A POCKET-BOOK FOR MECHANICAL ENGINEERS

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PREFACE

THE pages which have been revised in this edition concern the following subjects: Chains, Chain Slings, and Crane Hooks; Manila and Wire Ropes; Screw Threads; Pipes and Pipe Joints; Steam Tables; and Locomotives. Small changes and corrections have been made on other pages. Acknowledgments are made in the text, but the editor wishes to thank Messrs. Stewarts & Lloyds Ltd. and Messrs. E. Baylie & Co. Ltd. for so readily supplying information, and the British Standards Institution for permitting extracts to be made from specifications.

B. B. LOW.

September 1947.

EXTRACT FROM PREFACE TO NEW EDITION REVISED 1943

THE Equivalent Temperatures on the Fahrenheit and Centigrade Scales (pp. 541–543) now cover a wider range, and the notes on the Practical Measurement of Temperature (pp. 544–546) have been extended. The information on p. 594, in the Boiler section, replaces older matter, and the section on Compressed Air (pp. 712–719) has been revised.

B. B. LOW.

February 1943.

EXTRACT FROM PREFACE TO NEW EDITION REVISED 1942

In addition to some corrections, the chief alterations in this edition concern Weights and Measures. All the tables from p. 1 to p. 32 and some of the values on p. 342 have been entirely recalculated, using the most accurately known factors.

The editor wishes to thank the Director of the National Physical Laboratory and the Controller of the Standards Department of the Board of Trade for the considerable information they have given and for their kindness in answering a number of questions.

Information has also been obtained from *The Units and Standards of Measurement Employed at the National Physical Laboratory* (H.M. Stationery Office, 1929) and from *Measurement of Oil in Bulk, Part I, Standard Weights and Measures* (Institute of Petroleum, 1932).

In conclusion, thanks are due to C. J. Tranter, M.A.(Oxon.), for valuable assistance with the calculations.

B. B. LOW.

December 1941.

EXTRACT FROM PREFACE TO THE NEW (1938) EDITION

The main additions and alterations in this edition occupy about 240 pages.

Acknowledgments.—To individuals, institutions, and firms mentioned in the text, for information and assistance; British Standards Institution for extracts from specifications; Committee of Lloyd's Register of Shipping for extracts from rules relating to engines and boilers for vessels; Controller of H.M. Stationery Office for extracts from Board of Trade *Instructions as to the Survey of Passenger Steamships*, vol. i.—Text, and from *Magnesium and its Alloys*.

B. B. LOW.

October 1938.

PREFACE TO THE FIRST EDITION

THE preparation of this work has occupied the whole of the author's spare time during the past five years; and he has also had the services of several assistants in the calculation of tables, and in the preparation of the illustrations, which are unusually numerous for a work of this kind.

Many of the tables to be found in pocket-books and works of reference for engineers have been published for so many years that they may be regarded as public property, and the author might therefore have compiled a considerable portion of this work by the simple use of scissors and paste, but he had a suspicion that this had been done so often before that errors had found their way into tables which no one thought of verifying. He therefore decided to have all the older tables very carefully checked. Some of the tables were checked by simply comparing those given by the best authorities, English, American, and German, and wherever there was any difference the correct values were obtained by careful calculation. It would astonish many to learn how many errors there are in the tables in books which have been relied on for many years. Many of the tables have been calculated throughout and then compared with those of standard authorities.

Where the tables are new or have not before been published, and there are many such in this work, they have been calculated throughout at least twice, and generally by two different individuals, and the results compared, and wherever there was a difference fresh calculations were made until there was perfect agreement.

In nearly every case the formula used in calculating any table is given, and also the values of any constants which are used in it.

It will be found that a special feature of this pocket-book which distinguishes it from other pocket-books is the large space devoted to the proportions of machines and machine details, and in this section numerous rules and tables are given, which it is hoped may be of service to draughtsmen and students.

It is obvious that in a work of this kind use must be made of the experience of many men, and in this matter the author would record his great indebtedness to the numerous engineering journals, and the transactions of the principal engineering societies at home and abroad.

Great attention has been given by the Author, Printers, and Publishers to the arrangement of the matter and to the selection of type and paper; in fact, neither pains nor expense has been spared to make the book reliable, useful, and attractive to those engaged in mechanical engineering.

In conclusion, where there are so many tables, formulæ, and rules, the author can scarcely hope that there are no errors, notwithstanding the labour and care which have been bestowed on them, and the author will be grateful if those using this work will kindly inform him of any mistakes which they may discover.

D. A. LOW.

LONDON, *January* 1898.

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LOW'S

ECHANICAL ENGINEER'S POCKET-BOOK

BRITISH MEASURES.

Linear Measure.

Inches.	Feet.	Yards.	Poles.	Furlongs.	Mile
1	·08333..	·02777..	·00505....	·000126...	00001578...
12	1	33333...	·060606...	·001515...	·00018939...
36	3	1	·181818...	·004545...	·00056818...
198	16½	5½	1	·025	003125.
7920	660	220	40	1	·125
63360	5280	1760	320	8	1

mil = 0·001 inch. 1 chain = 100 links = 22 yards.
fathom = 6 feet. 1 knot = 1 nautical mile (6080 feet) per hour.

Square Measure.

. Inches.	Sq. Feet.	Sq. Yards.	Sq. Poles	Roods.	Acres.	Sq. Mile.
1	·006944...	0007716 ..	·0000255...	....	.	... .
144	1	·11111....	003673...	·0000918 ..	·0000229 .	.. .. .
1296	9	1	0330578. .	000826...	·0002066...	.. . .
39204	272¼	30¼	1	·025.	·00625.	·00000976..
1568160	10890	1210	40	1	25.	000390625.
6272640	43560	4840	160	4	1	·0015625
)14489600	27878400	3097600	102400	2560	640	1

1 square chain = 16 square poles = 484 square yards.
10 square chains = 1 acre.
1 circular inch = area of a circle 1 inch in diameter = ·7854 square inch.

Cubic Measure and Measures of Capacity.

1 cubic inch = ·00057870.. cub. foot = ·00002143. . cub. yard.
1728 cubic inches = 1 cub. foot = ·037037037... cub. yard.
46656 cubic inches = 27 cub. feet = 1 cub. yard.

Pints.	Quarts.	Gallons.	Pecks.	Bushels.	Quarters.	Cubic Inches.
1	·5	·125.	·0625.	·015625.	·001953125	34·678
2	1	·25.	·125.	·03125.	·00390625	69·355
8	4	1	·5.	·125.	·015625.	277·420
16	8	2	1	·25	·03125.	554·840
64	32	8	4	1	·125	2219·360
512	256	64	32	8	1	17754·880

4 gills = 1 pint.

One shipping ton (for measuring cargo) = 42 cubic feet.

The gallon is defined as the volume "containing ten Imperial standard pounds weight of distilled water weighed in air against brass weights, with the water and the air at the temperature of sixty-two degrees of Fahrenheit's thermometer and with the barometer at thirty inches."

There is no *legal* equivalent of the gallon expressed in cubic inches, but 1 gallon = 277·420 cubic inches is the equivalent normally adopted, and from this

1 cubic foot = 6·22882 gallons.

The weight and volume of water at various temperatures is given on p. 342.

Avoirdupois Weight.

Ounces.	Pounds.	Stones.	Quarters.	Hundred-weights.	Ton
1	·0625.	·004464...	·002232...	·000558...	·0000279...
16	1	·071428...	·035714 ..	·008928...	·0004464...
224	14	1	·5.	·125.	·00625.
448	28	2	1	·25.	·0125.
1792	112	8	4	1	·05.
35840	2240	160	80	20	1

1 dram = $\frac{1}{16}$ ounce is a legal avoirdupois weight.

Troy Weight.

24 grains = 1 pennyweight.
20 pennyweights = 1 ounce.

Apothecaries' Weight.

20 grains = 1 scruple.
3 scruples = 1 drachm.
8 drachms = 1 ounce.

1 ounce Troy = 1 ounce Apothecaries = 480 grains.

The grain is derived from the avoirdupois pound, which is equal to 7000 grains.

Measures of Velocity.

Feet per Second.	Feet per Minute.	Miles per Hour.
1	60	·681818....
·0166666....	1	·0113636....
1·4666666....	88	1

Measures of Work and Moments.

Inch-Pounds.	Foot-Pounds.	Inch-Tons.	Foot-Tons.
1	·08333...	·0004464..	·0000372....
12	1	·0053571...	·0004464 ...
2240	186·6666...	1	·0833333....
26880	2240	12	1

1 horse-power = 550 ft. lbs. per sec. = 33,000 ft. lbs. per min. = 1,980,000 ft. lbs. per hour = 746 watts. 1 kilowatt = 1·34 h.p. 1 Board of Trade unit = 1 kilowatt-hour.

Measures of Stress and Pressure.

Lbs per Sq. Inch	Lbs per Sq. Foot.	Tons per Sq. Inch.	Tons per Sq. Foot.
1	144	·0004464	·0642857....
·006944 .	1	·0000031....	·0004464 . ..
2240	322560	1	144
15·555	2240	·0069444	1

UNITED STATES MEASURES.

These are practically the same as the British measures.
A U.S. gallon = 231 cubic inches = ·83267 Imperial gallon.
An Imperial gallon = 1·20095 U.S. gallons.
One net or short ton = 2000 lbs. avoirdupois.
One shipping ton (for measuring cargo) = 40 cubic feet.

METRIC MEASURES.

Linear Measure.

Milli-metres.	Centi-metres.	Deci-metres.	Metres.	Deka-metres.	Hecto-metres.	Kilo-metre.
1	·1	·01	·001	·0001	·00001	·000001
10	1	·1	·01	·001	·0001	·00001
100	10	1	·1	·01	·001	·0001
1000	**100**	**10**	**1**	**·1**	**·01**	**·001**
10000	1000	100	10	1	·1	·01
100000	10000	1000	100	10	1	·1
1000000	100000	10000	1000	100	10	1

Square Measure.

Square Centimetres.	Square Decimetres.	Square Metres.	Ares.	Hectare.
1	·01	·0001	·000001	·00000001
100	1	·01	·0001	·000001
10000	**100**	**1**	**·01**	**·0001**
1000000	10000	100	1	·01
100000000	1000000	10000	100	1

1 centiare = 1 square metre.

Cubic Measure and Measures of Capacity.

Cubic Centimetres.	Cubic Decimetres.	Cubic Metre.
1	·001	·000001
1000	1	·001
1000000	1000	1

1 stere = 1 cubic metre. 1 litre * = 1·000027 cubic decimetres.

Millilitres.	Centilitres.	Decilitres.	Litres.	Dekalitres.	Hectolitres.	Kilolitre.
1	·1	·01	·001	·0001	·00001	·000001
10	1	·1	·01	·001	·0001	·00001
100	10	1	·1	·01	·001	·0001
1000	**100**	**10**	**1**	**·1**	**·01**	**·001**
10000	1000	100	10	1	·1	·01
100000	10000	1000	100	10	1	·1
1000000	100000	10000	1000	100	10	1

Weights.

Milligrammes.	Centigrammes.	Decigrammes.	Grammes.	Dekagrammes.	Hectogrammes.	Kilogramme.
1	·1	·01	·001	·0001	·00001	·000001
10	1	·1	·01	·001	·0001	·00001
100	10	1	·1	·01	·001	·0001
1000	**100**	**10**	**1**	**·1**	**·01**	**·001**
10000	1000	100	10	1	·1	·01
100000	10000	1000	100	10	1	·1
1000000	100000	10000	1000	100	10	1

1 myriagramme = 10 kilogrammes. 1 quintal = 100 kilogrammes.
1 millier or tonne = 1000 kilogrammes.

The International Prototype Kilogramme, a plain cylindrical mass of *platinum-iridium* alloy, differs slightly from the mass of one cubic decimetre of pure water at 4° C. and 760 mm., which was originally intended to be the kilogramme.

Equivalents of Metric and British Measures.

Linear Measure.

Metric Units.	British Equivalents.	British Units.	Metric Equivalents.
1 millimetre	·039370147 inch.	1 inch	25·399956 millimetres.
1 centimetre	·39370147 inch.	1 inch	2·5399956 centimetres.
1 decimetre	3·9370147 inches.	1 inch	·25399956 decimetre.
1 metre	39·370147 inches.	1 inch	·025399956 metre.
1 metre	3·2808456 feet.	1 foot	·30479947 metre.
1 metre	1·0936152 yards.	1 yard	·91439841 metre.
1 kilometre	·62137227 mile.	1 mile	1·6093412 kilometres.

The standard metre is the distance between two marks on a *platinum-iridium bar* when the temperature is 0° C. or 32° F. The standard yard is the distance between two marks on a *bronze bar* when the temperature is 62° F.

Conversion Factors.

N.P.L. (1922–4). 1 metre = 39·370147 inches from which 1 inch = 25·399956 mm.*
British legal (1898). 1 metre = 39·370113 inches from which 1 inch = 25·399978 mm.
For trade purposes, legal sanction (1898) is given to 1 inch = 25·4 mm.
American legal (1866). 1 metre = 39·370000 inches from which 1 inch = 25·400051 mm.

Square Measure.

Metric Units.	British Equivalents.	British Units.	Metric Equivalents.
1 square centimetre.	·1550008 square inch.	1 square inch	6·451578 sq. centimetres.
1 square decimetre	15·50008 square inches.	1 square inch	·06451578 sq. decimetre.
1 square decimetre	·1076395 square foot.	1 square foot	9·290272 sq. decimetres.
1 square metre	10·76395 square feet.	1 square foot	·09290272 sq. metre.
1 square metre	1·195994 square yards.	1 square yard	·8361245 sq. metre.
1 are	119·5994 square yards.	1 square yard	·008361245 are.
1 hectare	2·471062 acres.	1 acre	·4046842 hectare.
1 hectare	·003861035 sq. mile.	1 square mile	258·9979 hectares.

* Used in these tables.

Equivalents of Metric and British Measures.

Cubic Measure and Measures of Capacity.

Metric Units.	British Equivalents.	British Units.	Metric Equivalents.
1 cubic centimetre	·06102406 cubic inch.	1 cubic inch	16·38698 cubic centimetres.
1 cubic decimetre	61·02406 cubic inches.	1 cubic inch	·01638698 cubic decimetre.
1 cubic decimetre	·03531485 cubic foot.	1 cubic foot	28·31670 cubic decimetres.
1 cubic metre	35·31485 cubic feet.	1 cubic foot	·02831670 cubic metre.
1 cubic metre	1·307957 cubic yards.	1 cubic yard	·7645509 cubic metre.
1 millilitre	·06102571 cubic inch.	1 cubic inch	16·38654 millilitres.
1 centilitre	·6102571 cubic inch.	1 cubic inch	1·638654 centilitres.
1 centilitre	·07039217 gill.	1 gill	14·2061 centilitres.
1 decilitre	·7039217 gill.	1 gill	1·42061 decilitres.
1 decilitre	·1759804 pint.	1 pint	5·68245 decilitres.
1 litre	1·759804 pints.	1 pint	·568245 litre.
1 litre	·879902 quart.	1 quart	1·13649 litres.
1 litre	·2199755 gallon.	1 gallon	4·54596 litres.
1 dekalitre	2·199755 gallons.	1 gallon	·454596 dekalitre.
1 dekalitre	·2749694 bushel.	1 bushel	3·63677 dekalitres.
1 hectolitre	2·749694 bushels.	1 bushel	·363677 hectolitre.

The litre is the volume of 1 kilogramme of pure water at its maximum density (4° C.) and at 760 mm., the weighing being reduced to *in vacuo* conditions. 1 litre = 1·000027 cu. decimetres = 1000·027 cu. cm. = 61·02571 cu. inches. By legal sanction (1898) 1 gallon = 4·54596(31) litres. The N.P.L. states that 1 gallon = 4·54596 litres, which is reliable only to the fifth decimal place. Confusion occurs between the millilitre and the cubic centimetre which are nearly equal. The Joint Committee for the Standardisation of Scientific Glassware recommended: The litre (l.) and millilitre (ml.) shall be used as the standard units of volume and that standard volumetric glassware shall be graduated in terms of these units and marked "ml." instead of "c.c."

Equivalents of Metric and British Measures.

Measures of Weight.

Metric Units.	British Equivalents.	British Units.	Metric Equivalents.
	Avoirdupois.	*Avoirdupois.*	
1 milligramme . . .	·01543236 grain.	1 grain . . .	64·79891 milligrammes.
1 centigramme . . .	·1543236 grain.	1 grain . . .	6·479891 centigrammes.
1 decigramme . . .	1·543236 grains.	1 grain . . .	·6479891 decigramme.
1 gramme	15·43236 grains.	1 grain . . .	·06479891 gramme.
1 gramme	·0352740 ounce.	1 ounce . . .	28·34952 grammes.
1 dekagramme . . .	·352740 ounce.	1 ounce . . .	2·834952 dekagrammes.
1 hectogramme . .	3·52740 ounces.	1 ounce . . .	·2834952 hectogramme.
1 kilogramme . . .	35·27396 ounces.	1 ounce . . .	·02834952 kilogramme.
1 kilogramme . . .	2·20462275 pounds.	1 pound . . .	·453592343 kilogramme.
1 myriagramme . .	22·0462275 pounds.	1 pound . . .	·045359234 myriagramme.
1 quintal	1·96841 hundredweights.	1 hundredweight	·50802342 quintal.
1 millier or tonne . .	·98420658 ton.	1 ton	1·0160468 milliers or tonnes.
	Troy.	*Troy.*	
1 gramme	15·43236 grains.	1 grain . . .	·06479891 gramme.
1 gramme	·03215075 ounce.	1 ounce . . .	31·103475 grammes.
	Apothecaries.	*Apothecaries.*	
1 gramme	15·43236 grains.	1 grain . . .	·06479891 gramme.

Equivalents of Metric and British Measures.

Miscellaneous Compound Measures.

Metric Units.	British Equivalents.	British Units.	Metric Equivalents.
1 metre per second . .	3·2808 feet per second.	1 foot per second	·3048 metre per second.
1 metre per minute . .	3·2808 feet per minute.	1 foot per minute	·3048 metre per minute.
1 kilometre per hour. .	·6214 mile per hour.	1 mile per hour .	1·6093 kilometres per hour.
1 kilogramme per metre.	·67197 pound per foot.	1 pound per foot .	1·48817 kilogrms. per metre.
1 kilogramme per metre.	2·01590 pounds per yard.	1 pound per yard	·49606 kilogrm. per metre.
1 tonne per metre . .	·899957 ton per yard.	1 ton per yard .	1·11116 tonnes per metre.
1 tonne per kilometre .	1·5839 tons per mile.	1 ton per mile .	·63134 tonne per kilometre.
1 kilogramme per sq. centimetre . . .	14·22329 pounds per sq. in.	1 lb. per sq. inch .	·07031 kilogramme per sq. centimetre.
1 kilogrm. per sq. metre .	·20482 pound per sq. ft.	1 lb. per sq. foot .	4·88244 kilogrms. per sq. metre.
1 tonne per sq. metre .	·09144 ton per sq. foot.	1 ton per sq. foot	10·93668 tonnes per sq. metre.
1 kilogramme per cub. centimetre . .	36·1271 pounds per cub. in.	1 lb. per cub. inch	·02768 kilogramme per cub. centimetre.
1 kilogrm. per cub. metre	·0624276 lb. per cub. ft.	1 lb. per cub. foot	16·019 kilogrms. per cub. metre.
1 kilogrm. per cub. metre	1·68555 lbs. per cub. yard.	1 lb. per cub. yard	·5933 kilogrm. per cub. metre.
1 tonne per cub. metre .	·75248 ton per cub. yard.	1 ton per cub. yard	1·329 tonnes per cub. metre.
1 cub. centimetre per kilogramme . .	·02768 cub. in. per lb.	1 cub. inch per lb.	36·1271 cub. centimetres per kilogramme.
1 cub. metre per kilogrm.	16·019 cub. ft. per pound.	1 cub. foot per lb.	·06243 cub. metre per kilogrm.
1 cub. metre per kilogrm.	·5933 cub. yard per lb.	1 cub. yard per lb.	1·68555 cub. metres per kilogrm.
1 cub. metre per tonne .	1·329 cub. yards per ton.	1 cub. yard per ton	·75248 cub. metre per tonne.

Equivalents of Metric and British Measures.

Miscellaneous Compound Measures.

Metric Units.	British Equivalents.	British Units.	Metric Equivalents.
1 gramme per litre	70·155 grains per gallon.	1 grain per gallon	·01425 gramme per litre.
1 kilogramme per litre	10·0221 pounds per gallon.	1 pound per gallon	·09978 kilogrm. per litre.
1 kilogrammetre	7·23303 foot-pounds.	1 foot-pound	·1382547 kilogrammetre.
1 tonne-metre	3·22903 foot-tons.	1 foot-ton	·309691 tonne-metre.
1 force de cheval *	·986322 horse-power.	1 horse-power §	1·01387 force de cheval.
1 kilogrm. per force de cheval	2·23520 pounds per horse-power.	1 pound per horse-power	·44739 kilogramme per force de cheval.
1 sq. metre per force de cheval	10·9132 sq. feet per horse-power.	1 sq. foot per horse-power	·09163 sq. metre per force de cheval.
1 calorie †	3·96832 B.Th.U.	1 B.Th.U. ‖	·251996 calorie.
1 calorie per sq. metre	·36867 B.Th.U. per sq. foot.	1 B.Th.U. per sq. ft.	2·71247 calories per sq. metre
1 franc‡ per kilogrm.	·35999 shilling per pound.	1 shilling per pound	2·7778 francs per kilogrm.
1 franc per tonne	·04032 £ per ton.	1 £ per ton	24·802 francs per tonne.
1 franc per metre	·7257 shilling per yard.	1 shilling per yard	1·37796 francs per metre.
1 franc per kilometre	·06386 £ per mile.	1 £ per mile	15·6586 francs per kilometre.
1 franc per sq. metre	·07373 shilling per sq. ft.	1 shilling per sq. foot	13·563 francs per sq. metre.
1 franc per sq. metre	·66359 shilling per sq. yd.	1 shilling per sq. yd.	1·507 francs per sq. metre.
1 franc per cub. metre	·02247 shilling per cub. ft.	1 shilling per cub. ft.	44·497 francs per cub. metre.
1 franc per cub. metre	·60679 shilling per cub. yd.	1 shilling per cub. yd.	1·648 francs per cub. metre.
1 franc per litre	3·6079 shillings per gallon.	1 shilling per gallon	·2772 franc per litre.

* 1 force de cheval = 4500 kilogrammetres per minute.

† 1 calorie is the amount of heat required to raise one kilogramme of water 1° C. See p. 546. ‡ 25·2 francs = £1 (in 1914).

§ 1 horse-power = 33,000 foot-pounds per minute.

‖ 1 B.Th.U. is the amount of heat required to raise one pound of water 1° F. See p. 546.

Equivalents of Millimetres in Inches.

Inches = Millimetres × ·039370147

Mm.	Inches.	Mm.	Inches.	Mm.	Inches.	Mm.	Inches.
1	·0394	46	1·8110	91	3·5827	136	5·3543
2	·0787	47	1·8504	92	3·6221	137	5·3937
3	·1181	48	1·8898	93	3·6614	138	5·4331
4	·1575	49	1·9291	94	3·7008	139	5·4725
5	·1969	50	1·9685	95	3·7402	140	5·5118
6	·2362	51	2·0079	96	3·7795	141	5·5512
7	·2756	52	2·0472	97	3·8189	142	5·5906
8	·3150	53	2·0866	98	3·8583	143	5·6299
9	·3543	54	2·1260	99	3·8976	144	5·6693
10	·3937	55	2·1654	100	3·9370	145	5·7087
11	·4331	56	2·2047	101	3·9764	146	5·7480
12	·4724	57	2 2441	102	4·0158	147	5·7874
13	·5118	58	2·2835	103	4·0551	148	5·8268
14	·5512	59	2·3228	104	4 0945	149	5·8662
15	·5906	60	2·3622	105	4·1339	150	5·9055
16	·6299	61	2·4016	106	4·1732	151	5·9449
17	·6693	62	2·4409	107	4·2126	152	5·9843
18	·7087	63	2·4803	108	4·2520	153	6·0236
19	·7480	64	2·5197	109	4·2913	154	6·0630
20	·7874	65	2·5591	110	4·3307	155	6·1024
21	·8268	66	2·5984	111	4·3701	156	6·1417
22	·8661	67	2 6378	112	4·4095	157	6·1811
23	·9055	68	2·6772	113	4·4488	158	6·2205
24	·9449	69	2·7165	114	4·4882	159	6·2599
25	·9843	70	2·7559	115	4·5276	160	6·2992
26	1·0236	71	2·7953	116	4·5669	161	6·3386
27	1·0630	72	2·8347	117	4·6063	162	6·3780
28	1·1024	73	2·8740	118	4·6457	163	6·4173
29	1·1417	74	2·9134	119	4·6850	164	6·4567
30	1·1811	75	2·9528	120	4·7244	165	6·4961
31	1·2205	76	2·9921	121	4·7638	166	6·5354
32	1·2598	77	3·0315	122	4·8032	167	6·5748
33	1·2992	78	3·0709	123	4·8425	168	6·6142
34	1·3386	79	3·1102	124	4·8819	169	6·6536
35	1·3780	80	3·1496	125	4·9213	170	6·6929
36	1·4173	81	3·1890	126	4·9606	171	6·7323
37	1·4567	82	3·2284	127	5·0000	172	6·7717
38	1·4961	83	3·2677	128	5·0394	173	6·8110
39	1·5354	84	3·3071	129	5·0787	174	6·8504
40	1·5748	85	3·3465	130	5·1181	175	6·8898
41	1·6142	86	3·3858	131	5·1575	176	6·9291
42	1·6535	87	3·4252	132	5·1969	177	6·9685
43	1·6929	88	3·4646	133	5·2362	178	7·0079
44	1·7323	89	3·5039	134	5·2756	179	7·0473
45	1·7717	90	3·5433	135	5·3150	180	7·0866

Equivalents of Millimetres in Inches.

Mm.	Inches.	Mm.	Inches.	Mm.	Inches.	Mm.	Inches.
181	7·1260	226	8·8977	271	10·6693	316	12·4410
182	7·1654	227	8·9370	272	10·7087	317	12·4803
183	7·2047	228	8·9764	273	10·7481	318	12·5197
184	7·2441	229	9·0158	274	10·7874	319	12·5591
185	7·2835	230	9·0551	275	10·8268	320	12·5984
186	7·3228	231	9·0945	276	10·8662	321	12·6378
187	7·3622	232	9·1339	277	10·9055	322	12·6772
188	7·4016	233	9·1732	278	10·9449	323	12·7166
189	7·4410	234	9·2126	279	10·9843	324	12·7559
190	7·4803	235	9·2520	280	11·0236	325	12·7953
191	7·5197	236	9·2914	281	11·0630	326	12·8347
192	7·5591	237	9·3307	282	11·1024	327	12·8740
193	7·5984	238	9·3701	283	11·1418	328	12·9134
194	7·6378	239	9·4095	284	11·1811	329	12·9528
195	7·6772	240	9·4488	285	11·2205	330	12·9921
196	7·7165	241	9·4882	286	11·2599	331	13·0315
197	7·7559	242	9·5276	287	11·2992	332	13·0709
198	7·7953	243	9·5669	288	11·3386	333	13·1103
199	7·8347	244	9·6063	289	11·3780	334	13·1496
200	7·8740	245	9·6457	290	11·4173	335	13·1890
201	7·9134	246	9·6851	291	11·4567	336	13·2284
202	7·9528	247	9·7244	292	11·4961	337	13·2677
203	7·9921	248	9·7638	293	11·5355	338	13·3071
204	8·0315	249	9·8032	294	11·5748	339	13·3465
205	8·0709	250	9·8425	295	11·6142	340	13·3858
206	8·1103	251	9·8819	296	11·6536	341	13·4252
207	8·1496	252	9·9213	297	11·6929	342	13·4646
208	8·1890	253	9·9606	298	11·7323	343	13·5040
209	8·2284	254	10·0000	299	11·7717	344	13·5433
210	8·2677	255	10·0394	300	11·8110	345	13·5827
211	8·3071	256	10·0788	301	11·8504	346	13·6221
212	8·3465	257	10·1181	302	11·8898	347	13·6614
213	8·3858	258	10·1575	303	11·9292	348	13·7008
214	8·4252	259	10·1969	304	11·9685	349	13·7402
215	8·4646	260	10·2362	305	12·0079	350	13·7796
216	8·5040	261	10·2756	306	12·0473	351	13·8189
217	8·5433	262	10·3150	307	12·0866	352	13·8583
218	8·5827	263	10·3543	308	12·1260	353	13·8977
219	8·6221	264	10·3937	309	12·1654	354	13·9370
220	8·6614	265	10·4331	310	12·2047	355	13·9764
221	8·7008	266	10·4725	311	12·2441	356	14·0158
222	8·7402	267	10·5118	312	12·2835	357	14·0551
223	8·7795	268	10·5512	313	12·3229	358	14·0945
224	8·8189	269	10·5906	314	12·3622	359	14·1339
225	8·8583	270	10·6299	315	12·4016	360	14·1733

Equivalents of Millimetres in Inches.

Mm.	Inches.	Mm.	Inches.	Mm.	Inches.	Mm.	Inches.
361	14·2126	406	15·9843	451	17·7559	496	19·5276
362	14·2520	407	16·0236	452	17·7953	497	19·5670
363	14·2914	408	16·0630	453	17·8347	498	19·6063
364	14·3307	409	16·1024	454	17·8740	499	19·6457
365	14·3701	410	16·1418	455	17·9134	500	19·6851
366	14·4095	411	16·1811	456	17·9528	501	19·7244
367	14·4488	412	16·2205	457	17·9922	502	19·7638
368	14·4882	413	16·2599	458	18·0315	503	19·8032
369	14·5276	414	16·2992	459	18·0709	504	19·8426
370	14·5670	415	16·3386	460	18·1103	505	19·8819
371	14·6063	416	16·3780	461	18·1496	506	19·9213
372	14·6457	417	16·4174	462	18·1890	507	19·9607
373	14·6851	418	16·4567	463	18·2284	508	20·0000
374	14·7244	419	16·4961	464	18·2677	509	20·0394
375	14·7638	420	16·5355	465	18·3071	510	20·0788
376	14·8032	421	16·5748	466	18·3465	511	20·1181
377	14·8425	422	16·6142	467	18·3859	512	20·1575
378	14·8819	423	16·6536	468	18·4252	513	20·1969
379	14·9213	424	16·6929	469	18·4646	514	20·2363
380	14·9607	425	16·7323	470	18·5040	515	20·2756
381	15·0000	426	16·7717	471	18·5433	516	20·3150
382	15·0394	427	16·8111	472	18·5827	517	20·3544
383	15·0788	428	16·8504	473	18·6221	518	20·3937
384	15·1181	429	16·8898	474	18·6614	519	20·4331
385	15·1575	430	16·9292	475	18·7008	520	20·4725
386	15·1969	431	16·9685	476	18·7402	521	20·5118
387	15·2362	432	17·0079	477	18·7796	522	20·5512
388	15·2756	433	17·0473	478	18·8189	523	20·5906
389	15·3150	434	17·0866	479	18·8583	524	20·6300
390	15·3544	435	17·1260	480	18·8977	525	20·6693
391	15·3937	436	17·1654	481	18·9370	526	20·7087
392	15·4331	437	17·2048	482	18·9764	527	20·7481
393	15·4725	438	17·2441	483	19·0158	528	20·7874
394	15·5118	439	17·2835	484	19·0552	529	20·8268
395	15·5512	440	17·3229	485	19·0945	530	20·8662
396	15·5906	441	17·3622	486	19·1339	531	20·9055
397	15·6299	442	17·4016	487	19·1733	532	20·9449
398	15·6693	443	17·4410	488	19·2126	533	20·9843
399	15·7087	444	17·4803	489	19·2520	534	21·0237
400	15·7481	445	17·5197	490	19·2914	535	21·0630
401	15·7874	446	17·5591	491	19·3307	536	21·1024
402	15·8268	447	17·5985	492	19·3701	537	21·1418
403	15·8662	448	17·6378	493	19·4095	538	21·1811
404	15·9055	449	17·6772	494	19·4489	539	21·2205
405	15·9449	450	17·7166	495	19·4882	540	21·2599

Equivalents of Millimetres in Inches.

Mm.	Inches.	Mm.	Inches.	Mm.	Inches.	Mm.	Inches.
541	21·2992	586	23·0709	631	24·8426	676	26·6142
542	21·3386	587	23·1103	632	24·8819	677	26·6536
543	21·3780	588	23·1496	633	24·9213	678	26·6930
544	21·4174	589	23·1890	634	24·9607	679	26·7323
545	21·4567	590	23·2284	635	25·0000	680	26·7717
546	21·4961	591	23·2678	636	25·0394	681	26·8111
547	21·5355	592	23·3071	637	25·0788	682	26·8504
548	21·5748	593	23·3465	638	25·1182	683	26·8898
549	21·6142	594	23·3859	639	25·1575	684	26·9292
550	21·6536	595	23·4252	640	25·1969	685	26·9686
551	21·6930	596	23·4646	641	25·2363	686	27·0079
552	21·7323	597	23·5040	642	25·2756	687	27·0473
553	21·7717	598	23·5433	643	25·3150	688	27·0867
554	21·8111	599	23·5827	644	25·3544	689	27·1260
555	21·8504	600	23·6221	645	25·3937	690	27·1654
556	21·8898	601	23·6615	646	25·4331	691	27·2048
557	21·9292	602	23·7008	647	25·4725	692	27·2441
558	21·9685	603	23·7402	648	25·5119	693	27·2835
559	22·0079	604	23·7796	649	25·5512	694	27·3229
560	22·0473	605	23·8189	650	25·5906	695	27·3623
561	22·0867	606	23·8583	651	25·6300	696	27·4016
562	22·1260	607	23·8977	652	25·6693	697	27·4410
563	22·1654	608	23·9370	653	25·7087	698	27·4804
564	22·2048	609	23·9764	654	25·7481	699	27·5197
565	22·2441	610	24·0158	655	25·7874	700	27·5591
566	22·2835	611	24·0552	656	25·8268	701	27·5985
567	22·3229	612	24·0945	657	25·8662	702	27·6378
568	22·3622	613	24·1339	658	25·9056	703	27·6772
569	22·4016	614	24·1733	659	25·9449	704	27·7166
570	22·4410	615	24·2126	660	25·9843	705	27·7560
571	22·4804	616	24·2520	661	26·0237	706	27·7953
572	22·5197	617	24·2914	662	26·0630	707	27·8347
573	22·5591	618	24·3308	663	26·1024	708	27·8741
574	22·5985	619	24·3701	664	26·1418	709	27·9134
575	22·6378	620	24·4095	665	26·1811	710	27·9528
576	22·6772	621	24·4489	666	26·2205	711	27·9922
577	22·7166	622	24·4882	667	26·2599	712	28·0315
578	22·7559	623	24·5276	668	26·2993	713	28·0709
579	22·7953	624	24·5670	669	26·3386	714	28·1103
580	22·8347	625	24·6063	670	26·3780	715	28·1497
581	22·8741	626	24·6457	671	26·4174	716	28·1890
582	22·9134	627	24·6851	672	26·4567	717	28·2284
583	22·9528	628	24·7245	673	26·4961	718	28·2678
584	22·9922	629	24·7638	674	26·5355	719	28·3071
585	23·0315	630	24·8032	675	26·5748	720	28·3465

Equivalents of Millimetres in Inches.

