained in the last article. Now, as the stress in the top chord EF varies directly as this moment, it is evident that if ev, at (b), were equal to the stress produced in the chord by a unit load at D, the line $A'vB'$ would be the influence line for the stress in the chord EF. From the above it is evident that an influence line for the stress in any truss member can be drawn as readily as the influence lines for the moments and shears on the truss by computing the maximum stress in the member considered, due to a unit load and using this value as the ordinate in establishing the influence line. The stresses due to the unit load can be determined most readily by graphics.

As an example, let us first construct the influence line for the stress in the top chord EF. The first thing to do is to determine the stress in the member due to a unit load at D. Imagine OO' (drawn through the top chord) to be a beam, and each of the lines OD and DO' to be a rod or a rope. Then we would have a structure $OO'D$ such that the stress in the member OO', produced by a unit load at D, is the same as the stress produced in the top chord EF by a unit load at the same point. Now this structure $OO'D$ is very readily analyzed graphically. The reaction at O due to a unit load at D is given by the ordinate de, at (b). Then considering the point O and drawing from d a line parallel to OO' and from e a line parallel to OD intersecting the first line at r, we have the stress in OO' given by the line dr which is also the stress in the top chord EF. Then from e lay off ev equal to dr, just found, and the influence line $B'vA'$ for the stress in the top chord EF is drawn.

Next, let us construct the influence line for the stress in the diagonal ED. The stress in the diagonal corresponding to positive shear due to a unit load at D is given by the line rs, at (b), which is drawn from r parallel to the diagonal, and as bf is equal to the negative shear in the panel that would be produced by a unit load at C, by drawing the lines OC and CO' and br' and $r'f$ parallel, respectively, to OO' and CO', and $r's'$ parallel to the diagonal, we will have the stress in the diagonal that would be produced by a unit load at C given by this last line $r's'$. Then by laying off $e'd' = rs$, at (c), and $f'b' = r's'$, the influence line $A''b'd'B''$ is readily constructed. In like manner the influence line $A''t''r''B''$ for the stress in the post EC is readily constructed by laying off the ordinates $e'r'' = rt$ (given at (b)) and $t''f' = r't'$, as it will be readily seen that rt is equal to the stress in the post due to a unit load at D, and $r't'$ is equal to the stress in the same due to a unit load at C. The influence line for the stress in any member of any truss can be readily constructed in the same manner as shown for the above cases.

There are other methods employed to determine the stresses in the truss members due to the unit load, and these will be presented later as the practical application of influence lines is taken up.

CHAPTER X

DESIGN OF I-BEAMS AND PLATE GIRDERS

104. General Data.—

Specifications, A. R. E. Ass'n.* (For steel railroad bridges.)

Allowable bending stress on beam and girder flanges . 16,000 lbs. per sq. in.
Allowable shearing stress on shop rivets . . . . 12,000 lbs. per sq. in.
Allowable shearing stress on field rivets and webs. . . . 10,000 lbs. per sq. in.
Allowable bearing stress on shop rivets . . . . 24,000 lbs. per sq. in.
Allowable bearing stress on field rivets. . . . . 20,000 lbs. per sq. in.
Allowable bearing on masonry. 600 lbs. per sq. in.

DESIGN OF I-BEAMS

105. Outline of Usual Method of Procedure.—In designing I-beams the first thing to do is to determine the maximum bending moment in inch pounds. Then, using Formula C, Art. 53, we have the bending moment

$$M=\frac{fI}{y};$$

dividing through by f, we have

$$\frac{M}{f}=\frac{I}{y}.$$

The quantity I/y is known as the "Section Modulus," which is given in Tables 1 and 2 in the back of this book, for practically all I-beams, and the same will be found in structural handbooks, such as Carnegie, Cambria, etc. So the size of an I-beam required to resist a known bending moment can be determined by simply dividing the bending moment (in inch pounds) by the allowable stress, which is specified above as 16,000 lbs., thus obtaining the required section modulus, and from the tables we find the lightest I-beam having this or approximately this modulus and that will be the beam to use.

For example, suppose the bending moment for a given span is 433,000 inch pounds. Then the section modulus of the beam required for the span is

$$\frac{433,000}{16,000}=27.1.$$

Glancing over column 10 of Table 1 we find that a 10-inch by 30-pound beam, which has a section modulus of 26.8, is the nearest to the required beam, and hence would be used.

After an I-beam is designed to resist the maximum bending moment, it can be tested for shear, by dividing the maximum shear by the area of

* These specifications can be obtained from the American Railway Engineering Association, Chicago, Ill.

the cross-section of the beam. If this should exceed 10,000 lbs. per square inch, the permissible intensity, a heavier beam should be used or the web of the beam should be reinforced. However, there are but very few cases where the shear affects the design of an I-beam, owing to the webs of I-beams being comparatively thick.

An I-beam can be designed by using Formula D,

$$f=\frac{My}{I},$$

and finding values for y and I that will give $f=16,000$ (approximately). But, as is readily seen, this is not a very convenient method of procedure.

106. Example 1.—Design an I-beam of 15-ft. span to support a total uniform load of 800 lbs. per foot.

The maximum bending moment $(=pL^2/8) = \frac{1}{8}\times 800\times 15^2\times 12 =$ 270,000 inch lbs.

Then, $$\frac{270,000}{16,000}=16.8=\text{section modulus}.$$

The I-beam having a section modulus nearest this value is the 8-in. by 25.5-lb., but the 9-in. by 21-lb. would be used as it is lighter. It is seen that the 8-in. beams are all too small except the 25.5-lb. beam.

107. Example 2.—Determine the size of I-beam required in the case of the simple beam shown in Fig. 141, where AB represents the

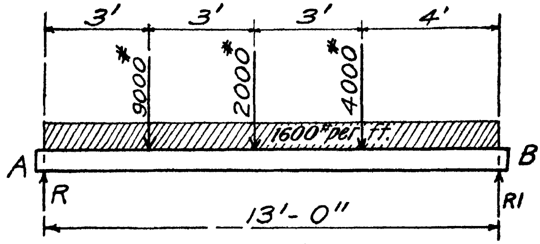


Fig 141

beam supporting the loads indicated. By taking moments about B we find $R=19,630$ lbs. Adding up the forces, beginning at A, we find that the shear passes through zero at the 2,000-lb. load, hence the maximum bending moment occurs at that load. (See Art. 88.) Then for the maximum bending moment we have

$$M=[19,630\times 6-(1,600\times 6\times 3)-(9,000\times 3)]12=743,760 \text{ inch lbs.}$$

Then, $$\frac{743,760}{16,000}=46.4=\text{Section Modulus}.$$

So a 15-in. by 42-lb. I-beam would be used. The 12-in. by 40-lb. is too light and 12-in. by 45-lb. is heavier than the 15-in. by 42-lb. beam.

108. Example 3.—Determine the size of I-beam required in the case of the simple beam shown in Fig. 142, where AB represents the beam supporting a load which varies from 0 at A to 2,000 lbs. at B, as indicated.

For convenience, let p (=2,000 lbs.) represent the intensity of the load at B. Then for the total load on the beam (neglecting the weight of the beam) we have

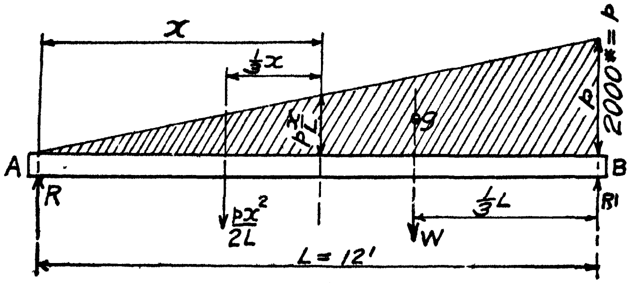


Fig 142

$$W=\frac{pL}{2}$$

and for the intensity of the load at any point x distance from A we have

$$\frac{px}{L}.$$

Now, taking moments about B we have

$$R=\frac{W\frac{L}{3}}{L}=\frac{W}{3}=\frac{pL}{6}$$

for the reaction at A.

Now for the bending moment at any point x distance from A, we have

$$M=\frac{pL}{6}x-\frac{px^3}{6L}\ldots\ldots(1).$$

When this moment is a maximum

$$\frac{dM}{dx}=\frac{pL}{6}-\frac{px^2}{2L}=0\ldots\ldots(2)$$

(which is obtained by differentiating (1)). This, as is readily seen, will occur when $L^2=3x^2$, from which we obtain

$$x=L\sqrt{\frac{1}{3}}=0.58L \text{ (approximately)}.$$

So the point of maximum bending moment is $0.58L$ from A. Then substituting $0.58L$ for x in equation (1) we have for the maximum bending moment

$$M=\left(\frac{p}{6}\right)0.58L^2-\left(\frac{p}{6}\right)0.19L^2=\frac{p}{6}\,(0.39L^2),$$

and substituting the numerical value of L and p, and multiplying by 12, to reduce the moment to inch pounds, we have

$$M=\frac{2{,}000}{6}(0.39\times144\times12)=224{,}600 \text{ inch lbs.}$$

for the maximum bending moment.

Then, $$\frac{224,600}{16,000} = 14 = \text{section modulus.}$$

So an 8″ x 18# I-beam would be used.

In case the weight of the beam were considered, we would simply include the moment of this weight in an equation for the bending moment, corresponding to (1). Thus, if the weight of the beam be w pounds per foot, the equation for the bending moment at any point x distance from A would then be

$$M = \frac{pL}{6}x - \frac{px^3}{6L} + \frac{wL}{2}x - \frac{wx^2}{2} \quad \text{(3).}$$

Then by placing $dM/dx = 0$, the point of maximum moment can be determined and then the maximum bending moment at that point due to the combined loads can be obtained and the beam designed accordingly.

109. **Example 4.**—Determine the size of I-beam required in the case of the overhanging beam shown in Fig. 143, where ABC represents the beam supporting the loads indicated.

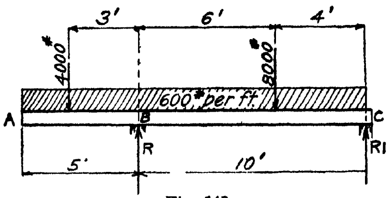


Fig. 143

In this case there are two bending moments to consider; one at B and the other at the point of zero shear in the span BC.

For the bending moment at B we have

$$M = (4,000 \times 3 + 600 \times 5 \times 2.5)12 = 234,000 \text{ inch lbs.}$$

Then taking moments about C we have

$$R = \frac{1}{10}(600 \times 15 \times 7.5 + 8,000 \times 4 + 4,000 \times 13) = 15,150 \text{ lbs.}$$

for the reaction at B. Then for the reaction at C we have the total load on the beam minus R, that is,

$$R1 = 21,000 - 15,150 = +5,850 \text{ lbs.}$$

Now beginning at C and adding up the forces toward the left we find that the shear changes signs, that is, passes through zero, at the 8,000-lb. load. So the maximum bending moment in span BC occurs at that load. Then taking moments about that load and considering the forces to the right of it, we have

$$M' = (5,850 \times 4 - 600 \times 4 \times 2)12 = 223,200 \text{ inch lbs.}$$

for the maximum bending moment in span BC. The bending moment at B is the greater and hence the I-beam must be designed for that moment.

So, using the moment at B, we have

$$\frac{234,000}{16,000} = 14.6 = \text{section modulus},$$

and hence an 8-in. by 18-lb. I-beam would be used.

110. Example 5.—Determine the size of I-beam required in the case of the continuous beam shown in Fig. 144, where ABC represents the beam supporting 4 ft. of uniform load in span AB and a single load of 18,000 lbs. in span BC.

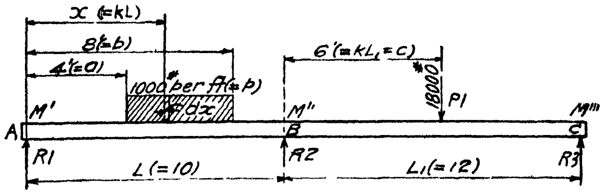


Fig 144

Applying the three-moment equation, (L),

$$M'L + 2M''(L+L_1) + M'''L_1 = -PL^2(k-k^3) - P'L_1^2(2k_1 - 3k_1^2 + k_1^3),$$

given in Art 70, we have both M' and $M''' = 0$, and the quantity

$$-PL^2(k-k^3) = -\int_a^b pdxL^2\left(\frac{x}{L} - \frac{x^3}{L^3}\right) = -p\left(\frac{b^2L}{2} - \frac{b^4}{4L} - \frac{a^2L}{2} + \frac{a^4}{4L}\right)$$

and the quantity

$$-P'L_1^2(2k_1 - 3k_1^2 + k_1^3) = -P'\left(2cL_1 - 3c^2 + \frac{c^3}{L_1}\right)$$

Then substituting these values in the general three-moment equation, we have

$$2M''(L+L_1) = -p\left(\frac{b^2L}{2} - \frac{b^4}{4L} - \frac{a^2L}{2} + \frac{a^4}{4L}\right) - P'\left(2cL_1 - 3c^2 + \frac{c^3}{L_1}\right) \ldots (1).$$

Now everything in this equation is known except M'', the bending moment at support B. Then by substituting the numerical values of the known quantities, as given in Fig. 144, and reducing, we have

$$M'' = -25,363 \text{ ft. lbs.} = -304,356 \text{ inch lbs.}$$

for the bending moment at B.

Then taking moments about B, we have

$$-25,363 = 10R1 - 4,000 \times 4,$$

from which we obtain

$$R1 = -936 \text{ lbs.}$$

for the reaction at A, and taking moments about B and considering span BC, we have

$$-25,363 = 12\,R3 - 18,000 \times 6,$$

from which we obtain

$$R3 = +6,884 \text{ lbs.}$$

for the reaction at C.

Now, as the three reactions must be equal to the total load on the beam, we have

$$22,000 = -936 + R2 + 6,886,$$

from which we obtain

$$R2 = +16,050 \text{ lbs.}$$

for the reaction at B.

Now, as all of the reactions are determined, the moments and stresses on the above beam can be determined as readily as for any beam. There are three bending moments to consider in the above case; the maximum in each of the spans and the one at support B.

The reaction $R1$ being minus, it is readily seen that the maximum bending moment in span AB will occur at support B, and as span BC supports only the one load it is evident that the maximum bending moment in that span will occur under the load. Then we really have only the moment under the 18,000-lb. load yet to determine as the moment at B is determined above.

Then, taking moments about the 18,000-lb. load, we have

$$M_1 = 6 \times R3 = 6 \times 6,884 = 41,316 \text{ foot lbs.} = 41,316 \times 12 = 495,792 \text{ inch lbs.}$$

for the bending moment at that load, which is greater than the moment at B and hence is the maximum bending moment on the beam, and consequently the beam must be designed to resist this moment. Thus we have

$$\frac{495,792}{16,000} = 31.0 = \text{section modulus.}$$

So a 12-in. by 31.5-lb. I-beam would be used.

DESIGN OF PLATE GIRDERS

111. Description.—A plate girder is, for the most part, a built I-beam composed of a plate web and angle flanges which are riveted to the edges of the plate as shown at (*a*), Fig. 145. Yet in addition to these parts there are usually vertical angles, known as stiffeners, riveted to the web, as shown at (*b*), to stiffen the web against buckling. The stiffeners are either bent around the flange angles as shown at (*c*) or filler plates are placed between them and the web, as shown at (*d*). When they are bent around the flange angles we say they are crimped. It is practice to limit the thickness of metal to about $\frac{5}{8}$ of an inch and when the area required in a flange is greater than that of two $6'' \times 6'' \times \frac{5}{8}''$ ∠s, plates are riveted to the backs of the outstanding legs of the flange angles as shown at (*e*), to provide for the additional area required. These plates are known as cover plates, or flange plates. The area of the cover plates in any flange should never be much greater than that of the two flange angles, and when they exceed the area of the angles very much, plates, known as side plates, are placed between the flange angles and the web as shown at (*f*) so that the area of the cover plates can be reduced. In this case the side plates are considered as part of the flange. There are other types of flanges used occasionally which will be shown later.

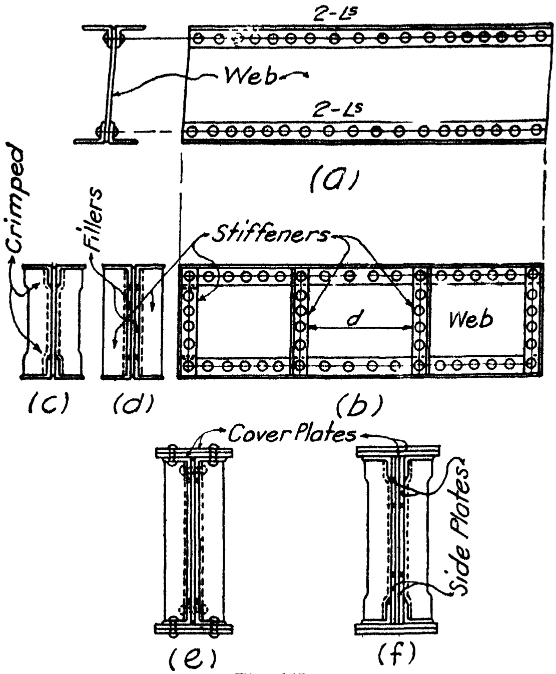


Fig 145

112. Stress and Area in Flange.—In case of a beam composed of one piece, as an I-beam, the stress due to cross bending is obtained through the application of Formula D, Art. 53, but in the case of a plate girder, while the same method would apply, quite a different one is used, wherein the web and flanges are treated separately. The cross bending is resisted by the flanges and web combined, but the resistance of the web is ignored by some engineers, while others consider it. So we really have two cases to consider.

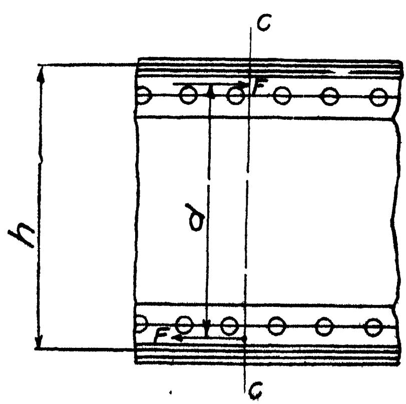


Fig. 146

In case the resistance of the web be ignored, the stresses in the two flanges form a couple which will be equal and opposite to the algebraic sum of the external couples on either side of any cross-section—which is the same thing as the bending moment.

Let Fig. 146 represent a portion of a plate girder:

Let M = bending moment at section cc;

F = total stress in each flange at that section;

d = distance between the centers of gravity of the flanges, which is known as the effective depth.

Then, in accordance with the condition of equilibrium, we have

$$M=Fd \text{ or } F=\frac{M}{d} \qquad (1).$$

If f be the allowable unit stress, the area required for each flange will be

$$A=\frac{F}{f}=\frac{M}{fd} \qquad (2).$$

This is assuming that the unit stress is the same over the entire cross-section of each flange—an assumption invariably made, although not absolutely true in any case, but quite accurate enough for practical designing.

In case the resistance of the web be considered, the bending moment will be resisted by the flanges, the same as shown above, aided by the web, which is really a rectangular beam. Then for the bending moment we have

$$M = Fd + \frac{f'I}{\frac{h}{2}} \qquad (3),$$

where f' represents the unit stress on the extreme elements of the web, and h and I represent, respectively, the height and moment of inertia of the web, while the other letters signify the same as they do in equation (2), except F, of course, is less.

Now, let t be the thickness of the web, A' its area of cross-section and A the area of each flange; then we have

$$A'=th, \quad I=\frac{1}{12}th^3$$

and as the unit stress on the extreme elements of the web is practically the same as that of the flanges, we have $f'=f$ and also $F=f'A=fA$. Then by substituting these values in (3), we have

$$M=fAd+\frac{fA'h}{6}.$$

But $h = d$, practically, so we have

$$M=fAd+\frac{fA'd}{6}=fd\left(A+\frac{A'}{6}\right),$$

from which we obtain

$$\left(A+\frac{A'}{6}\right)=\frac{M}{fd} \qquad (4),$$

which shows that one-sixth of the area of the web appears as flange area, but owing to the moment of inertia of the web being reduced in most cases by rivet holes, one-eighth is assumed in practice instead of one-sixth, in which case we have

$$A+\frac{A'}{8}=\frac{M}{fd}\qquad\text{(5)},$$

from which we obtain

$$A=\frac{M}{fd}-\frac{A'}{8}\qquad\text{(6)}$$

for the area required in each flange.

113. Economic Depth.—It is seen from the preceding article that the area, and consequently the weight, of the flanges of a plate girder varies inversely as the depth of the girder and it is evident that the weight of the web, stiffeners, and fillers varies directly as the depth. Then evidently a plate girder will have a theoretical economic depth when the weight of the two flanges equals the combined weight of the web, stiffeners, and fillers. There are really two cases to consider, which are as follows:

Case I. When the web is not considered to resist cross bending.

Let M = maximum bending moment in inch pounds;
L = length of girder in feet;
f = allowable unit stress on flanges;
t = thickness of web in inches;
W = total weight of girder in pounds;
x = depth of girder in inches back to back of flange angles.

Girders without cover plates. For the area of the cross-section of the two flanges we have

$$\frac{2M}{fx}$$

assuming depth and effective depth as being equal, and for the weight of the two flanges we have

$$2\frac{M}{fx}\times 3.4L.$$

As a bar of steel one square inch in cross-section and one foot long weighs 3.4 pounds, the total weight of the web alone is equal to $3.4Ltx$ and the total weight of the stiffeners, fillers, splices, etc., usually runs about 60 per cent of the weight of the web, so the total weight of the web, stiffeners, fillers, etc., can be taken as $1.6(3.4Ltx)$.

Now adding the weight of the web, stiffeners, fillers, etc., to that of the flanges, we have

$$W=2\frac{M}{fx}\times 3.4L+1.6(3.4Ltx)\qquad\text{(1)}$$

for the total weight of the girder.

This will be a minimum when

$$\frac{dW}{dx}=-2\frac{M}{fx^2}\times 3.4L+1.6(3.4Lt)=0,$$

which is obtained by differentiating (1).

From this we obtain for the economic depth

$$x = 1.12\sqrt{\frac{M}{ft}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Girders with cover plates. In this case the area of the flanges varies from the center to the ends of the span. If the cover plates have theoretical lengths, the average cross-section of each flange will be about 0.75 of the maximum area. So for the total weight of the two flanges we have

$$1.5\frac{M}{fx}(3.4L).$$

Now substituting this instead of $2(M/fx) \times (3.4L)$ in (1) and differentiating and reducing we obtain for the economic depth

$$x = .97\sqrt{\frac{M}{ft}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

Case II. When the web is considered to resist bending moment.

By assuming the web to resist bending moment each flange can be reduced in area to the amount of one-eighth of the area of the cross-section of the web, and hence the total weight of the girder would be reduced an amount equal to $2(\frac{1}{8}tx)3.4L$.

Then subtracting this from (1) we have

$$W = 2\left(\frac{M}{fx}\right)3.4L + 1.6(3.4Ltx) - \left(\frac{2}{8}tx\right)3.4L \quad \ldots\ldots\ldots\ldots (4)$$

for the total weight of the girder. Differentiating this equation and placing the first derivative $= 0$, and reducing, we obtain

$$x = 1.22\sqrt{\frac{M}{ft}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

for the economic depth of girders without cover plates, and by substituting $1.5(M/fx) \times (3.4L)$ for $2(M/fx) \times (3.4L)$ in (4) and differentiating and placing the first derivative $= 0$, and reducing, we obtain

$$x = 1.055\sqrt{\frac{M}{ft}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

for the economic depth of girders with cover plates.

The theoretical economic depth of plate girders can be computed from the above formulas. An inch or two either way from the theoretical depth will affect the design but little.

114. Length of Cover Plates.—A flange of a plate girder varies in area along the girder practically as the ordinates of a parabola, the same as the bending moment. (See Art. 56.) Then, if AB (Fig. 147) represents the length of a plate girder and OC the total area of the cross-section of one flange at the center of the span, any ordinate x to the parabola ABC will represent (approximately) the area of the flange at that point.

Suppose the flange of this girder to be composed of two angles and three cover plates.

Let $a1$ = area of top plate;
$a2$ = area of second plate from the top;
$a3$ = area of third plate from the top;
$a4$ = area of the two angles; and
A = total area of the flange.

Theoretically, net areas should be used throughout and one-eighth of the area of the web should be included with the area of the angles, but to provide against discrepancies that may result by assuming that the bending moment varies along the girder as the ordinates to a parabola (which is not absolutely true in the case of concentrated loads), we will use gross areas throughout and neglect the one-eighth of the web in determining the length of cover plates.

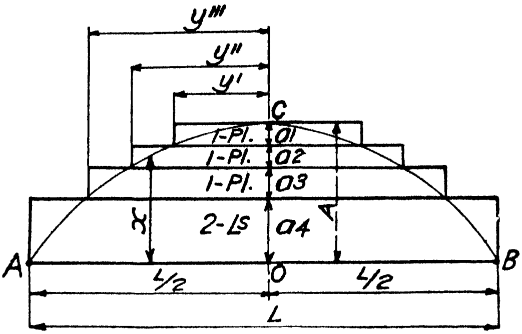


Fig. 147

Now, if these areas be laid off to scale, on the line OC (Fig. 147), the theoretical diagram of the cover plates can be drawn as shown and then the theoretical length of each plate can be determined by scale. In this manner the length of the cover plates on any plate girder can be graphically determined, but in practice the lengths are usually computed from the formulas given below, as that is the more convenient way.

In accordance with the properties of the parabola we have, referring to Fig. 147,

$$\frac{y_1^2}{\left(\frac{L}{2}\right)^2}=\frac{a_1}{A},$$

from which we obtain

$$y_1=\frac{L}{2}\sqrt{\frac{a_1}{A}}$$

for the half length of the top cover plate, and multiplying this by 2, we have

$$2y_1=L\sqrt{\frac{a_1}{A}} \quad \text{.................................} (1)$$

for the full theoretical length. Likewise we have

$$\frac{y''^2}{\left(\frac{L}{2}\right)^2}=\frac{a1+a2}{A},$$

from which we obtain

$$y''=\frac{L}{2}\sqrt{\frac{a1+a2}{A}}$$

for the half length of the second cover plate from the top, and multiplying this by 2, we have

$$2y''=L\sqrt{\frac{a1+a2}{A}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

for the full theoretical length of that plate.

In the same manner we obtain

$$2y'''=L\sqrt{\frac{a1+a2+a3}{A}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

for the full length of the third cover plate from the top, and further, we have

$$2\left(\frac{L}{2}\right)=L\sqrt{\frac{a1+a2+a3+a4}{A}}=L$$

for the full length of the angles.

Now from the above it is readily seen that the general formula for the theoretical length of cover plates can be written as

$$l=L\sqrt{\frac{a1+a2\ \ldots\ +an}{A}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a).$$

Then for the length of the first or outside plate, in either the top or bottom flange, we have

$$l'=L\sqrt{\frac{a1}{A}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (b),$$

for the second plate from the top or bottom we have

$$l''=L\sqrt{\frac{a1+a2}{A}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (c),$$

and for the third we have

$$l'''=L\sqrt{\frac{a1+a2+a3}{A}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (d),$$

and so on.

The theoretical lengths of cover plates can be determined very readily by the use of the ordinary slide rule. When a slide rule is used, Formula (a) should be squared, thereby obtaining

$$l^2=\frac{L^2}{A}(a1+a2\ \ldots\ +an).$$

Then, first the quantity L^2/A can be set off on the upper scale, once for all. By multiplying this by $a1$, $(a1+a2)$, $(a1+a2+a3)$, and so on, we obtain, respectively, the square of the lengths of the first, second, third, etc., cover plate, and at the same time the square root of this in each case (which is the length desired) is read off on the bottom scale. Thus the length of each cover plate on a girder can be determined by one setting of the rule.

115. Increment of the Flange Stress.— Imagine the web of the girder shown at (*a*), Fig. 145, to be made up of vertical strips each of

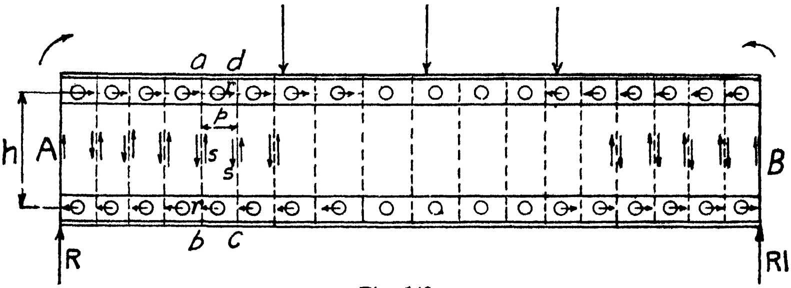


Fig 148

which is connected to each flange by one rivet, as shown in Fig. 148.

Suppose the girder supports a number of loads and let R and $R1$ be the reactions due to these loads and for the sake of simplicity suppose both of the reactions and all of the loads to be applied directly to the web.

The loads tend to move the girder downward as a whole; this motion is prevented by the reactions, and through the balancing of this activity results first a vertical shearing couple on each of the imaginary strips which tends to rotate each strip, but rotation of each is prevented by the rivets connecting it to the flange angles, whereby the flange stress results. To show this, let us first consider the end strip at A. The shearing couple on that strip tends to rotate it clock-wise which causes the strip to exert (through the connecting rivet) a force to the right upon the top flange angles and an equal force to the left upon the bottom flange angles, and the same is true of the second strip from the end A, and of the third, and so on, while, as is readily seen, the shearing couple on each of the strips near end B tends to rotate each of those strips in the opposite direction, or counter clock-wise, and hence the force exerted on the top flange angles by each will be to the left, while the equal force in each case will be exerted to the right upon the bottom flange angles.

The end rivet in the top flange angles at A and the corresponding end rivet at B, acting toward each other, produce a simple compressive stress in the flange angles throughout the distance between these rivets, and, similarly, the second rivets from the ends, acting toward each other, will produce a like stress in the angles throughout the distance between those rivets, and the third rivets from the ends will produce a like compressive stress in the flange angles from one rivet to the other, and so on. The same is true of the bottom flange except the direct stress produced there is tension.

Thus it is seen that the stress in the flanges of a simple plate girder is increased by each flange rivet as we pass from either end up to the point of maximum moment. (See Art. 60.) This increase at each rivet is the increment of the flange stress, which is often referred to as the "flange increment." The summation of these increments between any two points would be known as the increment of the flange stress between the two points.

116. Spacing of Rivets in the Vertical Legs of Flange Angles.—The rivets in a plate girder are practically always the same size throughout the girder, in which case the rivets in the vertical legs of the flange angles should be so spaced that the pressure against the flange angles would be the same for each rivet throughout the girder, and hence the stress on each rivet would be the same throughout. So the problem involved is to determine the pitch of the rivets so that the increment of the flange stress is just equal to the allowable stress on the flange rivet at all points along the flange.

Let S be the shear on any strip *abcd* (Fig. 148) of longitudinal length p, and let r represent the increment of the flange stress at that strip. Now as the couple rh is equal to the equal and opposite couple reacting on the strip and balancing the shearing couple Sp, we have

$$Sp = rh,$$

from which we obtain

$$p = \frac{rh}{S} \quad \ldots\ldots\ldots\ldots (1).$$

From this the length of any strip can be computed for any desired value of r. But the length of any strip can, in all cases, be taken as the pitch of the rivets at the same point and hence if r be taken as the allowable stress on one rivet, the required pitch of the rivets in the vertical legs of the flange angles at any point can be computed from this formula.

It is readily seen (from Fig. 148) that these rivets would fail either in double shear or in bearing on the web, and r, in any case, should be taken equal to whichever is the least. The bearing on the web is usually the least.

There are really four cases to consider:

Case I. When the loads are applied directly to the web and the resistance of the web to bending is neglected. In this case the pitch of the rivets at any point of the flange is determined from the above equation,

$$p = \frac{rh}{S} \quad \ldots\ldots\ldots\ldots (2)$$

where p = pitch;

r = allowable stress on one rivet in either double shear or bearing on the web, whichever is the least;

h = vertical distance between the rivets in the two flanges;

S = shear on the girder at the point considered.

Case II. When the loads are applied directly to the web and the resistance of the web to bending is considered. In this case a certain part of the shear couple Sp is resisted by the web.

Let S = shear on the girder at any point;

A = area of the cross-section of one flange at any point;

A' = area of the cross-section of the web;
f = stress per square inch transmitted to each flange by each rivet;
r = pressure exerted upon the flange by each rivet;
h = vertical distance between the rivets in the two flanges;
p = pitch of flange rivets.

It is shown in Art. 112 that the resistance of the web to cross bending can be accounted for by considering one-eighth of its area as being concentrated in each flange. So the portion of the shearing couple Sp resisted by the web can be approximately expressed as $fA'h/8$ and the remaining portion of the couple which is resisted by the flanges can be approximately expressed as fAh. Then adding these two expressions, we have

$$fAh + \frac{fA'h}{8} = Sp.$$

But, $$f = \frac{r}{A};$$

then substituting this value of f in the last equation and reducing, we have

$$p = \frac{rh}{S}\left(1 + \frac{A'}{8A}\right) \quad \text{. . . . .} \quad (3)$$

for the pitch of the rivets.

Case III. When the loads are applied directly to the flange angles and the resistance of the web to bending is neglected. In this case the loads will exert a vertical force upon the rivets in addition to the horizontal force considered above. Let v (Fig. 149) represent the vertical force per linear inch of flange and r the horizontal force on the rivet (the same as above). Then for the resultant force we have

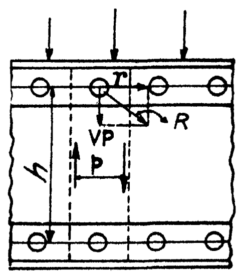


Fig. 149

$$R^2 = (vp)^2 + r^2.$$

But from (2) we have

$$r = \frac{Sp}{h}$$

and substituting this in the last equation, we have

$$R^2 = (vp)^2 + \left(\frac{Sp}{h}\right)^2.$$

Then transposing and extracting the square root, we have

$$p = \frac{R}{\sqrt{v^2 + \left(\frac{S}{h}\right)^2}} \quad \text{. . . . .} \quad (4)$$

for the pitch of the rivets, where

p = pitch;
S = shear on the girder at point considered;
v = load on the flange angles per linear inch;

R = stress on rivet, which should be taken as the allowable stress on one rivet;

h = vertical distance between the rivets in the two flanges.

Case IV. When the loads are applied directly to the flange angles and the resistance of the web to bending is considered. We have here, the same as in Case III,

$$R^2=(vp)^2+r^2,$$

except

$$r=\frac{Sp}{h\left(1+\frac{A'}{8A}\right)}$$

which is obtained from (3).
Then substituting this value of r, we have

$$R^2=(vp)^2+\left(\frac{Sp}{h\left(1+\frac{A'}{8A}\right)}\right)^2,$$

from which we obtain

$$p=\frac{R}{\sqrt{v^2+\left(\frac{S}{h}\times\frac{A}{A+\frac{A'}{8}}\right)^2}}\quad\ldots\ldots\ldots\ldots(5)$$

for the pitch, where the letters signify the same as specified above.

In case the flange angles have double rows of rivets, h should be taken as the mean vertical distance between the rivets in the two flanges.

117. Web Splice.—It is always desirable that the web of any plate girder be one continuous piece, but the length of such plates is limited by the steel mills and whenever the length of a web exceeds these limits it becomes necessary to splice it.

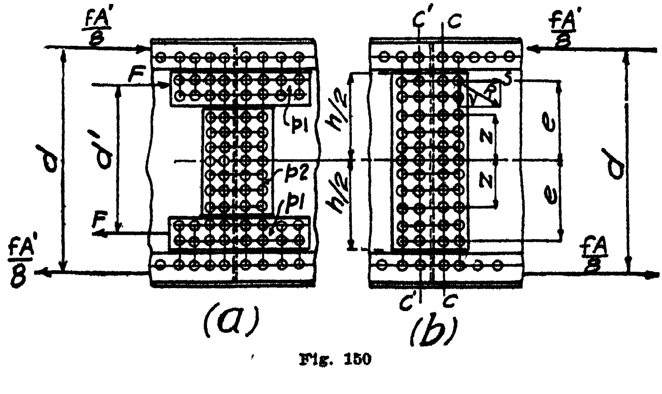


Fig. 150

In practice there are two standard ways of splicing a web, both of which are shown in Fig. 150. The splice shown at (*a*) is made up of six splice plates, three on each side of the web, while the splice shown at (*b*) is made up of only two plates, one on each side of the web. The splice plates should have sufficient material to transmit the shear at the point of splice, and also the cross bending on the web. The maximum shearing and maximum bending stress at any point in a simple girder do not occur at the same time, as is readily seen, yet in designing web splices it is practice to consider the two occurring simultaneously at the splice—which is an assumption entirely on the side of safety.

In the case of the splice shown at (*a*) the plates marked *p*1 are usually assumed to resist only the cross bending on the web, while the plates marked *p*2 are assumed to resist only the shear on the web. That is, the plates *p*1 and their connection to the web are designed as if no shear occurred at the splice and plates *p*2 and their connection to the web are designed as if no cross bending occurred. To illustrate this method of designing the splice:

Let f = allowable stress per square inch in each flange;
A' = area of the cross-section of the web;
d = vertical distance between the centers of gravity of the flanges;
d' = vertical distance center to center of plates *p*1;
F = stress in each pair of plates *p*1;
S = maximum shear at the splice.

Then the resistance of the web to bending is represented by the couple $(fA'/8)d$ which must be equal to Fd' and hence we have

$$F=\left(\frac{fA'}{8}\right)\frac{d}{d'}$$

for the direct stress in each pair of plates *p*1, due to the cross bending on the web. Now, evidently, these plates should be designed to take this stress F and the number of rivets on each side of the splice connecting each pair of these plates to the web should be sufficient to transmit the stress F in either double shear or bearing on the web, whichever requires the greater number of rivets. However, as the bending stress varies directly as the distance out from the center of the web, the intensities used in designing the plates *p*1 and also the stress allowed on the rivets connecting them to the web must be proportional to the intensities allowed in the flange. For example, if the allowable stress in the flange is f per square inch, the intensity to allow on the plates *p*1 would be $(f)\ d'/d$, and if r is the stress allowed on a rivet, say, in bearing on the web at the flange, the same size rivet through the plates would have an allowable bearing on the web of $(r)\ d'/d$.

Then for the net area of cross-section of each pair of plates *p*1 we have

$$a=\left(\frac{F}{f}\right)\frac{d}{d'}.$$

And for the number of rivets required on each side of the splice, we have

$$n=\frac{F}{\frac{(r)d'}{d}}.$$

In designing the plates $p2$, all we have to do is to select two plates that have sufficient net area along the vertical section to transmit the shear and wide enough to admit the necessary rivets on each side of the splice. However, as a matter of fact, there is, as a rule, more metal in the two splice plates than is necessary to carry the shear, as each plate is usually as thick as the web, in which case the two plates would have twice the net cross-section of the web. The number of rivets connecting the plates $p2$ to each side of the splice should be sufficient to transmit the shear. If r be the allowable stress on each rivet—and we will assume that each resists the same amount of stress—we have

$$n' = \frac{S}{r}$$

for the number of rivets on each side of the splice in plates $p2$. The value of r in this case is the full allowable intensity on a rivet in double shear or bearing on the web, using, of course, whichever is the least.

The assumption that plates $p1$ resist only the cross bending on the web and plates $p2$ only the shear is quite a reasonable assumption as plates $p2$ are too near the center of the web to resist so very much of the cross bending and plates $p1$ are really so narrow and too far from the center of the web to transmit so very much of the shear. As a matter of fact, however, the plates $p2$ do resist some of the cross bending and plates $p1$ resist some of the shear.

In the case of the splice shown at (b) the two plates should have sufficient net area of cross-section along the line cc or $c'c'$ to resist the cross bending on the web and also the shear.

Let f' = stress per square inch on the outer edges of the splice plates;
f = the allowable stress per square inch in the flanges;
A' = area of cross-section of the web;
h = height of each splice plate;
I = net moment of inertia of the two splice plates along the section cc or $c'c'$ (deducting the moment of inertia of the rivet holes). Then we have

$$f' = \left(\frac{fA'}{8}\right)d \times \frac{h}{2I}$$

for the stress per square inch on the outer edges of the splice plates. This stress should not exceed fh/d. As a matter of fact it never does, as the two splice plates, as a rule, have at least twice as much area of cross-section as the web. However, the stress f' should be considered in doubtful cases. The net area of the cross-section of the two splice plates along section cc or $c'c'$ should be sufficient to transmit the shear. That is, the shear divided by the net area should not exceed the allowable unit shearing stress on the web, say 10,000 lbs. per square inch, as specified in the A. R. E. Ass'n Specifications.

In case the web be spliced as shown at (b) we can not well consider other than that the maximum stress on each rivet is due to the cross bending on the web and shear combined. The cross bending produces a horizontal force on each rivet while the shear produces a vertical force and, as is evident, the maximum stress on each rivet will be the resultant of these two forces. The vertical shearing stress will be practically the same for each rivet, but the stress due to cross bending, which (as stated above) is transmitted to each horizontally, will vary directly as their distance out from the center of the web.

Let s be the horizontal stress due to cross bending on each of the rivets farthest out, as indicated at (*b*), and let e be the distance of these rivets from the center of the web. Then if there were a rivet out unit distance from the center of the web the stress on it would be equal to s/e and evidently the stress on any rivet out z distance from the center of the web would be $(s/e)z$ and the moment of this force or stress about the center of the web would be

$$z\left(\frac{s}{e}z\right)=\left(\frac{s}{e}\right)z^2.$$

Now it is evident that if this moment be determined for each and every rivet on one side of the splice, and these be added together, their sum would be equal to the couple $(fA'/8)d$. So we have

$$\Sigma\left(\frac{s}{e}\right)z^2=\left(\frac{fA'}{8}\right)d.$$

From this equation s can be determined and if v be the vertical shear on each rivet, which is readily computed, we have

$$R=\sqrt{v^2+s^2}$$

for the maximum resultant stress on each outer rivet which is the absolute maximum.

As an example, let a, b, c, d, and e represent, respectively, the distance of the first, second, third, fourth, and fifth horizontal row of rivets out from the center of the web, shown at (*b*), Fig. 150, then we have

$$2\left(\frac{2sa^2}{e}+\frac{2sb^2}{e}+\frac{2sc^2}{e}+\frac{2sd^2}{e}+2se\right)=\left(\frac{fA'}{8}\right)d.$$

From this we obtain

$$s=\frac{fA'd}{32\left(\frac{a^2}{e}+\frac{b^2}{e}+\frac{c^2}{e}+\frac{d^2}{e}+e\right)}.$$

Then combining this with the vertical shear, as shown above, the maximum stress on the rivets can be obtained. However, s should not exceed the allowable at the flange multiplied by $2e/d$.

The maximum stress on the rivets in the type of splice shown at (*a*) can be determined in the same manner. In fact, the rivets in all web splices should be tested in this manner.

118. The Stiffening Angles on a plate girder really have two functions: one is to stiffen the web against buckling, and the other is to transfer loads from the flanges directly to the web. It is obvious that the closer the stiffeners are spaced along a web the stronger the web is against buckling, and consequently the higher the web can be stressed. So the design of the web is influenced by the spacing of the stiffening angles.

There is no theoretical way of determining the spacing of stiffeners. A practical rule is to space them so that their distance apart along the girder is equal to the depth of the girder, but in no case farther apart than five feet or a little over. However, to satisfy modern requirements, it is usually necessary to space them closer together near the ends of the girders than this practical rule calls for in order to obtain an economic web. The allowable spacing of the stiffeners for a given stress on the web is specified as

$$d=\frac{t}{40}(12{,}000-s)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

in the Specifications prepared by the A. R. E. Ass'n,

where d = distance between the stiffeners (as shown at (b), Fig. 145);

t = thickness of web;

s = shearing stress per square inch in the web, which is obtained by dividing the shear at the section considered by the gross area of the cross-section of the web.

Equation (1) is really an empirical formula.

It is not economic, as a rule, to space stiffeners closer together than half the depth of the girder. When a spacing less than that at the ends of a girder is given by the above formula, a thicker web should be used.

There should always be stiffeners at any point where excessive concentrated loads are applied. These stiffeners should be designed as a column to support the load, the effective length being taken as half the depth of the girder.

119. Example 1.—Let it be required to design a girder 30 ft. long (*c.c.* end bearings), to support a uniform dead load of 400 lbs. per linear foot of girder, and a live load of 4,000 lbs. per foot.

For the maximum bending moment we have from dead load

$$M=\frac{1}{8}\times 400\times \overline{30}^2\times 12=540{,}000'' \text{ lbs.}$$

and from live load

$$M'=\frac{1}{8}\times 4{,}000\times \overline{30}^2\times 12=5{,}400{,}000'' \text{ lbs.},$$

making a total of 5,940,000″ lbs. bending moment.

Let us assume the web to be $\frac{3}{8}''$ thick, and that the loads are applied directly to the top flange, and that the web resists bending. Having made these assumptions we can proceed with the determination of the economic depth. Formula (5), Art. 113, will be used, as girders of such short lengths usually have no cover plates. So, substituting in this formula we have

$$x=1.22\sqrt{\frac{5{,}940{,}000}{16{,}000\times 3/8}}=38.4 \text{ ins.}$$

for the economic depth of the girder. So we will assume a $38''\times\frac{3}{8}''$ web. The next thing, we will test this web to see if it is satisfactory. For the maximum end shear we have from dead load

$$S=400\times 15=6{,}000 \text{ lbs.}$$

and from live load

$$S' = 4,000 \times 15 = 60,000 \text{ lbs.},$$

making a total of 66,000 lbs. end shear.

Now dividing this by the area of cross-section of the web ($= 38'' \times \frac{3}{8}''$), we have

$$\frac{66,000}{14.25} = 4,630 \text{ lbs.}$$

for the actual average unit shearing stress on the web.

Next, substituting this in Formula (1), Art. 117, we have

$$d = \frac{\frac{3}{8}}{40}(12,000 - 4,630) = 69.2 \text{ ins.}$$

for the maximum theoretical distance between stiffeners at the ends of the girder; and as this spacing is not less than half the depth of the girder the web itself is satisfactory, but common practice is not to permit the distance between the stiffeners to be greater than the depth of the girder. So we will space them so as not to violate this practice, that is, the clear horizontal distance between the stiffeners will not be made more than 38″ in any case.

Then substituting 38 for d in Formula (1), Art. 117, we have

$$38 = \frac{\frac{3}{8}}{40}(12,000 - s),$$

from which we obtain

$$s = 7,946 \text{ lbs.}$$

as the allowable unit shearing stress on the web.

Now dividing this into the shear we have

$$\frac{66,000}{7,946} = 8.3 \text{ sq. ins.}$$

for the required area (of cross-section) of the web, which is 5.95□″ ($= 14.25 - 8.3$) less than the area of the assumed web. So theoretically the web can be thinner than $\frac{3}{8}''$, but we will use the assumed web as the specifications limit the thickness to $\frac{3}{8}''$. However, in the cases of buildings and highway bridges, the web would likely be reduced in thickness, but in no case should it be thinner than $\frac{1}{4}''$.

As an example in such cases, let us assume a $38'' \times \frac{1}{4}''$ web which has an area of cross-section of 9.5□″ ($= 38'' \times \frac{1}{4}''$). Then for the actual unit shearing stress on the web we have

$$\frac{66,000}{9.5} = 6,952 \text{ lbs.}$$

Substituting this in Formula (1), Art. 117, we have

$$d = \frac{\frac{1}{4}}{40}(12,000 - 6,952) = 31.5 \text{ ins.}$$

for the required spacing of the stiffeners at the end of the girders, and as this is not less than half the depth of the girder the web is satisfactory and hence would be used.

We will next take up the designing of the flanges. Let us first try 2—Ls 6″ x 6″ x ½″ for each flange. From Table 6 we find that the distance from the back of these angles to their center of gravity is 1.68″. So if the girder be made 38¼″ deep back to back of flange angles, that is, ¼″ greater than the depth of the web, which is usual practice, we have $38.25 - 1.68 \times 2 = 34.89''$ for the effective depth. Now, by dividing this into the maximum bending moment we have

$$\frac{5,940,000}{34.89} = 170,000 \text{ lbs.}$$

for the stress in each flange, and by dividing this by 16,000, the allowable unit stress, we have

$$\frac{170,000}{16,000} = 10.6 \text{ sq. ins.}$$

for the required net flange area.

We have

2—Ls 6″ x 6″ x ½″ = 11.50 − 1	=	10.50□″ net
⅛ of web = (14.25 × ⅛) =		1.78□″ net
		12.28□″ net

This is too large, so let us try

2—Ls 6″ x 6″ x $\frac{7}{16}$″ = 10.12 − .87 =	9.25□″ net
⅛ of web =	1.78□″ net
	11.03□″ net

This flange section is about right, being only about 0.4□″ more than required, but the specifications require that the thickness of these flange angles be at least one-twelfth of the width of the outstanding legs which are 6″. So these $\frac{7}{16}$″ angles are too thin and consequently if 6″ x 6″ angles be used at all the 6″ x 6″ x ½″ angles would have to be used, which gives an excess of metal.

Let us try 2—Ls 6″ x 4″ x ½″ (the 4″ legs outstanding) for each flange.

The distance from the back of the 4″ leg to the center of gravity of each of these angles is (given in Table 4) 1.99″, say, 2″. Then for h effective depth we have $38.25 - 2 \times 2 = 34.25''$.

Now dividing this into the maximum bending moment we have

$$\frac{5,940,000}{34.25} = 173,000 \text{ lbs.}$$

for the stress in each flange, and dividing this by 16,000 we have

$$\frac{173,000}{16,000} = 10.8 \text{ sq. ins.}$$

for the required net flange area.

We have

2—Ls 6″ x 4″ x ½″ = 9.50 − 1	=	8.50□″ net
⅛ of web =		1.78□″ net
		10.28□″ net.

This flange section is 0.52□″ less than the required, so let us try the following:

$$
\begin{aligned}
&2\text{—Ls } 6'' \times 4'' \times \tfrac{9}{16}'' = 10.62 - 1.12 = 9.50\square'' \text{ net} \\
&\tfrac{1}{8} \text{ of web} = \qquad\qquad\qquad\qquad 1.78\square'' \text{ net} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad 11.28\square'' \text{ net.}
\end{aligned}
$$

This section is 0.48□″ more than required. So either of the last two sections may be used. We will use the latter (6″ x 4″ x $\frac{9}{16}$″ angles) as that is on the side of safety.

The size of the intermediate stiffeners is governed practically by the specifications which require that they be no less than $\frac{3}{8}$″ thick and the width of their outstanding legs be no less than $\frac{7}{86}$ of the depth of the girder plus 2 inches. So in this case we will use $3\frac{1}{2}$″ x $3\frac{1}{2}$″ x $\frac{3}{8}$″ angles for stiffeners throughout as this is the minimum size allowed. Thickness of the end stiffeners would very likely be $\frac{7}{16}$″ or $\frac{1}{2}$″, depending upon conditions. If the girder be supported upon masonry, these stiffeners would be designed as columns to take the end shear or reaction, one-half of the depth of the girder being taken as the length of the column.

If the girder be riveted at the ends to other girders or to columns which wholly support the girder, the end stiffeners in that case would be at least $\frac{7}{16}$″ thick to provide for the facing of the ends of the girder which is usually required in modern practice.

To obtain the spacing of the flange rivets at the ends of the girder, we have

$R = 7{,}880$ lbs. allowable bearing of a $\frac{7}{8}$″ rivet on the $\frac{3}{8}$″ web;
$v = 333$ lbs. $= 4{,}000 \div 12$;
$S = 66{,}000$ lbs.;
$h = 31.5 = (38.25 - 6.75)$.

Then substituting these values in Formula (4), Art. 116, (the formula required by the specifications), we obtain

$$p = \frac{7{,}880}{\sqrt{333^2 + \left(\dfrac{66{,}000}{31.5}\right)^2}} = 3.7 \text{ ins.}$$

for the pitch of the flange rivets at the ends of the girder. This pitch should be used for a distance out from the ends of the girder equal to the depth of the girder and varied from there on toward the center of the span to suit the shear at the different points, no pitch, however, being greater than 6″ in accordance with the specifications.

From the above calculations the necessary information for making the detail drawing of the girder is obtained, and when this drawing is finished the design of the girder is complete. All plate girders are designed in this manner.

CHAPTER XI

DESIGN OF SIMPLE RAILROAD BRIDGES

120. Types.—Simple railroad bridges can be divided into four general types: beam bridges, plate girders, viaducts, and truss bridges. Plate girders and truss bridges can be further divided into deck and through bridges; deck when the track is supported on the top of the structure; and through when the track is between the main girders or trusses. Beam bridges and viaducts are always deck bridges.

121. The Specifications prepared by the American Railway Engineering Association, referred to in the preceding chapter, will be used throughout this work.

122. The Live Load usually specified for railroad bridges consists of two typical consolidated locomotives and tenders coupled in tandem followed by a uniform train load, and a special load concentrated on two axles which is used in case of very short spans. The most common loading of this type is that known as "Cooper's Loading," devised by Theodore Cooper, M. A. Soc. C. E. Cooper's loadings vary in weight and are designated, beginning with the lightest, as $E30$, $E35$, $E40$, $E45$, $E50$, $E55$, and $E60$. These loadings vary by a certain ratio so that if any stress, shear, bending moment, etc., due to any one of the loadings be known the intensities of the same due to any one other of the loadings can be obtained by direct proportion, and hence any tables or diagrams giving the shears, moments, etc., for any one of the loadings can be used for any of the other loadings. The figures following the letter E are the indices of the ratio referred to above. Thus, for example, if any stress, shear, etc., due to $E40$ be known, the intensity of the same for $E50$ will be 50/40 of that due to $E40$; 60/40 for $E60$; 35/40 for $E35$; and so on. Most of the railroads in this country have adopted some one of Cooper's loadings, either exactly or slightly modified. The trend has been toward the using

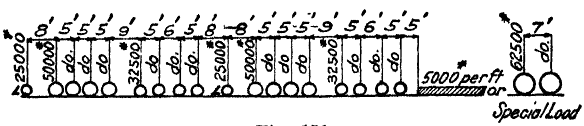


Fig. 151

of the heavier loadings. At present $E50$, represented in Fig. 151, is used quite extensively. The spacing of the wheels is the same for all of the loadings.

The $E40$ loading is the most convenient to use owing to the loads being in even thousands of pounds.

Table A* gives for this loading moments about any wheel of the wheels to the right or to the left of it, as will be seen upon inspection.

* The student should verify the results given in this table.

This table is quite convenient in determining the centers of gravity, shears, and moments, as will be shown later.

123. "An Equivalent Uniform Live Load" is sometimes used instead of the wheel loads described above. In the case of beams and deck girders this will give exactly the same results as the wheel loads, while in the case of trusses and through girders the results obtained are only approximately the same as for the wheels.

To obtain an equivalent uniform live load for the bending moment on a beam or deck girder of length L, the maximum bending moment M due to the wheel loads is computed, and then we have

$$M=\frac{pL^2}{8},$$

from which we obtain

$$p=\frac{8M}{L^2}$$

for the equivalent uniform load per foot which will produce the same maximum moment on the beam or girder as the wheels.

To obtain an equivalent uniform live load for the end shear or reaction on a beam or deck girder of length L, the maximum reaction R due to the wheel loads is computed and then we have

$$R=\frac{p'L}{2},$$

from which we obtain

$$p'=\frac{2R}{L}$$

for the equivalent uniform live load which will produce the same maximum end shear or reaction as the wheels.

In practice the equivalent uniform live load for trusses is usually taken either as the uniform load that will produce as great a bending moment at the quarter point of the span as the wheel loads or the uniform load that will produce as great a shear in the end panel as the wheel loads.

The first is obtained by computing the maximum bending moment M' at the quarter point due to the wheel loads, the span being considered as a simple deck beam, and then we have

$$M'=\frac{3pL^2}{32},$$

from which we obtain

$$p=\frac{32M'}{3L^2}$$

for the equivalent uniform live load per foot where L is the length of span in feet.

The second equivalent load is obtained by computing the maximum shear S in the end panel due to the wheel loads and then we have

$$S=p'\left(\frac{L-d}{2}\right),$$

from which we obtain

$$p' = \frac{S}{\frac{L-d}{2}}$$

for the equivalent uniform live load where L is the length of span and d the panel length in feet.

The first loading gives stresses too low for web members and chords near the end and too high for the chords near the center of the span, while the second loading gives stresses about correct in all web members, but considerably too high in the chord members, except the chords in the end panels. To correct for this discrepancy the author has proposed the reduction of the stresses in the chords (except the chords in the end panels) by the per cent given by the following empirical formula:*

$$\left(\frac{L}{100} + 2.5\right) \text{per cent}$$

where L is the length of span in feet.

For example, the chord stresses in a 300-ft. span would be reduced by 5.5 per cent. This does not apply to chords in end panels.

124. Dead Load consists of the weight of the metal in the structure (except the metal at the supports), and the weight of the track, known as the deck. The approximate weight of metal per foot of single-track bridges designed for Cooper's $E50$ loading can be obtained from the following formulas wherein

p = weight of metal per foot of span,
L = length of span in feet, c. c. end bearing:

For beam spans without lateral bracing, and stringers,
$p = 12L + 100$ .... (1),
For deck girder spans and beam spans with lateral bracing,
$p = 12L + 150$ .... (2),
For through plate girder spans,
$p = 13L + 600$ .... (3),
For truss spans,
$p = 7L + 660$ .... (4).

The weight obtained from the above formulas is for the metal alone, and to this must be added the weight of deck which in the case of ordinary wooden decks can be taken at 400 lbs. per foot of track. The weight obtained from the above formulas is usually correct enough for computing stresses, as 10 per cent variation is permissible, but it should not be used in making estimates of cost.

The approximate dead weight of metal in bridges designed to carry Cooper's $E60$ loading is about one-eighth more than given by the above formulas, and the weight of those designed to carry the $E40$ loading is about one-eighth less than that given by the above formulas.

The weight of metal in double-track bridges depends upon their construction. Their weight as for metal is about 70 per cent heavier than that of single-track bridges if three or two main girders or trusses are used, but about 100 per cent if four trusses or girders are used.

* See *Engineering News*, September 13, 1906.

In case of concrete or metal floors the floor should be designed and then the weight of it computed, and then the weight of the main girders or trusses can be assumed and added to this weight and in this way the approximate dead load can be determined, which should be within 10 per cent of the actual weight; if not, the dead load stresses should be redetermined, using a revised dead load.

125. Impact.—Rapidly moving trains will produce greater stresses in bridges than the same load when simply standing on a structure, as we consider it when computing the live-load stress, and to provide for this additional stress a certain amount of the live-load stress in each member is taken as the impact stress which is added to the corresponding live-load stress. The impact stress is obtained by simply multiplying the maximum live-load stress by a coefficient obtained from an empirical formula. The following formula is the one most used:

$$C=\frac{300}{L+300}.$$

Then for the impact stress in any member we have the formula

$$I=S\left(\frac{300}{L+300}\right)$$

as specified by the A. R. E. Ass'n in their Specifications referred to above, where

I = impact stress,
S = maximum computed live-load stress,
L = length of track (in feet) loaded when the maximum live-load stress occurs.

126. Wind Loads.—The horizontal pressure exerted on bridges by the wind is known as the "Wind Load." As a rule this load can be safely taken at 30 lbs. per square foot of the horizontal projection of both the structure and the live load carried. But, in addition to this, some provision should be made for lateral vibration due to the live load. To provide for this and the wind load a greater pressure than 30 lbs. probably should be used in some cases.

The lateral force, which includes the wind pressure, specified in the A. R. E. Ass'n Specifications, will be used in this book.

BEAM BRIDGES

127. Preliminary.—Beam bridges are used only for short spans, rarely ever exceeding 20 ft. in length. There are usually at least two I-beams under each rail connected to each other by diaphragms, similar to that shown in Figs. 158 and 159, thus forming a compound girder. These compound girders should be braced to each other by diagonal and transverse bracing, as shown in Fig. 159, whenever the span exceeds 12 ft. When the span length exceeds 15 ft., four panels of bracing should be used, in which case a horizontal diaphragm is needed at each lateral connection. In case there be more than two beams in each compound girder the beams can be placed somewhat closer together than in the case of two beams.

Complete Design of a 10-ft. Span

128. Data.—

Length = 10′ c.c. end bearings.

Dead load = 220 + 400 = 620 lbs. per foot of span.

(From (1), Art. 123.)

Live load, Cooper's $E50$ (shown in Fig. 151).

Specifications A. R. E. Ass'n.

129. Calculations.— For the maximum bending moment due to dead load we have

$$M = \frac{1}{8} \times \frac{620}{2} \times \overline{10}^2 \times 12 = 46,500 \text{ inch lbs.}$$

From mere inspection of the live-load diagram, in Fig. 151, it is seen that the maximum moment will occur either when one 62,500-lb. axle load (of the special load) is on the span or when two of the 50,000-lb. axle loads are on. So we have two cases to try.

The maximum moment due to the 62,500-lb. axle, one-half of which goes to each girder, will occur (according to Art. 88) when the load is at the center of the span, as shown in Fig. 152. Each of the reactions will

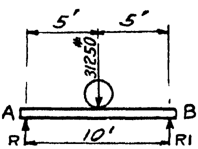


Fig 152

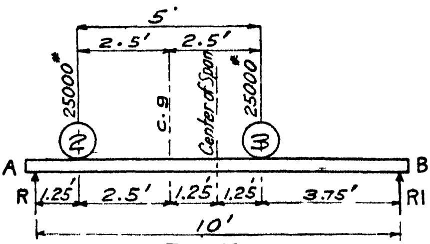


Fig 153

be equal to one-half of the load on each girder, or 15,625 lbs. Then we have

$$15,625 \times 5 \times 12 = 937,500 \text{ inch lbs.}$$

for the maximum bending moment due to this load.

The maximum due to the two 50,000-lb. axles will occur when the loads are in the position shown in Fig. 153 (according to Art. 88) and it will occur under the wheel marked 3.

Taking moments about A (Fig. 153) we have

$$R1 \times 10 - 25,000 \times 1.25 - 25,000 \times 6.25 = 0,$$

from which we obtain the reaction

$$R1 = 18,750 \text{ lbs.}$$

Then for the bending moment at wheel 3 (which is the maximum) we have

$$M' = 18,750 \times 3.75 \times 12 = 843,700 \text{ inch lbs.},$$

which is less than that produced by the one wheel of the special load, as is seen above, and hence the moment due to the special load will be used.

Then for the maximum impact moment we have

$$I = 937{,}500\left(\frac{300}{10+300}\right) = 907{,}000 \text{ inch lbs.}$$

Now adding the maximum dead- and live-load moments and impact together we have

$$46{,}500 + 937{,}500 + 907{,}000 = 1{,}891{,}000 \text{ inch lbs.},$$

which is the total moment which the girder under each rail must be designed to resist.

Dividing this by 16,000 (see Art. 105) we have

$$\frac{1{,}891{,}000}{16{,}000} = 118.2 \text{ for the section modulus.}$$

From Table I we find that this calls for 2—Is 15″ x 42# under each rail.

We can now make a preliminary estimate of the dead load. The weight of the four Is is 168 lbs. per foot of span, and the details can be neglected as the channel diaphragm (see Fig. 158) at the center of the span is all the detail there is to consider. Then adding this 168 lbs. to the weight of the deck we have 568 lbs., which is 52 lbs. less than the assumed dead load, but as this is within 10 per cent of the assumed load no recalculations are necessary.

For the end shear or reaction due to dead load we have

$$R = \frac{620}{2} \times \frac{10}{2} = 1{,}550 \text{ lbs.}$$

The maximum end shear or reaction due to the live load will occur when the two 62,500-lb. axles are on the span and in the position indicated in Fig. 154.

Taking moments about the support B we have

$$R_1 \times 10 - 31{,}250 \times 10 - 31{,}250 \times 3 = 0,$$

from which we obtain

$$R_1 = 40{,}600 \text{ lbs.},$$

which is the maximum live-load end shear or reaction.

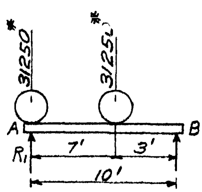


Fig. 154

For the impact we have

$$I = 40{,}600\left(\frac{300}{10+300}\right) = 39{,}300 \text{ lbs.}$$

Now adding the above reactions and impact together we have

$$1{,}550 + 40{,}600 + 39{,}300 = 81{,}450 \text{ lbs.}$$

for the total end shear or reaction. Dividing this by 600 we have

$$\frac{81{,}450}{600} = 136 \text{ sq. ins.}$$

for the required area of bearing on the masonry for each of the four supports. This completes the necessary calculations, and next the general drawing, as shown in Fig. 158, or a shop drawing can be made for the span. The details shown in Fig. 158 should be studied by the student until thoroughly understood.

Complete Design of a 15-Ft. Span

130. Data.—

Length = 15′ c.c. end bearings.

Dead load = 330 + 400 = 730 lbs. per ft. of span.

(From (2), Art. 123.)

Live load, Cooper's $F50$.

Specifications, A. R. E Ass'n.

131. Calculations.—For the maximum bending moment due to dead load we have

$$M = \frac{1}{8} \times \frac{730}{2} \times \overline{15}^2 \times 12 = 123{,}000 \text{ inch lbs.}$$

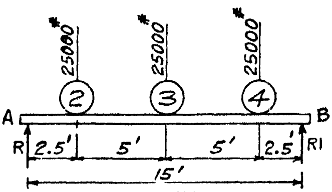


Fig 155

It is readily seen, from the diagram in Fig. 151, that the heaviest loading possible for this span is three 50,000-lb. axle loads, and, according to Art. 88, these will produce the maximum moment when placed in the position shown in Fig. 155, where the loads are indicated for one girder only.

Then taking moments about B (Fig. 155) we have

$$R \times 15 - 25{,}000 \times 2.5 - 25{,}000 \times 7.5 - 25{,}000 \times 12.50 = 0,$$

from which we obtain

$$R = 37{,}500 \text{ lbs.}$$

for the reaction at A. Then taking moments about the center load we have

$$M' = (37{,}500 \times 7.5 - 25{,}000 \times 5)\ 12 = 1{,}875{,}000 \text{ inch lbs.},$$

and for the impact we have

$$I = 1{,}875{,}000 \left(\frac{300}{15 + 300}\right) = 1{,}785{,}000 \text{ inch lbs.}$$

Now adding the dead and live moments and impact together we have 3,783,000 inch lbs. for the maximum bending moment. Then dividing by 16,000 we have

$$\frac{3{,}783{,}000}{16{,}000} = 236.4$$

for the section modulus which calls for 2—Is 20″ x 70# under each rail.

This beam appears a little heavy, from the section modulus, but the material cut out of the web along the vertical row of rivets at the center of the span must be taken into account. This is done by subtracting the moment of inertia of the material cut out of the web from the moment of inertia of the beam. Then substituting this net moment of inertia in the formula

$$f=\frac{My}{I}$$

which is Formula D, Art. 53, we obtain the maximum stress on the beam, which should not exceed 16,000 lbs.

The area of cross-section cut out of the web at each rivet hole, assuming that $\frac{3}{4}''$ rivets are used, is $\frac{13}{16}\times\frac{9}{16}=0.46$ square inches. Then taking moments about the neutral axis, which we will assume is at the third rivet from the bottom of the beam, which is only approximately true (see drawing, Fig. 160), we have

$$0.46\times\overline{6.75}^2+0.46\times\overline{2.5}^2=23.8$$

for the moment of inertia of the material cut out above the neutral axis, and

$$0.46\times\overline{2.5}^2+0.46\times 6^2=19.4$$

for the moment of inertia of the material cut out below the neutral axis. Then adding these together we have 43.2 for the total moment of inertia of the material cut out. Subtracting this from the moment of inertia of one beam we have

$$1219.9-43.2=1176.7$$

for the net moment of inertia of one beam.

Now substituting in the above formula we have

$$f=\frac{1}{2}\times\frac{3{,}783{,}000\times 10}{1176.7}=16{,}100 \text{ lbs.},$$

which is very nearly the allowable stress. So the beam has practically the correct section.

The rivet holes in the beams shown in Fig. 158 are so near the neutral axis that the moment of inertia is affected but little, and hence the material cut out was not considered in designing those beams.

We can now make a preliminary estimate of the dead weight.

The four beams will weigh 280 lbs. per foot of span and the laterals and details, including lateral connections and diaphragms, will be about 30 lbs. per foot of span, so the total effective weight of metal per foot is about $280+30=310$ lbs. Adding this to the weight of the deck we have 710 lbs., which is 20 lbs. less than the assumed dead weight, but this is much less than the allowable 10 per cent deviation, so recalculation is unnecessary.

The weight of details can be correctly assumed only by experienced designers. It is necessary that the student draw out the details to some extent in order to get the weight of metal to any reasonable degree of accuracy.

For the maximum end shear or reaction due to dead load we have

$$R=\frac{730}{2}\times 7.5=2{,}740 \text{ lbs.}$$

The maximum live-load end shear or reaction will occur at A when the wheels are in the position shown in Fig. 156.

Then taking moments about B we have

$$R' \times 15 - (25{,}000 \times 15 + 25{,}000 \times 10 + 25{,}000 \times 5) = 0,$$

from which we obtain

$$R' = 50{,}000 \text{ lbs.}$$

for the maximum live-load end shear or reaction. For the impact we have

$$I = 50{,}000 \left(\frac{300}{15 + 300}\right) = 47{,}600.$$

Adding these reactions and impact together we have

$$2{,}740 + 50{,}000 + 47{,}600 = 100{,}340 \text{ lbs.}$$

for the maximum end shear or reaction.

Dividing this by 600 we have

$$\frac{100{,}340}{600} = 167.2 \text{ sq. ins.}$$

for the required bearing on the masonry, for each of the four supports

According to the A. R. E. Ass'n Specifications the lateral force on the laterals will be

$$200 + 5{,}000 \times 0.10 = 700 \text{ lbs. per foot of span.}$$

This force is applied to the laterals only at the central connection. Let Fig. 157 represent the plan of the laterals. There will be a load of $700 \times 7.5 = 5{,}250$ lbs. applied at the central connection which must be

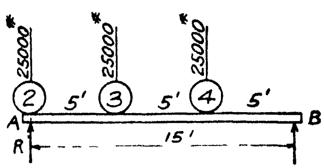


Fig 156

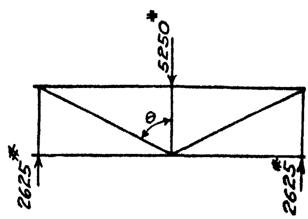


Fig. 157

transmitted to the ends by the laterals, one-half going each way. So, evidently, the maximum shear in each panel will be 2,625 lbs., which is one-half of 5,250 lbs. The stress in each lateral will then be equal to $2{,}625 \times \sec\theta$. $\sec\theta$ is equal to about 2.6. Then we have $2{,}625 \times 2.6 = 6{,}825$ lbs. for the stress in each lateral. If this stress be tension, as it would be in the case shown in Fig. 157, the net area required in each lateral would be equal to

$$\frac{6{,}825}{16{,}000} = 0.43 \text{ sq. ins.}$$

but when the lateral force is exerted in the opposite direction the laterals would be subjected to stress of the same intensity, but it would be compression, in which case the area required would be equal to

$$\frac{6{,}825}{\left(16{,}000 - 70\dfrac{L}{r}\right)}$$

So the laterals must be designed as compression members as well as tension members. Now as the stress is so very small the ratio of length to radius of gyration will really govern. This should not be over 120. As regards construction, a single angle for each lateral is the best section. Then if we use an angle the first question is—which radius of gyration shall we use in designing it? The laterals are fixed at their ends in the horizontal plane so that if the radius about the vertical axis be used, only one-half of the length of each lateral should be taken as the length. In the vertical plane the laterals are only partially fixed at their ends, as the connection plates resist bending upward and downward only to a limited extent so that if the radius about the horizontal axis be taken, the full length of the lateral should be taken, which, to be sure, is on the side of safety. As the laterals are fixed in the horizontal plane and partially fixed in the vertical plane they cannot fail readily about the 45° axis, so it would not be correct to use the least radius of gyration of an angle.

From this it is seen that the radius about the horizontal axis is the one to use. The length of each of the laterals is about 8 ft. or 96 ins. Then the radius of gyration about the horizontal axis must not be less than

$$\frac{96}{120}=0.8,$$

which permits the use of a 3″ x 3″ angle, but the specifications limit use to $3\frac{1}{2}$″ x $3\frac{1}{2}$″ x $\frac{3}{8}$″ angles, so this size will be used.

In the designing of the transverse struts, sense of fitness must govern to some extent as rigidity is the thing sought. However, the stress is readily determined. The one at the center of the span has the greatest load which is 5,250 lbs., which produces a compression stress in the strut of that amount, as is readily seen from Fig. 157. A 9″ [is the maximum size that will give us a convenient three-rivet connection. This size also looks about correct as far as rigidity is concerned.

The length of these struts is about 42 ins. and the least radius of gyration of the 9″ [is 0.64. Then we have

$$\frac{42}{0.64}=66 \text{ for } \frac{L}{r}$$

which is quite low. So the 9″ [appears to be satisfactory, and we will use 1—[9″ x 20# for each transverse strut.

This completes the necessary calculations and next a general drawing as shown in Fig. 159 can be made. This drawing shows the design only in a general way and the bridge could not be fabricated from it. Such drawings are usually made in railroad offices and in the offices of consulting engineers. A bridge company would make a shop drawing as shown in Fig. 160, and, in addition, the necessary shop bills from which the bridge would be fabricated.

The following shop bills are about what a bridge company would require for the work. However, bridge companies use printed forms.

Page No. 1 of the shop bills, which would accompany the shop drawing shown in Fig. 160, is principally for the templet and bridge shop.

Pages No. 2 and No. 5 are principally for the forge shop.

Page No. 3 is principally for the pattern shop and foundry.

Pages No. 4 and No. 5 would be used by the shipping department.

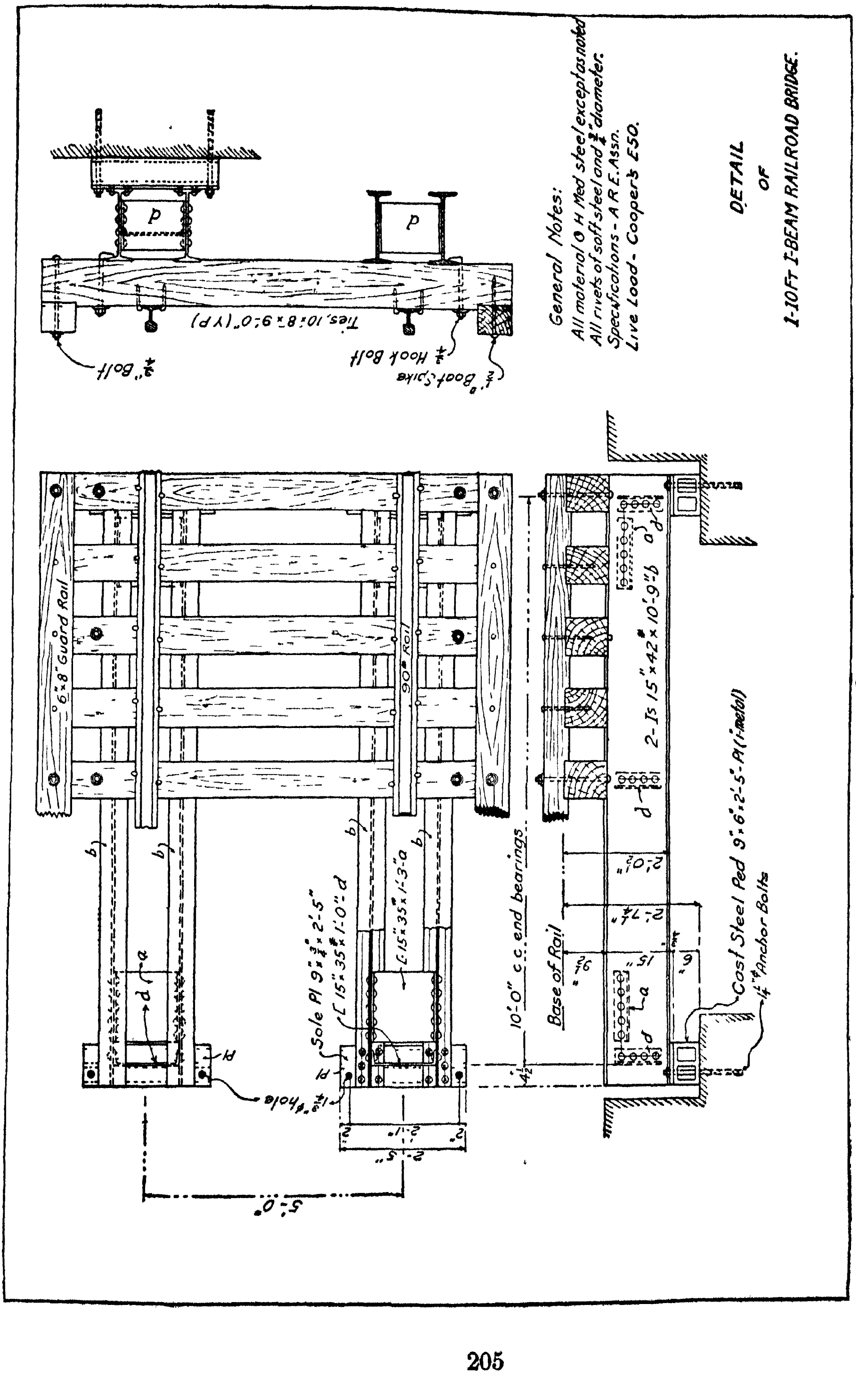

General Notes:
All material O H Med steel except as noted
All rivets of soft steel and 3/4" diameter.
Specifications - A R E Assn.
Live Load - Cooper's E50.
DETAIL
OF
1-10 Ft I-BEAM RAILROAD BRIDGE.
Ties, 10"x8"x9'-0" (Y P)
3/4" Bolt
3/4" Hook Bolt
3/4" Boat Spike
6"x8" Guard Rail
90# Rail
2-Is 15"x42#x10'-9"
Cast Steel Ped 9"x6"x2'-5" Pl (1-metal)
1 1/4" Anchor Bolts
Base of Rail
10'-0" c.c. end bearings
Sole Pl 9"x3/4"x2'-5"
[15"x35#x1'-0" d
[15"x35#x1'-3" a
1 3/8" hole
2'-5"
5'-0"

Fig. 158

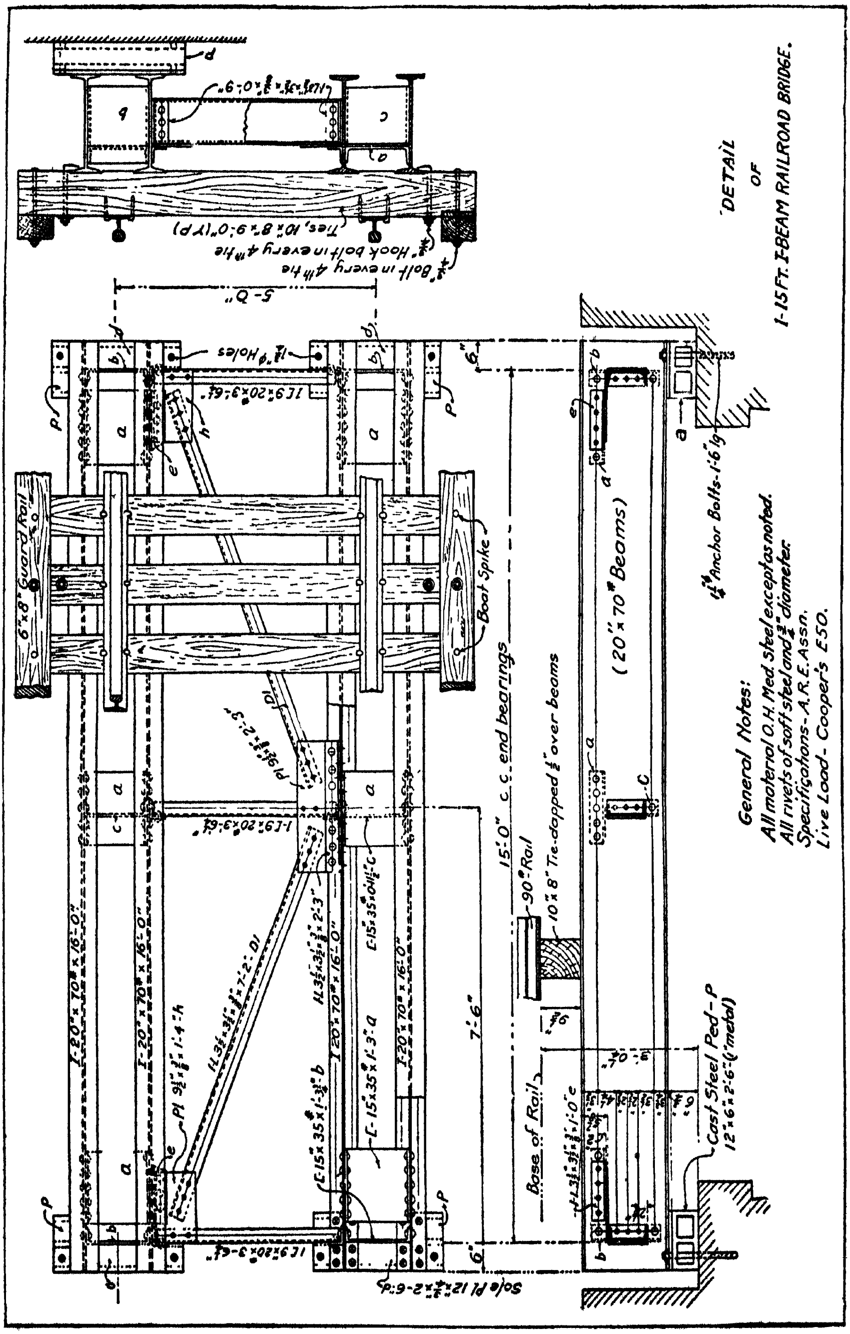

DETAIL
OF
1-15 Ft. I-BEAM RAILROAD BRIDGE.
General Notes:
All material O.H. Med. Steel except as noted.
All rivets of soft steel and ¾" diameter.
Specifications - A.R.E. Assn.
Live Load - Cooper's E50.
15'-0" c. c. end bearings
(20" x 70# Beams)
90# Rail
10" x 8" Tie-dapped ½" over beams
Base of Rail
Cast Steel Ped - P
12" x 6" x 2'-6" (1" metal)
1¼" Anchor Bolts - 1'-6" lg
6" x 8" Guard Rail
Boat Spike
Ties, 10" x 8" x 9'-0" (Y.P.)
¾" Hook bolt in every 4th tie
¾" Bolt in every 4th tie
5'-0"
7'-6"
1¼" Holes
I-20" x 70# x 16'-0"
C-15" x 35# x 1'-3" a
Sole Pl 12" x ¾" x 2'-6" d

Fig 159

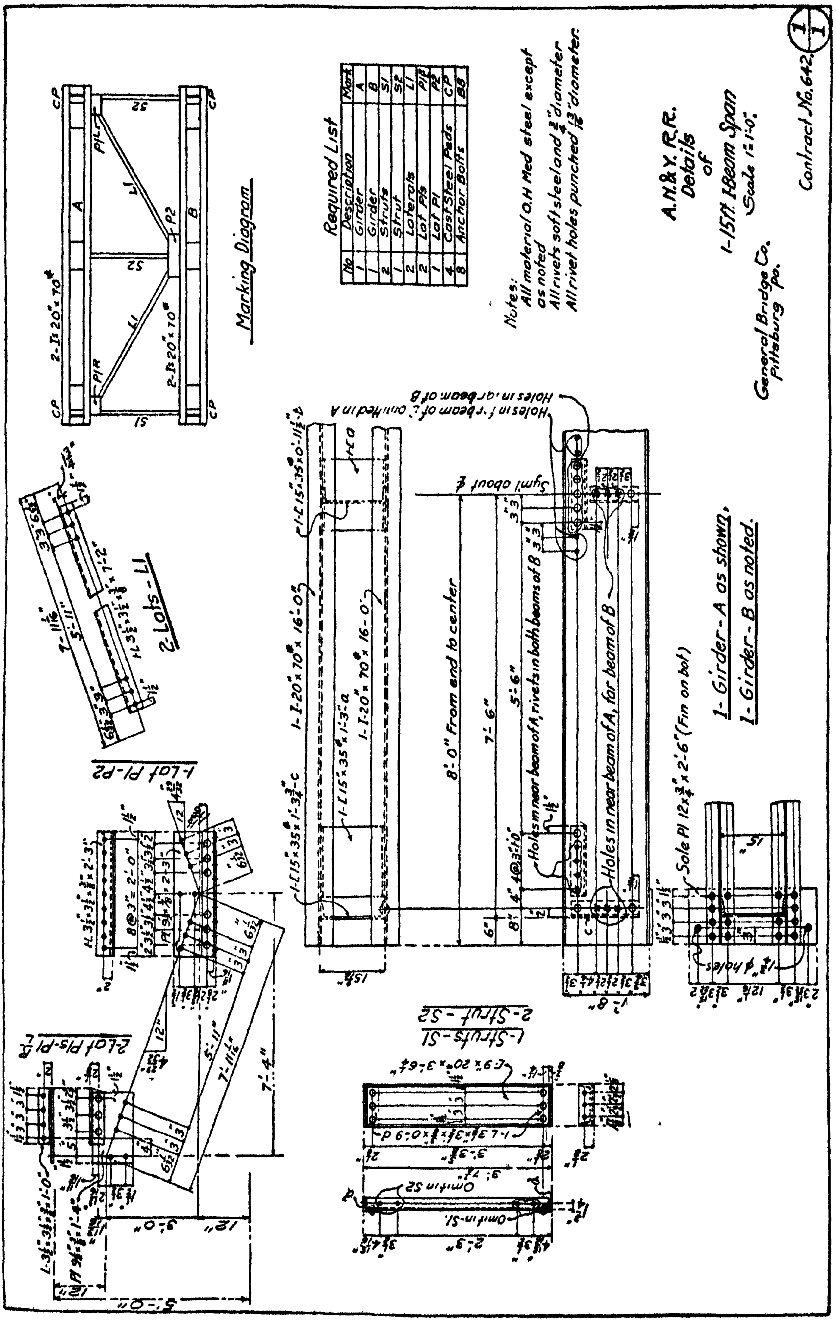

Marking Diagram
Required List
No | Description | Mark
1 | Girder | A
1 | Girder | B
2 | Strut | S1
1 | Strut | S2
2 | Laterals | L1
2 | Lat Pls | P1R
1 | Lat Pl | P2
4 | Cast Steel Pads | CP
8 | Anchor Bolts | BB
Notes:
All material O.H Med steel except as noted
All rivets soft steel and 3/4" diameter
All rivet holes punched 13/16" diameter.
A.N.&Y. R.R.
Details
of
1-15ft. I-Beam Span
Scale 1"=1'-0"
General Bridge Co.
Pittsburg pa.
Contract No. 642
1- Girder - A as shown,
1- Girder - B as noted.
2-Lots - L1
1-Lat Pl - P2
2-Lat Pls - P1
1-Struts - S1
2-Strut - S2
8'-0" From end to center

Fig. 160

GENERAL BRIDGE COMPANY
SHOP BILL

NAME OF STRUCTURE *I-Beam Bridge for A.N.&Y.R.R.*

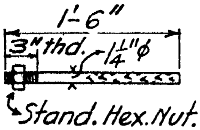
No of Pieces	Material: Kind	Material: Section	Material: Length ft.	Material: Length ins.	Piece Mark	Remarks	Calc'd Weight	Mill Order: No. of Pieces	Mill Order: Section	Mill Order: Length ft.	Mill Order: Length ins.	Item
	2-Beam Girders {1-A, 1-B}											
4	Is	20 70#	16	0			4480	4	20" 70#	16	0	1
6	[s	15 35#	1	3	a		262					S
4	[s	15 35#	1	3¾	c		184					S
2	[s	15 35#	0	11½	b	Fin. on bot.	68					S
4	Pls.	12 3/4	2	6		to 3/4"	306	4	12 7/8	2	6	2
							5300#					
	3-Struts {1-S1, 2-S2.}											
3	[s	9 20#	3	6¼			211	3	9" 20#	3	6¼	3
6	Ls	3½ 3½ 3/8"	0	9	d		38	1	3½" 3½" 3/8"	4	7	4
							249#					
	2-Lat. Pls.-Pl R/L.											
2	Pls.	9½" 3/8	1	4			32					S
2	Ls	3½ 3½ 3/8	1	0			17					S
							49#					
	1-Lat. Pl.-P2											
1	Pl.	9½ 3/8	2	3			28					S
1	L	3½ 3½ 3/8	2	3			19					S
							47#					
	2-Lats. L1.											
2	Ls	3½ 3½ 3/8	7	2			122	2	3½ 3½ 3/8	7	2	5
							122#					
							5767#					

Drawing No. 1 of 1 Contract No. 642

Made by C.D.M. 10/11 1910 Checked by C.D.R. 10/11/10 Page No. 1

GENERAL BRIDGE COMPANY
MISCELLANEOUS

NAME OF STRUCTURE *I-Beam Bridge for A.N.&Y.R.R.*

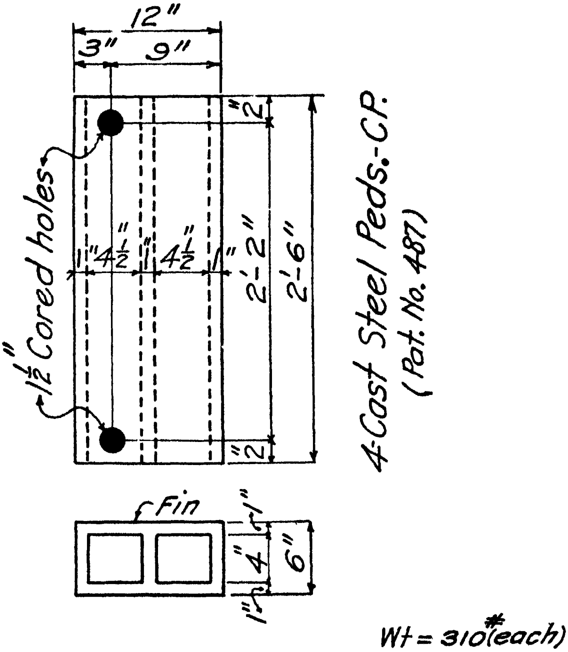
Sketch	No. Pc's	Description	Length ft.	Length ins.	Mark	Calc'd Weight	Order	Item
	8	1¼" Anchor Bolts	1	6	AA	56*		S
						56#		

Contract No. 642

Made by C.D.M. 10/11 1910 Checked by C.D.R. 10/14 10 Page No. 2

GENERAL BRIDGE COMPANY
SKETCH SHEET

NAME OF STRUCTURE *I-Beam Bridge for A.N. & Y. R.R.*



Contract No. 642

Made by C.D.M. 10/11 1910 Checked by C.D.R. 10/12 10 Page No. 3

GENERAL BRIDGE COMPANY

SHIPPING BILL

NAME OF STRUCTURE *I-Beam Bridge for A N&Y R.R.*

Member				Material				Calc'd Weight
No. Req'd	Name	Mark	Sheet No.	Size of Member	Length Ft.	Length In.	Remarks	
1	Beam Girder	A	1	22" x 20"	16	0	Incl'g rivets	2700
1	Beam Girder	B	1	22" x 20"	16	0		2700
2	Struts	S2	1	3" x 9"	3	6	"	168
1	Strut	S1	1	3" x 9"	3	6	"	84
2	Lat. Pls.	P1 R/L	1	10" x 3/8"	1	4		50
1	Lat. Pl.	P2	1	10" x 3/8"	2	3		47
2	Lats.	L1	1	3½" x 3½"	7	2		122
4	Cast Steel Peds.	CP	3	12" x 6"	2	6	Pat. No. 487.	1240
8	Anchor Bolts	AA	2	1¼ ⌀	1	6	Hex. Nuts.	56
							Tot. Wt. of Span =	7167

Contract No. 642

Made by C.D.M. 10/11 1910 Checked by B.D.R. 10/12/10 Page No. 4

GENERAL BRIDGE COMPANY

SUMMARY OF FIELD RIVETS AND BOLTS

NAME OF STRUCTURE *I-Beam Bridge for A.N.&Y. R.R.*

No. of Pieces	Dia.	Length of Rivet		Length of Bolt Under Head			Remarks	Calcd Weight
				Hex.	Square	Button		
							(Mat. Rivet Steel.)	
36	3/4"	3½"					About 10% added to	19
20	3/4"	2⅛"					number required.	8
5	3/4	2⅝"						2
								29
								# 29

Drawing No. *1 of 1* Contract No. *642*

Made by *C.D.M. 10/11 1910* Checked by *C.D.R. 10/12/10* Page No. *5*

General Bridge Company

Erectors List of Field Rivets and Bolts

NAME OF STRUCTURE *I-Beam Bridge for A.N.&Y. R.R.*

No. of Pieces	Grip	Dia.	Length of Rivet		Length of Bolt Under Head			Pieces Connected
					Hex.	Square	Button	
12	1 5/8"	3/4"	3 1/8"					S2 to Girders A & B.
6	1 5/8"	"	3 1/8"					S1 to " " ".
12	3/4"	"	2 1/8"					L1 to $P1^{R}_{L}$ & P2.
4	3/4"	"	2 1/8"					$P1^{R}_{L}$ to S2 & S1
2	3/4"	"	2 1/8"					P2 to S2.
8	1 5/8"	"	3 1/8"					$P1^{R}_{L}$ to Girder A.
5	1 5/8	"	3 1/8"					P2 to Girder B.
4	1"	"	2 5/8"					P2 " " ".

Drawing No. *1 of 1* Contract No. *642*

Made by *C.D.M. 10/11 1910* Checked by *C.D.R. 10/11/10* Page No. *6*

The superintendent of the shops, the inspectors, the erectors and others would receive the shop drawing and bills, depending upon the way the work is handled. The different companies do not have exactly the same kind of bills nor do they handle the work exactly in the same way. The above bills are intended only as a fair example of shop bills.

DRAWING ROOM EXERCISE NO. 2

Design a 12-ft. I-beam span and make a general drawing to a 1″ scale and a tracing of the same. The details to be similar to those shown in Fig. 159.

Data:

Length = 12′ 0″ c.c. end bearings.
Width = 5′ 0″ c.c. of girders.
Dead load to be assumed.
Live load, Cooper's $E50$.
Specifications, A. R. E. Ass'n.

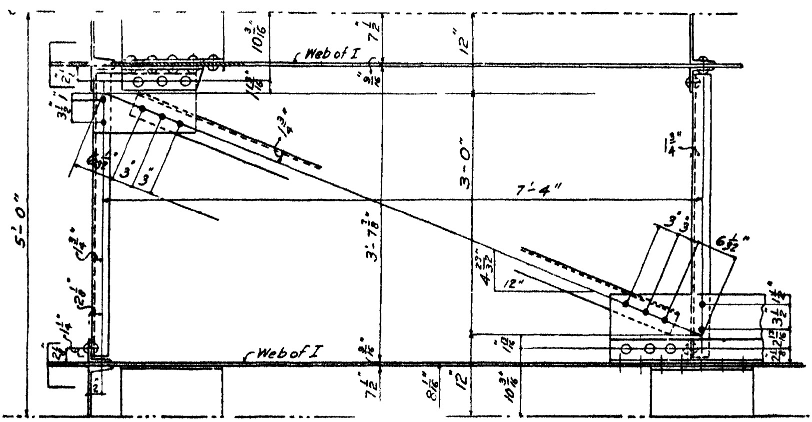


Fig. 161

A layout to a 1½″ or 3″ scale similar to the one shown in Fig. 161 should be made before beginning the final drawing for the span in order to determine the correct dimensions of details.

DECK PLATE GIRDER BRIDGES

132. Preliminary.—Deck plate girder bridges are used for spans ranging from 25 ft. to 110 ft. There have been a few spans longer than 110 ft. built, but such lengths are not economic, as a rule.

A single-track deck plate girder bridge is composed of two main girders connected to each other by cross frames and laterals. An isometric view of a typical single-track span is shown in Fig. 162 where the names of the different parts of the structure are given.

Double-track deck plate girder spans are, as a rule, composed of two single-track spans placed side by side.

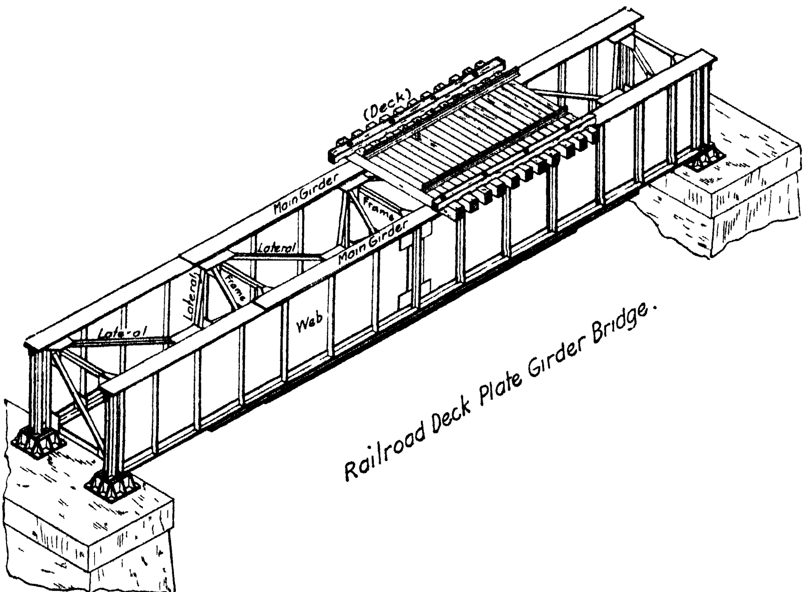


Fig. 162

Complete Design of 50-ft. Single-Track Deck Plate Girder Span

133. Data.—

Length = 50′-0″ c.c. end bearings.
Width = 6′-6″ c.c. girders.

Dead load = (750 + 400) = 1,150 lbs. per ft. of span (Art. 124).

Live load, Cooper's $E50$.

Specifications, A. R. E. Ass'n.

134. Calculations for Main Girders.—For the maximum bending moment due to dead load we have

$$M = \frac{1}{8} \times 575 \times \overline{50}^2 \times 12 = 2,156,000 \text{ inch lbs.}$$

From inspection of the diagram of the loading in Table A, we can see that the maximum moment due to live load will, very likely, occur under one of the heavy wheels, either wheel 12 or 13. Let us assume wheel 13, and let us consider wheels 9 to 16 on the span. Taking moments about wheel 16 (using Table A) we have

$$\bar{x} = \frac{2,740}{155-26} = 21.24 \text{ ft.}$$

for the distance that the center of gravity of these wheels (9 to 16 inclusive) is to the left of wheel 16. This shows that the center of gravity comes 2.24 ft. to the left of wheel 13. Then the maximum bending moment under wheel 13 will occur when the loads are in the position shown in Fig. 163. (See Art. 88.)

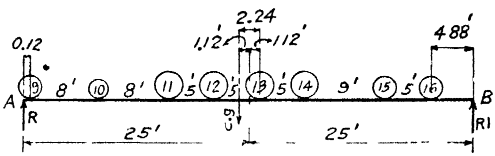


Fig 163

Taking moment about B (Fig. 163), and using Table A, we have

$$R \times 50 - 2,740 - (155 - 26)\ 4.88 = 0,$$

from which we obtain

$$R = 67,390 \text{ lbs.}$$

for the reaction at A.

Then taking moments about wheel 13, considering the forces to the left, we have

$$M' = 67,390 \times 26.12 - 818,000 = 942,200 \text{ foot lbs.}$$

or 11,306,000 inch lbs. for the bending moment at that wheel when wheels 9 to 16 are on the span. Next, suppose wheels 10 to 16 on the span. Taking moments of these wheels about 16 (using Table A) we have

$$\bar{x}_2 = \left(\frac{2,155}{142-26}\right) = 18.57 \text{ ft.}$$

for the distance that the center of gravity of the wheels (10 to 16 inclusive) is to the left of wheel 16. Therefore, the center of gravity is 0.43 ft. to the right of wheel 13. But when these wheels are placed on the span for maximum moment, wheel 17 comes on from the right. So we will consider wheels 10 to 17 on the span. Taking moments about wheel 17 we find that the center of gravity of wheels 10 to 17 is 2.9 ft. to the right of wheel 13. So this group of wheels will be in the position shown in Fig. 164 when the maximum under wheel 13 occurs.

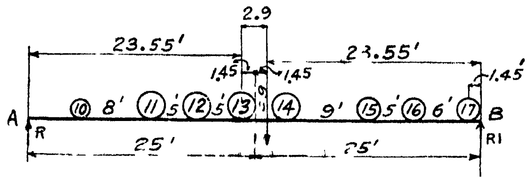


Fig 164

Then taking moments about B (Fig. 164) (using Table A) we have

$$R \times 50 - 2{,}851 - (142 - 13)\ 1.45 = 0,$$

from which we obtain

$$R = 60{,}760 \text{ lbs. for the reaction at } A.$$

Then taking moments about wheel 13 we have

$$M'' = 60{,}760 \times 23.55 - 480{,}000 = 950{,}900 \text{ foot lbs.}$$

or 11,411,000 inch pounds for the maximum bending moment under wheel 13 for this group of wheels. This is the absolute maximum as will be found by further trials. It is seen that the center of gravity is a little nearer to wheel 14 than it is to 13, yet the maximum moment occurs under wheel 13. This is one of the few cases where the maximum does not occur under the wheel nearer the center of gravity. (See Art. 88.) Generally, the position of the wheels for maximum moment can be ascertained the first trial. The above span is about as troublesome as any found.

For the maximum impact we have

$$I = 11{,}411{,}000 \times \left(\frac{300}{50 + 300}\right) = 9{,}780{,}000 \text{ inch lbs.}$$

Now multiplying the above maximum moment and impact each by 50/40, as the loading specified is $E50$, and adding these results to the dead-load moment, we have

$$2{,}156{,}000 + 14{,}263{,}000 + 12{,}220{,}000 = 28{,}639{,}000 \text{ inch lbs.}$$

for the total maximum moment on the span.

The next thing is to determine the economic depth. Assuming the thickness of the web to be $\frac{3}{8}''$ and substituting in equation (6), Art. 113, we have

$$x = 1.055 \sqrt{\frac{28{,}639{,}000}{16{,}000 \times \frac{3}{8}}} = 72.7 \text{ ins.,}$$

so we will use a $72'' \times \frac{3}{8}''$ web.

We can now determine the flange area required. It is readily seen that the effective depth will likely be a little less than the depth of the web, so let us assume 71″ as the effective depth. Then we have

$$28{,}639{,}000 \div 71 = 403{,}000 \text{ lbs.}$$

for the flange stress.

Then $$403{,}000 \div 16{,}000 = 25.2 \text{ sq. ins.},$$

the net area required for each flange.

Using the following:

2—∟s 6″ x 6″ x $\frac{9}{16}$″ = 12.86 − 2.25 = 10.61□″ net
1—cover plate 14″ x $\frac{1}{2}$″ = 7.00 − 1 = 6.00□″ net
1—cover plate 14″x$\frac{7}{16}$″ = 6.12 − 0.87 = 5.25□″ net
$\frac{1}{8}$ of the web = 3.37□″ net
25.23□″ net

for each flange we have practically the required area.

We can now determine the actual effective depth. Let Fig. 165 represent the cross-section of the above flange.

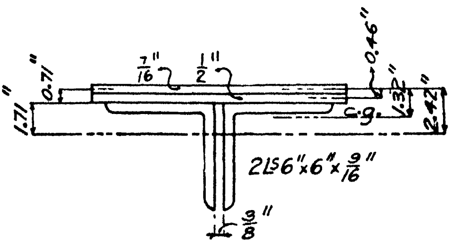


Fig. 165

Then taking moments about the center of the top cover plate (see Art. 47), we have

$$\bar{x} = \frac{12.86 \times 2.42 + 7.00 \times 0.46}{25.98} = 1.32 \text{ ins.}$$

for the distance from the center of the top cover plate to the center of gravity of the flange.

Then we have

$$1.32 - 0.71 = 0.61 \text{ ins.}$$

as the distance from the back of the angles to the center of gravity of the flange.

The web is assumed 72″ deep and according to practice the vertical distance from the back of top flange angles to the back of the bottom flange angles will be 0.25″ more or 72.25″. Then for the actual effective depth we have

$$d = 72.25 - (0.61 \times 2) = 71.03 \text{ ins.},$$

which is almost what we assumed, so recalculation of the flange is unnecessary. As a rule, the assumed effective depth does not come so close to the actual; however, $\frac{1}{2}$″ either way will not materially affect final results. If a greater difference than $\frac{1}{2}$″ is obtained the stress and section of the flange should be recalculated, using the computed effective depth.

The thickest cover plates should always be placed next to the flange angles. So in this case the $\frac{1}{2}''$ plate will be next to the angles, as shown in Fig. 165.

For the length of the $\frac{7}{16}''$ cover plate, or outside plate, we have

$$50\sqrt{\frac{6.12}{25.98}} = 24.3 \text{ ft.}$$

(see Art. 114) and for the length of the $\frac{1}{2}''$ cover plate we have

$$50\sqrt{\frac{13.12}{25.98}} = 35.5 \text{ ft.}$$

It is customary to make cover plates from two to three feet longer than the theoretical length, so we will make the above plates 27 and 37 feet long, instead of 24.3 and 35.5.

The next thing to determine is the stiffening angles and the web. To do this the first thing is to calculate the maximum end shear.

For maximum reaction or end shear due to dead load we have

$$R = \frac{1,150}{2} \times \frac{50}{2} = 14,400 \text{ lbs.}$$

The maximum live-load reaction or end shear will occur when the wheels are in the position shown in Fig. 166. (See Art. 86.)

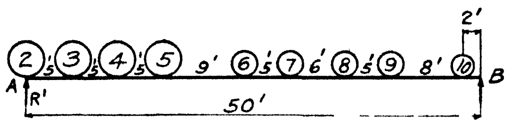


Fig 166

Taking moments about B, Fig. 166, and using Table A, we have

$$R' \times 50 - 4,072 - 142 \times 2 = 0,$$

from which we obtain

$$R' = 87,100 \text{ lbs.}$$

for the maximum reaction or end shear at A due to $E40$ loading. Then for $E50$ we have

$$87,100 \times \frac{50}{40} = 108,900 \text{ lbs.}$$

For impact we have

$$I = 108,900 \times \frac{300}{350} = 93,400 \text{ lbs.}$$

Now adding the above dead- and live-load reactions and impact together we have

$$14,400 + 108,900 + 93,400 = 216,700 \text{ lbs.}$$

for the maximum reaction or end shear on each girder.

According to the specifications the outstanding legs of intermediate stiffeners must not be less than one-thirtieth of the depth of the girder plus two inches. So we have

$$\frac{72}{30} + 2 = 4.4 \text{ ins.}$$

for the required width of their outstanding legs. The standard angle coming nearest to this requirement is a 5″ x 3½″, which will be used throughout.

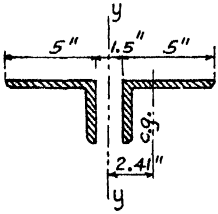


Fig. 167

The end stiffeners must be designed to transmit the total maximum reaction, each pair being considered as a column having a length equal to one-half the depth of the girder. (See specifications.) Then assuming ∟s – 5″ x 3½″ x ½″ used, we have for each pair a column having a cross-section as shown in Fig. 167. For the radius of gyration of this column in reference to axis y-y, we have

$$r = \sqrt{\frac{8. \times 2.41^2 + 19.98}{8}} = 2.88.$$

Then substituting this radius and 36″ for the length in Formula *Q*, Art. 73, we have

$$16{,}000 - 70\,\frac{36}{2.88} = 15{,}100 \text{ lbs.}$$

for the allowable compressive unit stress on the end stiffeners.

Now dividing this stress into the maximum reaction we have

$$\frac{216{,}700}{15{,}100} = 14.35 \text{ sq. ins.}$$

for the required area of cross-section of the end stiffeners. So we will use two pairs, or

4—∟s 5″ x 3½″ x ½″ = 16.0 sq. ins.,

the 5 x 3½ x $\frac{7}{16}$ angles being a little too small.

There is no theoretical way of computing the area of the intermediate stiffeners. It is practice to make them as thin as the specifications will permit. So we will use 5″ x 3½″ x ⅜″ angles throughout for intermediate stiffeners.

Using the assumed web, 72″ x ⅜″, we have

$$s = \frac{216{,}700}{27} = 8{,}020 \text{ lbs.}$$

for the maximum average unit shearing stress in the web.

Then substituting in Formula (1), Art. 118, we have

$$d = \frac{3}{40}\,(12{,}000 - 8{,}020) = 37 \text{ ins. (about)}$$

for the required distance between the stiffeners near the ends of the girders. Now as this spacing is not less than half the depth of the web the assumed web is economic and hence will be used.

For the bearing on the masonry we have

$$\frac{216{,}700}{600} = 361 \text{ sq. ins.}$$

Each support must be so designed that there will be at least this much area of bearing on the masonry.

This completes the necessary calculations for the main girders, as far as the general design is concerned, and the next thing is the designing of the lateral bracing, that is, the laterals and frames.

135. Calculations for Lateral Bracing.—The lateral bracing should always be symmetrical about the center of the span. The laterals should have a slope as near 45° as is practicable. The distance between cross-frames should never be over 15 ft. In accordance with this there must be three intermediate cross-frames in a 50-ft. span, as a less number would place them more than 15 ft. apart. So the lateral bracing will be as shown in Fig. 168.

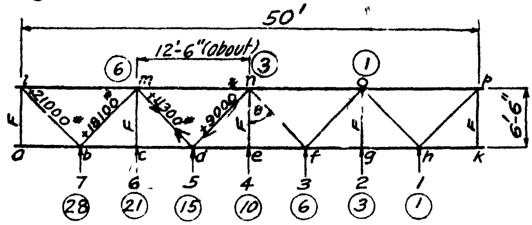


Fig. 168

According to the specifications the laterals must resist a uniform lateral force of $200 + 0.10\ (5,000) = 700$ lbs. per ft. of span, considered as a moving load.

Suppose this force or load acts from the direction indicated by the arrows, and suppose it moves on to the span from the right, as a uniform live load. The panel load at any point h, g, etc., will be

$$P = 700 \times 6.2 = 4,340 \text{ lbs. (about).}$$

Then for the maximum shear in the different panels, according to Art. 90, we have

$$\text{Shear in panel } hg = \frac{4340}{8} \times 1 = 540 \text{ lbs.;}$$

$$\text{" " " } gf = \frac{4340}{8} \times 3 = 1,630 \text{ lbs.;}$$

$$\text{" " " } fe = \frac{4340}{8} \times 6 = 3,260 \text{ lbs.;}$$

$$\text{" " " } ed = \frac{4340}{8} \times 10 = 5,420 \text{ lbs.;}$$

$$\text{" " " } dc = \frac{4340}{8} \times 15 = 8,140 \text{ lbs.;}$$

$$\text{" " " } cb = \frac{4340}{8} \times 21 = 11,400 \text{ lbs.;}$$

$$\text{" " " } ba = \frac{4340}{8} \times 28 = 15,200 \text{ lbs.}$$

Now if each of these shears be multiplied by the secant of the angle marked θ we shall obtain the stress in the corresponding diagonals (see Art. 92).

The tangent of angle θ, for the purpose of determining stresses, can be taken as

$$\frac{6.25}{6.50}=0.962 \text{ (about)},$$

and the corresponding secant can then be found in almost any table of natural trigonometrical functions (see Carnegie or Cambria handbook), or the secant can be determined directly by arithmetic.

For the above assumed figures we have

$$\text{Sec } \theta = 1.39.$$

Then for the maximum stress in the diagonals we have

$$5{,}420 \times 1.39 = -\ 7{,}500 \text{ lbs. for diagonal } nd,$$
$$8{,}140 \times 1.39 = +11{,}300 \text{ lbs. for diagonal } dm,$$
$$11{,}400 \times 1.39 = -15{,}800 \text{ lbs. for diagonal } mb,$$
$$15{,}200 \times 1.39 = +21{,}000 \text{ lbs. for diagonal } bl.$$

These are all of the stresses that we need determine with the load applied as indicated as the corresponding diagonals in the right half of the span will have the same stress when the force moves on to the span from left to right.

If the lateral force comes from the other direction than that indicated by the arrows, the panel loads will be twice as great as given above, as there are only half as many panels considered.

Then the stress in each of the diagonals *nd* and *dm* is

$$\frac{4{,}340 \times 2}{4} \times 3 \times 1.39 = \pm\ 9{,}050 \text{ lbs.}$$

and

$$\frac{4{,}340 \times 2}{4} \times 6 \times 1.39 = \pm\ 18{,}100 \text{ lbs.}$$

in each of the diagonals *mb* and *bl*.

We now have the maximum stresses in the diagonals determined which are indicated in the diagram in Fig. 168.

The plus and minus signs above signify compression and tension, respectively.

It is seen from the above that the diagonals have to resist both tension and compression. Compression will likely govern.

Let us try a single angle, say, 1—L $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$, as this is the smallest that the specifications will permit. From Table 6, or from a Carnegie or Cambria handbook, we have 1.07 for the radius of gyration of this angle about the horizontal axis (see Art. 130) and the length of each lateral will be about 8 ft. or 96 ins., which can be determined accurately enough by scale. Then substituting in Formula *Q*, Art. 73, we have

$$16{,}000 - 70\ \frac{96}{1.07} = 9{,}700 \text{ lbs.}$$

for the allowable unit stress.

Dividing the greatest compressive stress, which occurs in the lateral *bl*, by this intensity we have

$$\frac{21{,}000}{9{,}700} = 2.16 \text{ sq. ins.}$$

for the required cross-section of the lateral, and the assumed angle has 2.48□″. Then, as the ratio of L/r is only about 90, the assumed angle is safe in the case of the greatest compressive stress.

As is seen above the greatest tension is 18,100 lbs., which occurs also in lateral bl. Then we have

$$\frac{18,100}{16,000}=1.13 \text{ sq. ins.}$$

for the required net cross-section, and as the angle assumed has $[2.48-(\frac{3}{8}''\times 1'')]$ 2.11□″ it is quite safe for the greatest tension. So we will use 1—L $3\frac{1}{2}''\times 3\frac{1}{2}''\times \frac{3}{8}''$ for each end lateral. But as this angle is the smallest permitted in this work the laterals throughout will each be made of 1—$3\frac{1}{2}''\times 3\frac{1}{2}''\times \frac{3}{8}''$ angle.

The intermediate cross-frames are not subject to theoretical analyses and hence their design is governed principally by experience and sense of fitness.

The end cross-frames can be fairly well analyzed. The stress in the top angle of the frame can be taken equal to one-half of the lateral force per foot of span multiplied by one-half the length of the span, and this force multiplied by the secant of the slope of the diagonals in the frame is equal to the stress in each of these diagonals. The bottom angle of the frame has no stress (theoretically).

Then according to the above we have

$$\frac{700}{2}\times 25=8,750 \text{ lbs.}$$

for the stress in the top angle of the end frame, and as the diagonals of the frame slope about 45° with the horizontal we have

$$8,750\times 1.4=12,250 \text{ lbs.}$$

for the stress in the diagonals of the end frame. From this it is seen that very small angles are required theoretically for the end frames which are subjected to greater stresses (apparently) than the intermediate frames, but as $3\frac{1}{2}''\times 3\frac{1}{2}''\times \frac{3}{8}''$ angles are practically the minimum size used in railroad bridges we will use this size in all of the frames.

This completes the necessary preliminary calculations except for the preliminary estimate of the dead weight. The stress sheet for the span, as shown in Fig. 169, can be drawn from the information given in the above calculations. The estimate of the dead weight in this case will be deferred until after the detail shop drawing of the span is considered so as to familiarize the student with details before taking up preliminary estimates of dead weight.

136. Making of the Shop Drawing.—After the stress sheet (Fig. 169) is completed the shop drawing for the span, as shown in Fig. 171, can be made. This work is known as detailing and includes not only the drawing but the calculations for the details as well.

To evolve this drawing (Fig. 171) the details are first drawn in pencil to a $\frac{3}{4}''$ scale upon a $24''\times 36''$ sheet of ordinary drawing paper. After selecting the relative positions for the different views, so that no part of the sheet will be crowded, we start the drawing, as shown in Fig. 170, by drawing the center line CC of the span. Then scaling off 25 ft.,

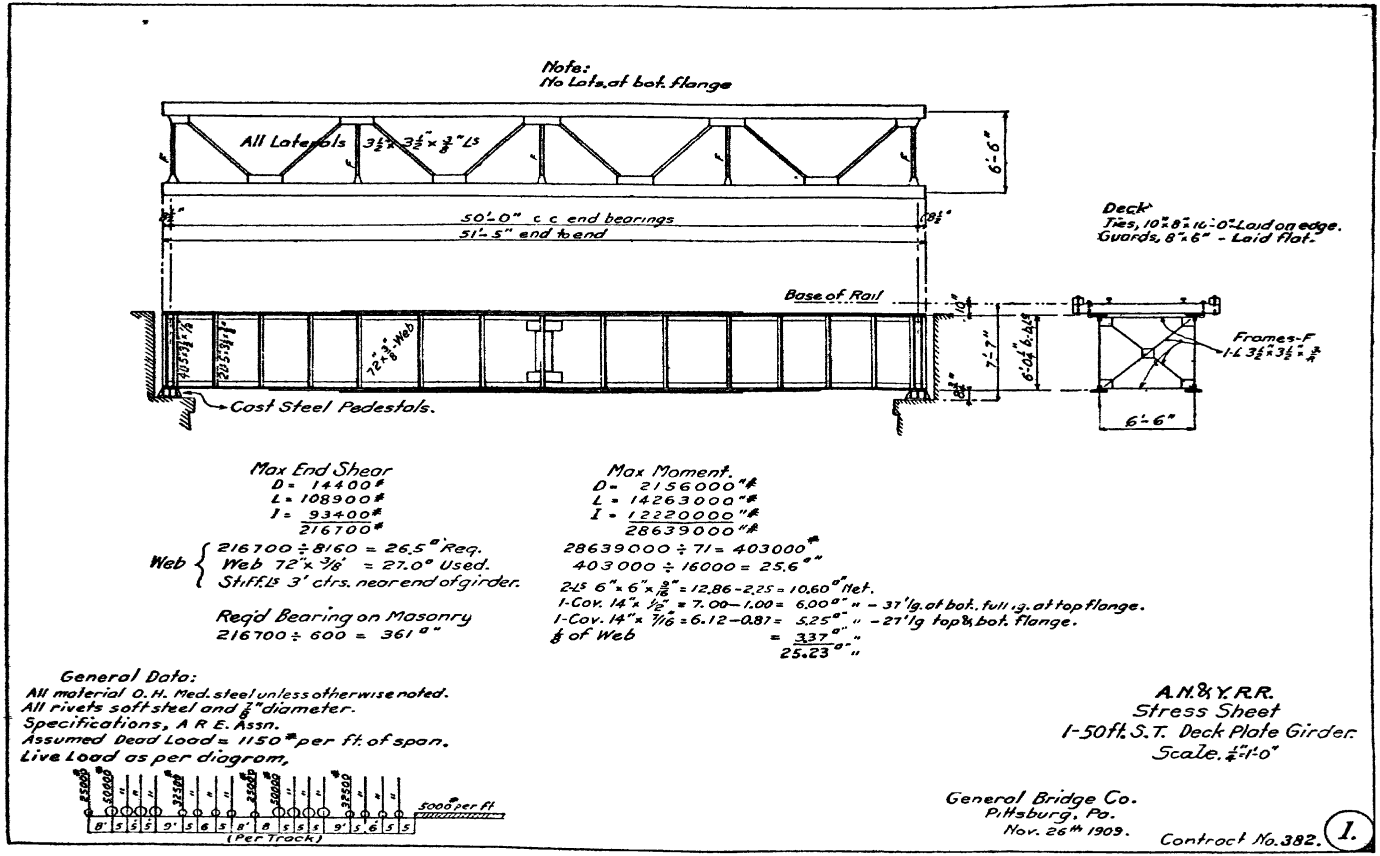

Note:
No Lats. at bot. flange
All Laterals 3½"×3½"×⅜" Ls
6'-6"
50'-0" c c end bearings
51'-5" end to end
Deck
Ties, 10"×8"×10'-0" Laid on edge.
Guards, 8"×6" - Laid flat.
Base of Rail
72"×⅜" Web
Cast Steel Pedestals.
Frames-F
1-L 3½"×3½"×⅜"
6'-6"
Max End Shear
D= 14400#
L= 108900#
I= 93400#
216700#
Web { 216700 ÷ 8160 = 26.5□" Req.
Web 72"×⅜" = 27.0□" Used.
Stiff.Ls 3' ctrs. near end of girder.
Req'd Bearing on Masonry
216700 ÷ 600 = 361□"
Max Moment.
D= 2156000"#
L= 14263000"#
I= 12220000"#
28639000"#
28639000 ÷ 71 = 403000#
403000 ÷ 16000 = 25.6□"
2-Ls 6"×6"×9/16" = 12.86-2.25 = 10.60□" Net.
1-Cov. 14"×½" = 7.00-1.00 = 6.00□" " - 37' lg. at bot., full lg. at top flange.
1-Cov. 14"×7/16 = 6.12-0.87 = 5.25□" " - 27' lg top & bot. flange.
⅛ of Web = 3.37□"
25.23□"
General Data:
All material O.H. Med. steel unless otherwise noted.
All rivets soft steel and ⅞" diameter.
Specifications, A R E. Assn.
Assumed Dead Load = 1150# per ft. of span.
Live Load as per diagram,
25000# 50000# 32500# 25000# 50000# 32500#
5000# per ft
8' 5 5 5 9' 5 6 5 8' 8 5 5 5 9' 5 6 5 5
(Per Track)
A.N.&Y.R.R.
Stress Sheet
1-50 ft. S.T. Deck Plate Girder.
Scale. ¼"=1'-0"
General Bridge Co.
Pittsburg, Pa.
Nov. 26th 1909.
Contract No. 382.
1.

Fig 169

the line *BB* through the center of bearing is drawn. Next, the line *EE*, through the end of the span, must be located and drawn. The end stiffeners should be placed 6″ back to back as shown, this being about the minimum distance permissible on account of driving rivets in the outstanding legs of the stiffeners connecting to the end cross-frames. Then allowing, say, 2″ from the end pair of stiffeners to the end of the girders, we have $3'' + 3\frac{1}{2}'' + 2'' = 8\frac{1}{2}''$ for the distance from the line *BB* to line

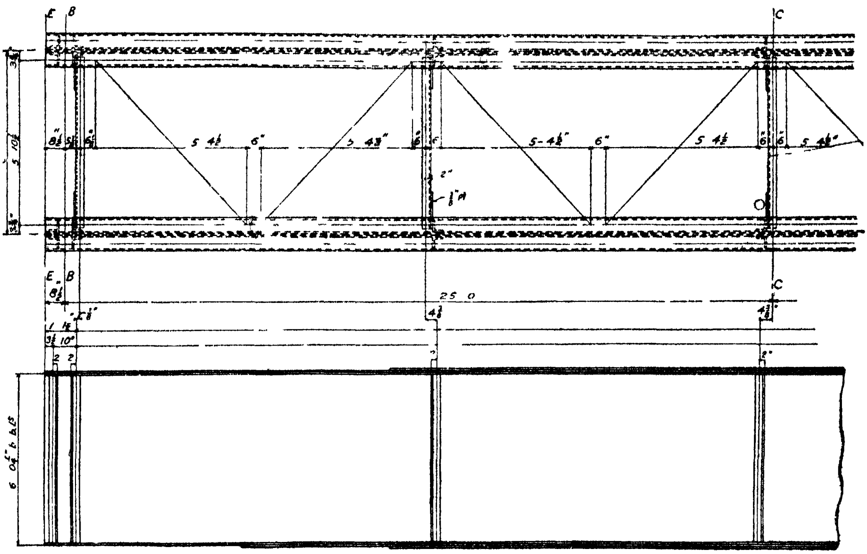


Fig 170

EE, and the line *EE* locating the end of the span can then be drawn. Then the top plan of each girder is drawn as shown in Fig. 170. The next thing after that is to locate the cross-frames as shown, so that the lateral bracing will be symmetrical about the center line *CC*, and the laterals will all have the same length (thus saving templets). Now there will be a pair of stiffeners on each girder at each cross-frame, one of which, in each case, will be connected to a frame, and when the position of the cross-frames is fixed these stiffeners to which the frames connect are also fixed in position. Then the elevation of one of the girders, always the far girder, can be drawn and the drawing completed as far as is shown in Fig. 170.

The next thing is to draw the laterals and the lateral plates connecting the laterals to the girders (as shown in Fig. 171). As the laterals are all of the same section and the strength of each is greater than is required by the stress, each end connection must contain enough rivets to develop the strength of the lateral in compression (see specifications). Now, as shown above, each lateral will safely resist 9,700 lbs. per square inch of cross-section. Then, as 2.48□″ is the area of cross-section of each lateral, we have $2.48 \times 9{,}700 = 24{,}000$ lbs. for the total allowable compressive stress in each lateral. Then using $\frac{7}{8}''$ field rivets for the

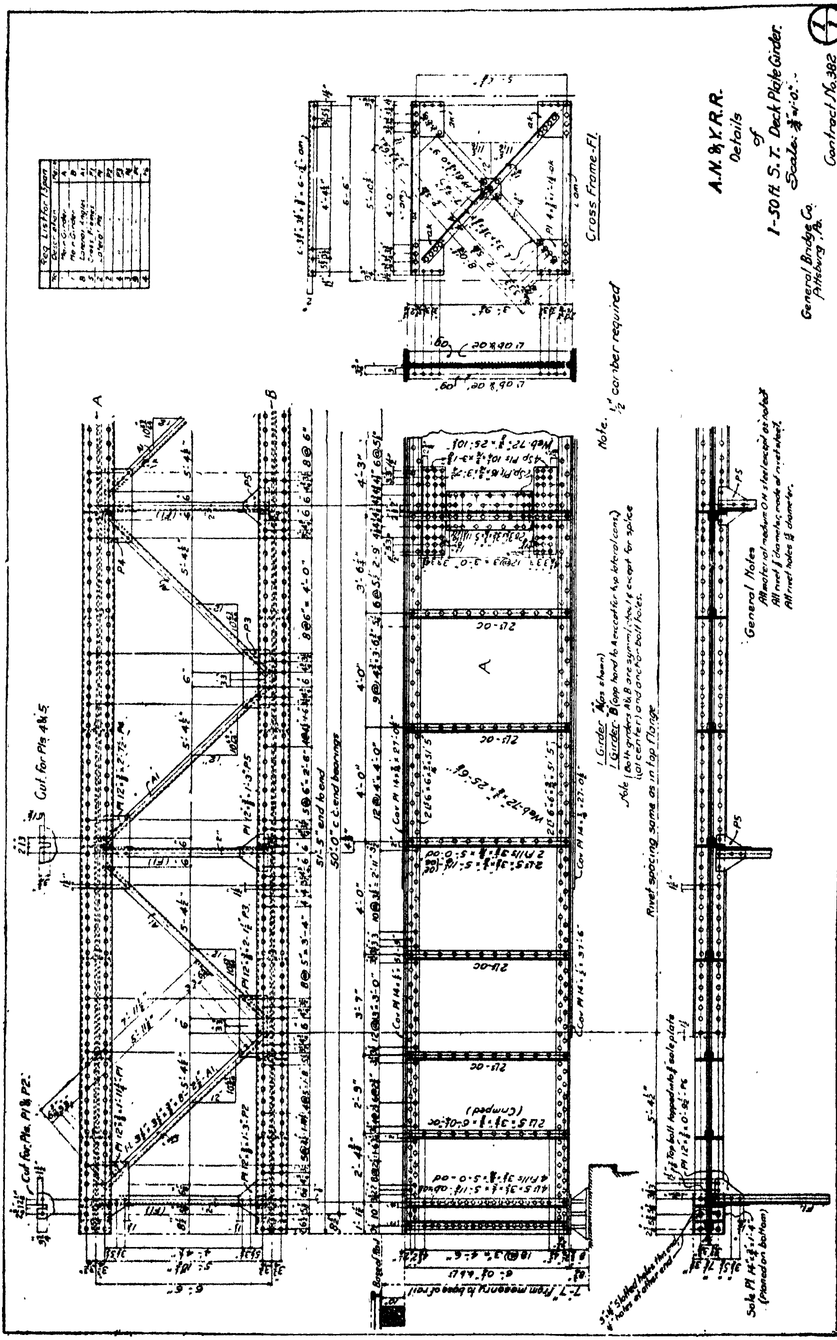

A.N. & Y.R.R.
Details
of
1-50 ft. S.T. Deck Plate Girder.
Contract No. 382
General Bridge Co.
Pittsburg, Pa.
Cross Frame-F1.
Note. 1/2" camber required
General Notes
Rivet spacing same as in top flange
51'-5" end to end
50'-0" c. c. end bearings

Fig 171

General Bridge Company
Shop Bill.

Name of Structure *50 ft. S.T. Deck Plate Girder Bridge, for A N & Y R.R.*

No. of Pieces	Material: Kind	Material: Section			Material: Length ft.	Material: Length ins.	Piece Mark	Remarks	Calc'd Weight	Mill Order: No. of Pieces	Mill Order: Section		Mill Order: Length ft.	Mill Order: Length ins.	Item
	2- Girders						{1- A. 1- B.								
2	Webs		72	3/8	25	6 1/8							25	6 1/8	
2	"		72	3/8	25	10 1/2							25	10 3/4	
2	Cov Pls		14	1/2	51	5							51	5 1/2	
2	"		14	1/2	37	6									
4	"		14	7/16	27	0 1/2									
8	Flg Ls	6	6	9/16	51	5							51	5 1/2	
16	Stiff. Ls	5	3 1/2	1/2	5	11 1/8	ab, ab'			4			23	9 1/2	
40	" "	5	3 1/2	3/8	6	0 1/4	ac	Crimped		10			24	2	
8	" "	5	3 1/2	3/8	5	11 1/8	ae, ae'			{ 3			23	9 1/2	
4	" "	5	3 1/2	3/8	5	11 1/8	af, af'								
24	Fills		3 1/2	9/16	5	0	ad & ad'			4			30	1	
8	Sp Pls.		10 1/4	9/16	3	1 3/8				1			24	11	
4	" "		16	9/16	3	2 3/4				1			12	11	
4	Sole Pls		14	3/4	1	4		Fin on Bot		1	14	13/16	5	4	
		8 - Laterals - A1													
8	Ls	3 1/2	3 1/2	3/8	8	2 1/2				2			32	11	

Drawing No. *1 of 1* Contract No. *382*
Made by *C. D. M. 11/2 1910* Checked by *G.D.R. 11/4 10* Page No. *1*

GENERAL BRIDGE COMPANY
SHOP BILL.

NAME OF STRUCTURE 50 ft. S.T. Deck Plate Girder Bridge for A.N.&Y.R.R.

No. of Pieces	Material: Kind	Material: Section			Length: ft.	Length: ins.	Piece Mark	Remarks	Calc'd Weight	Mill Order: No. of Pieces	Mill Order: Section	Mill Order: Length ft.	Mill Order: Length ins	Item
		5 - Cross Frames - F1												
10	Ls	$3\frac{1}{2}$	$3\frac{1}{2}$	$\frac{3}{8}$	6	$1\frac{1}{2}$	am			2		30	8	
10	Ls	$3\frac{1}{2}$	$3\frac{1}{2}$	$\frac{3}{8}$	7	$3\frac{1}{4}$				2		36	5	
20	Pls.		14	$\frac{3}{8}$	1	$1\frac{1}{2}$	ak			1		22	7	
5	Pls		$8\frac{1}{2}$	$\frac{3}{8}$	0	9								5
	24 - Lateral Plates													
2	Pls.		12	$\frac{3}{8}$	1	$11\frac{1}{4}$	—	P1						
2	"		12	$\frac{3}{8}$	1	3	—	P2		1		14	11	
4	"		12	$\frac{3}{8}$	2	$1\frac{1}{2}$	—	P3						
3	"		12	$\frac{3}{8}$	2	$7\frac{1}{2}$	—	P4						
9	"		12	$\frac{3}{8}$	1	3	—	P5		1		22	4	
4	"		12	$\frac{3}{8}$	0	$9\frac{1}{2}$	—	P6						

Drawing No. 1 of 1 Contract No. 382

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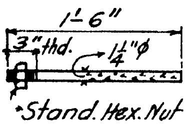
GENERAL BRIDGE COMPANY

MISCELLANEOUS

NAME OF STRUCTURE *50 ft. Deck Plate Girder Bridge for A.N.& Y.R.R.*

Sketch



Material					Calc'd Weight	Order	
No. Pcs.	Description	Length Ft.	Length Ins.	Mark			Item
8	1¼"φ Anchor Bolts	1	6	AA			S

Contract No. 382

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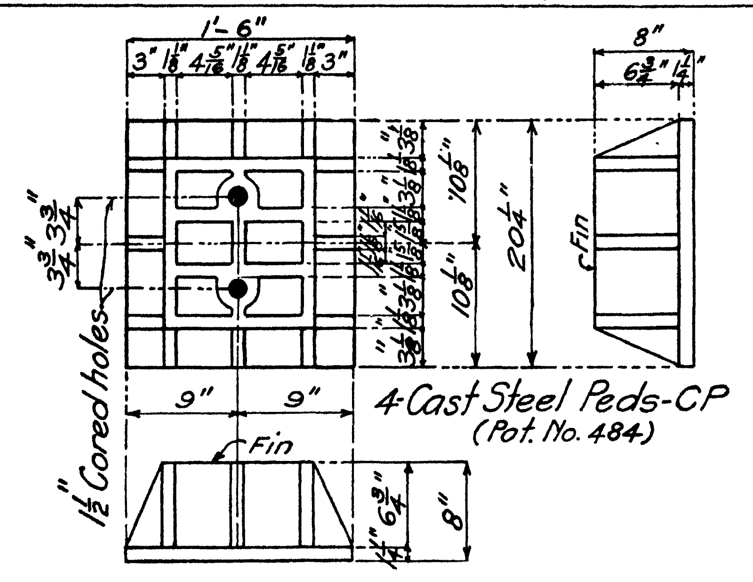
GENERAL BRIDGE COMPANY

SKETCH SHEET.

NAME OF STRUCTURE 50 ft. S.T. Deck Plate Girder Bridge, for A.N.&Y. R.R.



Contract No. 382

Made by G.D.M. 11/2 1910 Checked by E.D.R. 11/4-10 Page No. 4

GENERAL BRIDGE COMPANY

ERECTORS LIST OF FIELD RIVETS AND BOLTS

NAME OF STRUCTURE 50 ft. S.T. Deck Plate Girder Bridge for A.N.& Y.R.R.

No. of Pieces	Grip	Dia.	Length of Rivet		Length of Bolt Under Head			Pieces Connected
					Hex.	Square	Button	
48	3/4	7/8	2 1/4					Lats. A1 to Pls. P1, P3, & P4.
6	1 13/16	"	3 1/2"					Lats A1 to Pls. P1, P3 & Girders.
10	2 1/4	"	3 7/8					Lats. A1 to P3, P4 & Girder.
40	3/4	"	2 1/4"					Cross Frames F1 to Pls. P1, P2, P3, P4, P5 & P6.
8	1 7/16	"	3 1/8"					Plates P3 to Girder
4	1 13/16	"	3 1/2"					Cross Frames F1 to Pls. P1, P2 & Girders.
12	2 1/4"	"	3 7/8"					" " " " " P4, P5 & Girders.
10	1 7/16	"	3 1/8"					Plates P1 and P2 to Girders.
30	1 7/8	"	3 1/2"					" P3, P4, & P5 to Girders.
8	15/16	"	2 1/2"					" P6 to Girders.
24	7/8	"	2 3/8"					Cross Frames F1 to end stiff Ls.
8	1 1/4	"	2 3/4"					" " " " " " "
36	3/4	"	2 1/4"					" " " " Int stiff. Ls.
12	1 1/8	"	2 5/8"					" " " " " " "

Drawing No. 1 of 1 Contract No. 382

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GENERAL BRIDGE COMPANY

SUMMARY OF FIELD RIVETS AND BOLTS

NAME OF STRUCTURE *50 ft. S.T. Deck Plate Girder Bridge for A.N.& Y. R.R.*

No. of Pieces	Dia.	Length of Rivet		Length of Bolt Under Head			Remarks	Calc'd Weight
				Hex.	Square	Button		
	"	"						
136	7/8	2 1/4						
44	"	3 1/2"						
25		3 7/8"						
14		2 5/8"						
20		3 1/8"						
10		2 1/2"						
27		2 3/8"						
10		2 3/4"						

Drawing No. *1 of 1* Contract No. *382*

Made by *C.D.M. 11/2 1910* Checked by *C.D.R. 11/4/10* Page No. *6*

General Bridge Company

Shipping Bill

Name of Structure 50 ft. S.T. Deck Plate Girder Bridge for A.N.&Y.R.R.

Member				Material				Calc'd Weight
No. Req'd	Name	Mark	Sheet No	Size of Member	Length Ft.	Length Ins	Remarks	
1	Girder	A	1	76"× 14"	51	5		
1	Girder	B	1	76"× 14"	51	5		
8	Laterals	A1	1	3½"× 3½"	8	3		
5	Cross Frames	F1	2	73"× 7½"	5	11		
2	Lat. Plates	P1	2	12"× 3/8"	2	0		
2	Lat. Plates	P2	2	12"× 3/8"	1	3		
4	Lat Plates	P3	2	12"× 3/8	2	1½		
3	Lat. Plates	P4	2	12"× 3/8	2	7½		
9	Lat. Plates	P5	2	12"× 3/8"	1	3		
4	Lat. Plates	P6	2	12"× 3/8"	0	9½		
4	Cast Steel Peds.	CP	4	18"× 8"	1	8¼	Pat. No 484	
8	Anchor Bolts	AA	3	1¼ϕ"	1	6	1-Hex. Nut on each bolt.	

Contract No. 382

Made by C.D.M. 11/2 1910 Checked by C.D.R. 11/4 10 Page No. 7

lateral connections, the value of which in single shear will govern, 6,000 lbs. being the allowable value of each, we have

$$\frac{24,000}{6,000}=4$$

for the number of rivets required in each end of each lateral.

Now, the number of rivets connecting the lateral plates to the girders must be sufficient to take the component along the girder of the lateral or laterals. Twenty-four thousand pounds is the strength of each lateral, and using $\frac{7}{8}''$ field rivets in single shear, the number of rivets for intermediate points will be

$$2\left(\frac{24,000\times\sin\theta}{6,000}\right)=2\left(\frac{24,000\times .695}{6,000}\right)=5.5.$$

The number used would be not less than 6, or 3 for each lateral.

The laterals and lateral plates can now be drawn in the top plan (shown in Fig. 171), giving the spacing of the rivets at each connection, and determining clearances and the sizes of plates by scale as the work progresses, using separate sketches for each point drawn, say, to $1\frac{1}{2}''$ or $3''$ scale. After this work is completed the stiffeners between the cross-frames can be drawn on the elevation of the girder, care being taken to space the stiffeners so as to miss the lateral plates as much as possible. Then the longitudinal section showing the bottom flange can be drawn.

After this the next thing is to space the rivets in the vertical legs of the flange angles. As the live load is applied directly to the top flanges and the specifications ignore the one-eighth of the web in determining the spacing of flange rivets, the problem of determining the spacing comes under Case III, Art. 116.

We assume that each wheel of the live load is distributed over three ties and assuming ties 8″ wide and spaced 6″ apart (face to face) we have 42″ for the length over which each wheel is distributed. Then for the greatest vertical load per linear inch of flange due to the live load (Cooper's E50), when the maximum shear occurs, we have

$$\frac{25,000}{42}=595 \text{ lbs., say } 600 \text{ lbs.,}$$

per linear inch, and adding a 100 per cent for impact will make a total of 1,200 lbs. per inch.

Now referring to Case III, Art. 116, we have

$v=$ 1.200 lbs.

$R=$ 7,880 lbs., which is the allowable bearing of a $\frac{7}{8}''$ shop rivet on the $\frac{3}{8}''$ web.

$S=$ 216,700 lbs., which is the maximum shear at either end of each girder.

$h=$ 66 ins.

Then substituting these values in Formula (4), Art. 116, we have

$$p=\frac{7,880}{\sqrt{1,200^2+\left(\frac{216,700}{66}\right)^2}}=2.27 \text{ ins. (about)}$$

for the theoretical spacing of the rivets near the ends of the girders, and the theoretical spacing at other points along the girders can be determined by computing the maximum shear at each of the points and applying Formula (4), Art. 116, in each case. However, the following graphical method of determining the spacing is preferable as it saves time and is accurate enough for practical work:

The shear due to dead load is quite small compared with that due to live load and impact, so that the maximum shear can be assumed to vary across the span as the ordinates to a parabola (see Art. 55).

Now the above spacing (2.27″) gives about 5.3 rivets per foot of girder. Then if we lay off a vertical line *AB* (Fig. 172) equal to 5.3″, and taking a pair of dividers divide the line *AB* into any convenient

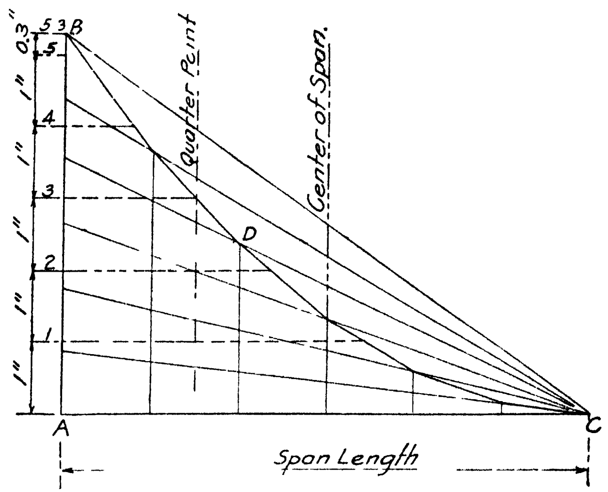


Fig 172

number of equal parts and from *A* step off the same number of equal parts on the line *AC* (the equal parts on the line *AC* can be taken any length, just so *A* to *C* is a convenient distance), and construct the parabola *CDB* (see Art. 58) and draw the horizontal lines to the curve through the points 1, 2, 3, etc., we have a diagram from which the required spacing of the rivets at any point between the end and the center of the span can be obtained, as the distance *AC* corresponds to the length of the span and the line *AB* gives the shear (correspondingly) in number of rivets per foot of girder. From the diagram (Fig. 172) it is seen that about one and one-third rivets per foot are required at the center of the span, and practically three rivets per foot at the quarter point. The required spacing at all other points from *A* to the center of the span is seen at a glance.

It is not practical, if not impossible, to space the rivets throughout a girder according to the theoretical requirements, as no two spaces would be the same (which is impractical) and the spaces next to stiffeners can not be less than about $3\frac{1}{2}''$, which in some cases is greater than the required; so the best we can do is to fit the rivets in between the stiffeners to about the theoretical spacing.

The theoretical spacing of the flange rivets as determined in the manner shown above will satisfy the requirements of the specifications, which are as follows:

"The flanges of plate girders shall be connected to the web with sufficient number of rivets to transfer the total shear at any point in a distance equal to the effective depth of the girder at that point combined with any load that is applied directly to the flange."

This is simply a practical manner of obtaining approximately the theoretical spacing. To show the justification of the above requirement given in the specifications let AB, Fig. 173 represent a plate girder. Let M and S be the bending moment and shear, respectively, at section C. Then for the bending moment at section D we have

$$M' = M + Sx - \Sigma Pz. \quad \text{(See Art. 66.)}$$

Now, $Sx - \Sigma Pz$ is the difference between the moments at the two sections. If the intervening loads be neglected, which is an error on the side of safety, the difference between the two moments is Sx, and if this be divided by the effective depth the result would be the difference between the flange stresses at the two sections, which would be equal to the shear when x is equal to the effective depth, as is readily seen. This difference of flange stress is simply the increment of the flange stress between the two sections and of course there should be a sufficient number of rivets to transfer this from the web to the flange and at the same time support whatever vertical load there is applied to the flange. This is entirely in accord with Arts. 115 and 116.

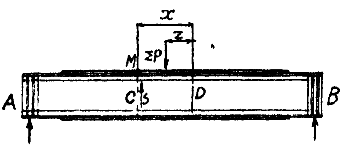


Fig 173

Now, having the stiffeners spaced as shown in Fig. 171 we can begin spacing the rivets in the vertical legs of the flange angles. Beginning at the end of the girder, the theoretical spacing as given above is 2.27″, but we will use 2¼″ in order to have practical spaces. So we obtain the spacing from the end of the girder to the first intermediate stiffener as shown in Fig. 171. Between the first and second stiffeners we can increase the spacing a little as shown by the diagram in Fig. 172, and between the second and third stiffeners we can increase the spacing a little more and so on towards the center of the span until the limit of 6″ spacing is reached, which according to the specifications must not be exceeded. It is practice to limit the fractions in the spacing to no less than ¼″. If necessary the stiffeners can be shifted slightly to suit the spacing.

After the rivets are spaced in the vertical legs of the flange angles the rivets in the horizontal legs, connecting the cover plates to the angles, can be spaced. The theoretical requirement in this case is that the rivets be spaced so that the number of rivets per foot will be sufficient to transmit the part of the flange increment taken by the cover plates. The part of the flange increment taken at any point by the cover plates will be to the total flange increment at that point as the area of the cross-section of the cover plates is to the total area of the cross-section of the flange at that point. As an example, let M be the bending moment found at section C (Fig. 173) and M' the bending moment at section D, which is a short distance to the right of C (say, a foot). Then for the total flange increment between the two sections we have

$$\frac{M - M'}{h} = F,$$

where h is the effective depth of the girder. Now it is obvious that there must be a sufficient number of rivets in the vertical legs of the flange angles between the two sections to take all of this increment as the total increment is transmitted through these rivets, but as the cover plates take only their proportional part of the increment, the number of rivets connecting them to the flange angles need be only sufficient to take their part. So if r be the allowable stress on each rivet connecting the cover plates to the flange angles, and A the total area of cross-section of the flange between section C and D. and A'', the area of the cover plates, we have

$$n = \frac{F}{r} \times \frac{A''}{A}$$

for the number of rivets required in the cover plates between the two sections and the number of rivets required to connect the cover plates to the flange angles at any other point along the girder can be determined in the same manner, but the same result can be obtained in any case with much less work by using such a diagram as shown in Fig. 172. So we will use the above diagram (Fig. 172) in this case. Beginning at the center of the span, the number of rivets per foot required to take the total flange increment is $1\frac{1}{3}$. Then as the net area of cross-section of the flange at that point, not including the one-eighth of the web, is 21.86□″, and the net area of the cover plates is 11.25□″, we have

$$1\tfrac{1}{3} \times \frac{11.25}{21.86} = 0.68 \text{ (about)}$$

for the number of rivets per foot in the cover plates, which gives a spacing of 17.6″. This would be the spacing if the allowable stress on the rivets in the horizontal legs were the same as in the vertical. But as the rivets in the horizontal legs must be considered in single shear and the rivets in the vertical legs in bearing on the $\frac{3}{8}$″ web, the above number (0.68) must be multiplied by 7,880/7,200. So we have

$$n = 1\tfrac{1}{3} \times \frac{11.25}{21.86} \times \frac{7{,}880}{7{,}200} = 0.75$$

for the number of rivets per foot in the cover plates at the center of the span, which gives (12/0.75) 16″ spacing, but as there are two rows of rivets in the cover plates the spacing of the rivets in each flange angle would really be twice this, or 32″.

Again, from the diagram (Fig. 172) the required number of the rivets in the vertical legs of the flange angles at the quarter point is 3 per foot. Then for the number of rivets required at that point in the cover plate (as the $\frac{7}{16}$″ plate only extends to about that point), we have

$$3 \times \frac{6.00}{16.61} \times \frac{7{,}880}{7{,}200} = 1.18 \text{ per foot,}$$

which gives a spacing of 10.1″ or 20.2″ along each angle. Now, if other points along the girder be considered it will likewise be found that the required spacing of the rivets in the cover plate or plates will exceed the 6″ maximum allowed. So that if we were to space the rivets in the cover plates, using the 6″ maximum allowable throughout, there would yet be an excess of rivets. Now the spacing of the rivets in the lateral plates is determined when the laterals are detailed, as stated above, and as these rivets must be spaced to suit those details it only remains to fit the rivets in between the lateral connections using as near the maximum 6″ spacing as will fit in nicely without interfering with the stiffeners. So we obtain the spacing shown in the plan of the top flanges (Fig. 171) and the same spacing can be used in the bottom flange, as shown in the longitudinal section below the girder. As is evident, the spacing selected at the different points depends entirely upon individual judgment and hence no two persons are likely to make the same selection. However, this is not a serious matter at all as any reasonable variation is permissible.

We shall next space the rivets in the stiffeners. Now according to shop practice the rivets in the stiffeners should all line up horizontally all the way across the girders.

There is no definite rule for determining the number of rivets required in the intermediate stiffeners other than that there should be a sufficient number to clamp the stiffeners firmly to the web, and the maximum 6″ spacing usually suffices for this, but the end stiffeners should be connected with a sufficient number to transmit the total end shear from the web to the masonry, as these stiffeners are really columns bearing against the bottom flange of the girder and really transmit this force. It is customary to space the rivets in the end stiffeners so that about every other rivet of this spacing can be omitted for the spacing in the intermediate stiffeners whereby an economic spacing that lines up horizontally throughout the full length of the girder is obtained. It is economic to crimp all stiffeners on all girders over three feet deep, but it is usual shop practice to put fillers under end stiffeners and under the stiffeners at all points where cross-frames connect. So we shall put fillers under the end stiffeners and under the stiffeners at cross-frames. The rivets in the stiffeners are in double shear and bearing on the $\frac{3}{8}$″ web. Then, using $\frac{7}{8}$″ shop rivets, we have

$$12{,}000 \times 0.6 \times 2 = 14{,}400 \text{ lbs.}$$

for the allowable shear on each rivet and

$$24{,}000 \times \tfrac{7}{8} \times \tfrac{3}{8} = 7{,}880 \text{ lbs.}$$

for the allowable bearing on each rivet. So the bearing governs the number of rivets.

Now, taking the case of the end stiffeners, the maximum end shear being 216,700 lbs., we have

$$\frac{216{,}700}{7{,}880} = 27.5, \text{ say, } 28,$$

for the number of rivets required in the two pairs of end stiffeners at each support. But, as the end stiffeners are on fillers there must be an excess of 50 per cent, according to the specification, so 42 rivets will be used as shown (Fig. 171), and by omitting every other rivet in the spacing shown in the end stiffeners we obtain the spacing shown in the intermediate stiffeners. The rivets in the stiffeners can be made to line up horizontally without any trouble if 3" spacing be used in the end stiffeners. In case this spacing gives more rivets in the end stiffeners than are required, some of the rivets can be omitted, thus making some 6" spaces.

The distance from the toe of any flange angle to the first rivet out (in the stiffener) from the flange, according to shop practice, should be equal to not less than twice the thickness of the flange angle plus $1\frac{1}{2}''$. So in this case we have $1\frac{1}{8}''+1\frac{1}{2}''+1\frac{1}{2}''=4\frac{1}{8}''$ for the minimum allowable distance from the outer gauge line in the flange angles to the first rivet out from the flange. These distances are given in the spacing at the end of the girder (Fig. 171) as $4\frac{5}{8}''$, which exceeds the minimum allowed by $\frac{1}{2}''$. These spaces should not be less in any case nor much more than $\frac{1}{2}''$ greater than the minimum allowed in conformity with the practice mentioned above.

After the rivets are spaced in the stiffeners the web splices can be calculated and drawn to conform to this spacing. The number of web splices is always limited in all cases to as few as is possible, which depends altogether upon the maximum length of the web plates obtainable from the steel mills. Each splice should be at a stiffener. The maximum length of $72''\times\frac{3}{8}''$ plates is given in Table 9 as 30 ft. So one splice is necessary in the above 50-ft. span.

Let us use the type of splice shown at (*a*), Fig. 150, Art. 117. The first thing to do is to determine the size of the longitudinal splice plates next to the flanges. These plates should have at least three horizontal rows or rivets in them in order to get a good hold on the web. This much is simply a matter of judgment. So let us assume that there are three horizontal rows of rivets in each pair, then the distance from the center of gravity of the rivets in the top pair of plates to the center of gravity of the rivets in the bottom pair is equal to the distance from the second rivet, from the top flange, to the second rivet from the bottom flange, which, as seen from the spacing on the girder (Fig. 171) is 4 ft. or 48". Now, according to Art. 117, using the figures given above, we have

$$\left(\frac{16,000\times 27}{8}\right)71=48\times F,$$

from which we obtain

$$F=79,600 \text{ lbs.}$$

for the direct longitudinal stress on each pair of plates.

Then we have $79,600\div(16,000)\ 48/71=7.35\square''$ for the required net area of each pair of plates, or about $3.67\square''$ for each plate. Allowing $\frac{1}{4}''$ clearance between these plates and the flange angles and the same clearance between them and the vertical splice plates, we obtain $10\frac{1}{4}''$ for their width. Now, let us assume that they are $\frac{1}{2}''$ thick, and that their cross-section is reduced by three rivet holes (for $\frac{7}{8}''$ rivets), then we have

$$(10\tfrac{1}{4}\times\tfrac{1}{2})-1\times\tfrac{1}{2}\times 3=3.63\square''$$

for the net area of cross-section of each plate, which is about equal to the required area. But if the plates be $\frac{1}{2}''$ thick a $\frac{1}{16}''$ filler would be required under each of the stiffeners at the splice and to avoid these thin fillers we will make the plates as thick as the flange angles, which is $\frac{9}{16}''$. This, of course, gives a little excess area, but the result obtained justifies its use.

The next thing is to determine the number of rivets in these longitudinal plates. As the number of rivets depends upon their allowable bearing upon the $\frac{3}{8}''$ web, which is

$$(7{,}880)\frac{48}{71}=5{,}330 \text{ lbs.},$$

we have

$$\frac{79{,}600}{5{,}330}=15 \text{ (about)}$$

for the number of rivets required on each side of the splice in each pair of plates; and spacing them in the plates, so as to conform to the spacing in the stiffeners and flange angles previously established, we obtain the spacing shown in Fig. 171.

The next thing is to determine the size of the vertical splice plates and the number of rivets required in them to transmit the maximum shear at the splice as explained in Art. 117.

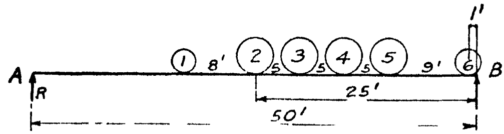


Fig. 174

Placing the wheel loads on the span so that wheel 2 will be at the splice, as shown in Fig. 174, and taking moments about the support B (using Table A), we have

$$R=\left(\frac{1{,}640+103\times 1}{50}\right)1{,}000=34{,}900 \text{ lbs.}$$

for the reaction at A, then, we have

$$(34{,}900-10{,}000)\frac{50}{40}=31{,}200 \text{ lbs.}$$

for the maximum live-load shear at the splice.

For the impact we have

$$31{,}200\left(\frac{300}{25+300}\right)=28{,}800 \text{ lbs.}$$

Now, as the dead-load shear at the splice is zero, we have

$$31{,}200+28{,}800=60{,}000 \text{ lbs.}$$

for the total maximum shear at the splice.

Then we have

$$\frac{60{,}000}{10{,}000}=6.0 \text{ sq. ins.}$$

for the required net area of the vertical section of the vertical splice plates, but as the splice plates are made $\frac{9}{16}''$ thick in order to avoid the $\frac{1}{16}''$ fillers under the stiffeners, referred to above, there will be considerable more metal in them than is required for shear, so they will be amply strong as far as shear is concerned. This is practically always the case, for the splice plates cannot, according to the specifications, be less than $\frac{3}{8}''$ in thickness and the two of them will always be thicker than the web they splice. The number of rivets required to connect these plates to the web will depend upon their allowable bearing on the $\frac{3}{8}''$ web as shown above. So we have

$$\frac{60,000}{7,880}=7.6,\text{ say, }8,$$

for the required number of rivets on each side of the splice, but a greater number will be used as there must be two rows of rivets on each side of the splice in order to securely clamp the plates to the web and the spacing must conform to that already established in the stiffeners. So following out these requirements without spacing the rivets farther apart than 6″, we obtain the spacing shown in Fig. 171.

Now to test the splice as a whole, let S = the horizontal stress on each of the rivets in the horizontal rows farthest out from the center of the web, that is, in the rows next to the flanges, and let V = the vertical shear on each rivet, which we will assume to be the same for each rivet in the splice, that is,

$$V=\frac{60,000}{34}=1,764\text{ lbs.}$$

Now taking moments about the center of the web, as explained in Art. 117, we have

$$2\frac{S}{27}[3^2+6^2+9^2+(12)^2+2(15)^2+2(18)^2+5(21)^2+5(24)^2+5(27)^2]=$$
$$16,000\times3.37\times71,$$

from which we obtain

$$S=5,118\text{ lbs.}$$

Then for the maximum resultant stress which occurs on the outer rivets, we have

$$R=\sqrt{5,118^2+1,764^2}=5,413\text{ (about)},$$

which is about 567 lbs. less than the allowable, which is $5,980=(7,880)\ 27\times2/71$. So the splice is all right and the drawing of it can be completed as shown.

It is seen from the above equation that the rivets near the center of the web are of little value, in resisting bending.

We will next take up the detailing of the cross-frames. The first thing to do is to draw the vertical lines representing the centers of the girders as shown in the detail to the right of the girder. Next the top and bottom horizontal angles should be located and the position of the diagonal determined. Then the next thing is to determine the size of each of the connection plates marked *ak*. The size of each of these plates will depend upon the number of rivets in the ends of the diagonals. As the diagonals take both tension and compression we consider them as compression numbers, and assuming them independent of each other, each has a length of 97″ (about). Then we have

$$16{,}000 - 70\,\frac{97}{1.07} = 9{,}650 \text{ lbs.}$$

for the allowable unit compressive stress on each diagonal,

and $$9{,}650 \times 2.48 = 23{,}900 \text{ lbs.}$$

for the allowable stress on each. Then using $\frac{7}{8}''$ shop rivets, we have

$$\frac{23{,}900}{7{,}220} = 3.3, \text{ say, } 4.$$

for the number of rivets required in each end of the diagonals.

Now a large scale drawing of one of the four plates *ak*, showing the entire detail at that point (girder and all) can be made from which the location of the rivets and all necessary clearances can be determined. Then the cross-frame and the cross-section of the girder can be drawn as shown and the pencil drawing is then complete and the tracing of the same can be made and by adding the required list and writing on the title and general notes we have the shop drawing for the span completed except for the anchor bolts and cast pedestals which are detailed on sketch sheets included in the above shop bills which are made after the shop drawing is completed. After the shop drawing and bills are checked and approved blueprints are made of them which are used in the fabricating, shipping, and erecting of the span.

137. Estimate of Dead Weight and Cost of Span.—From the shop drawing (Fig. 171) we have the following weight of metal:

Estimate of the weight of main girders.

1—web 72 x $\frac{3}{8}$ x 91.8 lbs. x 51.4′	4,718 lbs.
4—Ls 6 x 6 x $\frac{9}{16}$ x 21.9 lbs. x 51.4′	4,503 lbs.
2—cov. pls. 14 x $\frac{7}{16}$ x 20.82 lbs. x 27.0′ (top and bot.)	1,124 lbs.
1—cov. pl. 14 x $\frac{1}{2}$ x 23.80 lbs. x 51.4′ (top)	1,223 lbs.
1—cov. pl. 14 x $\frac{1}{2}$ x 23.80 lbs. x 37.0′ (bot.)	881 lbs.
8—Ls 5 x 3$\frac{1}{2}$ x $\frac{1}{2}$ x 13.6 lbs. x 6.0′ (end stiff.)	653 lbs.
26—Ls 5 x 3$\frac{1}{2}$ x $\frac{3}{8}$ x 10.4 lbs. x 6.0′ (int. stiff.)	1,622 lbs.
12—fills. 3$\frac{1}{2}$ x $\frac{9}{16}$ x 6.7 lbs. x 5.0′	402 lbs.
2—sp. pls. 16 x $\frac{9}{16}$ x 30.6 lbs. x 3.2′	196 lbs.
4—sp. pls. 10 x $\frac{9}{16}$ x 19.14 lbs. x 2.1′	161 lbs.
2—sole pls. 14 x $\frac{3}{4}$ x 35.71 lbs. x 1.3′	93 lbs.
	15,576 lbs.
1,015* rivets @ .44 lbs. per pair of heads ($\frac{7}{8}''$ rivets)	446 lbs.
Total weight of one main girder	16,022 lbs.
Total weight of two main girders	32,044 lbs.

Total weight of main girders per foot of span equals

$$\frac{32{,}044}{50} = 641 \text{ lbs.}$$

*For weight of rivet heads see Carnegie Handbook.

It will be seen from the above that the weight of the rivet heads is about 3 per cent of the weight of the other material in the girders.

Estimate of the weight of laterals and lateral plates.

8—Ls 3½ x 3½ x ⅜ x 8.5 lbs. x 8.2′	558 lbs.
7—pls. 12 x ⅜ x 15.3 lbs. x 2.6′	278 lbs.
2—pls. 12 x ⅜ x 15.3 lbs. x 2.0′	61 lbs.
2—pls. 12 x ⅜ x 15.3 lbs. x 1.3′	40 lbs.
9—pls. 12 x ⅜ x 15.3 lbs. x 1.0′	137 lbs.
4—pls. 12 x ⅜ x 15.3 lbs. x 0.8′	49 lbs.
Total weight of laterals and lateral plates	1,123 lbs.

Estimate of the weight of cross-frames.

2—Ls 3½ x 3½ x ⅜ x 8.5 lbs. x 6.2′	105 lbs.
2—Ls 3½ x 3½ x ⅜ x 8.5 lbs. x 7.3′	124 lbs.
4—pls. 14 x ⅜ x 17.86 lbs. x 1.1′	78 lbs.
1—pl. 8½ x ⅜ x 10.84 lbs. x 0.8′	9 lbs.
	316 lbs.
33 rivets @ 0.14 lbs	14 lbs.
Total weight of one frame	330 lbs.
Total weight of five frames	1,650 lbs.

Total weight of lateral system = 1,123 + 1,650 = 2,773 lbs.

Total weight of lateral system per foot $= \frac{2,773}{50} = 55$ lbs. per ft. of span.

Estimate of the total effective dead load per ft. of span.

2 girders	641 lbs.
Lateral system	55 lbs.
Deck (track)	400 lbs.
Total	1,096 lbs.

Difference between the assumed dead load and estimated = 1,150 − 1,096 = 54 lbs., which is less than 10 per cent, so no recalculation is necessary on account of error in assumed dead load.

Summary of total weight of metal in span.

2 girders	32,044 lbs.
Lateral system	2,773 lbs.
4 pedestals @ 360 lbs	1,440 lbs.
Anchor bolts and nuts	61 lbs.
Total	36,318 lbs.

Approximate Cost of Span Erected: 36,318 lbs. @ 3¼¢ = $1,180. Three and one-quarter cents is only a fair average price for this class of work erected. The price will vary from 2¼¢ to 4¼¢. It depends upon the market price of steel and the freight.

It will be seen from the above that the weight of rivet heads (=1,000 lbs., approximately) is equal to about 3 per cent of the weight of the other metal (=32,700 lbs.) in the span, not considering the pedestals. So in making preliminary estimates of such structures the weight of the rivet heads can be taken at about 3 per cent of the weight of the other metal.

138. Hints Regarding Shop Bills.—The above shop bills are for the most part self-explanatory. Pages Nos. 1 and 2 are for the structural material proper. The finished material is given on the material side of the bills exactly as it appears on the shop drawing. The length of material as obtained from the rolling mills is likely to vary slightly from the length ordered, so in case of long pieces where an under run would be objectionable a $\frac{1}{2}''$ or so is added in the length given on the mill order side as shown. In case of short pieces having a length of 10 ft. and under, and having the same section, they are ordered in multiple, that is, they are combined and ordered as one or more pieces, as is seen.

All pieces planed (marked fin.) on one side are ordered $\frac{1}{16}''$ thicker than the finished thickness and $\frac{1}{8}''$ thicker if planed on two opposite sides. All pieces planed on the ends are ordered $\frac{1}{4}''$ to $\frac{1}{2}''$ longer than the finished length. This is in addition to the amount added for under runs.

The dimensions appearing on the mill order side are only those differing from the finished dimensions. In cases where the material is to be ordered exactly as the finished material, it is not repeated on the mill order side, as a rule.

The cast-steel pedestals are detailed on page No. 4. The required area of bearing on the masonry of each pedestal is given in Art. 134 as $361\square''$. The actual area is

$$20\tfrac{1}{4} \times 18 = 364.5 \text{ sq. ins.,}$$

which is about the correct area. The thickness of metal in such pedestals should not be less than $1\frac{1}{4}''$ in order to insure the molds to properly fill. The top of each pedestal (where the girder rests) should be as small as is consistent with good details.

Everything shown on such bills as the above is, as a rule, written on by the draughtsman making the shop drawing. The item numbers are given by the order clerk as the final mill order blanks are made out in the order department, which, as a rule, is a separate department from the drawing room. The bills not mentioned are considered as being entirely self-explanatory.

Partial Design of a 75-Ft. Single-Track Deck Plate Girder Span

139. Data.—

Length = 75′ 0″ c.c. end bearings.

Dead Load = $12 \times 75 + 550$ = 1,450 lbs. per ft. of span.

Live Load, Cooper's $E50$.

Specifications, A. R. E. Ass'n.

140. Calculations.—In a manner similar to that shown in Art. 134 for the 50-ft. span, we obtain the following for the 75-ft. span:

Maximum End Shear:	Maximum Moment:
$D = 27,200$ lbs.	$D = 6,117,000$ in. lbs.
$L = 147,200$ lbs.	$L = 28,875,000$ in. lbs.
$I = 117,700$ lbs.	$I = 23,100,000$ in. lbs.
292,100 lbs.	58,092,000 in. lbs.

Assuming the web to be $\frac{7}{16}$ in. thick we have

$$x = 1.055\sqrt{\frac{58,092,000}{\frac{7}{16} \times 16,000}} = 96.1 \text{ ins. (about)}$$

for the economic depth. So we will try a $96 \times \frac{7}{16}''$ web. Using the ordinary flange the effective depth is about 95″ (merely an assumption). Then we have $58,092,000 \div 95 = 611,500$ lbs. for the flange stress, and $611,500 \div 16,000 = 38.2\square''$ for the net area of the flange. As stated before, about half of this area (minus $\frac{1}{8}$ of the area of the web) should be in each pair of flange angles. This would require angles which would be entirely too thick if 6″ x 6″ angles be used. So either 6″ x 6″ angles with side plates and cover plates, or 8″ x 8″ angles with cover plates should be used. The last-mentioned section is really the ordinary flange wherein the angles are 8″ x 8″.

Let us first try the flange made up of 6″ x 6″ angles, side plates and cover plates. As the effective depth in this case is less than in the case of ordinary flanges, the area of the flange will be a little larger than indicated above. So let us assume the following section:

	Gross Area	Net Area
2—Ls 6″ x 6″ x $\frac{5}{8}$″	$= 14.22\square'' - 2.5\ \square''$	$= 11.72\square''$
2—Side plates 13″ x $\frac{1}{2}$″	$= 13.00\square'' - 3.0\ \square''$	$= 10.00\square''$
1—Cover plate 16″ x $\frac{1}{2}$″	$= 8.00\square'' - 1.0\ \square''$	$= 7.00\square''$
1—Cover plate 16″ x $\frac{7}{16}$″	$= 7.00\square'' - 0.87\square''$	$= 6.13\square''$
$\frac{1}{8}$ of web		$= 5.25\square''$
	$42.22\square''$	$40.10\square''$

The flange is assembled as shown in Fig. 175.

By taking moments about the center of the top cover plate we obtain

$$\bar{x} = \frac{8 \times \frac{1}{2} + 14.22 \times 2.48 + 13 \times 7\frac{1}{2}}{42.22} = 3.24 \text{ ins.}$$

for the distance from the center of the top cover plate to the center of gravity of the flange. Then we have

$$3.24 - 0.75 = 2.49 \text{ ins.}$$

for the distance from the back of the flange angles to the center of gravity of the flange, and

$$96.25 - 4.98 = 91.27 \text{ ins.}$$

for the effective depth.

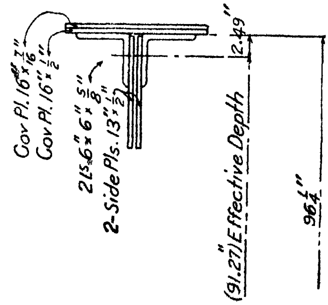


Fig. 175

Then using this effective depth we have

$$\frac{58{,}092{,}000}{91.27} = 637{,}000 \text{ lbs. (about)}$$

for flange stress, and

$$\frac{637{,}000}{16{,}000} = 39.8 \text{ sq. ins.}$$

for the required net area of each flange. This shows that the above assumed flange is about correct.

Next let us try a flange made up of 8″ x 8″ angles and cover plates. First assume the following section:

	Gross Area	Net Area
2—Ls 8″ × 8″ x $\frac{11}{16}$″	= 21.06□″ − 2.75□″	= 18.31□″
2—cov. pls. 18″ x $\frac{1}{2}$″	= 18.00□″ − 2.00□″	= 16.00□″
$\frac{1}{8}$ of web		= 5.25□″
	39.06□″	39.56□″

Taking moments about the center of the top cover plate we obtain

$$\bar{x} = \frac{9 \times \frac{1}{2} + 21 \times 3}{39} = 1.73 \text{ ins.}$$

for the distance from the center of the top cover plate to the center of gravity of the flange, and hence we have

$$1.73 - 0.75 = 0.98 \text{ ins., say 1 in.,}$$

for the distance from the back of the angles to the center of gravity of the flange. This gives 96.25 − 2 = 91.25″ for the effective depth. Then we have

$$\frac{58{,}092{,}000}{91.25 \times 16{,}000} = 38.55 \text{ sq. ins. (about)}$$

for the required net area of each flange. This shows that the section assumed above is a little too large. By making one of the cover plates $\frac{7}{16}$″ thick (instead of $\frac{1}{2}$″) the section will be about correct.

It is seen from the above that the flange made up of 8″ x 8″ angles and cover plates is more economic than the flange made up of 6″ x 6″ angles, side plates, and cover plates, and hence apparently should be used. However, this will depend mostly upon whether the 8″ x 8″ angles can be readily obtained from the rolling mills at the same price as the other sections. In case several spans are required it would very likely pay to use 8″ x 8″ angles, but in case of only one span it would likely be best to use the 6″ x 6″ angles, side plates, and cover plates.

The calculations for the remainder of this span would be quite similar to that shown above for the 50-ft. span, and when completed the corresponding stress sheet, detail shop drawing, and shop bills could be made. Fig. 176 shows a partial detail of the girder where the flanges are made up of 6″ x 6″ angles, side plates, and cover plates.

The student will have no trouble in designing and detailing this type of girder if the outline given above for the 50-ft. span be followed.

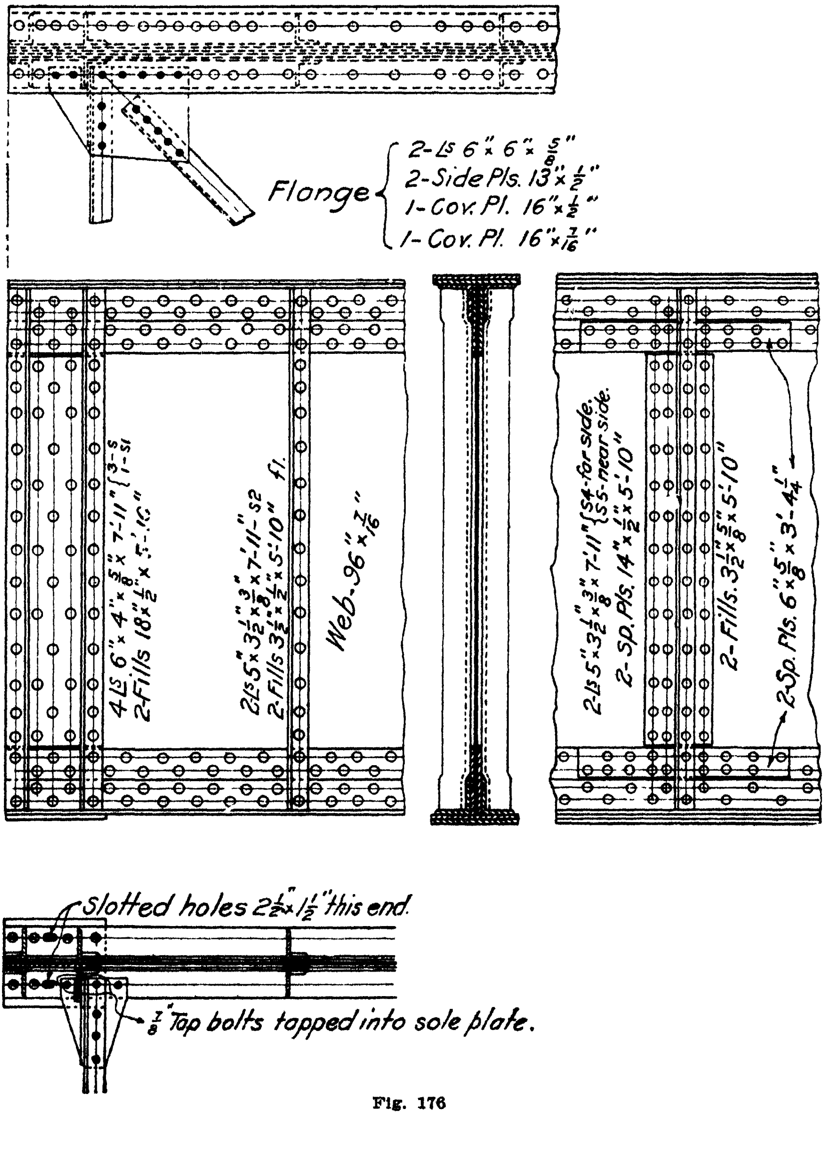
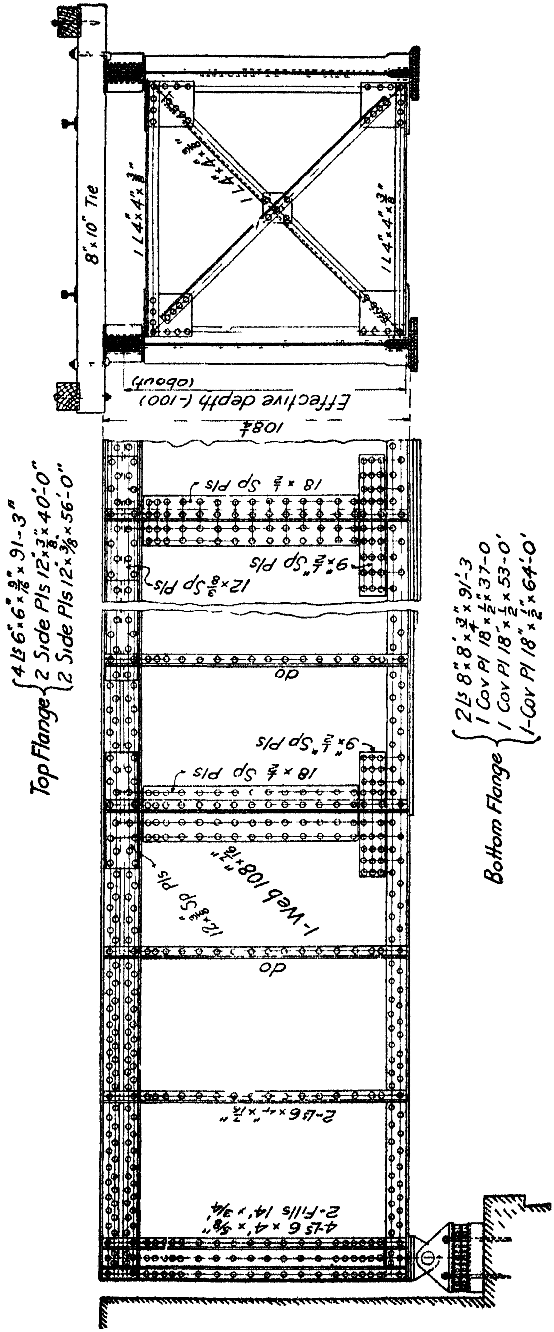

Flange
2-Ls 6" × 6" × 5/8"
2-Side Pls. 13" × 1/2"
1-Cov. Pl. 16" × 1/2"
1-Cov. Pl. 16" × 7/16"
4 Ls 6" × 4" × 5/8" × 7'-11" {3-S 1-S1
2-Fills 18" × 1/2" × 5'-10"
2 Ls 5" × 3 1/2" × 3/8" × 7'-11"-S2
2-Fills 3 1/2" × 1/2" × 5'-10" fl.
Web-96" × 7/16"
2 Ls 5" × 3 1/2" × 3/8" × 7'-11" {S4-far side. S5-near side.
2-Sp. Pls. 14" × 1/2" × 5'-10"
2-Fills 3 1/2" × 5/8" × 5'-10"
2-Sp. Pls. 6" × 5/8" × 3'-4 1/4"
Slotted holes 2 1/2" × 1 1/2" this end.
7/8" Tap bolts tapped into sole plate.

Fig. 176


8"x10" Tie
1 L 4"x4"x3/8"
Effective depth (-100) (about)
108¼
Top Flange
4 Ls 6"x6"x9/16"x 91-3"
2 Side Pls 12"x3/8"x 40'-0"
2 Side Pls 12"x3/8"x 56'-0"
18 x ½ Sp Pls
9"x ½"Sp Pls
12 x 3/8 Sp Pls
do
1-Web 108"x 7/16"
2-Ls 6 x 4 x 7/16"
4-Ls 6 x 4" x 5/8"
2-Fills 14" x 3/4
Bottom Flange
2 Ls 8"x 8"x 3/4"x 91'-3
1 Cov Pl 18"x ½"x 37-0
1 Cov Pl 18"x ½"x 53-0'
1-Cov Pl 18"x ½"x 64'-0'

Partial Design of a 90-Ft. Single-Track Deck Plate Girder Span

141. Data.—

Length = 90′–0″ c.c. end bearings.
Dead Load = 12 × 90 + 550 = 1,630 lbs. per ft. of span.
Live Load, Cooper's *E*50.
Specifications, A. R. E. Ass'n.

142. Calculations.—For the main girders of this span we have:

Maximum End Shear:	Maximum Moment:
D = 36,700 lbs.	D = 9,902,000 in. lbs.
L = 171,500 lbs.	L = 40,057,000 in. lbs.
I = 132,000 lbs.	I = 30,813,000 in. lbs.
340,200 lbs.	80,772,000 in. lbs.

Assuming the web to be $\frac{7}{16}''$ thick, we have

$$x = 1.055\sqrt{\frac{80,772,000}{\frac{7}{16} \times 16,000}} = 113 \text{ ins. (about)}$$

for the economic depth. But, as a few inches either way from the theoretical depth in case of such deep girders will not materially affect the design, it will be better to take 108″ as the depth, as this will give us a 108″ web, which is a very common plate, much more so than the 112′ or 113″. So we will assume a 108″ x $\frac{7}{16}''$ web.

There are three types of flanges suitable for this girder: One made up of 6″ x 6″ angles, side plates, and cover plates, as shown in Fig. 176; one made up of 8″ x 8″ angles and cover plates, as ordinary flanges; or one made up of 4—6″ x 6″ angles and side plates for the top flange and 8″ x 8″ angles and cover plates for the bottom flange, as shown in Fig. 177.

The flanges made up of 8″ x 8″ angles and cover plates the author thinks preferable. However, the design shown in Fig. 177 has some desirable features. For instance, there are no rivet heads on the top flange to interfere with the ties, as there are no cover plates and the top laterals are connected to the bottom angles of the top flange.

The stress sheet and detail shop drawing and bills for this span can be worked up in the same manner as shown above for the 50-ft. span.

All spans over 75 ft. long should be supported at one end upon rollers. Fig. 178 shows the general details for such a bearing, which is for the above 90-ft. span. Fig. 179 shows the general details for the corresponding fixed support for the same span. Figures 180 and 181 show the shop details for the girder shoe, roller shoe, roller nest, and pedestal.

In designing and drawing up such a bearing as that shown in Fig. 178, the first thing is to determine the space taken up by the rollers. For the above 90-ft. span we have 340,200# for the maximum reaction. The minimum size of rollers is limited by the specifications to 6″ diameter, and as plate girder bridges are comparatively small bridges we will use the minimum size roller. The allowable pressure per linear inch on

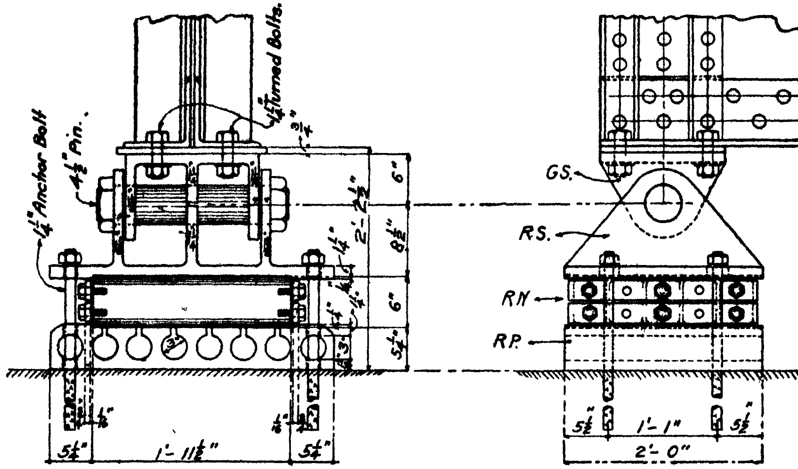


Fig. 178

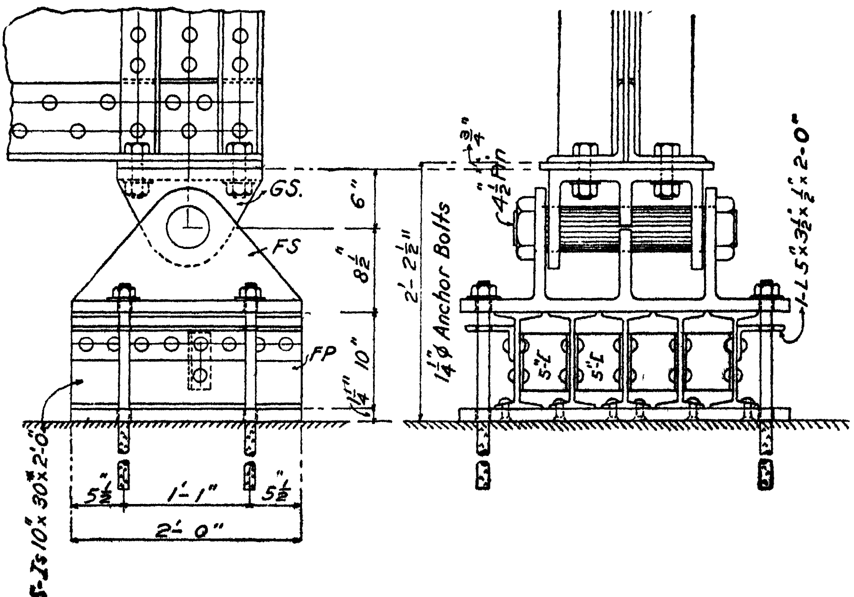


Fig. 179

these rollers is 600 × 6 = 3,600#. (See specifications, also Art. 84.) We then have

$$\frac{340,200}{3,600} = 94.5$$

for the linear inches of roller required. If we use five rollers we have

$$\frac{94.5}{5} = 19 \text{ ins. (about)}$$

for the required net length of each roller, and adding $4\frac{1}{2}''$ to each to provide for the slots in the pedestal we have $23\frac{1}{2}''$ for the total length of each roller, as shown in Fig 178. All rollers having a diameter of 6″ and over are usually flattened on the sides like those shown in Figs. 178 and 181, and are known as segmental rollers. These rollers should be placed quite close to each other so that their sides will come together whenever they are rolled beyond the amount desired. Thus the rollers are prevented from falling over. The distance between the rollers (between the sides) is usually about $\frac{1}{4}''$, as shown in Fig. 181. After determining the size of the roller nest, a preliminary drawing of the shoes and pedestal can be made similar to that shown in Fig. 178, wherein the general dimensions are made to conform to those of the roller nest, while the correct size of the pin passing through the shoes and the thickness of metal throughout are determined as the work progresses. We begin by deciding upon the general form of the shoes. Then we assume the size of the pin passing through them. Next we calculate the required thickness of the bearings of the shoes for the pin assumed. If this seems satisfactory we proceed with the work of calculating the stress on the pin to determine whether the pin assumed is of correct size, and if the size is found to be correct we proceed farther with the design, but if found to be incorrect we start over and make some new assumption, and so on, until the design is worked out satisfactorily.

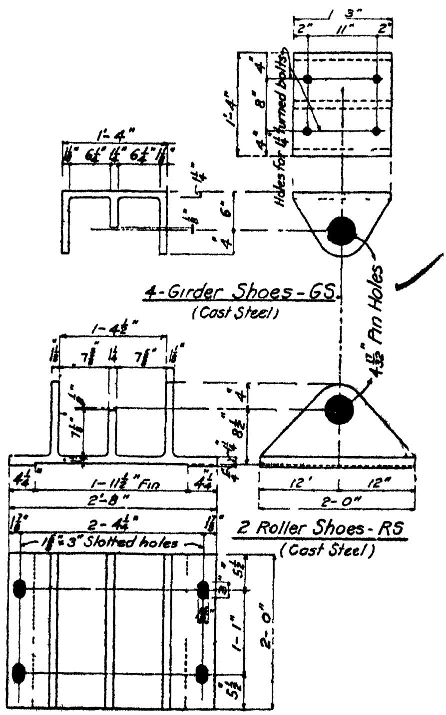


Fig 180

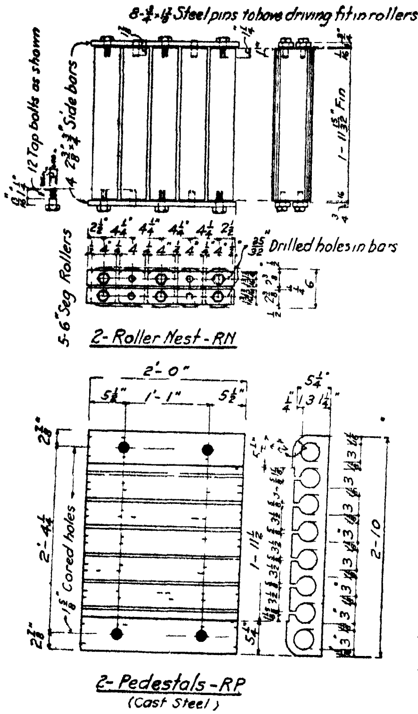


Fig 181

Taking the case shown in Fig. 178, let us assume the pin to be $4\frac{1}{4}''$ in diameter. The total thickness of the required bearing on each shoe is then

$$\frac{340,200}{24,000 \times 4\frac{1}{4}} = 3\frac{1}{3} \text{ ins.}$$

The central bearing of each shoe will likely be subjected to a little more pressure than either of the side bearings. So we will make them a little thicker than the side bearings. By making the central bearing of each shoe $1\frac{1}{4}''$ thick and each side bearing $1\frac{1}{8}''$ we have $3\frac{1}{2}''$ for the total thickness of bearing on each shoe, which is about equal to that required, and as the side bearings are of minimum thickness for such castings these thicknesses are quite satisfactory and hence will be used.

The maximum shear and cross bending on the pin occur at the side bearings. For the maximum shear we have

$$340,200 \times \frac{1\frac{1}{8}}{3\frac{1}{2}} = 109,300 \text{ lbs.}$$

and for the maximum shearing unit stress on the $4\frac{1}{4}''$ pin we have

$$v = \frac{109,300}{\pi\left(\frac{4\frac{1}{4}}{2}\right)^2} = \frac{109,300}{14.18} = 7,700 \text{ lbs.}$$

For the maximum cross bending on the pin we have

$$109,300 \times 1\frac{3}{8}'' = 150,000 \text{ inch lbs. (about)}$$

and for the maximum fiber stress due to cross bending we have

$$f = \frac{150,000 \times 2\frac{1}{8}}{\frac{\pi\ (4\frac{1}{4})^4}{64}} = 20,000 \text{ lbs. (about).}$$

Now, as the allowable shear on pins is 12,000 lbs. per square inch and the allowable fiber stress is 24,000 lbs. per square inch (see specifications), it is seen from the above that the $4\frac{1}{4}''$ pin assumed is amply strong; in fact it could be reduced in size, but as the saving in cost would be insignificant we will use the size assumed. It also appears to be about the correct size.

In addition to having sufficient bearing on the pin the shoes should be deep enough and contain enough metal to resist the cross bending to which they are subjected.

Considering the girder shoe (*GS*) we have a case of cross bending as indicated in Fig. 182. For the maximum moment we have

$$M_c = \frac{340{,}200}{15} \times 7\tfrac{1}{2} \times 3\tfrac{3}{4} = 638{,}000 \text{ inch lbs. (about),}$$

which occurs at the vertical transverse section *c-c* through the pin hole. The moment of inertia of this cross-section of the shoe (not subtracting the pin hole) is about 390. Then for the maximum compressive unit stress due to cross bending we have

$$f_c = \frac{628{,}000 \times 6\tfrac{5}{8}}{390} = 12{,}200 \text{ lbs.}$$

and for the maximum tensile unit stress we have

$$f_t = \frac{638{,}000 \times 3\tfrac{3}{8}}{390} = 5{,}500 \text{ lbs.}$$

This shows that the girder shoe is amply strong, as far as cross bending is concerned, as 16,000 lbs. unit stress is permissible. The bending on the roller shoe (*RS*) can be determined in the same manner. Further designing of the shoes is very much a matter of common judgment.

Regarding the above calculations it may first appear that the material cut out of the vertical section *c-c* by the pin hole should be deducted from the section in determining the moment of inertia. But this is far from being correct, as the top half of the pin is assumed to transmit 24,000 lbs. per square inch against the shoe—this we can rely upon—and owing to the lateral deformation of the pin there will be some horizontal pressure (perhaps equal to Poisson's ratio); so, taking all in all, the above assumption is fairly correct.

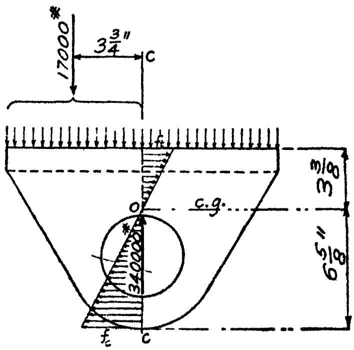


Fig. 182

The designing of the pedestal (*RP*) is very much a matter of common judgment. The pedestal should be at least 4 ins. or 5 ins. high in order to have the rollers above snow, slush, and dirt, and should be broad and long enough to support the load coming on them without producing a greater pressure upon the masonry than 600 lbs. per square inch. There should be open slots extending down from the top as shown in Fig. 178, so that dirt will not accumulate between the rollers. The longitudinal circular holes should be cored in the casting and the slots cut from the solid when the pedestals are being finished in the machine shop. This will guard against the pedestal warping when cooling. The details of the fixed bearing shown in Fig. 179 are considered to be self-explanatory.

143. Flange Splices.—The flanges of all plate girders over 90 ft. long must be spliced, as that is about the maximum length of angles and cover plates obtainable. These splices should preferably be symmetrically arranged in reference to the center of the span. The flange angle splices should, as a rule, be nearer to the ends of the girder than to the center of the span, and only one flange angle should be cut off at a splice. That is, one flange angle should be spliced on one side of the girder on one side of the center of the span and the other flange angle should be spliced on the other side of the girder at a corresponding point on the other side of the center of the span, as shown in Fig. 183. Here the near flange angle is spliced at *D*, one part extending from *D* to *B*, and the other part from *D* to *A*, while the far flange angle is spliced at *E*, one part extending from *E* to *A* and the other part from *E* to *B*.

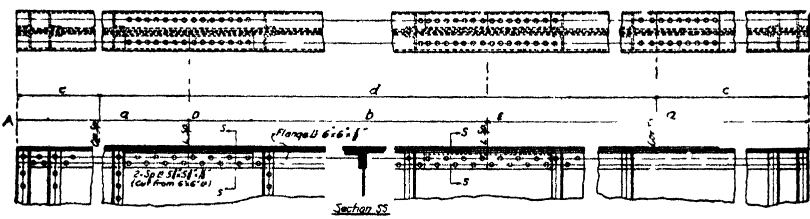


Fig. 183

The splice of a flange angle should be made by means of an angle having the same net area of cross-section, as the flange angle. The splice angle must be ground to fit the fillet of the flange angle and its legs cut so as not to project beyond the legs of the flange angle, as indicated at section *S-S*. (Fig. 183.) It is general practice to use two splice angles at each splice, one on each side of the girder. The splice angle on the side of the flange angle spliced should be of sufficient cross-section in every case to splice that angle. The splice angle on the other side of the girder is used simply to balance the flange section at that point.

The cover plates are usually spliced by extending the adjacent cover plates a short distance beyond their theoretical length, as shown in Fig. 183. As a rule, the cover plates next to the flange angles are the only ones that need to be spliced. Cover plates, of course, can be spliced by simply using a short plate having the same area of cross-section as the plate spliced, but usually this presents an unsightly appearance.

As an example in figuring splices, let the two flange angles shown in Fig. 183 be 2—∟s 6″ x 6″ x $\frac{5}{8}$″, as indicated. For the net area of either of these flange angles we have (using $\frac{7}{8}$″ rivets) $7.11 - 1.25 = 5.86$□″. Then for the allowable stress in each flange angle we have $5.86 \times 16{,}000 =$ 93,700#. Then for the number of $\frac{7}{8}$″ shop rivets required on each side of the splice (in the splice angle) we have $93{,}700 \div 7{,}200 = 13$ rivets (in single shear). As is seen, 14 are used—7 in the vertical leg and 7 in the horizontal leg. The splice angle at each splice should have 5.86□″ net. Using a 6″ x 6″ x $\frac{11}{16}$″ angle, and deducting 1.37□″ for rivet holes and 0.86□″ for the cutting of the legs and for the grinding to fit fillet, we have $7.78 - 1.37 - 0.86 = 5.55$□″ net, which is about correct.

The calculation for the splicing of a cover plate is simply a matter of determining the number of rivets required in single shear to develop the strength of the plate spliced. The student should have no trouble in doing that.

144. Graphical Determination of Live-Load Shear and Bending Moment on Deck Plate Girder Bridges.—The analytical method outlined in Art. 133 is generally used, as it is simple in application and the results obtained are absolutely numerically accurate, yet the graphical method is fully as simple in application and the results obtained are quite accurate enough. The graphical method at least affords an excellent means of checking results obtained analytically, and if for no other reason than this the engineer should be familiar with the method. In fact, there are two methods: that of influence lines and the one wherein the equilibrium polygon is used.