Mm.	Inches.	Mm.	Inches.	Mm.	Inches.	Mm.	Inches.
721	28·3859	766	30·1575	811	31·9292	856	33·7008
722	28·4252	767	30·1969	812	31·9686	857	33·7402
723	28·4646	768	30·2363	813	32·0079	858	33·7796
724	28·5040	769	30·2756	814	32·0473	859	33·8190
725	28·5434	770	30·3150	815	32·0867	860	33·8583
726	28·5827	771	30·3544	816	32·1260	861	33·8977
727	28·6221	772	30·3938	817	32·1654	862	33·9371
728	28·6615	773	30·4331	818	32·2048	863	33·9764
729	28·7008	774	30·4725	819	32·2442	864	34·0158
730	28·7402	775	30·5119	820	32·2835	865	34·0552
731	28·7796	776	30·5512	821	32·3229	866	34·0945
732	28·8189	777	30·5906	822	32·3623	867	34·1339
733	28·8583	778	30·6300	823	32·4016	868	34·1733
734	28·8977	779	30·6693	824	32·4410	869	34·2127
735	28·9371	780	30·7087	825	32·4804	870	34·2520
736	28·9764	781	30·7481	826	32·5197	871	34·2914
737	29·0158	782	30·7875	827	32·5591	872	34·3308
738	29·0552	783	30·8268	828	32·5985	873	34·3701
739	29·0945	784	30·8662	829	32·6379	874	34·4095
740	29·1339	785	30·9056	830	32·6772	875	34·4489
741	29·1733	786	30·9449	831	32·7166	876	34·4882
742	29·2126	787	30·9843	832	32·7560	877	34·5276
743	29·2520	788	31·0237	833	32·7953	878	34·5670
744	29·2914	789	31·0630	834	32·8347	879	34·6064
745	29·3308	790	31·1024	835	32·8741	880	34·6457
746	29·3701	791	31·1418	836	32·9134	881	34·6851
747	29·4095	792	31·1812	837	32·9528	882	34·7245
748	29·4489	793	31·2205	838	32·9922	883	34·7638
749	29·4882	794	31·2599	839	33·0316	884	34·8032
750	29·5276	795	31·2993	840	33·0709	885	34·8426
751	29·5670	796	31·3386	841	33·1103	886	34·8820
752	29·6064	797	31·3780	842	33·1497	887	34·9213
753	29·6457	798	31·4174	843	33·1890	888	34·9607
754	29·6851	799	31·4567	844	33·2284	889	35·0001
755	29·7245	800	31·4961	845	33·2678	890	35·0394
756	29·7638	801	31·5355	846	33·3071	891	35·0788
757	29·8032	802	31·5749	847	33·3465	892	35·1182
758	29·8426	803	31·6142	848	33·3859	893	35·1575
759	29·8819	804	31·6536	849	33·4253	894	35·1969
760	29·9213	805	31·6930	850	33·4646	895	35·2363
761	29·9607	806	31·7323	851	33·5040	896	35·2757
762	30·0001	807	31·7717	852	33·5434	897	35·3150
763	30·0394	808	31·8111	853	33·5827	898	35·3544
764	30·0788	809	31·8504	854	33·6221	899	35·3938
765	30·1182	810	31·8898	855	33·6615	900	35·4331

Equivalents of Millimetres in Inches.

Mm.	Inches.	Mm.	Inches.	Mm.	Inches.	Mm.	Inches.
901	35·4725	926	36·4568	951	37·4410	976	38·4253
902	35·5119	927	36·4961	952	37·4804	977	38·4646
903	35·5512	928	36·5355	953	37·5198	978	38·5040
904	35·5906	929	36·5749	954	37·5591	979	38·5434
905	35·6300	930	36·6142	955	37·5985	980	38·5827
906	35·6694	931	36·6536	956	37·6379	981	38·6221
907	35·7087	932	36·6930	957	37·6772	982	38·6615
908	35·7481	933	36·7323	958	37·7166	983	38·7009
909	35·7875	934	36·7717	959	37·7560	984	38·7402
910	35·8268	935	36·8111	960	37·7953	985	38·7796
911	35·8662	936	36·8505	961	37·8347	986	38·8190
912	35·9056	937	36·8898	962	37·8741	987	38·8583
913	35·9449	938	36·9292	963	37·9135	988	38·8977
914	35·9843	939	36·9686	964	37·9528	989	38·9371
915	36·0237	940	37·0079	965	37·9922	990	38·9764
916	36·0631	941	37·0473	966	38·0316	991	39·0158
917	36·1024	942	37·0867	967	38·0709	992	39·0552
918	36·1418	943	37·1260	968	38·1103	993	39·0946
919	36·1812	944	37·1654	969	38·1497	994	39·1339
920	36·2205	945	37·2048	970	38·1890	995	39·1733
921	36·2599	946	37·2442	971	38·2284	996	39·2127
922	36·2993	947	37·2835	972	38·2678	997	39·2520
923	36·3386	948	37·3229	973	38·3072	998	39·2914
924	36·3780	949	37·3623	974	38·3465	999	39·3308
925	36·4174	950	37·4016	975	38·3859	1000	39·3701

Equivalents of Fractions of an Inch in Millimetres.

Millimetres = Inches × 25·399956.

Inch.	Mm.	Inch.	Mm.	Inch.	Mm.	Inch.	Mm.
1/64	·3969	17/64	6·7469	33/64	13·0969	49/64	19·4468
1/32	·7937	9/32	7·1437	17/32	13·4937	25/32	19·8437
3/64	1·1906	19/64	7·5406	35/64	13·8906	51/64	20·2406
1/16	1·5875	5/16	7·9375	9/16	14·2875	13/16	20·6375
5/64	1·9844	21/64	8·3344	37/64	14·6843	53/64	21·0343
3/32	2·3812	11/32	8·7312	19/32	15·0812	27/32	21·4312
7/64	2·7781	23/64	9·1281	39/64	15·4781	55/64	21·8281
1/8	3·1750	3/8	9·5250	5/8	15·8750	7/8	22·2250
9/64	3·5719	25/64	9·9219	41/64	16·2718	57/64	22·6218
5/32	3·9687	13/32	10·3187	21/32	16·6687	29/32	23·0187
11/64	4·3656	27/64	10·7156	43/64	17·0656	59/64	23·4156
3/16	4·7625	7/16	11·1125	11/16	17·4625	15/16	23·8125
13/64	5·1594	29/64	11·5094	45/64	17·8593	61/64	24·2093
7/32	5·5562	15/32	11·9062	23/32	18·2562	31/32	24·6062
15/64	5·9531	31/64	12·3031	47/64	18·6531	63/64	25·0031
1/4	6·3500	1/2	12·7000	3/4	19·0500	1	25·4000

Equivalents of Inches and Fractions of an Inch in Millimetres.

Millimetres = Inches × 25·399956.

In.	·0	⅛	¼	⅜	½	⅝	¾	⅞
1	25·400	28·575	31·750	34·925	38·100	41·275	44·450	47·625
2	50·800	53·975	57·150	60·325	63·500	66·675	69·850	73·025
3	76·200	79 375	82·550	85·725	88·900	92·075	95·250	98·425
4	101·60	104·77	107·95	111·12	114·30	117·47	120·65	123·82
5	127·00	130·17	133·35	136·52	139·70	142·87	146·05	149·22
6	152·40	155·57	158·75	161·92	165·10	168·27	171·45	174·62
7	177·80	180·97	184·15	187·32	190·50	193·67	196·85	200·02
8	203·20	206·37	209·55	212·72	215·90	219·07	222·25	225·42
9	228·60	231·77	234·95	238·12	241·30	244·47	247·65	250·82
10	254·00	257·17	260·35	263·52	266·70	269·87	273·05	276·22
11	279·40	282·57	285·75	288·92	292·10	295·27	298·45	301·62
12	304·80	307·97	311·15	314·32	317·50	320·67	323·85	327·02
13	330·20	333·37	336·55	339·72	342·90	346·07	349·25	352·42
14	355·60	358·77	361·95	365·12	368·30	371·47	374·65	377·82
15	381·00	384·17	387·35	390·52	393·70	396·87	400·05	403·22
16	406·40	409·57	412·75	415·92	419·10	422·27	425·45	428·62
17	431·80	434·97	438·15	441·32	444·50	447·67	450·85	454·02
18	457·20	460·37	463·55	466·72	469·90	473·07	476·25	479·42
19	482·60	485·77	488·95	492·12	495·30	498·47	501·65	504·82
20	508·00	511·17	514·35	517·52	520·70	523·87	527·05	530·22
21	533·40	536·57	539·75	542·92	546·10	549·27	552·45	555·62
22	558·80	561·97	565·15	568·32	571·50	574·67	577·85	581·02
23	584·20	587·37	590·55	593·72	596·90	600·07	603·25	606·42
24	609·60	612·77	615·95	619·12	622·30	625·47	628·65	631·82
25	635·00	638·17	641·35	644·52	647·70	650·87	654·05	657·22
26	660·40	663·57	666·75	669·92	673·10	676·27	679·45	682·62
27	685·80	688·97	692·15	695·32	698·50	701·67	704·85	708·02
28	711·20	714·37	717·55	720·72	723·90	727·07	730·25	733·42
29	736·60	739·77	742·95	746·12	749·30	752·47	755·65	758·82
30	762·00	765·17	768·35	771·52	774·70	777·87	781·05	784·22
31	787·40	790·57	793·75	796·92	800·10	803·27	806·45	809·62
32	812·80	815·97	819·15	822·32	825·50	828·67	831·85	835·02
33	838·20	841·37	844·55	847·72	850·90	854·07	857·25	860·42
34	863·60	866·77	869·95	873·12	876·30	879·47	882·65	885·82
35	889·00	892·17	895·35	898·52	901·70	904·87	908·05	911·22
In.	·0	⅛	¼	⅜	½	⅝	¾	⅞

Equivalents of Metres in Feet. Feet = Metres × 3·2808456.

Metres.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Metres.
1	3·28085	3·60893	3·93701	4·26510	4·59318	4·92127	5·24935	5·57744	5·90552	6·23361	1
2	6·56169	6·88978	7·21786	7·54594	7·87403	8·20211	8·53020	8·85828	9·18637	9·51445	2
3	9·84254	10·1706	10·4987	10·8268	11·1549	11·4830	11·8110	12·1391	12·4672	12·7953	3
4	13·1234	13·4515	13·7796	14·1076	14·4357	14·7638	15·0919	15·4200	15·7481	16·0761	4
5	16·4042	16·7323	17·0604	17·3885	17·7166	18·0447	18·3727	18·7008	19·0289	19·3570	5
6	19·6851	20·0132	20·3412	20·6693	20·9974	21·3255	21·6536	21·9817	22·3098	22·6378	6
7	22·9659	23·2940	23·6221	23·9502	24·2783	24·6063	24·9344	25·2625	25·5906	25·9187	7
8	26·2468	26·5748	26·9029	27·2310	27·5591	27·8872	28·2153	28·5434	28·8714	29·1995	8
9	29·5276	29·8557	30·1838	30·5119	30·8399	31·1680	31·4961	31·8242	32·1523	32·4804	9
10	32·8085	33·1365	33·4646	33·7927	34·1208	34·4489	34·7770	35·1050	35·4331	35·7612	10

Equivalents of Feet in Metres. Metres = Feet × ·30479947.

Feet.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Feet.
1	·304799	·335279	·365759	·396239	·426719	·457199	·487679	·518159	·548639	·579119	1
2	·609599	·640079	·670559	·701039	·731519	·761999	·792479	·822959	·853439	·883918	2
3	·914398	·944878	·975358	1·00584	1·03632	1·06680	1·09728	1·12776	1·15824	1·18872	3
4	1·21920	1·24968	1·28016	1·31064	1·34112	1·37160	1·40208	1·43256	1·46304	1·49352	4
5	1·52400	1·55448	1·58496	1·61544	1·64592	1·67640	1·70688	1·73736	1·76784	1·79832	5
6	1·82880	1·85928	1·88976	1·92024	1·95072	1·98120	2·01168	2·04216	2·07264	2·10312	6
7	2·13360	2·16408	2·19456	2·22504	2·25552	2·28600	2·31648	2·34696	2·37744	2·40792	7
8	2·43840	2·46888	2·49936	2·52984	2·56032	2·59080	2·62128	2·65176	2·68224	2·71272	8
9	2·74320	2·77368	2·80416	2·83464	2·86512	2·89559	2·92607	2·95655	2·98703	3·01751	9
10	3·04799	3·07847	3·10895	3·13943	3·16991	3·20039	3·23087	3·26135	3·29183	3·32231	10

Equivalents of Kilometres in Statute Miles. Statute miles = Kilometres × ·62137227.

Kilometres.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Kilometres.
1	·621372	·683509	·745647	·807784	·869921	·932058	·994196	1·05633	1·11847	1·18061	1
2	1·24274	1·30488	1·36702	1·42916	1·49129	1·55343	1·61557	1·67771	1·73984	1·80198	2
3	1·86412	1·92625	1·98839	2·05053	2·11267	2·17480	2·23694	2·29908	2·36121	2·42335	3
4	2·48549	2·54763	2·60976	2·67190	2·73404	2·79618	2·85831	2·92045	2·98259	3·04472	4
5	3·10686	3·16900	3·23114	3·29327	3·35541	3·41755	3·47968	3·54182	3·60396	3·66610	5
6	3·72823	3·79037	3·85251	3·91465	3·97678	4·03892	4·10106	4·16319	4·22533	4·28747	6
7	4·34961	4·41174	4·47388	4·53602	4·59815	4·66029	4·72243	4·78457	4·84670	4·90884	7
8	4·97098	5·03312	5·09525	5·15739	5·21953	5·28166	5·34380	5·40594	5·46808	5·53021	8
9	5·59235	5·65449	5·71662	5·77876	5·84090	5·90304	5·96517	6·02731	6·08945	6·15159	9
10	6·21372	6·27586	6·33800	6·40013	6·46227	6·52441	6·58655	6·64868	6·71082	6·77296	10

Equivalents of Kilometres in Nautical Miles. Nautical miles * = Kilometres × ·53961276.

Kilometres.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Kilometres.
1	·539613	·593574	·647535	·701497	·755458	·809419	·863380	·917342	·971303	1·02526	1
2	1·07923	1·13319	1·18715	1·24111	1·29507	1·34903	1·40299	1·45695	1·51092	1·56488	2
3	1·61884	1·67280	1·72676	1·78072	1·83468	1·88864	1·94261	1·99657	2·05053	2·10449	3
4	2·15845	2·21241	2·26637	2·32033	2·37430	2·42826	2·48222	2·53618	2·59014	2·64410	4
5	2·69806	2·75203	2·80599	2·85995	2·91391	2·96787	3·02183	3·07579	3·12975	3·18372	5
6	3·23768	3·29164	3·34560	3·39956	3·45352	3·50748	3·56144	3·61541	3·66937	3·72333	6
7	3·77729	3·83125	3·88521	3·93917	3·99313	4·04710	4·10106	4·15502	4·20898	4·26294	7
8	4·31690	4·37086	4·42482	4·47879	4·53275	4·58671	4·64067	4·69463	4·74859	4·80255	8
9	4·85651	4·91048	4·96444	5·01840	5·07236	5·12632	5·18028	5·23424	5·28821	5·34217	9
10	5·39613	5·45009	5·50405	5·55801	5·61197	5·66593	5·71990	5·77386	5·82782	5·88178	10

* Based on 1 nautical mile = 6080 feet.

Equivalents of Statute Miles in Nautical Miles and in Kilometres

Nautical miles * = Statute miles × ·86842105.
Kilometres = Statute miles × 1·6093412.

Miles.	Fractions of a Mile.				Fractions of a Mile.			
	·0	·25	·5	·75	·0	·25	·5	·75
	Nautical Miles.				Kilometres.			
1	·8684	1·0855	1·3026	1·5197	1·609	2·012	2·414	2·816
2	1·7368	1·9539	2·1711	2·3882	3·219	3·621	4·023	4·426
3	2·6053	2·8224	3·0395	3·2566	4·828	5·230	5·633	6·035
4	3·4737	3·6908	3·9079	4·1250	6·437	6·840	7·242	7·644
5	4·3421	4·5592	4·7763	4·9934	8·047	8·449	8·851	9·254
6	5·2105	5·4276	5·6447	5·8618	9·656	10·058	10·461	10·863
7	6·0789	6·2961	6·5132	6·7303	11·265	11·668	12·070	12·472
8	6·9474	7·1645	7·3816	7·5987	12·875	13·277	13·679	14·082
9	7·8158	8·0329	8·2500	8·4671	14·484	14·886	15·289	15·691
10	8·6842	8·9013	9·1184	9·3355	16·093	16·496	16·898	17·300
11	9·5526	9·7697	9·9868	10·2039	17·703	18·105	18·507	18·910
12	10·4211	10·6382	10·8553	11·0724	19·312	19·714	20·117	20·519
13	11·2895	11·5066	11·7237	11·9408	20·921	21·324	21·726	22·128
14	12·1579	12·3750	12·5921	12·8092	22·531	22·933	23·335	23·738
15	13·0263	13·2434	13·4605	13·6776	24·140	24·542	24·945	25·347
16	13·8947	14·1118	14·3289	14·5461	25·749	26·152	26·554	26·956
17	14·7632	14·9803	15·1974	15·4145	27·359	27·761	28·163	28·566
18	15·6316	15·8487	16·0658	16·2829	28·968	29·370	29·773	30·175
19	16·5000	16·7171	16·9342	17·1513	30·577	30·980	31·382	31·784
20	17·3684	17·5855	17·8026	18·0197	32·187	32·589	32·991	33·394
21	18·2368	18·4539	18·6711	18·8882	33·796	34·199	34·601	35·003
22	19·1053	19·3224	19·5395	19·7566	35·406	35·808	36·210	36·613
23	19·9737	20·1908	20·4079	20·6250	37·015	37·417	37·820	38·222
24	20·8421	21·0592	21·2763	21·4934	38·624	39·027	39·429	39·831
25	21·7105	21·9276	22·1447	22·3618	40·234	40·636	41·038	41·441
26	22·5789	22·7961	23·0132	23·2303	41·843	42·245	42·648	43·050
27	23·4474	23·6645	23·8816	24·0987	43·452	43·855	44·257	44·659
28	24·3158	24·5329	24·7500	24·9671	45·062	45·464	45·866	46·269
29	25·1842	25·4013	25·6184	25·8355	46·671	47·073	47·476	47·878
30	26·0526	26·2697	26·4868	26·7039	48·280	48·683	49·085	49·487
31	26·9211	27·1382	27·3553	27·5724	49·890	50·292	50·694	51·097
32	27·7895	28·0066	28·2237	28·4408	51·499	51·901	52·304	52·706
33	28·6579	28·8750	29·0921	29·3092	53·108	53·511	53·913	54·315
34	29·5263	29·7434	29·9605	30·1776	54·718	55·120	55·522	55·925
35	30·3947	30·6118	30·8289	31·0461	56·327	56·729	57·132	57·534

* Based on 1 nautical mile = 6080 feet.

Equivalents of Nautical Miles in Statute Miles and in Kilometres.

1 Nautical mile = 6080 feet.
1 Statute mile = 5280 feet.
1 Kilometre = 3280·8456 feet.
Statute miles = Nautical miles × 1·1515152.
Kilometres = Nautical miles × 1·8531808.
N.M. = Nautical miles.

N.M.	Fractions of a Nautical Mile.				Fractions of a Nautical Mile.			
	·0	·25	·5	·75	·0	·25	·5	·75
	Statute Miles.				Kilometres.			
1	1·1515	1·4394	1·7273	2·0152	1·853	2·316	2·780	3·243
2	2·3030	2·5909	2·8788	3·1667	3·706	4·170	4·633	5·096
3	3·4545	3·7424	4·0303	4·3182	5·560	6·023	6·486	6·949
4	4·6061	4·8939	5·1818	5·4697	7·413	7·876	8·339	8·803
5	5·7576	6·0455	6·3333	6·6212	9·266	9·729	10·192	10·656
6	6·9091	7·1970	7·4848	7·7727	11·119	11·582	12·046	12·509
7	8·0606	8·3485	8·6364	8·9242	12·972	13·436	13·899	14·362
8	9·2121	9·5000	9·7879	10·0758	14·825	15·289	15·752	16·215
9	10·3636	10·6515	10·9394	11·2273	16·679	17·142	17·605	18·069
10	11·5152	11·8030	12·0909	12·3788	18·532	18·995	19·458	19·922
11	12·6667	12·9545	13·2424	13·5303	20·385	20·848	21·312	21·775
12	13·8182	14·1061	14·3939	14·6818	22·238	22·701	23·165	23·628
13	14·9697	15·2576	15·5455	15·8333	24·091	24·555	25·018	25·481
14	16·1212	16·4091	16·6970	16·9848	25·945	26·408	26·871	27·334
15	17·2727	17·5606	17·8485	18·1364	27·798	28·261	28·724	29·188
16	18·4242	18·7121	19·0000	19·2879	29·651	30·114	30·577	31·041
17	19·5758	19·8636	20·1515	20·4394	31·504	31·967	32·431	32·894
18	20·7273	21·0152	21·3030	21·5909	33·357	33·821	34·284	34·747
19	21·8788	22·1667	22·4545	22·7424	35·210	35·674	36·137	36·600
20	23·0303	23·3182	23·6061	23·8939	37·064	37·527	37·990	38·454
21	24·1818	24·4697	24·7576	25·0455	38·917	39·380	39·843	40·307
22	25·3333	25·6212	25·9091	26·1970	40·770	41·233	41·697	42·160
23	26·4848	26·7727	27·0606	27·3485	42·623	43·086	43·550	44·013
24	27·6364	27·9242	28·2121	28·5000	44·476	44·940	45·403	45·866
25	28·7879	29·0758	29·3636	29·6515	46·330	46·793	47·256	47·719
26	29·9394	30·2273	30·5152	30·8030	48·183	48·646	49·109	49·573
27	31·0909	31·3788	31·6667	31·9545	50·036	50·499	50·962	51·426
28	32·2424	32·5303	32·8182	33·1061	51·889	52·352	52·816	53·279
29	33·3939	33·6818	33·9697	34·2576	53·742	54·206	54·669	55·132
30	34·5455	34·8333	35·1212	35·4091	55·595	56·059	56·522	56·985

Equivalents of Square Centimetres in Square Inches. Sq. inches = sq. centimetres × ·1550008(5).

Sq. Cent.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Sq. Cent.
1	·155001	·170501	·186001	·201501	·217001	·232501	·248001	·263501	·279002	·294502	1
2	·310002	·325502	·341002	·356502	·372002	·387502	·403002	·418502	·434002	·449502	2
3	·465003	·480503	·496003	·511503	·527003	·542503	·558003	·573503	·589003	·604503	3
4	·620003	·635503	·651004	·666504	·682004	·697504	·713004	·728504	·744004	·759504	4
5	·775004	·790504	·806004	·821504	·837005	·852505	·868005	·883505	·899005	·914505	5
6	·930005	·945505	·961005	·976505	·992005	1·00751	1·02301	1·03851	1·05401	1·06951	6
7	1·08501	1·10051	1·11601	1·13151	1·14701	1·16251	1·17801	1·19351	1·20901	1·22451	7
8	1·24001	1·25551	1·27101	1·28651	1·30201	1·31751	1·33301	1·34851	1·36401	1·37951	8
9	1·39501	1·41051	1·42601	1·44151	1·45701	1·47251	1·48801	1·50351	1·51901	1·53451	9
10	1·55001	1·56551	1·58101	1·59651	1·61201	1·62751	1·64301	1·65851	1·67401	1·68951	10

Equivalents of Square Inches in Square Centimetres. Sq. centimetres = sq. inches × 6·451577(6).

Sq. In.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Sq. In.
1	6·45158	7·09674	7·74189	8·38705	9·03221	9·67737	10·3225	10·9677	11·6128	12·2580	1
2	12·9032	13·5483	14·1935	14·8386	15·4838	16·1289	16·7741	17·4193	18·0644	18·7096	2
3	19·3547	19·9999	20·6450	21·2902	21·9354	22·5805	23·2257	23·8708	24·5160	25·1612	3
4	25·8063	26·4515	27·0966	27·7418	28·3869	29·0321	29·6773	30·3224	30·9676	31·6127	4
5	32·2579	32·9030	33·5482	34·1934	34·8385	35·4837	36·1288	36·7740	37·4192	38·0643	5
6	38·7095	39·3546	39·9998	40·6449	41·2901	41·9353	42·5804	43·2256	43·8707	44·5159	6
7	45·1610	45·8062	46·4514	47·0965	47·7417	48·3868	49·0320	49·6771	50·3223	50·9675	7
8	51·6126	52·2578	52·9029	53·5481	54·1933	54·8384	55·4836	56·1287	56·7739	57·4190	8
9	58·0642	58·7094	59·3545	59·9997	60·6448	61·2900	61·9351	62·5803	63·2255	63·8706	9
10	64·5158	65·1609	65·8061	66·4512	67·0964	67·7416	68·3867	69·0319	69·6770	70·3222	10

Equivalents of Square Metres in Square Feet. Square feet = square metres × 10·76394(8).

Sq. M.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Sq. M.
1	10·7639	11·8403	12·9167	13·9931	15·0695	16·1459	17·2223	18·2987	19·3751	20·4515	1
2	21·5279	22·6043	23·6807	24·7571	25·8335	26·9099	27·9863	29·0627	30·1391	31·2154	2
3	32·2918	33·3682	34·4446	35·5210	36·5974	37·6738	38·7502	39·8266	40·9030	41·9794	3
4	43·0558	44·1322	45·2086	46·2850	47·3614	48·4378	49·5142	50·5906	51·6670	52·7433	4
5	53·8197	54·8961	55·9725	57·0489	58·1253	59 2017	60·2781	61·3545	62·4309	63·5073	5
6	64·5837	65·6601	66·7365	67·8129	68·8893	69·9657	71·0421	72·1185	73·1948	74·2712	6
7	75·3476	76·4240	77·5004	78·5768	79·6532	80·7296	81·8060	82·8824	83·9588	85·0352	7
8	86·1116	87·1880	88·2644	89·3408	90·4172	91·4936	92·5700	93·6463	94·7227	95·7991	8
9	96·8755	97·9519	99·0283	100·105	101·181	102·258	103·334	104·410	105·487	106·563	9
10	107·639	108·716	109·792	110·869	111·945	113·021	114·098	115·174	116·251	117·327	10

Equivalents of Square Feet in Square Metres. Square metres = square feet × ·09290271(8).

Sq. Ft.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Sq. Ft.
1	·092903	·102193	·111483	·120774	·130064	·139354	·148644	·157935	·167225	·176515	1
2	·185805	·195096	·204386	·213676	·222967	·232257	·241547	·250837	·260128	·269418	2
3	·278708	·287998	·297289	·306579	·315869	·325160	·334450	·343740	·353030	·362321	3
4	·371611	·380901	·390191	·399482	·408772	·418062	·427353	·436643	·445933	·455223	4
5	·464514	·473804	·483094	·492384	·501675	·510965	·520255	·529545	·538836	·548126	5
6	·557416	·566707	·575997	·585287	·594577	·603868	·613158	·622448	·631738	·641029	6
7	·650319	·659609	·668900	·678190	·687480	·696770	·706061	·715351	·724641	·733931	7
8	·743222	·752512	·761802	·771093	·780383	·789673	·798963	·808254	·817544	·826834	8
9	·836124	·845415	·854705	·863995	·873286	·882576	·891866	·901156	·910447	·919737	9
10	·929027	·938317	·947608	·956898	·966188	·975479	·984769	·994059	1·00335	1·01264	10

Equivalents of Cubic Centimetres in Cubic Inches. Cubic inches = cubic centimetres × ·06102406(2).

Cubic Cent.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Cubic Cent.
1	·061024	·067126	·073229	·079331	·085434	·091536	·097638	·103741	·109843	·115946	1
2	·122048	·128151	·134253	·140355	·146458	·152560	·158663	·164765	·170867	·176970	2
3	·183072	·189175	·195277	·201379	·207482	·213584	·219687	·225789	·231891	·237994	3
4	·244096	·250199	·256301	·262403	·268506	·274608	·280711	·286813	·292915	·299018	4
5	·305120	·311223	·317325	·323428	·329530	·335632	·341735	·347837	·353940	·360042	5
6	·366144	·372247	·378349	·384452	·390554	·396656	·402759	·408861	·414964	·421066	6
7	·427168	·433271	·439373	·445476	·451578	·457680	·463783	·469885	·475988	·482090	7
8	·488192	·494295	·500397	·506500	·512602	·518705	·524807	·530909	·537012	·543114	8
9	·549217	·555319	·561421	·567524	·573626	·579729	·585831	·591933	·598036	·604138	9
10	·610241	·616343	·622445	·628548	·634650	·640753	·646855	·652957	·659060	·665162	10

Equivalents of Cubic Inches in Cubic Centimetres. Cubic centimetres = cubic inches × 16·38697(9).

Cubic Inches.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Cubic Inches.
1	16·3870	18·0257	19·6644	21·3031	22·9418	24·5805	26·2192	27·8579	29·4966	31·1353	1
2	32·7740	34·4127	36·0514	37·6901	39·3287	40·9674	42·6061	44·2448	45·8835	47·5222	2
3	49·1609	50·7996	52·4383	54·0770	55·7157	57·3544	58·9931	60·6318	62·2705	63·9092	3
4	65·5479	67·1866	68·8253	70·4640	72·1027	73·7414	75·3801	77·0188	78·6575	80·2962	4
5	81·9349	83·5736	85·2123	86·8510	88·4897	90·1284	91·7671	93·4058	95·0445	96·6832	5
6	98·3219	99·9606	101·599	103·238	104·877	106·515	108·154	109·793	111·431	113·070	6
7	114·709	116·348	117·986	119·625	121·264	122·902	124·541	126·180	127·818	129·457	7
8	131·096	132·735	134·373	136·012	137·651	139·289	140·928	142·567	144·205	145·844	8
9	147·483	149·122	150·760	152·399	154·038	155·676	157·315	158·954	160·592	162·231	9
10	163·870	165·508	167·147	168·786	170·425	172·063	173·702	175·341	176·979	178·618	10

Equivalents of Cubic Metres in Cubic Feet. Cubic feet = cubic metres × 35·31485(0).

Cubic Metres.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Cubic Metres.
1	35·3149	38·8463	42·3778	45·9093	49·4408	52·9723	56·5038	60·0352	63·5667	67·0982	1
2	70·6297	74·1612	77·6927	81·2242	84·7556	88·2871	91·8186	95·3501	98·8816	102·413	2
3	105·945	109·476	113·008	116·539	120·070	123·602	127·133	130·665	134·196	137·728	3
4	141·259	144·791	148·322	151·854	155·385	158·917	162·448	165·980	169·511	173·043	4
5	176·574	180·106	183·637	187·169	190·700	194·232	197·763	201·295	204·826	208·358	5
6	211·889	215·421	218·952	222·484	226·015	229·547	233·078	236·609	240·141	243·672	6
7	247·204	250·735	254·267	257·798	261·330	264·861	268·393	271·924	275·456	278·987	7
8	282·519	286·050	289·582	293·113	296·645	300·176	303·708	307·239	310·771	314·302	8
9	317·834	321·365	324·897	328·428	331·960	335·491	339·023	342·554	346·086	349·617	9
10	353·149	356·680	360·211	363·743	367·274	370·806	374·337	377·869	381·400	384·932	10

Equivalents of Cubic Feet in Cubic Metres. Cubic metres = cubic feet × ·02831669(9).

Cubic Feet.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Cubic Feet.
1	·028317	·031148	·033980	·036812	·039643	·042475	·045307	·048138	·050970	·053802	1
2	·056633	·059465	·062297	·065128	·067960	·070792	·073623	·076455	·079287	·082118	2
3	·084950	·087782	·090613	·093445	·096277	·099108	·101940	·104772	·107603	·110435	3
4	·113267	·116098	·118930	·121762	·124593	·127425	·130257	·133088	·135920	·138752	4
5	·141583	·144415	·147247	·150079	·152910	·155742	·158574	·161405	·164237	·167069	5
6	·169900	·172732	·175564	·178395	·181227	·184059	·186890	·189722	·192554	·195385	6
7	·198217	·201049	·203880	·206712	·209544	·212375	·215207	·218039	·220870	·223702	7
8	·226534	·229365	·232197	·235029	·237860	·240692	·243524	·246355	·249187	·252019	8
9	·254850	·257682	·260514	·263345	·266177	·269009	·271840	·274672	·277504	·280335	9
10	·283167	·285999	·288830	·291662	·294494	·297325	·300157	·302989	·305820	·308652	10

Equivalents of Litres in Imperial Gallons. Gallons = Litres × ·2199755.

Litres.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Litres.
1	·219976	·241973	·263971	·285968	·307966	·329963	·351961	·373958	·395956	·417953	1
2	·439951	·461949	·483946	·505944	·527941	·549939	·571936	·593934	·615931	·637929	2
3	·659927	·681924	·703922	·725919	·747917	·769914	·791912	·813909	·835907	·857904	3
4	·879902	·901900	·923897	·945895	·967892	·989890	1·01189	1·03388	1·05588	1·07788	4
5	1·09988	1·12188	1·14387	1·16587	1·18787	1·20987	1·23186	1·25386	1·27586	1·29786	5
6	1·31985	1·34185	1·36385	1·38585	1·40784	1·42984	1·45184	1·47384	1·49583	1·51783	6
7	1·53983	1·56183	1·58382	1·60582	1·62782	1·64982	1·67181	1·69381	1·71581	1·73781	7
8	1·75980	1·78180	1·80380	1·82580	1·84779	1·86979	1·89179	1·91379	1·93578	1·95778	8
9	1·97978	2·00178	2·02377	2·04577	2·06777	2·08977	2·11176	2·13376	2·15576	2·17776	9
10	2·19976	2·22175	2·24375	2·26575	2·28775	2·30974	2·33174	2·35374	2·37574	2·39773	10

Equivalents of Imperial Gallons in Litres. Litres = Gallons × 4·54596.