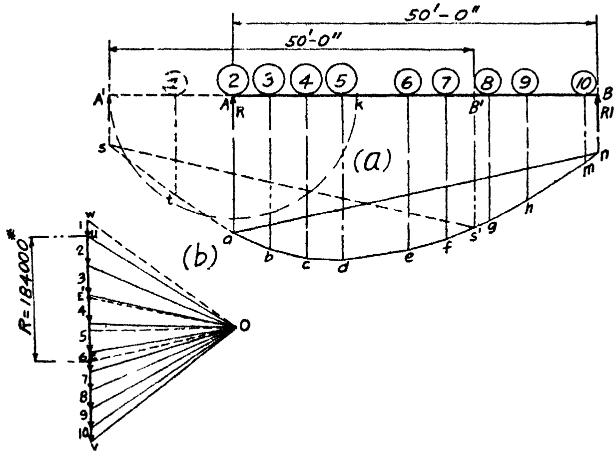


Fig. 184

The application of influence lines is fully given in Arts. 100 and 101. As an application of the equilibrium polygon let us take the case of the 50-ft. span treated analytically in Art. 133, where Cooper's *E*50 loading is used. Let *AB* (Fig. 184) represent the span drawn, say, to $\frac{1}{4}''$ scale. From inspection of the diagram in Table A (in the back of this book) we can see that the maximum end shear will occur when wheel 2 is at the end of the girder and wheels 2 to 10 (inclusive) are on the span, as shown in Fig. 184. After the loads are thus placed, the next thing to do is to lay off the load line *uv* (say, to a $\frac{1}{50}$-in. scale) at (*b*), and construct the ray diagram, as explained in Art. 95, and draw the corresponding equilibrium polygon *ab* . . . *na* at (*a*). Then by drawing from *O* (at *b*) the line *OE* parallel to the closing line *an*, we have the maximum end shear given by the part of the load line extending from *E* to *u* which can be scaled, using the same unit of scale as was used in laying off the load line.

To determine the shear at any point *K* of the girder, place the point *K* under wheel 2 by moving the girder, so to speak, to the left, so that the end A will be at A' and the other end will be at B' and wheels 1 to 7 will be on the span. Then add wheel 1 to the load line and draw the ray wO and extend the equilibrium polygon on to s and we have the equilibrium polygon $sta \ldots fs's$. Then draw OE' in the ray diagram parallel to the closing line ss' and we have the maximum shear at point K given by the part of the load line extending from E' to u. In this manner the maximum shear at any point on the span can be determined.

To determine the maximum bending moment place the loads on the span, as shown in Fig. 185, so as to satisfy the criterion for maximum bending moment as per Art. 88. Then construct the ray diagram as

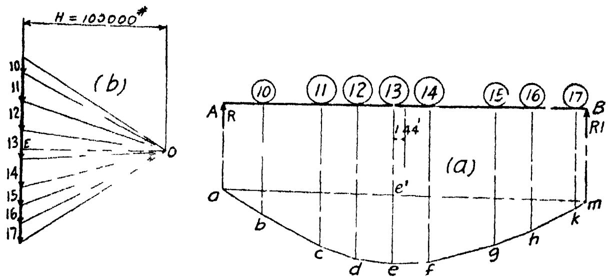


Fig 185

shown at (b) and draw the corresponding equilibrium polygon $ab \ldots ma$, and we obtain the maximum moment by multiplying the ordinate $e'e$ under wheel 13 ($e'e$ must be scaled off in feet or inches to the same scale as was used in laying off the length of span at (a) and spacing the loads) by H, the pole distance which is laid off in pounds to the same scale as the load line. If the ordinate $e'e$ is taken in feet the bending moment will be in foot pounds, and if taken in inches the bending moment will be in inch pounds.

Such a diagram as shown in Fig. 186 is very convenient if the work to be done is extensive enough to warrant the drawing of it. In offices where one loading is used for all bridges it is advisable to draw up such a diagram of the loading on a sheet of thick cardboard, inking in all lines shown in Fig. 186 except the closing lines. The closing lines can be drawn lightly in pencil as needed and then erased. In this way the diagram can be used in determining the shears and moments for any number of bridges. The diagram shown in Fig. 186 is for Cooper's $E40$ loading. The loads are spaced off to $\frac{1}{10}$-in. scale on the horizontal line at the top of the sheet. Then the load line LL is laid off to a $\frac{1}{50}$-in. scale and the ray diagram is constructed, as shown, and the corresponding equilibrium polygon ABC is drawn, and then the scale lines are drawn below this, as shown.

To show the manner of using the diagram in Fig. 186, let us take the case of a 50-ft. span. The maximum moment due to Cooper's loading in the case of most deck plate girder bridges will occur under one of the four wheels 11 to 14 (this we obtain from experience). So, beginning with zero at wheel 13 we lay off the scale line HH into 5-ft. units. To determine the maximum bending moment in this 50-ft. span let us start by assuming that the span extended out 25 ft. each side of wheel 13. Then drawing the closing line 1-1 we have the equilibrium polygon 1-*B*-1, and by scaling off the maximum ordinate between the closing line 1-1 and the curve 1-*B*-1 and multiplying it by the pole distance given in the ray diagram we obtain the maximum bending moment for the span in that position. By moving the span 5 ft. to the right and drawing the closing line 2-2 we have the equilibrium polygon 2-*B*-2 and by scaling off the maximum ordinate and multiplying it by the pole distance we obtain the maximum bending moment for the span in that position, and moving the span to the left the equilibrium polygon 3-*B*-3 is drawn and the maximum bending moment for the span in that position can be determined. Now, by simply drawing a few of these sub-equilibrium polygons the absolute maximum ordinate can be ascertained from the intersection of the closing lines with the verticals through the wheels. The maximum

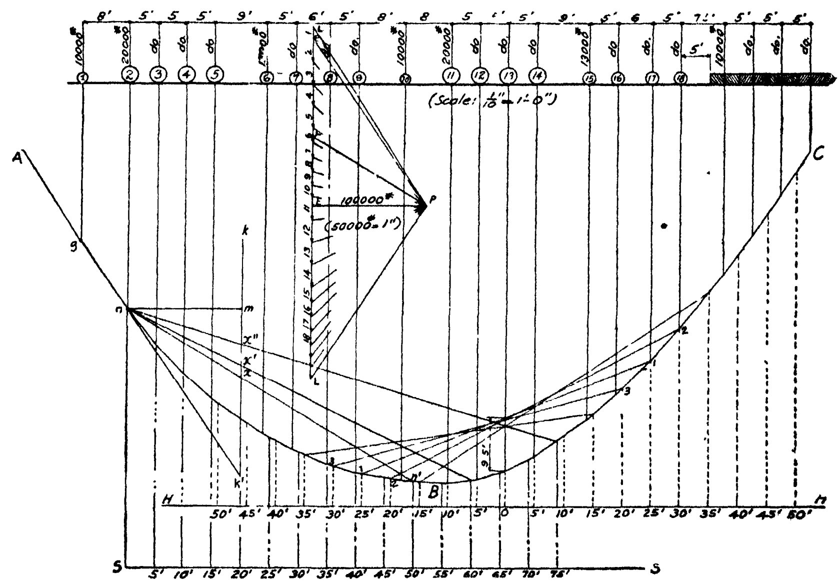


Fig. 186

ordinate will always be under a wheel, and after a sufficient number of closing lines are drawn the maximum ordinate can be quickly determined by the aid of a pair of dividers and then scaled off and multiplied by the pole distance. In this way the maximum bending moments are determined without bothering with the criterion for maximum moment, as is necessary when other methods are used.

To determine the maximum end shear on the 50-ft. span, place wheel 2 at the end of the span. Now using the scale line *SS*, the span will extend 50 ft. to the right of wheel 2 and we obtain an equilibrium polygon which has the closing line *nn'*. Then drawing *Pr* in the ray diagram parallel to this closing line we have the maximum end shear or reaction given by the part of the load line extending from *r* to *e*. By

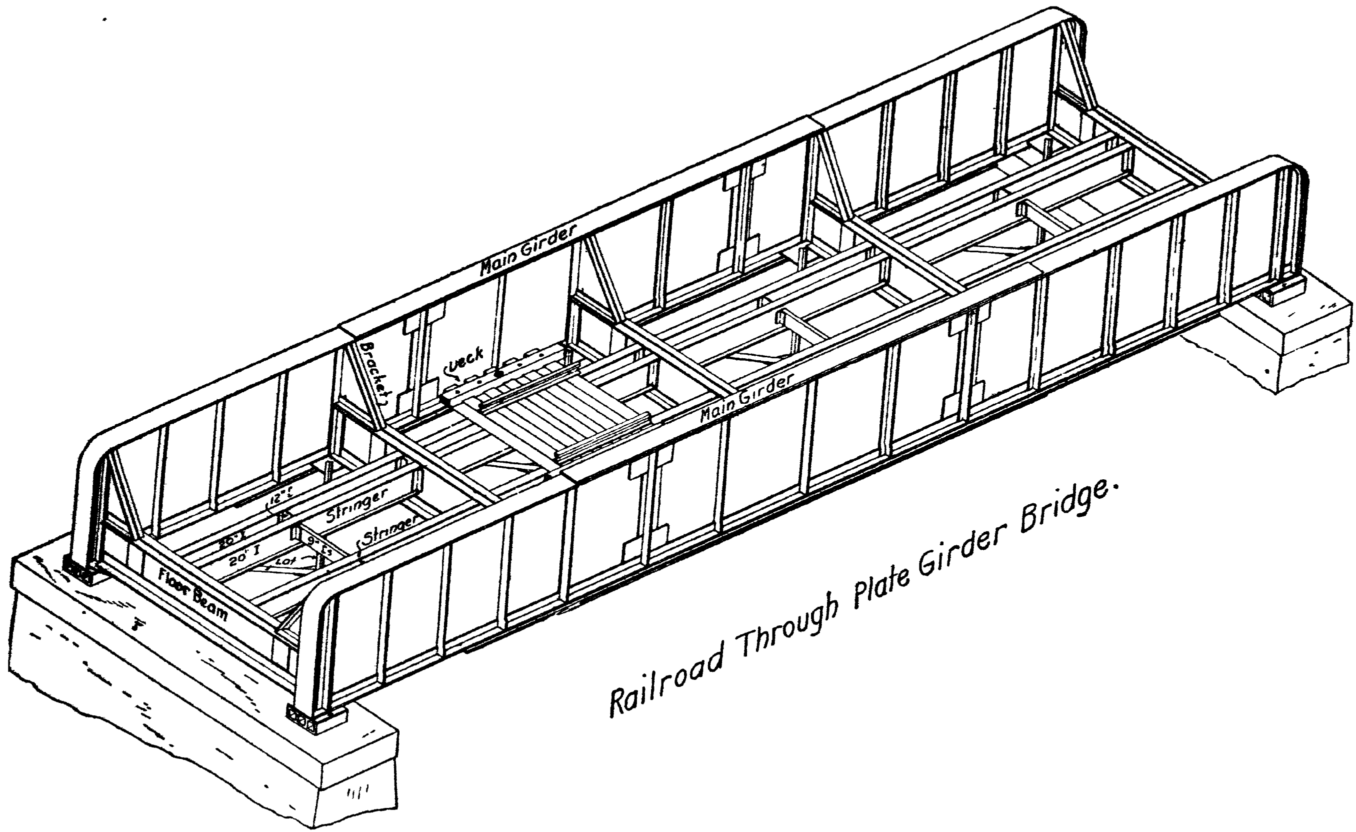

Main Girder
Main Girder
Bracket
Deck
Stringer
Stringer
Floor Beam
20" I
20" I
12" I
9" I
Railroad Through Plate Girder Bridge.

Fig 187.

the following construction the end shears can be obtained more quickly than if scaled from the ray diagram: From n (on the equilibrium polygon) lay off the horizontal line nm equal to the pole distance EP and draw the vertical line kk' through m and prolong the segment gn, of the equilibrium polygon, until it intersects this vertical line at k'. Then we have the triangle nxk' equal to the triangle Pre (in the ray diagram), and hence xk' is equal (by scale) to the maximum end shear on the 50-ft. span. Likewise, $x'k'$ and $x''k'$ are equal, respectively, to the maximum end shear on a 60-ft. and 75-ft. span.

The equilibrium polygon ABC can be used in general to determine the shear at any point in a span in the manner explained in the case shown in Fig. 184.

Problem 1. Construct a diagram for Cooper's $E50$ loading similar to the diagram shown in Fig. 186 and determine from it the maximum end shears and bending moments on a 40 ft., 50-ft., 60-ft., 70-ft., 80-ft., and 90-ft. span.

DRAWING ROOM EXERCISE NO. 3

Design a 60-ft. single-track deck plate girder railroad bridge and make a stress sheet for the same. The finished drawing, similar to that shown in Fig. 169, is to be upon an 18″ x 24″ sheet of tracing cloth.

Data:

Length = 60′-0″ c.c. end bearings.
Width = 6′-6″ c.c. girders.
Height, to be determined by student.
Dead Load, to be determined by the student.
Live Load, Cooper's $E50$.
Specifications, A. R. E. Ass'n.

DRAWING ROOM EXERCISE NO. 4

Make a shop drawing and shop bills for the 60-ft. bridge specified in Drawing Room Exercise No. 3. The finished drawing, similar to that shown in Fig. 171, is to be upon a 24″ x 36″ sheet of tracing cloth.

THROUGH PLATE GIRDER BRIDGES

145. Preliminary.—Through plate girder bridges are used, as a rule, only when the under clearance will not permit of the use of a deck span—owing to the cost of the through bridge being considerably more than the deck bridge. The ordinary through plate girder bridge is composed of two main girders and a floor system composed of longitudinal beams (or girders), which support the ties and are known as stringers, and cross beams, which support the stringers and are known as floor beams.

An isometric view of an ordinary single-track through plate girder span is shown in Fig. 187, where the names of the different parts of the structure are given.

Double-track through plate girder bridges are usually the same in construction as the single-track bridges, except the main girders in the double-track structures are farther apart and the floor system provides for the two tracks, which requires twice as many stringers.

Complete Design of a 60-Ft. Single-Track Through Plate Girder Span

146. Data.—

Length = 4 panels @ 15′-0″ = 60′-0″ c.c. end bearings.
Width = 15′-6″ c.c. main girders.
Assumed Dead Load:
For main girders, $(13\times 60+600+400) = 1{,}780$ lbs. per ft. of span.
For stringers, $(12\times 15+100+400) = 680$ lbs., say 700 lbs., per ft. of span.
Live Load, Cooper's $E50$ loading.
Specifications, A. R. E. Ass'n.

147. Design of 15-Ft. Stringers.—It is usually necessary to make the stringers in through plate girders just as shallow as good details will permit, owing to the under clearance being limited. This can be accomplished best by the use of I-beams, two or more under each rail. So we will use I-beams.

For dead-load moment, using the load assumed in Art. 116, we have

$$M=\frac{1}{8}\times\frac{700}{2}\times\overline{15}^2\times 12=118{,}000 \text{ inch lbs.}$$

For live-load moment we have

$$M'=1{,}875{,}000 \text{ inch lbs. (same as Art. 130).}$$

For impact we have

$$I=1{,}785{,}000 \text{ inch lbs. (same as Art. 130).}$$

Then we have for the total bending moment

$$118{,}000+1{,}875{,}000+1{,}785{,}000=3{,}778{,}000 \text{ inch lbs.}$$

Then for the section modulus we have

$$\frac{3{,}778{,}000}{16{,}000}=236. \quad \text{(See Art. 105.)}$$

This calls for 2—Is, 20″ x 70#, under each rail. That is, each stringer is to be composed of 2—20″ x 70# I-beams.

For dead-load end shear on each stringer we have

$$R=\frac{700}{2}\times\frac{15}{2}=2{,}625 \text{ lbs., say } 2{,}600 \text{ lbs.}$$

For live-load end shear we have

$$R'=50{,}000 \text{ lbs. (same as Art. 130)}$$

and for impact we have

$$I=47{,}600 \text{ lbs.}$$

Then, for the total end shear we have

$$2{,}600+50{,}000+47{,}600=100{,}200 \text{ lbs.}$$

Preliminary estimate of weight of stringers.

4—Is 20″ x 70# x 15′-0″	=	4,200 lbs.
1—[9″ x 20# x 3′-9″	=	75 lbs.
2—end con. Ls on 9″ [s	=	21 lbs.
2—[s 12″ x 25# x 1′-3″	=	62 lbs.
Total weight of one panel	=	4,358 lbs.

Weight of stringers per ft. of span = 4,358 ÷ 15 = 289 lbs. metal.
Weight of deck = 400 lbs. metal.

Total dead load for stringers per ft. of span = 689 lbs. metal.

This shows that the 700 lbs. dead load assumed above is about correct.

148. Design of Intermediate Floor Beams.—The dead load on these beams consists of the dead weight applied to them by the stringers and also the weight of the floor beams themselves.

The dead load from the stringers is concentrated on the floor beams at the points where the stringers are connected to them. This concentration at each point is equal to twice the dead-load end shear on one stringer which is given above as 2,600 lbs. So each concentration is 2,600 × 2 = 5,200 lbs. The weight of the floor beam is a uniform load. Such floor beams usually weigh about 3,600 lbs. each. So we will assume that weight. Then the dead load on any intermediate floor beam will be as shown in Fig. 187A, where *A-B* represents the beam.

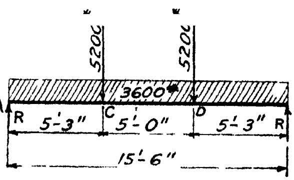


Fig. 187A

It is readily seen from Fig. 187A that zero shear occurs at the center of the beam and hence that is the point of maximum moment. As the bending moment on the floor beam due to the two concentrated loads is constant between the stringers it is customary to multiply one concentration by its distance from the end of the beam to obtain the bending moment due to these loads.

To show the correctness of this method let *AB*, Fig. 188, represent a floor beam supporting the two equal concentrated loads *P*, as shown. Taking moments about the center of the beam we have, since $R = P$,

$$M = R\left(a + \frac{b}{2}\right) - P\frac{b}{2} = Pa + P\frac{b}{2} - P\frac{b}{2} = Pa.$$

That the moment between the loads is constant can also be shown very readily by graphics: Lay off the load line *VWS* (Fig. 188) and construct the ray diagram *VOS* and draw the corresponding equilibrium polygon *efghe*. Now, as the reactions are equal for the two loads, the closing line *eh* must be parallel to the ray *OW*, and as the segment *fg* is parallel to this ray also, it is seen that the ordinates of the equilibrium polygon are constant between the two loads, and hence the moment between them must be constant. Now going back to the

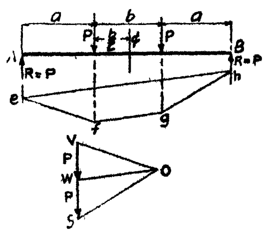


Fig. 188

problem in hand, we have the moment

$$M = 5{,}200 \times 63 = 327{,}600 \text{ inch lbs.}$$

from the concentrated load and

$$M' = \frac{1}{8} \times \frac{3{,}600}{15.5} \times \overline{15.5}^2 \times 12 = 83{,}700 \text{ inch lbs.}$$

from the weight of the floor beam, making a total maximum of 411,300 inch-lbs. dead-load bending moment.

It is obvious that the maximum live-load bending moment on the floor beam will occur when the maximum live-load concentrations from the stringers on to the floor beam occur. So it is first necessary to determine the position of the wheels when the maximum live-load concentrations occur.

Let Fig. 189 represent two adjacent panels of stringers with a floor beam at each of the ends, A, B, and C. Let l be the length of panel AB and l' *the length of* panel BC, as indicated. Let W be the total weight of the wheels in panel AB, the center of gravity of which we will assume is x distance from A, and let W' be the total weight of the wheels in panel BC, the center of gravity of which we will assume is z distance from C. Let r be the concentration on the floor beam at B due to the wheels in the panel AB, and let r' be the concentration due to the wheels in the panel BC, and let R be the concentration on the floor beam at B from the wheels in both panels. Then we have

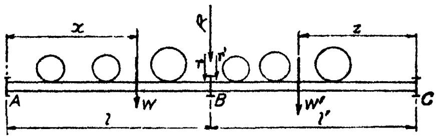


Fig 189

$$R = r + r' \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Taking moments about A we have

$$r = \frac{Wx}{l}$$

and taking moments about C we have

$$r' = \frac{W'z}{l'}$$

Then substituting these values in (1) we have the general expression

$$R = \frac{Wx}{l} + \frac{Wz}{l'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

for the concentration on the floor beam at B. Now, if the wheels roll to the left, r will decrease and r' increase provided no wheels pass B, and just the reverse is true if the wheels roll to the right. Now if the weight of the wheels in the two panels is such that the increment of r is just equal to the increment of r' for a very slight movement, it is evident that the increment of the concentration R will be zero and hence R will then be a maximum, for otherwise it would be either increasing or decreasing. If increasing, it evidently has not reached the maximum; and if decreasing, it has passed the maximum. So, differentiating equation (2) we have

$$dR = \frac{W\,dx}{l} + \frac{W'dz}{l'} = 0.$$

But $dz = dx$. Then we have

$$\frac{dR}{dx} = \frac{W}{l} - \frac{W'}{l'} = 0 \quad \text{or} \quad \frac{W}{l} = \frac{W'}{l'}.$$

That is, the unit load in one panel will be equal to the unit load in the other panel when the maximum concentration on the floor beam occurs.

If the panels are of equal length, as they are here, we have

$$W = W'.$$

That is, the maximum concentration on the floor beam will occur when the load in one panel is equal to the load in the other. Now the increment of R can change sign only when a wheel passes the floor beam at B. So there will be a load at that point when the maximum concentration occurs. It is evident that this criterion for maximum concentration on the floor beam, so far established, can be satisfied by most any group of wheels, but it is also evident that the absolute maximum concentration will occur when the heaviest group of wheels is near the floor beam. So the whole criterion for maximum concentration on an intermediate floor beam can be stated as follows:

The maximum concentration will occur when the heaviest group of wheels is near the floor beam and one wheel at it, and when the load in one panel is equal to the load in the other.

The next thing in our case is to satisfy this criterion. Referring to Table A we can see that wheels 1 to 5 are as heavy a group of wheels as can be placed on two adjacent panels.

Let us place them as shown in Fig. 190. Then the load in panel $AB = 30{,}000$ lbs., and that in $BC = 40{,}000$ lbs. This does not seem to be very close to the requirement—that the loads in the two panels be equal. But by trial it will be found to be as close as possible and still satisfy the first part of the criterion at the same time. It is not often that this criterion can be satisfied absolutely as to the loads in the panels being equal. We should, however, place the loads so that the criterion is as nearly satisfied as possible. In testing for the criterion the load at the floor beam can be considered as being equally divided between the two panels; that is, one-half of its weight being considered in each panel, or it can be ignored as was done above. It will make no difference which way it is considered.

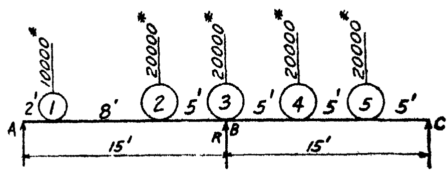


Fig. 190

So, taking the position shown in Fig. 190 as satisfying the criterion and taking moments about C, we have

$$20{,}000\left(\frac{5+10}{15}\right) = 20{,}000 \text{ lbs.,}$$

and taking moments about A we have

$$\frac{10{,}000 \times 2 + 20{,}000 \times 10}{15} = 14{,}660 \text{ lbs.}$$

Then adding these two concentrations to the 20,000-lb. load at the floor beam, we have

$$R = 20{,}000 + 14{,}660 + 20{,}000 = 54{,}600 \text{ lbs.}$$

for the maximum floor beam concentration due to Cooper's $E40$ loading, and multiplying this by 50/40 to reduce it to $E50$, we have

$$\frac{50}{40} \times 54{,}600 = 68{,}200 \text{ lbs. (about)}$$

for the maximum concentration.

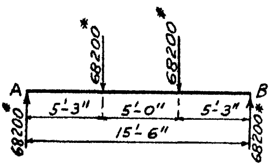


Fig. 191

Then the maximum live load on the floor beam will be as shown in Fig. 191.

For the maximum live-load bending moment, we have

$$M = 68{,}200 \times 63 = 4{,}297{,}000 \text{ inch lbs.},$$

and for the impact we have

$$I = \frac{300}{30 + 300} \times 4{,}297{,}000 = 3{,}906{,}000 \text{ inch lbs.}$$

Now adding the above maximum dead- and live-load bending moments and impact together we have

$$411{,}000 + 4{,}297{,}000 + 3{,}906{,}000 = 8{,}614{,}000 \text{ inch lbs.}$$

for the total maximum bending moment on the floor beam for which the beam must be designed to resist.

For the maximum end shear on the floor beam due to dead load we have

$$5{,}200 + \frac{3{,}600}{2} = 7{,}000 \text{ lbs. (See Fig. 187A.)}$$

From live load we have 68,200 lbs., as shown above (Fig. 191), and from impact we have

$$68{,}200 \times \frac{300}{30 + 300} = 62{,}000 \text{ lbs.}$$

Then adding the above dead- and live-load end shears and impact together, we have

$$7{,}000 + 68{,}200 + 62{,}000 = 137{,}200 \text{ lbs.}$$

for the total maximum end shear.

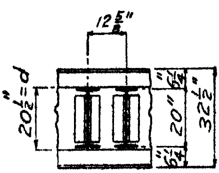


Fig. 192

The next thing is to design the floor beam. The flanges of such floor beams are usually composed of 6" x 6" angles, and the stringers should fit in between the flanges as shown in Fig. 192 so as to avoid fillers. So giving a ¼" clearance at both the top and bottom of the stringers, as shown, we obtain a floor beam 22½" deep. The distance from the back of the 6″ x 6″ flange angles to their center of gravity could be obtained from a handbook or from Table 6 provided the exact weight of them were known. However, we can obtain the average from this source. So let us take 1.78″ as the average. Then for the approximate effective depth we have

$$32.5 - 1.78 \times 2 = 28.94 \text{ ins.}$$

Now dividing this into the maximum bending moment given above we have

$$\frac{8{,}614{,}000}{28.94} = 298{,}000 \text{ lb. (about)}$$

for the flange stress. Then we have

$$\frac{298{,}000}{16{,}000} = 18.6 \text{ sq. ins.}$$

for the net flange area required minus one-eighth of the area of the cross-section of the web. Let us assume the web to be 32″ x ½″. Then one-eighth of the area of web = 2□″.

Then for the make-up of each flange we have

	Gross		Net
2—Ls 6″ x 6′ x $\frac{13}{16}$″	= 18.18□″	− 1.62	= 16.56□″
⅛ area of web			= 2.00□″
			18.56□″

Now for the actual effective depth (since we know the weight of the flange angles) we have

$$32.5 - 1.8 \times 2 = 28.9 \text{ ins.},$$

which is practically the same as assumed above.

As the floor beam has no regular stiffeners (the stringers, however, stiffen it) the vertical distance between the flange angles can be taken as d in Formula (1), Art. 118. Then we have

$$20.5 = \frac{\frac{1}{2}}{40}(12{,}000 - s),$$

from which we obtain

$$s = 10{,}360 \text{ lbs., say, } 10{,}000 \text{ lbs.}$$

Now dividing this into the maximum end shear given above we have

$$\frac{137{,}200}{10{,}000} = 13.72 \text{ sq. in.}$$

for the required area of the cross-section of the web. The web assumed above has 16□″, which is a little more than required, but if a thinner web be used the flange rivets would be quite close together, perhaps too close. This trouble often occurs in such shallow floor beams. So we will use the assumed web 32″ x ½″.

Preliminary estimate of the weight of an intermediate floor beam.*

1—web 32″ x ½″ x 15.5′ x 54.4#........................	= 843 lbs.
4—Ls 6″ x 6″ x $\frac{13}{16}$″ x 15.5′ x 31#........................	=1,922 lbs.
Gusset plates (1 plate 60″ x ½″ x 4′-0″ for the two) @ 102# per ft........................	= 408 lbs.
4—Ls 3½″ x 3½″ x ⅜″ x 4′-0″ x 8.5# (on gusset plates or brackets)........................	= 136 lbs.
2—Ls 3½″ x 3½″ x ⅜″ x 2′-6″ x 8.5# (end con. angles)......	= 42 lbs.
4—special plates 16″ x ½″ x 1′-8″ x 27.20#...............	= 181 lbs.
	3,532 lbs.
2.5% for rivets........................	88 lbs.
Total	3,620 lbs.

This shows that the 3,600 lbs. assumed weight of the floor beam is very close to the actual weight and hence no recalculations are necessary as 10 per cent variation either way is permissible.

149. Design of End Floor Beam.—The designing of end floor beams is very much the same as that of intermediate floor beams. The concentrations, however, are different and the weight of the beams themselves is a little less.

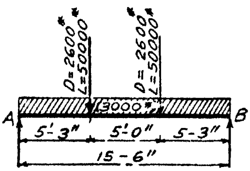


Fig. 193

Each concentration from dead load is equal to the dead-load end shear on a stringer and each live-load concentration is equal to the maximum live-load end shear on the same. Then assuming the weight of an end floor beam to be 3,000 lbs. we have the loading on the beam as shown in Fig. 193, using the end shears given in Art. 116. Then we have for the dead-load moment.

$$M = 2{,}600 \times 63 = 163{,}800 \text{ inch lbs.}$$

from the concentrated dead load and

$$M' = \tfrac{1}{8} \times 3{,}000 \times 15.5 \times 12 = 69{,}750 \text{ inch lbs.}$$

from the weight of the floor beam, making a total of 233,550 inch lbs. for the total maximum dead-load bending moment.

For the live-load bending moment we have

$$M'' = 50{,}000 \times 63 = 3{,}150{,}000 \text{ inch lbs.,}$$

and for the impact we have

$$I = 3{,}150{,}000 \left(\frac{300}{15 + 300}\right) = 3{,}000{,}000 \text{ inch lbs.}$$

Now adding these moments and impact together, we have

$$233{,}550 + 3{,}150{,}000 + 3{,}000{,}000 = 6{,}383{,}550 \text{ inch lbs.}$$

for the total maximum bending moment.

Assuming an effective depth of 29″ we have

$$\frac{6{,}383{,}550}{29} = 220{,}000 \text{ lbs. (about)}$$

* These preliminary estimates are made before the detail drawings are made and are intended to be only approximately correct.

for the flange stress. Then we have

$$\frac{220,000}{16,000} = 13.75 \text{ sq. ins.}$$

for the net area required for each flange minus one-eighth of the area of the cross-section of the web.

Assuming the web to be 32″ x $\frac{3}{8}$″ = 12□″, one-eighth of the area of the web is 1.5□″. Then for each flange we have

$$\begin{array}{ll} 2\text{—}\llcorner\text{s } 6'' \times 6'' \times \tfrac{9}{16}'' = 12.86\square'' - 1.12 = & 11.74\square'' \text{ net} \\ \tfrac{1}{8} \text{ area of web} = & \underline{1.50\square'' \text{ net}} \\ & 13.24\square'' \text{ net} \end{array}$$

As is seen, the net area of this flange is about 0.51□″ less than required, but the 6″ x 6″ x $\frac{5}{8}$″ angles give a flange about 0.72□″ too large, so the above will be used.

For the maximum end shear, as is readily seen from Fig. 193, we have

$$\begin{array}{lll} D = 1,500 + 2,600 & = & 4,100 \text{ lbs.} \\ L = & & 50,000 \text{ lbs.} \\ I = 50,000 \left(\frac{300}{315}\right) & = & \underline{47,600 \text{ lbs.}} \\ \qquad \text{Total} & & 101,700 \text{ lbs.} \end{array}$$

For the allowable stress on the web we have

$$20.5 = \frac{\frac{3}{8}}{40}(12,000 - s),$$

from which we obtain

$$s = 9,815 \text{ lbs., say, } 10,000 \text{ lbs.}$$

Then dividing this into the above shear we have

$$\frac{101,700}{10,000} = 10.17 \text{ sq. ins.}$$

for the required area of the web. So the assumed web 32″ x $\frac{3}{8}$″ = 12□″ is satisfactory, as the specifications permit of no thinner web.

The flange angles and web as specified above for the end floor beam weigh about 2,000 lbs., which is about 765 lbs. less than the flange angles and web in the intermediate floor beam. Then as the other parts are about the same as for an intermediate floor beam we have 3,621 − 765 = 2,856 lbs. for the weight of the end floor beam. So the assumed 3,000 lbs. for the weight of an end floor beam is within the 10 per cent limit.

150. Design of the Main Girders.—In the case of through plate girder bridges the live load is applied to the main girders only at the panel points, that is, where the floor beams connect to the main girders, just the same as in the case of through truss bridges. The dead load is applied to the main girders in the same way except for the weight of the girders themselves which is uniformly distributed along the girders. But it is usual practice to consider the dead load as wholly applied at the panel points as about the same result is obtained as if the more exact conditions were considered.

Taking the assumed dead load given in Art. 146 we have

$$\frac{1,780}{2} \times 15 = 13,350 \text{ lbs.}$$

for the panel load of dead load per girder. Then the dead load per girder will be as shown in Fig. 194, where AB represents the girder. Now it is obvious that the maximum bending moment due to this dead load will occur under the load at C. So taking moments at C we have

$$M = (20,025 \times 30 - 13.350 \times 15)\ 12 = 1,806,000 \text{ inch lbs.}$$

for the maximum bending moment due to dead load.

The live load will be applied at the panel points, but of course the panel loads will not be equal to each other as in the case of dead load. The maximum bending moment will occur at a panel point as in the case of a truss. It is readily seen that the maximum moment will occur at the panel point C, at the center of the span. Then the first thing to do is to place the wheel loads so as to obtain the maximum moment at that point which is simply the satisfying of the criterion for maximum moment as given in Art. 91. That is, the maximum live-load bending moment on the girder (which will occur at C) will occur when the average load to the left of C is equal to the average load on the span.

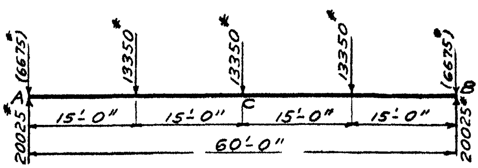


Fig 194

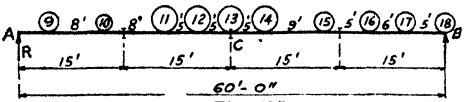


Fig 195

Now referring to Table A, let us try wheel 13 at C. Then the loads will be in the position shown in Fig. 195, and for the average unit load on the left, considering half of wheel 13 as being on the left, we have

$$\frac{73,000}{30} = 2,430 \text{ lbs. (about)},$$

and for the average unit load on the whole span, we have

$$\frac{142,000}{60} = 2,370 \text{ lbs.}$$

Now it is quickly seen that this position will give the maximum moment at C. For if wheel 14 be placed at C there would be 80,000 lbs. to the left which would give 2,660 lbs. average unit load to the left and the average unit load on the span would be the same as given above, and if wheel 12 be placed at C the average unit load on the left will be $53,000 \div 30 = 1,760$ lbs. and the average unit load on the span will remain the same as before. So the wheels when in the position shown in Fig. 195 will produce the maximum bending moment on the girder.

Then, the next thing is to determine the bending moment about wheel 13 as that is the moment desired. To do this we must first determine the reaction R at A which we do by taking moments about B. As wheel 18 just happens to come exactly at B we can obtain the moments of all the wheels on the span directly from Table A. Passing down the line through wheel 18 (in the table) until we come to the zigzag line, then passing to the left until we come to the vertical line through wheel 9, we find the moment 4,224 which is the moment in thousands of foot pounds of all the wheels, 9 to 17 inclusive, about wheel 18, which happens to be at the right support.

Then for the reaction R we have

$$\left(\frac{4{,}224}{60}\right) = 70.4 \text{ (thousands).}$$

Then taking moments about wheel 13 we have (using Table A)

$$M = (70.4 \times 30 - 818)\ 1{,}000 \times 12 = 15{,}528{,}000 \text{ inch lbs.}$$

Then multiplying this by 50/40 to reduce it to Cooper's $E50$, we have

$$15{,}528{,}000 \times \frac{50}{40} = 19{,}410{,}000 \text{ inch lbs.}$$

for the maximum live-load bending moment on the girder.

Then for the impact we have

$$19{,}410{,}000 \times \left(\frac{300}{60+300}\right) = 16{,}175{,}000 \text{ inch lbs.}$$

Now adding the above dead- and live-load moments and impact together we have

$$4{,}806{,}000 + 19{,}410{,}000 + 16{,}175{,}000 = 40{,}391{,}000 \text{ inch lbs.}$$

for the total maximum bending moment on the girder. Now, assuming the web will be $\frac{3}{8}''$ thick, we have

$$x = 1.055\sqrt{\frac{40{,}391{,}000}{16{,}000 \times \frac{3}{8}}} = 86.5 \text{ ins. (about)}$$

for the economic depth. So we will use a web 84″ deep (as this is a common plate) and assume it to be $\frac{3}{8}''$ thick.

The total depth of the girder back to back of angles will then be 84.25″. Then subtracting about an inch we have 83.2″ for an assumed effective depth. Dividing this into the maximum moment given above we have

$$\frac{40{,}391{,}000}{83.2} = 486{,}000 \text{ lbs. (about)}$$

for the flange stress, and then we have

$$\frac{486{,}000}{16{,}000} = 30.4 \text{ sq. ins. (about)}$$

for the required net area of the cross-section of each flange.

The area of the cross-section of the 84″ x $\frac{3}{8}$″ web = 31.5□″, and $\frac{1}{8}$ = 3.9□″. Then for the make-up of each flange we have

	Gross	($\frac{7}{8}$″ rivets)	Net
2—Ls 6″ x 6″ x $\frac{11}{16}$″	= 15.56□″	− 2.75□″	= 12.80□″
1—cov. pl. 14″ x $\frac{5}{8}$″	= 8.75□″	− 1.25□″	= 7.50□″
1—cov. pl. 14″ x $\frac{1}{2}$″	= 7.00□″	− 1.00□″	= 6.00□″
$\frac{1}{8}$ area of web			= 3.90□″
	31.31□″		30.20□″

Now taking moments about the center of the top cover plate (see Fig. 196) we have

$$\bar{x} = \frac{2.62 \times 15.56 + 8.75 \times 0.56}{31.3} = 1.46 \text{ ins.}$$

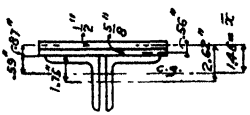


Fig. 196

for the distance from the center of the top cover plate to the center of gravity of the whole flange. Then we have

$$1.46 - 0.87 = 0.59 \text{ ins.}$$

for the distance from the back of the flange angles to the center of gravity of the flange. Then we have

$$84.25 - (0.59 \times 2) = 83.07 \text{ ins.}$$

for the actual effective depth of the girder, which is quite close to that assumed above, so no recalculation on account of the assumed effective depth is necessary.

For the length of the $\frac{1}{2}$″ cover plates, which will be the outside cover plates on the girder, we have

$$y = 60\sqrt{\frac{7}{31.3}} = 28'\text{-}6'' + 1'\text{-}6'' = 30'.$$

For the $\frac{5}{8}$″ cover plate we have

$$y' = 60\sqrt{\frac{15.7}{31.3}} = 42'\text{-}6'' + 1'\text{-}6'' = 44'.$$

The maximum end shear due to dead load is equal to the reaction shown in Fig. 194 plus the weight of the girder for half of the length of the end panel. The four flange angles of the girder, as given above, weight 106# per ft. of girder, the web weighs 108# per ft., and the weight of stiffeners and details at this point on the girder will be about 90 per cent of the weight of the web per ft., or 97#, and the two $\frac{5}{8}$″ cover plates weigh about 60#, making in all about 370#, say, 400# per ft. of girder. Then we have

$$S = 20{,}025 + 400 \times 7.5 = 23{,}000 \text{ lbs. (about)}$$

for the maximum end shear on the girder.

The maximum end shear on the girder due to live load is equal to the maximum shear in the end panel due to that load. Now according to Art. 90 the maximum shear in the end panel (the same as in a truss bridge) will occur when the load in the panel is equal to the total load on the bridge divided by the number of panels. So the first thing is to satisfy this criterion for maximum shear in the end panel.

Placing wheel 3 at the first panel point out from the end of the girder as shown in Fig. 197 we have (using Table A)

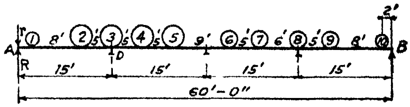


Fig. 197

$$10{,}000 + 20{,}000 + 10{,}000 = 40{,}000 \text{ lbs.}$$

for the load in the end panel AD, considering one-half of wheel 3 in the panel. With the wheels in this position the total load on the span is 152,000#. Then we have

$$\frac{152{,}000}{4} = 38{,}000 \text{ lbs.},$$

which to satisfy the criterion should be 40,000#. But this is as close as the criterion can be satisfied, as can be verified by trial.

The next thing is to determine the shear in the end panel AD with the wheels in the position shown in Fig. 197. First of all the shear in the panel is equal to the reaction R at A, due to all of the loads on the span, minus the concentration r from the stringers, due to the loads in the panel AD. In fewer words, the shear in the end panel AD is

$$S' = R - r.$$

Taking moments about the right support B (using Table A) we have

$$R = \frac{4{,}632{,}000 + 152{,}000 \times 2}{60} = 82{,}300 \text{ lbs.},$$

and taking moments about the panel point D we have

$$r = \frac{230{,}000}{15} = 15{,}300 \text{ lbs.}$$

Then for the maximum shear in the end panel AD we have

$$S' = 82{,}300 - 15{,}300 = 67{,}000 \text{ lbs.},$$

and multiplying this by 50/40 we have

$$67{,}000 \times \frac{50}{40} = 84{,}000 \text{ lbs. (about)}$$

for the maximum live-load shear on the girder.

Then for the impact we have

$$I = 84{,}000 \left(\frac{300}{45 + 300}\right) = 73{,}000 \text{ lbs.}$$

Now adding together the dead-load and live-load end shears, given above, and the impact, we have

$$23{,}000 + 84{,}000 + 73{,}000 = 180{,}000 \text{ lbs.}$$

for the total maximum end shear on the girder, which is practically the shear throughout the end panel.

Now for the unit-shearing stress on the assumed 84″ x $\frac{3}{8}$″ web we have

$$\frac{180,000}{31.5} = 5,710 \text{ lbs.}$$

Then substituting this for s in Formula (1), Art. 117, we have

$$d = \frac{\frac{3}{8}}{40}(12,000 - 5,710) = 59 \text{ ins.}$$

for the maximum spacing of the stiffeners at the ends of the girders. Now, as this is not less than the half depth of the girder, the assumed web is satisfactory. The specification will not permit of the web being made thinner.

The outstanding flanges or legs of the stiffeners, to satisfy the specifications, should not be less than $1/30 \times 84 + 2 = 4.6''$. This requires the use of 5″ x 3½″ angles, the 5″ leg outstanding. All intermediate stiffeners can be taken as 5″ x 3½″ x $\frac{3}{8}$″ angles without hesitating—this being the minimum thickness allowed—but the end stiffeners must have sufficient area of cross-section to take the maximum reaction on the girder when considered as columns, as per specifications. The cross-section of a pair of these angles, considered as a column, is shown in Fig. 198.

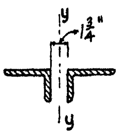


Fig. 198

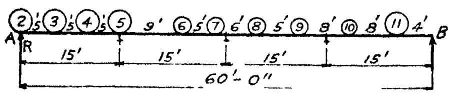


Fig. 199

Let us first assume them to be 5″ x 3½″ x $\frac{3}{8}$″ angles. Then for the radius of gyration about the axis y-y we have

$$r = \sqrt{\frac{2 \times 7.78 + \overline{2.48}^2 \times 6.1}{6.1}} = 2.96 \text{ ins.}$$

Then taking one-half the depth of the girder as the length, as per specification, we have

$$p = 16,000 - 70\frac{42}{2.96} = 15,000 \text{ lbs. (about)}$$

for the allowable unit stress on such a column.

Now from Fig. 194 it is readily seen that the total maximum dead-load reaction is

$$20,025 + 6,675 = 26,750 \text{ lbs., say, } 27,000 \text{ lbs.}$$

The maximum live-load reaction, as is readily seen, will occur when the wheels are in the position shown in Fig. 199, where AB represents the span.

Taking moments about B (using Table A) we have

$$R = \left(\frac{5{,}208 + 162 \times 4}{60}\right) 1{,}000 = 97{,}600 \text{ lbs.}$$

for the maximum live-load reaction at A due to Cooper's $E40$, and multiplying this by 50/40, to reduce it to Cooper's $E50$ loading, we have

$$97{,}600 \times \frac{50}{40} = 122{,}000 \text{ lbs.}$$

for the maximum live-load reaction desired.

Then for the impact we have

$$I = 122{,}000 \frac{300}{60+300} = 101{,}000 \text{ lbs.}$$

Now adding together the above maximum dead- and live-load reactions and the impact we have

$$27{,}000 + 122{,}000 + 101{,}000 = 250{,}000 \text{ lbs.}$$

for the total maximum reaction. Now dividing this by the allowable unit stress as found above for the end stiffeners we have

$$\frac{250{,}000}{15{,}000} = 16.6 \text{ sq. ins.}$$

for the required area of the end stiffeners.

It is necessary to have two pairs of end stiffeners at each end of the girders in order to properly distribute the pressure over the pedestal. Two pairs of the stiffeners assumed above have 12.2□″ cross-section, which is about 4□″ less than required. So thicker angles than those assumed will have to be used. The allowable unit stress will not be materially affected by using thicker angles as the radius of gyration varies but little with the thickness. So we can use any two pairs of 5″ x 3½″ angles that have the proper area. It is seen from a handbook, or from Table 4, that 4—Ls 5″ x 3½″ x ½″ have 16□″ cross-section, and that 4—Ls 5″ x 3½″ x 9/16″ have 17.88□″. As is seen, the ½″ angles have an area of cross-section that is 0.6□″ smaller than required, and the 9/16″ angles have about 1.28□″ more than required. So let us use 4—Ls 5″ x 3½″ x ½″ = 16.0□″ at each support for end stiffeners.

For area of bearing on the masonry we have required

$$\frac{250{,}000}{600} = 417 \text{ sq. ins.}$$

The pedestal should be made to suit.

Preliminary estimate of the weight of one main girder.

4—Ls 6″ x 6″ x 11/16″ x 69′-0″ x 26.5# (combined with end Ls)	7,314 lbs.	
1—web 84″ x 3/8″ x 62′-0″ x 107.12#	6,641 lbs.	
1—cov. pl. 14″ x ½″ x 60′-0″ x 23.80# (2 combined)	1,428 lbs.	
1—cov. pl. 14″ x 5/8″ x 118′-0″ x 29.75#	3,510 lbs.	18,893 lbs.

8—end stiff. Ls 5" x $3\frac{1}{2}$" x $\frac{1}{2}$" x 13.6# x 7'-0"......	762 lbs.	
26—int. stiff. Ls 5" x $3\frac{1}{2}$" x $\frac{3}{8}$" x 10.4# x 7'-0".....	1,893 lbs.	
7—fillers 8" x $\frac{11}{16}$" x 6'-0" x 18.7# (at floor beams).	785 lbs.	
8—splice pls. 10" x $\frac{11}{16}$" x 2'-3" x 23.38#..........	421 lbs.	
4—splice pls. 14" x $\frac{11}{16}$" x 4'-4" x 32.72#.........	568 lbs.	
3—fills. $3\frac{1}{2}$" x $\frac{11}{16}$" x 6'-0" x 8.18#...............	147 lbs.	4,576 lbs.
		23,469 lbs.
3% for rivet heads.......................................		700 lbs.
Total weight of one main girder.................		24,169 lbs.
Total weight of two main girders................		48,338 lbs.

Total weight of main girders per ft. of span = 48,338/60 = 805 lbs.

To make this estimate the student would have to sketch the details of the girders to some extent.

151. Design of Lateral System.—The laterals with the floor beams and main girders form a horizontal double truss as shown in Fig. 200, where AB represents one main girder and CD the other. The laterals, according to the specifications, must be designed to resist a lateral force of $200 + 5{,}000 \times 0.10 = 700\#$ per ft. of span considered as a live load. One system can be considered as taking the force when applied from one direction and the other system when it is applied from the other direction. So the laterals may be considered as taking tension only.

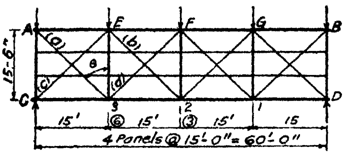


Fig 200

For a panel load we have

$$P = 700 \times 15 = 10{,}500 \text{ lbs.}$$

Then assuming the lateral force to be applied from the direction indicated by the arrows and suppose it moves onto the structure from right to left, loading panel points G and F, we have

$$S = \frac{3}{4} \times 10{,}500 = 7{,}800 \text{ lbs.}$$

for the maximum shear in panel FE (see Art. 90), and for the maximum shear in panel EA, we have

$$S' = \frac{6}{4} \times 10{,}500 = 15{,}700 \text{ lbs.}$$

By multiplying each of these shears by the secant of the angle θ we will obtain the stress in the corresponding laterals. As angle θ is about 45 degrees we can take the secant as 1.4. Then for the stress in the diagonal marked (a) we have

$$T = 15{,}700 \times 1.4 = 22{,}000 \text{ lbs.},$$

and for the stress in the diagonal marked (b) we have

$$T' = 7{,}800 \times 1.4 = 10{,}900 \text{ lbs.}$$

If the lateral force were applied in the opposite direction to that indicated by the arrows, the lateral marked (c) would be subjected to a maximum stress of 22,000# and the lateral marked (d) would be subjected to a maximum stress of 10,900# and laterals (*a*) and (*b*) in that case would have no stress. The same maximum stresses would be found in the laterals in the right half of the span if the lateral force were considered to move onto the span from left to right.

The stresses in the floor beams due to the lateral force (=700#) can be neglected, as the stress produced is simple compression in the bottom flanges, which are in tension from dead and live load. It is readily seen that the stresses calculated above are all that we need for designing the laterals.

For the laterals marked (*a*) and (*c*) we have

$$\frac{22,000}{16,000} = 1.37 \text{ sq. ins.}$$

for the required area of cross-section.

Use 1—L $3\frac{1}{2}''$ x $3''$ x $\frac{3}{8}''$ = 2.30 − 0.37 = 1.93□″ net. This is the smallest angle allowed by the specifications and therefore the same size will be used for each of the other laterals.

Preliminary estimate of weight of lateral system.

8—Ls $3\frac{1}{2}''$ x $3''$ x $\frac{3}{8}''$ x 7.9# x 20′	1,264 lbs.
6—lat. pls. $17''$ x $\frac{3}{8}''$ x 21.6# x 3′	389 lbs.
4—lat. pls. $17''$ x $\frac{3}{8}''$ x 21.6# x 2′	173 lbs.
4—sp. pls. $9''$ x $\frac{3}{8}''$ x 11.5# x 3′	138 lbs.
16—Ls $6''$ x $4''$ x $\frac{1}{2}''$ x 16.2# x 1′	259 lbs.
rivets	70 lbs.
	2,293 lbs.

Total weight of laterals per ft. of span = 2,293/60 = 38# per ft.

152. Preliminary Estimate of Weight and Cost.—

Estimates of effective dead weight per ft. of span.

Weight of stringers per ft. of span (Art. 147)	289 lbs.
Weight of intermediate floor beam per ft. of span (3,621/15) (Art. 148)	242 lbs.
Weight of main girders per ft. of span (Art. 150)	805 lbs.
Weight of laterals per ft. of span (Art. 151)	38 lbs.
Weight of deck per ft. of span	400 lbs.
Total weight of dead load per ft. of span	1,774 lbs.

As 1,780# was assumed no recalculations are necessary.

Estimate of total weight of metal in span.

2 main girders at 24,169#	48,338 lbs.
3 intermediate floor beams at 3,621#	10,863 lbs.
2 end floor beams at 2,856#	5,712 lbs.
4 panels of stringers and details at 4,358#	17,432 lbs.
8 laterals and details	2,293 lbs.
4 pedestals and sole plates	2,200 lbs.
	86,838 lbs.

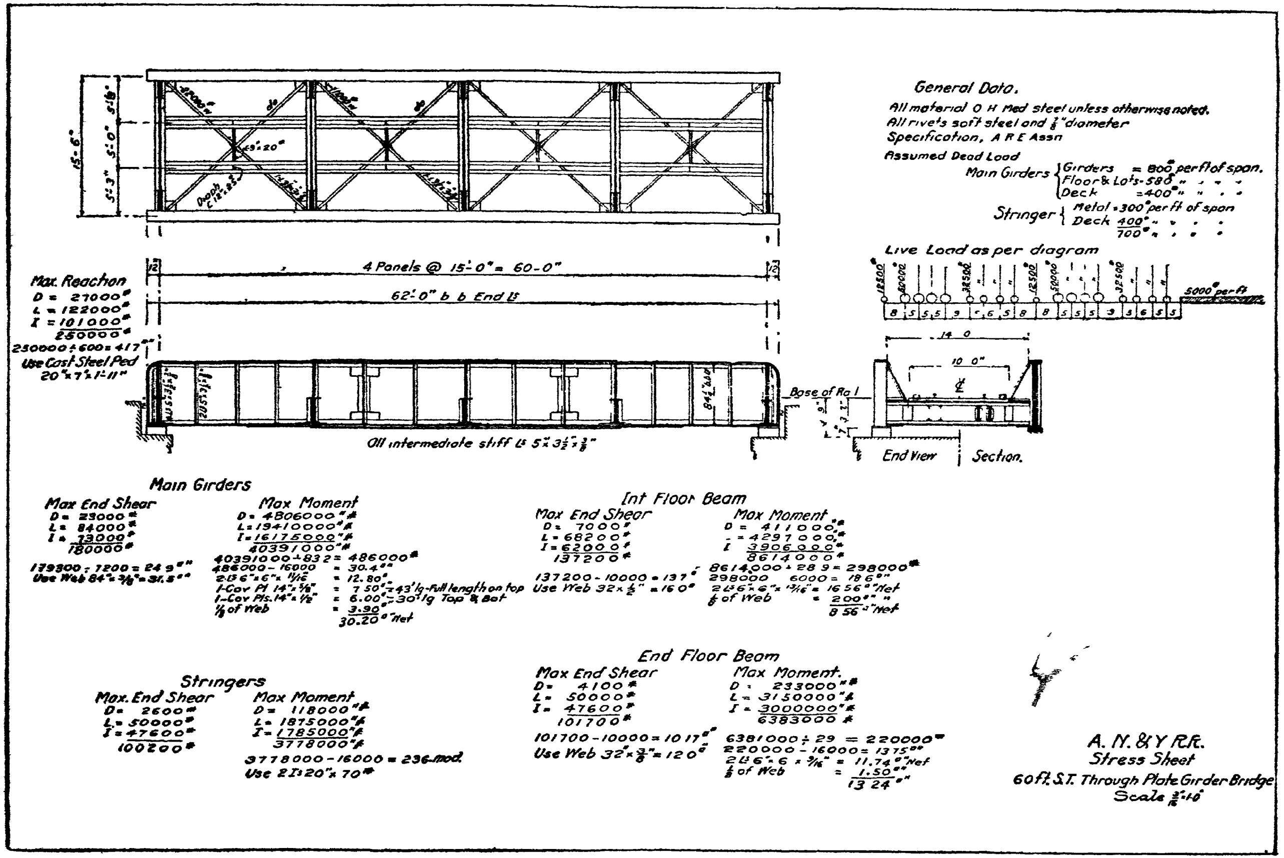

General Data.
All material O H Med steel unless otherwise noted.
All rivets soft steel and ⅞" diameter
Specification, A R E Assn
Assumed Dead Load
Main Girders { Girders = 800# per ft of span. Floor & Lats - 580# " " " Deck = 400# " " "
Stringer { Metal = 300# per ft of span Deck 400# " " " 700# " " "
Live Load as per diagram
5000# per ft
4 Panels @ 15'-0" = 60-0"
62'-0" b b End Ls
All intermediate stiff Ls 5"x3½"x⅜"
Base of Rail
End View
Section.
Max. Reaction
D = 27000#
L = 122000#
I = 101000#
250000#
250000 ÷ 600 = 417□"
Use Cast Steel Ped 20"x7"x1'-11"
Main Girders
Max End Shear
D = 23000#
L = 84000#
I = 73000#
180000#
179300 ÷ 7200 = 24.9□"
Use Web 84"x⅜" = 31.5□"
Max Moment
D = 4806000"#
L = 19410000"#
I = 16175000"#
40391000"#
40391000 ÷ 83.2 = 486000#
486000 - 16000 = 30.4□"
2 Ls 6"x6"x 11/16 = 12.80□"
1-Cov Pl 14"x⅝" = 7.50□" - 43' lg - full length on top
1-Cov Pls. 14"x½" = 6.00□" - 30' lg Top & Bot
⅛ of Web = 3.90□"
30.20□" Net
Int Floor Beam
Max End Shear
D = 7000#
L = 68200#
I = 62000#
137200#
137200 - 10000 = 13.7□"
Use Web 32"x½" = 16.0□"
Max Moment
D = 411000"#
L = 4297000"#
I = 3906000"#
8614000"#
8614000 ÷ 28.9 = 298000#
298000 ÷ 16000 = 18.6□"
2 Ls 6"x6"x 13/16 = 16.56□" Net
⅛ of Web = 2.00□"
8.56□" Net
Stringers
Max. End Shear
D = 2600#
L = 50000#
I = 47600#
100200#
Max Moment
D = 118000"#
L = 1875000"#
I = 1785000"#
3778000"#
3778000 - 16000 = 236 mod.
Use 2 Is 20"x70#
End Floor Beam
Max End Shear
D = 4100#
L = 50000#
I = 47600#
101700#
101700 - 10000 = 10.17□"
Use Web 32"x⅜" = 12.0□"
Max Moment
D = 233000"#
L = 3150000"#
I = 3000000"#
6383000"#
6381000 ÷ 29 = 220000#
220000 - 16000 = 13.75□"
2 Ls 6"x6"x 9/16" = 11.74□" Net
⅛ of Web = 1.50□"
13.24□"
A. N. & Y. R.R.
Stress Sheet
60 ft. S.T. Through Plate Girder Bridge
Scale 3/16" = 1'-0"

Fig. 201

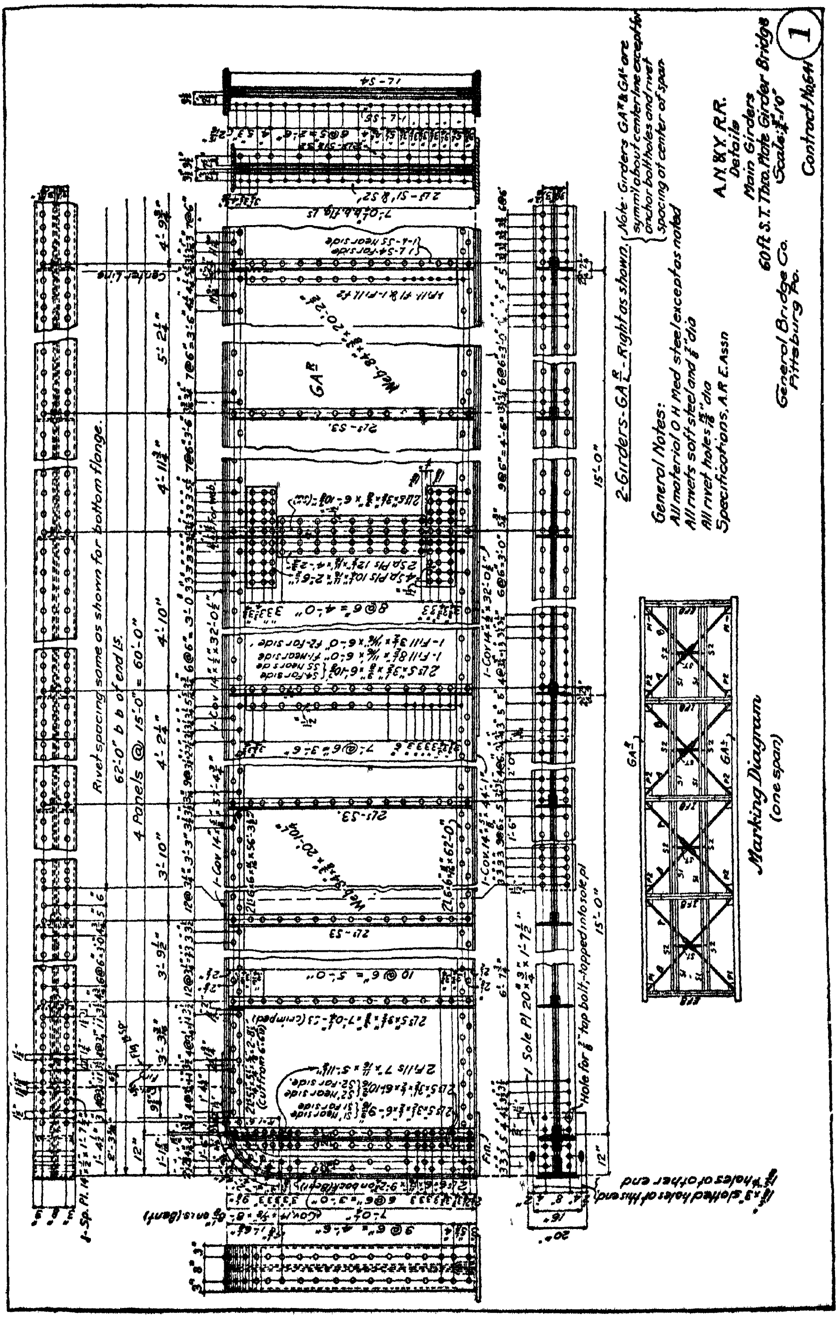
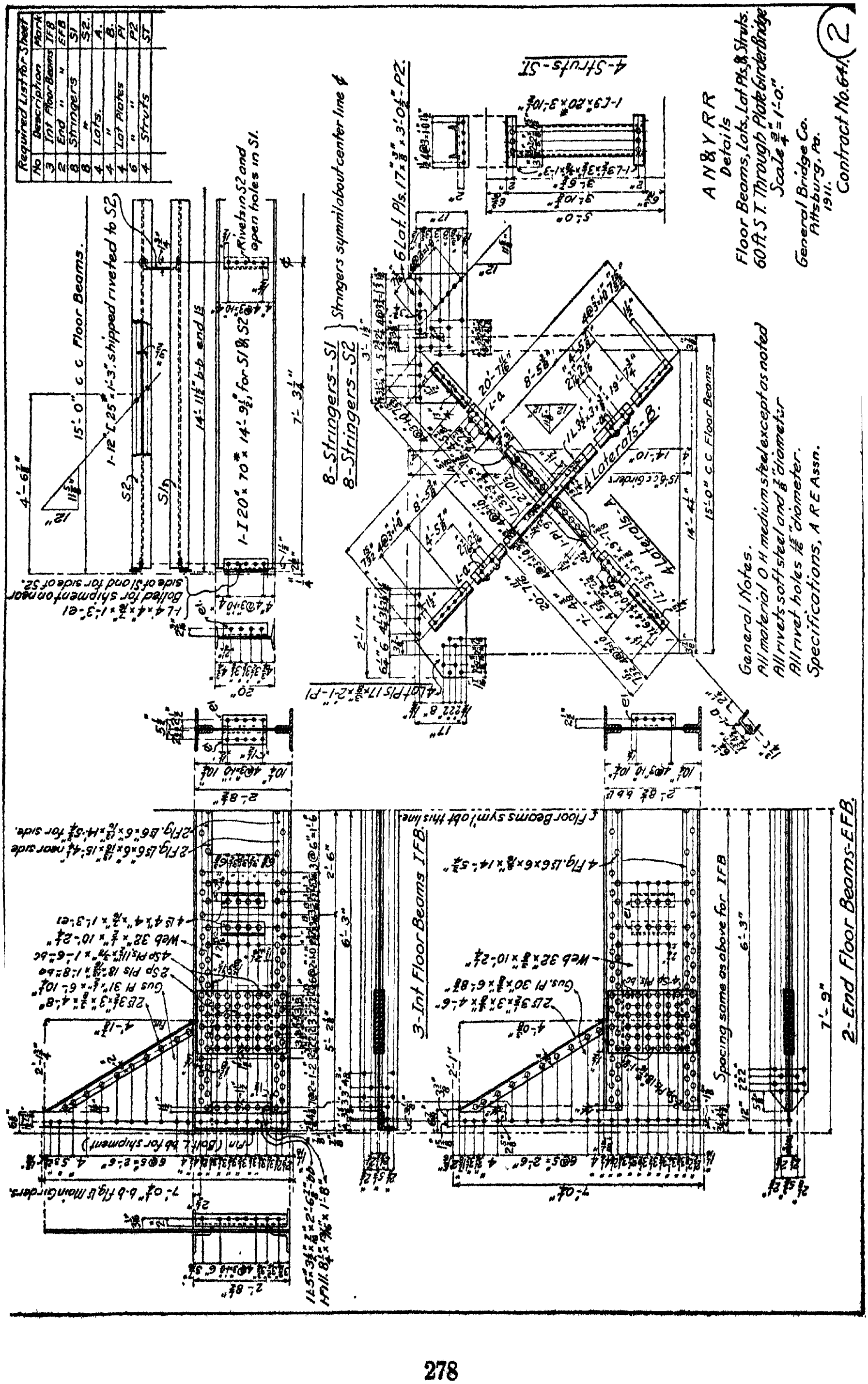

A.N.&Y. R.R.
Details
Main Girders
60 ft. S.T. Thro. Plate Girder Bridge
Contract No. 64
General Bridge Co.
Pittsburg Pa.
2-Girders-GA - Right as shown.
General Notes:
All material O.H. Med steel except as noted
All rivets soft steel
Specifications, A.R.E. Assn
Marking Diagram
(one span)
Center Line
Rivet spacing same as shown for bottom flange.
62'-0" b to b of end Ls.
4 Panels @ 15'-0" = 60'-0"
15'-0"
1

Fig. 202


Required List for Sheet
No | Description | Mark
3 | Int Floor Beams | IFB
2 | End " " | EFB
8 | Stringers | S1
8 | " | S2
4 | Lats | A
4 | " | B
4 | Lat Plates | P1
6 | " " | P2
4 | Struts | ST
15'-0" c c Floor Beams
8-Stringers-S1
8-Stringers-S2
Stringers symm'l about center line
4-Struts-ST
4 Laterals-A
4 Laterals-B
3-Int Floor Beams IFB
2-End Floor Beams-EFB
Spacing same as above for IFB
General Notes.
All material O H medium steel except as noted
All rivets soft steel and 7/8" diameter
All rivet holes 15/16" diameter.
Specifications, A R E Assn.
A N & Y R R
Details
Floor Beams, Lats, Lat Pls, & Struts.
60 ft. S. T. Through Plate Girder Bridge
Scale 3/4" = 1'-0"
General Bridge Co.
Pittsburg, Pa.
1911.
Contract No. 641.
2

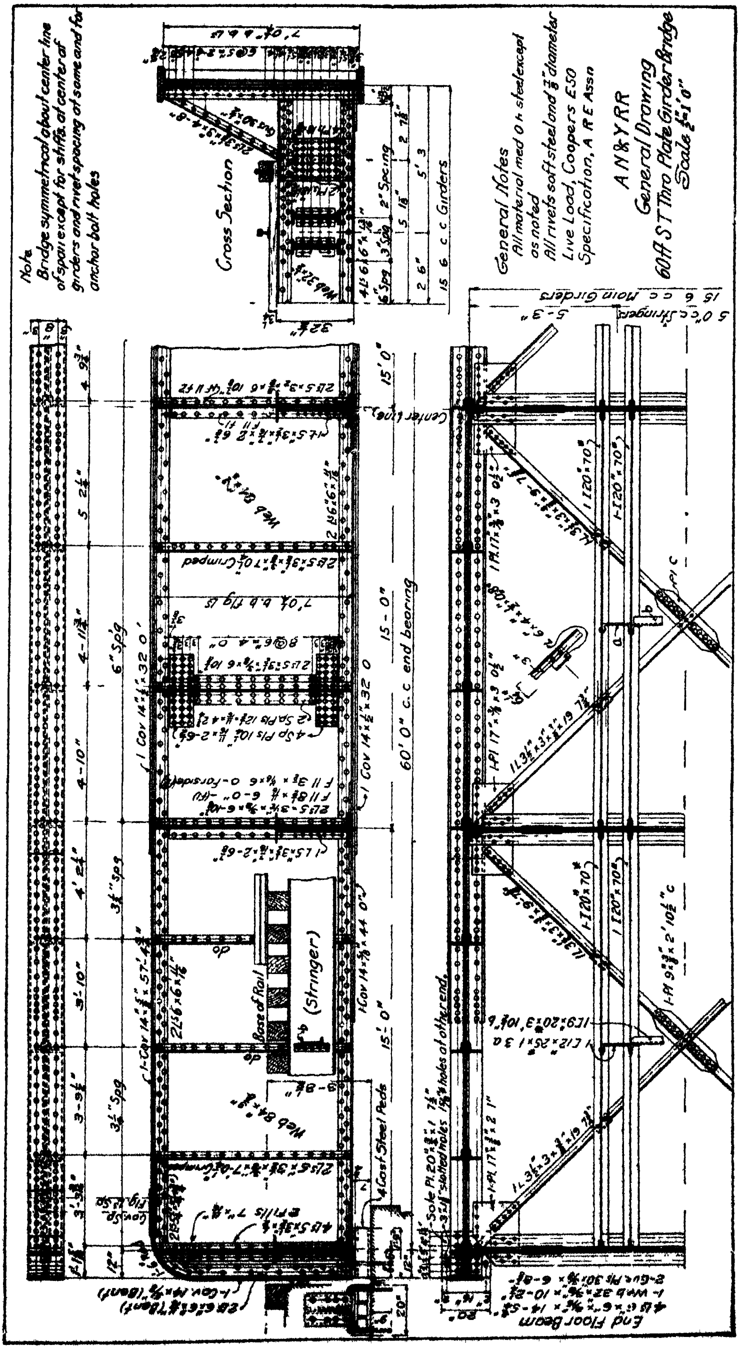

Cross Section
General Notes
End Floor Beam
(Stringer)
A N&Y RR
General Drawing

Fig 204

For the cost at $3\frac{1}{2}$¢ we have $86{,}838 \times 0.035 = \$3{,}039.33$. $3\frac{1}{2}$¢ per lb. is a common price for this class of work erected. However, the pound price will vary from $2\frac{1}{2}$¢ to $4\frac{1}{2}$¢. It depends upon the market price of metal and freight.

This completes the necessary preliminary calculations for the span. Next a stress sheet as shown in Fig. 201 can be drawn. Then the shop drawings (Figs. 202 and 203) for the span can be made.

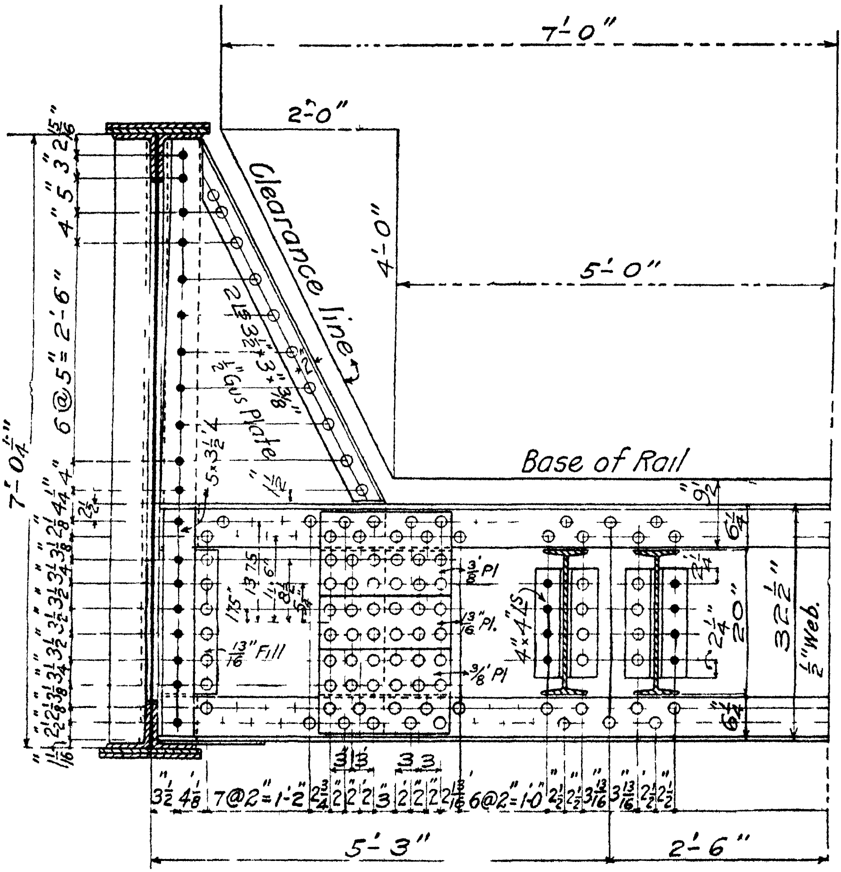


Fig 205

The work up to the completion of the stress sheet, as a rule, is done in the designing office of a bridge company, or in the office of a railroad company, or in the office of a consulting engineer. Consulting engineers as a rule (and sometimes railroad companies) make general drawings, similar to the one shown in Fig. 204, instead of stress sheets. In that case the general drawing for the span is submitted to the different bridge companies for use in preparing their bids for fabricating and (usually) erecting the structure. After the contract is awarded, the general drawing, instead of a stress sheet, is used as a guide in making the shop drawings by the bridge company obtaining the contract. As a rule bridge companies do not make general drawings of such structures—only the stress sheets and shop drawings.

153. Making of the Detail Drawings.—In beginning the work, the first thing to do is to draw a half cross-section of the span, as shown in Fig. 205, to a large scale, say $1\frac{1}{2}''$ scale. In making this sketch (Fig. 205) first draw the cross-section of the main girder and the stiffeners. Then the next thing to do is to draw the floor beam. Counting from the back of the bottom flange angles of the main girder, we have $\frac{11}{16}''$ for the thickness of the flange angle, and $\frac{3}{8}''$ for the thickness of the lateral plate, making in all $\frac{11}{16} + \frac{3}{8} = 1\frac{1}{16}''$ for the distance from the back of the flange angles of the main girder to the back of the bottom flange angles of the floor beam. This distance being determined, the bottom flange of the floor beam is located and can be drawn. The distance from the back of the bottom flange angles to the back of the top flange angles of such shallow floor beams is usually made $\frac{1}{2}''$ more than the depth of the web. So in this case, as the web is $32''$ deep, we have $32\frac{1}{2}''$ for the distance from the back of the bottom flange angles to the back of the top flange angles, and thus the top flange is located and can be drawn. Next the cross-sections of the two I-beams composing the stringer can be drawn at their proper location. Then by drawing a horizontal line $9\frac{1}{2}''$ above the top of these beams (using $10''$ ties dapped $\frac{1}{2}''$ on the stringer) we have the base of rail located, and having the base of rail located the clearance line can be drawn as shown. This is about as far as we can proceed with the drawing (Fig. 205) until the required spacing of the rivets in the floor beam is determined.

For the shear on the part of the floor beam between the end and where the stringer connects, we have 137,200# (see stress sheet). The vertical distance from the center of rivets in the top flange to center of rivets in the bottom flange is about 25.5″, and the allowable bearing of a $\frac{7}{8}''$ shop rivet on the $\frac{1}{2}''$ web is 10,500#. So, substituting 137,200 for S, 25.5 for h, and 10,500 for r in Formula (2) (Art. 116), we have

$$p = \frac{10{,}500 \times 25.5}{137{,}200} = 1.95 \text{ ins.},$$

say, 2″, for the theoretical spacing of the flange rivets in the part of the floor beam between the end and the stringer.

The shear on the part of the floor beam between the stringers is very slight. In fact there is no shear at all except that due to the weight of the intervening part of the floor beam itself. This being the case, the flange rivets in that part of the floor beam can be spaced 6″ apart, which is the maximum spacing allowed.

Now beginning at the end of the floor beam (Fig. 205) we space the rivets 2″ apart (as required) in the flange angles out to the splice (where the gusset and web meet) and the only break in the 2″ spacing there is that necessary to fit the details of the splice. The splice is located so as to bring the bracket just inside the clearance line as shown.

The rivets in the splice should line up with those in the stringer connection and at the same time with those in the end details of the floor beam.

There must be enough rivets in each stringer connection to transmit, in bearing on the web, the maximum floor beam concentration. For the concentration we have 5,200# from the dead load, 68,200# from the live load as given in Art. 148, and 62,000# [= 300 ÷ (30 + 300) × 68,200] from the impact, making a total of 135,400#. The allowable bearing of a $\frac{7}{8}''$ field rivet on the $\frac{1}{2}''$ web is $\frac{7}{8} \times 20{,}000 \times \frac{1}{2} = 8{,}750$#. Then for the number of rivets required in each stringer connection we have

$$\frac{135{,}400}{8{,}750} = 15.4, \text{ say } 16, \text{ rivets.}$$

It is seen from this that a single line of rivets in each of the angles connecting the stringers to the floor beam is sufficient. So we have this much data for later use.

Now the next thing to do is to determine the vertical spacing of the rivets in the floor beam. It appears that there is no question but that the rivets should be spaced as closely as is permissible in the vertical direction in the case of such shallow floor beams. So we will simply try putting in as many rivets in the vertical direction as will fit in. Beginning at the bottom of the floor beam, we will make the first gauge in the flange angle $2\frac{1}{2}''$, as the angle is quite thick ($\frac{13}{16}''$), and the second gauge $2\frac{1}{8}''$, which leaves $1\frac{3}{8}''$ edge distance on the angle. For the next space we have $1\frac{3}{8}''$ (edge distance on the angle) plus $\frac{1}{4}''$ (clearance) plus $1\frac{1}{2}''$ (edge distance for fillers and splice plates), making in all $3\frac{1}{8}''$. Now these same spaces will be used at the top of the floor beam in order to have symmetrical spacing. So we have

$$32\tfrac{1}{2} - 2(2\tfrac{1}{2} + 2\tfrac{1}{8} + 3\tfrac{1}{8}) = 17 \text{ ins.}$$

for the remaining distance in which rivets are to be spaced. It is readily seen that 3″ spacing will not fit in this distance and that five spaces is the maximum number possible. So we will space the rivets as shown.

We will now see how this spacing fits the detail at the end of the floor beam, at the splice, and at the stringer connection. There should be enough rivets in the end of the floor beam proper to transmit the greater part of the maximum end shear on the beam to the main girder. In the detail shown we have 8 field rivets in double shear or bearing on $1\frac{5}{16}''$ metal. The double shear being the least we have $8 \times 0.6 \times (10{,}000 \times 2) =$ 96,000 lbs. for the amount that these rivets will be considered to transmit. This leaves 41,200 lbs. (= 137,200 − 96,000) to be taken by the rivets in the bracket above the floor beam proper. These rivets are in single shear and hence the allowable stress on each is 6,000 lbs. Then we have

$$\frac{41{,}200}{6{,}000} = 6.8 \text{ rivets, say } 7,$$

required in the bracket above the beam proper. So the detail as shown for the end of the floor beam is quite satisfactory, as the extra rivets at the top of the bracket are really needed to hold the bracket which stiffens the top flange of the main girder. Now by prolonging the lines giving the vertical spacing on to the stringer connection, it is seen that this spacing fits nicely at that point, as we obtain the detail shown. The connecting angles between the I-beams are shop riveted to the floor beam to avoid difficult field riveting, as it would be difficult work to drive the rivets connecting these angles to the floor beams in the field.

Let us now turn our attention to the splice. The most satisfactory way of designing such splices is to draw in what looks to be about correct, and then determine the stress on the rivets assumed, and make whatever modifications are necessary. So let us assume we need three rows of rivets on each side of the splice as shown. First suppose that the $\frac{3}{8}''$ plates extending over the flange angles are omitted and that there is just a single $\frac{3}{8}''$ splice plate on each side of the web.

Now, for the vertical force on each rivet due to the vertical shear we have

$$v=\frac{137{,}200}{18}=7{,}620 \text{ lbs.},$$

and let s be the maximum horizontal force on each of the rivets farthest out from the center of the web due to cross bending. Then, according to Art. 117, we have the equation

$$2\times 3\ (\overline{1.75}^2+\overline{5.25}^2+\overline{8.5}^2)\times s\div 8.5=1/8 \text{ area of web}$$

multiplied by $16{,}000\times h$ $(=2\times 16{,}000\times 29)=928{,}000$ in. lbs., from which we obtain

$$s=\frac{928{,}000}{72.6}=12{,}800 \text{ lbs.}$$

This horizontal force should not exceed $(\frac{7}{8}\times 24{,}000\times \frac{1}{2})17/25.5 = 7{,}000$ lbs., so the splice as just considered would not be sufficient.

As one more line of rivets on each side of the splice will not be sufficient, it is obvious that it will be best to use the $\frac{3}{8}''$ plates extending over the flange angles as shown. So, considering the splice as it is shown in Fig. 205, we have

$$2\left[3(\overline{1.75}^2+\overline{5.25}^2+\overline{8.5}^2)+2\ (\overline{11.6}^2\ +\overline{13.75}^2)\right]\frac{s'}{13.75}=928{,}000 \text{ in. lbs.},$$

from which we obtain

$$s'=\frac{928{,}000}{139.0}=6{,}700 \text{ lbs. (about)}$$

for the horizontal force on the outer rivets (which are those in the outer gauge lines in the flange angles). These rivets are so near the top and bottom edge of the beam that the vertical shear on them can be neglected (see Art. 61).

The above 6,700# force can be considered as applied to the rivets by the web and transmitted from there to the angles and on to the $\frac{3}{8}''$ plates producing a shear of 3,330# on each rivet at each $\frac{3}{8}''$ plate. The flange increment can be considered as being transmitted by the $\frac{3}{8}''$ plates and web combined. Taking the flange increment as 10.500# (allowable bearing of a $\frac{7}{8}''$ rivet on the $\frac{1}{2}''$ web) and assuming that one-third is transmitted to the flange angles by the web and one-third by each of the $\frac{3}{8}''$ plates, we have

$$\frac{10{,}500}{3}+3{,}330=6{,}830 \text{ lbs.}$$

for the shear on each rivet at each $\frac{3}{8}''$ plate and

$$\frac{10{,}500}{3} + 6{,}700 = 10{,}200 \text{ lbs.}$$

for the bearing stress on the web. Now, as 7,200# is allowed in the former case and 10,500# in the latter, the top and bottom part of the splice is about correct.

The rivets connecting the splice plates to the web directly, not passing through the flange angles, are in double shear and bearing on the web. The allowable bearing on the web, which is 10,500 lbs., is less than double shear, and hence the maximum stress on these rivets results from the bearing on the web. Now, as a test for these rivets, let us consider the ones farthest out from the center of the web, which are $8\frac{1}{2}''$ out. Let s'' be the horizontal force on each. Then we have

$$2\left[3\left(\overline{1.75}^2 + \overline{5.25}^2 + \overline{8.5}^2\right) + 2\left(\overline{11.6}^2 + \overline{13.75}^2\right)\right]\frac{s''}{8.5} = 928{,}000 \text{ in. lbs.,}$$

from which we obtain

$$s'' = \frac{928{,}000}{224} = 1{,}140 \text{ lbs. (about)}$$

(this could be as great as 7,000 lbs.).

Then for the resultant force on each of these rivets we have

$$R' = \sqrt{1{,}110^2 + 7{,}620^2} = 8{,}670 \text{ lbs.,}$$

which is less than the allowable bearing of each on the $\frac{1}{2}''$ web, and hence the splice as shown is amply strong.

The splice, as shown in Fig. 205, is designed to develop the strength of the beam (as should be done), while as a matter of fact the flange at the point of splice is practically twice as strong as need be, as the bending moment at that point is only about one-half the maximum on the beam and if the splice were designed to carry only the shear no actual weakness would be detected in it.

This completes the necessary calculation in connection with the drawing of the general cross-section shown in Fig. 205. Next a sketch to $1\frac{1}{2}''$ scale for each of the lateral connections, and also a sketch (to a large scale) of the curved end of the main girder, should be made. After these preliminary sketches are made, we can proceed with the making of either the general drawing (Fig. 204) or the shop drawings (Figs. 202 and 203). We simply transfer the dimensions on these sketches to the final drawings.

Most of the calculations for the details shown on these drawings are quite similar to those given in Art. 135 for the 50-ft. deck plate girder. The main difference is in the determination of the flange rivets in the main girders. In the case of the through girder, here considered, the loads are applied directly to the web of the main girders (by the floor beams) instead of to the flanges as in the case of the deck girder treated in Art. 135.

So Formula (2) of Art. 116 is used to determine the pitch of the flange rivets instead of Formula (1). The shear in each panel is practically constant throughout the panel, and consequently the rivet spacing should be constant throughout each respective panel. Taking the case of the end panel, we have

$S = 180{,}000\#$ (maximum end shear);
$r = 7{,}880\#$ (= allowable bearing of a $\frac{7}{8}''$ rivet on the $\frac{3}{8}''$ web);
$h = 77''$.

Now substituting these values in Formula (2) (Art. 116), we have

$$p = \frac{7{,}880 \times 77}{180{,}000} = 3.36 \text{ ins., say, } 3\tfrac{1}{4} \text{ ins.,}$$

for the theoretical pitch of the flange rivets in the end panels. The shear on the girder is reduced some by the weight of the girder itself, as we pass from the end toward the center, and, consequently, the pitch can be increased slightly as the first intermediate floor beam is approached, as is shown in Fig. 202.

The dead-load shear in the second panel from the end, as seen from Fig. 194, Art. 150, is

$$20{,}025 \quad 13{,}350 = 6{,}670 \text{ lbs.}$$

The maximum live-load shear occurs in the second panel when the wheel loads are in the position shown in Fig. 206, as the criterion of Art. 90 is satisfied.

Taking moments about the end B of the span (Fig. 206) we have (using Table A)

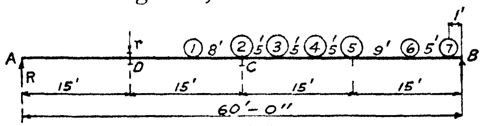


Fig 206

$$R = \frac{2{,}155{,}000 + 116{,}000 \times 1}{60} = 37{,}900 \text{ lbs.}$$

for the reaction at A. Then taking moments about the floor beam at C we have

$$r = \frac{80{,}000}{15} = 5{,}330 \text{ lbs.}$$

for the stringer concentration on the floor beam at D. Then we have $R - r = 37{,}900 - 5{,}330 = 32{,}570$ lbs. for the live-load shear in the second panel from the end of the span due to Cooper's $E40$; and for Cooper's $E50$ we have

$$32{,}570 \times \frac{50}{40} = 40{,}700 \text{ lbs.,}$$

and for impact we have

$$I = 40{,}700\left(\frac{300}{30 + 300}\right) = 37{,}000 \text{ lbs.}$$

Now adding the above dead- and live-load shears and the impact together, we have

$$6{,}670 + 40{,}700 + 37{,}000 = 84{,}370 \text{ lbs.}$$

for the total maximum shear in the second panel from the end of the span.

Then using Formula (2) (Art. 116) we have

$$p = \frac{7{,}880 \times 74}{84{,}370} = 6.9 \text{ ins. (about)}$$

for the theoretical spacing of the flange rivets in the main girders throughout the second panel from the end of the span. So 6″, the maximum spacing allowed, will be used.

Further analyses of the details of the above span are considered unnecessary, for if the details of the deck plate girder spans previously treated are thoroughly understood, the student should have little trouble in making the detail drawings for through plate girder spans, and especially if the drawings shown in Figs. 202, 203, and 204 be observed closely as the work progresses. The shop bills for this work are practically the same as for deck spans.

154. General Remarks.—The general discussion given above, regarding the design of flanges and end bearings for deck plate girder bridges, holds in the case of through plate girder bridges.

The floor beams for through plate girder bridges are sometimes made as shown in Fig. 207. This type of detail eliminates the web splice in the

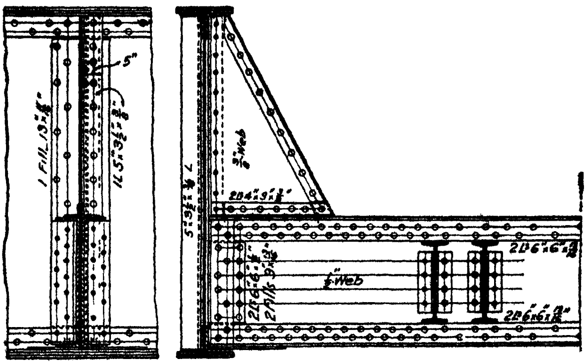


Fig 207

floor beam. The brackets are usually shipped loose and riveted to the floor beams and main girders in the field, especially when the main girders are quite deep. While this type simplifies the details of the floor beam, it complicates the details of the main girders and there are considerably more field rivets than there are in the type used above for the 60-ft. span. The details of the two types vary considerably in practice, and the details given here are intended only as a fair example.

The stringers for through plate girder bridges are sometimes plate girders (where the under clearance will permit), in which case two stringers per track are used.

155. Graphical Determination of Live-Load Shears and Bending Moments on Main Girders of Through Plate Girder Bridges.—The "influence line" method, outlined in Arts. 100 and 101, is the most convenient graphical method in the case of through plate girder bridges. However, the equilibrium polygon can be used if one so desires.

As an illustration, let it be required to determine the maximum reaction, end shear, and bending moment on a 75-ft. single-track through plate girder span of five 15-ft. panels, due to Cooper's $E50$ loading. (See diagram, Fig. 151.)

To determine the maximum reaction, lay off the span AB (Fig. 208) to, say, $\frac{1}{4}''$ scale, and place the loads as shown.

Then draw the base line ab and lay off $ac = 1'' = 1\#$ and draw the influence line cb, and draw the ordinates 1, 2, . . . 13 under the loads, as shown, and all is ready for determining the reaction. Take a pair of dividers and step off the ordinates 1 to 8 as explained in Example 1, Art. 100, and multiply their sum, in inches, by 25,000#. Next, in the same way, step off the ordinates 9 to 12 and multiply their sum by 16,250#

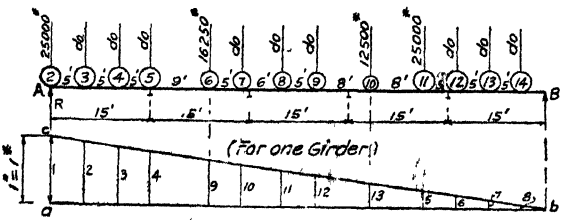


Fig. 208

Then multiply the length of ordinate 13 by 12,500 and add all three of the results together, and the desired reaction is thus obtained.

To determine the maximum shear in the end panel AD (Fig. 209), place the wheels as shown, thus satisfying the criterion for maximum shear (Art. 90). Then draw the reaction influence line cb for the span and cd for the panel AD. Then by multiplying the sum of the ordinates (obtained as explained above) ef, 1, 2, 3, 4, and 5 by 25,000#, 6 to 9 by 16,250#, and h and 10 by 12,500#, and adding these products together, the maximum shear in the end panel AD is obtained.

To determine the maximum shear in any of the intermediate panels

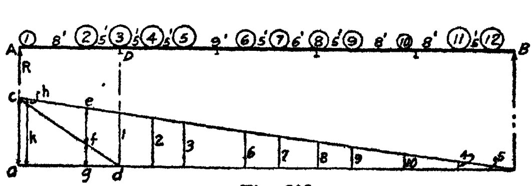


Fig. 209

as DE (Fig. 210), first draw the reaction influence lines eb and af and then the influence line $adcb$ for shear in the panel, as explained for trusses in Art. 102, and place the wheels as shown for maximum shear in the panel, as per Art. 90. Then by multiplying the sum of the ordinates 2 to 5 by 25,000#, 6 to 9 by 16,250#, and ordinate 1 by 12,500#, and adding the products together, the maximum shear in the panel DE is obtained.

It should be noted in the last case that the position of the wheels for maximum shear in the panel can be determined directly from the influence line. For, according to Art. 102, no load should pass the point O (Fig. 210), and there must be a load at E, so by placing wheel 1 as near O as possible, with a wheel at E, we have the position of the wheels for maximum shear in the panel. The position of the wheels for maximum shear in any of the intermediate panels can be determined in this manner.

The maximum bending moment will undoubtedly occur at the floor beam nearest the center of the span, and, as there are two floor beams

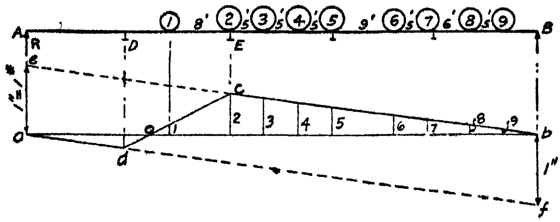


Fig. 210

equally near the center, the location of either beam can be taken as the point of maximum moment. So, if *AB* (Fig. 211) represents the span, either *C* or *D* can be taken as the point of maximum bending moment. Let us take point *C*. Then to determine the maximum moment, place the live load (in reference to *C*) as shown, thus satisfying the criterion for maximum moment given for trusses in Art. 91. Next construct the influence line *aOb* for the bending moment at *C* as explained in Art. 101, and also in Art. 102. Then by multiplying the sum of the ordinates 1 to 4 by 25,000#, 5 to 9 by 16,250#, 10 by 12,500#, and one-half of 11 by

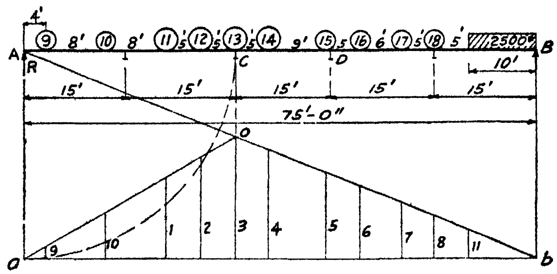


Fig 211

(2,500# × 10′) and adding these products together, we obtain the maximum bending moment on the main girders.

To determine the maximum shear in any panel, as *CD* (Fig. 212), by means of an equilibrium polygon, first place the load for maximum shear in the panel, as shown, and draw the ray diagram *MNO*, and the corresponding equilibrium polygon *a-c-b . . . a*. Then by drawing *OE*, in the ray diagram, parallel to the closing line *ac* we have the reaction (*R*) at *A* due to the loads given by the line *EM*. The shear in panel *CD* is equal to *R* minus the stringer reaction (*r*) at *C* due to the load in panel *CD*. By drawing the closing line *ek* we have the equilibrium polygon *ekhe* for the panel *CD*. Then by drawing *E′O* (in the ray diagram) parallel to the closing line *ek* we have the stringer reaction *r*, at *C*, due to wheel 1 (as that is the only load in the panel), given by the line *E′M*. Then, evidently, the maximum shear in panel *CD* is given by the line *EE′*.

The maximum shear in the other panels can be determined in the same manner.

To determine the maximum bending moment on the main girders by means of an equilibrium polygon, first place the loads on the span for

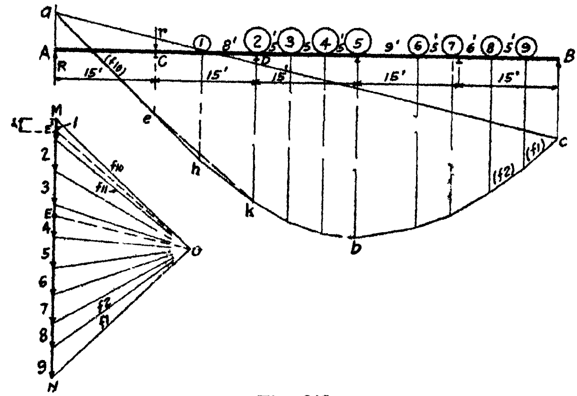


Fig 212

maximum moment at any panel point as C (Fig. 213), as explained above, and draw the ray diagram STP and the corresponding equilibrium polygon $acda$. Then the maximum moment at C is equal to the ordinate y multiplied by the pole distance H. H is measured in pounds to the same scale as used in drawing the ray diagram and y is measured in feet or inches to

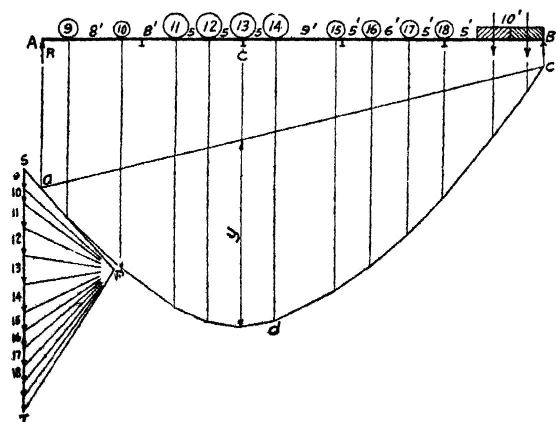


Fig 213

the same scale as used in drawing the span AB and in spacing the loads. If y is taken in feet the moment will be in foot pounds, and if taken in inches the moment will be in inch pounds.

In case an equilibrium polygon similar to the one shown in Fig. 186 (Art. 144) be drawn the shears and bending moments for any number of through plate girder spans can be determined from it.

The shear would be determined as shown in Fig. 212, the wheels being placed for maximum shear as per criterion, Art. 90. The maximum moment at any panel point C can, however, be determined without reference to the criterion for maximum bending moment, very much the same as the case of deck spans treated in Art. 144, the main difference being that the moment here is for a certain point on the girder.

As an illustration let UVW (Fig. 214) represent an equilibrium polygon, similar to the one shown in Fig. 186. We start, say, by first placing the point C, the panel point where the moment is desired, under

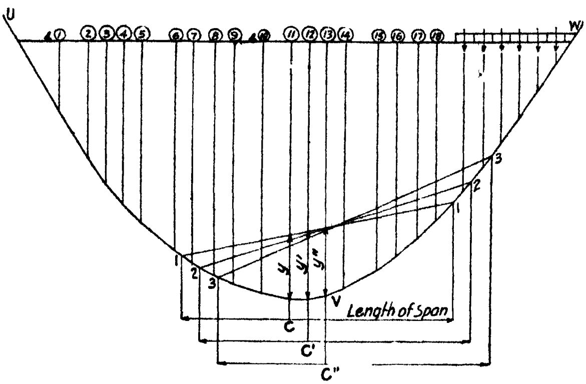


Fig 214

wheel 11 and drawing the closing line 1-1, we obtain the ordinate y. Next placing the point C under wheel 12 and drawing the closing line 2-2 we obtain the ordinate y', and placing C under wheel 13 and drawing the closing line 3-3 we obtain the ordinate y'', and so on. In this manner the maximum ordinate for point C is obtained by trial, and by multiplying this maximum ordinate by the pole distance the maximum bending moment is obtained.

The maximum bending moment at any panel point can be obtained in this manner.

156. Plate Girder Bridges with Solid Floors.—Plate girder bridges, especially through spans, sometimes have solid floors, particularly those over streets in cities where an ordinary open floor is undesirable owing to the dripping of water from the floor upon the sidewalk and street below due to rain and melting snow.

These structures, in reference to floor, taking the most common construction, can be divided into two general types: those having solid metal floors covered with concrete and ballast, and those having reinforced concrete floors covered with ballast. The most common of the solid metal floors is that known as the trough floor shown in Fig. 215 where the floor proper is made up of vertical and horizontal plates connected by angles so as to form successive troughs. In designing such bridges the first thing to do is to draw a longitudinal section of the floor, enough to include three ties 18″ on centers, as shown to a large scale in Fig. 215. At the start the whole thing is an assumption, that is, we draw in what looks to be correct.

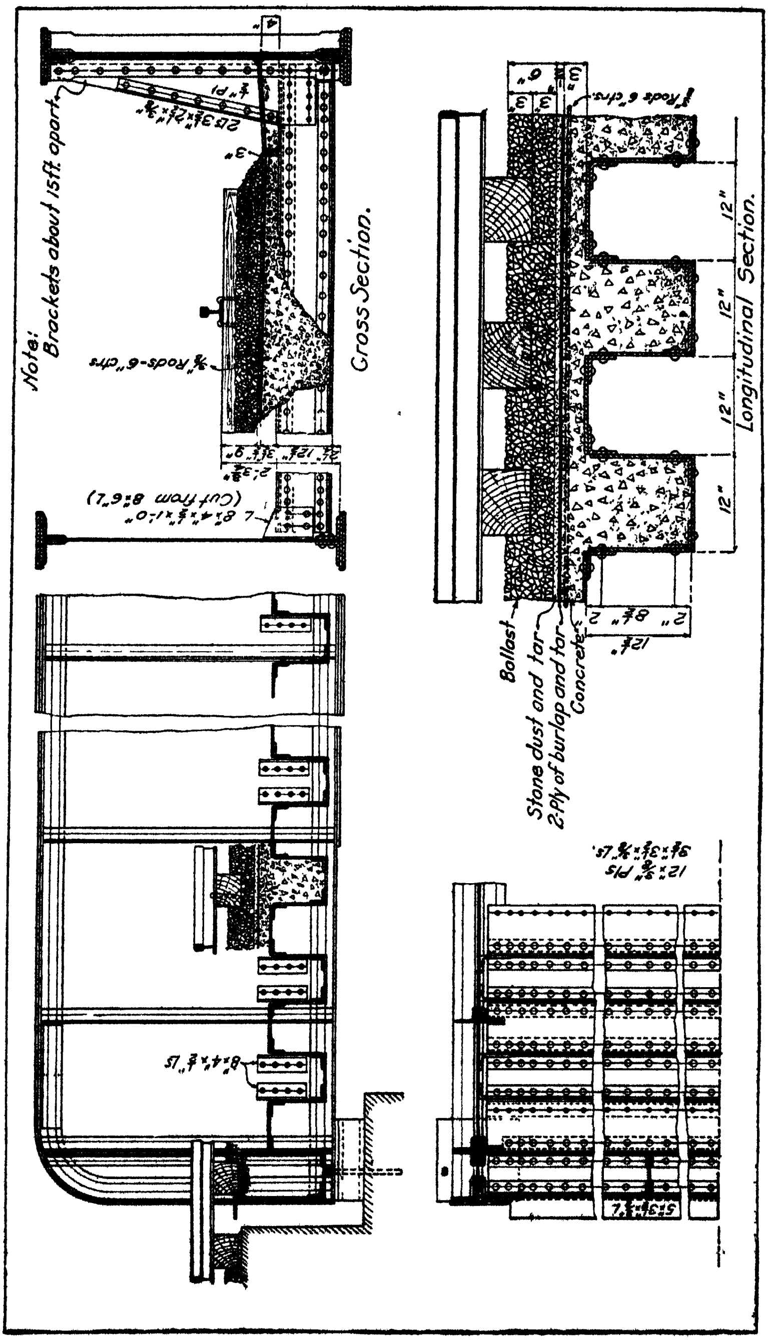

Note:
Brackets about 15ft. apart.
Cross Section.
Longitudinal Section.
Ballast
Stone dust and tar
2-Ply of burlap and tar
Concrete
12"
12"
12"
12"

Fig. 215

From this sketch we determine the dead weight of the floor, including metal, concrete, ballast and track. Then we compute the dead- and live-load stresses on this assumed floor and if necessary modify the design until it agrees with the final computation.

As an example let it be required to design the through plate girder span indicated in Fig. 215, using Cooper's $E50$ loading. Estimating from the longitudinal section of the floor (Fig. 215), taking concrete to weigh 150# and ballast 125# per cu. ft., we have for a strip through the floor one foot wide and two feet long, along the track, the following weights:

$1\frac{1}{2}$ cu. ft. of concrete at 150#......	225 lbs.
1 cu. ft. of ballast at 125#........	125 lbs.
Tie (6 ft. B. M.) at $6 \times 4\frac{1}{2}$#......	27 lbs.
$3\frac{1}{2}$ ft. of $12'' \times \frac{3}{8}''$ plates at 15.3#..	53 lbs.
4 ft. $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ Ls at 8.5#........	34 lbs.
Rails (90#) and spikes..........	12 lbs.
Tar, stone dust, and burlap.......	20 lbs.
Total......................	496 lbs.

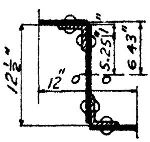


Fig. 216

for two square feet of floor, or 248# for each square foot (horizontal projection). The floor can be considered as being made up of independent Z-shape sections, each composed of one vertical plate, two angles and two halves of horizontal plates as indicated in Fig. 216. This section can be treated as an independent transverse beam. For the moment of inertia of this beam with reference to axis O-O, we have

$2 \times \overline{6.43}^2 \times 6'' \times \frac{3}{8}''$............	= 186	(hor. pls.)
$\frac{1}{12} \times \frac{3}{8} \times \overline{12}^3$....................	= 54	(vert. pl.)
$2\,(\overline{5.25}^2 \times 5 + 2.87)$.............	= 281	(angles)
	521	
$-(1.5\square'' \times \overline{6.25}^2)$ (rivets staggered)..	-60	
Total.....................	461	

If the main girders are 15′-6″ apart the dead load on our Z-beam will really be as indicated in Fig. 217, as the 248-lb. load computed above holds only for the part of the floor directly under the track. But as the final result will not be affected materially if we assume the 248 lbs. to extend the full length of the beam, we shall so consider it.

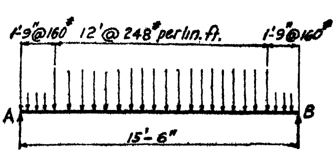


Fig 217

Then for the maximum bending moment due to dead load we have

$$M = \frac{1}{8} \times 248 \times \overline{15.5}^2 \times 12 = 89{,}370 \text{ inch lbs.}$$

The next thing is to determine the maximum bending moment due to live load. In accordance with the usual practice we will assume the maximum live load in this case to be the heaviest axle, distributed over three ties. So we have (special load, see diagram, Fig. 151)

$$\frac{62,500}{3} = 20,833 \text{ lbs.}$$

for the total load on each tie. Now from the longitudinal section of the floor (Fig. 215) we can see that our Z-beams carry from half of this load to practically all of it. So to be sure we will assume that each carries the 20,833 lbs. uniformly distributed along the beam by the tie.

Then for the maximum bending moment due to live load we have

$$M' = \tfrac{1}{8} \times 20,833 \times 15.5 \times 12 = 484,300 \text{ inch lbs.}$$

Now adding together the above dead- and live-load moments and allowing 100 per cent for impact we have

$$M'' = 89,370 + 2(484,300) = 1,057,970 \text{ inch lbs.}$$

for the total maximum bending moment on our Z-beam.

Then substituting the above values in Formula (D), (Art. 53), we have

$$f = \frac{1,057,970 \times 6.6}{161} = 15,100 \text{ lbs.}$$

for the maximum bending stress on our Z-beam which is the maximum bending stress on the floor. This shows that the floor is but little heavier than necessary for cross bending, 16,000 lbs. being the allowable bending stress.

For the end shear on our Z-beam we have

$$248 \times \frac{15.5}{2} = 1,920 \text{ lbs.}$$

from dead load and

$$\frac{20,833}{2} = 10,416 \text{ lbs.}$$

from live load and the same from impact, making 22,700 lbs. in all.

Then to resist this shear we should have

$$\frac{22,700}{10,000} = 2.27 \text{ sq. ins.}$$

of cross-section in the vertical plate of our Z-beam. This shows that the shear on the floor is amply provided for, as each vertical plate contains 1.5 sq. ins.

The above calculations show that the floor shown in Fig. 215 is about as correct as we can design it, as $\frac{3}{8}''$ metal is the minimum thickness allowed.

Having the floor designed, the dead weight coming onto the main girders from the floor can be computed and by adding this per foot of girder to the assumed weight of girder itself (per ft.) we obtain the dead load for determining the dead-load stress in the main girders. Then, using the formula $pL^2/8$, the maximum dead-load bending moment on the main girders can be determined.

To obtain the maximum live-load shear and bending moment on the main girders the live load is placed just the same as in the case of deck spans.

The trough floor shown in Fig. 215, while it is the most common type and a good design, is only one of several types of solid floors in use. For example, there is the buckle plate floor composed of buckle plates (see manufacturers' handbooks) laid upon I-beam or plate girder stringers, flat plates laid upon I-beams used either as stringers or transverse beams, the rolled trough section, etc. In addition to these types there are the I-beam and concrete floors shown in Fig. 218, in which case the I-beams are designed to carry all of the load. All of these types are used mostly

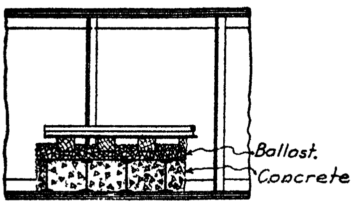


Fig. 218

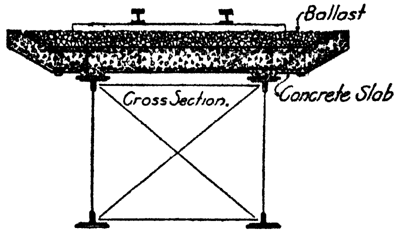


Fig 219

for through plate girder bridges. The trough floor is sometimes used on deck spans, in which case the troughs are placed transversely resting directly upon the main girders.

The out and out reinforced concrete floors are used on deck plate girders. These floors are simple concrete slabs resting directly upon the top of the main girders and turned up at each end so as to hold the ballast, as shown in Fig. 219.

In designing all plate girder bridges having solid floors, the first thing to do is to design the floor and determine its weight. Then the weight of the main girders can be assumed and added to the weight of the floor and we have the dead weight for determining the dead-load stresses in the main girders. The other work involved is practically the same as given above for ordinary bridges.

157. Double-Track Spans.—Double-track deck plate girder bridges are practically always composed of two simple single-track spans placed side by side, as previously stated. Double-track through spans are usually composed of two main girders placed far enough apart to admit the two parallel tracks 13-ft. centers. In the case of ordinary open floors there are usually four lines of stringers. The load in that case on each stringer is just the same as in the case of single-track spans, while the floor beams have four equal concentrations, each concentration being the same as for a single-track span. The dead load on the main girders, as stated in Art. 124, is about 70 per cent more than for single-track spans, while the live load is just twice as much, two trains being considered as moving abreast over the structure. In the case of double-track through plate girder bridges composed of three main girders, there are two independent single-track floor systems. The loads in that case carried by each floor and by each outside main girder are the same as for a single-track span, while the central girder carries practically twice as much as each of the outside girders. The objection to the three-girder type is that it is usually necessary to "spread" the tracks at the location of these bridges in order to obtain the necessary side clearance.

DRAWING ROOM EXERCISE NO. 5

Design a 64-ft. single-track through plate girder bridge and make a stress sheet for same upon a 24" x 18" sheet and tracing of same.

Length of span = 4 panels at 16'-0" = 64'-0" c.c. end bearings
Width = 15'-8" c.c. girders.
Live load, Cooper's $E50$ loading.
Dead load, to be assumed by student.
Specifications, A. R. E. Ass'n.

The required work consists of making the calculations and the drawing of a stress sheet for the span, similar to the one shown in Fig. 201 for the 60-ft. span.

VIADUCTS

158. Preliminary.—The ordinary railroad viaduct is a bridge composed of a series of deck plate girder spans supported upon towers. However, as a rule any bridge supported upon towers is classed as a viaduct. Viaducts are used where the crossings are so deep that ordinary concrete or masonry piers are impracticable as to cost. The most common type of viaduct is shown in Fig. 220(a). The type shown in Fig. 220(b) is built to some extent. The principal difference in the two types is in the

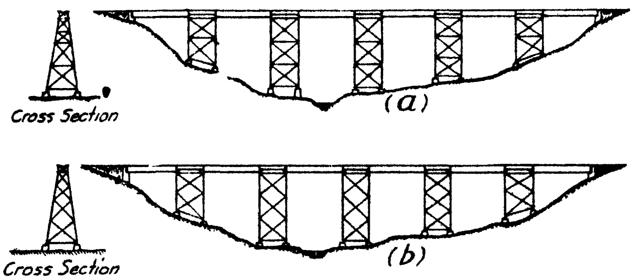


Fig. 220

tower bracing. Two columns connected together transversely by bracing constitutes what is known as a bent (see Fig. 221). Two bents connected together by longitudinal bracing constitutes what is known as a tower. It is usual practice to make the spans between the towers twice as long as the tower spans, except where the height of the viaduct is 40 ft. and under, in which case the spans are usually made equal in length.

From actual calculations the economic lengths of spans are found for ordinary cases to be as follows: 30' and 30' for viaducts 30 to 40 ft. high; 30' and 60' for a height of 40 to 80 ft.; and 40' and 80' for a height of 80 ft. and over.

The cost of erection governs the lengths of spans to quite an extent.

The approximate weight of metal in towers of ordinary single-track viaducts is given on the diagram in Fig. 222. This diagram is self-explanatory.

Complete Design of an Ordinary Single-Track Viaduct

159. Data.—Specifications, A. R. E. Ass'n.

Live Load, Cooper's $E50$.

160. General Layout of Structure.—The first thing the designer needs is a good profile of the crossing, which is usually furnished by the railroad company. Let the profile shown in Fig. 223 be such a profile of the crossing for which we are to design a viaduct.

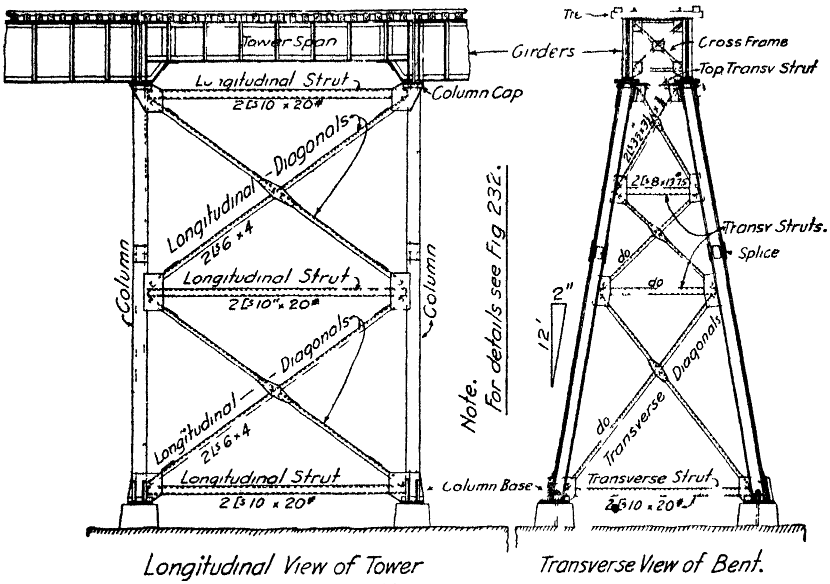


Fig 221

Usually (the first thing) we would redraw this profile carefully to a convenient scale (as a rule to a $\frac{1}{20}$ scale) so that it can be traced on the general stress sheet for the structure. Then upon this profile the positions of the towers are chosen, avoiding the stream and obtaining as many towers and columns of equal length as possible, using the length of spans that is economic for the height of the viaduct. Working in this manner we obtain the general layout of our structure as shown at the top of the general stress sheet, Fig. 224. After this preliminary work we start the detail design by taking up the spans first.

161. Designing of the Spans.—This work is just the same practically as previously outlined for deck plate girder bridges, except the 50-ft. spans in this case are made the same depth as the 60-ft. spans in order to simplify construction, the 60-ft. spans being of economic depth. The complete stress diagram, or stress sheet, as we might say, for the three different lengths of spans is given on the general stress sheet (Fig. 224).

162. Designing of the Columns and Tower Bracing.—In designing a column the first thing to do is to determine the maximum concentrations on the top of it due to dead and live load. The total dead load applied to the top of each column is equal to the sum of the dead-load reactions on the two girders supported and the greatest live load applied

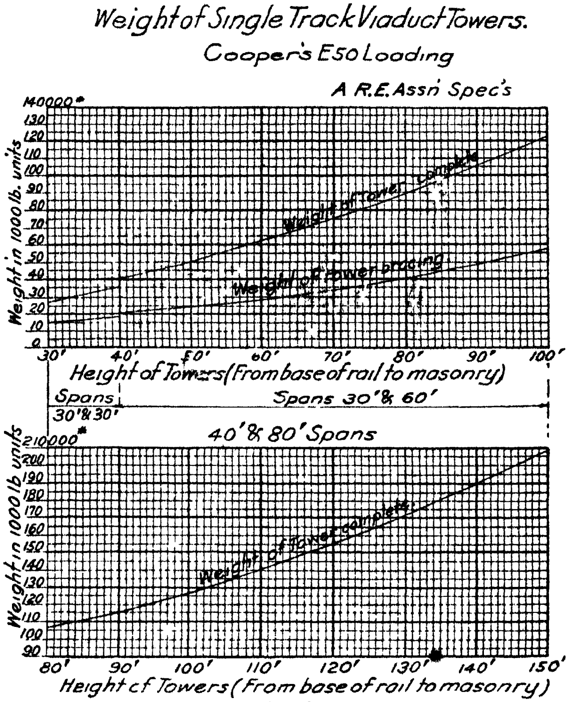


Fig 222

to the top of each is the maximum live-load concentration coming from the live load in the two adjacent spans, just the same as in the case of an intermediate floor beam. (Art. 148.) These loads are the same for all columns in bents 1 to 9 (Fig. 224), as each supports an end of a 30- and

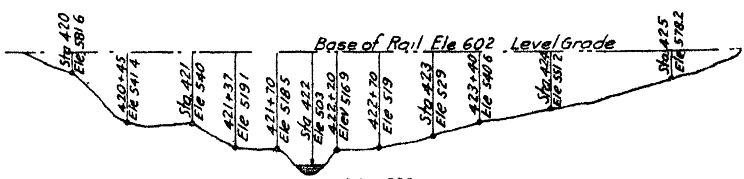


Fig 223

60-ft. span. The load on the columns in bent 10 is that coming from a 30- and 50-ft. span and from the two 50-ft. spans in the case of bent 11. Using the dead-load reactions given for the girders on the stress sheet, Fig. 224, we have

$$19{,}000+6{,}800=25{,}800 \text{ lbs.}$$

for the dead-load concentration on all columns supporting 30-ft. and

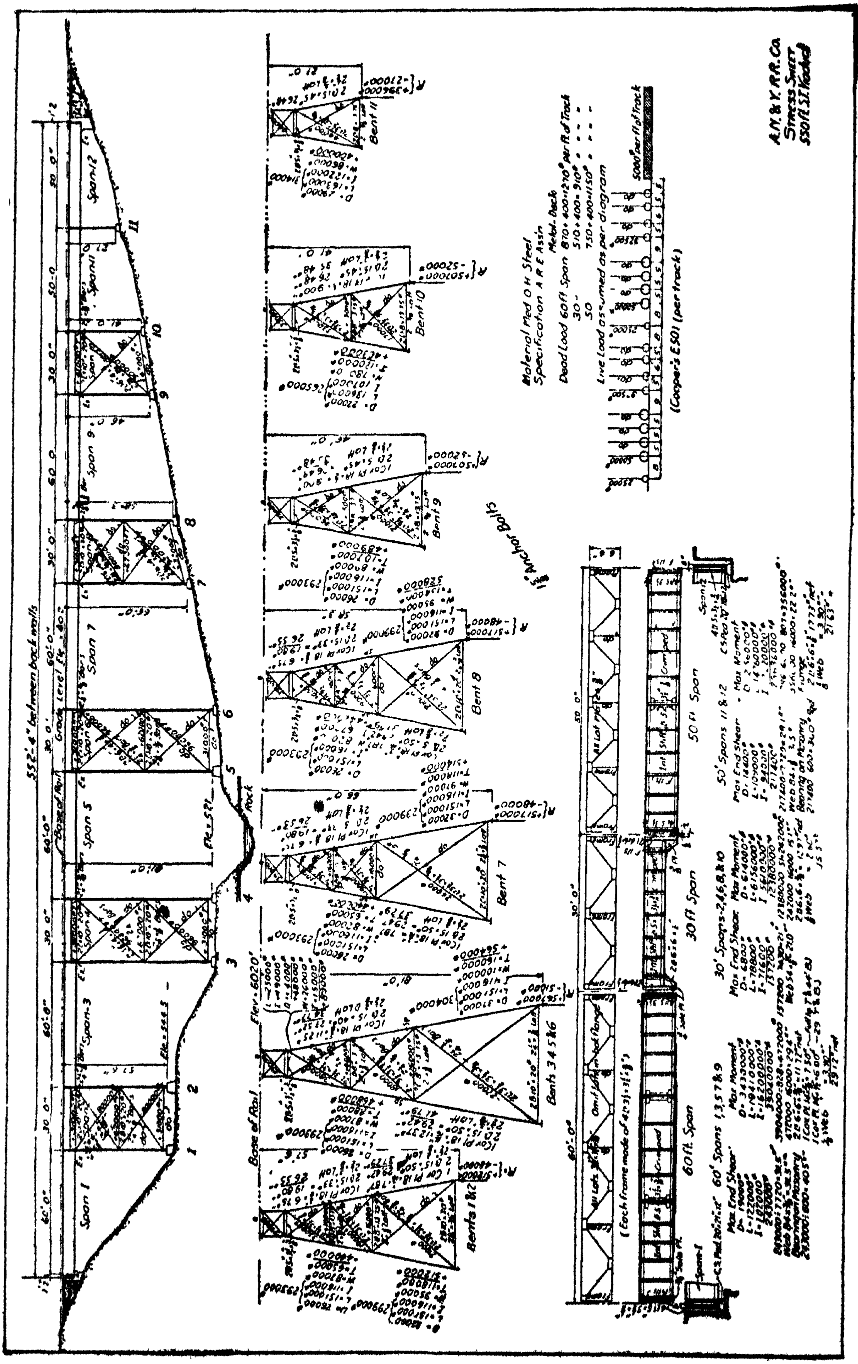

A.N. & Y. R.R. Co.
Stress Sheet
550 ft. St. Viaduct
552'-4" between back walls
Anchor Bolts
50 ft Span
30 ft Span
60 ft. Span
(Cooper's E.50) (per track)

Fig 224

60-ft. spans, which includes the columns in bents 1 to 9, and

$$6{,}800 + 14{,}400 = 21{,}200 \text{ lbs.}$$

for columns in bent 10 and

$$14{,}400 \times 2 = 28{,}800 \text{ lbs.}$$

for those in bent 11.

To obtain the maximum live-load concentration on a column we place the loading so that the heaviest loads (wheels) are near the column with one load at the column and the unit load in one adjacent span equal to

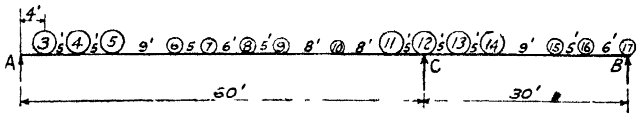


Fig. 225

the unit load in the other, which is in accordance with Art. 148; the spans in this case being of unequal length.

Taking first the columns supporting 30-ft. and 60-ft. spans, let C (Fig. 225) represent the column considered. By placing the loads as shown in Fig. 225, we have (see Table A) (considering one-half of wheel 12 in each span)

$$\frac{152{,}000}{60} = 2{,}533 \text{ lbs.}$$

for the average unit load in the 60-ft. span, and

$$\frac{76{,}000}{30} = 2{,}533 \text{ lbs.}$$

for the average unit load in the 30-ft. span.

This shows that the criterion is exactly satisfied, and we need go no farther with the work of satisfying the criterion. Taking moments about A (using Table A) we have

$$\frac{(3{,}154 + 142 \times 4)1{,}000}{60} = 62{,}000 \text{ lbs.}$$

for the reaction on the column at C due to loads 3 to 11 (inclusive), and taking moments about B we have

$$\frac{1{,}121 \times 1{,}000}{30} = 37{,}300 \text{ lbs.}$$

for the reaction on the column at C due to wheels 13 to 16 (inclusive). Now adding these two reactions and the weight of wheel 12 together we have

$$62{,}000 + 37{,}300 + 20{,}000 = 119{,}300 \text{ lbs.}$$

for the maximum live-load concentration on the column for Cooper's $E40$ loading and multiplying this by 50/40 we have

$$119{,}300 \times \frac{50}{40} = 149{,}000 \text{ lbs.}$$

for the maximum live-load concentration due to the $E50$ loading which is the concentration desired.

Then for the impact we have

$$149{,}000 \times \left(\frac{300}{90+300}\right) = 115{,}000 \text{ lbs.}$$

Now owing to the columns sloping transversely, known as the batter, the actual stress in the columns due to the above concentrations is equal to the concentrations multiplied by the secant of the slope angle. The slope, or batter, will be taken as 2 in 12, which is the usual batter for such columns. So we have 1.014 for the secant of the slope angle. Then we have

$$25{,}800 \times 1.014 = 26{,}000 \text{ lbs.}$$

in bents 1 to 9 (inclusive) for the dead-load stress in the top portion of the columns, and

$$149{,}000 \times 1.014 = 151{,}000 \text{ lbs.}$$

for the live-load stress in these columns throughout their whole length and

$$115{,}000 \times 1.014 = 116{,}000 \text{ lbs.}$$

for the impact.

Now adding these together we have

$$26{,}000 + 151{,}000 + 116{,}000 = 293{,}000 \text{ lbs.}$$

for the total maximum stress in the top sections of all columns in bents 1 to 9 (inclusive).

We will next determine the dead- and live-load stresses in the columns in bent 10. The dead-load concentration on each of these

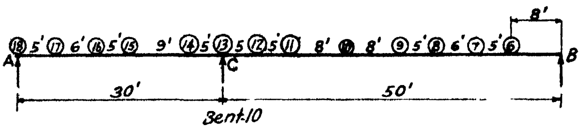


Fig. 226

columns is given above as 21,200 lbs., and we will proceed to determine the live-load concentration on them. Placing the loads as shown in Fig. 226 we have (considering one-half of wheel 13 in each span)

$$\frac{112{,}000}{50} = 2{,}240 \text{ lbs.}$$

for the average unit load in the 50-ft. span, and

$$\frac{69{,}000}{30} = 2{,}300 \text{ lbs.}$$

for the average unit load in the 30-ft. span.

This shows that the load in the 30-ft. span is a little too great. So let us place wheel 14 at column C. Then we have

$$\frac{132{,}000}{50} = 2{,}640 \text{ lbs.}$$

for the average unit load in the 50-ft. span, and

$$\frac{62,000}{30} = 2,066 \text{ lbs.}$$

for the average unit load in the 30-ft. span. So it is seen that the position of the wheels shown in Fig. 226 will give the maximum concentration on the column. So taking moments about A (using Table A) we have

$$\frac{916 \times 1,000}{30} = 30,500 \text{ lbs.}$$

for the reaction on the column at C due to loads 17 to 14 (inclusive), and taking moments about B we have

$$\frac{(2,036 + 102 \times 8)1,000}{50} = 57,000 \text{ lbs.}$$

for the reaction due to wheels 6 to 12 (inclusive).

Now adding these reactions and the weight of wheel 13 together we have

$$30,500 + 57,000 + 20,000 = 107,500 \text{ lbs.}$$

for the maximum live-load concentration on each column in bent 10 due to the $E40$ loading. Then for the $E50$ loading we have

$$\frac{50}{40} \times 107,500 = 134,000 \text{ lbs.}$$

and for the impact we have

$$134,000 \times \left(\frac{300}{80 + 300}\right) = 105,000 \text{ lbs.}$$

Now multiplying the dead- and live-load concentrations and impact by the secant of the slope angle of the column, as was done above, and adding all three of the results together we obtain 264,000 lbs. for the

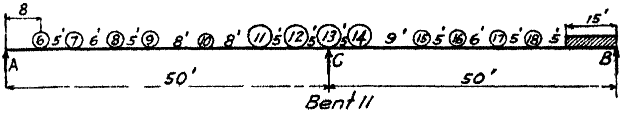


Fig. 227

total maximum stress in each of the columns in bent 10. Next placing the wheel loads as shown in Fig. 227 and proceeding in the same manner as above, in the case of the other columns, we obtain

$$128,600 \times \frac{50}{40} = 161,000 \text{ lbs.}$$

for the live-load concentration on each column in bent 11, and

$$161{,}000\times\left(\frac{300}{100+300}\right)=121{,}000 \text{ lbs.}$$

for the impact concentration.

Then, taking the dead-load concentration found above for these columns, we have

$$28{,}800\times 1.014=29{,}200 \text{ lbs.}$$

for the dead-load stress, and

$$161{,}000\times 1.014=163{,}000 \text{ lbs.}$$

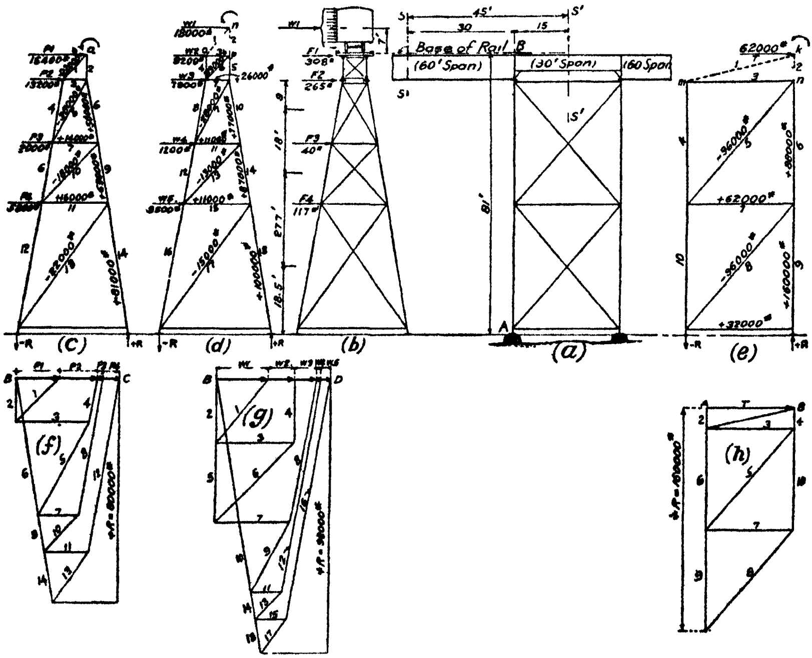


Fig. 228

for the live-load stress, and

$$121{,}000\times 1.014=122{,}000 \text{ lbs.}$$

for the impact, making in all 314,000 lbs. stress in each of the columns in bent 11.

Having the stresses in the columns this far determined we can now write the dead, live and impact stress on them at the cross-sections, as shown in Fig. 224, increasing the dead-load stress in the lower sections of the columns where the weight of the part of the tower above increases the stress an appreciable amount. The dead weight per ft. of height of towers 1-2 and 7-8 as seen from the diagram in Fig. 222 is about 1,000 lbs. So the weight per ft. of column would be 250 lbs. Then the dead-load stress in the lower portion (about one-half of the height) of the columns in these towers will be increased (in the worst case) about 250 × 28 = 7,000 lbs., about 11,000 lbs. in the case of towers 3-4 and 5-6. In tower 9-10 and bent 11 this increased dead weight is ignored as it is inappreciable.

We shall next determine the stresses in the columns and transverse bracing due to wind load. According to the specifications: "Viaduct towers shall be designed for a force of 50 lbs. per sq. ft. on one and one-half times the vertical projection of the structure unloaded; or 30 lbs. per sq. ft. on the same surface plus 400 lbs. per linear ft. of structure applied 7 ft. above the rail for assumed wind load on the train when the structure is fully loaded with empty cars assumed to weigh 1,200 lbs. per linear ft. of track."

Let us first consider the towers 3-4 and 5-6 (see Fig. 224), and let the diagram at (*a*), Fig. 228, represent the elevation of one of these towers drawn to, say, a $\frac{1}{10}$ scale. The wind load coming onto a single bent as *AB* would be that applied between the vertical sections *SS* and *S′S′*. This load will be considered (in accordance with practice) to be applied at the joints of the bents and on the girders and train as shown in the cross-section at (*b*), Fig. 228. To obtain the intensities of these forces it is first necessary to estimate the areas of the vertical projection of the structure at the different points. The area of the vertical projection of the train need not be considered as the load on it is given in the specification as 400 lbs. per ft. of track. However, the train is usually considered to be 10 ft. high.

For the area contributing to force *F*1 we have 1.5 × 45 = 67□′ in round numbers from ties and guard rail; 3.5 × 30 = 105□′ from the 60-ft. girder, and 2.25 × 15 = 33□′ from the 30-ft. tower girder, making in all 205□′ (= 67□′ + 105□′ + 33□′). (See Figs. 221 and 232.)

In the case of *F*2 we have the same area from the girders (= 105 + 33 = 138□′) and 9 ft. of tower. From the tower, we have 0.8 × 15 = 12□′ from the top longitudinal strut (see detail of tower, Fig. 232, also see Fig. 221), 0.5 × 24 = 12□′ from one-half of the longitudinal diagonal, and 1.5 × 9 = 14□′ from the column, making in all 176□′ (= 138□′ + 12□′ + 12□′ + 14□′).

For *F*3 we have 1.5 × 18 = 27□′ from the column (only).

For *F*4 we have 1.5 × 27.7 = 42□′ from the column; 0.8 × 15 = 12□′ from the longitudinal strut and 0.5 × 48 = 24□′ from the two longitudinal diagonals (one-half of each), making in all 78□′ (= 42□′ + 12□′ + 24□′).

Now by increasing each of the above areas by one-half (as per specification) we obtain the areas (to the nearest square feet) indicated at (*b*). The wind load on the lower part of the bent is transmitted directly to the masonry and hence is not considered.

Taking first the case of the 50-lb. load (when the train is not on the structure) we obtain the forces indicated at (*c*) by multiplying the given areas by 50. Then laying off these forces on the load line *BC*, at (*f*), we obtain the diagram of the stresses in the bent, as shown, by beginning at *a* and passing around each joint counter clock-wise. The intensities of the stresses thus obtained for the different members are given on the bent at (*c*). Next for the case of the 30-lb. load on the structure and the

400-lb. per linear ft. on train, we have the forces shown at (d). Those applied to the structure here are obtained by multiplying the corresponding areas given at (b) by 30 and the load on the train by multiplying 400 by 45′, the length of the train contributing pressure to bent AB. By drawing on and np we have the forces acting upon a continuous frame which can be graphically analyzed and the addition of the members on and np will not affect the stresses in the members of the bent in the least. Then by laying off the forces on the load line BD and beginning at joint n and passing around each joint counter clock-wise we obtain the diagram of the stresses shown at (g). The intensities of the stresses thus obtained for the different members are given on the bent at (d).

This work completes the determination of the wind stresses in the towers 3-4 and 5-6, and we shall next take up the determination of the stresses in these same towers due to the longitudinal force from the train. This force is known as traction. It is the force exerted along the rails when the brakes are applied to the wheels of a moving train causing the wheels to slide on the rails. The intensity of this force, as is evident, is

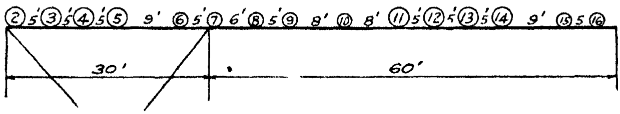


Fig. 229

equal to the coefficient of friction (of the wheels on the rails) times the vertical load on the wheels. This coefficient is designated in the specifications as 0.20. We will consider the traction on each of the towers, 3-4 and 5-6, as coming from the loads on the girders rigidly connected to the tower in each case, or, in other words, the loads between the expansion points of the girders. This, as is seen from Fig. 224 (the points of expansion being marked Ex), will be the loads on a 30-ft. and 60-ft. span, making in all 90 ft. of continuous load. The load will be a maximum when the wheels are in the position shown in Fig. 229. This gives a load of 248,000 lbs. (see Table A) for Cooper's $E40$ and 310,000 lbs. (= 248,000 × 50/40) for $E50$. Then for the traction force on each tower we have

$$T = 310{,}000 \times 0.2 = 62{,}000 \text{ lbs.},$$

which is applied at the top of the rail. By drawing mk and kn at the top of the tower as shown at (e) we have the load applied to a continuous frame which can be graphically analyzed. The diagram of stresses in the tower shown at (h), due to this 62,000-lb. force is obtained by laying off AB (representing the force) as a load line and then beginning at k and passing around each joint counter clock-wise. The intensities of the stresses thus obtained are given on the elevation of the tower at (e). The exact values of the stresses, however, are obtained by multiplying each of the given stresses by the secant of the slope angle of the columns, but as each stress is increased but a small amount we will ignore this correction (as is usual practice) and use the stresses shown.

We now have all of the stresses determined in the towers 3-4 and 5-6 except the dead, live, impact, and traction stresses in the top transverse struts (see Fig. 221) of the bents. These stresses are due to the columns being inclined. Owing to this inclination the columns exert a horizontal thrust toward each other upon this strut causing a compressive stress in it equal to the vertical load at the top of the column multiplied by the tangent of the slope angle of the columns. The slope of the columns is 2 in 12, so the tangent of the slope angle is 0.1666. Then for the top transverse strut we have the following stresses, in round numbers:

$$\left.\begin{aligned} D &= 25{,}800 \times 0.1666 = 4{,}000 \text{ lbs.} \\ L &= 149{,}000 \times 0.1666 = 25{,}000 \text{ lbs.} \\ I &= 115{,}000 \times 0.1666 = 19{,}000 \text{ lbs.} \end{aligned}\right\} +48{,}000 \text{ lbs.}$$

$$\begin{aligned} T &= 88{,}000 \times 0.1666 = 15{,}000 \text{ lbs.} \\ W &= 26{,}000 \text{ lbs.} \\ \hline & +89{,}000 \text{ lbs.} \end{aligned}$$

In order to design the column bases and anchorage it is necessary to know the positive and negative reactions on each column. The maximum positive reaction on each is equal to the maximum combined stress in the bottom section of the column divided by the secant of the slope angle of the column plus one-fourth of the weight of the lower half of the tower, which is about 11,000 lbs. So we have (see Fig. 224)

$$+R = 11{,}000 + \frac{564{,}000}{1.014} = +567{,}000 \text{ lbs.}$$

for the maximum positive reaction.

The maximum negative reaction, if there be such, will occur when the structure is loaded with a train of empty box cars which is assumed to weigh 1,200 lbs. per ft. of track, as per specification.

We have –98,000 lbs. for the negative reaction due to wind load as given at (*g*), Fig. 228. For the traction force along each rail due to the 1,200 lbs. live load we have

$$\frac{1{,}200}{2} \times 0.2 \times 90 = 10{,}800 \text{ lbs.}$$

Now this 10,800-lb. force can be applied to the top of the tower as was done with the 62,000-lb. force (at (*e*), Fig. 228) and the reaction due to it determined graphically. But as the reaction due to the 62,000-lb. force is already determined (at (*h*)), the one due to the 10,800-lb. force can be determined very readily by proportion as the reactions are directly proportional to the forces. Hence we have

$$-R' = \frac{10{,}800}{62{,}000} \times 160{,}000 = -28{,}000 \text{ lbs.}$$

for the negative reaction due to traction.

For the positive reaction due to the 1,200-lb. live load we have

$$+R'' = \frac{1{,}200}{2} \times 45 = +27{,}000 \text{ lbs.}$$

Now adding (algebraically) this and the dead-load reaction, which is

$$11{,}000 + \frac{37{,}000}{1.014} = 48{,}000 \text{ lbs.}$$

(see cross-section of bents, Fig. 224), to the above negative reactions we have

$$27{,}000 + 48{,}000 - 98{,}000 - 28{,}000 = -51{,}000 \text{ lbs.}$$

for the maximum negative reaction that can occur on each column and for which anchorage must be provided.

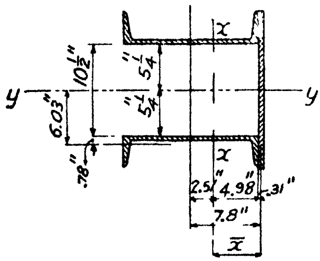


Fig. 230

This method of determining these negative reactions is not absolutely correct as the effect of the wind and traction forces being in different planes is not taken into account. The tendency of rotation of each tower, as is obvious, will really be about axes perpendicular to the resultant of the two kinds of forces. However, owing to the uncertainty of the intensities of the forces in the first place and to the resulting error being small, the more exact analysis is not really justifiable and hence will not be given.

As we now have all of the stresses determined for towers 3-4 and 5-6 we can write the same on the stress sheet (Fig. 224) as shown, the stresses in the longitudinal bracing on the elevation of the towers and those in the transverse bracing and columns on the cross-section of the bents, and we can then proceed with the designing of the sections of the different members in these towers.

Taking the columns first (bents 3, 4, 5, and 6) let us assume the following section for the top portion of each column:

$$\begin{aligned} &1\text{—cov. pl. } 18'' \times \tfrac{5}{8}'' = 11.25\square'' \\ &2\text{—}[\text{s } 15'' \times 40\# \quad = \underline{23.52\square''} \\ &\qquad\qquad\qquad\qquad\quad 34.77\square'' \end{aligned}$$

Taking moments about the center of the cover plate (see Fig. 230) we have

$$\bar{x} = \frac{23.52 \times 7.81}{34.77} = 5.29 \text{ ins.}$$

for the distance to the center of gravity of the assumed section from the center of the cover plate (axis x-x).

Now for the moment of inertia about axis x-x we have

$$I = 347.5 \times 2 + (\overline{2.52}^2 \times 23.52) + (\overline{5.29}^2 \times 11.25) = 1{,}159,$$

and hence for the radius of gyration about axis x-x we have

$$r = \sqrt{\frac{1{,}159}{34.77}} = 5.77.$$

For the moment of inertia about axis y-y we have

$$I' = 9.39 \times 2 + (\overline{6.03}^2 \times 23.52) + 303.75 = 1{,}178,$$

and for the radius of gyration about the same axis we have

$$r' = \sqrt{\frac{1{,}178}{34.77}} = 5.82.$$

The distance along the column between the points of connection of the longitudinal bracing is the greatest unsupported length (as is seen from Fig. 224) and hence will be taken as the length of column. This length (as obtained by scale) is about 36′ or 432″. Then substituting this length and the radius r' (as the column would fail about axis y-y) in the column formula, we have

$$p = 16{,}000 - 70\frac{432}{5.82} = 10{,}800 \text{ lbs.}$$

for the allowable unit stress in the case of the combined dead- and live-load and impact stresses. As the maximum stresses due to live load, wind and traction are not likely to occur simultaneously the unit stress can be increased 25 per cent for the case of combined dead- and live-load, impact, wind, and traction stresses, as per specifications.

So we have

$$\frac{293{,}000}{10{,}800} = 27.1 \text{ sq. ins.}$$

for the area required for the combined dead- and live-load and impact stresses, and for the combined dead, live, impact, wind, and traction stresses we have

$$\frac{468{,}000}{10{,}800\,(1+0.25)} = \frac{468{,}000}{13{,}500} = 34.66 \text{ sq. ins.},$$

which is the actual required area, being the greater. The section assumed is satisfactory as it is about as near the required area as we can get it.

Taking next the bottom portion of the column, let us assume the following section:

1—cov. pl. 18″ x $\frac{11}{16}$″ = 12.37□″
2—[s 15″ x 50# = 29.42□″
41.79□″

Taking moments about the center of the cover plate (see Fig. 231) we have

$$\bar{x} = \frac{29.42 \times 7.84}{41.79} = 5.52 \text{ ins.}$$

for the distance to the center of gravity of the section from the center of the cover plate.

For the moment of inertia about axis x-x we have

$$I = 402.7 \times 2 + (\overline{2.32}^2 \times 29.42) + (\overline{5.52}^2 \times 12.37) = 1{,}341,$$

and for the radius of gyration about the same axis we have

$$r=\sqrt{\frac{1,342}{41.79}}=5.66.$$

For the moment of inertia about axis y-y we have

$$I'=(\overline{6.05}^2\times 29.42)+(11.22\times 2)+334.13=1,433,$$

and for the radius of gyration we have

$$r'=\sqrt{\frac{1,433}{41.79}}=5.86.$$

The length of the bottom portion of the columns is about 36 ft., or 432 ins., and as this length is the same as regards the two axes, r the smaller radius will be used in the column formula. So we have

$$p=16,000-70\frac{432}{5.66}=10,680$$

for the allowable unit stress. Dividing this into the combined dead, live, and impact stress we have

$$\frac{304,000}{10,680}=28.46 \text{ sq. ins.}$$

for the required area of the column, and by increasing this unit stress 25 per cent and dividing it into the combined dead, live, impact, wind, and traction stress we have

$$\frac{564,000}{13,360}=42.2 \text{ sq. ins.}$$

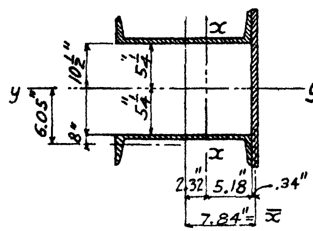


Fig. 231

for the required area which is the greater of the two. The assumed section is about as near to the required section as is possible to obtain, so it will be used.

We will next take up the designing of the transverse bracing. Beginning with the top diagonals (see cross-section of bents 3, 4, 5, and 6, Fig. 224) we have a stress of 38,000 lbs., which we will assume to be carried in tension by one system, so we have

$$\frac{38,000}{16,000}=2.37 \text{ sq. ins.}$$

for the required net area of each of the top diagonals. It is necessary to use two angles for each diagonal in order to obtain good details. It is seen that comparatively small angles could be used as far as the area of cross-section is concerned, but as $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angles are about as small as is consistent with good practice in the design of railroad bridges we will use 2—∟s $3\frac{1}{2}''\times 3\frac{1}{2}''\times\frac{3}{8}''=4.98-0.37=4.61$ sq. ins. net for each diagonal.

Now as these angles are the minimum size used and the stresses being less in the other transverse diagonal than in the ones just considered, 2—⌊s $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ will be used for each of the other transverse diagonals.

In designing the transverse struts L/r should not be greater than 120, and 100 would be better. The stresses, as is seen, are so small in these struts that the designing of the sections really consists in selecting sections that will be sufficiently rigid and provide good details.

The bottom strut is about 31′, or 372″, long. So we have

$$\frac{372}{100} = 3.72$$

for the required radius of gyration to give $L/r = 100$. We will use 2—[s 10″ x 20# which have a radius of 3.66 and considerable more section than required, but it is about as satisfactory a section as is obtainable and hence will be used for the bottom transverse strut.

The second transverse strut from the bottom is about 18′-6″, or 222″, long, so we have

$$\frac{222}{100} = 2.2$$

for the required radius. Here we will use 2—[s 8″ x 13.75# which have a radius of 2.98. We will use the same section for the next strut above also, in order to obtain uniform details.

Let us next take the case of the top strut. This strut has a length of about 78″. Let us assume 2—⌊s $5'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' = 6.10\square''$. The least radius of gyration of these two angles taken as a strut is 1.60. Then substituting in the column formula we have

$$p = 16{,}000 - 70\,\frac{78}{1.60} = 12{,}600 \text{ lbs.}$$

for the allowable unit stress for the combined dead, live, and impact stresses. So we have

$$\frac{48{,}000}{12{,}600} = 3.8 \text{ sq. ins.}$$

for the required area of cross-section, and increasing the above unit stress 25 per cent and dividing it into the combined dead, live, impact, wind, and traction stresses we have

$$\frac{89{,}000}{15{,}700} = 5.6 \text{ sq. ins.}$$

for the required area which is the greater. So our assumed section is about correct and hence will be used.

We will next take up the designing of the longitudinal bracing in towers 3-4 and 5-6. Here each diagonal will be designed to carry the 96,000-lb. stress in tension, assuming only one system to act at a time. So we have

$$\frac{96{,}000}{16{,}000} = 6 \text{ sq. ins. (net)}$$

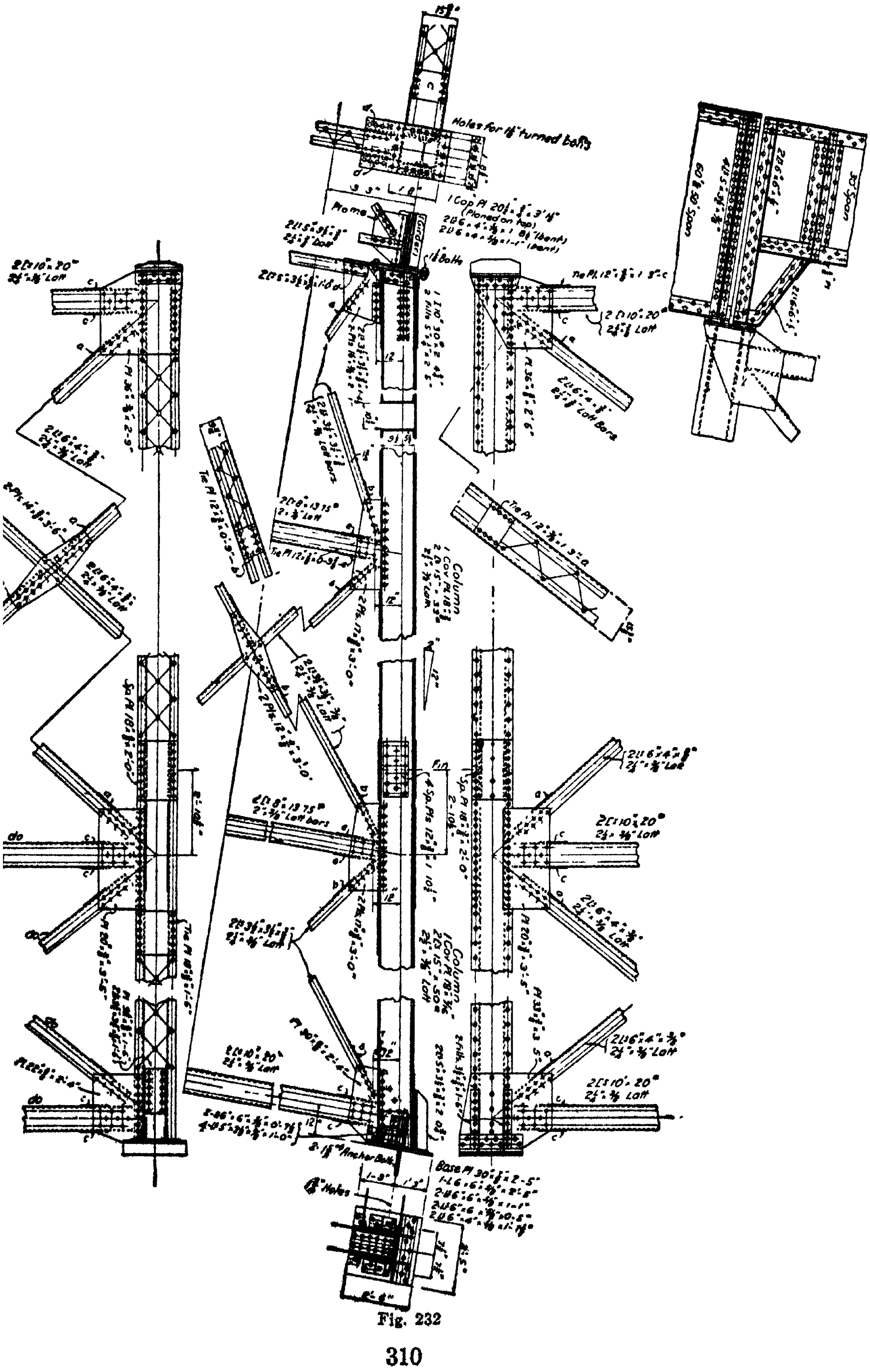


Fig. 232

for the required net area of cross-section of each diagonal. Then for each diagonal we can use (counting out of each angle one rivet hole) 2—Ls $6'' \times 4'' \times \frac{3}{8}'' = 7.22 - 0.75 = 6.47$ sq. ins. (net).

We will next consider the longitudinal struts, which are really columns 30′ or 360″ long. The maximum stress of 62,000# occurs on the intermediate one. Let us assume a section composed of 2—[s 10″x 20# = 11.76□″. Then we have

$$\frac{L}{r} = \frac{360}{3.66} = 98.5,$$

and we also have

$$p = 16{,}000 - 70\frac{360}{3.66} = 9{,}110 \text{ lbs.}$$

for the allowable unit stress. Dividing this into the stress we have

$$\frac{62{,}000}{9{,}110} = 6.79 \text{ sq. ins.}$$

for the required area which is much less than the area of the assumed section, yet we will use the assumed section for each of the longitudinal struts as the L/r is about correct. This completes the design for the towers 3-4 and 5-6, and the sections can be written on the stress sheet as shown (Fig. 224).

The other towers can be designed in the same manner as shown above for towers 3-4 and 5-6 and then the stress sheet can be finished as shown in Fig. 224.

163. Detail Drawings.—After the stress sheet (Fig. 224) is completed a general drawing of one (at least) of the towers should be made in order to show the kind of details desired unless standard drawings for such details, previously made, are available. The tower selected for detailing should be one of medium height so as to obtain average details. In this case we will select tower 1-2 (see stress sheet, Fig. 224). The general details for the tower will be sufficiently shown by drawing the details of one column and the principal details of the bracing connecting to the column. The first thing to do is to make large scale detail sketches of the column cap, base, and of the principal intermediate points. From these sketches we determine the exact slopes and clearances for the bracing, in fact all essential details are worked out on these sketches and simply transferred to the finished general drawing. Working in this manner we obtain the general drawing for the tower 1-2 shown in Fig. 232.

The calculations for these details are quite simple. The cap is made as small as is consistent with good details. The I-beam diaphragm at the top of the column is for the purpose of distributing the loads from the girders equally to the two channels. There should be a sufficient number of rivets in each side of this diaphragm to transmit one-half of the maximum end shear on the longer girder minus the dead-load end shear on the shorter girder. So in this case we have (see stress sheet, Fig. 224)

$$\frac{243{,}000}{2} - 6{,}800 = 114{,}700 \text{ lbs.}$$

for the shear that these rivets are to transmit. Dividing this by 7,200 (the value of a $\frac{7}{8}''$ shop rivet in single shear) we obtain practically 16 rivets. Sixteen is the number used.

The bolts connecting the girders to the columns should be sufficiently large to transmit the shear due to traction, neglecting the friction on the expansion end. Taking the case of the 60-ft. span, the heaviest load will occur when wheels 2 to 11 (inclusive) are on the span. This (for Cooper's $E50$, see Fig. 151) gives a load of 205,000 lbs. per girder.

Then, we have $0.2 \times 205{,}000 = 41{,}000$ lbs. for the horizontal thrust on each girder which must be taken by the bolts at the fixed end of the girder. We have four bolts taking this, so each must take one-fourth of it or 10,250 lbs. This calls for $1\frac{1}{8}''$ bolts, as the area of cross-section of each is 0.994□″, and stressing them 10,000 lbs. per square inch gives 9,940 lbs., which is about the required value. The same size bolts are used for the tower spans in order to have uniform sizes, although the shear due to traction in that case does not require them to be so large.

The column base should have sufficient area so as not to stress the masonry more than 600 lbs. per square inch (see specifications). The maximum reaction on bents 1 and 2 (see stress sheet, Fig. 224) is 512,000 lbs. Dividing this by 600 we obtain 853 sq. ins. for the required area of bearing on the masonry. The masonry plate or base plate used has ($30'' \times 29''$) 870 sq. ins., which is about the correct area. The masonry plate should be symmetrically placed in reference to the center of gravity of the column and the details should be such that the pressure from the column is quite uniformly distributed over the plate.

The anchor bolts should be large enough to take the maximum negative reaction (uplift) on the column at 16,000 lbs. per square inch. The splice in the column is considered to be a butt joint, that is, the top section of the column is considered to bear firmly against the bottom section so that the stress is transmitted from one to the other without the aid of the rivets in the splice. In that case there need be only enough rivets in the splice to hold the column in line. Just how many to use in such cases must be determined by mere judgment.

The calculation for the other details is mostly a matter of developing the sections in the bracing which the student should have no trouble in doing. Take, for example, the longitudinal diagonals. Each of these diagonals is composed of 2—∟s $6'' \times 4'' \times \frac{3}{8}'' = 7.22 - 0.75 = 6.47$ sq. ins. (net). Multiplying this net area (of the two angles) by 16,000 lbs. we have

$$6.47 \times 16{,}000 = 103{,}500 \text{ lbs.}$$

for the tensile strength of each diagonal. Dividing this by 6,000 lbs. we obtain about 17—$\frac{7}{8}''$ field rivets to develop the section. Eight on a side are used (in the end connections), which is about correct.

Take next the longitudinal struts which are composed of 2—[s $10'' \times 20\# = 11.76$□″. The strength of these struts as given in the last Article is 9,110 lbs. per square inch. Then for the total strength of each we have

$$11.76 \times 9{,}110 = 107{,}000 \text{ lbs.}$$

Dividing this by 6,000 lbs. we obtain about 18—$\frac{7}{8}''$ field rivets. Nine on a side are used (in the end connections), which is correct. The riveting in the end connections and splices of the transverse bracing is obtained in the same manner.

The stress sheet (Fig. 224) and the general drawing of the tower 1-2 (Fig. 232) are quite sufficient for general drawings in this case as the work is quite similar throughout. However, there should always be a sufficient number of general drawings to fully show the character of the work. All special towers should really be detailed.

From these general drawings the bridge company, obtaining the contract for fabricating the structure, works up the shop drawings for the work.

In working up the shop drawings usually the first thing, after large scale sketches of the principal details of the towers are made, is to make a drawing showing the location of the anchor bolts. This is known as the masonry plan. This drawing is used by the party building the substructure and is usually the first drawing called for. The material as a rule is next ordered and line drawings of the towers are made upon which the lengths of all members are given, which are usually computed by the aid of logarithmic tables. After this the making of the shop drawings proper (and bills) proceeds. The shop drawings for the girders are practically the same as previously shown for deck-plate girder bridges. The shop drawings for the towers are quite easy to make; the columns should be on separate sheets from the bracing. All should be drawn as much in their relative position as possible to aid in checking and also in identifying the members.

We will next make a preliminary estimate of the weight and cost of metal in the structure.

164. Preliminary Estimate of Weight of Metal and Cost.— Taking up the weight of the girders first we have the following:

Weight of metal in one 60-ft. span.

Weight of one girder.

1—web 84″ x $\frac{3}{8}''$ x 107.1# x 60′	6,426 lbs.
4—Ls 6″ x 6″ x $\frac{5}{8}''$ x 24.2# x 60′	5,808 lbs.
1—cov. pl. 14″ x $\frac{5}{8}''$ x 29.76# x 60′	1,786 lbs.
1—cov. pl. 14″ x $\frac{5}{8}''$ x 29.76# x 44′	1,309 lbs.
2—cov. pls. 14″ x $\frac{1}{2}''$ x 23.8# x 29′	1,380 lbs.
26—stiff. Ls 5″ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ x 10.4# x 7′	1,893 lbs.
8—end stiff. Ls 5″ x $3\frac{1}{2}''$ x $\frac{7}{16}''$ x 12.0# x 7′	672 lbs.
4—fillers 7″ x $\frac{5}{8}''$ x 14.88# x 6′	357 lbs.
8—sp. pls. 10″ x $\frac{5}{8}''$ x 21.25# x 2.4′	408 lbs.
4—sp. pls. 14″ x $\frac{5}{8}''$ x 29.76# x 4.3′	512 lbs.
2—sole pls. 12″ x $\frac{3}{4}''$ x 30.6# x 1.2′	74 lbs.
6—fillers $3\frac{1}{2}''$ x $\frac{5}{8}''$ x 7.44# x 6′	268 lbs.
	20,893 lbs.
rivets, 3% (= 20,890 × 0.03)	627 lbs.
Total weight of 1 girder	21,520 lbs.
	2
Total weight of 2 girders	43,040 lbs.

Weight of one frame.

2—Ls $3\frac{1}{2}''$ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ x 8.5# x 6.1′	104 lbs.
2—Ls $3\frac{1}{2}''$ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ x 8.5# x 8.2′	140 lbs.
4—pls. $12\frac{1}{2}''$ x $\frac{3}{8}''$ x 15.94# x 1.1′	70 lbs.
1—pl. 9″ x $\frac{3}{8}''$ x 11.48# x 0.8′	9 lbs.
	323 lbs.
rivets	22 lbs.
Total weight of 1 frame	345 lbs.
	5
Total weight of 5 frames	1,725 lbs.

Weight of laterals and lateral plates.

8—Ls $3\frac{1}{2}''$ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ x 8.5# x 9′	612 lbs.
7—pls. 12″ x $\frac{3}{8}''$ x 15.3# x 2.8′	300 lbs.
15—pls. 12″ x $\frac{3}{8}''$ x 15.3# x 1.0′	230 lbs.
2—pls. 12″ x $\frac{3}{8}''$ x 15.3# x 2.0′	61 lbs.
rivets	27 lbs.
Total weight of laterals and lateral plates	1,230 lbs.

Summary of weight.

2—girders	43,040 lbs.
5—frames	1,720 lbs.
laterals and plates	1,230 lbs.
Total weight of metal in 1—60-ft. span	45,990 lbs.

Weight of metal in one 30-ft. span.

Weight of one girder.

1—web 54″ x $\frac{3}{8}''$ x 68.85# x 25′	1,721 lbs.
4—Ls 6″ x 6″ x $\frac{5}{8}''$ x 24.2# x 30′	2,904 lbs.
2—end pls. 30″ x $\frac{3}{8}''$ x 38.25# x 6′	459 lbs.
4—sp. pls. 12″ x $\frac{5}{8}''$ x 25.5# x 3.5′	357 lbs.
4—Ls 6″ x 6″ x $\frac{1}{2}''$ x 19.6# x 4′ (bent)	314 lbs.
4—Ls 6″ x 6″ x $\frac{1}{2}''$ x 19.6# x 7′	549 lbs.
4—fillers 6″ x $\frac{5}{8}''$ x 12.7# x 6′	305 lbs.
14—Ls 5″ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ x 10.4# x 4.5′	655 lbs.
2—pls. 12″ x $\frac{5}{8}''$ x 25.5# x 2′	102 lbs.
2—sole pls. 12″ x $\frac{3}{4}''$ x 30.6# x 1.2′	74 lbs.
	7,440 lbs.
rivets, 2%	150 lbs.
Total weight of 1 girder	7,590 lbs.
	2
Total weight of 2 girders	15,180 lbs.

Weight of frames.

2—end frames = 345 × 2	690 lbs.	(from 60-ft. span)
1—interior frame	320 lbs.	(computed)
Total weight of 3 frames	1,010 lbs.	

Weight of laterals and plates $\frac{1,230}{60} \times 30 = 615\#$ (from 60-ft. span)

Summary of weight.

2—girders	15,180 lbs.
3—frames	1,010 lbs.
laterals and plates	615 lbs.
Total weight of metal in 1—30′ tower span	16,805 lbs.

Weight of metal in one 50-ft. span.

1—web 84″ x $\frac{3}{8}$″ x 107.1# x 50′	5,355 lbs.
4—Ls 6″ x 6″ x $\frac{7}{8}$″ x 33.1# x 50′	6,620 lbs.
22—Ls 5″ x $3\frac{1}{2}$″ x $\frac{3}{8}$″ x 10.4# x 7′	1,602 lbs.
4—Ls 5″ x $3\frac{1}{2}$″ x $\frac{7}{16}$″ x 12.0# x 7′	336 lbs.
4—fillers 7″ x $\frac{7}{8}$″ x 20.83# x 6′	500 lbs.
4—fillers $3\frac{1}{2}$″ x $\frac{7}{8}$″ x 10.42# x 6′	250 lbs.
splice plates (see 60-ft. span)	650 lbs.
sole plates (see 60-ft. span)	74 lbs.
	15,387 lbs.
rivets, 3%	450 lbs.
Total weight of 1 girder	15,837 lbs.
	2
Total weight of 2 girders	31,664 lbs.
Lateral system = (about same as 50-ft. span, Art. 137)	2,776 lbs.
Total weight of metal in 1—50-ft. span	34,440 lbs.

Weight of metal in tower 1-2.
(Detailed, Fig. 232.)
One Column.

2—[s 15″ x 33# x 23.75′	1,568 lbs.
1—cov. pl. 18″ x $\frac{3}{8}$″ x 22.95# x 23.7′	544 lbs.
2—[s 15″ x 50# x 26.6′	2,660 lbs.
1—cov. pl. 18″ x $\frac{7}{16}$″ x 26.78# x 26.6′	712 lbs.
Total weight of main section	5,484 lbs.

Cap.

1—cap plate $20\frac{1}{2}$″ x $\frac{3}{4}$″ x 52.28# x 3.1′	162 lbs.
1—I 10″ x 30# x 2.3′	69 lbs.
2—fills. 5″ x $\frac{1}{4}$″ x 4.25# x 2.3′	20 lbs.
2—Ls 6″ x 4″ x $\frac{5}{8}$″ x 20# x 1.7′	68 lbs.
2—Ls 6″ x 4″ x $\frac{5}{8}$″ x 20# x 1.0′	40 lbs.
*1—gus. pl. 29″ x $\frac{3}{8}$″ x 36.97# x 2.4′	89 lbs.
1—gus. pl. 36″ x $\frac{3}{8}$″ x 45.90# x 2.6′	120 lbs.
2—Ls $3\frac{1}{2}$″ x $3\frac{1}{2}$″ x $\frac{3}{8}$″ x 8.5# x 1.4′	24 lbs.
2—pls. 16″ x $\frac{3}{8}$″ x 20.4# x 1.75′	71 lbs.
2—Ls 5″ x $3\frac{1}{2}$″ x $\frac{3}{8}$″ x 10.4# x 1.0′	21 lbs.
	684 lbs.

* Average width and length of plates are given.

Intermediate details.

3—tie pls. 18″ x $\frac{3}{8}$″ x 22.95# x 1.5′	103 lbs.
2—lat. pls. 14″ x $\frac{3}{8}$″ x 17.85# x 3′	107 lbs.
4—sp. pls. 12″ x $\frac{3}{8}$″ x 15.3# x 1.85′	113 lbs.
2—sp. pls. 18″ x $\frac{3}{8}$″ x 22.95# x 2.0′	92 lbs.
2—lat. pls. 16″ x $\frac{3}{8}$″ x 20.4# x 3.0′	122 lbs.
2—lat. pls. 20″ x $\frac{3}{8}$″ x 25.5# x 3.4′	173 lbs.
128 ft. of $2\frac{1}{2}$″ x $\frac{3}{8}$″ latt. bars @ 3.19# per ft.	408 lbs.
	1,118 lbs.

Base.

1—base plate 30″ x $\frac{7}{8}$″ x 89.25# x 2.5′	223 lbs.
2—Ls 6″ x 6″ x $\frac{5}{8}$″ x 24.2# x 1.1′	53 lbs.
2—Ls 6″ x 4″ x $\frac{5}{8}$″ x 20# x 1.6′	64 lbs.
2—Ls 6″ x 6″ x $\frac{5}{8}$″ x 24.2# x 0.4′	19 lbs.
2—Ls 6″ x 6″ x $\frac{5}{8}$″ x 24.2# x 0.8′	39 lbs.
4—Ls 5″ x $3\frac{1}{2}$″ x $\frac{3}{8}$″ x 10.4# x 1.0′	42 lbs.
2—Ls 5″ x $3\frac{1}{2}$″ x 10.4# x 2.0′	42 lbs.
1—L 6″ x 6″ x $\frac{5}{8}$″ x 24.2# x 2.4′	58 lbs.
2—fills. $3\frac{1}{2}$″ x $\frac{5}{8}$″ x 7.44# x 1.5′	22 lbs.
1—pl. 9″ x $\frac{3}{8}$″ x 11.48# x 1.5′	17 lbs.
2—Ls $3\frac{1}{2}$″ x $3\frac{1}{2}$″ x $\frac{3}{8}$″ x 8.5# x 1.5′	26 lbs.
1—gus. pl. 30″ x $\frac{3}{8}$″ x 38.25# x 3.3′	126 lbs.
1—gus. pl. 28″ x $\frac{3}{8}$″ x 35.7# x 1.8′	64 lbs.
2—gus. pls. 30″ x $\frac{3}{8}$″ x 38.25# x 1.75′	134 lbs.
	929 lbs.

Summary of weight of details.

Cap	684 lbs.
Int. details	1,118 lbs.
Base	929 lbs.
Rivets	200 lbs.
	2,931 lbs.

$$\text{Percentage of details} = \frac{2{,}931}{5{,}484} = 53\% \text{ of main section.}$$

Summary of weight of one column (bents 1 and 2).

Details	2,931 lbs.	
Main section	5,484 lbs.	
	8,415 lbs.	= total weight of one column.
	4	
	33,660 lbs.	= total weight of four columns.

Longitudinal bracing.

Longitudinal strut.

2—[s 10″ x 20# x 28.4′	1,136 lbs.
4—tie pls. 12″ x ⅜″ x 15.3# x 1.25′	77 lbs.
164 ft. latt. bars of 2¼″ x ⅜″ @ 2.87# per ft.	470 lbs.
rivets	53 lbs.
(Details = 44%.)	1,736 lbs.
	6
Weight of metal in six struts	10,416 lbs.

Diagonal.

2—∟s 6″ x 4″ x ⅜″ x 12.3# x 35.5′	873 lbs.
2—tie pls. 12″ x ⅜″ x 15.3# x 1.25′	38 lbs.
48 ft. latt. bars 2¼″ x ⅜″ @ 2.87# per ft.	138 lbs.
rivets	20 lbs.
(Details = 22%.)	1,069 lbs.
	4
Weight of metal in four diagonals	4,276 lbs.

Diagonal (spliced).

4—∟s 6″ x 4″ x ⅜″ x 12.3# x 17.5′	861 lbs.
4—tie pls. 12″ x ⅜″ x 15.3# x 1.25′	77 lbs.
2—sp. pls. 13″ x ⅜″ x 16.57# x 3.5′	116 lbs.
44 ft. latt. bars 2¼″ x ⅜″ @ 2.87# per ft.	126 lbs.
rivets	24 lbs.
(Details = 40%.)	1,204 lbs.
	4
Weight of metal in four diagonals	4,816 lbs.
Total weight of longitudinal bracing	19,508 lbs.

Transverse bracing (bents 1 and 2).

Strut (beginning at top of bent).

2—∟s 5″ x 3½″ x ⅜″ x 10.4# x 4.0′	83 lbs.	
7 ft. 2¼″ x ⅜″ lat. bars @ 2.87#	20 lbs.	
rivets	2 lbs.	105 lbs.
	(Details, 27%.)	

Diagonal.

2—∟s 3½″ x 3½″ x ⅜″ x 8.5# x 12.75′	217 lbs.	
2—tie pls. 9″ x ⅜″ x 11.48# x 1.0′	23 lbs.	
16 ft. lat. bars 2¼″ x ⅜″ @ 2.87#	46 lbs.	
rivets	11 lbs.	297 lbs.
	(Details, 36%.)	

Diagonal.

2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 3.75′	64 lbs.	
2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 8.4′	143 lbs.	
4—tie pls. 9″ x ⅜″ x 11.48# x 1.0′	46 lbs.	
2—sp. pls. 12″ x ⅜″ x 15.3# x 3.0′	92 lbs.	
14 ft. latt. bars 2¼″ x ⅜″ @ 2.87#	40 lbs.	
rivets	16 lbs.	401 lbs.
	(Details, 93%.)	

Strut.

2—[s 8″ x 13.75# x 8.75′	240 lbs.	
4—tie pls. 12″ x ⅜″ x 15.3# x 0.8′	49 lbs.	
20 ft. lat. bars 2¼″ x ⅜″ @ 2.87#	57 lbs.	
rivets	12 lbs.	358 lbs.
	(Details, 49%.)	

Diagonal.

2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 17.2′	292 lbs.	
2—tie pls. 9″ x ⅜″ x 11.48# x 1.0′	23 lbs.	
34 ft. lat. bars 2¼″ x ⅜″ @ 2.87#	97 lbs.	
rivets	16 lbs.	428 lbs.
	(Details, 46%.)	

Diagonal.

2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 6.6′	112 lbs.	
2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 10.0′	170 lbs.	
4—tie pls. 9″ x ⅜″ x 11.48# x 1.0′	46 lbs.	
30 ft. lat. bars 2¼″ x ⅜″ @ 2.87#	86 lbs.	
2—sp. pls. 12″ x ⅜″ x 15.3# x 3.0′	92 lbs.	
rivets	20 lbs.	526 lbs.
	(Details, 87%.)	

Strut.

2—[s 8″ x 13.75# x 13.3′	366 lbs.	
4—tie pls. 12″ x ⅜″ x 15.3# x 0.8′	49 lbs.	
40 ft. lat. bars 2¼″ x ⅜″ @ 2.87#	115 lbs.	
rivets	40 lbs.	570 lbs.
	(Details, 56%.)	

Diagonal.

2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 27.2′	462 lbs.	
2—tie pls. 9″ x ⅜″ x 11.48# x 1.0′	23 lbs.	
48 ft. lat. bars 2¼″ x ⅜″ @ 2.87#	137 lbs.	
rivets	26 lbs.	648 lbs.
	(Details, 40%.)	

Diagonal.

2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 10.6′	180 lbs.	
2—Ls 3½″ x 3½″ x ⅜″ x 8.5# x 16.0′	272 lbs.	
4—tie pls. 9″ x ⅜″ x 11.48# x 1.0′	46 lbs.	
44 ft. lat. bars 2¼″ x ⅜″ @ 2.87#	126 lbs.	
2—sp. pls. 12″ x ⅜″ x 15.3# x 3.0′	92 lbs.	
rivets	30 lbs.	746 lbs.
	(Details, 65%.)	

Strut.

2—[s 10″ x 20# x 20.5′	820 lbs.	
4—tie pls. 12″ x $\frac{3}{8}$″ x 15.3# x 0.8′	49 lbs.	
72 ft. lat. bars $2\frac{1}{4}$″ x $\frac{3}{8}$″ @ 2.87#	206 lbs.	
rivets	50 lbs.	1,125 lbs.
	(Details, 37%.)	

Weight of transverse bracing in one bent = 5,204 lbs.
Weight of transverse bracing in two bents or tower = 10,408 lbs.

Summary of weight of metal in tower 1-2.

4 columns	33,660 lbs.
Long. bracing	19,508 lbs.
Trans. bracing	10,408 lbs.
Total	63,676 lbs.

Proceeding in the same manner the following weights are found:

Weight of metal in each of the towers 3-4 and 5-6.

4 columns (36% details)	52,200 lbs.
Long. bracing	21,820 lbs.
Trans. bracing	14,440 lbs.
Total	88,460 lbs.

Weight of metal in tower 7-8	64,900 lbs.
Weight of metal in tower 9-10	42,380 lbs.
Weight of metal in bent 11	8,420 lbs.

Summary of weight of metal in structure.

5—60-ft. spans @ 45,990 lbs	229,950 lbs.	
5—30-ft. spans @ 16,805 lbs	84,025 lbs.	382,855 lbs.
2—50-ft. spans @ 34,440 lbs	68,880 lbs.	
1—tower (1-2)	63,676 lbs.	
2—towers (3-4 and 5-6) @ 88,460 lbs	176,920 lbs.	
1—tower (7-8)	64,900 lbs.	356,296 lbs.
1—tower (9-10)	42,380 lbs.	
1—bent (10)	8,420 lbs.	
Total weight of metal	739,151 lbs.	

Cost of structure (superstructure).

Girders, 382,855 lbs. @ 3¢	$11,486
Towers, 356,296 lbs. @ $3\frac{1}{2}$¢	12,470
Total cost of steel work (except anchor bolts)	$23,956

This price is only a fair average pound price.

165. Double-Track Viaducts.—The ordinary double-track viaduct has a double line of spans, that is, four lines of track girders, which, as a rule, are supported upon cross-girders as shown in Fig. 233 instead of resting directly upon the top of columns. The towers, as for general design, are practically the same as for single-track viaducts. The concentrations at the top of the towers due to dead and live load and impact are just twice as much as for single-track viaducts and consequently the stresses in the columns due to the same will be twice as much, provided of course, the columns in the two cases have the same batter. We assume that two trains move abreast over the double-track structure. The traction is just twice as much for a double-track as it is for a single-track viaduct, while the wind pressure is practically the same for the two structures. (See specifications for wind load.)

The method of procedure in designing double-track viaducts is the same as for single-track viaducts except the transverse bracing is sub-

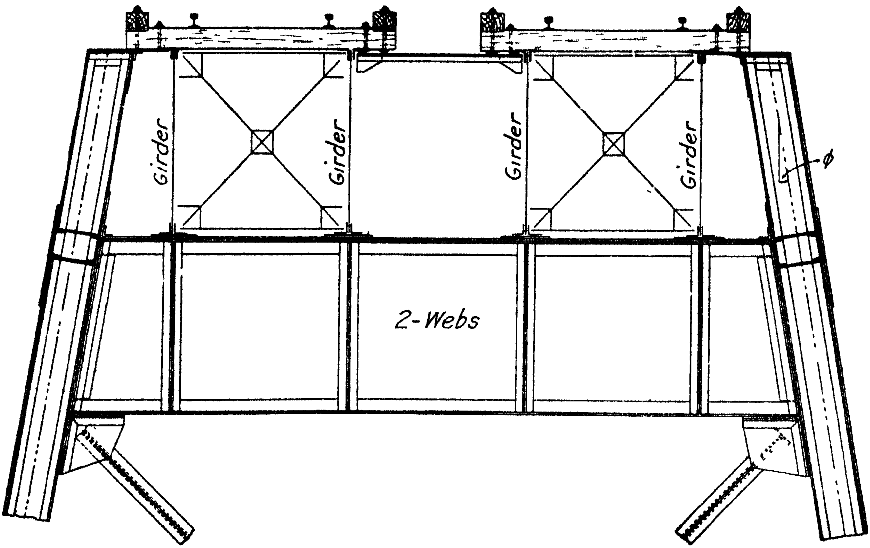


Fig. 233

jected to live-load stress and impact when only one track is loaded. This is due to the unequal thrusts exerted by the two columns upon the top strut, or cross-girder. As an illustration, suppose the right-hand track of a double-track viaduct (see Fig. 233) to be loaded with the maximum live load and no live load on the left-hand track. The right-hand column will receive most of the load, as is evident. Let V = the concentration on the right-hand column and V' the concentration on the left-hand column and let ϕ represent the slope angle of the columns. The horizontal thrust on the cross-girder exerted by the right-hand column is equal to $V\tan\phi$, and that exerted by the left-hand column is equal to $V'\tan\phi$. Now if these two thrusts were equal they would just balance each other and there would be no live-load stress in the transverse bracing. But as they are unequal the transverse bracing must transmit the difference (which produces a horizontal shear) down to the masonry. This force ($= V\tan\phi - V'\tan\phi$) can be assumed as applied at the bottom flange (considered as a strut) of the cross-girder (just exactly as a wind load) and the stresses in the transverse bracing due to it graphically determined.

The maximum stress in the columns occurs when the two tracks are fully loaded, at which time no live-load stress occurs in the transverse bracing.

166. Analytical Method of Determining Stresses in Tower Bracing.—Let Fig. 234 represent a transverse view of a bent of an ordinary viaduct, acted upon by forces as indicated. Let us first suppose that the horizontal wind forces F, $F1$, $F2$, and $F3$ alone are acting and that the stress in the diagonals AD, CN, and in the strut CD, due to these forces, is to be determined.

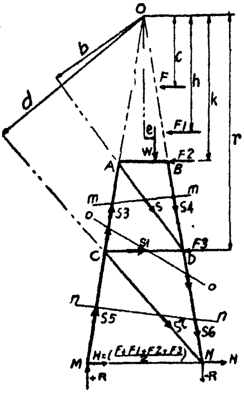


Fig. 234

To determine the stress in the diagonal AD, first assume the part of the bent (forces and all) below the section mm removed. Then the part above this section mm is held in equilibrium by the forces F, $F1$, $F2$, S, $S3$, and $S4$, where S, $S3$, and $S4$ represent the stress in members AD, AC, and BD, respectively. The stress S is what we desire. By prolonging the lines of action of the forces (stresses) $S3$ and $S4$ and taking moments about their point of intersection, O, we eliminate these forces from the equation of moments and we have

$$cF + hF1 + kF2 + bS = 0,$$

from which we obtain (in known quantities)

$$S = \frac{cF + hF1 + kF2}{b}.$$

Similarly, to determine the stress $S2$ in diagonal CN, assume the part of the bent below the section nn removed and taking moments about O we have

$$cF + hF1 + kF2 + rF3 + dS2 = 0,$$

from which we obtain the required stress

$$S2 = \frac{cF + hF1 + kF2 + rF3}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18).$$

To determine the stress $S1$ in the strut CD assume the part of the bent below the section oo removed and then taking moments about O we have

$$cF + hF1 + kF2 + rF3 + rS1 = 0,$$

from which we obtain the required stress

$$S1 = \frac{cF + hF1 + kF2 + rF3}{r}.$$

Instead of wind loads, as just considered, suppose there be an eccentrically applied load W acting upon the bent. Then taking moments about O, as before, we have

$$eW + bS = 0,$$

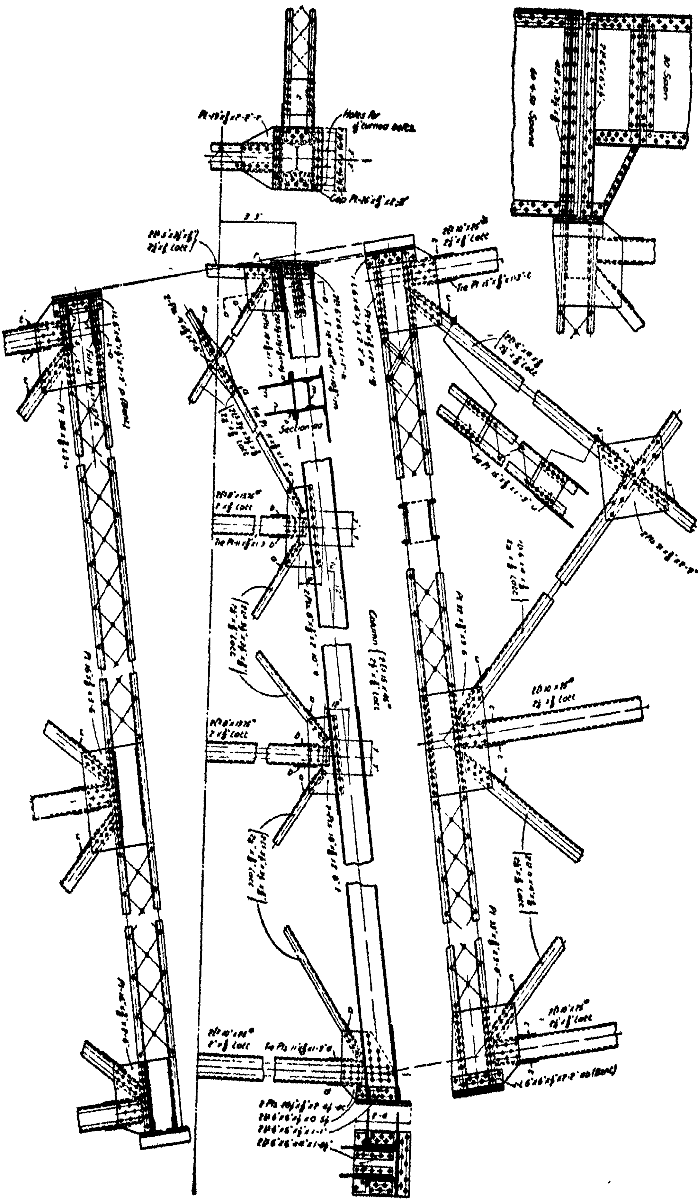


Fig. 225

from which we obtain

$$S = \frac{We}{b},$$

and similarly we obtain

$$S1 = \frac{We}{r} \text{ and } S2 = \frac{We}{d}.$$

Eccentrically applied loads, as just considered, occur mostly on viaducts built on curve and on double-track viaducts when only one track is loaded with live load.

If desired, the stresses in the columns can be determined by taking moments about panel points. For example, the stress $S4$ in BD can be obtained by passing the section mm and taking moments about A. By passing section oo and taking moments about D the stress $S3$ in AC can be determined, and, by taking moments at the same time about C the stress $S6$ in DN can be determined. Similarly, by passing the section nn and taking moments about N the stress $S5$ in CM can be determined.

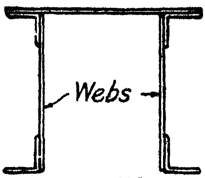


Fig 236

The only objection to the above method is that the length of some of the lever arms is troublesome to determine unless it be determined by scale, which, as a rule, is not a very satisfactory manner unless special care be taken.

167. General Remarks.—Viaduct columns are sometimes built without cover plates as shown in Fig. 235. There is no question but that a column with a cover plate is better than one without. Whenever a greater area is required than can be obtained by using two channels and a cover plate a section like the one shown in Fig. 236 should be used. However, in case there be just a few column sections in a structure requiring more area than is contained in two channels and a cover plate, the area may be increased so that that type of section can be used by riveting a plate to the back of each channel.

The batter of the columns in single-track viaducts varies from $1\frac{3}{4}$ in 12 to 3 in 12, for very low bents. Two in 12 is a very common batter. The batter of the columns in double-track viaducts, as a rule, is less than in the case of single-track structures. The batter should be sufficient to limit the negative reactions at the bases of the columns to a reasonable intensity, at least. In fact it would be better to have no negative reaction at all, but as a rule it is not practical to have that condition in the case of single-track viaducts.

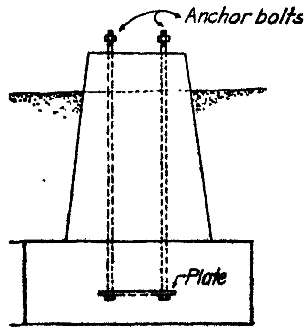


Fig. 237

There should be sufficient material in each pedestal (which is usually made of concrete) to properly distribute the positive reaction of the column supported and at the same time have sufficient weight to resist the negative reaction of the column. The anchor bolts should extend well down into the pedestals as indicated in Fig. 237.

In the case of viaduct towers having no horizontal struts, as shown in Fig. 221, the two systems of bracing are usually considered to act simultaneously, which requires that each member of the bracing be designed for compression as well as for tension, but at the same time, however, the stress in each is only half as much as obtains in the type of bracing shown in Fig. 220, assuming only one system to act at a time.

DRAWING ROOM EXERCISE NO. 6

Design and make stress sheet of a single-track viaduct for the following crossing:

Station	Elevation of Ground		Elevation of Base of Rail
400	640		648
400 + 81	588		648
401	579		648
401 + 70	568		648
402 + 13	564		648
402 + 20	554	Creek	648
402 + 40	554	Creek	648
402 + 49	566		648
403	567		648
403 + 41	564		648
404 + 5	608		648
404 + 41	613		648
404 + 80	626		648
405 + 2	646		648

Data:

Live load, Cooper's $E50$ loading.
Specifications, A. R. E. Ass'n.
Dead load, to be determined by student.

TRUSS BRIDGES

168. Preliminary.—Truss bridges are used for spans 100 ft. in length and over. With regard to the location of the track there are two types of truss bridges, the through and the deck bridge, the same as in

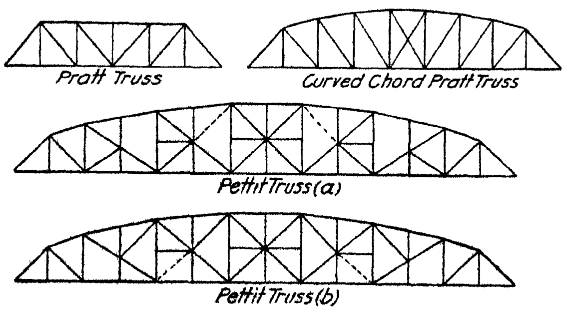


Fig. 238

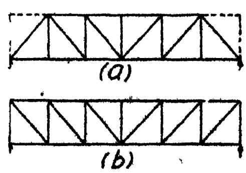


Fig. 239

the case of plate girders. The through type is used wherever the under clearance will not permit of the use of the deck type.

The four trusses shown in Fig. 238 are the most common types used for through bridges. Pratt trusses are used for spans up to 175 ft. in length and as a rule are riveted trusses. Curved chord Pratt trusses are used for spans from 200 ft. to 325 ft. long and as a rule are pin-connected

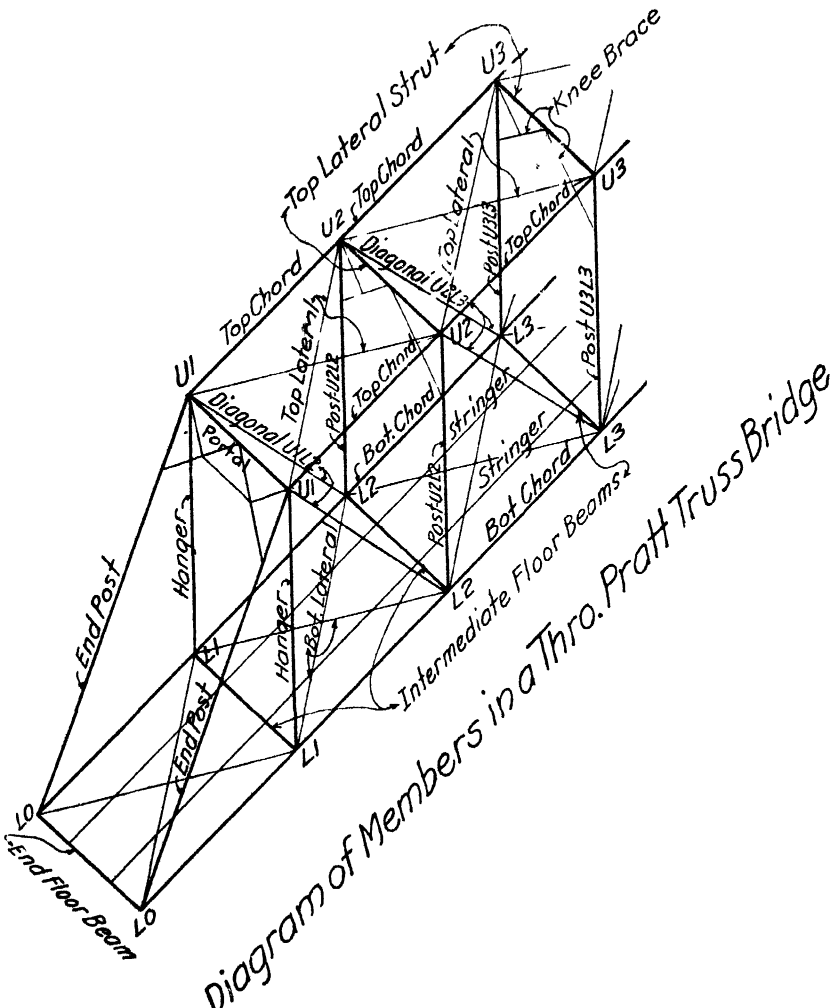


Fig. 240

trusses. Pettit trusses, both (*a*) and (*b*) types, are used for long spans, 350 ft. in length and over.

There are other types of bridge trusses which are used to some extent. These will be considered later.

In the case of deck bridges, the Pratt truss is practically always used. The end panels, however, are modified either as shown at (a) or (b), Fig. 239.

The diagram, Fig. 240, gives the names of the different parts of an ordinary Pratt truss bridge, but the same names hold for bridge trusses in general.

Complete Design of a 150-Ft. Single-Track Through Riveted Pratt Truss Span

169. Data.—

Length = 6 panels @ 25′–0″ = 150′–0″ c.c. end pins.
Width = 16′–0″ c.c. of trusses.
Height = 30′–0″ c.c. chords.
Stringers spaced, 6′–6″ c.c.
Live Load, Cooper's $E50$ loading.
Specifications, A. R. E. Ass'n.

170. Design of 25-Ft. Stringers.—(For detail of stringers, see Fig. 290.) For dead load on stringers we have, from (1) Art. 124, $w = 12 \times 25 + 100 + 400 = 800$ lbs. per ft. of span or 400 lbs. per ft. of stringer. Then, using this load, we have

$$M' = \tfrac{1}{8} \times 400 \times \overline{25}^2 \times 12 = 375{,}000 \text{ inch lbs.}$$

for the maximum bending moment due to dead load.

It is seen from Table A that the maximum live-load moment will likely occur when wheels 2 to 5 are on the stringer, and, according to Art. 88, they will be in the position shown in Fig. 241.

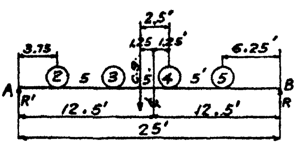


Fig. 241

Taking moments about A (Fig. 241), to find the reaction R (using Table A), we find first the moment of wheels 3, 4 and 5 about 2 by passing down the vertical line through wheel 2 to the zig-zag line, then to the right to the vertical line through wheel 5 and the figure 600, just to the right of this line, is the moment of wheels 3, 4 and 5 about 2, in thousands of foot pounds.

Then multiplying the total weight, in thousands of pounds, of the wheels, 2 to 5, by 3.75 (see Art. 46) and adding this product to the 600, we have the moment of the wheels, 2 to 5, about A. So we can write the equation of moment about A as

$$600 + (80 \times 3.75) - (R \times 25) = 0,$$

from which we obtain

$$R = \frac{600 + (80 \times 3.75)}{25} \times 1{,}000 = 36{,}000 \text{ lbs.},$$

for the reaction at B.

Then taking moments about 4, as the maximum moment occurs under that wheel (see Art. 88), we have the moment of R about wheel 4 minus the moment of wheel 5 about wheel 4 for the maximum bending moment; that is, we have

$$M'' = 36{,}000 \times 11.25 - 5 \times 20{,}000 = 305{,}000 \text{ ft. lbs.},$$

for the maximum live-load bending moment due to Cooper's $E40$ loading, and multiplying this by 50/40 and by 12 we have 4,575,000″# for the desired maximum live-load moment in inch pounds, provided the correct group of wheels were selected, which can be ascertained by trial, in case of doubt.

Then for the impact moment we have

$$I = 4{,}575{,}000 \times \frac{300}{325} = 4{,}223{,}000 \text{ inch lbs.}$$

Now adding the above moments together we have

$$M = 375{,}000 + 4{,}575{,}000 + 4{,}223{,}000 = 9{,}173{,}000 \text{ inch lbs.}$$

for the total maximum bending moment on the stringer.

Next let us assume the web as $\frac{3}{8}''$ thick. Then substituting in (5) of Art. 113 (stringers, as a rule, never have cover plates), we have

$$x = 1.22\sqrt{\frac{9{,}173{,}000}{\frac{3}{8} \times 16{,}000}} = 47.7 \text{ ins.}$$

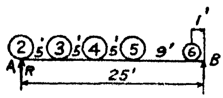


Fig 242

for the economic depth. So we will use a web 48″ deep.

For the dead-load end shear on the stringer, we have

$$400 \times 12.5 = 5{,}000 \text{ lbs.},$$

and placing the wheels as shown in Fig. 242 and taking moments about the support B (using Table A) we have

$$R = \frac{1{,}320 + (93 \times 1)}{25} \times 1{,}000 = 56{,}520 \text{ lbs.}$$

for the maximum end shear due to Cooper's $E40$ and

$$\frac{50}{40} \times 56{,}500 = 70{,}650 \text{ lbs., say } 71{,}000 \text{ lbs.,}$$

due to Cooper's $E50$ loading, and for impact we have

$$71{,}000 \times \frac{300}{325} = 66{,}000 \text{ lbs. (about)}$$

and adding we have

$$5{,}000 + 71{,}000 + 66{,}000 = 142{,}000 \text{ lbs.}$$

for the total maximum end shear on the stringer.