Galls.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Galls.
1	4·54596	5·00056	5·45515	5·90975	6·36434	6·81894	7·27354	7·72813	8·18273	8·63732	1
2	9·09192	9·54652	10·0011	10·4557	10·9103	11·3649	11·8195	12·2741	12·7287	13·1833	2
3	13·6379	14·0925	14·5471	15·0017	15·4563	15·9109	16·3655	16·8201	17·2746	17·7292	3
4	18·1838	18·6384	19·0930	19·5476	20·0022	20·4568	20·9114	21·3660	21·8206	22·2752	4
5	22·7298	23·1844	23·6390	24·0936	24·5482	25·0028	25·4574	25·9120	26·3666	26·8212	5
6	27·2758	27·7304	28·1850	28·6395	29·0941	29·5487	30·0033	30·4579	30·9125	31·3671	6
7	31·8217	32·2763	32·7309	33·1855	33·6401	34·0947	34·5493	35·0039	35·4585	35·9131	7
8	36·3677	36·8223	37·2769	37·7315	38·1861	38·6407	39·0953	39·5499	40·0044	40·4590	8
9	40·9136	41·3682	41·8228	42·2774	42·7320	43·1866	43·6412	44·0958	44·5504	45·0050	9
10	45·4596	45·9142	46·3688	46·8234	47·2780	47·7326	48·1872	48·6418	49·0964	49·5510	10

Equivalents of Kilogrammes in Pounds. Pounds = Kilogrammes × 2·20462275.

Kilo-grams.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Kilo-grams.
1	2·20462	2·42509	2·64555	2·86601	3·08647	3·30693	3·52740	3·74786	3·96832	4·18878	1
2	4·40925	4·62971	4·85017	5·07063	5·29109	5·51156	5·73202	5·95248	6·17294	6·39341	2
3	6·61387	6·83433	7·05479	7·27526	7·49572	7·71618	7·93664	8·15710	8·37757	8·59803	3
4	8·81849	9·03895	9·25942	9·47988	9·70034	9·92080	10·1413	10·3617	10·5822	10·8027	4
5	11·0231	11·2436	11·4640	11·6845	11·9050	12·1254	12·3459	12·5663	12·7868	13·0073	5
6	13·2277	13·4482	13·6687	13·8891	14·1096	14·3300	14·5505	14·7710	14·9914	15·2119	6
7	15·4324	15·6528	15·8733	16·0937	16·3142	16·5347	16·7551	16·9756	17·1961	17·4165	7
8	17·6370	17·8574	18·0779	18·2984	18·5188	18·7393	18·9598	19·1802	19·4007	19·6211	8
9	19·8416	20·0621	20·2825	20·5030	20·7235	20·9439	21·1644	21·3848	21·6053	21·8258	9
10	22·0462	22·2667	22·4872	22·7076	22·9281	23·1485	23·3690	23·5895	23·8099	24·0304	10

Equivalents of Pounds in Kilogrammes. Kilogrammes = Pounds × ·453592343.

Lbs.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Lbs.
1	·453592	·498952	·544311	·589670	·635029	·680389	·725748	·771107	·816466	·861825	1
2	·907185	·952544	·997903	1·04326	1·08862	1·13398	1·17934	1·22470	1·27006	1·31542	2
3	1·36078	1·40614	1·45150	1·49685	1·54221	1·58757	1·63293	1·67829	1·72365	1·76901	3
4	1·81437	1·85973	1·90509	1·95045	1·99581	2·04117	2·08652	2·13188	2·17724	2·22260	4
5	2·26796	2·31332	2·35868	2·40404	2·44940	2·49476	2·54012	2·58548	2·63084	2·67619	5
6	2·72155	2·76691	2·81227	2·85763	2·90299	2·94835	2·99371	3·03907	3·08443	3·12979	6
7	3·17515	3·22051	3·26586	3·31122	3·35658	3·40194	3·44730	3·49266	3·53802	3·58338	7
8	3·62874	3·67410	3·71946	3·76482	3·81018	3·85553	3·90089	3·94625	3·99161	4·03697	8
9	4·08233	4·12769	4·17305	4·21841	4·26377	4·30913	4·35449	4·39985	4·44520	4·49056	9
10	4·53592	4·58128	4·62664	4·67200	4·71736	4·76272	4·80808	4·85344	4·89880	4·94416	10

Equivalents of Milliers or Tonnes in Tons. Tons = Tonnes × ·98420658.

Tonnes.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Tonnes.
1	·984207	1·08263	1·18105	1·27947	1·37789	1·47631	1·57473	1·67315	1·77157	1·86999	1
2	1·96841	2·06683	2·16525	2·26368	2·36210	2·46052	2·55894	2·65736	2·75578	2·85420	2
3	2·95262	3·05104	3·14946	3·24788	3·34630	3·44472	3·54314	3·64156	3·73999	3·83841	3
4	3·93683	4·03525	4·13367	4·23209	4·33051	4·42893	4·52735	4·62577	4·72419	4·82261	4
5	4·92103	5·01945	5·11787	5·21629	5·31472	5·41314	5·51156	5·60998	5·70840	5·80682	5
6	5·90524	6·00366	6·10208	6·20050	6·29892	6·39734	6·49576	6·59418	6·69260	6·79103	6
7	6·88945	6·98787	7·08629	7·18471	7·28313	7·38155	7·47997	7·57839	7·67681	7·77523	7
8	7·87365	7·97207	8·07049	8·16891	8·26734	8·36576	8·46418	8·56260	8·66102	8·75944	8
9	8·85786	8·95628	9·05470	9·15312	9·25154	9·34996	9·44838	9·54680	9·64522	9·74365	9
10	9·84207	9·94049	10·0389	10·1373	10·2357	10·3342	10·4326	10·5310	10·6294	10·7279	10

Equivalents of Tons in Milliers or Tonnes Tonnes = Tons × 1·0160468.

Tons.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Tons.
1	1·01605	1·11765	1·21926	1·32086	1·42247	1·52407	1·62567	1·72728	1·82888	1·93049	1
2	2·03209	2·13370	2·23530	2·33691	2·43851	2·54012	2·64172	2·74333	2·84493	2·94654	2
3	3·04814	3·14975	3·25135	3·35295	3·45456	3·55616	3·65777	3·75937	3·86098	3·96258	3
4	4·06419	4·16579	4·26740	4·36900	4·47061	4·57221	4·67382	4·77542	4·87702	4·97863	4
5	5·08023	5·18184	5·28344	5·38505	5·48665	5·58826	5·68986	5·79147	5·89307	5·99468	5
6	6·09628	6·19789	6·29949	6·40109	6·50270	6·60430	6·70591	6·80751	6·90912	7·01072	6
7	7·11233	7·21393	7·31554	7·41714	7·51875	7·62035	7·72196	7·82356	7·92517	8·02677	7
8	8·12837	8·22998	8·33158	8·43319	8·53479	8·63640	8·73800	8·83961	8·94121	9·04282	8
9	9·14442	9·24603	9·34763	9·44924	9·55084	9·65244	9·75405	9·85565	9·95726	10·0589	9
10	10·1605	10·2621	10·3637	10·4653	10·5669	10·6685	10·7701	10·8717	10·9733	11·0749	10

Equivalents of Kilogrammes per Square Centimetre in Pounds per Square Inch.

Pounds per square inch = Kilogrammes per square centimetre × 14·22329(5).
K = Kilogrammes per square centimetre.

K.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	K.
1	14·2233	15·6456	17·0680	18·4903	19·9126	21·3349	22·7573	24·1796	25·6019	27·0243	1
2	28·4466	29·8689	31·2912	32·7136	34·1359	35·5582	36·9806	38·4029	39·8252	41·2476	2
3	42·6699	44·0922	45·5145	46·9369	48·3592	49·7815	51·2039	52·6262	54·0485	55·4709	3
4	56·8932	58·3155	59·7378	61·1602	62·5825	64·0048	65·4272	66·8495	68·2718	69·6941	4
5	71·1165	72·5388	73·9611	75·3835	76·8058	78·2281	79·6505	81·0728	82·4951	83·9174	5
6	85·3398	86·7621	88·1844	89·6068	91·0291	92·4514	93·8737	95·2961	96·7184	98·1407	6
7	99·5631	100·985	102·408	103·830	105·252	106·675	108·097	109·519	110·942	112·364	7
8	113·786	115·209	116·631	118·053	119·476	120·898	122·320	123·743	125·165	126·587	8
9	128·010	129·432	130·854	132·277	133·699	135·121	136·544	137·966	139·388	140·811	9
10	142·233	143·655	145·078	146·500	147·922	149·345	150·767	152·189	153·612	155·034	10
11	156·456	157·879	159·301	160·723	162·146	163·568	164·990	166·413	167·835	169·257	11
12	170·680	172·102	173·524	174·947	176·369	177·791	179·214	180·636	182·058	183·481	12
13	184·903	186·325	187·747	189·170	190·592	192·014	193·437	194·859	196·281	197·704	13
14	199·126	200·548	201·971	203·393	204·815	206·238	207·660	209·082	210·505	211·927	14
15	213·349	214·772	216·194	217·616	219·039	220·461	221·883	223·306	224·728	226·150	15
16	227·573	228·995	230·417	231·840	233·262	234·684	236·107	237·529	238·951	240 374	16
17	241·796	243·218	244·641	246·063	247·485	248·908	250·330	251·752	253·175	254·597	17
18	256·019	257·442	258·864	260·286	261·709	263·131	264·553	265·976	267·398	268·820	18
19	270·243	271·665	273·087	274·510	275·932	277·354	278·777	280·199	281·621	283·044	19
20	284·466	285·888	287·311	288·733	290·155	291·578	293·000	294·422	295·845	297·267	20
K.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	K.

Equivalents of Pounds per Square Inch in Kilogrammes per Square Centimetre.

Kilogrammes per square centimetre = Pounds per square inch × ·07030719(8).
P = pounds per square inch.

P.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	P.
1	·070307	·077338	·084369	·091399	·098430	·105461	·112492	·119522	·126553	·133584	1
2	·140614	·147645	·154676	·161707	·168737	·175768	·182799	·189829	·196860	·203891	2
3	·210922	·217952	·224983	·232014	·239044	·246075	·253106	·260137	·267167	·274198	3
4	·281229	·288260	·295290	·302321	·309352	·316382	·323413	·330444	·337475	·344505	4
5	·351536	·358567	·365597	·372628	·379659	·386690	·393720	·400751	·407782	·414812	5
6	·421843	·428874	·435905	·442935	·449966	·456997	·464028	·471058	·478089	·485120	6
7	·492150	·499181	·506212	·513243	·520273	·527304	·534335	·541365	·548396	·555427	7
8	·562458	·569488	·576519	·583550	·590580	·597611	·604642	·611673	·618703	·625734	8
9	·632765	·639796	·646826	·653857	·660888	·667918	·674949	·681980	·689011	·696041	9
10	·703072	·710103	·717133	·724164	·731195	·738226	·745256	·752287	·759318	·766348	10
11	·773379	·780410	·787441	·794471	·801502	·808533	·815563	·822594	·829625	·836656	11
12	·843686	·850717	·857748	·864779	·871809	·878840	·885871	·892901	·899932	·906963	12
13	·913994	·921024	·928055	·935086	·942116	·949147	·956178	·963209	·970239	·977270	13
14	·984301	·991331	·998362	1·00539	1·01242	1·01945	1·02649	1·03352	1·04055	1·04758	14
15	1·05461	1·06164	1·06867	1·07570	1·08273	1·08976	1·09679	1·10382	1·11085	1·11788	15
16	1·12492	1·13195	1·13898	1·14601	1·15304	1·16007	1·16710	1·17413	1·18116	1·18819	16
17	1·19522	1·20225	1·20928	1·21631	1·22335	1·23038	1·23741	1·24444	1·25147	1·25850	17
18	1·26553	1·27256	1·27959	1·28662	1·29365	1·30068	1·30771	1·31474	1·32178	1·32881	18
19	1·33584	1·34287	1·34990	1·35693	1·36396	1·37099	1·37802	1·38505	1·39208	1·39911	19
20	1·40614	1·41317	1·42021	1·42724	1·43427	1·44130	1·44833	1·45536	1·46239	1·46942	20
P.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	P.

Equivalents of Kilogrammetres in Foot-Pounds. Foot-pounds = Kilogrammetres × 7·2330268.

Kilogrammetres.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Kilogrammetres.
1	7·23303	7·95633	8·67963	9·40293	10·1262	10·8495	11·5728	12·2961	13·0194	13·7428	1
2	14·4661	15·1894	15·9127	16·6360	17·3593	18·0826	18·8059	19·5292	20·2525	20·9758	2
3	21·6991	22·4224	23·1457	23·8690	24·5923	25·3156	26·0389	26·7622	27·4855	28·2088	3
4	28·9321	29·6554	30·3787	31·1020	31·8253	32·5486	33·2719	33·9952	34·7185	35·4418	4
5	36·1651	36·8884	37·6117	38·3350	39·0583	39·7816	40·5050	41·2283	41·9516	42·6749	5
6	43·3982	44·1215	44·8448	45·5681	46·2914	47·0147	47·7380	48·4613	49·1846	49·9079	6
7	50·6312	51·3545	52·0778	52·8011	53·5244	54·2477	54·9710	55·6943	56·4176	57·1409	7
8	57·8642	58·5875	59·3108	60·0341	60·7574	61·4807	62·2040	62·9273	63·6506	64·3739	8
9	65·0972	65·8205	66·5438	67·2671	67·9905	68·7138	69·4371	70·1604	70·8837	71·6070	9
10	72·3303	73·0536	73·7769	74·5002	75·2235	75·9468	76·6701	77·3934	78·1167	78·8400	10

Equivalents of Foot-Pounds in Kilogrammetres. Kilogrammetres = Foot-pounds × ·13825471

Ft.-lbs.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Ft.-lbs.
1	·138255	·152080	·165906	·179731	·193557	·207382	·221208	·235033	·248858	·262684	1
2	·276509	·290335	·304160	·317986	·331811	·345637	·359462	·373288	·387113	·400939	2
3	·414764	·428590	·442415	·456241	·470066	·483891	·497717	·511542	·525368	·539193	3
4	·553019	·566844	·580670	·594495	·608321	·622146	·635972	·649797	·663623	·677448	4
5	·691274	·705099	·718924	·732750	·746575	·760401	·774226	·788052	·801877	·815703	5
6	·829528	·843354	·857179	·871005	·884830	·898656	·912481	·926307	·940132	·953957	6
7	·967783	·981608	·995434	1·00926	1·02308	1·03691	1·05074	1·06456	1·07839	1·09221	7
8	1·10604	1·11986	1·13369	1·14751	1·16134	1·17517	1·18899	1·20282	1·21664	1·23047	8
9	1·24429	1·25812	1·27194	1·28577	1·29959	1·31342	1·32725	1·34107	1·35490	1·36872	9
10	1·38255	1·39637	1·41020	1·42402	1·43785	1·45167	1·46550	1·47933	1·49315	1·50698	10

Equivalents of Force de Cheval in Horse-power. Horse-power = Force de cheval × ·98632184.

Force de Cheval.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Force de Cheval.
1	·986322	1·08495	1·18359	1·28222	1·38085	1·47948	1·57811	1·67675	1·77538	1·87401	1
2	1·97264	2·07128	2·16991	2·26854	2·36717	2·46580	2·56444	2·66307	2·76170	2·86033	2
3	2·95897	3·05760	3·15623	3·25486	3·35349	3·45213	3·55076	3·64939	3·74802	3·84666	3
4	3·94529	4·04392	4·14255	4·24118	4·33982	4·43845	4·53708	4·63571	4·73434	4·83298	4
5	4·93161	5·03024	5·12887	5·22751	5·32614	5·42477	5·52340	5·62203	5·72067	5·81930	5
6	5·91793	6·01656	6·11520	6·21383	6·31246	6·41109	6·50972	6·60836	6·70699	6·80562	6
7	6·90425	7·00289	7·10152	7·20015	7·29878	7·39741	7·49605	7·59468	7·69331	7·79194	7
8	7·89057	7·98921	8·08784	8·18647	8·28510	8·38374	8·48237	8·58100	8·67963	8·77826	8
9	8·87690	8·97553	9·07416	9·17279	9·27143	9·37006	9·46869	9·56732	9·66595	9·76459	9
10	9·86322	9·96185	10·0605	10·1591	10·2577	10·3564	10·4550	10·5536	10·6523	10·7509	10

Equivalents of Horse-power in Force de Cheval. Force de cheval = Horse-power × 1·0138678.

Horse-power.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Horse-power.
1	1·01387	1·11525	1·21664	1·31803	1·41941	1·52080	1·62219	1·72358	1·82496	1·92635	1
2	2·02774	2·12912	2·23051	2·33190	2·43328	2·53467	2·63606	2·73744	2·83883	2·94022	2
3	3·04160	3·14299	3·24438	3·34576	3·44715	3·54854	3·64992	3·75131	3·85270	3·95408	3
4	4·05547	4·15686	4·25824	4·35963	4·46102	4·56241	4·66379	4·76518	4·86657	4·96795	4
5	5·06934	5·17073	5·27211	5·37350	5·47489	5·57627	5·67766	5·77905	5·88043	5·98182	5
6	6·08321	6·18459	6·28598	6·38737	6·48875	6·59014	6·69153	6·79291	6·89430	6·99569	6
7	7·09707	7·19846	7·29985	7·40123	7·50262	7·60401	7·70540	7·80678	7·90817	8·00956	7
8	8·11094	8·21233	8·31372	8·41510	8·51649	8·61788	8·71926	8·82065	8·92204	9·02342	8
9	9·12481	9·22620	9·32758	9·42897	9·53036	9·63174	9·73313	9·83452	9·93590	10·0373	9
10	10·1387	10·2401	10·3415	10·4428	10·5442	10·6456	10·7470	10·8484	10·9498	11·0512	10

The C. G. S. System of Units.

This is the system of units recommended, for scientific purposes, by a committee of the British Association. The *centimetre* is the unit of *length*, the *gramme* is the unit of *mass*, and the *second* is the unit of *time*.

The unit of *area* is the *square centimetre*.

The unit of *volume* is the *cubic centimetre*.

The unit of *velocity* is a velocity of a *centimetre per second*.

The unit of *momentum* is the momentum of a *gramme* moving with a velocity of a *centimetre per second*.

The unit of *force* is that force which generates a *unit of momentum in a second*, and is therefore that force which, acting on a gramme for one second, generates a velocity of a centimetre per second. This unit of force is called the *dyne*.

The unit of *work* is the work done by a force of a *dyne* acting through a distance of a *centimetre*. This unit of work is called the *erg*.

Equivalents of ordinary British and C. G. S. Units.

1 foot = 30·47995 centimetres.
1 centimetre = ·03280846 foot.

1 square inch = 6·451578 square centimetres.
1 square foot = 929·0272 square centimetres.
1 square centimetre = ·1550008 sq. inch = ·00107639 sq. foot.

1 cubic inch = 16·386979 cubic centimetres.
1 cubic foot = 28316·7 cubic centimetres.
1 cubic centimetre = ·061024 cubic inch = ·00003531 cubic foot.

1 lb. avoirdupois = 453·59234 grammes.
1 gramme = ·00220462 lb. avoirdupois.

1 foot per second = 30·47995 centimetres per second.
1 mile per hour = 44·7039 centimetres per second.
1 centimetre per second = ·03280846 foot per second = ·02237 mile per hour.

1 lb. per cubic foot = ·016019 grammes per cubic centimetre.
1 gramme per cubic centimetre = 62·4276 lbs. per cubic foot.

Accelerating effect of gravity = 32·2 feet per sec. per sec. = 981 centimetres per sec. per sec., approximately.

In the equivalents below g is taken = 981 cm. per sec. per sec.

1 lb. avoirdupois = 444974 dynes.
1 gramme = 981 dynes.

1 foot-pound = 13562790 ergs.
1 kilogrammetre = 98100000 ergs.

1 lb. per square inch = 68971 dynes per square centimetre.
1 lb. per square foot = 478·97 dynes per square centimetre.
1 kilogramme per square centimetre = 981000 dynes per square centimetre.

Imperial or Legal Standard Wire Gauge.

Descriptive Number.	Equivalent in Parts of an Inch.	Equivalent in Millimetres.	Sectional Area of Wire in Sq. In.	Descriptive Number.	Equivalent in Parts of an Inch.	Equivalent in Millimetres.	Sectional Area of Wire in Sq. In.
7/0	·500	12·700	·196350	23	·024	·610	·00045239
6/0	·464	11·785	·169093	24	·022	·559	·00038013
5/0	·432	10·973	·146574	25	·020	·508	·00031416
4/0	·400	10·160	·125664	26	·018	·457	·00025447
3/0	·372	9·449	·108687	27	·0164	·4166	·00021124
2/0	·348	8·839	·095115	28	·0148	·3759	·00017203
0	·324	8·229	·082448	29	·0136	·3454	·00014527
1	·300	7·620	·070686	30	·0124	·3150	·00012076
2	·276	7·010	·059828	31	·0116	·2946	·00010568
3	·252	6·401	·049876	32	·0108	·2743	·00009161
4	·232	5·893	·042273	33	·0100	·2540	·00007854
5	·212	5·385	·035299	34	·0092	·2337	·00006648
6	·192	4·877	·028953	35	·0084	·2134	·00005542
7	·176	4·470	·024328	36	·0076	·1930	·00004536
8	·160	4·064	·020106	37	·0068	·1727	·00003632
9	·144	3·658	·016286	38	·0060	·1524	·00002827
10	·128	3·251	·012868	39	·0052	·1321	·00002124
11	·116	2·946	·010568	40	·0048	·1219	·00001810
12	·104	2·642	·008495	41	·0044	·1118	·00001521
13	·092	2·337	·006648	42	·0040	·1016	·00001257
14	·080	2·032	·005027	43	·0036	·0914	·00001018
15	·072	1·829	·004072	44	·0032	·0813	·00000804
16	·064	1·626	·003217	45	·0028	·0711	·00000616
17	·056	1·422	·002463	46	·0024	·0610	·00000452
18	·048	1·219	·001810	47	·0020	·0508	·00000314
19	·040	1·016	·001257	48	·0016	·0406	·00000201
20	·036	·914	·001018	49	·0012	·0305	·00000113
21	·032	·813	·000804	50	·0010	·0254	·00000079
22	·028	·711	·000616				

Birmingham Gauge.*

Descriptive Number.	Equiv. in Parts of an In.	Descriptive Number.	Equiv. in Parts of an In.	Descriptive Number.	Equiv. in Parts of an In.	Descriptive Number.	Equiv. in Parts of an In.
15/0 B.G.	1·000	3 B.G.	·2804	20 B.G.	·0392	37 B.G.	·0054
14/0 B.G.	0·9583	4 B.G.	·250	21 B.G.	·0349	38 B.G.	·0048
13/0 B.G.	·9167	5 B.G.	·2225	22 B.G.	·03125	39 B.G.	·0043
12/0 B.G.	·8750	6 B.G.	·1981	23 B.G.	·02782	40 B.G.	·00386
11/0 B.G.	·8333	7 B.G.	·1764	24 B.G.	·02476	41 B.G.	·00343
10/0 B.G.	·7917	8 B.G.	·1570	25 B.G.	·02204	42 B.G.	·00306
9/0 B.G.	·750	9 B.G.	·1398	26 B.G.	·01961	43 B.G.	·00272
8/0 B.G.	·7083	10 B.G.	·1250	27 B.G.	·01745	44 B.G.	·00242
7/0 B.G.	·6666	11 B.G.	·1113	28 B.G.	·015625	45 B.G.	·00215
6/0 B.G.	·625	12 B.G.	·0991	29 B.G.	·0139	46 B.G.	·00192
5/0 B.G.	·5883	13 B.G.	·0882	30 B.G.	·0123	47 B.G.	·00170
4/0 B.G.	·5416	14 B.G.	·0785	31 B.G.	·0110	48 B.G.	·00152
3/0 B.G.	·500	15 B.G.	·0699	32 B.G.	·0098	49 B.G.	·00135
2/0 B.G.	·4452	16 B.G.	·0625	33 B.G.	·0087	50 B.G.	·00120
1/0 B.G.	·3964	17 B.G.	·0556	34 B.G.	·0077	51 B.G.	·00107
1 B.G.	·3532	18 B.G.	·0495	35 B.G.	·0069	52 B.G.	·00095
2 B.G.	·3147	19 B.G.	·0440	36 B.G.	·0061		

* Board of Trade Standards, S.R. and O., 1914, No. 1095.

Birmingham Wire Gauge.

Mark or No.	Size. Inch.	Mark or No.	Size. Inch.	Mark or No.	Size. Inch.	Mark or No.	Size. Inch.
4/0	·454	7	·18	17	·058	27	·016
3/0	·425	8	·165	18	·049	28	·014
2/0	·38	9	·148	19	·042	29	·013
0	·34	10	·134	20	·035	30	·012
1	·3	11	·12	21	·032	31	·01
2	·284	12	·109	22	·028	32	·009
3	·259	13	·095	23	·025	33	·008
4	·238	14	·083	24	·022	34	·007
5	·22	15	·072	25	·02	35	·005
6	·203	16	·065	26	·018	36	·004

American Standard Wire Gauge (Brown and Sharpe).

Mark or No.	Size. Inch.	Mark or No.	Size. Inch.	Mark or No.	Size. Inch.	Mark or No.	Size. Inch.
4/0	·46	8	·12849	19	·03589	30	·010025
3/0	·40964	9	·11443	20	·031961	31	·008928
2/0	·3648	10	·10189	21	·028462	32	·00795
0	·32486	11	·090742	22	·025347	33	·00708
1	·2893	12	·080808	23	·022571	34	·006304
2	·25763	13	·071961	24	·0201	35	·005614
3	·22942	14	·064084	25	·0179	36	·005
4	·20431	15	·057068	26	·01594	37	·004453
5	·18194	16	·05082	27	·014195	38	·003965
6	·16202	17	·045257	28	·012641	39	·003531
7	·14428	18	·040303	29	·011257	40	·003144

ARITHMETICAL AND ALGEBRAICAL SIGNS.

$+$ (plus) is the sign of addition. $+$ placed between two quantities denotes that they are to be added together. $+$ placed in front of a quantity denotes that it is a *positive* quantity as distinguished from a *negative* quantity.

$-$ (minus) is the sign of subtraction. $-$ placed between two quantities denotes that the second is to be subtracted from the first. $-$ placed in front of a quantity denotes that it is a *negative* quantity as distinguished from a *positive* quantity.

A positive quantity may be considered as a quantity greater than nothing, and a negative quantity as a quantity less than nothing. Thus a man who has no debts and whose assets are valued at £100 has his wealth represented by + £100, while a man who owes £100 and has no assets has his wealth represented by - £100. The plus and minus signs are also used to show the direction in which a distance is measured along a line. If distances in one direction are positive (+), then distances measured in the opposite direction are negative (−). If + denotes compression, then − denotes tension.

$\sim$ placed between two quantities denotes that the smaller of the two is to be taken from the greater.

$=$ placed between two quantities or expressions denotes that they are equal to one another.

$>$ placed between two quantities denotes that the first is greater than the second.

$<$ placed between two quantities denotes that the first is less than the second.

$\times$ placed between two quantities denotes that they are to be multiplied together. In algebra $a.b = a \times b = ab$.

$\div$ placed between two quantities denotes that the first is to be divided by the second. Division is also indicated by a single line between the quantities, thus $\frac{12}{3} = {}^{12}/_{3} = 4$. In algebra $\frac{a}{b} = a/b = a \div b$.

$^\circ/_\circ$ means per cent., thus 21 $^\circ/_\circ$ means 21 per cent., or 21 in 100, or $\frac{21}{100}$.

In proportion $a : b :: c : d$ is read a is to b as c is to d, or the ratio of a to b is equal to the ratio of c to d, or $\frac{a}{b} = \frac{c}{d}$.

$3^2 = 3 \times 3 = 3$ squared, or 3 to the second power, or the second power of 3.

$3^3 = 3 \times 3 \times 3 = 3$ cubed, or 3 to the third power, or the third power of 3.

$3^n = n$ threes multiplied together, or 3 to the n^{th} power (n is here called the index of the power).

$\sqrt{9}$ = the square root of $9 = 3$. The square root of a quantity is another quantity whose second power equals the first quantity.

$\sqrt[3]{8}$ = the cube root of $8 = 2$. The cube root of a quantity is another quantity whose third power equals the first quantity.

$\sqrt[n]{a}$ = the n^{th} root of a. The n^{th} root of a quantity is another quantity whose n^{th} power equals the first quantity.

∞ (infinity) is used to denote a quantity which is so great that it cannot be represented by any number however large.

( ), { }, [ ] (brackets) are used to denote that the quantities between them are to be treated as one quantity. The vinculum ——— placed over two or more quantities is used for the same purpose as brackets. The following example illustrates the use of brackets and vincula:—

$$3[2+5\{23-2(2+8-\overline{6-4})\}+3\times\overline{7-2}]$$
$$=3[2+5\{23-2(2+8-2)\}+3\times 5]$$
$$=3[2+5\{23-2\times 8\}+15]=3[2+5\times 7+15]=3\times 52=156.$$

ALGEBRAICAL FORMULÆ.

Factors.

$x^2+(a+b)x+ab=(x+a)(x+b)$.

$x^2+(a-b)x-ab=(x+a)(x-b)$.

$a^2+2ab+b^2=(a+b)^2$. $\quad a^2-2ab+b^2=(a-b)^2$.

$a^2-b^2=(a+b)(a-b)$.

$a^3+b^3=(a+b)(a^2-ab+b^2)$.

$a^3-b^3=(a-b)(a^2+ab+b^2)$.

$a^n+b^n=(a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-\ldots\ldots+b^{n-1})$ where n is an odd number.

$a^n-b^n=(a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-\ldots\ldots-b^{n-1})$ where n is an even number.

$a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\ldots\ldots+b^{n-1})$ where n is either an odd or an even number.

$a^4+a^2b^2+b^4=(a^2+ab+b^2)(a^2-ab+b^2)$.

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$.

Ratio and Proportion.

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}$, then each of these ratios is equal to

$$\sqrt[n]{\left(\frac{pa^n+qc^n+re^n+sg^n}{pb^n+qd^n+rf^n+sh^n}\right)}.$$

If $p=q=r=s=n=1$, then $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\frac{g}{h}=\frac{a+c+e+g}{b+d+f+h}$.

If $a : b :: c : d$ or $\frac{a}{b}=\frac{c}{d}$, then,

(1.) $ad=bc$. (2.) $\frac{b}{a}=\frac{d}{c}$. (3.) $\frac{a}{c}=\frac{b}{d}$.

(4.) $\frac{a+b}{b}=\frac{c+d}{d}$. (5.) $\frac{a-b}{b}=\frac{c-d}{d}$. (6.) $\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

Indices.

$a^m \times a^n = a^{m+n}$. $\frac{a^m}{a^n}=a^{m-n}$. $a^{\frac{1}{n}}=\sqrt[n]{a}$.

$a^{\frac{m}{n}}=\sqrt[n]{a^m}$. $a^{-n}=\frac{1}{a^n}$. $\frac{1}{a^{-n}}=a^n$. $a^0=1$.

Quadratic Equations.—If $x^2+ax+b=o$, then $x=-\frac{a}{2}\pm\frac{\sqrt{a^2-4b}}{2}$.

The roots of an equation are the values of x which satisfy the equation.

If α and β are the roots of the equation $x^2+ax+b=o$, then $\alpha+\beta=-a$, and $\alpha\beta=b$.

Cubic Equations.—If $x^3+ax+b=o$, then *Cardan's solution* gives,

$$x=\left\{-\frac{b}{2}+\sqrt{\frac{a^3}{27}+\frac{b^2}{4}}\right\}^{\frac{1}{3}}+\left\{\frac{b}{2}-\sqrt{\frac{a^3}{27}+\frac{b^2}{4}}\right\}^{\frac{1}{3}}.$$

The equation $x^3+px^2+qx+r=o$ may be reduced to the form $x^3+ax+b=o$ by substituting $x-\frac{p}{3}$ for x in the given equation.

Arithmetical Progression.—Quantities are said to be in *arithmetical progression* when they increase or decrease by a common difference.

The common difference is found by subtracting any term of the series from the one which follows it.

EXAMPLES.—2, 4, 6, 8, &c., common difference $=2$.
13, 10, 7, 4, &c., " " $=-3$.
$a, (a+b), (a+2b), (a+3b)$, etc., " $=b$.

Let $a=1^{st}$ term, and $b=$common difference.

The r^{th} term from the beginning $=a+(r-1)b$.

Sum of n terms $=\frac{n}{2}\{2a+(n-1)b\}$.

If M, A, and N are in arithmetical progression, then $A=\frac{M+N}{2}$, and A is the *arithmetical mean* of M and N.

Geometrical Progression.—Quantities are said to be in *geometrical progression* when each is equal to the product of the preceding and some constant factor. The constant factor is called the common ratio of the series.

EXAMPLES.—2, 4, 8, 16, etc., common ratio $=2$.
27, 9, 3, 1, etc., " " $=\frac{1}{3}$.
a, ar, ar^2, ar^3, etc., " " $=r$.

Let $a=1^{st}$ term, and $r=$common ratio.
The n^{th} term from the beginning $=ar^{n-1}$.

Sum of n terms $=\frac{a(r^n-1)}{r-1}=\frac{a(1-r^n)}{1-r}$.

If r is less than 1, the sum of an infinite number of terms (called the *sum to infinity*) $=\frac{a}{1-r}$.

If M, G, and N are in geometrical progression, $G=\sqrt{MN}$, and G is the *geometrical mean* of M and N.

Harmonical Progression.—Three quantities M, H, and N are said to be in *harmonical progression* when $M:N::M-H:H-N$.

The reciprocals of quantities which are in harmonical progression are in arithmetical progression.

If M, H, and N are in harmonical progression, then $H=\frac{2MN}{M+N}$, and H is the *harmonical mean* of M and N.

Miscellaneous Series.—In the following formulæ S_n denotes the sum of n terms of the series, and S_∞ the sum to infinity.

$$S_n=1+2+3+\ldots\ldots+n=\frac{n}{2}(n+1).$$

$$S_n=1^2+2^2+3^2+\ldots\ldots+n^2=\frac{n(n+1)(2n+1)}{6}.$$

$$S_n=1^3+2^3+3^3+\ldots\ldots+n^3=\left\{\frac{n(n+1)}{2}\right\}^2.$$

$$S_n=(1\times2)+(2\times3)+(3\times4)+\ldots\ldots+n(n+1)=\frac{1}{3}n(n+1)(n+2).$$

$$S_n=(1\times2\times3)+(2\times3\times4)+(3\times4\times5)+\ldots\ldots+n(n+1)(n+2)$$
$$=\frac{1}{4}n(n+1)(n+2)(n+3).$$

$$S_n=\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots+\frac{1}{n(n+1)}=1-\frac{1}{n+1}.$$

$$S_\infty=\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots=1.$$

$$S_n=\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots\ldots+\frac{1}{n(n+1)(n+2)}$$
$$=\frac{1}{4}-\frac{1}{2(n+1)(n+2)}.$$

$$S_\infty=\frac{1}{1\times2\times3}+\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\ldots\ldots=\frac{1}{4}.$$

$$S_n = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots + nx^{n-1}$$
$$= \frac{1 - x^n}{(1-x)^2} - \frac{nx^n}{1-x}.$$

Permutations and Combinations.—"Each of the *arrangements* which can be made by taking some or all of a number of things is called a *permutation*."

Thus the permutations which can be made with the digits 1, 2, and 3, taking them two at a time, are 12, 13, 21, 31, 23, and 32.