Now for the unit shear on the assumed web we have

$$\frac{142{,}000}{\frac{3}{8} \times 48} = 7{,}888 \text{ lbs.},$$

and substituting this in (1) of Art. 118, we have

$$d = \frac{\frac{3}{8}}{40}(12{,}000 - 7{,}888) = 39 \text{ ins. (about)}$$

for the maximum distance allowed between the stiffeners near the ends of the stringer, and, as this is more than half the depth of the girder and as the minimum thickness of web allowed by the specifications is $\frac{3}{8}$ in., we will use a 48″ x $\frac{3}{8}$″ web.

To determine the area of flanges the first thing to do is to approximate the effective depth of the stringer. Assuming that 6″ x 6″ angles will be required we find from Table 6, or from some handbook, that the average distance from the back to the center of gravity of these angles is about 1.75 ins., and taking the depth of the girder as $\frac{1}{4}''$ more than the web, we have

$$48.25 - 1.75 \times 2 = 44.75, \text{ say } 45 \text{ ins.},$$

for the approximate effective depth. Dividing this into the maximum total bending moment, we have

$$\frac{9,173,000}{45} = 204,000 \text{ lbs. (about)},$$

for the stress in each flange (see Art. 112), and dividing this by 16,000 we have

$$\frac{204,000}{16,000} = 12.75 \text{ sq. ins.},$$

for the net area of each flange. Let us try

$$\begin{aligned} &2\text{—}\llcorner\text{s } 6'' \times 6'' \times \tfrac{1}{2}'' = (5.75 \times 2) - 1 = 10.50\square'' \text{ net} \\ &\tfrac{1}{8} \text{ area of web} \qquad\qquad\qquad\quad = \underline{\ 2.25\square'' \text{ net}} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad 12.75\square'' \text{ net} \end{aligned}$$

This gives exactly the area indicated. Now checking back, we have

$$48.25 - 1.68 \times 2 = 44.89 \text{ ins.}$$

for the actual effective depth, which differs so little from the assumed that the area of the flanges would be changed but very little, and hence recalculation of flanges is unnecessary.

The stiffeners on stringers are usually made of $3\frac{1}{2}'' \times 3\frac{1}{2}''$ angles. There is no rational way of determining them. Owing to the planing off of the ends of the stringers to obtain exact length, the end stiffeners are, as a rule, made $\frac{1}{16}''$ to $\frac{1}{8}''$ thicker than the intermediate stiffeners, which are of minimum ($\frac{3}{8}''$) thickness.

The laterals—bracing for stringers—are designed the same as for any deck plate girder. (See Art. 135.)

171. Design of Intermediate Floor Beams.—(For details of floor beams, see Fig. 291.) The dead load on an intermediate floor beam consists of the dead-load concentrations from the stringers and the weight of the floor beam itself.

Each concentration from the stringers is equal to twice the end shear on one stringer and such floor beams weigh, depending on details, from 3,000# to 3,800# each. So let us, in this case, assume 3,600# as the weight of one intermediate floor beam complete, or 225# per ft. of floor beam. Then, for dead load we have the case shown in Fig. 243, and for the maximum bending moment due to dead load, we have

$$\begin{aligned} 10,000 \times 4.75 \times 12 &= 570,000 \text{ in. lbs.} \\ \tfrac{1}{8} \times 225 \times 16^2 \times 12 &= \underline{\ 86,000 \text{ in. lbs.}} \\ &\ \ \ 656,000 \text{ in. lbs.} \end{aligned}$$

and for the maximum end shear, due to the same load, we have

$$\frac{3,600}{2}+10,000=11,800 \text{ lbs.}$$

To determine the maximum live-load bending moment and end shear on the floor beam we must first satisfy the criterion of Art. 148 for maximum live-load concentration on the floor beam. That is, the loads in the two adjacent panels must be as nearly equal as possible and at the same time the heaviest loads must be near the floor beam, with one load exactly at it.

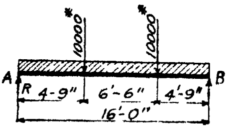


Fig 243

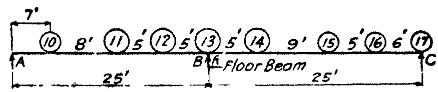


Fig 244

By trial (using Table A) it will be found that the maximum concentration on the floor beams will occur when the wheels are in the position shown in Fig. 244, as the criterion is most nearly satisfied with them in that position, that is, the heaviest wheels are near the floor beam, wheel 13 at it, and the load in panel BC, consisting of wheels 14, 15, and 16 (wheel 17 is exactly over the floor beam at C and hence is not in the panel BC), weighs 46,000 lbs., while the load in panel AB, consisting of wheels 10, 11, and 12, weighs 50,000 lbs. This is as close as the criterion can be satisfied, as will be found by further investigation. So, taking moments (using Table A) about A to find the concentration at B due to wheels 10, 11, and 12, we have

$$r=(420+50\times 7)\frac{1,000}{25}=30,800 \text{ lbs.},$$

and taking moments about C to find the concentration at B due to wheels 14, 15, and 16, we have

$$r'=\frac{621}{25}\times 1,000=24,850 \text{ lbs.}$$

Now adding these concentrations and the weight of wheel 13 together, we have

$$R=r+r'+20,000=30,800+24,850+20,000=75,650 \text{ lbs.},$$

for the maximum live-load concentration due to Cooper's $E40$ loading, and multiplying this by 50/40 we have 94,562#, say 94,500#, for the maximum live-load concentration due to Cooper's $E50$ loading.

Then for the live load on each intermediate floor beam we have the case shown in Fig. 245, and for the maximum live-load moment, taking moment about either C or D, we have

$$4.75\times 94,500\times 12=5,386,000 \text{ inch lbs.},$$

and for the impact we have

$$5,386,000\times\frac{300}{350}=4,616,800, \text{ say, } 4,620,000 \text{ inch lbs.}$$

(The load extends over two panel lengths, or 50 ft.)

Now adding the above dead, live, and impact moments, we have

$$M = 656{,}000 + 5{,}386{,}000 + 4{,}620{,}000 = 10{,}662{,}000 \text{ inch lbs.},$$

for the total maximum bending moment on the floor beam, and, as is seen from Fig. 245, the maximum live-load shear is 94,500# and the impact $= 94{,}500 \times 300/350 = 81{,}000$#. So by adding these to the above dead-load shear we have $11{,}800 + 94{,}500 + 81{,}000 = 187{,}300$# for the total maximum end shear on the floor beam.

The depth of the floor beam is governed practically by the depth of the stringers. The bottom laterals and the bottom flanges of the floor beams should be in the same plane, as the bottom flanges of the beams act as struts in the bottom lateral system, and there should be sufficient distance from the bottom of the stringers down to the bottom of the floor beam to permit the laterals to pass beneath the stringers, which usually requires from 4″ to 6″, depending on the size of the laterals, and the distance from the top of the floor beams down to the top of the stringers is usually about 3″. In this case we will assume 3″ as the distance from the top of the floor beams down to the top of the stringers and 5″ from the bottom of the stringers down to the bottom of the floor beams, which gives 56¼″ for the depth of the floor beam as shown in Fig. 246. [It will be found upon investigation that floor beams, as a rule, are deeper than the economic depth.]

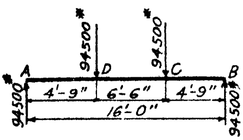


Fig. 245

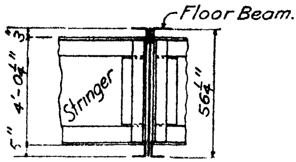


Fig. 246

After having decided upon the depth of the floor beam, the next thing is to approximate the effective depth. Assuming that each flange of the floor beam will be composed of 2—6″ x 6″ angles, we can take about the average distance from the back of the angles to their centers of gravity, which is given in Table 6 as, say, 1.75″. Then, for the approximate effective depth, we have

$$56.25 - (1.75 \times 2) = 52.75, \text{ say } 53 \text{ ins.}$$

Dividing this into the above maximum bending moment, we have

$$10{,}662{,}000 \div 53 = 201{,}000 \text{ lbs. (about)}$$

for the flange stress, and dividing this by 16,000 we have

$$201{,}000 \div 16{,}000 = 12.56 \text{ sq. in.}$$

for the required net area of each flange.

Assuming a 56″ x $\frac{7}{16}$″ web which has a section of 24.5□″ ($= 56 \times \frac{7}{16}$), let us try

2—Ls 6″ x 6″ x ½″ = (5.75 × 2) − 1	= 10.50□″ net
⅛ area of cross-section of web	= 3.06□″ net
	13.56□″ net

This gives 1□″ more than is required. 2—Ls 6″ x 6″ x $\frac{7}{16}$″ have a section more nearly equal to the required area, but the specifications require the angles to be $\frac{1}{2}$″ thick if the outstanding leg is 6″, so, to comply with the specifications, the above section will be used.

It will be seen that the true effective depth here is slightly less than the assumed, as the distance from the back of the 6″ x 6″ x $\frac{1}{2}$″ angles is 1.68″. So, for the actual effective depth, we have

$$56.25 - (1.68 \times 2) = 52.89 \text{ ins.}$$

This, however, is so near the assumed effective depth that recalculations are unnecessary.

For the maximum unit shear on the floor beam we have

$$187{,}300 \div 24.5 = 7{,}644 \text{ lbs. per sq. in.}$$

Now substituting this value for s in (1), Art. 118, we have

$$d - \frac{\frac{7}{16}}{40}(12{,}000 - 7{,}644) = 47.6 \text{ ins.},$$

for the allowable distance between stiffeners. The shear is practically zero between the stringers, so that no stiffeners are required there, and the clear distance from stringer to truss is 4′ – 9″ minus about 7″ for half width of truss and 6″ (at least) for connections, leaving 3′ – 8″, or 44″, for the actual value of d, while 47.6″ is allowed, so the assumed web is quite thick enough. In fact it could be thinner, but the rivets, spaced along the flanges between the stringers and trusses, would be closer than desired, if a thinner web be used, and hence the assumed 56″ x $\frac{7}{16}$″ web will be used, even though it is a little thicker than required.

The end stiffeners on the floor beams are usually 6″ x 6″ x $\frac{1}{2}$″ angles, placed upon fillers as shown in Fig. 291.

172. Design of End Floor Beams.—The dead load on an end floor beam consists of the two dead-load concentrations from the stringers,

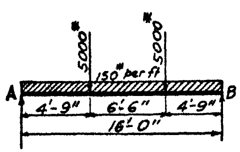


Fig 247

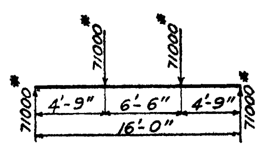


Fig. 248

each of which is equal to the dead-load end shear on one stringer, and the weight of the floor beam itself. The dead-load end shear on a stringer, as given in Art. 170, is 5,000#, and such end floor beams weigh about 2,400#, or, say, 150# per foot. Then the dead weight on each end floor beam is as shown in Fig. 247.

For the maximum bending moment, due to this dead load, we have

$$5{,}000 \times 4.75 \times 12 = 285{,}000 \text{ inch lbs.}$$

$$\tfrac{1}{8} \times 150 \times \overline{16}^2 \times 12 = \underline{57{,}600} \text{ inch lbs.}$$

342,600, say 343,000 inch lbs.,

and for the maximum end shear, due to the same dead load, we have

$$5{,}000 + 150 \times 8 = 6{,}200 \text{ lbs.}$$

The maximum live-load concentrations from the stringers on an end floor beam are each equal to the maximum live-load end shear on one stringer, which is given in Art. 170 as 71,000#. So the live load on an end floor beam is as shown in Fig. 248.

For maximum bending moment, due to this live load, we have

$$71{,}000 \times 4.75 \times 12 = 4{,}047{,}000 \text{ inch lbs.,}$$

and for the impact we have

$$4{,}047{,}000 \times \frac{300}{325} = 3{,}735{,}000 \text{ inch lbs.,}$$

and for maximum end shear, due to live load, we have 71,000, and for end shear, due to impact, we have

$$71{,}000 \times \frac{300}{325} = 66{,}000 \text{ lbs. (about).}$$

Now adding the above moments, we have

$$343{,}000 + 4{,}047{,}000 + 3{,}735{,}000 = 8{,}125{,}000 \text{ inch lbs.,}$$

for the total maximum bending moment, and adding together the above end shears we have

$$6{,}200 + 71{,}000 + 66{,}000 = 143{,}200 \text{ lbs.,}$$

for the total maximum end shear on an end floor beam.

Now assuming 53″ for the effective depth, the same as for the intermediate floor beam, and dividing it into the above moment, we have

$$\frac{8{,}125{,}000}{53} = 153{,}000 \text{ lbs.}$$

for the flange stress, and dividing this by 16,000, we have

$$\frac{153{,}000}{16{,}000} = 9.56 \text{ sq. in.,}$$

for the required net area of each flange. Now it is readily seen, from Table 6, that 6″ x 6″ angles are too large for these flanges and hence we will use 6″ x 4″ angles with the 6″ leg along the web to provide for the flange rivets.

Let us try, assuming a 56″ x $\frac{3}{8}$″ web (same depth as the intermediate floor beams),

2—Ls 6″ x 4″ x $\frac{7}{16}$″	$= 4.18 \times 2 - 0.87 =$	7.49□″ net
$\frac{1}{8}$ area of web	$= (56 \times \frac{3}{8}) \div 8$	$=$ 2.62□″ net
		10.11□″ net

This flange section is about correct. It is 0.55□″ greater than called for above, but the assumed effective depth is greater than the actual, which is really $56.25 - (2 \times 1.96) = 52.33$″.

Dividing the moment by this, we have

$$8{,}125{,}000 \div 52.33 = 155{,}000,$$

and dividing this by 16,000#, we have

$$\frac{155{,}000}{16{,}000} = 9.68 \text{ sq. ins.}$$

for the actual flange area required, which is 0.43□″ less than the above assumed section, but the above section will be used, as 6″ x 4″ x $\frac{3}{8}$″ angles are too small, giving a net area of 0.59□″ less than required.

Dividing the end shear by the area of cross-section of the web, we have

$$\frac{143{,}200}{21} = 6{,}819 \text{ lbs.},$$

for the actual unit shear on the web. Substituting this value for s in (1), Art. 118, we have

$$d = \frac{\frac{3}{8}}{40}\,(12{,}000 - 6{,}819) = 48.5 \text{ ins.},$$

for the allowable distance between stiffeners, but as the unsupported distance between stringers and truss is about 3′ − 8″, or 44″, according to the preceding article, no stiffeners are needed and the assumed 56″ x $\frac{3}{8}$″ web will be used. The actual allowable stress on it is determined by substituting 44″ for d in (1), Art. 118, and solving for s. Thus we have

$$44 = \frac{\frac{3}{8}}{40}\,(12{,}000 - s),$$

from which we obtain

$$s = 7{,}306 \text{ lbs.},$$

for the actual allowable unit stress. Now dividing this into the maximum end shear, we have

$$\frac{143{,}200}{7{,}306} = 19.57 \text{ sq. ins.},$$

for the required area of cross-section of the web. The area of the 56″ x $\frac{3}{8}$″ web, which we propose to use, is 21□″, which, as is seen, is more than is required, but the area of a 56″ x $\frac{5}{16}$″ web, which is the next in size, contains only 17.5□″. Therefore, a thinner web could not be used even if the specifications would permit it. From this it is seen that the assumed 56″ x $\frac{3}{8}$″ web is as near the correct section as is possible to obtain and hence will be used.

This completes the design of the floor system, and next we will take up the design of the trusses.

173. Determination of Dead-Load Stresses in Trusses.—The dead load of an ordinary bridge is considered as uniformly distributed along the span but applied to the trusses only at the panel points or joints, one-third at the top joints and two-thirds at the bottom joints, in the case of through bridges, and just the reverse in the case of deck spans.

This load consists of the weight of metal in the span and the weight of the deck.

From (4), Art. 124, we have

$$p = 7 \times 150 + 660 = 1{,}710 \text{ lbs.},$$

for the approximate weight of the metal per foot of span, and adding 400# for the weight of the deck, we have

$$1{,}710 + 400 = 2{,}110 \text{ lbs.},$$

for the total assumed dead load per foot of span, or 2,110/2 = 1,055# per foot of truss.

Multiplying this by 25, the panel length in feet, we have

$$W = 1{,}055 \times 25 = 26{,}375 \text{ lbs.},$$

for the panel load on each truss. One-third of this is considered as applied at each top joint and two-thirds at each bottom joint. However, this assumption affects only the verticals, and, as far as the stresses in the other members are concerned, the full panel load could be considered at the bottom joints, or top either, as for that.

After having thus computed the panel load, the next thing to do is to draw a sketch of the truss, as shown in Fig. 249 (as a guide), and then compute the values of tan θ and sec θ.

$$\text{Tan } \theta = \frac{25}{30} = 0.8333 \text{ and sec } \theta = 1.299, \text{ say } 1.3.$$

In determining the dead-load stresses in the web members (diagonals and verticals), we can begin either at the center of the span or at the end. The dead-load stresses in these members can really be determined by mere inspection.

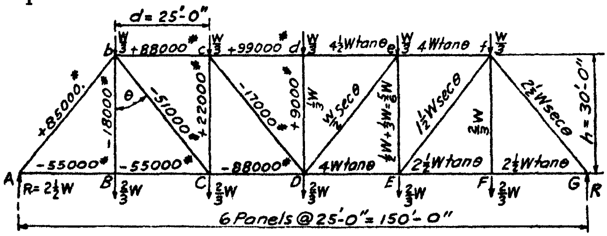


Fig. 249

Beginning at the center of the span and considering the post *dD*, it is obvious that this post can do nothing more than carry the load applied to the top of it, which is one-third of the panel load; so, evidently, the stress in it is 26,375 ÷ 3 = 8,791#, say 9,000#, compression.

The one-third of a panel load transmitted by the post *dD* down to joint *D* combines with the two-thirds at *D* and undoubtedly one-half of this combination is transmitted to the two ends of the truss. Therefore, the shear in panel *CD* is one-half of a panel load and the stress in the diagonal *cD* is

$$\frac{W}{2} \sec \theta = \frac{26{,}375}{2} \times 1.3 = 17{,}143, \text{ say } 17{,}000 \text{ lbs. tension.}$$

The stress in the post cC is equal to the half panel load coming from panel point D, or the vertical pull of diagonal cD, plus the one-third panel load at c. So, for the stress in post cC, we have

$$\frac{W}{2}+\frac{W}{3}=\frac{5W}{6}=\frac{5}{6}\times 26{,}375=21{,}980, \text{ say } 22{,}000 \text{ lbs.}$$

Likewise, it is evident that the diagonal bC must transmit, so to speak, the $\frac{1}{2}W$ coming from the central panel point D, the $\frac{1}{3}W$ from c, and the $\frac{2}{3}W$ from C, making $1\frac{1}{2}W$ in all, which is the shear in panel BC and must, undoubtedly, be equal to the vertical component of the stress in the diagonal bC, and hence the stress in the diagonal is equal to $1\frac{1}{2}\times 26{,}375\times\sec\theta=51{,}430\#$, say $51{,}000\#$, tension.

The member bB, known as a hanger or hip vertical, as is readily seen, can carry nothing more than the load hung on (so to speak) at the joint B, and hence the stress in it is $\frac{2}{3}\times 26{,}375=17{,}582\#$, say $18{,}000\#$, tension.

It is evident that the end post bA must transmit $\frac{1}{2}W$ from the central panel point D, the $\frac{1}{3}W$ from c, the $\frac{2}{3}W$ from C, the $\frac{2}{3}W$ from B, and the $\frac{1}{3}W$ from b, making in all $2\frac{1}{2}W$, which is the shear in panel AB and equal to the vertical component of the stress in this member bA. So the stress in it is equal to $2\frac{1}{2}\times 26{,}375\times\sec\theta=85{,}718\#$, say $85{,}000\#$, compression.

Now, if we begin at the end of the span instead of at the center, as we did above, we have first the reaction $R=2\frac{1}{2}W$ at A. (This is obtained from mere observation.) As this reaction acts vertically the end post bA must resist it, as the end post is the only member at point A having a vertical component. So we have $2\frac{1}{2}W\sec\theta$ for the stress in the end post, the same as found above for this member. The stress in the hanger bB is $\frac{2}{3}W$, as found before. The shear in panel BC is equal to the reaction R $(=2\frac{1}{2}W)$ minus the $\frac{1}{3}W$ at b and $\frac{2}{3}W$ at B, so that we have $2\frac{1}{2}W-W=1\frac{1}{2}W$ for the shear in panel BC. This, evidently, must be equal to the vertical component of the stress in diagonal bC, as that member is the only member in that panel having a vertical component force, and hence the stress in diagonal bC is equal to $1\frac{1}{2}W\sec\theta$, the same as found before. Likewise, the stress in diagonal cD is equal to the shear in panel CD times $\sec\theta$. The shear is equal to $R-2W=\frac{1}{2}W$, and hence the stress is equal to $\frac{1}{2}W\sec\theta$, the same as found before. The stress in the post cC is equal to the vertical component of the diagonal cD, which is $\frac{1}{2}W$ (the shear in panel CD) plus the $\frac{1}{3}W$ at c, making $\frac{5}{6}W$, the same as found before, or it is equal to the vertical component of diagonal bC, which is $1\frac{1}{2}W$ (the shear in panel BC), minus the $\frac{2}{3}W$ at C.

The expression for the stresses in the web members can be written on the members, as shown on the right half of the truss (Fig. 249), and the numerical results of the same may be found very quickly by the use of the slide rule.

For the chord stresses we can assume the full panel loads as applied at the bottom panel points, in order to simplify the work.

Let us first write out the formulas for the chord stresses. For the stress S in bottom chords AB and BC, we have, taking moments about point b (see Fig. 249),

$$Rd-Sh=2\tfrac{1}{2}Wd-Sh=0.$$

Transposing, we have

$$S = 2\tfrac{1}{2}\,W\frac{d}{h} = (2\tfrac{1}{2})\,W\tan\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

For the stress S_2, in chord bc, we have, taking moments about C,

$$R\,(2d) - Wd - S_2 \times h = 2 \times 2\tfrac{1}{2}\,Wd - Wd - (S_2 \times h) = 0,$$

from which we obtain

$$S_2 = 5\,W\frac{d}{h} - W\frac{d}{h} = 5\,W\tan\theta - W\tan\theta = 4\,W\tan\theta \ldots\ldots\ldots\ldots(2)$$

for the stress in chord bc. We obtain the same thing for bottom chord CD by taking moments about c.

For the stress S_3, in top chord cd, we have, taking moments about D,

$$R \times 3d - (W \times 2d) - Wd - (S_3 \times h) = 3 \times 2\tfrac{1}{2}Wd - 2Wd - Wd - (S_3 \times h) = 0,$$

from which we obtain

$$S_3 = 7\tfrac{1}{2}W\frac{d}{h} - 2W\frac{d}{h} - W\frac{d}{h} = 7\tfrac{1}{2}W\tan\theta - 3W\tan\theta = 4\tfrac{1}{2}\,W\tan\theta\ (3).$$

We thus have an expression for the stress in each chord member.

Substituting the numerical values in (1), we have

$$S = 2\tfrac{1}{2} \times 26{,}375 \times 0.8333 = 54{,}945, \text{ say } 55{,}000 \text{ lbs.}$$

tension for the stress in chords AB and BC, and by substituting, likewise, the numerical values in (2) and (3) we obtain 87,672#, say 88,000#, for the stress in the chord members bc and CD, and 98,631#, say 99,000#, for the stress in the chord member cd. This completes the calculations for the dead-load stresses in the trusses, as the structure is symmetrical about the center of the span.

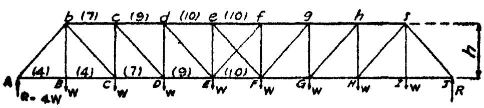


Fig. 250

It will be seen from Formulas (1), (2), and (3) that $W\tan\theta$ is a constant, and after the coefficients are written out the stresses in the chords could be quickly determined by the use of the slide rule.

These coefficients can be written down on the chord members by mere inspection. For example, let Fig. 250 represent a truss of nine equal panels.

Taking moments about b for the stress in the bottom chords AB and BC, we have $4Wd/h$, so the coefficient for chords AB and BC is 4.

Taking moments about C for stress in top chord bc, we have $(4W)2d/h - Wd/h$, so the coefficient for chord bc is 7. It is the same for the bottom chord CD, as the center of moments in that case is at c, which is in the same vertical line as point C. Taking moments about either d or D, for the stress in chords cd and DE, we have the 4 at A multiplied by $3d$ minus the W at B multiplied by $2d$ and minus the W at C multiplied by $1d$, so the coefficients for chords cd and DE are each $(4\times 3)-2-1=9$.

Similarly, taking moments about e or E, for the stress in chords de and EF, also ef, we have $(4\times 4)-3-2-1=10$ for the coefficient to be used in determining the stress in these chords.

After a little practice the student can write these coefficients down without hesitating.

If the slide rule is used, about all that need be written down, outside of the stresses (as they are determined), are the numerical values of the chord coefficients and that of W, $\tan\theta$, and $\sec\theta$, and if found convenient or necessary, $W\tan\theta$ and $W\sec\theta$.

174.—Determination of Live-Load Stresses in Trusses.—First, draw a sketch of the truss, as shown in Fig. 251, for reference.

Member bA. Suppose we start by determining the maximum stress in the end post bA. As is readily seen, the stress in this member will be

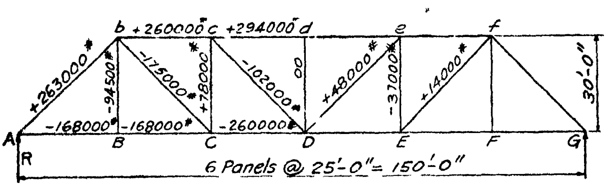


Fig. 251

a maximum when the shear in the panel AB is a maximum, and this will occur when the wheels are in the position that most nearly satisfies the criterion of Art. 90, which is: The load in the panel AB must be as nearly equal as possible to the total load on the bridge divided by the number of panels, and, at the same time, a wheel must be at B. Now placing wheel 4 at B, as shown in Fig. 252, we have (using Table A) $125-91=34$ ft. of uniform load on the bridge, making a total of 352,000# $=(284+34\times 2)\,1{,}000$ per truss, and dividing by the number of panels we have

$$\frac{352{,}000}{6}=58{,}666\text{ lbs.}$$

The load in the panel including one-half of wheel 4 is 60,000#. So this position comes as near to satisfying the criterion for shear as is possible, as will be found by trying the other positions.

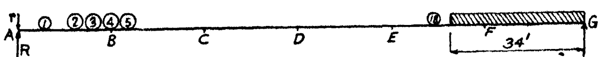


Fig. 252

Taking moments about G (Fig. 252), we have (using Table A)

$$R=\left(16{,}364+284\times 34+2\times\frac{\overline{34}^2}{2}\right)\frac{1{,}000}{150}=181{,}172\text{ lbs.}$$

for the total reaction at A. Then taking moments about B, we have

$$r=\left(\frac{480}{25}\right)1{,}000=19{,}200\text{ lbs.}$$

for the floor beam concentration at A. So we then have

$$R - r = 181{,}172 - 19{,}200 = 161{,}972, \text{ say } 162{,}000 \text{ lbs.}$$

for the maximum shear in the end panel AB, due to Cooper's $E40$ loading and for $E50$ we have

$$162{,}000 \times \frac{50}{40} = 202{,}500 \text{ lbs.}$$

Then for the maximum live-load stress in the end post bA we have

$$202{,}500 \times 1.3 = 263{,}250, \text{ say } 263{,}000 \text{ lbs. (compression)}$$

Member bB. The maximum live-load stress on the hanger bB, as is obvious, is equal to the maximum floor beam concentration found in Art. 171 to be 94,500#, and hence we will not recalculate it.

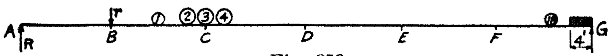


Fig. 253

Member bC. The maximum stress in the diagonal bC will occur when the shear in panel BC is a maximum, as this member carries the shear in that panel. Placing wheel 3 at panel point C, as shown in Fig. 253, we have $100 - 96 = 4$ ft. of uniform load on the bridge, making in all $292{,}000\# = (284 + 2 \times 4)1{,}000$ on one truss.

Dividing this by the number of panels, we have $292{,}000 \div 6 = 48{,}666\#$, and the load in panel BC is 40,000#, including one-half of the wheel 3, at C (see Fig. 253). This does not seem to satisfy the criterion very closely, so we will try wheel 4 at C, in which case we have 9 ft. of uniform load on the bridge, making a total load of 302,000# per truss, and dividing by the number of panels we have $302{,}000 \div 6 = 50{,}333\#$, and the load in the panel is 60,000. This position does not satisfy the criterion as closely as the first, and hence the first position, with wheel 3 at C, will be taken. It is evident that no other positions need be tried, as the load in the panel in the first case was too light and in the second it was too heavy.

Then taking moments about G (Fig. 253), we have (using Table A)

$$R = \left(16{,}364 + 284 \times 4 + 4^2\right)\frac{1{,}000}{150} = 116{,}773$$

for the reaction at A, and taking moments about C we have

$$r = 230 \times \frac{1{,}000}{25} = 9{,}200 \text{ lbs.}$$

for the concentration at point B.

Then, for the maximum shear in the panel BC, due to the $E40$ loading, we have

$$R - r = 116{,}773 - 9{,}200 = 107{,}573,$$

and for the $E50$ loading we have 134,466, say 134,500#. Then multiplying this by $\sec\theta$, we have

$$134{,}500 \times 1.3 = 174{,}850, \text{ say } 175{,}000 \text{ lbs.}$$

for the maximum tensile stress in the diagonal bC.

Member cD. The maximum live-load stress in the diagonal cD will occur when the live-load shear in the panel CD is a maximum. To satisfy the criterion for maximum shear in that panel, let us try wheel 3 at point D (see Fig. 254). This position of the wheels brings wheel 15 exactly at G, the right support.

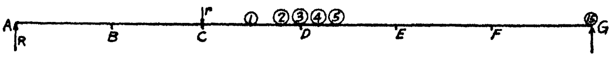


Fig. 254

Then the load on the bridge (not including wheel 15) is 232,000#. Dividing this by the number of panels, we have $232{,}000 \div 6 = 38{,}666$#, and the load in the panel is 40,000# (including one-half of wheel 3). This comes as near satisfying the criterion as is possible, as will be seen by trying other positions.

Now, by taking moments about G (using Table A) we obtain

$$R = 10{,}816 \times \frac{1{,}000}{150} = 72{,}107 \text{ lbs.}$$

for the reaction at A, and taking moments about D we obtain

$$r = 230 \times \frac{1{,}000}{25} = 9{,}200 \text{ lbs.}$$

for the concentration at C.

Then, for the shear in panel CD, due to the $E40$ loading, we have

$$R - r = 72{,}107 - 9{,}200 = 62{,}907 \text{ lbs.}$$

and for the $E50$ loading we have 78,633#. Multiplying this by $\sec\theta$, we have

$$78{,}633 \times 1.3 = 102{,}223, \text{ say } 102{,}000 \text{ lbs.}$$

for the maximum live-load tensile stress in the diagonal cD.

Member cC. As the live load is applied wholly on the bottom chord joints, the post cC will have a maximum compression stress when the diagonal cD has maximum tension, as the post takes only the vertical component of the stress in that diagonal. The maximum vertical component of the stress in the diagonal cD, as seen above, is equal to the maximum shear in the panel CD and given there as 78,633. So the maximum live-load compressive stress in the post cC is, say, 78,000#, using round numbers.

Member dD. There is no live-load stress in the post dD at all, as there is no member (as a diagonal) connecting to the top of it that will resist vertical action. For the sake of illustration, let us assume a live-load stress, S, in this post. Then the components of this stress must be taken by the top chords, and we would have

$$S \cos 90^\circ = C = 0, \text{ as } \cos \text{ of } 90^\circ = 0.$$

Members eE and eD. If the span is loaded from the right end so that maximum shear occurs in panel DE, the post eE will be subjected to maximum live-load tensile stress and the diagonal eD will be subjected to maximum compressive stress.

If wheel 2 be placed at E, wheel 10 will be just 2 ft. from the right end of the span, as shown in Fig. 255, and the total load on the bridge will be 152,000#.

Dividing this by the number of panels we have $152{,}000 \div 6 = 25{,}333\#$, and the load in the panel DE is equal to 20,000#, including one-half of wheel 2. Now, if wheel 3 be placed at E, the total load on the bridge

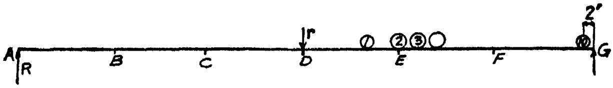


Fig. 255

will be 152,000#. Dividing this by the number of panels, we have $152{,}000 \div 6 = 25{,}333\#$, and the load in the panel DE is then 40,000#. So it is seen that the position of the wheels, shown in Fig. 255, comes nearest to satisfying the criterion for shear in panel DE.

Taking moments about G (Fig. 255), we obtain

$$R = \left(4{,}632 + 152 \times 2\right)\frac{1{,}000}{150} = 32{,}906 \text{ lbs.}$$

for the reaction at A, and taking moments about E we obtain

$$r = 80 \times \frac{1{,}000}{25} = 3{,}200 \text{ lbs.}$$

for the concentration at D.

Then, for the shear in panel DE, due to the $E40$ loading, we have

$$R - r = 32{,}906 - 3{,}200 = 29{,}706 \text{ lbs.},$$

and multiplying by 50/40 we have 37,100# for the shear due to the $E50$ loading, which is the maximum live-load tensile stress in post eE, and multiplying this by $\sec\theta$ we obtain 48,230#, say 48,000#, for the maximum live-load compression stress in diagonal eD.

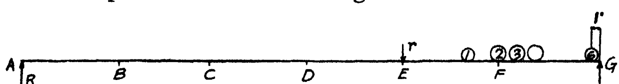


Fig. 256

Member fE. Now placing wheel 2 at F, wheel 6 will be just 1 ft. to the left of the right end of the span as shown in Fig. 256. Then the load on the span is 103,000#. Dividing this by 6 we have 17,166#, and the load in panel EF is 20,000#. This position comes the nearest to satisfying the criterion, as will be found by trial.

Taking moments about G (Fig. 256), we obtain

$$R = \left(1{,}640 + 103\right)\frac{1{,}000}{150} = 11{,}620 \text{ lbs.}$$

for the reaction at A, and taking moments about F we obtain

$$r = 80 \times \frac{1{,}000}{25} = 3{,}200 \text{ lbs.}$$

for the concentration at E.

Then, for the shear in panel EF, due to the $E50$ loading, we have

$$\left(R-r\right)\frac{50}{40}=\left(11{,}620-3{,}200\right)\frac{50}{40}=10{,}525 \text{ lbs.}$$

Multiplying this by $\sec\theta$ we have 13,682#, say 14,000#, for the maximum live-load compressive stress in diagonal fE.

It is seen, from Fig. 251, that diagonal bC has 175,000# tension in it, while the corresponding diagonal fE, on the right half of the span, has 14,000# compression, that the post cC has 78,000# compression, while the corresponding post eE has 37,000# tension and diagonal cD has 102,000# tension while the corresponding diagonal on the right half of the span has 48,000# compression. Now it is evident that, if the span is loaded from the left end instead of the right end, as it was above, and the maximum stresses in the web members computed, just the opposite results will be obtained, that is, for example, diagonal bC would be subjected to 14,000# compression, while diagonal fE would be subjected to 175,000# tension.

So it is seen that we have now determined both maximum tensile and compressive live-load stresses in the web members throughout the structure, and we will next determine the chord stresses.

Members AB and BC. It is obvious that, if either of the members AB or BC (Fig. 251) be cut, rotation would take place instantly about

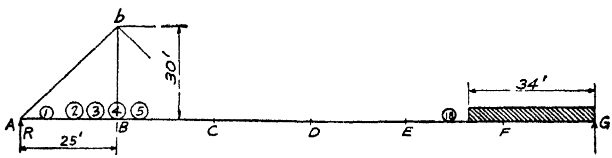


Fig. 257

joint b. Therefore these members undoubtedly prevent rotation about that joint, and hence the greater the tendency of rotation the greater the stress in these two members; so undoubtedly the maximum live-load stress in them will occur when the live-load bending moment about b is a maximum.

According to Art. 91, the moment about b will be a maximum when the average unit load on the left of the panel point is equal to the average unit load on the bridge.

Placing wheel 4 at B (Fig. 257), we have 34 ft. of uniform load on the bridge (see Table A). Then for the average unit load on the span we have

$$\left(284+34\times2\right)\frac{1{,}000}{150}=2{,}346 \text{ lbs.}$$

and for the average unit load on the left of B (or b) we have, including one-half of wheel 4, $60{,}000\div25=2{,}400$#.

This position of the wheels satisfies the criterion as nearly as possible, as will be found by trying other positions of the wheels.

Now, taking moments about G, we have

$$R=181{,}173 \text{ lbs.}$$

for the reaction at A. Then taking moments about b we have, using Table A,

$$181{,}173 \times 25 - 480 \times 1{,}000 = 4{,}049{,}300 \text{ ft. lbs.}$$

for the maximum bending moment about b, and multiplying this by 50/40 and dividing by the height of span (see Art. 92), we obtain 168,000# tension for the maximum live-load stress in member AB, and also in BC.

Members bc and CD. By imagining the top chord bc (Fig. 258) to be cut, it is readily observed that this member resists rotation about joint C. So the live-load stress in the top chord bc will, evidently, be a maximum when the load is in the position for maximum moment about panel point C, and a maximum in the bottom chord CD when the load is in the position for maximum moment about panel point c. But as joints C and c are in the same vertical line the position of the wheels will be the same in the two cases, and it will make no difference which joint be taken as the center of moments.

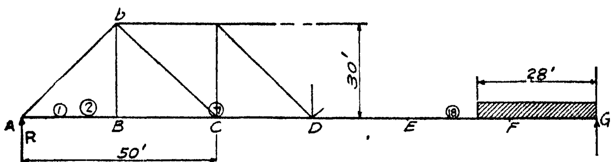


Fig. 258

By placing wheel 7 at panel point C, there will be 28 ft. of uniform load on the bridge, as shown in Fig. 258, and we have

$$(284{,}000 + 56{,}000) \div 150 = 2{,}266 \text{ lbs.}$$

for the average unit load on the bridge, and

$$\left(103{,}000 + \frac{13{,}000}{2}\right) \div 50 = 2{,}190 \text{ lbs.}$$

for the average unit load to the left of joint C. This position of the wheels comes as near to satisfying the criterion for maximum moment about joint C or c as possible. Then, taking moments about G, we have (using Table A)

$$R = \left(16{,}364 + 284 \times 28 + \overline{28}^2\right)\frac{1{,}000}{150} = 167{,}333 \text{ lbs.}$$

for the reaction at A (Fig. 258), and then taking moments about C we have

$$167{,}333 \times 50 - (2{,}155 \times 1{,}000) = 6{,}211{,}650 \text{ ft. lbs.}$$

for the bending moment about joint C. Now, it is a question as to whether the actual maximum moments occur at C or at the corresponding joint E on the right half of the structure, as the load passes to the left over the bridge. Therefore, to make sure, we will test for joint E.

By placing wheel 14 at E, we have 20 ft. of uniform load on the bridge, as shown in Fig. 259, and for the average unit load on the span

we have

$$\left(284+40\right)\frac{1,000}{150}=2,160,$$

and for the average unit load to the left of E we have

$$\frac{222,000}{100}=2,220.$$

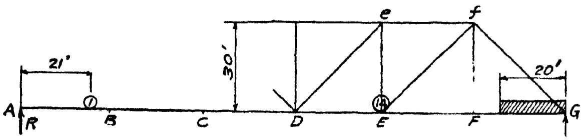


FIG. 259

This position of the wheels comes nearest to satisfying the criterion for maximum moment about E. Taking moments about G, with the wheels in this position, we obtain

$$R=\left(16,364+284\times 20+\overline{20}^2\right)\frac{1,000}{150}=149,600$$

for the reaction at A, and taking moments about E of the forces to the left we obtain

$$149,600\times 100-8,728\times 1,000=6,232,000 \text{ ft. lbs.}$$

for the maximum moment about E, which is greater than found above for point C, and consequently will be taken as the maximum bending moment for determining the stress in chords bc and CD or for the corresponding chords ef and DE on the right half of the bridge—which amounts to the same. Multiplying this moment by 50/40 and dividing by 30, the depth of the truss (in feet), we have 259,666, say 260,000#, for the stress in top chord bc and also in bottom chord CD, being compression in bc and tension in CD.

It is obvious from Fig. 259 that the bending moment about E would be increased if a uniform load be placed to the left of wheel 1, in which case a uniform load would be preceding the engines as well as following them. This loading is used in some cases, but, when such is used, the impact added should be materially less than when the engines are not preceded by a uniform load, for the reason that the speed of the train is not likely to be very high when the engines are pushing as well as pulling a load. In other words, a train so loaded is not likely to reach the "critical speed."

As an illustration, let us assume wheel 13 at E and 18 ft. of uniform load (2,000# per ft.) ahead of the engines, as indicated in Fig. 260.

Then, for the average unit load on the bridge, we have

$$\left(36+284+30\right)\frac{1,000}{150}=2,333 \text{ lbs.},$$

and for the average unit load to the left of E we have

$$\left(36+202\right)\frac{1,000}{100}=2,380\text{ lbs.}$$

This position of the loading comes nearest to satisfying the criterion for the maximum moment about E. Taking moments about G, we obtain

$$R=\left(36\times 141+16,364+284\times 15+\overline{15}^2\right)\frac{1,000}{150}=172,833\text{ lbs.}$$

for the reaction at A.

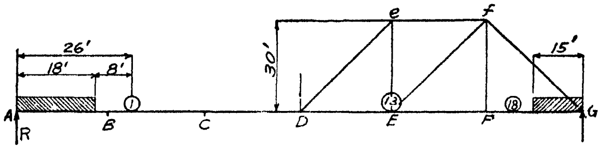


Fig. 260

Now taking moments about E, of the forces to the left, we obtain

$$172,833\times 100-7,668\times 1,000-36\times 91\times 1,000=6,339,300\text{ ft. lbs.},$$

for the maximum bending moment due to Cooper's $E40$ loading. Multiplying by 50/10 and dividing by 30 we obtain 264,137#, say 264,000#, for the stress in chords bc and CD, which, as is seen from the above, is about 4,000# greater than for the usual load, but if the impact is reduced, say, 50 per cent below the usual amount, the final result would be mucl less than that obtained from using the usual loading.

Member cd. It is evident that the maximum live-load stress will occur in the top chord cd when the live-load moment about joint D is a maximum. By placing wheel 12 at D we have 35 ft. of uniform load, as shown in Fig. 261, and for the average unit load on the bridge we have

$$\left(284+70\right)\frac{1,000}{150}=2,360\text{ lbs.},$$

and for the average unit load on the left of D we have (including one-half of wheel 12)

$$\frac{182,000}{75}=2,427\text{ lbs.}$$

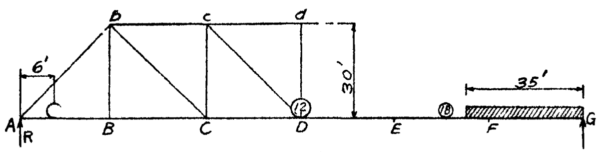


Fig. 261

This position of the loading comes the nearest to satisfying the criterion for maximum moment about joint D. Taking moments about G, we obtain

$$R=\left(16,364+284\times 35+\overline{35}^2\right)\frac{1,000}{150}=183,526\text{ lbs.}$$

for the reaction at A, and taking moments about joint D we obtain

$$183{,}526 \times 75 - 6{,}708 \times 1{,}000 = 7{,}056{,}450 \text{ ft. lbs.}$$

for the maximum live-load moment about that point, due to the $E40$ loading. Then multiplying this by 50/40 and dividing by 30 we have 294,018, say 294,000# compression stress in top chord *cd*.

As the bridge is symmetrical about the center of span we have all of the live-load stresses now determined and we will next determine the impact and make a summary of all the stresses in the trusses.

175. Summarizing of Stresses and Determination of Impact in the Trusses.—The dead- and live-load stresses can be written on the members, as shown in Fig. 262, directly from Figs. 249 and 251, and we have only the impact to determine.

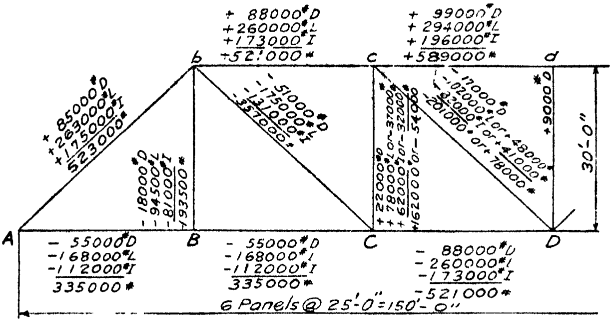


Fig. 262

The live load extends practically over the entire length of the bridge, as seen above, when the chords and end posts receive their maximum live-load stress. So, in determining the impact stress in these members, L in the impact formula (Art. 125) will be taken as the total length of the span, or 150 ft.

Then we have

$$C = \frac{300}{150 + 300} = 0.6666$$

for the coefficient of impact for these members, and multiplying the live-load stress in each by this coefficient we obtain the impact given on them in Fig. 262. As an example, the impact in the end post bA is $263{,}000 \times 0.6666 = 175{,}315$, say 175,000#; in chord bc it is $260{,}000 \times 0.6666 = 173{,}316$, say 173,000#; and so on for the other chord members.

The maximum live-load stress in hanger bB results from loads in the two adjacent panels, AB and BC, alone. So the L in the impact formula would be taken as 50 ft., the sum of the lengths of the two panels, in determining the impact stress in that member. Then for the impact stress in hanger bB we have

$$94{,}500 \times \left(\frac{300}{50 + 300}\right) = 81{,}000 \text{ lbs.}$$

When the maximum live-load tensile stress occurs in the diagonal bC the live load extends from the right support to a little beyond panel point C (see Fig. 253). In the case of a uniform live load the distance from the right support C would be taken as L in the impact formula, as in that case the loads are considered to be only at panel points, and as the result is affected but little we will use the same distance in the case of the concentrated loads. So for the impact stress in diagonal bC we have

$$175{,}000\times\left(\frac{300}{100+300}\right)=131{,}250,\text{ say }131{,}000\text{ lbs. tension.}$$

The maximum live-load compression occurs in post cC when the load extends from the right support to a little beyond panel point D, and the maximum tension occurs in diagonal cD (see Fig. 254) at the same time. So the distance from the right support to panel point D (75 ft.) will be taken as L in determining the impact in these members. Then for the impact (compression) in post cC we have

$$78{,}000\times\left(\frac{300}{75+300}\right)=62{,}400,\text{ say }62{,}000\text{ lbs.,}$$

and for the impact (tension) in diagonal cD we have

$$102{,}000\times\left(\frac{300}{75+300}\right)=81{,}600,\text{ say }82{,}000\text{ lbs.}$$

The maximum live-load tension in post cC and maximum compression in diagonal cD occur when the load extends from the left support to a little beyond panel point C. (See case of corresponding members (eE and eD) on right half of truss, Fig. 255.) So the distance from the left support to panel point C will be taken as L in determining the impact in this case, and we have

$$37{,}000\times\left(\frac{300}{50+300}\right)=31{,}685,\text{ say }32{,}000\text{ lbs.}$$

for the impact (tension) in post cC and

$$48{,}000\times\left(\frac{300}{50+300}\right)=41{,}146,\text{ say }41{,}000\text{ lbs.}$$

for the impact (compression) in diagonal cD.

The impact stress found above for each respective member can now be written on each member, and adding them in each case to the dead- and live-load stresses the total combined stresses, as shown in Fig. 262, are obtained.

In the case of members cC and cD we have what is known as reversal of stress. The dead-load stress, which acts at all times, is compression in post cC and tension in diagonal cD. When the live load moves onto the structure from the left a tensile stress of 37,000# occurs in post cC and a compression stress of 48,000# in diagonal cD, as shown for the corresponding members (eE and eD) on the right half of the bridge (Fig. 251). Now it is evident that, if the dead-load compression in post cC were just equal to the 37,000# live-load tension, the two stresses would just balance each other and consequently the actual stress in the post would be zero when the maximum live-load tension occurred in the post; but, if the dead-load stress were less than the live, the actual stress would be the difference between the two. In any case, before the dead load is subtracted the impact should be added to the live-load stress. To be sure that reversal is well provided for, as a rule not all of the dead-load stress is subtracted from the sum of the live-load and impact stresses. The A. R. E. Ass'n specification, the one we are using, calls for $\frac{2}{3}$. Therefore, for the tensile stress in post cC, we have

$$-37,000-32,000+\frac{2}{3}\times 22,000=-54,334, \text{ say } 54,000 \text{ lbs.},$$

and for the compressive stress in diagonal cD we have

$$+48,000+41,000-\frac{2}{3}\times 17,000=+77,667, \text{ say } 78,000 \text{ lbs.}$$

In case $\frac{2}{3}$ of the dead-load stress in any member is greater than the sum of the reverse live-load stress and impact, the member is not reversed and, consequently, the reverse stress is ignored, as in the case of diagonal bC. In bC we have 14,000# (see Fig. 251) live-load compression and $14,000\times 300\div(25+300)=13,000$# (about) impact, making in all a stress of $14,000+13,000=27,000$# compression, but $\frac{2}{3}$ of the dead-load tension is $51,000\times 2/3=33,666$# and hence no reversal takes place, that is, the diagonal is always in tension, due to dead load, and, consequently, the reversal stress can be ignored. The same is true in all such cases.

176. Designing of Members in Trusses.—After all of the maximum stresses are computed in the members of the trusses and combined as shown in Fig. 262, the next thing, logically, to do is to design the sections of these members. The first thing to do in such work is to glance, so to speak, over the structure and associate the details of the different members and, as far as possible, ascertain the governing members.

In this case, as in all such bridges, the intermediate posts are the governing members, as their width governs the width of practically all of the other members of the trusses. So we will first design the intermediate post cC (Fig. 262).

Member cC. (See Fig. 262.) Let us assume each of these posts to be composed of 2—15″ [s placed, in reference to each other, as shown in Fig. 263. The distance d between the toes of the flanges is really the governing distance. This distance should not be less than $5\frac{1}{2}$″ in any case, and a little more is preferable. If this distance is less than $5\frac{1}{2}$″ it is impossible to get the jaw of an ordinary riveter inside of the member and consequently the rivets would have to be hand driven, which is expensive work. From table 3 it is seen that the average width of flange of 15″ [s is about $3\frac{1}{2}$″, making 7″ for the two flanges, and if we make $d=6\frac{1}{2}$″, we have

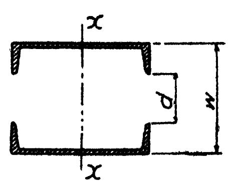


Fig. 263

$$w=6\tfrac{1}{2}+7=13\tfrac{1}{2} \text{ ins.}$$

for the width of the posts. The least average radius of gyration, which is in reference to axis x-x, as seen from Table 3, is about 5.5″. Then substituting this value of r in the column formula of Art. 73, and taking

L as 30 ft., or 360 ins., we have

$$16{,}000 - 70\,\frac{360}{5.5} = 11{,}420 \text{ lbs.}$$

for the allowable unit compressive stress on the post.

For the stress in the post we have 162,000# compression and 54,000# tension, as given in Fig. 262. Now, according to the specifications, each of these stresses must be increased by 0.5 of the lesser. So we have $162{,}000 + 0.5 \times 54{,}000 = 189{,}000$# compression and $54{,}000 + 0.5 \times 54{,}000 = 81{,}000$# tension which the post must be designed to carry.

Taking, first, the case of compression, we have

$$\frac{189{,}000}{11{,}420} = 16.6 \text{ sq. ins. (about)}$$

for the area required for compression and

$$\frac{81{,}000}{16{,}000} = 5.06 \text{ sq. ins.}$$

for the net area required for tension. The area, 16.6□″, required for compression governs. The nearest to this (considering 15″ channels) is 2—[s 15″ x 33#, which have an area of 19.8□″. These channels have 3.2□″ more area than required, so let us try 2—[s 12″ x 30#. Then we have

$$16{,}000 - 70\,\frac{360}{4.28} = 10{,}100 \text{ lbs.}$$

for the allowable unit stress on the post, and dividing this into the stress we have

$$189{,}000 \div 10{,}100 = 18.7 \text{ sq. ins.}$$

for the required area. This shows that the 12″ x 30# channels are too light and that the 12″ x 35# channels would have to be used, which are heavier than the 15″ x 33# channels; and, besides, better details are obtained by using the 15″ channels, so we will use the 2—[s 15″ x 33#. The radius of gyration of these channels is 5.62, within 0.12 of the assumed radius, and hence recalculations are unnecessary.

Member dD. The post dD carries nothing but the 9,000# of dead-load compression and theoretically could be made of very light section, but to keep the floor beams the same throughout the span and other details constant, as well as for general appearance, we will make this post of 2—[s 15″ x 33# (the same as post cC), that is, each of these posts will be made of 2—[s 15″ x 33# (the lightest 15″ channels) regardless of the excess of metal.

Member bB. The hangers bB, or hip verticals as they are often called, are not posts but wholly tension members, and it is not necessary that they be of the same type of section as the posts; however, they are often made so, but more often they are made of four angles built into an I-section. (See Fig. 298.) The total stress, as given in Fig. 262, divided by the allowable unit intensity gives $193{,}500 \div 16{,}000 = 12.09$□″ (net) for the required area of cross-section. We will use 4—∟s 6″ x 4″ x $\frac{3}{8}$″ =

14.44□″ − 1.5□″ = 12.94□″ (net), deducting a one-inch hole (for ⅞-in. rivet) out of each angle. This type of member is cheaper than the two channel sections, as used in the posts, as there are fewer details and, consequently, less shop work, but it is not an economic column section, as the radius of gyration is small in comparison with the area of cross-section contained, as in comparison with two channels of an equal cross-section.

Member bC. The diagonal is subjected to tension only, and, using the maximum stress given in Fig. 262, we have

$$357{,}000 \div 16{,}000 = 22.31 \text{ sq. ins. (net)}$$

for the required area of cross-section. We could use either four angles, as in the case of hanger *bB*, or two channels, as the member is subjected to tension only, but the two-channel section is better in this case as the member is subjected to some cross bending due to its own weight, and as the channel section is the more capable of resisting this it will be used. Let us try 2—[s 15″ x 40#. We can consider either two rivet holes cut out of each web or one hole cut through each flange, whichever is the greater. The web of the 15″ x 40# [is $\frac{17}{32}$″ thick (see Table 3) and the grip thickness (g) on the flange is $\frac{21}{32}$″, hence the holes through the flanges govern. Then we have 2—[s 15″ x 40# = 23.52 − 4 × 21/32 = 20.90□″. As is seen, these two channels are too light. So let us try 2—[s 15″ x 45#. In this case the thickness of the web and the grip through the flanges are the same, and hence either can be taken in determining the net section of member. So we have 2—[s 15″ x 45# = 26.48 − 4 × ⅝ = 23.98□″ (net), which is 1.67□″ more than required, but this is the best we can do if we use channels, so we will use this section for diagonal *bC*.

Member cD. This diagonal is subjected to 201,000# tension and 78,000# compression, as given in Fig. 262. Now increasing each by 0.5 of the lesser, as we did above in the case of post *cC* (according to the specifications), we have

$$201{,}000 + 0.5 \times 78{,}000 = 240{,}000 \text{ lbs. tension}$$

and

$$78{,}000 + 0.5 \times 78{,}000 = 117{,}000 \text{ lbs. compression.}$$

Then dividing the tensile stress by 16,000# (the allowable unit stress) we have

$$240{,}000 \div 16{,}000 = 15.0 \text{ sq. ins.}$$

for the net area of cross-section required to carry the tension. Now assuming the member to be made of two 15″ channels having a radius of gyration of 5.6 (a mere guess) and taking the length of the diagonal as 39 ft., or 468 ins., we have

$$16{,}000 - 70\,\frac{468}{5.6} = 10{,}150 \text{ lbs.}$$

for the allowable compressive unit stress. Dividing this into the compressive stress, we have

$$117{,}000 \div 10{,}150 = 11.5 \text{ sq. ins.}$$

for the required area of cross-section to carry the compression. Deducting for four rivet holes, out of the flanges of the two lightest 15″ channels, we have 2—[s 15″ x 33# = 19.8 − 4 × 21/32 = 17.2□″ (about) for the net area to carry tension, which is 2.2□″ more than required. The total, 19.8□″ is available for compression, so the section is larger than is necessary for compression as well as for tension, but this section will be used as it is desirable to have all of the diagonals of the same width. We now have all of the web members designed and we will next take up the designing of the chord members.

Members AB and BC. The bottom chords are purely tension members; so, dividing the maximum stress in chord AB or BC by 16,000, we have

$$335{,}000 \div 16{,}000 = 20.93 \text{ sq. ins.}$$

for the net area required. By glancing over the table of channels (see Table 3) we see that two 15″ channels can be used as section for these members. Deducting four rivet holes from the flanges we have 2—[s 15″ x 40# = 23.52 − 2.6 = 20.9□″, which is practically the section required, and hence these channels will be used.

Member CD. For the net area of cross-section required for chord CD, we have 521,000 ÷ 16,000 = 32.56 sq. ins. Now, from the table of channels (see Table 3 or any structural handbook, as Cambria or Carnegie) it is seen that no two 15″ channels will have the required net section. So, if channels are used, they will have to be reinforced by riveting plates to their webs. Let us try the following sections:

2—[s 15″ x 45#	= 26.48 − 4 × 5/8	= 23.98□″	net
2—pls. 12″ x 7/16	= 10.50 − 4 × 7/16	= 8.75□″	net
		32.73□″	net

The net area of this is practically equal to the area of the required section and hence will be used.

Top chords. The top chords and end posts of such bridges are usually made up of either two channels and a cover plate as shown at (*a*), Fig. 264, or of two built channels and a cover plate as shown at (*b*). The rolled channels are used whenever that section is sufficient. It is not desirable to use the heaviest channels, which are of $\frac{13}{16}$″ metal, as the shop work is rather expensive on account of the thick metal, and whenever the end post or heaviest chord members exceed the area of the cover plate and, say, two 15″ x 45# channels, a section similar to the one shown at (*b*), Fig. 264, is used throughout the structure, as it would be unsightly construction to build part of the chord members of a bridge of rolled channels and part of plates and angles. So, in this case, we will first ascertain as to whether we can or cannot use the rolled channel section as shown at (*a*), Fig. 264, for the top chord. The member *cd*, as is seen from Fig. 262, will be the heaviest member, that is, the one requiring the greatest area of

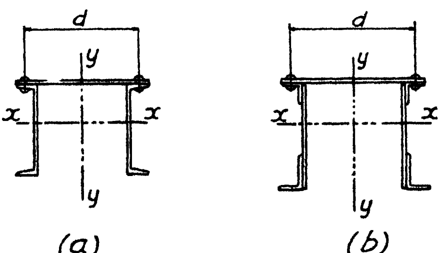


Fig. 264

cross-section. In preliminary calculations the least radius of gyration of this type of section can be taken as 0.40 of its depth. Then, assuming 15″ channels, we have $15\times 0.40=6.0$ for the approximate radius of gyration about axis x-x, which is usually the least radius. Now substituting this value of r and the length of the member, which is 25 ft., or 300 ins., into the column formula, we have

$$16{,}000-70\frac{300}{6.0}=12{,}500 \text{ lbs.}$$

for the allowable unit stress on the member. Then dividing the maximum stress, as given in Fig. 262, by this, we have

$$\frac{589{,}000}{12{,}500}=47.1 \text{ sq. ins.}$$

for the required area of cross-section of the member. The width of the intermediate posts, as found above, is $13\frac{1}{2}$″, and allowing for two $\frac{5}{8}$″ gusset plates and 7″ $(=3\frac{1}{2}\times 2)$ for the flanges of the channels, we have

$$13\tfrac{1}{2}+1\tfrac{1}{4}+7=21\tfrac{3}{4}, \text{ say } 22 \text{ ins.}$$

for the width of the cover plate. The cover plate should be as thin as is consistent with good practice. The specifications limit this to $\frac{1}{40}$ of the distance (shown as d at (a), Fig. 264) between the lines of rivets connecting the plate to the other parts of the member. That distance here, using 2″ gauge in channel flanges, is $13\frac{1}{2}+1\frac{1}{4}+4=18\frac{3}{4}$″, say 19″. One-fortieth of this is practically $\frac{1}{2}$″, so the cover plate would be 22″ x $\frac{1}{2}$″. Now, using the heaviest channels that it is practical to use, we have the following sections:

2—[s 15″ x 45#	= 26.48□″
1—cov. pl. 22″ x ½″	= 11.00□″
	37.48□″

This is about 10□″ less than the required area found above, and hence the rolled channel section has not sufficient area; and, therefore, the type of section shown at (b), Fig. 264, will be used.

In designing the top chords it is best to begin with the lightest section, which is the member having the least stress, and make it of minimum thickness of metal and then increase the area of cross-section of the other top chord members by increasing the thickness of webs, leaving the other parts about constant. For by so doing the centers of gravity of the chords will practically be in the same plane throughout. Therefore, we will take up the designing of chord bc first—it being the lightest.

Member bc. The maximum stress in chord bc, as given in Fig. 262, is 521,000#. Then assuming the allowable unit stress to be 12,500″ (see Example 4, Art. 74), we have

$$\frac{521{,}000}{12{,}500}=41.68 \text{ sq. ins.}$$

for the approximate required area. Next draw a sketch of the cross-section of the chord as shown in Fig. 265. The cover plate and top angles should be made as light as the specifications will permit in order that the gravity axis x-x be as near the center of the web as possible. As seen above, the cover plate must be about $\frac{1}{2}$″ thick and the smallest angles that can be used at the top, considering details and all, are $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$. The bottom angles should be as heavy as is practical to use for the same reason that the top ones are made light. So for the bottom angles we will use $6'' \times 4'' \times \frac{5}{8}''$, as that is about the maximum size and thickness used for such work. The cover plate, top angles, and the bottom angles are usually about constant throughout the chord.

Allowing for 2—$\frac{5}{8}''$ gusset plates and for 2—$\frac{1}{2}''$ webs (a mere guess), we have

$$13\frac{1}{2} + 1\frac{1}{4} + 1 + 7 = 22\frac{3}{4} \text{ ins.}$$

for the width of cover plate. To insure a neat finish, the cover plate should project $\frac{1}{8}''$ or $\frac{1}{4}''$ beyond the angles. So we will make the cover plate 23″ wide. However, the details should always be such that the width of cover plate would be in even inches.

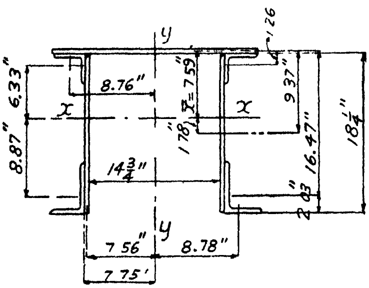


Fig. 265

The depth of webs should be such that the radius of gyration about the horizontal axis (x-x, Fig. 265) is about equal to the radius about the vertical axis (y-y). For preliminary calculations the radius of gyration in reference to axis x-x of such chords can be taken as 0.4 of the depth of the web, and the radius in reference to axis y-y can be taken as the horizontal distance out from the axis to the outer face of the web. So, in this case, we have (assuming $\frac{1}{2}''$ webs)

$$\frac{13\frac{1}{2} + 1\frac{1}{4} + 1}{2} = 7.87$$

for the approximate radius about axis y-y. Then for equal radii about the two axes, we have $0.4h = 7.87$, from which we obtain for the depth of the web

$$h = \frac{7.87}{0.4} = 19.6 \text{ ins.}$$

As the work here is only fairly approximate this figure shows only about what the depth of the web should be, and hence any web about this depth can be used. A 19″ web is an odd width and a 20″ web appears a little deep for this section, so we will use an 18″ web. Now the radius of gyration of the section about the horizontal gravity axis, which is usually the least radius, is about 0.4 of the depth of the web, as stated above, so we have

$$0.4 \times 18 = 7.2 \text{ ins.}$$

for the approximate value of the least radius of gyration of the section. Then substituting this value of r in the column formula, we have

$$16{,}000 - 70\frac{300}{7.2} = 13{,}084, \text{ say } 13{,}000 \text{ lbs.}$$

for the approximate allowable unit stress. Then dividing this into the stress we have

$$521{,}000 \div 13{,}000 = 40.07 \text{ sq. ins.}$$

for the approximate required area. Taking this as a guide, let us assume the following section for chord *bc:*

1—cov. pl. 23″ x ½″	= 11.50□″
2—web pls. 18″ x ⅜″	= 13.50□″
2—Ls 3½″ x 3½″ x ⅜″	= 4.96□″
2—Ls 6″ x 4″ x $\frac{9}{16}$″	= 10.62□″
	40.58□″

The next thing in order is to determine the center of gravity, moment of inertia and radius of gyration of this section and check back to see if the section is actually the correct one to use.

Taking moments about the center of the cover plate (Fig. 265), we have

$$\bar{x} = \frac{13.50 \times 9.37 + 4.96 \times 1.26 + 10.62 \times 16.49}{40.58} = 7.59 \text{ ins.}$$

for the distance from the center of the cover plate to the horizontal gravity axis *x-x*, and for the moment of inertia about this axis we have the following:

$$\begin{aligned}
&\text{cover plate } 11.50 \times \overline{7.59}^2 + 0 &&= 662.49\\
&\text{webs } 13.50 \times \overline{1.78}^2 + 182.25 \times 2 &&= 407.27\\
&\text{top angles } 4.96 \times \overline{6.33}^2 + 2.87 \times 2 &&= 204.48\\
&\text{bot. angles } 10.62 \times \overline{8.87}^2 + 19.26 \times 2 &&= \underline{874.07}\\
& &&\ 2{,}148.31
\end{aligned}$$

Then for the radius of gyration about this same axis *x-x* we have

$$r = \sqrt{\frac{2{,}148.31}{40.58}} = 7.28$$

which is within 0.08 of 0.4 of the depth of the web.

As the section is symmetrical about the vertical plane through the center of the cover plate, the vertical gravity axis *y-y* will pass through the center of the cover plate and for the moment of inertia of the section about this axis *y-y* we have the following:

$$\begin{aligned}
&\text{cover plate } 0 + 506.96 &&= 506.96\\
&\text{webs } 13.50 \times \overline{7.56}^2 + 0 &&= 771.57\\
&\text{top angles } 4.96 \times \overline{8.76}^2 + 2.87 \times 2 &&= 386.36\\
&\text{bot. angles } 10.62 \times \overline{8.78}^2 + 6.91 \times 2 &&= \underline{832.50}\\
& &&\ 2{,}497.39
\end{aligned}$$

Then for the radius of gyration about this axis y-y we have

$$r' = \sqrt{\frac{2{,}497.39}{40.58}} = 7.84 \text{ ins.},$$

which is a little larger than the radius about the x-x axis.

Now substituting the value of r (the least radius) in the column formula, we have

$$16{,}000 - 70\,\frac{300}{7.28} = 13{,}116 \text{ lbs.}$$

for the actual allowable unit stress on the chord. Dividing this into the stress, we have

$$\frac{521{,}000}{13{,}116} = 39.72 \text{ sq. ins.},$$

which is 0.86□″ less than the section assumed above, but as this is about as close as we can obtain we will use the assumed section for chord bc.

Member cd. There will be such a small difference between the allowable unit stress for the other top chord members and that found above for chord bc (as can be verified by actual calculations) that the allowable unit stress found for that member will be used in designing the others. So, dividing the maximum stress in chord cd, as given in Fig. 262, by that unit stress, we have

$$\frac{589{,}000}{13{,}116} = 44.9 \text{ sq. ins.}$$

for the required area of cross-section of the member cd.

Then using the same size angles and cover plate as for chord bc and increasing the thickness of the web to $\frac{1}{2}''$, we have

1—cover pl. 23″ x ½″	= 11.50□″
2—web pls. 18″ x ½″	= 18.00□″
2—Ls 3½″ x 3½″ x ⅜″	= 4.96□″
2—Ls 6″ x 4″ x $\frac{9}{16}$″	= 10.62□″
	45.08□″

This section is quite close to the required section, being only 0.18□″ larger, and hence will be used for chord cd.

This completes the design of the main truss members, as the bridge is symmetrical about the center of the span, except the end posts, which will be designed later, after the bending moment on them due to the wind pressure is determined.

177. Designing of the Bottom Lateral System.—The bottom lateral system is really a horizontal truss in the plane of the bottom chord. It is for the purpose of resisting the wind pressure on the lower portion of the structure, and, in the case of through bridges (such as the one we are now designing), the wind pressure on the train as well as the vibration due to the swaying of the train. A double system of diagonals, or laterals, is practically always used, which is as indicated in Fig. 266, where the bottom lateral system is shown in plain view, one system of laterals being dotted to avoid confusion.

The bottom chords of the main trusses act as the chords in this lateral system and the floor beams as posts. The stress in the chords due to the wind and vibration is, as a rule, ignored unless it reverses the dead-load stress, taking the wind pressure in that case on the unloaded structure as 350# per foot of span or where the maximum exceeds 25 per cent of the maximum combined dead, live, and impact stresses (see specifications), in which case it is combined with the dead, live, and impact stresses as in the case of viaduct columns (Art. 162). This combination, however, is very rarely necessary in ordinary bridges. The stress in the floor beams (acting as columns), due to wind and vibration, is ignored entirely, as the laterals are connected to the bottom flange of the beams,

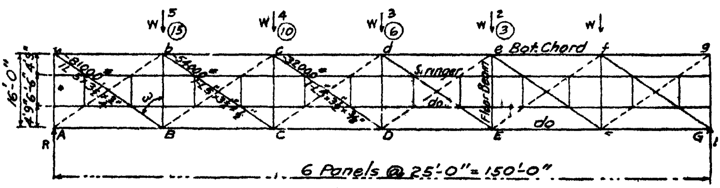


Fig. 266

which are in tension due to dead and live load, and hence the compression in these same flanges, due to wind and vibration, only tends to reverse the dead- and live-load stress in them.

There are two ways of considering the bottom lateral system: One is to consider the double system, in which case the two diagonals in any panel resist equally the shear in the panel, that is, one diagonal is considered as having the same intensity of stress as the other, one being in compression and the other in tension; the other way is to consider only a single system, one system being in action when the pressure comes from one direction and the other system acting when the pressure comes from the opposite direction. We will consider the single system, in which case the diagonals carry only tension, and then investigate for rigidity after the sections are determined for the single system.

According to the specifications, the pressure or horizontal load on the bottom lateral system, here considered, is $200+0.10\times 5{,}000=700\#$ per foot of span. So, for a panel load we have

$$W=700\times 25=17{,}500 \text{ lbs.}$$

This is, according to the specifications, to be considered a moving load (live load). Let us assume that this load acts in the direction indicated by the arrows (Fig. 266) and that it moves over the structure from right to left, that is, from G to A. Now, from Art. 90, we have

$$S=\frac{W}{n}\Big[1+2\ldots.(n-1)\Big]$$

for the maximum shear in the different panels, and hence for the maximum stress in the corresponding diagonal (considering single system) we have

$$T=\frac{W}{n}\Big[1+2\ldots.(n-1)\Big]\sec\omega.$$

Tan $\omega = 25/16 = 1.56$, and sec $\omega = 1.85$, which is most readily obtained from a table of natural functions after the tangent is computed.

Then for the maximum stress in diagonal cD we have

$$S = \frac{W}{n}(1+2+3)\sec\omega = \frac{17{,}500}{6} \times 6 \times 1.85 = 32{,}375,$$

say 32,000 lbs. (tension),

and for the maximum stress in bC we have

$$S' = \frac{W}{n}(1+2+3+4)\sec\omega = \frac{17{,}500}{6} \times 10 \times 1.85 = 53{,}958,$$

say 54,000 lbs. (tension),

and for the maximum stress in aB we have

$$S'' = \frac{W}{n}(1+2+3+4+5)\sec\omega = \frac{17{,}500}{6} \times 15 \times 1.85 = 80{,}937,$$

say 81,000 lbs. (tension).

This completes the necessary determination of stress in the diagonals or laterals (as they are called) due to wind and vibration, as the structure is symmetrical about the center of the span, and we will next design the sections for these members.

Taking the first lateral aB, we have

$$\frac{81{,}000}{16{,}000} = 5.06 \text{ sq. ins.}$$

for the net area of cross-section required. Using 1—∟ 5″ x 3½″ x ¾″ = 5.81 − 0.75 = 5.06□″, we have exactly the correct net area for that diagonal. For lateral bC, we have

$$\frac{51{,}000}{16{,}000} = 3.37 \text{ sq. ins.}$$

for the net area of cross-section required and by using 1—∟ 5″ x 3½″ x ½″ = 4 − 0.5 = 3.5□″, we have about the correct area, at least about as near the correct area as is possible to obtain. For lateral cD, we have

$$\frac{32{,}000}{16{,}000} = 2 \text{ sq. ins.}$$

for the net area of cross-section required, and by using 1—∟ 5″ x 3½″ x ⅜″ = 3.05 − 0.37 = 2.68□″ we have 0.68□″ more net area than required, but we will use this angle so as to have the bottom laterals made of 5″ x 3½″ angles throughout.

As the structure is symmetrical about the center of the span, we now have all of the laterals designed, considering the single system, and we will now investigate the double system. The laterals are connected to the bottom of the stringers at the points of intersection and hence the longest unsupported length of lateral is between the stringer and truss. This maximum unsupported length is about 8′-0″, or 96″. Now taking the end lateral aB, which is composed of 1—∟ 5″ x 3½″ x ¾″, we have $L/r = 96/0.96 = 100$, which is quite satisfying as the maximum value for L/r is 120, and substituting the value of L and r in the column formula we have

$$16{,}000 - 70\frac{96}{0.96} = 9{,}000 \text{ lbs.}$$

for the allowable compressive unit stress, and dividing this into one-half of the stress found above (the stress that it would be considered to resist if acting in the double system) we have

$$\frac{40{,}500}{9{,}000} = 4.5 \text{ sq. ins.}$$

for the required area of cross-section, which is considerably less than the cross-section of the angle. So this lateral is sufficient, considering either a single or double system, and the same is true for the other laterals, as will be found upon investigation. So the bottom laterals as designed are capable of resisting the forces coming on them from wind and vibration when considered either as a single or double system.

The bottom laterals are subjected to stress from traction in addition to the stress resulting from wind and vibration. As an illustration, suppose a rapidly moving train to come onto the structure and the brakes to

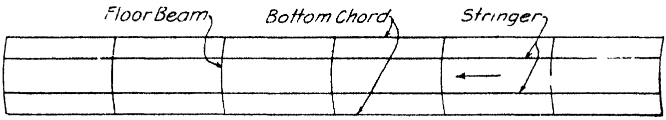


Fig. 267

be suddenly applied—the wheels would skid or tend to skid along the rails, whereby a longitudinal thrust would be exerted along the stringers which would tend to bend the floor beams transversely as indicated in Fig. 267 (where the train is assumed to be moving from right to left). Now this bending of the floor beams, which would cause serious stresses in them, is prevented by attaching the laterals to the bottom of the stringers so that the longitudinal thrust exerted by the stringers is transmitted directly to the laterals, and from there to the bottom chords of the trusses and on out to the end supports. As the stringers are connected to one another rigidly, end to end, throughout the length of the structure, it is evident that the traction from any loading would be transferred more or less from stringer to stringer, and hence would be distributed to some extent to all of the laterals; so in computing the stress in the laterals due to traction we will consider that the laterals in any panel resist the traction from an average panel load of fully loaded bridge. Then placing wheel 1 (see Table A) at one end of the bridge, we have

$$2\,(284 + 41 \times 2)\,1{,}000 \times \frac{50}{40} = 915{,}000 \text{ lbs.}$$

for the weight of the full live load (Cooper's $E50$ loading) on the bridge. Dividing this by the number of panels, we have

$$\frac{915{,}000}{6} = 152{,}500 \text{ lbs.}$$

for the average panel load. Two-tenths (0.2) of this vertical load is considered as traction and is exerted along the track, that is, the coefficient of friction of the wheels on the rails is taken as 0.2. So we have

$$152{,}500 \times 0.2 = 30{,}500 \text{ lbs.}$$

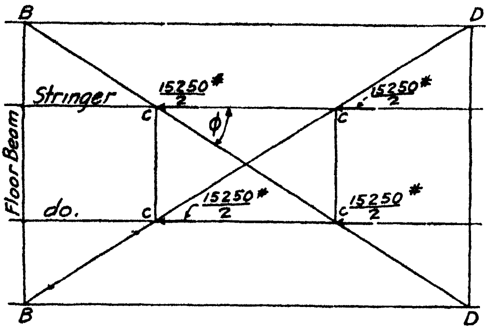


Fig. 268

as the traction force exerted on the two stringers in each panel, or 15,250# to each stringer. This force on each stringer is transmitted to the laterals at two points, where the laterals and stringers connect, so we can consider the case as indicated in Fig. 268. One component of the traction force, 15,250# ÷ 2 applied at each of the points *c* will, in each case, act along the lateral, causing compression in the case of the part *Bc* and tension in the part *Dc*, and the other component at each point *c* will act along member *cc*.

Without the members *cc*, there would be transverse bending on the stringers from traction, and hence *cc* is an essential member in the system.*

Then for the stress in a lateral, due to traction, we have

$$\frac{15{,}250}{2} \times \sec\phi = \frac{15{,}250}{2} \times 1.18 = 8{,}997, \text{ say } 9{,}000 \text{ lbs.},$$

which is compression in the part *cB* and tension in part *cD*, and just the reverse if the train were moving in the opposite direction. For the stress in member *cc*, which can be called a tie or strut, we have

$$\frac{15{,}250}{2} \times \tan\phi = \frac{15{,}250}{2} \times 0.64 = 4{,}880 \text{ lbs.},$$

which is compression in one and tension in the other.

These stresses are not very great in the first place, as is seen, and they are not very likely to occur at the same time that the maximum wind stress occurs and hence, as a rule, are ignored in the designing of the sections of the laterals. However, the traction should be provided for in so far as making the laterals capable of taking compression ($L/r < 120$) and the inserting of the ties *cc*. The tie *cc* does not necessarily have to be a very heavy section. They are usually made of 1—∟ $3\frac{1}{2}'' \times 3'' \times \frac{3}{8}''$, in which case we have

$$16{,}000 - 70\frac{78}{0.90} = 9{,}934 \text{ lbs.}$$

* As far as the author knows, Dr. J. A. L. Waddell, M.A.Soc.C.E., was the first to point out the necessity of these struts.

for the allowable unit stress for compression. Then we have

$$\frac{4,880}{9,934}=0.49 \text{ sq. ins.}$$

for the required area in the case of compression, and less is required for tension, as is obvious. So it is seen that the theoretical required area o' these struts (cc) is not the governing feature, but that it is a matter of obtaining sufficient rigidity. That means that L/r should not exceed 120 in any case. The L/r for a $3\frac{1}{2}'' \times 3'' \times \frac{3}{8}''$ angle is about 0.87, which is reasonably low. This angle is used as it is a very common angle in this class of work, and it is about the minimum size permitted and it fits the case wherein common judgment decides.

For the wind stress in the bottom chord ab or BC (Fig. 266) we have

$$2\tfrac{1}{2}\, W \tan \omega = 2\tfrac{1}{2} \times 17,500 \times 1.56 = 68,250 \text{ lbs.},$$

which is compression in ab and tension in BC, in this case. Now, as shown in Fig. 262, the maximum combined stress, dead, live and impact, is 335,000# tension. Twenty-five per cent of this is 83,750#, so the maximum wind stress in these chords can be ignored. (See specifications.) In case of combined dead, live, impact, and wind stresses, the allowable unit stress is raised 25 per cent, as the probability of all of these stresses being a maximum at the same time is very remote. The same is true of the other bottom chords, as will be found upon investigation. Now, so far, the wind stresses in the bottom chords can be ignored; but further, suppose the bridge to be unloaded, that is, no live load on it, the wind pressure of course will be less. Let us then assume a pressure of 350# per foot of span, which is about correct, instead of 700#. Then the stress in the chord ab or BC is

$$68,250 \times \frac{350}{700} = 34,125 \text{ lbs.},$$

which here, in the case of ab, is compression. But, as this does not reverse the dead-load tension in the member, which is 55,000#, the wind-load stress need not be considered. The same will be found to be true for the other bottom chords, so the stresses in the bottom chords of this bridge, due to wind—or lateral pressure, as the specifications call it—can be ignored. The same is true for most all ordinary railroad bridges.

178. Designing of the Top Lateral System.—The top lateral system is really a horizontal truss in the plane of the top chords. In the case of through bridges (such as we are designing) it is for the purpose of holding the top chords transversely and resisting the wind pressure coming on the top portion of the structure, but in the case of deck bridges, where the track is on the top of the structure, the wind pressure on the train and vibration due to the train must be provided for also, in which case the designing of the top lateral system would be the same as shown in the last article for the bottom lateral system. The top lateral system, as a rule, has a double system of diagonals, the same as the bottom laterals, except in the case of through bridges a portal is placed in each end panel in the plane of the inclined end posts which takes the place of diagonals in those panels. So, following the usual practice, the top lateral system in this case will be made as shown in Fig. 269, where one system is dotted to avoid confusion.

The lateral pressure or load which this system must be designed to carry, according to the specifications, is 200# per foot of span. This horizontal load should be considered as a moving load, that is, as a uniform live load. Then for a panel load we have

$$P = 25 \times 200 = 5{,}000 \text{ lbs.},$$

one-half of which is to be considered as applied to each truss, that is, wherever it is necessary to do so

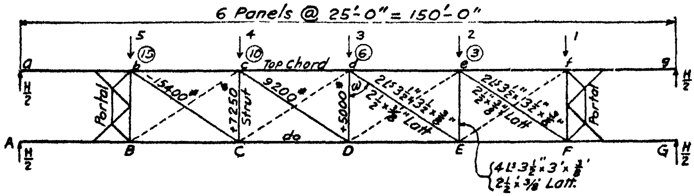


Fig. 269

Now, assuming this load as moving onto the bridge from the right end, and acting in the direction indicated by the arrows (Fig. 269), we obtain

$$6 \times \frac{P}{6} \sec \omega = 6 \times \frac{5{,}000}{6} \times 1.85 = 9{,}250, \text{ say } 9{,}200 \text{ lbs.}$$

for the maximum tensile stress in diagonal cD, and

$$10 \times \frac{5{,}000}{6} \times 1.85 = 15{,}416, \text{ say } 15{,}400 \text{ lbs.}$$

for the maximum tensile stress in diagonal bC.

Assuming a full panel load at each of the points F and E and a half panel load at d (none at D), we have

$$3 \times \frac{5{,}000}{6} + \frac{5{,}000}{2} = 5{,}000 \text{ lbs.}$$

for the maximum compressive stress in strut dD, and assuming a full panel load at each of the points F, E, and D, and a half panel load at c (none at C), we have

$$6 \times \frac{5{,}000}{6} + \frac{5{,}000}{2} = 7{,}550 \text{ lbs.}$$

for the maximum compression in strut cC.

This really completes the necessary calculation for the stresses in the top lateral system, as will be seen upon investigation. For example, when panel point F alone is loaded, and the forces acting as indicated, the stress in diagonal eF will be tension, as is obvious, but this diagonal has maximum tension when the panel points are loaded from the left end up to and including point E with forces acting in the opposite direction, in which case the stress in the diagonal would be 15,400#, the same as found for diagonal bC and loading points F and E, and the forces acting as indicated, diagonal dE would be in tension; but the maximum tension will occur in this diagonal when the span is loaded from the left end up to and including point D and forces acting in the opposite direction, in which case the tension in diagonal dE would be 9,200#, the same as found for diagonal cD. So it is seen that one system can be considered in action when the span is loaded from one direction and the other system when loaded from the opposite direction, and as the lateral system is symmetrical about the center of span it is obvious that the stresses determined above are sufficient for designing all the members in the system.

The members (diagonals and struts) should be as deep as the top chords, in order to hold the top chords rigidly. The diagonals, which are usually designed as tension members, are as a rule made of two angles latticed together, one angle being in the plane of the top of the chord and the other being in the plane of the bottom of the chord. The struts (members cC, dD, and eE) are compression members and are, as a rule, made of four angles, two at the top of the chord and two at the bottom. These are latticed together in the vertical plane so as to form an I-section.

Now, beginning with diagonal bC (Fig. 269), we have

$$\frac{15,100}{16,000}=0.9 \text{ sq. ins.}$$

for the required net area of cross-section of the member, considered as a tension member. This, as is seen, (theoretically) calls for two very small angles, but (practically) the angles should be such that L/r be not less than 120, to insure against vibration. The length of L can be taken as one-fourth of the total length of the member.

(The diagonals are connected to each other at their intersection; this alone reduces the length one-half, and each half can be considered as a fixed column, thus reducing the length to one-fourth of the total length.) The total length of the diagonal center to center of end connections is about 30 ft., or 360 ins. Then using one-fourth of this, we have

$$r=\frac{90}{120}=0.75$$

for the required radius of gyration.

According to this and to the required area of cross-section, found above, $2\frac{1}{2}''\text{x}\,2\frac{1}{2}''$ or $3''\text{x}\,3''$ angles could be used, but as much better details can be obtained by using $3\frac{1}{2}''\text{x}\,3\frac{1}{2}''$ angles—and the weight of the top lateral system is a small item—we will use 2—Ls $3\frac{1}{2}''\text{x}\,3\frac{1}{2}''\text{x}\,\frac{3}{8}''$ for each of the diagonals throughout. as cB, eF, and fE require the same section as bC, and the others, theoretically, require less.

The designing of the struts cC, dD, and eE is very similar to the designing of the diagonals; it is mostly a matter of obtaining rigidity and the selecting of sections that are satisfactory as to details. But in all such cases the theoretical requirement should always be determined before the selection of section is made.

These struts are more or less fixed at their ends, but as they are wholly compression members we will take the distance center to center of trusses, as their length, which is 16 ft., or 192 ins. In order to insure rigidity, L/r should not exceed 120. Then, taking L as 192, we have

$$r=\frac{192}{120}=1.6$$

for the allowable maximum radius of gyration. The most satisfactory section for these struts is four unequal leg angles, arranged in reference to one another as shown in Fig. 270. The long legs of the angles are turned out so as to make the radius of gyration about axis y-y as large as is possible, for, as is obvious, the least radius is about that axis. The lattice bars hold the angles about ½″ apart, and, as the radius of gyration about axis y-y is the same for one pair of angles as it is for the two pairs, we can select the angles from Table 5 by using the angles having $r3$ equal to 1.6—the required radius. As is seen in this table, the 3″ x 2½″ angles are too small, as $r3$ for them is less than 1.6; the 3½″ x 2½″ are large enough, but to use a 2½″ leg would necessitate the using of ¾″ rivets, while ⅞″ would be used in all the other members; so we will use the 3½″ x 3″ x ⅜″ angles, in which case $r3$ is a little more than 1.71, as seen in Table 5. Now substituting this value for r in the column formula, we have

$$16{,}000-70\frac{192}{1.71}=8{,}140 \text{ lbs. (about)}$$

for the allowable unit stress on a strut made of these angles. Dividing this into the greatest stress found in these struts, which is 7,250#, we obtain less than a square inch of metal for the required section. So it is seen that the stress does not really influence the design and that each

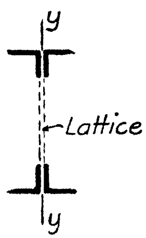


Fig. 270

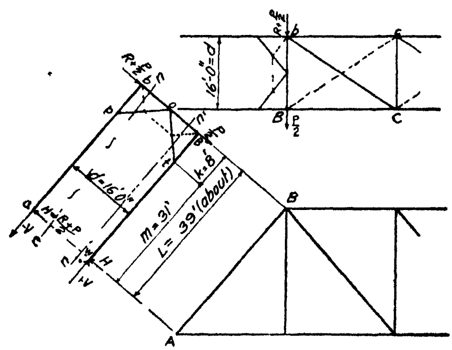


Fig. 271

strut will be made of 4—Ls 3½″ x 3″ x ⅜″ = 9.2□″, as this fits the case most satisfactorily as regards rigidity and details.

179. Design of Portals.—There are several types of portals in use. The type shown in Figs. 269 and 271 will be used in this case, as it is a type commonly used; it is quite simple, very rigid, and in fact quite satisfactory for ordinary bridges.

In determining the stresses in portals, the first thing to do, the same as in the case of all frames, is to locate all of the applied forces affecting it. As is obvious, the maximum stresses in the portals will occur when all of the panel points B to F, inclusive (Fig. 269), are loaded. As a specific case, let us consider the portal at the left end of the bridge (at bB) and let us assume that all of the panel points are loaded and that the loads act as indicated in Fig. 269. Then assuming that the diagonals carry only tension, the diagonal bC (the diagonal connecting to the portal at b) will be in tension and there will be no stress in diagonal cB. One component of the stress in bC will be taken by the portal and the other by the top chord bc. The component taken by the portal is applied (as is obvious) at b and is equal to the shear in panel BC, the panel next to the portal. This force we will designate as R. In addition, there will be one-half of a panel load at b and also one-half of a panel load at B. Designating the panel load as P, we then have the loading on the portal as shown in Fig. 271: $(R+P/2)$ at point b and $P/2$ at point B. If the pressure, or load, acted in the opposite direction, the loading of the portal would be just the reverse, that is, the force $(R+P/2)$ would be applied at B and $P/2$ at b. These forces applied to the portal are held horizontally by the two equal horizontal reactions, one at the bottom of each end post. Let each of these reactions be represented by H and we have

$$H=\left[\left(R+\frac{P}{2}\right)+\frac{P}{2}\right]\frac{1}{2}=\frac{R+P}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

These two horizontal reactions at the bottom of the end posts and the horizontal forces, $(R+P/2)$ and $P/2$, at the top of the portal form a couple which must be balanced by two equal and opposite reactions, V applied along and at the bottom of the end posts. Now, as the forces $(R+P/2)$, $P/2$, the two H's, and the two V's balance, evidently, the frame formed by the end posts and portal can be treated as an independent structure in equilibrium under the action of these forces. So, taking moments about u (Fig. 271), and considering the end posts and portal combined as a frame, we have

$$+V=\left[\left(R+\frac{P}{2}\right)+\frac{P}{2}\right]\frac{L}{d}=(R+P)\frac{L}{d},$$

or taking moments about w, we have

$$-V=(R+P)\frac{L}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

In the designing of portals for bridges, there are two cases: one when the end posts are considered as hinged at the bottom ends, and the other is when the end posts are considered as fixed at the bottom ends. The top ends of the end posts are practically always considered to be fixed by the portal. In the case of light bridges, and especially when end floor beams are not used, the end posts are not very rigidly fixed at the bottom ends and the actual condition is most closely met by considering the bottom ends of the end posts hinged, while in the case of heavy bridges, with end floor beams and with wide shoes, there is no question but that the bottom ends of the end posts are fixed.

In the case of the bridge we are designing, there is no question but that the end posts will be quite rigidly fixed at the bottom ends, but for the sake of illustration we will first determine the stresses in the portals.

assuming the bottom ends of the end posts to be hinged, and then determine them, assuming the bottom ends of the end posts to be fixed.

By imagining the part to the left of section *n-n* (Fig. 271), including the end post *bu* and a portion of the portal, moved bodily (forces and all) to a new position we obtain the structure shown in Fig. 272 which is acted upon by the known forces $(R+P/2)$, H, $-V$, and by the unknown forces S and $S1$, S being the stress in member *bo* and $S1$ the stress in *po*. The dotted members shown in the portal are assumed to have no stress in them as they are redundant members intended only to brace the main members of the portal, which are shown in full lines, and hence the dotted members are ignored in the calculations. By taking moments about *p* (Fig. 272) we eliminate $S1$ and $-V$ from the equation of moments, and as the moments of H and $(R+P/2)$ (about *p*) have the same sign, both tending to produce clock-wise rotation about *p*, it is evident that the sum of the moments of the two forces about *p* must be equal but of opposite sign to the moment of S about *p*. So we have

$$Sk=\left(R+\frac{P}{2}\right)k+Hm,$$

from which we obtain

$$S=R+\frac{P}{2}+\frac{Hm}{k} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

for the stress in portal member *bo*. Now it is readily seen from Fig. 272 that this stress S is compression, as H and $(R+P/2)$ both tend to produce clock-wise rotation about *p*, and as S alone prevents this rotation it must

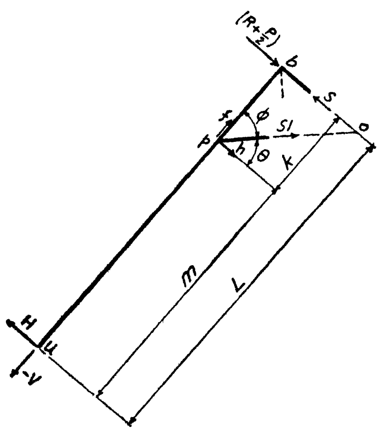


Fig. 272

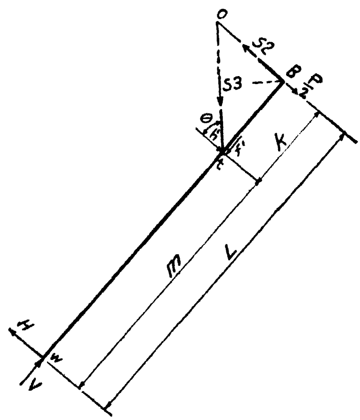


Fig. 273

evidently act to the left toward *b*, and hence is compression. In other words, the member *bo* pushes to the left against the end post and undoubtedly is in compression.

To determine the stress, $S1$, in the portal member *po*, let us consider it resolved at point *p* into two components, one (f) along the end post and the other (h) perpendicular to the end post. Then taking moments about *b* (Fig. 272), we eliminate $-V$, f, S, $(R+P/2)$, and $P/2$

from the equation of moments and we have

$$(H \times L) - (h \times k) = 0,$$

from which we obtain

$$h = \frac{H \times L}{k}$$

for the component of $S1$ perpendicular to the end post. Then we have

$$S1 = h \sec \theta = \frac{H \times L}{k} \sec \theta \quad \text{.............................(4)}$$

for the stress in the portal member (knee brace) pc Stress $S1$ is tension, as is seen by considering point b as the center of rotation; the force H acting to the left would necessitate h acting in the opposite direction or away from the post at point p, which indicates the component h to be tension, and hence the stress $S1$ is tension.

The stress $S1$ can also be determined from the summation of the vertical forces. In which case we have

$$-V + f = 0$$

($-V$ and f being the only vertical forces). From this we have

$$f = V,$$

and multiplying by $\sec \phi$ we obtain

$$S1 = f \sec \phi = V \sec \phi = (R + P)\frac{L}{d} \sec \phi \quad \text{...................(5).}$$

f equals V, and as these two forces must balance each other it is seen that f acts as indicated in Fig. 272, which shows $S1$ to be tension.

By imagining the part to the right of section n'-n' (Fig. 271), including end post wB and a portion of the portal, moved bodily (forces and all) to a new position we obtain the independent structure shown in Fig. 273 acted upon by the known forces H, V, and $P/2$ and by the unknown forces (stresses) $S2$ and $S3$. Taking t as the center of moments, it is seen that $P/2$ and H both tend to produce clock-wise rotation, and as this rotation is prevented by $S2$ alone, we have

$$S2 \times k = \frac{P}{2} k + Hm,$$

from which we obtain

$$S2 = \frac{P}{2} + \frac{Hm}{k} \quad \text{....................................(6)}$$

for the stress in portal member Bo. This stress is tension, as is readily seen, as it acts away from the end post at point B, that is, the member Bo pulls on the end post at point B and hence is in tension.

Resolving (at point t) the stress $S3$ into two components h' and f', and taking moments about B, we have

$$H \times L = h' \times k,$$

from which we obtain

$$h' = \frac{H \times L}{k}$$

and multiplying by $\sec\theta$ we have

$$S3 = h' \sec\theta = \frac{H \times L}{k} \sec\theta \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

for the stress in portal member *ot*.

By considering point *B* (Fig. 273) as the center of moments, the stress *S*3 is seen to be compressive, as *H* acting to the left requires the member *ot* to act to the right against the end post, and hence the member *ot* is in compression.

As is seen by comparison, (4) and (7) are exactly alike. That means that the stress in *po* is equal to the stress in *ot*, but, as stated above, one is tension and the other is compression.

Now, having derived a formula for the stress in each member of the portal, we will proceed with the determination of the stress in the members. To do this it is first necessary to determine the value of *P, R, H, L, m, k*, and $\sec\theta$. *P* is given in the last article as 5,000#. *R* is equal to the shear in the panel *BC* (Fig. 269) when all the panels are loaded. Beginning at the center of the span, we have one-half of a panel load at *D* and a full panel load at *C*, making in all one and one-half panel loads as the shear in the panel *BC*, hence we have

$$R = \frac{5{,}000}{2} + 5{,}000 = 7{,}500 \text{ lbs.}$$

for this shear. Then from (1) we have

$$H = \frac{7{,}500 + 5{,}000}{2} = 6{,}250 \text{ lbs.}$$

L is about 39 ft., as the height of the span is 30 ft. and the panel length is 25 ft. It will be close enough, for designing, to take θ as 45 degrees. Then, $k = 8$ ft., $m = 31$, and $\sec\theta = 1.4$. Now, using the above values, from (3) we obtain

$$S = 7{,}500 + \frac{5{,}000}{2} + 6{,}250 \times \frac{31}{8} = 34{,}218, \text{ say } 34{,}000 \text{ lbs.}$$

(compression) for the stress in portal member *bo*. From (4) or (7) we obtain

$$S1 = S3 = \frac{6{,}250 \times 39}{8} \times 1.4 = 42{,}655, \text{ say } 43{,}000 \text{ lbs.}$$

for the tensile stress in portal member *po* and compressive stress in *ot*. From (6) we obtain

$$S2 = \frac{5{,}000}{2} + \frac{6{,}250 \times 31}{8} = 26{,}718, \text{ say } 27{,}000 \text{ lbs.}$$

for the tensile stress in the portal member *Bo*. By comparing (3) and (6) it is seen that the stress in member *Bo* differs from the stress in member *bo* by the value of *R*.

This completes the calculations for the stresses in the portals under the assumption that the bottom ends of the end posts are hinged, and we will now determine the stress in the portal, assuming the bottom ends of the end posts fixed. To illustrate conditions, let e and g at (a) (Fig. 274) represent two thin wooden strips connected rigidly by another piece h. If this frame be supported as shown at (b) and a force F be applied, the pieces e and g would bend as indicated at (b), and we thus have an illustration of the case just considered where the end posts were assumed hinged at the bottom ends. Now, suppose the bottom ends of the pieces to be buried in concrete as indicated at (c) and the concrete be allowed to set and the force F be then applied. The pieces e and g would then bend as indicated at (c), and we thus have an illustration of the case to be considered where the bottom ends of the end posts are assumed to be fixed. The points o' are the points of contra-flexure, where the bending moment on the pieces e and g is zero, as is readily seen from the sketch. The points o' are at mid-distance from the concrete to the piece h, or $m/2$ from h, neglecting the portion buried in the concrete as being too small to materially change the length of the part marked m.

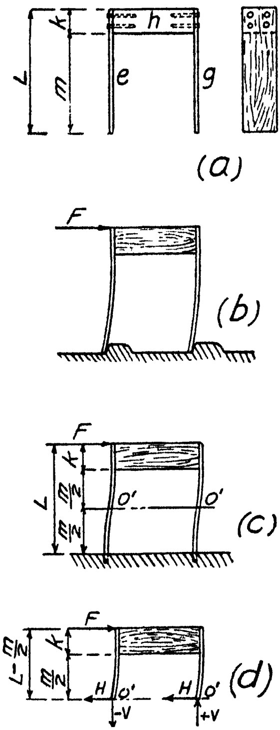


Fig. 274

Now, as there is no bending at the points o' (points of contra-flexure), the frame can be assumed as cut off at those points and we have the independent structure shown at (d). From this it is seen that the problem of determining the stresses in portals, when the bottom ends of the end posts are considered fixed, is the same as in the case of hinged ends, except the distance m (Figs. 271 and 273) is one-half as great, and hence the above formulas for determining the stresses in the portals are applicable if $m/2$ be substituted for m and $(L-m/2)$ or $(k+m/2)$ for L. In an imaginary sense, we simply move the bottom supports of the end posts half way up their unsupported length, and proceed with the determination of the stress in the portals in the same manner as if the bottom ends of the end posts were hinged. Thus substituting in (3), we have

$$S=7{,}500+\frac{5{,}000}{2}+6{,}250\times\frac{\frac{31}{2}}{8}=22{,}109, \text{ say } 22{,}000 \text{ lbs. (compression)}$$

for the stress in portal member bo (Fig. 271), assuming the bottom ends of the end posts fixed; and substituting in (4), we have

$$S1=\frac{6{,}250\times(39-\frac{31}{2})}{8}\times 1.4=25{,}703, \text{ say } 26{,}000 \text{ lbs.}$$

for the tensile stress in portal member po and also for the compressive stress in portal member *ot*, and likewise substituting in (6) we have

$$S2 = \frac{P}{2} + \frac{Hm/2}{k} = \frac{5{,}000}{2} + \frac{6{,}250 \times \frac{31}{2}}{8} = 14{,}609, \text{ say } 15{,}000 \text{ lbs.}$$

for the tensile stress in portal member *Bo*. Thus we have all the stresses in the portals determined when assuming the bottom ends of the end posts to be fixed. These stresses will be used in designing the sections of the portal members as the bridge considered is designed for a heavy loading; and we have provided end floor beams, consequently the bottom ends of the end posts will be fixed quite securely.

Each member of the portals will be as deep as the end posts and composed of two angles, latticed together vertically, similar to the top laterals.

As seen above, the members are subjected to one kind of stress when the wind load comes from one direction and just the reverse when it comes from the opposite direction, that is, each portal member is subjected to reversal of stress, and hence the members must be designed to carry both compression and tension.

The members *po* and *ot* (known as knee braces) are each subjected to 26,000# stress, considered to be either tension or compression. Taking the case of tension first, we have

$$\frac{26{,}000}{16{,}000} = 1.62 \text{ sq. ins.}$$

for the required net area of cross-section of each, which shows that two very small angles could be used (theoretically) as far as tension is concerned. As for compression, the length of each member is about 11.3 ft. ($\theta = 45°$), but as the redundant members (those dotted, in Fig. 271) support each at mid-point, the length L can be taken as $11.3/2 = 5.6$ ft., say 66 ins. As the members are subjected to reversal of stress, L/r should be reasonably low, say 80. Then we have

$$r = \frac{66}{80} = 0.82$$

for the required value of the radius of gyration.

Now, glancing over the table of angles (Tables 5 and 6) it is seen that this calls for very small angles. So the case here is the same as that of the top lateral struts—very much a matter of judgment. $3\frac{1}{2}''$ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ angles are as small as are used in this class of work (as it is desirable to use $\frac{7}{8}''$ rivets throughout), so we will try that section. Substituting the radius, found in Table 6, for this angle in the column formula, we have

$$16{,}000 - 70\frac{66}{1.07} = 11{,}682 \text{ lbs.}$$

for the allowable unit compressive stress in each of the members *po* and *ot*. Dividing this into the stress, we have

$$26{,}000 \div 11{,}682 = 2.22 \text{ sq. ins.}$$

for the required area, while 2—Ls $3\frac{1}{2}''$ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ have 4.96□", more than twice the amount required for compression, and, as found above, only 1.62□" is required for tension. So it is seen that $3\frac{1}{2}''$ x $3\frac{1}{2}''$ x $\frac{3}{8}''$ angles are much heavier than required, theoretically, yet smaller angles would give the portals a flimsy appearance even if satisfactory details were obtained by their use. So we are now to the point where the selection must be made mostly upon appearance. As the stress in the members *bo* and *Bo* is less than in members *po* and *ot*, and their lengths are also less, it is not necessary to compute the required sections for those members, but use the same as used for members *po* and *ot*. As the $3\frac{1}{2}'' \times 3\frac{1}{2}''$ angles are our minimum size we could use two of these for each portal member, but for the sake of details and appearance we will use 2—∟s $4'' \times 4'' \times \frac{3}{8}''$ for each of the main members and 2—∟s $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ for each of the redundant members, thus giving the main members a distinctive outline.

In the case of large bridges these members can be designed theoretically correct, as the required area of cross-section in those cases is such that sections can be found which are both theoretically and practically correct, but in the case of small bridges it is very much a matter of judgment, as is seen from the above calculations.

180. Design of End Posts.—The designing of the section of the end posts is here taken as a special problem for the reason that, unlike the other main truss members, it is subjected to bending stress in addition to direct stress. To correspond with the top chord, it will be made of 18″ webs, 23″ cover plate and four angles. The length of the end

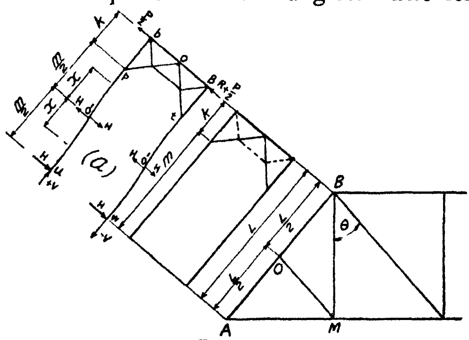


Fig. 275

post, considering the *x-x* axis (see Fig. 265) is the total length *L* (shown in Fig. 275 to equal *AB*) unless a collision strut *OM* be inserted, in which case the length is $L/2$ and the length, considering axis *y-y*, is equal to *m*. The collision strut should be used as a safeguard in case a derailed train should strike the end post, and therefore we will use one in this case. Then the length of the post in reference to the *x-x* axis is 19.5 ft. or 234 ins., and 31 ft. ($=m$) or 372 ins. in reference to axis *y-y*.

Using the radii found for top chord *bc* (Art. 176), which will be about the same in this case, we obtain

$$16{,}000 - 70\frac{234}{7.28} = 13{,}750 \text{ lbs.}$$

for the allowable unit direct compressive stress in the end post, considering axis *x-x*, and

$$16{,}000 - 70\frac{372}{7.84} = 12{,}678 \text{ lbs.}$$

for the allowable unit direct compressive stress considering axis y-y. Now dividing the latter, as it is the smaller, into the total direct stress in the end post, as given in Fig. 262, we obtain

$$\frac{523{,}000}{12{,}678}=41.2 \text{ sq. ins. (about)}$$

for the required area of cross-section of each end post, providing there were no cross bending, due to the wind load, on the posts. But as the bending stress on the compression side of the posts adds to the direct stress, the two unit stresses must be combined. However, in case these stresses be combined, the allowable unit stress can be increased 25 per cent, so the above area may be about correct. As preliminary let us try the section used for top chord bc, which contains $40.58\square''$.

In determining the bending moment on the end posts, which is due to wind—and consequently a transverse moment—we can consider each post as composed of two cantilevers as indicated at (a) in Fig. 275. Then the moment on each post at any point either above or below the point of contra-flexure (o') is Hx. Now it is seen that this moment is a maximum at the points p and t and at the bottom of each post, and that the maximum in each case is equal to $Hm/2$. As the points p and t are the farthest out from the ends of the member, where the column formula really applies, we will consider the bending at those points only. From the last article, we have $H=6{,}250$ lbs. and $m=31$ ft. or 372 ins. Then for the moment at p or t (Fig. 275 (a)) we have

$$M=6{,}250\times\frac{372}{2}=1{,}162{,}500 \text{ inch lbs.}$$

Now using the moment of inertia found in Art. 176 for chord bc in reference to axis y-y and taking the horizontal distance to the extreme fiber as $7.75+4=11.75''$ (see Fig. 265), we obtain

$$f=\frac{1{,}162{,}500\times 11.75}{2{,}497}=5{,}470 \text{ lbs.}$$

for the unit stress due to cross bending, assuming that the same section as used for chord bc is used for the end post.

Now dividing the area of the section into the total direct stress, we have

$$\frac{523{,}000}{40.58}=12{,}880 \text{ lbs. (about)}$$

for the actual direct unit stress, and adding this to the unit stress due to cross bending we have

$$5{,}470+12{,}880=18{,}350 \text{ lbs.}$$

for the combined unit stress. The allowable unit stress as given above is 12,678. But this can be increased 25 per cent in the case of combined stress, as per specifications. So we have

$$(12{,}678)\ 1.25=15{,}847 \text{ lbs.}$$

for the actual allowable combined unit stress, and as it is less than the above (18,350#) a larger section will have to be used for each end post than was just considered.

Let us assume the following sections:

1—cov. pl. $23'' \times \frac{1}{2}'' = 11.50\square''$
2—web pls. $18'' \times \frac{1}{2}'' = 18.00\square''$
2—Ls $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' = 4.96\square''$
2—Ls $6'' \times 4'' \times \frac{5}{8}'' = 11.72\square''$
$46.18\square''$

As the post is stronger in reference to the x-x axis than in reference to the y-y axis, owing to the length being greater in the last case (using the collision strut), we need consider the above section only in reference to the y-y axis.

For the moment of inertia about the y-y axis (see Fig. 276), we have

$$\text{cov. pl. } 507+0 = 507 \text{ (about)}$$
$$\text{web pls. } 0+\overline{7.62}^2 \times 9 \times 2 = 1{,}045$$
$$\text{top angles } 2.87 \times 2 + \overline{8.88}^2 \times 2.48 \times 2 = 397$$
$$\text{bot. angles } 7.52 \times 2 + \overline{8.90}^2 \times 5.86 \times 2 = 943$$
$$2{,}892$$

Then for the radius of gyration about axis y-y we have

$$r' = \sqrt{\frac{2{,}892}{46.18}} = 7.92.$$

Now substituting this radius in the column formula, we have

$$p = 16{,}000 - 70\frac{372}{7.92} = 12{,}710 \text{ (about)}$$

for the allowable unit stress for direct compression, and increasing this 25 per cent we obtain 15,890# for the allowable unit stress in the case of direct compression and bending stress combined. Then, using the last assumed section, we have

$$\frac{523{,}000}{46.18} = 11{,}320 \text{ lbs. (about)}$$

for the actual direct unit compressive stress, and

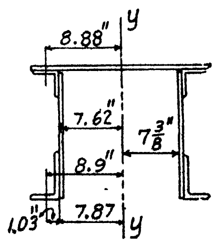


Fig. 276

$$f' = \frac{1{,}162{,}500 \times 11.75}{2{,}892} = 4{,}730 \text{ (about)}$$

for the unit stress due to bending. Now, adding these two stresses together, we have $11{,}320 + 4{,}730 = 16{,}050\#$ for the combined unit stress, which is only 160# $(= 16{,}050 - 15{,}890)$ greater than the 15,890# allowed, and as this is about as close as we can obtain we will use the above section for each end post.

181. Designing of Collision Struts.—These members should at least have sufficient section to resist the thrust that an ordinary loaded car moving at an ordinary rate would exert upon them in case the car were derailed. The problem involved is of such a nature that the entire premises must rest upon practical assumptions. We will assume that the car and its load weigh 100,000# and that it has a velocity of 30 ft. per second, which is a little over 20 miles per hour, and, further, we will assume the car brought to rest in one second. Then substituting in Formula A (Art. 23), we have

$$F=\frac{100,000\times 30}{32}=93,750,\text{ say }94,000\text{ lbs.},$$

for the constant force required to stop the car in one second. But at the beginning of contact the force would be zero and a maximum when the car was brought to rest, so the maximum force exerted by the car would really be twice the constant force required to stop it, or $94,000\times 2=188,000$#. This force would be exerted on the end post at a point about 9.5 ft. above the bottom chord (see Fig. 277), so that the horizontal force exerted against the collision strut would be about 9.5/15 of the 188,000, or 119,000#.

Then multiplying one-half of this by sec β, which we will take as 1.56, β being the complement of θ, we have

$$S=\frac{119,000}{2}\times 1.56=92,820,\text{ say }93,000\text{ lbs.}$$

for the stress in the collision strut. It may appear that the collision strut should connect to the end post at the point where the 188,000# is applied, but this is not the case, for if the blow of 188,000# were struck at the point where the strut connects to the end post there would be but little resilience and, consequently, the structure would be subjected to severe shock.

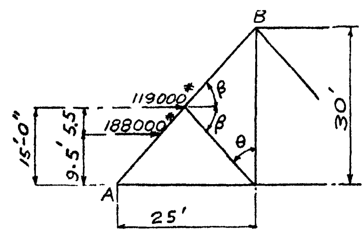


Fig. 277

It is seen from the amount of stress that the section of the collision struts need not be large as far as direct stress is concerned, but the value of L/r should not exceed 80 in order to obtain rigidity. The best section for these struts is two channels latticed together, and as 8″ channels are about the smallest used in this class of work we will try 2—[s 8″ x 16.25#. The length of each strut, center to center of end connections, is about 19.5 ft., or 234 ins., and, as seen from Table 3, the radius of gyration of the channels is 2.89, so we have

$$\frac{L}{r}=\frac{234}{2.89}=81.$$

Then for the allowable unit compressive stress on each strut we have

$$p=16,000-70\times 81=10,330\text{ lbs.}$$

Now, dividing this into the stress, we have

$$\frac{93,000}{10,330}=9.0\text{ sq. ins.}$$

for the section required. The area of the section of the two channels assumed, as seen from Table 3, is 9.56□″, which is 0.56□″ more than the required area, but we will use this section as it is about as close to the required area as we can come and at the same time obtain good details.

The above analysis is only an attempt to provide for reasonable accidents, as we might term it. The actual intensity of the blow struck by a derailed car is distinctively problematic for the reason that there could be so many different conditions assumed. As an illustration, suppose a derailed car in the middle of a train should strike the end post of a bridge. How much would the cars in the rear of the derailed car increase the blow? And how much would the engine pulling on the train increase it? And how much would the resistance of the wheels of the derailed car bumping over the ties diminish it? And, further, suppose a train traveling 50 miles per hour should jump the track just as it gets to the bridge and the engine should strike the end post. There is little doubt but that the bridge would be destroyed, yet it might glance off and do but little damage. It is obvious that it would be absolutely impracticable to provide for resisting the blow in the last case; and practically the same is true in several cases that could be assumed, yet it would not be good engineering to make no provision for reasonable accidents, as past experience has taught us, and that is what we have endeavored to do above.

182. Maximum Reaction on Shoe.—For the dead-load reaction on each shoe we have one-half of a panel load from D (Fig. 249), a full panel load from B and C each, and one-half (so considered) of a panel load from A, making in all three panel loads on each shoe. Then taking the dead-load panel load as found in Art. 173, we have

$$R = 3 \times 26{,}375 = 79{,}125, \text{ say } 80{,}000 \text{ lbs.}$$

for the dead-load reaction on each shoe.

As is evident, the maximum live-load reaction on a shoe will occur when the span is fully loaded and the heaviest wheels are near the shoe

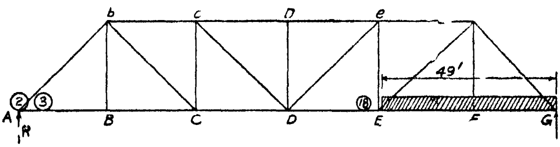


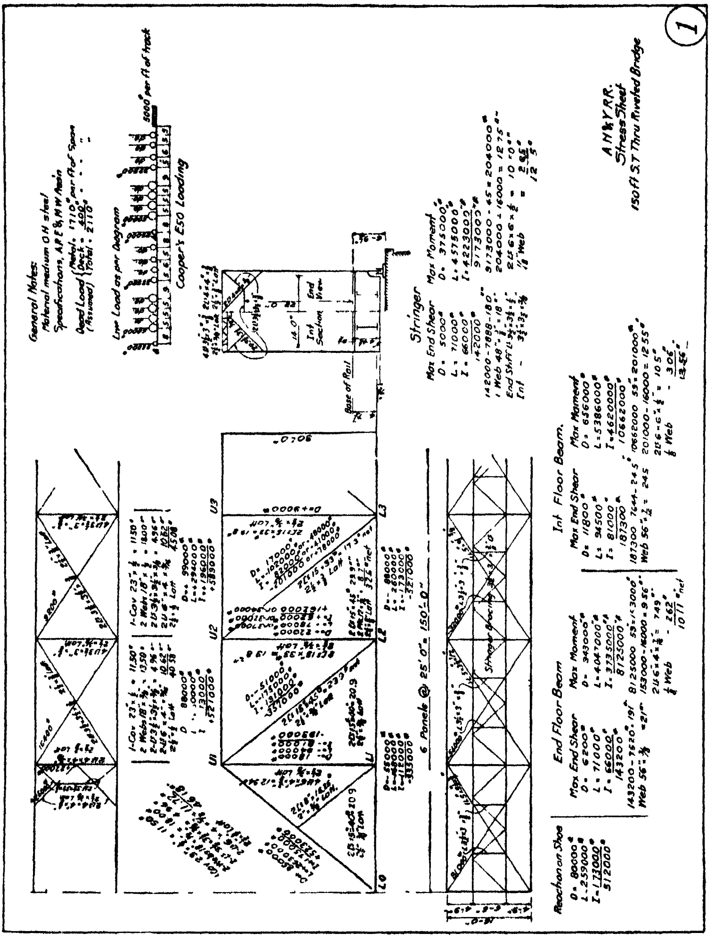
Fig. 278

and one directly over it—that is, over the end floor beam. So, placing the live load as shown in Fig. 278 with wheel (2) at A and taking moments about G, using Table A, we have

$$R' = \left(\frac{15{,}274 + 274 \times 49 + \overline{49}^2}{150}\right) 1{,}000 \times \frac{50}{40} = 259{,}175, \text{ say } 259{,}000 \text{ lbs.}$$

for the maximum live-load reaction on the shoe at A, which is the same as for the others, and, for the impact, we have

$$259{,}000 \times \frac{300}{450} = 172{,}666, \text{ say } 173{,}000 \text{ lbs.}$$



General Notes
Cooper's E50 Loading
6 Panels @ 25' 0" = 150' 0"
Stringer
End Floor Beam
Int Floor Beam
Reaction on Shoe
A.M.& Y.R.R.
Stress Sheet
150 ft S.T. Thru Riveted Bridge

Fig. 279

Now adding the above dead, live, and impact reactions together, we have

$$80{,}000+259{,}000+173{,}000=512{,}000 \text{ lbs.}$$

for the maximum reaction on each shoe.

183. Stress Sheet.—After having completed the calculations as above for the span the "stress sheet," Fig. 279, can be made. This drawing contains all the information resulting from the calculations; in fact, the drawing is really a summary of the calculations. After the stress sheet (Fig. 279) is completed the detail drawings can be made.

184. Calculations of Details.—In starting the detail drawings, a large scale pencil sketch ($1\frac{1}{2}''$, $2''$ or $3''$ scale) of each joint should be made first. Such a drawing for joint *LO* is shown in Fig. 282. The calculations of the details are made as the sketches are being drawn.

Joint LO. Taking first the case of joint *LO* (Fig. 282), the first thing to do is to locate the center line of the pin in the end post. The section of the end post is given on the stress sheet (Fig. 279), also in Art. 180.

Taking moments about the center of the cover plate of the section (see Fig. 280) we have

$$\bar{x}=\frac{1.26\times 4.96+18\times 9.37+11.72\times 16.47}{46.18}=\frac{367.9}{46.18}=7.96 \text{ ins.}$$

for the distance from the center of the cover plate down to the gravity axis x-x of the end post. Now, as far as the direct stress in the end post is concerned, the pin could be properly placed on this gravity axis x-x; but the member tends to bend downward as shown in Fig. 281, owing to its own weight, and it is necessary to place the pin a short distance below the gravity axis so as to counterbalance this bending. The pin being placed below the gravity axis x-x, the direct stress tends to bend the member upward as indicated by the dotted line, and, as is evident, if the pin be placed just the correct distance below the gravity axis the end post will be straight when the maximum stress in it occurs. For the weight of the end post per foot of length we have

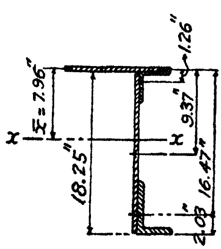


Fig. 280

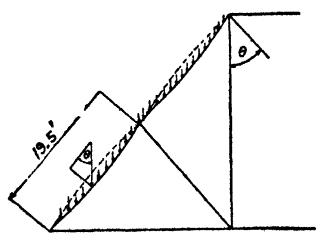


Fig. 281

$$W=46.18^{\square''}\times 3.4^{\#}=157 \text{ lbs.},$$

and increasing this one-third to provide for details we have 210# for the total weight of the member per foot of length. This weight acts vertically and, as the member is inclined, only the component perpendicular to the member causes transverse bending. So, for the bending moment, we have

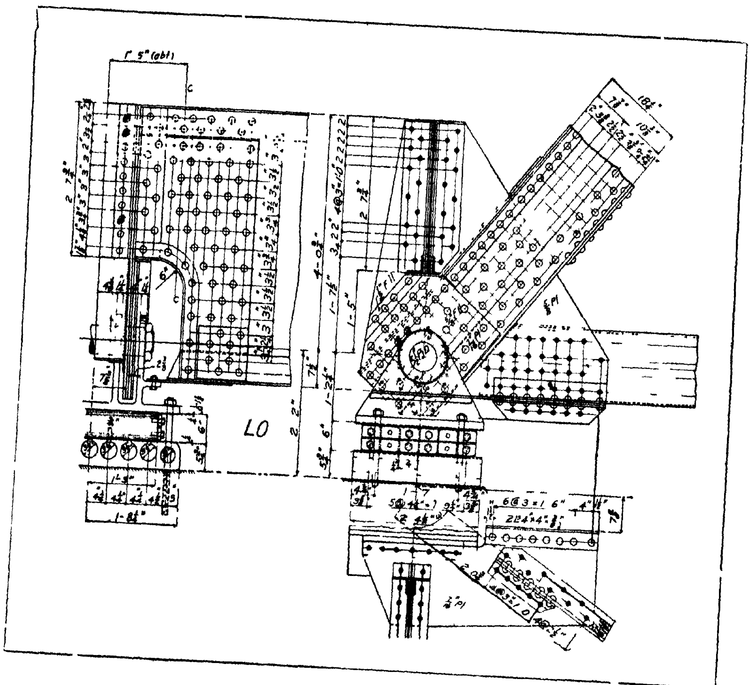


Fig 282

$$M = \frac{1}{8} \times 210 \times \sin\theta \times \overline{19.5}^2 \times 12 = 76{,}658 \text{ inch lbs.} \quad (\sin\theta = 0.64.)$$

Now by placing the pin a distance z below the gravity axis x-x, we have $z \times 523{,}000$ for the moment caused by the direct stress which tends to bend the end post upward, and hence the end post will be straight when $z \times 523{,}000 = 76{,}658$, from which we obtain

$$z = \frac{76{,}658}{523{,}000} = 0.1466 \text{ ins.},$$

which is the distance that the pin should be placed below the gravity axis x-x. Now adding this to $\bar{x}$ (given above) we have $7.96 + 0.1466 = 8.1066''$ for the distance from the center of the cover plate down to the center of the pin, and substracting one-half of the thickness of the cover plate from this we have $8.1066 - 0.25 = 7.8566''$ for the distance from the under side of the cover plate to the pin. The distance from the under side of the cover plate to the working line in the top chord, which would be the center line of the pins if the top chord were pin-connected, should be the same as for the end post in order to obtain uniform construction of these members, and hence at this juncture it is necessary to examine the top chord sections and select a distance suitable for all top chord members and end posts.

Considering the case of the top chord section $U2$-$U3$ (Fig. 279), and taking moments about the cover plate as was shown above in the case of the end post we obtain

$$\bar{x} = \frac{1.26 \times 4.96 + 18 \times 9.37 + 10.62 \times 16.49}{45.08} = 7.76 \text{ ins.}$$

for the distance from the center of the cover plate down to the horizontal gravity axis. For the weight of the section we have

$$45.08 \times 3.4 = 153 \text{ lbs.}$$

per foot and adding one-third for details we have 204# for the weight of the member per foot of length. Then we have

$$M = \frac{1}{8} \times 204 \times \overline{25}^2 \times 12 = 191{,}000 \text{ lbs. (about)}$$

for the bending on the chord due to its own weight, and for the distance z' below the horizontal gravity axis, where pins would have to be placed to balance the above moment, we have

$$z' = \frac{191{,}000}{589{,}000} = 0.325 \text{ ins.}$$

Adding this to the above $7.76''$ and subtracting one-half of the thickness of the cover plate we have

$$7.76 + 0.325 - 0.25 = 7.84 \text{ ins.}$$

for the distance down from the under side of the cover plate to the working line. In the same manner we find that the corresponding distance in the case of chord $U1$-$U2$ is $7.75''$. So it is seen that $7\frac{3}{4}''$ is about the correct value to use for the distance from the under side of the cover plate to the pin center or working line, as the case may be, and hence this distance will be taken in the case of all top chord sections and end posts throughout the structure. Thus we have the pin LO located and the outline of the portion of the end post shown (Fig. 282) can be drawn and the rivets spaced transversely in it as shown and then the drawing of the other details can proceed. The end post should extend far enough beyond the pin so that the bottom chord and end floor beam connections balance fairly well about the pin. This part is at first determined merely by appearance. After this the shoe can be sketched so as to clear the end post, the number of rollers required is determined by sketching roughly their length, and at the same time the length of the pedestal can be determined. After this is done the required pin bearing on the end post can be figured. This bearing must be sufficient to carry the maximum vertical reaction, which is 512,000# (see stress sheet, Fig. 279). Let us assume a $5\frac{1}{2}''$ pin. Then we have

$$t = 512{,}000 \div (5\tfrac{1}{2} \times 24{,}000) = 3.87 \text{ ins.}$$

for the required thickness of bearing on the two sides of the end post or $1.93''$ $(=1\frac{15}{16})$ for each side. As shown, we have a $\frac{5}{8}''$ gusset plate, $\frac{1}{2}''$ web, $\frac{5}{8}''$ filler, and a $\frac{7}{16}''$ plate, making in all, $\frac{5}{8} + \frac{1}{2} + \frac{5}{8} + \frac{7}{16} = 2\frac{3}{16}''$, which is $\frac{1}{4}''$ thicker than required. But this is about as close as we can come, as the gusset plate should be $\frac{5}{8}''$ thick (as will be seen later) and the $\frac{5}{8}''$ filler can not be reduced, as the bottom angles on the end post have that thickness and the $\frac{7}{16}''$ plate is as thin as should be used, owing to the countersunk rivets ($\frac{7}{8}''$ rivets should not be countersunk in metal less than $\frac{7}{16}''$ thick). So we will use the metal shown. There should be enough rivets above the pin to hold these plates. First, there should be enough rivets connecting the $\frac{5}{8}''$ filler and the $\frac{7}{16}''$ plate to the end post and gusset plate to transmit the pin pressure coming on the two. For this pressure we have $(\frac{5}{8} + \frac{7}{16})\ 5\frac{1}{2} \times 24{,}000 = 140{,}000\#$ (about). The rivets are $\frac{7}{8}''$ shop rivets in single shear, each of which is good for $12{,}000 \times 0.6 = 7{,}200\#$. So we have $140{,}000 \div 7{,}200 = 19.5$, say 20 rivets. We have just about 20 rivets above the pin, so the detail is satisfactory so far. The rivets in the $\frac{1}{2}''$ filler are not included, as they simply hold that narrow filler. There should be enough rivets in the $\frac{7}{16}''$ plate to take the pin pressure upon it, which is $\frac{7}{16} \times 5\frac{1}{2} \times 24{,}000 = 57{,}750\#$. Dividing this by 7,200 we have $57{,}750 \div 7{,}200 = 8$ rivets—whereas we have over twice that number, and that part is amply strong. To be on the safe side there should be enough rivets connecting the end post to the gusset plate to transmit, in single shear, one-half of the stress in the end post, which is $523{,}000 \div 2 = 261{,}500$ (see Fig. 279). Dividing this by 7,200 we have $261{,}500 \div 7{,}200 = 36.3$, say 37 rivets. We have about 38 above the pin, so that part of the detail is satisfactory. We will next consider the bottom chord connection. The net area of the bottom chord is $20.9^{\square}{}''$ (see Fig. 279), and for its strength we have $20.9 \times 16{,}000 = 334{,}400\#$. Then as the rivets are $\frac{7}{8}''$ field rivets, we have $334{,}400 \div 6{,}000 = 55.7$, say, 56 rivets, or 28 on each side. We have 30, so that detail is correct.

The end floor beam is cut, as shown (Fig. 282), to clear the shoe, pin, and the details on the end post. Dividing the end shear on the end floor beam (see Fig. 279) by 6,000 we have $143{,}200 \div 6{,}000 = 23.8$, say 24 rivets for the number of $\frac{7}{8}''$ field rivets required in each end connection. As is seen, 24 rivets are used. The thickness of pin bearing on the shoe need be (theoretically) but $1\frac{15}{16}''$ on a side, as found above for the end post, but there should be a point of bearing on each side of each web of the end post (as shown) so as to distribute the pressure well over the rollers and at the same time lighten the stress on the pin, and as every part of such castings should be at least $1\frac{1}{4}''$ in thickness, we will make each of these bearings $1\frac{1}{4}''$ thick, although we obtain an excess of bearing. However, as the shoe is subjected to both bending and shear from the direct load (see Art. 141) and wind, the metal is not so overly excessive as at first appears.

Assuming one-fourth of the maximum reaction on the shoe, which is equal to $512{,}000 \div 4 = 128{,}000\#$, as coming on each bearing of the shoe, and taking moments about the center of bearing of the end post, we have

$$128{,}000 \times 2\tfrac{3}{8} = 304{,}000 \text{ inch lbs.}$$

for the maximum bending moment on the $5\frac{1}{2}''$ pin. The lever arm $2\frac{3}{8}''$ can be either computed or scaled from the drawing. For the bending stress on the pin, due to this moment, we have

$$f = \frac{304{,}000 \times 2\frac{3}{4}}{(\pi/64)(5\frac{1}{2})^4} = 18{,}600 \text{ lbs. per sq. in. (about)}$$

which shows the pin to be amply strong, as $25{,}000\#$ per square inch is allowed. It requires a moment of $408{,}350''\#$ to stress the $5\frac{1}{2}''$ pin $25{,}000\#$ per square inch. The maximum bending moment allowed on the different sizes of pins is to be found in tables of practically all structural handbooks, and, in designing pins, usually these tables are consulted instead of computing the fiber stress as is done above.

For the maximum shearing stress per square inch on the pin we have

$$128{,}000 \div \pi r^2 = 128{,}000 \div 23.75 = 5{,}390 \text{ lbs. per sq. in.,}$$

which is quite low, as $12{,}000\#$ is allowed. As seen from above, the pin is really larger than required and, theoretically, should be made smaller; but if reduced in size the bearing would increase, little would be saved, and, judging from general appearance, a $5\frac{1}{2}''$ pin is as small as should be used in this class of work; hence the assumed $5\frac{1}{2}''$ pin will be used.

Six-inch rollers are the minimum size permitted by the specifications, and as that size appears to be about the correct size for this case, they will be used. Then, for the linear inches of roller required, we have

$$\frac{512{,}000}{600 \times 6} = 142.2 \text{ ins.}$$

Now, making each roller $30''$ long and deducting for the 7—$\frac{3}{4}''$ slots in pedestals, we have $(30 - 7 \times \frac{3}{4}) \times 6 = 148.5''$, which is a few inches more than is required, but is about as close as we can come and at the same time obtain good details all around. So 6—$6''$ rollers, each $30''$ long, as shown, will be used. The rollers are notched $\frac{1}{4}''$ into the cast shoe and pedestal, so as to prevent their sliding transversely (due to wind), and the side bars on their two ends control their relative longitudinal motion. It is necessary that the pedestal and the base of the shoe project to clear the side bars and bolt heads. In some cases it is necessary for the pedestal and the base to project out farther to obtain sufficient bearing on the masonry. In this case we have

$$A = \frac{512{,}000}{600} = 853 \text{ sq. ins.}$$

for the required area of bearing and we have, as shown, $(3' - 4\frac{1}{2}'') \times (2' - 4\frac{1}{2}'') = 1{,}154^{\square''}$, which is about $300^{\square''}$ more than required, but the roller nest requires about the same size of pedestal as shown and there would be but little saving, if any, in reducing it to exactly the theoretical size. So the size shown will be used.

In drawing the lateral connection at *LO*, we first determine the number of $\frac{7}{8}''$ field rivets necessary to develop the lateral in tension. The lateral is a $5'' \times 3\frac{1}{2}'' \times \frac{3}{4}''$ angle, which has a net area of $5.06^{\square''}$ (allowing for 1—1″ rivet hole). Multiplying this area by 16,000 and dividing by 6,000 we obtain 13.5, say 14 rivets. As is seen, 14 are used. The bolts connecting the $\frac{7}{16}''$ lateral plate to the shoe should be sufficient to carry the component (along the end floor beam) of the stress in the lateral. This component is equal to $81{,}000 \div \sec\omega = 81{,}000 \div 1.85 = 43{,}800\#$ (about) (see Art. 177). Dividing this by 6,000 we obtain 7.3 for the number of $\frac{7}{8}''$ turned bolts required. As is seen, 7 are used. Part of the component (43,800#) may be transferred along the floor beam to the other shoe, in which case less than 7 bolts would be required; but by using the 7 we are sure that the detail is sufficiently strong.

For the component of the stress in the lateral along the bottom chord we have

$$81{,}000 \times \sin\omega = 81{,}000 \times 0.8418 = 68{,}200 \text{ lbs. (about).}$$

Dividing this by 6,000 (the allowable shear on a $\frac{7}{8}''$ field rivet) we obtain 11.3, say 12 rivets. We have, as shown, 11. However, some of these are also needed to transmit the direct stress of the bottom chord into the gusset plate, but as the maximum stress in the bottom chord and lateral are not likely to occur at the same time, the detail as shown will be considered sufficient. The shop rivets connecting the $2—4'' \times 4''$ angles to the $\frac{7}{16}''$ lateral plate should be sufficient to transmit the 68,200# component in bearing on the $\frac{7}{16}''$ plate.

One $\frac{7}{8}''$ rivet at 24,000# bearing on a $\frac{7}{16}''$ plate will transmit $\frac{7}{8} \times \frac{7}{16} \times 24{,}000 = 9{,}188\#$. Then we have $68{,}200 \div 9{,}188 = 7.4$ for the number of rivets required, and, as is seen, 8 are used. The number of rivets required to connect the $\frac{7}{16}''$ lateral plate to the bottom of the end floor beam should be sufficient to take at least one-half of the component along the floor beam of the stress in the lateral. This requires about 4 rivets, but to obtain a good, rigid connection, more are needed. So 8 are used, wherein we are governed very much by appearance and sense of fitness.

The $3\frac{1}{2}''$ longitudinal holes in the cast-steel pedestal should be cored in casting and the $\frac{3}{4}''$ slots cut at the machine shop. If the slots were to be made in casting, the pedestal would be likely to warp in cooling. These slots and holes in the pedestal are for two purposes: one is to save metal; and the other one is to permit dust and dirt that would accumulate between the rollers to fall down into the holes. In other words, this arrangement permits the dirt that is blown into the roller nest to be blown out, and thus the roller nest is not likely to become clogged with dirt and rust. The side bars at each end of the rollers should be at least $\frac{5}{8}''$ thick to be of much service.

Joint L1. Taking the next lower chord joint *L*1 (Fig. 283), the cross-section of the floor beam and end connection angles of the same can be drawn as shown, and the outline of the bottom chord collision strut and

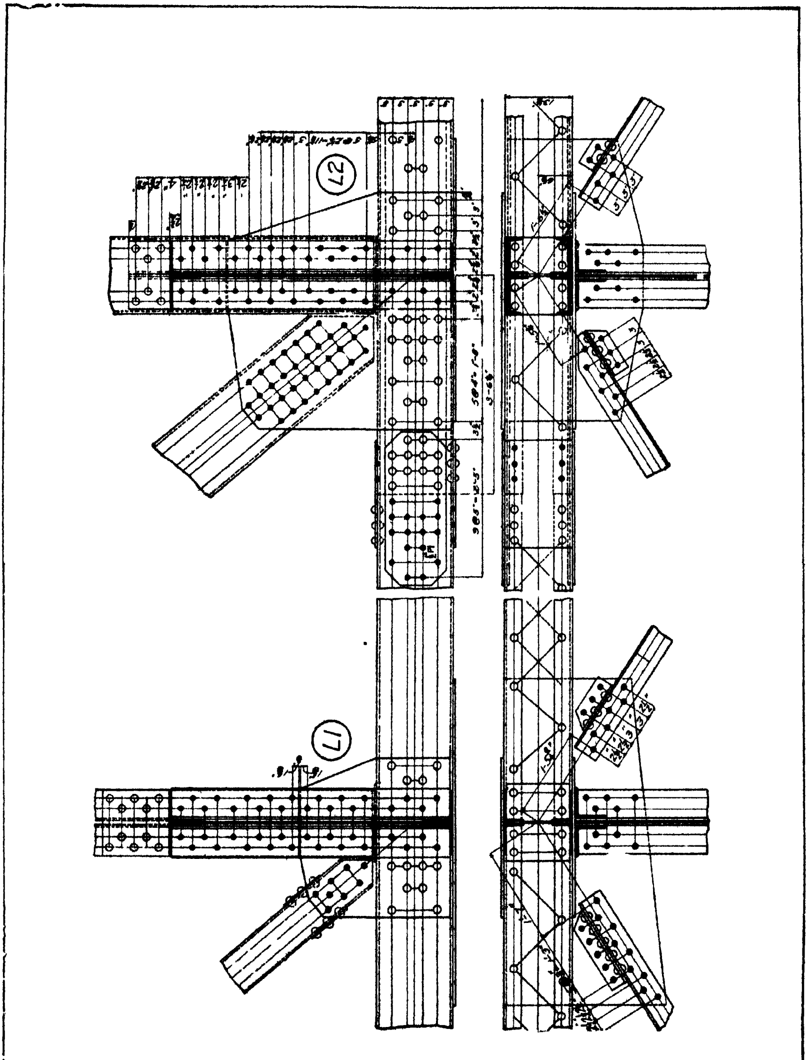


Fig. 288

the hanger can be sketched—care being taken to provide about ½″ clearance between these members. Then the rivet lines can be drawn and the rivets spaced as shown. The stress in the collision strut as given in Art. 181 is 93,000#. Dividing this by 6,000# we obtain 15.5, say, 16—⅞″ field rivets or bolts, as bolts should be used in this particular connection. As is seen, 16 are used. Dividing the maximum end shear on the floor beam, which is given on the stress sheet (Fig. 279) as 187,300, by 6,000 we obtain about 31 rivets for the number of ⅞″ field rivets required to connect the floor beam to the truss. All of these should be above the bottom chord, as there is no diaphragm in the bottom chord to transmit the pressure across to the other side of the hanger and, consequently, the rivets shown connecting the floor beam to the bottom chord must not be considered to carry any of the end shear on the floor beam. As is seen, there are 32 rivets connecting the floor beam directly to the hanger, which is really one more than required theoretically.

After spacing the rivets in the collision strut and floor beam, the gusset plate can be drawn to suit this spacing as shown. The rivets connecting the gusset plate to the bottom chord should at least be sufficient to transmit the component of the stress in the collision strut which is equal to $93{,}000 \times \sin\theta = 59{,}500$#. Dividing this by 7,200 we obtain a little over 8—⅞″ shop rivets, or 4 on a side, whereas 12 (not including the field rivets passing through the floor beam) are used, but the detail is satisfactory, as this number is necessary to obtain a well-balanced joint. Next, the bottom lateral connections can be drawn. The number of rivets required in the lateral to the left of $L1$ is 14—⅞″ field rivets, as determined at LO, and the detail can be drawn as shown. The lateral on the right-hand side of $L1$ (Fig. 283) is made of 1—5″ x 3½″ x ½″ angle, which will have a net area of 3.5□″. Then for the strength of this lateral we have $3.5 \times 16{,}000 = 56{,}000$#. Dividing this by 6,000 we obtain about 9—⅞″ field rivets. This, as is seen, is the number used. The number of rivets connecting the lateral plate to the floor beam should be sufficient to transmit the component of the lateral to the left of $L1$ along the floor beam. This component is equal to $5.06 \times 16{,}000 \times \cos\omega = 80{,}960 \times 0.5397 = 43{,}700$#. Dividing this by 6,000 we obtain about 7—⅞″ field rivets, whereas 10 are used. After the rivets are spaced in the laterals the lateral plate can be drawn to suit; that is, so the edges at no point will be less than 1½″ from the rivets. Then the rivets connecting the lateral plate to the floor beam can be spaced as shown.

Joint L2. We will next consider lower chord joint $L2$ (Fig. 283). The floor beam connection at this joint must be exactly the same as at $L1$, as the floor beams should be the same throughout. However, if the spacing established at $L1$ does not suit at $L2$, it must be changed so it will be satisfactory for the two joints and, likewise, for the other joints. In each case there must be 32 rivets above the chord as was determined at $L1$. After drawing the cross-section of the floor beam and the end connection, as shown (Fig. 283), the outlines of the bottom chord and diagonal can be drawn. The net area of the main diagonal, as given on the stress sheet (Fig. 279), is 23.9□″. Multiplying this by 16,000 and dividing by 6,000 we obtain about 64—⅞″ field rivets for the end connection, or 32 on a side, which, as is seen, is the number used. There should be a sufficient number of rivets connecting the gusset plates to the post to transmit the vertical component of the stress in the diagonal, which is equal to $23.9 \times 16{,}000 \div \sec\theta = 294{,}000\#$. Dividing this by 6,000 we obtain 49—$\frac{7}{8}''$ field rivets or, say, 24 on a side. As is seen, 24 on a side are used.

There should be a sufficient number of rivets to the left of the floor beam connecting the gusset plate to the chord (at $L2$) to transmit the horizontal component of the stress in the diagonal. For this component we have $23.9 \times 16{,}000 \times \sin\theta = 244{,}700\#$, and dividing by 7,200 we obtain 34—$\frac{7}{8}''$ rivets (shop), or 17 rivets on each side of the chord, whereas 16 are used, but the rivets passing through the floor beam and those to the right of the floor beam can be counted on for more than making up for the one rivet missing.

The splice in the bottom chord just to the left of $L2$ should be sufficient to develop the strength of the bottom chord member $L1$-$L2$. The chord is spliced, as is seen, by the $12'' \times \frac{7}{16}''$ inside plates, by the $12'' \times \frac{1}{2}''$ outside splice plates and the $\frac{3}{8}''$ tie plates. The rivets through the web of the channels to the left of the splice, which are field rivets, are in double shear and bearing on the web of the channels. The strength of the channels is equal to $20.9 \times 16{,}000 = 334{,}000\#$, or $167{,}000\#$ for each channel. Then considering one channel, the 16 field rivets in the web are good for $192{,}000\#$ $(= 2 \times 6{,}000 \times 16)$ in double shear, and $145{,}600\#$ $(= 0.52 \times \frac{7}{8} \times 16 \times 20{,}000)$ in bearing on the web of the channel; and hence the latter governs. The 6 rivets in the flanges of the channel are good for $6 \times 6{,}000 = 36{,}000\#$. Adding this to the $145{,}600\#$ we have $181{,}600\#$, which is $14{,}600\#$ more than is necessary to provide for; or, in other words, there is at least one rivet too many; but the rivets in the tie plates are none too numerous to hold those plates and to omit one rivet in the web would leave the spacing unbalanced; that is, unsymmetrical, and to omit two would be too much of a reduction, so the rivets as shown will be used. There should be enough rivets in the $12'' \times \frac{1}{2}''$ splice plate on each side of the splice to develop the strength of that plate. The net section of the plate is equal to $12 \times \frac{1}{2} - 1 = 5\square''$. Multiplying this by 16,000 we have $80{,}000\#$ for the strength of the plate. Dividing this by 7,200 we obtain 11 for the number of required shop rivets, and dividing it by 6,000 we obtain 13.3, say 14, for the number of required field rivets. As is seen, we have 12 shop rivets on one side of the splice and 16 field rivets on the other side, so that the riveting as far as the outside splice plates are concerned is quite satisfactory.

The net area of the $12'' \times \frac{7}{16}''$ plate is $5.25 - 0.87 = 4.38\square''$, and the net area of the $12'' \times \frac{1}{2}''$ plate is $5\square''$, making a total of $4.38 + 5 = 9.38\square''$ in the splice plates, while the net area of the $15'' \times 40\#$ channel is $10.45\square''$; but the area of the tie plates can be counted to the extent of 6 field rivets. This is equal to $6{,}000 \times 6 \div 16{,}000 = 2.25\square''$. Adding this to the area of the splice plates we obtain $11.63\square''$ for the total net area of the plates splicing each channel—which is quite sufficient.

The number of rivets in the laterals, shown at $L2$ (Fig. 283), is obtained by developing the laterals. For example, the lateral to the left of the floor beam is a $5'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angle which has a net section of $4 - 0.5 = 3.5\square''$. Multiplying this by 16,000 we obtain $56{,}000\#$ for the strength of the lateral, and dividing this by 6,000 we obtain 9—$\frac{7}{8}''$ field

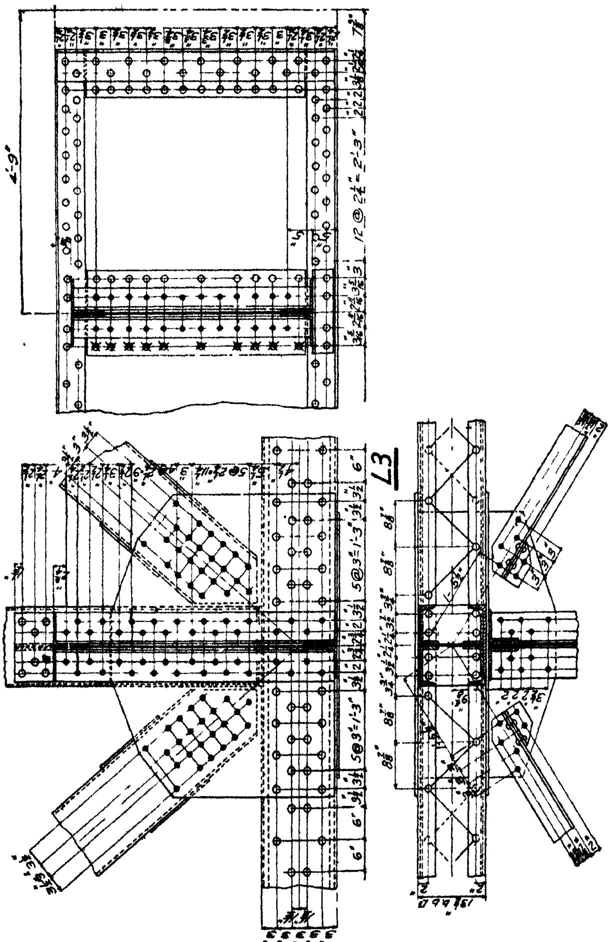


Fig. 284

rivets, whereas 9 are used. The number of rivets in the other laterals is obtained in the same manner.

Joint L3. Next we will make a sketch of the details at the joint $L3$, as shown in Fig. 284. In drawing this sketch, we can draw the outline of the bottom chord, diagonals and post. Then we can draw the end connections of the floor beam onto the sketch and draw the rivet lines in all the members, and then we are ready to determine the number of rivets and spacing of same for each member connecting at that joint.

The rivets in the connection of the two diagonals will be the same for each. These diagonals are subjected to both tension and compression, and, according to the specifications, the sum of the two stresses should be used in determining the required number of rivets in the end connections. So we have $(201{,}000+78{,}000)=279{,}000\#$ for the stress to be considered (see Fig. 279). Dividing this by 6,000 we obtain 46.5—$\frac{7}{8}''$ field rivets or, say, 24 on each side of each diagonal, and, as is seen, 24 are used. The rivets on each side of the floor beam connecting the gusset plate to the bottom chord should be sufficient to transmit the component of the stress in the diagonal along the bottom chord. This component is equal to $279{,}000\# \cdot \sin\theta = 279{,}000 \times 0.64 = 178{,}000\#$ (about). Dividing this by $7{,}200\#$ (the allowable single shear on a $\frac{7}{8}''$ shop rivet) we obtain 24.8 rivets or, say, 13 on a side and, as is seen, 14 are used. The rivets in the diagonals should be spaced first and the edge of the gusset plate located and drawn down to the bottom chord, and the rivets arranged in the bottom chord to suit the gusset plate. Then the next thing to do is to determine the rivets connecting the floor beam to the truss, as was explained above in the case of panel points $L1$ and $L2$.

After having the rivets spaced in the floor beam connection, the gusset plate can be drawn completely; however, the spacing in the floor beam connection selected for this joint must be the same as for the other intermediate lower chord joints. The bottom lateral connections can next be drawn. The rivets in each lateral are determined by developing the strength of each lateral as was shown above for the other joints. After the rivets are spaced in the laterals the lateral plate can be drawn to suit the spacing; however, the rivets in the bottom of the floor beam should be the same as required at $L1$, in order to have all of the intermediate floor beams alike.

Referring to the sketch of the intermediate floor beam, drawn just to the right of $L3$ (Fig. 284), the rivets connecting the end connection angles to the floor beam must be sufficient to transmit the maximum end shear on the floor beam in double shear or bearing, whichever requires the greater number. These end angles are placed upon $\frac{1}{2}''$ fillers (the fillers being just as thick as the flange angles). These fillers are held by rivets placed outside of the connection angles, and hence the thickness of bearing on the rivets through the angles can be taken as the total thickness of the 2—$\frac{1}{2}''$ fillers and web, making $1\frac{7}{16}''$, or the combined thickness of the two angles, which is the least thickness—as it is $1''$. So for the allowable bearing on each $\frac{7}{8}''$ shop rivet in this connection we have $\frac{7}{8}\times 1 \times 24{,}000 = 21{,}000\#$, and for the allowable double shear on the same we have $2\times 0.6\times 12{,}000 = 14{,}400\#$, which is less than the bearing and hence governs. Then dividing the maximum end shear by the last figure we have $187{,}300 \div 14{,}400 = 13$ rivets, and, as is seen, 13 rivets are used, not counting those passing through the flange angles.

There should be enough rivets connecting the stringer to the floor beam to transmit the maximum end shear on a stringer. So we have $142{,}000 \div 6{,}000 = 23.6$ rivets in single shear, and, as is seen, 24 are used. The bearing of these rivets is upon the 2—$\frac{1}{2}''$ fillers and the $\frac{7}{16}''$ web, so that the shear governs. There should be enough rivets through these fillers, outside of the stringer connection, to carry the total end shear on a stringer and also the concentration on the floor beam. In the first case, the rivets are in single shear and for the number required we have $142{,}000 \div 7{,}200 =$ about 20 shop rivets, whereas we have 20. In the second case, these rivets are in bearing on the $\frac{7}{16}''$ web and for the number required we have $187{,}300 \div 9{,}180 =$ about 20 rivets, so the correct number is used. As a matter of fact, the field rivets in the stringer connection (theoretically) bear on the web also and hence we have an excess of 24 field rivets in bearing on the web in the connection shown, but it is a question in the case of such long rivets, and especially when field driven, just how much bearing they will exert on the web, and there is some question as to the bending, and to be on the safe side it is advisable to compute the number on this important connection as is here shown.

It will be seen that in the case of the end connection of the floor beam there is an excess of 5 rivets in bearing on the web (not considering the ones passing through the flange—they resist the flange increment) and consequently 5 rivets could (theoretically) be omitted from the fillers, but this would make the rivets look rather sparing, so the number shown will be used.

The rivets through the fillers of the stringer connection are countersunk on one side of the connection to permit the swinging of the stringers in position during the erecting of the structure. The small angles under each stringer (connected to the floor beam) are for erection purposes and are not assumed to carry any load from the stringers.

Joint U1. We will next consider the drawing of the hip joint *U*1 shown in Fig. 285. The first thing to do in making this sketch is to select the center point of the joint and draw the center lines of the end post and top chord. Then bisect the angle between these lines, and this bisector will be the joint line—where the two members meet. Now, using the same transverse spacing as was used in the end post at *LO* (Fig. 282), the outlines of the end post and top chord can be drawn and the rivets spaced. There should be enough rivets connecting the end post to the gusset plates to transmit the total stress in the member. So we have

$$523{,}000 \div 6{,}000 = 87, \text{ say } 88,\text{—}\tfrac{7}{8}'' \text{ field rivets,}$$

or 44 on a side. As is seen, there are 38 in single shear, which provides for $6{,}000 \times 38 = 228{,}000\#$ of the stress, and 4 passing through the small outside splice plate which can be considered for bearing on the $\frac{1}{2}''$ web and hence provide for $8{,}750 \times 4 = 35{,}000\#$ of the stress in the end post. Adding these two stresses we have $228{,}000 + 35{,}000 = 263{,}000\#$, and multiplying this by 2 we have 526,000, which is 3,000# more than the stress in the end post, and hence the riveting as shown in the end post is sufficient. There should be enough rivets connecting the top chord to the

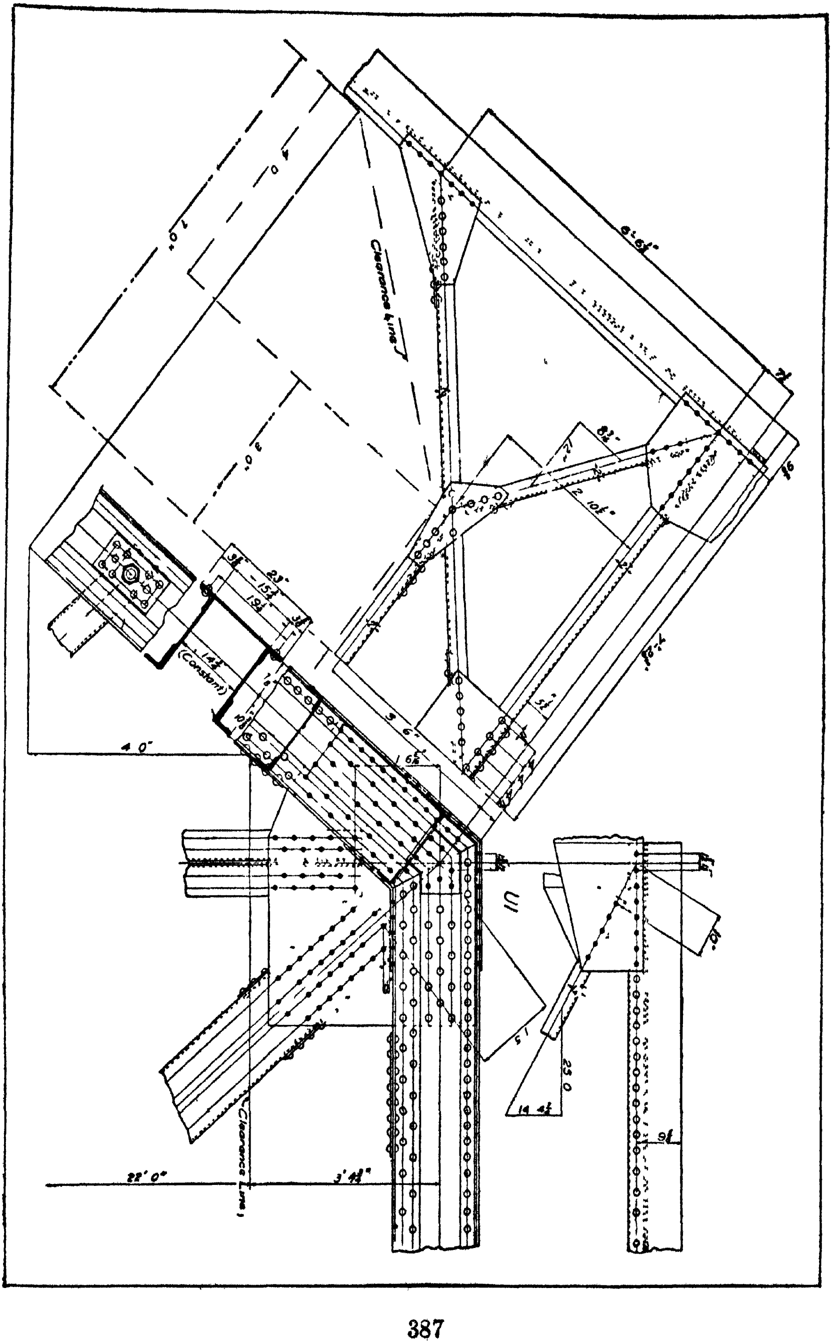


Fig 283

gusset plates to transmit the total stress in that member. As is shown, there are 33 shop rivets ($\frac{7}{8}''$ diameter) which provide for $33 \times 7{,}200 = 237{,}600\#$ of the stress in the top chord, and the 4 field rivets in bearing on the $\frac{3}{8}''$ web which provide for $6{,}563 \times 4 = 26{,}250\#$ stress. Adding these two quantities together and multiplying by 2 we have $263{,}850 \times 2 = 527{,}700\#$, which shows this riveting to be satisfactory, as the stress in the top chord $U1$-$L2$ is $521{,}000\#$ (see Fig. 279). Next, the hanger $U1$-$L1$ and the diagonal $U1$-$L2$ can be drawn and the rivets connecting them to the gusset plates spaced as shown. The rivets connecting the hanger to the gusset plates should be sufficient to develop the strength of the hanger, which is $12.94 \times 16{,}000 = 207{,}000\#$ (see Fig. 279). Then we have

$$207{,}000 \div 6{,}000 = 34.5, \text{ say } 36\text{—}\tfrac{7}{8}'' \text{ field rivets,}$$

or 18 on a side. As is seen, 18 are used. The number of rivets connecting the diagonal $U1$-$L2$ to the gusset plates should be sufficient to develop the strength of the diagonal, which is $23.9 \times 16{,}000 = 382{,}000\#$. Dividing this by $6{,}000\#$ we obtain 64—$\frac{7}{8}''$ field rivets, or 32 on a side; whereas 32 are used, as is seen.

There should be enough metal in the gusset plates to properly transmit all of the forces acting in the plane of the truss at the joint.

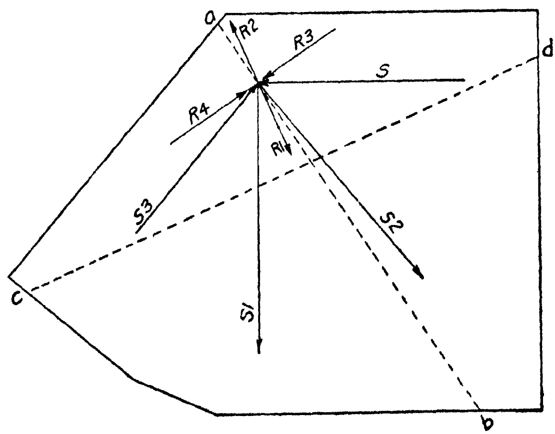


Fig. 286

These forces are as indicated in Fig. 286. As is readily seen, these forces will not all be a maximum at the same time. About the most severe stress will occur on the plates when the end post has a maximum stress, as the top chord $U1$-$U2$ has about the maximum stress at that time and the hanger $U1$-$L1$ has quite a large stress, and the diagonal $U1$-$L2$ has a low stress. Combining S and $S3$ (Fig. 286) we obtain the resultant $R2$, and combining $S1$ and $S2$ we obtain the resultant $R1$, which is equal and opposite to $R2$. So, evidently, the maximum tension on the plates will be on same section as cd, and combining S and $S2$ we obtain the resultant $R3$; and, likewise, combining $S1$ and $S3$ we obtain the resultant $R4$, which will be equal and opposite to $R3$. So, evidently, the maximum compression on the gusset plates will be on some section as *ab* perpendicular to these resultants ($R3$ and $R4$), due mostly to S and $S3$. A very satisfactory approximation can be obtained by making the plates thick enough so that two-thirds of the metal along the section *ab* is sufficient to transmit the maximum stress in the end post. By making the plate $\frac{5}{8}''$ thick, two-thirds of the section along section *ab* is about $48 \times \frac{5}{8} \times \frac{2}{3} \times 2 = 40\square''$. Dividing this into the maximum stress in the end post, we have $523{,}000 \div 40\square'' = 13{,}100\#$ (about) for the maximum compressive unit stress in the plates, which is about the correct value, as it is about the same as allowed in the chord $U1$-$U2$. So the gusset plates will be made $\frac{5}{8}''$ thick. The same thickness will be used at all of the other joints, whether required or not, in order to have the sides of the truss members in the same plane throughout without the use of fillers.

After the connections of the main members at joint $U1$ are drawn as shown (Fig. 285), the portal can be drawn. In drawing the portal, the first thing to do is to draw a top plan of the end post and locate the center of the portal. Then locate the clearance line as shown and draw the cross-section of the portal on the elevation of the end post (as shown), and then draw the plan of the portal, keeping inside the clearance line at every point.

The required number of rivets in the end connections of the portal members is obtained by developing the strength of each of the members. After the portal is drawn the top lateral and the bent lateral plates can be drawn as shown, and thus the sketch of joint $U1$ is completed.

Joint $U2$. The details of this joint are shown in Fig. 287. The outlines of the top chord as seen in elevation should be drawn first, and then the outlines of the diagonal ($U2$-$L3$) and the post ($U2$-$L2$) can be drawn as shown. The next thing after this is to locate the rivets in the end connection of the diagonal. As this diagonal is subjected to both tension and compression, the number of rivets in each end connection must, according to the specifications, be sufficient to transmit the sum of the two stresses. So we have

$$(78{,}000 + 201{,}000) \div 6{,}000 = 16.5\text{—}\tfrac{7}{8}'' \text{ rivets}$$

for the required number, or say 24 on a side, and, as is seen, 24 are used, the same as in the end connection of this same diagonal at $L3$ (see Fig. 284).

Next, the cross-section of the transverse strut can be drawn on to the chord and then the transverse view, shown to the right, can be drawn. In drawing this view it is best to draw first the cross-section of the top chord, as shown, and then the elevation of the transverse strut. Next the clearance line should be located and dotted in, as shown, and then the knee brace and sub-strut can be drawn, at which time the connections of the knee brace and sub-strut to the post are determined and can be projected over to the other view of the post as shown. The next thing to do is to determine the number of rivets required to connect the post ($U2$-$L2$) to the gusset plates.

This member is subjected to $162{,}000\#$ compression and $54{,}000\#$ tension. So, according to the specifications, there should be a sufficient number of rivets connecting the post to the gusset plates at $U2$ to transmit the sum of these two stresses. So we have $(162{,}000 + 54{,}000) \div$

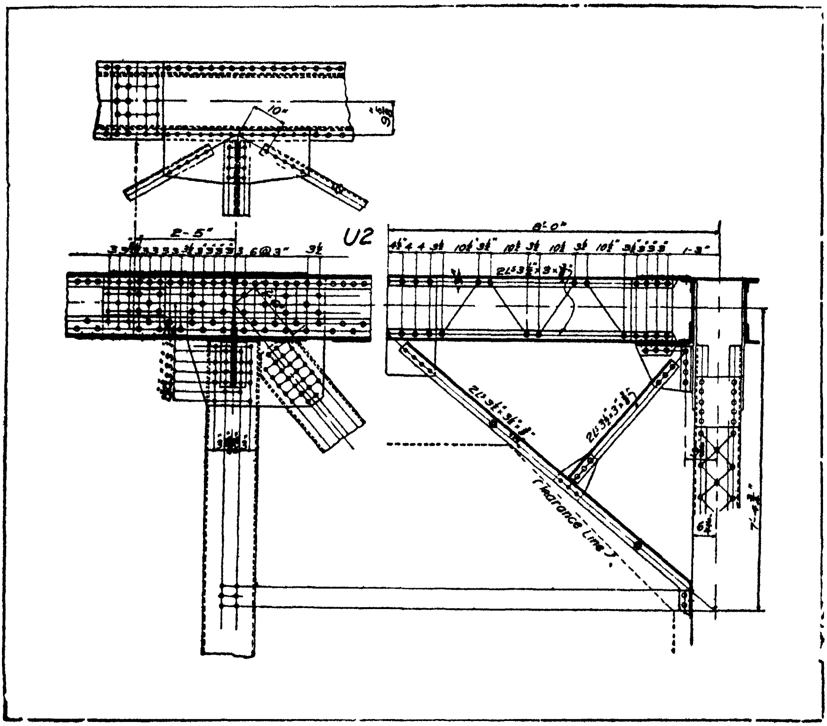


Fig. 287

6,000 = 36—$\frac{7}{8}''$ field rivets, or 18 on each side of the post, whereas 20 are used.

There should be enough rivets (at least) connecting the gusset plates to the top chord to transmit the horizontal component of diagonal *U*2-*L*3. This component, considering the sum of the stresses in the diagonal, is equal to $(201{,}000+78{,}000)\times \sin\theta = 279{,}000\times 0.64 = 178{,}560\#$. Dividing this by 7,200 we obtain about 25 shop rivets, say 13 on a side, whereas there are almost three times this number shown (Fig. 287). But the number cannot well be reduced, as the spacing of the rivets in the chord angles is about what the specifications require, and the rivets between the chord angles cannot be more sparingly spaced. So the riveting of the gusset plates to the chord will be considered satisfactory as shown.

The next thing, the plan of the top chord, strut and laterals can be drawn. There should be enough rivets in each lateral to develop the strength of the lateral in tension. Each lateral is composed of 2—⌊s $3\frac{1}{2}''\times 3\frac{1}{2}''\times\frac{3}{8}''$. So for the strength of each we have $(4.96-0.75)\times 16{,}000 = 67{,}360\#$. Dividing this by 6,000 we obtain about 11 field rivets, say 5 in each angle, whereas 5 are used—5 in the angle shown and 5 in the angle at the bottom of the chord.

There should be enough rivets in each transverse strut to develop the strength of the strut in compression. Each of these struts is composed of 4—⌊s $3\frac{1}{2}''\times 3''\times\frac{3}{8}''$ arranged so as to form an I-section. So, considering the $3\frac{1}{2}''$ legs turned horizontally and the vertical legs $\frac{1}{2}''$, back to back, we have

$$p = 16{,}000 - 70\,\frac{192}{1.71} = 8{,}140 \text{ lbs.}$$

for the allowable unit stress. Then multiplying this by the area of the 4 angles in each strut we obtain $8{,}140\times 9.2 = 74{,}888$. Dividing this by 6,000 we obtain about 13 field rivets, say 7 in the top angles and 7 in the bottom angles, whereas 8 are used so as to have symmetrical spacing.

The chord splice just to the left of joint *U*2 (Fig. 287) is what is known as a butt joint. The chord sections are planed so they fit perfectly against each other and consequently the stress can be considered as being transmitted from one chord segment to the other without the aid of the splice plates, and hence the splice plates are considered as merely holding the chord segments in position; and, as is evident, the size of these plates and the number of rivets connecting them to the chord is mostly a matter of judgment.

*Joint U*3. The details at this joint will be practically the same as at *U*2, except the connection of the diagonal and chord splice are omitted, and consequently no larger scale sketch is necessary.

This completes the necessary large scale sketches and next the general drawing, Fig. 288, and the shop drawings, Figs. 289 to 303, can be made, wherein the details are, as we may say, simply transferred from the above sketches to these drawings.

185. Camber.—To prevent bridge trusses from sagging they are "cambered," that is, they are built so that they curve upward. In the case of an ordinary truss bridge the cambering of the trusses is accom-

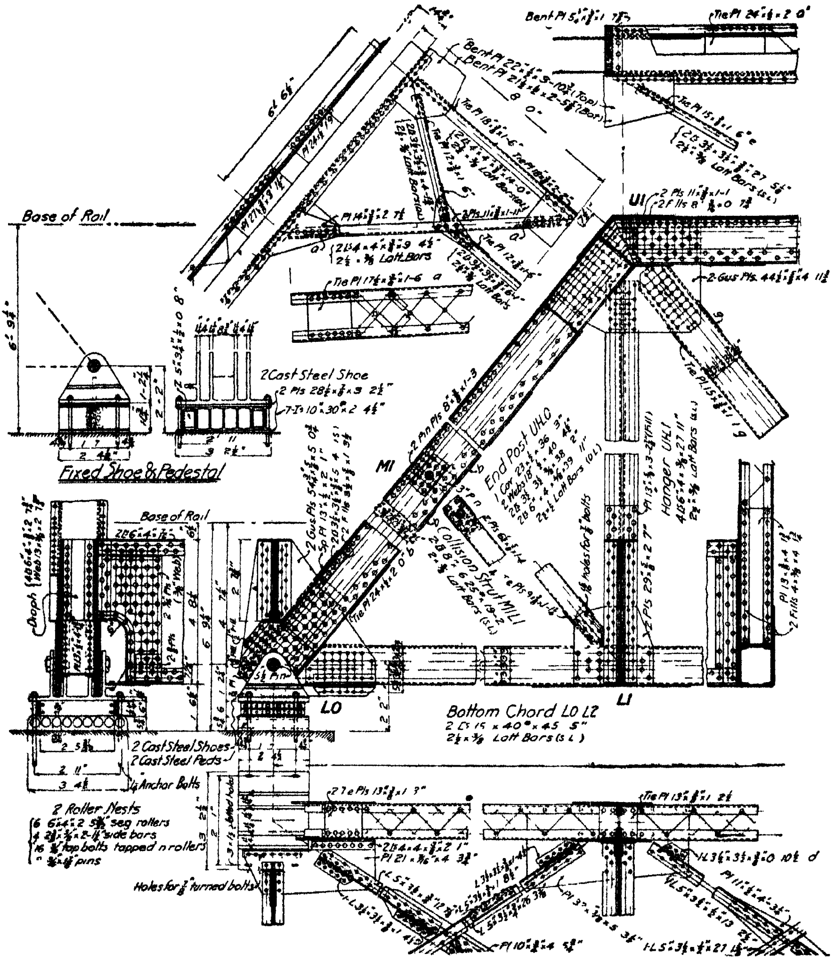

Base of Rail
Fixed Shoe & Pedestal
2 Cast Steel Shoe
End Post U1L0
Hanger U1L1
Collision Strut M1L1
Bottom Chord L0 L2
2 Cast Steel Shoes
2 Cast Steel Peds
Anchor Bolts
2 Roller Nests
Holes for 7/8" turned bolts
U1
M1
L0
L1

Fig. 288

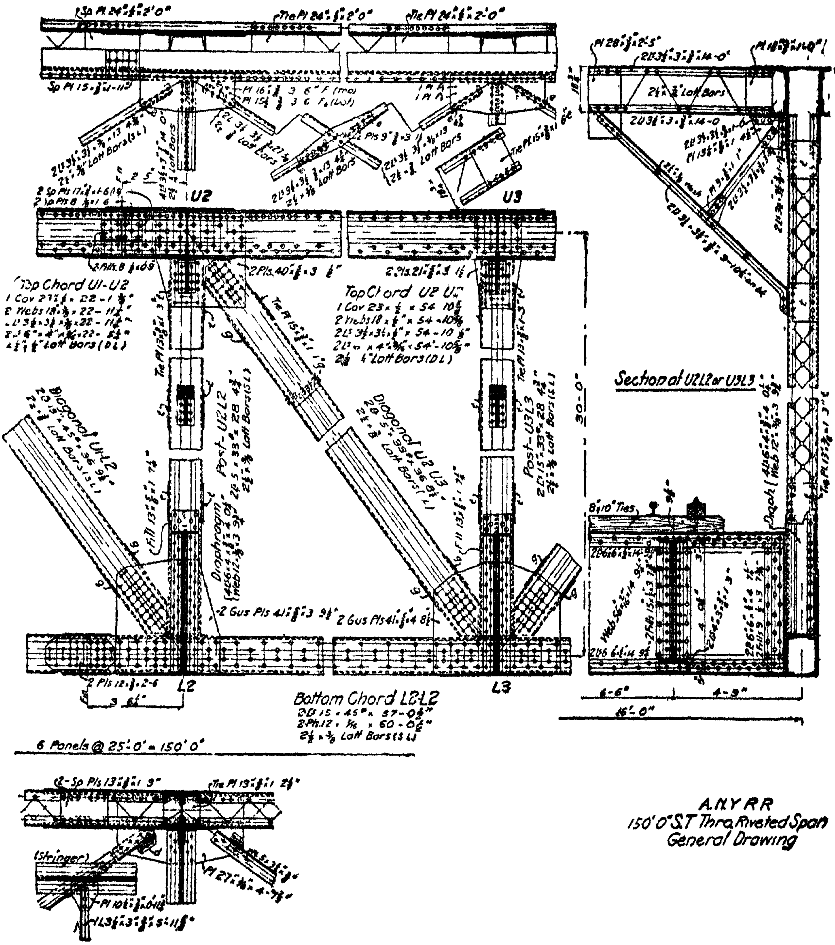

U2
U3
L2
L3
Top Chord U1-U2
Top Chord U2 U3
Diagonal U1-L2
Diagonal U2 U3
Post-U2L2
Post-U3L3
Diaphragm
Section at U2L2 or U3L3
Bottom Chord L2-L2
6 Panels @ 25'-0" = 150'-0"
30'-0"
16'-0"
(Stringer)
A.N.Y.R.R
150'0" S.T. Thro. Riveted Span
General Drawing

Fig 288

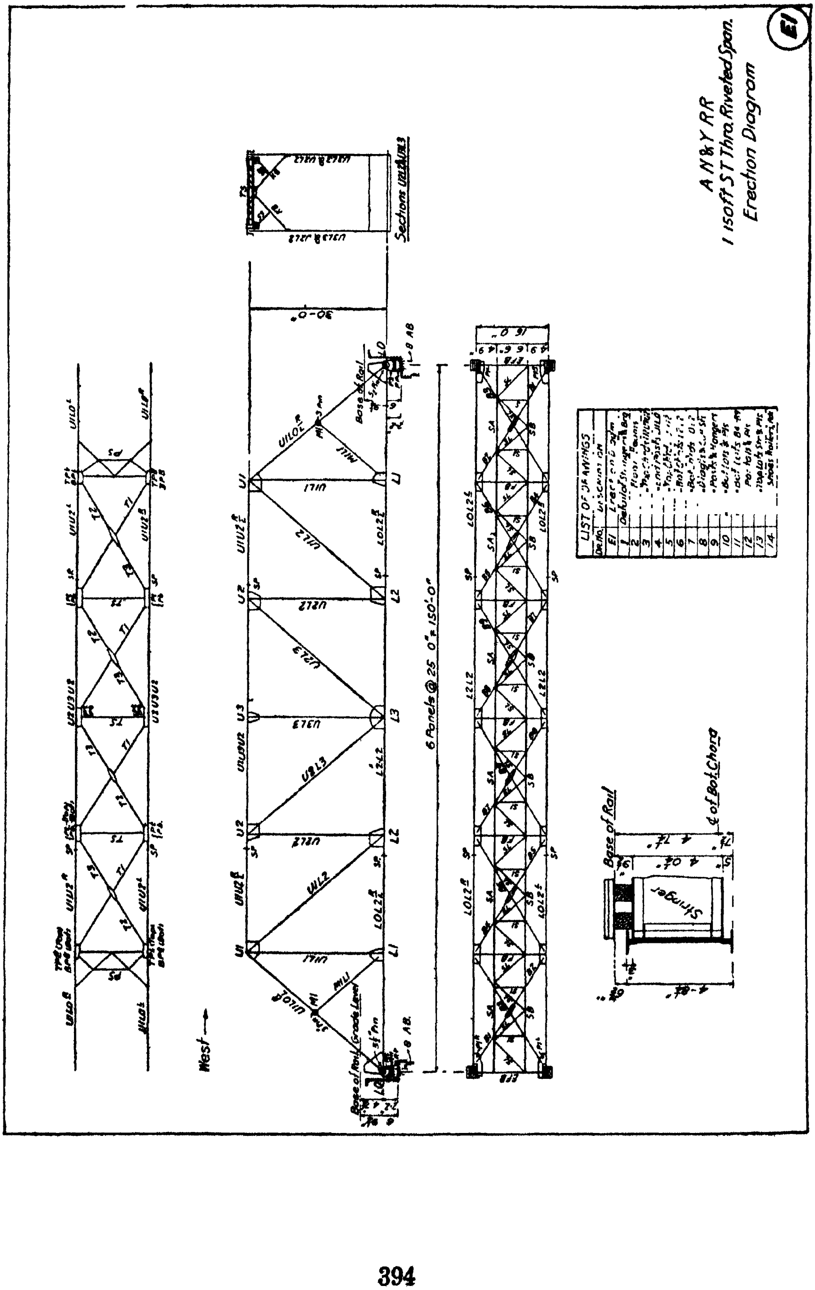
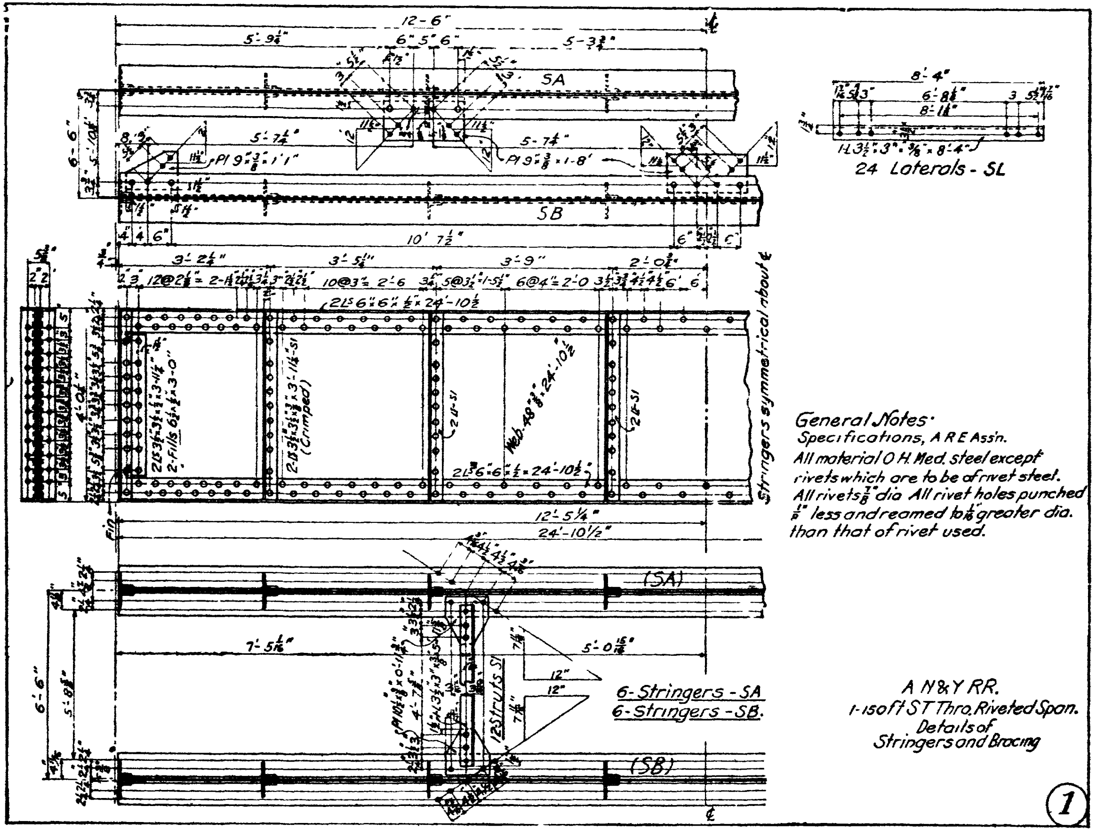

A N&Y RR
1 150 ft S T Thro. Riveted Span.
Erection Diagram
E1
Sections U2L2 & U3L3
30'-0"
6 Panels @ 25' 0" = 150'-0"
West
Base of Rail
Stringer
LIST OF DRAWINGS

Fig. 289



General Notes:
Specifications, A R E Ass'n.
All material O. H. Med. steel except rivets which are to be of rivet steel.
All rivets 7/8" dia. All rivet holes punched 1/8" less and reamed to 1/16" greater dia. than that of rivet used.
A N & Y RR.
1-150 ft S T Thro. Riveted Span.
Details of Stringers and Bracing
24 Laterals - SL
6-Stringers - SA
6-Stringers - SB.
Stringers symmetrical about ℄
SA
SB
(SA)
(SB)
12 Struts S1
(Crimped)
1

Fig. 290

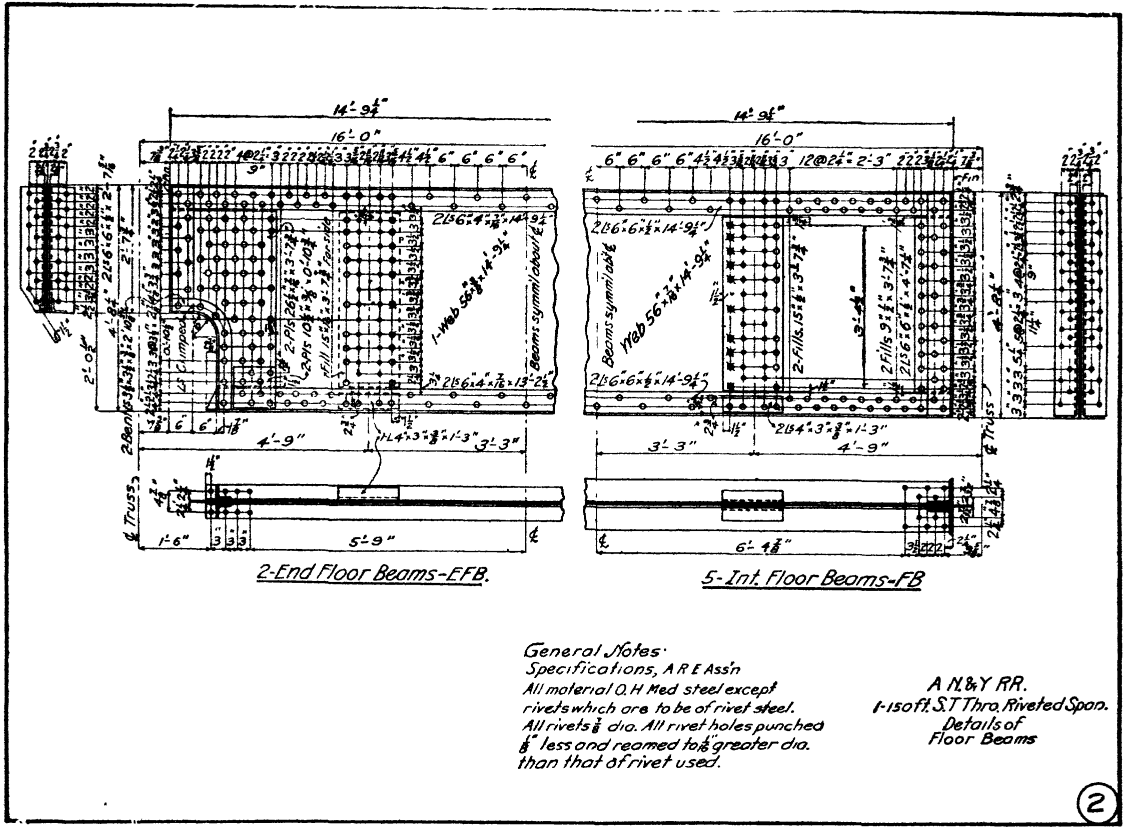

2-End Floor Beams-EFB.
5-Int. Floor Beams-FB
General Notes:
Specifications, A R E Ass'n
All material O.H Med steel except
rivets which are to be of rivet steel.
All rivets 7/8 dia. All rivet holes punched
1/8" less and reamed to 1/16" greater dia.
than that of rivet used.
A N.&Y RR.
1-150 ft. S.T Thro. Riveted Span.
Details of
Floor Beams
2

Fig. 291

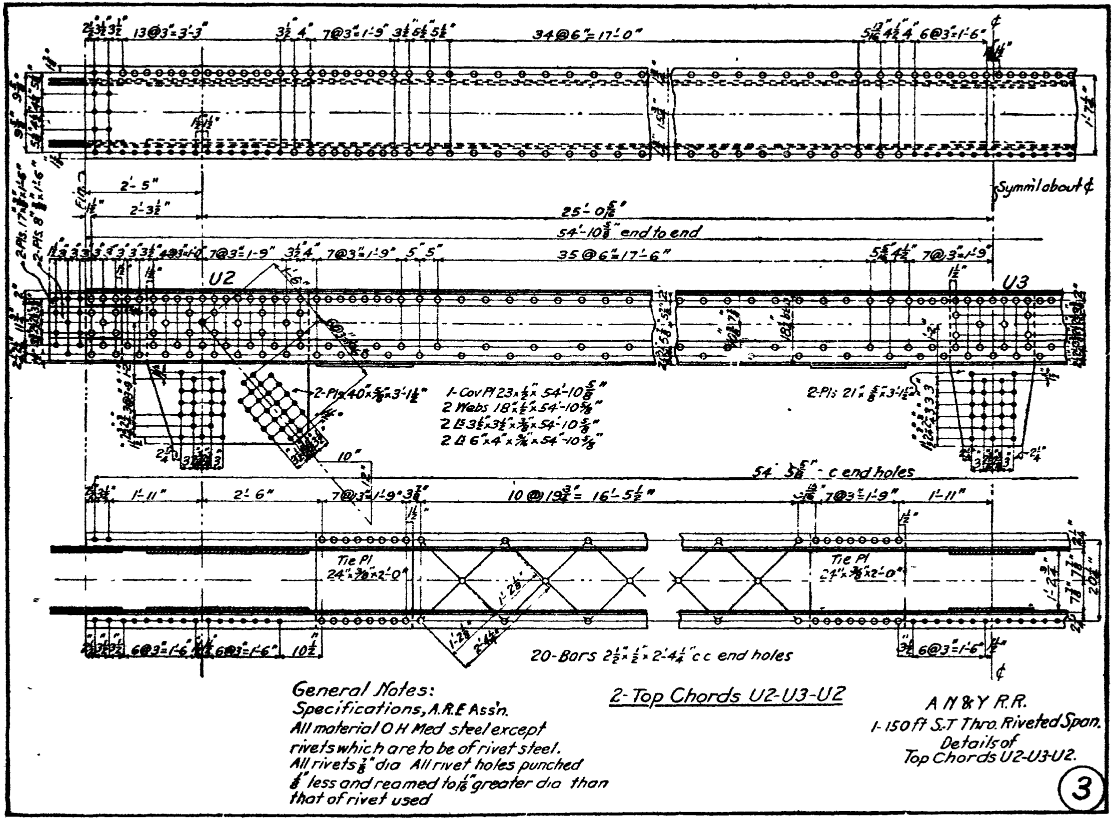
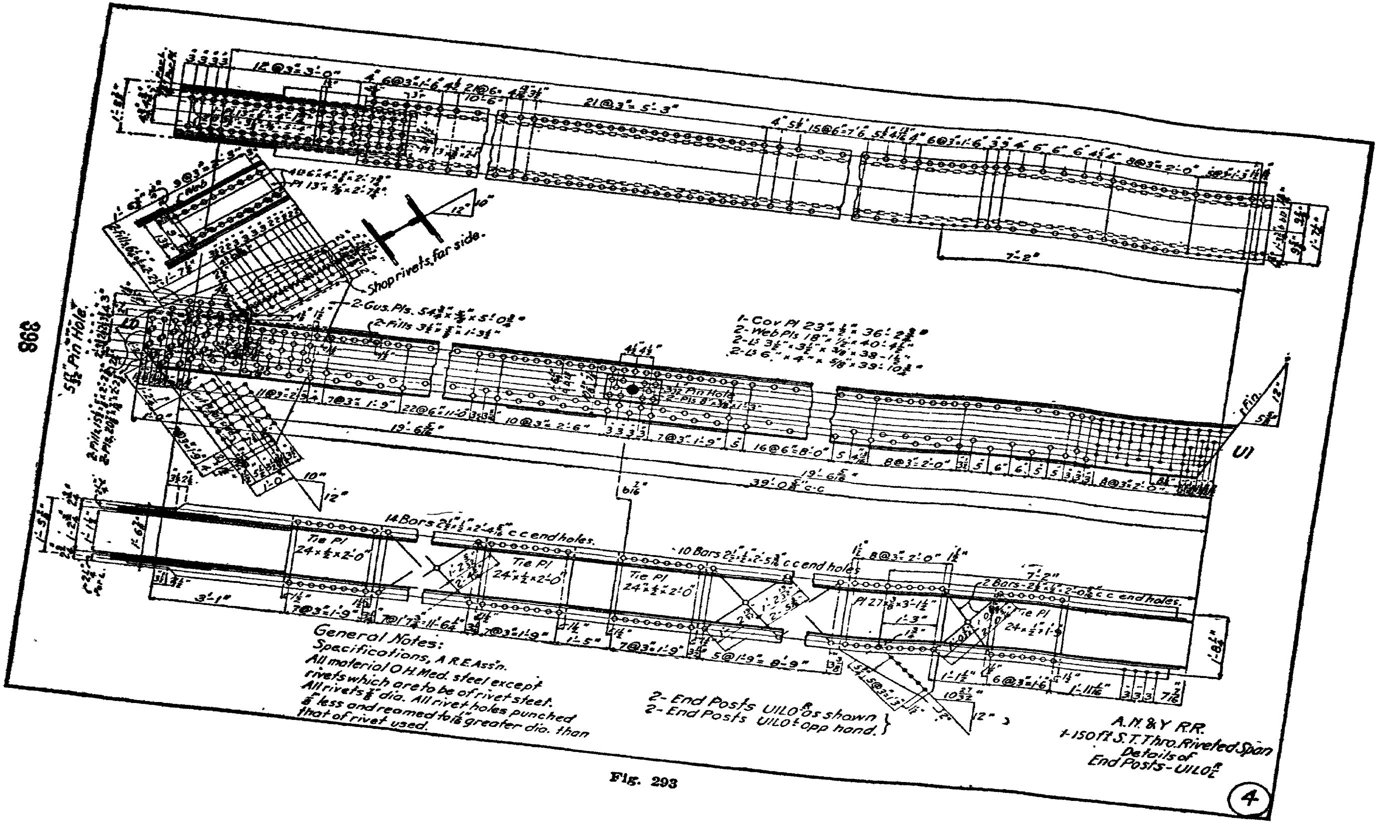

U2
U3
Symm'l about ℄
25'-0 5/16"
54'-10 5/8" end to end
35 @ 6" = 17'-6"
34 @ 6" = 17'-0"
2-Pls 40" x 5/8" x 3'-1 1/2"
2-Pls 21" x 5/8" x 3'-1 1/2"
1-Cov Pl 23 x 1/2" x 54'-10 5/8"
2 Webs 18" x 1/2" x 54'-10 5/8"
2 ∟s 3 1/2" x 3 1/2" x 3/8" x 54'-10 5/8"
2 ∟ 6" x 4" x 7/16" x 54'-10 5/8"
54' 5 5/8" - c end holes
10 @ 19 3/4" = 16'-5 1/2"
Tie Pl 24" x 3/8" x 2'-0"
20-Bars 2 1/2" x 1/2" x 2'-4 1/4" c c end holes
General Notes:
Specifications, A.R.E. Ass'n.
All material O H Med steel except rivets which are to be of rivet steel.
All rivets 7/8" dia All rivet holes punched 1/8" less and reamed to 1/16" greater dia than that of rivet used
2-Top Chords U2-U3-U2
A N & Y R.R.
1-150 ft S.T Thro. Riveted Span.
Details of
Top Chords U2-U3-U2.
3

Fig. 292



General Notes:
Specifications, A.R.E.Ass'n.
All material O.H.Med. steel except rivets which are to be of rivet steel.
All rivets 7/8" dia. All rivet holes punched 3/16" less and reamed to 1/16" greater dia. than that of rivet used.
2- End Posts U1L0R as shown
2- End Posts U1L0L opp. hand.
A.N.&Y. R.R.
1-150 ft. S.T. Thro. Riveted Span
Details of
End Posts - U1L0
4
1-Cov Pl 23" x 1/2" x 36'-2 3/8"
2-Web Pls 18" x 1/2" x 40'-4 1/2"
2-Ls 3 1/2" x 3 1/2" x 3/8" x 38'-1 1/2"
2-Ls 6" x 4" x 9/16" x 39'-10 1/2"
2-Gus. Pls. 54 3/4" x 5/8" x 5'-0 1/2"
Shop rivets, far side.
5 1/2" Pin Hole
U1
L0
39'-0 5/8" c-c
14 Bars 2 1/2" x 1/2" x 2'-4 5/8" c c end holes.
10 Bars 2 1/2" x 1/2" x 2'-5 1/4" c c end holes
Tie Pl 24 x 1/2 x 2'-0"

Fig. 293

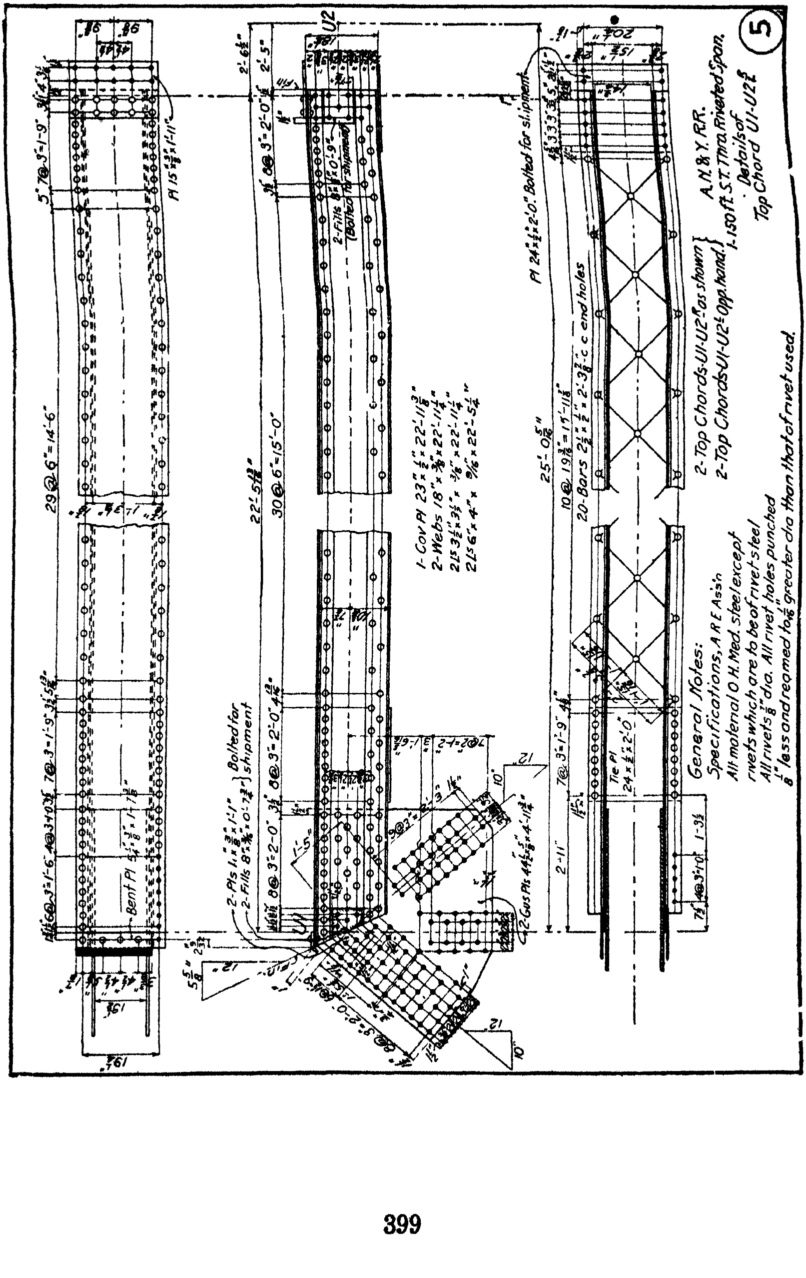
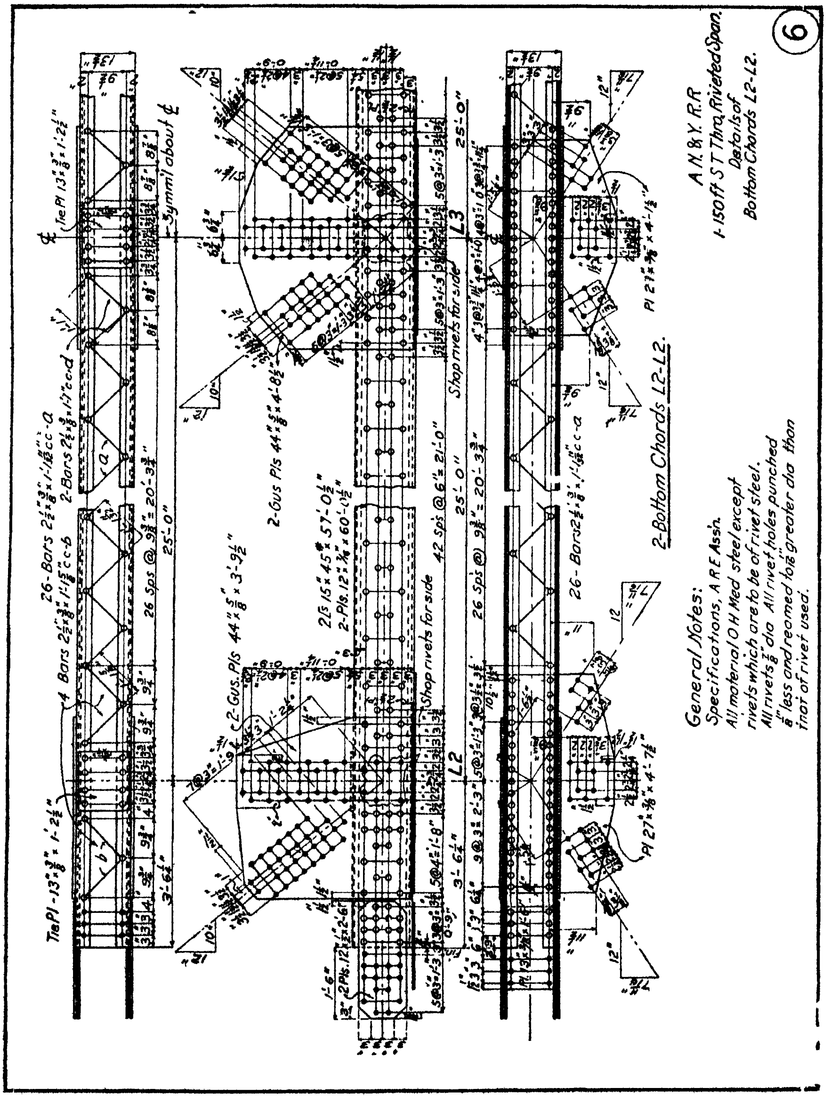

A. N. & Y. RR.
1-150 ft. S.T. Thro. Riveted Span.
Details of Top Chord U1-U2
2-Top Chords-U1-U2 as shown
2-Top Chords-U1-U2 Opp. hand.
General Notes:
Specifications, A R E Ass'n
All material O.H. Med. steel except rivets which are to be of rivet steel
All rivets 7/8" dia. All rivet holes punched 1/8" less and reamed to 1/16" greater dia than that of rivet used.