"Each of the *groups* or *selections* which can be made by taking some or all of a number of things is called a *combination*."

Thus the combinations which can be made with the digits 1, 2, and 3, taking them two at a time, are 12, 23, and 31.

12 and 21 are different permutations, but one combination of the digits 1 and 2.

The number of permutations of n different things taken r at a time is $n(n-1)(n-2) \ldots (n-r+1)$.

The number of permutations of n different things taken n at a time is $n(n-1)(n-2) \ldots\ldots 1$, or $1 \times 2 \times 3 \ldots\ldots n$.

The expression $n(n-1)(n-2) \ldots\ldots 1$ is denoted by $\underline{|n}$, which is called "factorial n."

The number of combinations of n different things taken r at a time is $\dfrac{n(n-1)(n-2) \ldots\ldots (n-r+1)}{\underline{|r}}$ or $\dfrac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}$.

Binomial Theorem.—$(a+x)^n =$

$$a^n + na^{n-1}x + \frac{n(n-1)}{1 \cdot 2}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}a^{n-3}x^3 + \ldots\ldots + x^n.$$

EXAMPLE.—$(a+x)^5 =$

$$a^5 + 5a^4x + \frac{5 \cdot 4}{1 \cdot 2}a^3x^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3}a^2x^3 + \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4}ax^4 + x^5$$
$$= a^5 + 5a^4x + 10a^3x^2 + 10a^2x^3 + 5ax^4 + x^5.$$

The $(r+1)^{\text{th}}$ term of $(a+x)^n = \dfrac{n(n-1)(n-2) \ldots\ldots (n-r+1)}{\underline{|r}}a^{n-r}x^r$.

Exponential and Logarithmic Series.

$$a^x = 1 + Ax + \frac{A^2x^2}{\underline{|2}} + \frac{A^3x^3}{\underline{|3}} + \frac{A^4x^4}{\underline{|4}} + \ldots\ldots$$

where $A = \log_e a$.

Put $a = e$, then since $\log_e e = 1$,

$$e^x = 1 + x + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \frac{x^4}{\underline{|4}} + \ldots\ldots$$

$$e = 1 + 1 + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \frac{1}{\underline{|4}} + \ldots\ldots$$

e is the base of the Napierian system of logarithms.

$e = 2{\cdot}7182818284 \ldots\ldots$

$$\frac{1}{e} = e^{-1} = \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} - \frac{1}{\lfloor 5} + \ldots\ldots$$

$$\log_e (1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots$$

$$\log_e m = 2\left\{\frac{m-1}{m+1} + \frac{1}{3}\left(\frac{m-1}{m+1}\right)^3 + \frac{1}{5}\left(\frac{m-1}{m+1}\right)^5 + \ldots\ldots\right\}$$

$$\log_e (n+1) - \log_e n = 2\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\ldots\right\}$$

$$\log_{10} (n+1) - \log_{10} n = 2\mu\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots\ldots\right\}$$

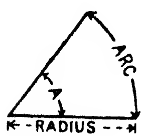
$$\mu = \frac{1}{\log_e 10} = {\cdot}43429448 \ldots\ldots$$

TRIGONOMETRY.

Measurement of Angles.

In *Sexagesimal Measure*, one right angle = 90 degrees, one degree = 60 minutes, and one minute = 60 seconds.

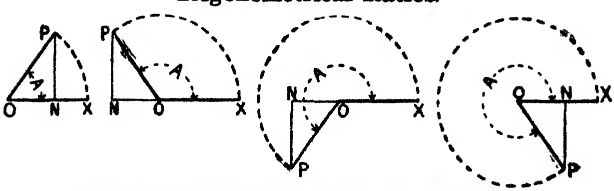
In *Radian* or *Circular Measure*, an angle is measured by the ratio, $\frac{\text{arc}}{\text{radius}}$.



n = number of degrees in angle A.
θ = circular measure of angle A.
π = ratio of half the circumference of a circle to its radius.
= circular measure of an angle of 180 degrees
= $3{\cdot}1416$ nearly.
(For functions of π, see pp. 139 and 140.)

$$\frac{n}{180} = \frac{\theta}{\pi}. \quad n = \frac{180\theta}{\pi} = 57{\cdot}2958\theta. \quad \theta = \frac{\pi n}{180} = {\cdot}017453n.$$

Trigonometrical Ratios.



Angle *A* is contained by the lines *OP* and *OX*.

PN is perpendicular to OX.
PN is positive (+) when it is above OX.
PN is negative (−) when it is below OX.
ON is positive (+) when it is to the right of O.
ON is negative (−) when it is to the left of O.
OP is always positive (+).

sine $A = \sin A = \frac{PN}{OP}$. cosine $A = \cos A = \frac{ON}{OP}$.

cosecant $A = \operatorname{cosec} A = \frac{OP}{PN}$. secant $A = \sec A = \frac{OP}{ON}$.

tangent $A = \tan A = \frac{PN}{ON}$ cotangent $A = \cot A = \frac{ON}{PN}$.

versed sine $A = \operatorname{vers} A = 1 - \cos A$.
coversed sine $A = \operatorname{covers} A = 1 - \sin A$.

Trigonometrical Formulæ.

$\operatorname{cosec} A = \frac{1}{\sin A}$. $\sec A = \frac{1}{\cos A}$.

$\tan A = \frac{\sin A}{\cos A} = \frac{1}{\cot A}$. $\cot A = \frac{\cos A}{\sin A} = \frac{1}{\tan A}$.

$$\sin^2 A + \cos^2 A = 1.$$

$\sec^2 A = 1 + \tan^2 A$. $\operatorname{cosec}^2 A = 1 + \cot^2 A$.

$\sin(360° - A) = -\sin A$. $\cos(360° - A) = \cos A$.
$\tan(360° - A) = -\tan A$. $\cot(360° - A) = -\cot A$.
$\sin(180° - A) = \sin A$. $\cos(180° - A) = -\cos A$.
$\tan(180° - A) = -\tan A$. $\cot(180° - A) = -\cot A$.
$\sin(180° + A) = -\sin A$. $\cos(180° + A) = -\cos A$.
$\tan(180° + A) = \tan A$. $\cot(180° + A) = \cot A$.
$\sin(90° - A) = \cos A$. $\cos(90° - A) = \sin A$.
$\tan(90° - A) = \cot A$. $\cot(90° - A) = \tan A$.
$\sin(90° + A) = \cos A$. $\cos(90° + A) = -\sin A$.
$\tan(90° + A) = -\cot A$. $\cot(90° + A) = -\tan A$.

$\sin(A + B) = \sin A \cos B + \cos A \sin B$.
$\sin(A - B) = \sin A \cos B - \cos A \sin B$.
$\cos(A + B) = \cos A \cos B - \sin A \sin B$.
$\cos(A - B) = \cos A \cos B + \sin A \sin B$.

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

$$\sin 2A = 2 \sin A \cos A.$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A.$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.$$
$$\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}. \qquad \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}.$$

$$\sin 3A = 3 \sin A - 4 \sin^3 A.$$
$$\cos 3A = 4 \cos^3 A - 3 \cos A.$$
$$\tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}.$$

$$\sin (A + B) \sin (A - B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A.$$
$$\cos (A + B) \cos (A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A.$$

$$\sin \frac{A}{2} = \pm \sqrt{\tfrac{1}{2}(1 - \cos A)}.$$
$$\cos \frac{A}{2} = \pm \sqrt{\tfrac{1}{2}(1 + \cos A)}.$$
$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}.$$

$$\sin \frac{A}{2} + \cos \frac{A}{2} = \pm \sqrt{1 + \sin A}.$$
$$\sin \frac{A}{2} - \cos \frac{A}{2} = \pm \sqrt{1 - \sin A}.$$

$$2 \sin A \cos B = \sin (A + B) + \sin (A - B).$$
$$2 \cos A \sin B = \sin (A + B) - \sin (A - B).$$
$$2 \cos A \cos B = \cos (A + B) + \cos (A - B).$$
$$-2 \sin A \sin B = \cos (A + B) - \cos (A - B).$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}.$$
$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}.$$
$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}.$$
$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}.$$

a, b, and c are the sides of a triangle, and A, B, and C are the opposite angles.

$$a+b+c=2s \qquad A+B+C=180^\circ.$$

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}. \qquad a=b\cos C+c\cos B.$$

$$a^2=b^2+c^2-2bc\cos A. \qquad \cos A=\frac{b^2+c^2-a^2}{2bc}.$$

$$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}.$$

$$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}. \qquad \tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

$$\sin A=\frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}.$$

$$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}.$$

Area of triangle $\Delta=\frac{1}{2}bc\sin A=\sqrt{s(s-a)(s-b)(s-c)}$.

R = radius of the circumscribing circle of a triangle.
r = radius of the inscribed circle.
r_1 = radius of the escribed circle which touches the side a and the sides b and c produced.
r_2 = radius of the escribed circle which touches the side b and the sides c and a produced
r_3 = radius of the escribed circle which touches the side c and the sides a and b produced.

$$R=\frac{a}{2\sin A}=\frac{abc}{4\Delta}. \qquad r=\frac{2\Delta}{a+b+c}=\frac{\Delta}{s}=(s-a)\tan\frac{A}{2}.$$

$$r_1=\frac{\Delta}{s-a}. \qquad r_2=\frac{\Delta}{s-b} \qquad r_3=\frac{\Delta}{s-c}.$$

$$r_1=s\tan\frac{A}{2}. \qquad r_2=s\tan\frac{B}{2}. \qquad r_3=s\tan\frac{C}{2}.$$

l = length of each side of a regular polygon.
n = number of sides.
R = radius of the circumscribing circle.
r = radius of the inscribed circle.

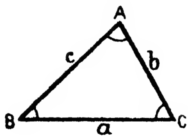
Perimeter of polygon $=nl=2nr\tan\frac{180^\circ}{n}=2nR\sin\frac{180^\circ}{n}$.

Area of polygon $=\frac{1}{4}nl^2\cot\frac{180^\circ}{n}=nr^2\tan\frac{180^\circ}{n}=\frac{1}{2}nR^2\sin\frac{360^\circ}{n}$.

$$l=2r\tan\frac{180^\circ}{n}=2R\sin\frac{180^\circ}{n}.$$

Solution of Triangles

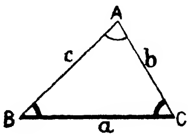
In the figures the given parts are shown by thick lines.



The three sides given

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

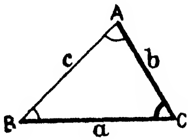
or $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$. Similarly B is determined, and $C = 180° - (A+B)$



Two angles and a side given.

$A = 180° - (B + C)$.

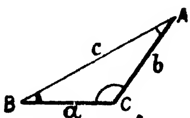
$$b = \frac{a \sin B}{\sin A}. \qquad c = \frac{a \sin C}{\sin A}.$$

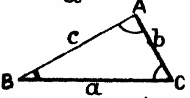


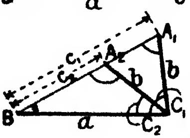
Two sides and the included angle given.

$$\tan \tfrac{1}{2}(A - B) = \frac{a-b}{a+b} \cot \tfrac{1}{2}C.$$

$\frac{1}{2}(A+B) = 90° - \frac{1}{2}C$. From the values of $\frac{1}{2}(A-B)$ and $\frac{1}{2}(A+B)$, A and B are determined, $c = \dfrac{a \sin C}{\sin A}$.







Two sides and the angle opposite to one of them given.

CASE I b not less than a. One solution only. $\sin A = \dfrac{a \sin B}{b}$.

$C = 180° - (A + B)$ $\quad c = \dfrac{a \sin C}{\sin A}$.

CASE II. $\dfrac{a \sin B}{b} = 1$. One solution only.

$A = 90°$. $C = 90° - B$. $c = a \sin C$.

CASE III b less than a, and $\dfrac{a}{b} \sin B$ less than 1. Two solutions.

$\sin A_1 = \dfrac{a \sin B}{b}$. $\quad A_2 = 180° - A_1$.

$C_1 = 180° - (A_1 + B)$. $\quad C_2 = 180° - (A_2 + B)$.

$c_1 = \dfrac{a \sin C_1}{\sin A_1}$. $\quad c_2 = \dfrac{a \sin C_2}{\sin A_2}$.

Sines, Cosines, Tangents, and Cotangents.

Angle in Degrees	Sine.	Tangent		Angle in Degrees	Sine.	Tangent.	
0	·00000	·00000	90	20	·34202	·36397	70
0·5	·00873	·00873	89·5	20 5	·35021	·37388	69·5
1	·01745	·01746	89	21	·35837	·38386	69
1·5	·02618	·02619	88·5	21·5	·36650	·39391	68·5
2	·03490	·03492	88	22	·37461	·40403	68
2·5	·04362	·04366	87·5	22·5	·38268	41421	67·5
3	·05234	·05241	87	23	·39073	·42447	67
3·5	·06105	·06116	86·5	23 5	·39875	·43481	66·5
4	·06976	·06993	86	24	·40674	·44523	66
4·5	·07846	·07870	85·5	24·5	·41469	·45573	65·5
5	·08716	·08749	85	25	·42262	·46631	65
5·5	·09585	·09629	84·5	25·5	·43051	·47698	64·5
6	·10453	·10510	84	26	·43837	·48773	64
6·5	·11320	·11394	83·5	26·5	·44620	·49858	63·5
7	·12187	·12278	83	27	·45399	·50953	63
7·5	·13053	·13165	82·5	27·5	·46175	·52057	62·5
8	·13917	·14054	82	28	·46947	·53171	62
8·5	·14781	·14945	81·5	28·5	·47716	·54296	61·5
9	·15643	·15838	81	29	·48481	·55431	61
9·5	·16505	·16734	80 5	29·5	·49242	·56577	60·5
10	·17365	·17633	80	30	·50000	·57735	60
10 5	18224	·18534	79·5	30·5	·50754	·58905	59·5
11	·19081	·19438	79	31	·51504	·60086	59
11·5	·19937	·20345	78·5	31·5	·52250	·61280	58·5
12	·20791	·21256	78	32	·52992	·62487	58
12·5	·21644	·22169	77·5	32·5	·53730	·63707	57·5
13	·22495	·23087	77	33	54464	·64941	57
13·5	·23345	·24008	76·5	33·5	·55194	·66189	56·5
14	·24192	·24933	76	34	·55919	·67451	56
14·5	·25038	·25862	75·5	34·5	·56641	·68728	55·5
15	·25882	·26795	75	35	·57358	·70021	55
15·5	·26724	·27732	74 5	35·5	·58070	·71329	54·5
16	·27564	·28675	74	36	·58779	·72654	54
16·5	·28402	·29621	73·5	36·5	·59482	·73996	53·5
17	·29237	·30573	73	37	·60182	·75355	53
17·5	·30071	·31530	72·5	37·5	·60876	·76733	52·5
18	·30902	·32492	72	38	·61566	·78129	52
18·5	·31730	·33460	71·5	38·5	·62251	·79544	51·5
19	·32557	·34433	71	39	·62932	·80978	51
19·5	·33381	·35412	70·5	39·5	·63608	·82434	50·5
	Cosine.	Cotangent.	Angle in Degrees		Cosine.	Cotangent.	Angle in Degrees.

Sines, Cosines, Tangents, and Cotangents.

Angle in Degrees.	Sine.	Tangent.		Angle in Degrees.	Sine.	Tangent	
40	·64279	·83910	50	60	·86603	1 73205	30
40·5	·64945	·85108	49 5	60·5	·87036	1·76749	29·5
41	·65606	·86929	49	61	·87462	1·80405	29
41·5	·66262	·88173	48·5	61·5	·87882	1 84177	28·5
42	·66913	·90040	48	62	·88295	1 88073	28
42 5	·67559	·91633	47·5	62·5	88701	1·92098	27·5
43	·68200	·93252	47	63	·89101	1·96261	27
43·5	·68835	·94896	46·5	63·5	·89493	2·00569	26·5
44	·69466	·96569	46	64	·89879	2·05030	26
44·5	·70091	·98270	45·5	64 5	·90259	2·09654	25 5
45	·70711	1·00000	45	65	·90631	2·14451	25
45·5	·71325	1·01761	44 5	65·5	·90996	2·19430	24·5
46	·71934	1·03553	14	66	·91355	2·24604	24
46·5	·72537	1 05378	43·5	66·5	·91706	2·29984	23·5
47	·73135	1·07237	43	67	·92050	2·35585	23
47 5	·73728	1·09131	42·5	67·5	·92388	2·41421	22·5
48	·74314	1·11061	42	68	·92718	2·47509	22
48·5	·74896	1 13029	41·5	68·5	93042	2·53865	21·5
49	·75471	1 15037	41	69	·93358	2·60509	21
49·5	·76041	1·17085	40·5	69·5	·93667	2·67462	20·5
50	·76604	1·19175	40	70	93969	2·74748	20
50·5	·77162	1·21310	39·5	70·5	·94264	2·82391	19·5
51	·77715	1·23490	39	71	·94552	2·90421	19
51·5	·78261	1·25717	38·5	71·5	·94832	2·98868	18·5
52	·78801	1·27994	38	72	·95106	3·07768	18
52 5	·79335	1·30323	37·5	72·5	·95372	3·17159	17 5
53	·79864	1 32704	37	73	·95630	3·27085	17
53·5	·80386	1·35142	36·5	73·5	·95882	3·37594	16·5
54	·80902	1·37638	36	74	·96126	3·48741	16
54·5	·81412	1·40195	35 5	74·5	·96363	3·60588	15·5
55	·81915	1·42815	35	75	·96593	3·73205	15
55·5	·82413	1·45501	34·5	75·5	·96815	3·86671	14·5
56	·82904	1·48256	34	76	·97030	4·01078	14
56·5	·83389	1·51084	33 5	76·5	·97237	4·16530	13·5
57	·83867	1·53987	33	77	·97437	4·33148	13
57·5	·84339	1·56969	32·5	77·5	·97630	4·51071	12·5
58	·84805	1·60033	32	78	·97815	4·70463	12
58·5	·85264	1·63185	31·5	78·5	·97992	4·91516	11·5
59	·85717	1·66428	31	79	·98163	5·14455	11
59·5	·86163	1·69766	30·5	79·5	·98325	5·39552	10·5
	Cosine.	Cotangent.	Angle in Degrees.		Cosine.	Cotangent.	Angle in Degrees.

Sines, Cosines, Tangents, and Cotangents.

Angle in Degrees	Sine.	Tangent		Angle in Degrees.	Sine.	Tangent	
80	·98481	5·67128	10	85·5	·99692	12·70620	4·5
80·5	·98629	5·97576	9·5	86	·99756	14·30067	4
81	·98769	6·31375	9	86·5	·99813	16·34985	3·5
81·5	·98902	6·69116	8·5	87	·99863	19·08114	3
82	·99027	7·11537	8	87·5	·99905	22·90377	2·5
82·5	·99144	7·59575	7·5	88	·99939	28·63625	2
83	·99255	8·14435	7	88·5	·99966	38·18846	1·5
83·5	·99357	8·77689	6·5	89	·99985	57·28996	1
84	·99452	9·51436	6	89·5	·99996	114·58865	0·5
84·5	·99540	10·38540	5·5	90	1·00000	Infinite	0
85	·99619	11·43005	5				
	Cosine	Cotangent	Angle in Degrees		Cosine.	Cotangent.	Angle in Degrees.

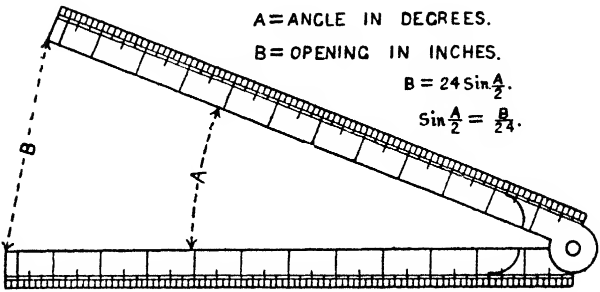
Trigonometrical Ratios of Certain Angles.

A Angle in degrees

A	Sine	Cosecant	Tangent.	
15	$\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{2\sqrt{2}}{\sqrt{3}-1}=\sqrt{6}+\sqrt{2}$	$2-\sqrt{3}$	75
18	$\frac{\sqrt{5}-1}{4}$	$\frac{4}{\sqrt{5}-1}=\sqrt{5}+1$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	72
30	$\frac{1}{2}$	2	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	60
36	$\frac{\sqrt{(10-2\sqrt{5})}}{4}$	$\frac{4}{\sqrt{(10-2\sqrt{5})}}=\sqrt{\frac{10+2\sqrt{5}}{5}}$	$\sqrt{5-2\sqrt{5}}$	54
45	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\sqrt{2}$	1	45
54	$\frac{\sqrt{5}+1}{4}$	$\frac{4}{\sqrt{5}+1}=\sqrt{5}-1$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$	36
60	$\frac{\sqrt{3}}{2}$	$\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$	$\sqrt{3}$	30
72	$\frac{\sqrt{(10+2\sqrt{5})}}{4}$	$\frac{4}{\sqrt{(10+2\sqrt{5})}}=\sqrt{\frac{10-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$	18
75	$\frac{\sqrt{3}+1}{2\sqrt{2}}=\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{2\sqrt{2}}{\sqrt{3}+1}=\sqrt{6}-\sqrt{2}$	$2+\sqrt{3}$	15
	Cosine.	Secant.	Cotangent.	A.

The trigonometrical ratios of angles greater than 90° are found by means of the formulæ on page 41, and the values given in the above tables.

Measurement of Angles by "Two-Foot" Rule.



ANGLE A IN DEGREES AND DECIMALS OF A DEGREE.

B	·0	⅛	¼	⅜	½	⅝	¾	⅞	B
0	—	6	1 2	1 8	2 4	3·0	3·6	4·2	0
1	4·8	5 4	6 0	6·6	7·2	7·8	8·4	9·0	1
2	9·6	10·2	10·8	11·4	12·0	12 6	13·2	13·8	2
3	14·4	15·0	15·6	16·2	16·8	17·4	18·0	18·6	3
4	19·2	19·8	20·4	21·0	21·6	22·2	22·8	23·4	4
5	24·1	24·7	25·3	25·9	26·5	27·1	27·7	28·3	5
6	29 0	29·6	30·2	30·8	31·4	32·0	32·7	33·3	6
7	33 9	34·5	35·2	35·8	36·4	37·0	37·7	38·3	7
8	38·9	39·6	40·2	40·8	41·5	42·1	42·8	43·4	8
9	44·0	44·7	45·3	46·0	46·6	47·3	47·9	48·6	9
10	49·2	49·9	50·6	51·2	51·9	52·6	53·2	53·9	10
11	54·6	55·2	55·9	56·6	57 3	57·9	58·6	59·3	11
12	60·0	60·7	61·4	62·1	62·8	63·5	64·2	64·9	12
13	65·6	66·3	67·0	67·7	68·5	69·2	69·9	70·6	13
14	71·4	72·1	72·8	73·6	74·3	75·1	75·8	76·6	14
15	77·4	78·1	78·9	79·7	80·5	81·2	82·0	82·8	15
16	83·6	84·4	85·2	86·0	86·9	87·7	88·5	89·4	16
B	·0	⅛	¼	⅜	½	⅝	¾	⅞	B

LOGARITHMS OF NUMBERS.

Logarithms are auxiliary numbers, by means of which the simple operations of addition and subtraction may be substituted for the more cumbrous operations of multiplication and division, and easy cases of multiplication and division for involution and evolution.

The logarithm of a number to a given base is the index of the power to which the base must be raised to be equal to the number. Thus, if $a^x = N$, then x is the logarithm of N to the base a.

There are two systems of logarithms in use, namely, the *common system*, in which the base is 10, and the *Napierian system*, in which the base (denoted by e) is 2·718281828 . Napierian logarithms are also called *natural* and also *Hyperbolic* logarithms.

When logarithms are spoken of without any qualification, common logarithms are to be understood.

The logarithm of a number usually consists of two parts, namely, an integral part and a fractional part ; thus log. 789 = 2·897077. The integral part is called the *characteristic*, and the fractional part is called the *mantissa*. Thus the characteristic of the logarithm of 789 is 2, and the mantissa is ·897077.

The characteristic of the logarithm of a number is determined by inspection. The rules for the characteristic are : (1.) If the number has an integral part, the characteristic is one less than the number of figures in the integral part, and it is *positive*. Thus the characteristic of the logarithm of 6873·45 is 3. (2.) If the number is wholly a decimal, the characteristic is one more than the number of noughts to the right of the decimal point, and is *negative*. Thus the characteristic of the logarithm of ·00567 is − 3, and is written $\bar{3}$.

The mantissa of the logarithm of a number depends only on the figures which occur in the number and their order, and is independent of the position of the decimal point. *The mantissa is always positive.*

The above rules for the characteristic and mantissa do not apply to Napierian logarithms.

In tables of common logarithms of numbers the mantissæ or decimal parts only of the logarithms are given.

Examples.

log. 7391 = 3·868703	log. ·7391 = $\bar{1}$·868703
log. 739100 = 5·868703	log. ·07391 = $\bar{2}$·868703
log. 739·1 = 2·868703	log. ·007391 = $\bar{3}$·868703
log. 73·91 = 1·868703	log. ·0007391 = $\bar{4}$·868703
log. 7·391 = 0·868703	log. ·00007391 = $\bar{5}$·868703

Computation of Negative Characteristics.

To add two negative characteristics, take their sum and make it negative. Thus $\bar{5}+\bar{2}=\bar{7}$.

To add a positive to a negative characteristic take their difference and make its sign the sign of the greater; thus, $\bar{3}+5=2$, and $3+\bar{5}=\bar{2}$.

Examples on the Addition of Logarithms.

	(1)	(2)	(3)	(4)
To	$\bar{3}$·458912	$\bar{1}$·516089	$\bar{2}$·189720	2·691856
Add	$\bar{4}$·319805	3·301562	$\bar{3}$·999118	$\bar{4}$·764865
Answer	$\bar{7}$·778717	2·817651	$\bar{4}$·188838	$\bar{1}$·456721

In examples (3) and (4) the sum of the decimal parts is greater than 1, and the integral part of this sum, which is positive, is added to the sum of the characteristics.

To subtract a negative characteristic, change its sign to plus and proceed as in addition; thus, $4-\bar{3}=4+3=7$, and $\bar{4}-\bar{3}=\bar{4}+3=\bar{1}$.

To subtract a positive characteristic, change its sign to minus and proceed as in addition; thus, $4-3=4+\bar{3}=1$, and $\bar{4}-3=\bar{4}+\bar{3}=\bar{7}$.

Examples on Subtraction of Logarithms.

	(1)	(2)	(3)
From	3·461735	$\bar{4}$·513829	$\bar{2}$·685609
Take	$\bar{1}$·214809	$\bar{2}$·608051	3·776152
Answer	4·246926	$\bar{3}$·905778	$\bar{6}$·909457

In examples (2) and (3) the lower decimal part is greater than the upper decimal part, hence 1 is borrowed from the upper characteristic, causing the characteristic $\bar{4}$ in example (2) to become $\bar{5}$, and the characteristic $\bar{2}$ in example (3) to become $\bar{3}$.

To multiply a negative characteristic, multiply as if positive and make the product negative; thus, $\bar{2}\times 3=\bar{6}$.

Examples on Multiplication of Logarithms.

	(1)	(2)
Multiply	$\bar{2}$·310311	$\bar{1}$·810698
By	3	6
Answer	$\bar{6}$·930933	$\bar{2}$·864188

In example (2) the decimal part multiplied by 6 gives 4 and a decimal, and this 4 is added to $6\times\bar{1}$, thus $6\times\bar{1}+4=\bar{6}+4=\bar{2}$.

To divide a logarithm having a negative characteristic.—If the characteristic is divisible by the divisor without a remainder, write the quotient with a negative sign and divide the decimal part in the usual way; thus, $\bar{6}{\cdot}458938 \div 2 = \bar{3}{\cdot}229469$. If the characteristic is not divisible by the divisor without a remainder, add such a negative number to it as will make it divisible without a remainder and prefix an equal positive number to the decimal part of the logarithm, then divide the increased negative characteristic and the other part of the logarithm separately; thus, $\bar{7}{\cdot}135718 \div 3 = (\bar{2} + \bar{7} + 2{\cdot}135718) \div 3 = (\bar{9} + 2{\cdot}135718) \div 3 = \bar{3}{\cdot}711906$.

To find the Logarithm of a Number by the Tables.

If the number contains *one*, or *two figures* only, the mantissa of its logarithm will be found on page 53, and to this must be prefixed the proper characteristic by the rules already given; thus, log. $87 = 1{\cdot}939519$, log. $8{\cdot}7 = 0{\cdot}939519$, log. $\cdot 87 = \bar{1}{\cdot}939519$, and log $\cdot 087 = \bar{2}{\cdot}939519$.

If the number contains *three figures*, the mantissa of its logarithm will be found in the column headed 0 in the table given on pages 54 to 98; thus, log. $549 = 2{\cdot}739572$, log. $54900 = 4{\cdot}739572$, and log. $\cdot 00549 = \bar{3}{\cdot}739572$.

If the number contains *four figures*, the mantissa of its logarithm will be found in the column headed by the last figure of the number, and in a line with the first three figures of the number to be found in the column headed N; thus, log. $3865 = 3{\cdot}587149$, and log. $38{\cdot}65 = 1{\cdot}587149$.

If the number contains more than four figures, its logarithm is determined by assuming that the increase in the logarithm is proportional to the corresponding increase in the number, which is approximately the case.

EXAMPLE.—To find the logarithm of 689412.

N.	Log.	N.	Log.
689500	838534	689412	838471 + x
689400	838471	689400	838471
Diff. 100	Diff. 63	Diff 12	Diff. x

$100 : 12 :: 63 : x.$

$$x = \frac{12 \times 63}{100} = 7{\cdot}56, \text{ say } 8. \quad 838471 + 8 = 838479.$$

Therefore log. $689412 = 5{\cdot}838479$.

In practice the above calculation is abbreviated by taking the difference between the logarithms from the column headed D in the table.

To find the Number corresponding to a given Logarithm.

If the mantissa is found in the table, the first three figures of the number will be found in a line with it in the column headed N, and the fourth figure will be that at the head of the column containing the mantissa. The characteristic will fix the position of the decimal point.

If the mantissa is not found exactly in the table, proceed as in the following example :—

EXAMPLE.—To find the number corresponding to the logarithm 2 027529.

Log	N.	Log.	N.
027757	106600	027529	106500 + x
027350	106500	027350	106500
Diff. 407	Diff. 100	Diff. 179	Diff. x

$$407 : 179 : : 100 : x.$$

$$x = \frac{179 \times 100}{407} = 44 \text{ very nearly.} \quad 106500 + 44 = 106544.$$

Therefore, the number corresponding to the logarithm 2·027529 is 106·544.

Applications of Logarithms.

Multiplication.—The logarithm of a product is equal to the sum of the logarithms of its factors; thus, log. $(A \times B \times C) =$ log. $A +$ log. $B +$ log. C.

EXAMPLE.—To find the product of 853·7, 99·18, and 6·437.

log. 853·7 = 2·931305
log. 99·18 = 1·996424
log. 6·437 = 0·808684

log of product = 5·736413. Product = 545020.

Division.—The logarithm of a quotient is equal to the logarithm of the dividend diminished by the logarithm of the divisor; thus, log. $\frac{A}{B} =$ log. $A -$ log. B.

EXAMPLE.—To find the value of $\frac{87·65 \times 3914}{8733 \times 47·19}$.

log. 87·65 = 1·942752
log. 3914 = 3·592621

log. of dividend = 5·535373
log. of divisor = 5·615013

log. of quotient = $\overline{1}$·920360

log. 8733 = 3·941163
log. 47·19 = 1·673850

log. of divisor = 5·615013

quotient = ·83245

Involution.—The logarithm of a power of a number is equal to the logarithm of the number multiplied by the index of the power; thus, log. $A^n = n$ log A.

EXAMPLE.—To find the fourth power of 8·79.

$$\text{log. } 8{\cdot}79^4 = 4 \text{ log. } 8{\cdot}79 = 4 \times {\cdot}943989 = 3{\cdot}775956$$
$$8{\cdot}79^4 = 5969{\cdot}7.$$

Evolution.—The logarithm of a root of a number is equal to the logarithm of the number divided by the index of the root; thus, log. $\sqrt[n]{A} = \frac{\text{log. } A}{n}$.

EXAMPLE.—To find the cube root of 2998.

$$\text{log. } \sqrt[3]{2998} = \frac{\text{log. } 2998}{3} = \frac{3{\cdot}476832}{3} = 1{\cdot}158944.$$
$$\sqrt[3]{2998} = 14{\cdot}419$$

To find x from the equation $a^x = b$, where a and b are known numbers.

$$x \text{ log. } a = \text{log. } b. \quad x = \frac{\text{log. } b}{\text{log. } a}.$$

Logarithms of Numbers.

N	Log	N	Log	N	Log	N	Log
1	000000	26	414973	51	707570	76	880814
2	301030	27	431364	52	716003	77	886491
3	477121	28	447158	53	724276	78	892095
4	602060	29	462398	54	732394	79	897627
5	698970	30	477121	55	740363	80	903090
6	778151	31	491362	56	748188	81	908485
7	845098	32	505150	57	755875	82	913814
8	903090	33	518514	58	763428	83	919078
9	954243	34	531479	59	770852	84	924279
10	000000	35	544068	60	778151	85	929419
11	041393	36	556303	61	785330	86	934498
12	079181	37	568202	62	792392	87	939519
13	113943	38	579784	63	799341	88	944483
14	146128	39	591065	64	806180	89	949390
15	176091	40	602060	65	812913	90	954243
16	204120	41	612784	66	819544	91	959041
17	230449	42	623249	67	826075	92	963788
18	255273	43	633468	68	832509	93	968483
19	278754	44	643453	69	838849	94	973128
20	301030	45	653213	70	845098	95	977724
21	322219	46	662758	71	851258	96	982271
22	342423	47	672098	72	857332	97	986772
23	361728	48	681241	73	863323	98	991226
24	380211	49	690196	74	869232	99	995635
25	397940	50	698970	75	875061	100	000000

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
101	004321	004751	005181	005609	006038	006466	006894	007321	007748	008174	428
102	008600	009026	009451	009876	010300	010724	011147	011570	011993	012415	424
103	012837	013259	013680	014100	014521	014940	015360	015779	016197	016616	420
104	017033	017451	017868	018284	018700	019116	019532	019947	020361	020775	416
105	021189	021603	022016	022428	022841	023252	023664	024075	024486	024896	412
106	025306	025715	026125	026533	026942	027350	027757	028164	028571	028978	408
107	029384	029789	030195	030600	031004	031408	031812	032216	032619	033021	404
108	033424	033826	034227	034628	035029	035430	035830	036230	036629	037028	400
109	037426	037825	038223	038620	039017	039414	039811	040207	040602	040998	397
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
111	045323	045714	046105	046495	046885	047275	047664	048053	048442	048830	390
112	049218	049606	049993	050380	050766	051153	051538	051924	052309	052694	386
113	053078	053463	053846	054230	054613	054996	055378	055760	056142	056524	383
114	056905	057286	057666	058046	058426	058805	059185	059563	059942	060320	379
115	060698	061075	061452	061829	062206	062582	062958	063333	063709	064083	376
116	064458	064832	065206	065580	065953	066326	066699	067071	067443	067815	373
117	068186	068557	068928	069298	069668	070038	070407	070776	071145	071514	370
118	071882	072250	072617	072985	073352	073718	074085	074451	074816	075182	367
119	075547	075912	076276	076640	077004	077368	077731	078094	078457	078819	364
120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	361
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
121	082785	083144	083503	083861	084219	084576	084934	085291	085647	086004	358
122	086360	086716	087071	087426	087781	088136	088490	088845	089198	089552	355
123	089905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
124	093422	093772	094122	094471	094820	095169	095518	095866	096215	096562	349
125	096910	097257	097604	097951	098298	098644	098990	099335	099681	100026	346
126	100371	100715	101059	101403	101747	102091	102434	102777	103119	103462	343
127	103804	104146	104487	104828	105169	105510	105851	106191	106531	106871	341
128	107210	107549	107888	108227	108565	108903	109241	109579	109916	110253	338
129	110590	110926	111263	111599	111934	112270	112605	112940	113275	113609	335
130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
131	117271	117603	117934	118265	118595	118926	119256	119586	119915	120245	330
132	120574	120903	121231	121560	121888	122216	122544	122871	123198	123525	328
133	123852	124178	124504	124830	125156	125481	125806	126131	126456	126781	325
134	127105	127429	127753	128076	128399	128722	129045	129368	129690	130012	323
135	130334	130655	130977	131298	131619	131939	132260	132580	132900	133219	321
136	133539	133858	134177	134496	134814	135133	135451	135769	136086	136403	318
137	136721	137037	137354	137671	137987	138303	138618	138934	139249	139564	316
138	139879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
139	143015	143327	143639	143951	144263	144574	144885	145196	145507	145818	311
140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
141	149219	149527	149835	150142	150449	150756	151063	151370	151676	151982	307
142	152288	152594	152900	153205	153510	153815	154120	154424	154728	155032	305
143	155336	155640	155943	156246	156549	156852	157154	157457	157759	158061	303
144	158362	158664	158965	159266	159567	159868	160168	160469	160769	161068	301
145	161368	161667	161967	162266	162564	162863	163161	163460	163758	164055	299
146	164353	164650	164947	165244	165541	165838	166134	166430	166726	167022	297
147	167317	167613	167908	168203	168497	168792	169086	169380	169674	169968	295
148	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
149	173186	173478	173769	174060	174351	174641	174932	175222	175512	175802	291
150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
151	178977	179264	179552	179839	180126	180413	180699	180986	181272	181558	287
152	181844	182129	182415	182700	182985	183270	183555	183839	184123	184407	285
153	184691	184975	185259	185542	185825	186108	186391	186674	186956	187239	283
154	187521	187803	188084	188366	188647	188928	189209	189490	189771	190051	281
155	190332	190612	190892	191171	191451	191730	192010	192289	192567	192846	279
156	193125	193403	193681	193959	194237	194514	194792	195069	195346	195623	278
157	195900	196176	196453	196729	197005	197281	197556	197832	198107	198382	276
158	198657	198932	199206	199481	199755	200029	200303	200577	200850	201124	274
159	201397	201670	201943	202216	202488	202761	203033	203305	203577	203848	272
160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556	271
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers *(continued)*

N	0	1	2	3	4	5	6	7	8	9	D
161	206826	207096	207365	207634	207904	208173	208441	208710	208979	209247	269
162	209515	209783	210051	210319	210586	210853	211121	211388	211654	211921	267
163	212188	212454	212720	212986	213252	213518	213783	214049	214314	214579	266
164	214844	215109	215373	215638	215902	216166	216430	216694	216957	217221	264
165	217484	217747	218010	218273	218536	218798	219060	219323	219585	219846	262
166	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
167	222716	222976	223236	223496	223755	224015	224274	224533	224792	225051	259
168	225309	225568	225826	226084	226342	226600	226858	227115	227372	227630	258
169	227887	228144	228400	228657	228913	229170	229426	229682	229938	230193	256
170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	255
171	232996	233250	233504	233757	234011	234264	234517	234770	235023	235276	253
172	235528	235781	236033	236285	236537	236789	237041	237292	237544	237795	252
173	238046	238297	238548	238799	239049	239299	239550	239800	240050	240300	250
174	240549	240799	241048	241297	241546	241795	242044	242293	242541	242790	249
175	243038	243286	243534	243782	244030	244277	244525	244772	245019	245266	248
176	245513	245759	246006	246252	246499	246745	246991	247237	247482	247728	246
177	247973	248219	248464	248709	248954	249198	249443	249687	249932	250176	245
178	250420	250664	250908	251151	251395	251638	251881	252125	252368	252610	243
179	252853	253096	253338	253580	253822	254064	254306	254548	254790	255031	242
180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
181	257679	257918	258158	258398	258637	258877	259116	259355	259594	259833	239
182	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
183	262451	262688	262925	263162	263399	263636	263873	264109	264346	264582	237
184	264818	265054	265290	265525	265761	265996	266232	266467	266702	266937	235
185	267172	267406	267641	267875	268110	268344	268578	268812	269046	269279	234
186	269513	269746	269980	270213	270446	270679	270912	271144	271377	271609	233
187	271842	272074	272306	272538	272770	273001	273233	273464	273696	273927	232
188	274158	274389	274620	274850	275081	275311	275542	275772	276002	276232	230
189	276462	276692	276921	277151	277380	277609	277838	278067	278296	278525	229
190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
191	281033	281261	281488	281715	281942	282169	282396	282622	282849	283075	227
192	283301	283527	283753	283979	284205	284431	284656	284882	285107	285332	226
193	285557	285782	286007	286232	286456	286681	286905	287130	287354	287578	225
194	287802	288026	288249	288473	288696	288920	289143	289366	289589	289812	223
195	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
196	292256	292478	292699	292920	293141	293363	293584	293804	294025	294246	221
197	294466	294687	294907	295127	295347	295567	295787	296007	296226	296446	220
198	296665	296884	297104	297323	297542	297761	297979	298198	298416	298635	219
199	298853	299071	299289	299507	299725	299943	300161	300378	300595	300813	218
200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
201	303196	303412	303628	303844	304059	304275	304491	304706	304921	305136	216
202	305351	305566	305781	305996	306211	306425	306639	306854	307068	307282	215
203	307496	307710	307924	308137	308351	308564	308778	308991	309204	309417	213
204	309630	309843	310056	310268	310481	310693	310906	311118	311330	311542	212
205	311754	311966	312177	312389	312600	312812	313023	313234	313445	313656	211
206	313867	314078	314289	314499	314710	314920	315130	315340	315551	315760	210
207	315970	316180	316390	316599	316809	317018	317227	317436	317646	317854	209
208	318063	318272	318481	318689	318898	319106	319314	319522	319730	319938	208
209	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
211	324282	324488	324694	324899	325105	325310	325516	325721	325926	326131	205
212	326336	326541	326745	326950	327155	327359	327563	327767	327972	328176	204
213	328380	328583	328787	328991	329194	329398	329601	329805	330008	330211	203
214	330414	330617	330819	331022	331225	331427	331630	331832	332034	332236	202
215	332438	332640	332842	333044	333246	333447	333649	333850	334051	334253	202
216	334454	334655	334856	335057	335257	335458	335658	335859	336059	336260	201
217	336460	336660	336860	337060	337260	337459	337659	337858	338058	338257	200
218	338456	338656	338855	339054	339253	339451	339650	339849	340047	340246	199
219	340444	340642	340841	341039	341237	341435	341632	341830	342028	342225	198
220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196	197
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
221	344392	344589	344785	344981	345178	345374	345570	345766	345962	346157	196
222	346353	346549	346744	346939	347135	347330	347525	347720	347915	348110	195
223	348305	348500	348694	348889	349083	349278	349472	349666	349860	350054	194
224	350248	350442	350636	350829	351023	351216	351410	351603	351796	351989	193
225	352183	352375	352568	352761	352954	353147	353339	353532	353724	353916	193
226	354108	354301	354493	354685	354876	355068	355260	355452	355643	355834	192
227	356026	356217	356408	356599	356790	356981	357172	357363	357554	357744	191
228	357935	358125	358316	358506	358696	358886	359076	359266	359456	359646	190
229	359835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	188
231	363612	363800	363988	364176	364363	364551	364739	364926	365113	365301	188
232	365488	365675	365862	366049	366236	366423	366610	366796	366983	367169	187
233	367356	367542	367729	367915	368101	368287	368473	368659	368845	369030	186
234	369216	369401	369587	369772	369958	370143	370328	370513	370698	370883	185
235	371068	371253	371437	371622	371806	371991	372175	372360	372544	372728	184
236	372912	373096	373280	373464	373647	373831	374015	374198	374382	374565	184
237	374748	374932	375115	375298	375481	375664	375846	376029	376212	376394	183
238	376577	376759	376942	377124	377306	377488	377670	377852	378034	378216	182
239	378398	378580	378761	378943	379124	379306	379487	379668	379849	380030	181
240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*)

N	0	1	2	3	4	5	6	7	8	9	D
241	382017	382197	382377	382557	382737	382917	383097	383277	383456	383636	180
242	383815	383995	384174	384353	384533	384712	384891	385070	385249	385428	179
243	385606	385785	385964	386142	386321	386499	386677	386856	387034	387212	178
244	387390	387568	387746	387923	388101	388279	388456	388634	388811	388989	178
245	389166	389343	389520	389698	389875	390051	390228	390405	390582	390759	177
246	390935	391112	391288	391464	391641	391817	391993	392169	392345	392521	176
247	392697	392873	393048	393224	393400	393575	393751	393926	394101	394277	176
248	394452	394627	394802	394977	395152	395326	395501	395676	395850	396025	175
249	396199	396374	396548	396722	396896	397071	397245	397419	397592	397766	174
250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
251	399674	399847	400020	400192	400365	400538	400711	400883	401056	401228	173
252	401401	401573	401745	401917	402089	402261	402433	402605	402777	402949	172
253	403121	403292	403464	403635	403807	403978	404149	404320	404492	404663	171
254	404834	405005	405176	405346	405517	405688	405858	406029	406199	406370	171
255	406540	406710	406881	407051	407221	407391	407561	407731	407901	408070	170
256	408240	408410	408579	408749	408918	409087	409257	409426	409595	409764	169
257	409933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
258	411620	411788	411956	412124	412293	412461	412629	412796	412964	413132	168
259	413300	413467	413635	413803	413970	414137	414305	414472	414639	414806	167
260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
261	416641	416807	416973	417139	417306	417472	417638	417804	417970	418135	166
262	418301	418467	418633	418798	418964	419129	419295	419460	419625	419791	166
263	419956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
264	421604	421768	421933	422097	422261	422426	422590	422754	422918	423082	164
265	423246	423410	423574	423737	423901	424065	424228	424392	424555	424718	164
266	424882	425045	425208	425371	425534	425697	425860	426023	426186	426349	163
267	426511	426674	426836	426999	427161	427324	427486	427648	427811	427973	162
268	428135	428297	428459	428621	428783	428944	429106	429268	429429	429591	162
269	429752	429914	430075	430236	430398	430559	430720	430881	431042	431203	161
270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809	161
271	432969	433130	433290	433450	433610	433770	433930	434090	434249	434409	160
272	434569	434729	434888	435048	435207	435367	435526	435685	435844	436004	159
273	436163	436322	436481	436640	436799	436957	437116	437275	437433	437592	159
274	437751	437909	438067	438226	438384	438542	438701	438859	439017	439175	158
275	439333	439491	439648	439806	439964	440122	440279	440437	440594	440752	158
276	440909	441066	441224	441381	441538	441695	441852	442009	442166	442323	157
277	442480	442637	442793	442950	443106	443263	443419	443576	443732	443889	157
278	444045	444201	444357	444513	444669	444825	444981	445137	445293	445449	156
279	445604	445760	445915	446071	446226	446382	446537	446692	446848	447003	155
280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552	155
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
281	448706	448861	449015	449170	449324	449478	449633	449787	449941	450095	154
282	450249	450403	450557	450711	450865	451018	451172	451326	451479	451633	154
283	451786	451940	452093	452247	452400	452553	452706	452859	453012	453165	153
284	453318	453471	453624	453777	453930	454082	454235	454387	454540	454692	153
285	454845	454997	455150	455302	455454	455606	455758	455910	456062	456214	152
286	456366	456518	456670	456821	456973	457125	457276	457428	457579	457731	152
287	457882	458033	458184	458336	458487	458638	458789	458940	459091	459242	151
288	459392	459543	459694	459845	459995	460146	460296	460147	460597	460748	151
289	460898	461048	461198	461348	461499	461649	461799	461948	462098	462248	150
290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
291	463893	464042	464191	464340	464490	464639	464788	464936	465085	465234	149
292	465383	465532	465680	465829	465977	466126	466274	466423	466571	466719	148
293	466868	467016	467164	467312	467460	467608	467756	467904	468052	468200	148
294	468347	468495	468643	468790	468938	469085	469233	469380	469527	469675	148
295	469822	469969	470116	470263	470410	470557	470704	470851	470998	471145	147
296	471292	471438	471585	471732	471878	472025	472171	472318	472464	472610	146
297	472756	472903	473049	473195	473341	473487	473633	473779	473925	474071	146
298	474216	474362	474508	474653	474799	474944	475090	475235	475381	475526	146
299	475671	475816	475962	476107	476252	476397	476542	476687	476832	476976	145
300	477121	477266	477411	477555	477700	477844	477989	478133	478278	478422	145
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
301	478566	478711	478855	478999	479143	479287	479431	479575	479719	479863	144
302	480007	480151	480294	480438	480582	480725	480869	481012	481156	481299	144
303	481443	481586	481729	481872	482016	482159	482302	482445	482588	482731	143
304	482874	483016	483159	483302	483445	483587	483730	483872	484015	484157	143
305	484300	484442	484585	484727	484869	485011	485153	485295	485437	485579	142
306	485721	485863	486005	486147	486289	486430	486572	486714	486855	486997	142
307	487138	487280	487421	487563	487704	487845	487986	488127	488269	488410	141
308	488551	488692	488833	488974	489114	489255	489396	489537	489677	489818	141
309	489958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
311	492760	492900	493040	493179	493319	493458	493597	493737	493876	494015	139
312	494155	494294	494433	494572	494711	494850	494989	495128	495267	495406	139
313	495544	495683	495822	495960	496099	496238	496376	496515	496653	496791	139
314	496930	497068	497206	497344	497483	497621	497759	497897	498035	498173	138
315	498311	498448	498586	498724	498862	498999	499137	499275	499412	499550	138
316	499687	499824	499962	500099	500236	500374	500511	500648	500785	500922	137
317	501059	501196	501333	501470	501607	501744	501880	502017	502154	502291	137
318	502427	502564	502700	502837	502973	503109	503246	503382	503518	503655	136
319	503791	503927	504063	504199	504335	504471	504607	504743	504878	505014	136
320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
321	506505	506640	506776	506911	507046	507181	507316	507451	507586	507721	135
322	507856	507991	508126	508260	508395	508530	508664	508799	508934	509068	135
323	509203	509337	509471	509606	509740	509874	510009	510143	510277	510411	134
324	510545	510679	510813	510947	511081	511215	511349	511482	511616	511750	134
325	511883	512017	512151	512284	512418	512551	512684	512818	512951	513084	133
326	513218	513351	513484	513617	513750	513883	514016	514149	514282	514415	133
327	514548	514681	514813	514946	515079	515211	515344	515476	515609	515741	133
328	515874	516006	516139	516271	516403	516535	516668	516800	516932	517064	132
329	517196	517328	517460	517592	517724	517855	517987	518119	518251	518382	132
330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
331	519828	519959	520090	520221	520353	520484	520615	520745	520876	521007	131
332	521138	521269	521400	521530	521661	521792	521922	522053	522183	522314	131
333	522444	522575	522705	522835	522966	523096	523226	523356	523486	523616	130
334	523746	523876	524006	524136	524266	524396	524526	524656	524785	524915	130
335	525045	525174	525304	525434	525563	525693	525822	525951	526081	526210	129
336	526339	526469	526598	526727	526856	526985	527114	527243	527372	527501	129
337	527630	527759	527888	528016	528145	528274	528402	528531	528660	528788	129
338	528917	529045	529174	529302	529430	529559	529687	529815	529943	530072	128
339	530200	530328	530456	530584	530712	530840	530968	531096	531223	531351	128
340	531479	531607	531734	531862	531990	532117	532245	532372	532500	532627	128
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
341	532754	532882	533009	533136	533264	533391	533518	533645	533772	533899	127
342	534026	534153	534280	534407	534534	534661	534787	534914	535041	535167	127
343	535294	535421	535547	535674	535800	535927	536053	536180	536306	536432	126
344	536558	536685	536811	536937	537063	537189	537315	537441	537567	537693	126
345	537819	537945	538071	538197	538322	538448	538574	538699	538825	538951	126
346	539076	539202	539327	539452	539578	539703	539829	539954	540079	540204	125
347	540329	540455	540580	540705	540830	540955	541080	541205	541330	541454	125
348	541579	541704	541829	541953	542078	542203	542327	542452	542576	542701	125
349	542825	542950	543074	543199	543323	543447	543571	543696	543820	543944	124
350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
351	545307	545431	545555	545678	545802	545925	546049	546172	546296	546419	124
352	546543	546666	546789	546913	547036	547159	547282	547405	547529	547652	123
353	547775	547898	548021	548144	548267	548389	548512	548635	548758	548881	123
354	549003	549126	549249	549371	549494	549616	549739	549861	549984	550106	123
355	550228	550351	550473	550595	550717	550840	550962	551084	551206	551328	122
356	551450	551572	551694	551816	551938	552060	552181	552303	552425	552547	122
357	552668	552790	552911	553033	553155	553276	553398	553519	553640	553762	122
358	553883	554004	554126	554247	554368	554489	554610	554731	554852	554973	121
359	555094	555215	555336	555457	555578	555699	555820	555940	556061	556182	121
360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
361	557507	557627	557748	557868	557988	558108	558228	558349	558469	558589	120
362	558709	558829	558948	559068	559188	559308	559428	559548	559667	559787	120
363	559907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
364	561101	561221	561340	561459	561578	561698	561817	561936	562055	562174	119
365	562293	562412	562531	562650	562769	562887	563006	563125	563244	563362	119
366	563481	563600	563718	563837	563955	564074	564192	564311	564429	564548	119
367	564666	564784	564903	565021	565139	565257	565376	565494	565612	565730	118
368	565848	565966	566084	566202	566320	566437	566555	566673	566791	566909	118
369	567026	567144	567262	567379	567497	567614	567732	567849	567967	568084	118
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
371	569374	569491	569608	569725	569842	569959	570076	570193	570309	570426	117
372	570543	570660	570776	570893	571010	571126	571243	571359	571476	571592	117
373	571709	571825	571942	572058	572174	572291	572407	572523	572639	572755	116
374	572872	572988	573104	573220	573336	573452	573568	573684	573800	573915	116
375	574031	574147	574263	574379	574494	574610	574726	574841	574957	575072	116
376	575188	575303	575419	575534	575650	575765	575880	575996	576111	576226	115
377	576341	576457	576572	576687	576802	576917	577032	577147	577262	577377	115
378	577492	577607	577722	577836	577951	578066	578181	578295	578410	578525	115
379	578639	578754	578868	578983	579097	579212	579326	579441	579555	579669	114
380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*)

N	0	1	2	3	4	5	6	7	8	9	D
381	580925	581039	581153	581267	581381	581495	581608	581722	581836	581950	114
382	582063	582177	582291	582404	582518	582631	582745	582858	582972	583085	114
383	583199	583312	583426	583539	583652	583765	583879	583992	584105	584218	113
384	584331	584444	584557	584670	584783	584896	585009	585122	585235	585348	113
385	585461	585574	585686	585799	585912	586024	586137	586250	586362	586475	113
386	586587	586700	586812	586925	587037	587149	587262	587374	587486	587599	112
387	587711	587823	587935	588047	588160	588272	588384	588496	588608	588720	112
388	588832	588944	589056	589167	589279	589391	589503	589615	589726	589838	112
389	589950	590061	590173	590284	590396	590507	590619	590730	590842	590953	111
390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
391	592177	592288	592399	592510	592621	592732	592843	592954	593064	593175	111
392	593286	593397	593508	593618	593729	593840	593950	594061	594171	594282	111
393	594393	594503	594614	594724	594834	594945	595055	595165	595276	595386	110
394	595496	595606	595717	595827	595937	596047	596157	596267	596377	596487	110
395	596597	596707	596817	596927	597037	597146	597256	597366	597476	597586	110
396	597695	597805	597914	598024	598134	598243	598353	598462	598572	598681	110
397	598791	598900	599009	599119	599228	599337	599446	599556	599665	599774	109
398	599883	599992	600101	600210	600319	600428	600537	600646	600755	600864	109
399	600973	601082	601191	601299	601408	601517	601625	601734	601843	601951	109
400	602060	602169	602277	602386	602494	602603	602711	602819	602928	603036	108
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers *(continued)*

N	0	1	2	3	4	5	6	7	8	9	D
401	603144	603253	603361	603469	603577	603686	603794	603902	604010	604118	108
402	604226	604334	604442	604550	604658	604766	604874	604982	605089	605197	108
403	605305	605413	605521	605628	605736	605844	605951	606059	606166	606274	108
404	606381	606489	606596	606704	606811	606919	607026	607133	607241	607348	107
405	607455	607562	607669	607777	607884	607991	608098	608205	608312	608419	107
406	608526	608633	608740	608847	608954	609061	609167	609274	609381	609488	107
407	609594	609701	609808	609914	610021	610128	610234	610341	610447	610554	107
408	610660	610767	610873	610979	611086	611192	611298	611405	611511	611617	106
409	611723	611829	611936	612042	612148	612254	612360	612466	612572	612678	106
410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
411	613842	613947	614053	614159	614264	614370	614475	614581	614686	614792	106
412	614897	615003	615108	615213	615319	615424	615529	615634	615740	615845	105
413	615950	616055	616160	616265	616370	616476	616581	616686	616790	616895	105
414	617000	617105	617210	617315	617420	617525	617629	617734	617839	617943	105
415	618048	618153	618257	618362	618466	618571	618676	618780	618884	618989	105
416	619093	619198	619302	619406	619511	619615	619719	619824	619928	620032	104
417	620136	620240	620344	620448	620552	620656	620760	620864	620968	621072	104
418	621176	621280	621384	621488	621592	621695	621799	621903	622007	622110	104
419	622214	622318	622421	622525	622628	622732	622835	622939	623042	623146	104
420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179	103
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
421	624282	624385	624488	624591	624695	624798	624901	625004	625107	625210	103
422	625312	625415	625518	625621	625724	625827	625929	626032	626135	626238	103
423	626340	626443	626546	626648	626751	626853	626956	627058	627161	627263	103
424	627366	627468	627571	627673	627775	627878	627980	628082	628185	628287	102
425	628389	628491	628593	628695	628797	628900	629002	629104	629206	629308	102
426	629410	629512	629613	629715	629817	629919	630021	630123	630224	630326	102
427	630428	630530	630631	630733	630835	630936	631038	631139	631241	631342	102
428	631444	631545	631647	631748	631849	631951	632052	632153	632255	632356	101
429	632457	632559	632660	632761	632862	632963	633064	633165	633266	633367	101
430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
431	634477	634578	634679	634779	634880	634981	635081	635182	635283	635383	101
432	635484	635584	635685	635785	635886	635986	636087	636187	636287	636388	100
433	636488	636588	636688	636789	636889	636989	637089	637189	637290	637390	100
434	637490	637590	637690	637790	637890	637990	638090	638190	638290	638389	100
435	638489	638589	638689	638789	638888	638988	639088	639188	639287	639387	100
436	639486	639586	639686	639785	639885	639984	640084	640183	640283	640382	100
437	640481	640581	640680	640779	640879	640978	641077	641177	641276	641375	99
438	641474	641573	641672	641771	641871	641970	642069	642168	642267	642366	99
439	642465	642563	642662	642761	642860	642959	643058	643156	643255	643354	99
440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	99
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
441	644439	644537	644636	644734	644832	644931	645029	645127	645226	645324	98
442	645422	645521	645619	645717	645815	645913	646011	646110	646208	646306	98
443	646404	646502	646600	646698	646796	646894	646992	647089	647187	647285	98
444	647383	647481	647579	647676	647774	647872	647969	648067	648165	648262	98
445	648360	648458	648555	648653	648750	648848	648945	649043	649140	649237	97
446	649335	649432	649530	649627	649724	649821	649919	650016	650113	650210	97
447	650308	650405	650502	650599	650696	650793	650890	650987	651084	651181	97
448	651278	651375	651472	651569	651666	651762	651859	651956	652053	652150	97
449	652246	652343	652440	652536	652633	652730	652826	652923	653019	653116	97
450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
451	654177	654273	654369	654465	654562	654658	654754	654850	654946	655042	96
452	655138	655235	655331	655427	655523	655619	655715	655810	655906	656002	96
453	656098	656194	656290	656386	656482	656577	656673	656769	656864	656960	96
454	657056	657152	657247	657343	657438	657534	657629	657725	657820	657916	96
455	658011	658107	658202	658298	658393	658488	658584	658679	658774	658870	95
456	658965	659060	659155	659250	659346	659441	659536	659631	659726	659821	95
457	659916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
458	660865	660960	661055	661150	661245	661339	661434	661529	661623	661718	95
459	661813	661907	662002	662096	662191	662286	662380	662475	662569	662663	94
460	662758	662852	662947	663041	663135	663230	663324	663418	663512	663607	94
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
461	663701	663795	663889	663983	664078	664172	664266	664360	664454	664548	94
462	664642	664736	664830	664924	665018	665112	665206	665299	665393	665487	94
463	665581	665675	665769	665862	665956	666050	666143	666237	666331	666424	94
464	666518	666612	666705	666799	666892	666986	667079	667173	667266	667360	94
465	667453	667546	667640	667733	667826	667920	668013	668106	668199	668293	93
466	668386	668479	668572	668665	668759	668852	668945	669038	669131	669224	93
467	669317	669410	669503	669596	669689	669782	669875	669967	670060	670153	93
468	670246	670339	670431	670524	670617	670710	670802	670895	670988	671080	93
469	671173	671265	671358	671451	671543	671636	671728	671821	671913	672005	92
470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
471	673021	673113	673205	673297	673390	673482	673574	673666	673758	673850	92
472	673942	674034	674126	674218	674310	674402	674494	674586	674677	674769	92
473	674861	674953	675045	675137	675228	675320	675412	675503	675595	675687	92
474	675778	675870	675962	676053	676145	676236	676328	676419	676511	676602	92
475	676694	676785	676876	676968	677059	677151	677242	677333	677424	677516	91
476	677607	677698	677789	677881	677972	678063	678154	678245	678336	678427	91
477	678518	678609	678700	678791	678882	678973	679064	679155	679246	679337	91
478	679428	679519	679610	679700	679791	679882	679973	680063	680154	680245	91
479	680336	680426	680517	680607	680698	680789	680879	680970	681060	681151	91
480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
481	682145	682235	682326	682416	682506	682596	682686	682777	682867	682957	90
482	683047	683137	683227	683317	683407	683497	683587	683677	683767	683857	90
483	683947	684037	684127	684217	684307	684396	684486	684576	684666	684756	90
484	684845	684935	685025	685114	685204	685294	685383	685473	685563	685652	90
485	685742	685831	685921	686010	686100	686189	686279	686368	686458	686547	89
486	686636	686726	686815	686904	686994	687083	687172	687261	687351	687440	89
487	687529	687618	687707	687796	687886	687975	688064	688153	688242	688331	89
488	688420	688509	688598	688687	688776	688865	688953	689042	689131	689220	89
489	689309	689398	689486	689575	689664	689753	689841	689930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
491	691081	691170	691258	691347	691435	691524	691612	691700	691789	691877	88
492	691965	692053	692142	692230	692318	692406	692494	692583	692671	692759	88
493	692847	692935	693023	693111	693199	693287	693375	693463	693551	693639	88
494	693727	693815	693903	693991	694078	694166	694254	694342	694430	694517	88
495	694605	694693	694781	694868	694956	695044	695131	695219	695307	695394	88
496	695482	695569	695657	695744	695832	695919	696007	696094	696182	696269	87
497	696356	696444	696531	696618	696706	696793	696880	696968	697055	697142	87
498	697229	697317	697404	697491	697578	697665	697752	697839	697926	698014	87
499	698101	698188	698275	698362	698449	698535	698622	698709	698796	698883	87
500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
501	699838	699924	700011	700098	700184	700271	700358	700444	700531	700617	87
502	700704	700790	700877	700963	701050	701136	701222	701309	701395	701482	86
503	701568	701654	701741	701827	701913	701999	702086	702172	702258	702344	86
504	702431	702517	702603	702689	702775	702861	702947	703033	703119	703205	86
505	703291	703377	703463	703549	703635	703721	703807	703893	703979	704065	86
506	704151	704236	704322	704408	704494	704579	704665	704751	704837	704922	86
507	705008	705094	705179	705265	705350	705436	705522	705607	705693	705778	86
508	705864	705949	706035	706120	706206	706291	706376	706462	706547	706632	85
509	706718	706803	706888	706974	707059	707144	707229	707315	707400	707485	85
510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
511	708421	708506	708591	708676	708761	708846	708931	709015	709100	709185	85
512	709270	709355	709440	709524	709609	709694	709779	709863	709948	710033	85
513	710117	710202	710287	710371	710456	710540	710625	710710	710794	710879	85
514	710963	711048	711132	711217	711301	711385	711470	711554	711639	711723	84
515	711807	711892	711976	712060	712144	712229	712313	712397	712481	712566	84
516	712650	712734	712818	712902	712986	713070	713154	713238	713323	713407	84
517	713491	713575	713659	713742	713826	713910	713994	714078	714162	714246	84
518	714330	714414	714497	714581	714665	714749	714833	714916	715000	715084	84
519	715167	715251	715335	715418	715502	715586	715669	715753	715836	715920	84
520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*)

N	0	1	2	3	4	5	6	7	8	9	D
521	716838	716921	717004	717088	717171	717254	717338	717421	717504	717587	83
522	717671	717754	717837	717920	718003	718086	718169	718253	718336	718419	83
523	718502	718585	718668	718751	718834	718917	719000	719083	719165	719248	83
524	719331	719414	719497	719580	719663	719745	719828	719911	719994	720077	83
525	720159	720242	720325	720407	720490	720573	720655	720738	720821	720903	83
526	720986	721068	721151	721233	721316	721398	721481	721563	721646	721728	82
527	721811	721893	721975	722058	722140	722222	722305	722387	722469	722552	82
528	722634	722716	722798	722881	722963	723045	723127	723209	723291	723374	82
529	723456	723538	723620	723702	723784	723866	723948	724030	724112	724194	82
530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
531	725095	725176	725258	725340	725422	725503	725585	725667	725748	725830	82
532	725912	725993	726075	726156	726238	726320	726401	726483	726564	726646	82
533	726727	726809	726890	726972	727053	727134	727216	727297	727379	727460	81
534	727541	727623	727704	727785	727866	727948	728029	728110	728191	728273	81
535	728354	728435	728516	728597	728678	728759	728841	728922	729003	729084	81
536	729165	729246	729327	729408	729489	729570	729651	729732	729813	729893	81
537	729974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
538	730782	730863	730944	731024	731105	731186	731266	731347	731428	731508	81
539	731589	731669	731750	731830	731911	731991	732072	732152	732233	732313	80
540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
541	733197	733278	733358	733438	733518	733598	733679	733759	733839	733919	80
542	733999	734079	734160	734240	734320	734400	734480	734560	734640	734720	80
543	734800	734880	734960	735040	735120	735200	735279	735359	735439	735519	80
544	735599	735679	735759	735838	735918	735998	736078	736157	736237	736317	80
545	736397	736476	736556	736635	736715	736795	736874	736954	737034	737113	80
546	737193	737272	737352	737431	737511	737590	737670	737749	737829	737908	79
547	737987	738067	738146	738225	738305	738384	738463	738543	738622	738701	79
548	738781	738860	738939	739018	739097	739177	739256	739335	739414	739493	79
549	739572	739651	739731	739810	739889	739968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
551	741152	741230	741309	741388	741467	741546	741624	741703	741782	741860	79
552	741939	742018	742096	742175	742254	742332	742411	742489	742568	742647	79
553	742725	742804	742882	742961	743039	743118	743196	743275	743353	743431	78
554	743510	743588	743667	743745	743823	743902	743980	744058	744136	744215	78
555	744293	744371	744449	744528	744606	744684	744762	744840	744919	744997	78
556	745075	745153	745231	745309	745387	745465	745543	745621	745699	745777	78
557	745855	745933	746011	746089	746167	746245	746323	746401	746479	746556	78
558	746634	746712	746790	746868	746945	747023	747101	747179	747256	747334	78
559	747412	747489	747567	747645	747722	747800	747878	747955	748033	748110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
561	748963	749040	749118	749195	749272	749350	749427	749504	749582	749659	77
562	749736	749814	749891	749968	750045	750123	750200	750277	750354	750431	77
563	750508	750586	750663	750740	750817	750894	750971	751048	751125	751202	77
564	751279	751356	751433	751510	751587	751664	751741	751818	751895	751972	77
565	752048	752125	752202	752279	752356	752433	752509	752586	752663	752740	77
566	752816	752893	752970	753047	753123	753200	753277	753353	753430	753506	77
567	753583	753660	753736	753813	753889	753966	754042	754119	754195	754272	77
568	754348	754425	754501	754578	754654	754730	754807	754883	754960	755036	76
569	755112	755189	755265	755341	755417	755494	755570	755646	755722	755799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
571	756636	756712	756788	756864	756940	757016	757092	757168	757244	757320	76
572	757396	757472	757548	757624	757700	757775	757851	757927	758003	758079	76
573	758155	758230	758306	758382	758458	758533	758609	758685	758761	758836	76
574	758912	758988	759063	759139	759214	759290	759366	759441	759517	759592	76
575	759668	759743	759819	759894	759970	760045	760121	760196	760272	760347	75
576	760422	760498	760573	760649	760724	760799	760875	760950	761025	761101	75
577	761176	761251	761326	761402	761477	761552	761627	761702	761778	761853	75
578	761928	762003	762078	762153	762228	762303	762378	762453	762529	762604	75
579	762679	762754	762829	762904	762978	763053	763128	763203	763278	763353	75
580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
581	764176	764251	764326	764400	764475	764550	764624	764699	764774	764848	75
582	764923	764998	765072	765147	765221	765296	765370	765445	765520	765594	75
583	765669	765743	765818	765892	765966	766041	766115	766190	766264	766338	74
584	766413	766487	766562	766636	766710	766785	766859	766933	767007	767082	74
585	767156	767230	767304	767379	767453	767527	767601	767675	767749	767823	74
586	767898	767972	768046	768120	768194	768268	768342	768416	768490	768564	74
587	768638	768712	768786	768860	768934	769008	769082	769156	769230	769303	74
588	769377	769451	769525	769599	769673	769746	769820	769894	769968	770042	74
589	770115	770189	770263	770336	770410	770484	770557	770631	770705	770778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
591	771587	771661	771734	771808	771881	771955	772028	772102	772175	772248	73
592	772322	772395	772468	772542	772615	772688	772762	772835	772908	772981	73
593	773055	773128	773201	773274	773348	773421	773494	773567	773640	773713	73
594	773786	773860	773933	774006	774079	774152	774225	774298	774371	774444	73
595	774517	774590	774663	774736	774809	774882	774955	775028	775100	775173	73
596	775246	775319	775392	775465	775538	775610	775683	775756	775829	775902	73
597	775974	776047	776120	776193	776265	776338	776411	776483	776556	776629	73
598	776701	776774	776846	776919	776992	777064	777137	777209	777282	777354	73
599	777427	777499	777572	777644	777717	777789	777862	777934	778006	778079	72
600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*)