Fig. 204


A.N.&Y. R.R.
1-150 ft S.T. Thro. Riveted Span.
Details of
Bottom Chords L2-L2.
2-Bottom Chords L2-L2.
General Notes:
Specifications, A.R.E. Ass'n.
All material O.H. Med. steel except rivets which are to be of rivet steel.
All rivets 7/8" dia. All rivet holes punched 1/8" less and reamed to 1/16" greater dia. than that of rivet used.
2-Gus. Pls 44" x 5/8" x 4'-8 1/2"
2-Gus. Pls 44" x 5/8" x 3'-9 1/2"
Shop rivets for side
26-Bars 2 1/2" x 3/8" x 1'-1 11/16" c-c-a
Symm'l about ℄
L2
L3
6

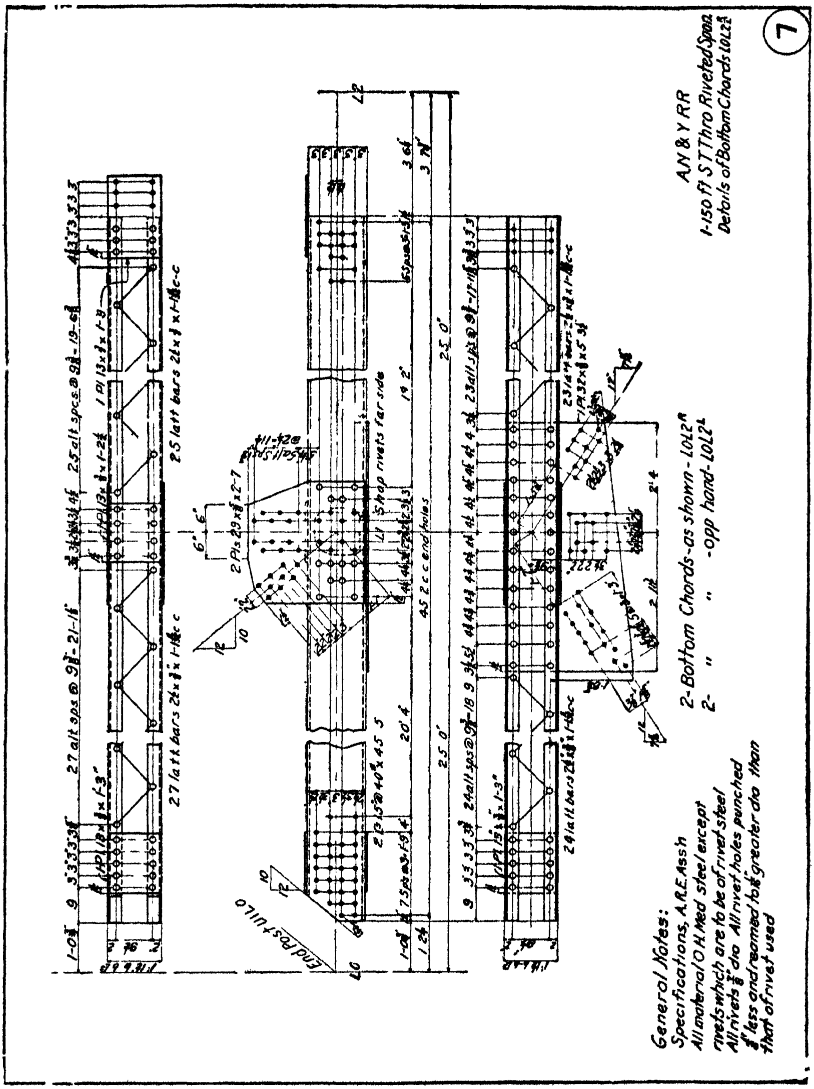

AN & Y RR
1-150 ft S T Thro Riveted Span
Details of Bottom Chords L0L2
General Notes:
Specifications, A.R.E.Ass'n
All material O.H. Med steel except rivets which are to be of rivet steel
All rivets 7/8" dia All rivet holes punched 3/16" less and reamed to 1/16" greater dia than that of rivet used
2-Bottom Chords-as shown-L0L2R
2- " " -opp hand-L0L2L
End Post U1L0
25 0'
7

Fig. 20f

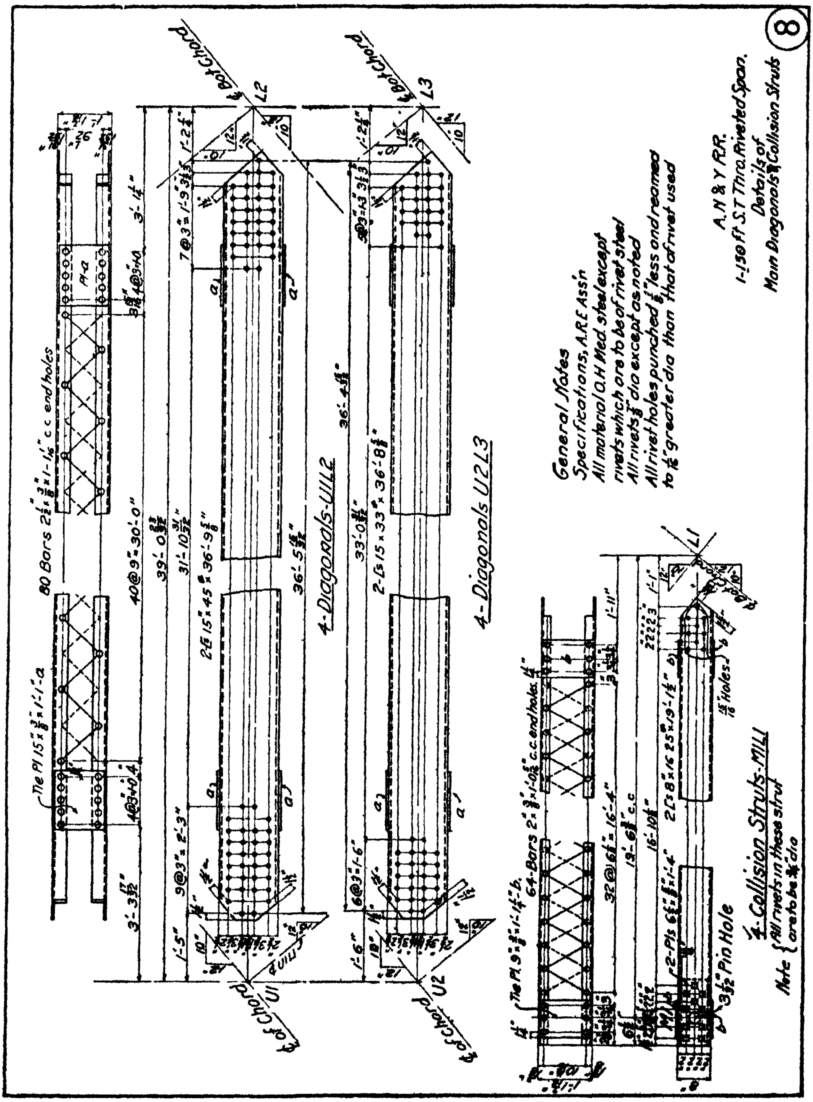

4-Diagonals-U1L2
4-Diagonals U2L3
4-Collision Struts-M1L1
General Notes
Specifications, A.R.E. Ass'n
All material O.H. Med. steel except rivets which are to be of rivet steel
All rivets 7/8" dia except as noted
All rivet holes punched 1/8" less and reamed to 1/16" greater dia than that of rivet used
A.N.& Y. RR.
1-150 ft. S.T. Thro. Riveted Span.
Details of
Main Diagonals & Collision Struts
8

Fig. 297

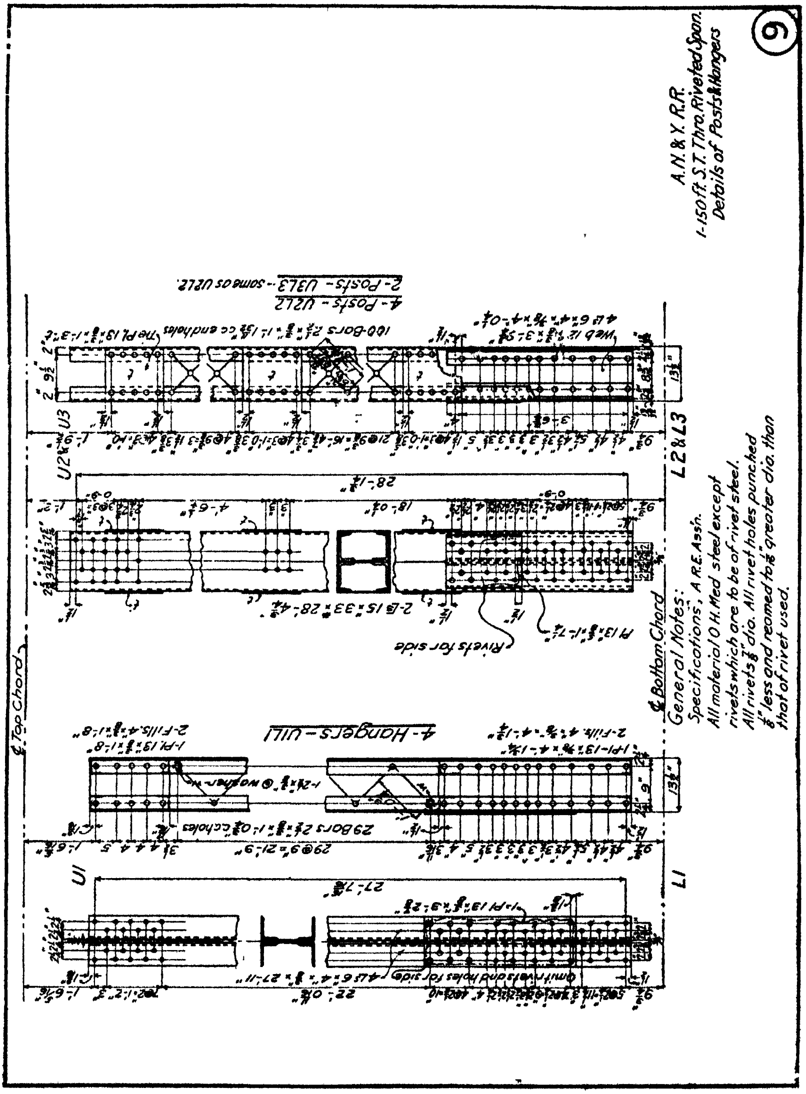

A.N.&Y.R.R.
1-150 ft. S.T. Thro. Riveted Span.
Details of Posts & Hangers
4-Posts-U2L2
2-Posts-U3L3--same as U2L2.
4-Hangers-U1L1
Top Chord
Bottom Chord
General Notes:
Specifications; A.R.E. Assn.
All material O.H. Med. steel except rivets which are to be of rivet steel.
All rivets 7/8" dia. All rivet holes punched 1/8" less and reamed to 1/16" greater dia. than that of rivet used.
Rivets for side
U1
L1
U2
U3
L2 & L3

Fig. 208

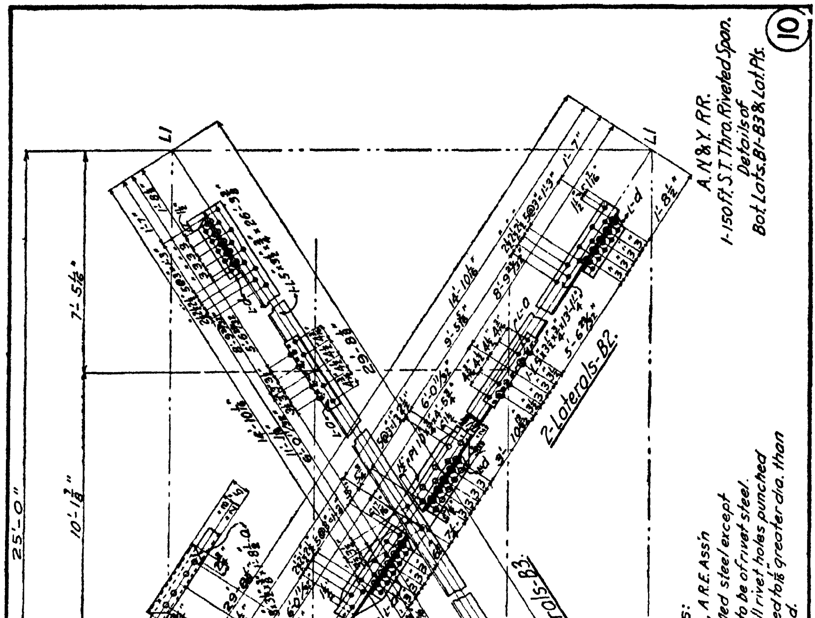

A. N. & Y. RR.
1-150 ft. S.T. Thro. Riveted Span.
Details of
Bot. Lats. B1-B3 & Lat. Pls.
10
General Notes:
Specifications, A.R.E. Ass'n
All material O.H. Med. steel except
rivets which are to be of rivet steel.
All rivets 7/8" dia. All rivet holes punched
1/8" less and reamed to 1/16" greater dia. than
that of rivet used.
2-Laterals-B1
2-Laterals-B2
2-Laterals-B3
2-Lat. Pls. Pl. as shown
2-Lat. Pls. Pl. opp. hand
25'-0"
16'-0"

Fig. 299

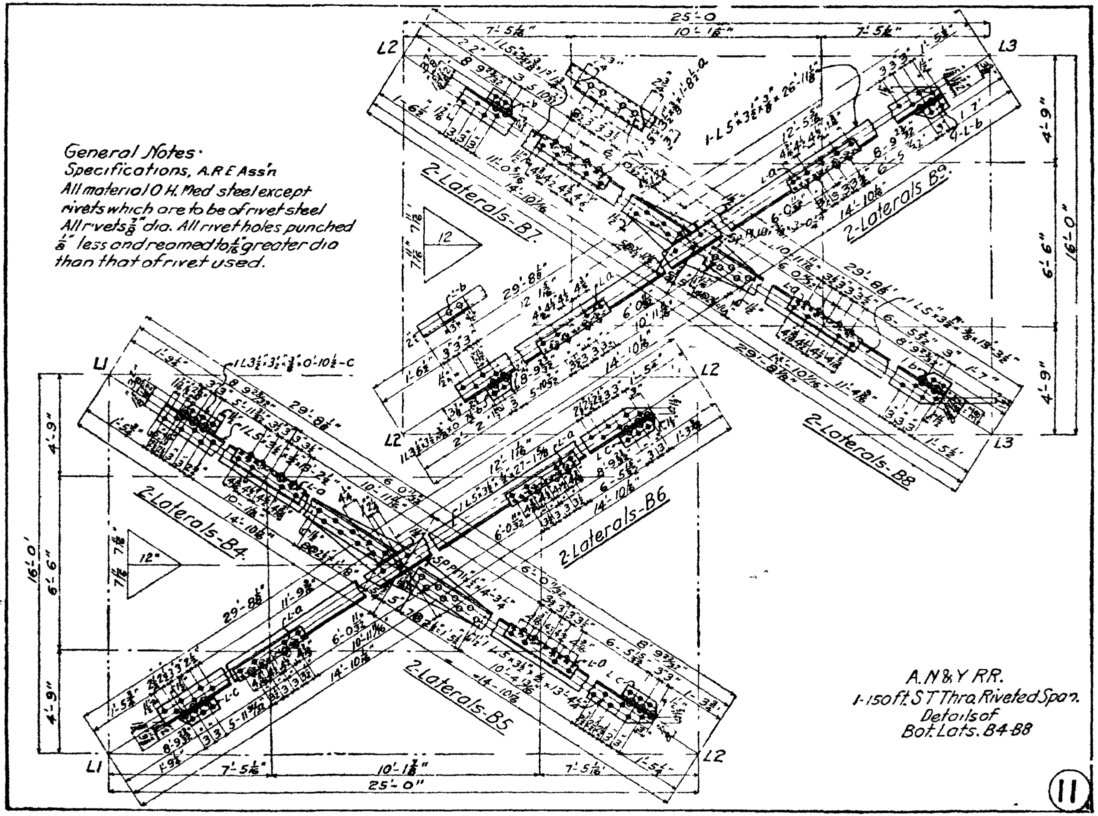

General Notes·
Specifications, A.R.E.Ass'n
All material O.H. Med steel except
rivets which are to be of rivet steel
All rivets 7/8" dia. All rivet holes punched
1/8" less and reamed to 1/16" greater dia
than that of rivet used.
2-Laterals-B4.
2-Laterals-B5
2-Laterals-B6
2-Laterals-B7.
2-Laterals-B8
2-Laterals B9.
L1
L2
L3
25'-0"
10'-1 1/8"
16'-0"
4'-9"
6'-6"
A.N.& Y RR.
1-150 ft. S.T. Thro. Riveted Span.
Details of
Bot. Lats. B4-B8

Fig. 300

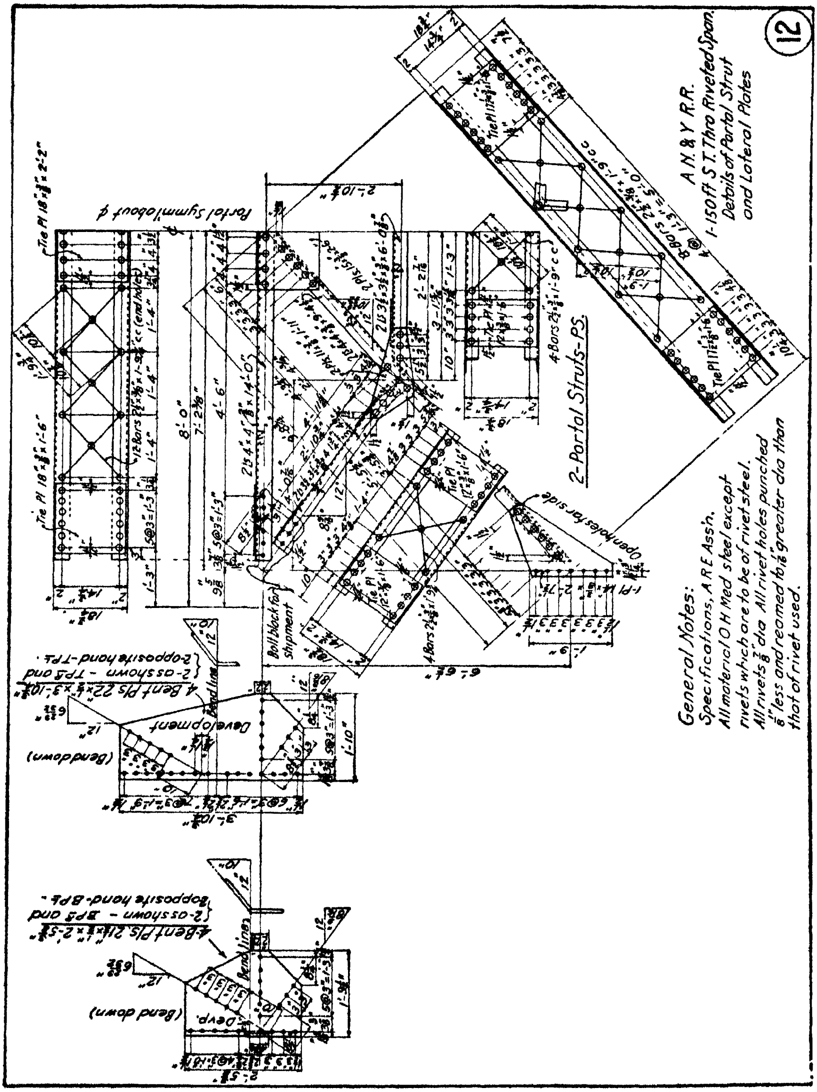
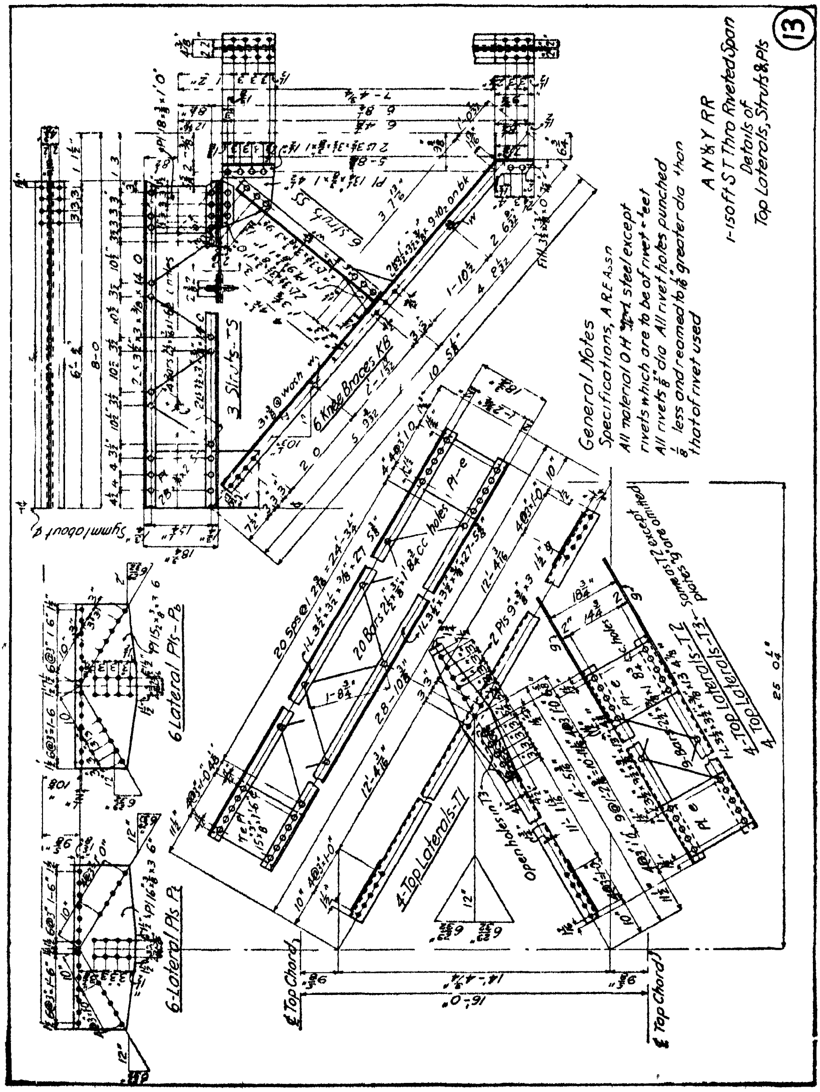

A.N.&Y. R.R.
1-150 ft. S.T. Thro Riveted Span.
Details of Portal Strut
and Lateral Plates
12
2-Portal Struts-PS.
General Notes:
Specifications, A.R.E. Ass'n.
All material O.H. Med steel except
rivets which are to be of rivet steel.
All rivets 7/8" dia. All rivet holes punched
1/8" less and reamed to 1/16" greater dia than
that of rivet used.

Fig. 301



A N & Y RR
1-150 ft S T Thro Riveted Span
Details of
Top Laterals, Struts & Pls
13
General Notes
3 Struts-S
6 Knee Braces-KB
6 Lateral Pls-P6
6-Lateral Pls-P
4-Top Laterals-T1
4-Top Laterals-T2
4 Top Laterals-T3
Top Chord
Top Chord

Fig 302

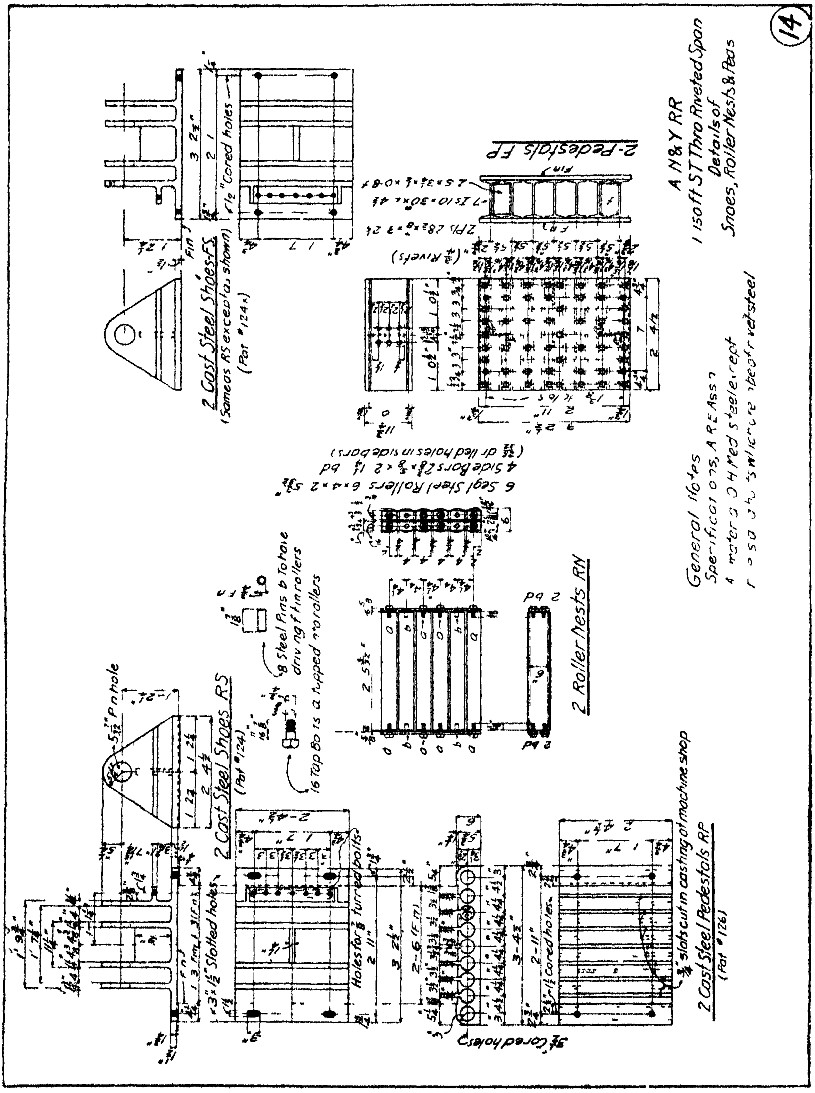

A N&Y RR
150 ft ST Thro Riveted Span
Details of
Shoes, Roller Nests & Peds
2 Cast Steel Shoes FS
2 Cast Steel Shoes RS
2 Roller Nests RN
2 Cast Steel Pedestals RP
2-Pedestals FP
General Notes

Fig 303

plished by simply making the top chord longer than the bottom chord and increasing the length of the diagonals to correspond.

The top chords are usually increased $\frac{1}{8}''$ for each 10 feet of length (horizontal). The length of each diagonal is computed by taking the mean of the top and bottom chord lengths (in the corresponding panel) as the base of a right-angled triangle, and the height of the adjoining post as the altitude, and the diagonal as the hypotenuse. This method of cambering is satisfactory for trusses up to 300 feet in length, and hence is used in the case of the above bridge. The panel length here is 25′-0″, so by increasing the top chord $\frac{1}{8}''$ for each 10 feet of length we obtain 25′-0$\frac{5}{16}$″ for the cambered length of the top chord in each panel as is shown for the chord sections in Figs. 292 and 294. The end posts are not increased in length. For the mean lengths of the top and bottom chords in each panel we have

$$\left[(25'-0'')+(25'-0\tfrac{5}{16}'')\right]\tfrac{1}{2}=25'-0\tfrac{5}{32}''$$

Then for the cambered length of each diagonal we have

$$\sqrt{(30)^2+(25'-0\tfrac{5}{32}'')^2}=39'-0\tfrac{3}{32}''.$$

This length is used for the diagonals, as is seen in Fig. 297.

The camber affects the lengths of only the top chords, diagonals, and top laterals.

186. Hints Regarding Shop Drawings.—*Drawing E1* (Fig. 289) is the erecting diagram. This drawing is intended principally for the use of the party erecting the structure. The mark (as is seen) of each piece, or member, and the general dimensions of the structure are shown and in addition a list of the shop drawings is given.

Drawing 1 (Fig. 290) shows the complete details of the stringers and stringer bracing. The work of detailing in this case is practically the same as for deck plate girders (Art. 136). There should be enough rivets in the flanges to transmit the flange increment and at the same time support the vertical load transmitted to them from the ties. To obtain the spacing of the flange rivets at the ends of the stringers we have

$R = 7{,}880\#$, which is the allowable bearing of a $\frac{7}{8}''$ rivet on the $\frac{3}{8}''$ web;

$v = 1{,}200\# = \dfrac{25{,}000}{42} + 100\%$ impact (see Art. 136);

$S = 142{,}000\#$;

$h = 41.5$.

Then substituting these values in Formula (4), Art. 116 (the formula required by the specifications), we obtain

$$p = \frac{7{,}880}{\sqrt{1{,}200^2 + \left(\dfrac{142{,}000}{41.5}\right)^2}} = 2.18 \text{ (about), say } 2\tfrac{1}{8}, \text{ ins.}$$

for the spacing of the flange rivets near the ends of the stringers, and, as is seen, this spacing is used. The spacing at intermediate points is obtained as explained in Art. 136.

The rivets connecting each pair of end stiffeners to the stringer should be sufficient to transmit the maximum end shear on the stringer. The two angles have a combined thickness of 1″ in bearing on the rivets, and the two fillers and web combined have a thickness of $1\frac{3}{8}$″ in bearing. Then, using $\frac{7}{8}$″ rivets, we have

$$24{,}000 \times 1 \times \tfrac{7}{8} = 21{,}000 \text{ lbs.}$$

for the allowable bearing on each rivet. The rivets are in double shear, so for the allowable shear on each rivet we have $12{,}000 \times 0.6 \times 2 = 14{,}400$. As is seen, double shear governs. Then for the number of rivets required in the angles we have $142{,}000 \div 14{,}400 = 9.8$ (about), whereas 11 are used, but the rivets in the flanges should not be counted for very much, as they take the flange increment. So the riveting, as shown, is about correct. There should be a sufficient number of rivets connecting the $\frac{1}{2}$″ fillers to the web to transmit the maximum end shear in bearing on the $\frac{3}{8}$″ web. Hence, for the number required we have $142{,}000 \div 7{,}880 = 18$ (about), and, as is seen, 18 are used.

The intermediate stiffeners are spaced in accordance with Art. 118 (see Art. 170). The spacing of the rivets in the intermediate stiffeners, as previously stated, is mostly a matter of judgment (see Art. 136).

There should be a sufficient number of rivets in the end of each lateral to develop the strength of the lateral in compression. The length of each lateral is about 98″. Then for the allowable unit stress on each lateral we have

$$p = 16{,}000 - 70\,\frac{98}{90} = 8{,}400 \text{ lbs. (about).}$$

Then for the strength of each we have $8{,}400 \times 2.48 = 20{,}800\#$. Dividing this by 6,000 we obtain a little over 3—$\frac{7}{8}$″ field rivets, and, as is seen, 3 are used. There should be enough rivets connecting each lateral plate to the stringer to transmit the sum of the components of the two laterals along the stringers, as the two act in the same direction, one being in compression and the other in tension. Each field rivet passing through the flange will take the component exerted on it directly and hence it is a matter of taking care of the component of the rivets in the lateral plate outside of the flange angle. Two of these are good for 12,000#. Resolving this along the stringer we have $12{,}000 \times 0.69 = 8{,}280\#$. This requires about 1 shop rivet, and, as is seen, 1 shop rivet is used.

The connection of the bottom laterals and struts to the bottom flanges of the stringers is mostly a matter of judgment, as the stresses due to traction are quite low (see Art. 177). However, there should be enough rivets in these connections to insure rigidity.

Drawing 2 (Fig. 291) shows the complete details of the floor beams. The end connections and the stringer connections, in the case of the intermediate floor beams, were previously designed (see Art. 184) and will not be considered here. The number of flange rivets in the intermediate floor beams should be sufficient to transmit the flange increment. To obtain the spacing of these rivets between the truss and the stringer

we have

$r = 9{,}190\#$, which is the allowable bearing of a $\frac{7}{8}''$ rivet on the $\frac{7}{16}''$ web;
$S = 187{,}300\#$;
$h = 49.5$.

Substituting these values in Formula (2) (Art. 116) we obtain

$$p = \frac{9{,}190 \times 49.5}{187{,}300} = 2.4 \text{ ins. (about)}$$

for the required spacing, whereas $2\frac{1}{4}''$ spacing is used.

Apparently this spacing could be a little more than $2\frac{1}{4}''$, but as the flange stress is zero at the end of the beam and practically a maximum at the stringer connection, it is obvious that there should be a sufficient number of rivets in each flange, between the end of the beam and the stringer connection, to transmit the flange stress. Then dividing the flange stress (see stress sheet, Fig. 279) by 9,190 we have $201{,}000 \div 9{,}190 = 22$ (about) for the number of rivets required, whereas 20 are used. From this it is seen that the number shown is about the correct number. In all such cases, where the distance is short, the rivet spacing should be verified in this manner. In most girders, however, the spacing required by the flange increment governs.

The spacing of the flange rivets between the stringer connections can be 6″, the maximum spacing allowed, as the shear between the stringers is (theoretically) zero except for the small amount due to the weight of the part of the floor beam intervening.

To obtain the spacing of the flange rivets in the end floor beam to transmit the flange increment, between the end of the beam and stringer connection, we have

$r = 7{,}880\#$, which is the allowable bearing of a $\frac{7}{8}''$ rivet on the $\frac{3}{8}''$ web;
$S = 143{,}200\#$ (see stress sheet, Fig. 279);
$h = 49.5$.

Then substituting these values in Formula (2) (Art. 116) we obtain

$$p = \frac{7{,}880 \times 49.5}{143{,}200} = 2.7 \text{ ins. (about)}$$

for the required spacing. For the number required to transmit the flange stress we have $153{,}000 \div 7{,}880 = 20$ (about), and, as is seen, 20 rivets are used in the top flange and hence the spacing shown is correct, although it is less than required by the flange increment.

In the bottom flange there are 14 rivets between the end of the beam and stringer connection, all of which can be considered as bearing on the $\frac{3}{8}''$ web, and those passing through the plates extending over the flange angles can be considered as being in double shear in addition to their bearing on the $\frac{3}{8}''$ web. So we have

$$(14 \times 7{,}880) + (4 \times 14{,}400) = 167{,}900 \text{ lbs.}$$

for the value of the rivets transmitting the flange stress, whereas this stress is 153,000#, and hence the rivets shown in the bottom flange, between the end of the beam and stringer connection, are quite satisfactory.

The spacing of the flange rivets between the stringer connections can be 6″, as the shear between the stringers is practically zero.

There should be sufficient metal through the reduced portion, near the end of the end floor beam, to resist the cross bending at that point. Taking a section *c-c* (Fig. 282) we have the metal shown in Fig. 304.

Taking moments about the back of the top flange angles we have

$$\bar{x} = \frac{(23.62 \times 19.75) + (12.37 \times 16.62) + (8.36 \times 1.96)}{44.35} = 15.52 \text{ ins.}$$

for the distance from the back of the angles to the center of gravity of the section.

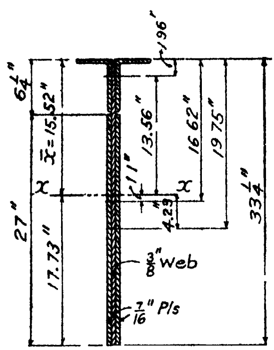


Fig. 304

Then for the moment of inertia about the gravity axis *x-x* we have the following:

$$8.36 \times \overline{13.56}^2 + 30.9 \quad = 1{,}568 \text{ for the angles;}$$

$$12.37 \times \overline{1.1}^2 + 1{,}122 \quad = 1{,}137 \text{ for the } \tfrac{3}{8}'' \text{ web;}$$

$$23.62 \times \overline{4.23}^2 + 1{,}435 = \frac{1{,}858}{4{,}565} \text{ for the } \tfrac{7}{16}'' \text{ plates;}$$

$$\text{Total moment of inertia} = \frac{-\ 295}{4{,}268} \text{ for rivet holes.}$$

Then for the maximum stress on the outer element we have

$$f = \frac{(143{,}200 \times 17)\ 17.7}{4{,}268} = 10{,}090 \text{ lbs.}$$

This shows that the metal at the reduced portion of the end floor beams is quite sufficient, as the allowable stress is 16,000#.

Drawings 3 to 14 (Figs. 292 to 303) are self-explanatory. The making of these drawings is mostly a matter of copying the details from the large scale sketches (Figs. 282 to 285 and 287).

187. Summary of Weight of Metal and Cost of the Above 150-Ft. Span.—

12—Stringers	@ 4,300#..	51,600 lbs.
Stringer bracing		2,120 lbs.
2—End floor beams	@ 3,010#..	6,020 lbs.
5—Intermediate floor beams	@ 3,470#..	17,350 lbs.
4—End posts *U*1-*LO*	@ 8,850#..	35,400 lbs.
4—Top chords *U*1-*U*2	@ 4,830#..	19,320 lbs.
2—Top chords *U*2-*U*2	@ 10,890#..	21,780 lbs.
4—Bottom chords *LO*-*L*2	@ 4,730#..	18,920 lbs.
2—Bottom chords *L*2-*L*2	@ 10,740#..	21,480 lbs.
4—Hangers *U*1-*L*1	@ 1,780#..	7,120 lbs.
6—Int. posts *U*2-*L*2 and *U*3-*L*3	@ 2,840#..	17,040 lbs.
4—Diagonals *U*1-*L*2	@ 3,760#..	15,040 lbs.
4—Diagonals *U*2-*L*3	@ 2,880#..	11,520 lbs.
4—Collision struts *M*1-*L*1	@ 950#.	3,800 lbs.
2—Portal struts	@ 2,270#..	4,540 lbs.
Top laterals		5,630 lbs.
Transverse struts		2,770 lbs.
Bottom laterals		6,460 lbs.
4—Shoes	@ 940#..	3,760 lbs.
2—Roller nests	@ 1,180#..	2,360 lbs.
2—Cast pedestals	@ 1,130#..	2,260 lbs.
2—Pedestals	@ 1,090#..	2,180 lbs.
Pins		640 lbs.
Anchor bolts		130 lbs.
Total weight of bridge		279,240 lbs.

Cost of span @ $3\frac{1}{4}$¢ per pound is $279,240 \times 0.0325 = \$9,075.30$. This price ($3\frac{1}{4}$¢) per pound is only a fair average price for this class of work. The price will vary from $2\frac{3}{4}$¢ to 4¢ per pound.

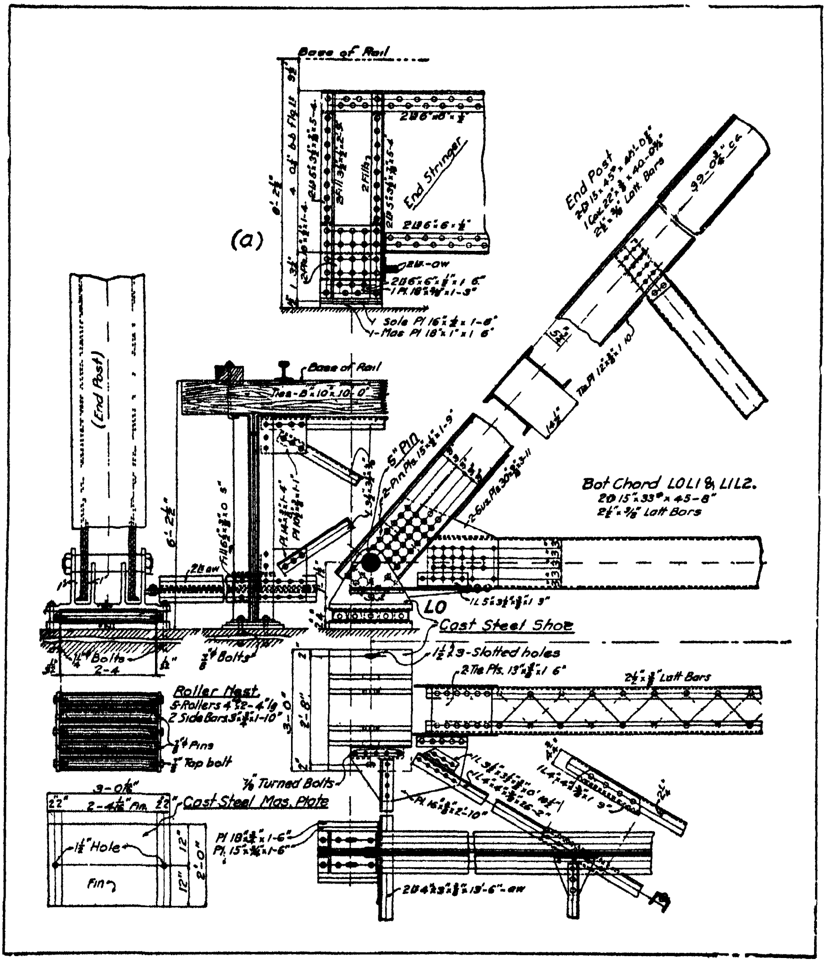
The effective dead load from the metal is equal to the total weight of metal in the span minus the weight of the end floor beams, shoes, roller nests, pedestals, pins, and anchor bolts. So we have $279,240 - 17,350 = 261,890$# for the effective dead weight of metal in the span. Dividing this by 150, the length of the span, we have $261,890 \div 150 = 1,745$# per foot of span, which is only 35# more than assumed above (see stress sheet, Fig. 279). So the dead load assumed is quite satisfactory, as 10 per cent variation would not materially affect the design.

DRAWING ROOM EXERCISE NO. 7

Design a single-track through riveted railroad bridge and make a stress sheet for same—the stress sheet to be upon tracing cloth.

Data:—

Length of span = 6 panels at 26′-0″ = 156′-0″.
Height of trusses = 31′-0″ c.c. of chords.
Live load, Cooper's *E*50.
Dead load, to be assumed.
Specifications, A. R. E. Ass'n.



Base of Rail
End Stringer
(a)
(End Post)
End Post
Bot Chord L0L1 & L1L2.
5" Pin
L0
Cast Steel Shoe
Roller Nest
Cast Steel Mas. Plate

Fig. 805

DRAWING ROOM EXERCISE NO. 8

Make a general drawing of the above 156-ft. bridge—the finished drawing to be upon tracing cloth and similar to the one shown in Fig. 288.

188. Remarks.—End floor beams are sometimes omitted in through truss bridges in which case the end stringer rests directly upon the piers as shown in Fig. 305. The ends of the stringers are usually constructed as shown at (a). The stringers, as is seen, are braced to each other by a cross-frame and they are connected to the shoes and trusses by a transverse strut which forms the lower part of the cross-frame.

Circular rollers are sometimes used. The roller nests in that case are usually constructed as shown in Fig. 305. The A. R. E. Association Specifications practically prohibit the use of circular rollers as the minimum diameter is specified as 6″ and this size of rollers would place them so far apart that the shoes would be unduly large. It is not considered good practice in any case to use rollers less than 4″ in diameter.

There is no question but that it is better practice to use end floor beams, as the bottom ends of the end posts are held better than when they are omitted and the thermal expansion and contraction are better provided for, as the entire structure is supported upon the shoes and will move as a whole. In fact, there is no excuse for omitting the end floor beams other than the slight saving in cost.

189. Dead-Load Stresses in Curved Chord Pratt Trusses.—As previously stated, the dead load is considered as uniformly distributed over the span. The approximate dead weight per foot of span can be obtained from (4), Art. 124, and adding 400 lbs. to this to provide for the weight of the deck (track) the total dead weight per foot of span for single-track bridges is obtained, and by dividing this by 2 and multiplying by the panel length the panel load per truss is obtained. One-third of this is considered applied at the top chord joints and two-thirds at the bottom chord joints.

Let it be required to determine the dead-load stresses in the truss shown at (a), Fig. 306, where P represents the panel load per truss.

Members bA and AB. Considering the part of the truss to the left of section 1-1, as shown at (b), and resolving the forces vertically we have

$$S1 \cos\theta - R = 0,$$

from which we obtain

$$S1 = R \sec\theta$$

for the stress in the end post bA, and resolving the forces horizontally we have

$$S1 \sin\theta - S2 = R \sec\theta \times \sin\theta - S2 = 0,$$

from which we obtain

$$S2 = R \tan\theta = 3P \tan\theta$$

for the stress in the bottom chord AB.

Members BC, bc and bC. Considering the part of the truss to the left of section 2-2, as shown at (c), and taking moments about joint b, we obtain

$$S5 = R\frac{d}{h} = 3P\frac{d}{h}$$

for the stress in bottom chord BC. Next, resolving the stress $S3$ (in top chord bc) into vertical and horizontal components at c and taking moments about C we have

$$R \times 2d - \frac{2}{3}Pd - \frac{1}{3}Pd = H3 \times h1,$$

from which we obtain

$$H3 = \frac{3P \times 2d}{h1} - \frac{Pd}{h1} = 5P\frac{d}{h1}$$

for the horizontal component of the stress $S3$ in the top chord bc, and multiplying this by $\sec\phi$ we obtain

$$S3 = H3 \times \sec\phi$$

for the stress in top chord bc.

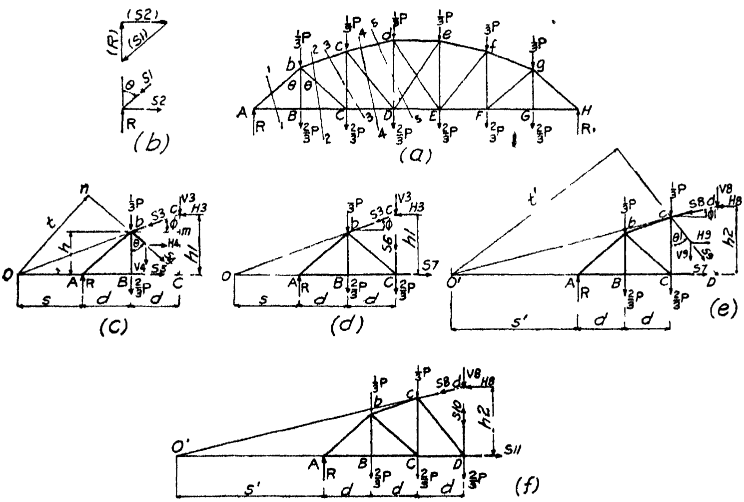


Fig 306

Resolving the forces vertically and horizontally and summing up the vertical we have

$$R - P - V3 - V4 = 0,$$

from which we obtain

$$V4 = (R - P) - V3$$

for the vertical component of the stress $S4$ in diagonal bC and multiplying this by $\sec\theta$ we have

$$S4 = [(R-P) - V3]\sec\theta$$

for the stress in that member. As $(R-P)$ is the shear in panel BC, it is seen that the vertical component of the stress in diagonal bC is equal to the shear in panel BC minus the vertical component of the stress in the top chord bc. Another way to obtain the stress in diagonal bC is to prolong the top chord bc until it intersects the bottom chord at O [see the diagram at (c)] and take moments about O. Thus, taking moments about O we have

$$Rs - P\ (s+d) = S4 \times t,$$

from which we obtain

$$S4 = R\frac{s}{t} - P\left(\frac{s+d}{t}\right)$$

for the stress in the diagonal bC.

In case the last method be used, the distances s and t can be determined in the following manner: Triangles cOC and cbm being similar we have

$$\frac{s+2d}{d} = \frac{h1}{h1-h},$$

from which we obtain

$$s = \left(\frac{2h-h1}{h1-h}\right)d.$$

Similarly, since triangles OnC and bBC are similar we have

$$\frac{t}{h} = \frac{s+2d}{bC},$$

from which we obtain

$$t = \left(\frac{s+2d}{bC}\right)h.$$

Member cC. Considering the part of the truss to the left of section 3-3, as shown at (d), and resolving the forces vertically we obtain

$$S6 = R - P - \frac{2}{3}P - V3 = (R - 1\frac{2}{3}P) - V3$$

for the stress in the post cC. Also, taking moments about O [at (d)] we have

$$S6\ (s+2d) = Rs - P\ (s+d) - \frac{2}{3}P\ (s+2d),$$

from which we obtain

$$S6 = \frac{Rs - P\ (s+d) - \frac{2}{3}P\ (s+2d)}{s+2d}$$

for the stress in post cC.

Members CD, cd and cD. Considering the part of the truss to the left of section 4-4, as shown at (e), and taking moments about c we have

$$S7 \times h1 = R \times 2d - Pd,$$

from which we obtain

$$S7 = \frac{R \times 2d}{h1} - \frac{Pd}{h1} = 5P\frac{d}{h1} = H3$$

(as seen above) for the stress in the bottom chord CD. Next, resolving the stress in the top chord cd into horizontal and vertical components at d and taking moments about D, we obtain

$$H8 = \frac{R \times 3d - P \times 2d - P \times d}{h2} = 6P\frac{d}{h2},$$

for the horizontal component and multiplying this by $\sec\phi1$ we obtain

$$S8 = \frac{6Pd}{h2}\sec\phi1$$

for the stress in the top chord cd.

Resolving the forces horizontally and vertically and summing up the vertical we have

$$R - 2P - V8 - V9 = 0,$$

from which we obtain

$$V9 = R - 2P - V8$$

for the vertical component of the stress in diagonal cD and hence multiplying this by $\sec\theta1$ we obtain

$$S9 = [(R - 2P) - V8]\sec\theta1$$

for the stress in that member. Also, taking moments about O' we obtain

$$S9 = \frac{Rs' - P(s' + d) - P(s' + 2d)}{t'}.$$

Members dE and eD. Adding up from either end of the truss we have

$$R - 3P = 0$$

for the shear in panel DE and as the top chord de is parallel to the bottom DE it is evident that the diagonals dE and eD have no stress from dead load and hence in determining the dead-load stresses in the truss we can ignore these members altogether.

Members dD, DE and de. Considering the part of the truss to the left of section 5-5, as shown at (f), (ignoring diagonal eD) and summing up the vertical components and forces we have

$$R - 2\frac{2}{3}P - V8 \pm S10 = 0,$$

from which we obtain

$$\pm S10 = R - 2\frac{2}{3}P - V8$$

for the stress in post dD.

If $R - 2(2/3\,P) - V8$ is minus $S10$ will be tension, as it would have to act in the same direction as R in that case in order that the part of the truss to the left of section 5-5 be in equilibrium, while if $R - 2(2/3\,P) - V8$ is plus $S10$ will be compression, as it would be acting in the opposite direction to that of R.

The stress in dD can also be obtained by taking moments about O' (see diagram at (f)). Thus, taking moments about O' we obtain

$$S10 = \frac{Rs' - P(s'+d) + P(s'+2d) + \frac{2}{3}P(s'+3d)}{(s'+3d)}.$$

Taking moments about d we obtain

$$S11 = 3(R-P)\frac{d}{h2} = 6P\ \frac{d}{h2}.$$

for the stress in the bottom chord DE, which, as seen above, is equal to $H8$, the horizontal component of the stress in the top chord cd.

Considering the diagonals eD and dE omitted from panel DE, as there is no dead-load stress in them, it is obvious that the stress in top chord de must be equal and opposite to the stress in the bottom chord DE as these are the only horizontal forces in the panel and hence must balance each other. So we have

$$S12 = S11 = S8$$

for the stress in the top chord de.

Member bB. The only load on the bridge that could affect the hanger bB is the one applied at B. This load, as is obvious, is supported directly by the hanger and hence the dead-load stress in it is equal to $\frac{2}{3}P$.

As the truss is symmetrical about the center of the span and symmetrically loaded we need not consider further the dead-load stresses, as we have now fully considered one-half of the span.

It is usual practice to determine graphically the dead-load stresses in curved chord Pratt trusses.

190. Live-Load Stresses in Curved Chord Pratt Trusses.—*Chords.* The criterion for the placing of wheel loads for maximum stress in the chords is exactly the same as given in Art. 91 for simple parallel chord trusses. That is, the maximum moment about any panel point will occur when the average unit load to the left of the point is equal to the average unit load on the bridge. For example, to determine the maximum live-load stress in chord CD (Fig. 307) we would place a wheel at C such that the average unit load to the left of joint C would be equal (as nearly as possible) to the average unit load on the bridge. Then by taking moments about c of the forces to the left and dividing this moment by h we would obtain the maximum live-load stress in the bottom chord CD and by multiplying this stress by $\sec\phi$ we would obtain the maximum live-load stress in the top chord bc. The live-load stresses in the other chord members are obtained in a similar manner.

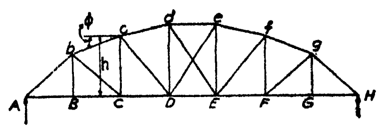


Fig. 307

Web Members. Let it be required to place the wheel loads so that maximum stress will occur in diagonal cD (Fig. 308). If the top chord cd and bottom chord CD were parallel maximum stress would occur in diagonal cD when the shear in panel CD was a maximum, as the diagonal in that case would carry all of the shear in the panel and hence the criterion for maximum shear given in Art. 90 would apply; but as the top chord *cd* is inclined, and consequently carries some of the shear in panel *CD*, the shear criterion will not apply exactly and hence we shall proceed to determine a criterion for the placing of the wheels for maximum stress in the diagonal *cD*. Suppose the span is loaded from *H* (Fig. 308) up to panel *CD* as shown, which is about the same position as for maximum shear in panel *CD*. Let *P* be the total load in panel *CD*, the center of gravity of which is *z* distance from *D*, and let *W* be the total load on the bridge, the center of gravity of which is *x* distance from *H*.

Considering the part of the truss to the left of section 1-1 and taking moments about *O* we obtain

$$S1=\frac{Rs}{t}-\frac{r}{t}(s+a)$$

for the stress in diagonal *cD*.

But
$$R=\frac{Wx}{L}$$

and
$$r=\frac{Pz}{d}.$$

Substituting these values of *R* and *r* we obtain

$$S1=\frac{Ws}{Lt}x-\frac{Ps}{td}z-\frac{Pa}{td}z \qquad (1).$$

Now suppose that there be a slight movement of the loads to the right or to the left, say, to the left (*W* and *P* remaining constant), then $S1$, x, and z will each receive an increment, $\Delta S1$, Δx, and Δz, respectively. Now adding these increments in (1) we have

$$S1+\Delta S1=\left(\frac{Ws}{Lt}x+\frac{Ws}{Lt}\Delta x\right)-\left(\frac{Ps}{td}z+\frac{Ps}{td}\Delta z\right)-\left(\frac{Pa}{td}z+\frac{Pa}{td}\Delta z\right).$$

Subtracting (1) we have

$$\Delta S1=\frac{Ws}{Lt}\Delta x-\frac{Ps}{td}\Delta z-\frac{Pa}{td}\Delta z \qquad (2)$$

for the increment of the stress in the diagonal *cD* due to the slight movement of the loads. But $\Delta z=\Delta x$, as is obvious. So substituting Δx for Δz in (2) we have

$$\Delta S1=\frac{Ws}{Lt}\Delta x-\frac{Ps}{td}\Delta x-\frac{Pa}{td}\Delta x \qquad (3).$$

Now $S1$, the stress in the diagonal *cD*, will be a maximum when $\Delta S1=0$. So placing (3) equals 0 and reducing and transposing we obtain

$$\frac{W}{L}=\frac{P}{d}\left(1+\frac{a}{s}\right) \qquad (4).$$

Expressing this equation in words we have: *The average unit load on the bridge is equal to the average unit load in panel CD multiplied by*

$(1+a/s)$ *when the maximum stress in diagonal cD occurs.* This can be taken as the criterion for placing the loads.

The increment of the stress in diagonal cD (W remaining constant) can change signs only when a wheel passes the panel point D, so it is seen that a load will be at that point when the maximum stress in the diagonal occurs. One-half of this load should be considered in panel CD in computing the value of P.

Next, let it be required to place the wheel loads so that maximum stress will occur in post cC (Fig. 308). Considering the part of the

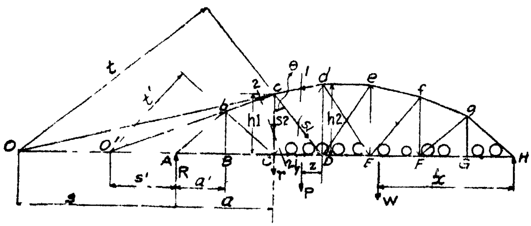


Fig. 308

truss to the left of section 2-2 with the wheel loads in about the position shown and taking moments about O' we have

$$S2(s'+a)=Rs'-r(s'+a),$$

from which we obtain

$$S2=\frac{Rs'}{(s'+a)}\quad r$$

for the stress in the post cC. Substituting $(W/L)x$, and $(P/d)z$ for R and r, respectively, we have

$$S2=\frac{Ws'}{L(s'+a)}x-\frac{P}{d}z \text{ ....(5).}$$

Now, $S2$, x, and z will each receive a small increment if the loads move slightly to the right or left. Let $\Delta S2$, Δx, and Δz be the increment, respectively, that each receives due to a slight movement to the left. Then substituting these increments in (5) we have

$$S2+\Delta S2=\frac{Ws'}{L(s'+a)}x+\frac{Ws'}{L(s'+a)}\Delta x-\frac{Pz}{d}-\frac{P}{d}\Delta z$$

and subtracting (5) from this we have

$$\Delta S2=\frac{Ws'}{L(s'+a)}\Delta x-\frac{P}{d}\Delta z$$

and substituting Δx for Δz, since $\Delta x=\Delta z$, we obtain

$$\Delta S2=\frac{Ws'}{L(s'+a)}\Delta x-\frac{P}{d}\Delta x \text{ ....(6)}$$

for the increment of the stress $S2$ in the post cC due to any slight movement of the loads. Now, $S2$ will be a maximum when the increment

$\Delta S2=0$ (see discussion of Art. 90). So placing (6) equal to zero and reducing we obtain

$$\frac{W}{L}=\frac{P}{d}\left(1+\frac{a}{s'}\right) \quad \ldots \ldots (7)$$

which is exactly the same as (4) except s' appears instead of s. Again, suppose it be required to place the wheel loads so that maximum stress will occur in diagonals bC. By loading panel BC, very much the same as we did CD (above), and taking moments about O' and adding increments, in the equation of moments we would obtain

$$\frac{W}{L}=\frac{P}{d}\left(1+\frac{a'}{s'}\right),$$

which expresses the criterion for the placing of the wheel loads for maximum stress in the diagonal bC. This last equation is of exactly the same form as (4); the only difference is the symbols have not the same value as they have in (4). From this it is seen that equation (4) expresses the general criterion for the placing of the wheel loads for maximum stress in diagonals and intermediate posts.

In case the chords are parallel, $a/s=0$ as s in that case is equal to infinity. The equation (4) then becomes

$$\frac{W}{L}=\frac{P}{d}(1+0).$$

Reducing and substituting nd for L we obtain

$$\frac{W}{n}=P$$

which is equation (5) of Art. 90.

It is evident that a live load moving onto the bridge from the right would cause compression in diagonal fE (Fig. 308) and that this com-

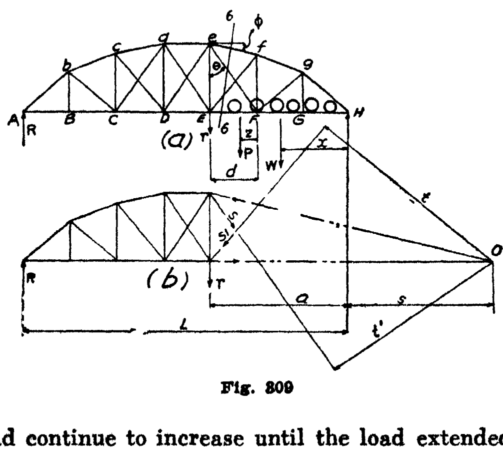


Fig. 309

pression would continue to increase until the load extended from H to a short distance beyond F. Now if this compression were greater than the dead-load tension in the diagonal we would have what is known as a reversal of stress and the diagonal fE in that case would have to be designed to carry both tension and compression or the member eF, which would be known as a counter, would have to be inserted to carry the reverse stress. In case the diagonal fE be composed of eye-bars, which are often used in the case of long span bridges, the counter eF would be used, as the bars would not carry compression; but if the diagonal fE be a rigid member—that is, capable of carrying compression—the counter would be omitted and the diagonal would be made sufficient to carry both the tensile and compressive stresses in it, previously explained in Art. 176 for diagonal cD of the 150-ft. span.

To determine the maximum live-load tension in the counter eF let the bridge be loaded as shown at (a), Fig. 309. Let P represent the load in the panel EF and W the total load on the bridge, and z the distance from F to the center of gravity of P and x the distance from H to the center of gravity of W. Now considering the part of the truss to the left of section 6-6 as shown at (b) and taking moments about O we have

$$R(L+s)-r(a+s)-St'-S1t=0,$$

from which we obtain

$$S=R\left(\frac{L+s}{t'}\right)-r\left(\frac{a+s}{t'}\right)-S1\frac{t}{t'} \dots\dots\dots\dots (8)$$

for the stress in the counter eF. Substituting $(W/L)x$ and $(P/d)z$ for R and r, respectively, we obtain

$$S=\frac{W}{L}x\left(\frac{L+s}{t'}\right)-\frac{P}{d}z\left(\frac{a+s}{t'}\right)-S1\frac{t}{t'} \dots\dots\dots\dots (9).$$

$S1$, as is readily seen, is equal and opposite to the dead-load stress in diagonal fE and hence is a known quantity.

Now, adding increments to S, x, and z in (9) and proceeding in the same manner as shown above in the case of equations (1) and (5) we obtain

$$\Delta S=\frac{W}{L}\left(\frac{L+s}{t'}\right)\Delta x-\frac{P}{d}\left(\frac{a+s}{t'}\right)\Delta z \dots\dots\dots\dots (10)$$

for the increment of the stress in the counter eF. The stress in the counter will be a maximum when this increment is equal to zero, as previously explained. So placing (10) equal to zero and transposing we obtain

$$\frac{W}{L}=\frac{P}{d}\left(\frac{a+s}{L+s}\right) \dots\dots\dots\dots (11).$$

Expressing this in words we have: *The average unit load on the bridge is equal to the average unit load in the panel EF multiplied by* $(a+s)\div(L+s)$ *when the maximum stress in the counter occurs.* This can be taken as the criterion for placing the loads.

To determine the maximum stress in the counter we would first determine the value of s, also of t and t', which can be determined sufficiently accurately by scale—in case of counters. Then we would place the loading so as to satisfy equation (11) (as nearly as possible) and determine R and r by taking moments about H and F. Then the stress S in the counter is readily found by substituting in equation (8). It can also be determined by summing up the vertical forces and components to the left of section 6-6. To determine it in this way, we would first take moments about F (Fig. 309) and determine the horizontal component of the stress in the top chord ef. Then multiplying this by $\tan\phi$ we would have the vertical component of the stress in chord ef, which we will designate as $V1$. Then by determining the vertical component of the dead-load stress in diagonal fE, which we will designate as $V2$, we would have all of the vertical forces to the left of section 6-6 determined except the vertical component of the stress in the counter eF. Let $V3$ represent this vertical component. Then summing up the vertical forces and components to the left of section 6-6 we have

$$R - r + V1 + V2 - V3 = 0,$$

from which we obtain

$$V3 = R - r + V1 + V2$$

for the vertical component of the stress in the counter eF and multiplying this by $\sec\theta$ we would obtain the desired stress S in the counter. The stress in other counters is determined in a like manner.

The maximum live-load stress in hanger bB (Fig. 310) is equal to the maximum floor beam concentration at B. This, as explained in Arts. 148 and 171, is obtained by placing a wheel at B, such that the load in panel AB will be equal (as nearly as possible) to the load in panel BC. After having the loads thus placed the concentration at B, which is equal to the stress in hanger bB, is obtained by taking moments about both A and C. The maximum live-load stress in hanger gG is obtained in the same manner as for bB.

Owing to the top chord segments having different slopes at the joints, the intermediate posts of curved chord bridges are subjected to relatively greater tensile stress from live load than the intermediate posts of parallel chord bridges. For example, let us consider the case of post cC (Fig.

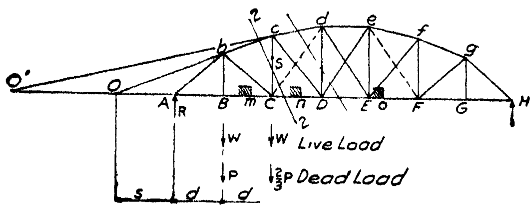


Fig 310

310). If panel points B and C alone were loaded with live load (ignoring counter dC) it is readily seen that post cC would be in tension—very much the same as in the case of parallel chord bridges—and it is obvious that this tensile stress would be greater than it would be if the chords bc and cd had the same slope at c, as the chords, owing to their slopes being different, pull upward (so to speak) on the post cC and also on diagonal cD. Now, it is evident that the tensile stress in post cC will increase as the stress in the top chords bc and cd increases, and that it will be a maximum when the stress in diagonal cD is zero, for in that case the post alone would resist the upward pull from the chords bc and cd. So the problem in placing the live load for maximum tension in post cC, is to place it so as to obtain zero stress in diagonal cD and at the same time as great a stress as possible in the top chords bc and cd. It is customary in practice to use an equivalent uniform live load (see Art. 123) in determining the tension in intermediate posts of curved chord bridges, as the work is very tedious if wheel loads be used. So we will consider a uniform live load in this case. Suppose that this live load moves onto the bridge from the left and loads it from A to m, just so the dead-load tension in diagonal cD is reversed. Then the stress in diagonal cD and also in counter dC would be zero. Now, as the load continues to move on to the right past m, the counter dC will be in tension and this tension will increase steadily, and likewise the stress in the top chords bc and cd (the stress in diagonal cD remaining zero) until some point n is reached when the counter dC will have maximum tension and the stress in diagonal cD will still be zero. Then, as the load continues to move on to the right (past n), while the stress in the top chords bc and cd will steadily increase, the tension in counter dC will steadily decrease until some point o is reached when the stress in the counter dC is zero. This position is the one for maximum tension in the post cC as any further movement of the load to the right would produce tension in diagonal cD (which has zero stress when the load extends from A to o) and compression in post cC and hence the tension in the post would be rapidly reduced.

The determination, as to location, of point o (the head of the uniform load) in any case is very much a matter of trial. We could first load from A to D, for instance. Then compute the stress in the counter dC (or diagonal cD in case no counter is used) and if a stress occurs with the load in that position we could move the load to the right or left, as the case may require, until the stress in the counter is found to be zero and this position of the load is the one required.

After the position of the load for maximum tension in the post is found, as Ao, we can take moments about H and obtain R, the live-load reaction at A. Then we can readily obtain the maximum live-load tension in the post cC by taking moments about O and considering all of the forces to the left of section 2-2. Thus, taking moments about O, we have

$$Rs - W(s+d) - W(s+2d) + S(s+2d) = 0,$$

from which we obtain

$$S = \frac{W(s+d) + W(s+2d) - Rs}{(s+2d)}$$

for the maximum live-load tension in post cC, where W = panel of live load.

To this tension should be added the dead-load tension that occurs in the post at the same time. The dead-load tension in the post is readily obtained in the same manner as shown above for the live-load tension. Thus, taking moments about O we have

$$R's - P(s+d) - \tfrac{2}{3}P(s+2d) + S'(s+2d) = 0,$$

from which we obtain

$$S' = \frac{P(s+d) - \frac{2}{3}P(s+2d) - R's}{(s+2d)}$$

for the dead-load tension that occurs in the post at the same time the maximum live-load tension occurs in it. P = panel of dead load and R' = total dead-load reaction at A—the same as occurs at H.

The maximum live-load tension in any other intermediate post is obtained in a manner similar to that shown above for post cC. For example, to obtain the maximum live-load tension in post dD, we would load the bridge from A on to the right, until we obtained zero stress in eD and then we would determine the maximum tension in the post by taking moments about O'. In the case of post dD the maximum tension would occur in the member when the span was fully loaded, for in that case both diagonals, eD and dE, would have zero stress.

Complete Design of a 225-Ft. Single-Track Through Pin-Connected Curved Chord Pratt Truss Span

191. Data.—

Length = 9 panels @ 25′-0″ = 225′-0″ c.c. end pins.
Width = 17′-0″ c.c. trusses.
Height = 45′-0″ at center and 31′-0″ at hip.
Stringers spaced 6′6″ c.c.
Live load, Cooper's $E50$ loading.
Specifications, A. R. E. Association.

192. Design of 25-Ft. Stringers.—Proceeding in the manner outlined in Art. 170 we obtain the following for the stringers:

Maximum End Shear:	Maximum Moment:
D = 5,000 lbs.	D = 375,000 in. lbs.
L = 71,000 lbs.	L = 4,575,000 in. lbs.
I = 66,000 lbs.	I = 4,223,000 in. lbs.
142,000 lbs.	9,173,000 in. lbs.
142,000 ÷ 7,888 = 18□″	9,173,000 ÷ 45 = 204,000#
1—web 48″ x $\frac{3}{8}$″ = 18□″	204,000 ÷ 16,000 = 12.75□″
End stiff—⌊s 3½″ x 3½ x ½″	2—⌊s 6″ x 6″ x ½″ = 10.50□″
Int. stiff—⌊s 3½″ x 3½ x $\frac{3}{8}$″	$\frac{1}{8}$ area of web = 2.25□″
	12.75□″

193. Design of the Intermediate Floor Beams.—Proceeding in the manner outlined in Art. 171 we obtain the following for the intermediate floor beams:

Maximum End Shear	Maximum Moment
D = 12,000 lbs.	D = 721,000 in. lbs.
L = 94,500 lbs.	L = 5,953,000 in. lbs.
I = 81,000 lbs.	I = 5,104,000 in. lbs.
187,500 lbs.	11,778,000 in. lbs.

187,500 ÷ 7,644 = 24.5□″	11,778,000 ÷ 53 = 222,000#
1—web 56″ x $\frac{7}{16}$″ = 24.5□″	222,000 ÷ 16,000 = 13.87□″
	2—Ls 6″ x 6″ x $\frac{1}{2}$″ = 10.50□″
	$\frac{1}{8}$ area of web = 3.06□″
	13.56□″

194. Design of End Floor Beams.—Proceeding in the manner outlined in Art. 172 and assuming the length of each beam to be 14′-6″. as they will rest upon the shoes in this case, we obtain the following for the end floor beams:

Maximum End Shear:	Maximum Moments:
D = 6,500 lbs.	D = 305,000 in. lbs.
L = 71,000 lbs.	L = 3,408,000 in. lbs.
I = 66,000 lbs.	I = 3,168,000 in. lbs.
143,500 lbs.	6,881,000 in. lbs.
143,500 ÷ 8,266 = 17.37□″	6,881,000 ÷ 52.37 = 131,000#
1—web 56″ x $\frac{3}{8}$″ = 21.00□″	131,000 ÷ 16,000 = 8.19□″
$35 = \frac{3/8}{40}(12{,}000 - s)$	2—Ls 6″ x 4″ x $\frac{3}{8}$″ = 7.22 − 0.75 = 6.47□″
	$\frac{1}{8}$ area of web = 2.62□″
	9.09□″

195. Determination of Dead-Load Stresses in Trusses.—From (4), Art. 124, we have

$$p = 7 \times 225 + 660 = 2{,}235 \text{ lbs.}$$

for the approximate weight of metal per ft. of span and adding 400 lbs. for the weight of the deck, we have

$$2{,}235 + 400 = 2{,}635 \text{ lbs.}$$

for the total assumed dead load per ft. of span or 2,635 ÷ 2 = 1,318 lbs. per ft. of truss.

Multiplying this by 25, the panel length in feet, we have

$$W = 1{,}318 \times 25 = 32{,}950, \text{ say, } 33{,}000 \text{ lbs.}$$

for the panel load on each truss. One-third of this will be considered at the top joints and two-thirds at the bottom joints. The dead-load stresses can be determined most readily by graphics.

Before we can proceed farther it is necessary that we determine the exact outline of the truss. The panel points of the top chord will be on the arc of a parabola. The height of the truss at the hip is 31 ft. and 45 ft. at the center of the span, making a drop of 14 ft. from the center of the span to the hip. Laying off the panel lengths along the bottom chord $LO - D$ (Fig. 311) and taking C as the vertex of the parabola, CD as the x – axis and considering half panel lengths we have

$$\frac{x}{14} = \frac{1^2}{7^2}$$

from which we obtain

$$x = \frac{14}{49} \times 1 = 0.2857 \text{ ft.}$$

for the vertical distance from the horizontal line through C down to panel point $U4$.

Similarly, we have

$$\frac{x'}{14} = \frac{3^2}{7^2}$$

from which we obtain

$$x' = \frac{14}{49} \times 9 = 2.5713 \text{ ft.}$$

for the vertical distance down to panel point $U3$, and similarly we have

$$\frac{x''}{14} = \frac{5^2}{7^2}$$

from which we obtain

$$= 7.1428 \text{ ft}$$

for the vertical distance down to panel point $U2$, and thus we have all of the panel points or joints of the top chord located and the outline of the

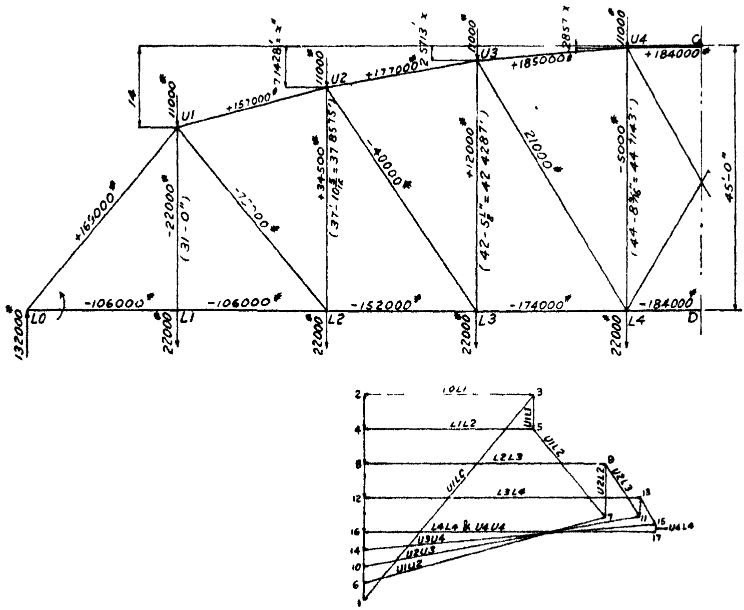


Fig. 311

truss can be drawn as shown (Fig. 311). After the outline of the truss is carefully drawn (say $\frac{3}{16}$ scale), the dead-load stress in each member can be graphically determined as shown in Fig. 311; one-third of a panel load of dead load, which is $33,000 \times \frac{1}{3} = 11,000\#$, being considered as applied at each top joint, and two-thirds, which is $33,000 \times \frac{2}{3} = 22,000\#$, at each bottom joint.

The reaction $= 4 \times 33,000 = 132,000\#$, which is applied at LO. We obtain the diagram of the stresses shown (Fig. 311) by first laying off 1-2 (to, say, a $\frac{1}{40}$ scale) equal to the reaction and passing around joint *LO* counter clock-wise we obtain the polygon 1-2-3-1 for joint *LO*. Then starting with *LO-L*1, at *L*1, and passing around joint *L*1 counter clock-wise we obtain the polygon 3-2-4-5-3. Considering *U*1 and passing around counter clock-wise, beginning with the 11,000# load, we obtain the diagram 6-1-3-5-7-6, and so on for the other joints as fully explained in Example 2, Art. 96.

196. Determination of Live-Load Stresses and Impact in the Trusses.—In beginning the work of determining the live-load stresses in the truss, a diagram showing the entire truss should be drawn carefully to scale (say, to a $\frac{1}{20}$ scale) and upon this diagram the important lengths and the calculated values of the different angles should be given as shown in Fig. 312.

End Post bA. The maximum stress will occur in this member when the loading is placed for maximum shear in panel *AB*. By placing wheel

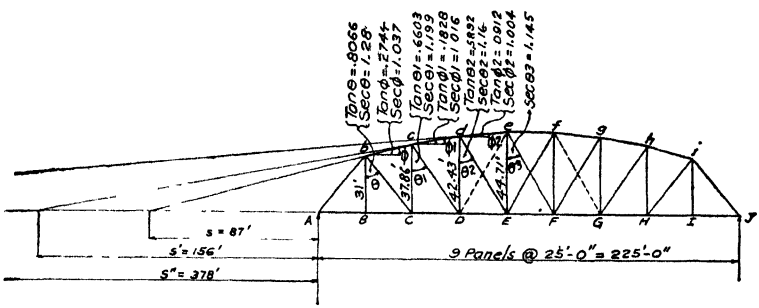


Fig. 312

4 at *B* (using Table A) we obtain 2,231 lbs. for the average unit load on the bridge and, considering one-half of the load at *B* as being in panel *AB*, we obtain 2,400 lbs. for the average unit load in the panel *AB*. This position comes nearest to satisfying the criterion of Art. 90. So taking moments about *J* (Fig. 312) we obtain (using Table A)

$$R = [16,364 + (284 \times 109) + \overline{109}^2] \frac{1,000}{225} = 263,100 \text{ lbs.}$$

for the reaction at *A*, and taking moments about *B* we obtain

$$r = (480)\frac{1,000}{25} = 19,200 \text{ lbs.}$$

for the stringer reaction at *A*. Then for the maximum shear in panel *AB* we have

$$S = 263,100 - 19,200 = 243,900 \text{ lbs.}$$

for Cooper's *E*40 loading and

$$243,900 \times \frac{50}{40} = 304,875, \text{ say, } 305,000 \text{ lbs.}$$

for Cooper's $E50$. Now multiplying this by $\sec\theta$ we have

$$305{,}000 \times 1.28 = 390{,}400, \text{ say, } 390{,}000 \text{ lbs.}$$

for the maximum live-load stress in end post bA.

The load extends over practically the entire span when this maximum stress occurs, and hence the L in the impact formula (Art. 125) will be taken as 225. So we have

$$I = \left(\frac{300}{300+225}\right) 390{,}000 = 223{,}000 \text{ lbs.}$$

for the maximum impact stress in bA.

Now adding the above live-load stress and impact and the dead-load stress, given in Fig. 311, together we have

$$390{,}000 + 223{,}000 + 169{,}000 = 782{,}000 \text{ lbs.}$$

for the total maximum stress in the end post bA.

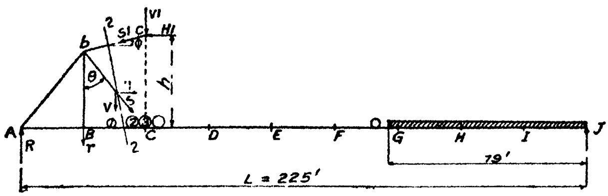


Fig. 313

Diagonal bC. By placing wheel 3 at C (Fig. 313) and applying Formula 4 of Art. 190 we obtain

$$\frac{442}{225} = 1.97 \text{ and } \frac{40}{25}\left(1 + \frac{25}{87}\right) = 2.06.$$

By placing wheel 2 at C we obtain

$$\frac{432}{225} = 1.92 \text{ and } \frac{20}{25}\left(1 + \frac{25}{87}\right) = 1.03.$$

From this it is seen that the criterion for placing the wheels for maximum stress in diagonal bC is nearest satisfied when wheel 3 is at C. Then placing wheel 3 at C as shown in Fig. 313 and taking moments about J we obtain

$$R = (16{,}364 + 22{,}436 + \overline{79}^2)\frac{1{,}000}{225} = 200{,}180 \text{ lbs.}$$

for the reaction at A. Then taking moments about C of the forces to left we obtain

$$H1 = [(200{,}180 \times 50) - 230{,}000] \div 37.86 = 258{,}290 \text{ lbs.}$$

for the horizontal component of the stress in the top chord bc and multiplying this by $\tan\phi$ we have

$$V1 = 258{,}290 \times 0.2744 = 70{,}875 \text{ lbs.}$$

for the vertical component of the stress in the top chord bc.

Taking moments about C of wheels 1 and 2 we obtain

$$r = 9{,}200 \text{ lbs.}$$

Now adding up all of the vertical forces and components to the left of sections 2-2 we have

$$R - r - V - V1 = 0,$$

from which we obtain

$$V = R - r - V1$$

and substituting in the numerical values given above we have

$$V = 200{,}180 - 9{,}200 - 70{,}875 = 120{,}105 \text{ lbs.}$$

for the vertical component of the stress in diagonal bC and multiplying this by $\sec\theta$ we obtain

$$S = 120{,}105 \times 1.28 = 153{,}734 \text{ lbs.}$$

for the stress in the diagonal due to Cooper's $E40$ and multiplying this by 50/40 we obtain 192,167, say 192,000 lbs. for the maximum live-load stress in diagonal bC.

When this maximum stress occurs the span is loaded practically from C to J and hence the L in the impact formula will be taken as 175 ft. So we have

$$I = \left(\frac{300}{175 + 300}\right) 192{,}000 = 121{,}000 \text{ lbs.}$$

for the impact stress in diagonal bC.

Now adding together the above live-load stress and impact and the dead-load stress, given in Fig. 311, we have

$$192{,}000 + 121{,}000 + 73{,}000 = 386{,}000 \text{ lbs.}$$

for the total maximum tension in diagonal bC.

Intermediate Post cC. Placing wheel 3 at D and applying Formula 4 of Art. 190 we obtain

$$\frac{392}{225} = 1.74 \text{ and } \frac{40}{25}\left(1 + \frac{50}{87}\right) = 2.52$$

and placing wheel 2 at D we obtain

$$\frac{382}{225} = 1.7 \text{ and } \frac{20}{25}\left(1 + \frac{50}{87}\right) = 1.26.$$

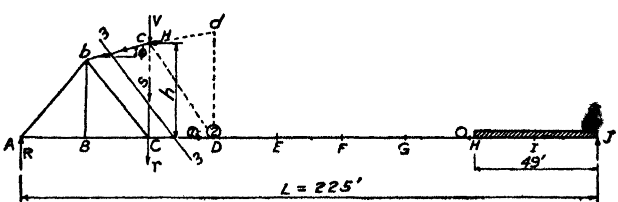


Fig. 314

From this it is seen that the criterion for placing the wheel loads for maximum stress in post cC is nearest satisfied when wheel 2 is at D. Then placing wheel 2 at D, as shown in Fig. 314, and taking moments about J we obtain

$$R = 145{,}200 \text{ lbs.}$$

for the reaction at A and taking moments about D of wheel 1 we obtain

$$r = (10{,}000 \times 8) \div 25 = 3{,}200 \text{ lbs.}$$

for the floor beam concentration at C. Then taking moments about C of all the forces and components to the left of section 3-3 we obtain

$$H = (R \times 50) \div h,$$

and substituting in the numerical values given above we have

$$H = (145{,}200 \times 50) \div 37.86 = 191{,}700 \text{ lbs.}$$

for the horizontal component of the stress in the top chord bc. Then multiplying this component by $\tan\phi$ we obtain

$$V = 191{,}700 \times 0\ 2744 = 52{,}600 \text{ lbs.},$$

for the vertical component of the stress in the top chord bc. Now adding up (algebraically) all of the vertical forces and components to the left of section 3-3 we have

$$R - r - V - S = 0,$$

and substituting in the numerical values given above and transposing we obtain

$$S = 145{,}200 - 3{,}200 - 52{,}600 = 89{,}400 \text{ lbs.}$$

for the stress in post cC due to Cooper's $E40$, and multiplying this by 50/40 we obtain 111,700, say, 112,000 lbs. for the maximum live-load compression in post cC.

When this maximum stress occurs the span is loaded practically from D to J and hence the L in the impact formula will be taken as 150. So we have

$$I = \left(\frac{300}{150 + 300}\right) 112{,}000 = 75{,}000 \text{ lbs. (about)}$$

for the impact stress in post cC. Now adding together the above live-load stress and impact and the dead-load stress, given in Fig. 311, we have

$$112{,}000 + 75{,}000 + 34{,}000 = 221{,}000 \text{ lbs.}$$

for the total maximum compression in post cC.

Diagonal cD. Placing wheel 3 at D (Fig. 312) and applying equation 4 of Art. 190 we obtain

$$\frac{392}{225} = 1.74 \text{ and } \frac{40}{25}\left(1 + \frac{50}{156}\right) = 2.11.$$

This position, as can be verified by trial, comes nearest to satisfying the criterion for maximum stress in diagonal cD. So placing wheel 3 at D, as shown in Fig. 315, and taking moments about J we obtain

$$R = (16{,}364 + 15{,}336 + \overline{54}^2)\frac{1{,}000}{225} = 153{,}700 \text{ lbs.}$$

for the reaction at A. Taking moments about D of the wheels to the left we obtain

$$r=(230)\frac{1{,}000}{25}=9{,}200\text{ lbs.}$$

for the floor beam concentration at C. Next, taking moments about D of the forces and components to the left of section 4-4 we obtain

$$H1=\frac{(153{,}700\times75)-(9{,}200\times25)}{12.13}=266{,}200\text{ lbs.}$$

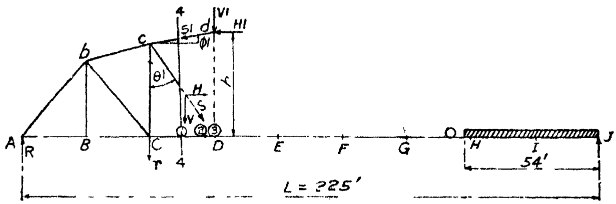


Fig. 315

for the horizontal component of the stress in the top chord cd. Multiplying this by $\tan\phi1$ we obtain

$$V1=H1\times\tan\phi1=266{,}200\times0.1828=18.660\text{ lbs.}$$

for the vertical component of the stress in top chord cd. Now summing up the vertical forces and components to the left of section 4-4 we have

$$R-r-V-V1=0.$$

Substituting the numerical values given above and transposing we obtain

$$V=153{,}700-9{,}200-18{,}660=95{,}840\text{ lbs.}$$

for the vertical component of the stress in diagonal cD and multiplying this by $\sec\theta1$ we have

$$S=V\sec\theta1=95{,}840\times1.199=114{,}912\text{ lbs.}$$

for the stress in diagonal cD due to Cooper's E 40 loading. Then by multiplying this by 50/40 we obtain 143,635, say, 144,000 lbs. for the maximum tension in diagonal cD due to the E 50 loading.

When this maximum stress occurs the span is practically loaded from D to J and hence the L in the impact formula will be taken as 150 ft. So we have

$$I=\left(\frac{300}{150+300}\right)144{,}000=96{,}000\text{ lbs.}$$

for the impact stress in diagonal cD.

Now adding together the above live-load stress and impact and the dead-load stress, given in Fig. 311, we obtain

$$144{,}000+96{,}000+40{,}000=280{,}000\text{ lbs.}$$

for the total maximum tension in diagonal cD.

Intermediate Post dD. Placing wheel 2 at E (Fig. 312) and applying equation 4 of Art. 190 we obtain

$$\frac{332}{225}=1.47\text{ and }\frac{20}{25}\left(1+\frac{75}{156}\right)=1.18.$$

This position of the wheels practically satisfies the criterion for maximum stress in post dD. So placing wheel 2 at E, as shown in Fig. 316, and taking moments about J we obtain (using Table A)

$$R = 105{,}500 \text{ lbs.}$$

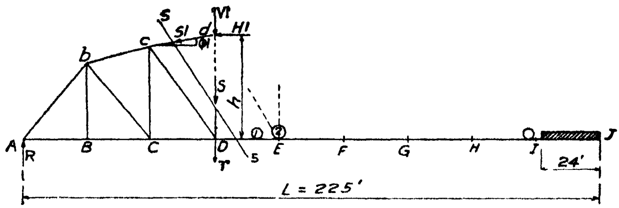


Fig 316

for the reaction at A, and taking moments about E of wheel 1 we obtain

$$r = 3{,}200 \text{ lbs.}$$

for the floor beam concentration at D. Then taking moments about D of the forces and components to the left of section 5-5 we obtain

$$H1 = 186{,}500 \text{ lbs.}$$

for the horizontal component of the stress in top chord cd and multiplying this by $\tan\phi 1$ we obtain

$$V1 = 186{,}500 \times 0.1828 = 34{,}092 \text{ lbs.}$$

for the vertical components of the stress in top chord cd.

Now, by summing up (algebraically) the vertical forces to the left of section 5-5 we obtain

$$S = 105{,}500 - 3{,}200 - 34{,}092 = 68{,}208 \text{ lbs.}$$

for the stress in post dD due to Cooper's $E40$ and multiplying this by 50/40 we obtain 85,260, say, 85,000 lbs. for the maximum compression in post dD due to the $E50$ loading.

When this maximum stress occurs the span is practically loaded from E to J and hence the L in the impact formula will be taken as 125 ft. So we have

$$I = \left(\frac{300}{125+300}\right)85{,}000 = 60{,}000 \text{ lbs.}$$

for the impact stress in post dD.

Now, adding together the above live-load stress and impact, and the dead-load stress, given in Fig. 311, we obtain

$$85{,}000 + 60{,}000 + 12{,}000 = 157{,}000 \text{ lbs.}$$

for the total maximum compression in post dD.

Diagonal dE. Placing wheel 3 at E (Fig. 312) we obtain

$$\frac{342}{225} = 1.52 \text{ and } \frac{40}{25}\left(1+\frac{75}{378}\right) = 1.9.$$

This position of the load comes nearest to satisfying the criterion for maximum stress in diagonal dE. So placing wheel 3 at E, as shown in Fig. 317, and taking moments about J we obtain

$$R = 113{,}200 \text{ lbs.}$$

for the reaction at A, and taking moments about E of the wheels to the left we obtain

$$r = 9{,}200 \text{ lbs.}$$

for the floor beam concentration at D. Then taking moments about E of the forces and components to the left we obtain

$$H1 = 248{,}100 \text{ lbs.}$$

for the horizontal component of the stress in the top chord de and multiplying this by $\tan\phi 2$ we obtain

$$V1 = 248{,}100 \times 0.0912 = 22{,}626 \text{ lbs.}$$

for the vertical component of the stress in top chord de.

Now, summing up the vertical forces and components to the left of section 6-6 we obtain

$$V = 113{,}200 - 9{,}200 - 22{,}626 = 81{,}374 \text{ lbs.}$$

for the vertical component of the stress in diagonal dE, and multiplying this by $\sec\theta 2$ we obtain 94,393 lbs. for the stress in diagonal dE due to

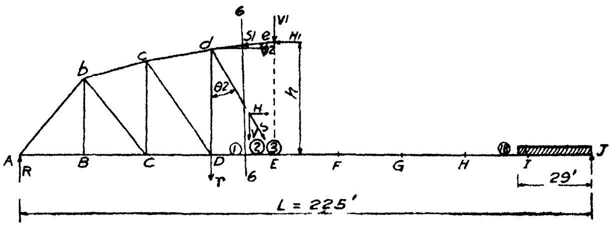


Fig 317

$E40$ loading; and multiplying this by 50/40 we obtain 117,991, say, 118,000 lbs. for the maximum live-load stress in the member. When this maximum stress occurs, the span is loaded practically from E to J and hence the L in the impact formula will be taken as 125 ft. So we have

$$I = \left(\frac{300}{125 + 300}\right) 118{,}000 = 83{,}200, \text{ say, } 83{,}000 \text{ lbs.}$$

for the impact stress in diagonal dE.

Now, adding together the above live-load stress and impact and the dead-load stress given in Fig. 311, we obtain

$$118{,}000 + 83{,}000 + 21{,}000 = 222{,}000 \text{ lbs.}$$

for the total maximum stress in diagonal dE.

Intermediate Post eE. Placing wheel 2 at F (Fig. 312) we obtain

$$\frac{284}{225}=1.26 \text{ and } \frac{20}{25}\left(1+\frac{100}{378}\right)=1.08$$

and placing wheel 3 at F we obtain

$$\frac{292}{225}=1.3 \text{ and } \frac{40}{25}\left(1+\frac{100}{378}\right)=2.16.$$

From this it is seen that the criterion (Art. 190) for maximum stress in post eE is the nearest satisfied with wheel 2 at F. So placing wheel 2 at F, as shown in Fig. 318, and taking moments about J we obtain (using Table A)

$$R=71{,}466 \text{ lbs.}$$

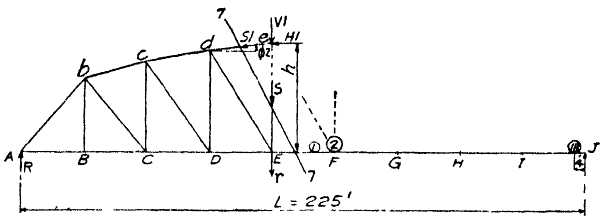


Fig. 318

for the reaction at A and taking moments about F of the wheel to the left we obtain

$$r=3{,}200 \text{ lbs.}$$

for the floor beam concentration at E. Then taking moments about E of the forces and components to the left of section 7-7 we obtain

$$H1=159{,}843 \text{ lbs.}$$

for the horizontal component of the stress in the top chord de and multiplying this by $\tan\phi 2$ we obtain

$$V1=14{,}577 \text{ lbs.}$$

for the vertical component of the stress in top chord de. Then adding algebraically the vertical forces and components to the left of section 7-7 we obtain

$$S=71{,}466-3{,}200-14{,}577=53{,}689 \text{ lbs.}$$

for the maximum compression in post eE due to the $E40$ loading and multiplying this by 50/40 we obtain 67,111, say, 67,000 lbs. for the maximum live-load compression in post eE.

When this maximum stress occurs the span is loaded practically from F to J and hence the L in the impact formula will be taken as 100. So we have

$$I=\left(\frac{300}{100+300}\right)67{,}000=50{,}250, \text{ say, } 50{,}000 \text{ lbs.}$$

for the impact stress in post eE.

Now, adding the above live-load stress and impact, and the dead-load stress given in Fig. 311 we obtain

$$67{,}000 + 50{,}000 - 5{,}000 = 112{,}000 \text{ lbs.}$$

for the total maximum compression in post eE.

Diagonal eF. As the top chord ef (Fig. 312) is parallel to bottom chord EF the diagonal eF will carry all of the maximum shear in panel EF when the load moves onto the bridge from the right and hence the ordinary criterion of Art. 90 for the maximum shear in panel EF applies. According to this criterion, the maximum shear will occur in panel EF when the average unit load on the bridge is equal to the average unit load in the panel.

This is the nearest satisfied when wheel 3 is at F. So placing wheel 3 at F as shown in Fig. 319 and taking moments about J we obtain

$$R = 77{,}851 \text{ lbs.}$$

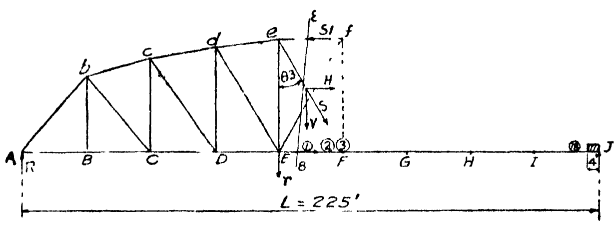


Fig 319

for the reaction at A and taking moments about F of the wheels to the left we obtain

$$r = 9{,}200 \text{ lbs.}$$

for the floor beam concentration at E. Then for the maximum shear in panel EF we have

$$(R - r) = (77{,}851 - 9{,}200) = 68{,}651 \text{ lbs.}$$

This is assumed to be carried altogether by the diagonal eF, so, evidently the vertical component of the stress in that member must be equal to this shear; that is,

$$V = 68{,}651 \text{ lbs.}$$

Then multiplying this by $\sec\theta 3$ we obtain

$$S = 68{,}652 \times 1.145 = 78{,}605 \text{ lbs.}$$

for the stress in the diagonal eF due to $E40$ loading and multiplying this by 50/40 we obtain 98,256, say 98,000 lbs. for the maximum live-load stress in diagonal eF.

When this maximum stress occurs the span is loaded practically from F to J and hence the L in the impact formula will be taken as 100.

Then we have

$$I = \left(\frac{300}{100 + 300}\right) 98{,}000 = 73{,}500, \text{ say } 73{,}000 \text{ lbs.}$$

for the impact in diagonal eF.

Now, adding the above live-load stress and impact, and the dead-load stress given in Fig. 311 we obtain

$$98{,}000 + 73{,}000 + 00{,}000 = 171{,}000 \text{ lbs.}$$

for the total maximum stress in diagonal eF.

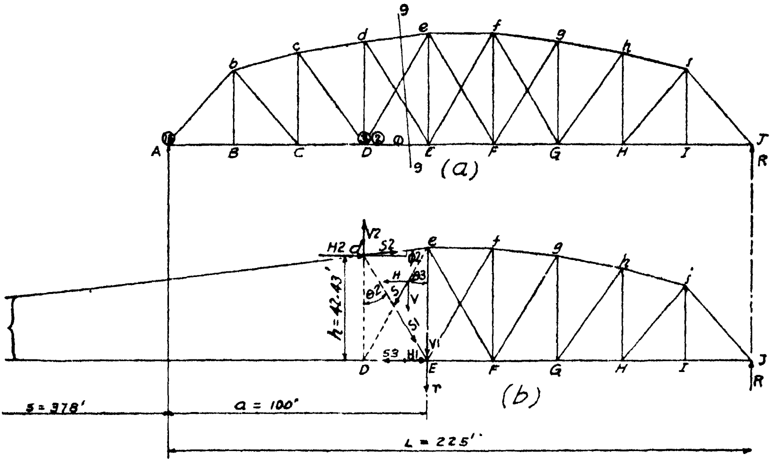


Fig. 320

Counter eD. Placing wheel 3 at D as shown at (a), Fig. 320, and applying Formula 11 of Art. 190 we have (not considering wheel 15 on the bridge)

$$\frac{232}{225} = 1.03 \text{ and } \frac{40}{25}\left(\frac{100 + 378}{225 + 378}\right) = 1.26.$$

This position of the wheels comes nearest to satisfying the criterion for maximum stress in counter eD. So placing wheel 3 at D and taking moments about A we obtain

$$R = (10{,}816)\frac{1{,}000}{225} = 48{,}071 \text{ lbs.}$$

for the reaction at J and taking moments about D of the wheels to the right we obtain

$$r = 9{,}200 \text{ lbs.}$$

for the floor beam concentration at E.

$S1$ is equal and opposite to the dead-load stress in diagonal dE which is given in Fig. 311 as 21,000 lbs. Then, resolving $S1$ into horizontal and vertical components at E, we obtain

$$V1 = S1 \div \sec\theta 2 = 21{,}000 \div 1.16 = 18{,}103 \text{ lbs.}$$

for the vertical component of the stress in diagonal dE.

Taking moments about D of the forces and components to the right we obtain

$$(R \times 150) - (V1 \times 25) - (r \times 25) - (H2 \times 42.43) = 0.$$

Substituting in the numerical values given above and transposing and reducing we obtain

$$H2 = 153{,}853 \text{ lbs.}$$

for the horizontal component of the stress in top chord *de* and multiplying this by $\tan\phi 2$ we obtain

$$V2 = 153{,}853 \times 0.0912 = 14{,}031 \text{ lbs.}$$

for the vertical component of the stress in top chord *de*.

Now, summing up all the vertical forces and components to the right of section 9-9 we obtain

$$V = R - r - V1 + V2$$

and substituting in the numerical values given above we have

$$V = 48{,}071 - 9{,}200 - 18{,}102 + 14{,}031 = 34{,}799 \text{ lbs.}$$

for the vertical component of the stress in counter *eD* and multiplying this by $\sec\theta 3$ we have

$$S = 34{,}799 \times 1.145 = 39{,}844 \text{ lbs.}$$

for the maximum stress in counter *eD* due to the *E*40 loading and multiplying this by 50/40 we obtain 49,805, say, 50,000 lbs. for the maximum live-load stress in counter *eD* due to *E*50 loading.

When this maximum stress occurs the span is loaded practically from *D* to *A* and hence the *L* in the impact formula will be taken as 75.

Then we have

$$I = \left(\frac{300}{75 + 300}\right) 50{,}000 = 40{,}000 \text{ lbs.}$$

for the impact stress in counter *eD*.

Now, adding the above live-load stress and impact together we have (counters carry no dead load)

$$50{,}000 + 40{,}000 = 90{,}000 \text{ lbs.}$$

for the total maximum stress in counter *eD*.

No counter is needed in panel *CD* as the live-load shear is not sufficient to reverse the dead-load shear; or, in other words, the dead-load tension in diagonal *cD* is greater than the live-load compression in it.

Tension in Post dD. The maximum shear in end panel *AB* was found above (in determining the stress in end post *bA*) to be 305,000 lbs.

Substituting this shear in the formula given in Art. 123 we obtain

$$p' = \left(\frac{305{,}000}{\frac{225 - 25}{2}}\right) = 3{,}050 \text{ lbs.}$$

for the equivalent uniform live load per foot of truss for determining the stress in the web members and hence we can use this load for determining the tension in intermediate posts. Then for the panel load we have

$$W = 3{,}050 \times 25 = 76{,}250, \text{ say } 76{,}000 \text{ lbs.}$$

The first part of our problem now is to place this uniform load so as to obtain zero stress in counter *eD* and also in diagonal *dE* and at the same time as great a stress in chords *cd* and *de* as possible (see Art. 190).

The equivalent uniform live load will produce practically the same stress in the counter *eD* (Fig. 320) as the wheel loads. So, if panel points *B*, *C*, and *D* are loaded with the above uniform load the counter *eD* will have a maximum tensile stress of 50,000 lbs., the same as previously found for wheel loads. Dividing this by $\sec\theta 3$ we obtain

$$50{,}000 \div 1.145 = 44{,}000 \text{ lbs. (about)}$$

for the vertical component of the maximum tensile stress in counter *eD*. Now, any load at *E*, *F*, or any panel point to the right of panel *DE* will tend to reduce the tension in counter *eD*. What we desire is to place the load just so that the 50,000 lbs. tension in the counter is exactly counteracted, for then the counter *eD* and diagonal *dE* will have zero stress and hence the post *dD* will have maximum live-load tension.

By loading panel point *E* alone we obtain a reaction at *A* of 76,-000 × 5/9 = 42,000 lbs. Now, as the vertical component of the 50,000 lbs.

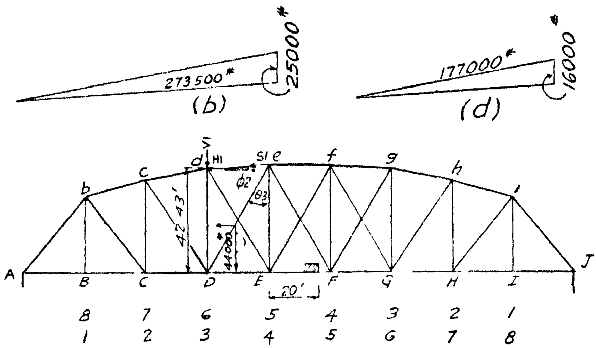


Fig. 321

tensile stress in the counter *eD* alone is about 44,000 lbs., it is obvious that a panel load at *E* will not reverse the 50,000 lbs. tension in the counter. So next, let us place a panel load at both *E* and *F*. Then for the reaction at *A*, which is equal to the negative shear in panel *DE* due to the loads at *E* and *F*, we obtain

$$76{,}000 \times \frac{5}{9} + 76{,}000 \times \frac{4}{9} = 76{,}000 \text{ lbs. (about).}$$

Taking moments about *D* (panel points *E* and *F* alone being loaded) we obtain

$$H1 = (76{,}000 \times 75) \div 42.43 = 135{,}000 \text{ lbs. (about)}$$

for the horizontal component of the stress in top chord *de*. Then multiplying this by $\tan\phi 2$ we obtain

$$V1 = 135{,}000 \times 0.0912 = 12{,}300 \text{ lbs. (about)}$$

for the vertical component of the stress in top chord *de* due to the panel loads at *E* and *F*.

Then subtracting this from the reaction we obtain

$$76{,}000 - 12{,}300 = 63{,}700 \text{ lbs.}$$

for the vertical component in counter eD. This is too great, as the counter would be more than reversed and hence the diagonal dE would then be in tension.

Next, suppose the load extends just up to panel point F so that the load at F will be only a half panel load or 38,000 lbs. Then for the reaction at A we have

$$76{,}000 \times \frac{5}{9} + 38{,}000 \times \frac{4}{9} = 59{,}000 \text{ lbs. (about).}$$

Now, taking moments about D we obtain

$$V1 = \left(\frac{59{,}000 \times 75}{42.43}\right) 0.0912 = 9{,}500 \text{ lbs. (about)}$$

for the vertical component of the stress in the top chord de.

Then subtracting this from the reaction at A we have

$$59{,}000 - 9{,}500 = 49{,}500 \text{ lbs.}$$

for the vertical component of the compressive stress in counter eD. As is seen, this is too large. So let us extend the load just 20 ft. beyond panel point E as shown. Then the load at E will be 75,000 lbs. and at F it will be 24,000 lbs. The reaction at A due to these two loads will be

$$\left(75{,}000 \times \frac{5}{9}\right) + \left(24{,}000 \times \frac{1}{9}\right) = 52{,}300 \text{ lbs. (about).}$$

Then taking moments about D we obtain

$$V1 = \left(\frac{52{,}300 \times 75}{42.43}\right) 0.0912 = 8{,}400 \text{ lbs. (about)}$$

for the vertical component of the stress in the top chord de. Now, subtracting this from the reaction at A we obtain

$$52{,}300 - 8{,}400 = 43{,}900 \text{ lbs.}$$

for the vertical component of the compressive stress in the counter eD due to the loads at E and F which just reverses the maximum tension in the counter. So this is the position of the load being sought for. That is, by placing the load so that it extends from A to 20 ft. beyond E the stress in both the counter eD and diagonal dE will be zero and the chords cd and de will have the greatest stress possible with that condition and hence the tension in post dD will be a maximum.

So placing the load in that position and taking moments about A we obtain

$$R' = (3{,}050 \times 120)\frac{60}{225} = 97{,}644 \text{ lbs.}$$

for the reaction at J.

Then taking moments about D we obtain

$$H1 = \frac{97{,}644 \times 150 - 3{,}050 \times 45 \times 45/2}{42.43} = 272{,}400 \text{ lbs. (about)}$$

for the horizontal component of the stress in top chord *de* and multiplying this by $\sec\phi 2$ we obtain

$$272{,}400 \times 1.004 = 273{,}500 \text{ lbs. (about)}$$

for the stress in top chord *de*. Now as the stress in diagonal *dE* is zero, the tensile stress in the post *dD* can be obtained very quickly by drawing the diagram shown at (b). From this diagram the tension in post *dD* is found to be about 25,000 lbs.

As is seen, when this maximum tension occurs in the post *dD* there is 120 ft. of load on the bridge, so we have

$$I = \left(\frac{300}{120 + 300}\right) 25{,}000 = 18{,}000 \text{ lbs. (about)}$$

for the impact stress.

Now, assuming diagonal *dE* as having zero stress we can obtain the dead-load tension in post *dD* by drawing the diagram shown at (d).

The stress in top chord *cd* is given in Fig. 311 as 177,000 lbs. Then drawing 1-3 equal (by scale) to 177,000 and parallel to *cd*, and 1-2 and 3-2 parallel to *de* and *dD*, respectively, we have the dead-load tension in the post given by the line 3-2. This tension is found in this manner to be 16,000 lbs.

Now adding together the above live- and dead-load stresses and impact we obtain

$$25{,}000 + 18{,}000 + 16{,}000 = 59{,}000 \text{ lbs.}$$

for the total maximum tension in post *dD*.

Tension in Post eE. As is obvious, the maximum tension will occur in post *eE* when the span is fully loaded, for in that case both of the diagonals *eD* and *fE* have zero stress. The counter *eD* also has zero stress at the same time.

Now, as we are using a uniform live load we can readily obtain the live-load tension in post *eE* by direct proportion as it will be proportional to the dead-load tension given in Fig. 311.

So, letting S represent the live-load tension, we have

$$\frac{S}{5{,}000} = \frac{3{,}050}{1{,}318},$$

from which we obtain

$$S = 11{,}570, \text{ say, } 12{,}000 \text{ lbs.}$$

for the maximum live-load tension in post *eE* where 3,050 is the uniform live load and 1,318 the dead load per ft. of truss.

For the impact we have

$$I = \left(\frac{300}{225 + 300}\right) 12{,}000 = 6{,}857, \text{ say, } 7{,}000 \text{ lbs.}$$

Now, adding together the above live-load stress and impact, and the dead-load stress given in Fig. 311, we obtain

$$12{,}000 + 7{,}000 + 5{,}000 = 24{,}000 \text{ lbs.}$$

for the total maximum tension in post *eE*.

Bottom Chords AB and BC. The maximum stress can be obtained in these members by placing a load at B such that the average unit load to the left is equal to the average unit load on the bridge (see Art. 90) and taking moments about b; but the same can be found by simply multiplying the maximum shear in panel AB by $\tan\theta$. The maximum shear was found above (in determining the stress in the end post bA) to be 305,000 lbs. So we have

$$305{,}000 \times 0.8066 = 246{,}013, \text{ say, } 246{,}000 \text{ lbs.}$$

for the live-load stress in each of the bottom chords AB and BC.

When this maximum stress occurs, the load extends practically over the entire span, so for the impact stress in each of these chords we have

$$I = \left(\frac{300}{225 + 300}\right) 246{,}000 = 140{,}571, \text{ say, } 141{,}000 \text{ lbs.}$$

Now, adding together the above live-load stress and impact, and the dead-load stress, given in Fig. 311, we obtain

$$246{,}000 + 141{,}000 + 106{,}000 = 493{,}000 \text{ lbs.}$$

for the total maximum stress in each of the bottom chords AB and BC.

Bottom Chord CD. The stress in this member (see Fig. 312) is found by taking moments about panel point c. Then, evidently, the stress will be a maximum when the load is placed so that this moment is a maximum. According to Art. 90, the moment will be a maximum when a load is at C such that the average unit load to the left is equal

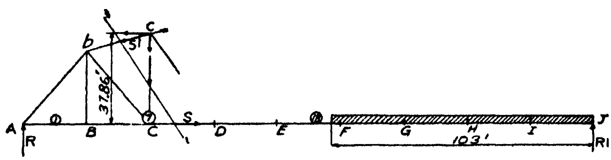


Fig. 322

to the average unit load on the bridge. Placing wheel 7 at C, as shown in Fig. 322, we have

$$(103 + 6.5)\frac{1{,}000}{50} = 2{,}190 \text{ lbs.}$$

for the average unit load to the left of C (considering one-half of wheel 7 as being to the left of C) and

$$[284 + (103 \times 2)]\frac{1{,}000}{225} = 2{,}177 \text{ lbs.}$$

for the average unit load on the bridge. This position of the load comes nearest to satisfying the criterion for maximum moment about c (or C).

Then by taking moments about J (using Table A) we obtain

$$R = [16{,}364 + (284 \times 103) + \overline{103}^2]\frac{1{,}000}{225} = 249{,}880 \text{ lbs. (about)}$$

for the reaction at A. Next, taking moments about c of the forces to the left and dividing by the height of the truss at that point we obtain

$$S = [(249{,}880 \times 50) - (2{,}155 \times 1{,}000)]\frac{1}{37.86} = 273{,}000 \text{ lbs. (about)}$$

for the maximum stress in the bottom chord CD due to the $E40$ loading and multiplying this by 50/40 we obtain 341,250, say 341,000 lbs. due to the $E50$ loading which is the maximum live-load stress desired.

When this maximum stress occurs the live load extends over practically the entire span and hence the L in the impact formula will be taken as 225. So for the impact stress we have

$$I = \left(\frac{300}{225 + 300}\right) 341{,}000 = 194{,}000 \text{ lbs. (about)}.$$

Now, adding together the above live-load stress and impact, and the dead-load stress given in Fig. 311, we obtain

$$341{,}000 + 194{,}000 + 152{,}000 = 687{,}000 \text{ lbs.}$$

for the total maximum stress in bottom chord CD.

Top Chord bc. Considering the forces to the left of section 1-1 (Fig. 322) it is readily seen that the horizontal component of the maximum stress in top chord bc is equal to the maximum stress in bottom chord CD. So by multiplying the stress in CD (found above) by $\sec\phi$ (see Art. 190) we obtain

$$S1 = 341{,}000 \times 1.037 = 353{,}617, \text{ say, } 354{,}000 \text{ lbs.}$$

for the maximum live-load stress in top chord bc. For the impact we have

$$I = \left(\frac{300}{225 + 300}\right) 354{,}000 = 202{,}000 \text{ lbs. (about)}.$$

Now, adding together the above live-load stress and impact, and the dead-load stress given in Fig. 311, we obtain

$$354{,}000 + 202{,}000 + 157{,}000 = 713{,}000 \text{ lbs.}$$

for the total maximum stress in top chord bc.

Bottom Chord DE. The stress in this member is found by taking moments about panel point d and hence the stress in the member will be a maximum when the moment about d is a maximum. Placing wheel 11 at D, as shown in Fig. 323, we have

$$(152 + 10)\frac{1{,}000}{75} = 2{,}160$$

for the average unit load to the left of D (or d) and

$$(284 + 210)\frac{1{,}000}{225} = 2{,}195$$

for the average unit load on the bridge. This position of the load comes the nearest to satisfying the criterion for maximum moment about d (or D).

Then by taking moments about J we obtain

$$R = [(16{,}364 \times 75) + (284 \times 105) + 105^2]\frac{1{,}000}{225} = 254{,}260 \text{ lbs. (about)}$$

for the reaction at A. Next, taking moments about d of the forces to the left and dividing by the height of the truss at that point we obtain

$$S = [\,(254{,}260 \times 75) - (5{,}848 \times 1{,}000)\,]\frac{1}{42.43} = 311{,}600 \text{ lbs. (about)}$$

for the maximum stress in chord DE due to the $E10$ loading and multiplying this by 50/10 we obtain 389,500, say 390,000 lbs. for the maximum stress due to the $E50$ loading.

For the impact we have

$$I = \left(\frac{300}{225 + 300}\right) 390{,}000 = 222{,}856, \text{ say, } 223{,}000 \text{ lbs.}$$

Now, adding together the above live-load stress and impact, and the dead-load stress given in Fig 311, we obtain

390,000 + 223,000 + 174,000 = 78[illegible],000 lbs.

for the total maximum stress in bottom chord DE.

Top Chord cd. Considering the forces to the left of section 2-2 (Fig. 323) it is readily seen that the horizontal component of the maximum

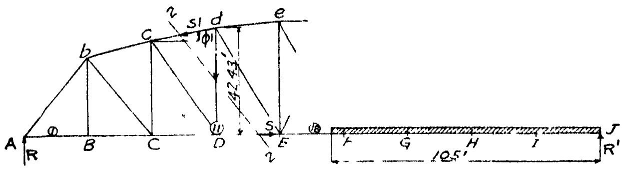


Fig. 323

stress in top chord cd is equal to the maximum stress in bottom chord DE.

So by multiplying the maximum stress in DE by $\sec\phi 1$ we obtain

$$390{,}000 \times 1.016 = 396{,}240, \text{ say, } 396{,}000 \text{ lbs.}$$

for the maximum live-load stress in top chord cd.

For the impact we have

$$I = \left(\frac{300}{225 + 300}\right) 396{,}000 = 226{,}000 \text{ lbs. (about).}$$

Now, adding together the above live-load stress and impact, and the dead-load stress given in Fig. 311, we obtain

$$396{,}000 + 226{,}000 + 177{,}000 = 799{,}000 \text{ lbs.}$$

for the total maximum stress in top chord cd.

Bottom Chord EF. The stress is found in this member by taking moments about panel point e and hence the stress in the member will be a maximum when the moment about e is a maximum.

Placing wheel 13 at E, as shown in Fig. 324, we have 2,020 lbs. for average unit load to the left of E (or e) and 2,062 lbs. for the average unit load on the bridge. This position of the load comes the nearest to satisfying the criterion for maximum moment about e (or E).

Then taking moments about J we obtain

$$R=[16{,}364+(284\times 90)\times \overline{90}^{2}]\ \frac{1{,}000}{225}=222{,}320\text{ lbs. (about)}$$

for the reaction at A. Next, taking moments about e of the forces to the left and dividing by the height of the truss at that point we obtain

$$S=[(222{,}320\times 100)-(7{,}668\times 1{,}000)]\frac{1}{44.71}=325{,}743\text{ lbs.}$$

for the maximum stress in bottom chord EF due to the $E40$ loading and

$$325{,}743\times\frac{50}{40}=407{,}178\text{, say, }407{,}000\text{ lbs.}$$

for the maximum stress due to the $E50$ loading.

For the impact we have

$$I=\left(\frac{300}{225+300}\right)407{,}000=232{,}571\text{, say }233{,}000\text{ lbs.}$$

Now adding together the above live-load stress and impact, and the dead-load stress given in Fig. 311 we obtain

$$407{,}000+233{,}000+184{,}000=824{,}000$$

for the total maximum stress in bottom chord EF.

Top Chord de. Considering the forces to the left of section 3-3 (Fig. 324) it is readily seen that the horizontal component of the maximum

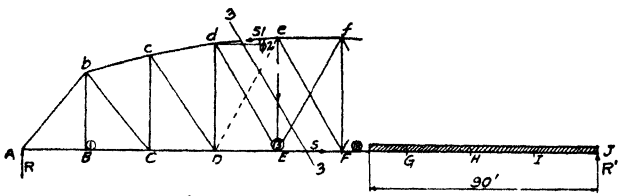


Fig. 324

stress in top chord de is equal to the maximum stress in bottom chord EF. So by multiplying the maximum stress in the bottom chord EF, which, as found above, is 407,000 lbs., by $\sec\phi 2$ we obtain

$$407{,}000\times 1.004=408{,}628\text{, say, }409{,}000\text{ lbs.}$$

for the maximum live-load stress in top chord de.

For the impact we have

$$I=\left(\frac{300}{225+300}\right)409{,}000=234{,}000\text{ lbs. (about).}$$

Now, adding together the above live-load stress and impact, and the dead-load stress given in Fig. 311, we obtain

$$409{,}000+234{,}000+185{,}000=828{,}000\text{ lbs.}$$

for the total maximum stress in top chord de.

Top Chord ef. In accordance with usual practice we will consider the stress in top chord ef to be equal and opposite to the stress in bottom chord EF. This would be absolutely true if the shear in panel EF were zero. If a uniform load be used the shear would be zero and in case wheel loads are used the shear is slight and hence the usual practice is not far wrong in any case.

We have now determined all of the stresses in one-half of the truss and as the structure is symmetrical about the center of span these are all the stresses necessary for designing the trusses. We can next write the above stresses on the stress sheet, Fig. 328.

197. Designing of Members in Trusses.— As the intermediate posts are the governing members (see Art. 176) we will consider them first. We will first ascertain as to whether channels can be used. The longest post is $U4$-$L4$ (Fig. 328), which has a length of about 536 ins. The average radius of gyration of 15″ channels is about 5.4 (see Table 3). Then for the maximum L/r we have

$$536 \div 5.4 = 99.2.$$

The maximum allowed for L/r for main members by the specifications is 100. So, as far as l/r is concerned, 15″ channels can be used. We will next ascertain as to whether the area of the channels is sufficient.

Post $U2$-$L2$ has the greatest stress and hence will require the greatest area. This post is about 454 ins. long. Now substituting in the column formula, using the average radius, we have

$$p = 16{,}000 - 70\frac{454}{5.4} = 10{,}115 \text{ lbs.}$$

for the allowable unit stress. Dividing this into the stress in the column we obtain

$$221{,}000 \div 10{,}115 = 21.74 \text{ sq. ins.}$$

for the required area. From this it is seen that channels can be used.

Post $U2$-$L2$. The area just found for this post is 21.74□″. Use 2—[s 15″ x 40# = 23.52□″. (The 15″ x 35# channels have 1.16□″ less area than required.)

Post $U3$-$L3$. This member has a maximum compressive stress of 157,000 lbs. and a maximum tensile stress of 80,000 lbs. Combining, according to the specifications, we have

$$157{,}000 + \frac{80{,}000}{2} = 197{,}000 \text{ lbs.}$$

for the maximum compression and

$$80{,}000 + \frac{80{,}000}{2} = 120{,}000 \text{ lbs.}$$

for the maximum tension to be considered in designing the member.

The length of the member is about 509 ins. So substituting in the column formula, using the average radius (as preliminary), we have

$$p = 16{,}000 - 70\frac{509}{5.4} = 9{,}400 \text{ lbs. (about)}$$

for the allowable compressive unit stress. Then we have

$$197{,}000 \div 9{,}400 = 20.96 \text{ sq. ins.}$$

for the area of cross-section required for compression, and for the area required for tension we have

$$120{,}000 \div 16{,}000 = 7.5 \text{ sq. ins.}$$

As is seen, the compression governs. Referring to Table 3, it is seen that 2—[s 15″ x 35# = 20 58□″ have the nearest to the required area for compression. So substituting the radius of these channels in the column formula we have

$$16{,}000 - 70\frac{509}{5.58} = 9{,}615 \text{ lbs.}$$

for the required unit stress. Dividing this into the maximum compression we obtain

$$197{,}000 \div 9{,}615 = 20.49 \text{ sq. ins.}$$

for the required area. This is very close to the area of the 2—[s 15″ x 35# and hence these channels will be used.

Post U4-L4. This member has a maximum compressive stress of 112,000 lbs. and a maximum tensile stress of 24,000 lbs. Combining, we have 124,000 lbs. compression and 36,000 lbs. tension to be considered in designing the member. The length of the member is about 536 ins. Then, using the radius of the lightest 15″ channel (as the stresses are low), we have

$$16{,}000 - 70\frac{536}{5.62} = 9{,}324 \text{ lbs.}$$

for the allowable unit compressive stress on the member. Dividing this into the above combined stress we have

$$124{,}000 \div 9{,}324 = 13.3 \text{ sq. ins. (about)}$$

for the required area for compression. For the area required for tension we have

$$36{,}000 \div 16{,}000 = 2.25 \text{ sq. ins.}$$

As is seen, the compression governs. 2—[s 15″ x 33# = 19.8□″ (which are the lightest 15″ channels) will be used for this member. As is seen, there is an excess of 6.5□″ of metal, yet we cannot do better if channels are used. The L/r would be too great if 12″ channels were used instead of the 15″ channels.

Hanger U1-L1. This is entirely a tension member. For the required net area of cross-section we have

$$198{,}000 \div 16{,}000 = 13.37 \text{ sq. ins.}$$

We will use 2—[s 15″ x 33# = 19.8 − (21/32 × 4) = 17.18□″ net.

(In this case the metal cut-out of the flanges is considered.) This is more section than needed but 15″ channels are used for the intermediate posts and for the sake of appearance they will be used for this member.

Diagonal U1-L2. For the required net area we have 386,000 ÷ 16,000 = 24.12□″ net. Use 2—bars 8″ x $1\frac{1}{2}$″ = 24.0□″.

Diagonal U2-L3. For the required net area we have 280,000 ÷ 16,000 = 17.5□″ net. Use 2—bars 7″ x $1\frac{1}{4}$″ = 17.5□″.

Diagonal U3-L4. For the required net area we have 222,000 ÷ 16,000 = 13.87□″ net. Use 2—bars 7″ x 1 = 14□″ net.

Diagonal U4-L4. For the required net area we have 171,000 ÷ 16,000 = 10.68□″ net. Use 4—Ls 5″ x $3\frac{1}{2}$ x $\frac{3}{8}$ = 12.20 − 1.50 = 10.7□″ net

Counter U4-L3. For the required area of cross-section we have 90,000 ÷ 16,000 = 5.62□″. Use 1—bar $2\frac{3}{8}$″ x $2\frac{3}{8}$″ = 5.64□″ (standard bar).

Bottom Chords L0-L1 and L1-L2. For the required net area we have 493,000 ÷ 16,000 = 30.8□″. Use the following section:

2—pls. 20″ x $\frac{5}{8}$″ = 25.0 − 3.75	= 21.25□″ net	
4—Ls $3\frac{1}{2}$″ x $3\frac{1}{2}$″ x $\frac{7}{16}$″ = 11.48 − 1.75 =	9.73□″ net	
	30.98□″ net	

Bottom Chord L2-L3. For the required area of cross-section we have 687,000 ÷ 16,000 = 42.92□″. Use the following section:

2—bars 8″ x $1\frac{3}{8}$″	= 22.00□″
2—bars 8″ x $1\frac{5}{16}$″	= 21.00□″
	43.00□″

Bottom Chord L3-L4. For the required area of cross-section we have 787,000 ÷ 16,000 = 49.18□″. Use the following section:

2—bars 8″ x $1\frac{1}{2}$″	= 24.00□″
2—bars 8″ x $1\frac{9}{16}$″	= 25.00□″
	49.00□″

Bottom Chord L4-L4. For the required area of cross-section we have 824,000 ÷ 16,000 = 51.5□″. Use 4—bars 8″ x $1\frac{5}{8}$ = 52.0″.

In designing the top chords, it is best to design the lightest section first, using minimum thicknesses of web (if sufficient) so that the area of cross-section can be increased for the others by merely increasing the thickness of the webs. So in this case we will first consider top chord *U1-U2*, as it has the least stress and consequently will have the lightest section.

Top Chord U1-U2. The first thing to do is to determine the general dimensions of the section. The width of the posts will be $12\frac{1}{2}$″ (see Art. 176) and the maximum thickness of eye-bars is 3″ (2—8″ x $1\frac{1}{2}$″ bars at *U1*) and allowing, say, 2″ for pin plates and clearance we have $12\frac{1}{2} + 3 + 2 = 17\frac{1}{2}$″ for the required distance between the webs. Then if the webs be $\frac{5}{8}$″ thick (each) and the top angles be 4″ x 4″ we obtain $17\frac{1}{2} + 1\frac{1}{4} + 8 = 26\frac{3}{4}$, say, 27″ for the width of the cover plate. The radius of gyration in reference to the *y-y* axis (see Art. 176) is approximately equal to the distance from the center of the cover plate out to the outer face of the web, which in this case is $(17\frac{1}{2}+1\frac{1}{4})\div 2=9.37$. Now, in order that the section be of economic depth, that is, so that the radius of gyration about the x-x axis is the same as about the y-y axis, the webs should have a depth 0.4 of which should equal the above radius. So for the economic depth of the web we have

$$h=\frac{9.37}{0.4}=23.4 \text{ ins.}$$

So we will make the webs 24″ deep. According to the specifications, the thickness of the cover plate should not be less than $\frac{1}{40}$ of the distance between the rivet lines. The distance between the rivet lines in this case will be about $17\frac{1}{2}+1\frac{1}{4}+4\frac{1}{2}=23\frac{1}{4}''$. $\frac{1}{40}$ of this distance is about $\frac{9}{16}$ of an inch. So we will make the cover plate $\frac{9}{16}''$ thick. The thickness of the webs, according to the specifications, should not be less than $\frac{1}{30}$ of the distance between the flanges. If the top angles be $4''\times 4''$ and the bottom angles $6''\times 4''$ (the 6″ leg along the web), we have $24-(4+6)=14''$ for the distance between the flanges. Then for the minimum thickness of the webs we have $14\times 1/30=0.46$, which is about $\frac{7}{16}$ of an inch.

The top angles should be of minimum thickness and the bottom angles should be of maximum thickness in order that the center of gravity of the section be as near the center of the web as possible.

The length of the member ($U1$-$U2$) is about 312 ins. Now substituting this length and the approximate radius given above in the column formula we obtain

$$p=16{,}000-70\frac{312}{9.37}=13{,}670 \text{ lbs.}$$

for the approximate allowable unit stress. Dividing this into the maximum stress in the member we obtain

$$713{,}000\div 13{,}670=52.15 \text{ sq. ins.}$$

for the approximate required area of cross-section.

In accordance with the above we obtain the following section:

1—cover pl. $27''\times\frac{9}{16}''$	$=15.18\square''$
2—web pls. $24''\times\frac{7}{16}''$	$=21.00\square''$
2—Ls $4''\times 4''\times\frac{3}{8}''$	$=5.72\square''$
2—Ls $6''\times 4''\times\frac{9}{16}''$	$=10.62\square''$
	$52.52\square''$

This section is really the minimum and is very near the approximate section found above.

Now taking moments about the cover plate, in the manner shown in Art. 176 we obtain 9.38″ for the distance from the cover plate down to the horizontal gravity axis of the section.

The next thing is to see if the eye-bars will fit into the top chord. The largest pin, which will be at $U1$, will be about 7″ in diameter. From the table of eye-bars, to be found in practically all structural handbooks, it is seen that the 8″ bars require $17\frac{1}{2}$″ head for a 7″ diameter pin. Half of this is $8\frac{3}{4}$″. So it is seen that the eye-bars fit into the chord satisfactorily, as it is 9.38″ from the gravity axis to the cover plate and hence the bars will not interfere with the plate.

In the same manner as shown in Art. 176 the radius of gyration about the horizontal gravity axis, or x-x axis, is found to be about 9.5 and 9.3 about the y-y axis. Now using the least radius we obtain

$$16{,}000 - 70\frac{312}{9.3} = 13{,}652 \text{ lbs.}$$

for the actual allowable unit stress.

Then dividing this into the stress we obtain

$$713{,}000 \div 13{,}652 = 52.22$$

for the required area of cross-section which is practically equal to the above assumed section and hence that section will be used.

Top Chord U2-U3. Dividing the maximum stress in this member by the allowable unit stress found for $U1$-$U2$ (which will be practically the same as for this member) we obtain

$$799{,}000 \div 13{,}652 = 58.53 \text{ sq. ins.}$$

for the approximate area of cross-section required. The following section has about this area:

1—cov. pl. 27″ x $\frac{9}{16}$″	= 15.18□″
2—web pls. 24″ x $\frac{9}{16}$″	= 27.00□″
2—Ls 4″ x 4″ x $\frac{3}{8}$″	= 5.72□″
2—Ls 6″ x 4″ x $\frac{9}{16}$″	= 10.62□″
	58.52□″

By taking moments about the cover plate we obtain 9.66″ for the distance from the cover plate down to the horizontal gravity, or x-x axis. The radius of gyrations (found as previously explained) in reference to the x-x axis is 9.3 and the radius in reference to the y-y axis is 9.2.

The length of the member is about 305 ins. Now, using the least radius we obtain

$$16{,}000 - 70\frac{305}{9.2} = 13{,}680 \text{ lbs.}$$

for the allowable unit stress. Dividing this into the maximum stress in the member we obtain

$$799{,}000 \div 13{,}680 = 58.40 \text{ sq. ins.}$$

for the required area of cross-section. This is practically the same as the approximate section found above and hence that section will be used.

Top Chord U3-U4. Dividing the maximum stress in the member by the allowable unit stress found for $U2$-$U3$ we obtain

$$828{,}000 \div 13{,}680 = 60.52 \text{ sq. ins.}$$

for the approximate area of cross-section required. The following section has about this area:

1—cov. pl.	$27'' \times \frac{9}{16}''$	$=15.18\square''$
2—web pls.	$24'' \times \frac{5}{8}''$	$=30.00\square''$
2—Ls	$4'' \times 4'' \times \frac{3}{8}''$	$=\ \ 5.72\square''$
2—Ls	$6'' \times 4'' \times \frac{9}{16}''$	$=10.62\square''$
		$61.52\square''$

The allowable unit stress for this member is practically the same as for *U2-U3* and the above section is as near the required section as we can obtain without changing the thickness of the bottom angles which would be an objectionable thing to do, as the center of gravity would be shifted, and hence the above section will be used.

Top Chord U4-U4. As the allowable unit stress for this member is practically the same as for *U3-U4* and the stress is practically the same the same section will be used as for *U3-U4*.

198. Designing of the Bottom Lateral System.—The load specified by the specifications for the bottom laterals is 700 lbs. per foot of span. This load, according to the specifications, is to be considered as a live load.

For the panel load we have $700 \times 25 = 17{,}500$ lbs., and for determining the stresses we have the following:

$$17{,}500 \div 9 = 1{,}944.44.$$
$$\sec \omega = 1.78.$$
$$1{,}944.44 \times 1.78 = 3{,}461.1.$$

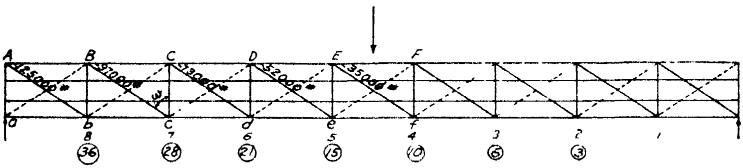


Fig. 325

Lateral Ab. For the maximum tensile stress in this member (see Fig. 325, also Art. 177) we have

$$36 \times 3{,}461.1 = 124{,}600, \text{ say } 125{,}000 \text{ lbs.}$$

For the area of cross-section required we have

$$125{,}000 \div 16{,}000 = 7.81 \text{ sq. ins.}$$

Use 2—Ls $6'' \times 3\frac{1}{2}'' \times \frac{1}{2}'' = 8.0\square''$ net.

Lateral Bc. For the maximum tensile stress in this member we have

$$28 \times 3{,}461.1 = 96{,}910, \text{ say } 97{,}000 \text{ lbs.}$$

For the area of cross-section required we have

$$97{,}000 \div 16{,}000 = 6.06 \text{ sq. ins.}$$

Use 2—Ls $6'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' = 6.09''$ net

Lateral Cd. For the maximum tensile stress in this member we have

$$21 \times 3{,}461.1 = 72{,}683, \text{ say } 73{,}000 \text{ lbs.}$$

For the area of cross-section required we have

$$73{,}000 \div 16{,}000 = 4.56 \text{ sq. ins.}$$

Use 2—∟s $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{7}{16}'' = 4.87\square''$ net.

Lateral De. For the maximum tensile stress in this member we have

$$15 \times 3{,}461.1 = 51{,}916, \text{ say } 52{,}000 \text{ lbs.}$$

For the area of cross-section required we have

$$52{,}000 \div 16{,}000 = 3.25 \text{ sq. ins.}$$

Use 2—∟s $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' = 4.21\square''$ net.

Lateral Ef. For the maximum tensile stress in this member we have

$$10 \times 3{,}461.1 = 34{,}611, \text{ say } 35{,}000 \text{ lbs.}$$

For the area of cross-section required we have

$$35{,}000 \div 16{,}000 = 2.18 \text{ sq. ins.}$$

Use 2—∟s $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' = 4.21\square''$ net.

This completes the designing of the bottom laterals as the structure is symmetrical about the center of span.

The stringer bracing is designed as previously explained for deck plate girder bridges and the transverse struts connecting to the stringers at the points of intersection of the laterals are designed as explained in Art. 177 for the 150-ft. riveted bridge.

199. Designing of Top Lateral System.—The load specified by the specifications for the top laterals is 200 lbs. per foot of span. This load, according to the specifications, is to be considered as a live load.

For the panel load we have $P = 25 \times 200 = 5{,}000$ lbs., and for determining the stresses we have the following:

$$5{,}000 \div 9 = 555.55.$$
$$\text{Sec}\,\omega 1 = 1.82.$$
$$\text{Sec}\,\omega 2 = 1.79.$$
$$\text{Sec}\,\omega 3 = 1.78.$$
$$\text{Sec}\,\omega \ \ = 1.78.$$

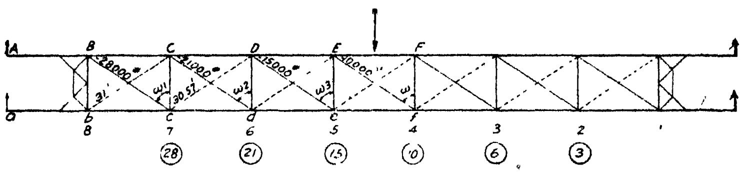


Fig. 326

Lateral Bc. For maximum tensile stress in this member (see Fig. 326) we have (see Art. 178)

$$\frac{P}{9} \sec\omega 1 \times 28 = 555.55 \times 1.82 \times 28 = 28{,}310, \text{ say. } 28{,}000 \text{ lbs.}$$

For the area of cross-section required we have

$$28{,}000 \div 16{,}000 = 1.75 \text{ sq. ins.}$$

Use 2—Ls $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' = 4.22\square''$ net. These are about the smallest angles used in railroad work and hence are used in this case.

Lateral Cd. For the maximum tensile stress in this member we have

$$\frac{P}{9} \sec\omega 2 \times 21 = 555.55 \times 1.79 \times 21 = 20{,}883, \text{ say } 21{,}000 \text{ lbs.}$$

For the area of cross-section required we have

$$21{,}000 \div 16{,}000 = 1.31 \text{ sq. ins.}$$

Use 2—Ls $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}'' = 4.22\square''$ net—same as for *Bc*.

Lateral De. For the maximum tensile stress in this member we have

$$\frac{P}{9} \sec\omega 3 \times 15 = 555.55 \times 1.78 \times 15 = 14{,}883, \text{ say } 15{,}000 \text{ lbs.}$$

Use the same section as used for *Bc*—2—Ls $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$.

Lateral Ef. For the maximum tensile stress in this member we have

$$\frac{P}{9} \sec\omega \times 10 = 555.55 \times 1.78 \times 10 = 9{,}888, \text{ say } 10{,}000 \text{ lbs.}$$

Use the same section as used for *Bc*—2—Ls $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$.

Strut Cc. For the maximum compression in this member we have (see Art. 178)

$$21 \times 555.55 + \left(\frac{5{,}000}{2}\right) = 14{,}166, \text{ say, } 14{,}000 \text{ lbs.}$$

The length of member (*c.c.* of chords) is 17 ft., or 204 ins. If the strut be composed of 4—Ls $3\frac{1}{2}'' \times 3'' \times \frac{3}{8}''$ the least radius of gyration will be about 1.7. Then $L/r = 204 \div 1.7 = 120$, which is just the maximum limit permitted by the specifications and hence the above angles will be used. It is seen that the stress in this strut is so low that it does not really influence the design, and, as the stress in the other struts—*Dd, Ee,* etc.—are less yet, there is really no need of computing it. We will simply make each of the other struts of 4—Ls $3\frac{1}{2}'' \times 3'' \times \frac{3}{8}''$ and let it go at that.

200. Design of Portal.—Assuming the ends of the end posts fixed and using the same symbols as used in Art. 179 (see Fig. 271) we have in this case

$$P = 5{,}000 \text{ lbs.},$$

$$R = \frac{5{,}000}{9} \times 28 = 15{,}555 \text{ lbs.},$$

$$H = 10{,}277 \text{ lbs.},$$

$$\pm V = (20{,}555 \times 24.16) \div 17 = 29{,}212, \text{ say } 29{,}000 \text{ lbs.}$$

$$\sec\phi = 1.42.$$

Taking moments about p (Fig. 327) we have

$$Sk = \left(R + \frac{P}{2}\right)k + H\frac{m}{2}$$

and substituting the numerical values given above and reducing we obtain

$$S = 18{,}055 + (10{,}277 \times 15{,}665) \div 8.5 = 36{,}994, \text{ say } 37{,}000 \text{ lbs.}$$

for the compressive stress in the part Bo of the portal.

In a similar manner taking moments about t we obtain

$$S1 = 2{,}500 + 18{,}939 = 21{,}439, \text{ say } 21{,}000 \text{ lbs.}$$

for the tensile stress in the part ob of the portal.

For the stress in the parts po and ot of the portal we have

$$S2 = V \sec\phi = 29{,}000 \times 1.42 = 41{,}180, \text{ say } 41{,}000 \text{ lbs.}$$

This is compression in one and tension in the other. With the applied forces acting as indicated in Fig. 327, op would be in tension and ot in compression.

The designing of the sections of the members of the portal in this case is mostly a matter of obtaining rigidity as the stresses are relatively low. For each of the members Bb, op and ot we will use 2—∟s 4″ x 4″ x 3/8″ and for each of the secondary members, the ones shown dotted in Fig. 327, we will use 2—∟s 3½″x3½″x3/8″. These sections are obtained in the manner shown in Art. 179.

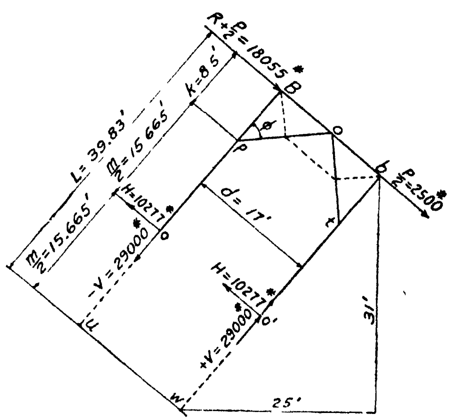


Fig. 327

201. Design of End Post.—As stated in Art. 180, collision struts are desirable, but they are sometimes omitted and for the sake of variety we will omit them in this bridge. The cross-section of the end posts will be the same in form as that of the top chords. The length of the member as regards the x-x axis (see Fig. 265) will be taken as the full length, which is 478 ins., and as regards the y-y axis the length will be taken as the distance from the lower end of the end post to the portal (distance pu, Fig. 327). This distance is about 376 ins.

Using the radius in reference to the x-x axis found for the chord section $U2$-$U3$ as preliminary we obtain

$$p = 16{,}000 - 70\,\frac{478}{9.3} = 12{,}403$$

for the approximate allowable unit stress, provided there were no cross bending on the member. Dividing this unit stress into the maximum stress in the member we obtain $782,000 \div 12,403 = 63.04\square''$. Now as the member is subjected to cross bending, due to wind, the section should very likely be larger than this. Let us assume the following section:

1—cov. pl. $27'' \times \frac{9}{16}''$	$= 15.18\square''$
2—web pls. $24'' \times \frac{11}{16}''$	$= 33.00\square''$
2—$\llcorner$s $4'' \times 4'' \times \frac{1}{2}''$	$= 7.50\square''$
2—$\llcorner$s $6'' \times 4'' \times \frac{5}{8}''$	$= 11.72\square''$
	$67.40\square''$

Taking moments about the cover plate of this section it is found that the distance from the cover plate to the gravity axis x-x is 10.14 ins.

The radius of gyration in reference to the x-x axis is 9.22. The moment of inertia in reference to the y-y axis is 5,712 and the radius in reference to the same axis is 9.23.

Then for the allowable unit stress, provided there were no cross bending we would have

$$16{,}000 - 70\frac{478}{9.22} = 12{,}371 \text{ lbs.}$$

But in case of direct and bending stresses combined this can be increased 25 per cent (see specifications). So we have

$$12{,}371 \times 1.25 = 15{,}464 \text{ lbs.}$$

for the allowable unit stress.

The maximum bending moment on the end post due to wind, considering the posts fixed at the ends, will occur at the bottom of the portal; that is, at points p and t (Fig. 327). For this moment we have

$$M = 10{,}277 \times 15.665 \times 12 = 1{,}931{,}870 \text{ inch-lbs.}$$

(see Fig. 327).

Then for the maximum bending stress we have

$$f = \frac{1{,}931{,}870}{5{,}742} \times 13.5 = 4{,}542 \text{ lbs. per sq. in.}$$

For the actual direct unit stress we have

$$782{,}000 \div 67.4 = 11{,}602 \text{ lbs.}$$

Now adding the bending stress to the direct stress we have

$$4{,}542 + 11{,}602 = 16{,}144 \text{ lbs.}$$

for the actual unit stress on the member due to direct stress and cross bending combined. As this stress is very near the allowed, being only 680 lbs. more $(16{,}144 - 15{,}464 = 680)$, the above assumed section will be used.

202. Maximum Reaction on Shoe.—The dead-load reaction on the shoe is the same as for the truss except for the half panel load at the end.

The panel load of dead load is 33,000 lbs. (see Art. 195). Then for the dead-load reaction on the shoe we have

$$4\tfrac{1}{2} \times 33,000 = 148,500 \text{ lbs.}$$

Placing wheel 2 at one end of the span, as shown in Fig. 278 (Art. 182) for the 150-ft. span, and taking moments about the other end we obtain 365,000 lbs. for the maximum live-load reaction on the shoe. Multiplying this by 300÷(225+300) we obtain 208,500 lbs. for the impact. Then adding together the above dead- and live-load reactions and impact we obtain

$$148,500 + 365,000 + 208,500 = 692,000 \text{ lbs.}$$

for the total maximum reaction on the shoe.

The stress sheet, Fig. 328, can now be completed and then the work of designing the details can proceed.

203. Details.—The general details of the entire span are shown in Figs. 329, 330 and 331. These details are drawn to a $\frac{3}{4}''$ scale on $\frac{3}{8}''$ layout; that is, the outline of the truss is drawn to a $\frac{3}{8}''$ scale and the details are drawn on this layout to a $\frac{3}{4}''$ scale.

Joint LO. The detail of this joint is shown in Fig. 329. In drawing this joint, first the joint line should be located, which is done by bisecting the angle between the end post and shoe. Then the distance from the cover plate of the end post to the center line of pin should be determined as explained in Art. 181. Then the outline of the lower end of the end post can be drawn and next the outline of the shoe can be sketched. The shoe should be deep enough to resist the cross bending on it, as explained in Art. 112, and it should be long enough to cover the rollers. The general dimensions of the shoe and number of rollers are first assumed and then modified, if necessary, to agree with the calculated requirements.

By drawing the end view and part cross-section of the joint, as shown, to the left of *LO*, about the desired length of the rollers is obtained. The size of the rails in the pedestal is selected, the rails are spaced and then a suitable diameter of roller can be determined. In this case, as is seen, 70-lb. (per yd.) rails are used. Allowing for one-eighth of an inch to be planed off the top of these rails, the width of bearing of each against a roller is $2\frac{1}{4}''$ (see Carnegie and the Illinois Steel Company catalogue).

Resolving the stress in the end post vertically (which can be done very rapidly by graphics) we obtain 612,000 lbs. (about). One-half of this, which is 306,000 lbs., should be supported by the rails to the left of the center line of the end post. Assuming 7″ rollers (as shown), the allowable bearing per inch of length of roller is $7 \times 600 = 4,200$ lbs. Then as there are five rails (not considering the 90-lb. guide rail at the center line) to the left of the center line of the end post and there being seven rollers, we have

$$(5 \times 2\tfrac{1}{4})\ 7 \times 4,200 = 330,700 \text{ lbs.}$$

for the allowable pressure on the part of the rollers to the left of the center line of the end post, which, as is seen, is about 24,000 lbs. more than necessary. The first five rails to the right of the center line of the end post will support the other half of the vertical component of the end post and the other two rails, directly under the end floor beam, should be about sufficient to support the maximum reaction of the beam. This reaction (see Fig. 328) is 143,500 lbs. For the allowable pressure on the two rails we have

$$(2 \times 2\tfrac{1}{4})7 \times 4{,}200 = 132{,}300 \text{ lbs.},$$

which is 11,200 lbs. less than required, but as the maximum stress in the end post and the maximum reaction on the end floor beam do not occur at the same time (and the other rails are understressed), the design of the roller pedestal, as shown, is satisfactory.

The shoe proper must be made to conform with the details of the end post. Assuming a 7″ pin, we obtain

$$782{,}000 \div (24{,}000 \times 7) = 4.65 \text{ ins.}$$

for the required thickness of pin bearing on the end post, or 2.32″ on each side. We have on each side of the post, as shown, a $\frac{9}{16}$″, $\frac{1}{2}$″ and $\frac{5}{8}$″ plate and an $\frac{11}{16}$″ web, making in all $2\frac{3}{8}$″ (2.375) of bearing, which is the thickness required.

For the required thickness of bearing on the shoe we have

$$612{,}000 \div (24{,}000 \times 7) = 3.64 \text{ ins.},$$

or 1.82″ at each side. We have at each side, as shown, 3—$\frac{5}{8}$″ plates, making $1\frac{7}{8}$″ (1.875) bearing which is about the required bearing. It will be seen that in addition to this bearing there is a $\frac{3}{8}$″ filler and a $\frac{7}{16}$″ jaw plate, but as these extend only a short distance below the pin, they will not be considered for bearing.

The bottom edges of the vertical bearing plates of the shoe are planed so that they bear firmly against the sole plate and hence the rivets passing through the plates simply hold them together. However, there should be as many rivets in the vertical legs of the angles connecting these plates to the sole plate as is possible to put in so that the pressure will be well distributed over the sole plate by the outstanding legs of the angles.

The pin plates on the lower end of the end post should be arranged so that they firmly grip the post. For the allowable bearing on the $\frac{9}{16}$″ inside plate against the 7″ pin we have

$$\tfrac{9}{16} \times 7 \times 24{,}000 = 94{,}500 \text{ lbs.}$$

Then the number of $\frac{7}{8}$″ shop rivets required in single shear to hold this plate is

$$94{,}500 \div 7{,}200 = 13.$$

As is seen, there are 21. For the allowable bearing on the $\frac{1}{2}$″ outside plate, we have

$$\tfrac{1}{2} \times 7 \times 24{,}000 = 84{,}000 \text{ lbs.}$$

Then for the number of $\frac{7}{8}$″ shop rivets required in single shear to hold this plate we have

$$84{,}000 \div 7{,}200 = 12.$$

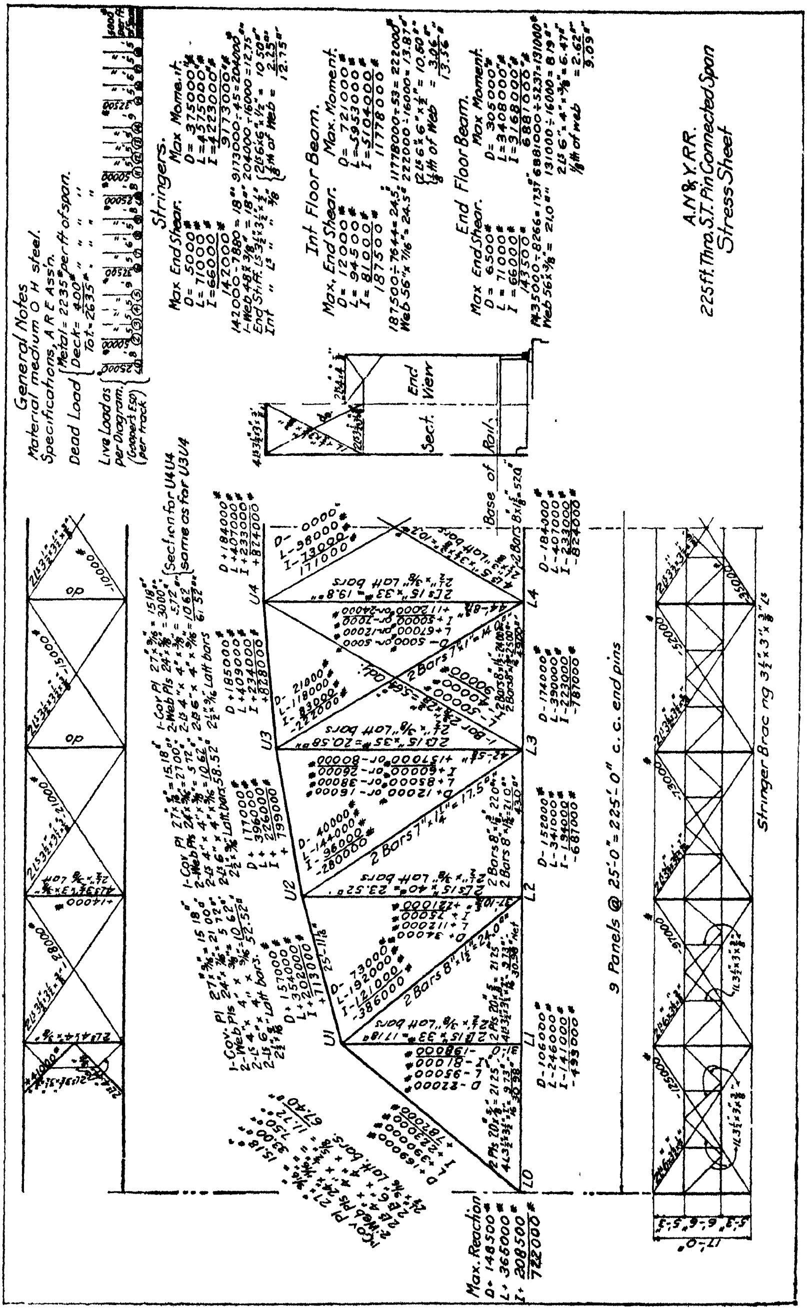

General Notes
Material medium O H steel.
Specifications, A R E Ass'n.
Dead Load
Live Load as per Diagram. (Cooper's E50) (per track)
Stringers.
Max End Shear.
Max Mome.t.
Int Floor Beam.
Max. Moment.
End Floor Beam.
Max Moment.
End View
Sect. of Rail
Base of Rail
A.N.& Y.R.R.
225 ft. Thro. S.T. Pin Connected Span
Stress Sheet
L0
L1
L2
L3
L4
U1
U2
U3
U4
Section for U4U4 same as for U3U4
9 Panels @ 25'-0" = 225'-0" c. c. end pins
Max. Reaction
D+ 148500#
L+ 365000#
I+ 208500#
722000#
Stringer Brac'ng 3½" x 3" x 3/8" L's

Fig. 328

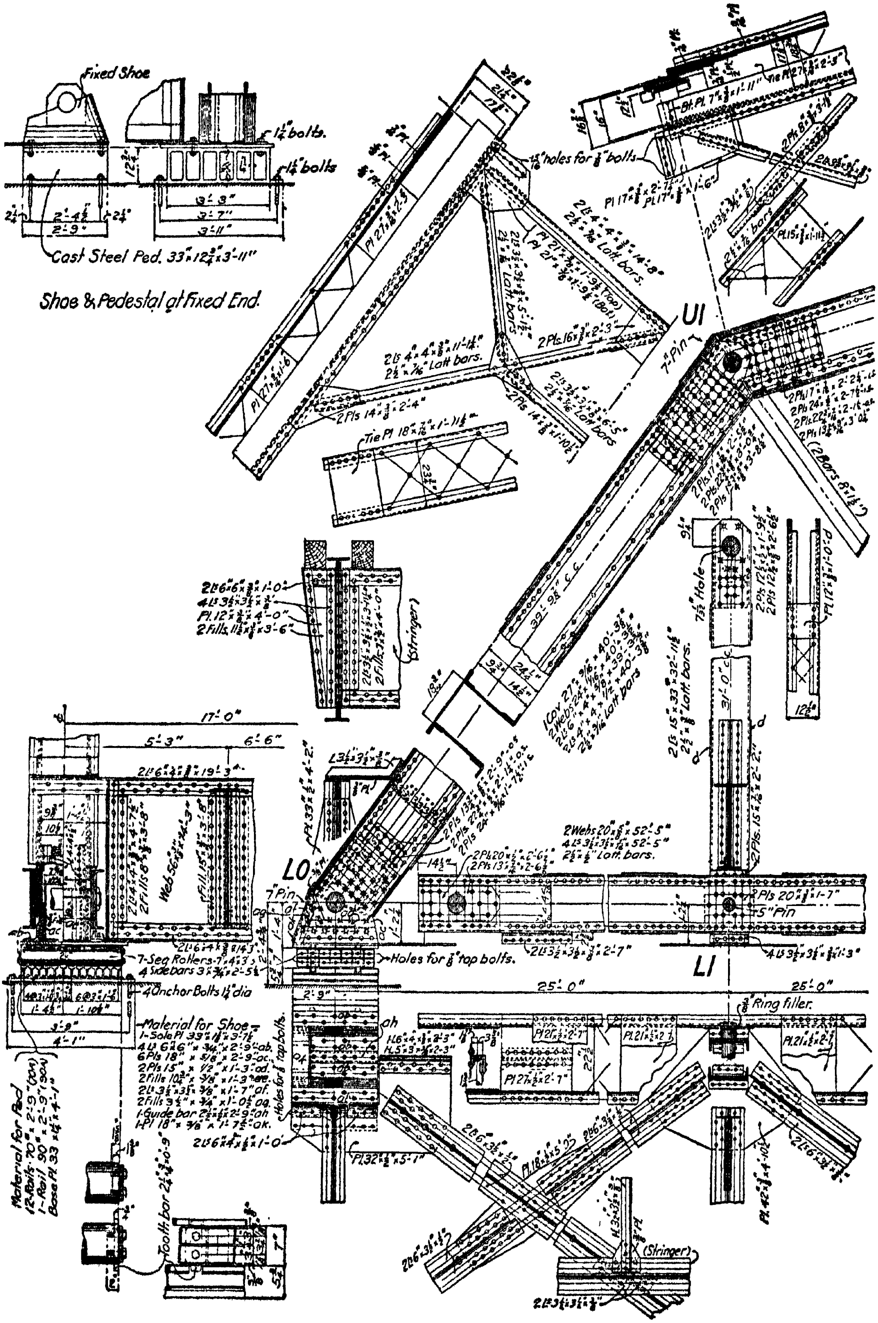

Fixed Shoe
1¼" bolts
Cast Steel Ped. 33"x12¾"x3'-11"
Shoe & Pedestal at Fixed End
holes for ⅞ bolts
Lat. bars
U1
7" Pin
Tie Pl
Stringer
Lott. bars
L0
7" Pin
5" Pin
L1
Holes for ⅞ tap bolts
Holes for ⅞ tap bolts
Ring filler
7-Seg. Rollers
4 Anchor Bolts 1¼" dia
Material for Shoe
Material for Ped.
Tooth bar
(Stringer)
17'-0"
5'-3"
6'-6"
25'-0"
25'-0"

Fig. 329

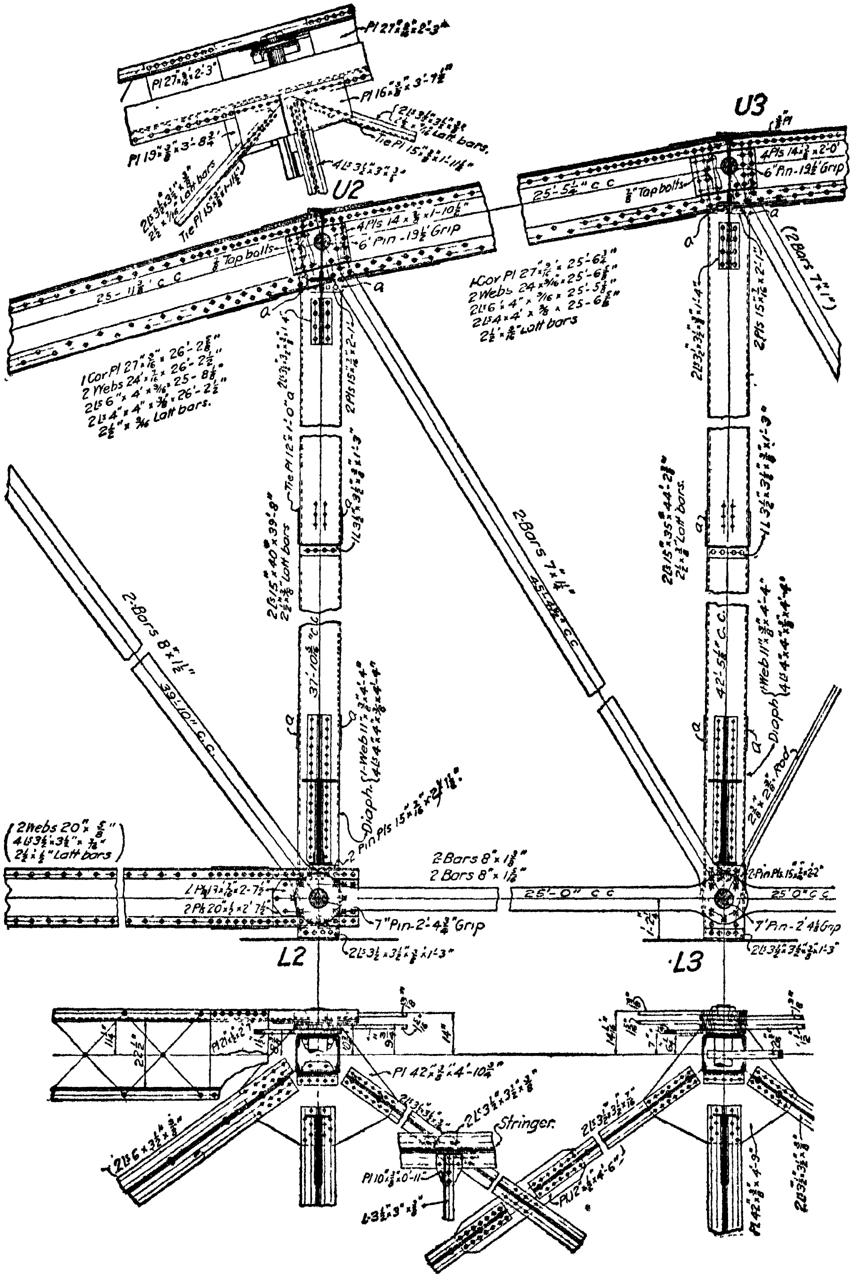


Fig. 380

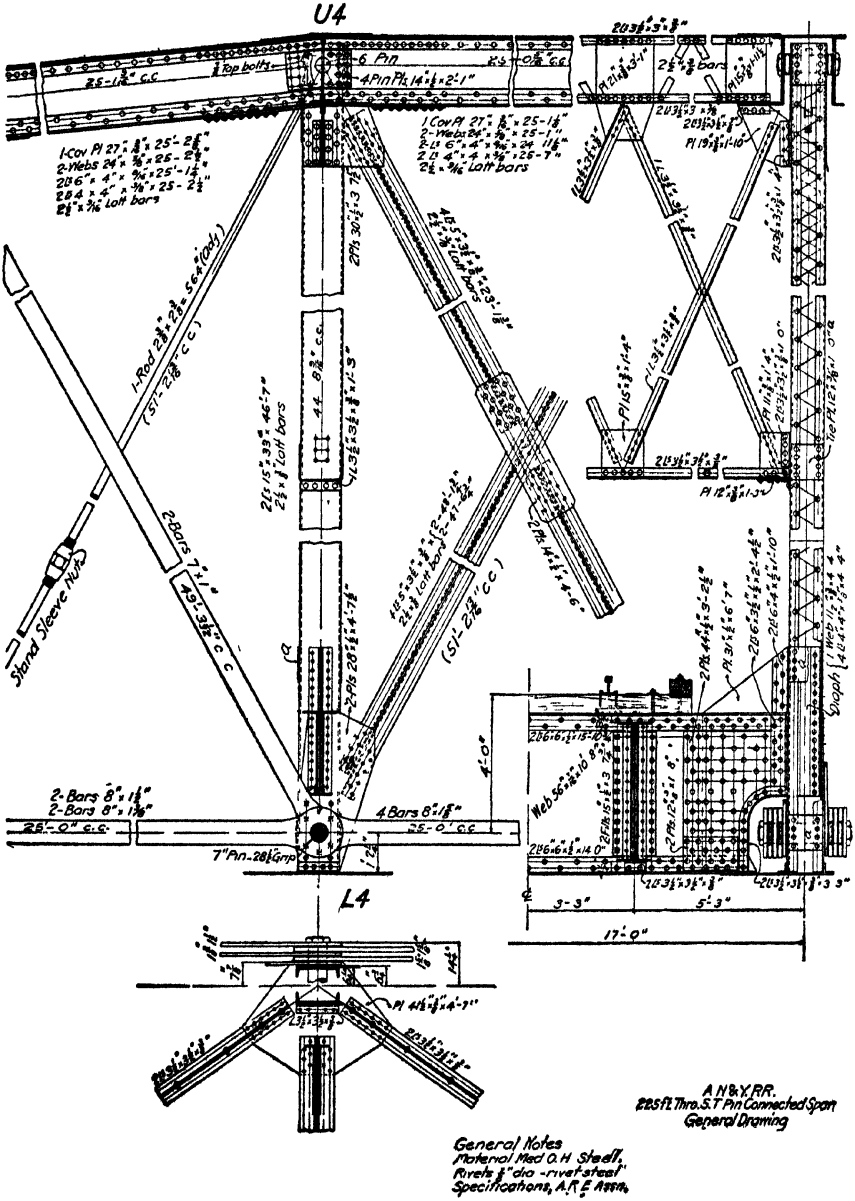

U4
L4
6 Pin
4 Pin Pl's 14"×½"×2'-1"
7" Pin-28½" Grip
2-Bars 8"×1½"
2-Bars 8"×1⅞"
25'-0" c.c.
4 Bars 8"×1⅝"
2-Bars 7"×1"
49'-3½" c c
1-Rod 2⅜"×2⅜"=56'4" (adj)
Stand Sleeve Nuts
4'-0"
3-3"
5'-3"
17'-0"
A N&Y RR.
225 ft. Thro. S.T Pin Connected Span
General Drawing
General Notes
Material Med O.H Steel.
Rivets ⅞" dia -rivet steel
Specifications, A.R.E. Assn.

Fig. 331

This plate should have a sufficient number of rivets connecting it directly to the top and bottom angles of the end post to transmit the total pin pressure on it to these angles. There are 14 passing through it and the angles, so the riveting is satisfactory, although counting the other rivets passing through the plate there is an excess of 15 rivets. For the allowable bearing on the $\frac{5}{8}''$ filler we have

$$\tfrac{5}{8} \times 7 \times 24{,}000 = 105{,}000 \text{ lbs.}$$

Then for the number of $\frac{7}{8}''$ shop rivets required in single shear to hold the filler we have

$$105{,}000 \div 7{,}200 = 15.$$

As is seen, there are 23 passing through the filler, which is an excess of 8 rivets. The $\frac{1}{2}''$ outside plate and the $\frac{5}{8}''$ filler acting together tend to shear the rivets off against the web and angles of the end post. The combined allowable bearing of these two plates against the 7″ pin is

$$84{,}000 + 105{,}000 = 189{,}000 \text{ lbs.}$$

Then for the number of $\frac{7}{8}''$ shop rivets required to hold these two plates when considered as acting together we have

$$189{,}000 \div 7{,}200 = 27.$$

As is seen, there are 37 — an excess of 10 rivets.

Although from the above there appears to be an excess of rivets in these pin plates, nevertheless, the rivets are spaced about as sparingly as possible, while yet obtaining good details, so the riveting is satisfactory.

According to the specifications, the net area of cross-section through the pin hole of the bottom chord (at LO) should be 25 per cent more than the net section of the member. The net section of the member (see Fig. 328) is 30.98□″. Then the area of section through the pin hole should be 38.72□″.

As is seen, the $\frac{7}{16}''$ plates have a net area through the pin hole of 5.25□″, the $\frac{1}{2}''$ plates have 13.00□″, and the web has 16.25□″, making a total of 34.51□″ for the plates. In addition, each of the angles can be considered to the extent of four rivets in shear. Each angle then is equivalent to $(4 \times 7{,}200) \div 16{,}000 = 1.8$□″, or 7.2□″ for the four angles. Adding this to the net area of the plates we obtain $34.51 + 7.2 = 41.71$□″, which is 2.99□″ more than required.

According to the specifications (A. R. E. Ass'n), the net area along the center line between the pin and the end of the member must be equal to the net area of cross-section of the member. As is seen, we have

$$2(\tfrac{1}{2} + \tfrac{7}{16} + \tfrac{5}{8})\,(14\tfrac{1}{2} - 3\tfrac{1}{2}) - (1 \times 3\tfrac{1}{8}) = 31.25 \text{ sq. ins.}$$

for the net area along the center line between the pin hole and the end of the member. As this is practically equal to the net area of cross-section of the member, which is 30.98□″, the detail of the end of the bottom chord, as shown, is satisfactory as far as sections are concerned.

Considering the net area of cross-section through the pin hole, the strength of the $\frac{7}{16}''$ outside plate in tension is $6 \times \frac{7}{16} \times 16{,}000 = 42{,}000$

lbs. Then for the number of $\frac{3}{4}''$ shop rivets required to the right of the pin hole to transmit this stress in single shear we have

$$42{,}000 \div 7{,}200 = 5.8, \text{ say } 6.$$

As is seen, 6 rivets are used.

The strength of the $\frac{1}{2}''$ inside plate in tension is $13 \times \frac{1}{2} \times 16{,}000 = 104{,}000$ lbs. Then for the number of $\frac{3}{4}''$ shop rivets required to the right of the pin hole to transmit this stress in single shear, we have

$$104{,}000 \div 7{,}200 = 15.$$

As is seen, 16 rivets are used. From the above, it is seen that the detail of the end of the bottom chord at LO is correct both as to section and riveting and hence the detail is satisfactory.

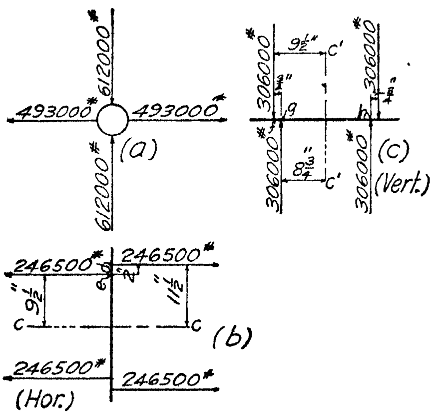


Fig. 332

Resolving the forces in the end post horizontally and vertically, we have the loading on the pin at LO shown at (a), Fig. 332. Considering first the bending moment on the pin due to the horizontal forces, shown at (b), the bending moment at d is zero and at c it is equal to $0 + 246{,}500 \times 2 = 193{,}000''\#$, which is the maximum horizontal moment, the forces being symmetrical in reference to the center line cc.

The maximum bending moment on the pin due to the vertical forces shown at (c), as is readily seen, occurs at both g and h and is equal to $306{,}000 \times \frac{3}{4} = 229{,}500''\#$. Then for the resultant maximum bending moment we have

$$M = \sqrt{493{,}000^2 + 229{,}500^2} = 543{,}000 \text{ inch lbs.}$$

Applying Formula (1), Art. 83, taking f as 25,000 lbs., we find that a $6\frac{1}{8}''$ pin is required for bending. For the maximum shear on this pin we have

$$306{,}000 \div \pi \left(\frac{6\frac{1}{8}}{2}\right)^2 = 10{,}385 \text{ lbs. per sq. in.}$$

So the $6\frac{1}{8}''$ is large enough as far as shear is concerned, as 12,000 lbs. per square inch is allowed. The bearing as computed above is for the $7''$ pin and to use a $6\frac{1}{8}''$ pin would necessitate increasing the bearing by about one-fourth, which would increase the weight of metal twice as much as would be saved on the pin by reducing it to $6\frac{1}{8}''$ diameter. So there is really no economy in using the smaller pin and therefore the $7''$ pin will be used.

The other details at LO are considered to be self-explanatory, especially after Arts. 184 and 186 are carefully read.

Joint U1. In drawing this joint the first thing to do is to locate the joint line by bisecting the angle between the end post and top chord and then the outlines of the members can be drawn and the portal located as explained in the case given in Art. 184; and next the required bearing on the pin can be calculated.

We will assume a 7″ pin—the size used at *LO;* then for the required thickness of pin bearing on the end post we have

$$782{,}000 \div (24{,}000 \times 7) = 4.65 \text{ ins.},$$

which is about $4\frac{11}{16}$″. As is seen, we have on each side of the post a $\frac{7}{16}$″ and a $\frac{5}{8}$″ plate and a $\frac{5}{8}$″ filler. This with the $\frac{11}{16}$″ web makes $2\frac{3}{8}$″ bearing on a side or $4\frac{3}{4}$″ for the member, which is practically the required bearing.

For the required thickness of pin bearing on the top chord we have

$$713{,}000 \div 168{,}000 = 4.24 \text{ ins.}$$

As is seen, we have on each side of the chord 1—$\frac{1}{2}$″ and 2—$\frac{7}{16}$″ plates and 1—$\frac{9}{16}$″ filler. This, with the $\frac{7}{16}$″ web, makes $2\frac{3}{8}$″ bearing on a side or $4\frac{3}{4}$″ for the member. This is about $\frac{1}{2}$″ more than required, but it is necessary to use this thickness in order to pack the joint.

At least one pin plate in each case should extend at least 6 ins. beyond the end of the tie plate.

The number of rivets required to connect the pin plates to the end post is determined in the same manner as shown above for joint *LO*. For the allowable pin pressure on the $\frac{7}{16}$″ plate we have

$$7 \times \tfrac{7}{16} \times 24{,}000 = 73{,}500 \text{ lbs.}$$

Then for the number of $\frac{7}{8}$″ shop rivets required to transmit this in single shear, we have

$$73{,}500 \div 7{,}200 = 11 \text{ (about)}.$$

It is obvious that most of the pin pressure on this $\frac{7}{16}$″ outside plate will be first transmitted to the $\frac{5}{8}$″ plate, then to the $\frac{5}{8}$″ filler, and on to the web and angles of the end post instead of being transmitted directly to the web and angles by the rivets, as the rivets are quite long and consequently will bend to some extent. Let us assume that the total pin pressure on the $\frac{7}{16}$″ plate is transmitted to the $\frac{5}{8}$″ plate. Then for the total pressure on the $\frac{5}{8}$″ plate we have

$$73{,}500 + 7 \times \tfrac{5}{8} \times 24{,}000 = 178{,}500 \text{ lbs.}$$

For the number of $\frac{7}{8}$″ shop rivets required to transmit this force, we have

$$178{,}500 \div 7{,}200 = 25 \text{ (about)}.$$

There should be about enough rivets connecting this $\frac{5}{8}$″ plate to the angles of the end post to transmit this 178,500 lbs. directly to the angles in order to distribute the pin pressure well over the cross-section of the member. As is seen, there are 22 rivets, which is about the correct number. For the allowable pin pressure (bearing) on the $\frac{5}{8}$″ filler we have

$$7 \times \tfrac{5}{8} \times 24{,}000 = 105{,}000 \text{ lbs.}$$

Then for the number of $\frac{7}{8}''$ shop rivets required to transmit this pressure in single shear, we have

$$105{,}000 \div 7{,}200 = 14.5, \text{ say } 15.$$

As is seen, there are 16 rivets between the lower end of the filler and the $\frac{7}{16}''$ plate, which we can consider as holding the filler.

As is seen from the above, if the rivets just below the pin be considered only to take the pressure on the $\frac{7}{16}''$ outside plate, the riveting of the pin plates to the end post at $U1$ is about correct.

The allowable pin pressure on the $\frac{7}{16}''$ inside pin plate of the top chord is $7 \times \frac{7}{16} \times 24{,}000 = 73{,}500$ lbs. It requires about 10—$\frac{7}{8}''$ shop rivets to transmit this in singular shear. As is evident, practically all of the 73,500 lbs. will be transmitted first to the $\frac{1}{2}''$ inside pin plate and from there on to the web, and hence there should be a sufficient number of rivets connecting the $\frac{1}{2}''$ plate to the web to transmit the combined pin pressure exerted on both the $\frac{7}{16}''$ and $\frac{1}{2}''$ plate.

For the allowable combined pin pressure of the two, we have

$$73{,}500 + (7 \times \tfrac{1}{2} \times 24{,}000) = 157{,}500 \text{ lbs.}$$

It requires about 22—$\frac{7}{8}''$ shop rivets to transmit this in single shear. As is seen, there are 38 passing through the $\frac{1}{2}''$ inside plate. But 10 of these should be considered as holding the $\frac{7}{16}''$ inside plate, thus really leaving 28, which is 6 more than required.

The allowable pin pressure on the $\frac{7}{16}''$ outside pin plate is 73,500 lbs. To transmit this in single shear requires about 10—$\frac{7}{8}''$ shop rivets. As this plate connects directly to the angles of the chord, there should be enough rivets connecting it to the angles to transmit the entire 73,500 lbs. As is seen, there are 14 rivets connecting the plate to the angles. So the riveting is satisfactory as far as the $\frac{7}{16}''$ outside plate is concerned. The allowable pin pressure on the $\frac{9}{16}''$ filler is

$$7 \times \tfrac{9}{16} \times 24{,}000 = 94{,}500 \text{ lbs.}$$

To transmit this in single shear requires about 13—$\frac{7}{8}''$ shop rivets. As is seen there are 26—twice as many as needed. As the web of the top chord is only $\frac{7}{16}''$ thick, it is a question as to whether the rivets connecting the pin plates to the member are not over-stressed in bearing on the web.

The $\frac{1}{2}''$ inside plate has a total pressure, as given above, of 157,500 lbs. There are $38-10$ (the 10 being considered to hold only the $\frac{7}{16}''$ inside plate) to transmit this, which stresses each rivet $157{,}500 \div 28 = 5{,}625$ lbs. The pin pressure on the $\frac{9}{16}''$ outside filler is 94,500, as given above, and this stresses each rivet $94{,}500 \div 26 = 3{,}634$ lbs. Adding these values we have $5{,}625 + 3{,}634 = 9{,}259$ lbs. for the maximum rivet bearing on the $\frac{7}{16}''$ web. For the maximum allowable bearing of a $\frac{7}{8}''$ shop rivet on $\frac{7}{16}''$ metal, we have

$$\tfrac{7}{16} \times \tfrac{7}{8} \times 24{,}000 = 9{,}180 \text{ lbs.},$$

which is about the bearing exerted. So, taking all in all, the riveting of the pin plates of the top chord at $U1$ is about correct.

The calculations for the details at the end of the hanger at $U1$ are made in the manner as shown above for the end of the bottom chord at LO.

The net section of the member is 17.18□″. Then the net section through the pin hole should be $17.18\,(1+0.25)=21.57$□″.

The net area of the two channels through the pin hole is $19.9-(7\times 0.4\times 2)=14.3$□″. The net area of the 2 — $\frac{1}{2}''$ plates through the pin hole is $(12.5\times\frac{1}{2}\times 2)-(7\times\frac{1}{2}\times 2)=5.5$□″ and of the $\frac{5}{8}''$ plates it is 6.87□″, making in all $14.3+5.5+6.87=26.67$□″, which is about 5□″ more than necessary.

The net section along the center line between the pin hole and the end of the hanger should be equal to the net section of the member, or 17.18□″, according to the specifications. We have $(3.05\times 9.25)-(3.05\times 3.5)=17.54$□″, which is about the section required, and as the distance shown as $9\frac{1}{4}''$ can not be increased, on account of clearance, the detail of the end of the hanger at $U1$ is satisfactory as far as the

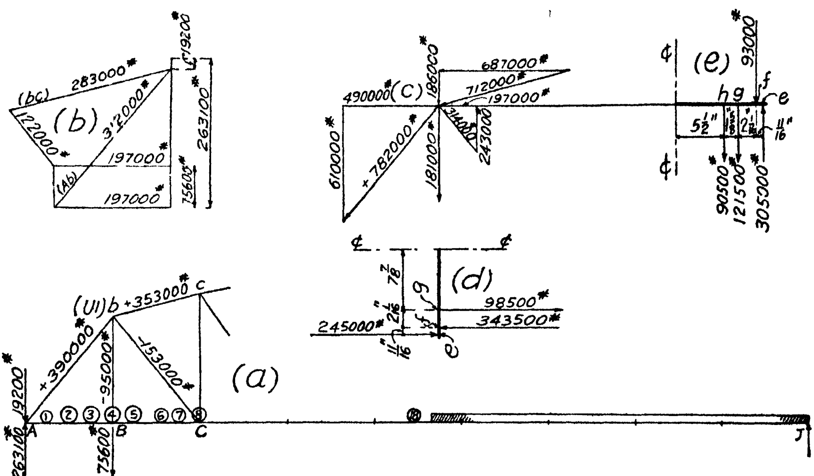


Fig. 333

section is concerned. The strength of the $\frac{1}{2}''$ plate in tension is $16{,}000\times 5.5=88{,}000$ lbs. To transmit this in single shear requires about 12 — $\frac{7}{8}''$ shop rivets, or 6 on a side, which should be below the pin. As seen, there are 8. The strength of the $\frac{5}{8}''$ plate is $16{,}000\times 6.87=109{,}900$ lbs. This requires about 15 — $\frac{7}{8}''$ shop rivets, or 7.5 on a side. As seen, there are 8 passing through this plate outside of the $\frac{1}{2}''$ plate. So, taking all in all, the detail of the end of the hanger, as shown at $U1$, is satisfactory.

In determining the bending moment on the pin at LO, there is no question concerning the live-load stress in the members as the maximum in the end post and bottom chord occurs simultaneously, but at $U1$ the case is different, for at that joint the maximum stress in the members does not occur simultaneously, and, consequently, it is necessary to ascertain the position of the live load for maximum bending moment on the pin at that joint. This can be done only by trial, as the bending on the pin due to any member not only depends upon the stress in the member, but upon its lever arm as well.

It is seen from Fig. 329 that the hanger and the diagonal (at $U1$) have the longest lever arms and hence most likely the pin will be subjected to the maximum moment when these members have about the maximum simultaneous stress. As this can be ascertained only by trial, let us place the loads as shown at (a), Fig. 333, in which case wheel 4 is at B. Taking moments about J (using Table A) we obtain the reaction 263,100 lbs. at A, and taking moments about B we obtain the concentration 19,200 lbs. at A. Further, taking moments about C and A we obtain the concentration 75,600 lbs. at B—all due to Cooper's $E40$ loading. Then beginning at joint A and drawing the stress diagram shown at (b) we obtain the stresses in all the members at joint b ($U1$) due to Cooper's $E40$ loading and multiplying these by $\frac{50}{40}$ we obtain the live-load stresses shown at (a), which are due to the $E50$ loading. Next, multiplying each of these stresses given at (a) (except the stress in the hanger) by $300 \div (225 + 300)$ we obtain the impact stress in each member. Then adding together these live-load stresses and the impact and the dead-load stresses (given in Fig. 328), we obtain the stresses shown at (c).

There is some question as to how much impact should be added to the hanger. Should L in the impact formula be taken as 50 or 225? As is seen, the impact really added is just sufficient to balance up the vertical components on the pin. This is a fair average for the impact and seems to be a rational value.

The stresses given at (c) are resolved graphically into horizontal and vertical components as shown. The horizontal components for half of the pin are shown at (d) and the vertical components for half of the pin are shown at (e).

Proceeding as outlined in Art. 83, for the bending moment on the pin due to the horizontal components, we have

$$M_e = 0$$
$$M_f = 0 + 245{,}000 \times \tfrac{11}{16} = +168{,}400 \text{ inch lbs.}$$
$$M_g = +168{,}400 + (-98{,}500 \times 2\tfrac{1}{16}) = -34{,}700 \text{ inch lbs.}$$

Similarly, for the bending moment, due to the vertical components, we have

$$M_e = 0$$
$$M_f = 0 + 305{,}000 \times \tfrac{11}{16} = +209{,}700 \text{ inch lbs.}$$
$$M_g = 209{,}700 + (212{,}000 \times 2\tfrac{1}{16}) = +646{,}900 \text{ inch lbs.}$$
$$M_h = 646{,}900 + (90{,}500 \times 1\tfrac{5}{8}) = +793{,}900 \text{ inch lbs.}$$

As the members are symmetrically arranged in reference to the center line of the chord, the moment at g, due to the horizontal forces, is constant between the two diagonals and hence for the resultant maximum moment at h, we have

$$M = \sqrt{\overline{34{,}700}^2 + \overline{793{,}900}^2} = 797{,}200 \text{ inch lbs.}$$

This, as is readily seen, is the maximum resultant bending moment on the pin when the live load is in the position shown at (a), Fig. 333.

In fact, it is about the maximum moment that can occur on the pin. Applying (1), Art. 83, taking f as 25,000 lbs., we find that the above moment (797,200″#) requires a $6\frac{7}{8}$″ pin and hence the 7″ pin assumed is about the correct size as far as the bending moment is concerned.

In case of doubt, in any case, as to the position of the live load, the loading can be placed in different positions and the moment on the pin calculated for each position in the same manner as shown above for that one position. This method of procedure, as is evident, is tedious. As a rule, the position of the live load can be ascertained near enough (as the impact is questionable) by mere *inspection*.

The maximum shear on the pin at each side of the chord is about 330,000 lbs., which is the resultant of the stresses in the hanger and diagonal. Then for the maximum shearing stress on the 7″ pin we have $330,000 \div 38.4 = 8,590$ lbs. per sq. in.

From this it is seen that the 7″ pin is amply large as far as shear is concerned, 12,000 lbs. being permissible, and as it is the correct size for bending it will be used.

The details of the portal and laterals at $U1$ are considered to be self-explanatory. The number of rivets required in each connection is obtained by developing the section of the member connected, as explained in Art. 184.

Joint L1. All of the details at this joint are practically self-explanatory. The pin supports only the weight of the bottom chord. The pin plates on the bottom chord are intended solely to replace the metal cut from the chord by the pin hole. The rivets connecting the bottom of the hanger to the lateral plate should be sufficient to transmit the component of the lateral in panel LO-$L1$ along the bottom chord. This component is about 105,000 lbs. This requires

$$105,000 \div 6,000 = 17\text{—}\tfrac{7}{8}\text{″ field rivets.}$$

16 are used.

The component of the stress in the same lateral along the floor beam is about 70,000 lbs. This requires about 12—$\frac{7}{8}$″ field rivets. 12 are used.

The pin plates on the hanger are for the purpose of reinforcing the hanger for bending. The bending on the hanger about the pin (at $L1$), which is due to the longitudinal component of the stress in the bottom lateral, is $105,000 \times 14.25 = 1,496,250$″#. The moment of inertia of the two channels is 625.2, and the moment of inertia of the 2 — $\frac{7}{16}$″ plates is 246.1, making a total moment of inertia of 871.3. Then for the maximum fiber stress in the hanger at $L1$, due to bending, we have

$$f = (1,496,250 \times 7) \div 871.3 = 12,020 \text{ lbs. per sq. in.}$$

From this it is seen that the hanger is amply strong to resist the bending at $L1$.

The rivets connecting the lug angles to the bottom of the hanger should be sufficient to transmit the longitudinal component of the stress in the lateral. These rivets are in double shear and bearing on about $\frac{13}{16}$″ of metal. For the number of $\frac{7}{8}$″ shop rivets required in double shear, we have

$$105,000 \div (7,200 \times 2) = 7.3, \text{say } 8.$$

For the number required in bearing on the $\frac{13}{16}''$ metal, we have

$$105{,}000 \div 17{,}060 = 6.$$

As is seen, 8 are used—the number required for shear.

Joint L2. The details of this joint are shown in Fig. 330. The calculations for the end of the bottom chord are just the same as given above for the other end of that member (at *LO*).

The pin bearing on the post should be sufficient to transmit the vertical component of the maximum stress in the diagonal *U*1-*L*2. This component is equal to about 300,000#. Then for the required bearing on the post, assuming a 7″ pin, we have

$$300{,}000 \div (7 \times 24{,}000) = 1.78 \text{ ins.}$$

or 0.89″ on a side. The thickness of the web of the 40# channel is 0.52″ (about $\frac{17}{32}''$). This with the $\frac{7}{16}''$ pin plate makes $\frac{31}{32}''$ or 0.96″, which is about $\frac{3}{32}$ more than needed, but as counter-sinking (as a rule) is not permitted in metal less than $\frac{7}{16}''$ thick the $\frac{7}{16}''$ plates shown will be used.

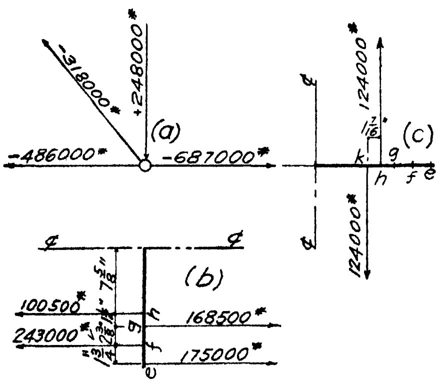


Fig. 334

About the maximum bending moment on the pin will occur when the bottom chord *L*2-*L*3 has maximum stress. (This can be determined in the manner shown above for joint *U*1.) The wheels are in the position shown in Fig. 322 when this stress occurs. Taking moments about *J* (Fig. 322) with wheel 7 at *C* we can obtain the reaction *R* at *A* and taking moments about *B* we can obtain the concentration at *A*. Then the stress in the bottom chord *AB* can be obtained quickly by analyzing graphically the joint *A*. Then the stress in chord *BC* is known, as it is equal to the stress in chord *AB*. Next the stress in the diagonal *bC* (*U*1-*L*2) is readily determined, as the horizontal component of its stress is equal to the difference between the stress in chord *BC* and *CD* and as the vertical component of the stress in the diagonal is equal to the stress in the post, we can readily determine the stress in the post and thus we would have all the live-load stresses in the members at *L*2 determined.

After determining these live-load stresses, the impact is determined in each member by multiplying the live-load stress in it by $300 \div (225 + 300)$. Then by adding together the live-load stress and impact and the dead-load stress, we obtain the total stress in each member as given at (a), Fig. 334. Two-thirds of a panel load of dead load is added to the post to balance up the vertical component of the diagonal. However, this should really be added to the post as the pin is below the floor beam connection and hence most of the weight at the joint is transmitted to the lower end of the post.

The horizontal components of the stresses on one-half of the pin are shown at (b) (Fig. 334), and the vertical components on one-half of the pin are shown at (c).

For the bending moment on the pin, due to the horizontal components, we have

$$M_e = 0$$
$$M_f = 0 + (175{,}000 \times 1\tfrac{3}{4}) = 306{,}250 \text{ inch lbs.}$$
$$M_g = +306{,}250 + (-68{,}000 \times 2\tfrac{3}{8}) = +144{,}750 \text{ inch lbs.}$$
$$M_h = +144{,}750 + (100{,}500 \times 1\tfrac{1}{2}) = +295{,}500 \text{ inch lbs.}$$

For the bending moment on the pin, due to the vertical component, we have

$$M_h - 0$$
$$M_k = 0 + (124{,}000 \times 1\tfrac{7}{16}) = 178{,}200 \text{ inch lbs.}$$

Then for the maximum resultant moment on the pin, we have

$$M = \sqrt{295{,}500^2 + 178{,}200^2} = 345{,}000 \text{ inch lbs.}$$

Then by applying (1), Art. 83, taking f as 25,000, we find that the above moment (345,000″#) calls for a $5\frac{1}{4}''$ pin (about).

The maximum shear on the pin is about 175,000 lbs., which requires that the pin be only about $4\frac{5}{16}''$ diameter.

From the above it is seen that the assumed 7″ pin is larger than necessary, but the 7″ will be used, for little would be saved by reducing the size, as the thickness of pin bearing on the bottom chord (*L2-LO*) and also on the post *U2-L2* would have to be increased if a smaller pin were used.

The other details at *L2* are readily understood. The calculation of the details of the bottom laterals, shown at this joint, is mostly a matter of developing the members. The rivets connecting the bottom of the post to the lateral plate should be sufficient to transmit the longitudinal component of the stress in the lateral in panel *L1-L2*.

Joint U2. The top chord has a butt joint at this point; that is, the ends of the members are planed so that they bear tightly against each other and hence the stress is transmitted from one chord member to the other chord member without passing through the pin. The splice plates on the chord are intended only to hold the chords in line. However, there must be sufficient pin bearing on the chord to take the component (along the chord *U2-U3*) of the stress in diagonal *U2-L3*. The maximum stress in the diagonal (see Fig. 328) is 280,000 lbs. Resolving this along the chord *U2-U3* (which can be done graphically) we obtain a component of 159,000 lbs. To transmit this component, assuming a 6″ pin, requires $159{,}000 \div (24{,}000 \times 6) = 1.1''$ thickness of bearing, or about $\frac{1}{2}''$ on each side of the chord. As is seen, there are $2 - \frac{3}{8}''$ plates and a $\frac{9}{16}''$ web, making in all $1\frac{5}{16}''$ bearing on each side of the chord, which is quite excessive.

The pin bearing on the post should be sufficient to transmit the maximum stress in the post, which is 221,000 lbs. (See Fig. 328.) This requires a bearing (assuming a 6″ pin) of $221{,}000 \div (24{,}000 \times 6)$

$=1.54''$ or $0.72''$ on a side. The web of the 40# channel is $\frac{17}{32}''$ thick. This with the $\frac{7}{16}''$ plate makes $\frac{31}{32}''$ thickness of bearing on each side of the post, which is excessive.