N	0	1	2	3	4	5	6	7	8	9	D
601	778874	778947	779019	779091	779163	779236	779308	779380	779452	779524	72
602	779596	779669	779741	779813	779885	779957	780029	780101	780173	780245	72
603	780317	780389	780461	780533	780605	780677	780749	780821	780893	780965	72
604	781037	781109	781181	781253	781324	781396	781468	781540	781612	781684	72
605	781755	781827	781899	781971	782042	782114	782186	782258	782329	782401	72
606	782473	782544	782616	782688	782759	782831	782902	782974	783046	783117	72
607	783189	783260	783332	783403	783475	783546	783618	783689	783761	783832	71
608	783904	783975	784046	784118	784189	784261	784332	784403	784475	784546	71
609	784617	784689	784760	784831	784902	784974	785045	785116	785187	785259	71
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
611	786041	786112	786183	786254	786325	786396	786467	786538	786609	786680	71
612	786751	786822	786893	786964	787035	787106	787177	787248	787319	787390	71
613	787460	787531	787602	787673	787744	787815	787885	787956	788027	788098	71
614	788168	788239	788310	788381	788451	788522	788593	788663	788734	788804	71
615	788875	788946	789016	789087	789157	789228	789299	789369	789440	789510	71
616	789581	789651	789722	789792	789863	789933	790004	790074	790144	790215	70
617	790285	790356	790426	790496	790567	790637	790707	790778	790848	790918	70
618	790988	791059	791129	791199	791269	791340	791410	791480	791550	791620	70
619	791691	791761	791831	791901	791971	792041	792111	792181	792252	792322	70
620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*)

N	0	1	2	3	4	5	6	7	8	9	D
621	793092	793162	793231	793301	793371	793441	793511	793581	793651	793721	70
622	793790	793860	793930	794000	794070	794139	794209	794279	794349	794418	70
623	794488	794558	794627	794697	794767	794836	794906	794976	795045	795115	70
624	795185	795254	795324	795393	795463	795532	795602	795672	795741	795811	70
625	795880	795949	796019	796088	796158	796227	796297	796366	796436	796505	69
626	796574	796644	796713	796782	796852	796921	796990	797060	797129	797198	69
627	797268	797337	797406	797475	797545	797614	797683	797752	797821	797890	69
628	797960	798029	798098	798167	798236	798305	798374	798443	798513	798582	69
629	798651	798720	798789	798858	798927	798996	799065	799134	799203	799272	69
630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
631	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
632	800717	800786	800854	800923	800992	801061	801129	801198	801266	801335	69
633	801404	801472	801541	801609	801678	801747	801815	801884	801952	802021	69
634	802089	802158	802226	802295	802363	802432	802500	802568	802637	802705	68
635	802774	802842	802910	802979	803047	803116	803184	803252	803321	803389	68
636	803457	803525	803594	803662	803730	803798	803867	803935	804003	804071	68
637	804139	804208	804276	804344	804412	804480	804548	804616	804685	804753	68
638	804821	804889	804957	805025	805093	805161	805229	805297	805365	805433	68
639	805501	805569	805637	805705	805773	805841	805908	805976	806044	806112	68
640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
641	806858	806926	806994	807061	807129	807197	807264	807332	807400	807467	68
642	807535	807603	807670	807738	807806	807873	807941	808008	808076	808143	68
643	808211	808279	808346	808414	808481	808549	808616	808684	808751	808818	67
644	808886	808953	809021	809088	809156	809223	809290	809358	809425	809492	67
645	809560	809627	809694	809762	809829	809896	809964	810031	810098	810165	67
646	810233	810300	810367	810434	810501	810569	810636	810703	810770	810837	67
647	810904	810971	811039	811106	811173	811240	811307	811374	811441	811508	67
648	811575	811642	811709	811776	811843	811910	811977	812044	812111	812178	67
649	812245	812312	812379	812445	812512	812579	812646	812713	812780	812847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
651	813581	813648	813714	813781	813848	813914	813981	814048	814114	814181	67
652	814248	814314	814381	814447	814514	814581	814647	814714	814780	814847	67
653	814913	814980	815046	815113	815179	815246	815312	815378	815445	815511	66
654	815578	815644	815711	815777	815843	815910	815976	816042	816109	816175	66
655	816241	816308	816374	816440	816506	816573	816639	816705	816771	816838	66
656	816904	816970	817036	817102	817169	817235	817301	817367	817433	817499	66
657	817565	817631	817698	817764	817830	817896	817962	818028	818094	818160	66
658	818226	818292	818358	818424	818490	818556	818622	818688	818754	818820	66
659	818885	818951	819017	819083	819149	819215	819281	819346	819412	819478	66
660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
661	820201	820267	820333	820399	820464	820530	820595	820661	820727	820792	66
662	820858	820924	820989	821055	821120	821186	821251	821317	821382	821448	66
663	821514	821579	821645	821710	821775	821841	821906	821972	822037	822103	65
664	822168	822233	822299	822364	822430	822495	822560	822626	822691	822756	65
665	822822	822887	822952	823018	823083	823148	823213	823279	823344	823409	65
666	823474	823539	823605	823670	823735	823800	823865	823930	823996	824061	65
667	824126	824191	824256	824321	824386	824451	824516	824581	824646	824711	65
668	824776	824841	824906	824971	825036	825101	825166	825231	825296	825361	65
669	825426	825491	825556	825621	825686	825751	825815	825880	825945	826010	65
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
671	826723	826787	826852	826917	826981	827046	827111	827175	827240	827305	65
672	827369	827434	827499	827563	827628	827692	827757	827821	827886	827951	65
673	828015	828080	828144	828209	828273	828338	828402	828467	828531	828595	64
674	828660	828724	828789	828853	828918	828982	829046	829111	829175	829239	64
675	829304	829368	829432	829497	829561	829625	829690	829754	829818	829882	64
676	829947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
677	830589	830653	830717	830781	830845	830909	830973	831037	831102	831166	64
678	831230	831294	831358	831422	831486	831550	831614	831678	831742	831806	64
679	831870	831934	831998	832062	832126	832189	832253	832317	832381	832445	64
680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
681	833147	833211	833275	833338	833402	833466	833530	833593	833657	833721	64
682	833784	833848	833912	833975	834039	834103	834166	834230	834294	834357	64
683	834421	834484	834548	834611	834675	834739	834802	834866	834929	834993	64
684	835056	835120	835183	835247	835310	835373	835437	835500	835564	835627	63
685	835691	835754	835817	835881	835944	836007	836071	836134	836197	836261	63
686	836324	836387	836451	836514	836577	836641	836704	836767	836830	836894	63
687	836957	837020	837083	837146	837210	837273	837336	837399	837462	837525	63
688	837588	837652	837715	837778	837841	837904	837967	838030	838093	838156	63
689	838219	838282	838345	838408	838471	838534	838597	838660	838723	838786	63
690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
691	839478	839541	839604	839667	839729	839792	839855	839918	839981	840043	63
692	840106	840169	840232	840294	840357	840420	840482	840545	840608	840671	63
693	840733	840796	840859	840921	840984	841046	841109	841172	841234	841297	63
694	841359	841422	841485	841547	841610	841672	841735	841797	841860	841922	63
695	841985	842047	842110	842172	842235	842297	842360	842422	842484	842547	62
696	842609	842672	842734	842796	842859	842921	842983	843046	843108	843170	62
697	843233	843295	843357	843420	843482	843544	843606	843669	843731	843793	62
698	843855	843918	843980	844042	844104	844166	844229	844291	844353	844415	62
699	844477	844539	844601	844664	844726	844788	844850	844912	844974	845036	62
700	845098	845160	845222	845284	845346	845408	845470	845532	845594	845656	62
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
701	845718	845780	845842	845904	845966	846028	846090	846151	846213	846275	62
702	846337	846399	846461	846523	846585	846646	846708	846770	846832	846894	62
703	846955	847017	847079	847141	847202	847264	847326	847388	847449	847511	62
704	847573	847634	847696	847758	847819	847881	847943	848004	848066	848128	62
705	848189	848251	848312	848374	848435	848497	848559	848620	848682	848743	62
706	848805	848866	848928	848989	849051	849112	849174	849235	849297	849358	61
707	849419	849181	849542	849604	849665	849726	849788	849849	849911	849972	61
708	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
709	850646	850707	850769	850830	850891	850952	851014	851075	851136	851197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
711	851870	851931	851992	852053	852114	852175	852236	852297	852358	852419	61
712	852480	852541	852602	852663	852724	852785	852846	852907	852968	853029	61
713	853090	853150	853211	853272	853333	853394	853455	853516	853577	853637	61
714	853698	853759	853820	853881	853941	854002	854063	854124	854185	854245	61
715	854306	854367	854428	854488	854549	854610	854670	854731	854792	854852	61
716	854913	854974	855034	855095	855156	855216	855277	855337	855398	855459	61
717	855519	855580	855640	855701	855761	855822	855882	855943	856003	856064	61
718	856124	856185	856245	856306	856366	856427	856487	856548	856608	856668	60
719	856729	856789	856850	856910	856970	857031	857091	857152	857212	857272	60
720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers *(continued)*

N	0	1	2	3	4	5	6	7	8	9	D
721	857935	857995	858056	858116	858176	858236	858297	858357	858417	858477	60
722	858537	858597	858657	858718	858778	858838	858898	858958	859018	859078	60
723	859138	859198	859258	859318	859379	859439	859499	859559	859619	859679	60
724	859739	859799	859859	859918	859978	860038	860098	860158	860218	860278	60
725	860338	860398	860458	860518	860578	860637	860697	860757	860817	860877	60
726	860937	860996	861056	861116	861176	861236	861295	861355	861415	861475	60
727	861534	861594	861654	861714	861773	861833	861893	861952	862012	862072	60
728	862131	862191	862251	862310	862370	862430	862489	862549	862608	862668	60
729	862728	862787	862847	862906	862966	863025	863085	863144	863204	863263	59
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
731	863917	863977	864036	864096	864155	864214	864274	864333	864392	864452	59
732	864511	864570	864630	864689	864748	864808	864867	864926	864985	865045	59
733	865104	865163	865222	865282	865341	865400	865459	865519	865578	865637	59
734	865696	865755	865814	865874	865933	865992	866051	866110	866169	866228	59
735	866287	866346	866405	866465	866524	866583	866642	866701	866760	866819	59
736	866878	866937	866996	867055	867114	867173	867232	867291	867350	867409	59
737	867467	867526	867585	867644	867703	867762	867821	867880	867939	867998	59
738	868056	868115	868174	868233	868292	868350	868409	868468	868527	868586	59
739	868644	868703	868762	868821	868879	868938	868997	869056	869114	869173	59
740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*)

N	0	1	2	3	4	5	6	7	8	9	D
741	869818	869877	869935	869994	870053	870111	870170	870228	870287	870345	59
742	870404	870462	870521	870579	870638	870696	870755	870813	870872	870930	58
743	870989	871047	871106	871164	871223	871281	871339	871398	871456	871515	58
744	871573	871631	871690	871748	871806	871865	871923	871981	872040	872098	58
745	872156	872215	872273	872331	872389	872448	872506	872564	872622	872681	58
746	872739	872797	872855	872913	872972	873030	873088	873146	873204	873262	58
747	873321	873379	873437	873495	873553	873611	873669	873727	873785	873844	58
748	873902	873960	874018	874076	874134	874192	874250	874308	874366	874424	58
749	874482	874540	874598	874656	874714	874772	874830	874888	874945	875003	58
750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
751	875640	875698	875756	875813	875871	875929	875987	876045	876102	876160	58
752	876218	876276	876333	876391	876449	876507	876564	876622	876680	876737	58
753	876795	876853	876910	876968	877026	877083	877141	877199	877256	877314	58
754	877371	877429	877487	877544	877602	877659	877717	877774	877832	877889	58
755	877947	878004	878062	878119	878177	878234	878292	878349	878407	878464	57
756	878522	878579	878637	878694	878752	878809	878866	878924	878981	879039	57
757	879096	879153	879211	879268	879325	879383	879440	879497	879555	879612	57
758	879669	879726	879784	879841	879898	879956	880013	880070	880127	880185	57
759	880242	880299	880356	880413	880471	880528	880585	880642	880699	880756	57
760	880814	880871	880928	880985	881042	881099	881156	881213	881271	881328	57
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
761	881385	881442	881499	881556	881613	881670	881727	881784	881841	881898	57
762	881955	882012	882069	882126	882183	882240	882297	882354	882411	882468	57
763	882525	882581	882638	882695	882752	882809	882866	882923	882980	883037	57
764	883093	883150	883207	883264	883321	883377	883434	883491	883548	883605	57
765	883661	883718	883775	883832	883888	883945	884002	884059	884115	884172	57
766	884229	884285	884342	884399	884455	884512	884569	884625	884682	884739	57
767	884795	884852	884909	884965	885022	885078	885135	885192	885248	885305	57
768	885361	885418	885474	885531	885587	885644	885700	885757	885813	885870	57
769	885926	885983	886039	886096	886152	886209	886265	886321	886378	886434	56
770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
771	887054	887111	887167	887223	887280	887336	887392	887449	887505	887561	56
772	887617	887674	887730	887786	887842	887898	887955	888011	888067	888123	56
773	888179	888236	888292	888348	888404	888460	888516	888573	888629	888685	56
774	888741	888797	888853	888909	888965	889021	889077	889134	889190	889246	56
775	889302	889358	889414	889470	889526	889582	889638	889694	889750	889806	56
776	889862	889918	889974	890030	890086	890141	890197	890253	890309	890365	56
777	890421	890477	890533	890589	890645	890700	890756	890812	890868	890924	56
778	890980	891035	891091	891147	891203	891259	891314	891370	891426	891482	56
779	891537	891593	891649	891705	891760	891816	891872	891928	891983	892039	56
780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
781	892651	892707	892762	892818	892873	892929	892985	893040	893096	893151	56
782	893207	893262	893318	893373	893429	893484	893540	893595	893651	893706	55
783	893762	893817	893873	893928	893984	894039	894094	894150	894205	894261	55
784	894316	894371	894427	894482	894538	894593	894648	894704	894759	894814	55
785	894870	894925	894980	895036	895091	895146	895201	895257	895312	895367	55
786	895423	895478	895533	895588	895644	895699	895754	895809	895864	895920	55
787	895975	896030	896085	896140	896195	896251	896306	896361	896416	896471	55
788	896526	896581	896636	896692	896747	896802	896857	896912	896967	897022	55
789	897077	897132	897187	897242	897297	897352	897407	897462	897517	897572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
791	898176	898231	898286	898341	898396	898451	898506	898561	898615	898670	55
792	898725	898780	898835	898890	898944	898999	899054	899109	899164	899218	55
793	899273	899328	899383	899437	899492	899547	899602	899656	899711	899766	55
794	899821	899875	899930	899985	900039	900094	900149	900203	900258	900312	55
795	900367	900422	900476	900531	900586	900640	900695	900749	900804	900859	55
796	900913	900968	901022	901077	901131	901186	901240	901295	901349	901404	55
797	901458	901513	901567	901622	901676	901731	901785	901840	901894	901948	54
798	902003	902057	902112	902166	902221	902275	902329	902384	902438	902492	54
799	902547	902601	902655	902710	902764	902818	902873	902927	902981	903036	54
800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
801	903633	903687	903741	903795	903849	903904	903958	904012	904066	904120	54
802	904174	904229	904283	904337	904391	904445	904499	904553	904607	904661	54
803	904716	904770	904824	904878	904932	904986	905040	905094	905148	905202	54
804	905256	905310	905364	905418	905472	905526	905580	905634	905688	905742	54
805	905796	905850	905904	905958	906012	906066	906119	906173	906227	906281	54
806	906335	906389	906443	906497	906551	906604	906658	906712	906766	906820	54
807	906874	906927	906981	907035	907089	907143	907196	907250	907304	907358	54
808	907411	907465	907519	907573	907626	907680	907734	907787	907841	907895	54
809	907949	908002	908056	908110	908163	908217	908270	908324	908378	908431	54
810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
811	909021	909074	909128	909181	909235	909289	909342	909396	909449	909503	54
812	909556	909609	909663	909716	909770	909823	909877	909930	909984	910037	53
813	910091	910144	910197	910251	910304	910358	910411	910464	910518	910571	53
814	910624	910678	910731	910784	910838	910891	910944	910998	911051	911104	53
815	911158	911211	911264	911317	911371	911424	911477	911530	911584	911637	53
816	911690	911743	911797	911850	911903	911956	912009	912063	912116	912169	53
817	912222	912275	912328	912381	912435	912488	912541	912594	912647	912700	53
818	912753	912806	912859	912913	912966	913019	913072	913125	913178	913231	53
819	913284	913337	913390	913443	913496	913549	913602	913655	913708	913761	53
820	913814	913867	913920	913973	914026	914079	914132	914184	914237	914290	53
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
821	914343	914396	914449	914502	914555	914608	914660	914713	914766	914819	53
822	914872	914925	914977	915030	915083	915136	915189	915241	915294	915347	53
823	915400	915453	915505	915558	915611	915664	915716	915769	915822	915875	53
824	915927	915980	916033	916085	916138	916191	916243	916296	916349	916401	53
825	916454	916507	916559	916612	916664	916717	916770	916822	916875	916927	53
826	916980	917033	917085	917138	917190	917243	917295	917348	917400	917453	53
827	917506	917558	917611	917663	917716	917768	917820	917873	917925	917978	52
828	918030	918083	918135	918188	918240	918293	918345	918397	918450	918502	52
829	918555	918607	918659	918712	918764	918816	918869	918921	918973	919026	52
830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
831	919601	919653	919706	919758	919810	919862	919914	919967	920019	920071	52
832	920123	920176	920228	920280	920332	920384	920436	920489	920541	920593	52
833	920645	920697	920749	920801	920853	920906	920958	921010	921062	921114	52
834	921166	921218	921270	921322	921374	921426	921478	921530	921582	921634	52
835	921686	921738	921790	921842	921894	921946	921998	922050	922102	922154	52
836	922206	922258	922310	922362	922414	922466	922518	922570	922622	922674	52
837	922725	922777	922829	922881	922933	922985	923037	923089	923140	923192	52
838	923244	923296	923348	923399	923451	923503	923555	923607	923658	923710	52
839	923762	923814	923865	923917	923969	924021	924072	924124	924176	924228	52
840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
841	924796	924848	924899	924951	925003	925054	925106	925157	925209	925261	52
842	925312	925364	925415	925467	925518	925570	925621	925673	925725	925776	52
843	925828	925879	925931	925982	926034	926085	926137	926188	926240	926291	51
844	926342	926394	926445	926497	926548	926600	926651	926702	926754	926805	51
845	926857	926908	926959	927011	927062	927114	927165	927216	927268	927319	51
846	927370	927422	927473	927524	927576	927627	927678	927730	927781	927832	51
847	927883	927935	927986	928037	928088	928140	928191	928242	928293	928345	51
848	928396	928447	928498	928549	928601	928652	928703	928754	928805	928857	51
849	928908	928959	929010	929061	929112	929163	929215	929266	929317	929368	51
850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
851	929930	929981	930032	930083	930134	930185	930236	930287	930338	930389	51
852	930440	930491	930542	930592	930643	930694	930745	930796	930847	930898	51
853	930949	931000	931051	931102	931153	931203	931254	931305	931356	931407	51
854	931458	931509	931560	931610	931661	931712	931763	931814	931865	931915	51
855	931966	932017	932068	932118	932169	932220	932271	932322	932372	932423	51
856	932474	932524	932575	932626	932677	932727	932778	932829	932879	932930	51
857	932981	933031	933082	933133	933183	933234	933285	933335	933386	933437	51
858	933487	933538	933589	933639	933690	933740	933791	933841	933892	933943	51
859	933993	934044	934094	934145	934195	934246	934296	934347	934397	934448	51
860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	51
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
861	935003	935054	935104	935154	935205	935255	935306	935356	935406	935457	50
862	935507	935558	935608	935658	935709	935759	935809	935860	935910	935960	50
863	936011	936061	936111	936162	936212	936262	936313	936363	936413	936463	50
864	936514	936564	936614	936665	936715	936765	936815	936865	936916	936966	50
865	937016	937066	937117	937167	937217	937267	937317	937367	937418	937468	50
866	937518	937568	937618	937668	937718	937769	937819	937869	937919	937969	50
867	938019	938069	938119	938169	938219	938269	938319	938370	938420	938470	50
868	938520	938570	938620	938670	938720	938770	938820	938870	938920	938970	50
869	939020	939070	939120	939170	939220	939270	939320	939369	939419	939469	50
870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
871	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
872	940516	940566	940616	940666	940716	940765	940815	940865	940915	940964	50
873	941014	941064	941114	941163	941213	941263	941313	941362	941412	941462	50
874	941511	941561	941611	941660	941710	941760	941809	941859	941909	941958	50
875	942008	942058	942107	942157	942207	942256	942306	942355	942405	942455	50
876	942504	942554	942603	942653	942702	942752	942801	942851	942901	942950	50
877	943000	943049	943099	943148	943198	943247	943297	943346	943396	943445	49
878	943495	943544	943593	943643	943692	943742	943791	943841	943890	943939	49
879	943989	944038	944088	944137	944186	944236	944285	944335	944384	944433	49
880	944483	944532	944581	944631	944680	944729	944779	944828	944877	944927	49
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
881	944976	945025	945074	945124	945173	945222	945272	945321	945370	945419	49
882	945469	945518	945567	945616	945665	945715	945764	945813	945862	945912	49
883	945961	946010	946059	946108	946157	946207	946256	946305	946354	946403	49
884	946452	946501	946551	946600	946649	946698	946747	946796	946845	916894	49
885	946943	946992	947041	947090	947140	947189	947238	947287	947336	947385	49
886	947434	947483	947532	947581	947630	947679	947728	947777	947826	947875	49
887	947924	947973	948022	948070	948119	948168	948217	948266	948315	948364	49
888	948413	948462	948511	948560	948609	948657	948706	948755	948804	948853	49
889	948902	948951	948999	949048	949097	949146	949195	949244	949292	949341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
891	949878	949926	949975	950024	950073	950121	950170	950219	950267	950316	49
892	950365	950414	950462	950511	950560	950608	950657	950706	950754	950803	49
893	950851	950900	950949	950997	951046	951095	951143	951192	951240	951289	49
894	951338	951386	951435	951483	951532	951580	951629	951677	951726	951775	49
895	951823	951872	951920	951969	952017	952066	952114	952163	952211	952260	49
896	952308	952356	952405	952453	952502	952550	952599	952647	952696	952744	48
897	952792	952841	952889	952938	952986	953034	953083	953131	953180	953228	48
898	953276	953325	953373	953421	953470	953518	953566	953615	953663	953711	48
899	953760	953808	953856	953905	953953	954001	954049	954098	954146	954194	48
900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
901	954725	954773	954821	954869	954918	954966	955014	955062	955110	955158	48
902	955207	955255	955303	955351	955399	955447	955495	955543	955592	955640	48
903	955688	955736	955784	955832	955880	955928	955976	956024	956072	956120	48
904	956168	956216	956265	956313	956361	956409	956457	956505	956553	956601	48
905	956649	956697	956745	956793	956840	956888	956936	956984	957032	957080	48
906	957128	957176	957224	957272	957320	957368	957416	957464	957512	957559	48
907	957607	957655	957703	957751	957799	957847	957894	957942	957990	958038	48
908	958086	958134	958181	958229	958277	958325	958373	958421	958468	958516	48
909	958564	958612	958659	958707	958755	958803	958850	958898	958946	958994	48
910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
911	959518	959566	959614	959661	959709	959757	959804	959852	959900	959947	48
912	959995	960042	960090	960138	960185	960233	960280	960328	960376	960423	48
913	960471	960518	960566	960613	960661	960709	960756	960804	960851	960899	48
914	960946	960994	961041	961089	961136	961184	961231	961279	961326	961374	48
915	961421	961469	961516	961563	961611	961658	961706	961753	961801	961848	47
916	961895	961943	961990	962038	962085	962132	962180	962227	962275	962322	47
917	962369	962417	962464	962511	962559	962606	962653	962701	962748	962795	47
918	962843	962890	962937	962985	963032	963079	963126	963174	963221	963268	47
919	963316	963363	963410	963457	963504	963552	963599	963646	963693	963741	47
920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
921	964260	964307	964354	964401	964448	964495	964542	964590	964637	964684	47
922	964731	964778	964825	964872	964919	964966	965013	965061	965108	965155	47
923	965202	965249	965296	965343	965390	965437	965484	965531	965578	965625	47
924	965672	965719	965766	965813	965860	965907	965954	966001	966048	966095	47
925	966142	966189	966236	966283	966329	966376	966423	966470	966517	966564	47
926	966611	966658	966705	966752	966799	966845	966892	966939	966986	967033	47
927	967080	967127	967173	967220	967267	967314	967361	967408	967454	967501	47
928	967548	967595	967642	967688	967735	967782	967829	967875	967922	967969	47
929	968016	968062	968109	968156	968203	968249	968296	968343	968390	968436	47
930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
931	968950	968996	969043	969090	969136	969183	969229	969276	969323	969369	47
932	969416	969463	969509	969556	969602	969649	969695	969742	969789	969835	47
933	969882	969928	969975	970021	970068	970114	970161	970207	970254	970300	46
934	970347	970393	970440	970486	970533	970579	970626	970672	970719	970765	46
935	970812	970858	970904	970951	970997	971044	971090	971137	971183	971229	46
936	971276	971322	971369	971415	971461	971508	971554	971601	971647	971693	46
937	971740	971786	971832	971879	971925	971971	972018	972064	972110	972157	46
938	972203	972249	972295	972342	972388	972434	972481	972527	972573	972619	46
939	972666	972712	972758	972804	972851	972897	972943	972989	973035	973082	46
940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*).

N	0	1	2	3	4	5	6	7	8	9	D
941	973590	973636	973682	973728	973774	973820	973866	973913	973959	974005	46
942	974051	974097	974143	974189	974235	974281	974327	974374	974420	974466	46
943	974512	974558	974604	974650	974696	974742	974788	974834	974880	974926	46
944	974972	975018	975064	975110	975156	975202	975248	975294	975340	975386	46
945	975432	975478	975524	975570	975616	975662	975707	975753	975799	975845	46
946	975891	975937	975983	976029	976075	976121	976167	976212	976258	976304	46
947	976350	976396	976442	976488	976533	976579	976625	976671	976717	976763	46
948	976808	976854	976900	976946	976992	977037	977083	977129	977175	977220	46
949	977266	977312	977358	977403	977449	977495	977541	977586	977632	977678	46
950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
951	978181	978226	978272	978317	978363	978409	978454	978500	978546	978591	46
952	978637	978683	978728	978774	978819	978865	978911	978956	979002	979047	46
953	979093	979138	979184	979230	979275	979321	979366	979412	979457	979503	46
954	979548	979594	979639	979685	979730	979776	979821	979867	979912	979958	46
955	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
956	980458	980503	980549	980594	980640	980685	980730	980776	980821	980867	45
957	980912	980957	981003	981048	981093	981139	981184	981229	981275	981320	45
958	981366	981411	981456	981501	981547	981592	981637	981683	981728	981773	45
959	981819	981864	981909	981954	982000	982045	982090	982135	982181	982226	45
960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**D**

Logarithms of Numbers (*continued*)

N	0	1	2	3	4	5	6	7	8	9	D
961	982723	982769	982814	982859	982904	982949	982994	983040	983085	983130	45
962	983175	983220	983265	983310	983356	983401	983446	983491	983536	983581	45
963	983626	983671	983716	983762	983807	983852	983897	983942	983987	984032	45
964	984077	984122	984167	984212	984257	984302	984347	984392	984437	984482	45
965	984527	984572	984617	984662	984707	984752	984797	984842	984887	984932	45
966	984977	985022	985067	985112	985157	985202	985247	985292	985337	985382	45
967	985426	985471	985516	985561	985606	985651	985696	985741	985786	985830	45
968	985875	985920	985965	986010	986055	986100	986144	986189	986234	986279	45
969	986324	986369	986413	986458	986503	986548	986593	986637	986682	986727	45
970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
971	987219	987264	987309	987353	987398	987443	987488	987532	987577	987622	45
972	987666	987711	987756	987800	987845	987890	987934	987979	988024	988068	45
973	988113	988157	988202	988247	988291	988336	988381	988425	988470	988514	45
974	988559	988604	988648	988693	988737	988782	988826	988871	988916	988960	45
975	989005	989049	989094	989138	989183	989227	989272	989316	989361	989405	44
976	989450	989494	989539	989583	989628	989672	989717	989761	989806	989850	44
977	989895	989939	989983	990028	990072	990117	990161	990206	990250	990294	44
978	990339	990383	990428	990472	990516	990561	990605	990650	990694	990738	44
979	990783	990827	990871	990916	990960	991004	991049	991093	991137	991182	44
980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
N	0	1	2	3	4	5	6	7	8	9	D

Logarithms of Numbers (*concluded*).

N	0	1	2	3	4	5	6	7	8	9	D
981	991669	991713	991758	991802	991846	991890	991935	991979	992023	992067	44
982	992111	992156	992200	992244	992288	992333	992377	992421	992465	992509	44
983	992554	992598	992642	992686	992730	992774	992819	992863	992907	992951	44
984	992995	993039	993083	993127	993172	993216	993260	993304	993348	993392	44
985	993436	993480	993524	993568	993613	993657	993701	993745	993789	993833	44
986	993877	993921	993965	994009	994053	994097	994141	994185	994229	994273	44
987	994317	994361	994405	994449	994493	994537	994581	994625	994669	994713	44
988	994757	994801	994845	994889	994933	994977	995021	995065	995108	995152	44
989	995196	995240	995284	995328	995372	995416	995460	995504	995547	995591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
991	996074	996117	996161	996205	996249	996293	996337	996380	996424	996468	44
992	996512	996555	996599	996643	996687	996731	996774	996818	996862	996906	44
993	996949	996993	997037	997080	997124	997168	997212	997255	997299	997343	44
994	997386	997430	997474	997517	997561	997605	997648	997692	997736	997779	44
995	997823	997867	997910	997954	997998	998041	998085	998129	998172	998216	44
996	998259	998303	998347	998390	998434	998477	998521	998564	998608	998652	44
997	998695	998739	998782	998826	998869	998913	998956	999000	999043	999087	44
998	999131	999174	999218	999261	999305	999348	999392	999435	999479	999522	44
999	999565	999609	999652	999696	999739	999783	999826	999870	999913	999957	44
N	0	1	2	3	4	5	6	7	8	9	D

Hyperbolic or Napierian Logarithms of Numbers.

N	Log	N	Log.	N	Log.	N	Log.
1·01	·0099	1·41	·3436	1·81	5933	2·21	·7930
1·02	·0198	1·42	·3507	1·82	·5988	2·22	·7975
1·03	·0296	1·43	·3577	1·83	·6043	2·23	·8020
1·04	·0392	1·44	·3646	1·84	·6098	2 24	·8065
1·05	·0488	1·45	·3716	1·85	·6152	2·25	·8109
1·06	·0583	1·46	·3784	1·86	·6206	2·26	·8154
1·07	·0677	1·47	·3853	1·87	·6259	2·27	·8198
1·08	·0770	1·48	·3920	1·88	·6313	2·28	·8242
1·09	·0862	1 49	·3988	1·89	·6366	2·29	·8286
1·10	·0953	1·50	·4055	1·90	·6419	2·30	·8329
1·11	·1044	1 51	·4121	1·91	·6471	2·31	·8372
1·12	·1133	1·52	·4187	1·92	·6523	2·32	·8416
1·13	·1222	1·53	·4253	1·93	·6575	2·33	·8459
1·14	·1310	1·54	·4318	1·94	·6627	2·34	·8502
1·15	·1398	1·55	·4383	1·95	·6678	2·35	·8544
1·16	·1484	1·56	·4447	1·96	·6729	2 36	8587
1·17	·1570	1·57	·4511	1 97	·6780	2·37	·8629
1·18	·1655	1·58	·4574	1·98	·6831	2 38	·8671
1·19	·1740	1·59	·4637	1·99	·6881	2·39	·8713
1·20	·1823	1·60	·4700	2·00	6931	2·40	·8755
1·21	·1906	1·61	·4762	2·01	·6981	2·41	·8796
1·22	·1988	1·62	·4824	2·02	·7031	2·42	·8838
1·23	·2070	1·63	·4886	2·03	·7080	2 43	·8879
1·24	2151	1·64	·4947	2·04	·7129	2·44	·8920
1·25	·2231	1·65	·5008	2 05	·7178	2·45	·8961
1·26	·2311	1·66	·5068	2·06	·7227	2·46	·9002
1·27	·2390	1·67	·5128	2·07	·7275	2·47	·9042
1·28	·2469	1·68	·5188	2·08	·7324	2·48	·9083
1·29	·2546	1·69	·5247	2·09	·7372	2·49	·9123
1·30	·2624	1·70	·5306	2·10	·7419	2·50	·9163
1·31	·2700	1·71	·5365	2·11	·7467	2 51	·9203
1·32	·2776	1·72	·5423	2·12	·7514	2·52	·9243
1·33	·2852	1·73	·5481	2·13	·7561	2·53	·9282
1·34	·2927	1·74	·5539	2·14	·7608	2·54	·9322
1·35	·3001	1·75	·5596	2·15	·7655	2·55	·9361
1·36	·3075	1·76	·5653	2·16	·7701	2·56	·9400
1·37	·3148	1·77	·5710	2·17	·7747	2·57	·9439
1·38	·3221	1·78	·5766	2·18	·7793	2·58	·9478
1·39	·3293	1·79	·5822	2·19	·7839	2·59	·9517
1·40	·3365	1·80	·5878	2·20	·7885	2·60	·9555

Hyperbolic Logarithms (*continued*).

N	Log	N	Log.	N	Log.	N	Log.
2·61	·9594	3·01	1·1019	3·41	1·2267	3·81	1·3376
2·62	·9632	3·02	1·1053	3·42	1·2296	3·82	1·3403
2·63	·9670	3·03	1·1086	3·43	1·2326	3·83	1·3429
2·64	·9708	3·04	1·1119	3·44	1·2355	3·84	1·3455
2·65	·9746	3·05	1·1151	3·45	1·2384	3·85	1·3481
2·66	·9783	3·06	1·1184	3·46	1·2413	3·86	1·3507
2·67	·9821	3·07	1·1217	3·47	1·2442	3·87	1·3533
2·68	·9858	3·08	1·1249	3·48	1·2470	3·88	1·3558
2·69	·9895	3·09	1·1282	3·49	1·2499	3·89	1·3584
2·70	·9933	3·10	1·1314	3·50	1·2528	3·90	1·3610
2·71	·9969	3·11	1·1346	3·51	1·2556	3·91	1·3635
2·72	1·0006	3·12	1·1378	3·52	1·2585	3·92	1·3661
2·73	1·0043	3·13	1·1410	3·53	1·2613	3·93	1·3686
2·74	1·0080	3·14	1·1442	3·54	1·2641	3·94	1·3712
2·75	1·0116	3·15	1·1474	3·55	1·2669	3·95	1·3737
2·76	1·0152	3·16	1·1506	3·56	1·2698	3·96	1·3762
2·77	1·0188	3·17	1·1537	3·57	1·2726	3·97	1·3788
2·78	1·0225	3·18	1·1569	3·58	1·2754	3·98	1·3813
2·79	1·0260	3·19	1·1600	3·59	1·2782	3·99	1·3838
2·80	1·0296	3·20	1·1632	3·60	1·2809	4·00	1·3863
2·81	1·0332	3·21	1·1663	3·61	1·2837	4·01	1·3888
2·82	1·0367	3·22	1·1694	3·62	1·2865	4·02	1·3913
2·83	1·0403	3·23	1·1725	3·63	1·2892	4·03	1·3938
2·84	1·0438	3·24	1·1756	3·64	1·2920	4·04	1·3962
2·85	1·0473	3·25	1·1787	3·65	1·2947	4·05	1·3987
2·86	1·0508	3·26	1·1817	3·66	1·2975	4·06	1·4012
2·87	1·0543	3·27	1·1848	3·67	1·3002	4·07	1·4036
2·88	1·0578	3·28	1·1878	3·68	1·3029	4·08	1·4061
2·89	1·0613	3·29	1·1909	3·69	1·3056	4·09	1·4085
2·90	1·0647	3·30	1·1939	3·70	1·3083	4·10	1·4110
2·91	1·0682	3·31	1·1969	3·71	1·3110	4·11	1·4134
2·92	1·0716	3·32	1·2000	3·72	1·3137	4·12	1·4159
2·93	1·0750	3·33	1·2030	3·73	1·3164	4·13	1·4183
2·94	1·0784	3·34	1·2060	3·74	1·3191	4·14	1·4207
2·95	1·0818	3·35	1·2090	3·75	1·3218	4·15	1·4231
2·96	1·0852	3·36	1·2119	3·76	1·3244	4·16	1·4255
2·97	1·0886	3·37	1·2149	3·77	1·3271	4·17	1·4279
2·98	1·0919	3·38	1·2179	3·78	1·3297	4·18	1·4303
2·99	1·0953	3·39	1·2208	3·79	1·3324	4·19	1·4327
3·00	1·0986	3·40	1·2238	3·80	1·3350	4·20	1·4351

Hyperbolic Logarithms (*continued*).

N	Log.	N	Log.	N	Log.	N	Log.
4·21	1·4375	4·61	1·5282	5·01	1·6114	5·41	1·6882
4·22	1·4398	4·62	1·5304	5·02	1·6134	5·42	1·6901
4·23	1·4422	4·63	1·5326	5·03	1·6154	5·43	1·6919
4·24	1·4446	4·64	1·5347	5·04	1·6174	5·44	1·6938
4·25	1·4469	4·65	1·5369	5·05	1·6194	5·45	1·6956
4·26	1·4493	4·66	1·5390	5·06	1·6214	5·46	1·6974
4·27	1·4516	4·67	1·5412	5·07	1·6233	5·47	1·6993
4·28	1·4540	4·68	1·5433	5·08	1·6253	5·48	1·7011
4·29	1·4563	4·69	1·5454	5·09	1·6273	5·49	1·7029
4·30	1·4586	4·70	1·5476	5·10	1·6292	5·50	1·7047
4·31	1·4609	4·71	1·5497	5·11	1·6312	5·51	1·7066
4·32	1·4633	4·72	1·5518	5·12	1·6332	5·52	1·7084
4·33	1·4656	4·73	1·5539	5·13	1·6351	5·53	1·7102
4·34	1·4679	4·74	1·5560	5·14	1·6371	5·54	1·7120
4·35	1·4702	4·75	1·5581	5·15	1·6390	5·55	1·7138
4·36	1·4725	4·76	1·5602	5·16	1·6409	5·56	1·7156
4·37	1·4748	4·77	1·5623	5·17	1·6429	5·57	1·7174
4·38	1·4770	4·78	1·5644	5·18	1·6448	5·58	1·7192
4·39	1·4793	4·79	1·5665	5·19	1·6467	5·59	1·7210
4·40	1·4816	4·80	1·5686	5·20	1·6487	5·60	1·7228
4·41	1·4839	4·81	1·5707	5·21	1·6506	5·61	1·7246
4·42	1·4861	4·82	1·5728	5·22	1·6525	5·62	1·7263
4·43	1·4884	4·83	1·5748	5·23	1·6544	5·63	1·7281
4·44	1·4907	4·84	1·5769	5·24	1·6563	5·64	1·7299
4·45	1·4929	4·85	1·5790	5·25	1·6582	5·65	1·7317
4·46	1·4951	4·86	1·5810	5·26	1·6601	5·66	1·7334
4·47	1·4974	4·87	1·5831	5·27	1·6620	5·67	1·7352
4·48	1·4996	4·88	1·5851	5·28	1·6639	5·68	1·7370
4·49	1·5019	4·89	1·5872	5·29	1·6658	5·69	1·7387
4·50	1·5041	4·90	1·5892	5·30	1·6677	5·70	1·7405
4·51	1·5063	4·91	1·5913	5·31	1·6696	5·71	1·7422
4·52	1·5085	4·92	1·5933	5·32	1·6715	5·72	1·7440
4·53	1·5107	4·93	1·5953	5·33	1·6734	5·73	1·7457
4·54	1·5129	4·94	1·5974	5·34	1·6752	5·74	1·7475
4·55	1·5151	4·95	1·5994	5·35	1·6771	5·75	1·7492
4·56	1·5173	4·96	1·6014	5·36	1·6790	5·76	1·7509
4·57	1·5195	4·97	1·6034	5·37	1·6808	5·77	1·7527
4·58	1·5217	4·98	1·6054	5·38	1·6827	5·78	1·7544
4·59	1·5239	4·99	1·6074	5·39	1·6845	5·79	1·7561
4·60	1·5261	5·00	1·6094	5·40	1·6864	5·80	1·7579

Hyperbolic Logarithms (*continued*).

N	Log.	N	Log.	N	Log.	N	Log.
5·81	1·7596	6·21	1·8262	6·61	1·8886	7·01	1·9473
5·82	1·7613	6·22	1·8278	6·62	1·8901	7·02	1·9488
5·83	1·7630	6·23	1·8294	6·63	1·8916	7·03	1·9502
5·84	1·7647	6·24	1·8310	6·64	1·8931	7·04	1·9516
5·85	1·7664	6·25	1·8326	6·65	1·8946	7·05	1·9530
5·86	1·7681	6·26	1·8342	6·66	1·8961	7·06	1·9544
5·87	1·7699	6·27	1·8358	6·67	1·8976	7·07	1·9559
5·88	1·7716	6·28	1·8374	6·68	1·8991	7·08	1·9573
5·89	1·7733	6·29	1·8390	6·69	1·9006	7·09	1·9587
5·90	1·7750	6·30	1·8405	6·70	1·9021	7·10	1·9601
5·91	1·7766	6·31	1·8421	6·71	1·9036	7·11	1·9615
5·92	1·7783	6·32	1·8437	6·72	1·9051	7·12	1·9629
5·93	1·7800	6·33	1·8453	6·73	1·9066	7·13	1·9643
5·94	1·7817	6·34	1·8469	6·74	1·9081	7·14	1·9657
5·95	1·7834	6·35	1·8485	6·75	1·9095	7·15	1·9671
5·96	1·7851	6·36	1·8500	6·76	1·9110	7·16	1·9685
5·97	1·7867	6·37	1·8516	6·77	1·9125	7·17	1·9699
5·98	1·7884	6·38	1·8532	6·78	1·9140	7·18	1·9713
5·99	1·7901	6·39	1·8547	6·79	1·9155	7·19	1·9727
6·00	1·7918	6·40	1·8563	6·80	1·9169	7·20	1·9741
6·01	1·7934	6·41	1·8579	6·81	1·9184	7·21	1·9755
6·02	1·7951	6·42	1·8594	6·82	1·9199	7·22	1·9769
6·03	1·7967	6·43	1·8610	6·83	1·9213	7·23	1·9782
6·04	1·7984	6·44	1·8625	6·84	1·9228	7·24	1·9796
6·05	1·8001	6·45	1·8641	6·85	1·9242	7·25	1·9810
6·06	1·8017	6·46	1·8656	6·86	1·9257	7·26	1·9824
6·07	1·8034	6·47	1·8672	6·87	1·9272	7·27	1·9838
6·08	1·8050	6·48	1·8687	6·88	1·9286	7·28	1·9851
6·09	1·8066	6·49	1·8703	6·89	1·9301	7·29	1·9865
6·10	1·8083	6·50	1·8718	6·90	1·9315	7·30	1·9879
6·11	1·8099	6·51	1·8733	6·91	1·9330	7·31	1·9892
6·12	1·8116	6·52	1·8749	6·92	1·9344	7·32	1·9906
6·13	1·8132	6·53	1·8764	6·93	1·9359	7·33	1·9920
6·14	1·8148	6·54	1·8779	6·94	1·9373	7·34	1·9933
6·15	1·8165	6·55	1·8795	6·95	1·9387	7·35	1·9947
6·16	1·8181	6·56	1·8810	6·96	1·9402	7·36	1·9961
6·17	1·8197	6·57	1·8825	6·97	1·9416	7·37	1·9974
6·18	1·8213	6·58	1·8840	6·98	1·9430	7·38	1·9988
6·19	1·8229	6·59	1·8856	6·99	1·9445	7·39	2·0001
6·20	1·8245	6·60	1·8871	7·00	1·9459	7·40	2·0015

Hyperbolic Logarithms (*continued*).

N	Log.	N	Log	N	Log	N	Log
7·41	2·0028	7·81	2·0554	8·21	2·1054	8·61	2·1529
7·42	2·0042	7·82	2·0567	8·22	2·1066	8·62	2·1541
7·43	2·0055	7·83	2·0580	8·23	2·1078	8·63	2·1552
7·44	2·0069	7·84	2·0592	8·24	2·1090	8·64	2·1564
7·45	2·0082	7·85	2·0605	8·25	2·1102	8·65	2·1576
7·46	2·0096	7·86	2·0618	8·26	2·1114	8·66	2·1587
7·47	2·0109	7·87	2·0631	8·27	2·1126	8·67	2·1599
7·48	2·0122	7·88	2·0643	8·28	2·1138	8·68	2·1610
7·49	2·0136	7·89	2·0656	8·29	2·1150	8·69	2·1622
7·50	2·0149	7·90	2·0669	8·30	2·1163	8·70	2·1633
7·51	2·0162	7·91	2·0681	8·31	2·1175	8·71	2·1645
7·52	2·0176	7·92	2·0694	8·32	2·1187	8·72	2·1656
7·53	2·0189	7·93	2·0707	8·33	2·1199	8·73	2·1668
7·54	2·0202	7·94	2·0719	8·34	2·1211	8·74	2·1679
7·55	2·0215	7·95	2·0732	8·35	2·1223	8·75	2·1691
7·56	2·0229	7·96	2·0744	8·36	2·1235	8·76	2·1702
7·57	2·0242	7·97	2·0757	8·37	2·1247	8·77	2·1713
7·58	2·0255	7·98	2·0769	8·38	2·1258	8·78	2·1725
7·59	2·0268	7·99	2·0782	8·39	2·1270	8·79	2·1736
7·60	2·0281	8·00	2·0794	8·40	2·1282	8·80	2·1748
7·61	2·0295	8·01	2·0807	8·41	2·1294	8·81	2·1759
7·62	2·0308	8·02	2·0819	8·42	2·1306	8·82	2·1770
7·63	2·0321	8·03	2·0832	8·43	2·1318	8·83	2·1782
7·64	2·0334	8·04	2·0844	8·44	2·1330	8·84	2·1793
7·65	2·0347	8·05	2·0857	8·45	2·1342	8·85	2·1804
7·66	2·0360	8·06	2·0869	8·46	2·1353	8·86	2·1815
7·67	2·0373	8·07	2·0882	8·47	2·1365	8·87	2·1827
7·68	2·0386	8·08	2·0894	8·48	2·1377	8·88	2·1838
7·69	2·0399	8·09	2·0906	8·49	2·1389	8·89	2·1849
7·70	2·0412	8·10	2·0919	8·50	2·1401	8·90	2·1861
7·71	2·0425	8·11	2·0931	8·51	2·1412	8·91	2·1872
7·72	2·0438	8·12	2·0943	8·52	2·1424	8·92	2·1883
7·73	2·0451	8·13	2·0956	8·53	2·1436	8·93	2·1894
7·74	2·0464	8·14	2·0968	8·54	2·1448	8·94	2·1905
7·75	2·0477	8·15	2·0980	8·55	2·1459	8·95	2·1917
7·76	2·0490	8·16	2·0992	8·56	2·1471	8·96	2·1928
7·77	2·0503	8·17	2·1005	8·57	2·1483	8·97	2·1939
7·78	2·0516	8·18	2·1017	8·58	2·1494	8·98	2·1950
7·79	2·0528	8·19	2·1029	8·59	2·1506	8·99	2·1961
7·80	2·0541	8·20	2·1041	8·60	2·1518	9·00	2·1972

Hyperbolic Logarithms (*concluded*).

N	Log	N	Log	N	Log.	N	Log.
9·01	2·1983	9·26	2·2257	9·51	2·2523	9·76	2 2783
9·02	2 1994	9 27	2·2268	9·52	2·2534	9·77	2·2793
9·03	2·2006	9·28	2·2279	9·53	2·2544	9·78	2·2803
9·04	2·2017	9·29	2·2289	9·54	2·2555	9·79	2·2814
9·05	2·2028	9·30	2·2300	9·55	2·2565	9·80	2 2824
9·06	2·2039	9·31	2·2311	9·56	2·2576	9·81	2·2834
9·07	2 2050	9·32	2·2322	9·57	2·2586	9·82	2·2844
9·08	2·2061	9·33	2·2332	9·58	2·2597	9·83	2·2854
9·09	2·2072	9·34	2·2343	9·59	2·2607	9·84	2·2865
9·10	2·2083	9·35	2·2354	9·60	2·2618	9·85	2·2875
9·11	2·2094	9·36	2·2364	9·61	2·2628	9·86	2·2885
9·12	2·2105	9·37	2·2375	9·62	2·2638	9·87	2·2895
9·13	2·2116	9·38	2·2386	9·63	2·2649	9·88	2·2905
9 14	2 2127	9·39	2·2396	9·64	2·2659	9·89	2·2915
9 15	2·2138	9·40	2·2407	9 65	2·2670	9·90	2·2925
9·16	2·2148	9 41	2·2418	9·66	2·2680	9·91	2·2935
9·17	2·2159	9·42	2·2428	9·67	2·2690	9·92	2·2946
9·18	2·2170	9·43	2·2439	9·68	2·2701	9·93	2·2956
9·19	2·2181	9·44	2·2450	9·69	2 2711	9·94	2 2966
9·20	2·2192	9·45	2·2460	9·70	2·2721	9·95	2·2976
9·21	2 2203	9·46	2·2471	9·71	2·2732	9·96	2·2986
9·22	2·2214	9·47	2·2481	9·72	2·2742	9 97	2·2996
9·23	2·2225	9·48	2·2492	9·73	2·2752	9·98	2·3006
9·24	2·2235	9·49	2 2502	9·74	2·2762	9·99	2·3016
9·25	2·2246	9·50	2·2513	9·75	2·2773	10·00	2·3026

The hyperbolic logarithm of a number may be obtained by multiplying the common logarithm of the number by 2·302585.

The hyperbolic logarithm of a number which is 10 times a number in the above table may be obtained by adding 2·3026 (the hyperbolic logarithm of 10) to the logarithm in the table.

Example.—To find the hyperbolic logarithm of 91·6

From the table $\log_e 9{\cdot}16 = 2{\cdot}2148$

$2{\cdot}3026$

$\log_e 91{\cdot}6 = 4{\cdot}5174$

The hyperbolic logarithm of a number which is 100 times a number in the above table may be obtained by adding 2·3026 twice, to the logarithm in the table.

Example.—To find the hyperbolic logarithm of 916

From the table $\log_e 9{\cdot}16 = 2{\cdot}2148$

$2{\cdot}3026$

$2{\cdot}3026$

$\log_e 916 = 6{\cdot}8200$

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers.

The use of the table which follows may be greatly extended by aid of the following notes and rules:—

The number in the first, or last column =	Square root of number in 2nd column	$n = \sqrt{n^2}$.
	Cube root of number in 3rd column	$n = \sqrt[3]{n^3}$.
	Square of number in 4th column	$n = (\sqrt{n})^2$.
	Cube of number in 5th column	$n = (\sqrt[3]{n})^3$.
	Reciprocal of number in 6th column	$n = 1 \div \frac{1}{n}$.
The number in the second column. =	Square of cube root of number in 3rd column	$n^2 = (\sqrt[3]{n^3})^2$.
	Fourth power of number in 4th column	$n^2 = (\sqrt{n})^4$.
	Sixth power of number in 5th column	$n^2 = (\sqrt[3]{n})^6$.
The number in the third column. =	Cube of square root of number in 2nd column	$n^3 = (\sqrt{n^2})^3$.
	Sixth power of number in 4th column.	$n^3 = (\sqrt{n})^6$.
	Ninth power of number in 5th column.	$n^3 = (\sqrt[3]{n})^9$.
The number in the fourth column. =	Fourth root of number in 2nd column.	$\sqrt{n} = \sqrt[4]{n^2}$.
	Sixth root of number in 3rd column.	$\sqrt{n} = \sqrt[6]{n^3}$.
	Square root of cube of number in 5th column	$\sqrt{n} = \sqrt{(\sqrt[3]{n})^3}$.
The number in the fifth column. =	Sixth root of number in 2nd column.	$\sqrt[3]{n} = \sqrt[6]{n^2}$.
	Ninth root of number in 3rd column.	$\sqrt[3]{n} = \sqrt[9]{n^3}$.
	Cube root of square of number in 4th column	$\sqrt[3]{n} = \sqrt[3]{(\sqrt{n})^2}$.

For numbers larger than those given in the table, the following rules are sometimes useful:—

The square or cube of a number is equal to the product of the squares or cubes respectively of its factors.

From Table.

Thus, $1778^2 = (2 \times 889)^2 = 2^2 \times 889^2 = 4 \times 790321 = 3161284$

$2895^2 = (3 \times 965)^2 = 3^2 \times 965^2 = 9 \times 931225 = 8381025$

$1396^3 = (2 \times 698)^3 = 2^3 \times 698^3 = 8 \times 340068392 = 2720547136$

The square root or cube root of a number is equal to the product of the square roots or cube roots respectively of its factors.

From Table.

Thus, $\sqrt{1964} = \sqrt{(4 \times 491)} = \sqrt{4} \times \sqrt{491} = 2 \times 22{\cdot}1585 = 44{\cdot}317$

$\sqrt{5058} = \sqrt{(9 \times 562)} = \sqrt{9} \times \sqrt{562} = 3 \times 23{\cdot}7065 = 71{\cdot}1195$

$\sqrt[3]{2768} = \sqrt[3]{(8 \times 346)} = \sqrt[3]{8} \times \sqrt[3]{346} = 2 \times 7{\cdot}0203 = 14{\cdot}0406$

Decimals.—The number of decimal places in the exact *square* of a number is equal to *twice* the number of decimal places in the number. Or, if the decimal point be moved *one* place in the number, it must be moved *two* places in the square of the number, in the same direction

The number of decimal places in the exact *cube* of a number is equal to *three times* the number of decimal places in the number. Or, if the decimal point be moved *one* place in the number, it must be moved *three* places in the cube of the number, in the same direction.

EXAMPLES.—$16^2 = 256$ $\quad 16^3 = 4096$

$1{\cdot}6^2 = 2{\cdot}56$ $\quad 1{\cdot}6^3 = 4{\cdot}096$

${\cdot}16^2 = {\cdot}0256$ $\quad {\cdot}16^3 = {\cdot}004096$

${\cdot}016^2 = {\cdot}000256$ $\quad {\cdot}016^3 = {\cdot}000004096$

Also, if the decimal point be moved *two* places in the number, it must be moved *one* place in the square root of the number, in the same direction; and if the decimal point be moved *three* places in the number, it must be moved *one* place in the cube root of the number, in the same direction.

EXAMPLES.— $\sqrt{16} = 4$ $\quad \sqrt{160} = 12{\cdot}6491 \ldots$

$\sqrt{{\cdot}16} = {\cdot}4$ $\quad \sqrt{1{\cdot}6} = 1{\cdot}26491 \ldots$

$\sqrt{{\cdot}0016} = {\cdot}04$ $\quad \sqrt{{\cdot}016} = {\cdot}126491 \ldots$

$\sqrt[3]{16} = 2{\cdot}5198 \ldots$ $\quad \sqrt[3]{160} = 5{\cdot}4288 \ldots$

$\sqrt[3]{{\cdot}016} = {\cdot}25198 \ldots$ $\quad \sqrt[3]{{\cdot}16} = {\cdot}54288 \ldots$

In the case of the reciprocal of a number, if the decimal point is moved in the number, it must be moved an equal number of places in the reciprocal, in the *opposite direction.*

EXAMPLES.—

Number	32	3·2	·32	·032	3200
Reciprocal	·03125	·3125	3·125	31·25	·0003125

The product of a number and its reciprocal is equal to 1.

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers.