The maximum bending moment on the pin will occur when the diagonal (*U2-L3*) has maximum stress, as the post (*U2-L2*) has about the maximum stress at the same time. By placing the live load for maximum stress in the diagonal (as shown in Fig. 315) and determining the reaction at *A* and the concentration at *C* (these are given on pages 432 and 433) the live-load stress in each of the members at *U2* can be readily determined by graphics. Then determining the impact by multiplying each stress by $300 \div (150 + 300)$ and adding together this impact and the live- and dead-load stresses and resolving the resulting stresses horizontally and vertically, the maximum bending moment on the pin can be determined in the same manner as shown above for the other joints. This maximum moment is about 190,000 inch lbs. This requires about a $4\frac{1}{4}''$ pin. From the above, it is seen that the $6''$ pin is larger than theoretically required. But it is usual practice to limit the minimum diameter of a pin to $\frac{8}{10}$ of the width of the smallest eye-bar connected. The eye-bars at *U2* (as seen) are $7''$ wide. So, according to this requirement, the pin should be $5.6''$ in diameter and hence the assumed $6''$ pin is about the correct size, and will be used. The other details at *U2* are readily understood.

The details at *U3* are similar to those at *U2* and the details at *L3* are very similar to those at *L2* and hence the calculations of the details at these joints are quite similar to those given above and consequently are omitted. The same can be said regarding joints *U1* and *L4* (Fig. 331). The details at these joints are readily understood.

The calculations of the details of the intermediate floor beams, shown in Fig. 331, are almost exactly as given in Art. 186 (pages 410 to 412) for the floor beams in the 150-ft. riveted span.

After the general drawings (Figs. 329, 330 and 331) are completed, the shop drawings and shop bills for the structure can be made by proceeding in the same manner as outlined in Art. 186 for the 150-ft. span.

204. Camber.—The camber of the trusses in this case is obtained by lengthening the top chord $\frac{1}{8}''$ for each 10 feet of horizontal length, as explained in Art. 185. The corresponding lengths of the diagonals are calculated by using the mean lengths of the top and bottom chords in each case. For example, the horizontal uncambered length of top chord *U2-U3* (Fig. 330) is 25 ft. So the cambered horizontal length would be $25'-0\frac{5}{16}''$. Taking this as the base of a right angle triangle, and the difference between the lengths of posts *U3-L3* and *U2-L2* as the altitude, the cambered length of the top chord *U2-U3* (given as $25'-5\frac{1}{4}''$) is computed as the hypotenuse of the triangle. Then taking $25'-0\frac{5}{32}''$ as the base of a right angle triangle and the length of post *U2-L2* as the altitude, the length of diagonal *U2-L3* (given as $45'-4\frac{1}{2}''$) is calculated as the hypotenuse of that triangle. The length of the end post is not increased.

The determination of actual amount of camber of bridge trusses and also the determination of the theoretical camber of the same will be given later.

205. Graphical Determination of Live-Load Stresses in Curved Chord Pratt Trusses.—The influence line method, outlined in Art. 103, is the most convenient graphical method to use in this case, as the stresses in the truss members are thus determined directly without considering either shears or moments.

Chord Members. Let it be required to determine the live-load stress in the top chord *cd* of the truss shown in Fig. 335, due to Cooper's $E40$ loading.

It is obvious that a single load moving from J to the left over the span will cause a stress in chord *cd* which will be zero when the load is at J and

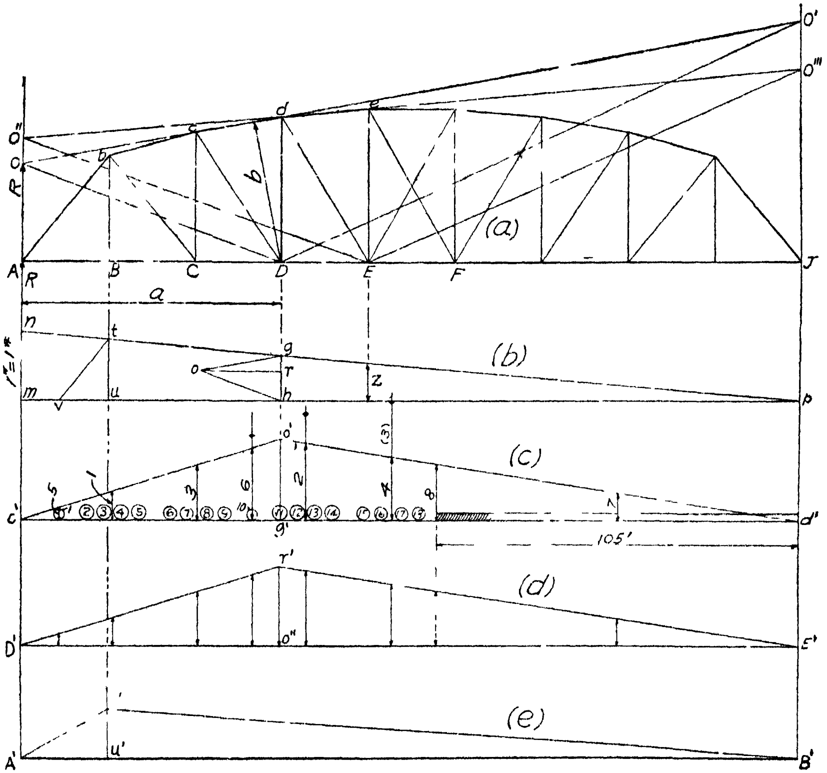


Fig. 335

increase constantly until the load reaches point D and that this stress will then decrease constantly as the load moves on to the left from D, becoming zero when the load reaches A. Then evidently the influence line for the stress in chord *cd* is of the form $c'o'd'$, shown at (c). Now, knowing the form of the influence line, the next thing is to construct it. This, as is readily seen, is a simple matter after the ordinate $o'g'$ is known. This ordinate $o'g'$, as we know, is the stress in the top chord *cd*, due to a unit load at D. Then, of course, the first thing to do in constructing the desired influence line is to determine the stress in the top chord *cd*, due to a unit load at D. This stress can be determined very readily either analytically or graphically. The graphical determination is the more convenient and hence that method will be used. By drawing, at (a), the lines OO', OD and $O'D$ we obtain, as we may say, the truss ODO', which will have end reactions exactly equal to those of the bridge truss, due to a unit load at D, and as the member OO' coincides with top chord cd, the stress, due to this unit load at D, will be the same in the two. This is readily seen, for, taking moments about D, we obtain

$$S = R\frac{a}{b}$$

for the stress in each, where R represents the reaction at either A or O due to a unit load at D.

By drawing mp (at (b)) and laying off nm (equal to one inch) equal to one pound and drawing np, we have the influence line for the reactions at A. Then gh is equal to the reaction at A and also at O, due to a unit load at D.

Then by drawing from h a line parallel to OD and from g a line parallel to OO', we obtain the line og, which is equal (in inches) to the stress in chord cd, due to the unit load at D. In other words, by constructing a diagram of the forces at O, we obtain the stress in OO', which is equal to the stress in the chord cd, due to a unit load at D.

Then by drawing $c'd'$ (at (c)) and laying off $o'g' = og$ and drawing $c'o'$ and $o'd'$, we have the desired influence line for the stress in top chord cd.

Then placing the loading for the maximum moment about D, according to Art. 91 (see Fig. 323), and drawing the ordinates 1, 2, ... 7, as shown, and multiplying each by the load or group of loads at each, and adding together the results, the desired maximum stress in top chord cd will be obtained.

Ordinates 1 and 2 are drawn at the center of gravity of equal groups. Then by adding together the ordinates 1 and 2 and multiplying by the weight of one of the groups, the stress in top chord cd, due to the two groups, will be obtained. Similarly, by adding together the ordinates 3 and 4 and multiplying by the weight of one of the groups (at either of the ordinates), the stress in the top chord cd, due to those two groups, will be obtained. Next, adding together ordinates 5 and 6 and multiplying by the weight of one of the wheels, the stress in top chord cd, due to the two wheels (1 and 10), will be obtained. Multiplying the ordinate 7 (which is equal to one-half of ordinate 8) by the total uniform load the stress in chord cd, due to the uniform load, is obtained. Then adding together all of these results, the total maximum live-load stress in top chord cd is obtained.

The combined length of ordinates 1 and 2 (which can be quickly combined by the use of a pair of dividers, as shown in Example 1 of Art. 100) scales 1.54 ins. This means that if (as $nm = 1'' = 1\#$) one unit load (1 lb.) be placed on the truss exactly over ordinate 1 and another exactly over ordinate 2, these two unit loads would produce a stress of 1.54 lbs. in top chord cd. Then evidently the two groups of wheels at these ordinates, each weighing 80,000 lbs., will produce a stress in the top chord *cd* of

$$1.54 \times 80{,}000 = 123{,}000 \text{ lbs.}$$

The combined length of ordinates 3 and 4 scales 1.73 ins. Then for the stress in top chord *cd* due to the two groups of wheels at these ordinates, we have

$$1.74 \times 52{,}000 = 90{,}480, \text{ say } 90{,}500 \text{ lbs.}$$

Similarly for the stress due to wheels 1 and 10, the combined length of ordinates 5 and 6 being 1.23 ins., we have

$$1.23 \times 10{,}000 = 12{,}300 \text{ lbs.}$$

Ordinate 7 scales 0.41 ins. Then for the stress in top chord *cd*, due to the uniform load, we have

$$0.41 \times (2{,}000 \times 105) = 86{,}100 \text{ lbs.}$$

Now, adding together the above, we obtain

$$123{,}000 + 90{,}500 + 12{,}300 + 86{,}100 = 311{,}900, \text{ say, } 312{,}000 \text{ lbs.}$$

for the maximum stress in top chord *cd* due to Cooper's $E40$ loading, and multiplying this by $\frac{50}{40}$ we obtain 390,000 lbs. for the stress due to the $E50$ loading. As is seen from Fig. 328, this is 6,000 lbs. less than the actual stress, but as this difference is less than 2 per cent, the result is accurate enough. That is, the change of section of the top chord resulting from the error would be negligible.

As the horizontal component of the stress in top chord *cd* is equal to the stress in bottom chord *DE*, as previously shown, and the position of the loading for maximum stress in the two members being the same, the influence line for the stress in bottom chord *DE* can be quickly constructed after the one for the top chord *cd* is completed. Thus: By drawing *or* we obtain the stress in bottom chord *DE*, due to a unit load at *D*. Then drawing $D'E'$ (at (d)) and laying off $r'o'' = or$ and drawing $D'r'$ and $r'E'$, we obtain the influence line for the stress in chord *DE*. The stresses can then be determined in this member in the same manner as shown above for top chord *cd*. However, the stress in bottom chord *DE* can be obtained more readily as the horizontal component of the stress in top chord *cd*. The method, of course, can be reversed; that is, the influence line for the stress in bottom chord *DE* could be constructed and the stress in *DE* determined and then the stress in top chord *cd* could be obtained by multiplying the stress found in *DE* by the secant of the slope angle of chord *cd*.

The construction of the influence lines and determination of the stresses in the other chord members are practically the same as shown above for chords *cd* and *DE*.

For example, by drawing $O''O'''$, $O''E$ and EO''' and constructing a diagram at ordinate *z* of the forces at O'', the stress in top chord *de*, due to a unit load at *E*, is obtained. Then the influence line for the stress in top chord *de*, as well as for bottom chord *EF*, is readily constructed.

As another example, by drawing *tv* (at (b)) parallel to the end post, we obtain vu, which is equal to the stress in bottom chord AB (also BC), due to a unit load at B. Then drawing $A'B'$ (at (c)) and laying off $v'u'=vu$ (which can be done by using a pair of dividers) and drawing $A'v'$ *and* $v'B'$, we obtain the influence line for the stress in bottom chords AB and BC, shown at (e).

Web Members. As an example, let it be required to determine the stress in diagonal cD. The influence line for the stress in this member, as explained in Art. 103, will be of the form $A's'o'B'$, shown at (c) in Fig. 336. To construct this influence line, first draw the influence lines for reactions, as shown at (b). Then gh represents the reaction at A, due to a unit load at D. Next, draw OO', OD and $O'D$. Then, drawing og and oh (at (b)) parallel, respectively, to OO' and OD, and ko parallel to diagonal cD, we have the stress in the diagonal cD, due to a unit load at D, represented by this line ko. Then, by drawing $A'B'$ (at (c)) and laying off $k'o'$ equal to ko, the part $o'B'$ of the influence line can be drawn.

Next, draw OI and IDO', as shown at (a). Then, drawing vs and su parallel, respectively, to IO' and OO' and ts parallel to the diagonal cD, we have the stress in the diagonal cD, due to a unit load at C, represented by this line ts. Then, by laying off $t's'$ (at (c)) equal to ts, the parts $s'A'$ and $s'o'$ of the influence line can be drawn, which will complete the construction of the influence line $A's'o'B'$.

Any load at any point to the right of N will produce tension in diagonal cD, and any load to the left will produce compression in the same. Then, evidently, to obtain the maximum tension in the diagonal, all the loads should be placed to the right of N. According to Art. 190, a wheel will be at D when the maximum tension in cD occurs. Then, evidently, the loading will be in the position for maximum stress in diagonal cD if the wheel be placed at k' that brings wheel 1 the closest to N, the limit being that wheel 1 should not be to the left of N. From this it is seen that the position of the loading for maximum stress in the diagonal can be determined directly from the influence line instead of applying the criterion of Art. 190.

The position of the loading for maximum tension in diagonal cD is shown at (c). By multiplying the ordinates by the loads, as explained above, the maximum tension in diagonal cD will be obtained.

In any case when a group of wheels comes at a point where the influence line changes direction, as wheels 2 . . . 5 do in this case, the ordinate at the center of gravity of the group can not be used. In such cases the weight of each wheel can be multiplied by the ordinate at it, or the ordinates at the center of gravity of the wheels to either side of the change of slope of the influence line can be used, or the sum of all of the ordinates can be obtained by the use of dividers (see Example 1, Art. 100), and this sum multiplied by the weight of one wheel. Thus, in the case shown at (c) the tensile stress in diagonal cD, due to wheels 2 . . . 5, can be obtained by multiplying ordinate 1 by the weight of wheel 2, and ordinate 3 by the weight of wheels 3, 4 and 5, or by multiplying the sum of all the ordinates 1, 2, 3 and 4 by the weight of one wheel.

The maximum compression in diagonal cD can be obtained by reversing the loading; that is, bringing it on from the left, and placing the wheel at t' that brings wheel 1 closest to N without passing that point.

The actual stress is then obtained by multiplying the loads by the ordinates, as previously explained.

The influence line for the maximum compression in post dD is shown at (d). To obtain this influence line OO', OD, $O'D$, ODI' and $O'I'$ should be drawn. Then, by drawing fy parallel to OO', yl parallel to OI', and yz parallel to post dD, we have the stress in post dD, due to a unit load at E, represented by yz. Then, drawing $C'D'$ at (d), and laying off $y'z'$ equal to yz, we can draw the part $y'D'$ of the influence line. Next, drawing qw parallel to OO', wh parallel to $O'D$, and ew parallel to post dD, we have the stress in post dD, due to a unit load at D. represented by line ew. Then, laying off $e'w'$, at (d), equal to ew, we can complete the influence line $C'w'y'D'$ by drawing $C'w'$ and $w'y'$.

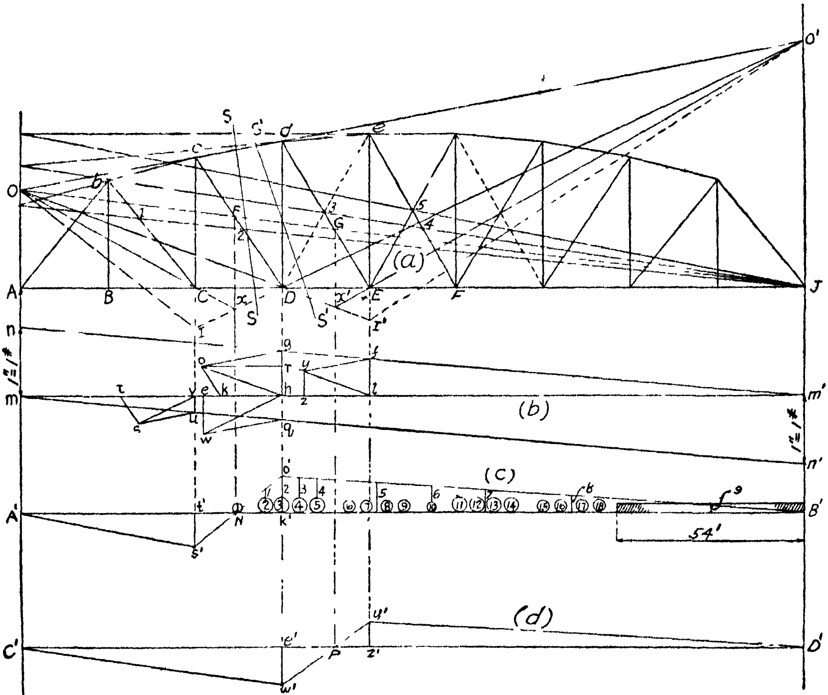


Fig. 336

The maximum compression in post dD can then be determined by placing the loading on PD' with the wheel at z' that brings wheel 1 the closest to P, and proceeding as explained above in the case of the diagonal dD. Influence lines for maximum tension in the posts will be considered later.

The influence lines for maximum stress in the other diagonals and posts and determination of the stresses are similar to the above.

The influence lines can be constructed more readily in the following manner, taking first the one shown at (c) in Fig. 336:

The ordinate $o'k'$ can be obtained in the manner explained above, and point N can be located by drawing OCx and $O'Dx$, as x is directly over N. Having the ordinate $o'k'$ determined and point N located, the influence line $A's'o'B'$ can be drawn. The ordinate $y'z'$, shown at (d), can be obtained in the manner explained above, and point P can be located by drawing ODx' and $O'Ex'$, as x' is directly over P. Having the ordinate $y'z'$ determined and the point P located, the influence line $C'w'y'D'$ can be drawn.

A better way to locate points N and P is to draw the line OJ. Then point F, where OJ intersects the diagonal cD, is directly over point N; and point G, where the line OJ intersects diagonal dE, is directly over point P.

It can be readily shown that this construction is correct. Referring to Fig. 337, the structure $OCDO'cO$, supporting the loads $P1$ and $P2$, is a rigid structure. It is evident that the loads $P1$ and $P2$ could have

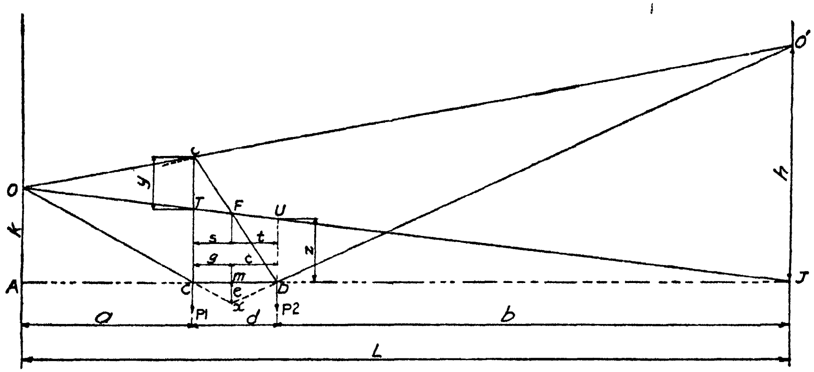


Fig. 337

such relative weights that the member cD (also cC) would have zero stress. In that case, $OCDO'$ would be an equilibrium polygon. Then by prolonging the segments $O'D$ and OC to intersection, the point x is obtained, which would be at the resultant, or center of gravity, of the loads $P1$ and $P2$ (see Art. 45). Then both of the loads could be placed at x and the stress in member cD (also cC) would be zero. Now, it is evident that as long as $P1$ and $P2$ have this same relative value, $OCDO'$ would remain an equilibrium polygon, regardless of the actual weights of the loads $P1$ and $P2$; so, evidently, the stress in cD due to any load at x will be zero, and hence x is the point of zero stress for diagonal cD.

From similar triangles Dmx and DJO', we have

$$\frac{c}{b}=\frac{e}{h},$$

from which we obtain

$$e=\frac{c}{b}h \quad \ldots\ldots (1).$$

From similar triangles OAC and xmC, we have

$$\frac{g}{a}=\frac{e}{k},$$

from which we obtain

$$e=\frac{g}{a}k \quad \text{.......... (2).}$$

Now equating (1) and (2) equal, we have

$$\frac{c}{b}h=\frac{g}{a}k,$$

from which we obtain

$$\frac{g}{c}=\frac{ah}{bk} \quad \text{.......... (3).}$$

From similar triangles cFT and DFU, we have

$$\frac{s}{t}=\frac{y}{z} \quad \text{.......... (4).}$$

But from similar triangles $O'JO$ and cOT, we have

$$\frac{y}{h}=\frac{a}{L},$$

from which we obtain

$$y=\frac{a}{L}h.$$

And from similar triangles JAO and JDU, we have

$$\frac{z}{k}=\frac{b}{L},$$

from which we obtain

$$z=\frac{b}{L}k.$$

Now substituting these values of y and z in (4), we obtain

$$\frac{s}{t}=\frac{ah}{bk} \quad \text{.......... (5).}$$

Equating (3) and (5) equal, we obtain

$$\frac{g}{c}=\frac{s}{t} \quad \text{.......... (6).}$$

But $$c=d-g \text{ and } t=d-s.$$

So, substituting these values in (6) and reducing, we obtain

$$g=s,$$

which proves that points F and x are on the same vertical line.

Any of the other cases can be proved in a similar manner. Referring to Fig. 336, point 1 is the point of zero stress for diagonal bC and point 2 is the point of zero stress for post *cC*. Point 3 is the point of zero stress for diagonal *dE* and point 4 is the point of zero stress for post *eE*. The point of zero stress for diagonal *eF* is at 5.

Tension in Posts. Let it be required to determine the maximum live-load tensile stress in post *dD* (Fig. 338). First construct the influence line, shown at (c), for stress in diagonal *dE*. As explained on pages 424 and 425 (Art. 190), the maximum live-load tension will occur in post *dD* when the load is so placed that the stress in diagonal *dE* is zero. The stress in the diagonal *dE* due to dead load is tension. A live load moving onto the bridge from the left would begin reversing this dead-load tension from the start and continue reversing it until a

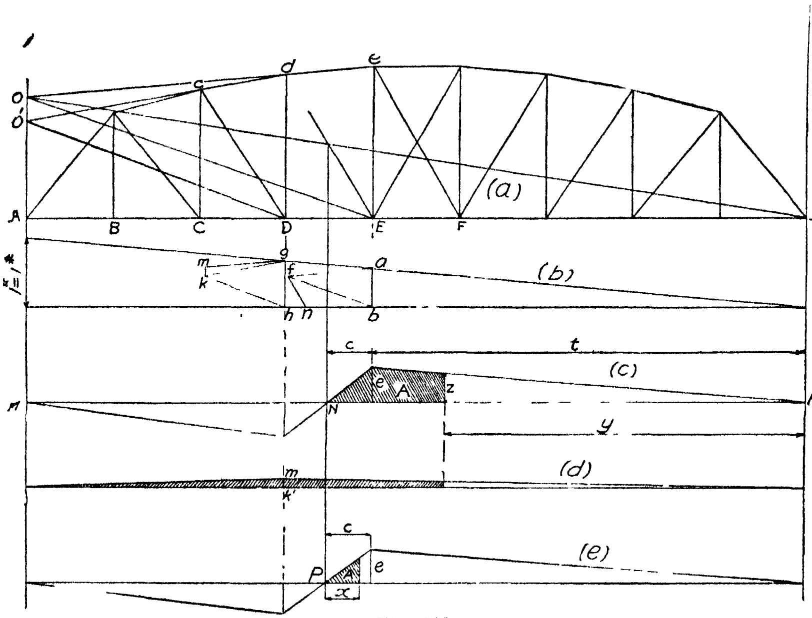


Fig. 338

point was reached where the stress in the diagonal would be zero and from there on, as the load moved on to the right, the stress in the diagonal would be compression and this compression would continue to increase until the live-load extended from *M* to *N* (see the influence line at (c)) at which time the live-load compression would be a maximum. Then as the load moved on past *N* the compression in the diagonal would decrease until a point was reached, as the load moved on to the right, where the stress in the diagonal *dE* would be zero for the second time. This last position is the one desired, as the maximum tension in post *dD* will occur when the load is in that position. The first thing is to find that position.

Let *S* represent the dead-load tension in diagonal *dE* and *S′* the live-load compression in the same when the load extends from *M* to *N*. Then the load to the right of *N* must be just sufficient to produce a tension of $S' - S$ pounds in diagonal dE. An equivalent uniform live load of w pounds per foot of truss will be used. Let y be the distance from H to the head of the uniform live load and let A represent the required shaded area. Then the required tension in the diagonal is Aw, which must be equal to $S' - S$, so we have

$$Aw = S' - S,$$

from which we obtain

$$A = \frac{S' - S}{w}$$

for the required shaded area in square feet. The next thing is to find the value of y when the shaded area is equal to $(S' - S)/w$.

Referring to the influence line at (c), we have

$$\left(\frac{c+t}{2}\right)e - A = \tfrac{1}{2}(yz) \dots\dots (7).$$

But

$$z = \frac{ye}{t}.$$

So, substituting this value of z, and $(S' - S)/w$ for A in (7), and reducing, we obtain

$$y = \sqrt{(c+t)t - \frac{2t}{ew}(S' - S)} \dots\dots (8).$$

From this equation (8) the head of the equivalent uniform live load can be located. Then, having the load located, the influence line for the maximum live-load tension in the post dD can be constructed as shown at (d), and by multiplying the shaded area by w the desired live-load tension in the post will be obtained.

To construct the influence line shown at (d): First assume zero stress in diagonal dE. Placing a unit load at D and drawing kh parallel to $O'D$ and kg parallel to $O'd$, we have the stress in top chord cd given by the line kg; and as the horizontal components of top chords cd and de must be equal (since the stress in dE is zero) by drawing from g a line parallel to top chord de intersecting the vertical mk at m, we have the stress in chord de represented by the line gm and the stress in the post dD represented by the line mk. Then laying off $m'k'$ at (d) equal to mk the influence line as shown can be constructed.

In case the uniform live load does not extend to ordinate e, as shown at (e), it is better to locate the head of the load with reference to P. Let x represent the distance from P to the head of the uniform live load. Then we have

$$A : \frac{ce}{2} :: x^2 : c^2,$$

from which we obtain

$$x = \sqrt{\frac{2Ac}{e}} \dots\dots (9).$$

206. Determination of Dead-Load Stresses in Pettit Trusses.—Let it be required to determine the dead-load stresses in the truss shown at (a), Fig. 339. Let $W1$, $W2$, and so on (all of which are equal) represent the loads at the lower panel points; $P1$, $P2$, and so on (all of which are equal) represent the loads at the upper panel points; and let R represent the reaction due to these loads.

Members U1-LO, LO-L1, L1-L2 and U1-L1. As previously shown (Art. 189), the dead-load stress in $U1$-LO is equal to $R\sec\theta$, $R\tan\theta$ in both LO-$L1$ and $L1$-$L2$, and $W1$ in $U1$-$L1$.

Member U1-U2. The stress in this member is readily obtained by taking moments about $L2$. Thus, we obtain

$$[R \times 2d - (W1 + P1)d]\frac{1}{h} = H$$

for the horizontal component of the stress in $U1$-$U2$. The stress in the member is then obtained by multiplying H by $\sec\phi$.

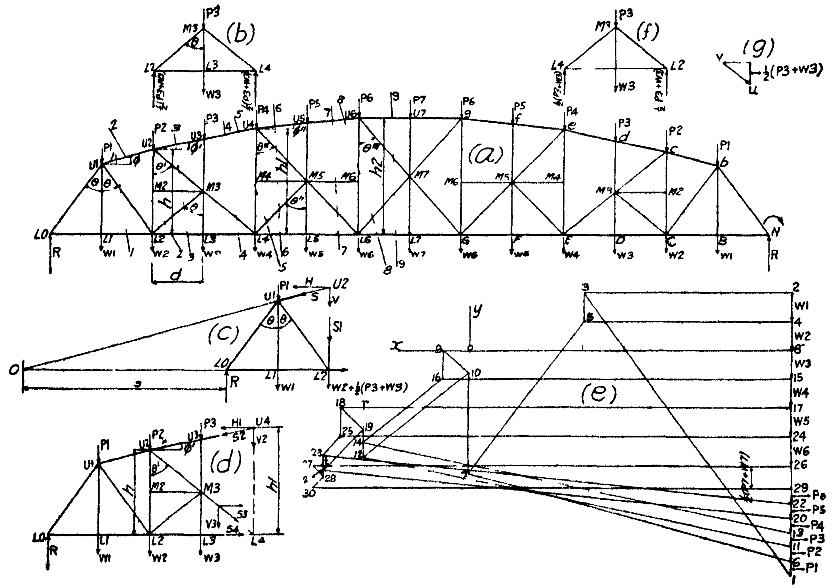


Fig 339

Member U1-L2. Having determined H, the horizontal component of the stress in top chord $U1$-$U2$, the vertical component of the same is equal to

$$H\tan\phi = V.$$

Now, considering the part of the structure to the left of section 1-1 and summing up the vertical forces, we obtain

$$V1 = R - (W1 + P1) - V$$

for the vertical component of the stress in $U1$-$L2$. Then we have $V1\sec\theta$ for the stress in $U1$-$L2$.

The stress in this diagonal can also be determined by taking moments about the intersection of the chords as explained in Art. 189.

Members M3-L2 and M3-L3. In determining the stresses in these two members the triangle $L2$-$M3$-$L4$ can be considered as a separate truss, as shown at (b). The stress in $M3$-$L2$ is equal to $\frac{1}{2}(P3+W3)\sec\theta'$, and the stress in the hanger $M3$-$L3$ is simply equal to $W3$.

Members U2-L2. Considering the part of the structure to the left of section 2-2 shown at (c) and resolving the stress in $L2$-$M3$ both vertically and horizontally at $L2$ and resolving the stress in $U1$-$U2$ and adding up the vertical forces, we have

$$R-[W1+W2+\tfrac{1}{2}(W3+P3)+V+P1]-S1=0,$$

from which we obtain

$$S1=R-(W1+W2+\tfrac{1}{2}(W3+P3)+V+P1)$$

for the dead-load stress in post $U2$-$L2$. This stress can also be determined by taking moments about O. Thus, taking moments about O, we have

$$Rs-(P1+W1)\ (s+d)-(W2+\tfrac{1}{2}W3+\tfrac{1}{2}P3)\ (s+2d)-S1(s+2d)=0,$$

from which we obtain

$$S1=[Rs-(P1+W1)\ (s+d)-(W2+\tfrac{1}{2}W3+\tfrac{1}{2}P3)\ (s+2d)]\frac{1}{s+2d}$$

for the stress in $U2$-$L2$.

Member M2-M3. This member is not subjected to any direct stress. It is for the purpose of stiffening post $U2$-$L2$.

Member U2-U3-U4. Considering the part of the structure to the left of section $4-4$, as shown at (d), and by taking moments about $L4$, we obtain

$$H1=\frac{1}{h1}[(R\times 4d)-(W1+P1)3d-(W2+P2)2d-(W3+P3)d]$$

for the horizontal component of the stress in top chord $U2$-$U3$-$U4$. Then by multiplying $H1$ by $\sec\phi'$ the stress in the member is obtained.

Member M3-L4. Having $H1$, the horizontal component of the stress in the top chord $U2$-$U3$-$U4$ determined, the vertical component $V2$ is equal to $H1\ \tan\phi'$. Then resolving the stress in $M3$-$L4$ vertically and horizontally and summing up the vertical forces shown at (d) we obtain

$$V3=R-(W1+W2+W3)-(P1+P2+P3)-V2$$

for the vertical component of the stress in diagonal $M3$-$L4$. Then for the stress in the member we have

$$S3=V3\times\sec\theta'.$$

Member L2-L3-L4. Considering the part of the structure shown at (d), and considering $S2$ and $S3$ instead of their components (see Art. 41), and taking moments about $U2$ of the forces shown at d, we obtain

$$S4=\frac{1}{h}[(R\times 2d)-(W1+P1)d+(W3+P3)d]$$

for the stress in bottom chord $L2$-$L3$-$L4$.

Member U2-M3. The vertical component of $M3\text{-}L2$ is equal to $\frac{1}{2}(P3+W3)$; the vertical component of the top chord $U2\text{-}U3\text{-}U4$, as found above, is equal to $V2$. Then considering the part of the structure to the left of section 3-3 and summing up the vertical forces and components on same, we obtain

$$V1 = R - (W1 + W2 + P1 + P2) - \tfrac{1}{2}(P3 + W3) - V2$$

for the vertical component of the stress in diagonal $U2\text{-}M3$. Then the stress is equal to $V1 \times \sec\theta'$.

Member U3-M3. This member simply supports the load $P3$ and hence the stress in it is equal to $P3$.

Members M5-L5 and M5-L4. The stress in hanger $M5\text{-}L5$ is equal to $W5$. The vertical component of the stress in $M5\text{-}L4$ is equal to $\frac{1}{2}(P5+W5)$. Then the stress in $M5\text{-}L4$ is equal to $\frac{1}{2}(P5+W5)\sec\theta''$.

Member U4-L4. Considering the part of the structure to the left of section 5-5 and resolving the stress in $U4\text{-}U3\text{-}U2$ and $M5\text{-}L4$ vertically and horizontally and summing up the forces and components, we obtain

$$V5 = R - (W1 + W2 \ldots W4) - (P1 + P2 + P3) - V2 - \tfrac{1}{2}(P5 + W5)$$

for the stress in post $U4\text{-}L4$.

Member U4-U5-U6. Considering the part of the structure to the left of section 7-7 and taking moments about $L6$ we can obtain $H2$, the horizontal component of the stress in top chord $U4\text{-}U5\text{-}U6$. Then the stress in the member is equal to $H2 \times \sec\phi''$.

Member L4-L5-L6. Considering again the part of the structure to the left of section 7-7 and taking moments about $U4$, we obtain

$$S5 = \frac{1}{h1}\left[(R \times 4d) - (W1 + P1)3d - (W2 + P2)2d - (W3 + P3)d + (W5 + P5)d\right]$$

for the stress in bottom chord $L4\text{-}L5\text{-}L6$.

Member M5-L6. Considering again the part of the structure to the left of section 7-7 and letting $V6$ represent the vertical component of the stress in top chord $U4\text{-}U5\text{-}U6$, we obtain

$$V7 = R - (W1 + W2 \ldots W5) - (P1 + P2 \ldots P5) - V6$$

for the vertical component of the stress in diagonal $M5\text{-}L6$. Then multiplying $V7$ by $\sec\theta''$ the stress in the member is obtained.

Members M4-M5 and M5-M6. These members have no direct stress in them. They are for the purpose of stiffening the posts $U4\text{-}L4$ and $U6\text{-}L6$.

Member U4-M5. Considering the part of the structure to the left of section 6-6 and letting $\frac{1}{2}(P5+W5)$ and $V6$ represent the vertical component of the stress in $M5\text{-}L4$ and $U4\text{-}U5\text{-}U6$, respectively, and summing up the vertical forces and components to the left of section 6-6, we obtain

$$V8 = R - (W1 + W2 \ldots W4) - (P1 + P2 \ldots P4) - \tfrac{1}{2}(P5 + W5) - V6$$

for the vertical component of the stress in diagonal $U4\text{-}M5$. Then the stress in the member is equal to $V8 \times \sec\theta''$.

Member U5-M5. This member supports the load $P5$ and hence the stress in it is equal to $P5$.

Members U7-M7 and M7-L7. As is evident, the stress in $U7$-$M7$ is equal to $P7$ and the stress in $M7$-$L7$ is equal to $W7$.

Members U6-M7 and M7-L6. The dead-load stress in these members is due to the loads $W7$ and $P7$. We will assume that each is equally stressed. Then the stress in each will be equal to $\frac{1}{4}(W7+P7)\sec\theta'''$.

Member U6-L6. Considering the part of the structure to the left of section 8-8 and letting $V6$ and $V9$ represent the vertical component of the stress in members $U4$-$U5$-$U6$ and $M7$-$L6$, respectively, and summing up all the forces and components to the left of section 8-8, we obtain

$$S6=R-(W1+W2\ldots W6)-(P1+P2\ldots P5)-V6-V9$$

for the stress in $U6$-$L6$.

Members U6-U7 and L6-L7. Considering the part of the structure to the left of section 9-9 and taking moments about $L7$, we obtain

$$S7=\frac{1}{h2}[(R\times 7d)-(W1+P1)6d-(W2+P2)5d-(W3+P3)4d-$$
$$(W4+P4)3d-(W5+P5)2d-(W6+P6)d]$$

for the stress in top chord $U6$-$U7$. It will be seen readily that the moments of the stresses in $U6$-$M7$ and $M7$-$L6$ about $L7$ are equal and of opposite signs and annul each other, and consequently are not given in the equation of moments.

The stress in $L6$-$L7$ is equal to the stress in top chord $U6$-$U7$. However, the stress in $L6$-$L7$ can be obtained by taking moments about $U7$.

As the truss is symmetrical in reference to the center of span and symmetrically loaded, we have now sufficiently considered the analytical determination of the dead-load stresses in the truss.

As a matter of fact, the dead-load stresses in such trusses are graphically determined. The diagram of the dead-load stresses for the right half of the truss is shown at (e). This diagram is obtained by beginning at N and passing around the joints clock-wise. Thus, beginning at point N, we draw 1-2 equal to R and then 1-3 and 2-3 parallel, respectively, to bN and BN. Then passing on to joint B we have the diagram 3-2-4-5-3 for that joint. Then passing on to joint b we have the diagram 6-1-3-5-7-6 for that joint. Now, at C there are three unknown forces, the same is true of joint c and, consequently, before we can go farther we must determine one of the unknown forces at one of these joints. By considering E-$M3$-C as a separate truss, as shown at (f), and constructing the diagram at (g), we have the stress in $M3$-C given by the line vu. Then considering joint C (at (a)) and passing around the joint clock-wise, we have the part (starting at 7) 7-5-4-8 of the stress diagram for that joint. We know that the stress in post cC will be represented by a vertical line from 7. So we can draw 7-y. We can also draw 8-x. Then laying off 0-10 equal to $\frac{1}{2}(P3+W3)$ and drawing 9-10 parallel to $M3$-C, we have the stress in $M3$-C represented by this line 9-10 (see diagram at (g)) and thus we have the complete stress diagram 7-5-4-8-9-10-7 for joint C. For joint c we obtain the stress diagram (beginning at 11) 11-6-7-10-12-11. For joint *d* we obtain the stress diagram (beginning at 13) 13-11-12-14-13. For joint *D* we obtain the diagram 9-8-15-16-9. For joint *M*3 we obtain the stress diagram 14-12-10-9-16-14. The line 16-14, which represents the stress in member *M*3-*E*, must be parallel to 12-10, which represents the stress in member *c*-*M*3. This is the first check on the work.

By considering *G*-*M*5-*E* as a separate truss, the stress in *M*5-*E* can be determined. Now, considering joint *E* and passing around clock-wise, we obtain the part 14-16-15-17 of the stress diagram for joint *E*.

Then drawing 14-*r* and laying off *r*-19 equal to $\frac{1}{2}(P5+W5)$ and drawing 19-18 parallel to the member *M*5-*E*, we have the complete

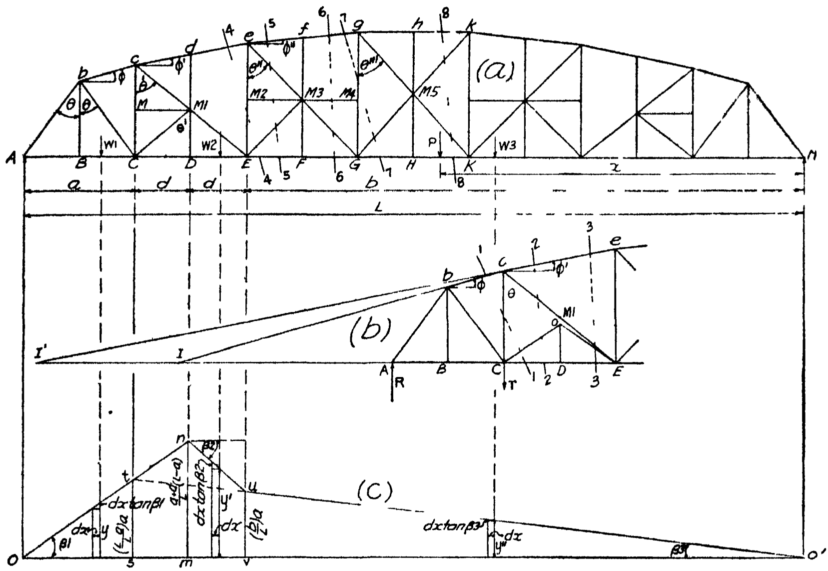


Fig 340

stress diagram 14-16-15-17-18-19-14 for joint *E*. For joint *e* we obtain the stress diagram 20-13-14-19-21-20. For joint *f* we obtain the stress diagram 22-20-21-23-22. For joint *F* we obtain the diagram 18-17-24-25-18. For joint *M*5 we obtain the diagram 23-21-19-18-25-23. The line 23-25 must be parallel to line 21-19. This is the second check on the work.

The dead-load stress is assumed to be the same in the four members, *M*7-*U*6, *M*7-*L*6, *M*7-*g* and *M*7-*G*. This stress is due to the loads *W*7 and *P*7. Then, by considering *L*6-*M*7-*G* as a separate truss, having a reaction at each end of $\frac{1}{4}(W7+P7)$, the stress in *M*7-*G* can be graphically determined.

Starting with 23, we have the part 23-25-24-26 of the stress diagram for joint *G*. Then drawing a vertical line from 23 and a horizontal line from 26 and laying off *t*-28 equal to $\frac{1}{4}(W7+P7)$ and drawing 28-27 parallel to *M*7-*G*, we obtain the complete stress diagram 23-25-24-26-27-

28-23 for joint G. For joint g we have the stress diagram 29-22-23-28-30-29.

This completes the graphical determination of the dead-load stresses, as the truss is symmetrical in reference to the center of span and the load is symmetrical in reference to same. The distance 26-29 should equal $\frac{1}{2}(P7+W7)$. This is the third and final check on the work.

207. Determination of Live-Load Stresses in Pettit Trusses.— Let it be required to determine the live-load stresses in the truss shown at (a), Fig. 340, due to Cooper's $E50$ loading.

Members bA, AB and BC. The maximum live-load stress will occur in these members when the wheel loads are placed for maximum shear in panel AB. The placing of the loading will be in accordance with (5), Art. 90. After the loads are thus placed, the next thing to do is to determine the reaction at A by taking moments about N. Let R represent this reaction. The next thing to do is to determine the concentration at A due to the loads in panel AB, which can be done by taking moments about B. Let r represent this concentration. Then we have R-r for the shear in panel AB. Then we obtain $(R\text{-}r)\sec\theta$ for the maximum live-load stress in end post bA and $(R\text{-}r)\tan\theta$ for the maximum live-load stress in each of the members AB and BC (see Art. 174).

Member bC. The maximum live-load stress will occur in diagonal bC when the loads are placed according to the requirements of (4) of Art. 190. After the loads are thus placed, the next thing to do is to determine the reaction at A, which can be done by taking moments about N. Let R represent this reaction. Next, by taking moments about C, the horizontal component of the stress in top chord bc is readily obtained. Let H represent this component. Then the vertical component of the stress in bc is equal to $H\tan\phi$. Taking moments about C of the loads in panel BC, the concentration at B is readily obtained. Let r represent this concentration. Now, for the vertical component of the stress in diagonal bC, we have $V=R-r-H\tan\phi$, and multiplying this by $\sec\theta$, we obtain $(R-r-H\tan\phi)\sec\theta$ for the stress in the member bC.

Member bB. The maximum live-load stress in this hanger, as is obvious, is equal to the maximum live-load floor beam concentration, which is determined as explained in Art. 148.

Member bc. The maximum live-load stress will occur in this member when the loads are placed for maximum moment about C. The placing of the loads will be in accordance with (5), Art. 91. That is, a wheel must be at C and the average unit load to the left of C must equal (approximately) the unit load on the span. After the loads are thus placed the next thing to do is to determine the reaction at A, which can be done by taking moments about N. Then, by taking moments about C of the forces to the left, the horizontal component of the stress in bc is readily obtained, and multiplying this component by $\sec\phi$, the maximum live-load stress in bc is obtained.

Members cC and c-M1. It is obvious if C-$M1$-E be considered a separate truss, shown as C-o-E at (b), that the stress in cC and c-$M1$ will not be affected in the least. The truss C-o-E really acts as a stringer extending from C to E. Then, as is readily seen, the maximum live-load stress in cC can be obtained by loading the span from the right, placing a wheel at E and loading panel CE (no loads to the left of C), all in accordance with (4) of Art. 190.

In applying equation (4) s should be taken equal to the distance IA, shown at (b), and a should be taken equal to distance AC.

After the loading is placed, the next thing to do is to determine the reaction at A, which can be done by taking moments about N. Let R represent this reaction. The next thing to do is to determine the concentration at C due to the loads in panel CE, which can be done by taking moments about E, ignoring panel point D altogether; that is, CE would be considered as a stringer. Let r represent this concentration at C.

Next, by taking moments about C, the horizontal component of the stress in top chord bc is readily obtained, as previously explained. Let H represent this component. Then the vertical component of the stress in the top chord bc is equal to $H\tan\phi$. Now, considering the part of the structure to the left of section 1-1, and adding up the vertical components and forces, we obtain $R-r-H\tan\phi$ for the maximum live-load stress in member cC. The member oC is entirely ignored, as the vertical component of its stress is included in the concentration r.

To obtain the maximum live-load stress in c-$M1$, the loads would be placed very much the same as for cC. In applying equation (4) of Art. 190 s would be taken equal to the distance $I'A$ and a equal to AC. After the loads are properly placed, the next thing to do is to determine the reaction at A, which can be done by taking moments about N. Then the next thing to do is to determine the concentration at C due to the loads in panel CE, ignoring panel D. By taking moments about E, the horizontal component of the stress in top chord ce is readily obtained, and multiplying this component by $\tan\phi'$, the vertical component of the stress in top chord ce is obtained. Then considering the part of the structure to the left of section 2-2 and summing up the vertical components and forces to the left which consist of the reaction at A, concentration at C, the vertical component of the stress in ce, and the vertical component of the stress in $cM1$, the vertical component of the stress in $cM1$ can be obtained; and multiplying this component by $\sec\theta'$, the stress in $cM1$ is obtained.

Members D-M1 and C-M1. As is obvious, the maximum live-load stress in hanger D-$M1$ is equal to the maximum live-load floor beam concentration, which is determined as explained in Art. 148. The maximum live-load stress in sub-diagonal C-$M1$, as is readily seen, is equal to one-half of the maximum floor beam concentration multiplied by $\sec\theta'$.

Member ce. The maximum live-load stress in this member occurs when a load is at E and the average unit load to the left of E is equal to the average unit load on the span. This is in accordance with Art. 91. After the loads are properly placed the next thing to do is to determine the reaction at A, which can be done by taking moments about N. Then considering the part of the structure to the left of section 3-3 and taking moments about E, the horizontal component of the stress in top chord ce is readily obtained, and then multiplying this component by $\sec\phi'$, the maximum live-load stress in ce is obtained.

Member E-M1. It is readily seen that if oE (shown at (b)) were combined with E-$M1$, any load at D would cause compression in E-$M1$

and hence the member $E\text{-}M1$, as far as maximum stress is concerned, acts just the same as an ordinary diagonal where DE is the panel length. Then the maximum stress in $E\text{-}M1$ is obtained by loading the span from the right with a wheel at E, and loading panel DE (no loads to the left of D) according to (4) of Art. 190. In applying equation (4) s should be taken equal to the distance $I'A$ and a equal to the distance AD. After the loads are properly placed, the next thing to do is to determine the reaction at A, which can be done by taking moments about N. Then the next thing to do is to determine the concentration at D, due to the loads in panel DE. Then, considering the forces to the left of section 3-3 (oE and $E\text{-}M1$ being considered as combined, that is, one member) and taking moments about E, the horizontal component of the stress in top chord ce is readily obtained; and multiplying this horizontal component by $\tan\phi'$ the vertical component of the stress in the top chord ce is obtained. Then summing up the vertical components and forces to the left of section 3-3 the vertical component of the stress in diagonal $E\text{-}M1$ is readily determined; and multiplying this component by $\sec\theta'$ the maximum live-load stress $E\text{-}M1$ is obtained.

Member CE. It is readily seen that any load in panel CD or DE will cause tension in CE, owing to the member CE acting as the bottom chord of truss $C\text{-}M1\text{-}E$. This stress, as we may say, is in addition to the stress produced in the member acting as a main bottom chord section of the structure. Then it appears that the maximum live-load stress in bottom chord CE will occur when the loads in panels CD and DE have some certain value as compared to the other loads on the structure. This can be investigated most satisfactorily by the use of influence lines.

The stress in CE is finally obtained by considering the part of the structure to the left of section 3-3 and taking moments about c. Then evidently the stress in CE will vary as the moments about c and hence the stress in the member can be investigated by constructing an influence line for moments about c.

Let P represent a load at any point x distance from N. Then when P is at any point to the right of panel point E we have $M=(Px/L)a$ for the bending moment about c. If $x=b$, the last equation becomes $M=(Pb/L)a$, and if $P=1$ we have $M=(b/L)a$. So if we draw OO' at (c) and lay off uv equal to $(b/L)a$ we can draw uO', which is the influence line for moments about c for loads to the right of E.

If the load P moves to any point in panel DE, the moment about c is

$$M'=\left(P\frac{x}{L}\right)a+P(x-b).$$

If $x=b$ we have the same as given above for ordinate uv. If $x=b+d$ and $P=1$ the equation reduces to

$$M'=\left(\frac{b+d}{L}\right)a+d=\frac{a+d}{L}(L-a).$$

Then if the ordinate nm be laid off equal to this last value of M' the line nu can be drawn, which is the influence line for moments about c for loads in panel DE.

If the load P moves to any point in panel CD, the moment about c will be

$$M'' = \left(P\frac{x}{L}\right)a + (b + 2d - x)P.$$

This is the equation to the line tn, which is the influence line for moments about c for loads in panel CD.

If the load P moves to any point to the left of C, the moment about c will be

$$M''' = \left(P\frac{x}{L}\right)a - (x - b - 2d)P = \left(P\frac{x}{L}\right)a + (b + 2d - x)P,$$

which is the equation to line tO. But this value of M''' is exactly the same as found above for M''. Therefore, the lines tn and tO are in the same straight line On and hence On is the influence line for moments about c for loads to the left of D. Then evidently $OnuO'$ is the complete influence line for moments about c. As is seen, this influence line has three different slopes and consequently there will be three different moment increments to consider.

Let $W1$ represent the resultant of the loads to the left of D, $W2$ the resultant of the loads in panel DE, $W3$ the resultant of the loads to the right of E and let W represent the total load on the span; that is, $W = W1 + W2 + W3$. Then for the moment about c, we have

$$M = yW1 + y'W2 + y''W3 \qquad (1).$$

Now if all loads move to the left the distance dx, we have

$$\Delta M = -W1dx\tan\beta1 + W2dx\tan\beta2 + W3dx\tan\beta3 \qquad (2).$$

for the increment of the moment about c. If the loads are in the position for maximum moment about c this increment will be equal to 0 and, hence, (2) would become

$$0 = -W1\tan\beta1 + W2\tan\beta2 + W3\tan\beta3 \qquad (3).$$

But, $\tan\beta1 = \dfrac{L - a}{L}$,

$$\tan\beta2 = \left[\frac{a + d}{L}(L - a) - \frac{b}{L}a\right]\frac{1}{d} = \frac{L + a}{L},$$

$$\tan\beta3 = \frac{a}{L}.$$

Substituting these values of the tangents in (3), we obtain

$$0 = -W1\left(\frac{L - a}{L}\right) + W2\left(\frac{L + a}{L}\right) + W3\frac{a}{L},$$

from which we obtain

$$(W2 - W1)L + a(W1 + W2 + W3) = 0,$$

from which we obtain

$$\frac{W}{L} = \frac{W1 - W2}{a} \qquad (4).$$

Expressing equation (4) in words, *the average unit load on the span is equal to the load to the left of D, minus the load in panel DE, divided by a.* This can be taken as the criterion for placing the load for maximum stress in bottom chord *CE*.

After the loads are placed in accordance with equation (4), the next thing to do is to determine the reaction at *A*, which can be done by taking moments about *N*. Then the next thing is to determine the concentration at *D*, which can be done by taking moments about *C* and *E*.

Let R = the reaction at *A*, r = the concentration at *D* (due to the loads in panels *CD* and *DE*), m = moment of load to the left of *C* about *c*, and h = height of post *cC*. Then considering the part of the structure to the left of section 3-3 and taking moments about *c*, we obtain

$$S = (Ra - m + rd)\frac{1}{h}$$

for the maximum live-load stress in bottom chord *CE*.

The moment about *c* of course could be obtained by the use of influence line *OnuO′*.

If *W* remains constant, the increment of the moment would change signs only as a load passed joint *D*, so there will be a load at that joint when the maximum moment occurs at joint *c*. Then in placing the loading for maximum moment about *c*, that is, satisfying (4), a wheel must be at joint *D*.

Member eg. The maximum live-load stress in top chord *eg* is obtained by placing a load at *G* such that the unit load to the left of *G* is equal to the unit load on the span. Then by taking moments about *G* the horizontal component of the stress in *eg* is readily determined and multiplying this component by $\sec\phi''$ the stress in the member is obtained.

Member eE. The maximum live-load stress in this member is determined in the same manner as shown above for *cC*. The sub-diagonal *E-M3* and hanger *F-M3* are assumed to be omitted. The span is loaded from the right, a wheel at *G* and panel *EG* loaded in accordance with (4) of Art. 190.

The value of *s* (in equation (4)) is obtained by prolonging top chord *ce* and *a* is taken equal to the distance *AE*. After the loads are properly placed, the next thing to do is to determine the reaction at *A*, which can be done by taking moments about *N*. Next, the concentration at *E* can be determined by taking moments about *G*. Then, the next thing is to determine the vertical component in top chord *ce*, which can be done by first taking moments about *E*, thereby obtaining the horizontal component of the stress in the member and multiplying this component by $\tan\phi'$, we obtain the vertical component of the stress in *ce*.

Let *R* represent the reaction at *A*, *r* the concentration at *E*, and *V* the vertical component of the stress in top chord *ce*. Then considering the part of the structure to the left of section 4-4 and summing up the vertical components and forces, we have

$$S = R - r - V,$$

for the maximum live-load stress in post *eE*.

Member e-M3. The maximum live-load stress in this member is determined in the same manner as shown above for member *c-M1*. The members E-$M3$ and F-$M3$ are assumed to be omitted. The span is loaded from the right with a wheel at G and panel EG loaded in accordance with (4) of Art. 190. Top chord eg in that case would be prolonged to determine the value of s, and a would be the distance AE.

After the loads are properly placed for maximum stress in e-$M3$, the next thing to do is to determine the reaction at A, which can be done by taking moments about N. The next thing to do is to determine the concentration at E, which can be done by taking moments about G. Then the next thing in order is to determine the vertical component of the stress in top chord eg, which can be done by first taking moments about G, thus obtaining the horizontal component of the stress in top chord eg and multiplying this component by $\tan\phi''$, we obtain the vertical component of top chord eg. Then considering the part of the structure to the left of section 5-5, and summing up the vertical components and forces, we obtain

$$V1 = R - r - V$$

for the vertical component of the stress in diagonal e-$M3$, and multiplying this component by $\sec\theta''$, we obtain the maximum live-load stress in member e-$M3$.

Members F-$M3$ and E-$M3$. As is evident, the maximum live-load stress in hanger F-$M3$ is equal to the maximum floor beam concentration, which is determined as explained in Art. 118. The maximum live-load stress in sub-diagonal E-$M3$ is equal to one-half of this concentration multiplied by $\sec\theta''$.

Member G-$M3$. The maximum live-load stress in this member is determined in the same manner as explained above for diagonal E-$M1$. The span is loaded from the right, a wheel is placed at G and panel GF is loaded in accordance with (4) of Art. 190. In this case the value s is obtained by prolonging top chord eg (see Art. 190) and a is equal to the distance AF. The reaction at A is obtained by taking moments about N. The concentration at F is obtained by taking moments about G, and the horizontal component of the stress in top chord eg is obtained by taking moments about G, and the vertical component of same is then obtained by multiplying the horizontal component by $\tan\phi''$. After these are determined the vertical component of the stress in diagonal G-$M3$ is obtained by considering the part of the structure to the left of section 6-6 and summing up the vertical component and forces, and then the stress in G-$M3$ is obtained by multiplying this component by $\sec\theta''$.

Member EG. The maximum live-load stress in this member is determined in the same manner as explained above for bottom chord CE. In applying equation (4) (given above) a would be taken equal to distance AE. $W1$ would be the load between A and F, $W2$ the load in panel FG, $W3$ the load between G and N, and W the total load on the span.

After the loads are properly placed, that is, in accordance with equation (4), the stress in FG is readily obtained by considering the part of the structure to the left of section 6-6 and taking moments about e.

Member gG. The maximum live-load stress in this member is obtained by considering members Gk and Hh as being omitted. The span is then loaded from the right, a wheel at K and panel KG loaded in accordance with (4) of Art. 190. The value of s in equation (4) of Art. 190 is obtained by prolonging top chord eg and a is equal to the distance AG. After the loads are properly placed the reaction at A is obtained by taking moments about N. The concentration at G is obtained by taking moments about K. The horizontal component of the stress in top chord eg is obtained by taking moments about G and the vertical component of same is obtained by multiplying the horizontal component by $\tan\phi''$. Having the above determined, the stress in gG is obtained by summing up the vertical forces to the left of section 7-7 (ignoring member G-$M5$).

Member GK. The maximum live-load stress in this member is determined in the same manner as explained above for bottom chord CE. In applying equation (4) (given above) a would be taken equal to distance AG, $W1$ would be the load between A and H, $W2$ the load in panel HK, $W3$ the load between K and N. After the loads are properly placed, that is, in accordance with equation (4), the stress in GK is readily obtained by considering the part of the structure to the left of section 8-8 (ignoring member k-$M5$) and taking moments about g.

Member g-M5. To obtain the maximum live-load stress in this member, we assume the members Gk and Hh to be omitted, and load the span according to (5) of Art. 90. That is, the span would be loaded from the right with a wheel at K and the load in panel GK equal to the load on the bridge divided by the number of panels. The panels considered in this case would be double panels—that is, the number would be 7 instead of 14.

After having the loads thus placed, the next thing to do is to determine the reaction at A and the concentration at G. These can be obtained by taking moments about N and K, respectively. Let R represent the reaction at A and r the concentration at G. Then we have

$$S=(R-r)\sec\theta'''$$

for the maximum live-load stress in diagonal g-$M5$.

Member K-M5. To obtain the maximum live-load stress in this member we assume member k-$M5$ to be omitted and load the span according to (5) of Art. 90. That is, the span would be loaded from the right with a wheel at K, and the load in panel HK equal to the total load on the span divided by the number of panels. In this case the number of panels would be 14. After the loads are thus placed, the next thing to do is to determine the reaction at A and the concentration at H. Let R' represent the reaction at A, and r' the concentration at H. Then we have

$$S'=(R'-r')\sec\theta'''$$

for the maximum live-load stress in diagonal K-$M5$.

Members g-$M5$ and k-$M5$ would be assumed to have equal tensile stress and likewise members K-$M5$ and G-$M5$. The latter members would be subjected to compression from the floor beam concentration at H. Assuming the concentration at H to be transmitted to panel points G and K by sub-truss K-$M5$-G the maximum live-load compression in each of the members G-$M5$ and K-$M5$ would be equal to one-half of the maximum floor beam concentration at H multiplied by $\sec\theta'''$.

Member gk. The maximum live-load stress in this member is obtained by placing the load for maximum moments about K. Then taking moments about K (ignoring member k-$M5$) and dividing this moment by the height of post kK, we would obtain the greater part of the maximum stress in gk. Next the concentration at H, due to the loads in panels GH and HK, can be determined by taking moments about G and K. Then, multiplying one-fourth of this concentration at H by $\tan\theta'''$, and adding the result to the stress found in gk, by taking moments about K, the total maximum live-load stress in the member gk is obtained.

Maximum Tension in Post gG. The maximum live-load tension will occur in this post when the live load extends from A to some point beyond G, so that the stress in diagonal g-$M5$ is zero. The position of the load

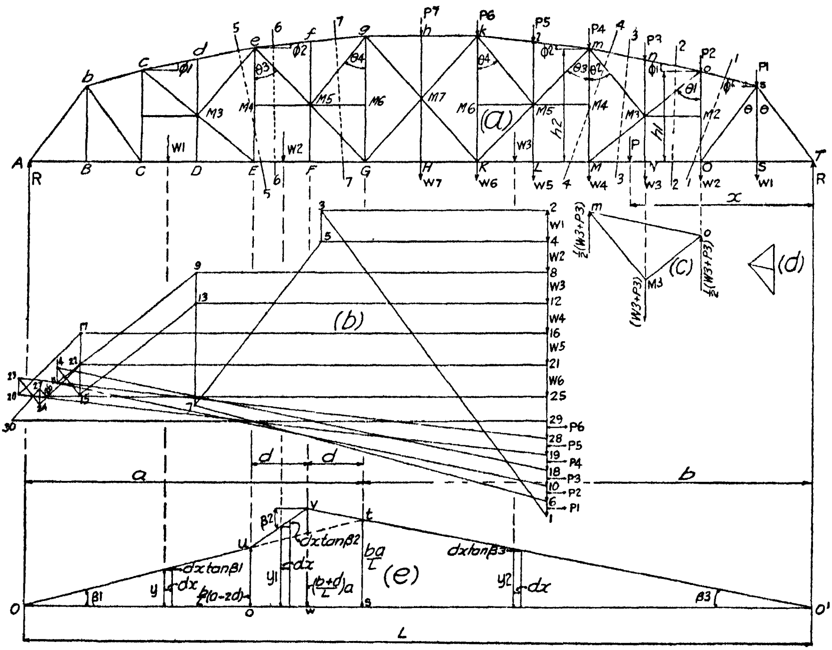


Fig. 341

can be determined by trial as explained in Art. 205 for curved chord Pratt trusses. After the position of the loading is found, the tension in the member is readily determined by considering the part of the structure to the left of section 7-7. The use of an equivalent uniform live-load will simplify the work very much and the result thus obtained will be sufficiently accurate.

Stress in Counters. In case a member g-$M3$ were inserted, it would be known as a counter. This member would not be stressed at all unless the dead-load tension in either or both of the members e-$M3$ and G-$M3$ were reversed. In case either, or both, of these members be subjected to reversal, the counter g-$M3$ would be used or members e-$M3$ and G-$M3$ would be designed to carry both tension and compression. In case the counter g-$M3$ is required, the maximum live-load stress (tension) in it can be determined by assuming members eG and F-$M3$ to be omitted and loading the span from the left with a load at E and loading panel EG in accordance with (4) of Art. 190. After the loading is thus placed, the next thing to do is to determine the reaction at N and the concentration at G, which can be done by taking moments about A and E, respectively. Then by considering the part of the structure to the right of section 6-6, the stress is readily determined. If counters are found necessary at other points the stress in them can be determined in a similar manner.

The determination of the stresses in trusses having the sub-paneling at the top of the truss, as shown at (a) in Fig. 341, is very similar to that given above for the truss shown in Fig. 339 wherein the sub-paneling is at the bottom of the truss.

Dead-Load Stresses. The graphical determination of the dead-load stresses in the type of truss shown in Fig. 341, is simpler than in the case of the type shown in Fig. 339, as the sub-paneling does not bother at all in the case of the truss shown in Fig. 341.

The graphical diagram of the dead-load stresses for the right half of the truss is shown at (b). This diagram is readily followed throughout.

The analytical determination of the dead-load stresses is just the same in some cases and similar in other cases to that given above for the truss shown in Fig. 339. To save space, just the formulas for the dead-load stresses in part of the members in the right half of the truss shown in Fig. 341 will be given:

Stress in sT equals $R\sec\theta$.

Stress in TS and OS equals $R\tan\theta$.

Stress in sS equals $W1$.

Stress in os equals $[(R\times 2d)-(W1+P1)d]\sec\phi/h1$ (d = panel lgth.).

Stress in Os equals $[R-(W1+P1)-V]\sec\theta$, where V represents the vertical component of the stress in top chord os.

The stress in member oO equals $R-(W1+W2+P1)-V$. In this case the part of the structure to the right of section 1-1 is considered.

The stress in member MO equals $[R\times 2d-(W1+P1)d]1/h1$. This is obtained by taking moments about o.

The stress in member mo equals $[(R\times 4d)-(3W1+3P1+2W2+2P2)d]\sec\phi 1/h2$. This is obtained by considering the part of the structure to the right of section 2-2 and taking moments about M.

The stress in member o-$M3$ equals $[R-(W1+W2+P1+P2)-V1]\sec\theta 1$, where $V1$ represents the vertical component of the stress in top chord mo. In this last case the part of the structure to the right of section 2-2 was considered.

The stress in sub-post n-$M3$ is equal to $P3$ and in N-$M3$ it is $W3$.

The stress in sub-diagonal m-$M3$ is determined by considering n-$M3$-o as an independent truss supporting the load $(W3+P3)$ at $M3$, as shown at (c) in Fig. 341.

The stress in the member m-$M3$ equals $(W3+P3)\div(\cos\theta 2+\sin\theta 2\cos\theta 1)$.

The stress in member m-$M3$ can be determined most readily by graphics (see the diagram at (d)).

The stress in M-$M3$ equals $[R-(W1+W2+W3+P1+P2+P3)-$

$V1+V2$ $\sec\theta 1$, where $V1$ and $V2$ represent, respectively, the vertical component of the stress in *mo* and *m-M3*. In this case the part of the structure to the right of section 3-3 is considered.

The stress in *mM* equals $R-(W1+W2+W3+W4+P1+P2+P3)-V1+V2$. In this case the part of the structure to the right of section 4-4 is considered.

Following this method the dead-load stresses in the other members of the truss are readily determined.

Live-Load Stresses. The determination of the live-load stresses in the type of truss shown in Fig. 341 is very much the same as given above for the truss shown in Fig. 340. The method of analysis will be sufficiently shown by considering the members in pan l *EG*.

Member eE. The maximum live-load stress will occur in this member when the span is loaded from the right, a wheel at *G* and panel *EG* loaded in accordance with (4) of Art. 190. In this case the top chord *ce* would be prolonged to determine the value of *s* to be used in equation (4) of Art. 190 and *a* would be taken equal to distance *AE*. After the loads are properly placed for maximum stress in *eE*, the next thing to do is to determine the reaction at *A* and the concentration at *E*, which can be done by taking moments about *T* and *G*, respectively. The next thing to do is to determine the vertical component of the stress in top chord *ce*. This can be done by taking moments about *E*, thus obtaining the horizontal component of the stress in *ce* (ignoring member *c-M3*, as it is not subjected to any live-load stress at this time, the live load not extending beyond *E*), and then multiplying this horizontal component by $\tan\phi 1$, the vertical component of the stress in *ce* is obtained. Let *R* represent the reaction at *A*, *r* the concentration at *E*, and *V* the vertical component of the stress in top chord *ce*. Then considering the part of the structure to the left of section 5-5 and summing up the vertical components and forces, we have

$$S=R-r-V$$

for the maximum live-load stress in post *eE*.

Member EG. The stress in this member is obtained, as is obvious, by taking moments about joint *e*. Then by placing the loads so that there is a wheel at *E* with the unit load to the left of *E* equal to the unit load on the span, which is in accordance with Art. 91, and taking moments about *e* of the forces to the left and dividing this moment by the height of post *eE*, the maximum live-load stress in *EG* is readily determined.

Member F-M5. The maximum live-load stress in this hanger is equal to the maximum live-load floor beam concentration at *F*, which is determined as explained in Art. 148.

Members M4-M5, M5-M6 and f-M5. These members are not subjected to live-load stress at all. *f-M5* is subjected to dead-load stress from the load at *f* only. The other two members are not subjected to any direct stress whatever. They are for the purpose of stiffening the posts *eE* and *gG*.

Member e-M5. The maximum live-load stress in this member will occur when the span is loaded from the right, a wheel at *F* and panel *EF* loaded in accordance with (4) of Art. 190.

The top chord *eg* would be prolonged to determine the value of *s* to use in (1) of Art. 190, and *a* would be taken equal to distance *AE*. After the loads are properly placed for maximum live-load stress in diagonal *e-M5*, the next thing to do is to determine the reaction at *A* and the concentration at *E*, which can be done by taking moments about *T* and *F*, respectively. Let *R* represent the reaction at *A* and *r* the concentration at *E*. Then considering the part of the structure to the left of section 6-6 and summing up the vertical components and forces, we have

$$S = (R - r - V)\sec\theta 3,$$

for the maximum live-load stress in diagonal *e-M5*. *V* here represents the vertical component of the stress in top chord *eg*, which can be determined by considering the part of the structure to the left of section 6-6, and taking moments about *G* and dividing this moment by the height of post *gG*, thus obtaining the horizontal component of the stress in top chord *eg*, then multiplying this component by $\tan\phi 2$, the vertical component *V* is obtained.

Member G-M5. In determining the maximum live-load stress in this member, the members *F-M5*, *g-M5* and *f-M5* are assumed to be omitted. The maximum stress will occur when the span is loaded from the right, a wheel at *G*, and panel *GE* loaded according to (1) of Art. 190. After the loads are properly placed for maximum stress in diagonal *G-M5*, the stress is obtained by considering the part of the structure to the left of section 7-7 (ignoring member *g-M5*) and summing up the vertical components and forces. Thus let *R* represent the reaction at *A*, *r* the concentration at *E* and *V*1 the vertical component of the stress in top chord *eg*, then we have

$$S1 = (R - r - V1)\sec\theta 3$$

for the maximum live-load stress in diagonal *G-M5*.

Member g-M5. The maximum live-load stress will occur in this member when the live-load concentration at *F* is a maximum. Let *r* represent this concentration. Then, considering *e-M5-g* as an independent truss, we obtain

$$S = r \div (\cos\theta 4 + \sin\theta 1 \cos\theta 3)$$

for the maximum live-load stress in sub-diagonal *g-M5*. This stress in *g-M5* can be determined most readily by graphics, considering *e-M5-g* as an independent truss.

Member eg. The live-load stress in this member can be investigated most satisfactorily by the use of the influence line.

The stress in *eg* is finally obtained by considering the part of the structure to the left of section 6-6 and taking moments about *G*. Then evidently the stress in *eg* will vary directly as the moments about *G* and, hence, the stress in the member can be investigated by constructing an influence line for moments about *G*.

Let *P* represent a load at any point *x* distance from *T*. Then when *P* is at any point to the right of panel point *G*, we have

$$M = \left(P\frac{x}{L}\right)a$$

for the bending moment about G. If $x=b$, the above equation becomes

$$M=\left(\frac{Pb}{L}\right)a,$$

and if $P=1$,

$$M=\frac{ba}{L}\cdot$$

So if we draw OO' at (e) and lay off $ts=ba/L$, we can draw tO' which is the influence line for moments about G for loads to the right of G.

If the load P moves to any point in panel FG, the moment about G, considering the part of the structure to the left of section 6-6, is

$$M'=\left(\frac{Px}{L}\right)a,$$

which is the same as found above for M, so the influence line vt for loads in panel FG is simply a continuation of the line tO'. If $x=b+d$ and $P=1$, we obtain

$$M'=\left(\frac{b+d}{L}\right)a,$$

which is equal to the ordinate vw.

If the load moves to any point in panel FE, the moment about G is

$$M''=\left(\frac{Px}{L}\right)a-P(x-b-d)2.$$

If $x=b+2d$ and $P=1$, we obtain

$$M''=(b+2d)\frac{a}{L}-2d=\frac{b}{L}(a-2d),$$

which is equal to ordinate uo, so the influence line vu for loads in panel EF can be drawn.

If the load P moves to any point to the left of E, the moment about G is $M'''=(Px/L)a-P(x-b)$. If $x=L$, we obtain $M'''=(PL/L)a-P(L-b)=Pa-Pa=0$. If $x=b+2d$ and $P=1$, we obtain $M'''=(b+2d)a/L-2d=b/L(a-2d)$, which is the same as found above for ordinate uo. So the line Ou can be drawn, which is the influence line for loads to the left of E. Then we have the complete influence line $OuvtO'$ for moments about G. As is seen, this influence line has three different slopes and consequently there will be three different moment increments to consider. Let $W1$ represent the resultant of the loads to the left of E, $W2$ the resultant of the loads in panel EF, $W3$ the resultant of the loads to the right of F, and let W represent the total load on the span; that is, $W=W1+W2+W3$. Then for the moment about G, we have

$$M=yW1+y1W2+y2W3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Now if all loads move to the right a distance dx, we have

$$\Delta M=W1dx\tan\beta1+W2dx\tan\beta2-W3dx\tan\beta3 \ldots\ldots\ldots\ldots (2),$$

for the increment of the bending moment about G. If the loads are in the position for maximum moment about G, this increment will be equal to zero and, hence, (2) would become

$$0 = W1\tan\beta1 + W2\tan\beta2 - W3\tan\beta3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3).$$

But, $\tan\beta1 = b/L(a-2d) \div (a-2d) = b/L,$

$$\tan\beta2 = \left(\frac{b+d}{L}\right)a - b/L(a-2d)\ 1/d = \frac{a+2b}{L} = \frac{L+b}{L},$$

$$\tan\beta3 = a/L.$$

Substituting these values of the tangents in (3), we have

$$0 = W1\frac{b}{L} + W2\frac{L+b}{L} - W3\frac{a}{L} = bW1 + LW2 + bW2 - aW3.$$

Substituting $L - a$ for b, we have

$$0 = LW1 - aW1 + LW2 + LW2 - aW2 - aW3 = L(W1 + 2W2) - a(W1 + W2 + W3),$$

from which we obtain

$$\frac{W1 + 2W2}{a} = \frac{W}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)'.$$

That is, the load to the left of E plus twice the load in the panel EF divided by the distance a must equal the total load on the span divided by the length of the span when the maximum stress in top chord eg occurs.

W remaining constant, the increment of the bending moment about G can change signs only as a load passes joint F, hence there will be a load at F when the maximum live-load stress in eg occurs. Then by placing the live load according to the above equation (4)′ with a load at F, and taking moments about G (considering the part of the structure to the left of section 6-6) the maximum live-load stress in eg can be readily determined.

The reaction at A would be determined first, then the concentration at E, due to the loads in panel EF, which can be done by taking moments about F. Let R = reaction at A, r = the concentration at E, due to the loads in panel EF, and m = the moment of all loads to the left of E about G. Then taking moments about G, we have

$$M = R(6d) - m - r(2d).$$

Dividing this moment by h, the height of post gG, we obtain the horizontal component of the stress in top chord eg and multiplying this component by $\sec\phi2$, we obtain the maximum live-load stress in top chord eg.

The stress in top chord ce is determined in a similar manner. The above equation (4)′ would be applied to determine the position of the loading. The value of a would be taken in that case equal to the distance AE.

208. Graphical Determination of Live-Load Stresses in Pettit Trusses.—The influence line method outlined in Art. 103 is the most convenient graphical method to use. The work as a whole is practically the same as shown in Art. 205 for curved chord Pratt trusses.

As an illustration, let it be required to determine the live-load stress in top chord eg of the truss shown at (a) in Fig. 342. The first thing to do is to draw the influence line for reaction, as shown at (b). The stress in eg can be determined by taking moments about G and considering the part of the structure to the left of section 2-2. Then evidently the influence line for the stress in eg will be of the form shown at (c). By placing a unit load at G and drawing ec (at (b)) and ed parallel, respectively, to SS' and SG, we have the stress in top chord eg, due to the unit load at G, represented by the line ec. Then drawing AB, at

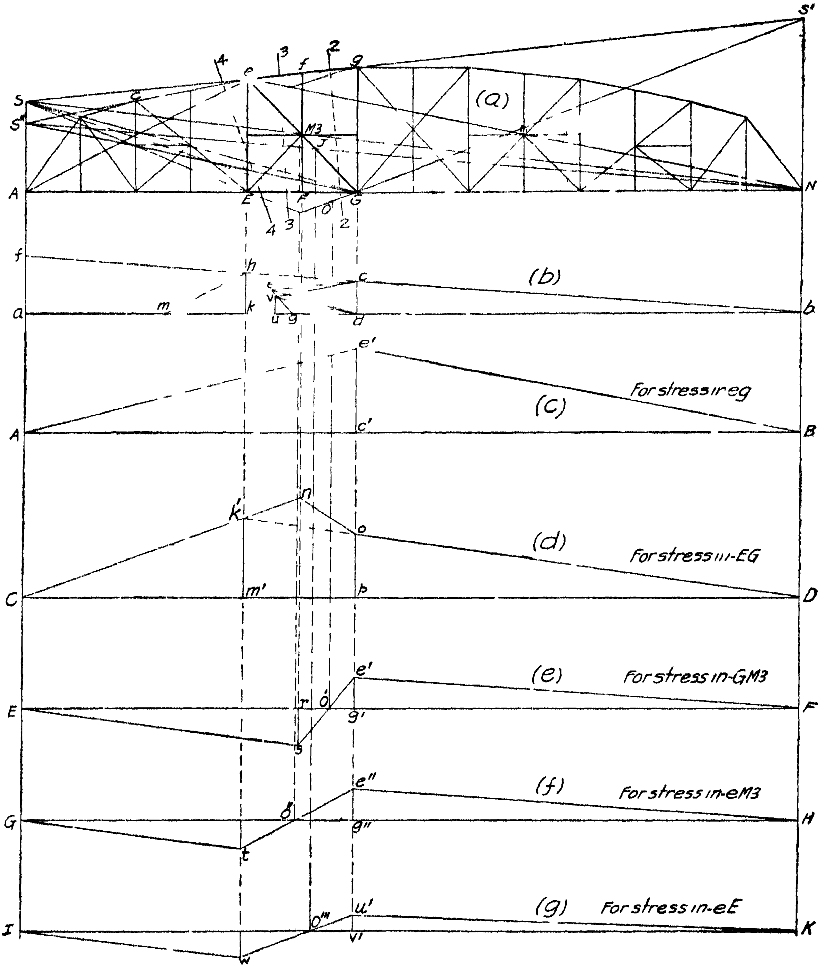


Fig 342

(c), and laying off $e'c' = ec$ and drawing $e'A$ and $e'B$, we obtain the influence line $Ae'B$ for the stress in top chord eg. Then placing the live load on the span so that there is a load at G and the unit load to the left of G equal to the unit load on the span, and multiplying the loads by their respective ordinates, as previously explained, the maximum live-load stress in eg is obtained. In case an equivalent uniform live load be used, the maximum stress in eg will be obtained by multiplying the area

of triangle $Ae'B$ by the uniform load per foot, as previously explained.
The stress in bottom chord EG can be determined by taking moments about e. Then evidently the influence line for the stress in EG will be partly of the form $Ck'D$, shown at (d). But we found in Art. 207 (see Fig. 340) that for moments the line $Ck'n$ was a straight line and undoubtedly $Ck'n$ will be a straight line if the influence line be constructed for

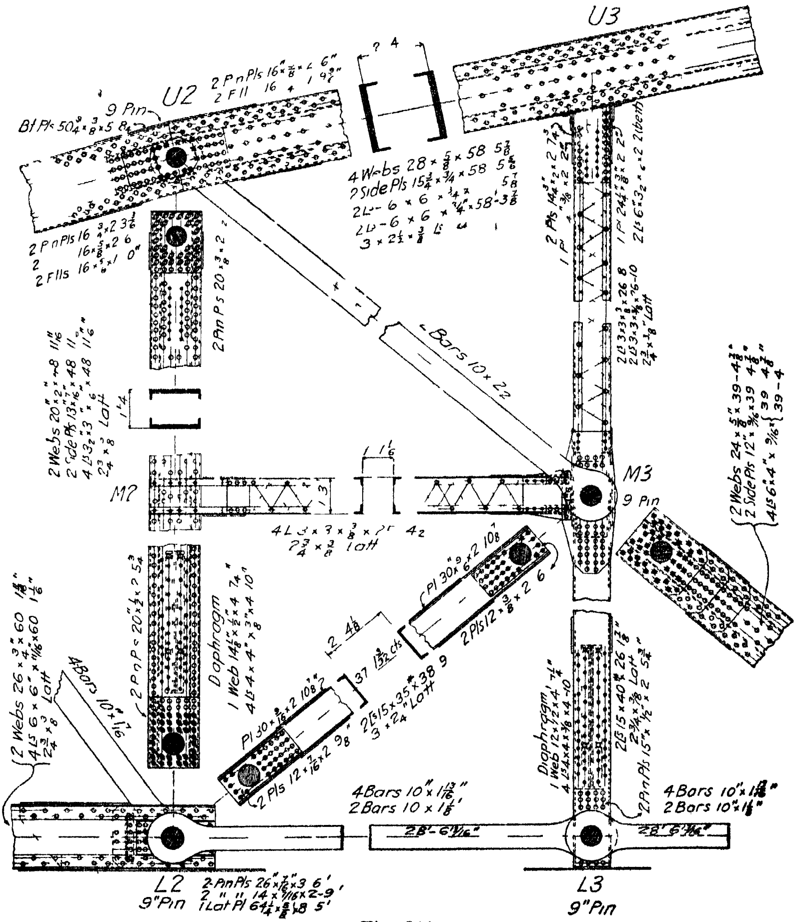


Fig 343

the stress in EG. So if $k'm'$ were known, $k'D$ could be drawn, thus locating point o, and next Cn could be drawn and then the line on could be drawn and thus we would have the complete influence line $CnoD$ for the stress in bottom chord EG. As is seen, the only thing needed to construct this influence line is the ordinate $k'm'$. By placing a unit load at E and taking moments about e (considering the part of the structure to the left of section 2-2) and dividing by the height of post eE the stress in bottom chord EG, due to the unit load at E, is obtained, which is the desired value of $k'm'$. By drawing Ae and eN we have the truss $AeNEA$. Now it is readily seen that a unit load at E will produce the same stress in AN as would be found in EG by taking moments about e. By drawing hm at (b), parallel to Ae, we have the stress in AN represented by the line km. Then by laying off $k'm'=km$, the influence line $CnoD$ can be drawn as explained above. Then placing the live load on the span in accordance with (4) of the last article, and multiplying the loads by the proper ordinates to the influence line at (d), the maximum live-load stress in bottom chord EG will be obtained. The other influence lines shown in Fig. 342 are considered self-explanatory, provided Art. 205 is thoroughly understood.

209. Remarks Concerning Pettit Trusses.—Pettit trusses, as previously stated, are used for long span bridges. These trusses should have economic heights, in which case, if single paneling were used, the floor system would be very heavy, or the diagonals would have an uneconomic and awkward slope. These undesirable features are eliminated by the sub-paneling.

All diagonals and sub-diagonals can be made out-and-out tension members (eye-bars) if the sub-paneling be at the top chord. This will result in a lighter truss than if the paneling be at the bottom chord. But experience seems to indicate that trusses with sub-paneling at the bottom chord are more rigid than trusses with sub-paneling at the top chord. It is a question whether the lack of rigidity, however, is not due more to poor details than to the form of the truss.

The stress sheets and the detail drawings for Pettit trusses are gotten out in the same manner as previously explained for parallel and curved chord bridges and the calculations of the details are similar to those previously shown for those bridges.

A fair idea of the details of a Pettit truss can be obtained from Fig. 343, where the details of a panel of the bridge indicated in Fig. 339 are shown. These details are of the Bismarck bridge designed by Ralph Modjeski and built by the American Bridge Company. It is a Northern Pacific Railway bridge over the Missouri River at Bismarck, North Dakota.

The designing of the floor and lateral systems and end bearings of Pettit truss bridges is exactly the same as for the curved chord Pratt truss bridges, previously given.

MISCELLANEOUS TRUSSES

210. Warren Trusses.—The truss shown in Fig. 344 is known as the Warren truss. In case of a through bridge, there would be a floor beam at each of the joints B, C, and D.

Dead-Load Stresses. Let P represent the dead load per panel at each of the upper joints and W the dead load per panel at each of the lower joints and let R represent the end reaction due to these loads. The dead-load stresses in the web members are indicated at (a) in Fig. 344. These expressions can be written from mere inspection. As is readily seen,

$$R=(1\tfrac{1}{2}W+2P).$$

Taking moments about b of the forces to the left of that joint and dividing this moment by h, we obtain

$$S = (1\tfrac{1}{2}W + 2P)\frac{d}{h} = (1\tfrac{1}{2}W + 2P)\tan\theta$$

for the stress in bottom chord AB. Taking moments about B of the forces to the left of that joint, we obtain

$$S1 = (1\tfrac{1}{2}W + 2P)2\frac{d}{h} - P\frac{d}{h} = 3(W + P)\tan\theta$$

for the stress in top chord bc.

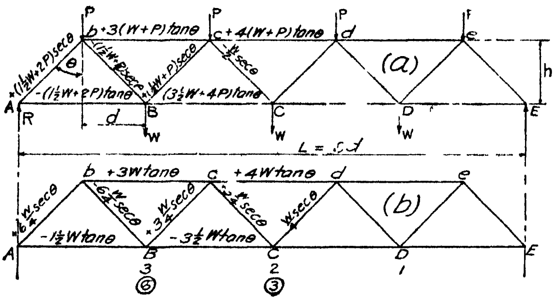


Fig 344

In a similar manner, taking moments about c, we obtain

$$S2 = (3\tfrac{1}{2}W + 4P)\tan\theta$$

for the stress in bottom chord BC and by taking moments about C, we obtain

$$S3 = 4(W + P)\tan\theta$$

for the stress in top chord cd.

From this it is seen that the dead-load stress in any Warren truss is easily determined.

Live-Load Stresses. In case the live load be a uniform load, the stresses are very easily determined. Suppose the live load to be w pounds per foot of truss. Then we have

$$2d \times w = W$$

for the live-load panel load.

Loading joint D (alone), we have

$$\frac{W}{4}\sec\theta$$

for the maximum live-load compression in diagonal dC (as indicated at (b)). Loading joints D and C, we have

$$3\frac{W}{4}\sec\theta$$

for the maximum live-load stress in each of the diagonals Cc and cB, tension in cC and compression in cB.

Loading joints D, C, and B, we have

$$6\frac{W}{4}\sec\theta$$

for the maximum live-load stress in diagonal bB and end post bA, tension in bB and compression in bA.

The chord stresses will be a maximum when the span is fully loaded; that is, when joints D, C, and B are loaded. The reaction at A is then equal to $1\frac{1}{2}W$. Then taking moments about b we have

$$1\tfrac{1}{2}W\times\frac{d}{h}=1\tfrac{1}{2}W\tan\theta$$

for the maximum live-load stress in bottom chord AB. Taking moments about B, we have

$$(1\tfrac{1}{2}W)2\frac{d}{h}=3W\tan\theta$$

for the maximum live-load stress in top chord bc. Taking moments about c, we have

$$(1\tfrac{1}{2}W)3\frac{d}{h}-W\frac{d}{h}=3\tfrac{1}{2}W\tan\theta$$

for the maximum live-load stress in bottom chord BC. Taking moments about joint C, we have

$$(1\tfrac{1}{2}W)4\frac{d}{h}-W2\frac{d}{h}=4W\tan\theta$$

for the maximum live-load stress in top chord cd.

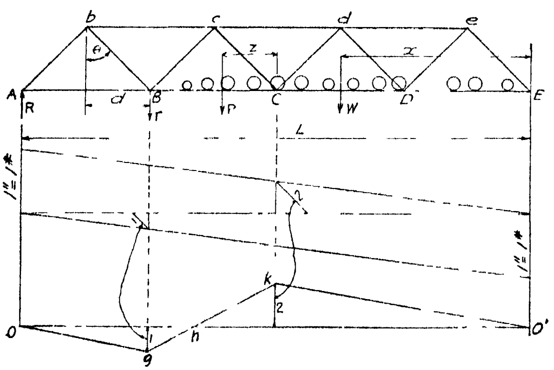


Fig. 345

From this it is seen that the stresses in any Warren truss due to a uniform live load are easily determined.

The stresses in Warren trusses, due to wheel loads, are very readily determined. As an example, let it be required to determine the maximum live-load tensile stress in diagonal cC. The span would be loaded as shown in Fig. 345. Let P represent the load in panel BC, the center of gravity of which is z distance from C; and let W represent the total load on the bridge, the center of gravity of which is x distance from E. Then by taking moments about E, we obtain

$$R = \frac{Wx}{L}$$

for the reaction at A and taking moments about C, we obtain

$$r = \frac{Pz}{2d}$$

for the concentration at B.

Then for the live-load tensile stress in cC, we have

$$S = (R - r)\sec\theta = \left(\frac{Wx}{L} - \frac{Pz}{2d}\right)\sec\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Now suppose the loads move a very short distance to the right or left, say to the left, we have

$$S \pm \Delta S = \left(\frac{Wx}{L} + \frac{W\Delta x}{L}\right)\sec\theta - \left(\frac{Pz}{2d} + \frac{P\Delta z}{2d}\right)\sec\theta$$

for the stress in cC.

Subtracting (1) from this last equation, we obtain

$$\Delta S = \left(\frac{W\Delta x}{L}\right)\sec\theta - \left(\frac{P\Delta z}{2d}\right)\sec\theta$$

for the increment of the stress in cC. Now this increment would be zero if the stress in cC were a maximum and hence the last equation would become

$$0 = \frac{W\Delta x}{L} - \frac{P\Delta z}{2d}.$$

But $\Delta x = \Delta z$, as is readily seen, so we have

$$0 = \frac{W}{L} - \frac{P}{2d} \text{ or } \frac{W}{L} = \frac{P}{2d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2),$$

when the stress in diagonal cC is a maximum. Expressing this in words, the unit load on the bridge is equal to the unit load in panel BC when the maximum live-load tensile stress in diagonal cC occurs. A load will be at C. The maximum compression in diagonal cB, which is equal to the tension in cC, occurs at the same time. To obtain maximum tension in bB and maximum compression in bA, the span would be loaded from the right with a wheel at B and panel AB loaded according to the above equation (2).

Let it be required to determine the maximum live-load stress in bottom chord BC. The span would be loaded as shown in Fig. 346. Let W represent the total load on the bridge, the center of gravity of which is x distance from E. Let $P1$ represent the load to the left of B, the center of gravity of which is z distance from B, and let $P2$ represent the load in panel BC, the center of gravity of which is y distance from C. Then we have

$$R = \frac{Wx}{L} \text{ and } r = (P2)\frac{y}{2d}.$$

Taking moments about c, we obtain

$$S = \left[\left(\frac{Wx}{L}\right)a - P1(z+d) - (P2)\frac{y}{2}\right]\frac{1}{h}$$

for the stress in BC.

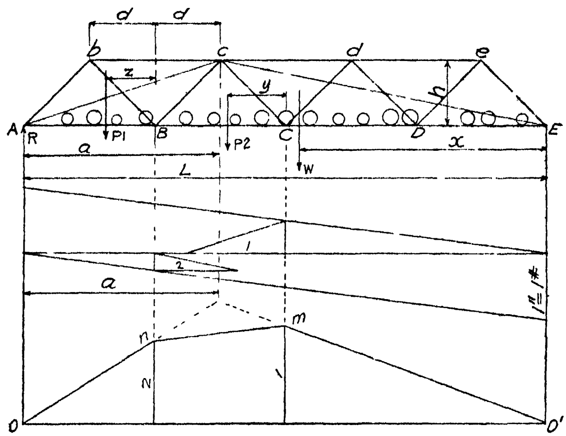


Fig. 346

Now, suppose the loads all move a short distance Δx, then we would have

$$\Delta S = \left[\left(\frac{W\Delta x}{L}\right)a - P1\Delta z - \tfrac{1}{2}(P2)\Delta y\right]\frac{1}{h}$$

for the increment of the stress in chord BC. But if the stress in BC were a maximum, this increment would be zero and we would have

$$0 = \left[\left(\frac{W\Delta x}{L}\right)a - P1\Delta z - \tfrac{1}{2}(P2)\Delta y\right]\frac{1}{h}.$$

But $\Delta x = \Delta z = \Delta y$, so this last equation reduces to

$$0 = \frac{Wa}{L} - P1 - \tfrac{1}{2}P2,$$

from which we obtain

$$\frac{W}{L} = \frac{\tfrac{1}{2}P2 + P1}{a} \qquad \text{. . . . . (3).}$$

This last equation shows clearly how the loads should be placed for maximum stress in bottom chord BC. The case of the other bottom chords is similar. The stress in the top chords is obtained by taking moments about the bottom chord joints. The placing of the loads in that case is simply a matter of applying (5) of Art. 91.

The influence line for stress in diagonal cC is shown in Fig. 345 and the influence line for stress in bottom chord BC is shown in Fig. 346. These are readily understood; in fact, the drawing of the influence line

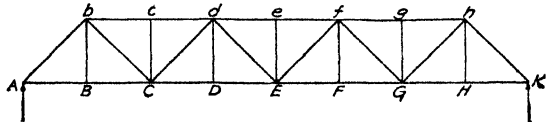


Fig 347

for any of the members in a Warren truss is a simple problem, provided the work previously given on influence lines is understood.

There are very few out-and-out Warren trusses (such as shown in Fig. 344) built. They usually have vertical posts and hangers, as shown in Fig. 347. In that case the loading for maximum stress and the determination of the stresses are the same as for an ordinary Pratt truss.

Referring to Fig. 347, the stress in hangers dD and fF is the same as in hangers bB and hH.

The posts cC, eE and gG in the case of through bridges have only dead-load stress, which is due to the load at the top chord joints. To obtain the maximum live-load stress in bottom chord CE, due to wheel loads, a wheel would be placed at D, such that the average unit load to the left of D would equal the average unit load on the bridge. The stress would then be obtained by taking moments about d. For maximum stress in top chord df the span would be loaded in reference to joint E.

That is, a wheel would be placed at E, such that the average unit load to the left of E would be equal to the average unit load on the bridge. The stress in df would then be determined by taking moments about E.

212. Double-System Warren Truss.—The truss shown at (a), Fig. 348, is known as a "double-system" Warren truss. The dead-load stresses and stresses due to a uniform live load can be determined very readily by considering the truss composed of two independent trusses, one of which is shown at (b) and the other one at (c).

The stresses are then determined in each of these trusses and combined; thus the stress in the structure as a whole is obtained.

Dead-Load Stresses. Let P represent the dead load per panel at each top joint and let W represent the dead load per panel at each bottom joint. The end reaction on the truss shown at (b) is

$$R1 = (2W + 1\tfrac{1}{2}P).$$

Then the dead-load stress in the web members is as indicated. Taking moments about b, we have

$$S = (2W + 1\tfrac{1}{2}P)\frac{d}{h} = (2W + 1\tfrac{1}{2}P)\tan\theta$$

for the stress in bottom chord AC. Taking moments about C, we have

$$S1 = (2W + 1\tfrac{1}{2}P)2\frac{d}{h} - \tfrac{1}{2}(W + P)\frac{d}{h} = (3\tfrac{1}{2}W + 2\tfrac{1}{2}P)\tan\theta$$

for the stress in top chord bd. Taking moments about d, we have

$$S2 = (2W + 1\tfrac{1}{2}P)3\frac{d}{h} - \tfrac{1}{2}(W + P)2\frac{d}{h} - \frac{Wd}{h} = (4W + 3\tfrac{1}{2}P)\tan\theta$$

for the stress in bottom chord CE. Taking moments about E, we have

$$S3 = (4\tfrac{1}{2}W + 3\tfrac{1}{2}P)\tan\theta$$

for the stress in top chord de. The dead-load stresses indicated on the truss shown at (c) are obtained in a similar manner. By combining the stresses given on the top and bottom chords and end posts at (b) and (c) the dead-load stresses in the truss as a whole are obtained.

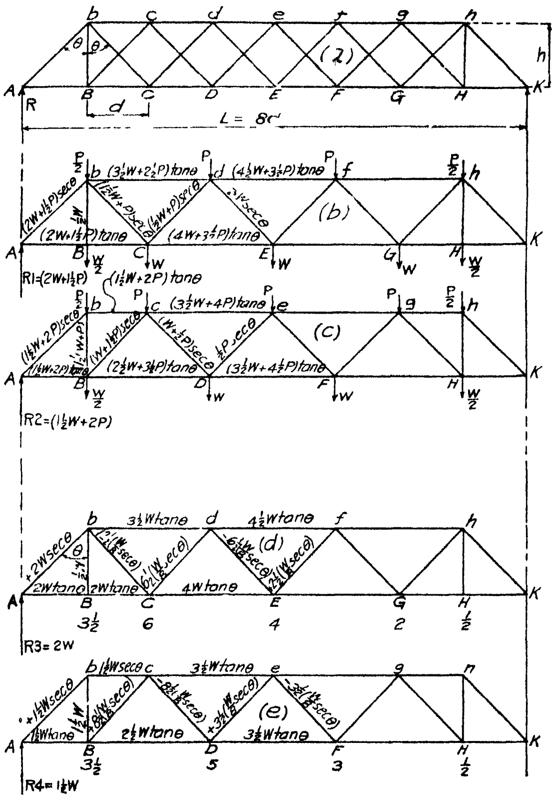


Fig 348

Live-Load Stresses.—Let w represent a uniform live load per foot of truss. Then we have

$$wd = W$$

for the live load per panel. Referring to the truss shown at (d) and considering $\frac{1}{2}W$ at H and W at G, we obtain

$$2\tfrac{1}{2}\left(\frac{W}{8}\right)\sec\theta$$

for the maximum live-load compressive stress in diagonal Ef, and by considering $\frac{1}{2}W$ at H and W at each of the joints G and E, we obtain $6\frac{1}{2}(W/8)\sec\theta$ for the maximum live stress in each of the diagonals dE and dC, tension in dE, and compression in dC. Considering $\frac{1}{2}W$ at H and W at each of the joints G, E, and C, we obtain $12\frac{1}{2}(W/8)\sec\theta$ for the maximum live-load tensile stress in diagonal bC. Considering $\frac{1}{2}W$ at H and W at each of the joints G, E, and C and $\frac{1}{2}W$ at B, we obtain $2W\sec\theta$ for the live-load stress in end post bA. When the truss at (d) is fully loaded, that is, W at each of the joints C, E, and G, and $\frac{1}{2}W$ at each of the joints H and B, we obtain, by taking moments about b,

$$(2W)\frac{d}{h}=2W\tan\theta$$

for the stress in bottom chords ABC. Taking moments about C, we obtain

$$(2W)2\frac{d}{h}-(\tfrac{1}{2}W)\frac{d}{h}=3\tfrac{1}{2}W\tan\theta$$

for the stress in top chord bd. Taking moments about d, we obtain

$$(2W)3\frac{d}{h}-(\tfrac{1}{2}W)2\frac{d}{h}-(W)\frac{d}{h}=4W\tan\theta$$

for the live-load stress in bottom chord CE and taking moments about E, we obtain $4\tfrac{1}{2}W\tan\theta$ for the live-load stress in top chord df. The stresses shown at (e) are obtained in a similar manner. Then combining the stresses given on the top and bottom chords and end post at (d) and (e) the live-load stresses in the truss as a whole are obtained.

In case wheel load be used, the stresses can be determined most readily by the use of influence lines. As an illustration, let it be required to determine the stress in chord de (Fig. 349) due to wheel loads. First construct the influence lines for shear as shown at (b). E is the center of moments when the system drawn in full is considered. Then drawing ms parallel to OE, the distance ns is obtained, which gives the stress in de, due to a unit load at E. Then, laying off at (c) the ordinate $b=ns$, the influence line C-4-B is obtained. D is the center of moments when the dotted system is considered. Then drawing ut parallel to OD the distance vt is obtained, which gives the stress in de due to a unit load at D. Then laying off the ordinate $a=vt$, the influence line C-3-B is obtained. Let us consider a single load passing over the span starting from K. When it reaches joint H half of it is transmitted to the two systems. Then, evidently, the point half way between the two influence lines at that point will be on the influence line for stress in de. When the load reaches joint G, the load will be supported by the system shown in full and, hence, the point 6 will be on the influence line for stress in de. When the load reaches joint F, the load will be supported by the dotted system and, hence, the point 5 will be on the influence line for stress in de. Tracing out in this manner, we obtain the influence line B-7-6-5-4-3-2-1-C for stress in de. Then placing the wheel loads (mostly by trial) for maximum stress in de and multiplying each load by its respective ordinate to this zigzag influence line the maximum stress in de, due to wheel loads is obtained. The maximum stress in the other top chord members is obtained in the same manner.

The influence line for the stress in bottom chord DE is shown at (d). Considering the system drawn in full, the center of moments in determining the stress in DE is at d. Drawing mx parallel to Ad, the distance nx is obtained, which gives the stress in DE, due to a unit load at E and drawing or parallel to SA, the distance ko is obtained, which gives the stress in DE, due to a unit load at C. Then laying off at (d) the ordinates d and c equal, respectively, to nx and ko, the influence line E-9-11-F is obtained. Considering the dotted system, e would be the center of moments in determining the stress in DE. Drawing wy parallel to *Ae*, the distance *ey* is obtained, which gives the stress in *DE* due to a unit load at *F*, and drawing *zc* parallel to *Ke*, the distance *vz* is obtained, which gives the stress in *DE*, due to a unit load at *D*. Now, laying off at (d) the ordinates *f* and *e* equal, respectively, to *ey* and *vz*, the influence line *E*-10-12-*F* is obtained. Then by considering a single load to move over the span as in the above case, the influence line *F*-14-13-12-11-10-9-8-*E* for the stress in bottom chord *DE* is obtained. By placing the wheel loads (by trial) for maximum stress in *DE* and

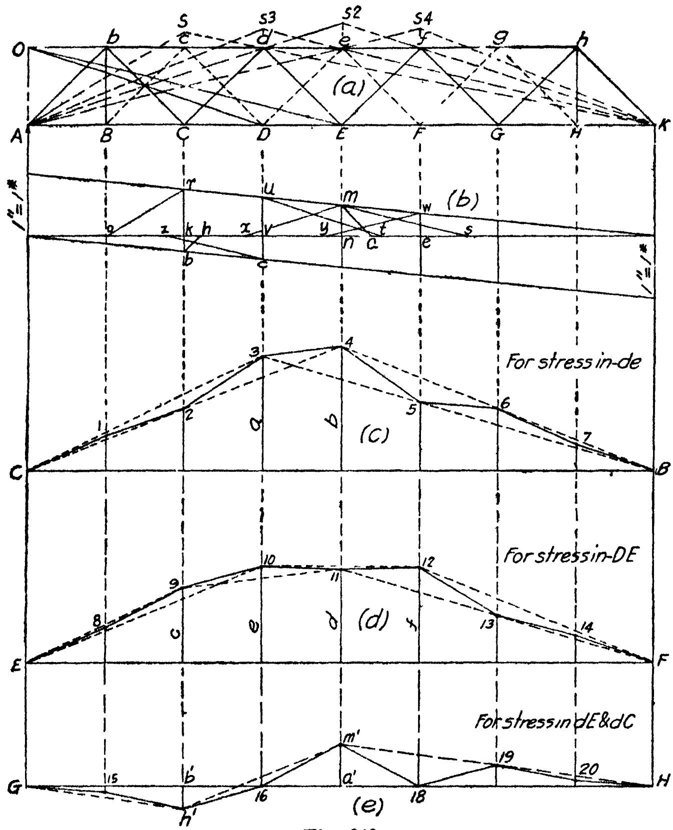


Fig. 349

multiplying each load by its respective ordinate to this influence line, the maximum live-load stress in bottom chord *DE* will be obtained. The stress in any of the other bottom chord members can be determined in the same manner.

Let it be required to determine the stress in diagonal *dE*. Drawing *am* parallel to diagonal *dE*, we have the stress in the diagonal due to a unit load at *E*, and drawing *bh* parallel to *dC*, we have the stress in diagonal *dC*, also diagonal *dE*, due to a unit load at *C*. Then laying off at (e) the ordinates *a'm'* and *b'h'* equal, respectively, to *am* and *bh*, the influence line *H-m′-h′-G* is obtained. Then by considering a single load to move over the span, the influence line *H*-20-19-18-*m′*-16-*h′*-15-*G* for the stress in diagonal *dE* is readily drawn. Then by placing the wheel loads (by trial) on the part of this influence line to the right of 16 and multiplying each load by its respective ordinate the maximum

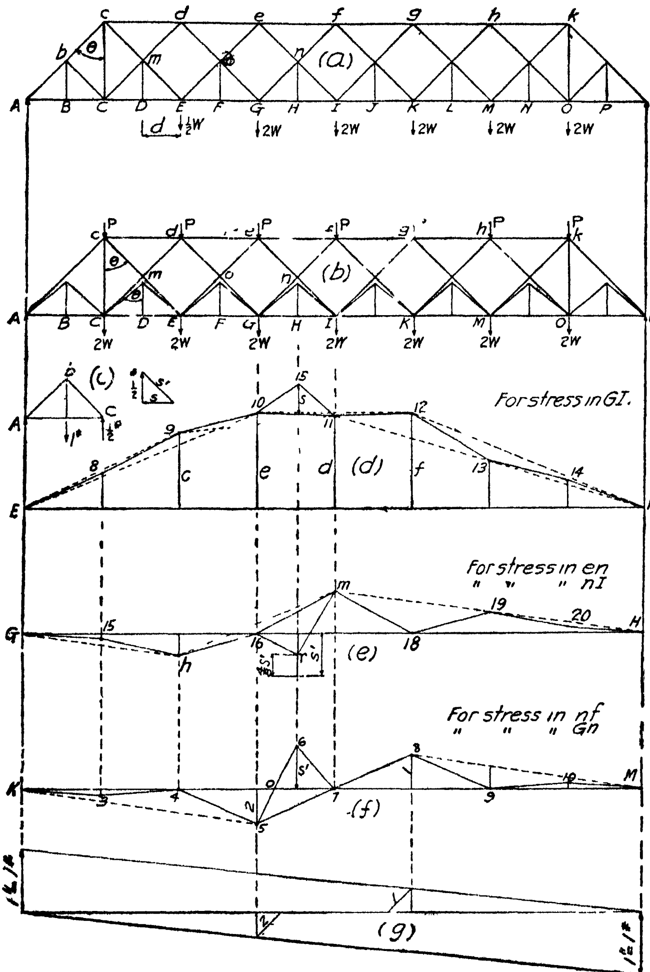


Fig. 350

tensile stress in diagonal *dE* is obtained. At the same time the maximum compression in diagonal *dC* is obtained, as the stress in *dE* and *dC* are equal and opposite. To obtain the maximum compression in *dE* (also maximum tension in *dC*) the wheel loads would be placed to the left of 16 and each load multiplied by its respective ordinate. The stress in any of the other web members due to wheel loads can be determined in the same manner.

In order to obtain short panels, double-system Warren trusses are sometimes sub-paneled as shown at (a), Fig. 350. The stresses are determined in very much the same manner as shown above for the trusses without sub-paneling.

Considering the case of dead load, let P represent the panel load at each upper panel point and W the load at each lower panel point. We can consider the sub-paneling as being independent sub-trusses or trussed stringers as indicated at (b) and that the load from these sub-trusses is transmitted directly to the main lower panel points, in which case there would be $2W$ on each.

Then by considering the truss to be composed of two separate trusses as shown in Fig. 348 the stress in all the main members can be readily determined and then considering the additional stress in each bottom chord and in the lower half of each diagonal due to the sub-trusses the dead-load stresses throughout the truss are obtained.

Let it be required to determine the dead-load stress in diagonal *eo*. The truss being symmetrical about *I* one-half of the load at that point can be considered as transmitted by *eo* and also the load P at *e*. Then we have $(W+P)\sec\theta$ for the dead-load stress in *eo*. This same stress will be in *oE* and, in addition, the load W at F will produce a compressive stress of $\frac{1}{2}W\sec\theta$ in the member which should be added to the above. So we have $(W+P)\sec\theta+\frac{1}{2}W\sec\theta$ for the total dead-load stress in diagonal *oE*.

For the dead-load stress in diagonal *od*, we have $\frac{1}{2}P\sec\theta$ from panel point *f* and $2W\sec\theta$ from panel point G. So for the total dead-load stress in *od*, we have $2W\sec\theta+\frac{1}{2}P\sec\theta$. This same stress would be in *oG* (so to speak) but the load W at F would produce a compressive stress of $\frac{1}{2}W\sec\theta$, which should be subtracted from the above and hence we obtain $2W\sec\theta+\frac{1}{2}P\sec\theta-\frac{1}{2}W\sec\theta$ for the total dead-load stress in diagonal *oG*.

The dead-load stress in each of the hangers *bB*, *mD*, *oF*, etc., is equal to W. The stress in the bottom chord of each sub-truss, as *CmE*, is equal to $\frac{1}{2}W\tan\theta$. This must be added to the stress found in each bottom chord when considering the truss as composed of two independent trusses. That is, the stress in each bottom chord is determined by considering the truss to be composed of two independent trusses ($2W$ at each lower joint), just the same as shown above in the case of trusses without sub-paneling, and then the stress ($\frac{1}{2}W\tan\theta$) in the bottom chord of the sub-truss is added to this.

The stress in the top chords and upper half of the diagonals is just the same as if there were no sub-paneling.

The stress in *bc* (the upper half of the end post) is equal to the shear in panel *AC* multiplied by $\sec\theta$. So we have $(7W+3\frac{1}{2}P)\sec\theta$ for the dead-load stress in *bc*. This same stress occurs in *Ab* and an additional amount of $\frac{1}{2}W\sec\theta$, due to the load at B. So for the stress in *Ab*, we have $(7W+3\frac{1}{2}P)\sec\theta+\frac{1}{2}W\sec\theta$. For the dead-load stress in bottom chord *AC*, we have $(7W+3\frac{1}{2}P)\tan\theta+\frac{1}{2}W\tan\theta$. The part, $\frac{1}{2}W\tan\theta$, is due to the load W at B.

In determining the stress in the hanger *cC* the load at *c*, *E*, *e*, and *I* can be considered as not affecting that member. Then we have $\frac{1}{2}P$ from *f*, $2W$ from G, P from *d* and $2W$ from C, making in all $(4W+1\frac{1}{2}P)$, which can be considered to be the stress in *cC*.

In determining the live-load stresses the work is greatly simplified by using an equivalent uniform load.

Let w represent the live load per foot of span. Then we have $wd = W$ for live load per panel. As an example, let it be required to determine the maximum live-load tension in diagonal od (shown at (a), Fig. 350). We would load all panels from P to F (inclusive). This would be equivalent to placing $2W$ at each of the joints O, M, K, I, and G and $\frac{1}{2}W$ at E. The load at O, K, and G are the only ones that produce stress in od. So we have $[(W)1/8 + (2W)3/8 + (2W)5/8]\sec\theta$ for the maximum live-load tension in diagonal od.

In determining the maximum live-load tensile stress in diagonal oG the load at F would be omitted as it would cause compression in that member. That is, the panels from P to G (inclusive) would be loaded which would be equivalent to placing $2W$ at each of the joints O, M, K, and I and $1\frac{1}{2}W$ at G. Then ignoring the load at I and M, we have $[(W)1/8 + (2W)3/8 + (1\frac{1}{2}W)5/8]\sec\theta$ for the maximum live-load tensile stress in diagonal oG.

To determine the maximum live-load compression in od, panel points B, C, and D would be loaded, in which case the stress in od would be $1/8(2W)\sec\theta$, which is due to the $2W$ at C.

In the case of diagonal oG, the maximum compression in it would occur when panels B to F (inclusive) were loaded.

The load at E would produce no stress in oG. One-half of the load at F would be transmitted to G. This would produce a compressive stress of $\frac{1}{2}W\sec\theta$ in oG but five-eighths of this would be transmitted back, so we would have $(\frac{1}{2}W\sec\theta)\frac{3}{8}$ for the compressive stress in oG due to the load W at F. The load $2W$ at C is the only load, except the one at F, that affects diagonal oG. So we have $[1/8W + (\frac{1}{2}W)3/8]\sec\theta$ for the maximum live-load compressive stress in diagonal oG.

The maximum live-load tensile and compressive stresses in any of the diagonals due to a uniform live load can be determined in the same manner as shown above for diagonals oG, od, eo, and oE. The maximum stress in the chord members due to a uniform live load occurs when all the joints from B to P (inclusive) are loaded. The sub-paneling is first ignored and a load of $2W$ is assumed to be at each of the joints C, E, G, I, K, M, and O. Then the stresses in the chords are determined by considering the truss to be composed of two independent trusses as previously explained. But to the stress in each bottom chord thus obtained the stress $\frac{1}{2}W\tan\theta$ must be added. $\frac{1}{2}W\tan\theta$ is the stress in the bottom chord of each sub-truss as CmE, EoG, etc.

In case wheel loads be used, the stresses can be determined most readily by the use of influence lines. The influence lines are easily drawn, some of which are shown in Fig. 350. The influence lines for the top chords are the same as if the truss were not sub-paneled. The influence line E-8-9-10-15-11-12-13-14-F, shown at (d), is for bottom chord GI. The influence line G-15-h-16-m-18-19-20-H, shown at (e), is for diagonal en, and G-15-h-16-r-m-18-19-20-H is for diagonal nI. The influence line K-3-4-5-7-8-9-10-M, shown at (f), is for diagonal nf and K-3-4-5-6-7-8-9-10-M is for diagonal Gn. The construction of these lines will be readily understood by referring to the diagrams at (c) and (g).

213. Whipple Trusses.—The truss shown at (a), Fig. 351, is known as a Whipple truss. Dead-load stresses and stresses due to uniform live load in this type of truss are readily determined by considering the truss as composed of two independent trusses. In determining the dead-load stresses in the truss shown at (a) the truss can be considered as composed of two independent trusses, one of which is shown at (b) and the other one at (c). Let W represent the dead load per panel, one-

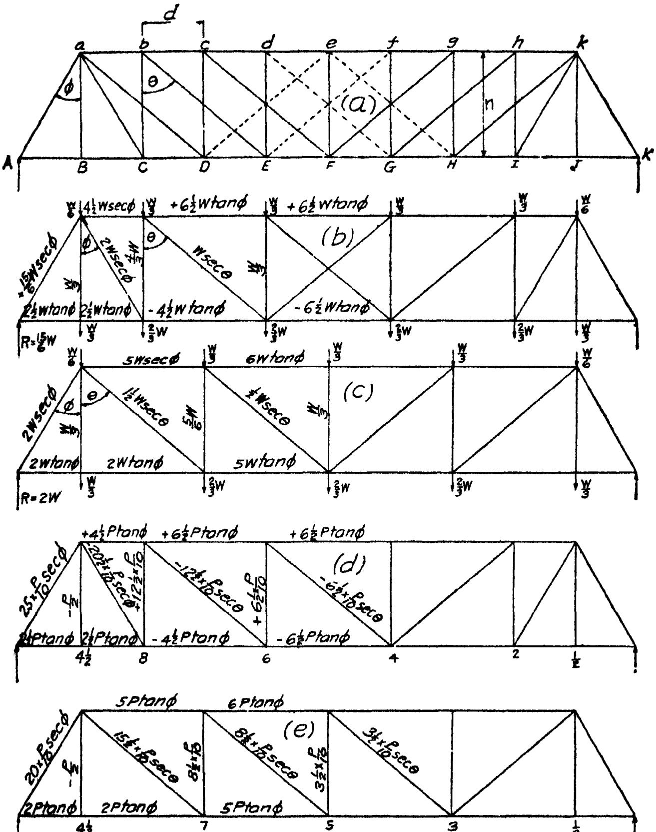


Fig 351

third applied at each top joint and two-thirds at each bottom joint. Taking moments about b and considering the truss at (b), we obtain

$$\left(\frac{15}{6}W\right)2\frac{d}{h}-\left(\frac{3}{6}W\right)\frac{d}{h}=4\frac{1}{2}W\tan\phi$$

for the stress in CE. Likewise taking moments about a and considering the truss at (c), we obtain

$$(2W)\frac{d}{h}=2W\tan\phi$$

for the stress in BD. Then the stress in bottom chord CD is equal to

$$\left(4\frac{1}{2}W\right)\tan\phi+2W\tan\phi=6\frac{1}{2}\,W\tan\phi.$$

Again taking moments about E and considering the truss at (b) we obtain

$$\left(\frac{15}{6}W\right)4\,\frac{d}{h}-\left(\frac{3}{6}W\right)3\,\frac{d}{h}-(W)2\,\frac{d}{h}=6\,\frac{1}{2}\,W\tan\phi$$

for the stress in bd and taking moments about D and considering the truss at (c), we obtain

$$(2W)3\frac{d}{h}-\left(\frac{1}{2}\,W\right)2\,\frac{d}{h}=5W\tan\phi$$

for the stress in ac. Then we have

$$\left(6\frac{1}{2}\,W\right)\tan\phi+5W\tan\phi=11\frac{1}{2}\,W\tan\phi$$

for the dead-load stress in top chord bc. The dead-load stress in any chord member can be readily determined in this manner.

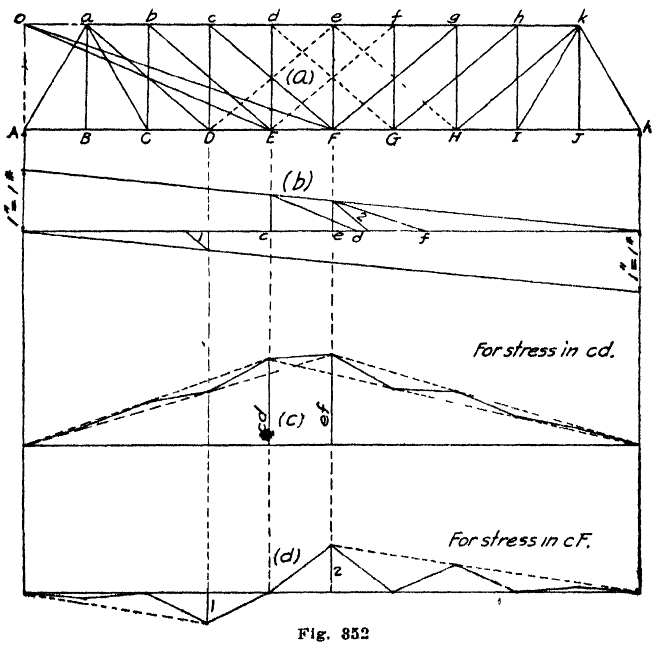


Fig. 852

Considering the trusses at (b) and (c) the dead-load stresses in the web members, as indicated, are readily determined.

Let P represent the uniform live load per panel. The live-load stresses in the chord members will then be as indicated on the trusses at (d) and (e). These are determined in exactly the same manner as explained above for the dead-load stresses in the chords. The live-load stresses in the web members are indicated at (d) and (e). These are determined as previously explained. For example, considering the truss at (d) and loading the panels from J to E (inclusive), we obtain

$$\frac{P}{10}\left(\frac{1}{2}+2+4+6\right)\sec\theta=12\frac{1}{2}P\sec\theta$$

for the maximum live-load tensile stress in diagonal bE and also $12\frac{1}{2}\times(P/10)$ for the maximum live-load stress in post bC. Likewise,

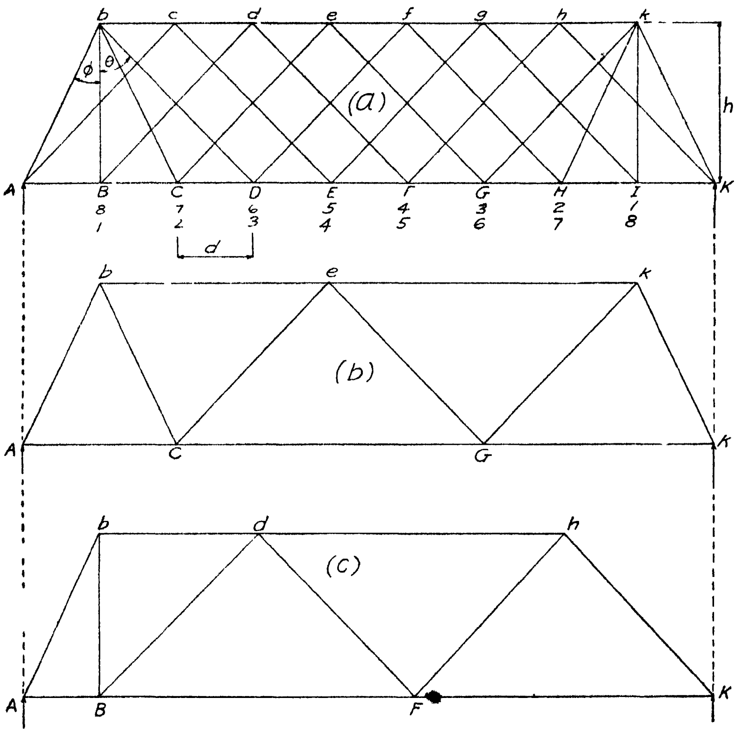


Fig. 858

loading the panels from J to F (inclusive) and referring to the truss at (e), we obtain

$$\frac{P}{10}\left(\frac{1}{2}+3+5\right)\sec\theta=8\frac{1}{2}P\sec\theta$$

for the maximum live-load stress in diagonal cF and also $8\frac{1}{2}\times(P/10)$ for the maximum live-load stress in post cD.

If wheel loads are used the stresses can be determined most readily by use of influence lines. Influence lines for stresses in Whipple trusses are constructed practically in the same manner as shown above for the double-system Warren trusses. Some of these influence lines are shown in Fig. 352. The one at (c) is for top chord *cd* and the one at (d) is for diagonal *cF*. The construction and use of influence lines in the case of the Whipple truss is quite simple provided the work on influence lines previously given is understood.

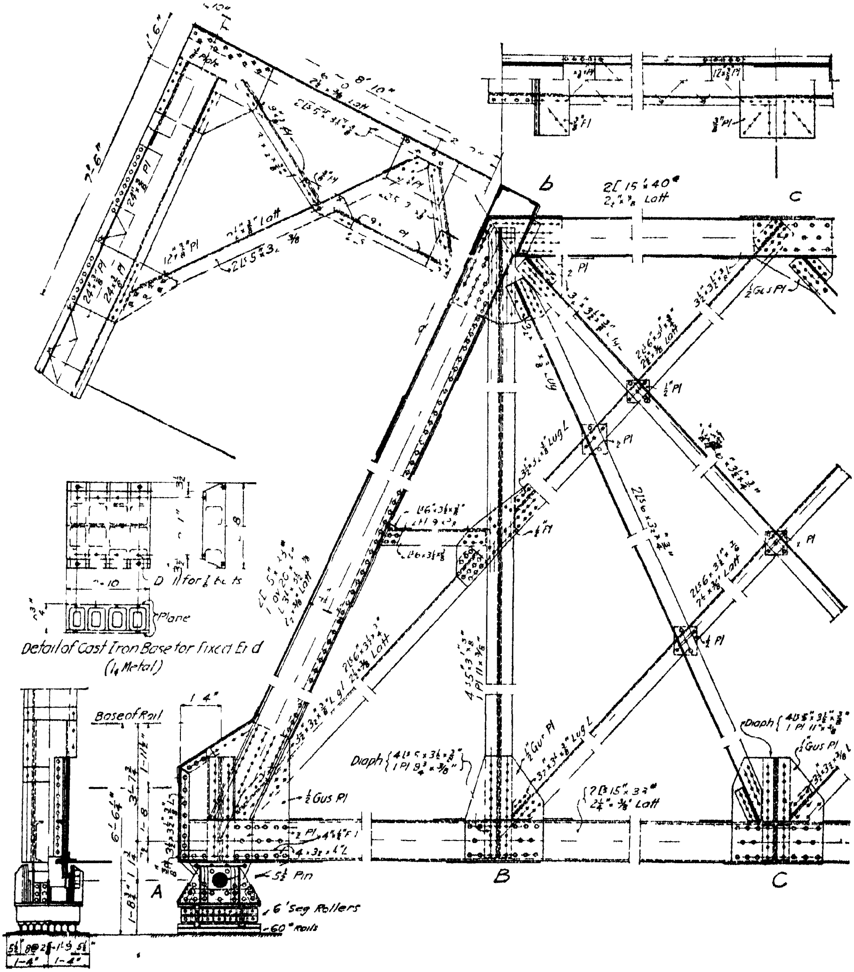


Fig 354

214. Lattice Truss.—The truss shown at (a), Fig. 353, is known as a lattice truss. It can be considered composed of four independent trusses, one of which is shown at (b) and another at (c) and by turning these two end for end, we obtain the other two trusses. By determining the dead and maximum live-load stress in each member of the trusses shown at (b) and (c) and combining the stresses, the stress in the truss shown at (a) is obtained. The determination of the dead-load stress is a simple problem and the determination of the live-load stresses is practically as simple provided a uniform live load be used. Let P represent the uniform live load per panel. Considering the truss at (b) and loading joint C (only) we obtain $\frac{8}{9}P\sec\theta$ for the live-load tension in eC and compression in eG. Loading joint G (only) we obtain $\frac{8}{9}P\sec\theta$ for the live-load tension in eG and compression in eC. Loading both joint C and G we obtain $\frac{10}{9}P\sec\phi$ for the maximum live-load tension in bC and $\frac{8}{9}P\sec\theta$ for the maximum live-load tension in kG, also $\frac{10}{9}P\sec\phi$ and $\frac{8}{9}P\sec\phi$ for the live-load compression in bA and kK, respectively. Loading joints C and G and taking moments about B, we obtain

$$\left(\frac{10}{9}P\right)\frac{d}{h}=\left(\frac{10}{9}P\right)\tan\phi$$

for the live-load stress in AC. Taking moments about C, we obtain

$$\left(\frac{10}{9}P\right)2\frac{d}{h}=2\left(\frac{10}{9}P\right)\tan\phi$$

for the live-load stress in be. Taking moments about k (considering reaction at K), we obtain

$$\left(\frac{8}{9}P\right)\frac{d}{h}=\left(\frac{8}{9}P\right)\tan\phi$$

for the live-load stress in GK. Taking moments about e (considering the forces to the left), we obtain

$$\left(\frac{10}{9}P\right)4\frac{d}{h}-P\times 2\frac{d}{h}=\frac{32}{9}P\tan\phi$$

for the live-load stress in CG. The stresses due to a uniform live load in the trusses shown at (b) and (c) are readily determined in this manner. Then by combining these carefully for the chords and end posts, the stresses throughout the truss are obtained.

In case wheel loads are used, the stress therefrom can be most readily determined by the use of influence lines. These are constructed very much the same as previously shown for double-system Warren and Whipple trusses.

Several railroad companies use the lattice truss shown at (a). The details of part of one of these trusses are shown in Fig. 354. These details are taken from the standard plans of the Chicago & North Western Railway Company (W. C. Armstrong, engineer of bridges).

DEFLECTION AND CAMBER OF TRUSSES

215. Analytical Determination of Deflection.—Let it be required to determine the deflection of joint C of the truss shown in Fig. 355, due to any loads. In the case of pin-connected trusses the deflection is due to the distortion of the members and to a small clearance between the pins and pin holes, while in the case of riveted trusses the deflection is due wholly to the distortion of the members. We will first consider the deflection of joint C, due entirely to the distortion of the members.

Let u represent the stress in any member as aB, due to a unit load placed at C, and let S represent the stress in the same member due to any loads placed at any panel points.

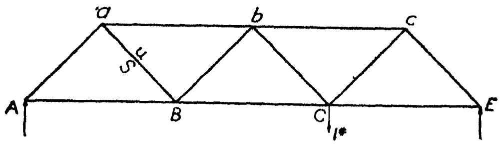


Fig. 355

Let L represent the length of member aB and A its gross area of cross-section. Then for the work done on the member aB by the stress u, we have $u \times \frac{1}{2}(uL/AE)$ which must undoubtedly be equal to $\frac{1}{2}(\Delta d)1\#$, where Δd represents the deflection of joint C, due to the stress u. So we have

$$\frac{1}{2}\left(\frac{uL}{AE}\right) u = \frac{1}{2}(\Delta d)1 \text{ lb.},$$

from which we obtain

$$\Delta d = \left(\frac{uL}{AE}\right) u$$

for the deflection of joint C, due to stress u. Now, evidently, the deflection of joint C, due to any stress S in member aB, will be directly proportional to the deflection Δd, due to stress u. Then let Δy represent the deflection of joint C, due to stress S and we have

$$\frac{\Delta y}{\Delta d} = \frac{S}{u},$$

and substituting $(uL/AE)u$ for Δd and reducing, we obtain

$$\Delta y = \frac{SuL}{AE}$$

for the deflection of joint C, due to any stress S in member aB.

Now, evidently, if u', u'', etc., represent the stress in the other members of the truss, due to a unit load at C, and S', S'', etc., the stress in these members, due to any loads placed at any joints, and $\Delta y'$, $\Delta y''$, etc., the deflection of joint C, due to the stresses S', S'', etc., we can write the formula

$$y = (\Delta y + \Delta y' + \Delta y'' \ldots \Delta y^n) = \left(\frac{SuL}{EA} + \frac{S'u'L'}{EA'} + \frac{S''u''L''}{EA''} \cdots\cdots \frac{S^n u^n L^n}{EA^n}\right)$$

for the total deflection of joint C, due to any loads producing the simultaneous stresses S, S', S'', etc.

By letting S = stress in any member due to any loading,

u = stress in any member due to a unit load at any point considered,

L = length of any member in inches,

A = gross area of cross-section of any member in square inches, the general formula for deflection can be written as

$$y=\Sigma\left(\frac{SuL}{EA}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Example 1. Let it be required to determine the deflection of joint $L3$ of the 150-ft truss shown in Fig. 279, due to the given dead and live loads.

The live-load stresses given for the truss members in Fig. 279 should not be used in determining the deflection as they do not occur simultaneously. The maximum deflection will occur when the span is fully loaded and a sufficiently accurate result will be obtained by using an equivalent uniform live load, in which case the live-load stresses will be directly proportional to the dead-load stresses, and hence they can readily be determined directly from the dead-load stresses by the use of a slide rule.

The maximum live-load stress in the end post (see Fig. 279) is 263,000 lbs. Dividing this by the secant of the slope of the post, we obtain

$$263{,}000 \div 1.3 = 203{,}000 \text{ lbs.}$$

for the maximum live-load shear in the end panel $L0$-$L1$ (see Art. 174). Then substituting this value for S in the formula of Art. 123, we obtain

$$P' = (203{,}000) \div \tfrac{1}{2}(L-d) = 203{,}000 \div 62.5 = 3{,}250 \text{ lbs.}$$

for the equivalent uniform live load per foot of truss for determining the live-load stress in the web members, but as the chords contribute most to the deflection we will use

$$3{,}250 - 3{,}250\left(\tfrac{150}{100} + 2.5\right)\% = 3{,}120 \text{ lbs.}$$

per foot of truss for the equivalent uniform live load, which is really the correct equivalent uniform live load for determining the maximum chord stresses. (See Art. 123.)

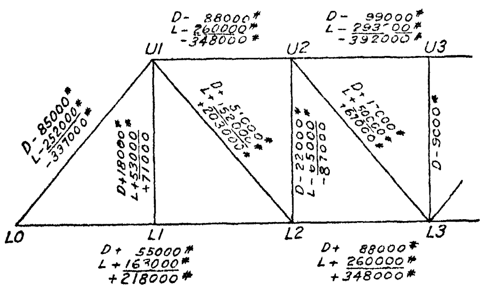


Fig. 356

As seen from Fig. 279, the dead load per foot of truss is $2{,}110 \div 2 = 1{,}055$ lbs. Then multiplying each of the dead-load stresses given for the truss members in Fig. 279 by $\frac{3{,}120}{1{,}055}$ the live-load stresses for determining the deflection of joint $L3$ are obtained. Then adding these to the dead-load stresses, the total stress in each truss member, as shown in Fig. 356, is obtained.

Then from Figs. 279 and 356 the following table can be computed:

Member	L Ins.	S Lbs.	A Sq. Ins.	$\frac{SL}{EA}$	u Lbs.	$\frac{SuL}{EA}$
$U1$-$L0$ ...	468	−337,000	46.18	−0.1138	−0.6500	0.0739
$L0$-$L2$ ...	600	+218,000	23.52	+0.1853	+0.4166	0.0772
$L2$-$L3$ ...	300	+318,000	36.98	+0.0941	+0.8332	0.0784
U1-U2 ...	300	−318,000	40.58	−0.0857	−0.8332	0.0714
$U2$-$U3$ ...	300	−392,000	45.08	−0.0869	−1.2198	0.1086
$U1$-$L2$ ...	168	+203,000	26.48	+0.1196	+0.6500	0.0777
$U2$-$L3$ ...	168	−67,000	19.80	+0.0527	+0.6500	0.0343
$U1$-$L1$ ...	360	+71,000	14.44	+0.0590	0	0
$U2$-$L2$ ...	360	−87,000	19.80	−0.0527	−0.5000	0.0263
$U3$-$L3$ ...	360	−9,000	19.80	−0.0054	0	0

($E = 30{,}000{,}000$)

$$\Sigma\left(\frac{SuL}{AE}\right) = 0.5498$$
$$\times 2$$

Total deflection of joint $L3 = 1.0996$ ins.

As $L3$ is at the center of the span and the bridge symmetrically loaded, the deflection of $L3$ can be obtained, as shown by the above table, by determining the deflection, due to just half of the truss members, and then multiplying this deflection by two. In case of the determination of the deflection of any other joint (except $U3$) all of the truss members would have to be considered. For example, let it be required to determine the deflection of joint $L2$ (see Fig. 279). The table in that case would be just the same as the above, except that the last two columns on the right would have two values in each case as the u for the members on one side of the center of span would be different from the u for the corresponding members on the other side of the center of the span. For example, placing a unit load at $L2$, we have four-sixths of a pound for the reaction at one end of the truss, and two-sixths for the reaction at the other end. Then for the u in one end post we would have $\frac{4}{6} \times 1.3 = 0.8333$, and $\frac{2}{6} \times 1.3 = 0.4166$ for the u in the other end post; and likewise $\frac{4}{6} \times 0.8333 = 0.5555$ for the u in one bottom chord LO-$L2$, and $\frac{2}{6} \times 0.8333 = 0.2777$ for the u in the other bottom chord LO-$L2$. Thus it is throughout the span. The last two right-hand columns in the table would simply have double numbers and the summation of the SuL/AE would not be multiplied by two. Otherwise, the table for each of the other joints would be just the same as shown above for joint $L3$.

In case the loading causing the stresses S were placed unsymmetrically, each member would be listed separately in the tables. In that case it is advisable not to have two or more members with the same mark, as in the case of the truss shown in Fig. 279.

It is customary to consider the distortion of compression members as negative, and that of the tension members as positive. In that case the tensile stresses, both u and S, should be indicated as plus, and the compressive stress as negative, as shown in the above table.

In case the longitudinal displacement of any joint be desired, it can be determined in the same manner as shown above for deflection, except that the unit load applied at the joint in question would be considered as acting horizontally instead of vertically. In that case it is necessary to pay strict attention to the signs of the u's and S's, as they may not have like signs, which they ordinarily have in the case of deflection.

As an example, let it be required to determine the horizontal displacement of joint $L3$ of the truss shown in Fig. 279, due to the stresses. The unit load would be applied at $L3$, as shown in Fig. 357. The end $L6$ being fixed and the end LO being on rollers, the only members that would really be stressed by the unit load would be $L3$-$L4$ and $L4$-$L6$, and hence we need consider only these members in determining the horizontal displacement of $L3$. The stress in each of these members, due to the unit force, as is obvious, is 1 lb. Then making the following table, the horizontal displacement of $L3$ is obtained:

Member	L Ins.	S Lbs.	A Sq. Ins.	$\frac{SL}{EA}$	u Lbs.	$\frac{SuL}{EA}$
$L3$-$L4$ ...	300	-348,000	36.98	-0.0911	-1.0000	0.0941
$L4$-$L6$ ...	600	-218,000	23.52	-0.1853	-1.0000	0.1853
		Total horizontal displacement of joint $L3 =$				0.2794 ins.

This shows that joint $L3$ will move about ¼″ to the left. It will be seen that this table is really compiled from the table given above in determining the deflection of $L3$.

The horizontal displacement of any of the other lower chord joints can be determined in a similar manner, and just as readily after the tables for the deflection of those joints are made.

The determination of the horizontal displacement of the top chord joints, while similar to that shown for the bottom chord joints, is not so simple, owing to the fact that more members are involved. As an illus-

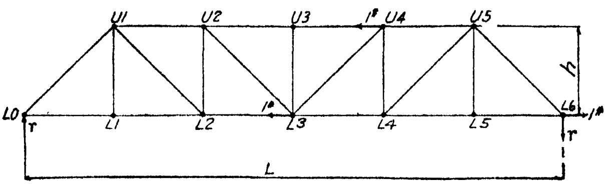


Fig 357

tration, let it be required to determine the horizontal displacement of joint $U4$ (Fig. 357). The unit load would be applied at $U4$ as indicated. The horizontal reaction of this load would be at $L6$, as indicated (the end $L6$ being fixed), but in addition, the unit load at $U4$ would produce a positive reaction of r at LO and an equal negative reaction r at $L6$. These vertical reactions and the 1 lb. horizontal reaction at $L6$ would have to be considered in determining the u's for the members throughout the truss. For each of the vertical reactions we have $r = h/L$. After the u's are determined for the members throughout the truss, the horizontal displacement of the top chord joints is determined by making a table for each as explained above.

In determining the deflection of pin-connected bridges, the distortion (SL/AE) of each member should be increased by $\frac{1}{32}$ in. to allow for the clearance between the pins and pin holes. Otherwise the work of determining the deflection of pin-connected bridges is exactly the same as shown above for the riveted bridge.

216. Graphical Determination of Deflection.—Let ACB at (a), Fig. 358, represent a cantilever frame and suppose a vertical load P be applied at C. This load will cause tension in member AC and compression in member BC and hence the distortion in AC will increase its length while the distortion in BC will decrease the length of that member. Let Ca represent the distortion of AC and Ce the distortion of BC and let us assume that the joints A, C, and B are pin connected so that the members AC and BC are free to turn about their ends. Now, undoubtedly, the distortion of the members AC and BC will cause the frame to deflect to some position as $AC'B$ As the joints A and B are fixed in position the deflection of the frame really consists of joint C moving to C' and, hence, the determination of the deflection is simply a matter of locating C' in reference to joints A and B, which remain fixed. By taking A as a center and Aa as a radius, the arc aC' could be described, and taking B as a center and Be as a radius the arc eC' could be described and thus the point C' would be located.

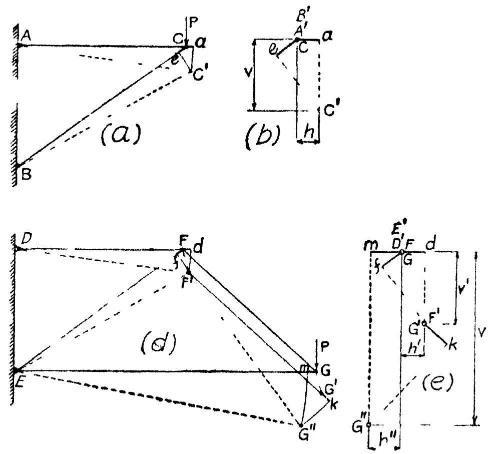


Fig. 358

As another example, let D-F-G-E at (d) represent a cantilever frame supporting a vertical load P at G. Let Fd, Ff, $G'k$, and Gm represent the distortions of the members DF, FE, FG, and GE, respectively. Now, owing to these distortions, the frame will deflect to some position as D-F'-G''-E. As is obvious, the problem of determining the deflection of the frame consists of locating F' and G'' in reference to the fixed joints D and E. First, taking D as a center and Dd as a radius, the arc dF' could be described; and taking E as a center and Ef as a radius the arc fF' could be described; and thus the position of F' would be determined, which is the deflected position of joint F. Now, having joint F located, the deflected position of joint G can be determined in reference to E and F. Suppose, for the time being, that joint G be disconnected and that member FG moves to the parallel position $F'G'$, while EG remains in position: Then, taking F' as a center and $F'k$ as a radius, the arc kG'' could be described; and taking E as a center and Em as a radius, the arc mG'' could be described; and thus the position of G'' would be determined, which is the deflected position of joint G; and hence the deflected position of the entire frame would be known.

Now it is evident that, theoretically, the deflection of any cantilever frame could be determined in the same manner as shown for the above two cases, and as any truss can be considered as being composed of one or more cantilevers, the same is true of any truss. But, in practice, the application of the method is practically impossible, as the distortions of the members are so small compared with their lengths that if the members be laid off to a reasonable scale it would be practically impossible to measure the distortions and deflections. This very fact, however, makes possible the graphical method in general use, in which it is assumed that the distortions are so small in comparison with the lengths of the members that all such arcs as aC'', eC', dF', mG'', etc., without appreciable error can be considered straight lines, that is, the arc and its tangent, in each case, are assumed to coincide. One can satisfy himself as to the accuracy of this assumption by trial. As a suggestion, suppose the distance AB (Fig. 358(a)) be laid off equal to 10 ft.; AC and BC equal to 15 ft. and 18.5 ft., respectively; and Ca and Ce each equal to ⅛ of an in. and the arcs AC' and eC' described, as explained above, and the difference between these arcs and their tangents noted: By assuming that all such arcs as AC' and eC', etc., coincide with their tangents, the deflection of any truss can be determined by simply laying off the distortions and drawing perpendiculars to these. As an example, let us consider the case shown at (a), Fig. 358. The joints A and B are considered fixed. Let us take any point A' at (b), as origin, to determine the deflection of the joints A, C, and B. Now, as both A and B are fixed, they will, as regards deflection, be at A' and hence we have A and B located. Joint C in reference to A moves to the right the distance Ca. Then at (b) lay off Ca from A' (the location of joint A) to the right as shown. Joint C in reference to B moves toward B the distance Ce. Then at (b) lay off Ce from A' (the location of joint B), as shown. Then, by drawing from e and a the perpendiculars aC' and eC' we have C' located, which is the deflected position of joint C. That is, as A' is the origin (point of zero deflection), V represents the deflection of joint C, and h represents the horizontal movement of that joint.

Next let us consider the case shown at (d), Fig. 358. Here joints D and E are considered fixed. Let D' at (e) be the origin to determine the deflection of joints D, E, F, and G. As D and E are fixed, they will both be at D', the origin. In reference to joint D joint F moves to the right the distance Fd. So from D' (the location of joint D) lay off Fd to the right as shown. In reference to E joint F moves toward E the distance Ff. So from D' (the location of E) lay off Ff as shown. Then by drawing the perpendiculars dF' and fF' the point F' is located, which is the deflected position of joint F. Now having F located, joint G can be located in reference to joints F and E. In reference to joint F joint G moves away from F the distance $G'k$, as shown. In reference to joint E, joint G moves toward E, or to the left, the distance mG. So from D' (the location of joint E) lay off mG to the left as shown. Then by drawing the perpendiculars kG'' and mG'', the point G'' is located, which is the deflected position of joint G. We then have the deflection of all the joints determined. The distances V' and V'' represent the deflection of joints F and G, respectively, and h' and h'' represent respectively the horizontal movement of these joints.

As another example, let it be required to determine the deflection of the joints of the cantilever frame shown at (a), Fig. 359, where $\Delta 1, \Delta 2 \ldots\ldots \Delta 10$ represent the distortions of the members. The plus sign signifies that the length of the member is increased by the distortion, and the minus sign signifies just the opposite. First, select any point A' at (b) as the origin. The joints A and B are fixed so they will both be at A'. In reference to A, joint C moves to the right the distance $\Delta 1$. So from A' (at (b)) draw $\Delta 1$ to the right. In reference to B, joint C moves toward B the distance $\Delta 4$. So from B' (at (b)) draw $\Delta 4$ as shown, and then drawing perpendiculars to these distortions the point C' is located, which shows the deflected position of joint C. That is, joint C has moved from A' to C'. Now, having C located, we can proceed to locate joint D in reference to joints B and C. In reference to C, joint D moves down the distance $\Delta 5$. So from C' lay off $\Delta 5$ downward as shown. In reference to B, joint D will move toward B the distance $\Delta 9$. So from B' (at (b)) lay off $\Delta 9$ to the left. Then, drawing perpendiculars to these distortions ($\Delta 5$ and $\Delta 9$), we obtain D' which shows the deflected position of joint D. That is, joint D has moved from A' to D'. Now, having both C and D located, we can next locate the joint E. In reference to C, joint E moves to the right the distance $\Delta 2$. So from C' draw $\Delta 2$ to the right. In reference to D, joint E moves toward D the distance $\Delta 6$. So from D', draw $\Delta 6$ as shown. Then by drawing perpendiculars to these distortions ($\Delta 2$ and $\Delta 6$), we obtain E', which shows the deflected position of E. That is, joint E has moved from A' to E'. Now having joints E and D located, we can next locate joint F. In reference to E, joint F moves downward the distance $\Delta 7$. So from E' draw $\Delta 7$ downward. In reference to joint D, joint F moves toward D the distance $\Delta 10$. So from D' draw $\Delta 10$ to the left. Then by drawing perpendiculars to these distortions ($\Delta 7$ and $\Delta 10$), we obtain point F', which shows the deflected position of joint F. That is, joint F has moved from A' to F'. Now, having joints E and F located, we can locate joint G. In reference to E, joint G moves to the right the distance $\Delta 3$. So from E' draw $\Delta 3$ to the right. In reference to F, joint G moves toward F the distance $\Delta 8$. So from F' draw $\Delta 8$ as shown. Then by drawing perpendiculars to these distortions ($\Delta 8$ and $\Delta 3$), we obtain the point G', which shows the deflected position of joint G. That is, joint G has moved from A' to G'.

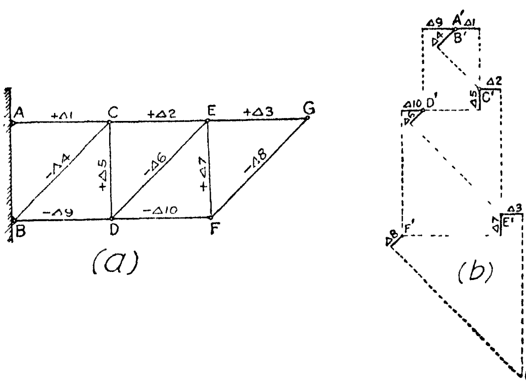


Fig. 359

Referring to the diagram at (b), we can consider all of the joints as being at A' before deflection, and that they deflect to the positions indicated. That is, C moves from A' to C', D from A' to D', E from A' to E', F from A' to F', and G from A' to G'. By measuring the vertical distance down from A' in each case, the deflections are obtained and the horizontal movements are obtained by measuring the horizontal distance in each case from a vertical line through A'.

As all trusses can be considered as being composed of one or more cantilever frames, it is obvious that the deflection of any truss can be determined by constructing diagrams similar to those shown at (b) and (e), Fig. 358, and at (b), Fig. 359.

These diagrams are known as "Williot diagrams," being named after the French engineer Williot, who proposed them.

Example 1. Let it be required to determine the deflection of the truss shown in Fig. 279, due to the stresses given in Fig. 356. The distortions of the members, due to these stresses are given in Example 1 of Art. 216 (see table).

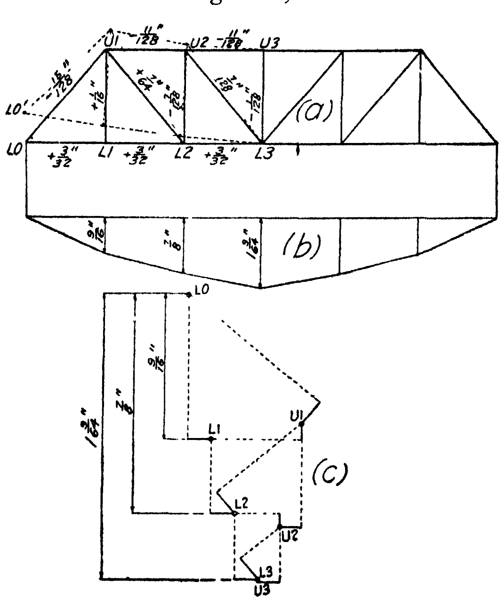


Fig. 360

First, draw the outline of the truss as shown at (a), Fig. 360. The distortions shown there for one-half of the members (they are the same for the other half of the truss) are expressed in fractional form for convenience.

As the truss is symmetrical about $U3$-$L3$ and symmetrically loaded the deflection will be fully known if it be determined for one-half of the truss. By considering joint $L3$ as fixed in position and member $U3$-$L3$ as fixed in direction either half of the truss can be considered as a cantilever and the deflection determined as explained above. Let us take $L3$ at (c) as a starting point. Then the position of joint $U3$ is obtained by laying off the distortion of $U3$-$L3$ (which is $\frac{1}{128}''$) downward, using a scale of $\frac{1}{16}'' = \frac{1}{64}''$. Then having joint $U3$ located, the position of $U2$ can be determined in reference to $U3$ and $L3$. Joint $U2$ moves towards $U3$ and away from $L3$. Then laying off the distortion of member $U2$-$U3$ to the right from $U3$ (at (c)) as shown, and the distortion of member $U2$-$L3$ upward and parallel to $U2$-$L3$ from $L3$ and drawing perpendiculars to these distortions, we obtain point $U2$, which shows the position of joint $U2$. That is, joint $U2$ has moved from $L3$ to $U2$. Joint $L2$ moves toward $U2$ and away from $L3$. Then laying off the distortion of member $U2$-$L2$ upward from $U2$ and the distortion of $L2$-$L3$ to the left from $L3$, and drawing perpendiculars to these distortions, we obtain point $L2$, which shows the position of joint $L2$. Joint $U1$ moves toward $U2$ and away from $L2$. Then laying off the distortion of member $U1$-$U2$ to the right from $U2$, as shown, and the distortion of member $U1$-$L2$

upward and parallel to $U1$-$L2$ from $L2$, as shown, and drawing perpendiculars to these distortions, we obtain point $U1$, which shows the location of joint $U1$. Joint $L1$ moves downward from $U1$ and to the left from $L2$. So laying off the distortion in $U1$-$L1$ downward from $U1$ (at (c)) and the distortion of $L1$-$L2$ to the left from $L2$ as shown and drawing perpendiculars to these distortions we obtain point $L1$, which shows the location of joint $L1$. Joint LO moves toward $U1$ and away from $L1$. Then laying off the distortion of member $U1$-LO upward and parallel to $U1$-LO from $U1$ and the distortion of member LO-$L1$ to the left from $L1$, and drawing perpendiculars to these distortions, we obtain the point LO, which shows the location of joint LO. We now have the position of the joints of the left half of the truss determined at (c) in reference to $L3$. As is obvious, in reference to that point, the left half of the truss would be bent upward, as indicated by the dotted outline, and of course the right half would be bent up the same amount. Now, it is evident that the deflection of the bottom chord joints is the distance they are below a horizontal line through LO'. These distances are obtained for each joint by measuring down from LO (at (c)). We thus obtain $\frac{9}{16}$″, $\frac{7}{8}$″, and $1\frac{9}{64}$″ for the deflection of joints $L1$, $L2$, and $L3$, respectively. Then a diagram of these deflections can be drawn as shown at (b). The deflec-

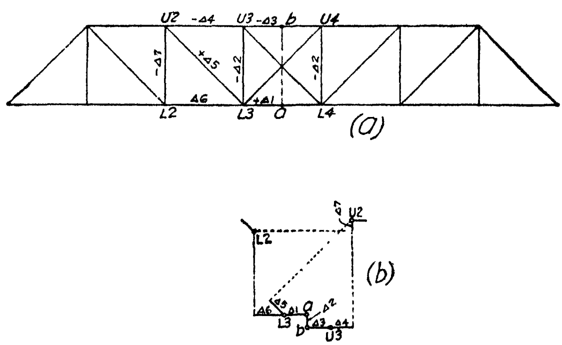


Fig. 361

tion of any ordinary bridge truss can be determined in this manner. In case of odd panels, it is best to begin at the middle of the center panel in constructing the Williot diagram. For example, point a (Fig. 361(a)) would be considered fixed in position and line ab fixed in direction. The diagonal $U3$-$L4$ and $U4$-$L3$ would be ignored, as they would have no stress in them—assuming that a uniform live- and dead-load extended over the entire span. The point b would move downward the distance $\Delta 2$, which is the distortion of each of the posts $U3$-$L3$ and $U4$-$L4$. Then taking a as the starting point (at (b)), the point b is located by laying off $\Delta 2$ downward from a, as shown. Then joint $L3$ is located by laying off $\Delta 1$, which is one-half of the distortion of $L3$-$L4$, to the left from a and joint $U3$ is located by laying off $\Delta 3$, which is one-half of the distortion of $U3$-$U4$, to the right from b. Then, having joints $L3$ and $U3$ located, the work of determining the deflection proceeds as previously explained.

The above manner of determining deflection of trusses is quite accurate as far as vertical movement is concerned, which is usually all that is desired, but if the horizontal movement be desired, the work must be carried out in a somewhat different manner. It will be seen that in the above case we assumed the center of the truss to remain fixed and that each end of the truss was free to move. This assumption is not true, as we know, as one end of the truss is fixed and the other end is free to move.

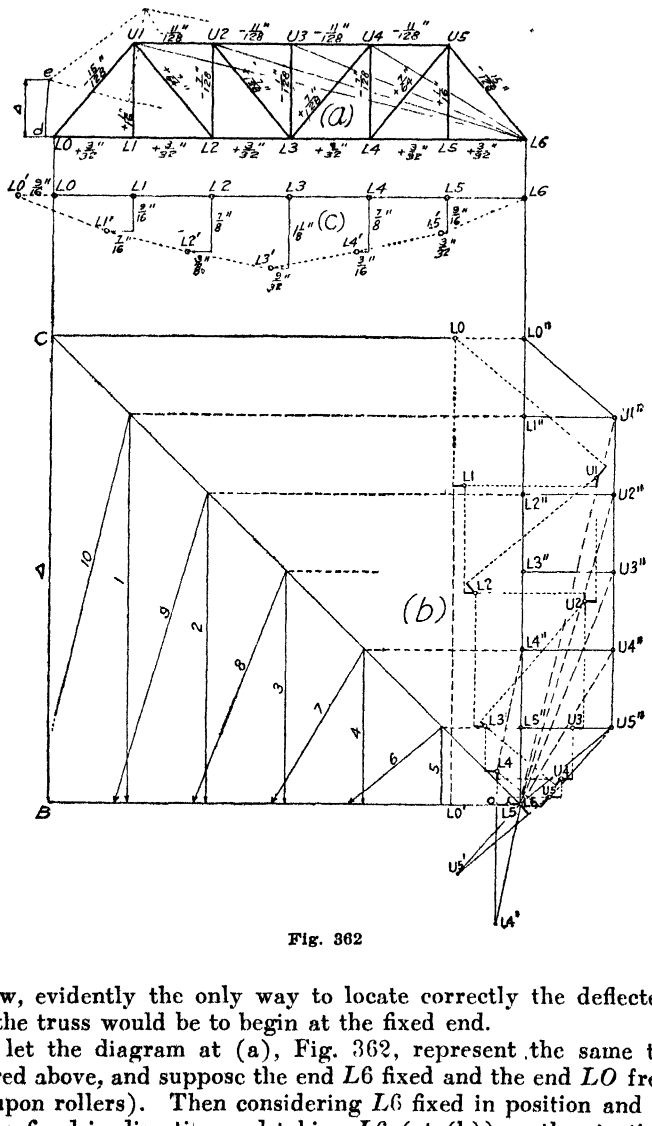


Fig. 362

Now, evidently the only way to locate correctly the deflected position of the truss would be to begin at the fixed end.

So let the diagram at (a), Fig. 362, represent the same truss as considered above, and suppose the end $L6$ fixed and the end LO free (supported upon rollers). Then considering $L6$ fixed in position and member $L6$-$L5$ as fixed in direction and taking $L6$ (at (b)) as the starting point and constructing the Williot diagram as shown, we obtain the relative position of the joints, but the truss will be bent upward (so to speak) as indicated by the dotted outline. So, to obtain the correct position of the joints, the truss must be rotated downward about *L6*, until the joint *LO* is again in the same horizontal line as *L6*. This means that joint *LO* will be rotated through the arc *ed*. This arc is so nearly equal to the vertical distance Δ that it can be considered equal to it. The distance Δ is equal to the vertical distance *LO-LO′*, shown at (b). Now, evidently, all of the other joints of the truss will be revolved through the same angle as *LO* and the arc they describe in each case will be directly proportional to their radius. These arcs are so small that the distance from *L6* to the undeflected position of any joint can be taken as its radius. Then the radius for any joint of the bottom chord is the horizontal distance from *L6* out to the joint. For example, the radius for joint *L1* is *L1-L6*; for *L2* it is *L2-L6*; and so on. The radius for joint *U1* is *U1-L6*; for *U2* it is *U2-L6*; and so on. The arcs can be considered as straight lines perpendicular to these radii.

Then the arcs through which the joints *LO, L1, L2, L3, L4*, and *L5* are rotated are represented, respectively, by the lines Δ, 1, 2, 3, 4, and 5 and the arcs through which joints *U5, U4, U3, U2*, and *U1* are rotated are represented, respectively, by the lines 6, 7, 8, 9, and 10. Now, having the lengths of the arcs through which the joints are rotated, it is an easy matter to determine the true deflected position of any joint. As an example, let us consider joint *L4*. By considering the member *L5-L6* fixed in direction the joints would all move upward to the positions shown on the Williot diagram at (b). That is, each would move from *L6* to the position indicated. Then laying off from *L4* downward the arc 4, we obtain the point *L4′*, which is the true deflected position of joint *L4*. That is, joint *L4* really deflects from *L6* to *L4′* and, hence, its vertical movement is represented by the distance *O-L4′* and its horizontal movement by the distance *L6-O*. As another case, let us consider joint *U5*. The line 6, which is perpendicular to the end post *U5-L6*, represents the arc through which this joint is rotated. So, by laying off this arc from *U5* the point *U5′* is obtained, which is the true deflected position of joint *U5*. That is, joint *U5* really moves from *L6* to *U5′*. The true deflected position of each of the other joints can be determined in this manner, but time can be saved by laying off all of the arcs upward from *L6*. In applying this method let us first consider joint *L4*. Laying off arc 4 upward from *L6*, we obtain point *L4″*. Now, the deflected position of joint *L4* is shown by the line *L4″-L4*. This is readily seen to be true as *L6-L4″* is the arc through which the joint is rotated, and as the joint was already deflected up to *L4*, the difference between the two positions would undoubtedly be the actual deflected distance. In the same manner, laying off arc 7, from *L6*, we obtain point *U4″*. Then the actual movement of joint *U4* is from *U4″* to *U4*. By laying off all the arcs described by the joints from *L6* we will obtain the points *LO″*, *L1″*, *U1″*, *U2″*, etc. If these be connected by lines, we obtain the truss shown, which is perpendicular to the truss shown at (a). We then have the deflected distance of each joint given. For example, the joint *LO* moves from *LO″* to *LO*, joint *L1* from *L1″* to *L1*, joint *U1* from *U1″* to *U1*, joint *U2* from *U2″* to *U2*, and so on. By measuring the vertical and horizontal components of these distances the true deflected position of each joint can be plotted as shown at (c), which in that case is for the bottom chord joints.

Now, it will be seen that all we need do to determine the deflection of the truss is to construct the Williot diagram at (b) and draw the truss *LO″-U1″ .....L6*. This truss can be drawn by dividing the line *L6-LO″* into as many equal divisions as there are panels in the bridge and then locating one of the points *U1″*, *U2″*, etc., which is easily done. For instance, *U1″* is located by drawing from *LO″* a line perpendicular to end post *LO-U1* and drawing from *L1″* a line perpendicular to *L6-LO″*. The entire truss *LO″-U1″ ...L6* of course can be constructed in the manner indicated at (b). That is, by projecting points *L1″*, *L2″*, etc., over from the line *C-L6* and then locating the points *U1″*, *U2″*, etc., by laying off from *L6* the arcs 10, 9, etc.

That the extremities of the arcs laid off from *L6* will form the truss shown at (b) is readily seen. In the first place, there is no question as to the location of the points *LO″*, *L1″*, *L2″ ...L6* and it is seen from the construction that the two points *L4″* and *U4″*, *L3″* and *U3″*, and so on, are in each case on the same horizontal line. So that in reality, the only thing in question is as to whether the points *U1″*, *U2″*, *U3″*, etc., are in the same vertical line. This, however, is readily shown to be true by comparing the construction at (b) with the diagram at (a). The arc *L6-U4″* is perpendicular to the radius *L6-U4*, and, as is seen from the construction *L4″* and *U4″*, is on the same horizontal line and, hence, the triangles *L6-U4″-L4″* and *L6-U4-L4* are similar, the corresponding sides being perpendicular each to each. In the same manner it can be shown that triangle *L6-U3″-L3″* is similar to *L6-U3-L3*, and triangle *L6-U2″-L2″* is similar to triangle *L6-U2-L2*. Then, evidently, as *U4-L4*, *U3-L3*, *U2-L2*, etc., are of equal length at (a), the lines *U4″-L4″*, *U3″-L3″* will be of equal length at (b) and, hence, the points *U5″*, *U4″*, etc., will be in the same vertical line.

The determination of deflection or displacement by the use of the Williot diagram and the rotation diagram combined, as shown at (b), was first proposed by Professor Mohr,* and the diagram *LO″-U1″ ...L6* is known as Mohr's rotation diagram."

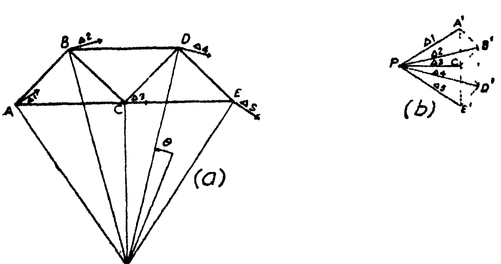


Fig. 363

The construction of the Mohr rotation diagram is based upon the fact that the lines joining the extremities of the arcs described by the joints of a truss when rotated through a small angle about any point will, when the arcs are laid off from a point, form a truss perpendicular to the first truss. For example, suppose the truss shown at (a), Fig. 363, be revolved about any point *O* through the small angle θ. Let $\Delta 1$, $\Delta 2 \ldots \Delta 5$ represent the arcs described by the joints. Then, by

* See Molitor's Kinetic Theory of Engineering Structures.

laying off these arcs from any point P, as shown at (b), and connecting their extremities, we obtain the truss shown dotted. This truss is perpendicular to the truss at (a). All of this is readily shown to be true by simple geometrical analysis.

217. Determination of Camber.—It is obvious that, if each compression member in a truss were lengthened an amount equal to its distortion and each tension member were shortened an amount equal to its distortion, that the truss when supporting no load would be cambered (curved upward) to an extent equal to the deflection and that the truss would be straight when supporting the maximum full load. Camber obtained in this manner is known as "exact camber."

In the case of trusses up to 300 ft. in length sufficient camber seems to be obtained by merely increasing the length of the top chord about $\frac{1}{8}$ of an inch for each 10 feet of length, as explained in Art. 185, but longer spans should have exact camber.

The position of the joints due to camber, in the case of exact camber, is determined exactly in the same manner as the deflection; in fact, the movement of the joints is exactly equal but opposite in direction. In case the camber is obtained by merely lengthening the top chord, the position of the joints is most readily obtained by the graphical method, and the work is practically the same as for the deflection. As an example, let it be required to locate the cambered position of the joints of the truss shown in Fig. 279, due to the cambered length specified for that structure. (See Art. 185.)

First draw the diagram of the truss shown at (a), Fig. 364, and write on each of the members the amount its length is changed to obtain the camber. As is seen, only the lengths of the top chords and diagonals are changed, and as the lengths are increased in every case, the plus sign is used throughout.

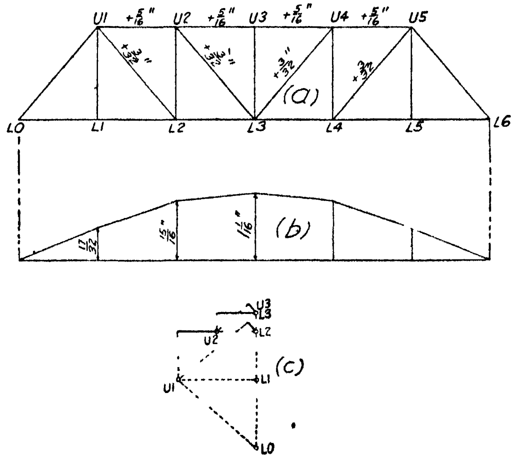


Fig. 364

We draw the diagrams at (c) by taking $L3$ as the starting point and assuming joint $L3$ as fixed in position, and member $U3$-$L3$ fixed in direction. Joint $U3$ does not move in reference to $L3$ (as the length of $U3$-$L3$ is not changed), so $U3$ will be at the same point as $L3$. Joint $U2$ in reference to $L3$ moves to the left $\frac{5}{16}$ in. So from $U3$ (marked $L3$ also) lay off $\frac{5}{16}$ in. to the left, as shown. Joint $U2$ in reference to $L3$ moves away from $L3$ the distance $\frac{3}{32}$ in. So from $U3$ lay off $\frac{3}{32}$ in. upward and to the left as shown. Then drawing perpendiculars to these, we obtain point $U2$, which shows the position of joint $U2$. Joint $L2$ does not move in reference to either joint *U2* or *L3*. So drawing from point *U2* a perpendicular to member *U2-L2* and from *L3* a perpendicular to member *L3-L2*, we obtain point *L2*, which shows the location of joint *L2*. To aid in understanding this step, we can imagine the change in length of each of the members *U2-L2* and *L2-L3* as being infinitesimal and the lines *U2-L2* and *L2-L3* as being perpendicular, respectively, to these changes.

Joint *U1* moves $\frac{5}{16}$ in. away from *U2* and $\frac{3}{32}$ in. from *L2*. Then from *U2* (at (c)) lay off $\frac{5}{16}$ in. to the left and from *L2* lay off $\frac{3}{32}$ in. upward and to the left, and then drawing perpendiculars we obtain point *U1*, which shows the position of joint *U1*. Joint *L1* does not move in reference to either joint *U1* or *L2*. Then by drawing from *U1* a line perpendicular to member *U1-L1* and from *L2* a line perpendicular to member *L1-L2*, we obtain the point *L1*, which shows the location of joint *L1*. Likewise, joint *LO* does not move in reference to either joint *U1* or *L1*. Then by drawing from *U1* a line perpendicular to member *U1-LO* and from *L1* a line perpendicular to member *LO-L1*, we obtain point *LO*, which shows the location of joint *LO*. Now, we have the position of the joints shown at (c) for the left half of the truss, and as the truss is symmetrical about the center of the span the diagram at (c) really shows the position of all the joints in the truss. The diagram at (b) shows the positions of the joints of the lower chord which are obtained from the diagram at (c) by measuring upward from *LO*.

It is interesting to note how near the camber shown at (b) comes to being equal to the deflection found for this same truss in Art. 215.

The cambered position of the joints, especially the lower chord joints, of long spans is required mostly for locating the tops of the camber blocks upon which the truss is supported while the structure is being erected. These supports or blocks, placed under each panel point, should have about the correct height, for otherwise the truss members will not fit into place.

CHAPTER XII

DESIGN OF SIMPLE HIGHWAY BRIDGES

218. Types.—Steel highway bridges can be divided into the following types: beam bridges, plate girders, viaducts, pony trusses, and high-truss bridges.

219. Live Load.—The actual maximum live load on roadway depends, as a rule, upon the location of the structure. It varies from interurban cars, street cars, heavy trucks, and dense crowds of people in and near cities and towns, to light trucks, slow moving traction engines, and droves of live stock in outlying country districts.

For convenience, standard loadings that very closely approximate the heaviest combination of actual loads are used, and the desired variation of the loading is obtained by varying the weight of the standard unit of loading.

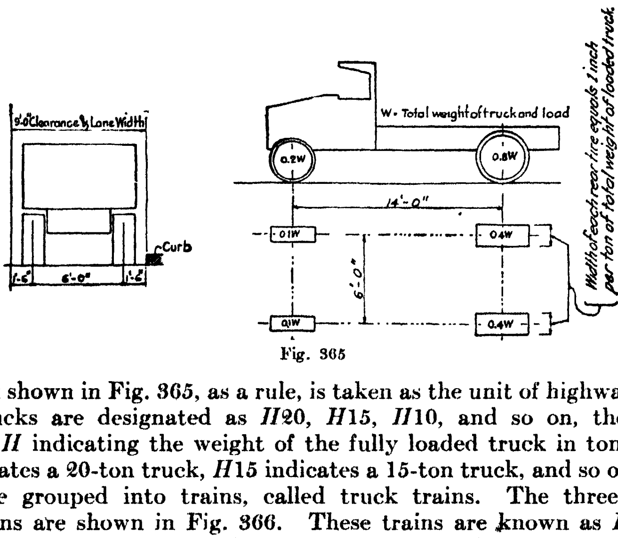


Fig. 365

The truck shown in Fig. 365, as a rule, is taken as the unit of highway loading. These trucks are designated as *H*20, *H*15, *H*10, and so on, the numeral following *H* indicating the weight of the fully loaded truck in tons; that is, *H*20 indicates a 20-ton truck, *H*15 indicates a 15-ton truck, and so on. These trucks are grouped into trains, called truck trains. The three standard truck trains are shown in Fig. 366. These trains are known as *H*20, *H*15, and *H*10 loadings, as indicated in Fig. 366. Each train consists of one truck (fully loaded) of the gross weight indicated by the loading class, followed by or preceded by or both followed and preceded by a line of trucks of indefinite length, each of the following or preceding trucks having a gross weight of three-fourths of the gross weight indicated by the loading class.

Each truck train is assumed to occupy a traffic lane 9 ft. wide along the roadway. Within the curb-to-curb width of roadway, the traffic lanes will be assumed to occupy any position which will produce maximum stress, but which will not involve overlapping of adjacent lanes, or place the center of the lane nearer than 4 ft. 6 ins. from the roadway face of the curb. Trucks in adjacent lanes will be considered as headed in the same direction.

The Wheel Loads shown in Figs. 365 and 366 are used in designing all spans less than 60 ft. in length.

The Equivalent Loading shown in Fig. 367 is used in designing spans 60 ft. in length and greater. The equivalent loading, which produces about the

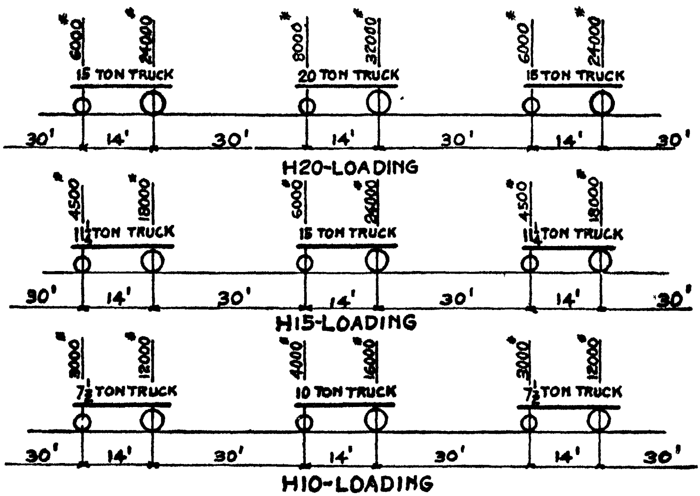


Fig. 366.

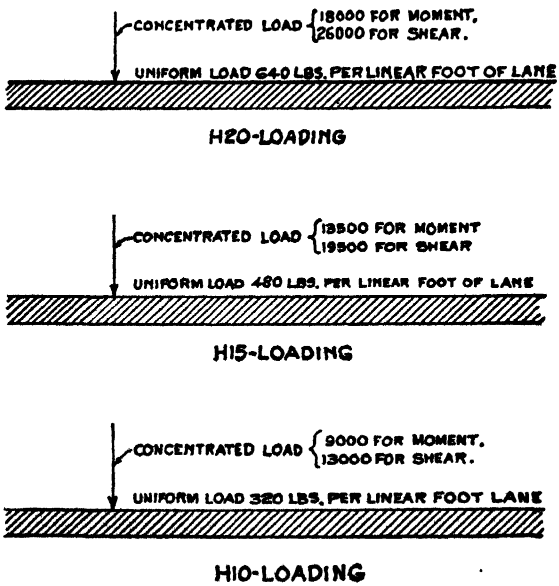


Fig. 367

same stresses as the wheel loads (truck trains), consists of a uniform load per linear foot of traffic lane combined with a single concentrated load so placed on the span as to produce maximum stress.

The concentrated load is considered as being uniformly distributed across the lane on a line normal to the center line of the lane. As indicated in Fig. 367, the heavier concentrated load is used in determining shear and the lighter one is used in determining moment. The equivalent loading is applied to the roadway in two ways, and the one producing maximum stress in the member is used:

(1) "Each traffic lane loading shall be considered as a unit and the number and position of loading lanes shall be such as will produce maximum stresses.

(2) "The roadway may be considered as loaded over its entire width with a load per foot of width equal to one-ninth of the load on one traffic lane reduced 1 per cent for each foot of loaded roadway width in excess of 18 ft. with a maximum reduction of 25 per cent, corresponding to a loaded roadway width of 43 ft."

220. Live Load on Sidewalks.—Sidewalk floors, sidewalk stringers, and their immediate supports are usually designed for a live load of not less than 100 lbs. per square foot of sidewalk floor.

Girders or trusses of bridges with sidewalks shall be designed for a sidewalk live load determined from the following formula:

$$P=\left(40+\frac{3,000}{L}\right)\times\left(\frac{55-W}{50}\right),$$

in which P = live load in pounds per square foot of sidewalk area, but not to exceed 100 lbs. per sq. ft.

L = loaded length of sidewalk in feet.

W = width of sidewalk in feet.

221. Impact.—All live-load stresses, except those due to sidewalk loads and centrifugal, traction and wind forces, are increased to provide for stresses that may be caused by heavy trucks moving rapidly over rough or uneven bridge floors or by the nosing or by uneven rails in the case of electric railway cars. These are known as stresses due to impact. The amount of such stress is expressed as a fraction of the corresponding live-load stress. This fraction is known as the coefficient of impact.

The coefficient of impact for both highway live-load and electric railroad loadings is obtained from the following empirical formula:

$$c=\frac{50}{L+125} \quad (1),$$

wherein c = coefficient of impact;

L = length in feet of the portion of span loaded to produce maximum live-load stress in the member considered.

The stress due to impact in a member is obtained by multiplying the maximum live-load stress in the member by the coefficient of impact for that member.

222. Traction.—Provision must be made for a longitudinal force, known as traction, due to sudden stopping of the live load on a bridge. This force will be taken as 10 per cent of the live load on the structure and it may be considered as acting 4 ft. above the floor surface.

223. Wind Load.—The wind pressure on a bridge is a lateral force and should be assumed to be a moving horizontal load equal to 30 lbs. per square foot on 1½ times the area of the surface of the structure as seen in elevation including the floor system and railings and on one-half of the area of all trusses or girders in excess of two in a span. This load is applied at the panel points of the top and bottom chords in proportion to the area of exposed surface attributed to each joint. To this pressure must be added the pressure of the wind upon the live load which is transferred laterally to the loaded chord, but in considering overturning it may be considered as acting 6 ft. above the roadway. The live load may be considered to be 8 ft. high, in which case the pressure upon it due to wind would be 8x30 = 240 lbs. per linear foot of loaded chord.

The panel loads at the top and bottom chord joints due to wind are computed by estimating the exposed surface area and multiplying $1\frac{1}{2}$ times this, in each case, by 30 lbs. and then adding the 240 lbs. times the panel length to the loaded chord joints.

In practice the total minimum wind pressure is assumed to be not less than 300 lbs. per linear foot of the loaded chord, 150 lbs. per linear foot of the unloaded chord on truss spans, and not less than 300 lbs. per linear foot on girder spans.

For Stringers

Kind of floor	Floors designed for one traffic lane		Floors designed for two traffic lanes	
	Fraction of wheel load to each stringer	Limiting stringers spacing, feet	Fraction of a wheel load to each stringer	Limiting stringer spacing, feet
Wood plank	$\frac{S}{4.0}$	4.0	$\frac{S}{3.5}$	5.0
Wood strips 4 ins. in thickness or wood blocks on 4-in. plank sub-floor	$\frac{S}{4.5}$	4.5	$\frac{S}{3.75}$	5.5
Wood strips 5 ins. or more in thickness	$\frac{S}{5.0}$	5.0	$\frac{S}{4.0}$	6.0
Concrete slab	$\frac{S}{6.0}$	6.0	$\frac{S}{4.5}$	10.0

S = transverse distance in feet between stringers.

224. Distribution of Loads.—In calculating end shears and end reactions in transverse floor beams and longitudinal beams and stringers, it is general practice not to consider any longitudinal distribution of wheel loads.

In calculating bending moments in longitudinal beams or stringers, no longitudinal distribution of wheel loads is considered. The lateral distributions in such cases are determined as follows:

Interior Stringers are proportioned for loads determined in accordance with the table, shown on page 536, except that when the limiting stringer spacings are exceeded, the stringer loads are determined by the reaction of the truck wheels, assuming the flooring between stringers to act as a simple beam.

Outside Stringers. The live load supported by outside stringers shall be the reaction of the truck wheels, assuming the flooring to act as a simple beam between stringers.

Floor Beams. In calculating bending moments in floor beams, no transverse distribution of wheel loads is assumed. If longitudinal stringers are omitted and the floor is supported directly on the floor beams, the latter are proportioned for a fraction of the wheel loads, as indicated in the following table, except that when the limiting floor-beam spacing is exceeded, the floor-beam loads are determined by the reaction of the truck wheels, assuming the flooring between the floor beams to act as a simple beam:

For Floor Beams

Kind of floor	Fraction of wheel loads to each floor beam	Limiting floor-beam spacing, feet
Plank	$\frac{S}{4.0}$	4.0
Wood strips 4 ins. in thickness or wood blocks on 4-in. plank sub-floor	$\frac{S}{4.50}$	4.5
Wood strips 5 ins. or more in thickness	$\frac{S}{5.0}$	5.0
Concrete	$\frac{S}{6.0}$	6.0

S = Spacing of floor beams in feet.

Concrete-Slab Floors. In calculating bending moment due to wheel loads on concrete slabs, no distribution in the direction of the span is considered. In the direction perpendicular to the span of slab, the wheel loads are considered to be distributed uniformly over a width of slab which is termed the effective width and is obtained from the following formulas in which:

S = span of slab in feet;
W = width of wheel on tire in feet;
D = distance in feet from the center of the nearest support to the center of the wheel;
E = effective width in feet for one wheel.

Case I. Main reinforcement parallel to direction of traffic.

$$E = 0.7S + W \qquad (1),$$

in which case E shall not exceed 7 ft. When two wheels are so located on a transverse element of a slab that their effective widths overlap, the effective width for each wheel is obtained from the following formula:

$\frac{1}{2}(E+C)$ .... (2),

in which E is the value obtained from Formula (1) and C is the distance between wheels in feet.

Case II. Main reinforcement perpendicular to direction of traffic.
In this case the following formula is used:

$$E=0.7(2D+W) \qquad (3).$$

The moment on a strip of slab 1 ft. wide is determined by placing the wheel loads in the position for maximum moment, assuming no distribution, determining the effective width (from above formulas) for each wheel, and assuming the load of each wheel on the 1-ft. wide strip to be the wheel load divided by its respective effective width.

225. Allowable Unit Stresses.—The unit stresses allowed depend upon the quality of material used. The following intensities are widely used:

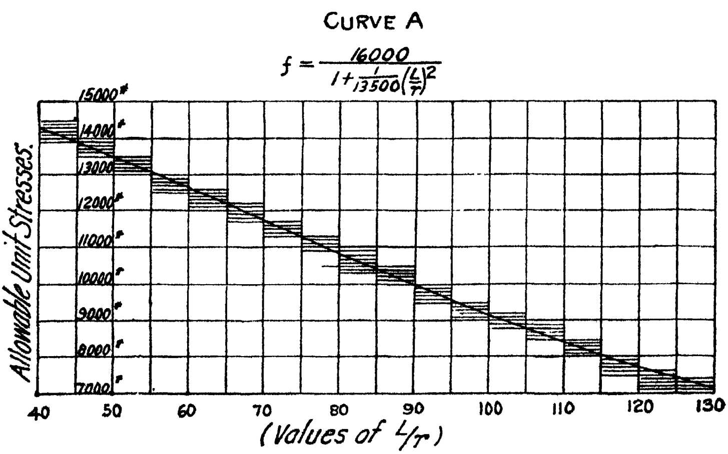
For Structural Grade Steel and Rivet Steel.

(*a*) *Tension:*

	Pounds per Square Inch
Axial tension, structural member	
Net section	16,000
Bolts, area at root of thread	10,000

(*b*) *Axial compression:*

Axial compression, gross section .... $\dfrac{16,000}{1+\dfrac{1}{13,500}\left(\dfrac{L}{r}\right)^2}$



but not to exceed the value obtained when $L/r=40$:

L = length of member in inches.
r = least radius of gyration of member in inches.

(*c*) *Compression splice material:*

	Pounds per Square Inch
Gross section	16,000

(*d*) *Bending on extreme fiber:*

Compression in flanges of beams and plate girders...... $\dfrac{16{,}000}{1+\dfrac{1}{2{,}000}\left(\dfrac{L}{b}\right)^2}$

L = length in inches of the unsupported flange between lateral connections or knee braces.
b = width of flange in inches.

Tension in rolled shapes, built sections and girders, net section	16,000
Pins (bending stress)	24,000

(*e*) *Diagonal tension:*

In web of girders and rolled beams at sections where maximum shear and bending occur simultaneously	16,000

(*f*) *Shear:*

Girder webs, gross section	10,000
Pins and shop-driven rivets	12,000
Power-driven field rivets and turned bolts	10,000
Hand-driven rivets and unfinished bolts	8,000

(*g*) *Bearing:*

Pins, steel parts in contact and shop-driven rivets	24,000
Power-driven field rivets and turned bolts	20,000
Hand-driven rivets and unfinished bolts	16,000
Expansion rollers and rockers, in pounds per linear inch	6,000

d = diameter of roller or rocker.

In proportioning rivets, the nominal diameter of the rivet shall be used.

The effective bearing area of a pin, bolt or rivet shall be its diameter multiplied by the thickness of the metal on which it bears.

The value of a pin or rivet in shear is equal to the allowable unit shearing stress given above multiplied by the area of cross-section of the pin or rivet.

For Steel Castings and Cast Iron.

(*h*) *For steel castings.* Three-fourths of the unit stresses specified above for structural grade steel shall apply.

(*i*) *For cast iron:*

	Pounds per Square Inch
Bending on extreme fiber	3,000
Shear	3,000
Direct compression (short column)	12,000
Bearing on bronze expansion bearings	2,000
Bearing on granite masonry	800
Bearing on sandstone and limestone masonry	400
Bearing on concrete masonry	600

(*j*) *For concrete:*

	Pounds per Square Inch
Tension	0
Compression	650
Shear—without stirrups (rods hooked)	60
Shear—with stirrups (maximum; rods hooked)	120
Punch shear	120

(*k*) *For timber* (fir, pine and oak):*

Tension (bending extreme fiber)	1,200
Compression parallel to grain	800
Compression perpendicular to grain	300
Horizontal shear due to bending	80

226. Clearances.—The horizontal clearance is the clear width, and the vertical clearance is the clear height available for the passage of vehicular traffic, as indicated in Figs. 368, 369, and 370.

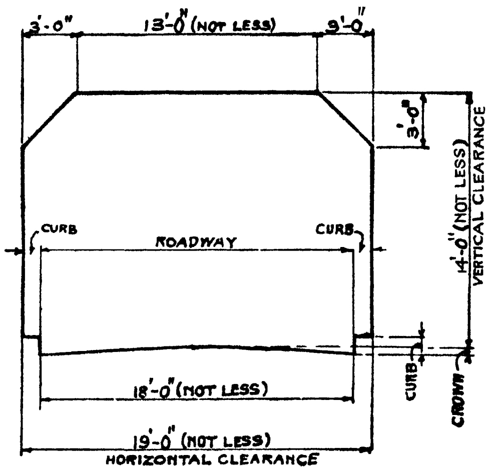


Fig. 368

227. Bridge Floors can be classed as wood and concrete floors. The wood floors consist of either plain planks laid flat or wood strips laid on edge and nailed together forming a continuous floor. Concrete floors consist of reinforced concrete slabs.

The floors are continuous over several supports and are analyzed as continuous beams.

The following formulas may be used for determining the maximum moment on a floor:

For Maximum Positive Moment in Foot Pounds.†

(1) Dead load . . . . . $M = \frac{1}{10}wL^2$.

(2) Live load . . . . . $M = \frac{1}{6}PL$.

* See American Institute of Steel Construction, Inc., "Handbook."

† See Kirkham, "Highway Bridges." McGraw Hill Book Company, Inc., New York.

For Maximum Negative Moment in Foot Pounds.

(3) Dead load. . . . . $M = -\frac{1}{9.5}wL^2$.

(4) Live load. . . . . $M = -\frac{1}{6}PL$ for floors on longitudinal stringers.

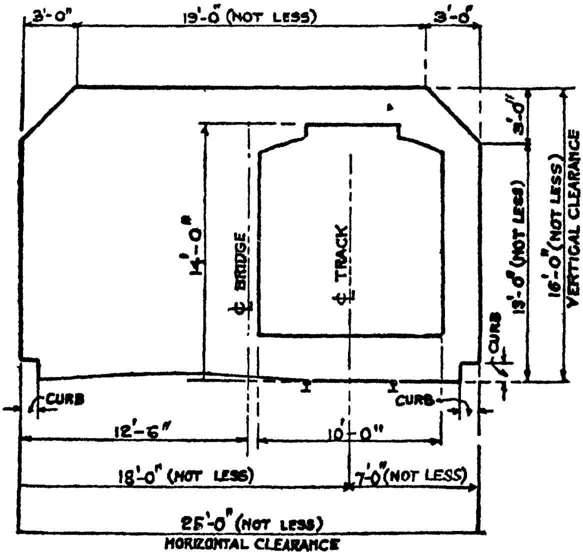


Fig. 369

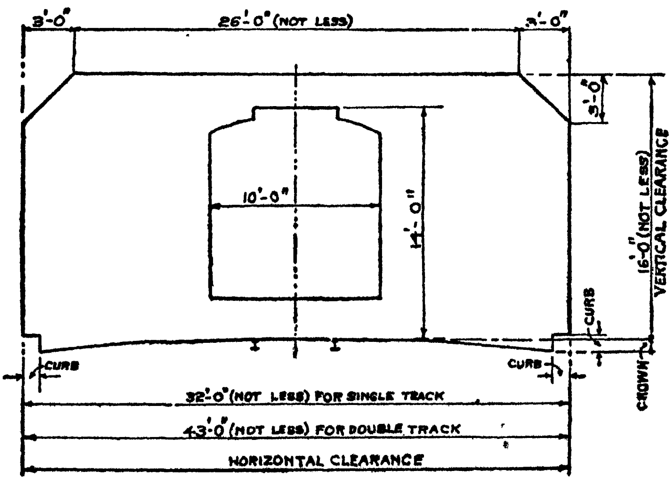


Fig. 370

(5) Live load. . . . . $M = -\frac{1}{8}PL$ for floors on transverse beams.

P = heaviest wheel load.

L = distance center to center of stringers or distance center to center of transverse beams in feet.

w = weight of floor and floor covering in pounds per square feet.

For convenience the following designs of concrete floors, supporting *H15* loading, are given.

Floors Supported upon Longitudinal Stringers.

Stringers spaced 2′-6″ apart.

Required thickness = 6½″. Make 6¾″ at curbs and 7¾″ at crown to provide wearing surface and crowning.

Bottom reinforcing.......................... ¾″ round rods spaced 9½″ ctrs.
Top reinforcing.......................... ⅝″ round rods spaced 7⅞″ ctrs.

Stringers spaced 3′-0″ apart.

Required thickness = 6¾″. Make 7″ at curbs and 8″ at crown.

Bottom reinforcing.......................... ¾″ round rods spaced 9½″ ctrs.
Top reinforcing.......................... ⅝″ round rods spaced 7¾″ ctrs.

Stringers spaced 4′-0″ apart.

Required thickness = 7″. Make 7¼″ at curbs and 8¼″ at crown.

Bottom reinforcing.......................... ¾″ round rods spaced 9″ ctrs.
Top reinforcing.......................... ⅝″ round rods spaced 7¼″ ctrs.

Floors Supported upon Transverse Beams.

(A strip about 5 to 6 ft. wide along the center of the roadway is considered to be the inner zone, and the longitudinal strips to each side of the inner zone are the outer zones.)

6′-0″ and 7′-0″ panels.

Thickness of slab at crown = 7¾″(+¾″ = 8½″).
Thickness of slab at curbs = 7″(+½″ = 7½″).

Reinforcing in inner zone { ⅝″ round rods @ 6″ ctrs. in top of slab.
¾″ round rods @ 7″ ctrs. in bottom of slab.

Reinforcing in outer zones { ⅝″ round rods @ 6¾″ ctrs. in top of slab.
¾″ round rods @ 8″ ctrs. in bottom of slab.

8′-0″ panels.

Thickness of slab at crown = 8″(+¾″ = 8¾″).
Thickness of slab at curbs = 7½″(+½″ = 8″).

Reinforcing in inner zone { ⅝″ round rods @ 6⅜″ ctrs. in top of slab.
¾″ round rods @ 6¾″ ctrs. in bottom of slab.

Reinforcing in outer zones { ⅝″ round rods @ 7¼″ ctrs. in top of slab.
¾″ round rods @ 8″ ctrs. in bottom of slab.

9′-0″ and 10′-0″ Panels.

Thickness of slab at crown = 8½″(+¾″ = 9¼″).
Thickness of slab at curbs = 7¾″(+½″ = 8¼″).

Reinforcing—inner zone { ⅝″ round rods @ 5½″ ctrs. in top of slab.
¾″ round rods @ 6¼″ ctrs. in bottom of slab.

Reinforcing in outer zones { ⅝″ round rods @ 6¾″ ctrs. in top of slab.
¾″ round rods @ 7¾″ ctrs. in bottom of slab.

DESIGN OF I-BEAM BRIDGES

228. Spacing of Beams.—It is seen from Art. 227 that the thickness of the concrete floor slabs and the area of the reinforcing steel in them are only slightly affected by the spacing of the stringers and it is also found that the spacing of the stringers has only a slight effect upon their total weight. This results from the fact that the distribution of the wheel loads increases directly with the spacing of the stringers. Then evidently the selection of

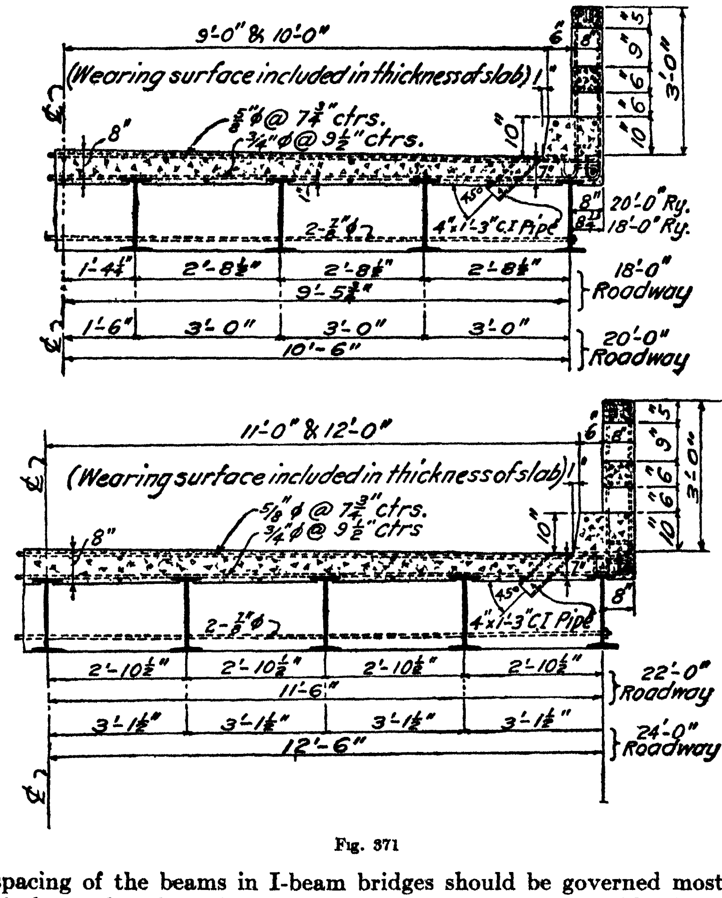


Fig. 371

the spacing of the beams in I-beam bridges should be governed mostly by practical considerations of rigidity and distribution of the wheel loads. There is no doubt but that the most even distribution of the wheel loads will be obtained when there are at least two beams for each line of wheels, but at the same time the fact that the beams should have sufficient depth for rigidity should not be overlooked.

The exact spacing of the beams depends upon the details of the curbs and railing. The spacing found to be satisfactory for 18-, 20-, 22-, and 24-ft. concrete roadways is shown in Fig. 371. In the case of wood floors, as a rule, the spacing of the stringers should be limited to about 2′-6″.

Design of a 24-Ft. I-Beam Bridge: 18-Ft. Wooden Roadway; H15 Loading

229. The Floor can be made of planks laid flat, as shown in Fig. 372*a*, or of 2″x4″ or 3″x5″ pieces stood on edge and nailed together forming a continuous floor as shown in Fig. 372*b*.

230. Spacing of Beams.—On account of the low fiber stresses allowed on wood, the spacing (distance between stringers) should be limited to about 2′-6″. The spacing shown in Fig. 372 is satisfactory for the 18-ft. roadway.

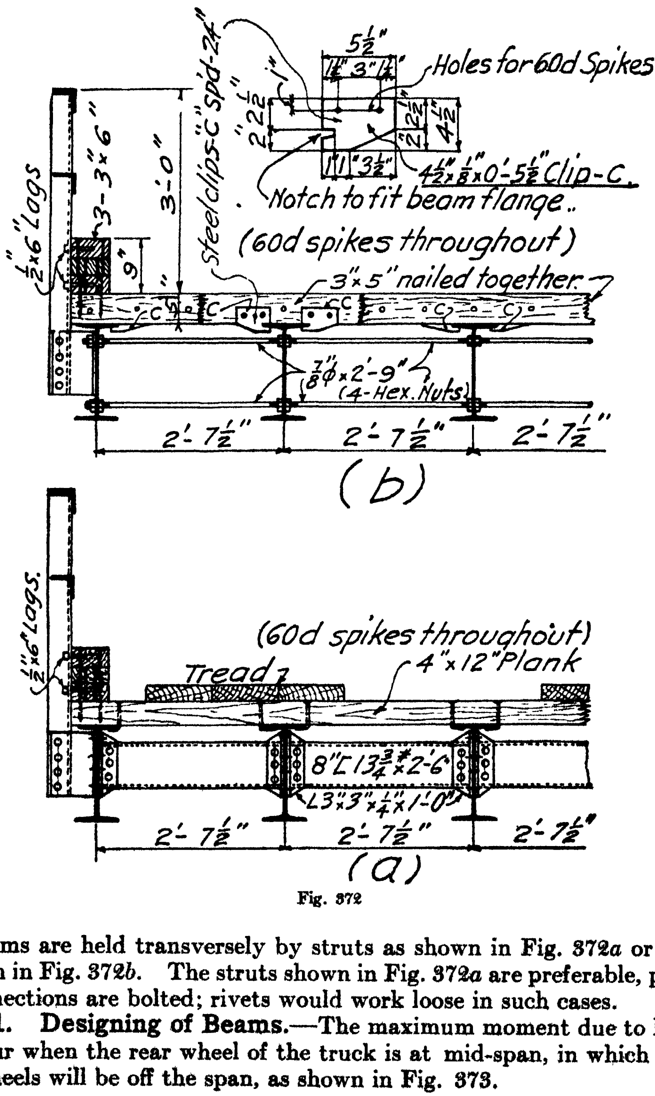


Fig. 372

The beams are held transversely by struts as shown in Fig. 372*a* or by rods as shown in Fig. 372*b*. The struts shown in Fig. 372*a* are preferable, provided the connections are bolted; rivets would work loose in such cases.

231. Designing of Beams.—The maximum moment due to live load will occur when the rear wheel of the truck is at mid-span, in which case the front wheels will be off the span, as shown in Fig. 373.

Taking moments about support *B*, we obtain

$$\frac{12{,}000 \times 12}{24} = 6{,}000 \text{ lbs.}$$

for the reaction at *A*. Then for the maximum moment due to the wheel we have

$$6{,}000 \times 12 \times 12 = 864{,}000 \text{ in. lbs.}$$

Now, using the distribution factor for plank floors, given in Art. 224 (page 536), we obtain

$$864{,}000 \times \frac{2.625}{3.5} = 648{,}000 \text{ in. lbs.}$$

for the maximum moment due to live load on each interior beam. For the corresponding impact (using (1) of Art. 221, page 535) we have

$$648{,}000\left(\frac{50}{24+125}\right) = 217{,}080 \text{ in. lbs.}$$

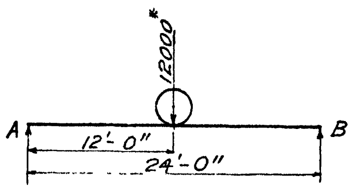


Fig. 373

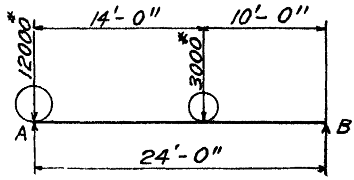


Fig. 374

Using 4"x12" plank and assuming wood to weigh 60 lbs. per cubic foot, we obtain

$$60 \times \tfrac{4}{12} \times 2.625 = 52.5 \text{ lbs.}$$

for the weight of floor per foot of interior beam. Assuming the beam to weigh 35 lbs. per foot, we have

$$52.5 + 35 = 87.5 \text{ lbs.}$$

for the total dead load per foot of interior beam. Then for the moment due to dead load on each interior beam we have

$$\frac{1}{8}(87.5 \times \overline{24}^2 \times 12) = 75{,}600 \text{ in. lbs.}$$

Adding this to the moment due to live load and impact, we obtain

$$648{,}000 + 217{,}080 + 75{,}600 = 940{,}680 \text{ in. lbs.}$$

for the total maximum moment on each interior beam. Dividing this by 16,000, we obtain

$$940{,}680 \div 16{,}000 = 58.7$$

for the section modulus. This calls for a 16-in.x38-lb. Carnegie beam, which will be used for interior beams.

The outside beams support a little less live load and a little more dead load than do the interior beams. As a rule, therefore, the outside beams have the same section as the interior beams. In fact, most specifications require that the outside beams have the same section as the interior beams.

Placing the wheel loads in the position on the span as shown in Fig. 374 and taking moments about support *B*, we obtain

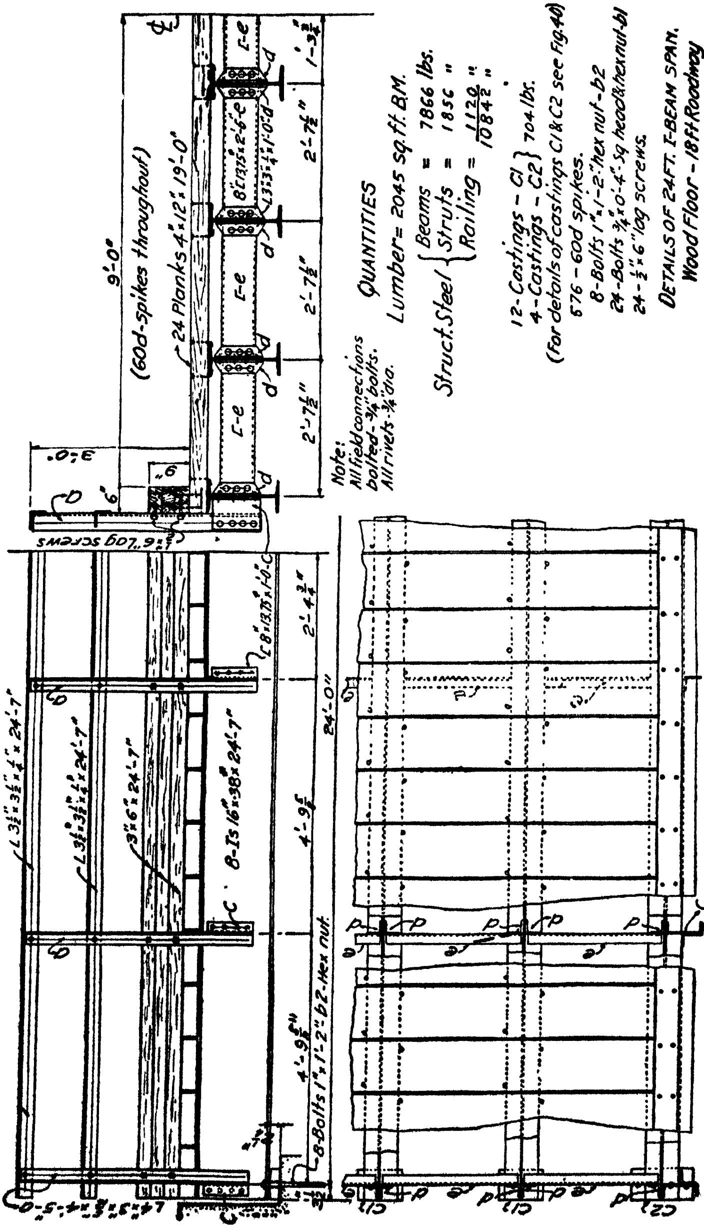

9'-0"
(60d-spikes throughout)
24 Planks 4" x 12" x 19'-0"
Note: All field connections bolted - 3/4" bolts. All rivets 3/4" dia.
QUANTITIES
Lumber = 2045 sq. ft. B.M.
Struct. Steel { Beams = 7866 lbs. Struts = 1856 " Railing = 1120 " 10842 "
12-Castings - C1
4-Castings - C2 } 704 lbs.
(For details of castings C1 & C2 see Fig. 40)
676 - 60d spikes.
8-Bolts 1" x 1'-2" hex nut - b2
24-Bolts 3/4" x 0'-4" sq head & hex nut - b1
24 - 1/2" x 6" log screws.
DETAILS OF 24 FT. I-BEAM SPAN.
Wood Floor - 18 ft Roadway
4/5 LOADING.
8-Is 16" x 38# x 24'-7"
24'-0"

Fig. 375

$$\frac{3{,}000\times 10}{24}+12{,}000=13{,}250 \text{ lbs.}$$

for the maximum reaction and end shear on a beam due to live load. For the corresponding impact we have (from Art. 121, page 535)

$$13{,}250\left(\frac{50}{24+125}\right)=4{,}450 \text{ lbs.}$$

For the end reaction due to dead load we have

$$87.5\times 12=1{,}050 \text{ lbs.}$$

Then for total end shear and reaction we have

$$13{,}250+4{,}450+1{,}050=18{,}750 \text{ lbs.}$$

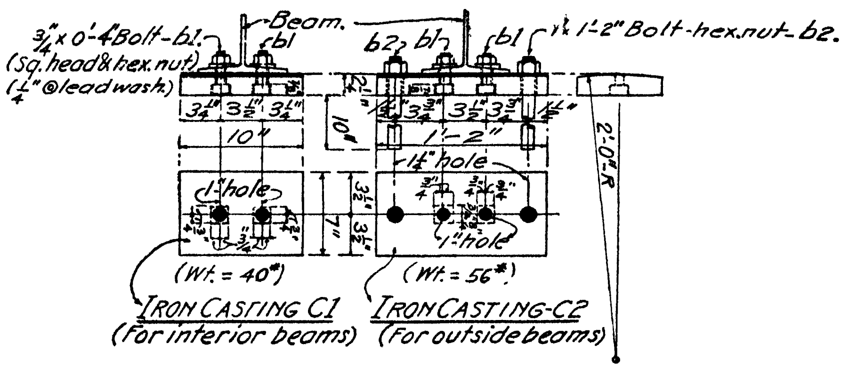


Fig. 376

From the handbook issued by the American Institute of Steel Construction, Inc. (A.I.S.C.) or from the Carnegie Company's handbook we find that the area of cross-section of each beam is 11.17 sq. ins.

Then for the maximum unit shear on each beam we have

$$18{,}750+11.17=1{,}678 \text{ lbs.}$$

which is low, as 10,000 lbs. is allowed. For the area of bearing on masonry at each end of each beam we have

$$18{,}750\div 600=31 \text{ sq. ins.}$$

The foregoing analysis is sufficient for detailing the structure as shown in Fig. 375. The details are for the most part self-explanatory. The railing is designed to support a horizontal load of 150 lbs. per foot of railing and a vertical load of 100 lbs. per foot of railing.

Experience has shown that end bearings should support the beams a sufficient distance above the bridge seat so that the beams will not come in contact with dirt or slush that may lodge on the bridge seat and that these bearings should provide for free movement of the beams due to deflection and thermal expansion and contraction.

The details of such bearings (*C*1 and *C*2) are shown in Fig. 376. The bearings are made of cast iron, since cast iron resists rusting better than does steel. Each bearing is 2¼ in. high and is curved on top to provide for movement of the beams due to deflection. The clearance in the bolt holes provides for the small longitudinal movement of the spans due to changes in temperature. The bearings (shown in Fig. 376) are made of sufficient size so that they are standard for all lengths of spans. They are designed to resist the stresses due to shearing, bearing, and bending in the case of a 40-ft. span supporting a concrete floor.

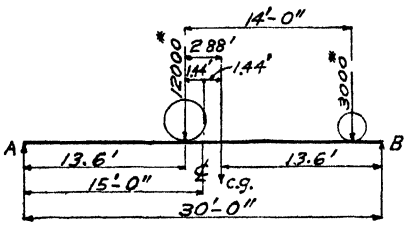


Fig. 377

Wood floors on I-beam bridges are generally an emergency construction; sooner or later the wood is replaced with concrete and, hence, the beams should be designed to support a concrete floor. However, the beams designed for a wood floor as a rule are sufficient for supporting a concrete floor, because the distribution on a concrete floor is greater than that allowed on a wood floor. The floor shown in Fig. 375 is not sufficient for supporting $H15$ loading without longitudinal tracks or treads made of 3″x12″ planks. Each tread should be at least three planks in width.

Design of a 30-Ft. I-Beam Bridge: 20-Ft. Concrete Roadway; H15 Loading

232. Designing of Beams.—The beams will be spaced 3′-0″ centers as indicated in Fig. 371. The maximum moment on the span due to live load will occur when the wheels are in the position indicated in Fig. 377 (see Art. 88). Taking moments about support B, we obtain

$$\frac{15{,}000 \times 13.6}{30} = 6{,}800 \text{ lbs.}$$

for the reaction at A. Then for the maximum moment we have

$$6{,}800 \times 13.6 \times 12 = 1{,}109{,}760 \text{ in. lbs.}$$

Considering distribution, we obtain (see table, Art. 224).

$$1{,}109{,}760 \times \frac{3}{4.5} = 737{,}822 \text{ in. lbs.}$$

for the maximum moment on the beam due to live load. Then for the corresponding impact we have

$$737{,}822 \times \left(\frac{50}{30+125}\right) = 238{,}006 \text{ in. lbs.}$$

We shall next consider dead load. As seen from Art. 227, the floor slab will be 8 ins. thick at crown and 7 ins. thick at curbs. So, to be on the safe side, we shall assume the slab to be 8 ins. thick. Then for the assumed dead load per foot of beam we have

Slab	$12.5 \times 8 \times 3 =$	300 lbs.
Extra floor covering	$15 \times 3 =$	45 lbs.
Beam (assumed)		50 lbs.
Total		395 lbs.

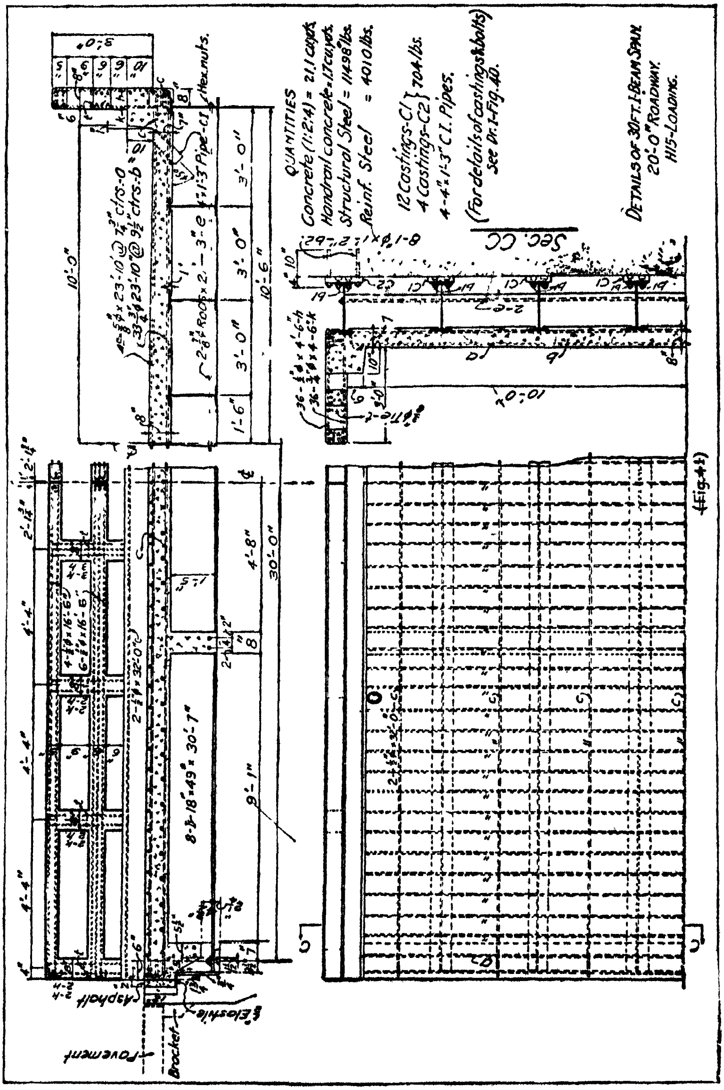

QUANTITIES
Concrete (1:2:4) = 21.1 cu.yds.
Handrail concrete = 1.7 cu.yds.
Structural Steel = 11498 lbs.
Reinf. Steel = 4010 lbs.
12 Castings-C1
4 Castings-C2 } 704 lbs.
4-4"x1-3" C.I. Pipes.
(For details of castings & bolts see Dr. 1-Fig. 40.)
DETAILS OF 30 FT. I-BEAM SPAN.
20'-0" ROADWAY.
H15-LOADING.
Sec. CC
Pavement
Asphalt
Bracket
Elastite
Hex. nuts.
8-I-18"-49# x 30'-7"
30'-0"
10'-0"
10'-6"

Fig. 378

Then for the maximum moment due to dead load we have

$$\frac{1}{8}(395\times\overline{30}^2\times12)=533,250 \text{ in. lbs.}$$

Now adding together the moments due to live load, impact, and dead load, we obtain

$$737,822+238,006+533,250=1,509,078 \text{ in. lbs.}$$

for the total maximum moment on each beam due to all loads. Dividing this by 16,000, we obtain

$$\frac{1,509,078}{16,000}=94.3$$

for the section modulus. This calls for 18-in.x52-lb. Carnegie beam, which will be used.

233. Details.—The details of the span are shown in Fig. 378. These details are mostly self-explanatory. The end bearings are the same as those shown in Fig. 376. The I-beams are held firmly transversely by concrete struts as shown. The intermediate struts aid in distributing the wheel loads to the I-beams, although this point was not considered in designing these beams. The end struts are constructed so that the forms can be easily removed after the concrete in the floor hardens and so that the ends of the beams and the cast bearings can be painted. These transverse concrete struts are designed as T-beams having a length of two beam spaces to support the maximum wheel loads at mid-span. The struts are considered to carry one-half of a wheel load in shear.

Experience shows unmistakably that the side of the railing next to the roadway should be in one vertical plane. The lower rail is of the correct height to catch the main body of a vehicle in case of collision with the railing, and the top rail will catch the upper part of the vehicle. The railing shown in Fig. 378 has an open joint at mid-span of bridge to provide for deflection of the bridge. The rails in the center panel are designed as cantilevers. Each post will withstand a horizontal force of 6,600 lbs. applied at the top of the lower rail and of about 3,000 lbs. at the top rail. The lower rail will support a horizontal force of about 7,000 lbs.

The curbs are constructed in accordance with standard specifications. The drain pipes are 4-in. cast-iron pipes. Four-inch cast-iron pipes can be obtained from any plumbing shop and can be readily cut and placed as shown.

PLATE GIRDER BRIDGES

234. Preliminary.—Plate girder highway bridges are either through or deck type, as are railroad plate girder bridges. The through type is the more common, especially for single-span bridges. The through type consists of two main side girders and floor beams, about 10 ft. apart, upon which a concrete floor slab is supported.

The deck-type bridges, in the case of wide roadways, are constructed very similar to I-beam bridges—the built-up girders (plate girders) simply taking the place of the I-beams. In the case of ordinary roadways, deck plate girder bridges often consist of three girders braced transversely by cross-frames about 12 ft. apart. The floor slab is then supported directly upon the top of the three girders and cantilevered out beyond the outside girders. It is necessary to place the girders a certain distance apart if the three girders are to be of equal section.

Design of a 60-Ft. Through Plate Girder Bridge

235. Data.—

Length = 60′-0″ c.c. end bearings.
Width = 20′-0″ clear roadway.
Floor, concrete slab supported upon floor beams.
Panel length = 10′-0″.
Live load, *H*15 loading.

236. Design of Floor Beams.—We shall assume the average thickness of the floor slab, including wearing surface, to be $8\frac{3}{4}$ ins. and the beam to weigh 91 lbs. per foot. Then for the dead load per foot of floor beam we have

Slab............ ....8.75×12.5×10 =	1,093 lbs.
Beam.......	91 lbs.
Total..........	1,184 lbs.

If each top flange is 12 ins. wide and the clear width of each curb is 6 ins., the required distance center to center of main girders will be 22′-0″, which will be considered to be the length of each floor beam.

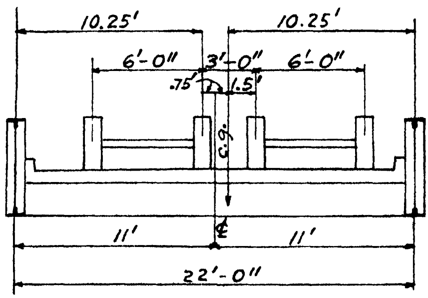


Fig. 379

Then for the maximum moment on an intermediate floor beam due to dead load we have

$$\tfrac{1}{8}(1{,}184\times\overline{22}^2\times12)=859{,}584 \text{ in. lbs.}$$

The maximum moment due to live load occurs when the rear wheels of the two trucks are directly over the floor beam and in the position shown in Fig. 379. Taking moments about the right-hand girder, we obtain

$$\frac{48{,}000\times10.25}{22}=22{,}364 \text{ lbs.}$$

for the reaction on the floor beam at the left-hand girder.

Next, taking moments about the wheel nearest the center of the floor beam and considering the forces to the left, we obtain

$$[(22{,}364\times10.25)-(12{,}000\times6)]12=1{,}886{,}770 \text{ in. lbs.}$$

for the maximum moment due to live load. For the corresponding impact we obtain

$$\left(\frac{50}{14+125}\right)1,886,770=678,688 \text{ in. lbs.}$$

Now, adding the moments together, we obtain

$$859,584+1,886,770+678,688=3,425,042 \text{ in. lbs.}$$

for the total maximum moment on the floor beam due to all loads. Dividing this by 16,000, we obtain

$$3,425,042 \div 16,000=214$$

for the section modulus. This calls for a 24-in.×90.5-lb. Bethlehem beam, which will be used for all intermediate floor beams.

For the end reaction on an interior floor beam due to dead load we obtain

$$11 \times 1,184=13,024 \text{ lbs.}$$

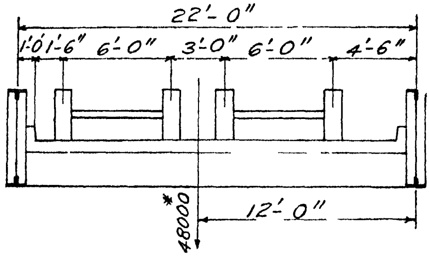


Fig. 380

Placing the wheel loads as shown in Fig. 380 and taking moments about the right-hand girder, we obtain

$$\frac{48,000 \times 12}{22}=26,180 \text{ lbs.}$$

for the maximum reaction on each interior floor beam. For the corresponding impact we obtain

$$\left(\frac{50}{22+125}\right)26,180=8,900 \text{ lbs.}$$

Then for the total maximum reaction (end shear) on each interior floor beam we have

$$13,024+26,180+8,900=48,104 \text{ lbs.}$$

The loads on the end floor beams are all the same as the loads on the intermediate floor beams, except that the dead load from the floor slab is only about two-thirds as great and the beam may be slightly lighter. Then for the maximum moment on the end floor beam due to dead load we have

$$\tfrac{2}{3} \times 859,584=573,056 \text{ in. lbs.}$$

Then for the total maximum moment on each end floor beam due to all loads we have

$$573,056+1,886,770+678,688=3,138,514 \text{ in. lbs.}$$

Dividing this by 16,000, we obtain 196.1 for the section modulus. This calls for a 24-in. × 84.5-lb. Bethlehem beam, which will be used for each end floor beam.

For the maximum reaction on each end floor beam we have

Dead load	$13{,}024\times\frac{2}{3}=$ 8,682 lbs.
Live load	26,180 lbs.
Impact	8,900 lbs.
Total	43,762 lbs.

237. Design of Main Girders.—Except for the weight of the main girders, the loads are all applied to the main girders through the floor beams, that is, through the end connections of the floor beams; or, in other words, the loads are applied at the panel points. The loads thus applied are known as

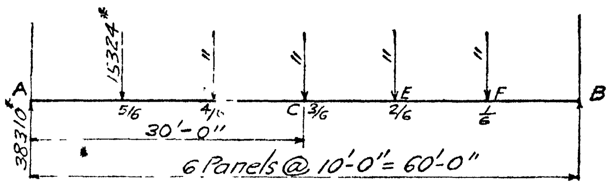


Fig. 381

panel loads. Without appreciable error we may also consider the weight of the main girders as applied at the panel points. Then, assuming each main girder to weigh 230 lbs. per foot, we have

$$(230\times 10)+13{,}024=15{,}324 \text{ lbs.}$$

for the dead-load panel load at each floor beam. The dead load on each main girder will then be as indicated in Fig. 381, where AB represents the girder.

Taking moments about C (the center of span) and considering the forces to the left, we obtain

$$[(38{,}310\times 30)-(15{,}324\times 2)15]12=8{,}274{,}960 \text{ in. lbs.}$$

for the maximum moment on each main girder due to dead load.

The span being 60 ft. long, the live load, according to Art. 219, consists of a uniform load and a single concentration, designated as $H15$ loading in Fig. 367.

To obtain the concentrations on the girders (see Art. 219) the 480-lb., (1) uniform load is considered to be on the two traffic lanes as shown in Fig. 382a; or (2) the same uniform load reduced 2 per cent is considered to be distributed over the entire roadway as shown in Fig. 382b. The loading that produces the greatest concentration on the main girder will be used.

Considering the case shown in Fig. 382a, we have

$$\frac{960\times 12}{22}=523.6, \text{ say } 524 \text{ lbs.,}$$

for the concentration on the main girder per foot of span. Next considering the case shown at Fig. 382b, we have

$$\tfrac{480}{9}=53.33 \text{ lbs. per sq. ft. of roadway.}$$

Then we obtain

$$53.33-(53.33\times 0.02)=52.27 \text{ lbs. per sq. ft. of roadway.}$$

For the concentration on the main girder due to uniform load we have

$$52.27\times 10=522.7, \text{ say } 522 \text{ lbs.,}$$

which is about 2 lbs. less than that obtained from the loading shown at Fig 382*a*. So the loading shown at Fig. 382*a* will be used.

Then for the panel load of uniform live load we have

$$524\times 10=5{,}240 \text{ lbs.}$$

The maximum moment on the girder due to this loading will occur at center of span (point *C*, Fig. 381) when all panel points are loaded. Then for the maximum moment due to this uniform live load we have

$$[(5{,}240\times 2.5)30-(5{,}240\times 2\times 15)]12=2{,}829{,}600 \text{ in. lbs.}$$

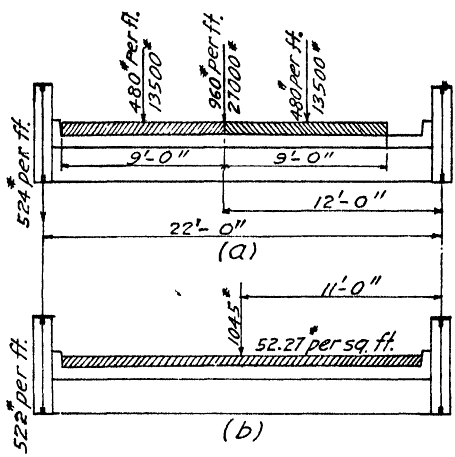


Fig. 382

The 13,500-lb. concentrated loads will be placed as indicated in Fig. 382*a*. Then we obtain

$$\frac{27{,}000\times 12}{22}=14{,}727 \text{ lbs.}$$

for the concentrated load on the girder. Placing this load at point *C* (Fig. 381) and taking moments about that point, we obtain

$$\frac{14{,}727}{2}\times 30\times 12=2{,}650{,}860 \text{ in. lbs.}$$

for the maximum moment due to the concentrated live load.

Now, adding the two moments together, we obtain

$$2{,}650{,}860+2{,}829{,}600=5{,}480{,}460 \text{ in. lbs.}$$

for the total maximum moment on the girder due to live load. Then for the corresponding impact we have

$$5{,}480{,}460\left(\frac{50}{60+125}\right)=1{,}481{,}205 \text{ in. lbs.}$$

Now, adding the above moments to the maximum moment due to dead load, we obtain

$$5{,}480{,}460+1{,}481{,}205+8{,}274{,}960=15{,}236{,}625 \text{ in. lbs.}$$

for the total maximum moment on the main girder due to all loads.

Substituting this total moment in (5) of Art. 113 (page 181) assuming the web to be $\frac{3}{8}$ in. thick we obtain 61.5 ins. for the economic depth of the girder. Since such wide plates vary in width by 6 ins. we shall use a 66-in. plate. The required thickness according to the latest specifications is given by the formula

$$t=\tfrac{1}{20}\sqrt{D},$$

where D is the distance between flange angles in inches.

Then, assuming that 5″x5″ angles will be used for flange angles, we obtain

$$t=\frac{1}{20}\sqrt{66-10}=0.374=\frac{3}{8}\text{ in.}$$

for the required thickness of web, which was assumed.

The maximum end shear will occur when all of the panel points are loaded with uniform live load and the concentrated live load of

$$2(19{,}500\times\tfrac{1}{2}\tfrac{3}{2})=21{,}272 \text{ lbs.}$$

is at the first interior panel point. Then we obtain the following maximum end shear:

Live load..............	$(2\tfrac{1}{2}\times5{,}240)+(21{,}272\times\tfrac{5}{6})=30{,}826$ lbs.	(see Fig. 381)
Impact..........................	$(50\div175)30{,}826=\ \ 8{,}807$ lbs.	
Dead load..........................	$2.5\times15{,}324=38{,}310$ lbs.	
Total..................................	77,943 lbs.	

The dead-load shear at mid-span is zero. To obtain the maximum shear at mid-span due to live load, we place the 5,240-lb. panel load at E, F and C (Fig. 381), and then, taking moments about B, we obtain 5,240 lbs. for the reaction at A, which is the maximum shear at mid-span due to the uniform live load. Then placing the 21,272 load at mid-span (at C), we obtain

$$5{,}240+(21{,}272)\frac{3}{6}=15{,}876 \text{ lbs.}$$

for the maximum shear at mid-span due to live load. For the corresponding impact we have

$$15{,}876\left(\frac{50}{30+125}\right)=5{,}121 \text{ lbs.}$$

Then we have

$$15{,}876+5{,}121=20{,}997 \text{ lbs.}$$

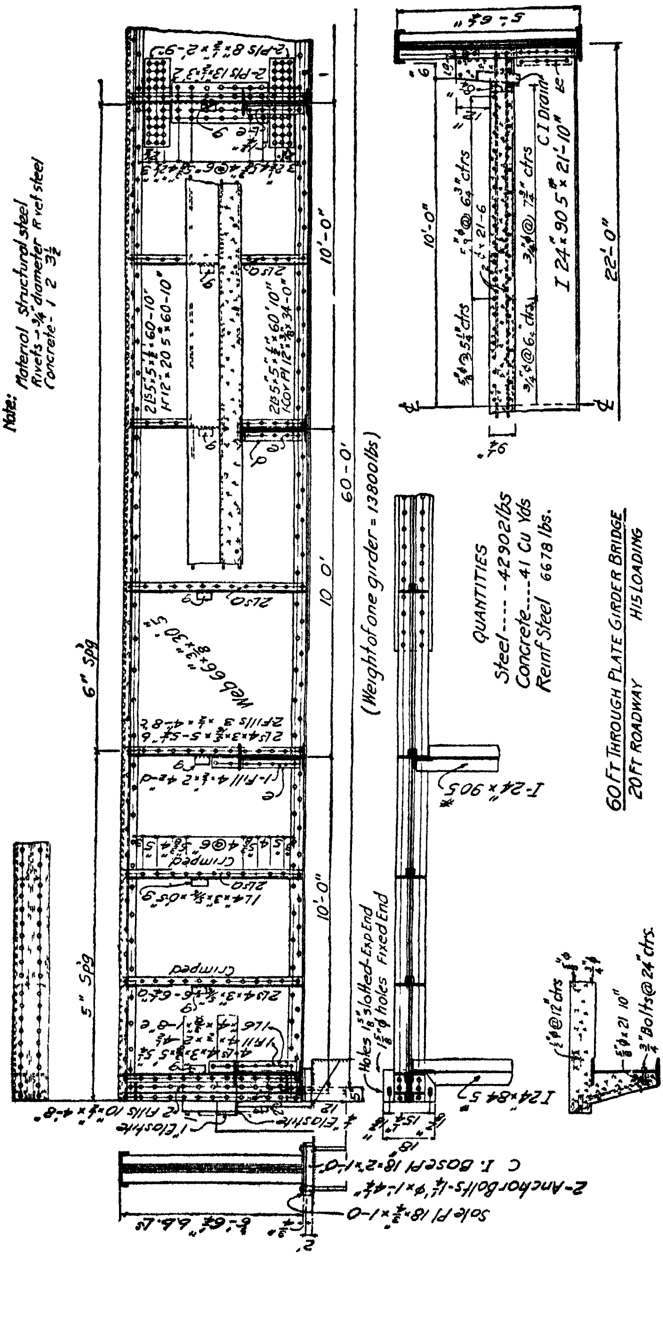

Note: Material structural steel
Rivets - 3/4" diameter Rivet steel
Concrete - 1 2 3½
(Weight of one girder = 13800 lbs)
QUANTITIES
Steel ---- 42902 lbs
Concrete.... 41 Cu Yds
Reinf Steel 6678 lbs.
60 Ft Through Plate Girder Bridge
20 Ft Roadway H15 Loading

Fig 383

for the total maximum shear at mid-span. The shear at any other point on the girder may be found in the same manner.

238. Details.—Having the moments and shears determined, the section and details may be worked out in the manner explained in Arts. 111

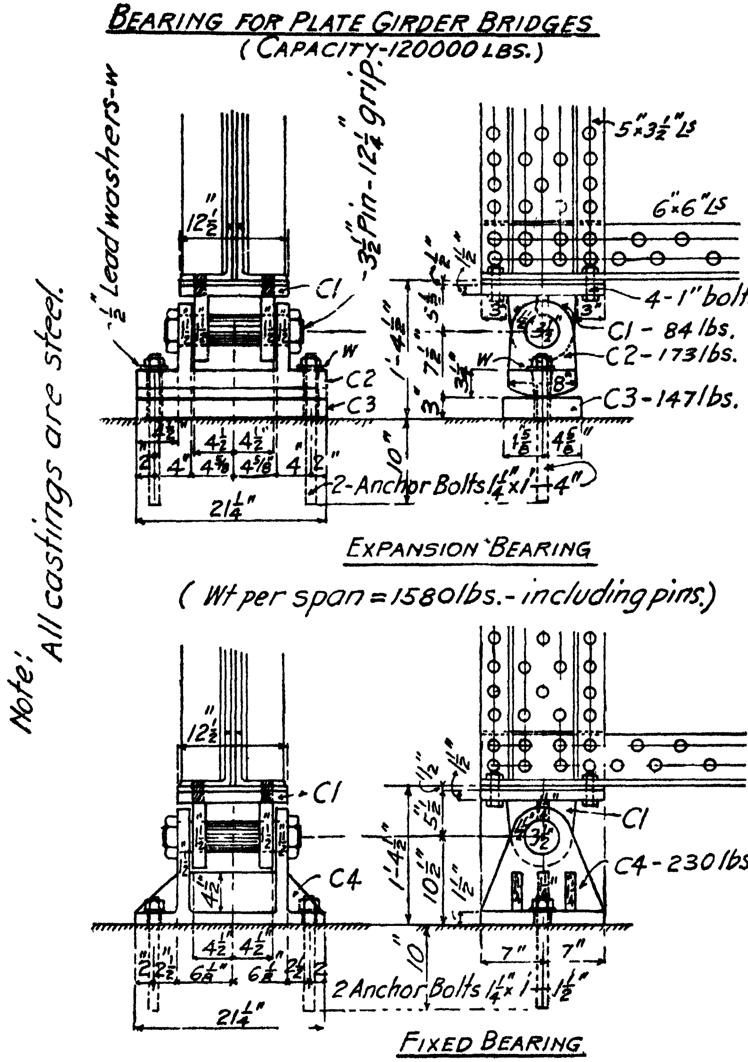


Fig 384

to 119 (pages 177–194) and then the details may be drawn as shown in Fig. 383. These details are practically self-explanatory. The concrete floor having a firm grip on the main girders, as seen, laterals are not needed.

239. Bearings.—For spans up to 60 ft. in length the end bearings shown in Fig. 383 are satisfactory, but for spans longer than 60 ft. the type of bearing shown in Fig. 384 should be used.

240. Deck Plate Girder Spans having roadways up to 24 ft. wide usually consist of three girders, as shown in the cross-sections in Fig. 384*a*.

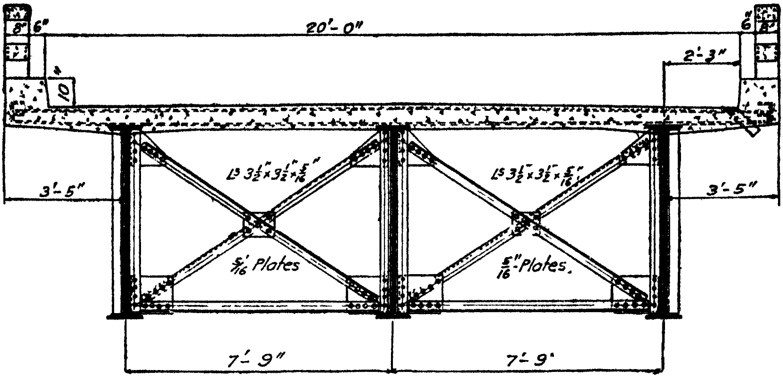


Fig 384*a*

The girders are usually spaced so that the sections of the girders are alike. This spacing of the girders can be obtained most readily by trial.

PONY-TRUSS BRIDGES

241. Description.—Pony-truss bridges in general are very similar to through plate girder bridges. The trusses take the place of the plate girders. The floor system is the same for the two types. The trusses may be of either the Warran or the Pratt type. The chords may be parallel, or the top chord may be curved. Usually the top chord is curved in spans 60 to 100 ft. in length.

242. Depth of Trusses.—Theoretically, the depth of truss should be such that the weight of the chords is equal to the weight of the web system, but details and rigidity should be considered in selecting the depth of truss.

The following depths have been found from experience to be very satisfactory:

Span Length	Depth	
Feet	Feet	Inches
50	7	6
60	8	0
70	9	0
80	9	6
90	10	0
100	12	0

243. Length of Panels.—The thickness of concrete-slab floors varies but slightly with the length of panel so that comparatively long panel lengths are required to obtain economic construction. The panels are usually about 10 ft. in length, as the details work out satisfactorily for this length.

244. Dead Load.—The dead load consists of the metal in the span and the weight of the concrete-slab floor plus any floor covering (wearing surface) that may be used.

The weight of the metal in a span varies with the length of span, width of roadway, and weight of live load used.

For the purpose of computing stresses the following formula may be used for obtaining the weight of metal in pony-truss bridges without stringers, having 20-ft. roadway and carrying $H15$ loading:

$$w = 5L + 525 \quad \text{(P)},$$

where w = weight of metal in pounds per foot of bridge (not including reinforcement in the concrete floor) and L = length of span in feet. Weight of metal in spans having roadways wider than 20 ft. may be obtained from Formula (P) by adding 50 lbs. for each additional foot of width over 20 ft. In case a loading lighter or heavier than $H15$ is used, the value obtained from Formula (P) should be reduced or increased according to the weight of the loading used.

The weight of the concrete floor may be obtained from the following formula:

$$w' = 124(R+2) \quad \text{(P')},$$

where w' = weight of concrete floor (including curbs) per foot of roadway and R = width of roadway in feet.

Complete Design of a 50-Ft. Pony-truss Bridge

245. Data.—

Length – 50′-0″ c. c. end bearings.
Width = 20′-0″ clear roadway
Panel length = 10′-0″.
Height = 7′-6″.
Floor concrete slab supported upon floor beams, without stringers.
Live load, $H15$ loading.

245a. Dead-load Stresses in Trusses.—Substituting in Formula (P′), given in Art. 244, we obtain

$$w' = 124(20+2) = 2{,}728 \text{ lbs.}$$

for the weight of the concrete floor per foot of span (including curbs), or 1,364 lbs. per foot of truss.

As a check on the formula, taking the average thickness of the slab (designed in Art 227, page 542) as $8\frac{3}{4}$ ins., we obtain

$$(8.75 \times 12.5) + 15 = 124 \text{ lbs.}$$

for the weight of the floor slab per square foot (the 15 lbs. is for floor covering). The curbs weigh 224 lbs. (computed from details) per foot of span. Then we obtain

$$\frac{(124 \times 20) + 224}{2} = 1{,}352 \text{ lbs.}$$

for the weight of the floor slab per foot of truss, which is only (1,364 – 1,352) 12 lbs. less than given by Formula (P′).

Next, from Formula (P), Art. 244, we obtain

$$w = \frac{(5 \times 50) + 525}{2} = 387 \text{ lbs.}$$

for the weight of metal in span per foot of truss. Now we have

$$1{,}352 + 387 = 1{,}739 \text{ lbs.}$$

for the total dead load per foot of truss. Then for the panel load (dead load), we have

$$W = 1{,}739 \times 10 = 17{,}390 \text{ lbs.}$$

Only a small part of the dead load is on the top chord; so for convenience we shall consider all of it on the bottom chord joints.

The dead-load stresses are as shown in Fig. 385. (See Art. 173, page 334.)

$$\tan\theta = 10 \div 7.5 = 1.333$$

and from tables we find

$$\sec\theta = 1.67.$$

Then we obtain

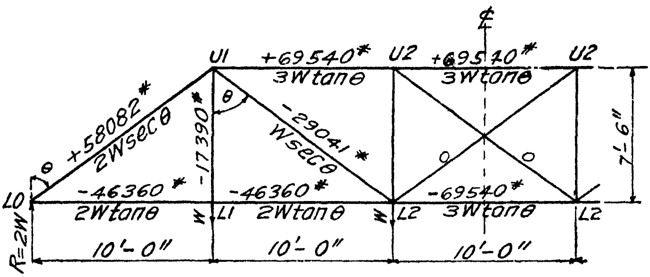


Fig. 385

$$W\tan\theta = 17{,}390 \times 1.333 = 23{,}180 \text{ lbs.}$$
$$W\sec\theta = 17{,}390 \times 1.67 \;\; = 29{,}041 \text{ lbs.}$$

Then we obtain the following stresses:

End post $U1L0 = 29{,}041 \times 2 = 58{,}082$ lbs. compression.
Diagonal $U1L2 = 29{,}041 \times 1 = 29{,}041$ lbs. tension.
Chords $U1U2\text{-}U2U2 = 23{,}180 \times 3 = 69{,}540$ lbs. compression.
Chord $L2L2 = 23{,}180 \times 3 = 69{,}540$ lbs. tension.
Chords $L0L1\text{-}L1L2 = 23{,}180 \times 2 = 46{,}360$ lbs. tension.

245b. Designing of Floor Beams.—If the distance from the center of truss to face of curb is 1'-3", the distance center to center of trusses will be

$$20 + 2.5 = 22.5 \text{ ft.},$$

which will be taken as the length of each floor beam.

The weight of the floor slab as previously found is 124 lbs. per square foot. Then for the dead load per linear foot of each intermediate floor beam we have

Slab........................	$124 \times 10 = 1{,}240$ lbs.
Beam (assumed).....................	90 lbs.
Total	1,330 lbs.

Then for the maximum moment on floor beam due to dead load, we have

$$\tfrac{1}{8} \times 1{,}330 \times (22.5)^2 \times 12 = 1{,}010{,}013 \text{ in. lbs.}$$

Placing the wheel loads in position for maximum moment, as explained in Art. 236, and taking moments, we obtain

$$[(22{,}400 \times 10.5) - (12{,}000 \times 6)]12 = 1{,}958{,}400 \text{ in. lbs.}$$

for the maximum moment on the floor beam due to live load. For the corresponding impact we have

$$\left(\frac{50}{20+125}\right)1,958,400=675,310 \text{ in. lbs.}$$

Then for the total maximum moment on floor beam due to all loads we have

$$1,010,013+1,958,400+675,310=3,643,723.$$

Then for the section modulus required we obtain

$$3,643,723 \div 16,000=227.7,$$

which calls for a 27-in.x91-lb Carnegie beam, which will be used. (See A.I.S.C. handbook.)

Placing the wheel loads in position for maximum end shear and taking moment as explained in Art. 236 (Fig. 380) we obtain

$$\frac{48,006 \times 12.25}{22.5}=26,133 \text{ lbs.}$$

for the maximum end shear on the floor beam due to live load. For the corresponding impact we have

$$\left(\frac{50}{20+125}\right)26,133=9,010 \text{ lbs.}$$

For the end shear due to dead load, we have

$$\frac{(1,240\times 20)+(224\times 10)+(91\times 21)}{2}=14,475 \text{ lbs.}$$

Then for the total maximum end shear due to all loads we have

$$26,133+9,010+14,475=49,618 \text{ lbs.}$$

The maximum moment and shear on an end floor beam due to live load and impact are about the same as on an intermediate floor beam; but, since the dead load is about one-third less, the moment and shear due to dead load will be less but to simplify details, the end floor beams are usually made the same size as the intermediate floor beams.

245c. Live-Load Stress in Trusses.—The maximum concentration on the truss from a floor beam due to the rear axles, as given in Art. 245*b*, is 26,133 lbs., and the concentration from the front axles will be one-fourth of this, or 6,533 lbs. The live-load stresses in the truss members will be due to these concentrations, and the stresses are determined in the same manner as previously shown for railroad bridges.

For example, by placing the wheels as shown in Fig. 386*a*, the criteria given in Arts. 90 and 91 (pages 134–139) are satisfied for maximum shear in panel $L0L1$ and also for the maximum moment about joint $U1$. Then taking moments about end A, we obtain

$$\frac{(40\times 26,133)+(26\times 6,533)}{50}=24,303 \text{ lbs.}$$

for the reaction at $L0$, which is also equal to the shear in panel $L0L1$. Then we obtain

$$24{,}303 \times \sec\theta = 24{,}303 \times 1.67 = 40{,}586 \text{ lbs.}$$

for the maximum live-load stress in end post $L0U1$. Taking $L = 40$ (from A to $L1$), we obtain

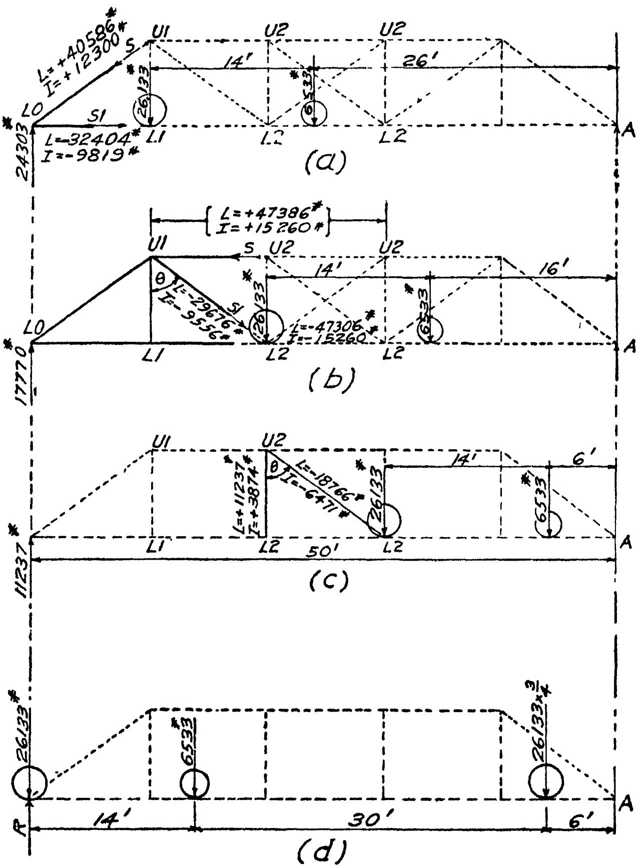


Fig. 386

$$\left(\frac{50}{40+125}\right)40{,}586 = 12{,}300 \text{ lbs.}$$

for the corresponding impact.

Next, taking moments about $U1$ (Fig. 386*a*), we obtain

$$S1 = \frac{24{,}303 \times 10}{7.5} = 32{,}404 \text{ lbs.}$$

for the maximum live-load stress in the bottom chords $L0L1$ and $L1L2$. For the corresponding impact we have

$$\left(\frac{50}{40+125}\right)32{,}404=9{,}819 \text{ lbs.}$$

As is obvious, the maximum live-load stress in hanger $U1L1$ is 26,133 lbs. and the corresponding impact is (L = two panels)

$$\left(\frac{50}{20+125}\right)26{,}133=9{,}011 \text{ lbs.}$$

The wheel loads placed as shown in Fig. 386*b* are in the position for maximum shear in panel $L1L2$ and also for maximum moment about joint $L2$.

Taking moments about end A, we obtain 17,770 lbs. for the reaction at $L0$, which is equal to the maximum shear in panel $L1L2$ due to live load. Then we obtain

$$17{,}770\times1.67=29{,}676 \text{ lbs.}$$

for the maximum live-load stress in diagonal $U1L2$. For the corresponding impact we have

$$\left(\frac{50}{30+125}\right)29{,}676=9{,}556 \text{ lbs.}$$

Taking moments about joint $L2$, we obtain

$$\frac{17{,}770\times20}{7.5}=47{,}386 \text{ lbs.}$$

for the maximum live-load stress in top chord $U1U2$. For the corresponding impact we have

$$\left(\frac{50}{30+125}\right)47{,}386=15{,}260 \text{ lbs.}$$

The maximum live-load stress occurs in post $U2L2$ and in diagonal $U2L2$ when the wheel loads are in the position shown in Fig. 386*c*.

Taking moments about end A (Fig. 386*c*), we obtain 11,237 lbs. for the reaction at $L0$, which is also equal to the maximum live-load shear in panel $L2L2$ and to the maximum live-load stress in post $U2L2$, as is obvious. For the corresponding impact in this post we obtain

$$\left(\frac{50}{20+125}\right)11{,}237=3{,}874 \text{ lbs.}$$

For the stress in diagonal $U2L2$ we have

$$11{,}237\times\sec\theta=11{,}237\times1.67=18{,}766 \text{ lbs.}$$

and for the corresponding impact we have

$$\left(\frac{50}{20+125}\right)18{,}766=6{,}471 \text{ lbs.}$$

Placing the wheel loads as shown in Fig. 386*d* and taking moments about end *A*, we obtain 33,188 lbs. for the maximum end reaction on the truss due to live load. For the corresponding impact we have

$$\left(\frac{50}{50+125}\right)33,188=9,483 \text{ lbs.}$$

For the end reaction due to dead load we have

Slab (see Art. 245*a*)	1,352(25+1) = 35,152 lbs.
Metal	387×25 = 9,675 lbs.
End floor beam and bearings (computed)	1,201 lbs.
Total	46,028 lbs.

Then for the total maximum reaction on truss we have

$$33,188+9,483+46,028=88,699 \text{ lbs.}$$

Then for the required area of bearing on masonry, we have

$$88,699 \div 600 = 147 \text{ sq. ins.}$$

245d. Wind Stresses.—Considering wind load, according to Art. 223, we have (by estimating) an area of about 4.6 sq. ft. per foot of span for the $1\frac{1}{2}$ times the vertical projection of the structure. Then for the wind pressure on the structure we obtain

$$4.6\times30=138 \text{ lbs. per ft. of span.}$$

For the pressure on the live load, we have

$$8\times30=240 \text{ lbs. per ft. of span.}$$

Then for the total wind pressure we have

$$138+240=378, \text{ say } 380 \text{ lbs. per ft. of span.}$$

Then the panel load equals

$$380\times10=3,800 \text{ lbs.}$$

For determining the stresses in the laterals, we have

$$\tan\phi=\frac{20}{22.5}=0.888 \text{ (see Fig. 387)}$$

and

$$\sec\phi=1.338$$

$$\tan\phi'=\frac{10}{22.5}=0.444$$

and

$$\sec\phi'=1.094.$$

Then for the wind stress in the end laterals we have

$$\tfrac{6}{5}\times3,800\times1.338=6,101 \text{ lbs.}$$

and for the wind stress in the laterals in the central panel we have

$$\tfrac{3}{5}\times 3{,}800\times 1.094=2{,}494 \text{ lbs.}$$

Now, by collecting the stresses found in the foregoing analysis, we obtain the stresses shown on the stress sheet (Fig. 387).

245e. Designing of the Sections.—All floor beams, as previously designed, are 27-in.x91-lb. Carnegie I-beams. The sections for the truss members must be selected with reference to details and rigidity. The most efficient top chord, as a rule, consists of two channels and a cover plate. The

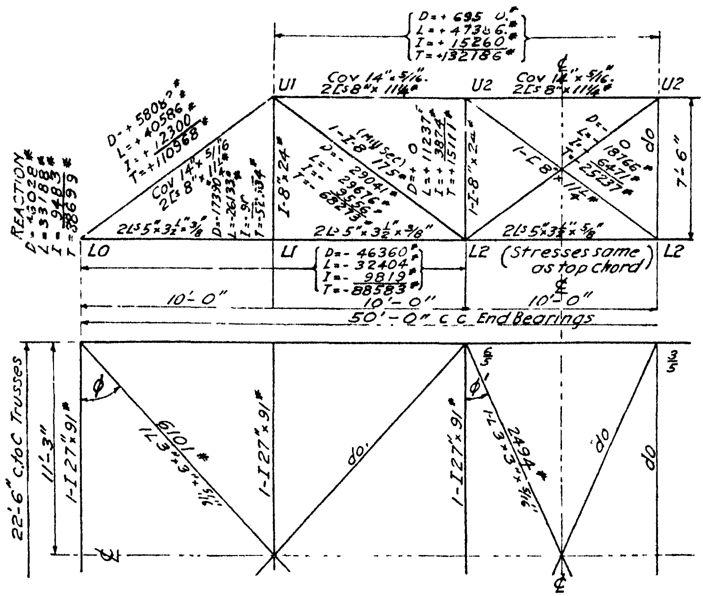


Fig. 387

bottom chord may be made of angles or channels, the diagonals may be angles or I-beams, and the posts and hangers may be four angles and web or I-beams.

The work of selecting the sections is the same as that previously shown for railroad bridges.

Bottom Chord L0L2. Required area $=88{,}583\div 16{,}000=5.5$ sq. ins. Two ∠s $5\text{x}3\tfrac{1}{2}\text{x}\tfrac{3}{8}$ have a net area of $2\text{x}3.05-0.33\text{x}2=5.44$ sq. ins. ($\tfrac{3}{4}$-in. rivets), which is about as near to the required sections as can be obtained and hence will be used.

Diagonal U1L2. To obtain rigidity and simplify details, we shall use I-beams. For the required area, we have $68{,}273\div 16{,}000=4.26$ sq. ins. net. One 8-in.x17.5-lb. Carnegie I-beam has a gross area of 5.14 sq. ins. Considering two rivet ($\tfrac{7}{8}$-in.) holes out of the flanges, we obtain $5.14-0.33\text{x}2\text{x}\tfrac{7}{8}=4.48$ sq. ins. net area (see Carnegie handbook).

This area is about correct and hence will be used.

Diagonals U2L2. For the required area we have $25{,}237\div 16{,}000=1.57$ sq. ins., which is a small area; so an 8-in.x$11\tfrac{1}{4}$-lb. channel will be used as a matter of detail. Considering two $\frac{7}{8}$-in. holes out of the flanges, this channel has a net section of 2.7 sq. ins.

Top Chord U1U2. It now seems that the diagonals will be 8 ins. wide and the verticals may be 8 ins. wide, so that the chord channels will be about $8\frac{3}{4}$ ins. back to back, assuming the gusset plates to be $\frac{3}{8}$ in. thick, which is the minimum thickness required by modern specifications. Eight-inch channels are the smallest channels permitting the use of $\frac{3}{4}$-in. rivets, which is a desirable-size rivet for highway bridges. So we shall assume a section composed of two 8-in.x$11\frac{1}{2}$-lb. channels and a 14-in.x$\frac{5}{16}$-in. cover plate which has a combined area of $6.72+4.38=11.10$ sq. ins. The approximate radius of gyration about the vertical axis is $r_v=0.55\text{x}8.75=4.8$ (see table in back of book) and for the approximate radius about the horizontal axis, we have

$$r_h=0.39\times8=3.12.$$

The author believes that the length, considering the transverse direction should be taken as two panel lengths in the case of pony-truss bridges. So we have

$$(\text{Vertical axis})\frac{L}{r_v}=\frac{240}{4.8}=50.$$

Now, from Curve *A* (Art. 225), we obtain, 13,500 lbs. for the allowable unit stress on the assumed section. Then for the required section we obtain

$$132{,}186\div13{,}500=9.79 \text{ sq. ins.},$$

which is reasonably close to the area of the section assumed.

Considering the vertical direction, the length can be taken as 10 ft., that is, one panel length, in which case we obtain

$$\frac{L}{r_h}=\frac{120}{3.12}=38.7.$$

Then, from Curve *A* (Art. 225), we obtain 14,300 lbs. for the allowable unit stress, which is the maximum allowed on any compression member.

From actual calculations it is found that $r_v=4.64$ and $r_h=3.16$, which is quite close to the values used. So it is seen that the assumed section of top chord *U1U2U2* is satisfactory.

End Post. As is obvious, the end posts could have a smaller section than top chord *U1U2*, but we are justified in using the same section to simplify details. So the same section will be used for the end posts as is used for top chords.

Hanger U1L1. For the required area of cross-section for tension we have

$$52{,}534\div16{,}000=3.28 \text{ sq. ins.}$$

About the smallest I-beam that can be used, on account of the floor beam connection, is an 8-in.x24-lb. Carnegie beam which has a section of 7.06 sq. ins. Deducting four rivet holes ($\frac{7}{8}$ in.) out of the flanges, we have 5.66 sq. ins. net section remaining, which is excessive. But usually the hanger would be designed to take a certain thrust from traffic applied perpendicularly to the

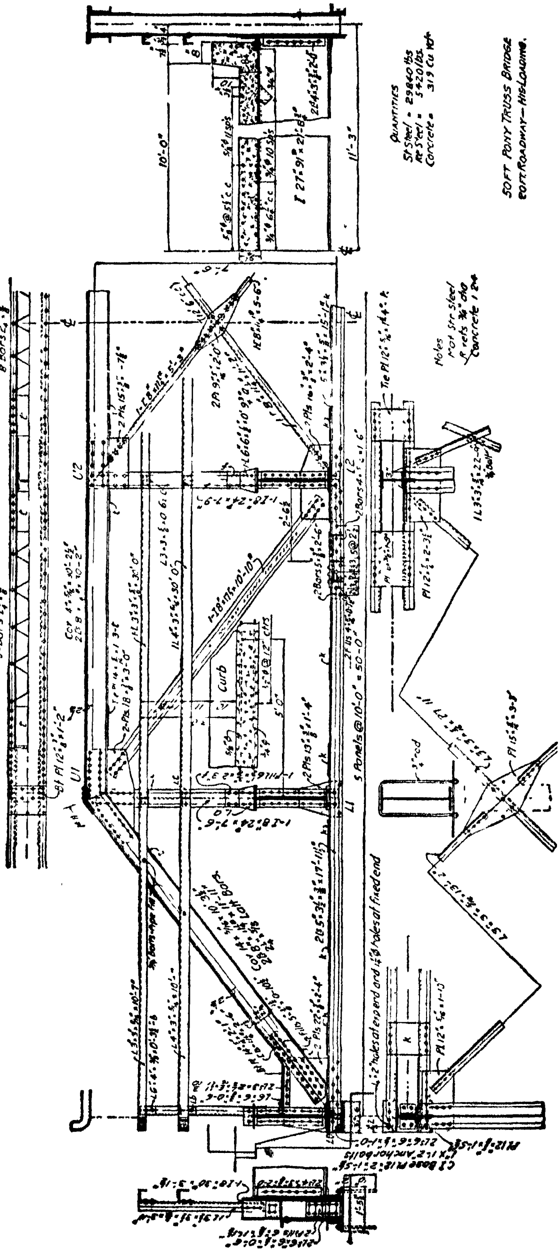

50 FT. Pony Truss Bridge
20 FT. Roadway—H15 Loading
Quantities
5 Panels @ 10'-0" = 50'-0"
Curb
U1
U2
L1
L2

Fig. 387a

side of the top chord and end post at joint $U1$. This thrust is usually determined from the following empirical formula:

$$R = 150(A + P),$$

where R = thrust in pounds, A = area of the cross-section of the top chord in square inches, and P = panel length in feet.

Then for the transverse thrust at $U1$ we have

$$R = 150(11.10 + 10) = 3,165 \text{ lbs.}$$

Taking moments about the top of the floor beam, we have

$$3,165 \times 60 = 189,900 \text{ in. lbs.}$$

for the maximum moment on the hanger due to transverse thrust. Then for the stress on the hanger (8-in.x24-lb. I) due to this moment we have

$$f = \frac{189,900 \times 4}{84.3} = 9,033 \text{ lbs. per sq. in.}$$

For the direct stress we have

$$52,534 \div 5.66 = 9,281 \text{ lbs. per sq. in.}$$

Adding these two stresses together, we obtain

$$9,033 + 9,281 = 18,314 \text{ lbs. per sq. in.}$$

for the maximum combined stress, which is satisfactory, since usually 20,000 lbs. per square inch is permitted in such combinations of stresses. So the 8-in.x24-lb. beam would be used.

Post $U2L2$. The same size beam as that for the hanger $U1L1$ will be used, to simplify details, although the combined stress is less.

Laterals. For the maximum area of cross-section required, we have

$$6,101 \div 16,000 = 0.38 \text{ sq. in.,}$$

which calls for a very small angle; but to obtain rigidity we shall use $3'' \times 3'' \times \frac{5}{16}''$ angles throughout.

Having the stress sheet (Fig. 387) completed, we may next draw the details shown in Fig. 387*a*. The details are calculated in the manner previously shown for railroad bridges.

All other pony-truss bridges are designed in the manner shown in the foregoing analysis except that, for spans having a length of 60 ft. and over, the equivalent loading is used instead of the wheel loads. As an illustration we shall consider a 60-ft. span, in the next article.

246. Sixty-foot Pony-truss Bridge.—From Art. 242 it is seen that the truss should be 8 ft. deep. We shall make it 8 ft. deep at mid-span and 7 ft. 3 ins. deep at the hip, as shown in Fig. 388. The roadway is to be 20 ft. wide. Taking the distance from the center of truss to the face of curb as 1 ft. 3 ins., we find the distance center to center of trusses to be 22.5 ft., the same as for the 50-ft. span analyzed in the foregoing articles. So the floor beams will be 27-in.×91-lb. Carnegie beams, the same as in the 50-ft. span.

246a. Dead-load Stresses in the Trusses.—From Formula (P) of Art. 244 (page 559), we obtain

$$w = 5 \times 60 + 525 = 825 \text{ lbs.}$$

for the weight of metal per foot of span, or 412 lbs. per foot of truss. Adding this to the weight of the floor given in Art. 245*a* (page 559), we obtain

$$1{,}352+412=1{,}764 \text{ lbs.}$$

for the total dead load per foot of truss. Then for the panel load we have

$$1{,}764\times 10=17{,}640, \text{ say } 18{,}000 \text{ lbs.}$$

We can now determine the stresses due to dead load, as shown in Fig. 388, most readily by graphics.

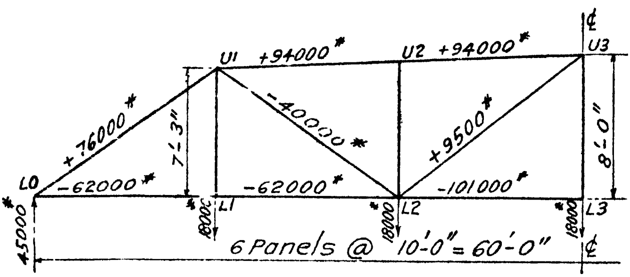


Fig. 388.

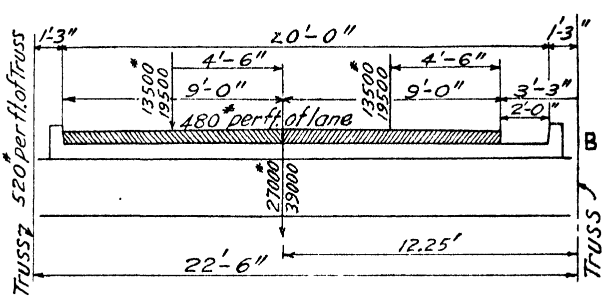


Fig. 388*a*

For the maximum end reaction on the truss due to dead load we have

$$(2\tfrac{1}{2}+\tfrac{2}{3})18{,}000=57{,}000 \text{ lbs.}$$

246b. Live-load Stresses.—The equivalent *H*15 loading (see Fig. 367, page 534) will be used. The maximum live load on the truss will occur when the loads are in the position shown in Fig. 388*a*. (For wider roadways the loading distributed over the entire roadway would be used, since the load on the truss would, in that case, be the greater. See Art. 219, page 535.)

Taking moments about *B* (Fig. 388*a*), we obtain

$$\frac{(480\times 2)12.25}{22.5}=520 \text{ lbs.}$$

for the maximum uniform live load per foot of truss. Then for the panel load due to the uniform live load we have

$$P=520\times 10=5{,}200 \text{ lbs.}$$

Again, taking moments about B (Fig. 388*a*), we obtain

$$P' = \frac{(2 \times 13{,}500)12.25}{22.5} = 14{,}688, \text{ say } 14{,}700 \text{ lbs.}$$

for the panel load due to the single live-load concentration to be used in determining maximum moments on the truss. In the same manner we obtain

$$P'' = \frac{(2 \times 19{,}500)12.25}{22.5} = 21{,}231, \text{ say } 21{,}230 \text{ lbs.}$$

for the panel load due to the single live-load concentration to be used in determining shears on the truss.

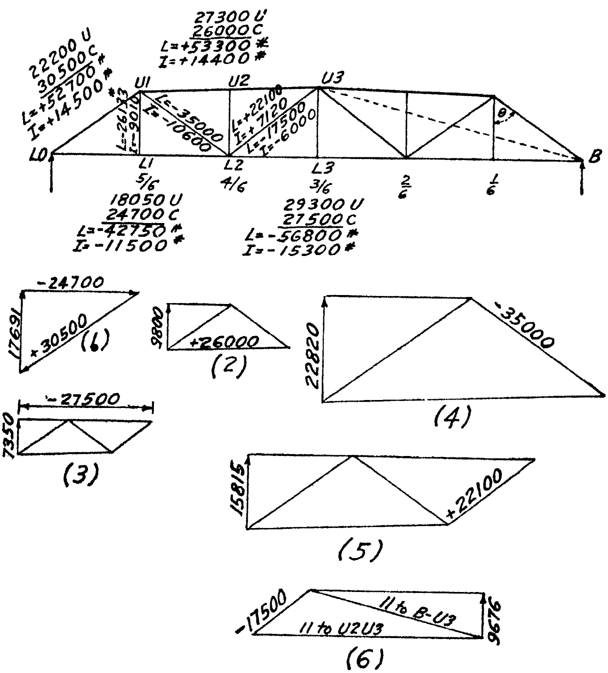


Fig. 388*b*

We now have the panel loads computed, and we shall next determine the stresses in the truss due to these live loads.

As is obvious, the maximum stress in the chords and end posts due to the uniform live load will occur when all of the panels are loaded—the same as dead load. So we can obtain the live load stresses due to the uniform live load (marked U in Fig. 388*b*) by multiplying the dead-load stresses (given in Fig. 388) by 5,200/18,000—the ratio of the panel loads. The slide rule is convenient in this case, as the stresses may be obtained by one setting of the rule.

As is obvious, the maximum stress in the end post $U1L0$ (Fig. 388*b*) and bottom chord $L0L1$ due to the single live-load concentration will occur when the single concentration is at $L1$, and the maximum stress in the end post will be due to shear in panel $L0L1$. So, by placing the 21,230-lb. load at $L1$, we obtain

$$\tfrac{5}{6}\times 21{,}230\times \sec\theta = S$$

for the stress in the end post $U1L0$ due to the single concentration and, at the same time, we obtain

$$\tfrac{5}{6}\times 21{,}230\times \tan\theta = S1$$

for the stress in the bottom chord $L0L1$. The stress throughout the truss due to this particular highway loading can be determined more easily by graphics than by algebraic resolution. With the 21,230-lb. load at $L1$ the reaction at $L0$ due to this load is

$$\tfrac{5}{6}\times 21{,}230 = 17{,}691 \text{ lbs.}$$

Then, by drawing diagram (1) (Fig. 388*b*) we obtain the stresses in the end post $U1L0$ and in the bottom chord $L0L1$ as indicated. Now, adding this live-load stress in the end post to the stress due to the uniform live load, as previously found, we obtain

$$22{,}200+30{,}500=52{,}700 \text{ lbs.}$$

for the maximum live-load stress in the end post $U1L0$. For the corresponding impact in the end post, taking $L=60$ ft. (total length of span), we obtain

$$\left(\frac{50}{60+125}\right)52{,}700=14{,}500 \text{ lbs. (about)}$$

and for the maximum live-load stress in bottom chord $L0L1$ (also $L1L2$) we have

$$18{,}050+24{,}700=42{,}750 \text{ lbs.}$$

For the corresponding impact in that member we have

$$\left(\frac{50}{60+125}\right)42{,}750=11{,}500 \text{ lbs. (about).}$$

The maximum stress in top chord $U1U3$ due to the 14,700 lbs. (concentration (P')) will occur when the concentration is at $L2$. Then, placing this load at $L2$, the reaction at $L0$ is

$$14{,}700\times\tfrac{4}{6}=9{,}800 \text{ lbs.}$$

and then, drawing the diagram (2) (Fig. 388*b*), we obtain the maximum stress in top chord $U1U3$ due to the concentrated live load. Now, adding this stress to the stress due to the uniform live load, we obtain

$$27{,}300+26{,}000=53{,}300 \text{ lbs.}$$

for the total maximum live-load stress in top chord $U1U3$. Then for the corresponding impact in top chord $U1U3$ we have

$$\left(\frac{50}{60+125}\right)53{,}300=14{,}400 \text{ lbs. (about).}$$

The maximum stress in bottom chord $L2L3$ due to the concentrated live load will occur when the 14,700-lb. load is at $L3$. The reaction at $L0$ then equals

$$14{,}700\times \tfrac{3}{6}=7{,}350 \text{ lbs.}$$

Then, drawing diagram (3) (Fig. 388*b*), we obtain the maximum stress in bottom chord $L2L3$ due to concentrated live load. Then, adding this stress to the stress due to the uniform live load, we obtain

$$29{,}300+27{,}500=56{,}800 \text{ lbs.}$$

for the total maximum live-load stress in bottom chord $L2L3$. For the corresponding impact we obtain

$$\left(\frac{50}{60+125}\right)56{,}800=15{,}300 \text{ lbs. (about).}$$

Diagonals. The maximum live-load stress in diagonal $U1L2$ will occur when panel point $L2$ and the three other panel points to the right are loaded with uniform live load and the 21,230-lb. concentration is at $L2$. With that loading, we obtain

$$\frac{10}{6}(5{,}200)+\frac{4}{6}(21{,}230)=22{,}820 \text{ lbs.}$$

for the reaction at $L0$. Then, by drawing diagram (4) (Fig. 388*b*), we obtain the maximum live-load stress in diagonal $U1L2$. For the corresponding impact we obtain

$$\left(\frac{50}{40+125}\right)35{,}000=10{,}600 \text{ lbs.}$$

The maximum compression due to live load will occur in diagonal $U3L2$ when panel point $L3$ and the two other panel points to the right are loaded with uniform live load and the 21,230-lb. concentration is at $L3$. With the loads in this position, we have

$$\frac{6}{6}(5{,}200)+\frac{3}{6}(21{,}230)=15{,}815 \text{ lbs.}$$

for the reaction at $L0$. Then, by drawing diagram (5) (Fig. 388*b*), we obtain the maximum live-load compression in diagonal $U3L2$. For the corresponding impact we obtain

$$\left(\frac{50}{30+125}\right)22{,}100=7{,}120 \text{ lbs.}$$

Maximum tension due to live load will occur in diagonal $U3L2$ when panel points $L1$ and $L2$ are loaded with uniform live load and the 21,230-lb. concentration is at $L2$. With the loads in that position we have

$$\frac{3}{6}(5{,}200)+\frac{2}{6}(21{,}230)=9{,}676 \text{ lbs.}$$

for the reaction at B.

Then, by drawing diagram (6) (Fig. 388*b*), we obtain the maximum live-load tension in diagonal *U3L2*. For the corresponding impact we have

$$\left(\frac{50}{20+125}\right)17{,}500=6{,}030, \text{ say } 6{,}000 \text{ lbs.}$$

The maximum stress in hanger *U1L1* (also *U3L3*) due to wheel loads, as found previously for the 50-ft. span, is 26,133 lbs. From the equivalent loading we obtain

$$21{,}230+5{,}200=26{,}430 \text{ lbs.},$$

which is practically the same value, but the stress due the wheel loads is usually taken, since the wheel loads are used in designing the floor beams.

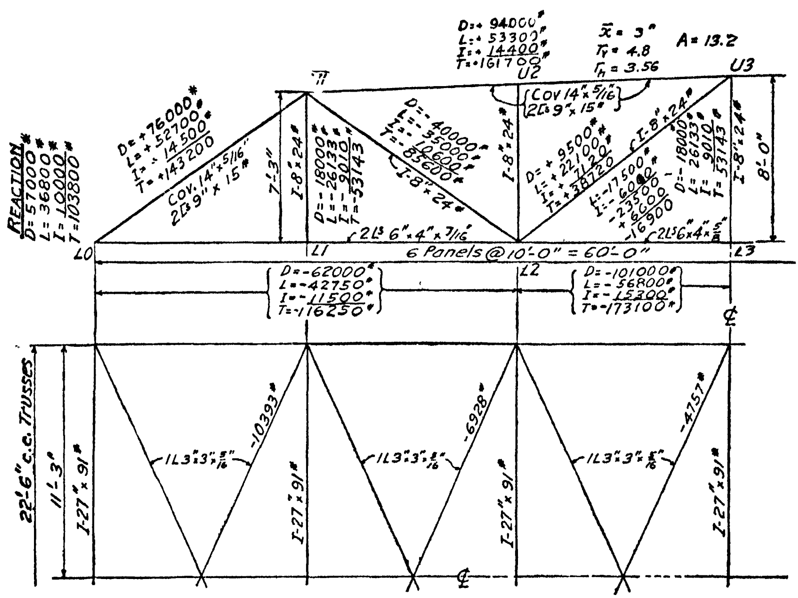


Fig. 388*c*

Post *U2L3* transmits no load except a small amount of dead load at panel point *U2*, which will be disregarded.

By collecting the stresses found in the foregoing analyses, we have the stresses that are given on the stress sheet (Fig. 388*c*). The sections shown on the stress sheet are obtained in the manner explained in the preceding articles for the 50-ft. span, and the wind stresses are obtained in the manner shown for the 50-ft. span. Having the stress sheet completed, the detail drawing of the 60-ft. span shown in Fig. 388*d* may be made, which completes the designing. The shop drawings may be made from the general detail drawing shown in Fig. 388*d*.

All pony-truss bridges may be designed in the manner outlined in the foregoing articles.

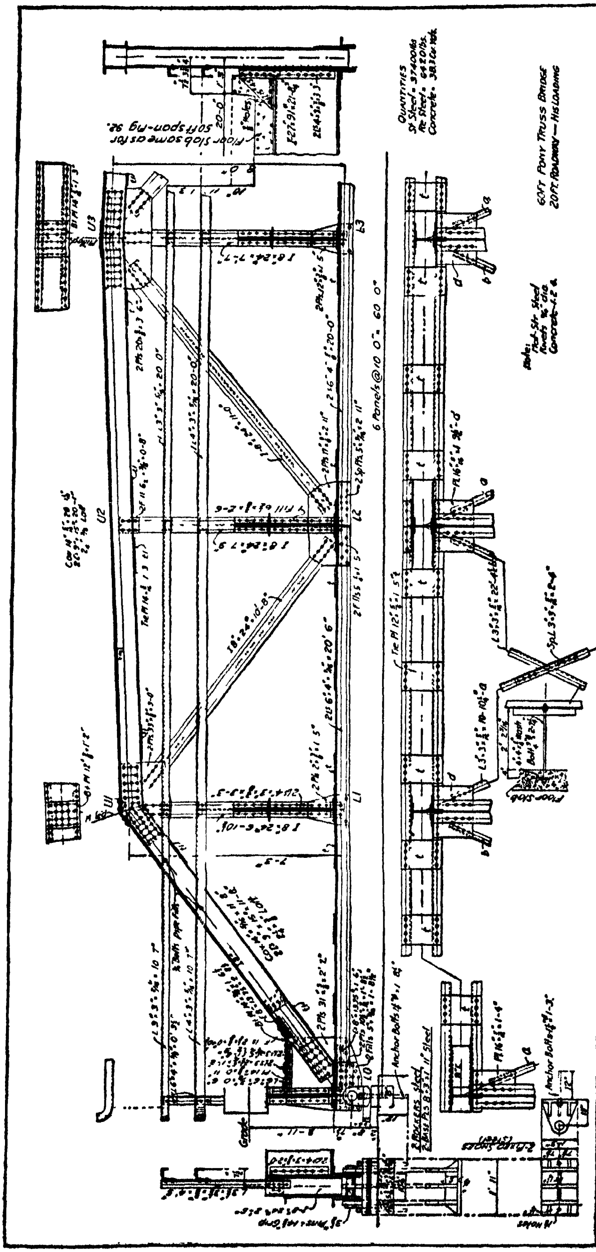


Fig. 388d

THROUGH TRUSS BRIDGES

247. Type.—The parallel-chord Pratt-truss type is generally used for short spans, that is, for spans not exceeding 130 ft. in length. The curved-chord Pratt-truss type is generally used for spans from 130 to 250 ft. in length. The Pettit-truss type is generally used for spans over 250 ft. in length.

248. Length of Panels.—In most cases economic construction is obtained by making comparatively short panels, owing to the weight saved in the stringers.

249. Depth of Trusses.—As is obvious, a truss will have an economic depth when the weight of the two chords is equal to the weight of the web members. However, this condition does not always conform to satisfactory details, and the depths used in practice are those that most closely meet all requirements, including economic proportions. The depths given in Art. 255 are found in practice to be satisfactory.

250. Dead Load.—The approximate weight of metal in through truss bridges having a 20-ft. concrete roadway may be obtained from the following formula:

$$w=4.5L+460 \quad \text{(T)},$$

where w = weight of metal per foot of span and L = length of span in feet. Formula (T) is for a 20-ft. roadway, and allowances must be made in case the formula is used for determining the weight of metal in spans having a different width of roadway.

The approximate weight of a concrete roadway may be obtained from the following formula:

$$w=124(W+1) \quad \text{(T')},$$

where w = weight of floor (including 25 lbs. per square foot of wearing surface) per foot of span in pounds and W = width of roadway in feet.

Design of 100-Ft. Through Parallel Chord Truss Bridge: 20-Ft. Roadway; H15 Loading

251. Description.—In order to obtain a suitable portal depth the trusses will be made 20 ft. deep, center to center of chords, and divided into five equal panels of 20 ft. each. We shall assume the cover plate on the end posts to be 15 ins. wide and each curb to be 6 ins. wide. Then we have

$$1.25+1+20=22.25 \text{ ft.}$$

for the distance center to center of trusses.

251a. Designing of Stringers.—The stringers will be 20 ft. long and spaced 4 ft. apart. Then, taking the weight of the slab as 124 lbs. per square foot, including about 25 lbs. of wearing surface, we obtain the following dead weight per foot of stringer

Floor	124×4 = 496	lbs. per ft. of stringer
Beam (assumed)	35	lbs. per ft. of stringer
Total	531	lbs. per ft. of stringer

Then for the maximum moment on the stringers due to dead load we have

$$\tfrac{1}{8}\times 531\times 20^2\times 12=318{,}600 \text{ in. lbs.}$$

The maximum live-load moment will occur when the 12,000-lb. rear wheel is at mid-span. Then (see Art. 224) we have

$$\frac{12{,}000}{2}\times\frac{4}{4.5}10\times12=640{,}000 \text{ in. lbs.}$$

for the maximum moment on the stringer due to live load. For the corresponding impact we have

$$\left(\frac{50}{20+125}\right)640{,}000=221{,}000 \text{ in. lbs. (about).}$$

Then for the total maximum moment on the stringer due to all loads we have

$$318{,}600+640{,}000+221{,}000=1{,}179{,}600 \text{ in. lbs.}$$

Then for the required section modulus we have

$$1{,}179{,}600\div16{,}000=73.6,$$

which calls for a 16-in.x45-lb. beam, which will be used throughout.

If the 12,000-lb. wheel is placed at one end of the stringer, the front wheel will be 6 ft. from the other end. Then, taking moments about the other end with the wheels in that position, we have

$$12{,}000+\left(\frac{3{,}000\times6}{20}\right)\frac{4}{4.5}=12{,}800 \text{ lbs.}$$

for the maximum end shear on the stringer due to live load. For the corresponding impact we have

$$\left(\frac{50}{20+125}\right)12{,}800=4{,}420 \text{ lbs. (about).}$$

Then for the total maximum end shear on the stringer due to all loads we have

$$(496+45)10+12{,}800+4{,}420=22{,}630 \text{ lbs.}$$

251b. Designing of Floor Beams.—The length of each floor beam will be considered to be 22.25 ft., which is the assumed distance, center to center of trusses.

For the dead load per foot of an interior beam we have the following:

Floor....	$124\times20=2{,}480$ lbs. per ft. of beam.
Stringers..........................	$\frac{6\times45\times20}{22.25}=$ 243 lbs. per ft. of beam.
Beam (assumed)................................	115 lbs. per ft. of beam.
Total....................................	2,838 lbs. per ft. of beam.

Then for the maximum moment on the floor beam due to dead load we have

$$\frac{1}{8}(2{,}838\times\overline{22.25}^2)12=2{,}107{,}464 \text{ in. lbs.}$$

The maximum moment on the floor beam due to live load will occur when the rear wheels of the two trucks are just over the floor beam and in the position shown in Fig. 389.

The front wheels will be 6 ft. from the adjacent floor beam. Taking moment about the adjacent floor beam, we obtain

$$\frac{3,000\times 6}{20}=900 \text{ lbs.}$$

for the part of each front wheel load transmitted to the floor beam supporting the rear wheels. This 900 lbs. is transmitted through the floor slab and stringers so that it is really more or less a uniform load as applied to the floor beam, but for convenience we shall consider it applied with each 12,000-lb. rear wheel, as indicated in Fig. 389.

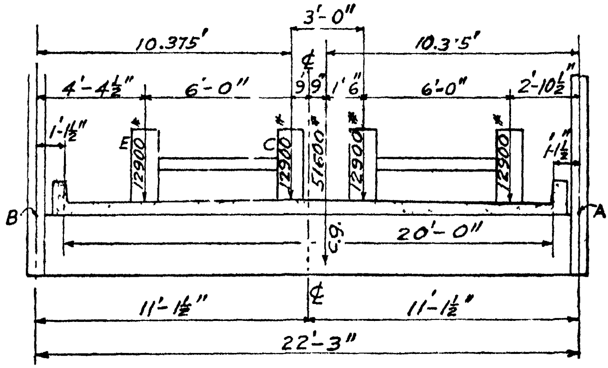


Fig. 389

Taking moments about A (Fig. 389), we obtain

$$\frac{51,600\times 10.375}{22.25}=24,060 \text{ lbs.}$$

for the reaction at B. Then, taking moments about wheel C, we obtain

$$[(24,060\times 10.375)-(12,900\times 6)]12=2,066,664 \text{ in. lbs.}$$

for the maximum moment due to live load. For the corresponding impact we have

$$\left(\frac{50}{(2\times 20)+125}\right)2,066,664=626,000 \text{ in. lbs. (about).}$$

Now for the total maximum moment on the floor beam we have

$$2,107,464+2,066,664+626,000=4,800,128 \text{ in. lbs.}$$

Then for the section modulus required we have

$$\frac{4,800,128}{16,000}=300.0,$$

which calls for a 30-in.x110-lb. Bethlehem beam or a 30-in.x115-lb. Carnegie beam. The Bethlehem beam will be used.

By moving the loads shown in Fig. 389 to the left so that wheel E is 1 ft. 6 in. from the left curb, and by taking moments about A, with the wheels in that position, we obtain 28,100 lbs. for the maximum live-load end shear on the floor beam. Taking $L=40$ ft. (two panel lengths), we obtain 8,500 lbs. for the corresponding impact.

For the end shear on the floor beam due to dead load we have

$$2{,}838\times(11.125-0.375)=30{,}508 \text{ lbs.}$$

Now, adding the above values for end shear together, we obtain

$$28{,}100+8{,}500+30{,}508=67{,}108 \text{ lbs.}$$

for the total maximum end shear on each interior floor beam due to all loads.

The maximum moment and shear on an end floor beam due to live load and impact are about the same as on an interior floor beam. As a rule the dead weight is about one-third less and, hence, the end floor beams could be a

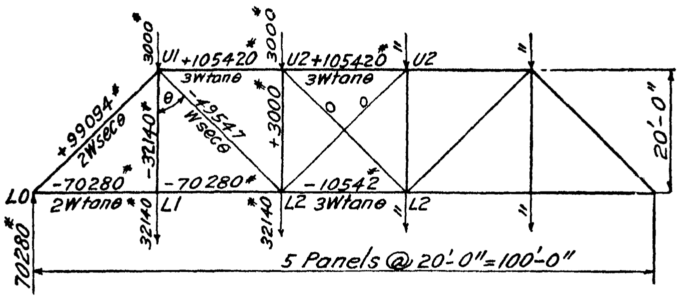


Fig. 389a

little smaller than the interior floor beams, but to simplify details, we shall use the same size as used for the interior beams—which is usually done in practice, the possible saving of metal being small.

251c. Dead-Load Stresses in Trusses.—From Formula (T′), Art. 250 (page 575), we obtain

$$124(21)=2{,}604 \text{ lbs.}$$

for the weight of the concrete floor per foot of span. Then for the panel load per truss due to the concrete floor alone we have

$$\frac{2{,}604}{2}\times 20=26{,}040 \text{ lbs.}$$

Next, from Formula (T), Art. 250, we obtain

$$(4.5\times 100)+460=910 \text{ lbs.}$$

for the weight of metal in bridge per foot of span. Then for the panel load due to metal alone we have

$$\tfrac{910}{2}\times 20=9{,}100 \text{ lbs.}$$

About one-third of this load is considered applied at the top chord joints as indicated in Fig. 389a. For the total dead-load panel load we have

$$W=26{,}040+9{,}100=35{,}140 \text{ lbs.}$$

For computing stresses algebraically we have

$$\tan\theta = \frac{20}{20} = 1$$

and

$$\sec\theta = 1.41$$

Then we have

$$W\sec\theta = 49{,}547 \text{ lbs.},$$

which is the stress in diagonal $U1L2$ (see Art. 173, page 333); twice this is the stress in the end post $U1L0$. The dead-load stress in diagonal $U2L3$ is zero, since the dead-load shear in panel $L2L2$ is zero. The dead-load stress in post

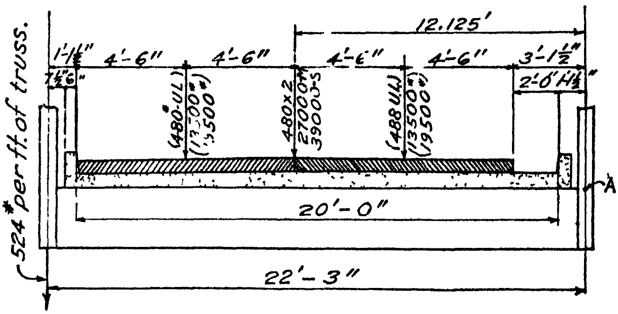


Fig. 389b

$U2L2$, as is obvious, is +3,000 lbs.; and in hanger $U1L1$ 32,140 lbs. As is readily seen, the stress in top chord $U1U2$ is

$$3W\tan\theta = 3\times 35{,}140 = 105{,}420 \text{ lbs.}$$

This stress also occurs in top chord $U2U2$ and in bottom chord $L2L2$. The stress in bottom chords $L0L1$ and $L1L2$ is

$$2W\tan\theta = 2\times 35{,}140 = 70{,}280 \text{ lbs.}$$

Considering the end panel load as being two-thirds of an interior panel load, we obtain

$$(\tfrac{2}{3}\times 35{,}140) + (2\times 35{,}140) = 93{,}706 \text{ lbs.}$$

for the maximum reaction on the truss due to dead load.

251d. Live-Load Stresses in Trusses.—The maximum loading on the truss will occur when the $H15$ loading is in the position shown in Fig. 389b.

Taking moments about A (Fig. 389b), we obtain

$$\frac{(480\times 2)12.125}{22.25} = 524 \text{ lbs.}$$

for the maximum uniform live load per foot of truss. Then we have

$$524\times 20 = 10{,}480 \text{ lbs.}$$

for the panel load due to the uniform live load.

The stresses in the chords and end posts due to this uniform live load occur when all of the panel points are loaded the same as the dead load. So the stress in the chords and end posts due to the uniform live load may be obtained quickly by multiplying the dead-load stresses by the ratio of the uniform live load to the dead load. The ratio in this case may be expressed as 10,480/35,140. Then, multiplying the stress in the chords and end posts, given in Fig. 389*a*, by this ratio we obtain the stresses due to the uniform live load, marked *U* in Fig. 389*c*.

For the panel load due to the concentration used for moments, we have (see Fig. 389*b*)

$$27{,}000 \times \frac{(12.125)}{22.25} = 14{,}710 \text{ lbs. (about)}$$

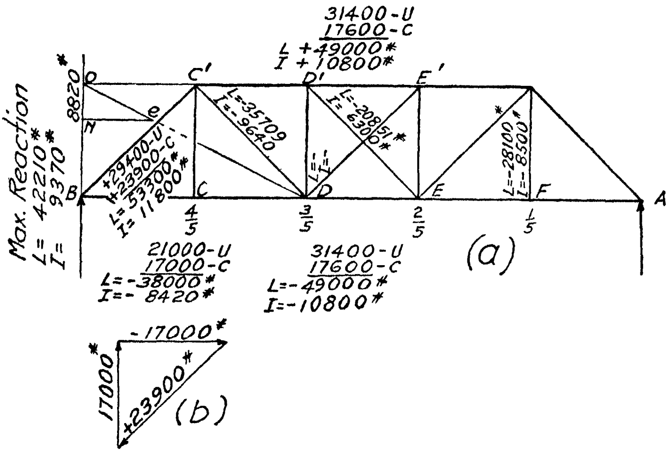


Fig. 389c

and for the panel load due to the concentration used for shear we have

$$39{,}000 \times \frac{(12.125)}{22.25} = 21{,}250 \text{ lbs. (about).}$$

The maximum stress in end post BC' (Fig. 389*c*) due to the concentrated live load will occur when the 21,250-lb. load is at panel point C. The reaction at B due to this concentration at C is

$$\tfrac{4}{5} \times 21{,}250 = 17{,}000 \text{ lbs.}$$

Then, by drawing the diagram at (*b*), Fig. 389*c*, we obtain the maximum stress in the end post BC' and bottom chord BCD due to the concentrated live load. Adding the stress in the end post BC' to the stress due to the uniform live load, we obtain 53,300 lbs. for the total maximum live-load stress in end post BC'. For the corresponding impact we have

$$\left(\frac{50}{100+125}\right) 53{,}300 = 11{,}800 \text{ lbs. (about).}$$

Adding the stress in the bottom chord BC due to the concentrated live load, given in the diagram at (b), we obtain 38,000 lbs. for the total maximum live-load stress in bottom chord BCD. For the corresponding impact, we obtain

$$\left(\frac{50}{100+125}\right)38,000=8,420 \text{ lbs. (about).}$$

The maximum stress in top chord $C'D'$ (Fig. 389c) due to the concentrated live load will occur when the 14,710-lb. concentration is at panel point D. For the reaction at B due to this load at D we have

$$\tfrac{3}{5}\times 14,710=8,820 \text{ lbs.}$$

Then prolong top chord $C'D'$ to O and draw OD, then lay off $ON=$ 8,820 lbs. and draw Ne. The length of Ne is the stress in top chord $C'D'$, also in top chord $D'F'$, and in bottom chord DE. Adding this stress to the stress due to the uniform live load, we obtain 49,000 lbs. for the total maximum live-load stress in the top chord and in bottom chord DE. For the corresponding impact we have

$$\left(\frac{50}{100+125}\right)49,000=10,800 \text{ lbs. (about).}$$

The maximum live-load stress in post $D'D$ and in diagonal $D'E$ will occur when the 21,250-lb. concentration is at E and 10,480 lbs. of uniform live load is at each of the panel points E and F. In other words, the uniform live load extends from A to E and the concentration for shear is at E. With the loads in this position we obtain

$$(\tfrac{3}{5}\times 10,480)+(\tfrac{2}{5}\times 21,250)=14,788 \text{ lbs.}$$

for the reaction at B, which is the maximum live-load stress in post $D'D$. For the corresponding impact we have ($L=$distance A to E)

$$\left(\frac{50}{40+125}\right)14,788=4,480 \text{ lbs. (about)}$$

and for the stress in diagonal $D'E$ we obtain

$$14,788\times\sec\theta=14,788\times 1.41=20,851 \text{ lbs.}$$

For the corresponding impact we have

$$\left(\frac{50}{40+125}\right)20,851=6,300 \text{ lbs. (about).}$$

The maximum live-load stress in diagonal $C'D$ will occur when the 21,250-lb. concentration is at D and the uniform live load extends from A to D. With the loads in this position we obtain

$$(\tfrac{6}{5}\times 10,480)+(\tfrac{3}{5}\times 21,250)=25,326 \text{ lbs.}$$

for the reaction at B. Then for the maximum live-load stress in diagonal $C'D$ we have

$$25,326\times 1.41=35,709 \text{ lbs.}$$

and for the corresponding impact we obtain

$$\left(\frac{50}{60+125}\right)35,709=9,640 \text{ lbs. (about).}$$

The maximum live-load stress in hanger $C'C$ is equal to the maximum end shear on the floor beam, which is given as 28,100 lbs. in Art. 251*b*, and the corresponding impact is 8,500 lbs.

The maximum live-load reaction at B will occur when the span is fully loaded with uniform live load and the 21,250-lb. concentration is at B. So we have

$$(10,480\times2)+21,250=42,210 \text{ lbs.}$$

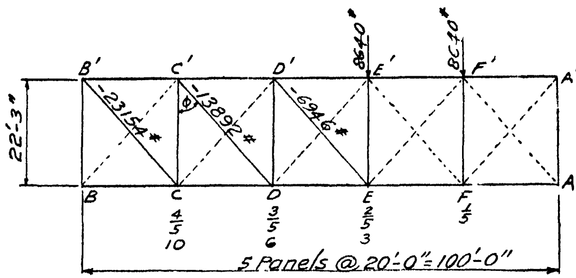


Fig. 389*d*

for the maximum reaction due to live load. For the corresponding impact we have

$$\left(\frac{50}{100+125}\right)42,210=9,370 \text{ lbs. (about).}$$

251e. Wind Stresses in Bottom Laterals.—From preliminary calculations it is found that $1\frac{1}{2}$ times the vertical projection of the lower half of the structure is about 6.4 sq. ft. per foot of span. (See Art. 223.) Then for the wind load per foot of bottom chord we have

Structure	$6.4\times30=192$	lbs. per ft. of bottom chord.
Live load	$8\times30=240$	lbs. per ft. of bottom chord.
Total	432	lbs. per ft. of bottom chord.

Then for the panel load we have

$$P=432\times20=8,640 \text{ lbs.}$$

The bottom lateral system will be double intersection as indicated in Fig. 389*d*. For determining the stresses in laterals we have

$$\tan\phi=\frac{20}{22.25}=0.90$$

and

$$\sec\phi=1.34.$$

Then $P\sec\phi=8,640\times1.34=11,577$ lbs.

Then, loading the panels from A to E, we obtain

$$\tfrac{3}{5}P\sec\phi=\tfrac{3}{5}\times11,577=6,946 \text{ lbs.}$$

for the maximum stress in lateral $D'E$ due to wind. Loading the panels from A to D, we obtain

$$\tfrac{6}{5}\times 11{,}577 = 13{,}892 \text{ lbs.}$$

for the maximum stress in lateral DC'. Loading the panels from A to C, we obtain

$$\tfrac{10}{5}\times 11{,}577 = 23{,}154 \text{ lbs.}$$

for the maximum stress in lateral CB'.

As the laterals are symmetrical with reference to center of span, the above stresses are all that are necessary for designing the laterals.

251f. Wind Stresses in Top Laterals.—The top laterals will be a double system, as shown in Fig. 389*e*.

From preliminary calculation it is found that the area of the vertical projection of the upper half of the truss is about 2.1 sq. ft. per foot of truss. So for the wind pressure on the top chord we have

$$2.1\times 1.5\times 30 = 94.5 \text{ lbs. per foot of span.}$$

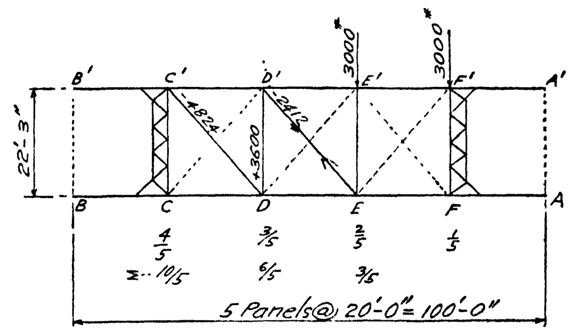


Fig. 389*e*

But since, according to Art. 223 the pressure is limited to 150 lbs., the 150 lbs. will be used. Then for the panel load we have

$$P = 150\times 20 = 3{,}000 \text{ lbs.}$$

$$\sec\phi = 1.34$$

the same as for the bottom laterals. Then we have

$$P\sec\phi = 4{,}020 \text{ lbs.}$$

Then, loading from A to E, we obtain

$$\tfrac{3}{5}\times 4{,}020 = 2{,}412 \text{ lbs.}$$

for the stress in top lateral $D'E$. Likewise, loading from A to D, we obtain

$$\tfrac{6}{5}\times 4{,}020 = 4{,}824 \text{ lbs.}$$

for the stress in lateral DC'. The compression in strut DD' is equal to

$$\frac{6}{5}\times 3{,}000 = 3{,}600 \text{ lbs.}$$

The above are all the stresses necessary for designing the top laterals.

251g. Wind Stresses in Portals.—As is seen from Fig. 389e, there can be two panel loads (3,000x2 = 6,000 lbs.) applied to the portal at either panel point C' or C. The end posts are usually considered fixed, in which case the point of contra-flexure in each post is mid-way between the bottom of the portal and the shoe joint B. By drawing the clearance line, it is found that the portal can be 6 ft. 0 in. deep in this case.

The total length of the end post is about 28 ft. $3\frac{1}{2}$ ins. So we have

$$28 \text{ ft. } 3\tfrac{1}{2} \text{ ins.} - 6 \text{ ft. } 0 \text{ ins.} = 22 \text{ ft. } 3\tfrac{1}{2} \text{ ins.}$$

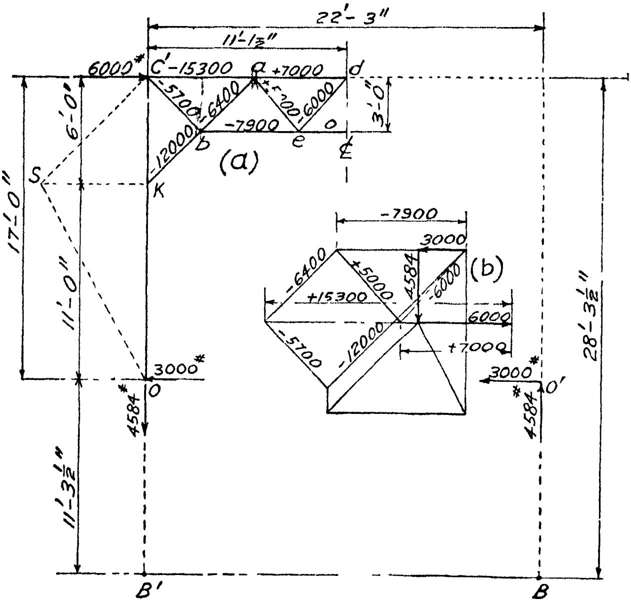


Fig. 389f

for the distance from the bottom of the portal to the shoe joint. Then the point of contra-flexure is about 11 ft. $1\frac{3}{4}$ ins. down from the portal; for convenience, we shall assume 11 ft. Then the case will be as in Fig. 389f, where O and O' are points of contra-flexure in the end posts.

The end posts are considered to have equal resistance to horizontal shear. Then each will have a horizontal shear of 3,000 lbs. at the point of contra-flexure, as indicated in Fig. 389f. The 6,000-lb. load applied to C' will cause a positive reaction at O' and an equal, but negative, reaction at O. By taking moments about either O or O', we obtain

$$\frac{6{,}000 \times 17}{22.25} = 4{,}584 \text{ lbs.}$$

for the reaction at O or O'. So it is seen that there is a horizontal and vertical force at each point of contra-flexure that holds the 6,000-lb. wind pressure in equilibrium. Knowing these forces and having the general dimensions of the portal, we may determine the stresses in the portal very readily by graphics.

We may first draw the substituted frame OSC'-K to any convenient proportions, and then, by beginning at O, we may draw the stress diagram shown at (*b*) in Fig. 389*f*, giving the stresses throughout.

Now, the stresses being computed throughout for the span, they may be collected and written on the stress sheet (Fig. 389*g*), and next the required sections may be determined.

251h. Sections.—*Intermediate Posts.* In designing the sections of through truss bridges, it is best to begin with the intermediate posts. In this case, as the depth of the truss is only 20 ft., I-beams with wide flanges can be used for intermediate posts, thereby greatly reducing shop work. An 8-in.x31-lb. Carnegie beam has a radius of gyration of 2.01 (least radius). Then

$$\frac{L}{r}=\frac{240}{2.01}=120,$$

which is just the limit. Hence this section is satisfactory as far as slender ratio is concerned. From Curve A (page 538) we find that the allowable unit stress is 7 750 lbs. Then we have

$$22,256 \div 7,750 = 2.87 \text{ sq. ins.}$$

for the area of cross-section actually required.

The beam has a cross-section of 9.10 sq. ins., which is excessive. Yet the beam is as economical a section as can be obtained and hence will be used.

Hanger $U1L1$. For the required section we have

$$68,740 \div 16,000 = 4.3 \text{ sq. ins.}$$

but to simplify details we shall use the 8-in.x31-lb. beam, the same as used for posts $U2L2$. In that case, considering four rivet holes ($\frac{3}{4}$-in. rivets) out of the flanges, we have

$$9.10-(\tfrac{7}{16}\times\tfrac{7}{8}\times 4)=7.57 \text{ sq. ins.}$$

for the net section, which is excessive, but considering the saving on details and shop work, however, the 8-in.x31-lb. beam is an economical section for the hanger $U1L1$ and will be used.

Diagonal $U1L2$. For the required section we have

$$94,871 \div 16,000 = 5.93 \text{ sq. ins.}$$

An 8-in.x27-lb. Carnegie beam, considering two holes out of the flanges, has a net section of

$$7.93-(\tfrac{7}{16}\times\tfrac{7}{8}\times 2)=6.4 \text{ sq. ins.}$$

This beam will be used, as it is as economical a section as can be obtained.

Diagonal $U2L2$. Two angles will be used in this case, as any beam that could be used has a very excessive section. For the required section we have

$$27,134 \div 16,000 = 1.69 \text{ sq. ins.,}$$

which is a very small section for two angles; but rigidity is required, that is, L/r for such tension members should not exceed 200. The unsupported length (in vertical plane) is about 14 ft. Then the angle used must have a radius equal to

$$\frac{200}{14\times 12}=1.1.$$

This requires the use of $3\frac{1}{2}$x$2\frac{1}{2}$x$\frac{5}{16}$ angles, which will be used, although the section is excessive.

Bottom Chord L0L2. Considering one rivet hole out ($\frac{7}{8}$-in. hole for $\frac{3}{4}$-in. rivet) of each angle, we find that two 6x4x$\frac{7}{16}$ angles are practically the required size; hence, they will be used.

Bottom Chord L2L2. Considering one rivet hole out of each angle, we find that two 6x4x$\frac{5}{8}$ angles are about the required size; hence they will be used.

Top Chord U1U2. The allowable unit stress on such top chords, as a rule, is 12,000 to 13,000 lbs. So, taking 12,000 lbs. as the allowable unit stress, we obtain

$$165,220 \div 16,000 = 10.3 \text{ sq. ins.}$$

for the required area of the cross-section. This gives us some idea as to what is required.

Let us assume the following section:

One cover plate	15 ins. × $\frac{5}{16}$ in. =	4.69 sq. ins.
Two channels	10 ins. × 15.3 lbs. =	8.94 sq. ins.
Total section		13.63 sq. ins.

For the approximate value of the radius of gyration about the horizontal axis, we have

$$r_h = 0.39 \times 10 = 3.9,$$

and for the radius of gyration about the vertical axis (assuming the distance back to back of channels to be $8\frac{3}{4}$ ins.) we have

$$r_v = 0.55 \times 8.75 = 4.81.$$

Then, using the least radius, we obtain

$$\frac{L}{r_v} = \frac{20 \times 12}{3.9} = 60 \text{ (about)}.$$

Then from Curve *A* (page 538) we find the allowable unit stress to be 12,600 lbs. So for the required section of the top chord we have

$$165,220 \div 12,600 = 13.2 \text{ sq. ins.},$$

which is practically the same as assumed. Hence the assumed section will be used.

End Post U1L0. The end post is subjected to direct stress from live and dead load and stress from cross-bending caused by wind load. The unit stress due to this cross-bending should be added to the direct stress.

For the section of the end post we shall assume,

One cover plate	15 ins. × $\frac{5}{16}$ in. =	4.69 sq. ins.
Two channels	10 ins. × 25 lbs. =	14.70 sq. ins.
Total		19.39 sq. ins.

Length of end post = 28.28 ft. Taking $r = 3.9$, the same as for the top chord, we obtain

$$\frac{L}{r_h} = \frac{28.28 \times 12}{3.9} = 87.$$

Then from Curve *A* (page 538) we find that 10,300 lbs. is the allowable stress per square inch.

For the actual direct stress, we have

$$164{,}194 \div 19.37 = 8{,}480 \text{ lbs. per sq. in.}$$

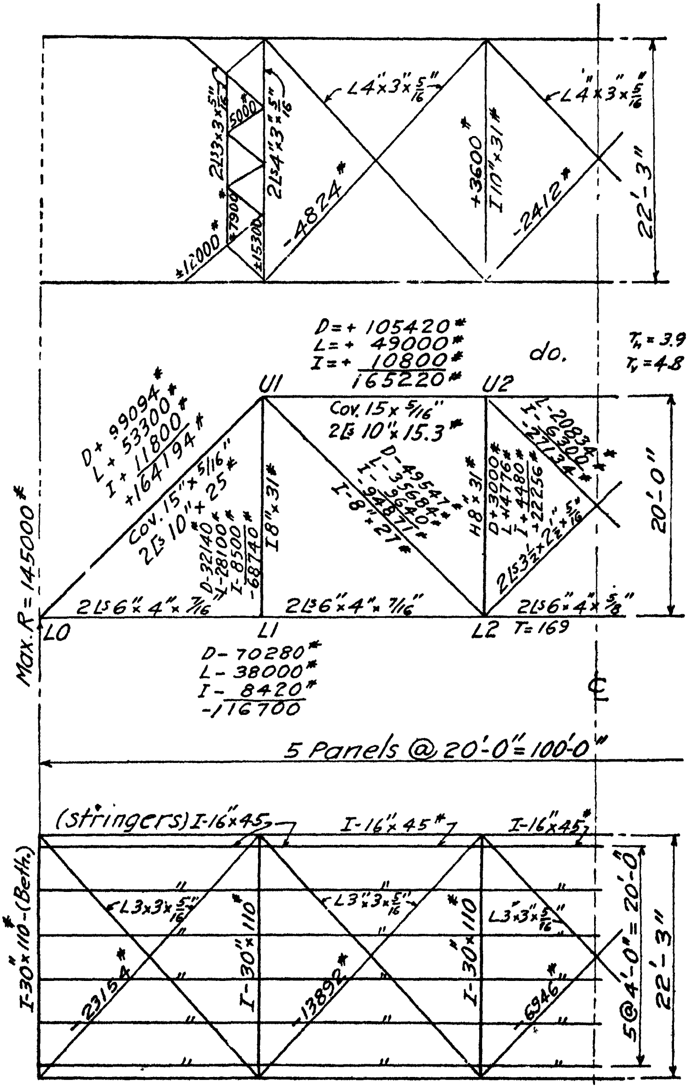


Fig. 389*g*

As seen from Fig. 389*f*, the maximum moment on the end post will occur at *K*, the knee-brace connection, and is equal to

$$3{,}000 \times 11 \times 12 = 396{,}000 \text{ in. lbs.}$$

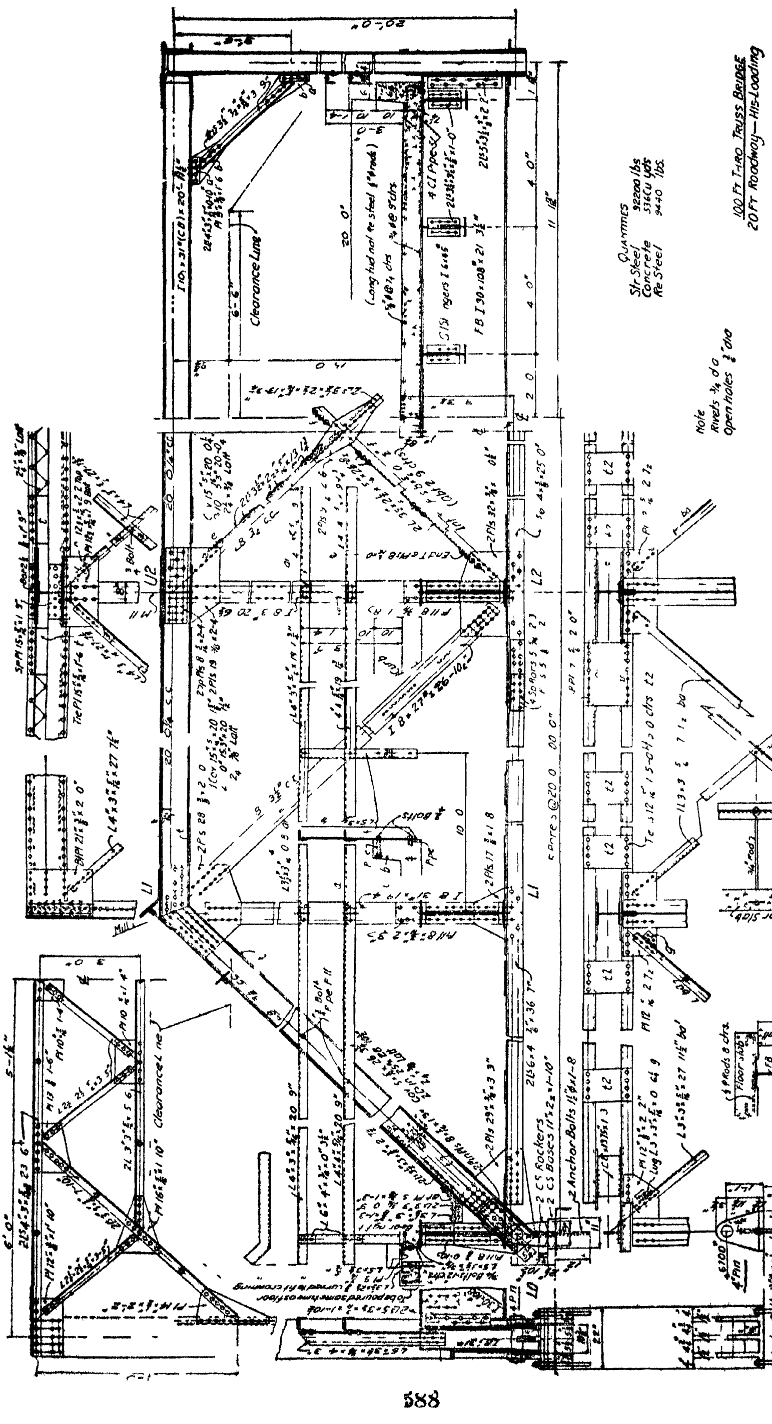

100 Ft Thro Truss Bridge
20 Ft Roadway—H15-Loading
Quantities
Str Steel 92200 lbs
Concrete 336 cu yds
Re Steel 9440 lbs
Note
Rivets ¾ dia
Open holes 13/16 dia
Clearance Line
Fixed End
2 Fixed Shoes (Steel)
2 CS Rockers
2 CS Bases
Anchor Bolts
Floor Slab

Fig 390

The moment of inertia of the section about the vertical axis (obtained from calculation) is 462.19 inch units.

Then for the stress due to cross-bending we have

$$\frac{396{,}000\times 7.5}{462.19}=4.827 \text{ lbs. per sq. in.}$$

on the outermost fiber. Adding this to the direct stress, we obtain

$$8{,}480+4{,}827=13{,}307 \text{ lbs.}$$

for the combined stress, which is about 3,000 lbs. greater than allowed for direct stress. But in such combined stresses the allowable unit stress is usually increased one-third. So the assumed section is satisfactory and will be used.

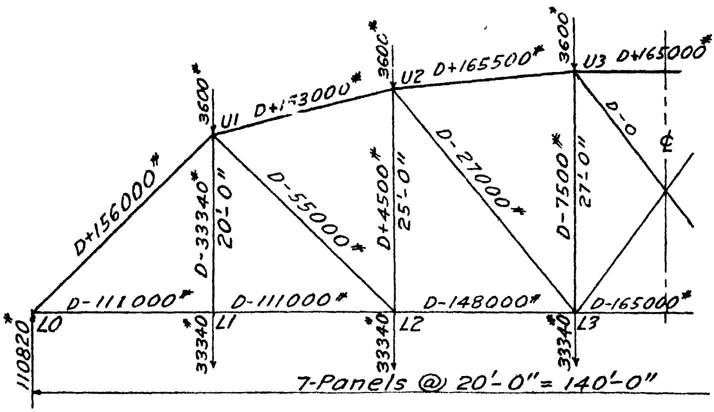


Fig. 391

The stress sheet, shown in Fig. 389*g*, can now be completed, and then the detail drawing shown in Fig. 390 can be made.

252. Design of Curve-Chord Through Pratt-Truss Bridges.— The dead-load stresses can be determined most readily by graphics. For the purpose of illustration we shall consider a 140-ft. truss span, 20-ft. roadway, and supporting *H*15 loading. The heights of truss are as shown in Fig. 391. The top chord is usually on the arc of a parabola with the *X*-axis passing vertically at mid-span, as in railroad bridges.

Dead-Load Stresses. From Formula (T), Art. 250 (page 575), we obtain

$$w=(4.5\times 140)+460=1{,}090 \text{ lbs.}$$

for the weight of metal per foot of span. Then for the panel load per truss we have

$$\frac{1{,}090}{2}\times 20=10{,}900 \text{ lbs.}$$

One-third of this, say 3,600 lbs., is considered at the top chord. From Formula (T′), Art. 250, we obtain

$$w'=124\times 21=2{,}604 \text{ lbs.}$$

for the weight of concrete floor per foot of span. Then for the panel load per truss we have

$$\frac{2,604}{2}\times 20=26,040 \text{ lbs.},$$

all of which is at the bottom chord joints. Then we have

$$(10,900-3,600)+26,040=33,340 \text{ lbs.}$$

for the loads on the bottom chord joints, and there is 3,600 lbs. on the top chord joints as shown in Fig. 391. The dead-load stresses shown can be determined graphically.

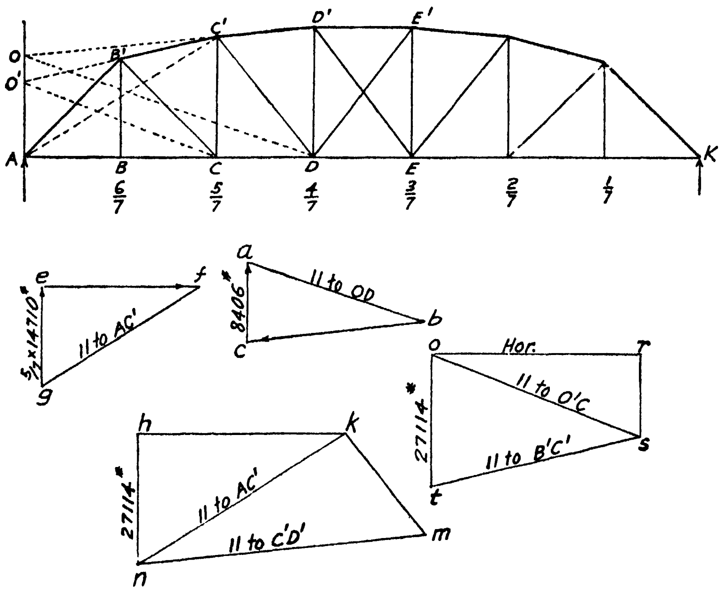


Fig. 391a

Live-Load Stresses. The live load on the 20-ft. roadway will be the same as on the 100-ft. span, but the value per foot of truss may be different as the distance center to center of trusses may be different. To determine this distance (center to center of truss), we begin by investigating the required width of the intermediate posts. We know, to begin with, that the stresses in these posts will be comparatively light so that L/r will mostly govern.

The longest post is

$$27\times 12=324 \text{ ins.}$$

Then, if the maximum value of L/r be limited to 120, we obtain

$$324\div 120=2.7$$

for the minimum value of the allowed radius of gyration of the post. By glancing over the tables of I-beams, we find that it is not practical to use

I-beams for these posts. Two channels will therefore be used. From tables of channels we find that 8-in. channels will be satisfactory for the posts, and to simplify details 8-in. channels will also be used for the hangers. Then, if $\frac{3}{8}$-in. gusset plates are used, the distance back to back of the top chord channels will be $8\frac{3}{4}$ ins.

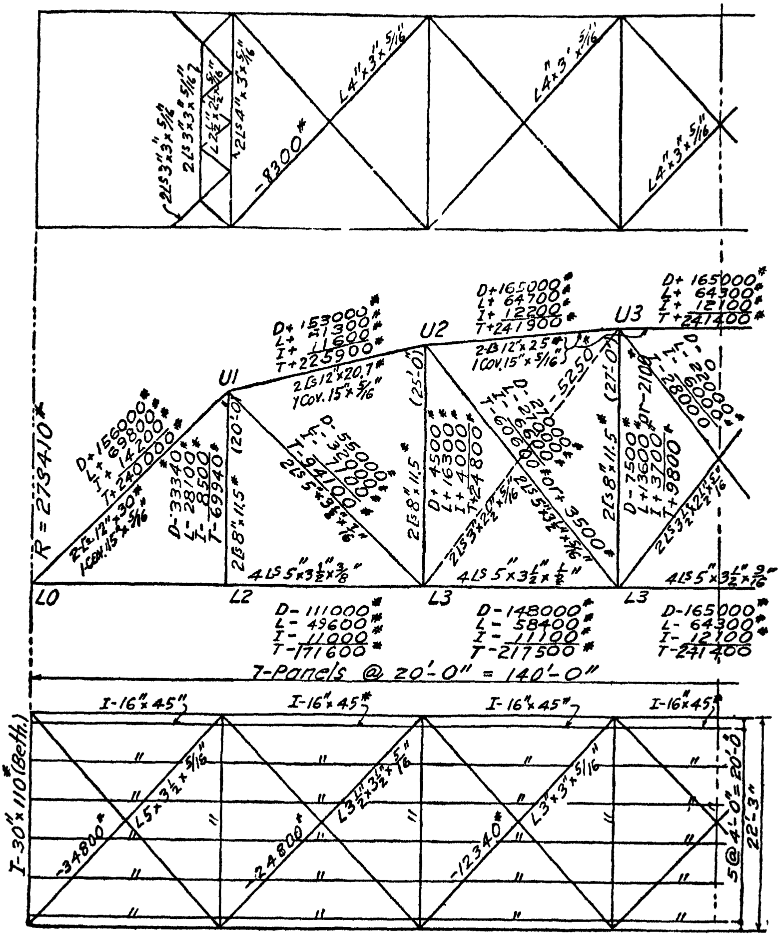


Fig. 392

If 12-in. channels are used in the top chords and end posts, the width of each channel flange will be about 3 ins. Then we obtain

$$8\tfrac{3}{4}+6=14\tfrac{3}{4}, \text{ say } 15 \text{ ins.}$$

for the width of cover plate on the top chords and end posts. Now, if each of the curbs be 6 ins. wide, we obtain

$$20 \text{ ft.}+(6 \text{ ins.}\times 2)+1 \text{ ft. } 3 \text{ ins.}=22 \text{ ft. } 3 \text{ ins.}$$

for the distance center to center of trusses, which is the same as for the 100-ft. span. Hence the live load will be the same as for the 100-ft. span. (See Art. 251*d*.)

The live-load panel loads are as follows:

Uniform live load	10,480 lbs.
Concentration for shear .	21,250 lbs.
Concentration for moment.	14,710 lbs.

By multiplying the chord stresses due to dead load by the ratio of the uniform live load to the dead load, the stresses in the chords and end posts due to the uniform live load can be obtained very quickly —using the slide rule. The stresses due to the single concentrations can be determined most readily by graphics.

For example, let it be required to determine the stress in top chord *C′ D′* (Fig. 391*a*).

Prolong top chord *C′ D′* to *O* and draw *OD*. Then, by placing the 14,710-lb. load at *D*, the reaction at *A* will be equal to

$$\tfrac{4}{7}\times 14{,}710 = 8{,}406 \text{ lbs.}$$

Then, by drawing diagram *abc* (Fig. 391*a*), the line *cb* represents the maximum stress in top chord *C′D′* due to the single concentration. To obtain the maximum stress in bottom chord *CD* due to the single live-load concentration, we place the 14,710-lb. load at *C* and draw line *C′A* and then construct the diagram *efg*; line *ef* represents the maximum stress in bottom chord *CD* due to the single concentration, and so on, for the chords.

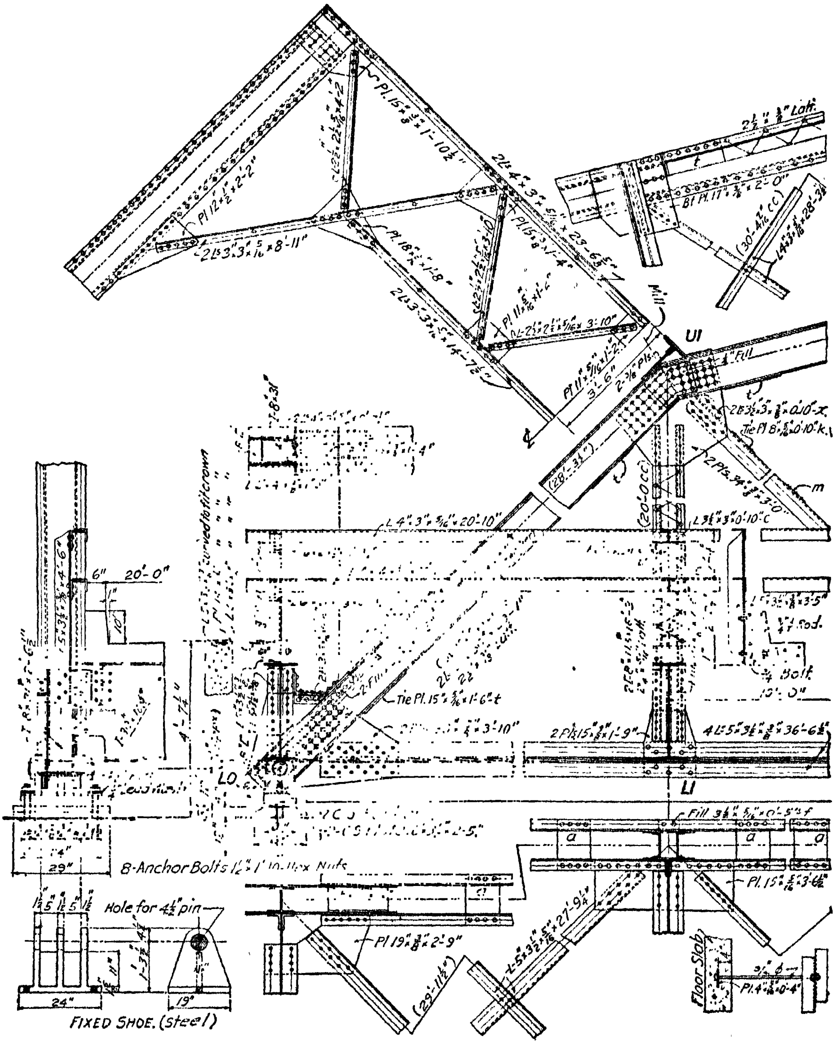
The stresses in the web members are determined by applying the uniform and the concentration at the same time. For example, let it be required to determine the maximum tension in diagonal *C′D*. The uniform load would extend from *K* to *D* (Fig. 391*a*) and the 21,250 lbs. would be at *D*. With the load in this position the reaction at *A* would be

$$(\tfrac{10}{7}\times 10{,}480) + (\tfrac{4}{7}\times 21{,}250) = 27{,}114 \text{ lbs.}$$

Then, by constructing the diagram *hkmn*, the line *km* represents the maximum stress in diagonal *C′D*. The maximum stress occurs in post *C′C* at the same time. This stress in post *C′C* is obtained by prolonging top chord *B′C′* to *O′* and drawing line *O′C*. Then, constructing the diagram *orst* (Fig. 391*a*), we obtain the line *rs*, which represents the maximum live-load compressive stress in post *C′C*, and so on, for the web members.

The complete stress sheet for the span is shown in Fig. 392 and the detail drawing for the span is shown in Figs. 393, 393*a*, and 393*b*. Typical details of a pin-connected span are shown in Figs. 393*c* and 393*d*.

Stresses in Pettit trusses can be determined in the manner outlined in the foregoing analysis of curved-chord bridges.



U1
L1
L0
Mill
Floor Slab
FIXED SHOE. (steel)
Hole for 4½" pin
20'-0"
29"
24"
19"

Fig. 393

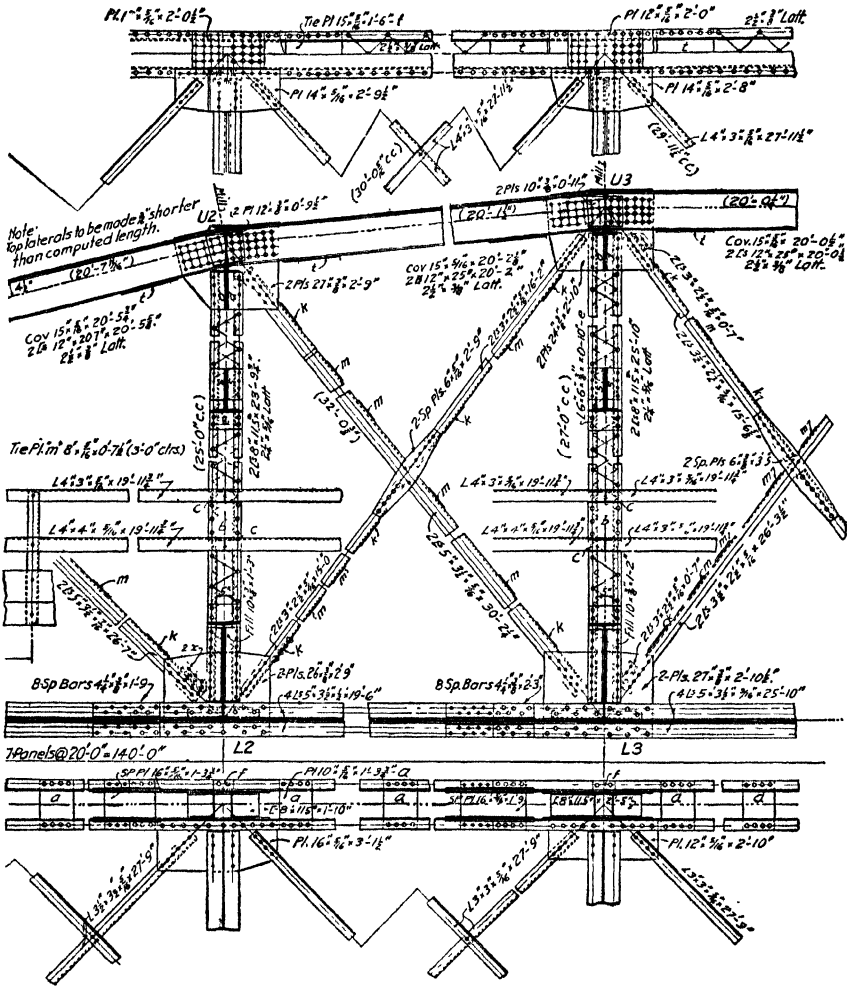
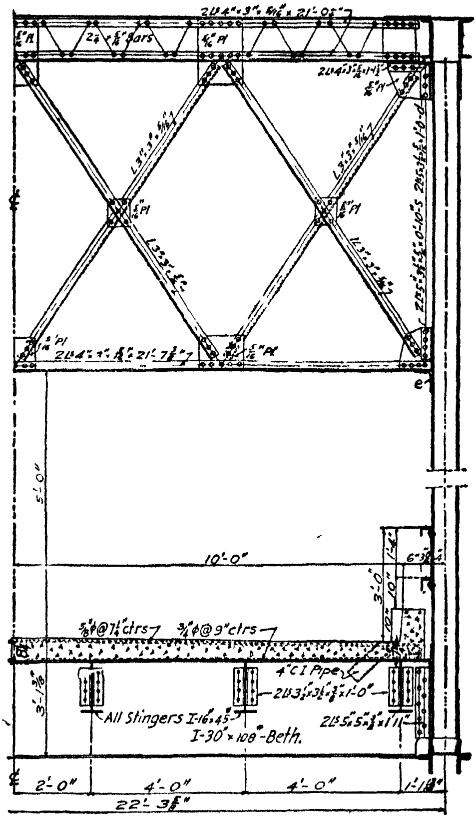

U2
U3
L2
L3
Note: Top laterals to be made ⅜" shorter than computed length.
7 Panels @ 20'-0" = 140'-0"
8 Sp Bars
8 Sp Bars
Tie Pl.
Latt.
Mill
Mill

Fig. 393a



Note:
Rivets - 3/4" dia.
Open holes - 13/16" dia

140 FT THRO. TRUSS BRIDGE
20 ft. Roadway H15 Loading

Fig. 393b

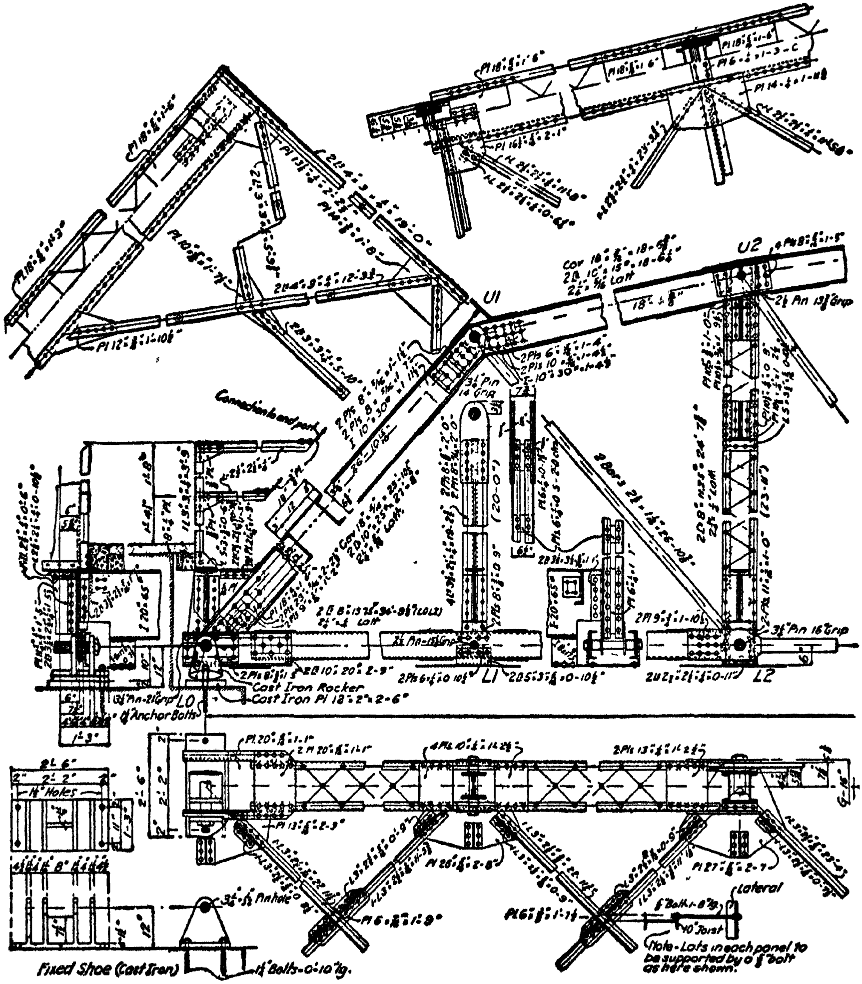
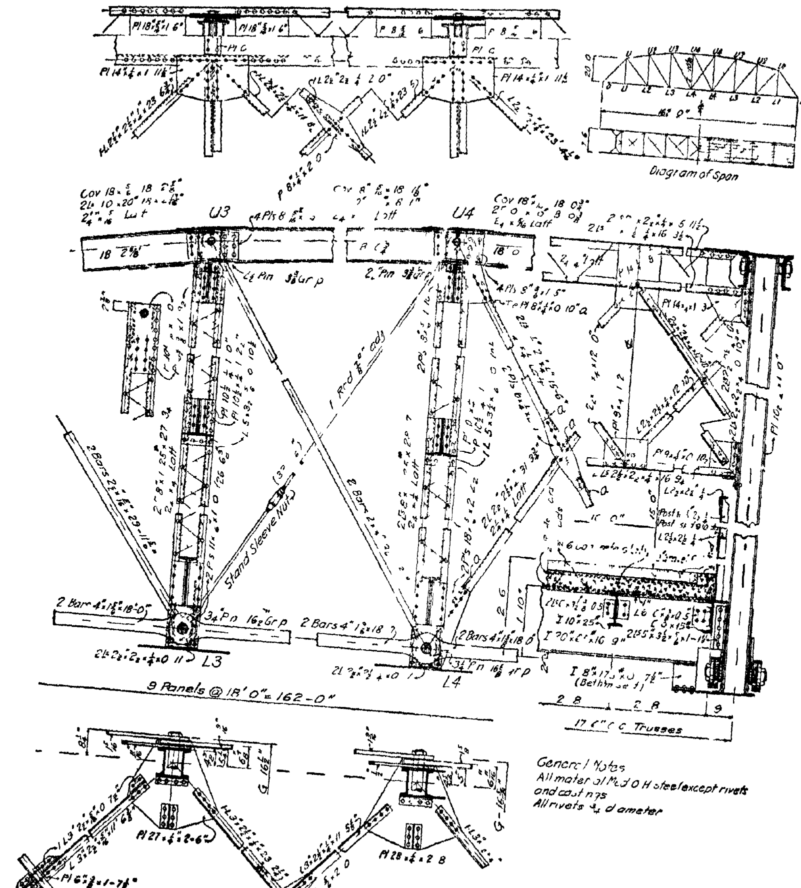


Fig. 393c



U3
U4
L3
L4
Diagram of Span
9 Panels @ 18' 0" = 162-0"
17' 6" C C Trusses
Stand Sleeve Nut
General Notes
All material Med O H steel except rivets and castings
All rivets ¾ diameter
STANDARD 162 ft THRO PIN CONN TRUSS HIGHWAY BRIDGE.
Concrete Floor - 16-0" Roadway
GENERAL DRAWING

Fig. 393d

CHAPTER XIII

SKEW BRIDGES, BRIDGES ON CURVES, ECONOMIC HEIGHT AND LENGTH OF TRUSSES, AND STRESSES IN PORTALS

253. Skew Bridges.—Bridges crossing over streams, railroads, streets, and roads at an angle are usually built on the skew. This is done in the case of crossings over streams in order to obtain piers parallel to the flow of the stream and in the other cases to obtain minimum span lengths.

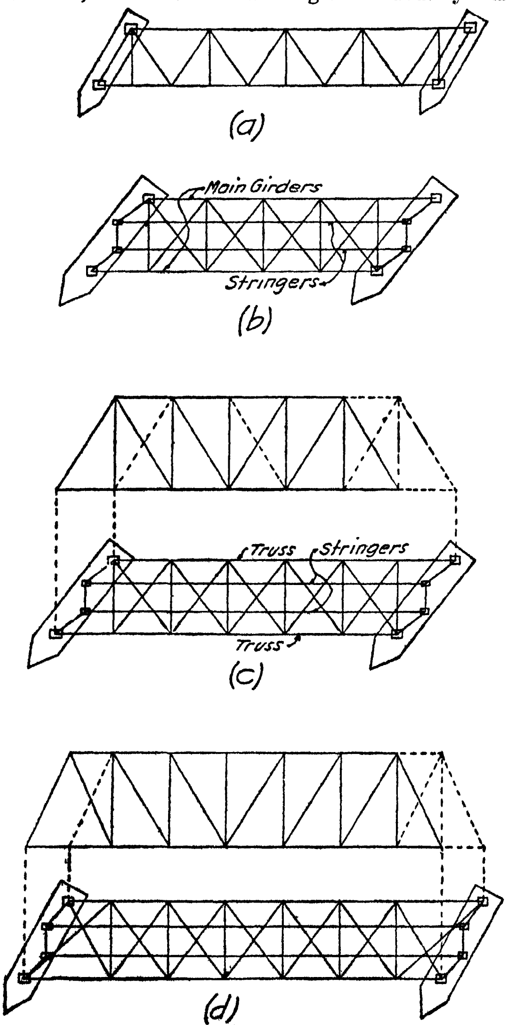


Fig. 394

Skew deck plate girder railroad bridges are usually constructed as shown at (*a*), Fig. 394, through girder spans as shown at (*b*), truss spans as shown at (*c*) and (*d*). The case shown at (*c*) is where the skew is a full panel length and the case shown at (*d*) is where the skew is less than a panel length.

Skew highway bridges are constructed very similar to skew railroad bridges, as far as the skew is concerned. End floor beams are, as a rule, used in the case of highway bridges, while in the case of railroad bridges as a rule they are omitted and details similar to those shown in Fig. 305 are used, except they are constructed to fit the skew.

The outer ends of the stringers in railroad bridges should be square with the track, as indicated in Fig. 394, so as to avoid having ties resting upon masonry at one end and upon a stringer at the other end. By this construction we also obtain equal stringer concentrations on the first intermediate floor beams, as is readily seen.

The end posts in each end panel of skew truss bridges should always be parallel. The stresses in skew bridges are determined practically in the same manner as previously shown for square spans; the only difference being that the skew must be taken into account, which is done by assuming the loads to be applied along the center line of the bridge. Let the diagram at (*m*) in Fig. 395 represent the plan of a skew truss bridge. We would assume the live load applied along the center line *gh*. The influence line for the reaction at *A* would be as shown at (*o*). The influence lines for the stress in chord *cd* and diagonal *cD* would be as shown at (*t*) and (*v*), respectively. To satisfy the criterion for maximum live-load moment about, say, point *D*, the distance *sg* would be taken as *k* in (5) of Art. 91 and as *gw* in case the maximum moment at *B* is desired, and so on. To satisfy the criterion (see Art. 90) for maximum live-load shear in, say, panel *BC*, the load would extend from *h* to and into panel *BC* so that the unit load in that panel (with a load at *C*) would be equal (or as nearly equal as is possible) to the total load on the span divided by the distance *gh*. In determining the dead-load stress the panel loads are readily computed for all panel points. This load will be the same for all panels except for points *a*, *f*, and *F*, in which case a slight correction on account of the skew must be made. After the panel loads are computed the dead-load stresses are readily determined either analytically or graphically. However, it must be borne in mind that the truss is unsymmetrical with reference to the center of span.

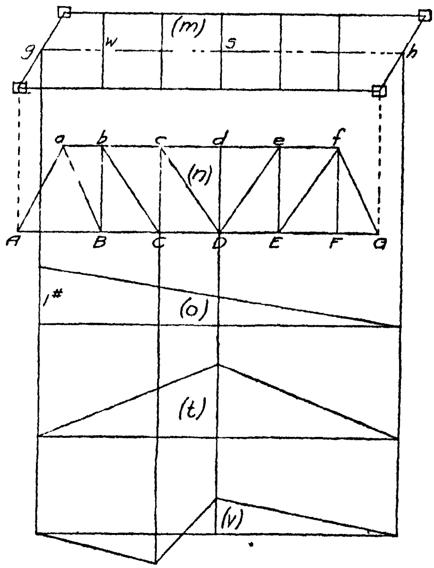


Fig. 395

The maximum moment and shears on skew through plate girder bridges are determined in exactly the same manner as shown above for truss spans. Skew deck plate girder bridges can be treated the same as square spans without any appreciable error.

254. Bridges on Curves.—Bridges supporting curved tracks are subjected to stresses due to centrifugal force and to the loads being eccentrically applied. These stresses are in addition to dead, live, and impact stresses.

Stresses Due to Centrifugal Force occur in the laterals system at the loaded chord. The laterals and chords constitute the system which is really a horizontal truss. For centrifugal force in general we have

$$F = \frac{Wv^2}{gr} \qquad (1).$$

where W represents the weight of the moving body, in pounds, v the velocity of the body in feet per second, g the acceleration due to gravity, and r the radius of curvature in feet.

Let D equal degree of curvature. Then we have

$$r=\frac{50}{\sin\frac{1}{2}D},$$

and hence for a 1° curve we have

$$r=\frac{50}{0.00873}=5{,}730 \text{ ft}.$$

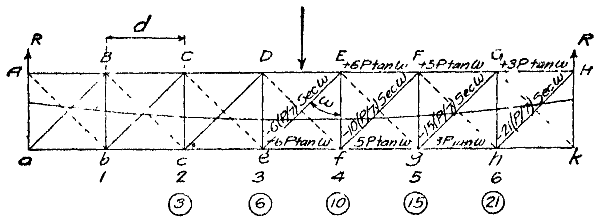


Fig. 396

Let V equal velocity in miles per hour. Then we have

$$v=\frac{5{,}280}{3{,}600}V=\frac{22}{15}V.$$

Now substituting these values in (1) we have

$$F=\frac{22^2}{15^2\times 32.2\times 5{,}730}WV^2=.0000117WV^2 \qquad (2)$$

for the centrifugal force for a 1° curve where W equals weight of the moving body and V equals the velocity in miles per hour. Therefore, for a 4° curve and 50-mile velocity, we have

$$F=.0000117\times W\times 50^2\times 4=0.117W.$$

Then if W be the uniform live load per ft. of span, $0.117W$ would be the centrifugal force per ft. of span which would be used to determine the stress in the chords and laterals. As an illustration, let Fig. 396 represent the plan of the lateral system at the loaded chord of a 7-panel bridge supporting a track on a 4° curve. Let W represent the live load per ft. of span. Then for a velocity of 50 miles per hour we have $0.117W$ for the centrifugal force per ft. of span, and for the panel load we have

$$P=0.117W\times d.$$

Then the maximum stress in the laterals and chords due to this horizontal load will be as indicated in Fig. 396. These stresses are determined in the same manner as previously shown for wind stresses. For example, to obtain the maximum stress in diagonal *eE* panel points *B*, *C*, and *D* would be loaded and in the case of diagonal *fF* panel points *B*, *C*, *D*, and *E* would be loaded,

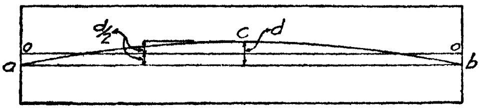


Fig. 397

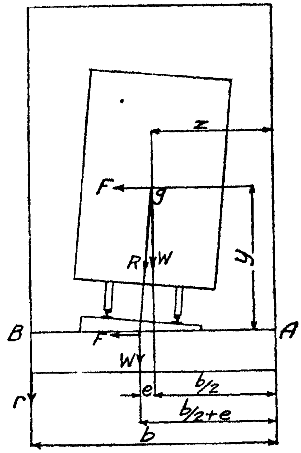


Fig. 398

and so on. The maximum chord stress would be obtained by loading all panel points from *B* to *G*. The compression in the floor beams due to centrifugal force is not considered, as the laterals are practically always connected to the tension flange of the beams and consequently the stress due to centrifugal force only tends to reverse the dead and live-load tensile stress in those flanges.

The stress in the chords due to centrifugal force is always added to the dead, live, and impact stresses in summing up the total maximum stress.

Stress Due to Eccentricity occurs in the trusses, stringers, and floor beams. Let the diagram shown in Fig. 397 represent the plan of a bridge supporting a curved track, where the curve *acb* represents the center line of track and the line *oo* represents the center line or axis of the bridge which always bisects the middle ordinate (d) of the curve as indicated. It is readily seen that loads near the center of the span will load the outer truss and stringer more than the inner truss and stringer and that just the reverse is true for loads near the ends of the span.

The diagram in Fig. 398 shows the conditions at or near the center of the span where g represents the center of gravity of the load. Taking moments above A we obtain

$$r = \frac{Wz}{b} + \frac{Fy}{b} \qquad (3)$$

for the vertical concentration on the truss at B where W represents the weight of the load and F the centrifugal force. If the super-elevation of the outer rail be in accord with the speed or velocity of the load the resultant of W and F will pass through the center of the track, in which case, taking moments about A, we have

$$r = W\left(\frac{b}{2} + e\right) \div b = \frac{W}{2} + \frac{We}{b} \qquad (4)$$

for concentration on the outer truss. As is seen, the part We/b is due to the eccentricity, and as is evident this is greater for a moving load than for a static load, for in the latter case F would be zero and e would be less, if not in reality shifted to the other side of the center line of the floor beam. If the velocity of the load be greater than the super-elevation of the outer rail provides, e will be increased. However, such a velocity is not likely to occur for the heavy freight trains which produce the maximum live-load stress in the trusses.

If it were possible to locate accurately the center of gravity (g) of the load, the concentration could be determined in any case on the outer truss by taking moments about A—which would be simply the application of equation (3)—and likewise the concentration on the inner truss could be determined by taking moments about B. But as it is practically impossible to locate the center of gravity of the load, as it varies so widely for the different engines and cars, it is more practical to consider the velocity to accord with the superelevation of the outer rail and simply consider the concentration on the outer truss to be as expressed by equation (4).

As is seen from Fig. 397, e is equal to one-half of the middle ordinate at the center and ends of the span, and fair results will be obtained by assuming e equal to one-half of the middle ordinate throughout the entire length of the span, as the loads near the center of span contribute the greater part of the chord stresses and the loads near the end contribute the greater part of the web stresses and hence the actual discrepancy is slight. Then, to obtain the maximum live-load stresses in the trusses, we would first calculate the maximum stress in each member the same as for ordinary bridges, assuming the load applied along the center line of the bridge, and then increase each of these stresses by the amount obtained by multiplying the stress in each case by e/b when e is taken as half the middle ordinate and b as the distance between trusses. That is, if S be the stress in a member due to the load considered applied along the center line of the bridge, the stress due to the eccentricity e would be Sxe/b, and hence the total stress in the member due to live load would be $S+Se/b$.

In case more accurate results be desired than are obtained by taking the eccentricity e equal to one-half the middle ordinate, the value of e at each floor beam can be computed and the concentration on the trusses at each of those points determined and the stresses in the trusses due to same computed. However, in that case an equivalent uniform live load should be used, as the work would be quite tedious if wheel loads be used.

When the stringers are symmetrically placed with reference to the center line of the bridge, the maximum moment and shear on the outer stringers are obtained by first determining the moment and shear for symmetrical load and then increasing each of these by the amount obtained by multiplying each by e/b' where b' is taken as the distance between the stringers. The moment and shear on the inner stringers would really be decreased by the same amount; however, it is usual practice to make the outer and inner stringers of equal section.

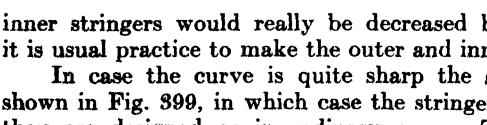


Fig. 399

In case the curve is quite sharp the stringers are usually off-set, as shown in Fig. 399, in which case the stringers are about equally loaded and they are designed as in ordinary cases. The stringer bracing should be designed sufficiently heavy to transmit the centrifugal force. The stress in this bracing is determined in the same manner as shown above for the lateral system of the structure.

In determining the maximum moment and shear on the floor beams, the eccentricity of the loading must be taken into account. Otherwise the work is the same as previously shown.

In the case of double-track bridges the determination of the stresses due to centrifugal force is the same as for single-track structures, except that both tracks must be considered.

According to the A.R.E.A. Specification for structures located on curves, the centrifugal force (assumed to act 6 ft. above the rail) shall be taken equal to a percentage of the live load, including impact, according to the following table:

Degree of curve..	0°-20′	0°-40′	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°
Percentage.......	$2\frac{1}{2}$	5	$7\frac{1}{2}$	10	10	10	10	10	10	10	10	10	10	10
Speed, miles per hour..........	80	80	80	65	53	46	41	38	35	33	31	29	28	27

255. Economic Depth of Trusses.—As is evident, a truss is of economic depth as regards weight and stiffeners when the weight of the web members is equal to the weight of the chord members. Of course the weight of the floor system, which varies with the length of panels, is part of the total weight of the structure, and hence the panel length is involved. There are so many variables involved that the theoretical determination of economic heights and panel lengths is practically impossible. The most satisfactory values for heights and panel lengths are obtained by actual calculations. The following values are recommended:

For Railroad Bridges

Length of Span in Feet	Length of Panels in Feet	Height of Truss in Feet
100	25	30
125	25	30
150	25	31
175	25	33
200	25	39
225	25	45
250	25	50
300	25	55
400 (14 panels)	28.58 (about)	62
500 (16 panels)	31.25	72

Heights for intermediate lengths can be obtained by interpolating.

For Highway Bridges

Span Length, Feet	Depth, Feet	Span Length, Feet	Depth, Feet
100	20	180	34
120	21	200	37
130	22	225	41
140	27	250	44
150	28	300	50
160	30	350	60

In order to obtain the required overhead clearance and a satisfactory depth of portal the height of truss is limited to about 30 ft. in the case of railroad bridges and to about 20 ft. in the case of highway bridges. The minimum economic panel length for railroad bridges is about 25 ft. and the maximum about 33 ft., while in the case of highway bridges the minimum panel length is about 15 ft. and the maximum about 20 ft.

256. Economic Span Length.—The shortest bridge in the case of a single span, as a rule, is the cheapest, as the cost of the two abutments in most cases is practically independent of the span length. However, in a few cases the profile of the crossing is such that the cost of the abutments is materially affected by their location. In such cases care must be taken in locating the abutments so that their cost is a minimum, the usual limit being when the cost of the span equals the cost of the two abutments. Often the required water way or local conditions govern the length of the span, in which cases economy of span length can not be considered. In case a bridge is composed of several spans the spans are practically of economic length when the cost of superstructure minus the floor system is equal to the cost of the substructure minus the cost of the end abutments. This is, of course, assuming that the total length of the bridge is fixed by either the required water way or local conditions. The problem can be solved only by trial. First a profile of the crossing should be drawn carefully to scale and the base of rail or roadway located thereon, and next the cost of an average pier computed, and then by using the formulas of Art. 124 (Arts. 244, and 250) in the case of highway bridges) the weight of metal in the spans of different lengths can be obtained (the weight of the floor system must be subtracted in each case) and by careful comparison of the different bridges possible we can readily ascertain the economic length of spans.

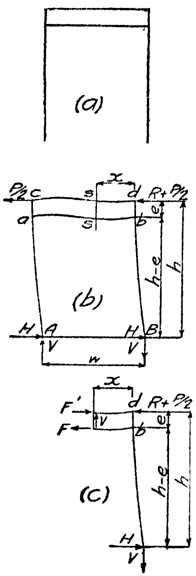


Fig. 400

257. Stresses in Portals.—Portals other than the one previously considered (Art. 179) are sometimes used. The plate girder portal shown at (*a*), Fig. 400, is occasionally used in case of bridges having shallow trusses.

Assuming the bottom ends of the end posts as hinged, the wind pressure tends to distort the end posts and portal as indicated at (*b*).

Considering the forces to the right of section *SS* and taking moments about *d* [see sketch at (*c*)] we obtain

$$F=\frac{Hh}{e}-\frac{Vx}{e} \qquad (1)$$

for the stress in the bottom flange of the portal.

But $$V=\frac{h}{w}(R+P) \text{ and } H=\frac{1}{2}(R+P).$$

Substituting these values in (1) we obtain

$$F=\frac{h}{2e}(R+P)-\frac{hx}{ew}(R+P).$$

When $x=0$

$$F=\frac{h}{2e}(R+P) \quad \ldots\ldots (2).$$

When $x=w/2$

$$F=0 \quad \ldots\ldots (3).$$

When $x=w$

$$F=-\frac{h}{2e}(R+P) \quad \ldots\ldots (4).$$

Now it is seen from the above equations that the stress in the bottom flange of the portal is zero at the center of the portal and a maximum at each end, being tension at one end and compression at the other.

Taking moments about b we obtain

$$F'=H\left(\frac{h-e}{e}\right)+\left(R+\frac{P}{2}\right)-\frac{Vx}{e} \quad \ldots\ldots (5)$$

for the stress in the top flange of the portal. Substituting $h/w(R+P)$ for V and $\frac{1}{2}(R+P)$ for H in equation (5) we obtain

$$F'=\tfrac{1}{2}(R+P)\left(\frac{h-e}{e}\right)+\left(R+\frac{P}{2}\right)-\frac{xh}{ew}(R+P)$$

$$=\frac{h}{2e}(R+P)+\frac{R}{2}-\frac{xh}{ew}(R+P) \quad \ldots\ldots (6).$$

When $x=0$

$$F'=\frac{h}{2e}(R+P)+\frac{R}{2} \quad \ldots\ldots (7).$$

When $x=w/2$

$$F'=\frac{R}{2} \quad \ldots\ldots (8).$$

When $x=w$

$$F'=\frac{h}{2e}(R+P)+\frac{R}{2} \quad \ldots\ldots (9).$$

From these equations it is seen that the maximum stress in the top flange of the portal occurs at the ends, being compression at one end and tension at the other, and that the minimum stress in the top flange occurs at the center of the portal where it is equal to $R/2$.

The shear on the portal is constant throughout its length and is equal to V. In case the bottom ends of the end posts be fixed the above will apply by substituting $\frac{1}{2}(h-e)$ for $(h-e)$ and $e+h/2$ for h (see Art. 179).

After the stresses in the portal are determined the required sections are readily computed. Each of the flanges should be such that L/r does not exceed 120. The portal shown at (a), Fig. 401, is used in case of limited clearance.

In order to obtain the required overhead clearance and a satisfactory depth of portal the height of truss is limited to about 30 ft. in the case of railroad bridges and to about 20 ft. in the case of highway bridges. The minimum economic panel length for railroad bridges is about 25 ft. and the maximum about 33 ft., while in the case of highway bridges the minimum panel length is about 15 ft. and the maximum about 20 ft.

256. Economic Span Length.—The shortest bridge in the case of a single span, as a rule, is the cheapest, as the cost of the two abutments in most cases is practically independent of the span length. However, in a few cases the profile of the crossing is such that the cost of the abutments is materially affected by their location. In such cases care must be taken in locating the abutments so that their cost is a minimum, the usual limit being when the cost of the span equals the cost of the two abutments. Often the required water way or local conditions govern the length of the span, in which cases economy of span length can not be considered. In case a bridge is composed of several spans the spans are practically of economic length when the cost of superstructure minus the floor system is equal to the cost of the substructure minus the cost of the end abutments. This is, of course, assuming that the total length of the bridge is fixed by either the required water way or local conditions. The problem can be solved only by trial. First a profile of the crossing should be drawn carefully to scale and the base of rail or roadway located thereon, and next the cost of an average pier computed, and then by using the formulas of Art. 124 (Arts. 244, and 250) in the case of highway bridges) the weight of metal in the spans of different lengths can be obtained (the weight of the floor system must be subtracted in each case) and by careful comparison of the different bridges possible we can readily ascertain the economic length of spans.

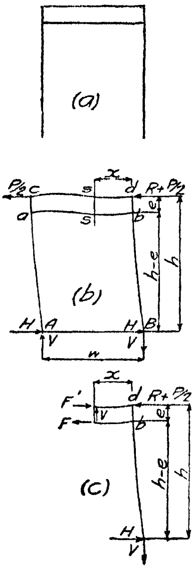


Fig. 400

257. Stresses in Portals.—Portals other than the one previously considered (Art. 179) are sometimes used. The plate girder portal shown at (*a*), Fig. 400, is occasionally used in case of bridges having shallow trusses.

Assuming the bottom ends of the end posts as hinged, the wind pressure tends to distort the end posts and portal as indicated at (*b*).

Considering the forces to the right of section *SS* and taking moments about *d* [see sketch at (*c*)] we obtain

$$F=\frac{Hh}{e}-\frac{Vx}{e} \quad \text{.......... (1)}$$

for the stress in the bottom flange of the portal.

But $$V=\frac{h}{w}(R+P) \text{ and } H=\frac{1}{2}(R+P).$$

Substituting these values in (1) we obtain

$$F=\frac{h}{2e}(R+P)-\frac{hx}{ew}(R+P).$$

When $x=0$

$$F=\frac{h}{2e}(R+P)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

When $x=w/2$

$$F=0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

When $x=w$

$$F=-\frac{h}{2e}(R+P)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Now it is seen from the above equations that the stress in the bottom flange of the portal is zero at the center of the portal and a maximum at each end, being tension at one end and compression at the other.

Taking moments about b we obtain

$$F'=H\left(\frac{h-e}{e}\right)+\left(R+\frac{P}{2}\right)-\frac{Vx}{e}\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

for the stress in the top flange of the portal. Substituting $h/w(R+P)$ for V and $\frac{1}{2}(R+P)$ for H in equation (5) we obtain

$$F'=\tfrac{1}{2}(R+P)\left(\frac{h-e}{e}\right)+\left(R+\frac{P}{2}\right)-\frac{xh}{ew}(R+P)$$

$$=\frac{h}{2e}(R+P)+\frac{R}{2}-\frac{xh}{ew}(R+P)\ldots\ldots\ldots\ldots\ldots(6).$$

When $x=0$

$$F'=\frac{h}{2e}(R+P)+\frac{R}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7).$$

When $x=w/2$

$$F'=\frac{R}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8).$$

When $x=w$

$$F'=\frac{h}{2e}(R+P)+\frac{R}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9).$$

From these equations it is seen that the maximum stress in the top flange of the portal occurs at the ends, being compression at one end and tension at the other, and that the minimum stress in the top flange occurs at the center of the portal where it is equal to $R/2$.

The shear on the portal is constant throughout its length and is equal to V. In case the bottom ends of the end posts be fixed the above will apply by substituting $\frac{1}{2}(h-e)$ for $(h-e)$ and $e+h/2$ for h (see Art. 179).

After the stresses in the portal are determined the required sections are readily computed. Each of the flanges should be such that L/r does not exceed 120. The portal shown at (*a*), Fig. 401, is used in case of limited clearance.

Assuming the bottom ends of the end posts to be hinged, the wind pressure tends to distort the end posts and the portal as is indicated at (*b*). The shear between *g* and *f* is constant and is equal to *V*. The moment at *g* and also at *f* is equal to $Vxb/2$ which is obtained by taking moments about either *g* or *f*. The moment at *o* is zero and at any point between *o* and *f* or between *o* and *g* it is Vz. Assuming the moment to be zero at *a* and *k* the strut *cfgk* can be considered as a continuous beam.

Then taking moments at either *f* or *g* we have $(Vxb/2) \div d$ for the reaction at *k* or *a* which is the shear in *gk* or *af*. The moment at any point in *gk* or *af* is equal to $[(Vxb/2) \div d]x$. If $x = d$ we have $Vxb/2$ for the moment at *g* or *f* which is the same as found above.

The horizontal component of the stress in knee brace *gm* is obtained by considering the forces to the right of section 2-2 and taking moments about *k*. Then the stress in *gm* is obtained by multiplying this component by $\sec\phi$. The stress in knee brace *cf* is obtained in the same manner—considering the forces to the left of section 1-1.

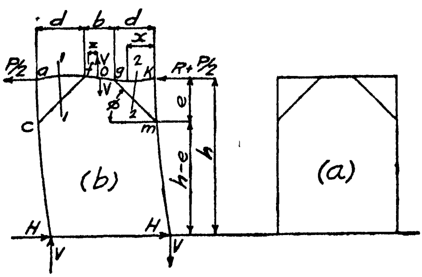


Fig. 401

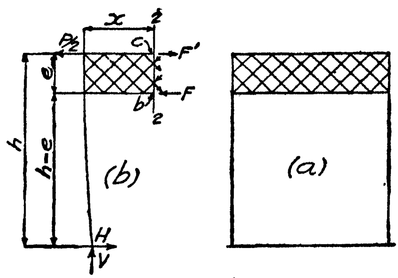


Fig. 402

In case the bottom ends of the end posts are fixed the above will apply by substituting $\frac{1}{2}(h-e)$ for $(h-e)$ and $e+\frac{1}{2}(h-e)$ for *h*.

The portal shown at (*a*), Fig. 402, is known as a lattice portal. It is distorted by the wind pressure very much the same as the plate girder portal.

As the shear is constant and equal to *V* throughout the length of the portal the diagonals are assumed to be equally stressed. As one system is in compression and the other in tension the moment of the stresses in the diagonals about any point on a vertical line through the intersection of the diagonals is equal to zero as the moments balance. Then by taking moments about *c* and considering the forces to the left of any section as 2-2, shown at (*b*), we have

$$F = \frac{Hh - Vx}{e}$$

for the stress in the bottom flange at *b* and taking moments about *b* we have

$$F' = \frac{H(h-e) - Vx}{e} + \frac{P}{2}$$

for the stress in the top flange at *c*. The flange stress throughout the portal can be obtained in this manner.

The stress in the diagonals is obtained by dividing the shear (V) by the number of diagonals cut by a vertical plane, and multiplying this by the secant of the slope of the diagonals.

The portal shown in Fig. 403 is used to some extent, especially for long span bridges. Let us first assume that the diagonals take tension only. Then diagonal *db* would have zero stress when the wind pressure is applied as indicated and diagonal *ac* would have zero stress if the wind pressure were applied from the opposite direction.

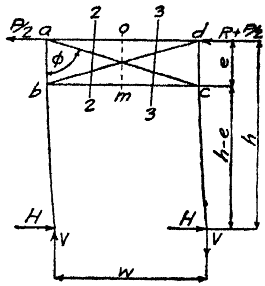


Fig. 403

Assuming the bottom ends of the end posts hinged, and taking moments about *a* (ignoring diagonal *bd*) and considering the forces to the left of section 2-2 we obtain

$$S=\frac{Hh}{e} \quad \text{(1)}$$

for stress (compression) in *bc*, and taking moments about *c* (ignoring diagonal *bd*) and considering the forces to the right of section 3-3 we obtain

$$S'=\frac{H}{e}(h-e)+\left(R+\frac{P}{2}\right)$$

for the stress (tension) in *ad*.

The stress in the diagonal *ac* is equal to $V\sec\phi$. If the wind pressure were applied from the opposite direction to that indicated diagonal *bd* would be stressed, and the stress in *bc* and *ad* would be determined by taking moments about *d* and *b*, respectively. The stress in diagonal *bd* would be $V\sec\phi$ and the stress in diagonal *ac* would be zero.

In case the diagonal were considered to take compression as well as tension we would assume the two equally stressed and then by taking moments about *o* and *m* the stress in *bc* and *ad* could be determined, as the sum of the moments of the stresses in the diagonals about these points would be zero as they would balance.

If the lower ends of the end posts be considered fixed, the above would apply if $\frac{1}{2}(h-e)$ be substituted for $(h-e)$ and $e+\frac{1}{2}(h-e)$ for h.

The depth of the portal, as a rule, is so shallow, as compared with the length of the end posts, that the bending of the posts along the ends of the portal can be ignored, and in case the lower ends of the posts are fixed the points of contra-flexure in the posts can be considered as being midway between the portal and the lower end of the posts, as has been done in all previous examples. But in a few cases the required depth of the portal is so great that the bending of the end posts along the ends of the portal must be taken into account in determining the points of contra-flexure in the posts. Let the diagram shown at (*a*), Fig. 404, represent such a case. The distortion of the portal members is so small as compared with the flexure of the posts that the points C and B can be assumed to remain in a vertical line and likewise the points D and E. The problem is to determine the value of x, which is the distance from the lower ends of the end posts up to the points of contra-flexure.

Let R represent the horizontal component of the forces acting upon the post at C, and R' the horizontal component of the forces at B. Then the post ABC may be considered as a beam fixed at A and acted upon by the forces R and R' as indicated at (b).

Then, for the bending moment at any point on this beam between A and B we have

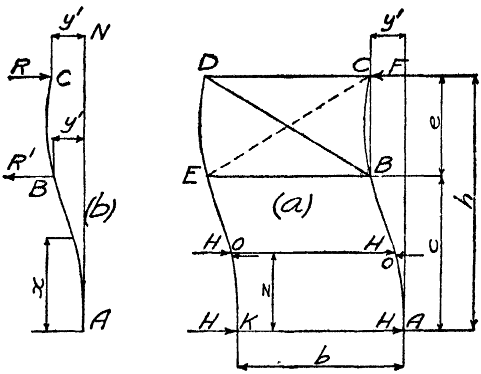


Fig. 404

$$M=EI\frac{d^2y}{dx^2}=R(h-x)-R'(c-x)\ldots\ldots(1).$$

Integrating, we have

$$EI\frac{dy}{dx}=R\left(hx-\frac{x^2}{2}\right)-R'\left(cx-\frac{x^2}{2}\right)+C.$$

But $\frac{dy}{dx}=0$ when $x=0$, therefore $C=0$, and we have

$$EI\frac{dy}{dx}=R\left(hx-\frac{x^2}{2}\right)-R'\left(cx-\frac{x^2}{2}\right)\ldots\ldots(2).$$

Integrating again, we obtain

$$EIy=R\left(h\frac{x^2}{2}-\frac{x^3}{6}\right)-R'\left(c\frac{x^2}{2}-\frac{x^3}{6}\right)+C'.$$

But $y=0$ when $x=0$, therefore $C'=0$, and we have

$$EIy=R\left(h\frac{x^2}{2}-\frac{x^3}{6}\right)-R'\left(c\frac{x^2}{2}-\frac{x^3}{6}\right)\ldots\ldots(3).$$

For the bending moment at any point between C and B we have

$$M'=EI\frac{d^2y}{dx^2}=R(h-x)\ldots\ldots(4).$$

Integrating, we have

$$EI\frac{dy}{dx}=R\left(hx-\frac{x^2}{2}\right)+C'' \qquad \text{(5).}$$

As is obvious (2) and (5) are equal when $x=c$ in each. So by substituting c for x in both (2) and (5), equating and reducing, we obtain

$$C''=-R'\frac{c^2}{2}$$

Substituting this value of C'' in (5) we obtain

$$EI\frac{dy}{dx}=R\left(hx-\frac{x^2}{2}\right)-R'\frac{c^2}{2}$$

Integrating again, we have

$$EIy=R\left(h\frac{x^2}{2}-\frac{x^3}{6}\right)-R'\frac{c^2}{2x}+C''' \qquad \text{(6).}$$

As is obvious, (3) and (6) are equal when $x=c$ in each. So by substituting c for x in both (3) and (6), equating and reducing, we obtain

$$C'''=+R'\frac{c^3}{6}$$

Substituting this value of C''' in (6) we obtain

$$EIy=R\left(h\frac{x^2}{2}-\frac{x^3}{6}\right)-R'\frac{c^2}{2}x+\frac{c^3}{6}R' \qquad \text{(7).}$$

As points C and B are in the same vertical line (3) and (7) are equal for $x=c$ in (3) and $x=h$ in (7). So substituting these values, equating and reducing, we obtain

$$\frac{R}{R'}=\frac{3c^3-3c^2b}{3hc^2-c^3-2h^3}=\frac{3c^2}{2hc+2h^2-c^2} \qquad \text{(8).}$$

When $x=z$ we have

$$M''=R(h-z)-R'(c-z)=0.$$

From this equation we obtain

$$\frac{R}{R'}=\frac{(c-z)}{(h-z)} \qquad \text{(9).}$$

Now equating (8) and (9) and reducing we obtain

$$z=\frac{c(c+2h)}{2(2c+h)} \qquad \text{(10).}$$

By applying equation (10) the points of contra-flexure in the posts can be determined and then the stresses in the portal can be computed as previously explained.

DRAWING ROOM EXERCISE NO. 9

(a) Design a single-track through pin-connected curve chord Pratt truss railroad bridge and make a complete stress sheet of same upon tracing cloth.

Data:

Length of span = 9 panels at 26′-0″ = 234′-0″ c.c. end pins.
Height of truss at center = 46′-0″.
Height of truss at hip = 31′-0″.
Top chord joints to be on the arc of a parabola.
Live load, Cooper's *E*50.
Dead load, to be assumed.
Specifications, A. R. E. Ass'n.

(b) Make a general detail drawing of the above bridge.

(c) Design a single-track through Pettit truss railroad bridge and make a stress sheet for same.

Data:

Length of span = 14 panels at 28′-0″ = 392′-0″ c.c. end pins.
Live load, Cooper's *E*50.
Dead load, to be assumed.
Specifications, A. R. E. Ass'n.

(d) Design a 180-ft. riveted highway bridge.

Data:

Length of span = 9 panels at 20′-0″ = 180′-0″.
Height of truss at center = 34′-0″.
Height of truss at hip = 22′-0″.
Width of roadway = 22′-0″.
Concrete floor.
Live load, Cooper's *H*15.

CHAPTER XIV

DESIGN OF BUILDINGS

MILL BUILDINGS

258. Preliminary.—The simplest type of mill building consists of a steel roof supported upon brick or concrete walls as shown at (*a*), Fig. 405. This type of building is satisfactory for power houses, pumping stations, and small manufacturing plants where light material is handled.

The type shown at (*b*) consists of a steel roof supported upon steel columns. The side and end walls for this type may be brick, hollow tile, concrete, metal lathing plastered, or corrugated iron. In case the material handled is heavy, an overhead crane would be installed. This crane would run lengthwise of the building and be supported upon girders riveted to the columns.

The type shown at (*c*) is practically the same as the one shown at (*b*) except a shed, known as a lean-to, is attached to each side and a monitor or ventilator is built onto the top of the roof for the purpose of ventilating and lighting the building.

The type shown at (*d*) is simply a double building of the same type as the one shown at (*c*) except, as a rule, it would be proportionately larger.

The type shown at (*e*) is known as the saw-tooth construction. This type of construction is used in the case of a very wide building in order to obtain light in the interior of the building.

All of the types shown above are for steep roof construction where the roof covering is corrugated iron, slate, concrete, or tile. In case of flat roof construction, where the roof covering is felt covered with tar and gravel, the types shown in Fig. 406 are used.

The type shown at (*f*) consists of a steel roof supported upon brick or concrete walls.

The type shown at (*g*) consists of a steel roof supported upon steel columns. The end and side walls may be corrugated iron, brick, concrete, hollow tile, or metal lathing plastered.

The type shown at (*h*) is the same as the one shown at (*g*) except that a lean-to is added to each side.

The type shown at (*k*) is practically the same as the one shown at (*h*) except that each lean-to is wider and the building is proportionately larger in every way.

The types of buildings shown in Figs. 405 and 406 are general types. These are often modified to suit certain special requirements; in fact it is impossible to indicate types suitable for all cases.

As a rule the nature of the material to be handled and the required location of the machinery govern the type of building used.

The maximum length of span for the trusses as a rule is limited by the length of the cranes which have a maximum length of about 80 ft.—more often 40 or 50 ft. is used.

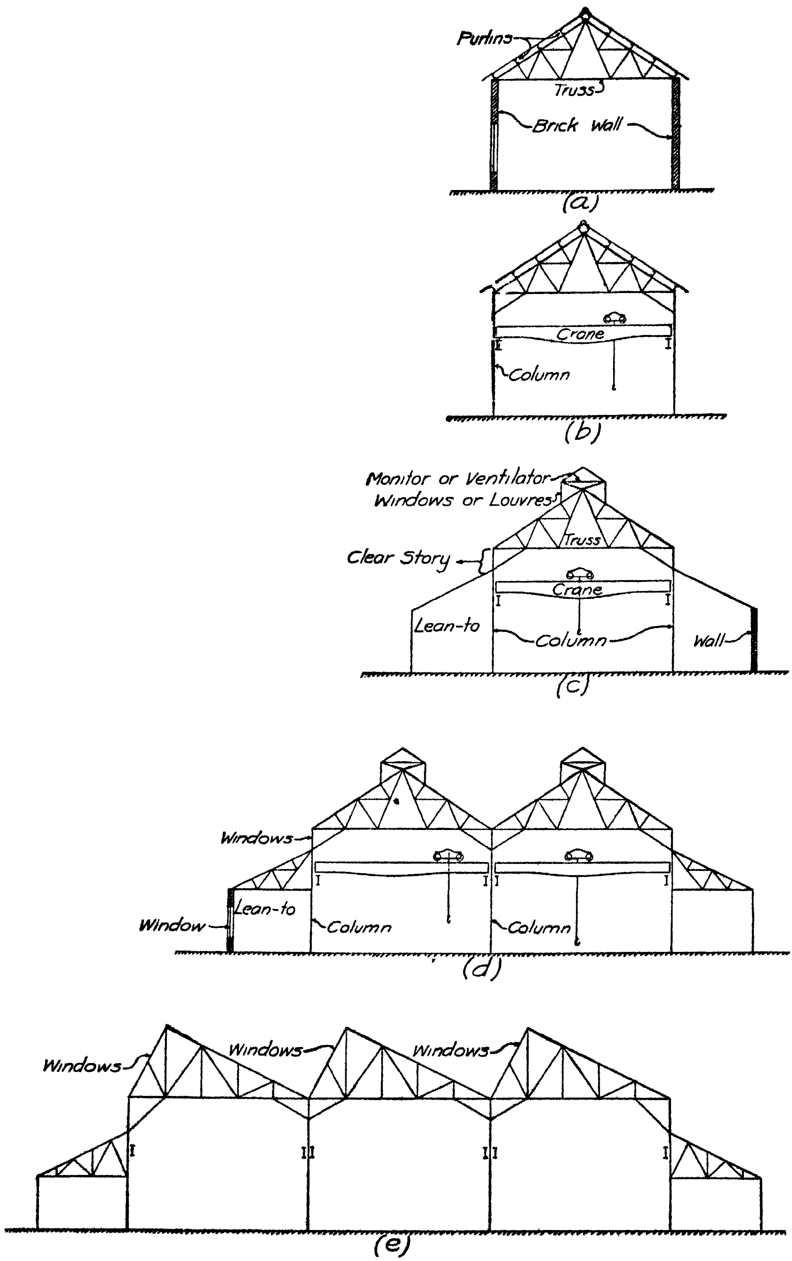

Purlins
Truss
Brick Wall
(a)
Crane
Column
(b)
Monitor or Ventilator
Windows or Louvres
Truss
Clear Story
Crane
Lean-to
Column
Wall
(c)
Windows
Lean-to
Window
Column
Column
(d)
Windows
Windows
Windows
(e)

Fig. 405

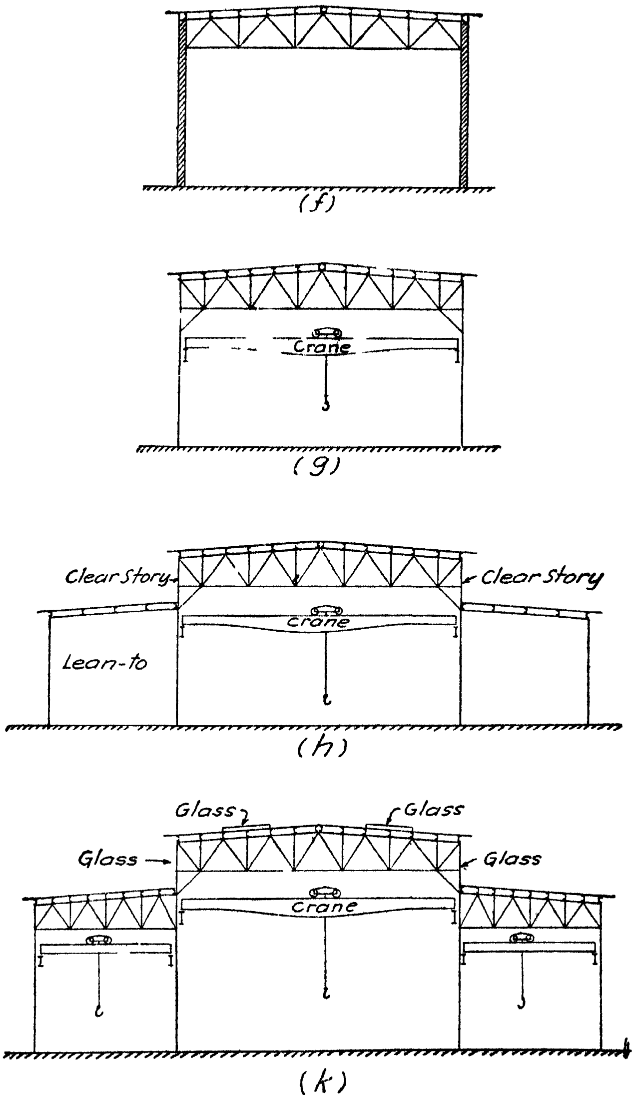

(f)
Crane
(g)
Clear Story
Clear Story
crane
Lean-to
(h)
Glass
Glass
Glass
Glass
crane
(k)

Fig 406

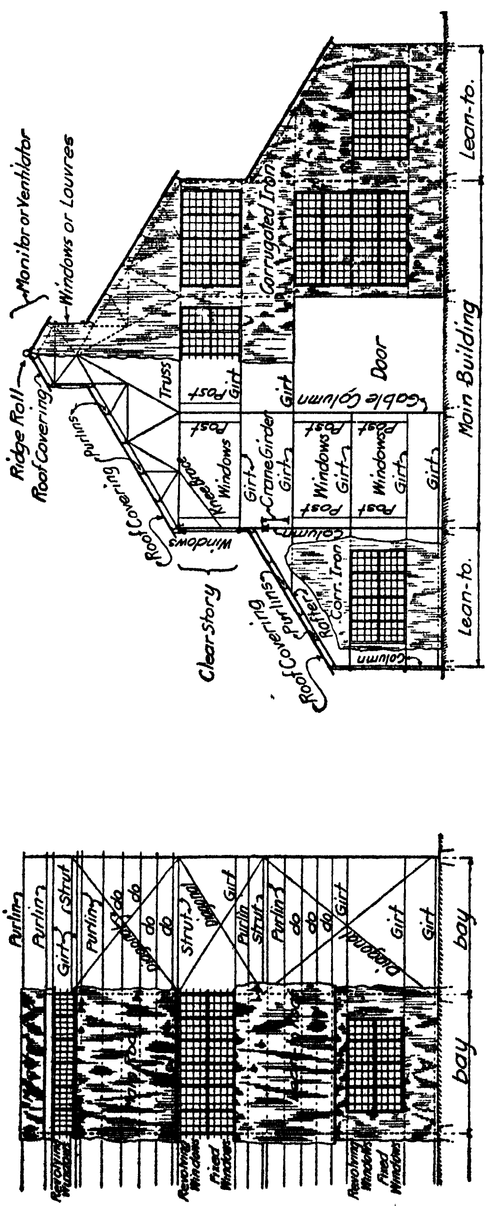

Monitor or Ventilator
Windows or Louvres
Ridge Roll
Roof Covering
Purlins
Roof Covering
Truss
Post
Girt
Knee Brace
Windows
Crane Girder
Gable Column
Door
Corrugated Iron
Clear Story
Roof Covering
Purlins
Rafter
Corr. Iron
Column
Lean-to.
Main Building
Lean-to.
Purlin
Strut
Diagonal
do
Revolving Windows
Fixed Windows
bay
bay

Fig. 407

The names of the different parts of an ordinary steel mill building are given in Fig. 407. These same names, however, apply in general to steel mill buildings.

The most common forms of roof trusses are shown in Fig. 408. The steep pitch roofs are used when the roof covering is corrugated iron, slate, or tile, and the flat roofs are used when the roof covering is felt covered with tar and gravel. Concrete roof covering may be used in either case.

The details shown in Fig. 409 are those of a truss supported upon columns. The purlins are channels which are supported on the roof truss at panel points. The roof covering is corrugated iron connected to the purlins by steel bands.

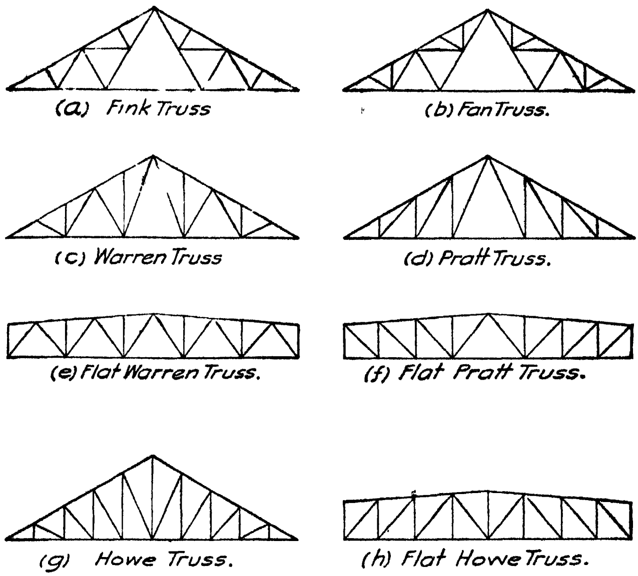


Fig. 408

The details shown in Fig. 410 are those of a roof truss supported upon walls. The roof covering is slate laid on 2″ sheathing. The purlins are channels which have 6″x3″ nailing strips bolted to them. In this case the purlins are not at panel points and the top chord must be designed heavy enough to resist, in addition to the direct stress, the cross bending caused by the purlin loads. Wooden purlins as shown at (*a*) are sometimes used, which is very good construction.

The details shown in Fig. 411 are those of a roof without purlins. The sheathing consists of 2″x4″ timbers laid on edge, acting as beams extending from truss to truss. The bay length in the case of such construction should be limited to about 14 or 15 ft. In case the roof load on the trusses is quite heavy and not supported at panel points, the top chords of the trusses are constructed of two angles and a plate as shown in Fig. 411. The plate is principally for transmitting shear.

The details shown in Fig. 412 are those of an ordinary monitor where louvre bars are used. The details shown in Fig. 413 are those of an ordinary monitor where pivoted windows are used.

Pitch of a roof truss is the ratio of its height to its length. Thus: A roof truss 60 ft. long and 20 ft. high would have $\frac{1}{3}$ pitch, and if its height were 30 ft. the pitch would be $\frac{1}{2}$. The terms "pitch" and "slope" are somewhat

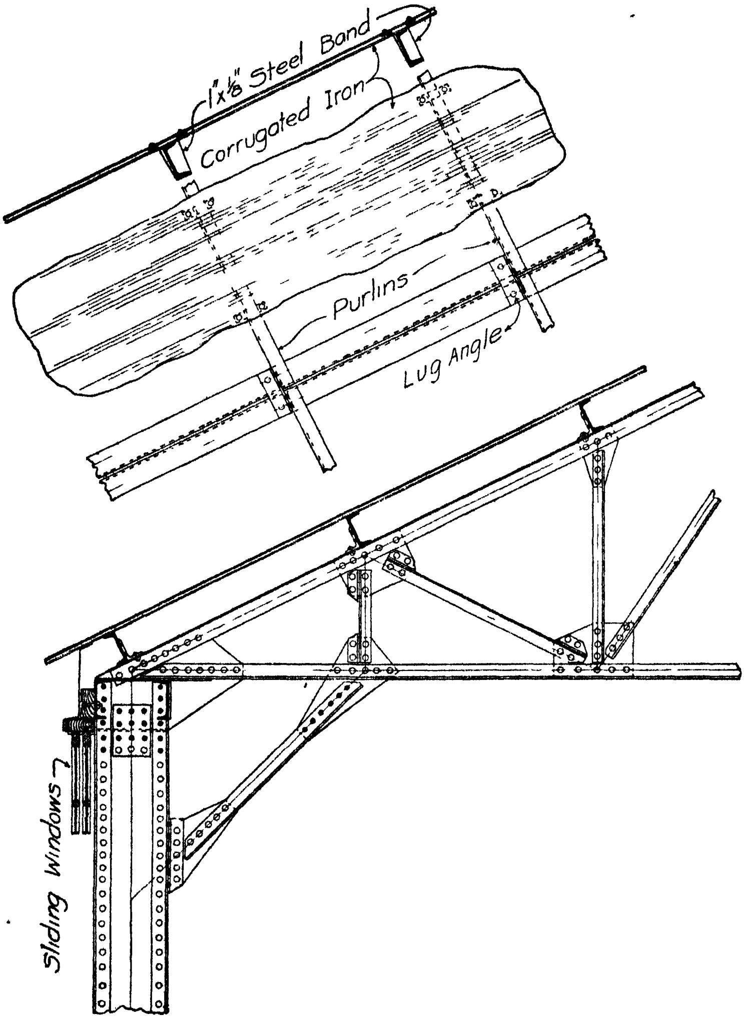


Fig. 409

confusing. The pitch of an ordinary truss is obtained by dividing its rise or height by its total length, while the slope is obtained by dividing its height by one-half of its length.

A lean-to truss, as shown at (*d*), Fig. 405, can be considered as a half truss.

The pitch of a roof depends upon the roof covering. The following is recommended:

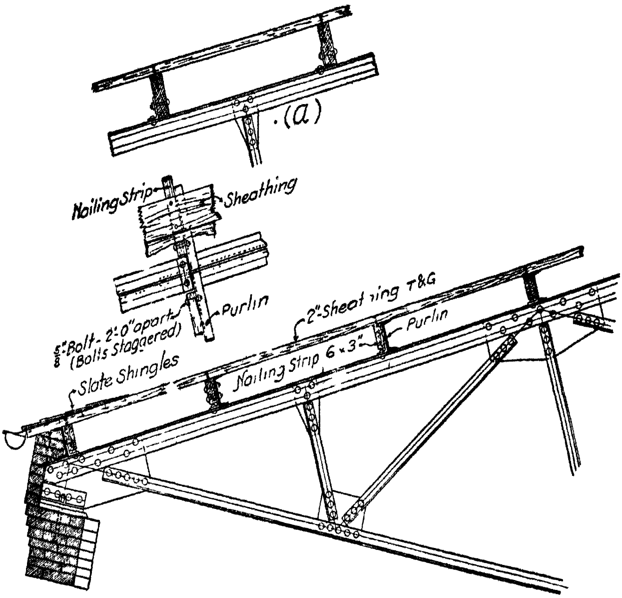


Fig. 410

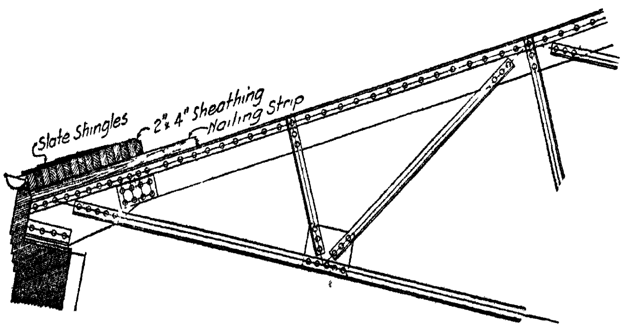


Fig. 411

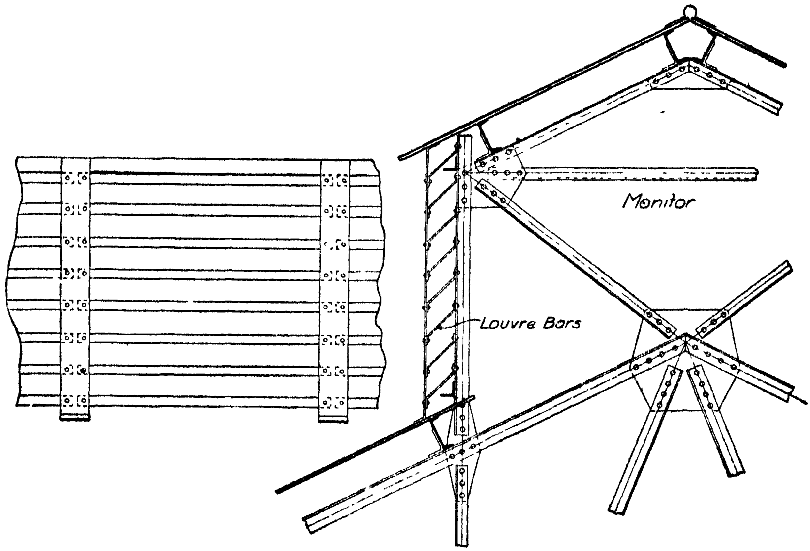


Fig. 412

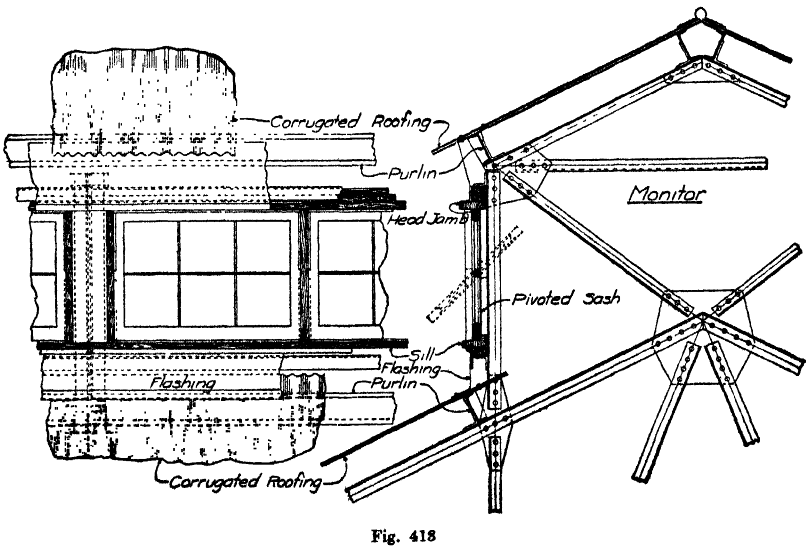


Fig. 413

Roof Covering	Pitch
Corrugated Iron	not less than $\frac{1}{4}$.
Slate	not less than $\frac{1}{4}$, preferably $\frac{1}{3}$ or $\frac{1}{2}$.
Tile	not less than $\frac{1}{2}$.
Felt, tar, and gravel	not more than $\frac{1}{32}$, preferably $\frac{1}{48}$.

259. Dead Load.—Except for the roof trusses, the dead load for mill buildings is readily estimated. In making these estimates the following units of weights may be used:

Timber	4.0 lbs. per sq. ft. B. M.
Concrete	150.0 lbs. per cu. ft.
Corrugated iron, #16	3.5 lbs. per sq. ft.
Corrugated iron, #18	2.7 lbs. per sq. ft.
Corrugated iron, #20	1.9 lbs. per sq. ft.
Corrugated iron, #22	1.5 lbs. per sq. ft.
Slate, $\frac{1}{8}''$ thick, 3″ double lap	4.5 lbs. per sq. ft.
Slate, $\frac{3}{16}''$ thick, 3″ double lap	6.5 lbs. per sq. ft.
Tile (plain)	18.0 lbs. per sq. ft.
Tile (Spanish)	8.5 lbs. per sq. ft.
Glass, $\frac{1}{8}''$ thick	2.0 lbs. per sq. ft.
Gravel, felt, and tar roof covering	5.0 lbs. per sq. ft.
Hollow tile (gross)	40.0 lbs. per cu. ft.

The approximate weight of roof trusses in pounds per sq. ft. of roof surface is as follows:

	Pitch				
	Flat	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$
20-ft. span	2.0	1.9	1.8	1.7	1.5
30-ft. span	2.6	2.5	2.3	2.1	1.8
40-ft. span	3.3	3.2	2.9	2.8	2.4
50-ft. span	3.8	3.6	3.4	3.2	2.7
60-ft. span	4.6	4.4	4.1	3.8	3.2
70-ft. span	5.2	4.9	4.6	4.3	3.7
80-ft. span	5.7	5.4	5.1	4.8	4.1
90-ft. span	6.4	6.1	5.7	5.3	4.6
100-ft. span	7.0	6.7	6.2	5.8	5.0
120-ft. span	8.0	7.6	7.1	6.7	5.7

260. Snow Load.—The snow load on roofs varies with the locality, slope of roof, and kind of roof covering. According to Trautwine, fresh fallen snow weighs from 5 to 12 lbs. per cu. ft., while moistened snow compacted by rain will weigh from 15 to 50 lbs. per cu. ft.

The following loads in pounds per sq. ft. of roof surface are considered to be sufficient allowance for snow:

	Pitch					
Locality	Flat	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$
Central States	30	30	25	20	15	10
Northwestern States	40	40	38	30	20	12
New England States	36	36	32	27	18	12
Rocky Mountain States	35	35	30	26	15	10

These loads should be reduced 20 per cent in case the roof covering be slate or corrugated iron.

No snow load need be considered south of latitude 35°. However, in some localities south of this latitude a load of 10 lbs. per sq. ft. of roof surface should be included in roof loads to allow for sleet. In fact, load due to sleet, in all cases, should be taken as 10 lbs. per sq. ft. of roof surface.

261. Wind Load.—The wind pressure against vertical surfaces varies with the velocity of the wind and to some extent with the area of the exposed surface. The wind load (pressure) on vertical surfaces can be safely considered as 30 lbs. per sq. ft. except for steel towers, where the exposed surfaces are small, in which case the load should be considered as 40 lbs. per sq. ft. This 30 lbs. pressure, in the case of mill buildings, is considered to be applied to the vertical sides and ends of the buildings.

In the case of wind load on sloping roofs the pressure is assumed to act perpendicularly or normally to the roof surface. This pressure can be determined only experimentally. Hutton derived, from his experiments in 1788, the following formula:

$$P_n = P(\sin a)^{1.842 \cos a - 1}$$

for the normal pressure due to wind upon inclined surfaces, where P_n represents the normal pressure, P the corresponding horizontal pressure, and a the slope of the surface.

Duchemin, in 1829, derived in the same manner the following formula:

$$P_n = P\left(\frac{2\sin a}{1+\sin^2 a}\right)$$

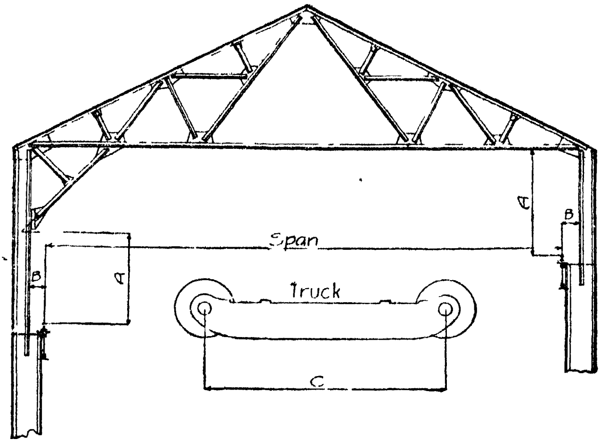
for the normal pressure on inclined surfaces, where the letters signify the same as in Hutton's formula.

These two formulas are in general use and the values for the normal pressures when P equals 30 lbs. are as follows:

Slope of Roof	Normal Pressure Hutton	Duchemin
5°	3.9	5.1
10°	7.2	10.2
15°	10.5	14.2
18°-26′ (1/6 pitch)	13.0	17.4
21°-48′ (1/5 pitch)	15.0	19.8
26°-34′ (1/4 pitch)	18.0	22.4
30°	19.9	24.0
33°-41′ (1/3 pitch)	22.0	25.5
40°	25.1	26.7
45° (1/2 pitch)	27.1	28.2
60°	30.0	30.0

262. Cranes.—In designing a mill building, it is absolutely necessary that the designer have complete data as regards the type, weight and capacity of the cranes to be used, and also the clearance required for their operation.

The data given in Fig. 414 will suffice in most cases for overhead cranes. However, the general requirements in each case should be carefully studied; and often it is advisable to consult the company that manufactures the crane to be used.



Capacity	Span	A	A′	B	C	Each Wheel	Total Net Wt of Crane
5 Tons	40 Ft	4′ 11″	5-7″	0 9′	8 8″	12800	22400
5	60	5′ 3″	5′ 11″	0′ 9″	8 8″	15500	31300
10	40	5-10″	6-6″	0-10″	9′-2′	19800	28400
10	60	6′-0″	6′-8″	0′ 10″	9′-2″	22700	37800
15	40	6-0″	6′-9″	0′ 10″	9′-2″	26500	33900
15	60	6′-2″	6′-11″	0-10″	9′ 2″	29800	44000
20	40	6′ 2″	7′-3″	0 11″	9′ 8″	32700	37600
20	60	6′ 5″	7′-6″	0′ 11′	9′-8′	37000	50700
25	40	6′-5″	7′-7″	0-11″	9′-8″	39300	43400
25	60	6′-9″	8′-0″	0′-11″	10′-10″	44500	58700
25	80	7′-2″	8-5′	0 11″	13′-4″	50800	79900
30	40	6′-8″	8′-0″	1′-0″	10′-4″	46200	49500
30	60	7′-0″	8-6″	1′-0′	11′-2″	51700	66600
30	80	7′-5″	8-11″	1′-0′	13′ 4″	58800	90700
40	40	7′-0″	8′-9″	1′-0″	11′-4″	60100	64200
40	60	7′-4″	9′-1″	1′-0″	11′-10″	67000	84300
40	80	7′-9″	9′-6″	1-0″	13′-4″	75600	112900
50	40	7′-1″	9′-5″	1-0″	11′-4″	74000	76100
50	60	7′-9″	9′-7″	1′-0″	11′ 0″ 3′ 10′	41200	98500
50	80	8′ 5″	10′-3″	1′-0″	13′ 2″ 5′-1″	45900	131200

Fig. 414

Complete Design of a Roof Supported upon Brick Walls

263. Data.—

Size of building = 80 ft. long by 40 ft. wide.
Length of roof trusses = 40′-0″ c.c. end bearings.
Height of roof trusses = 10′-0″ ($\frac{1}{4}$ pitch).
Length of bay = 20′-0″.
Roof covering, slate laid on 2″ sheathing.
Purlins, steel channels.

264. Design of Purlins.—As 2″ sheathing is specified, the first thing to do is to determine the distance that the purlins can be apart, so that the sheathing will not be overstressed or the deflection be too great.

The load on the sheathing includes snow, wind, and the weight of the slate and sheathing.

The maximum snow and wind load is not likely to occur at the same time as the snow would be blown off. Let us assume that the wind pressure is 18 lbs. (Hutton—see Art. 261) and the snow 10 lbs. per sq. ft. of roof surface. The 10 lbs. snow load provides for sleet, ice, or frozen snow. Using the weights given in Art. 259, we have the following dead load per sq. ft. of roof:

$\frac{3}{16}$″ slate	= 6.5	lbs. per sq. ft. of roof surface
2″ sheathing	= 8.0	lbs. per sq. ft. of roof surface
	14.5	lbs. per sq. ft. of roof surface

Adding this to the snow load we obtain 24.5 lbs. This load acts vertically while the wind load acts normally to the roof.

Let ϕ be the slope of the roof. Then for the normal pressure due to dead and snow load we have

$$24.5 \times \cos\phi = 24.5 \times 0.895 = 21.9, \text{ say } 22 \text{ lbs.}$$

Now adding this to the wind load we have

$$18 + 22 = 40 \text{ lbs.}$$

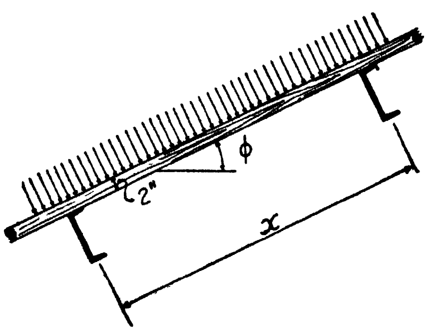


Fig. 415

for the total maximum normal pressure on the roof per sq. ft. of roof surface. Let x be the distance between purlins, as indicated in Fig. 415, and considering the sheathing to be a simple beam 1 ft. wide, we have

$$M = \tfrac{1}{8} \times 40 \times x^2 \times 12 = 60x^2 \text{ inch lbs.}$$

for the maximum bending moment. Taking 1,000 lbs. as the allowable unit stress on the sheathing and applying Formula (C) of Art. 53, we have

$$60x^2 = 1{,}000 \times 8,$$

from which we obtain

$$x = 11.5 \text{ ft.}$$

for the distance that the purlins could be apart and the sheathing would not be overstressed from the 40 lbs. load. The sheathing should be capable of supporting a 200-lb. man in addition to the 12.5 lbs. dead load. Considering the man to be midway between purlins (center of span) and resolving the loads normally to the roof, we have

$$M' = \tfrac{1}{8}(14.5 \times 0.895 \times x^2 \times 12) + \frac{200}{2} \times 0.895 \times \frac{x}{2} \times 12 = 19.4x^2 + 537x$$

for the bending moment. Then we have

$$19.4x^2+537x=1{,}000\times 8,$$

from which we obtain

$$x=10.8 \text{ ft.}$$

for the distance between the purlins which is only 0.7 ft. less than found above for the 40 lbs. load. But we know from experience that such spacing would not be practical as the deflection of the sheathing would be so great that the slate would be broken. From this it is seen that the spacing of the purlins depends entirely upon the maximum allowable deflection of the sheathing. This deflection should not exceed $\frac{1}{800}$ of the distance between the purlins. For the deflection due to the 200-lb. man at mid span (see Art. 65) we have the following expansion:

$$\Delta=\frac{Px^3}{48EI}.$$

Placing this equal to $x/800$ and taking E as 1,000,000 and P as 200, we have

$$\frac{200\times 0.895\times x^3}{48\times 1{,}000{,}000\times 8}=\frac{x}{800},$$

from which we obtain

$$x=51.8'' \text{ or about } 4'\text{-}4''.$$

If the deflection due to the dead load of 14.5 lbs. per sq. ft. of roof surface be taken into account the value of x will be a little less than 4′-4″, so the correct distance will be about 4′-0″. The actual spacing of the purlins depends to some extent upon the design of the roof truss. In this case a Fink truss will be used and the most suitable spacing obtainable of the purlins is shown in Fig. 416. This gives a distance of about 3′-8″ between purlins.

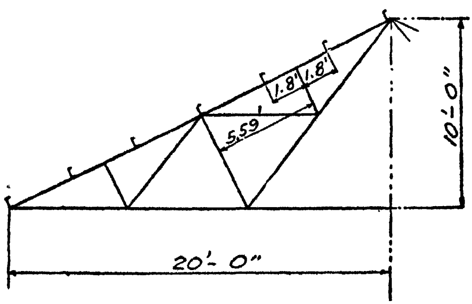


Fig. 416

The dead and snow load as given above is 24.5 lbs. per sq. ft. of roof surface. Adding to this the wind load of 18 lbs., which we shall consider to act the same as the dead load on the purlins, we have

$$24.5+18=42.5, \text{ say } 43 \text{ lbs.}$$

for the total load per sq. ft. of roof surface.

Now, as the purlins are 3′-8″ apart, we have

$$43\times 3.66=157.38, \text{ say } 158 \text{ lbs.}$$

for the roof load per ft. of purlin. Assuming the purlin to weigh 12 lbs. per ft. we have

$$158+12=170 \text{ lbs.}$$

for the total load per ft. of purlin. Then for the maximum bending moment on the purlin we have

$$M' = \frac{1}{8} \times 170 \times \overline{20}^2 \times 12 = 102{,}000 \text{ inch lbs.}$$

Dividing this by 16,000 we obtain a section modulus of 6.3 which calls for a 7"x12¼# [or an 8"x11¼# [. The 8"x11¼# [will be used for purlins throughout.

Theoretically the slope of a purlin should be taken into account in determining the maximum stress on it, but as the above result would not be changed by so doing (especially if the stiffening effect of the nailing strips and sheathing be taken into account) we are really justified in ignoring the slope, as was done in the above calculations.

265. Determination of Stresses in the Trusses Due to Dead and Snow Load.—There are really two loads to consider: The dead and snow loads that can be combined with the maximum wind load, and the dead and maximum snow load that could be combined with one-third of the maximum wind load. The first is given above as 24.5 lbs. per sq. ft. of roof surface

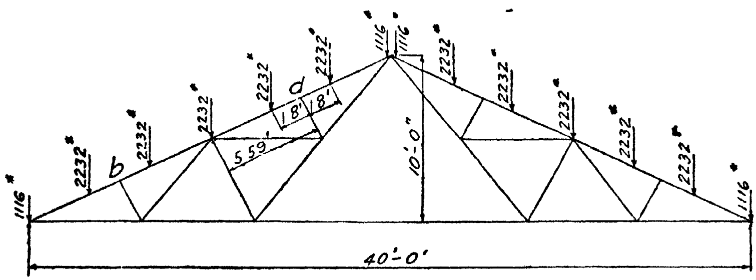


Fig 417

where the snow (sleet) is taken as 10 lbs. Combining this with the maximum wind load (Hutton) we obtained the 43 lbs. given above. For the other load, taking the maximum snow for the Central States, we have

Snow		Dead Load		Wind	
20	+	14.5	+	6	= 40.5 lbs.

which is less than the combination first considered. So we shall use the 24.5 lbs. dead and snow load. The end reaction on each purlin, except the eave and ridge purlins, including the weight of truss and purlins, is $[(24.5+2.9) \times 3.66+11.25]\ 10 = 1{,}116$ lbs. about, and hence the concentration on the truss at each of these purlins is 1,116x2 = 2,232 lbs. Then the load on the truss will be as shown in Fig. 417.

The concentration on the panel points, considering the top chord in each panel as a simple beam, is shown at (*a*), Fig. 418. The diagram of the stresses is shown at (*b*). This diagram is constructed by beginning at joint *A* and passing around each joint clock-wise. Thus, laying off 1-2 equal to the reaction and 2-3 equal to the 1,834 lbs. load and closing on 1 and 3 we obtain 1-2-3-4-1 for the stress diagram for joint *A*. Starting at 4 and passing around joint *a* clock-wise, we obtain 4-3-5-6-4 for the stress diagram for that joint. Starting at 1 and passing around joint *B* clock-wise we obtain 1-4-6-7-1 for the stress diagram for that joint. The stress diagrams for joints *b* and *C* can not be drawn as the structure stands as there are three unknown forces at each joint. We get around this difficulty by inserting the temporary member *Cc* and omitting members *be* and *ce*. Having made this modification we obtain the stress diagram 7-6-5-9-8-7 for joint *b* and diagram 8-9-11-12-8 for joint *c* and next beginning at 1 and passing around joint *C* clock-wise we obtain 1-7-8-12-*s*-1 for the stress diagram for that joint. The only correct stress obtained by drawing the last three diagrams is the stress in *CD* which is represented by the line *s*-1. This stress, as is readily seen, is not affected by the changing of the web members. Having determined the stress in *CD* we

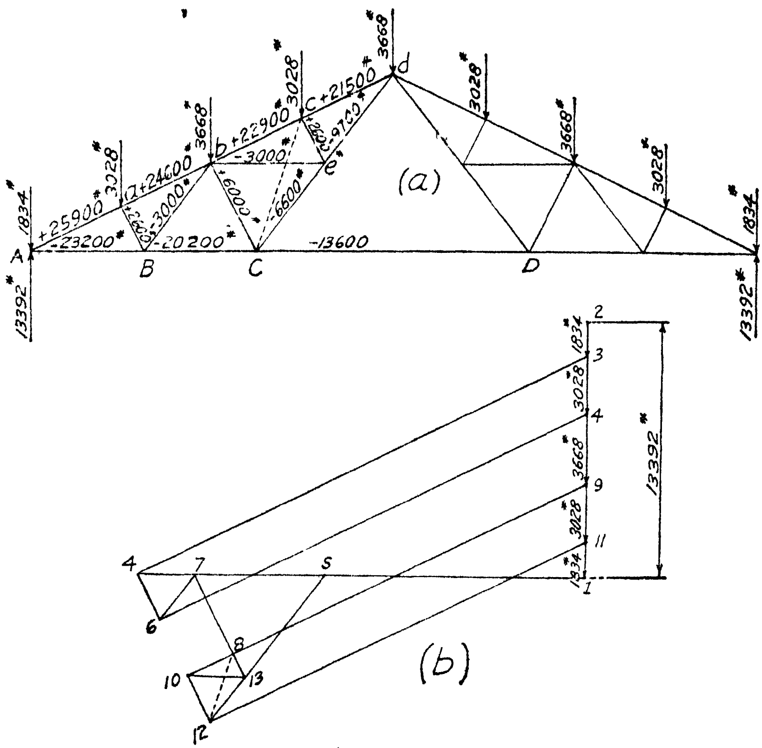


Fig. 418

can consider members *be* and *ce* in place and *Cc* omitted, and proceed with the analysis. Considering joint *C* and passing around clock-wise we obtain *s*-1-7-13-*s* for the stress diagram for that joint. Having the stress in *bC* determined, which is represented by the line 7-13, we can draw the stress diagram for joint *b* which is 13-7-6-5-9-10-13.

The remainder of the diagram at (*b*) is readily constructed. This diagram represents the stresses in the left half of the truss. The stresses in the right half are the same, for corresponding members, and hence the diagram at (*b*) is sufficient for determining the stress in all the truss members. By scaling the diagram at (*b*) the desired stresses in the truss members are obtained. These are shown on the members at (*a*).

266. Determination of the Stresses in the Trusses Due to Wind Load.—The wind load (see table of Art. 261) is 18 lbs. (Hutton) per sq. ft. of roof surface. Then for the load due to wind at each purlin we have 18x3.66 x20 = 1,318 lbs. These loads produce the panel loads on the roof shown in Fig. 419. These panel loads are obtained by taking moments about the joints in the top chord. (See Fig. 417 for distances.)

The two ends of the trusses are considered to be equally fixed. Then the reactions will be parallel to the loads. Laying off the load line *MN*, shown at (*b*), and drawing the ray diagram *MON*, and constructing the corresponding equilibrium polygon *ab'* . . . *g*, and drawing the line *OE* parallel to the closing line *ag*, we obtain *ME*, which represents the reaction *R* at *A*, and *EN*, which represents the reaction *R*1 at *F*.

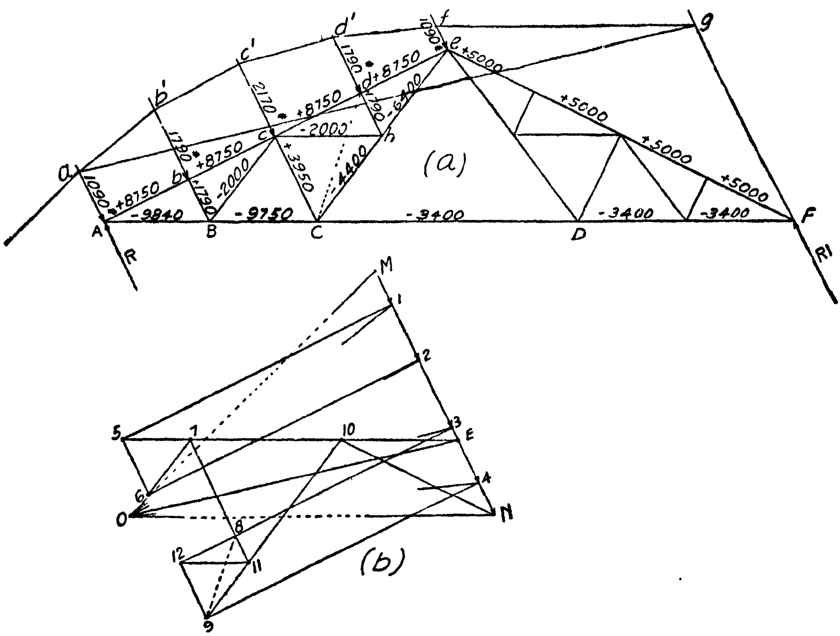


Fig. 419

Then beginning at joint *A* and passing around each joint clock-wise, as explained in the last article, the stress diagram shown at (*b*) is readily drawn. By scaling this diagram the desired stresses, which are given at (*a*), are obtained.

267. Designing of the Truss Members.—Combining the stresses shown in Figs. 418 and 419 we obtain the stresses shown in Fig. 420, which are maximum stresses, and which the truss members must be designed to transmit.

The members will be designed so that ¾" rivets can be used throughout, which requires that the flanges of the angles through which rivets pass be not less than 2½".

Bottom Chord AB. This is a tension member and the required net area of cross-section is equal to 33,040 ÷ 16,000 = 2.06 □". Two angles will make the most satisfactory sections for this member. The least size permissible is 2½"x2½", as ¾" rivets are to be used, and besides they should be at least this size to secure the necessary rigidity. So we shall use 2—⌊s $2\frac{1}{2}''$x$2\frac{1}{2}''$x$\frac{1}{4}''$ = 2.38 − 0.42 = 1.96□″, which has 0.10□″ less section than required, but this section is as near the required as is possible to obtain. This same section will be used for the bottom chord throughout, as it is the minimum.

Members ck and cB. These are tension members and the required section is only about 0.31□″, so we will use 1—⌊ $2\frac{1}{2}''$x2″x$\frac{1}{4}''$ = 1.06 − 0.21 = 0.85□″, which is the smallest angle permissible if $\frac{3}{4}''$ rivets are used.

Members ek and kC. These are tension members and the required section for *ek* is 1.0□″, and for *kC* it is 0.7□″. We will use 2—⌊s $2\frac{1}{2}''$x2″x$\frac{1}{4}''$ = 1.7□″ net, which are the smallest angles permissible.

Member cC. This is a compression member which has a length of 67″. The value of L/r should not exceed 120. Then for the allowable r we

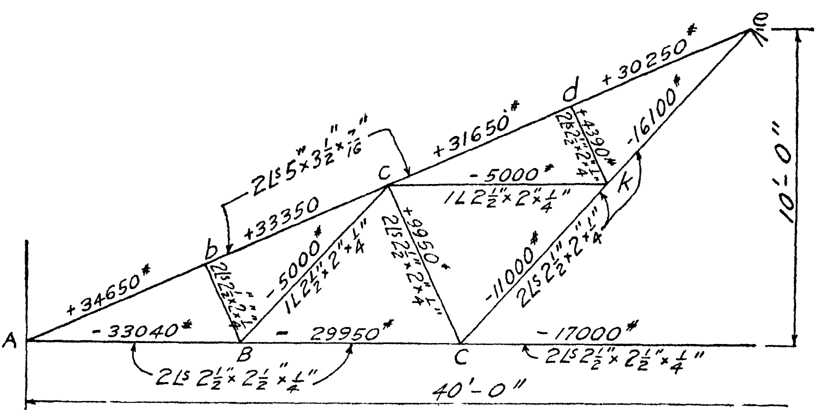


Fig. 420

have 67/120 = 0.55. Let us try 2—⌊s $2\frac{1}{2}''$x2″x$\frac{1}{4}''$ = 2.12□″. If the 2″ flanges are turned out the least radius of gyration will be 0.79. Then we have

$$16{,}000 - 70 \times \frac{67}{0.79} = 10{,}060 \text{ lbs.}$$

for the allowable unit stress. Dividing this into the stress we obtain

$$9{,}950 \div 10{,}060 = 0.99 \text{ sq. in.}$$

for the required area of cross-section, which is about one-third of the area contained in the above angles, but these angles will be used as they are as small as we can use. The $L/r = 67/0.79 = 85$, which is comparatively low.

Members dk and bB. We will make each of these members of 2—⌊s $2\frac{1}{2}''$x2″x$\frac{1}{4}''$, which are minimum size angles, the same as used for *cC*. The actual requirement, however, can be ascertained in the same manner as shown above for *cC* but, as is obvious, this work is not necessary.

Top Chord. This is a compression member. Considered in the vertical direction the length would be a panel length, and considered in the transverse direction the length should be taken as two panel lengths, owing to there being some question as to the actual support given to the chord by the purlins.

The length, considering the transverse direction, is from *A* to *c*, which is about 11′-2″ or 134″. Owing to the purlin concentration being applied between panel points there will be cross bending on the chord which it must resist in addition to the direct stress. Let us assume 2—⌊s 5″x3½″x$\frac{7}{16}$″ = 7.06□″. The 5″ legs will be placed vertically to resist cross bending. Assuming that the angles are ⅜″ apart, (b-b), the least radius of gyration, is 1.50. Then we have

$$16{,}000-70\times\frac{134}{1.50}=9{,}750 \text{ lbs.}$$

for the allowable unit stress in compression. The concentrations on the top chord due to dead and snow load are given in Fig. 417. The chord in each panel can be considered as a fixed beam. The maximum moment will occur at supports b and d. The loads in panel Ab (the same as in the other panels) are shown in Fig. 421. This load (at g is) obtained by resolving the 1,800 lbs. purlin concentration due to dead and snow load (see Fig. 417) normally to the roof and adding the concentration due to the 18 lbs. wind load, which is equal to 18x3.66x20 = 1,317 lbs. Hence, for the concentration at g we have

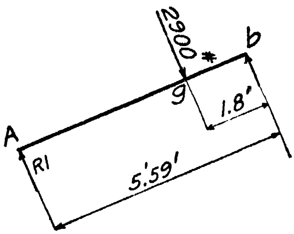


Fig. 421

$$1{,}800\times0.895+18\times3.66\times20=2{,}928, \text{ say } 2{,}900 \text{ lbs.}$$

Applying (7) of Art. 69, we have

$$M'=2{,}900\times67(2k^2-k^3-k) \qquad (1)$$

for the maximum moment which occurs at b.

$$k=\frac{1.8}{5.59}=0.32.$$

Substituting this value of k in (1), we obtain

$$M'=2{,}900\times67(2\times0.102-0.033-0.32)=28{,}215, \text{ say } 28{,}000 \text{ inch lbs.}$$

for the maximum bending moment on the top chord.

The moment of inertia of the 2—⌊s 5″x3½″x$\frac{7}{16}$″ about the gravity axis perpendicular to the long legs is 8.9x2 = 17.8, and the distance from the top of the chord down to this same axis is 1.63. Then applying C of Art. 53, we have

$$M'=28{,}000=\frac{f\times17.8}{3.37}$$

from which we obtain

$$f=5{,}300 \text{ lbs.}$$

for the maximum compressive unit stress due to cross bending. For the maximum unit stress due to direct compression, we have

$$34{,}650\div7.06\square''=4{,}910 \text{ lbs.}$$

Now adding this to the stress due to cross bending, found above, we have

$$5{,}300+4{,}910=10{,}210 \text{ lbs. (about)}$$

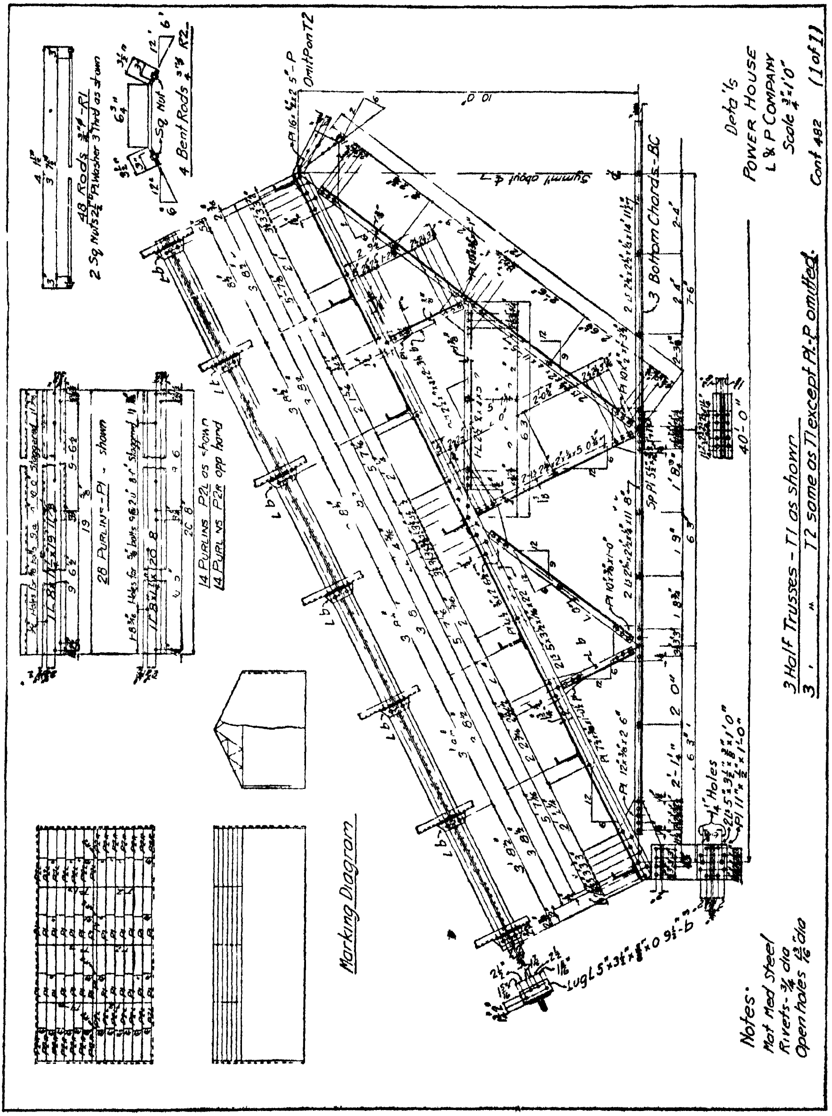

3 Half Trusses - T1 as shown
3 " " T2 same as T1 except Pl. P omitted.
Details
POWER HOUSE
L & P COMPANY
Scale 3/4"=1'0"
Cont 482 (1 of 1)
Notes:
Mat Med Steel
Rivets - 3/4 dia
Open holes 13/16 dia
Marking Diagram
3 Bottom Chords - BC
40'-0"
48 Rods 3/4"-R1
4 Bent Rods 3/4" R2
28 Purlins - P1 - shown
14 Purlins P2L as shown
14 Purlins P2R opp hand

Fig. 422

for the actual maximum unit stress on the top chord. The allowable, as found above, is 9,750 lbs. So the 2—∟s 5″x3½″x$\frac{7}{16}$″ = 7.06□″, assumed above, are about the correct size, and hence will be used.

This completes the necessary calculations and next the details can be drawn. Complete shop drawings for the building are shown in Fig. 422. The details shown here are considered to be self-explanatory. However, the student should verify the riveting.

268. Determination of Stresses in Trusses Supported upon Columns.—*Stresses due to dead and snow load* are determined just the same as if the trusses were supported upon walls, and are fully treated in Art. 265. The knee braces are ignored in the analysis.

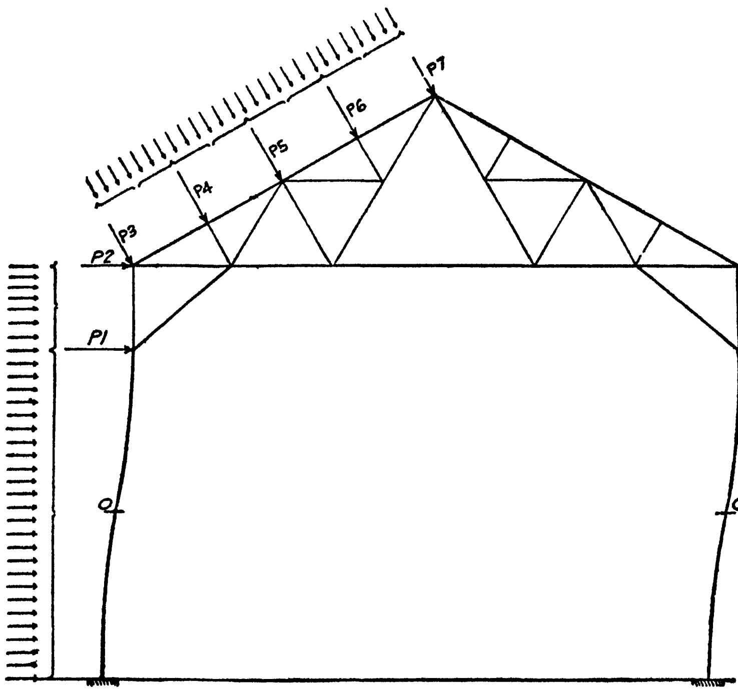


Fig. 423

Stresses Due to Wind Load. The columns are fixed to the masonry by anchor bolts, which should extend well into the masonry, and by the addition of knee braces the columns are quite firmly connected to the trusses, so that the columns can really be considered as fixed, in which case the wind pressure will cause them to bend as indicated in Fig. 423, where *O* and *O′* are the points of contra-flexure.

In determining the stresses in the truss due to wind pressure, we first locate the points of contra-flexure in the columns by applying (10) of Art. 257. After the points of contra-flexure are located, we next consider the ground moved up to that level as indicated at (*a*) in Fig. 424.

We next compute the value of the wind loads, *P*1, *P*2, etc. In computing the horizontal loads (*P*1 and *P*2) we take the wind pressure at 30 lbs per sq. ft. of vertical surface, and in computing the value of the loads ($P3$ . . . $P7$) normal to the roof we use the pressure per sq. ft. of roof surface given in Art. 261, the actual intensity of the pressure used, of course, being in accord with the slope of the roof.

After the intensities of the loads ($P1$ . . . $P7$) are computed, we next determine the horizontal and vertical components of the reactions on the columns at O and O'. If the columns are of equal moment of inertia we assume that the horizontal components are equal and that each is equal to one-half of the horizontal component of all of the loads. These horizontal components can be graphically determined by laying off the loads as shown at (b), drawing the vertical AP, and the horizontal PB. Then the horizontal component in each case is represented by one-half of the line PB. Let H represent this component. To determine the vertical components of the reactions at O and O' we complete the ray diagram at (b), then construct the corresponding equilibrium polygon z n-o . . . y as shown at (a). Then by prolonging the segments zn and uy to intersect at I and from I drawing a line parallel to the line AB, which represents the resultant F of all of the loads, we obtain the line of action of the resultant, and prolonging this resultant to intersection at N with line OO', and then laying off $O'M$ equal to the vertical component ($=AP$) of all of the loads, and drawing the line MO and the ordinates NT and TU, we have the vertical components $V2$ of the reaction at O' represented by the line NT and the vertical component $V1$ of the reaction at O by the line TU.

The line MO is really an influence line for the vertical component of reaction at O'. To show that this is true, let us assume that the resultant F passes through point O'. Then, as is readily seen, the total vertical component of all of the loads would be taken by column kO', in which case the vertical component on column Oc would be zero, and if the resultant F intersected the line OO' midway between the two columns the vertical components of the reactions at O and O' would be equal and each to one-half of the ordinate $O'M$. So, evidently, the line MO is the influence line for the vertical component of the reaction at O'. That is, if the resultant F intersects the line OO' at any point C between O and O' the vertical components of the reaction at O' and O are represented by the ordinate CD and DG, respectively.

If the resultant F intersects the line OO' at any point between O and O' the vertical components of the reactions will be positive; but if it intersects at any point E to the right of O' the vertical component at O would be negative, and the vertical component at O' positive. In that case the ordinate EK (MO prolonged to K) would represent the positive vertical component of the reaction at O', while MS would represent the negative vertical component of the reaction at O. This last construction is readily proven: Taking moments about O we have

$$V \times OE = V2 \times OO',$$

from which we obtain

$$V2 = (V \times OE) \div OO'.$$

But from similar triangles we have

$$\frac{OE}{OO'} = \frac{EK}{O'M} = \frac{EK}{V},$$

from which we obtain

$$EK = (V \times OE) \div OO',$$

and therefore

$$EK = V2.$$

After the horizontal and vertical components of the reactions at O and O' are thus determined, the actual resultant reaction at each of the

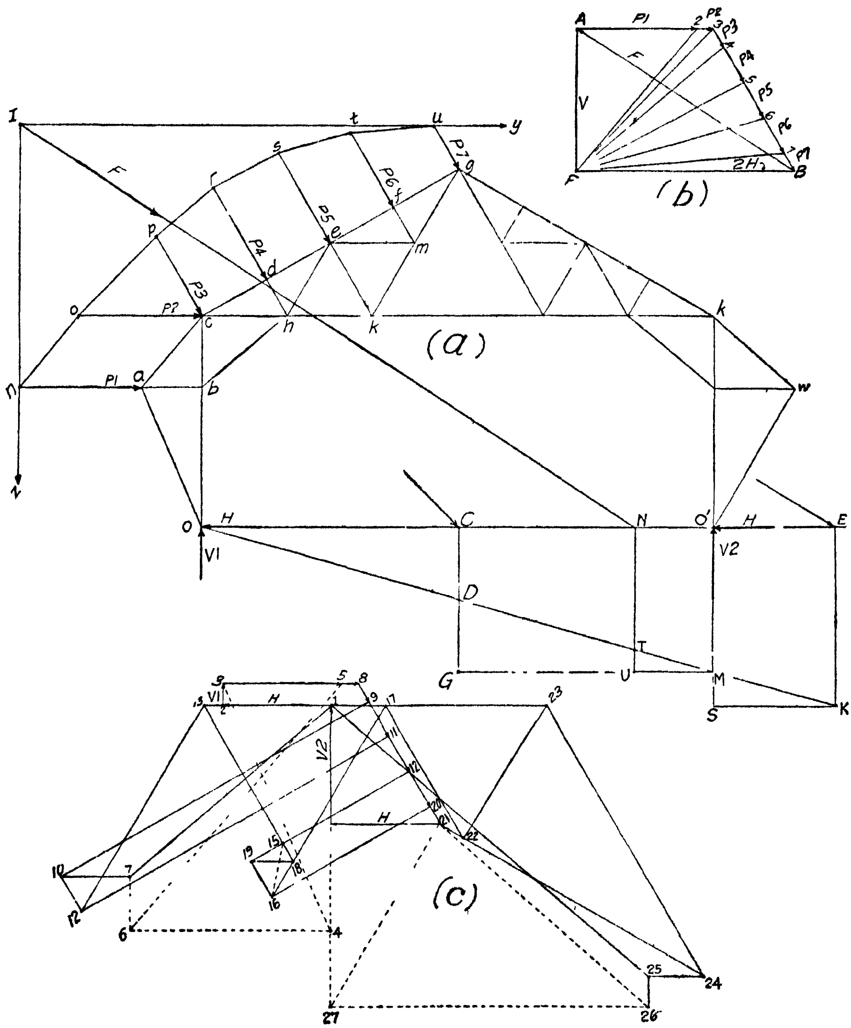


Fig. 424

points is readily determined, but it is really just as convenient to use the components in determining the stresses in the truss.

Having determined the horizontal and vertical components of the reactions at O and O' in the above manner, we can readily determine graphically the stresses in the truss due to the wind loads by first drawing the auxiliary frames Oac and $O'wk$ and beginning at either O or O' and passing around each joint in the same direction. The stress diagram at (c) is obtained by starting at O and passing around each joint clock-wise. The stresses obtained in the auxiliary trusses, including the stresses in the columns, are to be ignored, but the stresses in the knee braces and truss proper are not affected by adding the auxiliary trusses and hence they are correct stresses.

Buildings with Lean-tos. In case the lean-tos are of simple construction, as indicated at (f), Fig. 425, each lean-to column should be considered as acting only as a simple beam in resisting the wind pressure, in which case the wind pressure on the side of a lean-to would be transmitted equally to the top and bottom of the columns. Then, considering the case shown in Fig. 425, the pressure on the side AB would be transmitted equally to points A and B, producing the loads $P1$ and $P2$ (each = 3,000 lbs.). The horizontal pressure of the loads from B to C would be transmitted directly to the column at C. The horizontal pressure represented as $F(=8{,}600$ lbs.) is obtained by constructing the force diagram shown at (b). Part of the force F is transmitted to point D and part to E. The results obtained will be accurate enough if we consider the part DE of the main column as a simple beam. Then taking moments about E we have

$$f=\frac{Fk}{h}$$

for the part of F transmitted to point D. Then adding this force f to the 3,200 lbs. at D, we obtain the 9,650 lbs. force shown at (d). The part of F transmitted to D can also be determined by constructing the influence line shown at (e).

After the force or load at D (all other loads are assumed as known) is determined, the ray diagram shown at (e) can be drawn and the corresponding equilibrium polygon a-b-c-d-e-f-g-h . . . m at (d) constructed. Then prolonging lm and ab to intersection at I and drawing from I a line parallel to the resultant GK we obtain $O''I$ for the line of action of the resultant of all the wind loads affecting the roof.

By laying off $O'P(=V=8{,}400)$ equal to the vertical component of all of these loads, and drawing ON and NO'', we have the vertical component $V2$ of the reaction on the column at O' represented by the distance NO'', and the vertical component $V1$ of the reaction on the column at O represented by the distance MP. $V2$ is positive and $V1$ negative. Each of the horizontal components H on each column is equal to one-half of the distance SK.

Now, having determined all of the loads and the horizontal and vertical components of the reactions at O and O' we can determine the stresses in the roof truss in exactly the same manner as shown above for the building without lean-tos.

In case the lean-tos are of the construction shown in Fig. 426, the wind pressure will cause the columns to bend as indicated. If the shear, indicated as $H1$, $H2$, etc., and the direct stress on the columns at the points of contraflexure—that is, the horizontal and vertical components of the reactions at these points—were determined, there would be no difficulty in determining the stresses in the main and lean-to trusses. The actual values of the horizontal and vertical components of the reactions at the points of contra-flexure depend not only upon the relative stiffness of the columns but upon the stiffness

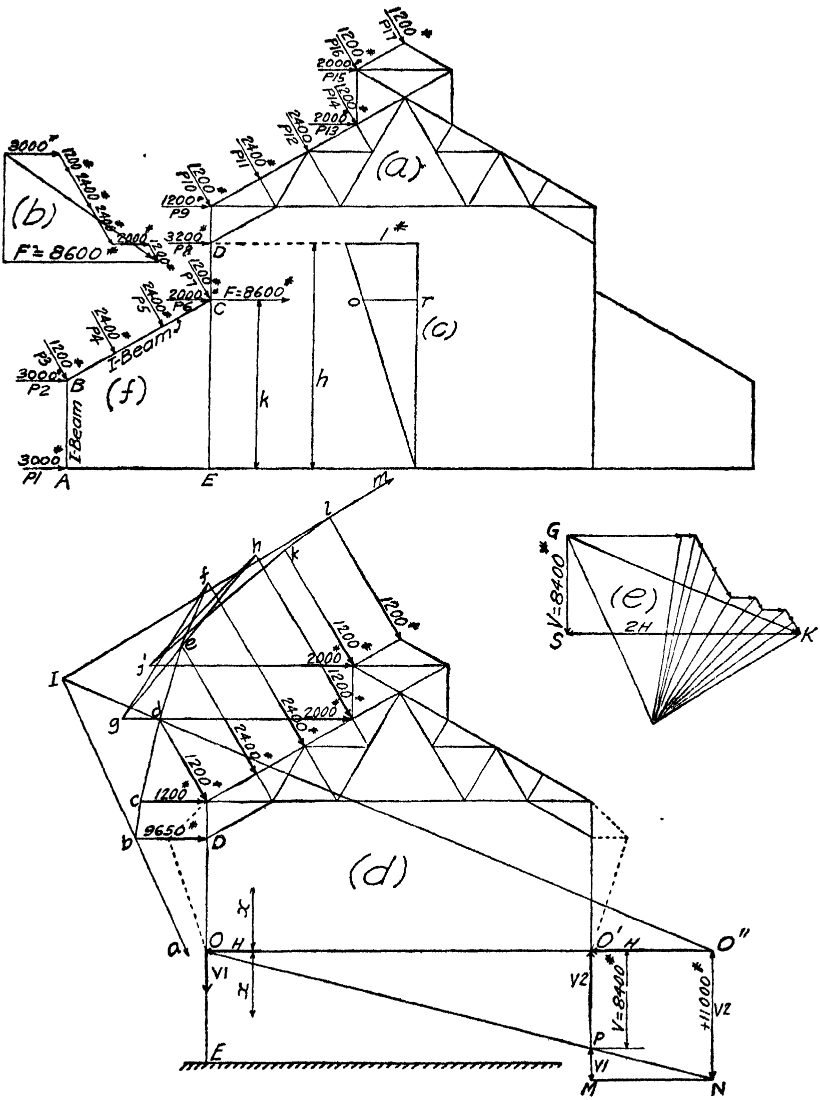

(a)
(b)
(c)
(d)
(e)
(f)
F = 8600*
F=8600*
I-Beam
P1
P2
P3
P4
P5
P6
P7
P8
P9
P10
P11
P12
P13
P14
P15
P16
P17
A
B
C
D
E
h
k
V=8400*
2H
G
S
K
O
O'
O''
H
V1
V2
M
N
P
+11000*
9650*

Fig. 425

of the trusses as well. But as this relative stiffness cannot be determined at all until the structure is fully designed, and then only to a limited extent, it is evident that the method used for determining the wind stresses must necessarily be based to some extent upon practical assumptions.

By considering the main truss and the parts of the columns above the points of contra-flexure, O and O', as an independent structure, shown at (*a*), Fig. 427, we can determine the horizontal and vertical components of the reactions at O and O' by drawing the ray diagram at (*b*) and the corresponding equilibrium polygon at (*a*) and the influence line OJ, all of which has been previously explained.

Considering the lean-to and the part of the main column below the point of contra-flexure O as an independent structure, shown at (*c*), we know H' and V' which are applied at O, as they are equal and opposite, respectively,

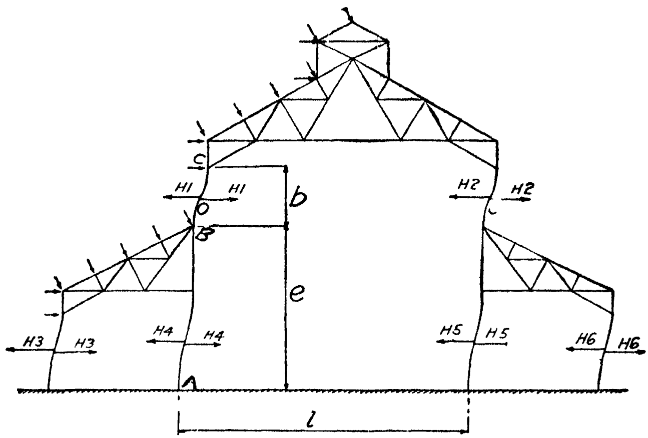


Fig. 426

to the H' and V' found at (*a*), and we also know the wind loads, but the components of the reactions at the points of contra-flexure S and S' and likewise the force H'' are unknown. The force H'' is the reaction from the part of the structure above O due to the wind loads on the lean-to, and hence is equal to the part of these loads that is transmitted through the main truss to the other side of the building. The actual value of H'' depends upon the rigidity of the columns and trusses. This rigidity can be ascertained only by tedious analysis of the structure after it is designed. However, a sufficiently accurate value of H'' can be obtained from the following empirical formula:

$$H''=H\left(\frac{e}{b+e}-\frac{L}{150}\right)\qquad(1)$$

where H represents the horizontal component of the wind loads on the lean-to and L the length of the main roof truss and e the distance from the ground to the top of the lean-to and b the distance from the top of the lean-to to the knee brace (see Fig. 426).

The values of e, b, and L are known (always) and H can be determined by constructing the force polygon CDE shown at (*e*). By substituting in (1)

H'' can be determined and then constructing the ray diagram at (*e*), and drawing the corresponding equilibrium polygon at (*c*) and the influence line ST, we obtain the vertical components $V1$ and $V2$ of the reactions at S and S', and then the only unknown forces remaining are the horizontal components of these reactions which are represented at $H3$ and $H4$. These forces ($H3$ and $H4$) would be considered to be equal if the two columns were of equal moment of inertia, but as they never are we must assume relative values.

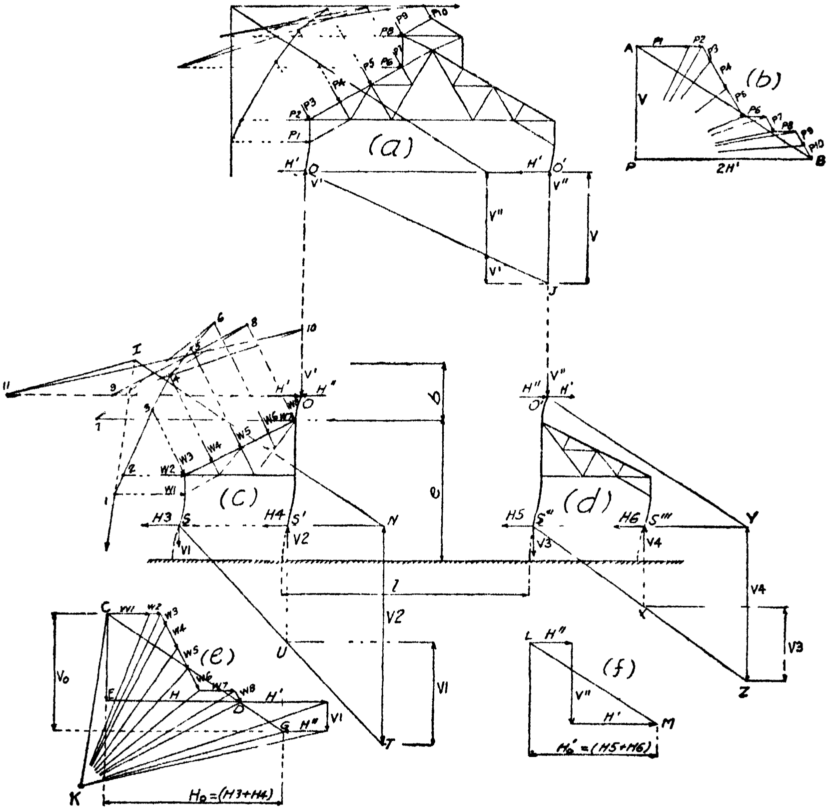


Fig. 427

We shall assume the lean-to column to be one-fourth as rigid as the main column. Then as $H_o = H3 + H4$, we have

$$H3 = \frac{1}{4}H_o, \text{ and } H4 = \frac{3}{4}H_o,$$

where H_o represents the total horizontal component of all the forces on the structure shown at (*c*), including those at O.

Considering the lean-to and part of the main column shown at (*d*) as an independent structure, the only applied forces are H'', V'', and H' at O'. These forces having been previously determined, we can determine the vertical component of the reactions at the points of contra-flexure, S'' and S''', by constructing the force polygon at (*f*) and drawing from O' the line $O'Y$ parallel to the resultant LM and then the influence line $S''Z$.

Assuming the rigidity of the lean-to column to be one-fourth of that of the main column, we have

$$H6 = \frac{1}{4}H'_o \text{ and } H5 = \frac{3}{4}H'_o.$$

We now have all of the components of the reactions at the points of contra-flexure determined, and we can now proceed with the determination of the stresses in the structure.

By considering the part of the structure above the points of contra-flexure O and O' and adding the auxiliary trusses to the sides as indicated at (*a*), Fig. 428, the stresses in knee braces and main truss can be graphically determined very readily by beginning at either O or O' and passing in the same direction around each joint.

By considering the part of the structure at (*c*), Fig. 428, as an independent structure and adding the auxiliary frames, shown dotted, the stresses in the knee brace and lean-to truss can be graphically determined very readily by

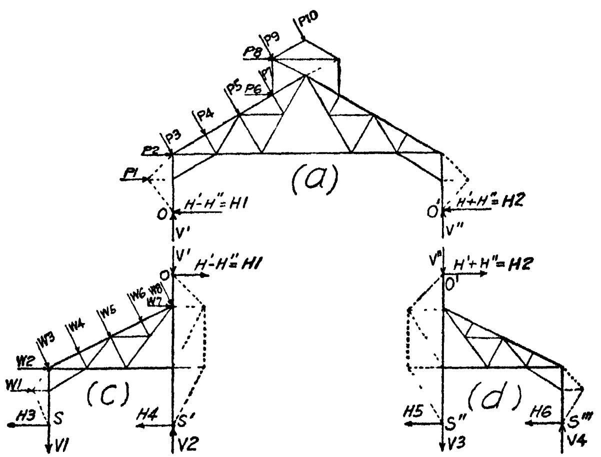


Fig. 428

beginning at S and passing in the same direction around each joint. Likewise the stresses in the knee brace and the lean-to truss shown at (*d*), Fig. 428, can be determined.

269. Determination of the Stresses in the Columns.—The columns are subjected to direct stress and cross bending similar to the case of the end posts in bridges. (See Art. 180.)

After the horizontal components of the reactions at the points of contra-flexure are determined, the bending moment at any point in a column is readily computed. For example, the bending moment at any point in column DE (Fig. 425) is equal to Hx, being a maximum at D and E.

Complete Design of a Mill Building

270. Data.—

Nature of building, machine shop.
Length of building = 11 bays @ 20′-0″ = 220′-0″.
Width of main shop = 60′-0″ c.c. of main columns.
Width of lean-tos = 24′-0″ (lean-to on each side of building).
Roof covering, No. 20 corrugated iron.
Allowable intensities on metal—
Tension......16,000 lbs. per sq. in.
Compression..16,000 − 70 L/r where L is the length of member in ins. and r the least radius of gyration of the cross-section in ins.
Crane........................20-ton overhead in main shop.

271. Preliminary.—In starting the designing the very first thing to do is to draw a sketch of the cross-section of the building as shown in Fig. 429, showing the general requirements as to type and pitch of roof, spacing of purlins, height of lean-to, clearance for crane, height of clear story, etc.

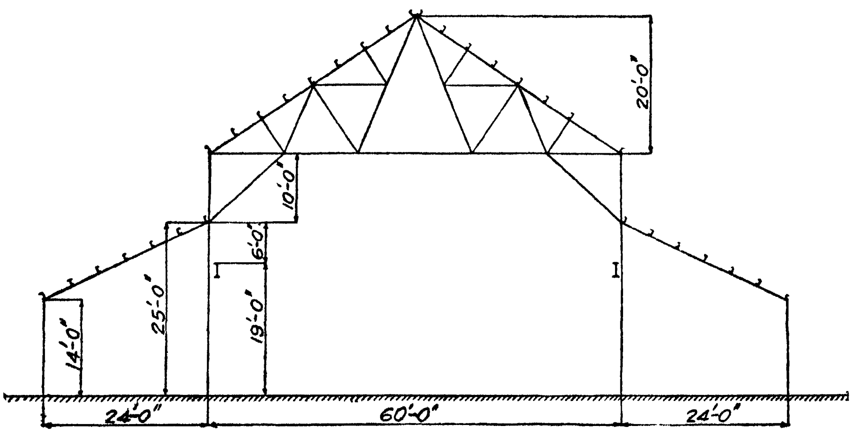


Fig. 429

The roof covering is to be of corrugated iron. The pitch of the main roof will be $\frac{1}{3}$ and the lean-tos about $\frac{1}{4}$ (see Art. 258). The maximum spacing of the purlins is governed by the kind of roof covering. In this case No. 20 corrugated iron will be used and, as stated in the Carnegie handbook, the maximum span must not be more than 6 ft., and less is preferable. In this case the purlins will be spaced about 4′-6″ centers. The distance from the knee brace to the crane girder, as specified in Fig. 414, is 6′-2″.

272. Designing of the Purlins.—The weight of the corrugated iron, as per Art. 259, is 1.9 lbs. per sq. ft. of roof surface. The weight of the purlin will be assumed to be 12 lbs. per ft. of length. The snow load (frozen snow and ice) will be taken as 10 lbs. per sq. ft. of roof surface and the wind load as 22 lbs. per sq. ft. of roof surface as per Art. 261 for roof having $\frac{1}{3}$ pitch.

Then for the total load on each intermediate purlin per ft. of length we have the following:

$$
\begin{aligned}
&\text{Dead load} = 1.9\times 4.5+12 = 20.5 \\
&\text{Snow load} = 10 \times 4.5 \quad = 45.0
\end{aligned}\Bigg\} 65.5
$$

$$
\text{Wind load} = 22 \times 4.5 \quad = 99.0
$$

$$
\overline{164.5}, \text{ say } 165 \text{ lbs.}
$$

Then for the maximum moment on the purlin we have

$$
M = \tfrac{1}{8}\times 165\times \overline{20}^2\times 12 = 99{,}000 \text{ inch lbs.}
$$

Dividing this moment by 16,000 we obtain 99,000 ÷ 16,000 = 6.2 for the section modulus, which calls for a 7″x12.25# channel or an 8″x11.25# channel. Use 1—[8″x11.25#] for each purlin.

273. Designing of the Lean-to Rafters.—The wind load acts perpendicularly to the rafter while the dead and snow load acts vertically. So, for the concentration on the rafter at each intermediate purlin, using the same

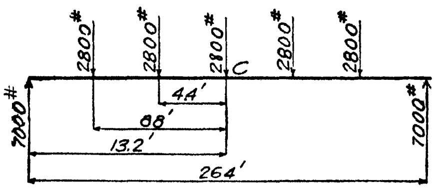


Fig. 430

unit loads as used in the last article, except the wind load which is taken as 18 lbs. (the lean-to roof has $\frac{1}{4}$ pitch), we have:

$$
\begin{aligned}
\text{Dead load} &= 20.5\times 20\times 0.908 = 372 \text{ lbs.}\\
\text{Snow load} &= 45.0\times 20\times 0.908 = 817 \text{ lbs.}\\
\text{Wind load} &= (18\times 4.5)\times 20 \quad = \underline{1{,}620} \text{ lbs.}\\
& \qquad\qquad 2{,}809 \text{ lbs., say } 2{,}800 \text{ lbs.}
\end{aligned}
$$

Then applying this concentration at each purlin and considering the rafter as a simple beam we have the case fully represented in Fig. 430. The maximum moment on the rafter due to these loads, as is obvious, will occur at C, the center of span. So taking moments about C we have

$$
M = [7{,}000\times 13.2 - (2{,}800\times 8.8) - 2{,}800\times 4.4]12 = 665{,}280 \text{ inch lbs.}
$$

for the maximum moment on the rafter due to concentrated loads. Assuming the rafter to weigh 42 lbs. per ft. of length we have

$$
M' = \tfrac{1}{8}\times 42\times \overline{26.4}^2\times 12 = 43{,}900 \text{ inch lbs.}
$$

for maximum moment on the rafter due to its own weight. Adding together the above moments we have

$$
665{,}280 + 43{,}900 = 709{,}180 \text{ inch lbs.}
$$

for the total maximum moment on the lean-to rafter. Dividing this by 16,000 we obtain 44.3 for the section modulus which calls for a 12″x40# or 15″x42# I. We will use the 15″x42# I for each lean-to rafter.

274. Determination of Stresses in Trusses Due to Snow and Dead Load.—The snow and dead load per ft. of purlin as given in Art. 272 is 65.5 lbs. Then for the concentration on the truss at each intermediate purlin we have 65.5x20 = 1,310 lbs. Then, as two of these concentrations are transmitted to each panel point of the top chord of the truss (see Fig. 429), we obtain 1,310x2 = 2,620 lbs. for the part of the panel load due to snow, roof covering, and purlins. The weight of the roof truss as per Art. 259 is 3.8 lbs. per sq. ft. of roof surface. Then, for the part of the panel load due to the weight of the roof truss we have 3.8x9x20 = 684 lbs. Now, adding this to the part due to the roof load we have 2,620+684 = 3,304, say 3,300 lbs. (1,800 lbs. of this is due to snow) for the panel load due to snow and dead load, and hence the snow and dead load on the truss will be as shown at (*a*), Fig. 431. By beginning at *A* and constructing the stress diagram shown at (*b*) the stresses shown on the truss are obtained. Multiplying each of these stresses by 1,500/3,300 the stresses marked *D* which are due to dead load alone are obtained.

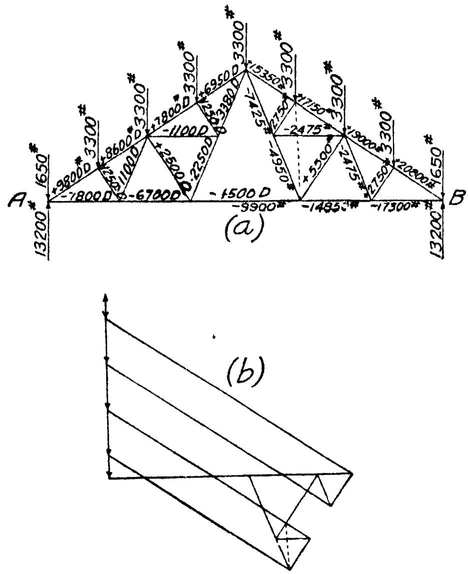


Fig. 431

275. Determination of Stresses in Trusses Due to Wind Load. The wind load per sq. ft. of roof surface is 22 lbs. (Hutton), as per Art. 261. Then for the panel load on the roof we have 22x9x20 = 3,960 lbs. and hence the wind loads on the truss will be as shown in Fig. 432. The horizontal load at *a* is equal to 30x5x20 = 3,000 lbs. The load at *b* will be the same as at *a* plus the horizontal component of the wind load from the lean-to. The horizontal load at the top and bottom of the lean-to column is equal to 30x7x20 = 4,200 lbs., as shown at (*b*). The pitch of the lean-to roof is about $\frac{1}{4}$ ($\frac{1}{4}\frac{1}{8}$), so the wind pressure per sq. ft. of roof is 18 lbs., as per Art. 261. Then the concentration on the rafter at each intermediate purlin is equal to 18x 4.5x20 = 1,620 lbs. and hence the wind loads on the lean-to roof will be as shown at (*b*). Then by constructing the force polygon at (*c*) we obtain

$$F = 8{,}250 \text{ lbs.}$$

for the horizontal component of the forces on the lean-to. Then adding this to the 30x5x20 = 3,000 lbs. from the clear story we obtain 3,000+8,250 = 11,250 lbs. for the total wind load at *b*. Now having all the wind loads determined we can proceed with the determination of the stresses in the knee braces and truss due to same.

The points of contra-flexure in each column will be considered as being midway between the bottom of the column and knee brace. By constructing the ray diagram shown at (*d*), and the corresponding equilibrium polygon *c-d-e* . . . *n-m* at (*a*), and prolonging the segments *cd* and *nm* to intersection at *I*, and drawing from *I* a line parallel to the resultant *AB*, we obtain the

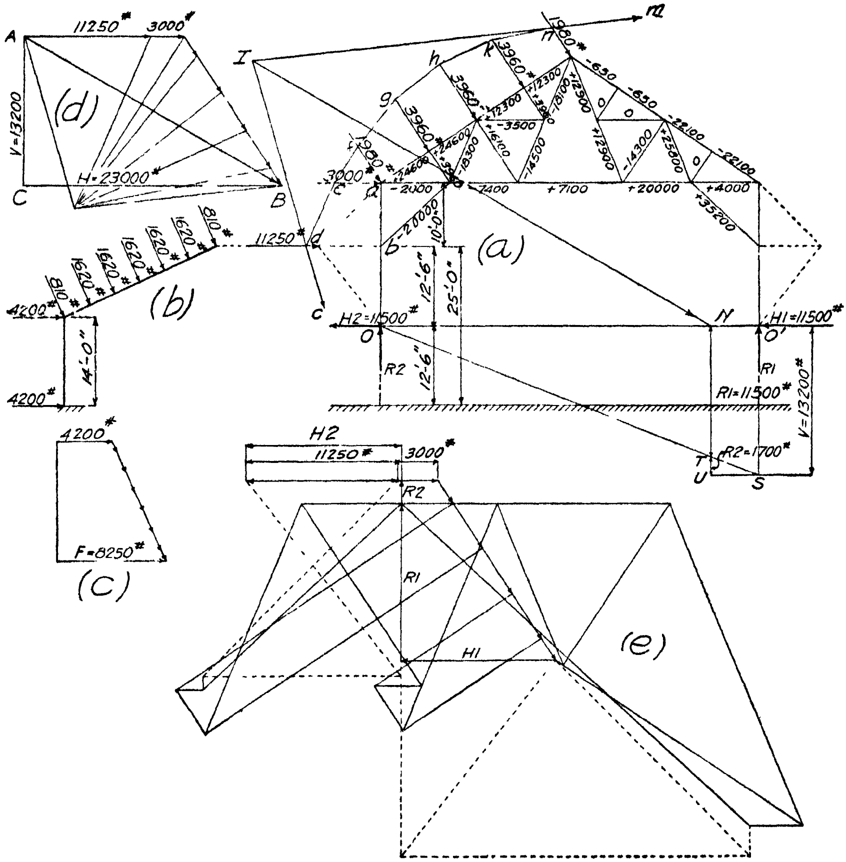


Fig. 432

line of action of the resultant of all of the forces, which intersects the horizontal line *OO′* at *N*.

Then laying off *O′S* equal to the vertical component of all of the forces and drawing the influence line *OS* and dropping a vertical from *N* we have the vertical component of the reaction at *O′* given by the ordinate *NT* and the vertical component of the reaction at *O* by the ordinate *TU*. The horizontal components *H*1 and *H*2 of the reactions at *O′* and *O* are equal and each is equal to one-half of the horizontal component of all the forces, which is represented at (*d*) by the line *CB*.

Now having the components of the reactions at the points of contraflexure, O and O', determined, the stresses in the knee braces and truss due to wind loads, which are given at (*a*), are determined by adding the auxiliary trusses, shown dotted, and constructing the stress diagram shown at (*e*). The diagram shown at (*e*) is obtained by beginning at O and passing around each joint clock-wise. The diagram at (*e*) should be verified by the student.

276. Designing of the Truss Members.—The maximum stresses in each truss member (as given in Figs. 431 and 432), and also the approximate lengths of the members, are shown on the one diagram in Fig. 433. Having these data we can proceed with the designing of the members.

Member KF. This member is subjected to 35,200 lbs. compression and 20,000 lbs. tension. Let us assume 2—⌊s $5''\text{x}3\frac{1}{2}''\text{x}\frac{5}{16}''=5.12$ □″ as section.

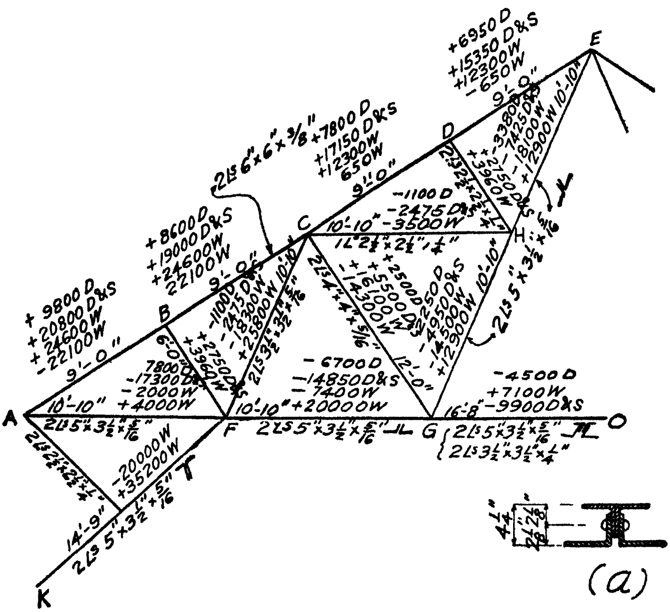


Fig. 433

Assuming the 5″ legs vertical and ½″ apart we have $L/r=177/1.50=118$, and hence for the allowable compressive stress we have $16{,}000-70\text{x}118=7{,}740$ lbs. per sq. in. Then we have $35{,}200\div7{,}740=4.5$ □″ for the area required for compression, which is 0.61 □″ less than the assumed section.

For tension the net area of cross-section of the assumed section is $5.12-0.55=4.57$ □″. The actual area required for tension is equal to $20{,}000\div16{,}000=1.25$ □″. From the above it is seen that the assumed section is somewhat larger than required, but owing to the fact that it is about as satisfactory as can be obtained (the value of L/r being about correct) it will be used.

Members AF and FG. The member *AF* is subjected to 19,300 lbs. tension while member *FG* is subjected to 22,250 lbs. tension and 18,800 lbs. compression. The truss is held transversely at *G* and *A* by struts, and as member *FG* is in compression it is necessary to consider (horizontally) *AG* as one member subjected to the 13,300 lbs. compression. As the stresses are small the designing is simply a matter of obtaining the proper value of L/r. Let us assume 2—⌊s $5''\text{x}3\frac{1}{2}''\text{x}\frac{5}{16}''=5.12\square''$; the 5″ legs horizontal and the vertical legs $\frac{1}{2}''$ apart. Then $L/r=260/2.44=106$ in the horizontal plane and $130/1.03=126$ in the vertical plane. The first value (106) of L/r is quite satisfactory, but the last value is a little high (125 being the limit), but as the angles are fixed more or less at the joints in the vertical plane the above assumed section will be used. There should be enough rivets in these angles at points A and G to develop the angles in tension in order to obtain rigidity.

Members BF and DH. Each of these members is subjected to 6,710 lbs. compression. This stress is so small that the designing is simply a matter of obtaining the proper value of L/r. Let us assume 2—⌊s $2\frac{1}{2}''\text{x}2\frac{1}{2}''\text{x}\frac{1}{4}''=2.38\square''$ for section. Then $L/r=72/0.78=92$, which is quite satisfactory, and hence the assumed section will be used.

Member CF. This member is subjected to 20,775 lbs. tension and 24,700 lbs. compression. Let us assume 2—⌊s $3\frac{1}{2}''\text{x}3\frac{1}{2}''\text{x}\frac{5}{16}''=4.18\square''$ as section. $L/r=130/1.08=120$. Then for the allowable compressive stress we have $16{,}000-70\text{x}120=7{,}600$ lbs. Dividing this into the compressive stress we obtain $24{,}700\div7{,}600=3.25\square''$ for the area required for compression. For the net area of cross-section required for tension we have $20{,}775\div16{,}000=1.30\square''$. From the above it is seen that the assumed section is larger for section than required, but as the value of L/r is about as great as is permissible the assumed section will be used.

Member CG. This member is subjected to 11,800 lbs. tension and 21,600 lbs. compression. Let us assume 2—⌊s $4''\text{x}4''\text{x}\frac{5}{16}''=4.80\square''$. Then $L/r=144/1.25=115$, and hence for the allowable compressive stress we have $16{,}000-70\text{x}115=7{,}950$ lbs. per sq. in. Dividing this into the compressive stress we obtain $21{,}600\div7{,}950=2.7\square''$ for the area of cross-section required for compression. For the net area of cross-section required for tension we have $11{,}800\div16{,}000=0.73\square''$. From the above it is seen that the assumed section, as far as area is concerned, is larger than required, but as L/r is of about the correct value the assumed section will be used.

Member CH. This member is subjected to 5,975 lbs. tension. The area of cross-section required is about $0.37\square''$. We shall use 1—⌊ $2\frac{1}{2}''\text{x}2\frac{1}{2}''\text{x}\frac{1}{4}''=2.38-0.44=1.94\square''$ net for section. This is about as small an angle as can be used and the member be in harmony with the other members of the truss.

Members GH and HE. The member GH is subjected to 19,450 lbs. tension and 10,650 lbs. compression, while member HE is subjected to 25,525 lbs. tension and 9,520 lbs. compression. As the truss is not supported transversely at H it is necessary to design EG as one compression member. Let us assume 2—⌊s $5''\text{x}3\frac{1}{2}''\text{x}\frac{5}{16}''=5.12\square''$ for section. The maximum radius of gyration is 2.44 and the least is 1.03. Then $L/r=260/2.44=106$ in one case and $130/1.03=126$ in the other. For the maximum allowable compressive unit stress we have $16{,}000-70\text{x}126=7{,}180$. Dividing this into the maximum compressive stress we obtain $10{,}650\div7{,}180=1.48\square''$ for the required area of cross-section. For the required area of cross-section for tension we have $25{,}525\div16{,}000=1.59\square''$ net. From the above it is seen that the assumed section is larger than it need be, as far as section is concerned, but that the value of L/r is about as large as allowed, so the assumed section will be used.

Member GO. This member is subjected to 9,900 lbs. tension and 2,600 lbs. compression. These stresses are so low that the designing of the member is simply a matter of obtaining the correct value for L/r. The length of the member is 200 ins. and hence the radius of gyration of the member must not be less than $200/125 = 1.60$. By using 2—∟s 5″x3½″x$\frac{5}{16}$″ and 2—∟s 3½″x 3½″x¼″ riveted together as shown at (*a*), Fig. 433, we obtain about the correct radius and hence this section will be used.

Top Chord AB . . . E. This member should be of the same section throughout. It is subjected to a maximum compression of 43,600 lbs. and to a maximum tension of 22,100 − 8,600 = 13,500 lbs. The purlins hold the truss transversely more or less, but as there is some question as to the rigidity thus obtained the top chord will be considered unsupported transversely for half of its length, which is about 18 ft. or 216 ins. The length as regards the vertical direction is 9 ft. or 108 ins. Then, as regards the transverse direction, the radius of gyration must not be less than $216/125 = 1.73$, and as regards the vertical direction the radius of gyration must not be less than $108/125 = 0.86$.

There is a purlin concentration midway between panel points of $\frac{1}{2}(3{,}960 + 3{,}300 \times 0.83) = 3{,}350$ lbs. (see Figs. 431 and 432) which causes a bending moment on the chord (considered as a fixed beam) of $\frac{1}{8} \times 3{,}350 \times 108 = 45{,}225$ inch lbs.

Now, it is seen that the section of the top chord must be sufficient to transmit the direct stress and the cross bending given above and that the radii must not be less than indicated above.

Let us assume 2—∟s 6″x4″x½″ = 9.50□″ for section; the 6″ legs vertical and ½″ apart. The radii are about 1.71 and 1.91, which are satisfactory values. For the allowable compression we have $16{,}000 - 70 \times 216/1.70 = 7{,}100$ lbs. per sq. in. The actual direct stress is $43{,}600 \div 9.5 = 4{,}600$ lbs. per sq. in. and the stress due to cross bending is $(45{,}225 \times 1.99) \div 34.8 = 2{,}590$ lbs. per sq. in. Now adding these two stresses together we obtain 7,190, which is larger than the allowed, so the assumed section is too small. Let us try 2—∟s 6″x6″x⅜″ = 8.72□″. The radii are about 2.6 and 1.8. Then L/r in one case is equal to $216/2.6 = 83$, and in the other $108/1.8 = 60$. Then for the allowable unit stress we have $16{,}000 - 70 \times 83 = 10{,}190$.

For the stress due to direct compression we have $43{,}600 \div 8.72 = 5{,}000$ lbs. per sq. in., and for the stress due to cross bending we have $(45{,}225 \times 1.64) \div 30.78 = 2{,}410$ lbs. Now adding these two stresses together we have $5{,}000 + 2{,}410 = 7{,}410$ lbs., which is 2,780 lbs. less than the allowable, so the 2—∟s 6″x 6″x⅜″ will be satisfactory for the top chord. We now have all of the truss members designed and the sections can be written on the diagram as shown in Fig. 433.

277. Designing of the Lean-to Columns.—We assume that these columns are not fixed. The load on each purlin due to snow and dead load, as given in Art. 272, is 65.5 lbs. per ft. of purlin. The vertical component of the wind load is equal to $(18 \times 4.5) \times 0.908 = 73.5$ lbs. per ft. of purlin. Then for the vertical load on the rafter at each intermediate purlin we have $(65.5 + 73.5) \times 20 = 2{,}780$ lbs. Then the concentration on the lean-to column due to the roof load is equal to $2{,}780 \times 3 = 8{,}340$ lbs. Adding the weight of one-half of the rafter we have $8{,}340 + (13.2 \times 42) = 8{,}894$, say 8,900 lbs., for the total direct load on the column.

The wind load acting perpendicularly to the column is (considering it uniformly distributed) equal to $30 \times 20 = 600$ lbs. per ft. of column. Then for the moment on the column due to this load we have $\frac{1}{8} \times 600 \times \overline{14}^2 \times 12 = 176{,}400$ inch lbs. Let us assume an 8″x34# H-beam (Carnegie) for section, in which case $L/r=168/1.87=89.8$. Then for the allowable unit stress we have $16,000-70x89.8=9,714$ lbs. and for the actual direct unit stress we have $8,900 \div 10=890$ lbs. For the maximum unit stress due to cross bending we have

$$f=(176,400\times 4)\div 115.4=6,110 \text{ lbs.}$$

Now adding the direct unit stress to this bending stress we have $6,110+890=7,000$ lbs. for the maximum unit stress on the column, which shows that the assumed H-beam is larger than required, but as this beam seems to be the most satisfactory section obtainable it will be used throughout for the lean-to columns.

278. Designing of the Crane Girders.—The spacing and load on the wheels of a 20-ton crane having a 60-ft. span are given in Fig. 414. The maximum moment on the crane girder will occur when the wheels are in the position shown at (*a*), Fig. 434. Taking moments about *B* the reaction shown at *A* is obtained. Then taking moments about wheel *C* we have

$$28,046\times 7.58\times 12=2,551,000 \text{ inch lbs.}$$

for the maximum moment on the crane girder due to the crane. Assuming the crane girder to weigh 80 lbs. per ft. and the rail 60 lbs. per yd., making in all 100 lbs. per ft. of girder, we have $\frac{1}{8}x100x20^2x12=60,000$ inch lbs. for the maximum bending moment on the girder due to the weight of the girder and rail. Adding this moment to the moment due to the crane, we have $2,551,000+60,000=2,611,000$ inch lbs. for the total maximum bending moment on the crane girder. Dividing this by 16,000 we obtain 163 for the section modulus which calls for a 24″x80# I, which will be used for each girder.

The maximum reaction on the crane girder will occur when the crane wheels are in the position shown at (*b*), Fig. 434. Taking moments about *E* we obtain the reaction 56,100 lbs. shown at *D*, which is the maximum end shear on the crane girder due to the crane wheels. The end shear due to the weight of the crane girder and rail is equal to $100x20/2=1,000$ lbs. Then for the total end shear or reaction on the crane girder we have $56,100+1,000=57,100$ lbs.

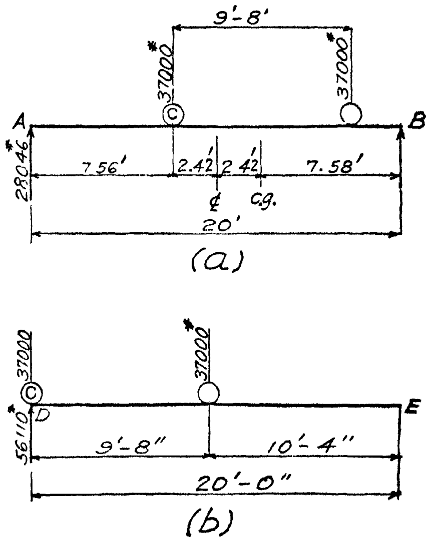


Fig. 434

279. Designing of the Main Columns.—The load applied to the top of each intermediate column due to snow and dead load is 13,200 lbs., as given in Fig. 431. The load from the lean-to rafter due to snow and dead load is equal to $65.5x20x3=3,930$, say 4,000 lbs. The direct stress due to the wind load is equal to the vertical component of the reaction at the point of contra-flexure, the maximum of which is given in Fig. 432 as 11,500 lbs. Then for maximum direct stress on the column above the crane girder we have 13,200+4,000+11,500=28,700 lbs. This load comes on the leeward column. As is seen from Fig. 432, the maximum moment on the column due to wind load occurs at the knee brace and is equal to (H2x12.5) 12=11,500x 12.5x12=1,725,000 inch lbs. Let us assume the part of the column above the crane girders to be a built I-section composed of 4—L s 6″x3½″x$\frac{9}{16}$″ and a web 18″x⅜″. The distance from the truss down to the crane girder, which is 16 ft. or 192 ins. (see Fig. 429), will be taken as the length of this column. The least radius of gyration of the section is 2.5. Then we have $L/r=$ $192/2.5=76.8$, and hence for the allowable unit stress we have 16,000−(70x76.8)= 10,624 lbs. The moment of inertia of the section with reference to the axis perpendicular to the web is 1,576. Then for the maximum unit stress due to cross bending we have

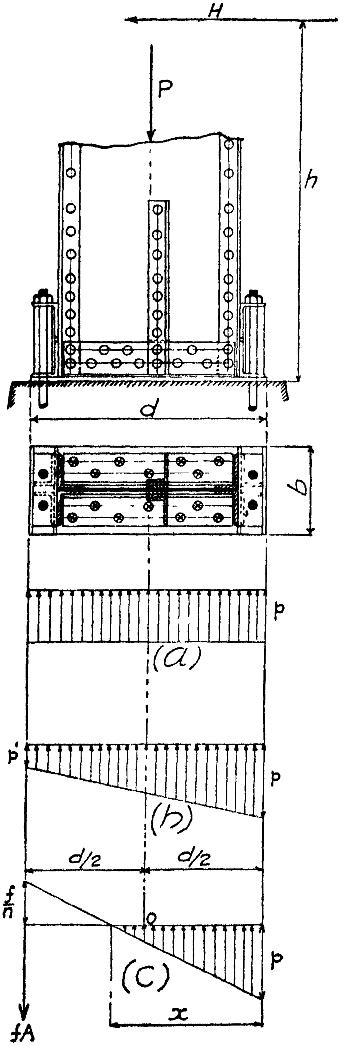


Fig. 435

$$f=(1,725,000\times 9)\div 1,576=9,850 \text{ lbs.}$$

The area of the cross-section is 26.87□″. For the direct stress on the column we have 28,700÷26.87=1,068 lbs. per sq. in. Adding this to the bending stress we obtain 9,850+1,068=10,918 lbs. for the total maximum unit stress on the column (above crane girder), which is only 294 lbs. greater than the 10,624 lbs. allowable, so the above assumed section will be used.

The part of the column below the crane girder will be at least 11 ins. wider than the part above (see Fig. 414), so that the crane girder can be connected directly to the column. This requires that the part below the crane girder be 30″ wide. So let us assume the part of the column below the crane girder to be composed of 4—Ls 6″x3½″x$\frac{9}{16}$″= 20.12□″ and a web 30″x$\frac{5}{16}$″=9.38, making in all 29.5□″ of cross-section. The least radius of gyration is 2.55. The distance from the crane girder to the base of the column is 19′-0″ or 228″. So we have $L/r=228/2.55=89.4$, and hence for the allowable compression we have 16,000−70x89.4=9,742 lbs. per sq. in. The direct load, considering the windward column, applied from the part of the column above the crane girder, is equal to 13,200+4,000+ 1,700=18,900 lbs. This causes a direct unit stress of 18,900÷29.5=641 lbs. The two angles of the column directly under the crane girders should be sufficient to carry the maximum concentration from these girders. As the crane wheels are equal in weight the maximum concentration due to the wheels is equal to the maximum end shear on the crane girder, which is given in Art. 272 as 56,100 lbs. The concentration due to the weight of the crane girders and struts is equal to (80+20)20=2,000 lbs. Then for the total concentration on the column from the crane girders we have 56,100+2,000=58,100 lbs. The area of cross-section of the two angles is 10.06□″. Then we have 58,100÷10.06=5,776 lbs. for the direct stress on the two angles due to the concentration from the crane girders. Adding to this the direct stress transmitted from the part of the column above the crane girder we obtain 5,776+641=6,417 lbs. for the maximum direct stress on the two column angles directly under the crane girders.

As is seen from Fig. 432, the maximum bending moment on the part of the column below the crane girder due to wind is equal to 12.5x11,500x12=1,725,000 inch lbs., which occurs at the base of the column. The moment of inertia of the column in reference to the gravity axis perpendicular to the web is about 4,800. Then we have

$$f=(1,725,000\times 15)\div 4,800=5,390 \text{ lbs. per sq. in.}$$

for the maximum stress due to cross bending. Adding this to the above direct stress we have 6,417+5,390=11,807 lbs. for the total maximum unit stress on the two angles directly under the crane girders. The allowable stress is given above as 9,742 lbs. per sq. in.; but as the maximum crane load and maximum wind load are not likely to occur at the same time this stress could safely be increased 25 per cent. So we have 1.25x9,742=12,179 lbs. for the unit stress permissible for the combined loading. This shows that the assumed section for the part of the column below the crane girder is about correct and hence will be used. The two outside angles and web of the column are stressed less than the angles directly under the crane girders, but it is desirable to have a symmetrical section and hence the main section of the column as given above is satisfactory throughout.

280. Designing of Column Base.—*Case I. When column is subjected to direct load only.* In that case the pressure on the base will be uniformly distributed as indicated at (*a*), Fig. 435.

Let P = direct load on column,
p = pressure per square in.,
b = width of base in ins.,
d = length of base in ins.
Then we have

$$p=\frac{P}{bd} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1).$$

The value of p should not exceed 600 lbs., which is the allowable pressure on concrete masonry.

Case II. When the column is subjected to both direct load and cross bending but the bending not sufficient to reverse the direct pressure. In that case the pressure will vary as indicated at (*b*), Fig. 435.

Let P = direct load on column,
p = maximum pressure per sq. in.,
p' = minimum pressure per sq. in.,
M = bending moment in in. lbs.,

b = width of base in ins.,
d = length of base in ins.

Then we have

$$p=\frac{P}{bd}+\frac{6M}{bd^2} \qquad (2).$$

and

$$p'=\frac{P}{bd}-\frac{6M}{bd^2} \qquad (3).$$

Case III. When the column is subjected to both direct load and cross bending and the bending reverses the direct pressure. In that case, assuming linear variation, the forces will be as indicated at (c), Fig. 435.

Let P = direct load on column,
p = maximum pressure per sq. in.,
f = stress per sq. in. on the anchor bolt = 10,000 lbs.,
A = area of cross-section of anchor bolt or bolts on each side of column,
M = bending moment in in. lbs.,
n = ratio of the modulus of elasticity of steel to masonry = 15,
b = width of column base in ins.,
d = length of column base in ins.

The problem is to determine p and A. As is seen from the diagram at (c) these could readily be determined if x were known. So we shall first derive an expression for the value of x.

It is obvious that the moments of the forces about o (center of the column) must be equal to M.

Then we have

$$\frac{p}{2}xb\left(\frac{d}{2}-\frac{x}{3}\right)+fA\frac{d}{2}=M \qquad (4).$$

As the summation of the vertical forces must equal 0, we have

$$\frac{p}{2}xb-fA=P \qquad (5).$$

From the stress diagram at (c) we have

$$\frac{p}{\left(\frac{f}{n}\right)}=\frac{x}{d-x},$$

from which we obtain

$$p=\frac{f}{n}\left(\frac{x}{d-x}\right) \qquad (6).$$

From (5) we obtain

$$A=\frac{p}{2f}\times b-\frac{P}{f} \qquad (7).$$

From (4), (5), and (6) the following expression for x can be obtained:

$$x^3 - 3dx^2 - \frac{3n}{fb}(2M+Pd)x + \frac{3n}{bf}(2M+Pd)d = 0 \ldots\ldots\ldots\ldots\ldots\ldots (8).$$

The value of x for any case can be determined from (8) and then the value of p can be obtained from (6) and A from (7). For convenience the anchor bolts are assumed to be exactly at the edge of the base plate, which of course is not absolutely true, but the error resulting from the assumption is small.

Now, considering the windward column, the direct load above the crane girder as previously given is 28,700 lbs. and the load from the crane girder is 58,100 lbs., and assuming the column to weigh 2,000 lbs. we have

$$P = 28{,}700 + 58{,}100 + 2{,}000 = 88{,}800 \text{ lbs.}$$

for the direct load on the column. As previously found the bending moment

$$M = 1{,}725{,}000 \text{ inch lbs.}$$

Let us assume d equals 40″ and b equals 16″. Substituting these values in (8) and assuming f equals 10,000 we obtain

$$x = 20 \text{ ins. (about).}$$

Substituting in (6) we obtain

$$p = 666 \text{ lbs.,}$$

which is quite satisfactory, as the pressure is due to direct load and cross bending which are not likely to occur simultaneously. In fact p could safely be 800 or 900 lbs.

Substituting in (7) we obtain

$$A = \left(\frac{666}{2} \times 20 \times 16 - 88{,}800\right) \div 10{,}000 = 2.77 \text{ sq. ins.}$$

for the area of cross-section of anchor bolts. Use two $1\frac{3}{8}$″ anchor bolts on a side.

281. Drawings.—The data given in Articles 270 to 280 are sufficient for making the stress sheet, Fig. 436, except for the gable framing, girts, and bracing, which can be considered more conveniently as the work on the stress sheet progresses. The gable framing is designed to resist a wind pressure of 30 lbs. per sq. ft. on the end of the building. Each gable column can be considered as a vertical beam 35 ft. long and, as they are about 11 ft. apart, the load on each is 11x30 = 330 lbs. per ft. of length. Then the bending moment on each is $\frac{1}{8}$x330x35²x12 = 606,370″#. Dividing this by 16,000 we obtain 37.9 for the section modulus. This calls for a 12″x35# I which will be used for each gable column.

The bracing between the trusses and main columns in the end bays should be sufficient to resist the wind pressure on the end of the building. The stresses in this bracing are determined as indicated in Fig. 437. The diagram at (*a*) represents the bracing between the main columns and the diagram at (*b*) represents the stresses in the same. The diagram at (*c*) represents the bracing in the plane of the roof and the diagram at (*d*) represents the bracing in the plane of the bottom chord of trusses. The designing of the bracing in

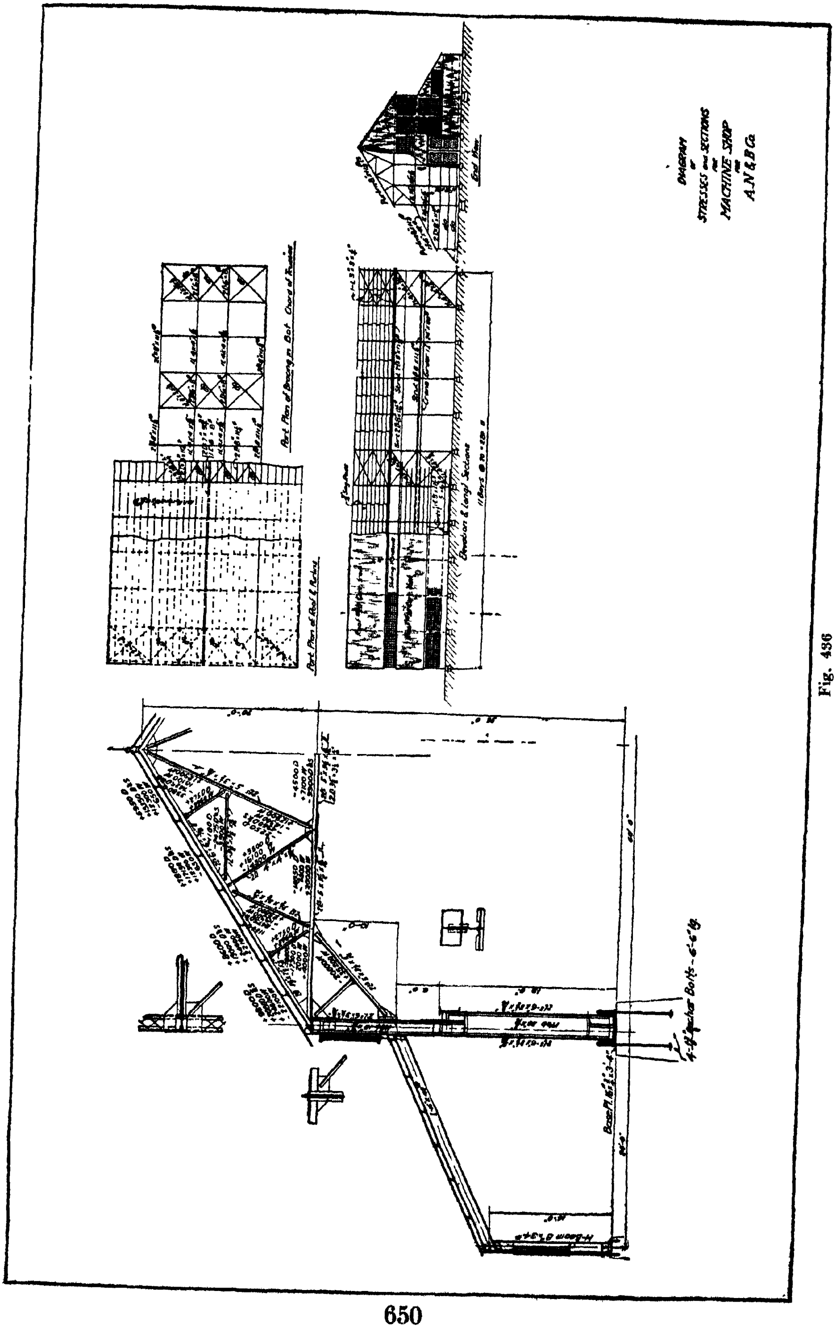


DIAGRAM OF STRESSES AND SECTIONS FOR MACHINE SHOP FOR A.N.&B.Co.
Part Plan of Bracing in Bot. Chord of Trusses
Part Plan of Roof & Purlins
Elevation & Longl. Sections
End View

Fig. 436

intermediate bays is mostly a matter of judgment as it is intended mainly for rigidity.

The girts are designed to resist a horizontal wind pressure of 30 lbs. per sq. ft. and in some cases they should act as struts, L/r should not exceed 125. After the stress sheet, Fig. 436, is completed the shop drawings and

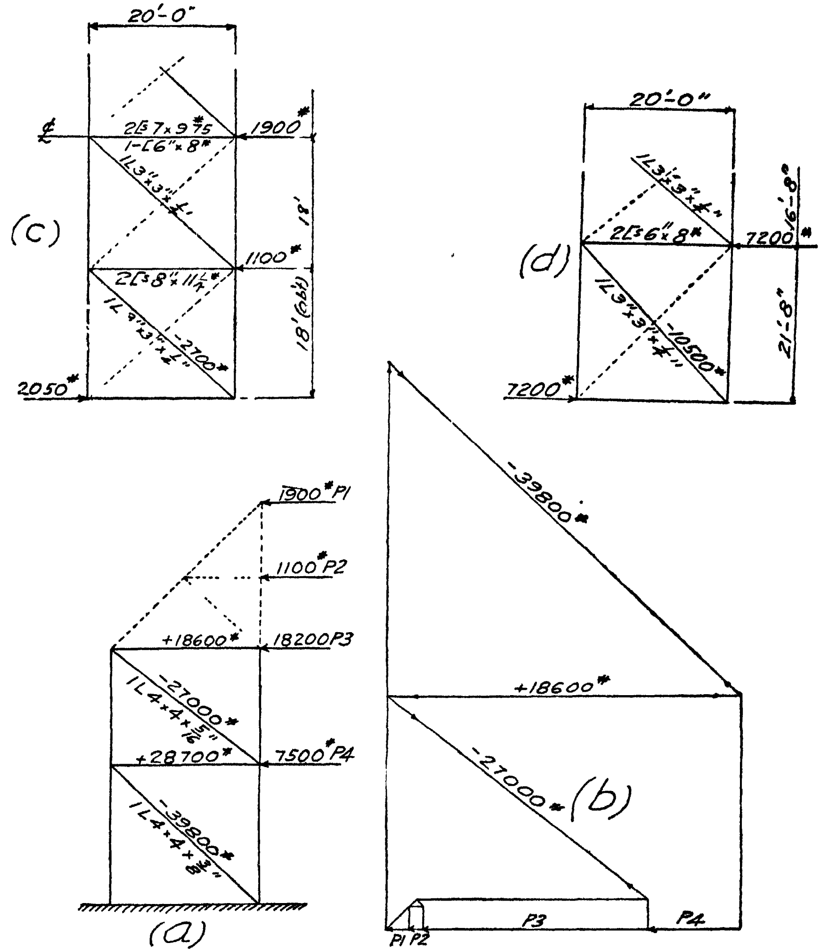


Fig. 437

bills for the structure can be made. A fair idea of the details may be obtained from the general drawing, Fig. 438.

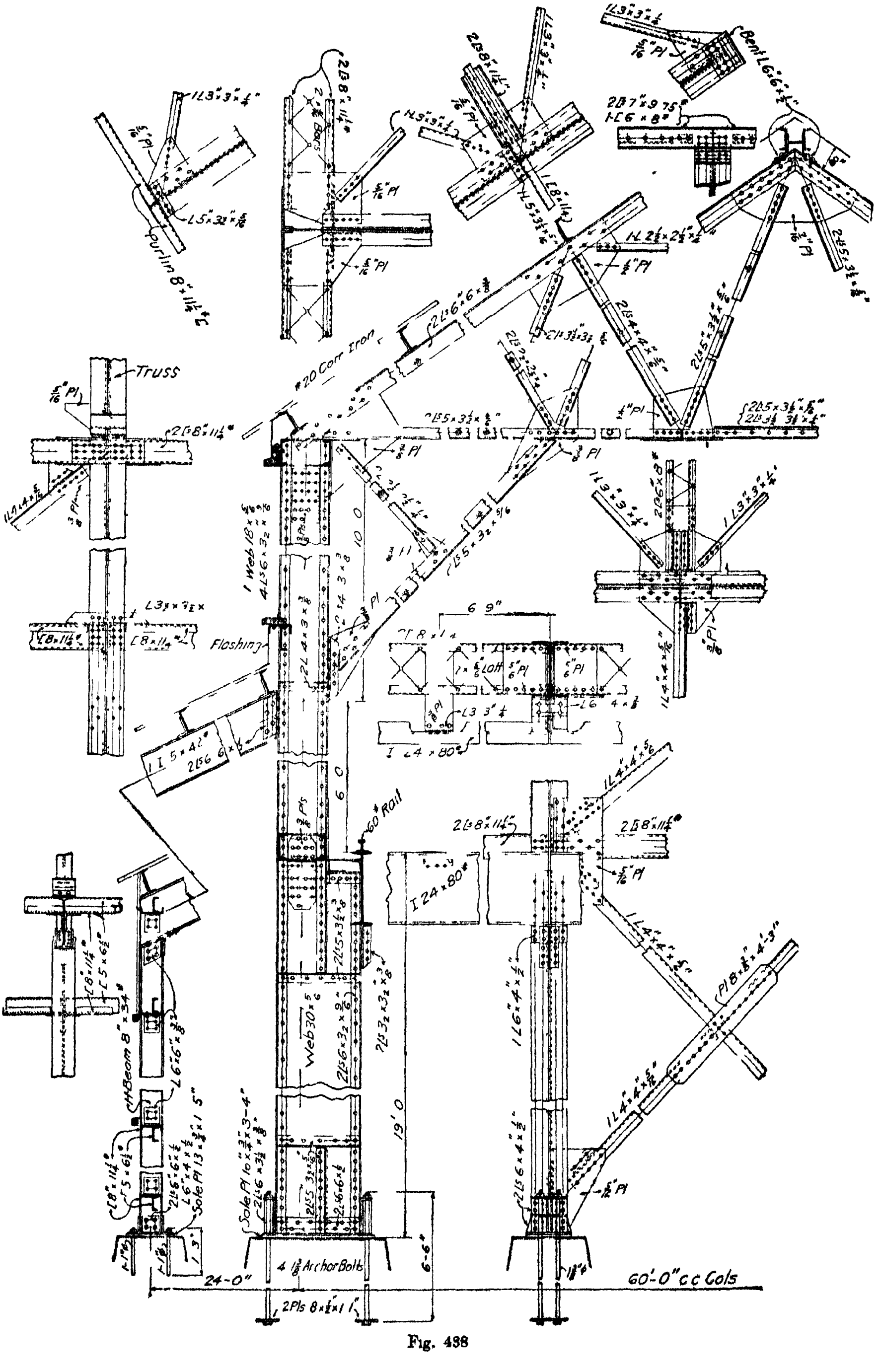


Fig. 438

HIGH BUILDINGS

282. Description.—In the case of modern high buildings such as office buildings, hotels, and storage houses, all loads including floors, exterior walls, partitions, and live loads on floors are supported upon a continuous rectangular frame consisting of columns and beams. Though the designing of these buildings as a whole is architectural work, the designing of the frame is purely a structural engineering problem and hence will be here treated as such.

When a building is comparatively small and not closely surrounded by other buildings, the frame is usually rectangular in plan, as shown in Fig. 439.

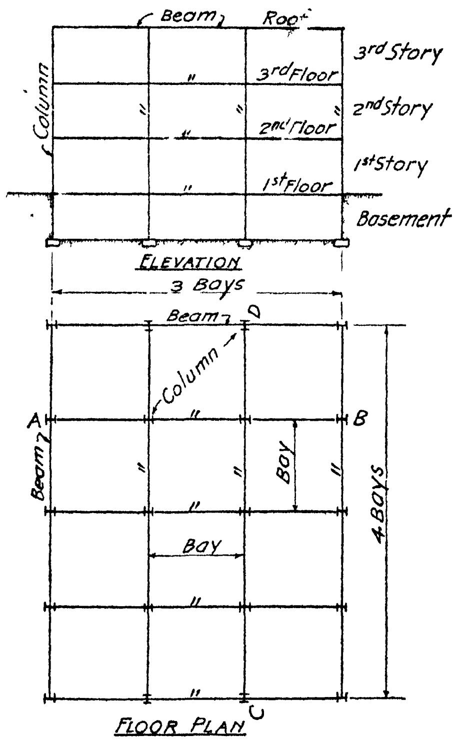


Fig. 439

In the case of a building adjacent to other buildings, an opening (known as light court) is necessary to provide light in the rooms on the side next to the adjacent building, as indicated in Fig. 439*a*. In the case of a large building, the light court extends well into the body of the building, as indicated in Fig. 439*b*. The outside walls of high buildings abutting on wide streets are usually constructed in a continuous vertical plane, especially in buildings not over twenty stories in height, but in buildings of extreme heights the general floor plan is decreased in area from the bottom of the building up to the top by setbacks, as indicated in Fig. 439*c*, so as to obtain sufficient daylight and air at the street level.

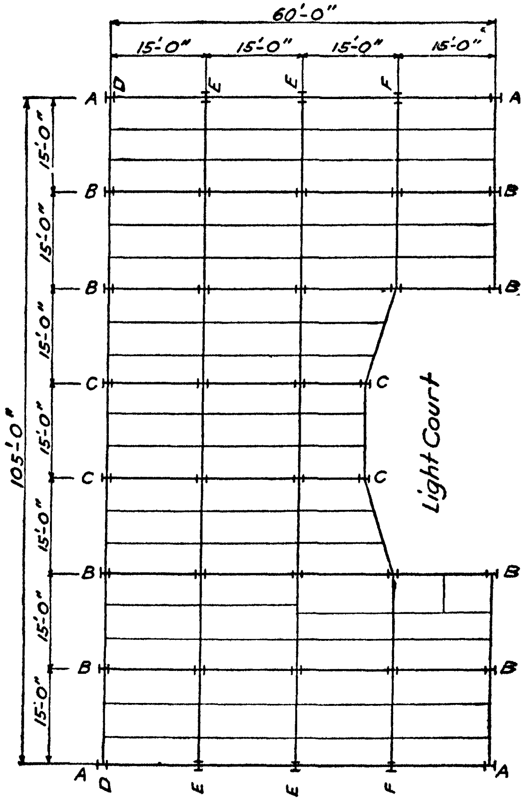
283. Dead Load includes all permanent parts of a building and also all stationary fixtures and mechanisms supported by the building.

In estimating dead loads the following units of weights may be used:

Steel	490	lbs. per cu. ft.
Concrete	150	lbs. per cu. ft.
Brick walls	140	lbs. per cu. ft.
Hollow-tile walls	40	lbs. per cu. ft.
Hollow-tile floors (tile alone)	50	lbs. per sq. ft.
Plaster	5	lbs. per sq. ft.
Wood	4.5	lbs. per sq. ft. B. M.

The weights of other materials can be obtained from manufacturers' handbooks.

284. Live Load in buildings consists of crowds of people, movable equipment, and storage material. A live load of a stated number of pounds



General Plan of Floor Framing.

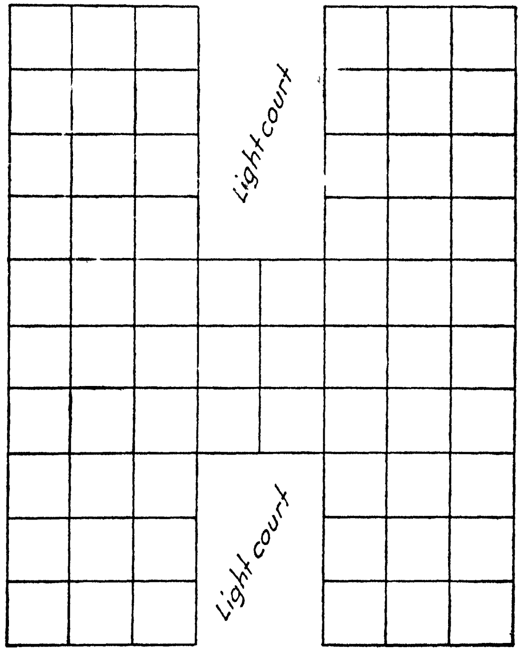
Fig. 489a

per square foot of floor is usually used in designing. This uniform load is considered to be equivalent to any live loads that may come upon the floors. In the case of office buildings and hotels, the live load on the first floor is usually assumed to be 100 to 150 lbs. per square foot of floor; on all other floors from 50 to 75 lbs. per square foot; and on the roof 25 lbs. per square foot.

The live load for storage buildings varies from 150 to 250 lbs. per square foot of floor. The building ordinances of different cities specify what the minimum live load can be in each case and, of course, designers are governed accordingly.

285. Wind Pressure.—In addition to supporting live and dead loads, the frame of a high building must be designed to resist a horizontal wind pressure applied upon the entire vertical surface exposed to the wind.

The intensity of the wind pressure depends upon the velocity of the wind and upon the area of the surface exposed. It has been found by experiment that when the velocity of the wind is 100 miles per hour, the pressure is about



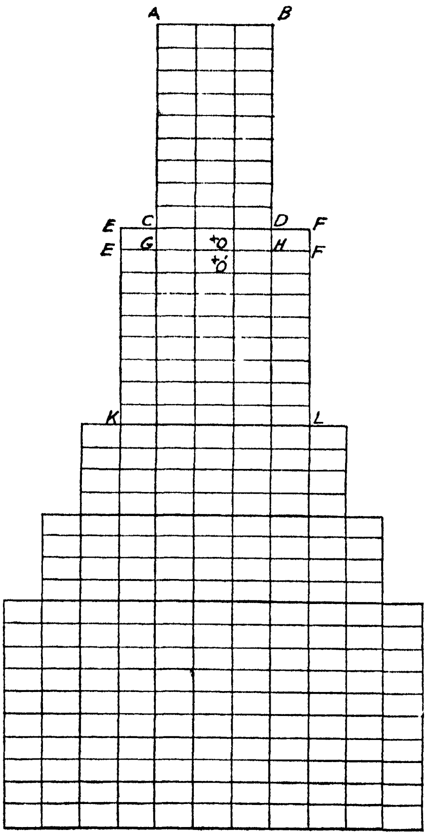
FLOOR PLAN
Fig. 439b

30 lbs. per square foot of surface on small surfaces and considerably less on large surfaces. So for large buildings the horizontal wind pressure is usually assumed to be 20 lbs. per square foot of the entire surface exposed to the wind. It is well known that the intensity of the pressure increases from the ground upward. This variation in intensity is due to the retarding effect on the velocity of the wind by the ground and by various objects at and near the ground surface. Owing to this observed fact some designers assume the wind pressure to vary from the ground upward taking about 30 lbs. per square foot as the maximum at the top of the building and about 10 lbs. at or near the ground level. This variable pressure, as a rule, produces about the same stresses as the 20-lb. pressure over the entire surface. So the 20 lbs. is usually used, since the analysis is simplified by so doing. However, city building ordinances practically always govern the final decision as to the pressure used, especially the minimum pressures.

One often hears the claim that the partitions and floors resist the wind pressure on buildings. This is true to some extent, especially in case of pressures due to gusts, for in that case the inertia of the floors and partitions, as well as all other parts of the building, offers resistance to the wind pressure. Engineering judgment, however, dictates that at least the greater part of the wind pressure be considered to be resisted by the rigid frame alone and that the inertia of the building be considered nothing more than an unknown safety factor.

286. Stresses in Frame Due to Wind Pressure.—The stresses in the columns and beams of any bent of a building frame due to wind pressure could be computed by the *method of slope deflection* or by the *method of least work*, provided the sections of all of the members were known; but as the sections are not known, to start with, it is evident that these so-called exact methods cannot be used, at least not until the stresses and section are determined approximately by using some method based upon assumed static conditions that approach the actual conditions of the case considered.

The method most commonly used for determining stresses due to wind is known as the *cantilever method*, in which the building as a whole is considered to act as a vertical contilever beam fixed at the foundation, and the point of contra-flexure of each beam in the frame is considered to be at mid-span and at mid-story of each column.

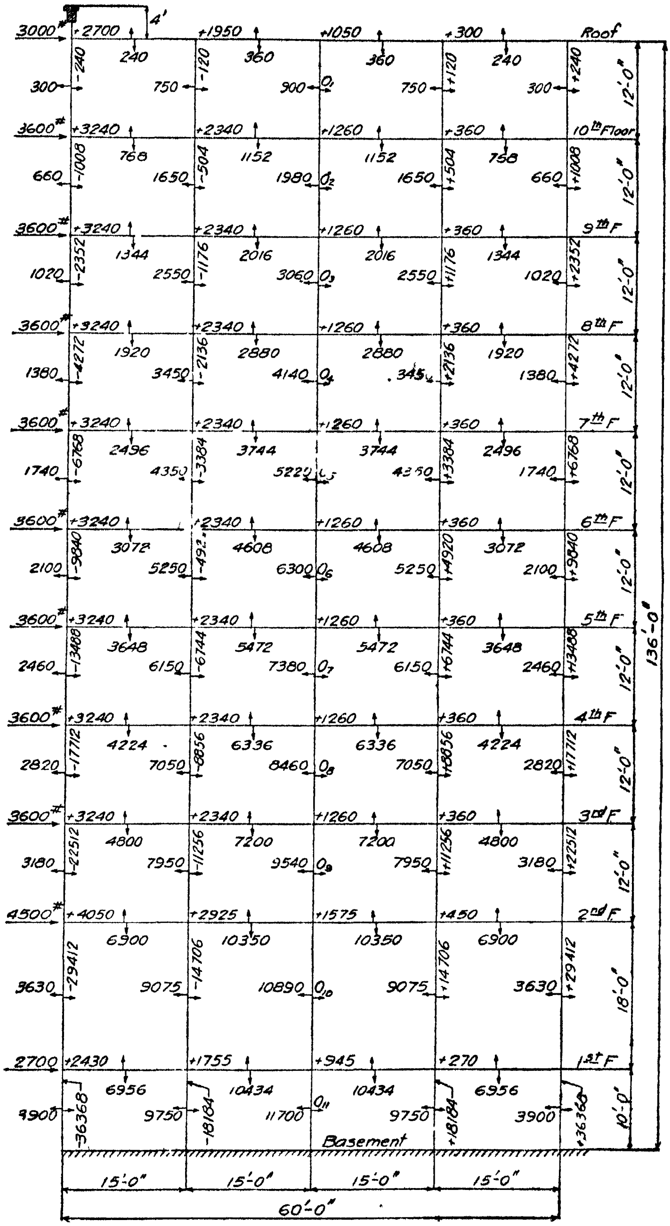


ELEVATION
Fig. 439c

For the purpose of illustration we shall consider the bents in a ten-story building the floor plan of which is shown in Fig. 439*a*. It is seen from Fig. 439*a* that it would be necessary to compute the wind stresses in bents *BB*, *CC*, and *EE*, at least. The wind stresses in bents *AA* may be considered to be one-half those in bents *BB*, and likewise the stresses in bent *DD* may be considered to be one-half those in bents *EE* and *FF*. Let us first consider bents *BB*, the elevation of which is shown in Fig. 440. The wind load at the roof is equal to

$$20 \times 15 \times (6+4) = 3{,}000 \text{ lbs.}$$

At each floor from the tenth down to the third (inclusive) the wind load is equal to 20x15x12 = 3,600 lbs., as shown, and at the second floor it is equal to 20x15x(6+9) = 4,500 lbs., and at the first floor it is equal to 20x15x9 = 2,700



Bents – BB

Fig. 440

lbs. Having the wind loads determined, we can proceed with the determination of the stresses due to them by beginning at the roof and working downward story after story.

The columns and the floor beams connecting to them will be considered fixed and the point of contra-flexure in each column of each story will be considered to be at mid-story, while the point of contra-flexure in each floor beam will be considered to be at mid-span. Any part of the structure between points of contra-flexure can be considered as an independent structure.

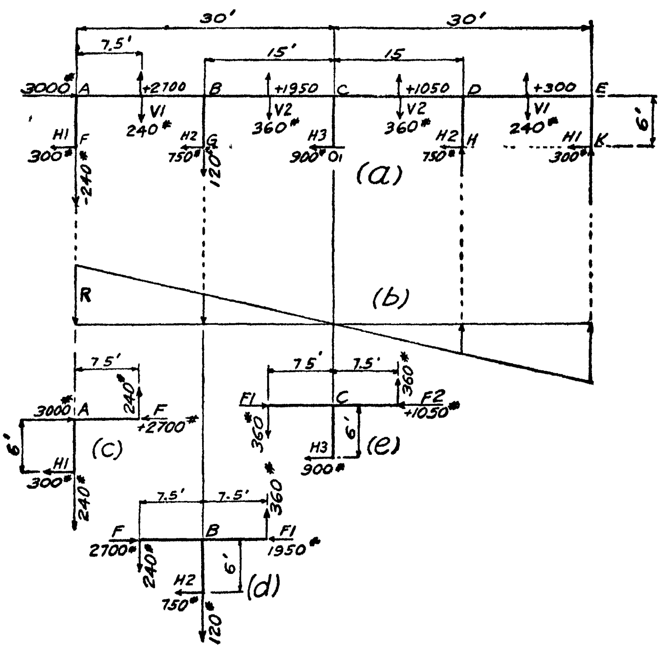


Fig. 441

In determining the stresses, the building as a whole will be considered to act as a vertical cantilever beam, in which case the direct stress on the columns will vary directly as their distance from the neutral axis.

Considering the part of the structure above the points of contra-flexure of the columns in the tenth story, we have the independent structure shown at (*a*), Fig. 441. As is obvious, the neutral axis will be at O_1, the center of the building. Then the vertical reaction on the columns will vary directly as their distance out from O_1 as indicated at (*b*).

For the moment of the wind load about O_1 we have

$$M = 3{,}000 \times 6 = 18{,}000 \text{ ft. lbs.}$$

This moment must be balanced by the moment of the vertical reactions on the columns. Let R represent the reaction at F. Then the reaction at a point one foot (unit) out from O_1 would be $R/30$, and hence the reaction at G and H would be $(R/30)15$ and similarly at F and K it would be $(R/30)30$. Then taking moments about O_1 we have

$$2\left[\left(\frac{R}{30}\right)\overline{30}^2+\left(\frac{R}{30}\right)\overline{15}^2\right]-18{,}000=0,$$

from which we obtain

$$R=\frac{18{,}000}{75}=240 \text{ lbs.}$$

The number 75 is a constant that can be used in determining the R in all the other stories.

Now having the vertical reaction R at F determined, we can readily determine the reactions on the other columns, as the intensity in each case is directly proportionate to the distance of the column from O_1. Then, for the reaction at G and H, we have $(15/30)\text{x}240=120$ lbs. and for the reaction at K we have $(30/30)\text{x}240=240$ lbs., which of course is the same as at F.

Having the vertical reactions on the columns determined, we can next determine the vertical shear on the floor beams by simply adding up the vertical forces beginning at either A or E. For example, beginning at A we have

$$V1=240$$

for the shear on girder AB. For the shear on girder BC we have

$$V2=240+120=360.$$

The structure being symmetrical about O_1 the shears and reactions on one side of O_1 will be equal and opposite to those on the other side, and hence only one-half of the structure need be considered in determining these stresses.

Having the vertical reactions on the columns and the vertical shear on the floor beams determined, we can compute the horizontal shears $H1$, $H2$, and $H3$ on the columns. Considering the part of the structure shown at (c) as an independent structure and taking moments about A, we have

$$240\times7.5-H1\times6=0,$$

from which we obtain

$$H1=300 \text{ lbs.}$$

Then summing up the horizontal forces, we have

$$3{,}000-300-F=0,$$

from which we obtain

$$F=2{,}700 \text{ lbs.}$$

for the direct compression in beam AB. Considering the part of the structure shown at (d) as an independent structure and taking moments about B, we have

$$240\times7.5+360\times7.5-H2\times6=0,$$

from which we obtain

$$H2=750 \text{ lbs.}$$

Then summing up the horizontal forces, we have

$$2{,}700-750-F1=0,$$

from which we obtain

$$F1 = 1{,}950 \text{ lbs.}$$

for the direct compression in floor beam BC. Likewise, considering the part of the structure shown at (e) as an independent structure and taking moments about C, we have

$$360 \times 15 - H3 \times 6 = 0,$$

from which we obtain

$$H3 = 900 \text{ lbs.}$$

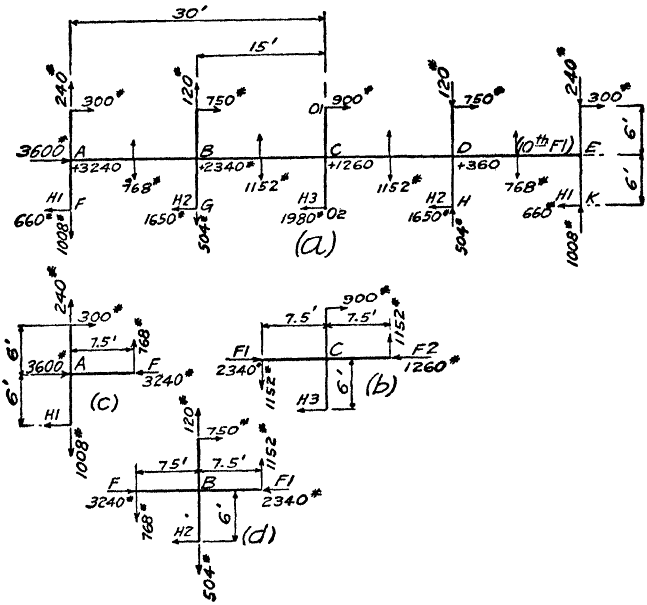


Fig. 442

Then adding up the horizontal forces, we have

$$1{,}950 - 900 - F2 = 0,$$

from which we obtain

$$F2 = 1{,}050 \text{ lbs.}$$

for the direct stress in beam CD. The direct stress in beam DE is equal to $1{,}050 - 750 = 300$ lbs., which is and should be equal to $H1$.

These shears and stresses can now be written on the diagram in Fig. 440.

Next considering the part of the structure between the points of contraflexure in the columns of the tenth and ninth stories, we obtain the independent structure at (a), Fig. 442. Taking moments about O_2 of the wind forces on the building above this point (see Fig. 440), we have

$$M = 3{,}000 \times 18 + 3{,}600 \times 6 = 75{,}600 \text{ ft. lbs.}$$

Letting R represent the vertical reaction on the column at F, we have

$$2\left[\left(\frac{R}{30}\right)\overline{30}^2+\left(\frac{R}{30}\right)15^2\right]=75{,}600,$$

from which we obtain

$$R=\frac{75{,}600}{75}=1{,}008 \text{ lbs.}$$

Then the reaction on the column at G is equal to $1{,}008(15/30)=504$ lbs.

Beginning at A and adding up the vertical forces (algebraically), we have $1{,}008-240=768$ lbs. for the shear on beam AB, $1{,}008-240+504-120=1{,}152$ lbs. for the shear on beam BC. Now considering the part of the structure shown at (c) as an independent structure and taking moments about A, we have

$$H1\times 6+300\times 6-768\times 7.5=0,$$

from which we obtain

$$H_1=660 \text{ lbs.}$$

for the horizontal shear on the column at F.

Similarly, considering the part of the structure shown at (d) as an independent structure and taking moments about B, we have

$$H2\times 6+750\times 6-768\times 7.5-1{,}152\times 7.5=0,$$

from which we obtain

$$H2=1{,}650 \text{ lbs.}$$

Considering the part of the structure shown at (b) as an independent structure and taking moments about C, we have

$$H3\times 6+900\times 6-1{,}152\times 15=0,$$

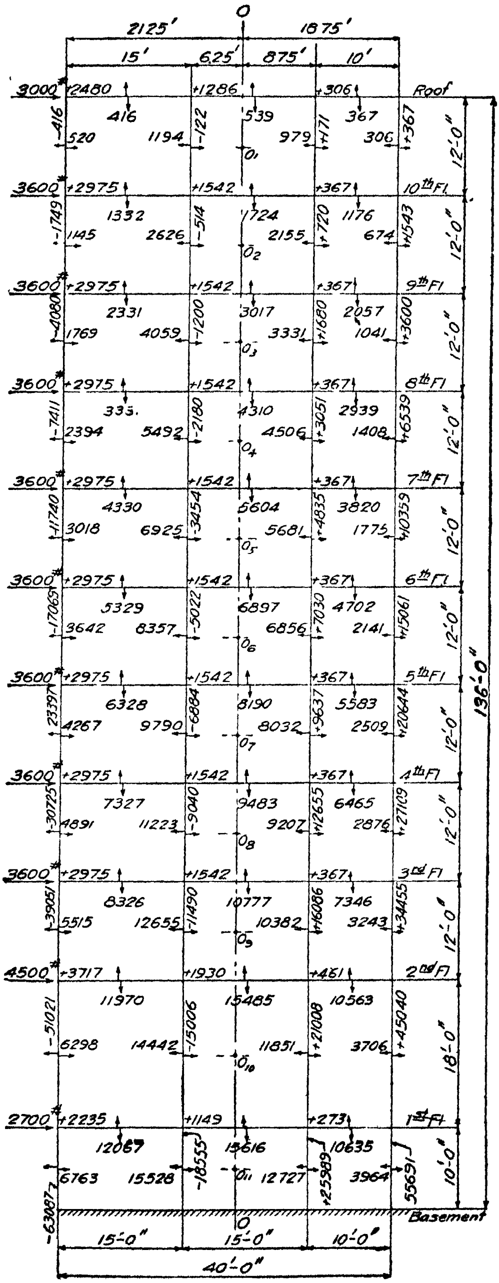
from which we obtain

$$H3=1{,}980 \text{ lbs.}$$

for the shear on the column at O_2. Now adding up the horizontal forces beginning at A, we have $3{,}600+300-660=3{,}240$ lbs. for the direct compression in beam AB, $3{,}600+300-660-1{,}650+750=2{,}340$ lbs. for the direct compression in beam BC, $2{,}340+900-1{,}980=1{,}260$ lbs. for the direct compression in beam CD, and $1{,}260+750-1{,}650=360$ lbs. for the direct compression in beam DE.

Continuing in the manner shown above, considering the parts of the structure between points of contra-flexure in the column of consecutive stories on down to the base of the building, we can determine the direct stresses and shears on the columns and floor beams, as shown in Fig. 440.

It will be seen from Fig. 440 that the horizontal shear on each column varies from the ninth down to the second story by a constant and that the vertical shear on the floor beams in each bay varies from the tenth down to the third floor by a constant, and hence these stresses can be quickly computed by the use of the constants, which can be determined after the stresses are computed in the three top stories. Also, it will be seen (see Fig. 440) that the direct stress in the floor beams in each bay is the same from the tenth down to the third floor. After the shears on the columns and floor beams are computed by the use of the constants, the direct stress on the columns can be



Bents – CC.
Fig. 443

determined by beginning at the ninth story and adding up the vertical forces included between the points of contra-flexure. As an example, for the direct stress in the outer left-hand column of the sixth story, we have 4,272+2,496= 6,768 lbs. For the next column to the right we have 2,496+2,136+3,744= 3,384 lbs., and so on. To obtain the stresses in the second-story floor beams and first-story columns, we first take moments about O_{10} of the wind forces above that point. For this moment (see Fig. 440), we have

$$M = 3{,}000 \times 117 + 28{,}800 \times 63 + 4{,}500 \times 9 = 2{,}205{,}900 \text{ ft. lbs.}$$

Then dividing this by the constant 75 (previously determined), we obtain 29,412 lbs. for the reaction or direct stress in the outer left-hand column. The reactions on the other columns can then be quickly determined by direct proportion, and the remainder of the work of determining the stress is the same as previously explained.

To obtain the stresses in the floor beams of the first floor and in the basement columns, we should first take moments about O_{11} (see Fig. 440) of the wind forces above this point. For this moment, we have

$$M = 3{,}000 \times 131 + 28.800 \times 77 + 4{,}500 \times 23 + 2{,}700 \times 5 = 2{,}727{,}600 \text{ ft. lbs.}$$

Then dividing this by the constant 75, we obtain 36,368 lbs. for the reaction or direct stress on the outer left-hand column. The reactions on the other columns are readily determined by direct proportion, and the remainder of the work of determining the stresses is the same as previously explained.

From the above it is seen that all of the direct stresses and shears in the columns and floor beams of the bent are obtained by fully analyzing only the three top stories and the first floor and basement.

We shall next consider the bents *CC* (through the light court), the elevation of which is shown in Fig. 443. Considering the part of the structure above the points of contra-flexure of the columns in the tenth story, we obtain the independent structure which is shown at (*a*), Fig. 444. As these bents are unsymmetrical, we have to determine the location of the neutral axis *OO*. The reactions on the columns will vary directly as their distance from this neutral axis, as indicated at (*b*), and the sum of the reactions on one side must equal the sum of those on the other.

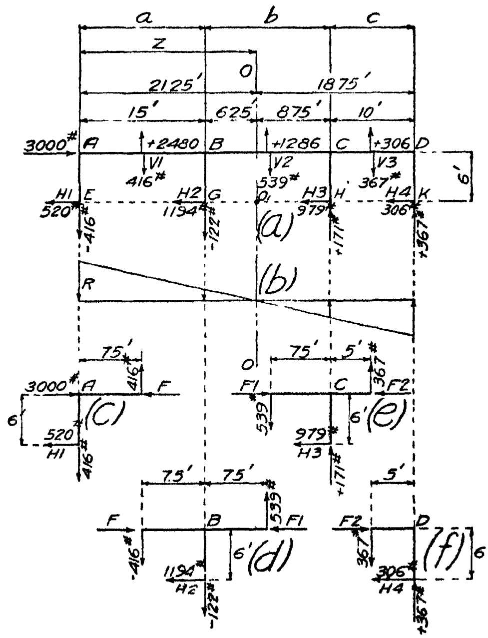


Fig. 444

Let z = distance from A to the neutral axis, and for the sake of simplicity let a, b, and c represent the bay lengths as indicated, and let R represent the reaction at E.

Then we have R for the reaction at E, $(R/z)(z-a)$ for the reaction at G, $(R/z)(a+b-z)$ for the reaction at H, and $(R/z)(a+b+c-z)$ for the reaction at K. Then, as the sum of the reactions on one side of the neutral axis must be equal to the sum of those on the other side of the neutral axis, we have

$$R+\left(\frac{R}{z}\right)(z-a)=\left(\frac{R}{z}\right)(a+b-z)+\left(\frac{R}{z}\right)(a+b+c-z),$$

from which we obtain

$$z=\frac{3a+2b+c}{4} \qquad \text{(1)}$$

for the distance from A to the neutral axis. Now, substituting the numerical values of the bay lengths in (1), we obtain

$$z=\frac{45+30+10}{4}=21.25 \text{ ft.},$$

which gives the location of the neutral axis.

Taking moments about O_1 we have

$$\left(\frac{R}{21.25}\right)21.25^2+\left(\frac{R}{21.25}\right)6.25^2+\left(\frac{R}{21.25}\right)8.75^2+\left(\frac{R}{21.25}\right)18.75^2-3{,}000\times 6=0,$$

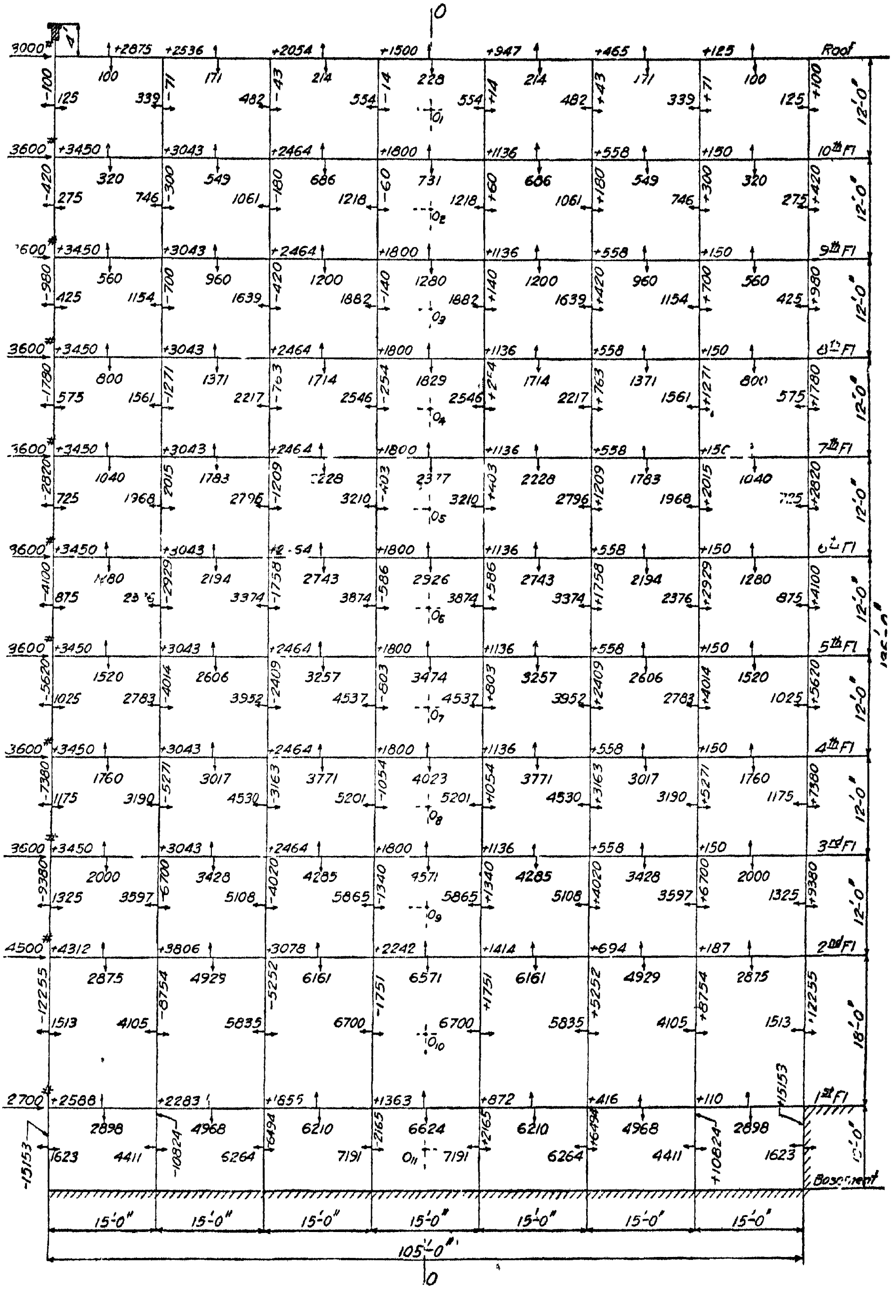
from which we obtain

$$R=\frac{18{,}000}{43.23}=416 \text{ lbs.}$$

for the reaction on the column at E. The number 43.23 is the constant for determining the reaction R for all the other stories. Having the reaction at E determined, the reaction on the other columns is readily determined by direct proportion. For example, the reaction at H is $(416/21.25)8.75=171$ lbs. After the reactions on the columns are determined, the shears on the columns and the shears and direct stresses on the floor beams can be determined, as previously explained, by considering the independent portions shown at (*c*), (*d*), (*e*), and (*f*). In fact, the remainder of the work of determining the direct stresses and shears on the columns and floor beams shown in Fig. 443 is the same as previously explained for bents *BB*.

The shears and direct stresses in the columns and floor beams for bents *EE* and *FF* are given in Fig. 445. These are determined in the same manner as previously shown for bents *BB*. It will be seen that the neutral axis for bents *EE* and *FF* is midway between the central columns. After the shears and direct stresses on the columns and floor beams, as given in Figs. 440, 443, and 445, are determined the moments at the column connections are readily obtained by multiplying the shear at the point of contra-flexure by the distance to the connection in each case. After the moments at the column connection are computed, the fiber stresses are obtained in the usual manner.

The approximate stresses in any building frame due to wind can be determined in the manner outlined in the foregoing analysis. However, there



Bents-EE & FF.

Fig. 445

are some cases where practical assumptions are required, owing to offsetting of columns and special details. In the case of such buildings, as indicated in Fig. 439c, we can start at the top (AB) and work down to CD in the same manner as shown in the foregoing analysis. Then we should next take moments about O and consider the total width EF of the building. We should not consider the entire computed direct stress to come on the outside columns EE and FF, but should consider these columns to take, say, one-half of the computed stress and should transfer the required amount to the other columns in proportion to their distance out from O. Next taking moments about O', we can consider the entire width of the building from there on down to KL in the usual manner, and so on. Just how much stress to assume on the outside columns, such as EE and FF, is at first a matter of judgment, and later readjustments of the stresses on the columns throughout the story can be made if found necessary. However, such readjustments affect the building as a whole but slightly.

Some of the largest buildings in the United States have been analyzed in the manner outlined in the foregoing article.

287. Stresses in Frame Due to Dead and Live Load.—The connections of the floor beams to the columns must practically fix the beams in order that the frame be capable of resisting satisfactorily the wind pressure on the building. This being done, it is evident that the floor beams should be considered fixed beams in computing the moments and shear on them due to dead and live load. If the load on the floor beams be uniformly distributed, the maximum moment at the end of a floor beam would be

$$M' = -\tfrac{1}{12}wL^2 \qquad (1)$$

and at mid-span of the beam it would be

$$M'' = \tfrac{1}{24}wL^2,$$

where w = uniform load per foot of beam and L = length of span in feet. To allow for some distortion that may occur in the connections at the ends of the beams, which would tend to increase the moment at mid-span, the formula

$$M'' = \tfrac{1}{16}wL^2 \qquad (2)$$

is often used instead of $M'' = \frac{1}{24}wL^2$; but Formula (1) ($M' = -\frac{1}{12}wL^2$) can always be used for determining the moment at the end of floor beams when the dead and live loads on them are considered to be uniformly distributed. The actual distribution of the dead and live loads upon floor beams depends upon the type of floor used. For floors reinforced in one direction, the load as a rule will be uniformly distributed along the beam; but for floors reinforced in two directions, the dead and live load will not be distributed uniformly along the beam but as a rule will vary along the beam approximately as the ordinate to a parabola having a maximum value at mid-span and zero at each end.

The stress on columns due to dead load is mostly direct compression except on outside columns where a moment is exerted on each of the columns because the floor load is applied to only one side of each column. This statement is true for live load also when the live load is considered to be continuously distributed over each floor of the building—like the dead load—but when the live load is considered to be non-continuous, that is, when say every other panel is loaded with live load, there will as a rule be bending on the columns throughout due to such loading. The moment on the columns in that case is known in general as the moment due to unsymmetrical live load.

288. General Details of Frame.—Columns generally consist of H-beams, as shown in Figs. 446, 447, and 448. The flanges of the floor

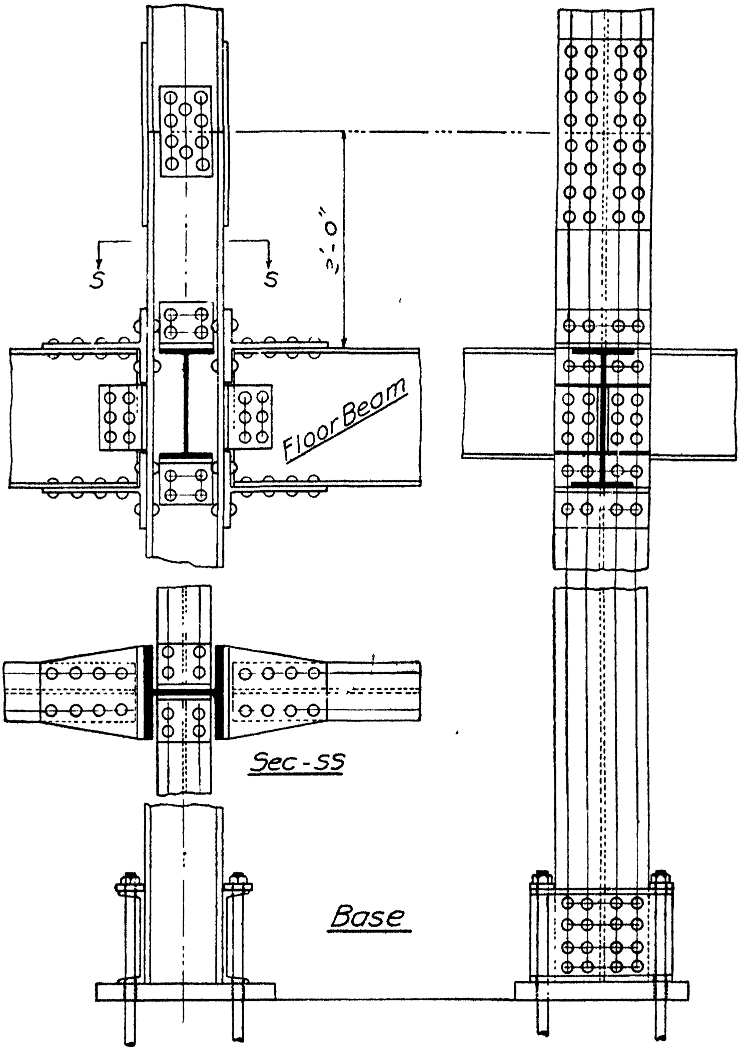


Fig. 446

beams are often connected to the columns by parts of I-beams sheared from full beams, as shown in Figs. 446 and 447. Such connections are known as "split I-beam connections." These connections transmit the end moments on the floor beams, and the angle connections on the web transmit the end shear.

The details shown in Fig. 448 are all for typical welded connections.

A typical one-way hollow clay tile floor is shown in Fig. 449. The tile is for the purpose of lessening the weight of floor and of saving on forms.

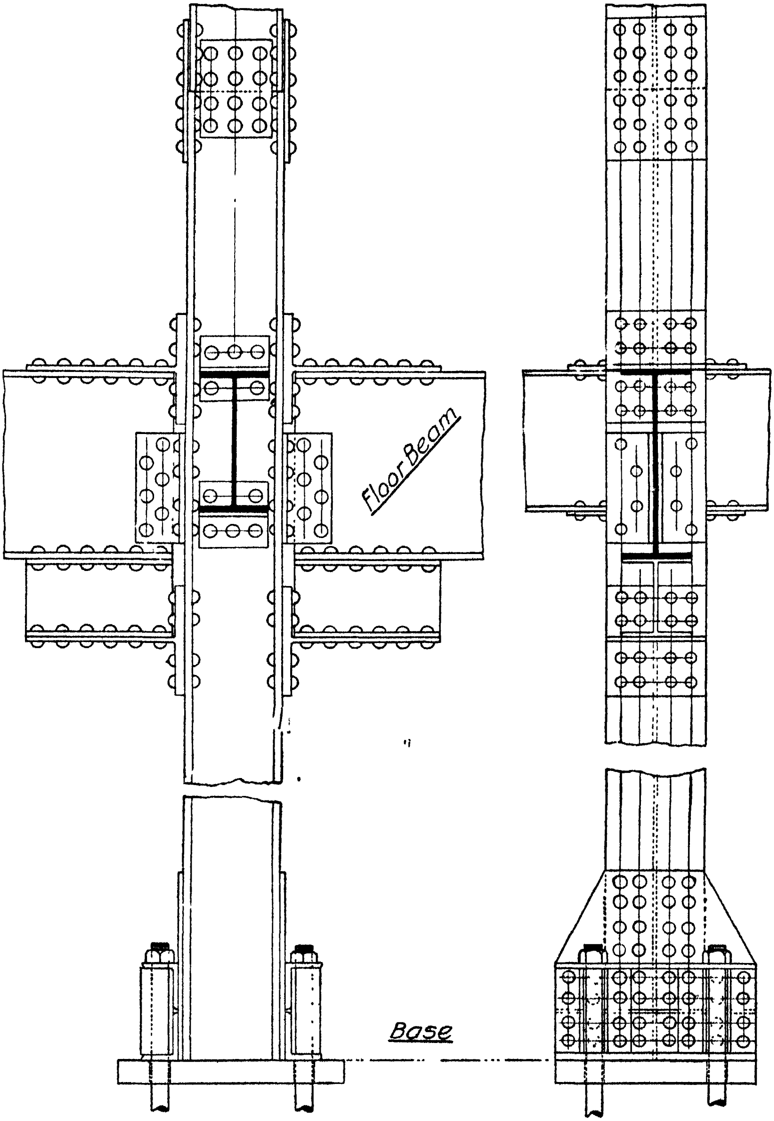


Fig. 447

A typical one-way hollow steel tile (steel forms) floor is shown in Fig. 450. These two types of floors are used more often than are any other type of floor.

A typical hollow clay tile flat arch floor and a typical concrete slab floor are shown in Fig. 451. The flat arch floors require intermediate beams as shown, and tie rods are required for resisting the horizontal thrust of the arch.

Design of a Three-Story Office Building

289. Data.—

The plan and elevation will be similar to that shown in Fig. 439.

Length = 5 bays @ 18′-0″ = 90′-0″.

Width = 3 bays @ 18′-0″ = 48′-0″.

Heights { Basement = 10′-0″.
3 stories @ 12′-0″ = 36′-0″.

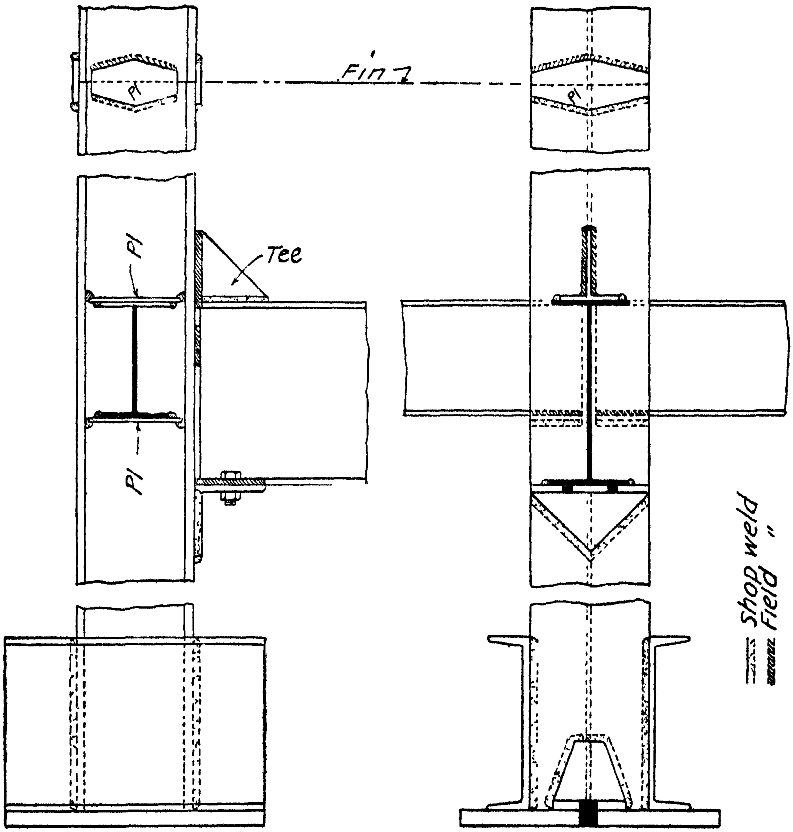


Fig. 448

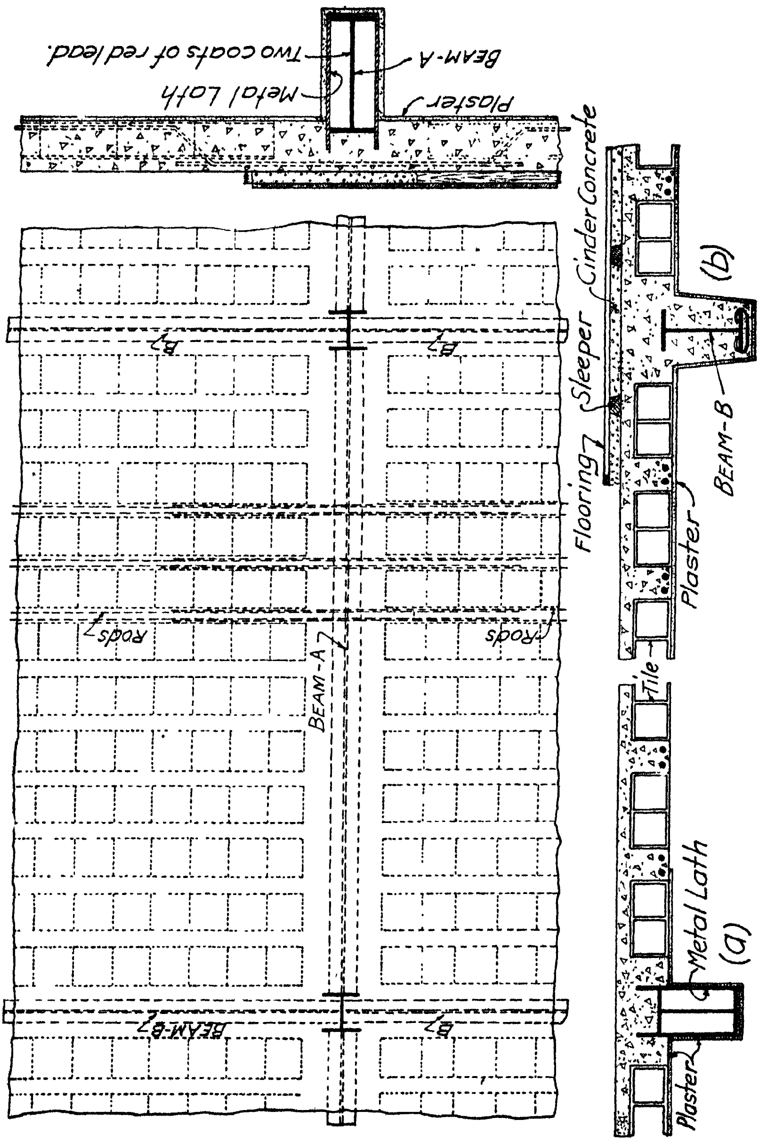
Dead load, to be computed.

Live load { All floors = 80 lbs. per sq. ft. of floor.
Roof = 25 lbs. per sq. ft. of roof.

Wind load = 20 lbs. per sq. ft. of vertical exposed surface.

Type of floor, one-way clay tile construction as shown in Fig. 449.

All metal to be incased in concrete.



Two coats of red lead.
BEAM-A
Metal Lath
Plaster
B7
Rods
BEAM-A
BEAM-B7
Flooring
Sleeper
Cinder Concrete
(b)
Plaster
BEAM-B
Tile
Metal Lath
(a)
Plaster

Fig. 449

290. Allowable Stress per Square Inch—Structural Steel.—

(*a*) *Tension:*

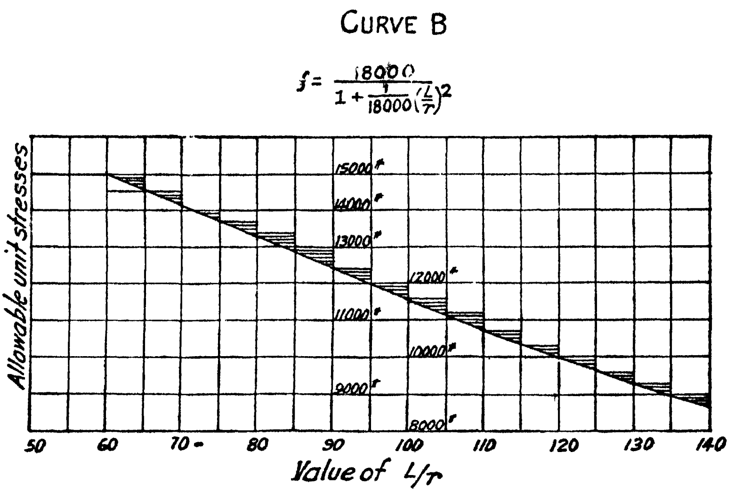
Axial stress on *main members and details* = 18,000 lbs. per square inch net section.

Bolts and rivets = 13,500.

(*b*) *Compression:*

$$\textit{Main Members} = \frac{18{,}000}{1+\frac{1}{18{,}000}\left(\frac{L^2}{r^2}\right)}, \text{ with a maximum of 15,000 lbs.,}$$

wherein L = unsupported length of member in inches and r = least radius of gyration of the cross-section of the member in inch units.



L/r for main members shall not exceed 120 and for bracing and other secondary members 200.

(*c*) *Bending:*

On extreme fiber of rolled beams and plate girders, if laterally supported = 18,000.

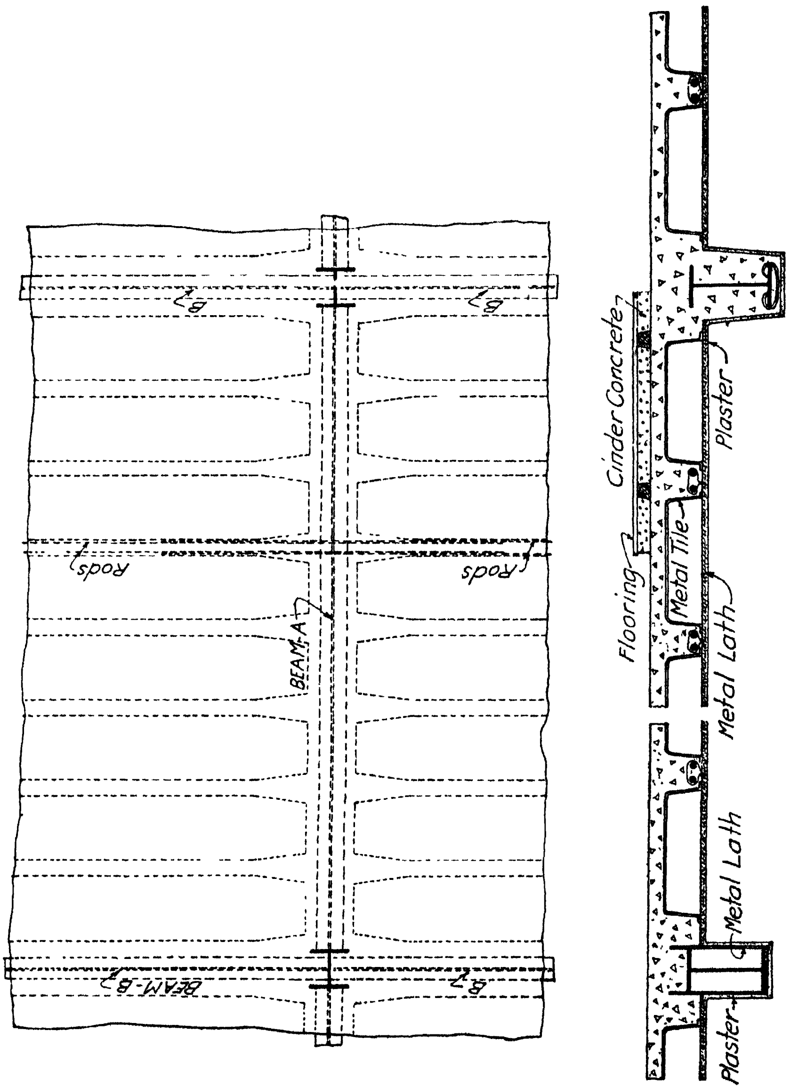
On pins = 27,000

When the unsupported length of the compression flange exceeds 15 times the width, the stresses in the compression flange shall not exceed

$$\frac{20{,}000}{1+\frac{L^2}{2{,}000b^2}},$$

wherein L = unsupported length of the flange in inches and b = width of flange in inches.

The laterally unsupported length of beams and girders shall not exceed 40 times b, the width of the compression flange.


Cinder Concrete
Plaster
Flooring
Metal Tile
Metal Lath
Metal Lath
Plaster
BEAM-A
BEAM-B
Rods
Rods
Fig. 450

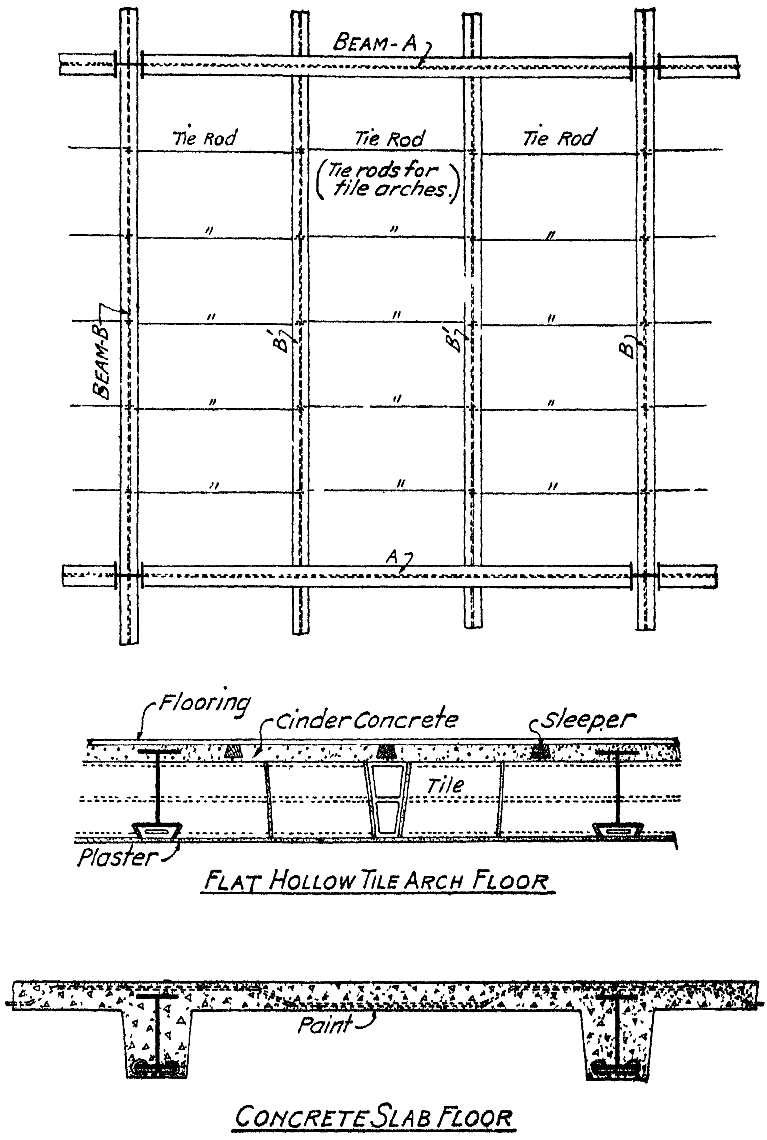

BEAM-A
Tie Rod
Tie Rod
Tie Rod
(Tie rods for tile arches.)
BEAM-B
B'
B'
B
A
Flooring
Cinder Concrete
Sleeper
Tile
Plaster
FLAT HOLLOW TILE ARCH FLOOR
Paint
CONCRETE SLAB FLOOR

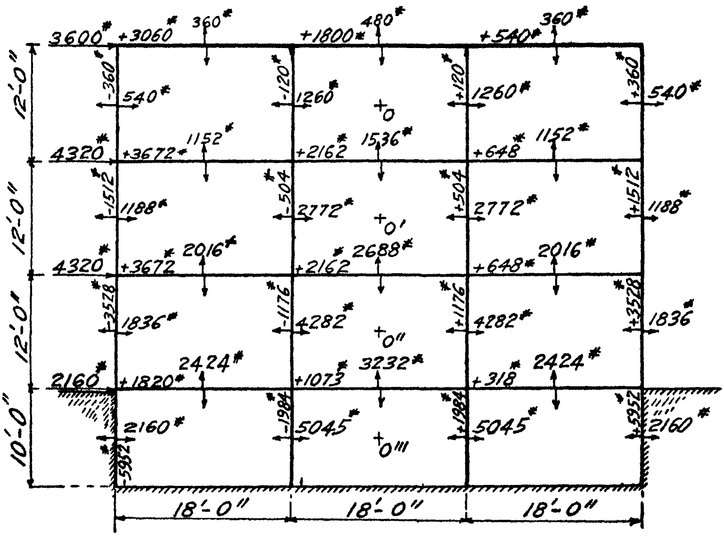
Fig. 451

(*d*) *Shear:*

On pins	13,500
On rivets (power driven)	13,500
On rivets (hand driven)	10,000
On turned bolts	13,500
On unfinished bolts	10,000

(*e*) *Bearing:*

	Double Shear	Single Shear
On pins	30,000	24,000
On rivets (power driven)	30,000	24,000
On rivets (hand driven)	20,000	16,000
On turned bolts	30,000	24,000



Shears and Thrusts on Transverse Bent-AB Due to Wind.

Fig. 452

(*f*) *Combined Stresses.*

For the combined stresses due to wind and other loads the permissible working stresses may be increased one-third, provided the section thus found is not less than required for live and dead load.

291. Wind Stresses.—If the fire wall extends 4 ft. above the roof, the wind load per bay at the roof is

$$20 \times (4+6)18 = 3,600 \text{ lbs.}$$

and at the second and at the third floor it is

$$20 \times 12 \times 18 = 4,320 \text{ lbs.}$$

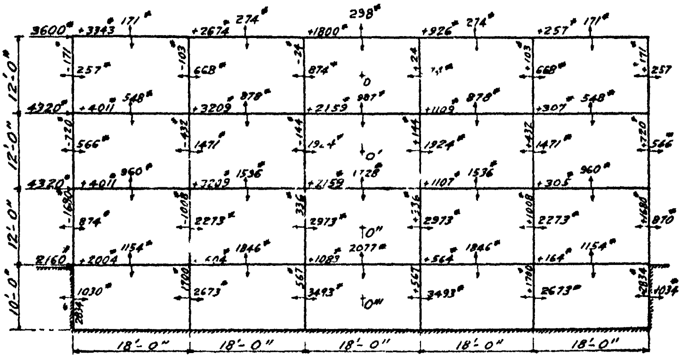
and at the first floor it is

$$20 \times 6 \times 18 = 2{,}160 \text{ lbs.},$$

as shown in Figs. 452 and 453.

Having the wind loads at each floor and roof, we can readily determine the shears and thrusts shown in Figs. 452 and 453, as explained in Art. 286.

292. Design of Roof Slab.—As preliminary we shall assume the tile to be 6 ins. deep and $4\frac{1}{2}$ in. apart; and 2 in. of concrete to be above the tile, as indicated in Fig. 454.



Shears and Thrusts on Longitudinal Bent CD Due to Wind

Fig. 453

As is seen, the roof slab consists of a series of small T-beams. For the weight of the concrete on each of these small T-beams, we have

$$\left[\frac{4.5}{12} \times \frac{6}{12} \times 150\right] + [2 \times 12.5 \times 1.37] = 62 \text{ lbs. (about) per foot.}$$

For the weight of the tile per linear foot of small T-beam we have

$$40 \times \tfrac{1}{2} = 20 \text{ lbs.}$$

Allowing 5 lbs. per square foot for plaster and 10 lbs. per square foot for roof covering, we have

$$62 + 20 + (5 + 10)1.37 = 103 \text{ lbs.}$$

for total dead load per linear foot of small T-beam. Adding the live load to this, we obtain

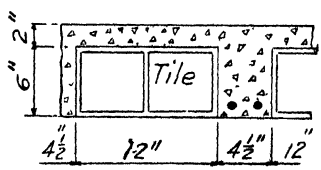


Fig. 454

$$103 + (25 \times 1.37) = 137, \text{ say } 140 \text{ lbs.}$$

for the total load per linear foot on each small T-beam.

Then for the maximum moments due to this load we have

$$\tfrac{1}{12} \times 140 \times \overline{18}^2 \times 12 = 45{,}360 \text{ in. lbs.}$$

at each support and

$$\tfrac{1}{16} \times 140 \times \overline{18}^2 \times 12 = 34{,}020 \text{ in. lbs.}$$

at mid-span.

Taking $j=0.89$ and $d=7$ ins. we obtain

$$F=\frac{34{,}020}{jd}=\frac{34{,}020}{0.89\times 7}=5{,}375 \text{ lbs.}$$

for the stress in steel at mid-span. Then for the area of steel at mid-span we obtain

$$5{,}375\div 16{,}000=0.336 \text{ sq. in.,}$$

which calls for two $\frac{1}{2}$-in. round rods per beam.

In the same manner we obtain

$$F=\frac{45{,}360}{0.89\times 7}=7{,}161 \text{ lbs.}$$

for the stress in the steel over the support. Then for the area of the steel over the support we have

$$7{,}161\div 16{,}000=0.45 \text{ sq. in.,}$$

which calls for three $\frac{7}{16}$-in. round rods, but this steel can be obtained by bending up the $\frac{1}{2}$-in. rods over the supports.

For the end shear on each small T-beam, say 1 ft. from the support, we have

$$(9-1)140=1{,}120 \text{ lbs.}$$

Then we obtain

$$\frac{8}{7}\left(\frac{1{,}120}{4.5\times 7}\right)=51 \text{ lbs.}$$

for the maximum shear on the concrete per square inch, which is satisfactory, since 60 lbs. is allowed. For this it is seen that the tile can extend to within 1 ft. of each support.

The compression on the concrete at mid-span is practically always low. For the compression on the web of the small T-beam at the end of the tile, we have

$$\tfrac{1}{12}\times 140\times 16^2\times 12=35{,}840 \text{ in. lbs.}$$

for the moment. Then taking $k=0.38$ and $j=0.89$, we obtain

$$f_c=\frac{35{,}840\times 2}{0.38\times 0.89\times 7^2\times 4.5}=960 \text{ lbs. per square in.}$$

on the concrete at the bottom of the small web. This stress is too high but it is reduced sufficiently by the steel in the bottom of the beam.

As seen from the foregoing analysis, the roof weighs $137\div 1.37=100$ lbs. per square foot of roof including the live load.

293. Design of Floors.—The live load on each floor is considered to be 80 lbs. per square foot of floor. Let us assume the tile to be 8 in. deep, the concrete above the tile to be $2\frac{1}{2}$ ins. thick, and the distance between the tile to be 6 ins. Then the width of the flange of each small T-beam will be 1.5 ft.

Then for the dead load on each small T-beam per foot of span we have (see Fig. 449)

Flange ..	$2.5\times 12.5\times 1.5=$	46.8 lbs.
Web.....	$0.5\times 0.66\times 150=$	50.0 lbs.
Tile.....	40×0.66 $=$	26.4 lbs.
Plaster ..	5×1.5 $=$	7.5 lbs.
		130.7 lbs., say 131 lbs. per ft.

Weight of flooring

$\frac{3}{4}$-in. flooring.........	$4\times0.75\times1.5=$	4.5 lbs.
2-in. cinder concrete..	$8.33\times2\times1.5=$	25.0 lbs.
Sleepers (2×2).......	(computed) $=$	1.0 lb.
		30.5 lbs. per ft.

If flooring is used we have

$$131+30.5=161.5, \text{ say } 162 \text{ lbs.}$$

for the total dead load per foot on each small T-beam. Adding the 80 lbs. live load, we obtain

$$162+(80\times1.5)=282 \text{ lbs.}$$

for the total load per foot of span on each small T-beam.

Then for the moment at mid-span we have

$$\tfrac{1}{16}\times282\times18^2\times12=68{,}526 \text{ in. lbs.}$$

and for the moment at support we have

$$\tfrac{1}{12}\times282\times18^2\times12=91{,}366 \text{ in. lbs.}$$

Considering the small T-beam at mid-span and assuming $d=9$ ins., and $j=0.9$, we obtain

$$F=\frac{68{,}526}{9\times0.9}=8{,}460 \text{ lbs.}$$

for the total stress on the steel in the bottom of the T-beam. Then we have

$$8{,}460\div16{,}000=0.53 \text{ sq. in.}$$

for the area of steel required. This calls for two $\frac{5}{8}$-in. round rods.

For the area of steel in the top of the slab over the supports, assuming $d=8$ ins., $j=0.87$ (rectangular beam), we have

$$\frac{91{,}366}{9\times0.87\times16{,}000}=0.73 \text{ sq. in.}$$

This calls for two $\frac{3}{4}$-in. round rods.

For the shear 1 ft. from the support we have

$$(9-1)282=2{,}256 \text{ lbs.}$$

Then for the shear per square inch on the small T-beam at this point we have

$$\frac{8}{7}\left(\frac{2{,}256}{6\times9}\right)=48.0 \text{ lbs.,}$$

which is satisfactory, since 60 lbs. is allowed. The moment 1 ft. from the support is (see Art. 66)

$$-91{,}366+12[(9\times282)\times1-(282\times0.5)]=62{,}602 \text{ in. lbs.}$$

Now, taking $j=0.87$ and $k=0.38$ we obtain

$$F=\frac{62{,}602}{0.87\times9}=8{,}000 \text{ lbs. (about)}$$

and

$$f_c=\frac{2\times8{,}000}{0.38\times9\times6}=770 \text{ lbs. per sq. in.}$$

on the concrete at the bottom of the web of the small T-beam. The two ⅝-in. round rods in the bottom will reduce this stress if they extend the full length of the span.

If 700 lbs. per square inch is permitted on the concrete, we have

$$\left(\frac{700}{2}\right)(0.38\times 9)6 = 7{,}182 \text{ lbs.}$$

of the 8,000 lbs. that the concrete will take. So we have

$$8{,}000 - 7{,}182 = 818 \text{ lbs.}$$

that the steel will be required to take.

Assume the steel up 1.25 ins. from the bottom of the web. Then for the stress on the concrete at the location of the steel we have

$$700\left(\frac{3.42-1.25}{3.42}\right) = 444 \text{ lbs. per sq. in.}$$

Then for the allowable compression on the steel we have

$$444\times 14 = 6{,}216 \text{ lbs. per sq. in.}$$

Then for the required area of steel we have

$$818 \div 6{,}216 = 0.14 \text{ sq. in.}$$

The two ⅝-in. round rods have an area of 0.61 sq. in., which is more than enough. So we shall consider the floor slabs as designed satisfactory.

For the weight on these floor slabs per square foot we have

Dead load	...... 162 ÷ 1.5 =	108 lbs. per sq. ft.
Live load	....	80 lbs. per sq. ft.
Total	....	188 lbs. per sq. ft.

The type of floor shown in Fig. 450, which is extensively used, is designed in the manner shown in the foregoing analysis of the clay tile floor. The analysis of other types of floors can be found in the manufacturers' handbooks.

294. Design of Roof Beams.—These beams will be considered to be fixed beams.

The Transverse Beams carry most of the load, as the reinforcing and the small T-beams in the slabs extend perpendicularly to these beams. (See Fig. 449.)

The Interior Transverse Beam will be considered first.

For the load per foot of beam (including live load), assuming the beam and extra concrete around the beam to weigh 210 lbs. per foot of beam, we have

$$(100\times 18) + 210 = 2{,}010 \text{ lbs.}$$

Then for the moment at each end we have

$$\tfrac{1}{12}\times 2{,}010\times \overline{18}^2\times 12 = 651{,}240 \text{ in. lbs.}$$

As seen from Fig. 452, the maximum shear on these beams due to wind is 480 lbs. Then for the moment at each end of the beam due to wind we have

$$480\times 9\times 12 = 51{,}840 \text{ in. lbs.,}$$

which is less than one-third of the moment due to dead and live load; hence the moment due to wind will be ignored.

So we obtain

$$651{,}240 \div 18{,}000 = 36.1$$

for the required section modulus, which calls for a 12-in.x28-lb. beam, which has a deflection of less than $\frac{1}{360}$ of the span. This beam is satisfactory for all transverse interior roof beams.

The End Transverse Roof Beams have a load of about

$$(100 \times 9) + 210 = 1{,}110 \text{ lbs. per ft.}$$

from the roof slab. In addition to this load these beams carry the fire wall, which we shall assume is composed of 8-in. hollow tile veneered with 4-in. brick. Then we obtain the following weight per square foot of wall:

Tile	$\frac{2}{3} \times 40 = 26.6$	lbs. per sq. ft.
Brick	.. $\frac{1}{3} \times 140 = 46.6$	lbs. per sq. ft.
Total	. 73.2	lbs. per sq. ft.

Then for the weight of the fire wall per foot of beam we have

$$73.2 \times 4 = 292.8, \text{ say } 293 \text{ lbs.}$$

Adding this to the weight from the roof slab, we have

$$1{,}110 + 293 = 1{,}403 \text{ lbs.}$$

for the total load per foot on the end transverse beams.

Then we have

$$\tfrac{1}{12} \times 1{,}403 \times 18^2 \times 12 = 454{,}572 \text{ in. lbs.}$$

for the maximum moment at each end of these end transverse beams.

Then we obtain

$$454{,}572 \div 18{,}000 = 25.3$$

for the required section modulus, which calls for 12-in.x25-lb. beam.

Interior Longitudinal Roof Beams, as seen from Fig. 449, carry only a small strip of live load—about 2.5 ft. wide. The only dead load carried is the weight of the beam and the concrete surrounding the beam and the floor extending out about 1.25 ft. to each side of the beam. This dead load can be estimated, which in this case amounts to about 390 lbs. per linear foot of beam. Then, adding the live load to this dead load, we obtain

$$390 + (25 \times 2) = 440 \text{ lbs.}$$

for the total load per linear foot of beam. Then we obtain

$$\tfrac{1}{12} \times 440 \times 18^2 \times 12 = 142{,}560 \text{ in. lbs.}$$

for the maximum moment at each end of an interior longitudinal roof beam due to live and dead load.

The maximum shear on these beams at mid-span due to wind, as seen in Fig. 453, is 298 lbs.

Then for the moment at the end of each beam due to wind we have

$$298 \times 9 \times 12 = 32{,}184 \text{ in. lbs.,}$$

which is less than one-third of the moment due to live and dead load; hence the moment due to wind will be ignored.

Then we obtain

$$142,560 \div 18,000 = 7.9$$

for the required section modulus. This calls for a very small beam, but the one used must not have a deflection greater than $\frac{1}{360}$ of the span length. The length of the span is 18x12 = 216 in. Then for the maximum deflection allowed we have

$$216 \div 360 = 0.6 \text{ in.}$$

From tables found in manufacturers' handbooks* we find that a 10-in.x23-lb. beam is satisfactory as regards deflection. This will be used, although the section modulus is 24.4 (Carnegie), which is excessive.

The Exterior Longitudinal Roof Beams support the fire wall, which, as previously given, weighs 293 lbs. per foot of beam, and the surrounding concrete, which weighs about 230 lbs. (estimated). So for the total dead load on each beam we have

$$293 + 230 = 523 \text{ lbs. per foot}$$

of span. The live load is not considered in this case because the fire wall occupies the space above the beam.

Then we obtain

$$\tfrac{1}{12} \times 523 \times 18^2 \times 12 = 169,452 \text{ in. lbs.}$$

for the maximum moment at each end of the beam, which is due to dead load.

From Fig. 453 it is seen that the maximum shear due to wind is

$$298 \div 2 = 149 \text{ lbs.}$$

So for the moment at each end due to wind we have

$$149 \times 9 \times 12 = 16,092 \text{ in. lbs.}$$

but, as this is less than one-third of the moment due to dead load, it will be ignored.

Then we have

$$169,452 \div 18,000 = 9.4$$

for the required section modulus. This calls for a very small beam, but to provide against excessive deflection we shall use the 10-in.x23-lb. beam, the same as was used for the interior longitudinal roof beams.

This completes the designing of the roof beams. The diagram of the roof framing can be drawn as shown in Fig. 456.

295. Design of Third-Floor Beams.—*The Interior Transverse Beam* will be considered first. As previously shown, the floor load is 188 lbs. per square foot.

Then for the toal load per linear foot of beam we have

Floor load (live+dead)	188×18 = 3,384 lbs. per ft.
Beam (assumed)	36 lbs. per ft.
Concrete surrounding beam (estimated)	220 lbs. per ft.
Total	3,640 lbs. per ft.

* See handbook issued by the American Institution of Steel Construction Inc.

So for the maximum moment at each end due to dead and live load we have

$$\tfrac{1}{12}\times 3{,}640\times 18^2\times 12=1{,}179{,}360 \text{ in. lbs.}$$

As seen from Fig. 452, the maximum shear on these beams due to wind is 1,536 lbs. Then we obtain

$$1{,}536\times 9\times 12=165{,}888 \text{ in. lbs.}$$

for the moment at each end of the beam due to wind; but, as this is less than one-third of the moment due to dead and live load, it will be ignored.

Then we have

$$1{,}179{,}360\div 18{,}000=65.4$$

for the required section modulus. This calls for 16-in.x40-lb. beam, which will be used.

The End (or Exterior) Transverse Floor Beams support the exterior end wall, which we shall assume to be composed of 8-in. hollow clay tile veneered with 4-in. brick. This wall, as shown in the preceding article, weighs about 73.2 lbs. per square foot. Adding 5 lbs. for plaster, we obtain 78.2, say 78 lbs. for the total weight of the wall per square foot of surface.

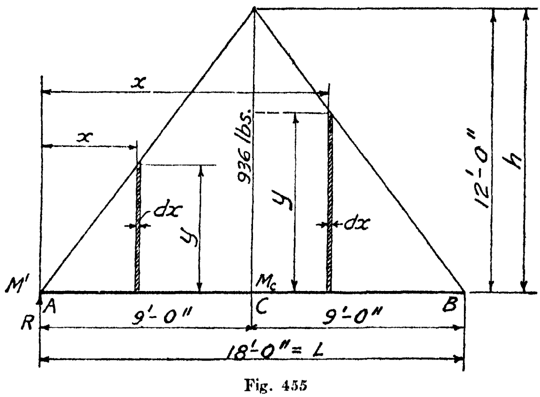


Fig. 455

Considering that windows are omitted, this load will not be uniformly distributed along the beam on account of the arch action of the wall. Experiments seem to indicate that the load will be triangular, as indicated in Fig. 455, that is, the load varies from 0 at each end to

$$78\times 12=936 \text{ lbs.}$$

at mid-span.

Let $w=78$. Then, taking A as the origin, the load at any point between A and B is

$$P=wydx \quad\text{.............. (1)}$$

Between A and C we have

$$\frac{y}{h}=\frac{x}{L/2},$$

from which we obtain

$$y=\frac{2xh}{L}.$$

Then, substituting this value of y in (1), we obtain

$$P=\frac{2whxdx}{L} \qquad \qquad (2)$$

for the value of the load at any point between A and C.

For the value of y between C and B we obtain

$$y=\frac{2h-2xh}{L}.$$

Substituting this value of y in (1), we obtain

$$P_1=2whdx-2w\left(\frac{h}{L}\right)xdx \qquad \qquad (3)$$

for the load at any point between C and B.

Now, $kL=x$ and hence $k=\frac{x}{L}$.

Now, substituting in (7), page 92, we obtain

$$M'=2wh\int_0^{\frac{L}{2}}\left(\frac{2x^3}{L^2}-\frac{x^4}{L^3}-\frac{x^2}{L}\right)dx-2wh\int_{\frac{L}{2}}^{L}\left(\frac{2x^3}{L^2}-\frac{x^4}{L^3}-\frac{x^2}{L}\right)dx+ 2wh\int_{\frac{L}{2}}^{L}\left(\frac{2x^2}{L}-\frac{x^3}{L^2}-x\right)dx,$$

from which we obtain

$$M'=-\frac{whL^2}{30}+\frac{whL^2}{30}-10\frac{whL^2}{192}=-0.052\ whL^2. \qquad \qquad (4)$$

for the maximum moment at each end of the beam where there are no windows in the wall.

The end reaction, as seen from Fig. 455, is equal to $\frac{1}{4}whL$. Then according to Art. 66 (page 84), taking moments about C, we obtain

$$M_c=-0.052whL^2+\frac{whL^2}{8}-\frac{whL^2}{24}=+0.032whL^2 \qquad \qquad (5)$$

for the maximum moment at C (mid-span.)

Now, substituting in (4), we obtain

$$-0.052\times 78\times 12\times 18^2\times 12=-189{,}237 \text{ in. lbs.}$$

for the moment at the end of the beam due to the weight of the wall alone.

The floor slab including live load weighs 188 lbs. per square foot as previously shown. We shall assume the beam to weigh 40 lbs. per foot and the surrounding concrete to weigh 220 lbs. (estimated) per foot of beam.

Then we have

$$188\times 9+40+220=1{,}952 \text{ lbs.}$$

for the total uniform load per linear foot of beam.

For the moment at each end due to this load we obtain

$$-\tfrac{1}{12}\times 1{,}952\times \overline{18}^2\times 12 = -622{,}448 \text{ in. lbs.}$$

Adding the moment due to the wall to this moment, we obtain

$$-(622{,}448+189{,}237) = -811{,}685 \text{ in. lbs.}$$

for the total maximum moment at the end of the beam, which is the maximum moment on the beam due to live and dead load. The moment due to wind is one-half of that on the interior beams (given above), which is much less than one-third of the moment due to live and dead load. So the moment due to wind will be neglected.

Then we obtain

$$811{,}685\div 18{,}000 = 45.2$$

for the required section modulus. This calls for a 14-in.x33-lb. beam, which will be used.

If there are windows in the bay and the window frames are practically continuous for the length of the bay, we should consider the wall to extend up to about 3 ft. above the beam, in which case the wall would weigh 78x3 = 234 lbs. per foot of beam; and we should consider the wall surface above, including glass and frames, to weigh 8 lbs. per square foot of surface, which makes 8x9 = 72 lbs. per foot of beam.

Then for the moment at the end of the beam due to the entire wall we obtain

$$-\tfrac{1}{12}\times(234+72)18^2\times 12 = -99{,}144 \text{ in. lbs.}$$

Adding this to the moment due to the load from the floor slab, surrounding concrete, and beam, we obtain

$$-(622{,}448+99{,}144) = -721{,}592 \text{ in. lbs.}$$

For the moment due to wind (see Fig. 452) we have

$$\frac{1{,}536}{2}\times 108 = \mp 82{,}944 \text{ in. lbs.,}$$

which is less than one-third of the moment due to dead and live load; hence the moment due to wind will be neglected.

Then for the required section modulus we have

$$721{,}592\div 18{,}000 = 40.2,$$

which calls for a 14-in.x30-lb. beam, which may be used where windows are continuous throughout the bay.

However, no exact stipulated value of the weight of the wall can be taken in all cases because the weight varies with the size and number of windows in the bay. Hence the weight should be carefully computed in each case and any part of the weight applied as a uniform load should be considered as uniformly distributed and any part applied as a concentrated load should be considered concentrated in computing the moment on the beam.

Interior Longitudinal Beams, as seen from Fig. 449, support only a narrow strip (2.5 ft. wide) of the floor. Then for the total load on the beam we obtain

Floor slab............................	10.5×12.5×2.5=328 lbs. per ft.
Beam (assumed).......................	30 lbs. per ft.
Concrete surrounding beam (computed)..	182 lbs. per ft.
Live load............................	80×2.5=200 lbs. per ft.
Total................................	740 lbs. per ft.

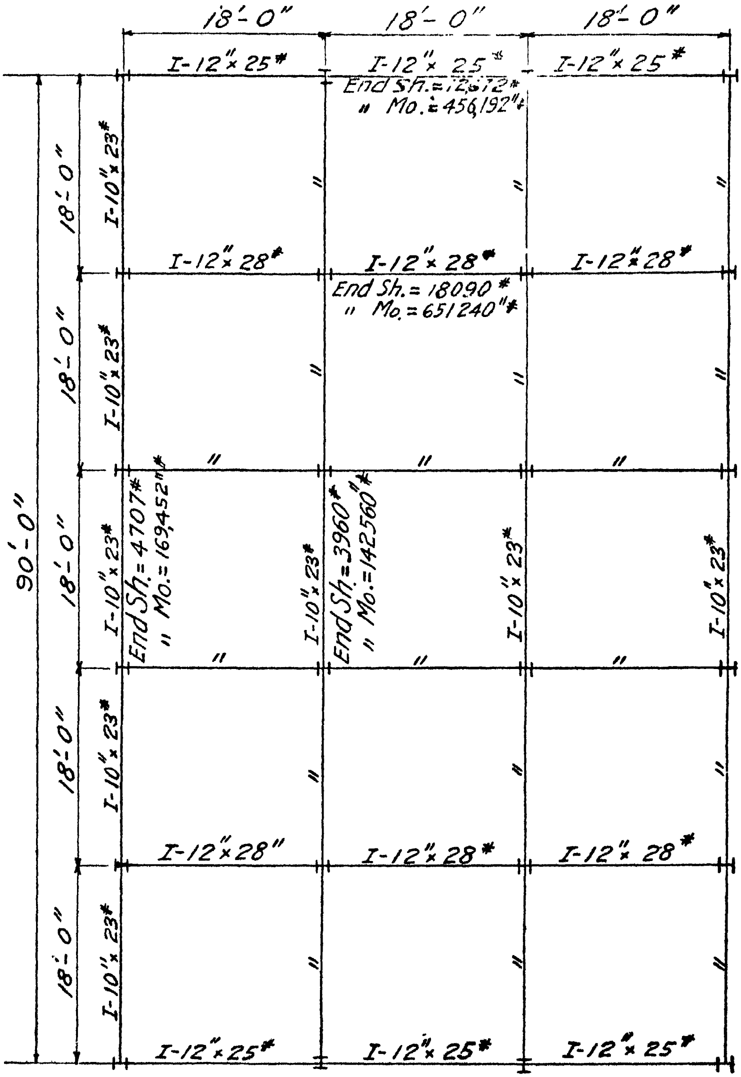
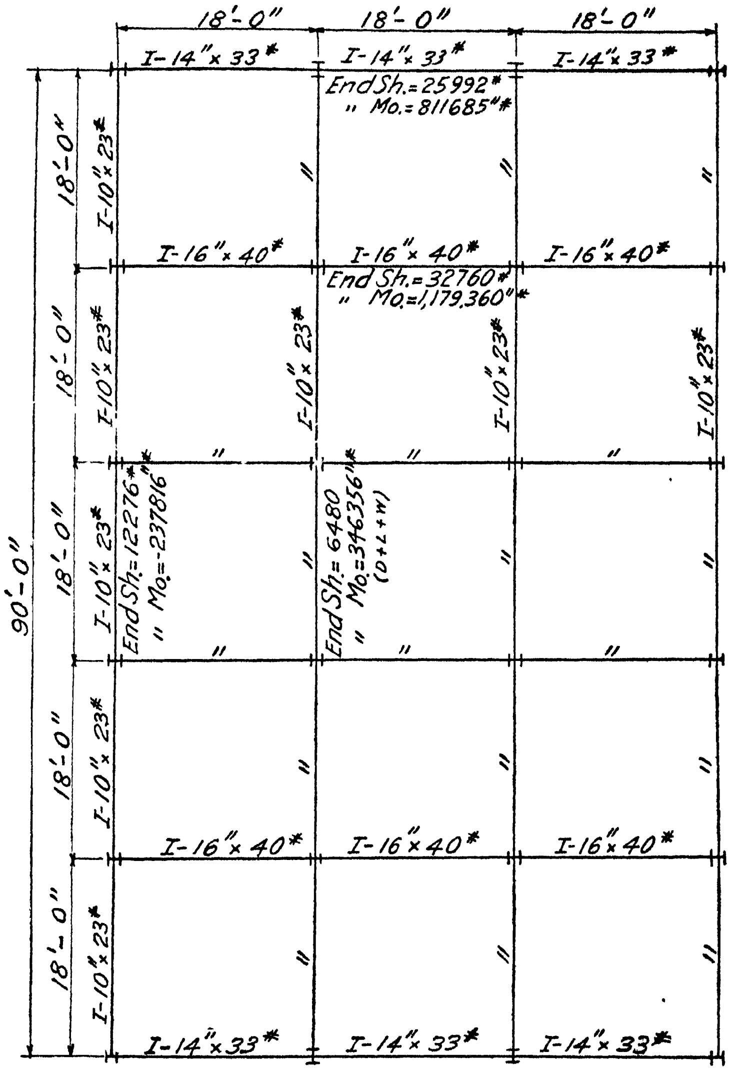


Fig. 456—Roof plan

Then for the moment at each end of the beam due to dead and live load we have

$$-\frac{1}{12}\times 740\times 18^2\times 12=-239,760 \text{ in. lbs.}$$



THIRD FLOOR PLAN
(First and second Floor Plan same as Third Floor)

Fig. 457

For the moment due to wind, using the greatest shear (see Fig. 453), we have

$$987 \times 9 \times 12 = 106{,}596 \text{ in. lbs.,}$$

which is greater than one-third of the moment due to dead and live load. So we obtain

$$(239{,}760 + 106{,}596) \div 18{,}000 \times 1\tfrac{1}{3} = 14.4$$

for the required section modulus, which calls for an 8-in.x17.5-lb. beam. But we shall use a 10-in.x23-lb. beam to limit the deflection to $\frac{1}{360}$th of the span length.

The Exterior Longitudinal Floor Beams in third floor support the exterior wall, the weight of the beam, the concrete surrounding the beam, and a strip of the slab about 1.5 ft. wide. Then, considering the windows to be continuous, we have −99,144 in. lbs. (as found above for the end transverse beams) for the moment at each end of the beam due to the weight of the wall.

For the other dead load we have

Slab..........................	$10.5 \times 12.5 \times 1.5 =$	197 lbs. per ft. of beam.
Beam (assumed)...............		21 lbs. per ft. of beam.
Concrete surrounding the beam (computed)...............		210 lbs. per ft. of beam.
Total		428 lbs. per ft. of beam.

Then for the moment at each end of the beam due to this load we have

$$-\tfrac{1}{12} \times 428 \times \overline{18}^2 \times 12 = -138{,}672 \text{ in. lbs.}$$

Adding this to the moment due to the wall, we obtain

$$-(99{,}144 + 138{,}672) = -237{,}816 \text{ in. lbs.}$$

for the total maximum moment at the end of the beam due to dead load.

For the moment due to wind load we have

$$\frac{987}{2} \times 9 \times 12 = 53{,}298 \text{ in. lbs.,}$$

which is less than one-third of the moment due to dead load. (There is no live load on these beams.) So the moment due to wind load will be ignored.

Then we obtain

$$237{,}816 \div 18{,}000 = 13.2$$

for the required section modulus, which calls for an 8-in.x17.5-lb. beam. But to limit the deflection we shall use a 10-in.x23-lb. beam.

The third-floor plan shown in Fig. 457 can now be drawn.

296. Design of Second-Floor Beams.—The moments due to dead and live load will be the same as found in Art. 295 for the floor beams in the third floor.

The Transverse Interior Floor Beams have a maximum shear of 2,688 lbs. due to wind, as seen in Fig. 452. Then for the moment at each end of the beam due to wind we have

$$2{,}688 \times 9 \times 12 = \pm 290{,}304 \text{ in. lbs.}$$

The moment at the end of each beam due to live and dead load, as given in Art. 295, is 1,179,360 in. lbs. As seen, the moment due to wind is less than one-third of this, hence the moment due to wind will not be considered and the 16-in.x40-lb. beam used in the third floor will be used.

The Exterior Transverse Floor Beams on the Second Floor have a maximum shear of

$$2{,}688 \div 2 = 1{,}344 \text{ lbs.}$$

due to wind. Then for the maximum moment at each end due to wind we have

$$1{,}344 \times 9 \times 12 = 145{,}152 \text{ in. lbs.}$$

As shown in Art. 295, the moment due to dead and live load is 721,592 in. lbs., considering windows to be continuous. The moment due to wind will not be considered, since it is less than one-third of the moment due to dead and live load. Hence the 14-in.x33-lb. beams will be used, as for the third floor.

The Interior Longitudinal Floor Beams on the Second Floor have a maximum shear of 1,728 lbs. due to wind, as shown in Fig. 453. Then for the maximum end moment due to wind we have

$$1{,}728 \times 9 \times 12 = \pm 186{,}624 \text{ in. lbs.}$$

As shown in Art. 295, the maximum end moment due to live and dead load is $-239{,}760$ in. lbs. Adding this to the moment due to wind, we obtain

$$-(186{,}624 + 239{,}760) = -426{,}384 \text{ in. lbs.}$$

Then we obtain

$$426{,}384 \div 18{,}000 \times 1\tfrac{1}{3} = 17.8,$$

which calls for 9-in.x20.5-lb. beam. But we shall use a 10-in.x23-lb. beam on account of deflection—as on the third floor.

The Exterior Longitudinal Floor Beams on the Second Floor have a maximum shear of $1{,}728 \div 2 = 864$ lbs. due to wind. Then for the end moment due to wind we have

$$864 \times 9 \times 12 = \pm 73{,}872 \text{ in. lbs.}$$

As shown in Art. 295, the end moment due to dead load (considering the windows to be continuous) is $-237{,}816$ in. lbs. Now, as seen, the moment due to wind is less than one-third of this. So we shall use the 10-in.x23-lb. beam, as on the third floor.

297. Design of First-Floor Beams.—The moments due to dead and live load will be the same as found in Art. 295 for the corresponding beams in the third floor.

The Interior Transverse Floor Beams on the First Floor have a maximum shear due to wind of 3,232 lbs., as shown in Fig. 452. Then for the end moment due to wind we have

$$3{,}232 \times 9 \times 12 = \pm 349{,}056 \text{ in. lbs.}$$

As previously shown, the end moment due to dead and live load is 1,179,360 in. lbs. As seen, the moment due to wind is less than one-third of this, so the 16-in.x40-lb. beam will be used—as on the second and third floors.

The Exterior Transverse Floor Beams on the First Floor have a maximum shear of $3{,}232 \div 2 = 1{,}616$ lbs. due to wind. Then for the maximum end moment due to wind we have

$$1{,}616 \times 9 \times 12 = \pm 174{,}528 \text{ in. lbs.}$$

As previously shown, the moment due to dead and live load is −721,592 in. lbs., considering windows continuous. As is seen, the moment due to wind is less than one-third of this; so the 14-in.x33-lb. beams will be used, as on the second and third floors.

The Interior Longitudinal Floor Beams on the First Floor have a maximum shear of 2,077 lbs. due to wind (see Fig. 453). Then, for the end moment due to wind we have

$$2{,}077 \times 9 \times 12 = \pm 224{,}316 \text{ in. lbs.}$$

As previously shown, the maximum end moment due to dead and live load is −239,760 in. lbs. Adding this to the moment due to wind, we obtain −464,076 in. lbs. Then we obtain

$$464{,}076 \div 18{,}000 \times 1\tfrac{1}{3} = 19.3$$

for the required section modulus, which calls for 10-in.x23-lb. beams, which is the same as were used in the second and third floors.

The Exterior Longitudinal Floor Beams on the First Floor have a maximum shear of 2,077 ÷ 2 = 1,038 lbs. due to wind. Then for the end moment due to wind we have

$$1{,}038 \times 9 \times 12 = \pm 112{,}104 \text{ in. lbs.}$$

As previously shown, the maximum end moment due to dead and live load is −237,816 in. lbs. Adding this to the moment due to wind, we have −349,920 in. lbs. for the total maximum end moment.

Then we obtain

$$349{,}920 \div 18{,}000 \times 1\tfrac{1}{3} = 14.5$$

for the required section modulus, which calls for an 8-in.x17.5-lb. beam. But on account of deflection the 10-in.x23-lb. beam will be used, as on the second and third floors.

The floor plan for the first and second floors would be the same as the plan of the third floor shown in Fig. 457.

298. Design of Interior Columns.—

Third-Story Interior Columns support the dead and live load from the roof. The maximum load (live+dead) from the roof, as seen from Fig. 456, is

$$(2 \times 18{,}090) + (2 \times 3{,}960) = 44{,}100 \text{ lbs. per column.}$$

For the part of this load duet o live load alone, (doubling live load to provide for extra load as tanks and so forth) we have

$$50 \times 18 \times 18 = 16{,}200 \text{ lbs. per column.}$$

As the direct load is light and the moment on these columns due to wind and to unsymmetrical live load will be comparatively small, it is evident that quite a small column section is required (theoretically), but on account of the roof beam connections and desired rigidity in general an 8-in. beam is about the smallest section that can be used satisfactorily.

So we shall assume each interior column in the third story to be an 8-in.x31-lb. beam, the properties of which are as follow (see manufacturers' handbooks):

Area of cross-section............	9.10 sq. ins.
I (axis perpendicular to web).....	110.9
r (axis perpendicular to web).....	3.49
I (axis parallel to web)..........	36.7
r (axis parallel to web)..........	2.01
Width of flange.................	8 ins.

Taking the story depth of 12 ft. as L, we obtain

$$\frac{L}{r}=\frac{12\times 12}{2.01}=71.6.$$

Now, from Curve B, Art. 290 (page 671), we find that the allowable unit stress on this assumed column is 14,000 lbs. per square inch. Then we obtain

$$14{,}000\times 9.10=127{,}400 \text{ lbs.}$$

for the direct load that the column could support, which shows that the cross-section of the column as assumed is larger than need be as far as direct load is concerned.

The maximum shear on each column due to wind, given in Fig. 452, is 1,260 lbs. Then for the moment at the top (at connection of roof beams) of each column, due to wind, we have

$$1{,}260\times 6\times 12=90{,}720 \text{ in. lbs.}$$

In addition to this moment due to wind there can be a moment due to unsymmetrical distribution of the live load, as explained in Appendix B. The moment at the top of the column due to the unsymmetrical live load can be obtained from Formula (K), Fig. 458, where

C'_{ab} = moment at the end of beam AB due to live load in span AB.
C_{fg} = moment at the end of beam FG due to live load in span FG.
S'_2 = stiffness factor (I/L) of column AF.
S'_3 = stiffness factor (I/L) of beam AB.
S_3 = stiffness factor (I/L) of beam FG.
S_2 = stiffness factor (I/L) of column FD.

For substituting in Formula (K), Fig. 458, we have the following values:

$$C'_{ab}=\tfrac{1}{12}\times(50\times 18)\times\overline{18}^2\times 12=291{,}600 \text{ in. lbs.}$$

$$C_{fg}=\tfrac{1}{12}\times(80\times 18)\times\overline{18}^2\times 12=466{,}560 \text{ in. lbs.}$$

$$S'_2=\frac{I}{L}=110.9\div(12\times 12)=0.77.$$

$$S'_3=\frac{I}{L}=213.4\div(18\times 12)=0.988.$$

$$S_3=\frac{I}{L}=524.6\div(18\times 12)=2.42.$$

$$S_2=0.80 \text{ (assumed).}$$

Now, substituting the above values in Formula (K), Fig. 458, we obtain

$$M_{ab}=127{,}800+34{,}500=162{,}300 \text{ in. lbs.}$$

for the moment at the top of the column due to unsymmetrical live load.

This moment occurs when one bay on the roof and one bay on the third floor are not loaded with live load. These unloaded spans are adjacent to the column considered and in the same bay one above the other as explained in Appendix *B*.

The moment on the column due to wind will occur regardless of the distribution of the live load. So we shall first consider the stress due to full live, dead, and wind load combined.

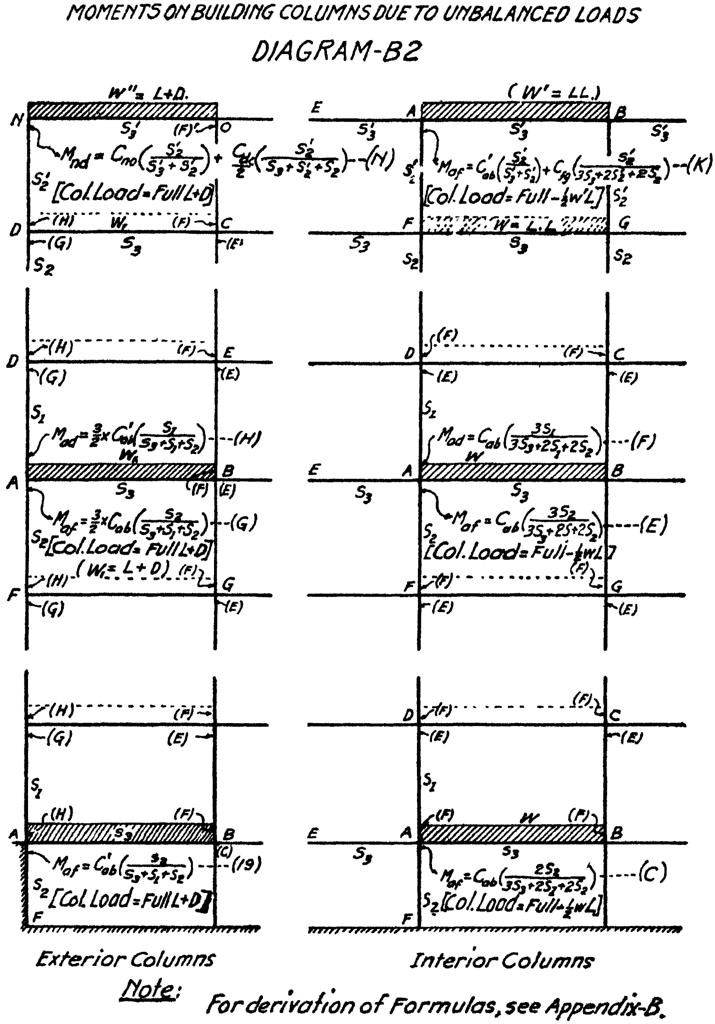


Fig. 458

For the direct load on the column, we have

Dead + live............	44,100 lbs.	(given above)
Wind..................	120 lbs.	(see Fig. 452)
Total...............	44,220 lbs.	

(This direct wind load is usually ignored, as it has relatively small value.)

Then for the direct compressive stress per square inch on the column we have

$$44,220 \div 9.10 = 4,860 \text{ lbs.}$$

For the stress due to the moment from wind we have

$$f = \frac{My}{I} = \frac{90,720 \times 4}{110.9} = 3,272 \text{ lbs. per sq. in.}$$

Then, we obtain

$$4,860 + 3,272 = 8,132 \text{ lbs.}$$

for the maximum stress per square inch on the column due to wind and full dead and live load. As is seen, this is low.

We shall now include the stress due to the moment resulting from unsymmetrical live load. In this case we obtain

$$f' = \frac{(90,720 + 162,300)4}{110.9} = 9,130 \text{ lbs.}$$

for the maximum stress per square inch due to moment.

For the direct stress from live, dead, and wind load in this case we have

$$f'' = \frac{44,200 - \frac{1}{2}(16,200)}{9.10} = 3,970 \text{ lbs.}$$

Then, collecting the unit stresses, we have

$$9,130 + 3,970 = 13,100 \text{ lbs.}$$

for the maximum stress per square inch on the column, considering the transverse direction of the building.

Considering the longitudinal direction of the building, it is obvious that the moment due to unsymmetrical live load is negligible owing to the live loads being carried mostly to the transverse beams by the roof and floor slabs.

The maximum shear on a column (in the third story) due to wind, as given in Fig. 453, is 874 lbs.

Then for the moment due to wind (longitudinal direction) we obtain

$$874 \times 6 \times 12 = 62,928 \text{ in. lbs.}$$

For the stress per square inch on the column (at roof), due to this moment, we have

$$\frac{62,928 \times 4}{36.7} = 6,860 \text{ lbs.}$$

For the direct stress per square inch we have

$$\frac{44,100 + 24}{9.10} = 4,850 \text{ lbs.}$$

Then we obtain $6,860 + 4,850 = 11,710$ lbs.

for the maximum stress per square inch on the column, considering the longitudinal direction of the building.

It is seen from the foregoing analysis that the 8-in.x31-lb. beam is larger than required theoretically, as the allowable unit stress from the column formula is 14,000 lbs. and the greatest combined unit stress obtained is 13,100 lbs., but the 13,100-lb. stress is at the roof connection where column action is not involved. Hence the 14,000-lb. stress could be safely increased at least one-third, which would increase the allowable unit stress for the combined stress to $14,000 \times 1\frac{1}{3} = 18,666$ lbs.

So we shall investigate smaller sections.

The greatest value of L/r permissible is 120, and a lesser value is preferable, as a more rigid structure would be obtained. Then for the minimum radius of gyration allowed we have

$$\frac{144}{120} = 1.2.$$

This means that no section having a radius less than 1.2 can be used.

Glancing over the tables of I-beams in manufacturers' handbooks we find that the lighter sections have a radius of gyration less than 1.2 or that the flanges are too narrow for properly connecting the roof beams to the columns, unless we change to a 10-in.x26-lb. beam, in which case

$$\frac{L}{r} = \frac{144}{1.43} = 101.$$

Then from Curve B (page 671) we find the allowable unit stress on the column to be 11,550 lbs.

In the recalculation the value of $S_2' = 139.5/144 = 0.968$ and the same value is assumed for S_2. Then, substituting these values in Formula (K), Fig. 458, along with the other values used before, we obtain

$$M_{ab} = 144,300 + 40,600 = 184,900 \text{ in. lbs.}$$

for the maximum moment on the column due to unsymmetrical distribution of the live load. Adding this to the moment due to wind load, we obtain

$$184,900 + 90,720 = 275,620 \text{ in. lbs.}$$

for the total maximum moment on the column (at the roof). Then we obtain

$$\frac{275,620 \times 5}{139.5} = 9,880 \text{ lbs.}$$

for the stress per square inch on the column due to moment.

For the stress per square inch due to direct load we have

$$\frac{44,220 - \frac{1}{2}(16,200)}{7.64} = 4,730 \text{ lbs.}$$

Then we obtain

$$9,880 + 4,730 = 14,610 \text{ lbs.}$$

As shown above, the allowable unit stress obtained from the column formula is 11,550 lbs. This can be increased to $11,550 \times 1\frac{1}{3} = 15,400$ lbs. for the allowable stress due to direct load and cross-bending combined.

So, it is seen, the column section is satisfactory. This last section is the best we can obtain if beams are used for the columns, since the radius of gyration and flange width are about the minimum allowed.

Now, suppose we try four angles (latticed). The radius of gyration must not be less than 1.2. From the tables in the back of the book we find that the lightest angles that can be used are four-4-in.x3-in.x$\frac{5}{16}$-in., which weigh 7.2x4 = 28.4 lbs. per foot, which is more than the beam weighs; but to the cost of these angles must be added the additional shop work. Moreover, the lattice does not carry the shear from the wind satisfactorily. To obtain a section as satisfactory as the beam, a web plate would have to be used with the angles, which would also increase the weight and the shop work. So it is seen that the most satisfactory section for the interior columns in the third story is the 10-in.x26-lb. beams; hence they will be used.

Second-story Interior Columns support the load from the roof and the load from the third floor. This total load, as seen from Figs. 456 and 457, is equal to

$$2(18,090+3,960)+2(32,760+6,840)=123,300 \text{ lbs.},$$

which is applied at the column just below the third floor.

We shall assume that a 10-in x31-lb. beam will be used for the column section. So we have

$$\frac{L}{r}=\frac{144}{1.89}=76.2.$$

Then from Curve *B* (page 671) we find that 13,600 lbs. is the allowable unit stress on the column. For the direct unit stress we have

$$123,300 \div 9.1 = 13,560 \text{ lbs.},$$

which is about equal to the allowable direct stress alone. We shall need a heavier section, most likely, to provide for the moment due to wind and unsymmetrical live load. Let us try a 10-in.x42-lb. beam. Then we have

$$\frac{L}{r}=\frac{144}{1.73}=83.$$

Then from Curve *B* (page 671) we find that 13,200 lbs. is the allowable unit stress on the column. For the direct stress we have

$$123,200 \div 12.35 = 9,985 \text{ lbs. per sq. in.}$$

The maximum shear on the column due to wind, as given in Fig. 452, is 2,772 lbs. Then for the moment on the column (at the third floor) due to wind we have

$$2,772 \times 6 \times 12 = 199,584 \text{ in. lbs.}$$

For the stress on the column due to this moment we have

$$\frac{199,584 \times 5}{190.4} = 5,230 \text{ lbs. per sq. in.}$$

Then we obtain

$$9,985 + 5,230 = 15,215 \text{ lbs.}$$

for the combined unit stress due to dead, live, and wind load. For the allowable we have

$$13,200 \times 1\tfrac{1}{3} = 17,600 \text{ lbs.},$$

which shows the section to be satisfactory for dead, live, and wind load.

We shall next include the stress due to unsymmetrical live load. For the direct load in that case we have

$$123{,}300-(80\times 18\times 9)=110{,}340 \text{ lbs.}$$

Then for the direct unit stress we have

$$110{,}340\div 12.35=8{,}934 \text{ lbs.}$$

The moment due to unsymmetrical live load can be obtained from Formula (E), Fig. 458, where

$$C_{ab}=\tfrac{1}{12}(80\times 18)\times \overline{18}^2\times 12=466{,}560 \text{ in. lbs.}$$
$$S_2=190.4\div(12\times 12)=1.32 \text{ in. lbs.}$$
$$S_1=139.5\div(12\times 12)=0.968 \text{ in. lbs.}$$
$$S_3=524.6\div(18\times 12)=2.42 \text{ in. lbs.}$$

Substituting the above values in Formula (E), Fig. 458, we obtain

$$M_{af}=466{,}560\times 0.3344=156{,}170 \text{ in. lbs.}$$

for the moment on column due to unsymmetrical live load. Then we obtain

$$\frac{156{,}170\times 5}{190.4}=4{,}100 \text{ lbs.}$$

for the stress per square inch on the column due to the moment caused by the unsymmetrical live load.

Then we have

$$5{,}230+8{,}934+4{,}100=18{,}264 \text{ lbs.}$$

for the total combined stress on the column due to dead, wind, and unsymmetrical live load. As seen, this is only 664 lbs. greater than the allowable stress, so the 10-in.x42-lb. beams will be considered satisfactory, considering the transverse direction of the building.

Considering the longitudinal direction, we find from Fig. 453 that the maximum shear on the columns due to wind is 1,924 lbs. Then for the moment due to wind we have

$$1{,}924\times 6\times 12=138{,}530 \text{ in. lbs.}$$

Then for the stress due to this moment we have

$$\frac{138{,}530\times 4.16}{36.8}=15{,}660 \text{ lbs.}$$

Adding this to the direct stress, we obtain

$$8{,}934+15{,}660=24{,}594 \text{ lbs.,}$$

which is too high, as 17,600 lbs. is allowed. Let us try a 10-in.x49-lb. beam. Then, for the direct stress, we have

$$123{,}300\div 14.4=8{,}560 \text{ lbs.}$$

and for the stress due to moment, we have

$$\frac{138{,}530\times 5}{93}=7{,}460 \text{ lbs. per sq. in.}$$

Then we have

$$8{,}560+7{,}460=16{,}020 \text{ lbs.}$$

for the combined stress. Now

$$\frac{L}{r}=\frac{144}{2.54}=56.7.$$

From Curve *B* (page 671) we find that the allowable unit stress as a column is 15,000 lbs. For combined stress this can be increased to 15,000x1⅓= 20,000 lbs. This shows that the 10-in.x49-lb. beam is a little larger than need be, but it is as near the required section as can be obtained if H-beams are to be used. This section will be used, since it is a standard column used in practice.

The moment on the column due to unsymmetrical live load is negligible considering the longitudinal direction, owing to the detail of the floor.

First-Story Interior Columns.—For the maximum direct load on each column, we have

From roof		44,100 lbs.
From third floor	2(32,760+6,840) =	79,200 lbs.
From second floor		79,200 lbs.
Total		202,500 lbs.

We shall assume a 10-in.x54-lb. beam as section. Then, we obtain

$$202{,}500\div 15.87=12{,}760 \text{ lbs.}$$

for the direct unit stress. Now

$$\frac{L}{r}=\frac{144}{2.48}=58.1.$$

Then, from Curve *B* (page 671) we find that 15,000 lbs. is the allowable unit stress, which can be increased to 15,000x1⅓=20,000 lbs. for stresses due to direct load and cross-bending combined.

The shear on the column due to wind, as given in Fig. 452, is 4,282 lbs. Then for the moment on the column due to wind we have

$$4{,}282\times 6\times 12=308{,}204 \text{ inch lbs.}$$

For the stress on the column due to this moment from wind, we have

$$\frac{308{,}204\times 5}{284.3}=5{,}420 \text{ lbs. per sq. in.}$$

Then we obtain

$$12{,}760+5{,}420=18{,}180 \text{ lbs.}$$

for the unit stress due to full live, dead, and wind load. The allowable as shown above is 20,000 lbs.

We shall next include the stress caused by unsymmetrical live load. The direct load in that case is

$$202{,}500-(80\times 18\times 9)=189{,}540 \text{ lbs.}$$

Then for the direct stress we have

$$189{,}540 \div 15.87 = 11{,}950 \text{ lbs. per sq. in.}$$

The moment due to the unsymmetrical live load can be obtained from Formula (E), Fig. 458. In this case

$$C_{ab} = \frac{1}{12} \times (80 \times 18) \times \overline{18}^2 \times 12 = 466{,}560 \text{ in. lbs.}$$

$$S_1 = 190.4 \div 144 \qquad = 1.32$$
$$S_2 = 284.3 \div 144 \qquad = 1.97$$
$$S_3 = 524.6 \div 18 \times 12 \qquad = 2.42$$

Substituting the above values in Formula (E), Fig. 458, we obtain

$$M_{af} = 198{,}500 \text{ in. lbs.}$$

for the moment on the column due to the unsymmetrical live load. Then for the unit stress due to this moment we have

$$\frac{198{,}500 \times 5}{284.3} = 3{,}490 \text{ lbs. per sq. in.}$$

Then we have for the total maximum unit stress

$$5{,}420 + 11{,}950 + 3{,}490 = 20{,}860 \text{ lbs.,}$$

which is only 860 lbs. greater than the allowable stress. Hence the 10-in.x 54-lb. beam will be considered satisfactory, considering the transverse direction of the building.

Considering the longitudinal direction, we find from Fig. 453 that the maximum shear due to wind on these columns is 2,973 lbs. Then for the moment on the column due to wind we have

$$2{,}973 \times 6 \times 12 = 214{,}050 \text{ in. lbs.}$$

and for the stress on the column due to this moment we obtain

$$\frac{214{,}050 \times 5.07}{97.3} = 11{,}150 \text{ lbs.}$$

Adding this to the direct stress, we have

$$11{,}150 + 12{,}760 = 23{,}910 \text{ lbs.}$$

for the combined unit stress, which is too great, as 20,000 lbs. is the maximum allowable stress.

Let us try a 10-in.x64-lb. beam. Then we have

$$202{,}500 \div 18.81 = 10{,}760 \text{ lbs.}$$

for the direct stress. For the stress due to moment we have

$$\frac{214{,}050 \times 5.22}{106.3} = 10{,}300 \text{ lbs.}$$

Then for the combined stress we have

$$10{,}760 + 10{,}530 = 21{,}260 \text{ lbs.,}$$

which is near enough to the allowable (20,000 lbs.) so that the 10-in.x64-lb. beam will be used.

Basement Interior Columns. For the maximum direct load, we have

$$44,100+3(79,200)=281,800 \text{ lbs.}$$

For the section, we shall assume a 10-in.x70-lb. beam. Then for the direct unit stress we have

$$281,800 \div 20.59 = 13,700 \text{ lbs.}$$

The shear on the column due to wind, as shown in Fig. 452, is 5,045 lbs. Then for the moment (at first floor) due to wind we obtain

$$5,045 \times 5 \times 12 = 302,700 \text{ in. lbs.}$$

and for the unit stress on the column due to this moment we have

$$\frac{302,700 \times 5}{369.3} = 4,100 \text{ lbs.}$$

Then we have

$$13,700+4,100=17,800 \text{ lbs.}$$

for the unit stress due to full live, dead and wind load. Now

$$\frac{L}{r}=\frac{120}{2.55}=47.$$

We find from Curve B (page 671) that 15,000 lbs. is the allowable stress for direct load. Increasing this one-third, we obtain 20,000 lbs. for the allowable stress due to direct load and cross-bending combined.

The moment on the column (at the first floor) due to unsymmetrical live load can be obtained from Formula (C), Fig. 458. In this case, we have

$$C_{ab}=\tfrac{1}{12}\times(80\times18)\times18^2\times12=466,560 \text{ in. lbs.}$$
$$S_1=308.8\div144 \qquad =2.143$$
$$S_2=369.3\div120 \qquad =3.075$$
$$S_3= \qquad 2.42$$

Substituting these values in Formula (C), Fig. 458, we obtain

$$M_{af}=162,200 \text{ in. lbs.}$$

for the moment (at first floor) on the interior basement column due to unsymmetrical live load.

For the stress on the column due to this moment we obtain

$$\frac{162,200\times5}{369.3}=2,200 \text{ lbs. per sq. in.}$$

The direct load on the column in this case is

$$281,800-(80\times18\times9)=268,840 \text{ lbs.}$$

Then for the stress on the column due to this load we obtain

$$\frac{268,840}{20.59}=13,050 \text{ lbs. per sq. in.}$$

So for the total maximum combined unit stress on the column we have

$$4{,}100+2{,}200+13{,}050=19{,}350 \text{ lbs.},$$

which is only 650 lbs. less than the allowable stress, showing that the 10-in.x 70-lb. beam is satisfactory, considering the transverse direction.

Now, considering the longitudinal direction of the building, we find from Fig. 453 that the maximum shear on these columns due to wind is 3,493 lbs. Then for the moment due to wind we have

$$3{,}493\times5\times12=209{,}580 \text{ in. lbs.}$$

For the stress on the column due to this moment we have

$$\frac{209{,}580\times5}{13'.3}=7{,}800 \text{ lbs.}$$

Adding this to the direct stress previously found, we obtain

$$13{,}050+7{,}800=20{,}850 \text{ lbs.},$$

which is only 850 lbs. more than the allowable stress. So the 10-in.x70-lb. beam is satisfactory and will be used.

299. Design of Exterior Columns.—

Third-Story Intermediate Columns support the roof load and fire wall. This load, as seen from Fig. 456, is

$$18{,}090+2(4{,}707)=27{,}500 \text{ lbs.}$$

Let us assume a 10-in.x31-lb. beam as column section. Then we have

$$\frac{L}{r}=\frac{144}{1.89}=76.2.$$

Then from Curve B (page 671), we find that 13,600 lbs. is the allowable stress per square inch (at mid-story). Increasing this one-third, we obtain

$$13{,}600\times1\tfrac{1}{3}=18{,}133 \text{ lbs}$$

for the allowable unit stress for combined stresses at the joints.

For the direct stress we have

$$27{,}500\div9.1=3{,}025 \text{ lbs. per sq. in.}$$

Considering the transverse direction of the building, we find the shear on the columns due to wind to be 540 lbs. (See Fig. 452.) Then for the moment on the column (at the roof) due to wind we have

$$540\times6\times12=38{,}880 \text{ in. lbs.}$$

and for the stress on the column, due to this moment, we have

$$\frac{38{,}800\times5}{163.4}=1{,}190 \text{ lbs. per sq. in.}$$

The moment on the column at the roof due to live and dead load can be obtained from Formula (N), Fig. 458.

For substituting in Formula (N), we have the following values:

$$C_{no} = 651{,}240 \text{ (see Fig. 456)}$$
$$\frac{C_{dc}}{2} = 589{,}680 \text{ (see Fig. 457)}$$
$$S'_3 = 213.4 \div 216 = 0.989$$
$$S_3 = 524.6 \div 216 = 2.42$$
$$S'_2 = 163.4 \div 144 = 1.134$$
$$S_2 = 279 \div 144 = 1.937 \text{ (assumed).}$$

Substituting the above values in Formula (N), we obtain

$$348{,}700 + 121{,}700 = 470{,}400 \text{ in. lbs.}$$

For the stress on the column due to this moment we have

$$\frac{470{,}400 \times 5}{163.4} = 14{,}400 \text{ lbs. per sq. in.}$$

Then for the total combined stress (at the roof) due to live, dead and wind load we have

$$3{,}025 + 1{,}190 + 14{,}400 = 18{,}615 \text{ lbs.,}$$

which is near enough to the allowable stress, so that the 10-in.x31-lb. beam is satisfactory, considering the transverse direction of the building.

Considering the longitudinal direction, we find the shear on the column to be $257 \div 2 = 128$ lbs. Then we obtain

$$128 \times 6 \times 12 = 9{,}216 \text{ in. lbs.}$$

for the moment due to wind. Then for the stress on the column due to this moment we have

$$\frac{9{,}216 \times 4}{32.5} = 1{,}135 \text{ lbs. per sq. in.}$$

Adding this to the direct stress, given above, we have

$$3{,}025 + 1{,}135 = 4{,}160 \text{ lbs. per sq. in.}$$

So the 10-in.x31-lb. beam is satisfactory and, hence, will be used.

Second-Story Exterior Columns. For the direct load on each of these columns, we have

$$27{,}500 + 32{,}760 + 2(12{,}276) = 84{,}812 \text{ lbs. (see Fig. 457)}$$

Let us assume a 10-in.x49-lb. beam as column section. Then

$$\frac{L}{r} = \frac{144}{2.54} = 56.7.$$

Now from Curve B (page 671) we find that 15,000 lbs. is the allowed unit stress for column action alone. Increasing this one-third, we obtain 20,000 lbs. for the allowable combined unit stress.

For the direct stress on the column we have

$$84{,}812 \div 14.4 = 5{,}890 \text{ lbs. per sq. in.}$$

For the moment due to wind (see Fig. 452) we have

$$1,188 \times 6 \times 12 = 85,530 \text{ in. lbs.}$$

For the stress on the column due to this moment we have

$$\frac{85,530 \times 5}{272} = 1,570 \text{ lbs. per sq. in.}$$

The moment on the column due to live and dead load can be obtained from Formula (G), Fig. 458.

For substituting in Formula (G), we have the following values:

$$\begin{aligned} \tfrac{3}{2}C_{ab} &= 1,179,360 \times \tfrac{3}{2} = 1,769,040 \text{ in. lbs.} \\ S_1 &= 163.4 \div 144 \quad = 1.134 \\ S_2 &= 272 \div 144 \quad = 1.890 \\ S_3 &\qquad\qquad\quad = 2.430 \\ & \qquad\qquad\qquad\ \ \overline{5.454} \end{aligned}$$

Substituting the above values in Formula (G), we obtain 614,000 in. lbs. for the moment on the column (at third floor) due to live and dead load.

Then for the stress on the column due to this moment we have

$$\frac{614,000 \times 5}{272} = 11,280 \text{ lbs. per sq. in.}$$

Then we have

$$5,890 + 1,570 + 11,280 = 18,740 \text{ lbs.}$$

for the total combined unit stress, which is 1,260 lbs. less than the allowable stress.

Considering the longitudinal direction of the building, we obtain

$$\frac{1,924}{2} \times 6 \times 12 = 69,264 \text{ in. lbs.}$$

for the moment on the column due to wind. For the stress on the column due to this moment we have

$$\frac{69,264 \times 5}{93} = 3,724 \text{ lbs. per sq. in.}$$

Adding this to the direct stress, previously given, we obtain

$$5,890 + 3,724 = 9,614 \text{ lbs.}$$

for the combined stress. As seen, the 10-in.x49-lb. beam is satisfactory; it will be used as the next smaller section would be overstressed.

First-Story Exterior Columns. For the direct load (at second floor) we have

$$27,500 + 2(32,760) + 4(12,276) = 142,124 \text{ lbs.}$$

Let us assume a 10-in.x59-lb. beam as column section. Then we have

$$\frac{L}{r} = \frac{144}{2.42} = 59.5.$$

Then the allowable column stress is 15,000 lbs. per square inch. Increasing this one-third, we obtain 20,000 lbs. for the allowable unit stress for combined stresses.

For the direct stress on the column, we have

$$142{,}124 \div 17.34 = 8{,}200 \text{ lbs. per sq. in.}$$

Considering the transverse direction of the building, we obtain

$$1{,}836 \times 6 \times 12 = 132{,}192 \text{ in. lbs.}$$

for the moment on the column due to wind. For the stress on the column due to this moment we have

$$\frac{132{,}192 \times 5}{296.5} = 2{,}230 \text{ lbs. per sq. in.}$$

The moment on the column due to dead and live load can be obtained from Formula (G), Fig. 458.

For substituting into Formula (G), we have the following values:

$$C_{ab} = 1{,}769{,}040 \text{ in. lbs.}$$
$$S_1 = 1.89$$
$$S_2 = 296.5 \div 144 = 2.06$$
$$S_3 = 2.43$$

Substituting these values in Formula (G), we obtain 571,000 in. lbs. for the moment on the column due to dead and live load.

For the stress due to this moment we have

$$\frac{571{,}000 \times 5}{296.5} = 9{,}630 \text{ lbs. per sq. in.}$$

Then, collecting the unit stresses, we obtain

$$8{,}200 + 2{,}230 + 9{,}630 = 20{,}060 \text{ lbs.,}$$

which is practically equal to the allowable stress.

Considering the longitudinal direction of the building, we find the moment due to wind to be

$$\frac{2{,}973}{2} \times 6 \times 12 = 107{,}028 \text{ in. lbs.}$$

For the stress due to this moment we have

$$\frac{107{,}028 \times 5.14}{101.7} = 5{,}420 \text{ lbs. per sq. in.}$$

Then, adding this to the direct stress, we have

$$8{,}200 + 5{,}420 = 13{,}620 \text{ lbs. per sq. in.}$$

From the preceding it is seen that the 10-in.x59-lb. beam is satisfactory; it will be used.

Basement Exterior Columns. For the direct load on each of these columns we have

$$142{,}124 + 32{,}760 + 2(12{,}276) = 199{,}436 \text{ lbs.}$$

Let us assume a 10-in.x70-lb. beam for column section. Then for the direct stress we have

$$199{,}436 \div 20.59 = 9{,}680 \text{ lbs. per sq. in.}$$

Now

$$\frac{L}{r} = \frac{120}{2.55} = 47.$$

Then from Curve B (page 671) we find that 15,000 lbs. is the allowable column unit stress. Increasing this one-third, we obtain 20,000 lbs. for the allowable unit stress for combined stresses.

For the moment due to wind we have

$$2{,}160 \times 5 \times 12 = 129{,}600 \text{ in. lbs.}$$

For the stress on the column due to this moment we have

$$\frac{129{,}600 \times 5}{369.3} = 1{,}750 \text{ lbs. per sq. in.}$$

The moment due to dead and live load can be obtained from Formula (19), Fig. 458. For substituting in this formula, we have the following values:

$$\begin{aligned} C_{ab} &= 1{,}179{,}360 \text{ in. lbs.} \\ S_1 &= 296.5 \div 144 = 2.06 \\ S_2 &= 369.3 \div 120 = 3.08 \text{ constants} \\ S_3 &= = 2.42 \end{aligned}$$

Substituting the above values in Formula (19), Fig. 458, we obtain 480,000 in. lbs. for the moment on the column (at first floor) due to dead and live load.

For the stress on the column due to this moment we have

$$\frac{480{,}000 \times 5}{369.3} = 6{,}500 \text{ lbs. per sq. in.}$$

Now, collecting the unit stresses, we obtain

$$9{,}680 + 1{,}750 + 6{,}500 = 17{,}930 \text{ lbs.}$$

for the total maximum unit stress on the column, which, as seen, is 2,070 lbs. less than the allowable stress.

Let us try a 10-in.x64-lb. beam. Then for the direct stress we have

$$199{,}436 \div 18.81 = 10{,}620 \text{ lbs.}$$

For the stress due to wind we have

$$\frac{129{,}600 \times 5}{308.8} = 2{,}100 \text{ lbs.}$$

For the stress due to moment from live and dead load we have

$$\frac{480{,}000 \times 5}{308.8} = 7{,}780 \text{ lbs.}$$

Now, collecting the above unit stresses, we obtain

$$10{,}620 + 2{,}100 + 7{,}780 = 20{,}500 \text{ lbs.,}$$

which is only 500 lbs. more than the allowable stress. So the 10-in.x64-lb. beam will be used.

300. Design of Intermediate End Columns.—

Third-Story End Columns support a direct load of

$$(12{,}672\times 2)+3{,}960=29{,}304 \text{ lbs.}$$

We shall assume a 10-in.x31-lb. beam as column section placed so that the web is perpendicular to the end bent.

Considering the transverse direction of the building, we can obtain the moment on the columns due to unsymmetrical live load from Formula (K) (Fig. 458).

Here we have (see Figs. 456 and 457)

$$\begin{aligned}
C'_{ab} &= \tfrac{1}{12}\times(25\times 9)\times 18^2\times 12 = 72{,}900 \text{ in. lbs.}\\
C_{fg} &= \tfrac{1}{12}\times(80\times 9)\times \overline{18}^2\times 12 = 233{,}280\\
S'_2 &= 32.5\div 144 = 0.225\\
S_2 &= 93\div 144 \text{ (assumed)} = 0.646\\
S'_3 &= 183\div 216 = 0.847\\
S_3 &= 333.4\div 216 = 1.544
\end{aligned}$$

Substituting the above values in Formula (K), Fig. 458, we obtain

$$15{,}300+8{,}220=23{,}520 \text{ in. lbs.}$$

for the moment due to unsymmetrical live load.

For the moment due to wind (see Fig. 452) we have

$$\frac{1{,}260}{2}\times 6\times 12=45{,}360 \text{ in. lbs.}$$

The direct load in this case is

$$29{,}304-(25\times 9\times 9)=27{,}279 \text{ lbs.}$$

Then for the stress due to direct load we have

$$27{,}279\div 9.11=2{,}993 \text{ lbs. per sq. in.}$$

For the stress due to moment we have

$$\frac{(23{,}520+45{,}360)\times 4}{32.5}=8{,}482 \text{ lbs. per sq. in.}$$

Then for the maximum unit stress on the column, considering the transverse direction of the building, we have

$$2{,}993+8{,}482=11{,}475 \text{ lbs. per sq. in.}$$

Considering the longitudinal direction of the building, we can obtain the moment on the column due to dead and live load from Formula (N), Fig. 458. In that case

$$\begin{aligned}
C_{no} &= 142{,}560 \text{ in. lbs. (see Fig. 456)}\\
C_{dc} &= 364{,}356 \text{ in. lbs. (see Fig. 457)}\\
S'_2 &= 163.4\div 144 = 1.134\\
S_2 &= 272\div 144 \text{ (assumed)} = 1.889\\
S'_3 &= 107.6\div 216 = 0.498\\
S_3 &= 107.6\div 216 = 0.498
\end{aligned}$$

Substituting the above values in Formula (N), Fig. 458, we obtain

$$99{,}100+58{,}600=157{,}700 \text{ in. lbs.}$$

for the moment on the column in the longitudinal direction of the building.

For the unit stress due to this moment we have

$$\frac{157{,}700\times 5}{163.4}=4{,}825 \text{ lbs. per sq. in.}$$

Now, adding the unit stresses that act simultaneously, we obtain

$$11{,}475+4{,}825=16{,}300 \text{ lbs. per sq. in.}$$

for the maximum combined unit stress on the column.

Now

$$\frac{L}{r}=\frac{144}{1.89}=76.$$

Then from Curve B (page 671) we find that 13,600 lbs. is the allowable column stress. Increasing this one-third, we obtain

$$13{,}600\times 1\tfrac{1}{3}=18{,}100 \text{ lbs.}$$

for the allowable unit stress for combined stresses. As seen, the column is stressed 1,833 lbs. less than the allowable stress; but the 10-in.x31-lb. beam will be used, as it has as near the required section as can be obtained.

In the preceding analysis it was considered that the roof and floor slabs caused no moment on the columns in the end bents. This will be true if each slab is simply supported (as it should be) at the end bent; but if constructed so that it is fixed to the transverse beams in the end bents a very heavy moment on the columns will be exerted by the roof and floor slabs.

For the purpose of illustration let us assume that the roof slabs and floor slabs are constructed so as to be fixed to the transverse beams in the end bents. Then, from Art. 292, we find that the moment at the end of the roof slab will be

$$45{,}360\div 1.375=33{,}000 \text{ in. lbs. (about)}$$

per linear foot of transverse beam. This moment produces a torque on the transverse beam, which in turn is transmitted to the column causing a moment at each joint (at roof) of

$$33{,}000\times 18=594{,}000 \text{ in. lbs.}$$

This moment must be resisted by the column and by the longitudinal roof beam, thus causing a high stress in each. Similarly, from Art. 293, it is found that the moment at the end of a floor slab will be

$$91{,}366\div 1.5=60{,}910 \text{ in. lbs.}$$

per foot of transverse beam. This moment produces a torque on the transverse beam, which in turn is transmitted to the column causing a moment at each joint of

$$60{,}910\times 18=1{,}096{,}380 \text{ in. lbs.}$$

This moment (as is obvious) must be resisted by the column (above and below the joint) and by the longitudinal floor beam, thus causing a high stress in each.

Now, taking

$$C_{no} = 594,000$$

and

$$C_{dc} = 1,096,380$$

and substituting in Formula (N), Fig. 458, using the sections found in the foregoing analysis, we obtain

$$413,000 + 176,700 = 589,700 \text{ in. lbs.}$$

for the moment on the column at the roof due to the action of the roof and floor slabs.

This moment alone would cause a stress on the column of

$$\frac{589,700 \times 5}{163.4} = 18,040 \text{ lbs. per sq. in.}$$

As is seen, the size of the column would have to be greatly increased to provide for the full torque from the roof and floor slabs; that is not all, as the transverse beams would have to be designed to transmit the torque. This would require a horizontal beam (additional) at the bottom of each transverse beam. The need of all this additional material is eliminated by constructing the slabs so that they are simply supported at the end bents. This can be done in several ways. The author suggests the simple detail shown in Fig. 459, where the continuity of the slab is broken by the sheet steel, which is placed just above the flange of the transverse beam and extends the full length of the beam—from column to column in each end bay.

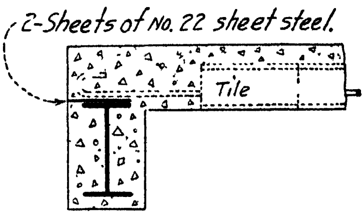


Fig. 459

Second-Story End Columns support a direct load of

$$29,304 + (2 \times 25,992) + 6,480 - (80 \times 9 \times 9) = 81,288 \text{ lbs.}$$

Let us assume a 10-in.x49-lb. beam for column section. Then for the direct stress we have

$$81,288 \div 14.4 = 5,650 \text{ lbs. per sq. in.}$$

For the moment on the column due to wind in the transverse direction of the building (see Fig. 452), we have

$$\frac{2,772}{2} \times 6 \times 12 = 99,792 \text{ in. lbs.}$$

The moment on the column in the transverse direction of the building due to unsymmetrical live load can be obtained from Formula (E), Fig. 458. Here, we have

$$C_{ab} = \tfrac{1}{12} \times (80 \times 9) \times \overline{18}^2 \times 12 = 233,280 \text{ in. lbs.}$$
$$S_1 = 32.5 \div 144 = 0.225$$
$$S_2 = 93 \div 144 = 0.646$$
$$S_3 = 333.4 \div 216 = 1.544$$

Substituting the above values in Formula (E), Fig. 458, we obtain 70,900 in. lbs. for the moment due to unsymmetrical live load.

Then for the stress on the column due to moment in the transverse direction of the building we have

$$\frac{(99{,}792+70{,}900)\times 5}{93}=9{,}180 \text{ lbs. per sq. in.}$$

The moment in the longitudinal direction of the building due to dead and live load can be obtained from Formula (G), Fig. 458. In this case, we have

$C_{ab}=346{,}356$ in. lbs. (see Fig. 453)
$S_2=272\div 144=1.889$
$S_1=163.4\div 144=1.134$
$S_3=107.6\div 216=0.498$

Substituting the above values in Formula (E), Fig. 458, we obtain 278,500 in. lbs. for the moment on the column ir the longitudinal direction of the building due to dead and live load.

For the stress on the column due to this moment we have

$$\frac{278{,}500\times 5}{272}=5{,}120 \text{ lbs.}$$

Now, collecting the unit stresses, we have

$$5{,}650+9{,}180+5{,}120=19{,}950 \text{ lbs.}$$

for the total maximum unit stress on the column:

$$\frac{L}{r}=\frac{144}{2.42}=59.5.$$

Then from Curve B (page 671) we find that the allowable column stress is 15,000 lbs. Increasing this one-third, we obtain 20,000 lbs. for the allowable unit stress in the case of combined stresses. This shows that the 10-in.x49-lb. beam assumed is satisfactory.

First-Story End Columns support a direct load of

$$81{,}288+(2\times 25{,}992)+6{,}480=139{,}752 \text{ lbs.}$$

Let us assume a 10-in.x70-lb. beam as column section. Then for the direct stress, we have

$$139{,}752\div 20.59=6{,}790 \text{ lbs. per sq. in.}$$

For the moment on the column in the transverse direction of the building due to wind (see Fig. 452) we have

$$\frac{4{,}282}{2}\times 6\times 12=154{,}152 \text{ in. lbs.}$$

For determining the moment due to unsymmetrical live load in the transverse direction of the building, we now have

$C_{ab}=\frac{1}{12}\times(80\times 9)\times \overline{18}^2\times 12=233{,}280$ in. lbs.
$S_2=134.3\div 144=0.933$
$S_1=93\div 144=0.646$
$S_3=333.4\div 216=1.544$

Substituting the above values in Formula (E), Fig. 458, we obtain 65,200 in. lbs. for the moment on the column in the transverse direction of the building due to unsymmetrical live load. Then for the total unit stress on the column due to moment in the transverse direction of the building we have

$$\frac{(154,152+65,200)5}{134.3}=8,170 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the longitudinal direction of the building due to dead and live load, we have

$$C_{ab}=346,356 \text{ (see Fig. 457)}$$
$$S_2=369.3\div144=2.564$$
$$S_1=272\div144=1.889$$
$$S_3=107.6\div216=0.498$$

Substituting the above values in Formula (G), Fig. 458, we obtain 268,900 in. lbs. for the moment on the column in the longitudinal direction of the building due to dead and live load

For the stress on the column due to this moment we have

$$\frac{268,900\times5}{369.3}=3,640 \text{ lbs. per sq. in.}$$

Then, collecting the unit stress, we obtain

$$6,790+8,170+3,640=18,600 \text{ lbs.}$$

for the total maximum combined unit stress on the column. This is so near the 20,000 lbs. allowable stress that the assumed 10-in.x70-lb. beam will be considered satisfactory for column section.

Basement End Columns support a direct load of

$$139,752+(2\times25,992)+6,480=198,216 \text{ lbs.}$$

Let us assume a 10-in.x70-lb. beam as column section. Then for the direct stress we have

$$198,216\div20.59=9,630 \text{ lbs. per sq. in.}$$

For the moment on the column in the transverse direction of the building due to wind we have

$$\frac{5,045}{2}\times5\times12=151,350 \text{ in. lbs.}$$

For determining the moment on the column in the transverse direction of the building due to unsymmetrical live load, we have

$$C_{ab}=\tfrac{1}{12}\times(80\times9)\times\overline{18}^2\times12=233,280 \text{ in lbs.}$$
$$S_2=134.3\div120=1.11$$
$$S_1=134.3\div144=0.933$$
$$S_3=333.4\div216=1.544$$

Substituting the above values in Formula (C), Fig. 458, we obtain 59,400 in. lbs. for the moment on the column in the transverse direction of the building due to unsymmetrical live load.

Then for the unit stress on the column due to moment in the transverse direction of the building we have

$$\frac{(151,350+59,400)5}{134.3}=7,840 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the longitudinal direction of the building due to dead and live load, we have

$C'_{ab}=364,456$ in. lbs. (see Fig. 457)
$S_2=369.3\div120=3.077$
$S_1=369.3\div144=2.564$
$S_3=107.6\div216=0.498$

Substituting the above values in Formula (19), Fig. 458, we obtain 182,300 in. lbs. for the moment on the column in the longitudinal direction of the building due to live and dead load.

For the unit stress on the column due to this moment we have

$$\frac{182,300\times5}{369.3}=2,470 \text{ lbs. per sq. in.}$$

Now, collecting the unit stresses, we obtain

$$9,630+7,840+2,470=19,940 \text{ lbs.},$$

which shows that the assumed 10-in.x70-lb. beam is about the size required, as 20,000 lbs. is the allowable stress.

301. Design of Corner Columns.—

Third-Story Corner Column supports a direct load (see Fig. 456) of

$$4,707+12,672=17,379 \text{ lbs.}$$

Let us assume a 10-in.x49-lb. beam as column section. Then for the direct stress we have

$$17,379\div14.4=1,206 \text{ lbs. per sq. in.}$$

For determining moment on the column in the transverse direction of the building due to dead and live load, we have

$C_{no}=456,192$ in. lbs. (see Fig. 456)
$C_{dc}=811,685$ in. lbs. (see Fig. 457)
$S'_2=272\div144=1.889$
$S_2=272\div144=1.889$
$S'_3=183\div216=0.847$
$S_3=333.4\div216=1.544$

Substituting the above values in Formula (N) Fig. 458, we obtain

$$314,700+143,600=458,300 \text{ in. lbs.}$$

for the moment on the column in the transverse direction of the building due to dead and live load.

For the moment due to wind (see Fig. 452) we have

$$\frac{540}{2}\times6\times12=19,440 \text{ in. lbs.}$$

Then for the total maximum unit stress on the column due to moments in the transverse direction of the building we have

$$\frac{(458,300+19,440)5}{272}=8,780 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the longitudinal direction of the building due to dead and live load, we have

$$C_{nv}=169,452 \text{ in. lbs. (see Fig. 456)}$$
$$C_{dc}=237,816 \text{ in. lbs. (see Fig. 457)}$$
$$S'_2=93\div 144=0.646$$
$$S_2=93\div 144=0.646$$
$$S'_3=107.6\div 216=0.498$$
$$S_3=107.6\div 216=0.498$$

Now, substituting values in Formula (N), Fig. 458, we obtain

$$95,700+42,900=138,600 \text{ in. lbs.}$$

for the moment on the column in the longitudinal direction of the building due to dead and live load.

For the unit stress due to this moment we have

$$\frac{138,600\times 5}{93}=7,450 \text{ lbs.}$$

Collecting the unit stresses, we obtain

$$1,206+8,780+7,450=17,436 \text{ lbs. per sq. in.,}$$

which is 2,564 lbs. less than the 20,000 lbs. allowed. The 10-in.x49-lb. beam will be used, however, as it has as near the required section as can be obtained.

Second-Story Corner Column supports a direct load of

$$17,379+12,276+25,992=55,647 \text{ lbs.}$$

Let us assume a 10-in.x49-lb. beam as column section. Then for the direct stress, we have

$$55,647\div 14.4=3,860 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the transverse direction of the building due to dead and live load, we have

$$C_{ab}=811,685 \text{ in. lbs. (see Fig. 457)}$$
$$S_1=272\div 144=1.889$$
$$S_2=272\div 144=1.889$$
$$S_3=333.4\div 216=1.544$$

Substituting the above values in Formula (G), Fig. 458, we obtain 432,000 in. lbs. for the moment on the column in the transverse direction of the building due to dead and live load.

For the moment on the column due to wind (see Fig. 452), we have

$$\frac{1,188}{2}\times 6\times 12=42,768 \text{ in. lbs.}$$

Now for the combined unit stress on the column due to moments in the transverse direction of the building, we have

$$\frac{(432,000+42,768)5}{272}=8,720 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the longitudinal direction of the building due to dead and live load, we have

$$\begin{aligned} C_{ab} &= 237,816 \text{ in. lbs. (see Fig. 457)} \\ S_1 &= 93 \div 144 = 0.646 \\ S_2 &= 93 \div 144 = 0.646 \\ S_3 &= 107.6 \div 216 = 0.498 \\ & \qquad\qquad\qquad 1.790 \end{aligned}$$

Substituting the above values in Formula (G), Fig. 458, we obtain 128,600 in. lbs. for the moment on the column in the longitudinal direction of the building due to dead and live load.

For the stress due to this moment, we have

$$\frac{128,600\times 5}{93}=6,920 \text{ lbs. per sq. in.}$$

Now, collecting the unit stresses, we obtain

$$3,860+8,720+6,920=19,500 \text{ lbs. per sq. in.}$$

for the total combined stress on the column. This shows that the 10-in.x 49-lb. beam (assumed) is satisfactory, as 20,000 lbs. is allowed.

First-Story Corner Column supports a direct load of

$$55,647+12,276+25,992=93,915 \text{ lbs.}$$

Let us assume a 10-in.x64-lb. beam as column section. Then for the direct stress we have

$$93,915 \div 18.81=4,990 \text{ lbs.}$$

For determining the moment on the column in the transverse direction of the building due to dead and live load, we have

$$\begin{aligned} C_{ab} &= 811,685 \text{ in. lbs. (see Fig. 457)} \\ S_1 &= 272 \div 144 = 1.889 \\ S_2 &= 308.8 \div 144 = 2.143 \\ S_3 &= 333.4 \div 216 = 1.544 \end{aligned}$$

Substituting the above values in Formula (G), Fig. 458, we obtain 468,500 in. lbs. for the moment on the column in the transverse direction of the building due to dead and live load.

For the moment due to wind we have

$$\frac{1,836}{2}\times 6\times 12=66,096 \text{ in. lbs.}$$

Now for the total combined unit stress due to the moments in the transverse direction of the building we have

$$\frac{(468,500+66,096)5}{308.8}=8,660 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the longitudinal direction of the building, we have

$$C_{ab}=237,816 \text{ in. lbs. (see Fig. 457)}$$
$$S_1=93\div 144=0.646$$
$$S_2=106.3\div 144=0.739$$
$$S_3=107.6\div 216=0.498$$

Substituting the above values in Formula (G), Fig. 458, we obtain 139,800 in. lbs. for the moment on the column in the longitudinal direction of the building due to dead and live load.

For the stress on the column due to this moment we have

$$\frac{139,800\times 5.22}{106.3}=6,870 \text{ lbs. per sq. in.}$$

Collecting the unit stresses, we obtain

$$4,990+8,660+6,870=20,520 \text{ lbs. per sq. in.}$$

for the total maximum unit stress on the column, which shows that the 10-in.x 64-lb. beam is satisfactory, as 20,000 lbs. is the allowable stress.

Basement Corner Column supports a direct load of

$$93,915+12,276+25,992=132,183 \text{ lbs.}$$

Let us assume a 10-in.x64.-lb. beam as column section.

Then for the direct unit stress we have

$$132,183\div 18.81=7,020 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the transverse direction of the building due to dead and live load, we have

$$C'_{ab}=811,685 \text{ in. lbs. (see Fig. 457)}$$
$$S_1=308.8\div 144=2.142$$
$$S_2=308.8\div 120=2.572$$
$$S_3=333.4\div 216=1.544$$

Substituting the above values in Formula (19), Fig. 458, we obtain 332,500 in. lbs. for the moment on the column in the transverse direction of the building due to dead and live load.

For the moment due to wind (see Fig. 452) we have

$$\frac{2,160}{2}\times 5\times 12=64,800 \text{ in. lbs.}$$

Then for the total maximum unit stress on the column due to moments in the transverse direction of the building we obtain

$$\frac{(332,500+64,800)5}{308.8}=6,440 \text{ lbs. per sq. in.}$$

For determining the moment on the column in the longitudinal direction of the building due to dead and live load, we have

$$C'_{ab}=237,816 \text{ in. lbs. (see Fig. 457)}$$
$$S_1=106.3\div 144=0.738$$
$$S_2=106.3\div 120=0.886$$
$$S_3=107.6\div 216=0.498$$

Substituting the above values in Formula (19), Fig. 458, we obtain 99,200 in. lbs. for the moment on the column in the longitudinal direction of the building.

For the stress on the column due to this moment we have

$$\frac{99{,}200\times 5.22}{106.3}=4{,}870 \text{ lbs. per sq. in.}$$

Now, collecting the unit stresses, we obtain

$$7{,}020+6{,}430+4{,}870=18{,}320 \text{ lbs.}$$

for the total maximum combined unit stress on the column, which shows that the 10-in.x64-lb. beam is satisfactory, as 20,000 lbs. is the allowable stress.

The calculations for the columns being completed, the diagrams showing the sections of the columns can be drawn, as shown in Fig. 460, and then the details of the frame throughout can be made.

302. Details.—Typical riveted connections throughout the building are shown on the column details in Figs. 461 and 462. To show the procedure in designing, we shall consider the details shown on the interior columns (Fig. 461).

Footing for Interior Column. For the direct load on the footing, we have

Roof....................		44,100
Floors....................	3(79,200) =	237,600
Column (steel+concrete)...	36×150 =	5,400
Total....................		287,100

For the moment at footing due to wind (transverse direction of building) we have

$$5{,}045\times 60=302{,}700 \text{ in. lbs.}$$

The moment at footing due to unsymmetrical live load is one-half that at the top of the basement column (see Appendix *B*). So we have

$$165{,}000\div 2=82{,}500 \text{ in. lbs.}$$

for the moment at the footing due to unsymmetrical live load.

Then for the total moment at the footing, we have

$$302{,}700+82{,}500=385{,}200 \text{ in. lbs.}$$

Let us assume a 24-in.x26-in. footing—the 26-in. in the transverse direction of the building.

Then for the pressure on the masonry due to direct load we have

$$287{,}100\div(24\times 26)=460 \text{ lbs. per sq. in.}$$

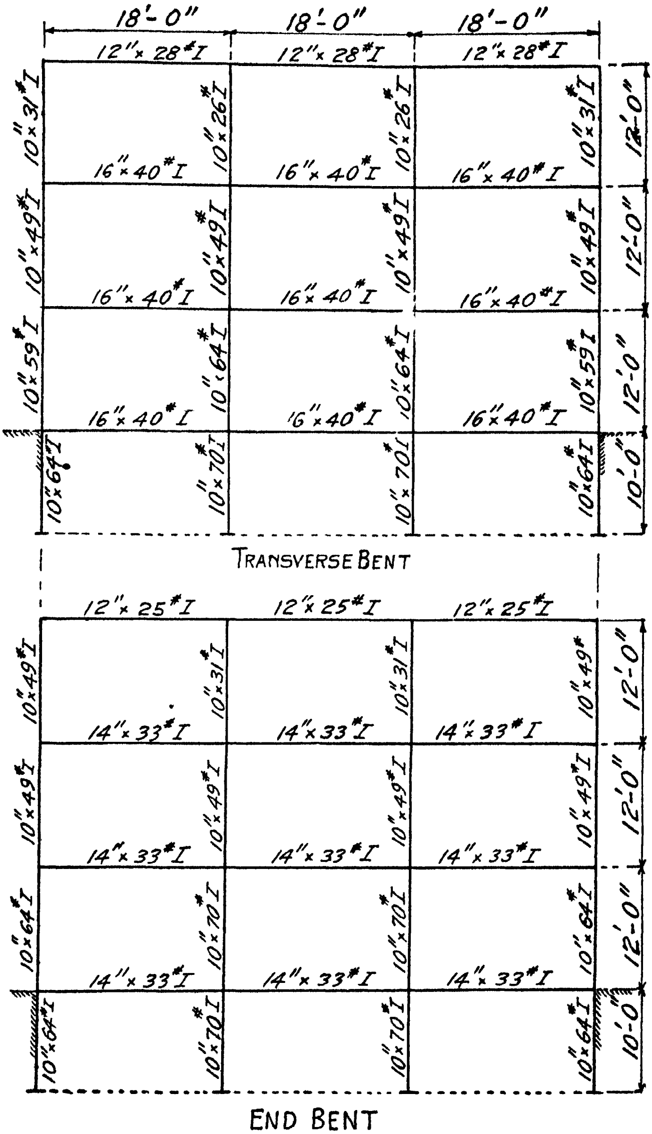
For the pressure due to moment we have

$$f=\frac{My}{I}=\frac{385{,}200\times 13}{\frac{1}{12}(24\times 26^3)}=142 \text{ lbs. per sq. in.}$$

Then for the maximum pressure on the masonry or footing we have

$$460+142=602 \text{ lbs. per sq. in.,}$$

which is satisfactory, as 600 lbs. is usually allowed.


18'-0"
18'-0"
18'-0"
12"×28#I
12"×28#I
12"×28#I
10"×31#I
10"×26#I
10"×26#I
10"×31#I
12'-0"
16"×40#I
16"×40#I
16"×40#I
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10"×49#I
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10"×64#I
10"×70#I
10"×70#I
10"×64#I
10'-0"
TRANSVERSE BENT
12"×25#I
12"×25#I
12"×25#I
10"×49#I
10"×31#I
10"×31#I
10"×49#
12'-0"
14"×33#I
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12'-0"
14"×33#I
14"×33#I
14"×33#I
10"×64#I
10"×70#I
10"×70#I
10"×64#I
12'-0"
14"×33#I
14"×33#I
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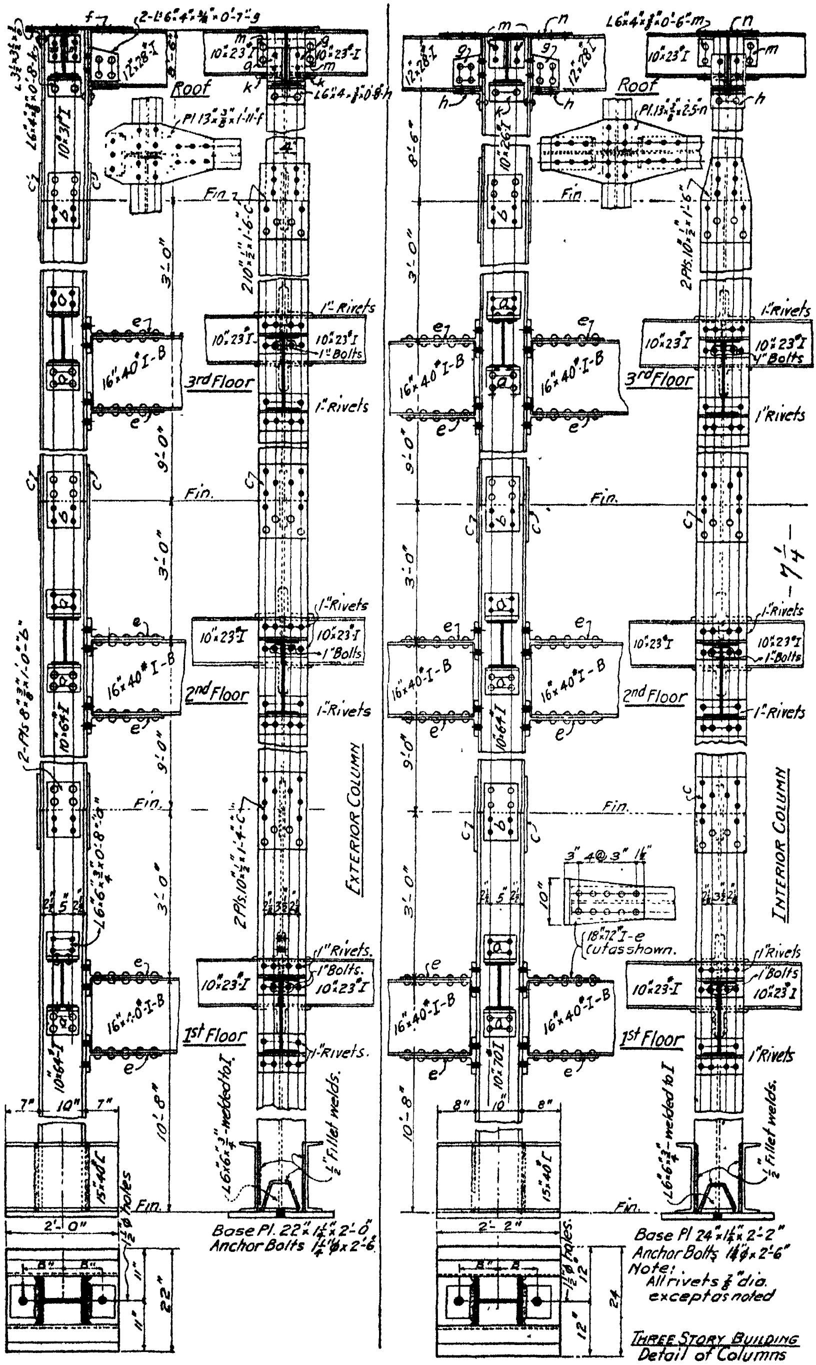
10"×64#I
10"×70#I
10"×70#I
10"×64#I
10'-0"
END BENT

Fig. 460



Roof
3rd Floor
2nd Floor
1st Floor
Fin.
16"×40# I-B
10"×23# I
1" Rivets
1" Bolts
10"×31# I
10"×64# I
10"×26# I
10"×70# I
12"×28# I
15"×40# [
½" Fillet welds.
18"×72# I-e cut as shown.
Exterior Column
Interior Column
Base Pl. 22"×1¼"×2'-0"
Anchor Bolts 1¼"φ×2'-6"
Base Pl 24"×1¼"×2'-2"
Anchor Bolts 1¼"φ×2'-6"
Note: All rivets ⅞" dia. except as noted
Three Story Building
Detail of Columns

Fig. 461

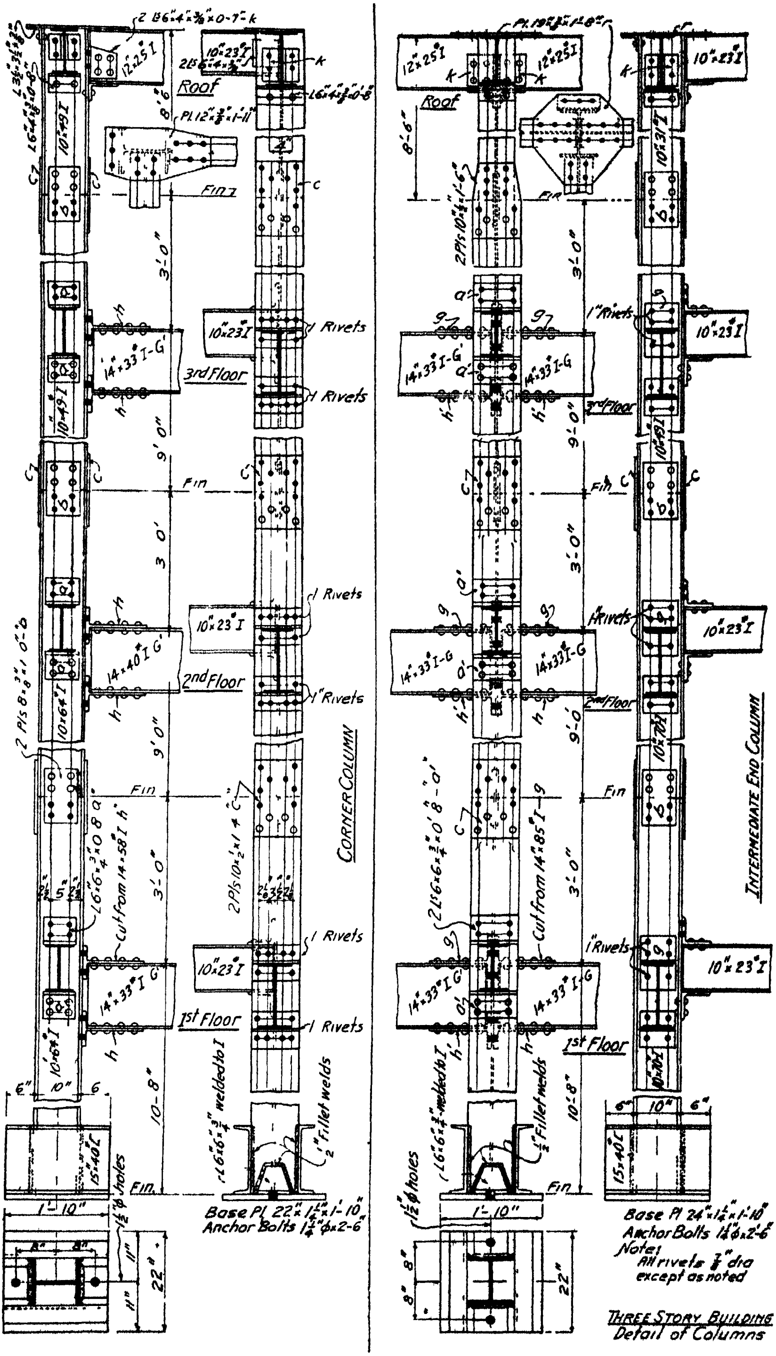

Roof
3rd Floor
2nd Floor
1st Floor
Corner Column
Intermediate End Column
Base Pl 22"×1¼"×1'-10"
Anchor Bolts 1¼"φ×2-6"
Base Pl 24"×1¼"×1-10"
Anchor Bolts 1¼φ×2-6
Note:
All rivets ⅞" dia
except as noted
Three Story Building
Detail of Columns

Fig. 462

Details at First Floor (see Figs. 461 and 462). From Fig. 457 we find that the end moment on the 16-in.x40-lb. beam is 1,179,360 in. lbs. Taking the depth of the beam as the effective depth of the end connection (which is practically correct), we have

$$1,179,360 \div 16 = 73,300 \text{ lbs.}$$

for the tension on the top connection. The eight 1-in. field rivets (shown) have an allowable resistance, in tension, of

$$8(0.7854 \times 13,500) = 84,600 \text{ lbs.},$$

which is excessive by about one rivet.

There are ten $\frac{7}{8}$-in. rivets connecting the "split" I-beam (*e*) to the 16-in. beam. These rivets have an allowable resistance of

$$10(0.60132 \times 13,500) = 81,180 \text{ lbs.},$$

which is excessive by almost one rivet.

The same number of $\frac{7}{8}$-in. rivets is used to connect the "split" I-beam (*e*) at the bottom of the 16-in. beam—as there should be. The 1-in. rivets connecting the "split" beam at the bottom of the 16-in. beam (*B*) to the column take mostly shear, as the compression is transmitted directly to the column, as is obvious. But since the connection should be rigid, the 1-in. rivets are used. The shear on beam *B*, as given in Fig. 457, is 32,760 lbs. So we have

$$32,760 \div 14 = 2,340 \text{ lbs.}$$

for the shear on each of the 1-in. rivets connecting the 16-in.x40-lb. beam to the column, which is quite low.

We shall next consider the connections of the 10-in.x23-lb. beams to the column. We find from Fig. 457 that the maximum end moment on these beams is 346,356 in. lbs. Then taking 10 ins. as the effective depth of the end connections, we obtain

$$346,356 \div 10 = 34,635 \text{ lbs.}$$

for the tension on the four $\frac{7}{8}$-in. rivets in the top connection.

The allowable stress on them is

$$4(0.60132 \times 13,500) = 32,472 \text{ lbs.},$$

which is only a little less than the stress. Hence the four $\frac{7}{8}$-in. rivets will be considered satisfactory.

The Column Splices are placed 3 ft. above the floors to avoid excessive bending at the splices. At the point 3 ft. above a floor (the stories being 12 ft. high) the moment due to wind is only one-half of what it is at the floor, and we can consider, without being much in error, that the moment at each splice due to unsymmetrical live load is also one-half of what it is at the floor.

Then, considering the column splice in the first story (see Art. 298), we have

$$308,204 \div 2 = 154,102 \text{ in. lbs.}$$

for the moment at the splice due to wind and

$$198,500 \div 2 = 99,250 \text{ in. lbs.}$$

for the moment at the splice due to unsymmetrical live load, making a total moment of

$$154{,}102+99{,}250=253{,}352 \text{ in. lbs.}$$

at the splice in the transverse direction of the building. Taking the width of the column as the effective depth for the splice plates, we obtain

$$253{,}352 \div 10 = 25{,}335 \text{ lbs.}$$

for the stress on each splice plate—marked c in Fig. 461. As is seen, this (theoretically) requires only a very thin plate, but to obtain sufficient stiffness the plates are made $\frac{1}{2}$ in. thick. It requires about three $\frac{7}{8}$-in. rivets to transmit the stress on each plate, but properly to hold the plates rigidly six $\frac{7}{8}$-in. rivets are used. As is seen, this is the least number of rivets that could be used if a rigid detail is to be obtained.

The web of the column beam should be spliced sufficiently to resist the shear due to wind. As seen in Fig. 452, the maximum shear due to wind is 4,282 lbs. This shear, as is obvious, requires less than one $\frac{7}{8}$-in. rivet; but four $\frac{7}{8}$-in. rivets are the least that can be used if a rigid detail is to be obtained. Hence the four $\frac{7}{8}$-in. rivets as shown are used. Of course, the direct stress on the column reverses the tension on the splice plates, hence the splices shown are (theoretically) quite excessive. But to obtain the desired rigidity of the structure the splices as shown are desirable, especially since the additional cost is small.

The other details throughout the structure are worked out in the manner shown in the foregoing analysis. As a whole the details are self-explanatory.

303. Welded Connections.—Typical welded connections for the three-story building considered in the preceding articles are shown in Fig. 463.

The usual stress in tension and shear allowed on welds are as follows:

$\frac{5}{16}$-in. full fillet weld . . .	2,500 lbs. per lin. in.
$\frac{3}{8}$-in. full fillet weld.	3,000
$\frac{1}{2}$-in. full fillet weld	3,500

The above values can be increased 20 per cent for compression on a welded connection.

Welded details are very readily designed. For example, in the connections at the first floor, in Fig. 463, the tension on the top connection of the 16-in.x40-lb. beam is 73,300 lbs., as previously given. The length of weld required (using $\frac{1}{2}$-in. fillet welds) is

$$73{,}300 \div 3{,}500 = 21 \text{ ins.}$$

By using the two triangular plates, it is seen that the length of shop weld connecting these plates to the beam is about 24 ins. and that the length of the field weld connecting the triangular plates to the column is about 24 ins., both of which are excessive (by a few inches). By welding the top flange of the beam directly to the column as indicated, the connection at the top of the 16-in. beam, as is seen, is quite ample.

At the bottom of the 16-in.x40-lb. beam (B) there are 12 ins. of field weld along the flange of the beam connecting the beam to the 6-in.x6-in. angle and 7 ins. of field weld connecting the flange of the beam directly to the column. The 7 ins. of weld, being in compression, is equivalent to $7 \times 1.20 = 8.4$ ins., making in all $12+8.4=20.4$ ins. of weld, which is about equal to the required length. The bolts shown are for the purpose of holding the beam B in place for welding.

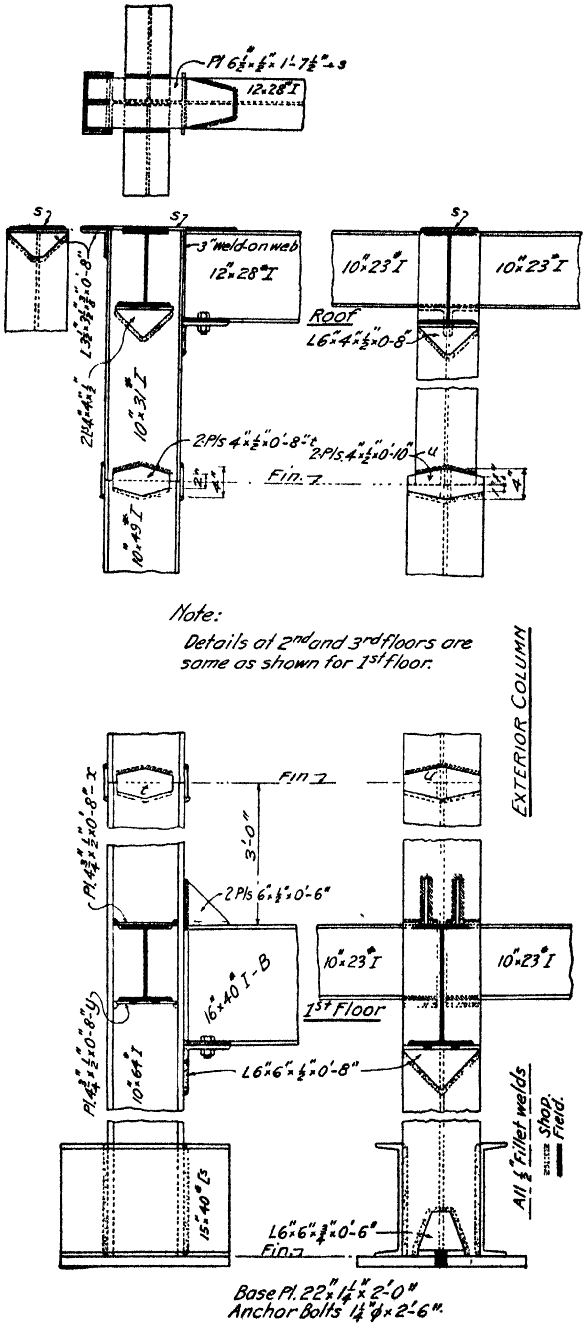

Pl 6½"×½"×1'-7½"-s
12×28#I
3" weld on web
12"×28#I
2Ls 4"×4"×½"
L3½"×3½"×⅜"×0'-8"
10"×31#I
2Pls 4"×½"×0'-8"-t
Fin.
10"×49#I
10"×23#I
10"×23#I
Roof
L6"×4"×½"×0'-8"
2Pls 4"×½"×0'-10"-u
Note:
Details at 2nd and 3rd floors are same as shown for 1st floor.
EXTERIOR COLUMN
Pl 4¾"×½"×0'-8"-x
3'-0"
2Pls 6"×½"×0'-6"
16"×40#I-B
10"×23#I
10"×23#I
1st Floor
Pl 4¾"×½"×0'-8"-y
10"×64#I
L6"×6"×½"×0'-8"
All ½" Fillet welds
Shop.
Field.
15"×40#Ls
L6"×6"×¾"×0'-6"
Fin.
Base Pl. 22"×1¼"×2'-0"
Anchor Bolts 1¼"φ×2'-6"

Fig. 463

The 10-in.x23-lb. beam is connected to the column by plates x and y. Plate y is welded to the column in the shop and plate x is welded to the beam in the shop as indicated.

The maximum end moment on the 10-in.x23-lb. beams as given in Fig. 457 is 346,356 in. lbs. Taking the depth of the beam as the effective depth of the welded connections, we obtain a stress at each flange of 34,635 lbs. Then for the length of weld required at each flange we have

$$34{,}635 \div 3{,}500 = 9.9 \text{ ins.}$$

As is seen, there is 9.5 ins. at each flange, which is about correct.

The holes in the bottom flange of the beam and in plate y are for bolting the beam in place while the field welding is being done.

The welds connecting the 15-in.x40-lb. channels in the footing should be sufficient to transmit the pressure coming on the channels. Approximately, the pressure on each channel could be

$$7 \times 24 \times 600 = 100{,}800 \text{ lbs.}$$

This requires

$$100{,}800 \div 3{,}500 = 29 \text{ ins. of weld.}$$

As seen, there are 30 ins

The footing is the same as shown in Fig. 461. The connections at the other floors are the same as shown for the first floor.

All of the other welded details throughout the building are worked out in the same manner.

The $\frac{1}{2}$-in. fillet weld is the largest that should be used.

304. Special Details.—The three-story building designed in the preceding articles is of sufficient magnitude for illustrating all of the fundamental principles involved in the designing of high buildings. It is obvious that the designing of buildings higher than three stories is simply the continuation of the work outlined in the designing of the three-story building. However, there are many special details that occur owing to particular requirements of room space. The most common case encountered is where columns are discontinued for a story to obtain a large room clear of columns, as required for auditoriums. In case the enlargement of room space is near the top of the building, the columns discontinued can be supported upon plate girders, as shown in Fig. 464. In that case the deflection of the plate girders would be small and consequently the stress on the frame above due to deflection of the plate girders would not be serious, but if there are several stories above the room enlargement the columns discontinuous should be supported upon trusses having a depth at least equal to the depth of a story, as indicated in Fig. 465. The trusses can be concealed in partition walls. Such girders, or trusses, should be designed so that the deflection in each case is a minimum. The effect of the deflection can be relieved to some extent by placing the proper thickness of shims under the columns where they connect to the girders or trusses.

Wherever columns are discontinued in a story, the remaining columns must be designed to take the shear from the wind at that story and to take also the moment and transferred load caused by the wind. In the case shown in Fig. 464 the shears on the beams and columns due to wind are obtained by beginning at the top of the building and continuing in the usual manner (previously explained) on down to the story in which the columns are discontinued. Then the shear and transferred load on the remaining columns in that story due to wind are obtained by analyzing the frame as an unsymmetrical bent, as previously explained (see Fig. 443). On the story below the wide opening, after the transferred loads are judiciously distributed to the columns, the shears on the beams and columns due to wind can be determined on down the building in the usual manner.

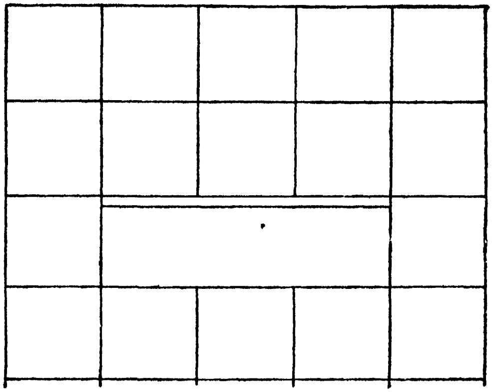


Fig. 464

In cases such as that shown in Fig. 465 it is usually advisable to construct a braced tower, in which case the tower is designed to take all of the horizontal shear in the story due to wind, thus eliminating the moment on the columns due to wind. The transferred load on the columns due to wind is determined by treating the frame in the story in which the columns are discontinued as an unsymmetrical bent, as previously explained.

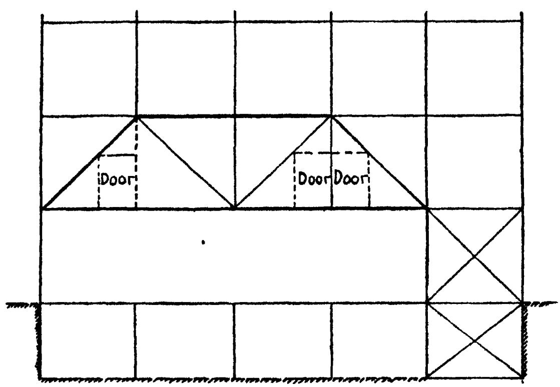


Fig. 465

In many cases where columns are discontinuous, the columns as well as the adjoining beams are subjected to moments due to unsymmetrical live and dead loads. These moments can, as a rule, be determined most satisfactorily by the "method of slope-deflection," the theory of which is given in Appendix *B*. However, there are many special details required in the construction of high buildings that the engineer must work out according to sense of fitness and best judgment, as no general rule can be given to cover the entire field.

APPENDIX A

SPECIFICATIONS FOR RAILWAY BRIDGES

(For Designing)

1. Introduction.—Specifications for railway bridges have passed through the formative stage so that few pronounced changes have been made during the past twenty years. The "General Specifications for Steel Railway Bridges" published by the American Railway Engineering Association* are used to quite an extent. The following paragraphs conform in general to these specifications.

2. Clearances on straight track shall be not less than those shown in Fig. 1a.

3. Floors.—Timber cross-ties shall be designed for the maximum wheel load distributed over three ties and with 100 per cent impact added. The fiber stress shall not exceed 2,000 lbs. per square inch. The ties shall be not less than 10 ft. in length and they shall be spaced with openings not exceeding 4 ins.

4. Live Load.—Cooper's *E*60, which is obtained by multiplying the loads shown in Fig. 151 by 60/50 (page 195).

5. Impact.—The formula

$$I=S\left(\frac{300}{L+300}\right) \quad \text{(page 198)} \ldots\ldots \quad (1)$$

has been used in designing most of the existing railway bridges and is still used to some extent. It has been recently replaced in the A. R. E. A. specifications by the formula

$$I=S\left(\frac{300}{300+\frac{L^2}{100}}\right) \ldots\ldots\ldots\ldots (2)$$

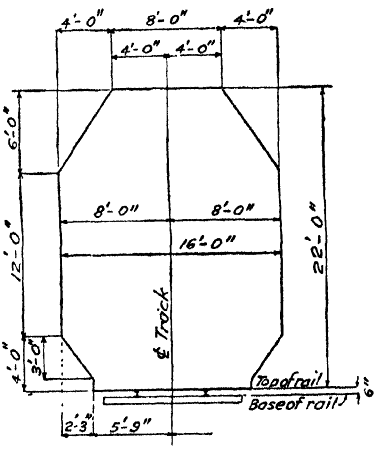


Fig. 1a

In the above formulas I = stress due to impact, S = maximum computed live-load stress, L = length of track (in feet) loaded when the maximum live-load stress occurs.

Formula (1) gives slightly less values than Formula (2) when L is less than 100 ft.—the values being equal when $L = 100$ ft. But when L is greater than 100 ft., Formula (1) gives greater values than Formula (2).

6. Column Formula.—The column formula

$$16{,}000-70\frac{L}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where L = length of member in inches and r = least radius of gyration of the cross-section of the member, has been used for the last twenty-five years and is still used to some extent. The above formula has been recently replaced in the A. R. E. A. specifications by the formula

$$15{,}000-50\frac{L}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

* 59 East Van Buren St., Chicago, Ill.

Formulas (1) and (2) give so nearly the same values that either can be used in the design of railroad bridges.

7. Allowable Unit Stresses.—The several parts of structures shall be so proportioned that the unit stresses shall not exceed the following:

Axial tension, net section	16,000
Tension in extreme fibers of rolled shapes, built sections and girders, net section	16,000
Tension in extreme fibers of pins	24,000
Shear in power-driven rivets	12,000
Bearing on power-driven rivets	24,000
Bearing on rollers	$600d$ lbs.

per linear inch of roller, where d = diameter of roller in inches.

8. Longitudinal Force (usually known as traction).—Provision shall be made in the design for the effect of a longitudinal force of 20 per cent of the live load on one track only, applied 6 ft. above the top of rail.

9. Slenderness Ratio.—The ratio of length to least radius of gyration shall not exceed:

100 for main compression members.
120 for wind and sway bracing.
140 for single lacing.
170 for double lacing.
200 for riveted tension members.

10. Reversal of Stress.—Members subjected to reversal of stress under the passage of the live load shall be proportioned as follows:

Determine the resultant tensile stress and the resultant compressive stress and increase each by 50 per cent of the lesser; then proportion the member so that it will be capable of resisting either increased resultant stress. The connections shall be proportioned for the sum of the resultant stresses.

11. Net Section at Pin Holes.—In pin-connected riveted tension members, the net section along the center line between the pin and the end of the member must not be less than the net area of the cross-section of the member and the net cross-section through the pin hole shall not be less than the net area of the member plus 25 per cent. The above has been changed in the latest specifications to read as follows:

"In pin-connected tension members, the net section across the pin hole shall not be less than 140 per cent and the net section back of the pin hole not less than 100 per cent of the net section of the body of the member." As is seen, the required net section across the pin hole has been increased.

12. Compression Members.—In built compression members, the metal shall be concentrated in the webs and flanges. The thickness of each web shall be not less than one-thirtieth of the distance between the lines of rivets connecting it to the flanges. The thickness of cover plates shall be not less than one-fortieth of the distance between the nearest rivet lines.

13. Tie Plates.—Open sides of compression members shall be provided with lacing bars and shall have tie plates as near each end as practicable, and tie plates shall also be provided at intermediate points where the lacing is interrupted.

In main members the length of the end tie plates shall be not less than 1¼ times the distance between the lines of rivets connecting them to the outer flanges, and the length of intermediate tie plates shall be not less than three-quarters of that distance. The thickness of tie plates shall not be less than one-fiftieth of that distance.

Tie plates on tension members shall have a length not less than two-thirds of the lengths specified for tie plates on compression members.

14. Lacing Bars.—The minimum width of lacing bars shall be 3 ins. for 1-in. rivets, 2¾ ins. for ⅞-in. rivets and 2½ ins. for ¾-in. rivets. Lacing bars of compression members shall be so spaced that the L/r of the portion of the flange included between their connections will be not greater than 40 and not greater than two-thirds of L/r of the member.

The angle of lacing bars with the axis of the member shall be not less than 45 deg. for double lacing and 60 deg. for single lacing. If the distance between rivet lines in the flanges is more than 15 ins. and a single rivet lacing bar is used, the lacing shall be double and riveted at the intersections. The lacing of compression members shall be proportioned to resist a shearing stress of $2\frac{1}{2}$ per cent of the direct stress where the bars are considered as compression members.

In members composed of side segments and a cover plate, with open side laced, one-half the shear shall be considered as taken by the lacing. Where double lacing is used, the shear in the plane of the lacing shall be distributed equally between the two systems.

As a rule the thickness of the lacing bars should not be less than the thickness of the adjoining tie plates.

15. Splices.—Abutting joints in compression members faced for bearing shall have their component parts spliced. The gross area of the splice material shall be not less than 50 per cent of the gross area of the smaller member. In determining the number of rivets in compression splices, the stress i the splice material shall be taken as 15,000 lbs. per square inch of gross area.

16. Size of Rivets.—Rivets $\frac{7}{8}$ ins. in diameter are usually used in railroad bridges. However, $\frac{3}{4}$-in. and 1-in. diameter rivets are used in a few cases to provide for special requirements.

17. Pitch of Rivets at Ends.—The pitch of rivets at ends of built compression members shall not exceed four diameters of rivet used for a distance equal to $1\frac{1}{2}$ times the maximum width of the member.

18. Expansion Bearing.—Spans more than 70 ft. in length shall have rollers at one end. Shorter spans will be arranged to slide on smooth surfaces.

19. Rollers.—Expansion rollers shall be not less than 6 ins. in diameter. They shall be coupled together with substantial side bars, which shall be so arranged that the rollers can be cleaned readily.

20. Minimum Thickness of Metal.—Metal shall not be less than $\frac{3}{8}$ ins. thick, except for fillers.

21. Thickness of web plates shall not be less than given by the formula:

$$\frac{1}{20}\sqrt{D},$$

where D represents the distance between flanges in inches.

22. Intermediate Stiffeners.—The webs of plate girders shall be stiffened by angles at intervals not greater than given by the formula:

$$d=\frac{t}{40}(12{,}000-s),$$

where d = the distance between stiffeners in inches, t = thickness of the web in inches, s = web shear in pounds per square inch at the point considered.

Intermediate stiffeners shall be riveted in pairs to the web of the girder. The outstanding leg of each angle shall not be less than 2 ins. plus one-thirtieth of the depth of the girder or more than 16 times its thickness.

APPENDIX B

DETERMINATION OF MOMENTS IN RECTANGULAR FRAMES BY METHOD OF SLOPE DEFLECTION

1. Preliminary.—For the purpose of illustration, let $ABCD$, shown at (*a*) in Fig. 1*b* represent a simple unloaded rectangular frame wherein the members AB, BC, CD, and DA are straight stiff beams rigidly connected together at joints A, B, C, and D. Now, if a horizontal load P is applied at B, the frame (as can be seen from observation) will be bent by load P into a position indicated at (*b*), Fig. 1*b*. Joints A and B, as seen, are deflected a horizontal distance Δ (ignoring the effect of direct stress in members as

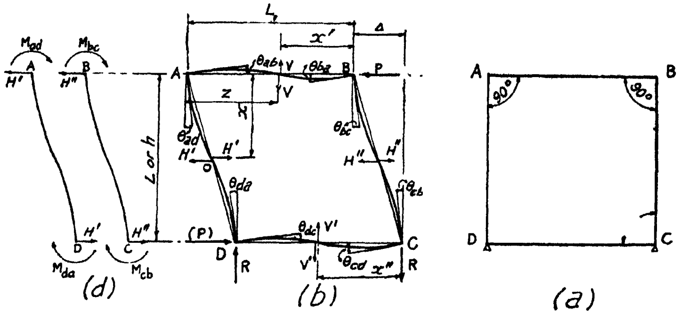


Fig. 1*b*

being negligible); each member is bent into a reverse curve, as shown at (*b*), Fig. 1*b*; the angles subtended by the members may no longer be 90-deg. angles; and the ends of each member have a slope with reference to their position before being bent by the application of load P.

These slopes at the ends of the members are known as slope angles. There are two (one at each end) for each member and they are designated at (*b*), Fig. 1*b*, as θ_{ab}, θ_{ba}, θ_{bc}, and so on. The subscripts indicate location, as is readily seen. R is the reaction at D and C; V and V' represent the vertical shear on members AB and DC, respectively; and H' and H'' represent the horizontal shear on members AD and BC, respectively, all of which is due to load P applied at joint B.

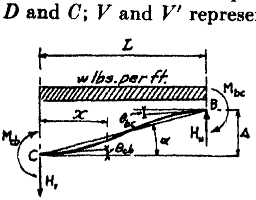


Fig. 2*b*

The problem, of course, is to determine the stresses in the members (composing the frame) due to load P. We know of no point on the frame where the moment is zero; so the frame is statically indeterminate. In fact, we know the location of only two joints, that is, joints D and C; but we do know that the slope angles and the deflections of the joints are functions of the moments on the members and that, by determining the slope angles and the deflection of the joints, the moments on the members can be found. This we do in applying the Method of Slope Deflection.

2. Slope-Deflection Formulas.—As a general case we shall consider the member BC (Fig. 1*b*) placed in a horizontal position (so as not to confuse algebraic symbols) as shown in Fig. 2*b*.

To make the case still more general we shall assume a uniform load of w lbs. per linear foot applied directly to the member as shown. We shall consider that w is a light load compared to P, so that the curvature of the member is affected but slightly. The analysis will hold regardless of the weight of w, but the curvature of the member would be changed.

Let M_{cb} = moment on member BC at C and let M_{bc} = moment on member BC at B. These moments are exerted by the other members connected at B and C.

Referring to Fig. 1b, we see that the moment exerted upon BC at B (by other members) is exerted by member AB and is equal to Vx' and acts clock-wise, as indicated in Fig. 2b. (V is shown applied at the point of contra-flexure in member AB.) Likewise the moment exerted upon BC at C (by other members) is exerted by member CD and is equal to $V'x''$, which also acts clock-wise, as indicated in Fig. 2b.

Let us consider counter clock-wise positive and clock-wise negative for moments and let us consider angles formed by the tangent rotating counter clock-wise as positive, and angles formed by the tangent rotating clock-wise as negative. Then M_{bc} and M_{cb} are both negative, and slope angles θ_{bc} and θ_{cb} are both positive, considering rotation about B for θ_{bc} and about C for θ_{cb}.

It is obvious that the shear H_{11} exerted on the end of BC at B will act in the same direction as P and that the shear H_1 exerted on the end of BC at C will be opposite in direction to P, all of which is indicated in Fig. 2b.

Then for the moment at any point (Fig. 2b) x distance to the right of C, taking C as the origin, we have

$$M = -M_{cb} + H_1 x + \frac{wx^2}{2}.$$

Then we have

$$EI\frac{d^2y}{dx^2} = -M_{cb} + H_1 x + \frac{wx^2}{2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Integrating once, we obtain

$$EI\frac{dy}{dx} = -xM_{cb} + \frac{H_1 x^2}{2} + \frac{wx^3}{6} + C.$$

When $x=0$, $dy/dx = \tan\theta_{cb} = \theta_{cb}$ (angle θ_{cb} is very small) and hence we obtain $C = EI\theta_{cb}$. Substituting in this value of C, we obtain

$$EI\frac{dy}{dx} = -xM_{cb} + \frac{H_1 x^2}{2} + \frac{wx^3}{6} + EI\theta_{cb} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Integrating again, we obtain

$$EIy = -\frac{x^2 M_{cb}}{2} + \frac{H_1 x^3}{6} + \frac{wx^4}{24} + xEI\theta_{cb} + C'.$$

When $x=0$, $y=0$, and $C'=0$, we have

$$EIy = -\frac{x^2 M_{cb}}{2} + \frac{H_1 x^3}{6} + \frac{wx^4}{24} + xEI\theta_{cb} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3).$$

When $x=L$, $dy/dx = \theta_{bc}$. So, substituting L for x in (2), we obtain

$$EI\theta_{bc} = -LM_{cb} + \frac{H_1 L^2}{2} + \frac{wL^3}{6} + EI\theta_{cb} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

When $x=L$ in (3), $y=\Delta$. So, substituting L for x in (3), we obtain

$$EI\Delta = -\frac{L^2 M_{cb}}{2} + \frac{H_1 L^3}{6} + \frac{wL^4}{24} + LEI\theta_{cb} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5).$$

From (4) we obtain

$$H_1 = \frac{2M_{cb}}{L} - \frac{wL}{3} - \frac{2EI\theta_{cb}}{L^2} + \frac{2EI\theta_{bc}}{L^2}.$$

Substituting this value of H in (5) we obtain

$$-EI\Delta-\frac{L^2M_{cb}}{2}+\frac{L^2M_{cb}}{3}-\frac{wL^4}{18}-\frac{LEI\theta_{cb}}{3}+\frac{LEI\theta_{bc}}{3}+\frac{wL^4}{24}+LEI\theta_{cb}=0,$$

from which we obtain

$$M_{cb}=\frac{2EI}{L}\left(2\theta_{cb}+\theta_{bc}-3\frac{\Delta}{L}\right)-\frac{wL^2}{12}\quad\text{(A)}$$

for the moment at C.

This is a fundamental equation used in slope-deflection analysis. It expresses the moment at one end of a member in terms of the slope angles, deflection, and the load acting directly on the member.

Next, taking B as the origin, we obtain

$$M=-M_{bc}+H_{11}x-\frac{wx^2}{2}$$

for the moment at any point x distance to the left of B. Then we have

$$EI\frac{d^2y}{dx^2}=-M_{bc}+H_{11}x-\frac{wx^2}{2}\quad\text{(6).}$$

By integrating and reducing in the manner shown with origin at C, we obtain

$$M_{bc}=\frac{2EI}{L}\left(2\theta_{bc}+\theta_{cb}-3\frac{\Delta}{L}\right)+\frac{wL^2}{12}\quad\text{(B)}$$

for the moment at B. This is also a fundamental equation used in slope-deflection analysis.

It should be noted that $wL^2/12$ occurring in equations (A) and (B) is the moment at each end of a fixed beam uniformly loaded. Now, by referring to Fig. 2*b* (or Fig. 1*b*), it is readily seen that, if the member is bent as shown, the top is in compression at C. We know that it would be in tension acting as a fixed beam due to the uniform load w, so that $wL^2/12$ should be negative at C in summing up the resultant moment at C. It is likewise readily seen that, if the member is bent as shown, the top of the member at B will be in tension so that the moment $wL^2/12$ at B due to the uniform load w adds to the moment at B and hence should be positive, as shown in equation (B). However, the signs can be taken unreservedly as signifying direction of rotation.

3. General Analysis of Frames.—It is readily seen that members meeting at a joint must have equal slope angles as the slope angles result from a certain twisting of the joint as a whole through a certain angle. Hence the slope angle of each member meeting at the joint must be equal to this twist angle of the joint. Then, referring to the diagram shown at (*b*), Fig. 1*b*, we have

(1) $\theta_{ab}=\theta_{ad}=\theta_a$ (3) $\theta_{cb}=\theta_{cd}=\theta_c$

(2) $\theta_{ba}=\theta_{bc}=\theta_b$ (4) $\theta_{dc}=\theta_{da}=\theta_d$

From this it is seen that there are four slope angles to be determined and hence four equations are required for their determinations. Equations (A) and (B) apply in any case where uniform load or no load is applied directly to the member regardless of deflection or curvature of the member. In case there is no load w applied, the term $wL^2/12$ drops out of the equations; and if there is no deflection, Δ drops out of the equations.

Considering member AB and applying equations (A) and (B) we obtain at A

$$M_{ab}=\frac{2EI}{L_1}(2\theta_{ab}+\theta_{ba})\quad\text{(1)}$$

since $\Delta=0$ (the ends having the same elevation) and $w=0$. We obtain at B

$$M_{ba}=\frac{2EI}{L_1}(2\theta_{ba}+\theta_{ab})\quad\text{(2).}$$

Similarly, considering member *DC* and applying equations (A) and (B), we obtain the two equations

$$M_{dc}=\frac{2EI}{L_1}(2\theta_{dc}+\theta_{cd})\ldots\ldots(3)$$

and

$$M_{cd}=\frac{2EI}{L_1}(2\theta_{cd}+\theta_{dc})\ldots\ldots(4).$$

Next, considering member *AD* and applying equations (A) and (B), we obtain

$$M_{ad}=\frac{2EI}{L}\left(2\theta_{ad}+\theta_{da}-3\frac{\Delta}{L}\right)\ldots\ldots(w=0)\ldots\ldots(5)$$

and

$$M_{da}=\frac{2EI}{L}\left(2\theta_{da}+\theta_{ad}-3\frac{\Delta}{L}\right)\ldots\ldots(6).$$

Then, considering member *BC* and applying equations (A) and (B), we obtain

$$M_{cb}=\frac{2EI}{L}\left(2\theta_{cb}+\theta_{bc}-3\frac{\Delta}{L}\right)\ldots\ldots(7).$$

$$M_{bc}=\frac{2EI}{L}\left(2\theta_{bc}+\theta_{cb}-3\frac{\Delta}{L}\right)\ldots\ldots(8).$$

We thus have eight equations.

It is obvious that the value of the slope angles and deflection also depend upon the stiffness of the members involved. The stiffness of a member is directly proportional to the moment of inertia of the area of the cross-section of the member and inversely proportional to the length of the member. So, for the stiffness factor in general, we have

$$S=\frac{I}{L}.$$

Let S_1, S_2, S_3, and S_4 be the stiffness factor for members *AB*, *BC*, *CD*, and *DA*, respectively.

By substituting these stiffness factors and also substituting θ_a for θ_{ab} and θ_{ad} and θ_b for θ_{ba} and θ_{bc} and so on, in the preceding equations, we obtain the following equations:

$$M_{ab}=2ES_1(2\theta_a+\theta_b)\ldots\ldots(1)'.$$
$$M_{ba}=2ES_1(2\theta_b+\theta_a)\ldots\ldots(2)'.$$
$$M_{dc}=2ES_3(2\theta_d+\theta_c)\ldots\ldots(3)'.$$
$$M_{cd}=2ES_3(2\theta_c+\theta_d)\ldots\ldots(4)'.$$

$$M_{ad}=2ES_4\left(2\theta_a+\theta_d-3\frac{\Delta}{L}\right)\ldots\ldots(5)'.$$

$$M_{da}=2ES_4\left(2\theta_d+\theta_a-3\frac{\Delta}{L}\right)\ldots\ldots(6)'.$$

$$M_{cb}=2ES_2\left(2\theta_c+\theta_b-3\frac{\Delta}{L}\right)\ldots\ldots(7)'.$$

$$M_{bc}=2ES_2\left(2\theta_b+\theta_c-3\frac{\Delta}{L}\right)\ldots\ldots(8)'.$$

The joints are in equilibrium and hence the moments exerted by the members about each joint must balance, that is, the sum of the moments must be equal to zero. For example, at joint *A* (Fig. 1*b*) we have

$$H'x=Vz \text{ or } H'x+(-Vz)=0.$$

It would not make any difference if there were a dozen members. The sum of their moments would be zero. So we have

$$M_{ab}+M_{ad}=0,$$
$$M_{ba}+M_{bc}=0,$$
$$M_{cb}+M_{cd}=0,$$
$$M_{dc}+M_{da}=0$$

Then, combining equations (1)′ and (5)′, and so on, and substituting r for Δ/L and dividing through by 3, we obtain the following equations:

Joint A: $M_{ab}+M_{ad}=0=S_1(2\theta_a+\theta_b)+S_4(2\theta_a+\theta_d-3r)$ . . . . (a).
Joint B: $M_{ba}+M_{bc}=0=S_1(2\theta_b+\theta_a)+S_2(2\theta_b+\theta_c-3r)$ . . . . (b).
Joint C: $M_{cd}+M_{cb}=0=S_3(2\theta_c+\theta_d)+S_2(2\theta_c+\theta_b-3r)$ . . . . (c).
Joint D: $M_{dc}+M_{da}=0=S_3(2\theta_d+\theta_c)+S_4(2\theta_d+\theta_a-3r)$ . . . . (d).

As will be seen, there are five unknown quantities in these four equations, that is θ_a, θ_b, θ_c, θ_d, and r, and hence one more equation is required in order to determine the five unknown values. The fifth equation may be obtained by considering in static equilibrium the two vertical members AD and BC cut off just below the joints A and B and just above the joints C and D. In that case each member is held in equilibrium by the moment at each end, by the horizontal shear at each end, and by the direct

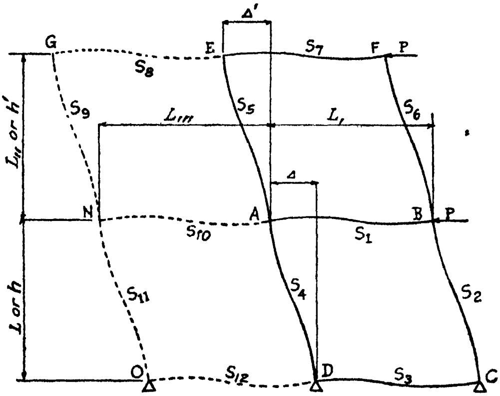


Fig. 3*b*

stress on each member as indicated at (*d*) in Fig. 1*b*. H' and H'' represent the horizontal shear at the ends of the member as indicated. Taking moment about the horizontal line through CD, we obtain

$$(M_{ad}+M_{da}+H'h)+(M_{bc}+M_{cb}+H''h)=0,$$
$$(H'+H'')=H=P$$

Then we have

$$M_{ad}+M_{da}+M_{bc}+M_{cb}=-Hh.$$

Adding the values of these moments given by equations (5)′, (6)′, (7)′, and (8)′ and taking $r=\Delta/L$ and reducing, we obtain

$$S_4(\theta_a+\theta_d-2r)+S_2(\theta_b+\theta_c-2r)=-\frac{Hh}{6E}. \quad \text{(e)}$$

for the required fifth equation.

The values of θ_a, θ_b, θ_c, θ_d and r can now be determined by solving the five preceding simultaneous equations (a), (b), (c), (d), and (e).

Then, substituting these values in equations (1)′, (2)′, (3)′, and so on, the moments on the members throughout are obtained. Then the stresses in the members can be found in the usual manner.

Now let us imagine that another frame of similar magnitude is connected to the top of the one just considered, as indicated in Fig. 3*b*. The frame now has three more members—*AE*, *EF*, and *FB*—and two more rigid joints—one at *E* and the other one at *F*. Let the additional loads be as indicated.

As is obvious, there will now be six equations similar to equations (a), (b), (c), and (d)—equations (c) and (d) without change of form. The six equations are as follows:

Joint *A*: $S_1(2\theta_a+\theta_b)+S_4(2\theta_a+\theta_d-3r)+S_5(2\theta_a+\theta_e-3r')=0$ .... (a)'.
Joint *B*: $S_1(2\theta_b+\theta_a)+S_2(2\theta_b+\theta_c-3r)+S_6(2\theta_b+\theta_f-3r')=0$ .... (b)'.
Joint *C*: $S_3(2\theta_c+\theta_d)+S_2(2\theta_c+\theta_b-3r)=0$ .... (c)'.
Joint *D*: $S_3(2\theta_d+\theta_c)+S_4(2\theta_d+\theta_a-3r)=0$ .... (d)'.
Joint *E*: $S_7(2\theta_e+\theta_f)+S_5(2\theta_e+\theta_a-3r')=0$ .... (e)'.
Joint *F*: $S_7(2\theta_f+\theta_e)+S_6(2\theta_f+\theta_b-3r')=0$ .... (f)'.

$r'=\Delta'/L_{11}$ in the above equations and $r=\Delta/L$.

There are two more equations required in order to obtain r and r' in addition to the six slope angles.

Let H represent the horizontal shear just below AB which now equals $2P$. Then we have

$$M_{ad}+M_{da}+M_{bc}+M_{cb}+Hh=0$$

from which we obtain

$$S_4(\theta_a+\theta_d-2r')+S_2(\theta_b+\theta_c-2r)=-\frac{Hh}{6E} \qquad \text{(g)}'$$

by substituting in the value of the moments and reducing, as shown in the preceding analysis.

Let H_1 represent the horizontal shear just below joints E and F, which equals P. Considering members EA and FB in equilibrium as previously shown for members AD and BC, we obtain

$$S_5(\theta_e+\theta_a-2r')+S_6(\theta_f+\theta_b-2r)=-\frac{H_1h'}{6E} \qquad \text{(k)}'.$$

By solving the eight preceding simultaneous equations (a)', (b)', (c)', (d)', (e)', (f)', (g)', and (k)', the value of θ_a, θ_b, θ_c, θ_d, θ_e, θ_f, r, and r' can be obtained. Then the moments on each of the members and the stresses in each can be obtained in the usual manner.

Equations (a)', (b)', (c)', (d)', (e)', (f)', (g)', and (k)' are obtained in the manner explained for equations (a), (b), (c), (d), and (e). For example, we obtain equation (a)' by considering first, say, member AD at joint A and writing

$$M_{ad}=2ES_4(2\theta_a+\theta_d-3r) \qquad (w=0)$$

(using Formula A) then considering member AB and writing

$$M_{ab}=2ES_1(2\theta_a+\theta_b)$$

and next considering member AE and writing

$$M_{ae}=2ES_5(2\theta_a+\theta_e-3r') \qquad (9)'.$$

Now, the sum of these three equations $=0$. So we obtain

$$M_{ab}+M_{ad}+M_{ae}=2ES_1(2\theta_a+\theta_b)+2ES_4(2\theta_a+\theta_d-3r)+2ES_5(2\theta_a+\theta_e-3r')=0$$

Dividing through by $2E$, we have

$$S_1(2\theta_a+\theta_b)+S_4(2\theta_a+\theta_d-3r)+S_5(2\theta_a+\theta_e-3r')=0,$$

which is equation (a)'.
Similarly for equation (b)', considering joint B, we have

Member *BA*: $M_{ba}=2ES_1(2\theta_b+\theta_a)$.
Member *BC*: $M_{bc}=2ES_2(2\theta_b+\theta_c-3r)$.
Member *BF*: $M_{bf}=2ES_6(2\theta_b+\theta_f-3r')$.

Placing the sum of these three equations equal to 0 and dividing through by $2E$, we obtain equation (b)′.

Next, for equation (e)′, considering joint E, we have

Member EA: $M_{ea}=2ES_5(2\theta_e+\theta_a-3r')$.
Member EF: $M_{ef}=2ES_7(2\theta_e+\theta_f)$.

Placing the sum of these two equations equal to zero and dividing through by $2E$, we obtain equation(e)′.

For equation (f)′, considering point F, we have

Member FE: $M_{fe}=2ES_7(2\theta_f+\theta_e)$.
Member FB: $M_{fb}=2ES_6(2\theta_f+\theta_b-3r')$.

Placing these two equations equal to zero and dividing through by $2E$, we obtain equation (f)′.

Next let us consider two more frames $EGNA$ and $ANOD$ (shown dotted in Fig. 3b) rigidly connected to the frame $EADCBFE$ just considered. Then we have the continuous frame $GNODCBFEG$. The applied loads are as before.

Beginning at any joint, say joint G, we can write the equations:

Member GE: $M_{ge}=2ES_8(2\theta_g+\theta_e)$.
Member GN: $M_{gn}=2ES_9(2\theta_g+\theta_n-3r')$.

Then we have for joint G

$$M_{ge}+M_{gn}=S_8(2\theta_g+\theta_e)+S_9(2\theta_g+\theta_n-3r')=0 \quad \text{(a)}''.$$

For joint N we obtain

Member NG: $M_{ng}=2ES_9(2\theta_n+\theta_g-3r')$.
Member NA: $M_{na}=2ES_{10}(2\theta_n+\theta_a)$.
Member NO: $M_{no}=2ES_{11}(2\theta_n+\theta_o-3r)$.

Then we obtain for joint N

$$M_{ng}+M_{na}+M_{no}=S_9(2\theta_n+\theta_g-3r')+S_{10}(2\theta_n+\theta_a)+S_{11}(2\theta_n+\theta_o-3r)=0 \quad \text{(b)}''$$

For joint A we obtain

Member AN: $M_{an}=2ES_{10}(2\theta_a+\theta_n)$.
Member AD: $M_{ad}=2ES_4(2\theta_a+\theta_d-3r)$.
Member AB: $M_{ab}=2ES_1(2\theta_a+\theta_b)$.
Member AE: $M_{ae}=2ES_5(2\theta_a+\theta_e-3r')$.

Then we obtain for joint A

$$M_{an}+M_{ad}+M_{ab}+M_{ae}=S_{10}(2\theta_a+\theta_n)+S_4(2\theta_a+\theta_d-3r)+S_1(2\theta_a+\theta_b)+S_5(2\theta_a+\theta_e-3r')=0 \quad \text{(c)}'',$$

and so on, for all the joints in the structure.

In this manner nine equations, one for each joint, are obtained. These nine equations contain eleven unknown terms, θ_a, θ_b, θ_c, θ_d, θ_e, θ_f, θ_g, θ_n, θ_o, r, and r'. So two more equations are required. These two additional equations can be obtained by utilizing the shear just below joints F, E, and G and just below joints B, A, and N as previously explained. For example, considering members FB, AE, and GN, we have

$$M_{gn}+M_{ea}+M_{fb}+M_{ng}+M_{ae}+M_{bf}=-H_1h',$$

where H_1 = shear just below joints G, E, and F ($H_1=P$). Then, by substituting $2ES_9(2\theta_g+\theta_n-3r)$ for M_{gn}, and so on, for each of the other moments, we obtain

$$S_9(\theta_g+\theta_n-2r')+S_5(\theta_e+\theta_a-2r')+S_6(\theta_f+\theta_b-2r')=\frac{H_1h'}{6E} \quad \text{(s)}''$$

for one additional equation. Considering members NO, AD, and BC in equilibrium (as previously explained), we obtain

$$M_{no}+M_{on}+M_{ad}+M_{da}+M_{bc}+M_{cb}=-Hh,$$

where H = shear just below joints N, A, and B ($H=2P$). Then, by substituting $2ES''(2\theta_n+\theta_o-3r)$ for M_{no}, and so on, we obtain

$$S_{11}(\theta_n+\theta_o-2r)+S_4(\theta_a+\theta_d-2r)+S_2(\theta_b+\theta_c-2r)=-\frac{Hh}{6E} \qquad \text{(t)}''$$

for the second additional equation required.

Now it is seen from the foregoing analysis that by utilizing the relations given in equations (A) and (B) the necessary equation involving the slope angle and the deflection at each end of each member for any rectangular frame of any magnitude can be written. Then, by solving these equations as simultaneous equations, all of the slope angles and deflections can be obtained. Then, by substituting these values in the moment equations given for each end of each member, the moments on each member can be obtained and, finally, the stresses in the members due to the moments can be computed in the usual manner.

The case so far considered is for horizontal loads applied at joints and is the same as wind loads on rectangular building frames.

If only vertical loads are applied to a rectangular frame, the deflection of the joints are so small (as a rule) that the value of the terms $3r$ is negligible. Hence these terms drop out of equations (A) and (B) and then we have

$$M_{ab}=\frac{2EI}{L}(2\theta_{ab}+\theta_{ba})-\frac{wL^2}{12} \qquad \text{(A)}'$$

and

$$M_{ba}=\frac{2EI}{L}(2\theta_{ba}+\theta_{ab})+\frac{wL^2}{12} \qquad \text{(B)}'.$$

Formulas (A)′ and (B)′ can be used when all the loads act vertically on the frame.

For the purpose of illustration, let $ABCD$, Fig. 4b, represent the same frame as shown in Fig. 1b. Suppose the only load now applied is a uniform load of w lbs. per linear foot on member AB. It is seen from observation that the frame will be bent as indicated in Fig. 4b. Now, considering joint A and writing out the equation for that end of AB, using Formula (A)′, we have

$$M_{ab}=2ES_1(2\theta_a+\theta_b)-\frac{wL^2}{12}$$

and for AD, we have

$$M_{ad}=2ES_4(2\theta_a+\theta_d) \quad (w=0 \text{ for member } AD)$$

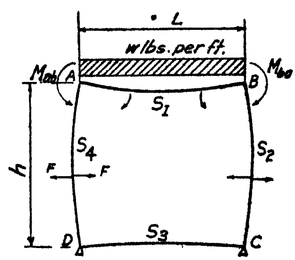


Fig. 4b

Then, as the sum of these moments must be equal to zero to have equilibrium of joint A, we obtain

$$M_{ab}+M_{ad}=2ES_1(2\theta_a+\theta_b)-\frac{wL^2}{12}+2ES_4(2\theta_a+\theta_d)=0,$$

which reduces to

$$S_1(2\theta_a+\theta_b)+S_4(2\theta_a+\theta_d)-\frac{wL^2}{24E}=0 \qquad (1)''.$$

Next considering joint B and using equation (B)′, we have

$$M_{ba}=2ES_1(2\theta_b+\theta_a)+\frac{wL^2}{12}$$

for that end of member AB. For member BC we have

$$M_{bc}=2ES_2(2\theta_b+\theta_c) \qquad (w=0)$$

Then we obtain

$$M_{ba}+M_{bc}=2ES_1(2\theta_b+\theta_a)+\frac{wL^2}{12}+2ES_2(2\theta_b+\theta_c)=0,$$

from which we obtain

$$S_1(2\theta_b+\theta_a)+S_2(2\theta_b+\theta_c)+\frac{wL^2}{24E}=0 \qquad \ldots\ldots (2)''.$$

Considering joint C, we obtain

$$M_{cb}=2ES_2(2\theta_c+\theta_b)$$

and

$$M_{cd}=2ES_3(2\theta_c+\theta_d).$$

Adding these two equations, we obtain

$$S_2(2\theta_c+\theta_b)+S_3(2\theta_c+\theta_d)=0 \qquad \ldots\ldots (3)''.$$

Next, considering joint D, we obtain

$$M_{dc}=2ES_3(2\theta_d+\theta_c)$$

and

$$M_{da}=2ES_4(2\theta_d+\theta_a).$$

Adding these two equations, we obtain

$$S_3(2\theta_d+\theta_c)+S_4(2\theta_d+\theta_a)=0 \qquad \ldots\ldots (4)''.$$

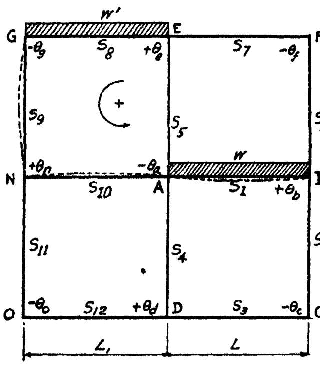


Fig. 5b

Now, the values of the four slope angles can be determined by solving the four simultaneous equations (1)'', (2)'', (3)'', and (4)''. Then, by substituting these values (observing the signs of the angles) in the moment equations, the moments on the members can be obtained and the stresses due to these moments can then be found in the usual manner.

The necessary equations for determining the slope angles throughout any rectangular frame of any magnitude can be written out in the manner shown above. There will be one equation for each joint. For example, let us consider the rectangular frame shown in Fig. 5b where the loads are as indicated, that is, vertical load on member AB and GE only. Beginning at any joint we can write out the following equations:

Joint G:
$$M_{ge}=2ES_8(2\theta_g+\theta_e)-\frac{w'L_1^2}{12}.$$
$$M_{gn}=2ES_9(2\theta_g+\theta_n).$$

Joint E:
$$M_{eg}=2ES_8(2\theta_e+\theta_g)+\frac{w'L_1^2}{12}.$$
$$M_{ef}=2ES_7(2\theta_e+\theta_f).$$
$$M_{ea}=2ES_5(2\theta_e+\theta_a).$$

Joint F:
$$M_{fe}=2ES_7(2\theta_f+\theta_e).$$
$$M_{fb}=2ES_6(2\theta_f+\theta_b).$$

Joint B:
$$M_{bf}=2ES_6(2\theta_b+\theta_f).$$
$$M_{ba}=2ES_1(2\theta_b+\theta_a)+\frac{wL^2}{12}.$$
$$M_{bc}=2ES_2(2\theta_b+\theta_c).$$

$$\text{Joint } C: \begin{cases} M_{cb} = 2ES_2(2\theta_c + \theta_b). \\ M_{cd} = 2ES_3(2\theta_c + \theta_d). \end{cases}$$

$$\text{Joint } O: \begin{cases} M_{on} = 2ES_{11}(2\theta_o + \theta_n). \\ M_{od} = 2ES_{12}(2\theta_o + \theta_d). \end{cases}$$

$$\text{Joint } D: \begin{cases} M_{dc} = 2ES_3(2\theta_d + \theta_c). \\ M_{do} = 2ES_{12}(2\theta_d + \theta_o). \\ M_{da} = 2ES_4(2\theta_d + \theta_a). \end{cases}$$

$$\text{Joint } N: \begin{cases} M_{ng} = 2ES_9(2\theta_n + \theta_g). \\ M_{no} = 2ES_{11}(2\theta_n + \theta_o). \\ M_{na} = 2ES_{10}(2\theta_n + \theta_a). \end{cases}$$

$$\text{Joint } A: \begin{cases} M_{ab} = 2ES_1(2\theta_a + \theta_b) - \dfrac{wL^2}{12}. \\ M_{ad} = 2ES_4(2\theta_a + \theta_d). \\ M_{an} = 2ES_{10}(2\theta_a + \theta_n). \\ M_{ae} = 2ES_5(2\theta_a + \theta_e) \end{cases}$$

Now, adding the values of the moments for each joint (separately) as given in the foregoing equations and placing the sum in each case equal to zero and reducing, we obtain the following nine equations:

Joint G: $S_8(2\theta_g + \theta_e) + S_9(2\theta_g + \theta_n) - \dfrac{w'L_1^2}{24E} = 0.$

Joint E: $S_8(2\theta_e + \theta_g) + S_7(2\theta_e + \theta_f) + S_5(2\theta_e + \theta_a) + \dfrac{w'L_1^2}{24E} = 0.$

Joint F: $S_7(2\theta_f + \theta_e) + S_6(2\theta_f + \theta_c) = 0.$

Joint B: $S_6(2\theta_b + \theta_f) + S_1(2\theta_b + \theta_a) + S_2(2\theta_b + \theta_c) + \dfrac{wL^2}{24E} = 0.$

Joint C: $S_2(2\theta_c + \theta_b) + S_3(2\theta_c + \theta_d) = 0.$

Joint O: $S_{11}(2\theta_o + \theta_n) + S_{12}(2\theta_o + \theta_d) = 0.$

Joint D: $S_3(2\theta_d + \theta_c) + S_{12}(2\theta_d + \theta_c) + S_4(2\theta_d + \theta_a) = 0.$

Joint N: $S_3(2\theta_n + \theta_g) + S_{11}(2\theta_n + \theta_o) + S_{10}(2\theta_n + \theta_a) = 0.$

Joint A: $S_1(2\theta_a + \theta_b) + S_4(2\theta_a + \theta_d) + S_{10}(2\theta_a + \theta_n) + S_5(2\theta_a + \theta_e) - \dfrac{wL^2}{24E} = 0.$

By solving the nine simultaneous equations given above, the values of the nine unknown slope angles can be determined. Then by substituting the values thus found into the moment equations, the moments on each member can be determined. Then the stresses in each member due to the moments on the member can be found in the usual manner.

The nine equations given above can be put into better form by collecting the coefficients. They were given in the present form to show the direct derivation. In substituting the value of the slope angles into the moment equations, care must be taken to use the correct sign. These signs can be obtained by studying the diagram of the frame with respect to the position of the loading. For example, referring to joint A (Fig. 5b) it is readily seen that angle θ_a on member AB is formed by the tangent rotating clock-wise and hence θ_a is minus, while at joint B just the opposite occurs. So we have $-\theta_a$ and $+\theta_b$. Again at joint N it is readily seen that angle θ_n is formed by the tangent rotating counter clock-wise about joint N as member NA is bent upward by the load on member AB. So we have $+\theta_n$. Considering joint G, it is readily seen that the load w' on member GE will bend the member downward and hence the angle θ_g is formed by the tangent at G rotating clock-wise and we have $-\theta_g$. The member GN is bent, as shown by the dotted line. From this it is seen that θ_n is positive. In this manner the signs of the slope angles throughout any frame can be ascertained.

4. General Slope-Deflection Formula.—So far two formulas, one for each end of the member considered, have been used for the sake of illustration and only uniform loads have been considered. However, only one general formula is necessary and it can be written

$$M_c = 2ES(2\theta_c + \theta_o - 3r) \pm C_c \qquad \text{(M)}$$

where the subscript c signifies the end that is being considered and o the other end of the member, and C represents the moment at the end that is being considered due to any kind of load. The member is considered to be a fixed beam in determining C. M_c = moment at the end that is being considered. $r = \frac{\Delta}{L} = \frac{\text{deflection}}{\text{length of member}}$. θ_c = slope angle at the end that is being considered. θ_o = slope angle at the other end of the member, $S = I/L$, where I is the moment of inertia of the cross-section of the member and L is the length of the member.

To illustrate the foregoing description, let AB (Fig. 6*b*) represent a horizontal restrained member supporting a load P at mid-span. Substituting in (M) and considering end A, we have

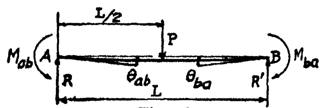


Fig. 6*b*

$$M_{ab} = 2ES(2\theta_{ab} + \theta_{ba}) + \tfrac{1}{8}PL \qquad \text{(a)}$$

for the moment on member AB at A.

Next, considering end B and substituting in (M), we have

$$M_{ba} = 2ES(2\theta_{ba} + \theta_{ab}) - \tfrac{1}{8}PL \qquad \text{(b)}$$

for the moment on member AB at B. $\frac{1}{8}PL$ is the moment at each end due to the load P at mid-span, considering the member as a fixed beam. *The sign of C will be opposite to that of the moments of the loads about the end considered.* For example, the load on member AB tends to rotate clock-wise about A (Fig. 6*b*) and therefore has a negative moment about A (considering counter clock-wise positive). As C must act in the opposite direction to balance this moment, C will be positive, as indicated in equation (a). Considering end B, the moment of the load about B is positive and hence C at B is negative as shown in (b). Further to illustrate the meaning of the signs, let us consider that member AB is equally restrained at A and B. Then $M_{ab} = M_{ba}$ and $\theta_{ab} = \theta_{ba}$. The tangent at A rotates clock-wise in forming angle θ_{ab}. So θ_{ab} is negative and the tangent at B rotates counter clock-wise in forming angle θ_{ba} so θ_{ba} is positive. Then

$$(2\theta_{ab} + \theta_{ba}) = (-2\theta_{ab} + \theta_{ab}) = -(\theta_{ab})$$

and

$$(2\theta_{ba} + \theta_{ab}) = (2\theta_{ba} - \theta_{ba}) = +(\theta_{ba}) = +(\theta_{ab})$$

Then, substituting in (a), we obtain

$$M_{ab} = -2ES\theta_{ab} + \frac{PL}{8} \qquad \text{(a)}'$$

and, substituting in (b), we obtain

$$(-)M_{ba} = +2ES\theta_{ab} - \frac{PL}{8} \qquad \text{(b)}'.$$

As is seen (a)′ and (b)′ are equal in value, which we knew must be, according to the assumptions made. But M_{ba} is negative (as seen from Fig. 6*b*). Hence, by multiplying equation (b)′ by -1, we obtain

$$M_{ba} = -2ES\theta_{ab} + \frac{PL}{8},$$

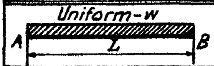
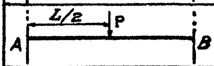
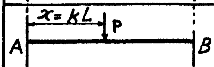
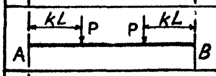
which is the same as given in (a)′.

The author selected counter clock-wise rotation as positive, as a matter (so he thinks) of convenience. If clock-wise was selected as positive the same final results would be obtained.

The sign of $r(=\Delta/L)$ is determined by the direction of rotation of the straight line joining the ends of the member. For example, if angle α (Fig. 2*b*) is formed by the line CB rotating counter clock-wise, the angle α is positive and, likewise, Δ and r are positive.

The following table gives the value of C for the different loadings.

Table 1 VALUE OF C.

Kind of Load	C_{ab}	C_{ba}
	$\frac{wL^2}{12}$	$\frac{wL^2}{12}$
	$\frac{PL}{8}$	$\frac{PL}{8}$
	$Pk(1-k)^2L$	$Pk^2(1-k)L$
	$Pk(1-k)L$	$Pk(1-k)L$
	$\frac{L^2}{60}(3w_a+2w_b)$	$\frac{L^2}{60}(2w_a+3w_b)$
	$\frac{w_aL^2}{20}$	$\frac{w_aL^2}{30}$

5. Analysis of a Single Bent.—Let $ABCD$ (Fig. 7b) represent a rigid bent supporting a uniform load of w lbs. per foot upon member AB and two equal concentrated loads P symmetrically placed upon member AB, as indicated in Fig. 7b. We shall consider member AD fixed at D and member BC fixed at C. Then $\theta_c=0$ and $\theta_d=0$. Let S_1 be stiffness factor I/L of member AB and S_2 be the stiffness factor of each of the members AD and BC.

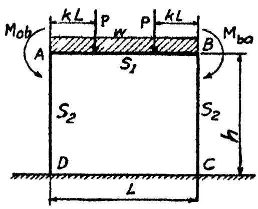


Fig. 7b

As the load is symmetrical with reference to the center of frame and members AD and BC have equal stiffness factors (S_2), we have $-\theta_a=\theta_b$. Then we can write the following equations:

$$M_{ab}=-2ES_1\theta_a+C_a \quad \text{(1).}$$
$$M_{ba}=+2ES_1\theta_a-C_b \quad \text{(2).}$$
$$M_{ad}=2ES_2(-2\theta_a+0)=-4ES_2\theta_a \quad \text{(3).}$$
$$M_{da}=2ES_2(0+(-\theta_a)=-2ES_2\theta_a \quad \text{(4).}$$
$$M_{bc}=2ES_2(2\theta_b+0)=4ES_2\theta_a \quad (\theta_b=\theta_a) \quad \text{(5).}$$
$$M_{cb}=2ES_2(0+\theta_b)=2ES_2\theta_a \quad \text{(6).}$$

Now, multiplying (3) by -1 and adding the result to equation (1), we have

$$-2ES_1\theta_a+C_a+4ES_2\theta_a=0,$$

from which we obtain

$$\theta_a=-C_a\left(\frac{1}{2E(2S_2-S_1)}\right).$$

Substituting this value of θ_a in equation (1), we obtain

$$M_{ab}=C_a\left(\frac{S_1}{2S_2-S_1}+1\right) \quad \text{(7)}$$

and substituting the same in (3), we obtain

$$M_{ad}=C_a\left(\frac{2S_2}{(2S_2-S_1)}\right) \quad \text{(8)}$$

and next, substituting in (4), we obtain [multiplying (4) by -1]

$$M_{da}=C_a\left(\frac{S_2}{(2S_2-S_1)}\right)\ldots\ldots\qquad(9).$$

Next, substituting the value of θ_a in (2) and multiplying by -1, we obtain

$$M_{ba}=C_b\left(\frac{S_1}{2S_2-S_1}+1\right)=C_a\left(\frac{S_1}{2S_2-S_1}+1\right)\ldots(\text{since } C_a=C_b)\ldots\ldots\qquad(10).$$

Then, substituting in (5), we obtain

$$M_{bc}=C_a\left(\frac{2S_2}{2S_2-S_1}\right)\ldots\ldots\qquad(11)$$

which is the same as (8). Next, substituting (value of θ_a) in (6) (multiplied by -1), we obtain

$$M_{cb}=C_a\left(\frac{S_2}{2S_2-S_1}\right)\ldots\ldots\qquad(12)$$

which is the same as (9).

It should be kept in mind that the moment around the end of a member (being considered) has the same sign as the moment on the adjoining member (or members). For example, M_{ad} (Fig. 1*b*) is negative because the forces acting on member AB causing the moment ($=V_z$) are acting clock-wise about joint A.

From Table 1 (Art. 4*b*) we obtain

$$C_a=C_b=\frac{wL^2}{12}+Pk(1-k)L$$

Now, by substituting numerical values in equations (7), (8), and (9), all the moments at the ends of the members will be obtained and then the moment at any point in any member is readily obtained. For example, the moment at mid-span of AB is equal to

$$-M_{ab}+V_a\frac{L}{2}-m,$$

where V_a = shear on AB at $A=\left(\frac{wL}{2}+P\right)$ and m = moment of intervening loads about mid-span.

If a horizontal load was applied at A or B, the term $3r$ would appear in the equations and θ_a and θ_b would not be equal and each should be determined, as can be done very readily, and it would be necessary to determine r. However, the problem presents no difficulties if the preceding analysis is understood.

6. Stresses in Building Frames.—The stresses in building frames are due to live and dead load, which are vertical loads, and to wind load, which is a horizontal load.

Stresses Due to Wind Load can be determined for each member of any frame by the slope-deflection method, as explained in the foregoing analysis (Art. 3), by writing out an equation for each joint, involving the slope angles (twist angle of the joint), and an equation for each story, involving r (the deflection factor of the story), and then by solving these simultaneous equations to obtain the slope angles and the deflection factors. Then, by substituting the values of these slope angles and the deflection factors thus found into the moment equation for each member, the moments on each member can be obtained.

It is obvious that this method requires an enormous amount of work, which can be done* but is not necessary, since simpler methods can be used which will give moments sufficiently accurate. It is true that by assuming that certain slope angles are equal and that certain other relations exist (which is only approximately true) the application of the method of slope deflection is greatly shortened and simplified, but the results

* See *Bulletin* 80, Engineering Experiment Station, University of Illinois, Wilson and Maney.

thus obtained are approximations. A simpler method giving approximate results is used because only approximate results can be obtained regardless of method used on account of the indeterminable resistance of the walls, partitions, and floor slabs.

We ignore this resistance and compute the moments and shears throughout the frame by using, instead of the method of slope deflection, a simple method that gives approximate results. The author does not wish to be understood as condemning the method of slope deflection in the least, for it is a very efficient method of determining moments in statically indeterminate frames whenever the uncertain effects of necessary details do not cause the values obtained to be only approximations.

There are several so-called approximate methods for determining wind stresses in building frames. The method given in Art. 286 has been used by many engineers for the past twenty-five years. It was presented in the *Engineering Record*, Sept. 5, 1908, by A. C. Wilson. W. M. Wilson and G. A. Maney and others admit* that this method gives fair results in comparison with results obtained by the slope-deflection method. There are other approximate methods giving slightly better results, but the method given in Art. 286 is the easiest to apply and is recommended.

Stresses Due to Dead and Live Load.—Both dead and live loads are usually considered to be uniform loads. Building frames if properly designed are sufficiently stiff

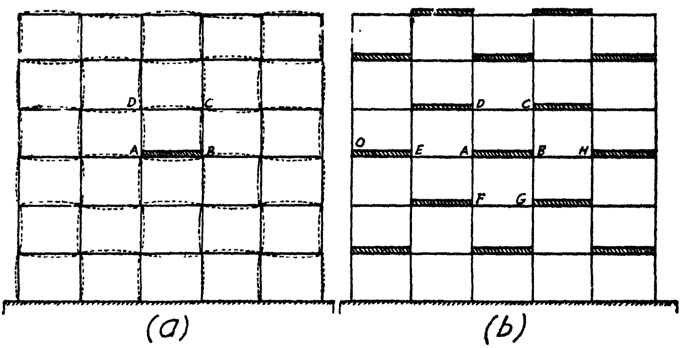


Fig. 8b

to warrant the assumption that the floor girders and beams are fixed at the ends. In this case, we have

$$M' = -\frac{wL^2}{12} \qquad (1)$$

for the moment at each end and

$$M = \frac{wL^2}{24} \qquad (2)$$

for the moment at mid-span of each floor girder or beam, where w = dead load or live load, or the sum of the two, per foot of span and L = length of span in feet.

Formula (1) gives the maximum moment that is likely to occur at the ends of the girders or beams and is therefore generally used. But any distortion at the ends of a span will cause an increase in the moment at mid-span over that given by Formula (2). To provide for this possible increase in the moment at mid-span, the following formula

$$M = \frac{wL^2}{16} \text{ or } \frac{wL^2}{12} \qquad (3)$$

* See *Bulletin* 80, Engineering Experiment Station, University of Illinois.

is generally used instead of Formula (2) to obtain the moment at mid-span of floor girders and beams in buildings. ($wL^2/16$ is used for intermediate spans and $wL^2/12$ is used for end spans.) The dead load, if the spans are equal or nearly equal in length, will cause no bending on the interior columns. The same statement can be made of the live load if it is continuous over the floors, but both loads cause bending on the exterior columns. The live load, if not continuous over the floors, will cause moments on interior columns as well as on exterior columns. These moments should be considered in designing the columns. Let the diagram at (*a*), Fig. 8*b*, represent a frame bent of a building. If span *AB* alone were loaded with live load, the members throughout the bent would deflect and bend (to some extent), as indicated by the dotted lines. By examining the general curvature of the members as indicated by the dotted lines it is readily seen that the maximum moment at joints *A* and *B* and at the center of span *AB* will occur when the spans are loaded as shown at (*b*), (Fig. 8*b*). However, in the first place, such a loading is improbable, and, in the second place, the loads in the remote panels from panel *AB* contribute only a negligible part of the moment at joints *A* and *B* and at the center of span *AB* compared with the moment caused by the load in panel *AB*. This has been found to be true by many engineers from experiments and from calculations for actual designs.* Owing to the small contribution to the moment at any given joint from loads in remote panels, it is found sufficient in practice to consider only a small portion of the frame as bound by the letters *E-D-C-H-G-F* as shown in Fig. 9*b*.

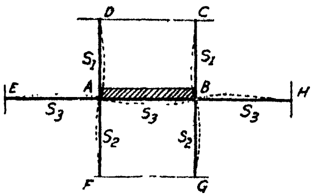


Fig. 9*b*

Moments on Interior Columns. Let us consider the portion shown in Fig. 9*b* where span *AB* (alone) is loaded with a uniform live load of w lbs. per foot of span. Let the stiffness factors be as indicated on the members and assume that the joints *E, D, C, H, G,* and *F* are all fixed and that joints *A* and *B* are partially fixed.

Then, from symmetry, it is evident that the slope angles at *A* and *B* are equal but of opposite signs, that is, $-\theta_a=\theta_b$, if counter clock-wise is taken as positive. The joints *E, D, C, H, G,* and *F* being fixed, the slope angle at each will be zero.

Now, writing out the slope-deflection equations for joint *A*, by substituting in equation (M), Art. 4, we obtain

$$M_{ab}=2ES_3(2\theta_a+\theta_b)+\frac{wL^2}{12}.$$

Substituting θ_a for θ_b, we obtain

Member AB: $M_{ab}=-2ES_3\theta_a+\dfrac{wL^2}{12}$ ....(1).

Member AE: $M_{ae}=-4ES_3\theta_a[=2ES_1(-2\theta_a+0)]$ ....(2).

Member AD: $M_{ad}=-4ES_1\theta_a[=2ES_3(-2\theta_a+0)]$ ....(3).

Member AF: $M_{af}=-4ES_2\theta_a[=2ES_2(-2\theta_a+0)]$ ....(4).

The sum of these moments about joint *A* is equal to zero in order that joint *A* may be in equilibrium. So, placing the sum of equations (1), (2), (3), and (4) equal to zero and reducing, we obtain

$$(3S_3+2S_1+2S_2)\theta_a=\frac{1}{2E}\times\frac{wL^2}{12}=\frac{C_{ab}}{2E}$$

where $C_{ab}=wL^2/12$, from which we obtain

$$\theta_a=\frac{C_{ab}}{2E}\left(\frac{1}{3S_3+2S_1+2S_2}\right)\ldots\ldots(5).$$

* See Norman M. Stineman (Editor *Concrete*), "Moments and Shear Charts for Continuous Beams and Rigid Building Frames," *American Concrete Institute,* Vol. 1, No. 3 reprinted by Portland Cement Association.

Substituting the value of θ_a (given in (5)) in (4), we obtain

$$M_{af}=-C_{ab}\left(\frac{2S_2}{3S_3+2S_1+2S_2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(C)}$$

for the moment on column AF just below joint A (Fig. 9b) due to the live load w on span AB. The negative sign simply refers to direction of rotation, clock-wise being assumed negative.

Next, substituting the value of θ_a [given in (5)] in (3), we obtain

$$M_{ad}=-C_{ab}\left(\frac{2S_1}{3S_3+2S_1+2S_2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(D)}$$

for the moment on column AD just above joint A due to the live load w on span AB (Fig. 9b).

By substituting the value of θ_a in (2), we obtain

$$M_{ae}=-C_{ab}\left(\frac{2S_3}{3S_3+2S_1+2S_2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

for the moment on girder AE just to the left of joint A due to the live load w on span AB (Fig. 9b).

By substituting the value of θ_a in (1), we obtain

$$M_{ab}=-C_{ab}\left(\frac{2S_3+2S_1+2S_2}{3S_3+2S_1+2S_2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

for the moment on girder AB just to the right of joint A due to the live load w on span AB (Fig. 9b).

The value of each of the moments M_{ae} and M_{ab} on the girders, as seen from (6) and (7), is less than $wL^2/12$. Equations (6) and (7) are therefore of no direct value, as $wL^2/12$ is used in designing the floor girders and beams.

For the moment at D (Fig. 9b), we have

$$M_{da}=2ES_1[0+(-\theta_a)]=-2ES_1\theta_a.$$

Substituting the value θ_a given in (5), we obtain

$$M_{da}=-C_{ab}\left(\frac{S_1}{3S_3+2S_1+2S_2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8).$$

It will be seen that (8) gives just one-half the value given by Formula (D).

Next, for the moment at F (in the way shown for D), we obtain

$$M_{af}=-C_{ab}\left(\frac{S_2}{3S_3+2S_1+2S_2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

which, as is seen, is one-half the value given by Formula (C).

We shall next consider a live load of w lbs. per foot on span CD in addition to the equal live load on AB. Then we have the case shown in Fig. 10b. It is readily seen that column AD will be bent by the two loads, as shown in Fig. 10b, and that the bending at D and A are in the same direction as in the case for the one load shown in Fig. 9b. So, evidently, the moment at each end of column AD is greater when span AB and DC are loaded than when only span AB is loaded. The moment at D on column AD can be obtained by writing out the equation for joint D involving stiffness factors S_1, S_4, and S_5, in the manner shown for joint A, and by then eliminating the two slope angles at joints D and A. But the small change in slope angle at A would have only a slight result at D and *vice versa* for joint A. So it is seen that we shall be on the safe side if we add equation (9) to Formula (C), thus obtaining

$$M_{af}=-C_{ab}\left(\frac{3S_2}{3S_3+2S_1+2S_2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(E)}$$

which is for the maximum moment at the top of any interior column (due to unbalanced live load) except at the roof and basement.

Likewise, by adding equation (8) to Formula (D), we obtain

$$M_{ad} = -C_{ab}\left(\frac{3S_1}{3S_3+2S_1+2S_2}\right) \qquad \text{(F)}$$

which is for the maximum moment at the bottom of any interior column (due to unbalanced live load) except at the footing.

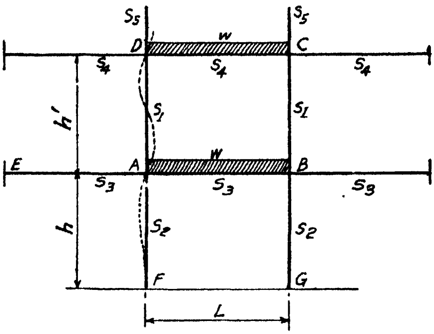


Fig. 10b

The subscripts are readily changed to conform to the story considered.

Top of Interior Column at Roof. The case is shown in Fig. 11b. Consider joints E, H, F, and G fixed, and joints A and B partially fixed. Then $-\theta_a = +\theta_b$.

Writing out the equations for joint A as previously shown for the intermediate joints, we obtain

$$M_{ab} = -2ES_3'\theta_a + C_{ab}' \qquad (10).$$

$$M_{af} = -4ES_2'\theta_a \qquad (11).$$

$$M_{ae} = -4ES_3'\theta_a \qquad (12).$$

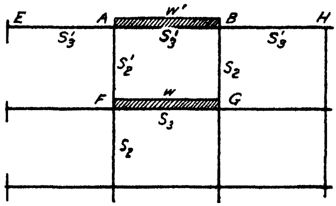


Fig. 11b

Placing the sum of these equations equal to zero and reducing, we obtain

$$\theta_a = \frac{C_{ab}'}{E}\left(\frac{1}{4S_3'+4S_2'}\right) \qquad (13).$$

Substituting this value of θ_a in (11), we obtain

$$M_{af} = -C_{ab}'\left(\frac{S_2'}{S_3'+S_2'}\right) \qquad (14).$$

Adding (9) (to provide for the load on FG) to (14), we obtain

$$M_{af} = -\left[\left(C_{ab}'\left(\frac{S_2'}{S_3'+S_2'}\right) + C_{fg}\left(\frac{S_2'}{3S_3+2S_2'+2S_2}\right)\right]\right. \qquad \text{(K)}$$

which is the maximum moment at the top of any interior column at the roof due to unbalanced live load.

For uniform load

$$C'_{ab}=\frac{w'L^2}{12} \text{ and } C_{fg}=\frac{wL^2}{12}.$$

At Top of Basement Column use Formula (C) for moment due to unbalanced live load on first floor and equation (9) for the moment on the footing.

Moments on Exterior Columns. The dead load in the different stories more or less balances the bending on the exterior columns due to dead load, but as we shall consider all joints fixed except the joint considered, we shall ignore this balancing action and include the dead load with the live load in computing the value of C.

Intermediate Joints. A typical case of an intermediate joint is shown in Fig. 12*b*. Let us first consider intermediate joint A. We shall consider joints D, B, and F fixed, and joint A partially fixed. Let θ_a = slope angle at A. (θ_a is negative.) Writing out the equation for joint A as previously explained, we obtain

$$M_{ab}=-4ES_3\theta_a+C'_{ab} \quad \text{(15)},$$

$$M_{ad}=-4ES_1\theta_a. \quad \text{(16)},$$

and

$$M_{af}=-4ES_2\theta_a. \quad \text{(17)}.$$

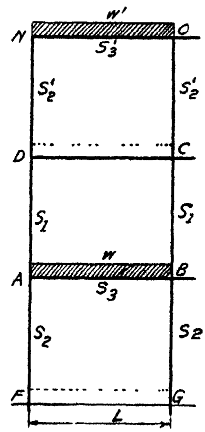


Fig. 12*b*

Placing the sum of these three equations equal to zero and reducing, we obtain

$$\theta_a=\frac{C_{ab}}{4E}\left(\frac{1}{S_3+S_1+S_2}\right) \quad \text{(18)}.$$

Substituting this value of θ_a in (17), we obtain

$$M_{af}=-C_{ab}\left(\frac{S_2}{S_3+S_1+S_2}\right) \quad \text{(19)}.$$

For the moment at F, we have

$$M_{fa}=-2ES_2\theta_a$$

Substituting the value of θ_a given in (18), we obtain

$$M_{fa}=\frac{C_{ab}}{2}\left(\frac{S_2}{S_3+S_1+S_2}\right). \quad \text{(20)}.$$

Adding this equation to (19) (to provide for load on FG), we obtain

$$M_{af}=\frac{3}{2}C_{ab}\left(\frac{S_2}{S_3+S_1+S_2}\right). \quad \text{(G)},$$

which is for the maximum moment on column AF just below joint A and hence is for the moment at the top of any exterior column except at the roof and basement.

Next, by substituting the value of θ_a in (16), we obtain

$$M_{ad}=-C_{ab}\left(\frac{S_1}{S_3+S_1+S_2}\right) \quad \text{(21)}.$$

Adding (20) to (21) (changing numerator to S_1), we obtain

$$M_{ad}=-\frac{3}{2}C_{ab}\left(\frac{S_1}{S_3+S_1+S_2}\right). \quad \text{(H)},$$

which gives the moment at the base of any exterior column except at the basement, where equation (19) can be used unless the basement wall joins to the column so that the moment can be ignored.

Joint at the Roof. Writing out the equations for joint N (Fig. 12*b*) (considering joints D and O fixed, and N partially fixed), we obtain

$$M_{no} = -4ES'_3\theta_n + C_{no} \quad \text{(22)},$$

and

$$M_{nd} = -4ES_2\theta_n \quad \text{(23)}.$$

Placing the sum of these two equations equal to zero and reducing, we obtain

$$\theta = \frac{C_{no}}{4E}\left(\frac{1}{S_3+S'_2}\right) \quad \text{(24)}.$$

Substituting this value for θ_n in (23), we obtain

$$M_{nd} = C_{no}\left(\frac{S'_2}{S'_3+S'_2}\right) \quad \text{(25)}.$$

Adding (20) ($S_1 = S'_2$) to provide for load on DC, we obtain

$$M_{nd} = -\left[C_{no}\left(\frac{S'_2}{S'_3+S'_2}\right) + \frac{C_{dc}}{2}\left(\frac{S'_2}{S_3+S'_2+S_2}\right)\right] \quad \text{(N)},$$

which is for the moment on exterior columns at roof due to unbalanced live and dead load.

Formulas (E), (F), (C), (K), (G), (H), (N) and equation (19) derived in the foregoing analysis are sufficient for determining the moment on building columns due to unbalanced loads. These formulas are shown on Diagram *B2*, Fig. 458.

MOMENT TABLE

2-142 Ton Engines Followed by 4000 lb per ft

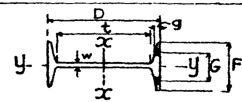
COOPERS CLASS - E-40

For one rail only

TABLE A

a	284	274	254	234	214	194	184	168	155	142	132	112	92	72	52	39	26	13	
b	10	30	50	70	90	103	116	129	142	152	172	192	212	232	245	258	271	284	
c	10	20	20	20	20	13	13	13	13	0	20	20	20	20	13	13	13	13	
	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮	⑯	⑰	⑱	2000 PLF
d	8	5	5	5	9	5	6	5	8	8	5	5	5	9	5	6	5	5	
e	0	8	13	18	23	32	37	43	48	56	64	69	74	79	88	93	99	104	109
f	109	101	96	91	86	77	72	66	61	53	45	40	35	30	21	16	10	5	0
	16364	15274	13254	11394	9514	7794	6793	5857	4999	4208	3676	2776	1976	1276	678	403	195	65	
⑱	14944	13904	11984	10184	8444	6824	5888	5017	4224	3498	3016	2216	1516	916	416	208	65		
⑰	13589	12599	10799	9059	7439	5919	5048	4292	3514	2851	2421	1721	1121	621	221	78		65	
⑯	12041	11111	9411	7811	6311	4911	4118	3390	2740	2155	1785	1205	725	345	65		78	221	
⑮	10816	9936	8336	6836	5436	4436	3408	2745	2160	1640	1320	840	460	180		65	208	416	
⑭	8728	7938	6518	5198	3978	2858	2247	1701	1233	830	800	300	100		117	299	559	884	
⑬	7668	6928	5608	4388	3268	2228	1702	1221	818	480	300	100		100	282	529	854	1244	
⑫	6708	6018	4798	3678	2658	1738	1257	841	503	230	100		100	300	547	859	1249	1704	
⑪	5848	5208	4088	3068	2148	1328	912	561	288	80		100	300	600	912	1289	1744	2264	
⑩	4632	4072	3112	2252	1492	832	520	273	104		160	420	780	1240	1656	2137	2896	3320	
⑨	3496	3016	2216	1516	916	416	208	65		80	400	820	1340	1960	2480	3065	3728	4456	
⑧	2851	2421	1721	1121	621	221	78		65	195	615	1135	1755	2475	3080	3710	4438	5231	
⑦	2155	1785	1205	725	345	65		78	221	411	951	1591	2331	3171	3834	4562	5368	6239	
⑥	1640	1320	840	460	180		65	208	416	656	1296	2036	2876	3810	4544	5337	6208	7124	
⑤	830	600	300	100		117	299	559	884	1214	2034	2954	3974	5094	5939	6849	7837	8890	
④	480	300	100		100	287	529	854	1244	1624	2544	3564	4684	5904	6814	7789	8842	9960	
③	230	100		100	300	547	859	1249	1704	2134	3154	4774	5494	6814	7789	8829	9947	11130	
②	80		100	300	600	912	1289	1744	2264	2744	3864	5084	6404	7824	8864	9989	11152	12200	
①		160	420	780	1240	1656	2137	2896	3320	3860	5160	6540	8020	9600	10844	11953	13240	14592	

Moments in 1000 ft lbs. for One Truss



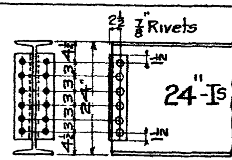
PROPERTIES OF I-BEAMS

Table-1

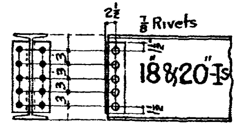
1 Depth of Beam in Inches D	2 Weight per Foot in Pounds Wt	3 Area of Cross Sec. in Sq. Ins A	4 Thickness of Web in Inches w	5 Width of Flange in Inches F	6 Moment of Inertia Axis-xx I	7 Moment of Inertia Axis-yy I'	8 Radius of Gyration Axis-xx r	9 Radius of Gyration Axis-yy r'	D	wt.	10 Section Modulus Axis xx m	11 Standard Gauge in Inches G	12 Clearance bt Fillets in Inches t	13 Grip in Inches g
24"	100.00	2941	3/4	7 1/4	23803	4856	900	128		100	1984	4	20 3/4	27/32
	95.00	2794	11/16	7 3/16	23096	4710	909	130		95	1925	4	20 3/4	27/32
	9000	26.47	5/8	7 1/8	22391	4570	920	1.31	24"	90	1866	4	20 3/4	27/32
	85.00	2500	9/16	7 1/16	21686	4435	9.31	133		85	180.7	4	20 3/4	27/32
	8000	2332	1/2	7	20879	4286	948	136		80	174.0	4	20 3/4	27/32
20"	10000	29.41	7/8	7 9/32	1655.8	5265	750	134		100	1656	4	16 1/2	29/32
	9500	2794	51/64	7 13/64	1606.8	5078	758	1.35		95	1607	4	16 1/2	29/32
	90.00	2646	47/64	7 9/64	1567.8	4898	767	136		90	1558	4	16 1/2	29/32
	85.00	25.00	21/32	7 1/16	15087	47.25	777	137		85	1509	4	16 1/2	29/32
	80.00	23.73	19/32	7	1466.5	4581	786	139	20"	80	1467	4	16 1/2	29/32
	7500	22.06	21/32	6 13/32	1268.9	3025	758	117		75	1269	3 1/2	17	25/32
	7000	2059	31/64	6 21/64	1219.9	2904	770	119		70	1220	3 1/2	17	25/32
	6500	1908	1/2	6 1/4	11696	27.86	783	121		65	1170	3 1/2	17	25/32
18"	7000	20.59	23/32	6 17/64	9213	2462	669	109		70	1024	3 1/4	15 1/4	11/16
	6500	19.12	3/8	6 11/64	8815	2347	679	111		65	979	3 1/4	15 1/4	11/16
	6000	1765	35/64	6 3/32	8418	2238	691	113	18"	60	935	3 1/4	15 1/4	11/16
	5500	15.93	29/64	6	7956	2119	7.07	115		55	884	3 1/4	15 1/4	11/16
15"	9500	27.94	1 5/64	6 41/64	872.9	4837	5.59	132		95	1164	3 3/4	11	1 1/32
	90.00	2647	63/64	6 37/64	845.4	4591	565	132		90	1127	3 3/4	11	1 1/32
	8500	25.00	57/64	6 31/64	8178	43.57	572	132		85	1090	3 3/4	11	1 1/32
	8000	2381	13/16	6 13/32	7955	4176	578	132		80	1061	3 3/4	11	1 1/32
	7500	2206	57/64	6 19/64	6912	3068	560	118		75	922	3 3/4	11 3/4	13/16
	7000	2059	25/32	6 5/16	663.6	29.00	568	119		70	885	3 1/2	11 3/4	13/16
	6500	19.12	11/16	6 7/32	6360	27.42	577	120	15	65	848	3 1/2	11 3/4	13/16
	6000	1767	19/32	6	609.0	2596	587	121		60	812	3 1/4	11 3/4	13/16
	5500	16.18	21/32	5 3/4	5110	1706	562	1.02		55	681	3	12 1/2	5/8
	5000	1471	35/64	5 41/64	4834	1604	573	104		50	645	3	12 1/2	5/8
	4500	13.24	29/64	5 35/64	4558	1500	587	107		45	608	3	12 1/2	5/8
	4200	12.48	13/32	5 1/2	4417	1462	595	108		42	589	3	12 1/4	5/8
12"	5500	1618	13/16	5 39/64	321.0	17.46	445	1.04		55	535	3	9 1/4	21/32
	5000	1471	11/16	5 31/64	3033	16.12	4.54	1.05		50	506	3	9 1/4	21/32
	4500	1324	9/16	5 23/64	2857	14.89	4.65	1.06		45	476	3	9 1/4	21/32
	4000	1184	29/64	5 1/4	2689	1381	477	108	12	40	448	3	9 1/4	21/32
	3500	1029	7/16	5 3/32	2283	1007	471	099		35	390	2 3/4	9 3/4	17/32
	3150	926	11/32	5	2158	950	4.83	1.01		31 1/2	360	2 3/4	9 3/4	17/32
10"	4000	11.76	3/4	5 3/32	1587	950	367	090		40	31.7	2 5/8	8	15/32
	3500	10.29	39/64	4 61/64	1464	852	377	091		35	293	2 5/8	8	15/32
	3000	8.82	15/32	4 13/16	134.2	765	390	0.93	10"	30	268	2 5/8	8	15/32
	2500	737	5/16	4 21/32	1221	6.89	407	097		25	244	2 5/8	8	15/32

14

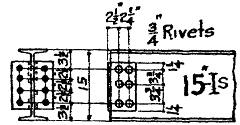
Standard Connections



2 Ls 4×4"×7/16×1-6"



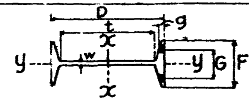
2 Ls 4"×4"×7/16×1'-3"



2 Ls 6"×4×7/16"×0-10"

PROPERTIES OF I-BEAMS

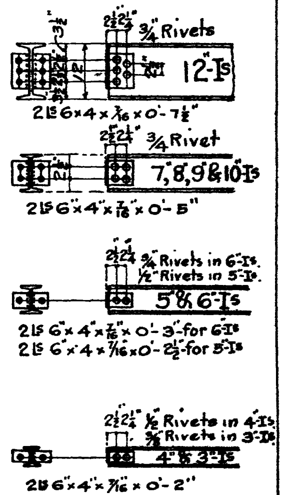
Table-2.



1	2	3	4	5	6	7	8	9			10	11	12	13
Depth of Beam in Inches	Weight per Foot in Pounds	Area of Cross Sec. in Sq. Ins.	Thickness of Web in in Inches	Width of Flange in Inches	Moment of Inertia Axis-xx.	Moment of Inertia Axis-yy.	Radius of Gyration Axis-xx	Radius of Gyration Axis-yy.			Section Modulus. Axis-xx	Standard Gauge in Inches	Clearance bt. Fillets in Inches.	Grip in Inches
D	Wt.	A	w	F	I	I'	r	r'	D	Wt.	m	G	t	g
9"	35.00	10.29	47/64	4 45/64	111.8	7.31	3.29	0.84	9"	35	24.8	2½	7	7/16
	30.00	8.82	37/64	4 33/64	101.9	6.42	3.40	0.85		30	22.6	2½	7	7/16
	25.00	7.35	27/64	4 29/64	91.9	5.65	3.54	0.88		25	20.4	2½	7	7/16
	21.00	6.31	19/64	4 21/64	84.9	5.16	3.67	0.90		21	18.9	2½	7	7/16
8"	25.50	7.50	17/32	4 17/64	68.4	4.75	3.02	0.80	8"	25½	17.1	2¼	6¼	13/32
	23.00	6.76	7/16	4 11/64	64.5	4.39	3.09	0.81		23	16.1	2¼	6¼	13/32
	20.50	6.03	23/64	4 3/32	60.6	4.07	3.17	0.82		20½	15.1	2¼	6¼	13/32
	18.00	5.33	17/64	4	56.9	3.78	3.27	0.84		18	14.2	2¼	6¼	13/32
7"	20.00	5.88	15/32	3 7/8	42.2	3.24	2.68	0.74	7"	20	12.1	2¼	5¼	3/8
	17.50	5.15	25/64	3 49/64	39.2	2.94	2.76	0.76		17½	11.2	2¼	5¼	3/8
	15.00	4.42	1/4	3 21/32	36.2	2.67	2.86	0.78		15	10.4	2¼	5¼	3/8
6"	17.25	5.07	31/64	3 37/64	26.2	2.36	2.27	0.68	6"	17¼	8.7	[illegible]	4½	11/32
	14.75	4.34	23/64	3 29/64	24.0	2.09	2.35	0.69		14¾	8.0	2	4½	11/32
	12.25	3.61	15/64	3 21/64	21.8	1.85	2.46	0.72		12¼	7.3	2	4½	11/32
5"	14.75	4.34	1/2	3 19/64	15.2	1.70	1.87	0.63	5"	14¾	6.1	1¾	3½	5/16
	12.25	3.60	11/32	3 9/64	13.6	1.45	1.94	0.63		12¼	5.4	1¾	3½	5/16
	9.75	2.87	13/64	3	12.1	1.23	2.05	0.65		9¾	4.8	1¾	3½	5/16
4"	10.50	3.09	13/32	2 7/8	7.1	1.01	1.52	0.57	4"	10½	3.6	1½	2¾	9/32
	9.50	2.79	11/32	2 13/16	6.7	0.93	1.55	0.58		9½	3.4	1½	2¾	9/32
	8.50	2.50	17/64	2 41/64	6.4	0.85	1.59	0.58		8½	3.2	1½	2¾	9/32
	7.50	2.21	3/16	2 21/32	6.0	0.77	1.64	0.59		7½	3.0	1½	2¾	9/32
3"	7.50	2.21	23/64	2 33/64	2.9	0.60	1.15	0.52	3"	7½	1.9	1 3/16	1¾	1/4
	6.50	1.91	17/64	2 27/64	2.7	0.53	1.19	0.52		6½	1.8	1 3/16	1¾	1/4
	5.50	1.63	11/64	2 21/64	2.5	0.46	1.23	0.53		5½	1.7	1 3/16	1¾	1/4

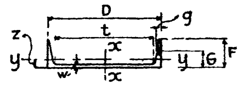
14

Standard Connections



PROPERTIES OF CHANNELS

Table 3



1	2	3	4	5	6	7	8	9			10	11	12	13	14
Depth of Channel in Inches	Weight per Foot in Pounds	Area of Gross Sec in Sq. Ins	Thickness of Web in Inches	Width of Flange in Inches	Moment of Inertia Axis-XX	Moment of Inertia Axis-YY	Radius of Gyration Axis-XX	Radius of Gyration Axis-YY			Section Modulus Axis-XX	Standard Gauge in Inches	Clearance bt Fillets in Inches	Grip in Inches	Ctr of Gravity from Back of Channel
D	Wt	A	w	F	I	I'	r	r'	D	Wt	m	G	t	g	Z-inches
15"	55.00	16.18	13/16	3 13/16	4302	12.19	516	868	15" [s	55	574	2 1/4	12 1/4	5/8	823
	5000	14.71	23/32	3 23/32	4027	11.22	523	873		50	537	2 1/4	12 1/4	5/8	803
	4500	1324	5/8	3 5/8	3751	1029	5.32	882		45	500	2 1/4	12 1/4	5/8	788
	4000	1176	17/32	3 17/32	3475	9.39	5.43	893		40	463	1 7/8	12 1/4	21/32	783
	3500	1029	27/64	3 27/64	3200	8.48	558	908		35	427	1 7/8	12 1/4	21/32	789
	3300	9.90	13/32	3 13/32	3126	823	562	912		33	417	1 7/8	12 1/4	21/32	794
12"	4000	1176	49/64	3 27/64	1970	663	409	751	12" [s	40	328	2	10	15/32	722
	3500	1029	41/64	3 19/64	1793	590	4.17	757		35	29.9	2	10	15/32	694
	3000	882	33/64	3 11/64	1617	521	428	768		30	269	2	10	15/32	677
	25.00	735	25/64	3 3/64	144.0	453	443	785		25	24.0	1 3/4	10	15/32	678
	2050	603	9/32	2 15/16	1281	391	461	805		20 1/2	214	1 3/4	0	15/32	704
10"	35.00	10.29	53/64	3 3/16	1155	466	335	672	10" [s	35	231	2	8 1/4	3/8	695
	3000	8.82	43/64	3 1/32	1032	390	342	672		30	206	2	8 1/4	3/8	651
	2500	735	17/32	2 57/64	910	340	3.52	680		25	182	2	8 1/4	3/8	620
	2000	5.88	3/8	2 47/64	78.7	285	3.66	696		20	157	1 1/2	8 1/4	7/16	609
	1500	4.46	15/64	2 19/32	669	230	387	718		15	13.4	1 1/2	8 1/4	7/16	639
9"	25.00	7.35	39/64	2 13/16	707	298	310	637	9" [s	25	157	1 3/4	7 1/4	3/8	615
	20.00	5.88	29/64	2 21/32	608	2.45	321	646		20	135	1 3/4	7 1/4	3/8	585
	15.00	4.41	9/32	2 31/64	50.9	195	340	665		15	113	1 5/8	7 1/4	13/32	590
	13.25	389	15/64	2 7/16	473	177	349	674		13 1/4	105	1 5/8	7 1/4	13/32	607
8"	2125	625	37/64	2 5/8	478	225	277	600	8" [s	21 1/4	119	1 1/2	6 1/4	3/8	587
	1875	5.51	31/64	2 17/32	438	201	282	603		18 3/4	110	1 1/2	6 1/4	3/8	567
	1625	4.78	25/64	2 7/16	399	178	289	610		16 1/4	100	1 1/2	6 1/4	3/8	556
	13.75	4.04	19/64	2 11/32	360	155	298	619		13 3/4	90	1 1/4	6 1/4	3/8	557
	1125	3.35	7/32	2 17/64	323	133	311	630		11 1/4	8.1	1 1/4	6 1/4	3/8	576
7"	1975	581	5/8	2 33/64	33.2	185	239	565	7" [s	19 3/4	95	1 1/2	5 1/2	3/8	583
	17.25	507	33/64	2 13/32	302	162	244	564		17 1/4	8.6	1 1/2	5 1/2	3/8	555
	14.75	4.34	13/32	2 19/64	272	140	2.50	568		14 3/4	7.8	1 1/4	5 1/2	11/32	535
	1225	360	5/16	2 13/64	242	119	259	575		12 1/4	69	1 1/4	5 1/2	11/32	528
	9.75	285	13/64	2 3/32	211	098	272	586		9 3/4	6.0	1 1/4	5 1/2	11/32	546
6"	15.50	4.56	9/16	2 9/32	195	128	2.07	529	6" [s	15 1/2	6.5	1 3/8	4 1/2	11/32	546
	13.00	3.82	7/16	2 5/32	173	107	2.13	529		13	5.8	1 3/8	4 1/2	11/32	517
	1050	309	5/16	2 1/32	15.1	088	2.21	534		10 1/2	50	1 1/8	4 1/2	11/32	503
	8.00	2.38	13/64	1 59/64	13.0	0.70	234	542		8	4.3	1 1/8	4 1/2	11/32	517
5"	1150	3.38	15/32	2 1/32	10.4	082	1.75	493	5" [s	11 1/2	4.2	1 1/4	3 3/4	5/16	508
	9.00	265	21/64	1 57/64	89	064	183	493		9	3.5	1 1/4	3 3/4	5/16	.481
	6.50	1.95	3/16	1 3/4	74	0.48	195	498		6 1/2	3.0	1	3 3/4	5/16	.489
4"	725	213	21/64	1 23/32	4.6	044	146	455	4" [s	7 1/4	23	1	2 3/4	9/32	463
	625	1.84	17/64	1 21/32	4.2	038	1.51	454		6 1/4	21	1	2 3/4	9/32	458
	5.25	155	3/16	1 37/64	3.8	0.32	156	453		5 1/4	19	1	2 3/4	9/32	.464
3"	6.00	176	3/8	1 39/64	21	031	108	421	3" [s	6	14	15/16	1 3/4	1/4	459
	5.00	147	17/64	1 1/2	18	025	1.12	415		5	12	15/16	1 3/4	1/4	.443
	4.00	119	11/64	1 13/32	16	020	117	409		4	1.1	15/16	1 3/4	1/4	443

15 — Standard Connections

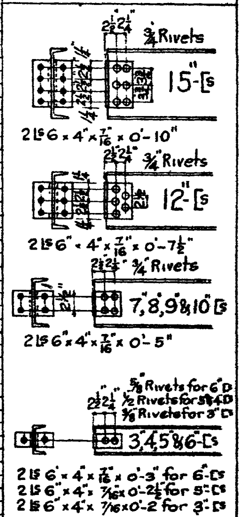
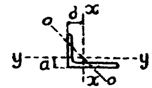


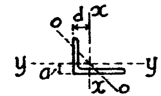
Table-4

PROPERTIES OF ANGLES.
For Unequal Flanges.



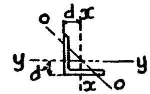
1 Size	2 Thickness in Inches	3 Weight per foot in Pounds	4 Area of Gross Section in Square Inches	5 Ctr of Gravity from Back of Long Flange	6 Ctr of Gravity from Back of Short Flange	7 Moment of Inertia Axis-yy	8 Moment of Inertia Axis-xx	9 Radius of Gyration Axis-yy	10 Radius of Gyration Axis-xx			11 Least Radius of Gyration Axis-oo	12 Section Modulus Axis-yy	13 Section Modulus Axis-xx	14 Radius of Gyration of Two Angles							
	t	Wt.	A	a-Inches	b-Inches	I	I₁	r	r₁		t	ro	m	m₁	rc	r2	r3	r4	rb	r5	r6	r7
6×4	1	30.6	9.00	1.17	2.17	10.75	30.75	1.09	1.85	6×4	1	.85	3.79	8.02	1.09	2.85	3.04	3.14	1.85	1.60	1.79	1.89
6×4	15/16	28.9	8.50	1.14	2.14	10.26	29.26	1.10	1.86		15/16	.85	3.59	7.59								
6×4	7/8	27.2	7.99	1.12	2.12	9.75	27.73	1.11	1.86		7/8	.86	3.39	7.15								
6×4	13/16	25.4	7.47	1.10	2.10	9.23	26.15	1.11	1.87		13/16	.86	3.18	6.10								
6×4	3/4	23.6	6.94	1.08	2.08	8.68	24.51	1.12	1.88		3/4	.86	2.97	6.25								
6×4	11/16	21.8	6.41	1.06	2.06	8.11	22.82	1.13	1.89		11/16	.86	2.76	5.78								
6×4	5/8	20.0	5.86	1.03	2.03	7.52	21.07	1.13	1.90		5/8	.86	2.54	5.31								
6×4	9/16	18.1	5.31	1.01	2.01	6.91	19.26	1.14	1.90		9/16	.87	2.31	4.83								
6×4	1/2	16.2	4.75	0.99	1.99	6.27	17.40	1.15	1.91		1/2	.87	2.08	4.33								
6×4	7/16	14.3	4.18	0.96	1.96	5.60	15.46	1.16	1.92		7/16	.87	1.85	3.83								
6×4	3/8	12.3	3.61	0.94	1.94	4.90	13.47	1.17	1.93		3/8	.88	1.60	3.32	1.17	2.74	2.92	3.01	1.93	1.50	1.67	1.76
6×3½	1	28.9	8.50	1.01	2.26	7.21	29.24	0.92	1.85	6×3½	1	.74	2.90	7.83	0.92	2.93	3.13	3.23	1.85	1.37	1.56	1.67
6×3½	15/16	27.3	8.03	0.99	2.24	6.88	27.84	0.93	1.86		15/16	.74	2.74	7.41								
6×3½	7/8	25.7	7.55	0.97	2.22	6.55	26.38	0.93	1.87		7/8	.75	2.59	6.98								
6×3½	13/16	24.0	7.06	0.95	2.20	6.20	24.89	0.94	1.88		13/16	.75	2.43	6.55								
6×3½	3/4	22.4	6.56	0.93	2.18	5.84	23.34	0.94	1.89		3/4	.75	2.27	6.10								
6×3½	11/16	20.6	6.06	0.90	2.15	5.47	21.74	0.95	1.89		11/16	.75	2.11	5.65								
6×3½	5/8	18.9	5.55	0.88	2.13	5.08	20.08	0.96	1.90		5/8	.75	1.94	5.[illegible]9								
6×3½	9/16	17.1	5.03	0.86	2.11	4.67	18.37	0.96	1.91		9/16	.75	1.77	4.72								
6×3½	1/2	15.3	4.50	0.83	2.08	4.25	16.59	0.97	1.92		1/2	.76	1.59	4.24								
6×3½	7/16	13.5	3.97	0.81	2.06	3.81	14.76	0.98	1.93		7/16	.76	1.41	3.75								
6×3½	3/8	11.7	3.42	0.79	2.04	3.34	12.86	0.99	1.94		3/8	.77	1.23	3.25	0.99	2.81	3.00	3.10	1.94	1.26	1.43	1.53
5×3½	7/8	22.7	6.67	1.04	1.79	6.21	15.67	0.96	1.53	5×3½	7/8	.75	2.52	4.88	0.96	2.36	2.55	2.65	1.53	1.42	1.61	1.71
5×3½	13/16	21.3	6.25	1.02	1.77	5.89	14.81	0.97	1.64		13/16	.75	2.37	4.58								
5×3½	3/4	19.8	5.81	1.00	1.75	5.55	13.92	0.98	1.55		3/4	.75	2.22	4.28								
5×3½	11/16	18.3	5.37	0.97	1.72	5.20	12.99	0.98	1.56		11/16	.75	2.06	3.97								
5×3½	5/8	16.8	4.92	0.95	1.70	4.83	12.03	0.99	1.56		5/8	.75	1.90	3.65								
5×3½	9/16	15.2	4.47	0.93	1.68	4.45	11.03	1.00	1.57		9/16	.75	1.73	3.32								
5×3½	1/2	13.6	4.00	0.91	1.66	4.05	9.99	1.01	1.58		1/2	.75	1.56	2.99								
5×3½	7/16	12.0	3.53	0.88	1.63	3.63	8.90	1.01	1.59		7/16	.76	1.39	2.64								
5×3½	3/8	10.4	3.05	0.86	1.61	3.18	7.78	1.02	1.60		3/8	.76	1.21	2.29								
5×3½	5/16	8.7	2.56	0.84	1.59	2.72	6.60	1.03	1.61		5/16	.76	1.02	1.94	1.03	2.26	2.44	2.54	1.61	1.33	1.50	1.59
5×3	13/16	19.9	5.84	0.86	1.86	3.71	13.98	0.80	1.55	5×3	13/16	.64	1.74	4.45	0.80	2.42	2.62	2.72	1.55	1.18	1.37	1.47
5×3	3/4	18.5	5.44	0.84	1.84	3.51	13.15	0.80	1.55		3/4	.64	1.69	4.16								
5×3	11/16	17.1	5.03	0.82	1.82	3.29	12.28	0.81	1.56		11/16	.64	1.51	3.86								
5×3	5/8	15.7	4.61	0.80	1.80	3.06	11.37	0.82	1.57		5/8	.64	1.39	3.55								
5×3	9/16	14.3	4.18	0.77	1.77	2.83	10.43	0.82	1.58		9/16	.65	1.27	3.23								
5×3	1/2	12.8	3.75	0.75	1.75	2.58	9.45	0.83	1.59		1/2	.65	1.15	2.91								
5×3	7/16	11.3	3.31	0.73	1.73	2.32	8.43	0.84	1.60		7/16	.65	1.02	2.58								
5×3	3/8	9.8	2.86	0.70	1.70	2.04	7.37	0.84	1.61		3/8	.65	0.89	2.24								
5×3	5/16	8.2	2.40	0.68	1.68	1.76	6.26	0.85	1.61		5/16	.66	0.75	1.89	0.85	2.33	2.51	2.61	1.61	1.09	1.26	1.35

Table-5. PROPERTIES OF ANGLES
For Unequal Flanges.



1	2	3	4	5	6	7	8	9	10		11	12	13	14							
Size	Thickness in Inches	Weight per foot in Pounds	Area of Gross Section in Square Inches	Ctr of Gravity from Back of Long Flange	Ctr of Gravity from Back of Short Flange	Moment of Inertia Axis yy	Moment of Inertia Axis xx	Radius of Gyration Axis yy	Radius of Gyration Axis xx		Least Radius of Gyration Axis oo	Section Modulus Axis yy	Section Modulus Axis xx	Radius of Gyration of Two Angles r2	r3 ½"	r4 ¾"		r5	r6 ½"	r7 ¾"	
	t	Wt.	A	a-Ins	b-Ins	I	I_1	r	r1		r0	m	m1	rc	r2	r3	r4	rb	r5	r6	r7
4×3	13/16	17.1	5.03	0.94	1.44	3.47	7.34	0.83	1.21	4" × 3"	0.72	2.30	2.92	0.83	1.88	2.08	2.18	1.21	1.25	1.45	1.55
4×3	3/4	16.0	4.65	0.92	1.42	3.28	6.93	0.84	1.22		0.72	2.15	2.75								
4×3	11/16	14.8	4.34	0.89	1.39	3.08	6.49	0.84	1.22		0.72	2.00	2.56								
4×3	5/8	13.6	3.98	0.87	1.37	2.87	6.03	0.85	1.23		0.72	1.84	2.35								
4×3	9/16	12.4	3.62	0.85	1.35	2.66	5.55	0.86	1.24		0.72	1.68	2.15								
4×3	1/2	11.1	3.25	0.83	1.33	2.42	5.05	0.86	1.25		0.72	1.52	1.93								
4×3	7/16	9.8	2.87	0.80	1.30	2.18	4.52	0.87	1.25		0.72	1.35	1.72								
4×3	3/8	8.5	2.48	0.78	1.28	1.92	3.96	0.88	1.26		0.73	1.18	1.50								
4×3	5/16	7.2	2.09	0.76	1.26	1.65	3.38	0.89	1.27		0.73	1.01	1.26	0.[illegible]9	1.79	1.97	2.07	1.27	1.17	1.35	1.44
3½×3	13/16	15.8	4.62	0.98	1.23	3.33	4.98	0.85	1.04	3½" × 3"	0.62	1.65	2.20	0.85	1.61	1.81	1.91	1.04	1.30	1.50	1.60
3½×3	3/4	14.7	4.31	0.96	1.21	3.15	4.70	0.85	1.04		0.62	1.54	2.05								
3½×3	11/16	13.6	4.00	0.94	1.19	2.96	4.41	0.86	1.05		0.62	1.44	1.91								
3½×3	5/8	12.5	3.67	0.92	1.17	2.76	4.11	0.87	1.06		0.62	1.33	1.76								
3½×3	9/16	11.4	3.34	0.90	1.15	2.55	3.79	0.87	1.07		0.62	1.21	1.61								
3½×3	1/2	10.2	3.00	0.88	1.13	2.33	3.45	0.88	1.07		0.62	1.10	1.45								
3½×3	7/16	9.1	2.65	0.85	1.10	2.09	3.10	0.89	1.08		0.62	0.98	1.29								
3½×3	3/8	7.9	2.30	0.83	1.08	1.85	2.72	0.90	1.09		0.62	0.85	1.13								
3½×3	5/16	6.6	1.93	0.81	1.06	1.58	2.33	0.90	1.10		0.63	0.72	0.96	0.90	1.52	1.71	1.80	1.10	1.22	1.40	1.49
3½×2½	11/16	12.5	3.65	0.77	1.27	1.72	4.13	0.67	1.06	3½" × 2½"	0.53	0.99	1.85	0.67	1.66	1.86	1.96	1.06	1.03	1.23	1.33
3½×2½	5/8	11.5	3.36	0.75	1.25	1.61	3.85	0.69	1.07		0.53	0.92	1.71								
3½×2½	9/16	10.4	3.06	0.73	1.23	1.49	3.55	0.70	1.08		0.53	0.84	1.56								
3½×2½	1/2	9.4	2.75	0.70	1.20	1.36	3.24	0.70	1.09		0.53	0.76	1.41								
3½×2½	7/16	8.3	2.43	0.68	1.18	1.23	2.91	0.71	1.09		0.54	0.68	1.26								
3½×2½	3/8	7.2	2.11	0.66	1.16	1.09	2.56	0.72	1.10		0.54	0.59	1.09								
3½×2½	5/16	6.1	1.78	0.64	1.14	0.94	2.19	0.73	1.11		0.54	0.50	0.93								
3½×2½	1/4	4.9	1.44	0.61	1.11	0.78	1.80	0.74	1.12		0.54	0.41	0.75	0.74	1.58	1.76	1.86	1.12	0.96	1.13	1.29
3×2½	9/16	9.5	2.78	0.77	1.02	1.42	2.28	0.72	0.91	3" × 2½"	0.52	0.53	1.30	0.72	1.37	1.56	1.66	0.91	1.05	1.25	1.35
3×2½	1/2	8.5	2.50	0.75	1.00	1.30	2.08	0.72	0.91		0.52	0.48	1.17								
3×2½	7/16	7.6	2.22	0.73	0.98	1.18	1.88	0.73	0.92		0.52	0.43	1.05								
3×2½	3/8	6.6	1.92	0.71	0.96	1.04	1.66	0.74	0.93		0.52	0.37	0.91								
3×2½	5/16	5.6	1.62	0.68	0.93	0.90	1.42	0.74	0.94		0.53	0.32	0.77								
3×2½	1/4	4.5	1.31	0.66	0.91	0.74	1.17	0.75	0.95		0.53	0.26	0.63	0.75	1.31	1.50	1.59	0.95	1.00	1.18	1.28
2½×2	1/2	6.8	2.00	0.63	0.88	0.64	1.14	0.56	0.75	2½" × 2"	0.42	0.46	0.70	0.56	1.16	1.35	1.46	0.75	0.84	1.04	1.15
2½×2	7/16	6.1	1.78	0.60	0.85	0.58	1.03	0.57	0.76		0.42	0.41	0.62								
2½×2	3/8	5.3	1.55	0.58	0.83	0.51	0.91	0.58	0.77		0.42	0.36	0.55								
2½×2	5/16	4.5	1.31	0.56	0.81	0.45	0.79	0.58	0.78		0.42	0.31	0.47								
2½×2	1/4	3.7	1.06	0.54	0.79	0.37	0.65	0.59	0.78		0.42	0.25	0.38								
2½×2	3/16	2.8	0.81	0.51	0.76	0.29	0.51	0.60	0.79		0.43	0.20	0.29	0.60	1.10	1.28	1.39	0.79	0.79	0.97	1.07

Table-6. PROPERTIES OF ANGLES
For Equal Flanges



1	2	3	4	5	6	7	8	9	10	
Size	Thickness in Inches	Weight per Foot in Pounds	Area of Gross Sec in Square Inches	Distance of Center of Gravity from Back	Moment of Inertia Axis yy or xx	Radius of Gyration Axis yy or xx	Least Radius of Gyration Axis oo	Section Modulus Axis yy or xx	Radius of Gyration of Two Angles	
	t	Wt	A	d	I	r	r1	m	r2	r3
8×8	1⅛	56.9	16.73	2.41	97.97	2.42	1.55	17.53	3.42	3.69
8×8	1 1/16	54.0	15.87	2.39	93.53	2.43	1.56	16.67		
8×8	1	51.0	15.00	2.37	88.98	2.44	1.56	15.80		
8×8	15/16	48.1	14.12	2.34	84.33	2.44	1.56	14.91		
8×8	7/8	45.0	13.23	2.32	79.58	2.45	1.57	14.01		
8×8	13/16	42.0	12.34	2.30	74.71	2.46	1.57	13.11		
8×8	3/4	38.9	11.44	2.28	69.74	2.47	1.57	12.18		
8×8	11/16	35.8	10.53	2.25	64.64	2.48	1.58	11.25		
8×8	5/8	32.7	9.61	2.23	59.42	2.49	1.58	10.30		
8×8	9/16	29.6	8.68	2.21	54.09	2.50	1.58	9.34		
8×8	1/2	26.4	7.75	2.19	48.63	2.50	1.58	8.37	3.32	3.58
6×6	1	37.4	11.00	1.86	35.46	1.80	1.16	8.57	2.59	2.87
6×6	15/16	35.3	10.37	1.84	33.72	1.80	1.16	8.11		
6×6	7/8	33.1	9.74	1.82	31.92	1.81	1.17	7.64		
6×6	13/16	31.0	9.09	1.80	30.06	1.82	1.17	7.15		
6×6	3/4	28.7	8.44	1.78	28.15	1.83	1.17	6.66		
6×6	11/16	26.5	7.78	1.75	26.19	1.83	1.17	6.17		
6×6	5/8	24.2	7.11	1.72	24.16	1.84	1.18	5.66		
6×6	9/16	21.9	6.43	1.71	22.07	1.85	1.18	5.14		
6×6	1/2	19.6	5.75	1.68	19.91	1.86	1.18	4.61		
6×6	7/16	17.2	5.06	1.66	17.68	1.87	1.19	4.07		
6×6	3/8	14.9	4.36	1.64	15.39	1.88	1.19	3.53	2.49	2.76
4×4	13/16	19.9	5.84	1.29	8.14	1.18	0.77	3.01	1.75	2.04
4×4	3/4	18.5	5.44	1.27	7.67	1.19	0.77	2.81		
4×4	11/16	17.1	5.03	1.25	7.17	1.19	0.77	2.61		
4×4	5/8	15.7	4.61	1.23	6.66	1.20	0.77	2.40		
4×4	9/16	14.3	4.18	1.21	6.12	1.21	0.78	2.19		
4×4	1/2	12.8	3.75	1.18	5.56	1.22	0.78	1.97		
4×4	7/16	11.3	3.31	1.16	4.97	1.23	0.78	1.75		
4×4	3/8	9.8	2.86	1.14	4.36	1.23	0.79	1.52		
4×4	5/16	8.2	2.40	1.12	3.71	1.24	0.79	1.29	1.67	1.94
3½×3½	13/16	17.1	5.03	1.17	5.25	1.02	0.67	2.25	1.55	1.85
3½×3½	3/4	16.0	4.69	1.15	4.96	1.03	0.67	2.11		
3½×3½	11/16	14.8	4.34	1.12	4.65	1.04	0.67	1.96		
3½×3½	5/8	13.6	3.98	1.10	4.33	1.04	0.67	1.81		
3½×3½	9/16	12.4	3.62	1.08	3.99	1.05	0.68	1.65		
3½×3½	1/2	11.1	3.25	1.06	3.64	1.06	0.68	1.49		
3½×3½	7/16	9.8	2.87	1.04	3.26	1.07	0.68	1.32		
3½×3½	3/8	8.5	2.48	1.01	2.87	1.07	0.69	1.15		
3½×3½	5/16	7.2	2.09	0.99	2.45	1.08	0.69	0.98	1.47	1.74
3×3	5/8	11.5	3.36	0.98	2.62	0.88	0.57	1.30	1.32	1.62
3×3	9/16	10.4	3.06	0.95	2.43	0.89	0.58	1.19		
3×3	1/2	9.4	2.75	0.93	2.22	0.90	0.58	1.07		
3×3	7/16	8.3	2.43	0.91	1.99	0.91	0.58	0.95		
3×3	3/8	7.2	2.11	0.89	1.76	0.91	0.58	0.83		
3×3	5/16	6.1	1.78	0.87	1.51	0.92	0.59	0.71		
3×3	1/4	4.9	1.44	0.84	1.24	0.93	0.59	0.58	1.25	1.53
2½×2½	1/2	7.7	2.25	0.81	1.23	0.74	0.47	0.73	1.10	1.40
2½×2½	[illegible]	[illegible]	2.00	0.78	1.11	0.74	0.48	0.65		
2½×2½	3/8	5.9	1.73	0.76	0.98	0.75	0.48	0.57		
2½×2½	5/16	5.0	1.47	0.74	0.85	0.76	0.49	0.48		
2½×2½	1/4	4.1	1.19	0.72	0.70	0.77	0.49	0.40		
2½×2½	3/16	3.1	0.90	0.69	0.55	0.78	0.49	0.30	1.04	1.32
2×2	7/16	5.3	1.56	0.66	0.54	0.59	0.39	0.40		
2×2	3/8	4.7	1.36	0.64	0.48	0.59	0.39	0.35		
2×2	5/16	4.0	1.15	0.61	0.42	0.60	0.39	0.30		
2×2	1/4	3.2	[illegible]	0.59	0.35	0.61	0.39	0.25		
2×2	3/16	2.5	0.72	0.57	0.28	0.62	0.40	0.19		
1½×1½	3/8	3.4	0.99	0.51	0.19	0.44	0.29	0.19		
1½×1½	5/16	2.9	0.84	0.49	0.16	0.44	0.29	0.16		
1½×1½	1/4	2.4	0.69	0.47	0.14	0.45	0.29	0.13		
1½×1½	3/16	1.8	0.53	0.44	0.11	0.46	0.29	0.10		
1½×1½	1/8	1.3	0.36	0.42	0.08	0.46	0.30	0.07		

I = Moment of inertia;
y = Distance from neutral axis to extreme fiber;
M = Bending moment in inch pounds;
f = Stress on extreme fiber.

Then, $M = \frac{fI}{y}$ from which we obtain

$\frac{M}{f} = \frac{I}{y}$ = Section Modulus (m)

Areas and Weights of Bars and Plates

Table-7

A = area of cross-section in sq inches
W = weight in pounds per lineal ft at 490 pounds per cu.ft.

Width in Inches	Thickness $\frac{1}{4}''$ A	$\frac{1}{4}''$ W	$\frac{5}{16}''$ A	$\frac{5}{16}''$ W	$\frac{3}{8}''$ A	$\frac{3}{8}''$ W	$\frac{7}{16}''$ A	$\frac{7}{16}''$ W	$\frac{1}{2}''$ A	$\frac{1}{2}''$ W	$\frac{9}{16}''$ A	$\frac{9}{16}''$ W	Extreme Length in Feet
1	0.250	0.85	0313	106	0375	128	0438	149	0500	170	0563	192	70
$1\frac{1}{2}$	0.375	128	0469	159	0563	192	0656	223	0750	255	0844	287	"
$1\frac{3}{4}$	0.438	149	0547	186	0656	223	0766	260	0875	298	0984	335	"
2	0.500	170	0625	212	0750	255	0875	298	100	340	113	383	"
$2\frac{1}{4}$	0563	191	0703	239	0844	287	0984	335	113	383	127	430	"
$2\frac{1}{2}$	0.625	212	0781	265	0938	319	109	372	125	425	141	478	"
3	0750	255	0938	319	113	3.83	131	446	150	510	169	574	"
$3\frac{1}{2}$	0875	298	109	372	131	447	153	520	175	595	197	670	"
4	100	340	125	425	150	5.10	175	595	200	6.80	225	765	"
5	125	425	156	531	188	638	219	744	250	850	281	957	"
6	150	510	188	638	225	765	263	893	300	1020	338	1148	"
7	175	595	219	744	263	893	306	1041	350	1190	394	13.39	"
8	200	680	250	850	300	1020	350	1190	400	13.60	450	1530	"
9	225	765	281	956	338	1148	394	1340	450	1530	506	1722	"
10	250	850	313	10.62	375	1275	438	1488	500	1700	563	1914	"
11	275	934	344	1168	413	1403	481	1636	550	1870	619	2102	"
12	300	1020	3.75	1275	450	1530	525	1785	600	2040	675	2295	"
13	3.25	1106	406	13.81	488	1658	569	1934	650	2210	731	2486	"
14	3.50	1190	4.38	1488	525	1786	6.13	2082	700	2380	788	2678	"
15	3.75	1275	469	1594	5.63	1914	656	2232	750	2550	844	2870	"
16	400	13.60	500	1700	600	2040	700	2380	800	2720	9.00	3060	"
17	425	1444	5.31	18.06	638	2168	744	2528	850	2889	956	3252	"
18	450	1530	563	1912	6.75	2296	788	2679	900	3060	1013	3444	"
20	500	1700	6.25	2124	750	2550	8.75	2975	1000	3400	1125	38.27	"
22	550	1869	688	2336	825	2806	963	3272	1100	3740	1238	4204	"
24	6.00	2040	750	2552	900	3060	1050	3572	1200	4080	1350	4592	"
26	650	2212	813	2762	975	33.16	1138	3868	1300	4420	1463	4973	"
28	700	2380	8.75	2976	1050	3572	1225	4165	1400	4760	1575	5356	"
30	750	2550	938	3188	1125	3828	1313	4464	1500	5100	1688	5740	"
32	800	2720	1000	3400	1200	4080	1400	4760	16.00	5440	1800	6122	"
34	850	2888	10.63	3612	1275	4336	1488	5057	1700	5778	19.13	6504	"
36	900	3059	1125	3824	1350	4592	15.75	5358	1800	6120	2025	6888	"
38	950	3232	11.88	4039	1425	4848	1663	56.56	1900	64.62	2138	7268	"
40	10.00	34.00	1250	4248	1500	5100	1750	5950	2000	68.00	2250	76.54	"

Table-8. Areas and Weights of Bars and Plates

A = area of cross-section in sq inches

W = weight in pounds per lineal ft. at 490 pounds per cu ft

Width in Inches	Thickness 5/8" A	W	11/16" A	W	3/4" A	W	13/16" A	W	7/8" A	W	15/16" A	W	Extreme Length in Feet
1	0.625	2.13	0.688	2.34	0.75	2.55	0.813	2.76	0.88	2.98	0.94	3.19	70
1½	0.938	3.19	1.031	3.51	1.125	3.83	1.219	4.14	1.31	4.46	1.41	4.78	"
1¾	1.094	3.72	1.203	4.09	1.313	4.46	1.422	4.83	1.53	5.21	1.64	5.58	"
2	1.25	4.25	1.375	4.68	1.50	5.10	1.625	5.53	1.75	5.95	1.88	6.38	"
2¼	1.406	4.78	1.547	5.26	1.688	5.74	1.828	6.22	1.97	6.69	2.11	7.17	"
2½	1.563	5.31	1.719	5.84	1.875	6.38	2.031	6.91	2.19	7.44	2.34	7.97	"
3	1.875	6.38	2.063	7.01	2.25	7.65	2.43	8.29	2.63	8.93	2.81	9.56	"
3½	2.188	7.44	2.406	8.18	2.625	8.93	2.844	9.67	3.06	10.41	3.28	11.16	"
4	2.5	8.50	2.75	9.35	3.00	10.20	3.25	11.05	3.50	11.90	3.75	12.75	"
5	3.125	10.63	3.438	11.69	3.15	12.75	4.063	13.81	4.38	14.88	4.69	15.94	"
6	3.75	12.75	4.125	14.03	4.50	15.30	4.875	16.58	5.25	17.85	5.63	19.13	"
7	4.375	14.88	4.813	16.36	5.250	17.85	5.688	19.34	6.13	20.83	6.56	22.31	"
8	5.00	17.0	5.50	18.70	6.00	20.40	6.50	22.10	7.00	23.80	7.50	25.5	"
9	5.625	19.13	6.188	21.04	6.75	22.95	7.313	24.86	7.88	26.78	8.44	28.69	"
10	6.25	21.25	6.875	23.38	7.50	25.50	8.125	27.63	8.75	29.75	9.38	31.88	"
11	6.875	23.38	7.563	25.71	8.25	28.05	8.938	30.39	9.63	32.73	10.31	35.06	"
12	7.50	25.50	8.25	28.05	9.00	30.60	9.75	33.15	10.50	35.70	11.25	38.25	"
13	8.13	27.63	8.94	30.4	9.75	33.2	10.56	35.9	11.38	38.7	12.19	41.4	"
14	8.75	29.75	9.63	32.7	10.50	35.7	11.38	38.7	12.25	41.7	13.13	44.6	"
15	9.38	31.88	10.31	35.1	11.25	38.3	12.19	41.4	13.13	44.6	14.06	47.8	"
16	10.00	34.0	11.00	37.4	12.00	40.8	13.00	44.2	14.00	47.6	15.00	51.0	"
17	10.63	36.13	11.69	39.7	12.75	43.4	13.81	47.0	14.88	50.6	15.94	54.2	"
18	11.25	38.25	12.38	42.1	13.50	45.9	14.63	49.7	15.75	53.6	16.88	57.4	"
20	12.50	42.50	13.75	46.8	15.00	51.0	16.25	55.3	17.50	59.5	18.75	63.8	"
22	13.75	46.75	15.13	51.4	16.50	56.1	17.88	60.8	19.25	65.5	20.63	70.1	"
24	15.00	51.0	16.50	56.1	18.00	61.2	19.50	66.3	21.00	71.4	22.50	76.5	"
26	16.25	55.25	17.88	60.8	19.50	66.3	21.13	71.8	22.75	77.4	24.38	82.9	"
28	17.50	59.50	19.25	65.5	21.00	71.4	22.75	77.4	24.50	83.3	26.25	89.3	"
30	18.75	63.75	20.63	70.1	22.50	76.5	24.38	82.9	26.25	89.3	28.13	95.6	"
32	20.00	68.0	22.00	74.8	24.00	81.6	26.00	88.4	28.00	95.2	30.00	102.0	"
34	21.25	72.25	23.38	79.5	25.50	86.7	27.63	93.9	29.75	101.2	31.88	108.4	"
36	22.50	76.50	24.75	84.2	27.00	91.8	29.25	99.5	31.50	107.1	33.75	114.8	"
38	23.75	80.75	26.13	88.8	28.50	96.9	30.88	105.0	33.25	113.1	35.63	121.1	"
40	25.00	85.00	27.50	93.5	30.00	102.0	32.50	110.5	35.00	119.0	37.50	127.5	"

Areas, Weights and Max. Lengths of Wide Plates.

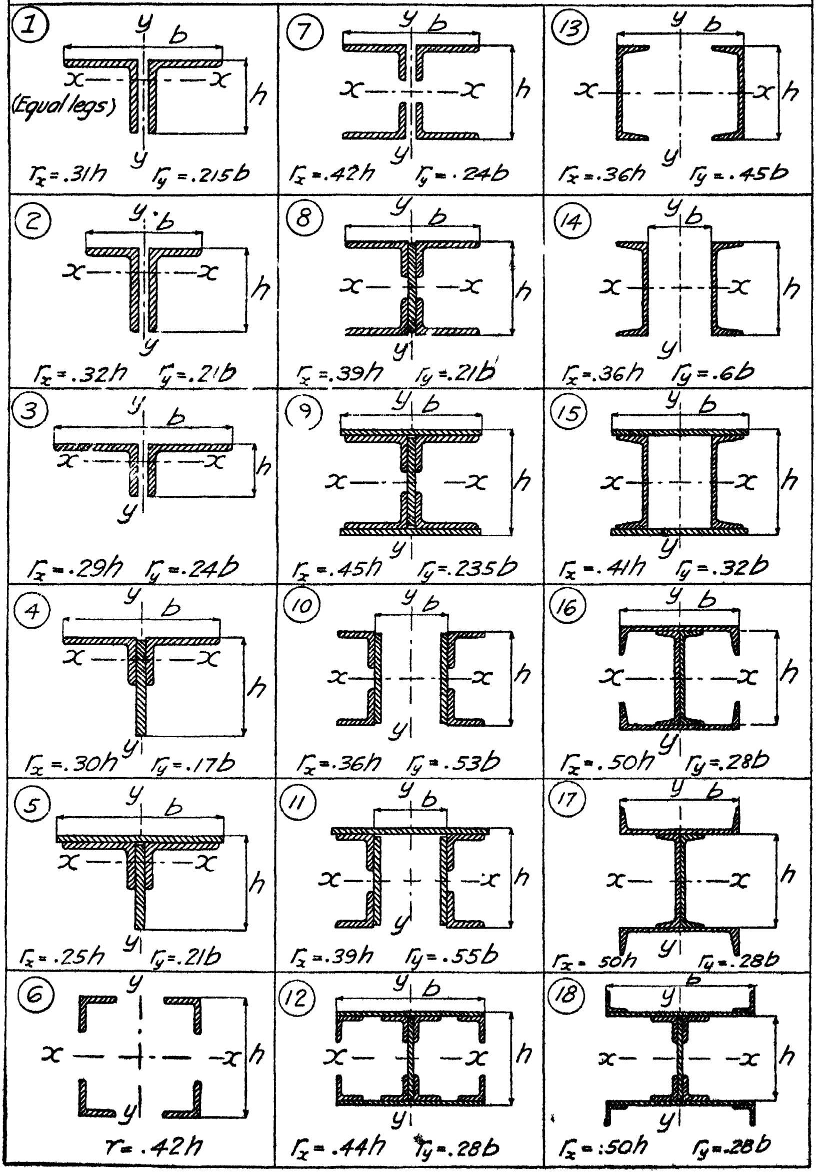
Table-9

A = area of cross section in sq. ins.
W = weight in pounds per ft.
L = maximum length in ft. obtainable.

Width in Inches	Thickness											
	5/16"			3/8"			7/16"			1/2"		
	A	W	L	A	W	L	A	W	L	A	W	L
42	13.13	44.63	37.5	15.75	53.55	41.6	18.38	62.48	45.8	21.00	71.40	45.8
46	14.38	48.88	37.5	17.25	58.65	41.6	20.13	68.43	45.8	23.00	78.20	45.8
48	15.00	51.00	41.6	18.00	61.20	45.0	21.00	71.40	46.6	24.00	81.60	46.6
50	15.63	53.10	37.5	18.75	63.80	45.0	21.88	74.40	46.6	25.00	85.00	46.6
52	16.25	55.30	37.5	19.50	66.30	45.0	22.75	77.40	46.6	26.00	88.40	46.6
56	17.50	59.50	37.5	21.00	71.40	45.0	24.50	83.30	46.6	28.00	95.20	46.6
60	18.75	63.80	37.5	22.50	76.50	45.0	26.25	89.30	46.6	30.00	102.0	46.6
66	20.63	70.10	33.3	24.75	84.20	41.6	28.88	98.20	46.6	33.00	112.2	46.6
68	21.25	72.30	33.3	25.50	86.70	41.6	29.75	101.2	46.6	34.00	115.6	46.6
72	22.50	76.50	31.6	27.00	91.80	32.5	31.50	107.1	43.3	36.00	122.4	43.6
76	23.75	80.80	31.6	28.50	96.90	38.3	33.25	113.1	39.6	38.00	129.2	43.6
80	25.00	85.00	31.6	30.00	102.0	38.3	35.00	119.0	39.6	40.00	136.0	41.6
84	26.25	89.30	27.1	31.50	107.1	36.6	36.75	125.0	38.7	42.00	142.8	39.6
90	28.13	95.60	25.0	33.75	114.8	33.3	39.38	133.9	36.6	45.00	153.0	37.5
96	30.00	102.0	22.9	36.00	122.4	29.1	42.00	142.8	31.6	48.00	163.2	33.3
100	31.25	106.3	21.6	37.50	127.5	25.8	43.75	148.8	29.1	50.00	170.0	30.0

Table 10

Approximate Radii of Gyration of Column Sections.



Shearing and Bearing Values of Rivets

Table 11

Diameter of Rivet in inches		Area of	Single Shear, 12000#	Bearing Values for Different Thicknesses of Plates @ 24000 lbs.										
Fraction	Decimal	Rivet	per sq in.	1/4"	5/16"	3/8"	7/16"	1/2"	9/16"	5/8"	11/16"	3/4"	13/16"	7/8"
3/8	0 375	0.1104	1320	2250	2820	3380								
1/2	0.500	0.1963	2360	3000	3750	4500	5250	6000						
5/8	0.625	0.3068	3680	3750	4690	5630	6560	7500	8440					
3/4	0.750	0.4418	5300	4500	5630	6750	7880	9000	10130	11250				
7/8	0 875	0.6013	7220	5250	6560	7880	9190	10500	11820	13120	14430	15750	17050	
1	1.000	0 7854	9430	6000	7500	9000	10500	12000	13500	15000	16500	18000	19500	21000

Diameter of Rivet in inches		Area of	Single Shear 10000#	Bearing Values for Different Thicknesses of Plates @ 20000 lbs.										
Fraction	Decimal	Rivet	per sq in	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8
3/8	0.375	0.1104	1100	1880	2340	2810								
1/2	0.500	0.1963	1960	2500	3130	3750	4380	5000						
5/8	0.625	0.3068	3070	3130	3910	4690	5470	6250	7030	7810				
3/4	0.750	0.4418	4420	3750	4690	5630	6560	7500	8440	9380	10310	11250		
7/8	0.875	0.6013	6010	4380	5470	6570	7660	8750	9840	10940	12030	13130	14220	
1	1.000	0.7854	7850	5000	6250	7500	8750	10000	11250	12500	13750	15000	16250	17500

Allowable Bearing and Moments on Pins

Table 12

Diameter of Pin	Bearing at 24000# on 1" of metal	Allowable Moment at 25000#	Diameter of Pin	Bearing at 24000# on 1" of metal.	Allowable Moment at 25000#
2"	48000#	19600"#	7"	168000#	841800"#
2¼	54000	28000	7¼	174000	935300
2½	60000	38300	7½	180000	1035400
2¾	66000	51000	7¾	186000	1142500
3	72000	66300	8	192000	1256600
3¼	78000	84300	8¼	198000	1378200
3½	84000	105200	8½	204000	1507300
3¾	90000	129400	8¾	210000	1644200
4	96000	157100	9	216000	1789200
4¼	102000	188400	9¼	222000	1942500
4½	108000	223700	9½	228000	2104300
4¾	114000	263000	9¾	234000	2274900
5	120000	306800	10	240000	2454400
5¼	126000	355200	10¼	246000	2643100
5½	132000	408300	10½	252000	2841200
5¾	138000	466600	10¾	258000	3049100
6	144000	530100	11	264000	3266800
6¼	150000	599200	11¼	270000	3494600
6½	156000	674000	11½	276000	3732800
6¾	162000	754800	11¾	282000	3981600

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