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
1	1	1	1	1	1	1
2	4	8	1·4142	1·2599	·500000	2
3	9	27	1·7321	1·4422	·333333	3
4	16	64	2	1·5874	·250000	4
5	25	125	2·2361	1·7100	·200000	5
6	36	216	2·4495	1·8171	·166667	6
7	49	343	2·6458	1·9129	·142857	7
8	64	512	2·8284	2	·125000	8
9	81	729	3	2·0801	·111111	9
10	100	1000	3·1623	2·1544	·100000	10
11	121	1331	3·3166	2·2240	·090909	11
12	144	1728	3·4641	2·2894	·083333	12
13	169	2197	3·6056	2·3513	·076923	13
14	196	2744	3·7417	2·4101	·071429	14
15	225	3375	3·8730	2·4662	·066667	15
16	256	4096	4	2·5198	·062500	16
17	289	4913	4·1231	2·5713	·058824	17
18	324	5832	4·2426	2·6207	·055556	18
19	361	6859	4·3589	2·6684	·052632	19
20	400	8000	4·4721	2·7144	·050000	20
21	441	9261	4·5826	2·7589	·047619	21
22	484	10648	4·6904	2·8020	·045455	22
23	529	12167	4·7958	2·8439	·043478	23
24	576	13824	4·8990	2·8845	·041667	24
25	625	15625	5	2·9240	·040000	25
26	676	17576	5·0990	2·9625	·038462	26
27	729	19683	5·1962	3	·037037	27
28	784	21952	5·2915	3·0366	·035714	28
29	841	24389	5·3852	3·0723	·034483	29
30	900	27000	5·4772	3·1072	·033333	30
31	961	29791	5·5678	3·1414	·032258	31
32	1024	32768	5·6569	3·1748	·031250	32
33	1089	35937	5·7446	3·2075	·030303	33
34	1156	39304	5·8310	3·2396	·029412	34
35	1225	42875	5·9161	3·2711	·028571	35
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
36	1296	46656	6	3·3019	·027778	36
37	1369	50653	6·0828	3·3322	·027027	37
38	1444	54872	6·1644	3·3620	·026316	38
39	1521	59319	6 2450	3·3912	025641	39
40	1600	64000	6·3246	3·4200	025000	40
41	1681	68921	6·4031	3·4482	·024390	41
42	1764	74088	6·4807	3·4760	·023810	42
43	1849	79507	6·5574	3·5034	023256	43
44	1936	85184	6·6332	3·5303	·022727	44
45	2025	91125	6·7082	3 5569	·022222	45
46	2116	97336	6·7823	3·5830	·021739	46
47	2209	103823	6·8557	3·6088	·021277	47
48	2304	110592	6·9282	3·6342	·020833	48
49	2401	117649	7	3·6593	020408	49
50	2500	125000	7·0711	3·6840	·020000	50
51	2601	132651	7·1414	3·7084	·019608	51
52	2704	140608	7·2111	3·7325	019231	52
53	2809	148877	7 2801	3·7563	·018868	53
54	2916	157464	7·3485	3·7798	·018519	54
55	3025	166375	7·4162	3·8030	018182	55
56	3136	175616	7·4833	3·8259	·017857	56
57	3249	185193	7·5498	3·8485	·017544	57
58	3364	195112	7·6158	3·8709	·017241	58
59	3481	205379	7·6811	3·8930	·016949	59
60	3600	216000	7·7460	3·9149	·016667	60
61	3721	226981	7·8102	3·9365	·016393	61
62	3844	238328	7·8740	3 9579	·016129	62
63	3969	250047	7·9373	3·9791	·015873	63
64	4096	262144	8	4	·015625	64
65	4225	274625	8·0623	4·0207	·015385	65
66	4356	287496	8·1240	4·0412	·015152	66
67	4489	300763	8·1854	4·0615	·014925	67
68	4624	314432	8·2462	4·0817	·014706	68
69	4761	328509	8·3066	4·1016	·014493	69
70	4900	343000	8·3666	4·1213	·014286	70
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
71	5041	357911	8·4261	4·1408	·014085	71
72	5184	373248	8·4853	4·1602	·013889	72
73	5329	389017	8·5440	4·1793	·013699	73
74	5476	405224	8·6023	4·1983	·013514	74
75	5625	421875	8·6603	4·2172	·013333	75
76	5776	438976	8·7178	4·2358	·013158	76
77	5929	456533	8·7750	4·2543	·012987	77
78	6084	474552	8·8318	4·2727	·012821	78
79	6241	493039	8·8882	4·2908	·012658	79
80	6400	512000	8·9443	4·3089	·012500	80
81	6561	531441	9	4·3267	·012346	81
82	6724	551368	9·0554	4·3445	·012195	82
83	6889	571787	9·1104	4·3621	·012048	83
84	7056	592704	9·1652	4·3795	·011905	84
85	7225	614125	9·2195	4·3968	·011765	85
86	7396	636056	9·2736	4·4140	·011628	86
87	7569	658503	9·3274	4·4310	·011494	87
88	7744	681472	9·3808	4·4480	·011364	88
89	7921	704969	9·4340	4·4647	·011236	89
90	8100	729000	9·4868	4·4814	·011111	90
91	8281	753571	9·5394	4·4979	·010989	91
92	8464	778688	9·5917	4·5144	·010870	92
93	8649	804357	9·6437	4·5307	·010753	93
94	8836	830584	9·6954	4·5468	·010638	94
95	9025	857375	9·7468	4·5629	·010526	95
96	9216	884736	9·7980	4·5789	·010417	96
97	9409	912673	9·8489	4·5947	·010309	97
98	9604	941192	9·8995	4·6104	·010204	98
99	9801	970299	9·9499	4·6261	·010101	99
100	10000	1000000	10	4·6416	·010000	100
101	10201	1030301	10·0499	4·6570	·009901	101
102	10404	1061208	10·0995	4·6723	·009804	102
103	10609	1092727	10·1489	4·6875	·009709	103
104	10816	1124864	10·1980	4·7027	·009615	104
105	11025	1157625	10·2470	4·7177	·009524	105
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
106	11236	1191016	10·2956	4·7326	·009434	106
107	11449	1225043	10·3441	4·7475	·009346	107
108	11664	1259712	10·3923	4·7622	·009259	108
109	11881	1295029	10·4403	4·7769	·009174	109
110	12100	1331000	10·4881	4·7914	·009091	110
111	12321	1367631	10·5357	4·8059	·009009	111
112	12544	1404928	10·5830	4·8203	·008929	112
113	12769	1442897	10·6301	4·8346	·008850	113
114	12996	1481544	10·6771	4·8488	·008772	114
115	13225	1520875	10·7238	4·8629	·008696	115
116	13456	1560896	10·7703	4 8770	·008621	116
117	13689	1601613	10·8167	4·8910	·008547	117
118	13924	1643032	10·8628	4·9049	·008475	118
119	14161	1685159	10·9087	4·9187	·008403	119
120	14400	1728000	10·9545	4·9324	·008333	120
121	14641	1771561	11	4·9461	·008264	121
122	14884	1815848	11·0454	4·9597	·008197	122
123	15129	1860867	11·0905	4·9732	·008130	123
124	15376	1906624	11·1355	4·9866	·008065	124
125	15625	1953125	11·1803	5	·008000	125
126	15876	2000376	11·2250	5·0133	·007937	126
127	16129	2048383	11·2694	5·0265	·007874	127
128	16384	2097152	11·3137	5·0397	007813	128
129	16641	2146689	11·3578	5·0528	·007752	129
130	16900	2197000	11·4018	5·0658	·007692	130
131	17161	2248091	11·4455	5·0788	·007634	131
132	17424	2299968	11·4891	5·0916	·007576	132
133	17689	2352637	11·5326	5·1045	·007519	133
134	17956	2406104	11·5758	5·1172	·007463	134
135	18225	2460375	11·6190	5·1299	·007407	135
136	18496	2515456	11·6619	5·1426	·007353	136
137	18769	2571353	11·7047	5·1551	·007299	137
138	19044	2628072	11·7473	5·1676	·007246	138
139	19321	2685619	11·7898	5·1801	·007194	139
140	19600	2744000	11·8322	5·1925	·007143	140
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
141	19881	2803221	11·8743	5·2048	·007092	141
142	20164	2863288	11·9164	5·2171	·007042	142
143	20449	2924207	11·9583	5 2293	·006993	143
144	20736	2985984	12	5·2415	·006944	144
145	21025	3048625	12 0416	5·2536	·006897	145
146	21316	3112136	12·0830	5·2656	·006849	146
147	21609	3176523	12·1244	5·2776	·006803	147
148	21904	3241792	12·1655	5·2896	006757	148
149	22201	3307949	12·2066	5·3015	·006711	149
150	22500	3375000	12·2474	5·3133	·006667	150
151	22801	3442951	12·2882	5·3251	·006623	151
152	23104	3511808	12·3288	5·3368	·006579	152
153	23409	3581577	12·3693	5·3485	·006536	153
154	23716	3652264	12·4097	5·3601	·006494	154
155	24025	3723875	12·4499	5·3717	·006452	155
156	24336	3796416	12 4900	5·3832	·006410	156
157	24649	3869893	12 5300	5·3947	·006369	157
158	24964	3944312	12·5698	5·4061	·006329	158
159	25281	4019679	12·6095	5·4175	006289	159
160	25600	4096000	12 6491	5·4288	006250	160
161	25921	4173281	12·6886	5·4401	·006211	161
162	26244	4251528	12·7279	5·4514	·006173	162
163	26569	4330747	12·7671	5·4626	·006135	163
164	26896	4410944	12·8062	5·4737	·006098	164
165	27225	4492125	12·8452	5·4848	·006061	165
166	27556	4574296	12·8841	5·4959	·006024	166
167	27889	4657463	12·9228	5·5069	·005988	167
168	28224	4741632	12·9615	5·5178	005952	168
169	28561	4826809	13	5·5288	005917	169
170	28900	4913000	13·0384	5·5397	·005882	170
171	29241	5000211	13·0767	5·5505	·005848	171
172	29584	5088448	13·1149	5·5613	·005814	172
173	29929	5177717	13·1529	5·5721	·005780	173
174	30276	5268024	13 1909	5·5828	·005747	174
175	30625	5359875	13·2288	5·5934	·005714	175
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
176	30976	5451776	13·2665	5·6041	·005682	176
177	31329	5545233	13·3041	5·6147	·005650	177
178	31684	5639752	13·3417	5·6252	·005618	178
179	32041	5735339	13·3791	5·6357	·005587	179
180	32400	5832000	13·4164	5·6462	·005556	180
181	32761	5929741	13·4536	5·6567	·005525	181
182	33124	6028568	13·4907	5·6671	·005495	182
183	33489	6128487	13·5277	5·6774	·005464	183
184	33856	6229504	13·5647	5·6877	·005435	184
185	34225	6331625	13·6015	5·6980	·005405	185
186	34596	6434856	13·6382	5·7083	·005376	186
187	34969	6539203	13·6748	5·7185	·005348	187
188	35344	6644672	13·7113	5·7287	·005319	188
189	35721	6751269	13·7477	5·7388	·005291	189
190	36100	6859000	13·7840	5·7489	·005263	190
191	36481	6967871	13·8203	5·7590	·005236	191
192	36864	7077888	13·8564	5·7690	·005208	192
193	37249	7189057	13·8924	5·7790	·005181	193
194	37636	7301384	13·9284	5·7890	·005155	194
195	38025	7414875	13·9642	5·7989	·005128	195
196	38416	7529536	14	5·8088	·005102	196
197	38809	7645373	14·0357	5·8186	·005076	197
198	39204	7762392	14·0712	5·8285	·005051	198
199	39601	7880599	14·1067	5·8383	·005025	199
200	40000	8000000	14·1421	5·8480	·005000	200
201	40401	8120601	14·1774	5·8578	·004975	201
202	40804	8242408	14·2127	5·8675	·004950	202
203	41209	8365427	14·2478	5·8771	·004926	203
204	41616	8489664	14·2829	5·8868	·004902	204
205	42025	8615125	14·3178	5·8964	·004878	205
206	42436	8741816	14·3527	5·9059	·004854	206
207	42849	8869743	14·3875	5·9155	·004831	207
208	43264	8998912	14·4222	5·9250	·004808	208
209	43681	9129329	14·4568	5·9345	·004785	209
210	44100	9261000	14·4914	5·9439	·004762	210
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
211	44521	9393931	14·5258	5·9533	·004739	211
212	44944	9528128	14·5602	5·9627	·004717	212
213	45369	9663597	14·5945	5·9721	·004695	213
214	45796	9800344	14·6287	5·9814	·004673	214
215	46225	9938375	14·6629	5·9907	·004651	215
216	46656	10077696	14·6969	6	·004630	216
217	47089	10218313	14·7309	6·0092	·004608	217
218	47524	10360232	14·7648	6·0185	·004587	218
219	47961	10503459	14·7986	6·0277	·004566	219
220	48400	10648000	14·8324	6·0368	·004545	220
221	48841	10793861	14·8661	6·0459	·004525	221
222	49284	10941048	14·8997	6·0550	·004505	222
223	49729	11089567	14·9332	6·0641	·004484	223
224	50176	11239424	14·9666	6·0732	·004464	224
225	50625	11390625	15	6·0822	·004444	225
226	51076	11543176	15·0333	6·0912	·004425	226
227	51529	11697083	15·0665	6·1002	·004405	227
228	51984	11852352	15·0997	6·1091	·004386	228
229	52441	12008989	15·1327	6·1180	·004367	229
230	52900	12167000	15·1658	6·1269	·004348	230
231	53361	12326391	15·1987	6·1358	·004329	231
232	53824	12487168	15·2315	6·1446	·004310	232
233	54289	12649337	15·2643	6·1534	·004292	233
234	54756	12812904	15·2971	6·1622	·004274	234
235	55225	12977875	15·3297	6·1710	·004255	235
236	55696	13144256	15·3623	6·1797	·004237	236
237	56169	13312053	15·3948	6·1885	·004219	237
238	56644	13481272	15·4272	6·1972	·004202	238
239	57121	13651919	15·4596	6·2058	·004184	239
240	57600	13824000	15·4919	6·2145	·004167	240
241	58081	13997521	15·5242	6·2231	·004149	241
242	58564	14172488	15·5563	6·2317	·004132	242
243	59049	14348907	15·5885	6·2403	·004115	243
244	59536	14526784	15·6205	6·2488	·004098	244
245	60025	14706125	15·6525	6·2573	·004082	245
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
246	60516	14886936	15·6844	6·2658	·004065	246
247	61009	15069223	15·7162	6·2743	·004049	247
248	61504	15252992	15·7480	6·2828	·004032	248
249	62001	15438249	15·7797	6·2912	·004016	249
250	62500	15625000	15·8114	6·2996	·004000	250
251	63001	15813251	15·8430	6·3080	·003984	251
252	63504	16003008	15·8745	6·3164	·003968	252
253	64009	16194277	15·9060	6·3247	·003953	253
254	64516	16387064	15·9374	6·3330	·003937	254
255	65025	16581375	15·9687	6·3413	·003922	255
256	65536	16777216	16	6·3496	·003906	256
257	66049	16974593	16·0312	6·3579	·003891	257
258	66564	17173512	16·0624	6·3661	·003876	258
259	67081	17373979	16·0935	6·3743	·003861	259
260	67600	17576000	16·1245	6·3825	·003846	260
261	68121	17779581	16·1555	6·3907	·003831	261
262	68644	17984728	16·1864	6·3988	·003817	262
263	69169	18191447	16·2173	6·4070	·003802	263
264	69696	18399744	16·2481	6·4151	·003788	264
265	70225	18609625	16·2788	6·4232	·003774	265
266	70756	18821096	16·3095	6·4312	·003759	266
267	71289	19034163	16·3401	6·4393	·003745	267
268	71824	19248832	16·3707	6·4473	·003731	268
269	72361	19465109	16·4012	6·4553	·003717	269
270	72900	19683000	16·4317	6·4633	·003704	270
271	73441	19902511	16·4621	6·4713	·003690	271
272	73984	20123648	16·4924	6·4792	·003676	272
273	74529	20346417	16·5227	6·4872	·003663	273
274	75076	20570824	16·5529	6·4951	·003650	274
275	75625	20796875	16·5831	6·5030	·003636	275
276	76176	21024576	16·6132	6·5108	·003623	276
277	76729	21253933	16·6433	6·5187	·003610	277
278	77284	21484952	16·6733	6·5265	·003597	278
279	77841	21717639	16·7033	6·5343	·003584	279
280	78400	21952000	16·7332	6·5421	·003571	280
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
281	78961	22188041	16·7631	6·5499	·003559	281
282	79524	22425768	16·7929	6·5577	·003546	282
283	80089	22665187	16·8226	6·5654	·003534	283
284	80656	22906304	16·8523	6·5731	·003521	284
285	81225	23149125	16·8819	6·5808	·003509	285
286	81796	23393656	16·9115	6·5885	·003497	286
287	82369	23639903	16·9411	6·5962	·003484	287
288	82944	23887872	16·9706	6·6039	·003472	288
289	83521	24137569	17	6·6115	·003460	289
290	84100	24389000	17·0294	6·6191	·003448	290
291	84681	24642171	17·0587	6·6267	·003436	291
292	85264	24897088	17·0880	6·6343	·003425	292
293	85849	25153757	17·1172	6·6419	·003413	293
294	86436	25412184	17·1464	6·6494	·003401	294
295	87025	25672375	17·1756	6·6569	·003390	295
296	87616	25934336	17·2047	6·6644	·003378	296
297	88209	26198073	17·2337	6·6719	·003367	297
298	88804	26463592	17·2627	6·6794	·003356	298
299	89401	26730899	17·2916	6·6869	·003344	299
300	90000	27000000	17·3205	6·6943	·003333	300
301	90601	27270901	17·3494	6·7018	·003322	301
302	91204	27543608	17·3781	6·7092	·003311	302
303	91809	27818127	17·4069	6·7166	·003300	303
304	92416	28094464	17·4356	6·7240	·003289	304
305	93025	28372625	17·4642	6·7313	·003279	305
306	93636	28652616	17·4929	6·7387	·003268	306
307	94249	28934443	17·5214	6·7460	·003257	307
308	94864	29218112	17·5499	6·7533	·003247	308
309	95481	29503629	17·5784	6·7606	·003236	309
310	96100	29791000	17·6068	6·7679	·003226	310
311	96721	30080231	17·6352	6·7752	·003215	311
312	97344	30371328	17·6635	6·7824	·003205	312
313	97969	30664297	17·6918	6·7897	·003195	313
314	98596	30959144	17·7200	6·7969	·003185	314
315	99225	31255875	17·7482	6·8041	·003175	315
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
316	99856	31554496	17·7764	6·8113	·003165	316
317	100489	31855013	17·8045	6·8185	·003155	317
318	101124	32157432	17·8326	6·8256	·003145	318
319	101761	32461759	17·8606	6·8328	·003135	319
320	102400	32768000	17·8885	6·8399	·003125	320
321	103041	33076161	17·9165	6·8470	·003115	321
322	103684	33386248	17·9444	6·8541	·003106	322
323	104329	33698267	17·9722	6·8612	·003096	323
324	104976	34012224	18	6·8683	·003086	324
325	105625	34328125	18·0278	6·8753	·003077	325
326	106276	34645976	18·0555	6 8824	·003067	326
327	106929	34965783	18·0831	6 8894	·003058	327
328	107584	35287552	18·1108	6·8964	·003049	328
329	108241	35611289	18·1384	6·9034	·003040	329
330	108900	35937000	18·1659	6·9104	·003030	330
331	109561	36264691	18·1934	6 9174	·003021	331
332	110224	36594368	18·2209	6·9244	·003012	332
333	110889	36926037	18·2483	6·9313	·003003	333
334	111556	37259704	18·2757	6·9382	·002994	334
335	112225	37595375	18·3030	6·9451	002985	335
336	112896	37933056	18·3303	6·9521	·002976	336
337	113569	38272753	18·3576	6·9589	·002967	337
338	114244	38614472	18·3848	6·9658	·002959	338
339	114921	38958219	18·4120	6·9727	·002950	339
340	115600	39304000	18·4391	6·9795	·002941	340
341	116281	39651821	18·4662	6·9864	·002933	341
342	116964	40001688	18·4932	6·9932	·002924	342
343	117649	40353607	18·5203	7	·002915	343
344	118336	40707584	18·5472	7·0068	·002907	344
345	119025	41063625	18·5742	7·0136	·002899	345
346	119716	41421736	18·6011	7·0203	·002890	346
347	120409	41781923	18·6279	7·0271	·002882	347
348	121104	42144192	18·6548	7·0338	·002874	348
349	121801	42508549	18·6815	7·0406	·002865	349
350	122500	42875000	18·7083	7·0473	·002857	350
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
351	123201	43243551	18·7350	7·0540	·002849	351
352	123904	43614208	18·7617	7·0607	·002841	352
353	124609	43986977	18·7883	7·0674	·002833	353
354	125316	44361864	18·8149	7·0740	·002825	354
355	126025	44738875	18·8414	7·0807	·002817	355
356	126736	45118016	18·8680	7·0873	·002809	356
357	127449	45499293	18·8944	7·0940	·002801	357
358	128164	45882712	18·9209	7·1006	002793	358
359	128881	46268279	18·9473	7 1072	·002786	359
360	129600	46656000	18·9737	7·1138	·002778	360
361	130321	47045881	19	7·1204	·002770	361
362	131044	47437928	19·0263	7·1269	·002762	362
363	131769	47832147	19·0526	7 1335	·002755	363
364	132496	48228544	19·0788	7·1400	·002747	364
365	133225	48627125	19·1050	7·1466	·002740	365
366	133956	49027896	19·1311	7·1531	·002732	366
367	134689	49430863	19·1572	7·1596	·002725	367
368	135424	49836032	19·1833	7·1661	·002717	368
369	136161	50243409	19·2094	7·1726	·002710	369
370	136900	50653000	19·2354	7·1791	·002703	370
371	137641	51064811	19·2614	7·1855	002695	371
372	138384	51478848	19·2873	7·1920	002688	372
373	139129	51895117	19·3132	7·1984	·002681	373
374	139876	52313624	19·3391	7·2048	·002674	374
375	140625	52734375	19·3649	7·2112	·002667	375
376	141376	53157376	19·3907	7·2177	·002660	376
377	142129	53582633	19·4165	7·2240	·002653	377
378	142884	54010152	19·4422	7·2304	·002646	378
379	143641	54439939	19·4679	7·2368	·002639	379
380	144400	54872000	19·4936	7·2432	·002632	380
381	145161	55306341	19·5192	7·2495	·002625	381
382	145924	55742968	19·5448	7·2558	·002618	382
383	146689	56181887	19·5704	7·2622	·002611	383
384	147456	56623104	19·5959	7·2685	·002604	384
385	148225	57066625	19·6214	7·2748	·002597	385
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
386	148996	57512456	19·6469	7·2811	·002591	386
387	149769	57960603	19·6723	7·2874	·002584	387
388	150544	58411072	19·6977	7·2936	·002577	388
389	151321	58863869	19·7231	7·2999	·002571	389
390	152100	59319000	19·7484	7·3061	·002564	390
391	152881	59776471	19·7737	7·3124	·002558	391
392	153664	60236288	19·7990	7·3186	·002551	392
393	154449	60698457	19·8242	7·3248	·002545	393
394	155236	61162984	19·8494	7·3310	·002538	394
395	156025	61629875	19·8746	7·3372	·002532	395
396	156816	62099136	19·8997	7·3434	·002525	396
397	157609	62570773	19·9249	7·3496	·002519	397
398	158404	63044792	19·9499	7·3558	·002513	398
399	159201	63521199	19·9750	7·3619	·002506	399
400	160000	64000000	20	7·3681	·002500	400
401	160801	64481201	20·0250	7·3742	·002494	401
402	161604	64964808	20·0499	7·3803	·002488	402
403	162409	65450827	20·0749	7·3864	·002481	403
404	163216	65939264	20·0998	7·3925	·002475	404
405	164025	66430125	20·1246	7·3986	·002469	405
406	164836	66923416	20·1494	7·4047	·002463	406
407	165649	67419143	20·1742	7·4108	·002457	407
408	166464	67917312	20·1990	7·4169	·002451	408
409	167281	68417929	20·2237	7·4229	·002445	409
410	168100	68921000	20·2485	7·4290	·002439	410
411	168921	69426531	20·2731	7·4350	·002433	411
412	169744	69934528	20·2978	7·4410	·002427	412
413	170569	70444997	20·3224	7·4470	·002421	413
414	171396	70957944	20·3470	7·4530	·002415	414
415	172225	71473375	20·3715	7·4590	·002410	415
416	173056	71991296	20·3961	7·4650	·002404	416
417	173889	72511713	20·4206	7·4710	·002398	417
418	174724	73034632	20·4450	7·4770	·002392	418
419	175561	73560059	20·4695	7·4829	·002387	419
420	176400	74088000	20·4939	7·4889	·002381	420
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
421	177241	74618461	20·5183	7·4948	·002375	421
422	178084	75151448	20·5426	7·5007	·002370	422
423	178929	75686967	20·5670	7·5067	·002364	423
424	179776	76225024	20·5913	7·5126	·002358	424
425	180625	76765625	20·6155	7·5185	·002353	425
426	181476	77308776	20·6398	7·5244	·002347	426
427	182329	77854483	20·6640	7·5302	·002342	427
428	183184	78402752	20·6882	7·5361	·002336	428
429	184041	78953589	20·7123	7·5420	·002331	429
430	184900	79507000	20·7364	7·5478	·002326	430
431	185761	80062991	20·7605	7·5537	·002320	431
432	186624	80621568	20·7846	7·5595	·002315	432
433	187489	81182737	20·8087	7·5654	·002309	433
434	188356	81746504	20·8327	7·5712	·002304	434
435	189225	82312875	20·8567	7·5770	·002299	435
436	190096	82881856	20·8806	7·5828	·002294	436
437	190969	83453453	20·9045	7·5886	·002288	437
438	191844	84027672	20·9284	7·5944	·002283	438
439	192721	84604519	20·9523	7·6001	·002278	439
440	193600	85184000	20·9762	7·6059	·002273	440
441	194481	85766121	21	7·6117	·002268	441
442	195364	86350888	21·0238	7·6174	·002262	442
443	196249	86938307	21·0476	7·6232	·002257	443
444	197136	87528384	21·0713	7·6289	·002252	444
445	198025	88121125	21·0950	7·6346	·002247	445
446	198916	88716536	21·1187	7·6403	·002242	446
447	199809	89314623	21·1424	7·6460	·002237	447
448	200704	89915392	21·1660	7·6517	·002232	448
449	201601	90518849	21·1896	7·6574	·002227	449
450	202500	91125000	21·2132	7·6631	·002222	450
451	203401	91733851	21·2368	7·6688	·002217	451
452	204304	92345408	21·2603	7·6744	·002212	452
453	205209	92959677	21·2838	7·6801	·002208	453
454	206116	93576664	21·3073	7·6857	·002203	454
455	207025	94196375	21·3307	7·6914	·002198	455
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
456	207936	94818816	21·3542	7·6970	·002193	456
457	208849	95443993	21·3776	7·7026	·002188	457
458	209764	96071912	21·4009	7·7082	·002183	458
459	210681	96702579	21·4243	7·7138	·002179	459
460	211600	97336000	21·4476	7·7194	·002174	460
461	212521	97972181	21·4709	7·7250	·002169	461
462	213444	98611128	21·4942	7·7306	·002165	462
463	214369	99252847	21·5174	7·7362	·002160	463
464	215296	99897344	21·5407	7·7418	·002155	464
465	216225	100544625	21·5639	7·7473	·002151	465
466	217156	101194696	21·5870	7·7529	·002146	466
467	218089	101847563	21·6102	7·7584	·002141	467
468	219024	102503232	21·6333	7·7639	·002137	468
469	219961	103161709	21·6564	7·7695	·002132	469
470	220900	103823000	21·6795	7·7750	·002128	470
471	221841	104487111	21·7025	7·7805	·002123	471
472	222784	105154048	21·7256	7·7860	·002119	472
473	223729	105823817	21·7486	7·7915	·002114	473
474	224676	106496424	21·7715	7·7970	·002110	474
475	225625	107171875	21·7945	7·8025	·002105	475
476	226576	107850176	21·8174	7·8079	·002101	476
477	227529	108531333	21·8403	7·8134	·002096	477
478	228484	109215352	21·8632	7·8188	·002092	478
479	229441	109902239	21·8861	7·8243	·002088	479
480	230400	110592000	21·9089	7·8297	·002083	480
481	231361	111284641	21·9317	7·8352	·002079	481
482	232324	111980168	21·9545	7·8406	·002075	482
483	233289	112678587	21·9773	7·8460	·002070	483
484	234256	113379904	22	7·8514	·002066	484
485	235225	114084125	22·0227	7·8568	·002062	485
486	236196	114791256	22·0454	7·8622	·002058	486
487	237169	115501303	22·0681	7·8676	·002053	487
488	238144	116214272	22·0907	7·8730	·002049	488
489	239121	116930169	22·1133	7·8784	·002045	489
490	240100	117649000	22·1359	7·8837	·002041	490
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
491	241081	118370771	22·1585	7·8891	·002037	491
492	242064	119095488	22·1811	7·8944	·002033	492
493	243049	119823157	22·2036	7·8998	·002028	493
494	244036	120553784	22·2261	7·9051	·002024	494
495	245025	121287375	22·2486	7·9105	·002020	495
496	246016	122023936	22·2711	7·9158	·002016	496
497	247009	122763473	22·2935	7·9211	·002012	497
498	248004	123505992	22·3159	7·9264	·002008	498
499	249001	124251499	22·3383	7·9317	·002004	499
500	250000	125000000	22·3607	7·9370	·002000	500
501	251001	125751501	22·3830	7·9423	·001996	501
502	252004	126506008	22·4054	7·9476	·001992	502
503	253009	127263527	22·4277	7·9528	·001988	503
504	254016	128024064	22·4499	7·9581	·001984	504
505	255025	128787625	22·4722	7·9634	·001980	505
506	256036	129554216	22·4944	7·9686	·001976	506
507	257049	130323843	22·5167	7·9739	·001972	507
508	258064	131096512	22·5389	7·9791	·001969	508
509	259081	131872229	22·5610	7·9843	·001965	509
510	260100	132651000	22·5832	7·9896	·001961	510
511	261121	133432831	22·6053	7·9948	·001957	511
512	262144	134217728	22·6274	8	·001953	512
513	263169	135005697	22·6495	8·0052	·001949	513
514	264196	135796744	22·6716	8·0104	·001946	514
515	265225	136590875	22·6936	8·0156	·001942	515
516	266256	137388096	22·7156	8·0208	·001938	516
517	267289	138188413	22·7376	8·0260	·001934	517
518	268324	138991832	22·7596	8·0311	·001931	518
519	269361	139798359	22·7816	8·0363	·001927	519
520	270400	140608000	22·8035	8·0415	·001923	520
521	271441	141420761	22·8254	8·0466	·001919	521
522	272484	142236648	22·8473	8·0517	·001916	522
523	273529	143055667	22·8692	8·0569	·001912	523
524	274576	143877824	22·8910	8·0620	·001908	524
525	275625	144703125	22·9129	8·0671	·001905	525
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
526	276676	145531576	22·9347	8·0723	·001901	526
527	277729	146363183	22·9565	8·0774	·001898	527
528	278784	147197952	22·9783	8·0825	·001894	528
529	279841	148035889	23	8·0876	·001890	529
530	280900	148877000	23·0217	8·0927	·001887	530
531	281961	149721291	23·0434	8·0978	·001883	531
532	283024	150568768	23·0651	8·1028	·001880	532
533	284089	151419437	23·0868	8·1079	·001876	533
534	285156	152273304	23·1084	8·1130	·001873	534
535	286225	153130375	23·1301	8·1180	·001869	535
536	287296	153990656	23·1517	8·1231	·001866	536
537	288369	154854153	23·1733	8·1281	·001862	537
538	289444	155720872	23·1948	8·1332	·001859	538
539	290521	156590819	23·2164	8·1382	·001855	539
540	291600	157464000	23·2379	8·1433	·001852	540
541	292681	158340421	23·2594	8·1483	·001848	541
542	293764	159220088	23·2809	8·1533	·001845	542
543	294849	160103007	23·3024	8·1583	·001842	543
544	295936	160989184	23·3238	8·1633	·001838	544
545	297025	161878625	23·3452	8·1683	·001835	545
546	298116	162771336	23·3666	8·1733	·001832	546
547	299209	163667323	23·3880	8·1783	·001828	547
548	300304	164566592	23·4094	8·1833	·001825	548
549	301401	165469149	23·4307	8·1882	·001821	549
550	302500	166375000	23·4521	8·1932	·001818	550
551	303601	167284151	23·4734	8·1982	·001815	551
552	304704	168196608	23·4947	8·2031	·001812	552
553	305809	169112377	23·5160	8·2081	·001808	553
554	306916	170031464	23·5372	8·2130	·001805	554
555	308025	170953875	23·5584	8·2180	·001802	555
556	309136	171879616	23·5797	8·2229	·001799	556
557	310249	172808693	23·6008	8·2278	·001795	557
558	311364	173741112	23·6220	8·2327	·001792	558
559	312481	174676879	23·6432	8·2377	·001789	559
560	313600	175616000	23·6643	8·2426	·001786	560
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
561	314721	176558481	23·6854	8·2475	·001783	561
562	315844	177504328	23·7065	8·2524	·001779	562
563	316969	178453547	23·7276	8·2573	·001776	563
564	318096	179406144	23·7487	8·2621	·001773	564
565	319225	180362125	23·7697	8·2670	·001770	565
566	320356	181321496	23·7908	8·2719	·001767	566
567	321489	182284263	23·8118	8·2768	·001764	567
568	322624	183250432	23·8328	8·2816	·001761	568
569	323761	184220009	23·8537	8·2865	·001757	569
570	324900	185193000	23·8747	8·2913	·001754	570
571	326041	186169411	23·8956	8·2962	·001751	571
572	327184	187149248	23·9165	8·3010	·001748	572
573	328329	188132517	23·9374	8·3059	·001745	573
574	329476	189119224	23·9583	8·3107	·001742	574
575	330625	190109375	23·9792	8·3155	·001739	575
576	331776	191102976	24	8·3203	·001736	576
577	332929	192100033	24·0208	8·3251	·001733	577
578	334084	193100552	24·0416	8·3300	·001730	578
579	335241	194104539	24·0624	8·3348	·001727	579
580	336400	195112000	24·0832	8·3396	·001724	580
581	337561	196122941	24·1039	8·3443	·001721	581
582	338724	197137368	24·1247	8·3491	·001718	582
583	339889	198155287	24·1454	8·3539	·001715	583
584	341056	199176704	24·1661	8·3587	·001712	584
585	342225	200201625	24·1868	8·3634	·001709	585
586	343396	201230056	24·2074	8·3682	·001706	586
587	344569	202262003	24·2281	8·3730	·001704	587
588	345744	203297472	24·2487	8·3777	·001701	588
589	346921	204336469	24·2693	8·3825	·001698	589
590	348100	205379000	24·2899	8·3872	·001695	590
591	349281	206425071	24·3105	8·3919	·001692	591
592	350464	207474688	24·3311	8·3967	·001689	592
593	351649	208527857	24·3516	8·4014	·001686	593
594	352836	209584584	24·3721	8·4061	·001684	594
595	354025	210644875	24·3926	8·4108	·001681	595
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
596	355216	211708736	24·4131	8·4155	·001678	596
597	356409	212776173	24·4336	8·4202	·001675	597
598	357604	213847192	24·4540	8·4249	·001672	598
599	358801	214921799	24·4745	8·4296	·001669	599
600	360000	216000000	24·4949	8·4343	·001667	600
601	361201	217081801	24·5153	8·4390	·001664	601
602	362404	218167208	24·5357	8·4437	·001661	602
603	363609	219256227	24·5561	8·4484	·001658	603
604	364816	220348864	24·5764	8·4530	·001656	604
605	366025	221445125	24·5967	8·4577	·001653	605
606	367236	222545016	24·6171	8·4623	·001650	606
607	368449	223648543	24·6374	8·4670	·001647	607
608	369664	224755712	24·6577	8·4716	·001645	608
609	370881	225866529	24·6779	8·4763	·001642	609
610	372100	226981000	24·6982	8·4809	·001639	610
611	373321	228099131	24·7184	8·4856	·001637	611
612	374544	229220928	24·7386	8·4902	·001634	612
613	375769	230346397	24·7588	8·4948	·001631	613
614	376996	231475544	24·7790	8·4994	·001629	614
615	378225	232608375	24·7992	8·5040	·001626	615
616	379456	233744896	24·8193	8·5086	·001623	616
617	380689	234885113	24·8395	8·5132	·001621	617
618	381924	236029032	24·8596	8·5178	·001618	618
619	383161	237176659	24·8797	8·5224	·001616	619
620	384400	238328000	24·8998	8·5270	·001613	620
621	385641	239483061	24·9199	8·5316	·001610	621
622	386884	240641848	24·9399	8·5362	·001608	622
623	388129	241804367	24·9600	8·5408	·001605	623
624	389376	242970624	24·9800	8·5453	·001603	624
625	390625	244140625	25	8·5499	·001600	625
626	391876	245314376	25·0200	8·5544	·001597	626
627	393129	246491883	25·0400	8·5590	·001595	627
628	394384	247673152	25·0599	8·5635	·001592	628
629	395641	248858189	25·0799	8·5681	·001590	629
630	396900	250047000	25·0998	8·5726	·001587	630
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
631	398161	251239591	25·1197	8·5772	·001585	631
632	399424	252435968	25·1396	8 5817	·001582	632
633	400689	253636137	25 1595	8·5862	·001580	633
634	401956	254840104	25 1794	8·5907	·001577	634
635	403225	256047875	25·1992	8·5952	·001575	635
636	404496	257259456	25·2190	8·5997	001572	636
637	405769	258474853	25·2389	8·6043	001570	637
638	407044	259694072	25·2587	8 6088	·001567	638
639	408321	260917119	25·2784	8·6132	·001565	639
640	409600	262144000	25 2982	8 6177	·001563	640
641	410881	263374721	25·3180	8 6222	·001560	641
642	412164	264609288	25 3377	8·6267	001558	642
643	413449	265847707	25·3574	8·6312	·001555	643
644	414736	267089984	25 3772	8 6357	001553	644
645	416025	268336125	25 3969	8·6401	001550	645
646	417316	269586136	25·4165	8·6446	·001548	646
647	418609	270840023	25·4362	8·6490	·001546	647
648	419904	272097792	25·4558	8 6535	·001543	648
649	421201	273359449	25·4755	8·6579	·001541	649
650	422500	274625000	25·4951	8·6624	001538	650
651	423801	275894451	25·5147	8·6668	·001536	651
652	425104	277167808	25 5343	8 6713	·001534	652
653	426409	278445077	25·5539	8·6757	·001531	653
654	427716	279726264	25·5734	8·6801	·001529	654
655	429025	281011375	25·5930	8·6845	·001527	655
656	430336	282300416	25·6125	8·6890	·001524	656
657	431649	283593393	25 6320	8·6934	001522	657
658	432964	284890312	25·6515	8·6978	001520	658
659	434281	286191179	25·6710	8·7022	·001517	659
660	435600	287496000	25·6905	8 7066	·001515	660
661	436921	288804781	25·7099	8·7110	·001513	661
662	438244	290117528	25·7294	8·7154	·001511	662
663	439569	291434247	25·7488	8·7198	·001508	663
664	440896	292754944	25·7682	8·7241	·001506	664
665	442225	294079625	25·7876	8·7285	·001504	665
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
666	443556	295408296	25 8070	8 7329	·001502	666
667	444889	296740963	25·8263	8·7373	·001499	667
668	446224	298077632	25·8457	8·7416	·001497	668
669	447561	299418309	25·8650	8·7460	·001495	669
670	448900	300763000	25 8844	8 7503	·001493	670
671	450241	302111711	25·9037	8·7547	·001490	671
672	451584	303464448	25·9230	8 7590	·001488	672
673	452929	304821217	25·9422	8·7634	·001486	673
674	454276	306182024	25·9615	8·7677	·001484	674
675	455625	307546875	25·9808	8·7721	·001481	675
676	456976	308915776	26	8·7764	001479	676
677	458329	310288733	26·0192	8·7807	001477	677
678	459684	311665752	26 0384	8·7850	001475	678
679	461041	313046839	26·0576	8 7893	·001473	679
680	462400	314432000	26 0768	8·7937	·001471	680
681	463761	315821241	26·0960	8·7980	·001468	681
682	465124	317214568	26·1151	8·8023	001466	682
683	466489	318611987	26 1343	8·8066	·001464	683
684	467856	320013504	26·1534	8·8109	·001462	684
685	469225	321419125	26·1725	8·8152	·001460	685
686	470596	322828856	26·1916	8·8194	·001458	686
687	471969	324242703	26·2107	8·8237	001456	687
688	473344	325660672	26·2298	8·8280	·001453	688
689	474721	327082769	26·2488	8·8323	·001451	689
690	476100	328509000	26·2679	8·8366	·001449	690
691	477481	329939371	26·2869	8·8408	·001447	691
692	478864	331373888	26·3059	8·8451	·001445	692
693	480249	332812557	26·3249	8·8493	·001443	693
694	481636	334255384	26·3439	8·8536	·001441	694
695	483025	335702375	26 3629	8·8578	·001439	695
696	484416	337153536	26 3818	8·8621	·001437	696
697	485809	338608873	26·4008	8·8663	·001435	697
698	487204	340068392	26·4197	8·8706	·001433	698
699	488601	341532099	26·4386	8·8748	·001431	699
700	490000	343000000	26·4575	8·8790	001429	700
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*)

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
701	491401	344472101	26·4764	8·8833	·001427	701
702	492804	345948408	26 4953	8·8875	·001425	702
703	494209	347428927	26·5141	8·8917	001422	703
704	495616	348913664	26·5330	8·8959	·001420	704
705	497025	350402625	26·5518	8 9001	·001418	705
706	498436	351895816	26·5707	8·9043	·001416	706
707	499849	353393213	26·5895	8 9085	·001414	707
708	501264	354894912	26 6083	8·9127	·001412	708
709	502681	356400829	26·6271	8·9169	·001410	709
710	504100	357911000	26·6458	8·9211	·001408	710
711	505521	359425431	26·6646	8·9253	001406	711
712	506944	360944128	26·6833	8 9295	001404	712
713	508369	362467097	26·7021	8·9337	001403	713
714	509796	363994344	26·7208	8 9378	001401	714
715	511225	365525875	26 7395	8·9420	001399	715
716	512656	367061696	26·7582	8·9462	·001397	716
717	514089	368601813	26·7769	8·9503	·001395	717
718	515524	370146232	26·7955	8·9545	·001393	718
719	516961	371694959	26·8142	8·9587	·001391	719
720	518400	373248000	26·8328	8·9628	001389	720
721	519841	374805361	26·8514	8·9670	001387	721
722	521284	376367048	26 8701	8·9711	001385	722
723	522729	377933067	26·8887	8 9752	·001383	723
724	524176	379503424	26 9072	8·9794	·001381	724
725	525625	381078125	26·9258	8·9835	·001379	725
726	527076	382657176	26 9444	8·9876	·001377	726
727	528529	384240583	26 9629	8·9918	·001376	727
728	529984	385828352	26·9815	8·9959	·001374	728
729	531441	387420489	27	9	001372	729
730	532900	389017000	27·0185	9 0041	001370	730
731	534361	390617891	27·0370	9·0082	·001368	731
732	535824	392223168	27·0555	9·0123	·001366	732
733	537289	393832837	27·0740	9·0164	·001364	733
734	538756	395446904	27·0924	9·0205	·001362	734
735	540225	397065375	27·1109	9·0246	·001361	735
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
736	541696	398688256	27·1293	9·0287	·001359	736
737	543169	400315553	27·1477	9·0328	·001357	737
738	544644	401947272	27·1662	9·0369	·001355	738
739	546121	403583419	27·1846	9·0410	·001353	739
740	547600	405224000	27·2029	9·0450	·001351	740
741	549081	406869021	27·2213	9·0491	·001350	741
742	550564	408518488	27·2397	9·0532	·001348	742
743	552049	410172407	27·2580	9·0572	·001346	743
744	553536	411830784	27·2764	9·0613	·001344	744
745	555025	413493625	27·2947	9·0654	·001342	745
746	556516	415160936	27·3130	9·0694	·001340	746
747	558009	416832723	27·3313	9·0735	·001339	747
748	559504	418508992	27·3496	9·0775	·001337	748
749	561001	420189749	27·3679	9·0816	·001335	749
750	562500	421875000	27·3861	9·0856	·001333	750
751	564001	423564751	27·4044	9·0896	·001332	751
752	565504	425259008	27·4226	9·0937	·001330	752
753	567009	426957777	27·4408	9·0977	·001328	753
754	568516	428661064	27·4591	9·1017	·001326	754
755	570025	430368875	27·4773	9·1057	·001325	755
756	571536	432081216	27·4955	9·1098	·001323	756
757	573049	433798093	27·5136	9·1138	·001321	757
758	574564	435519512	27·5318	9·1178	·001319	758
759	576081	437245479	27·5500	9·1218	·001318	759
760	577600	438976000	27·5681	9·1258	·001316	760
761	579121	440711081	27·5862	9·1298	·001314	761
762	580644	442450728	27·6043	9·1338	·001312	762
763	582169	444194947	27·6225	9·1378	·001311	763
764	583696	445943744	27·6405	9·1418	·001309	764
765	585225	447697125	27·6586	9·1458	·001307	765
766	586756	449455096	27·6767	9·1498	·001305	766
767	588289	451217663	27·6948	9·1537	·001304	767
768	589824	452984832	27·7128	9·1577	·001302	768
769	591361	454756609	27·7308	9·1617	·001300	769
770	592900	456533000	27·7489	9·1657	·001299	770
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*)

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
771	594441	458314011	27·7669	9·1696	·001297	771
772	595984	460099648	27·7849	9·1736	·001295	772
773	597529	461889917	27·8029	9·1775	·001294	773
774	599076	463684824	27·8209	9·1815	·001292	774
775	600625	465484375	27·8388	9·1855	·001290	775
776	602176	467288576	27·8568	9·1894	·001289	776
777	603729	469097433	27·8747	9·1933	·001287	777
778	605284	470910952	27·8927	9·1973	·001285	778
779	606841	472729139	27·9106	9·2012	·001284	779
780	608400	474552000	27·9285	9·2052	·001282	780
781	609961	476379541	27·9464	9·2091	·001280	781
782	611524	478211768	27·9643	9·2130	·001279	782
783	613089	480048687	27·9821	9·2170	·001277	783
784	614656	481890304	28	9·2209	·001276	784
785	616225	483736625	28·0179	9·2248	·001274	785
786	617796	485587656	28·0357	9·2287	·001272	786
787	619369	487443403	28·0535	9·2326	·001271	787
788	620944	489303872	28·0713	9·2365	·001269	788
789	622521	491169069	28·0891	9·2404	·001267	789
790	624100	493039000	28·1069	9·2443	·001266	790
791	625681	494913671	28·1247	9·2482	·001264	791
792	627264	496793088	28·1425	9·2521	·001263	792
793	628849	498677257	28·1603	9·2560	·001261	793
794	630436	500566184	28·1780	9·2599	·001259	794
795	632025	502459875	28·1957	9·2638	·001258	795
796	633616	504358336	28·2135	9·2677	·001256	796
797	635209	506261573	28·2312	9·2716	·001255	797
798	636804	508169592	28·2489	9·2754	·001253	798
799	638401	510082399	28·2666	9·2793	·001252	799
800	640000	512000000	28·2843	9·2832	·001250	800
801	641601	513922401	28·3019	9·2870	·001248	801
802	643204	515849608	28·3196	9·2909	·001247	802
803	644809	517781627	28·3373	9·2948	·001245	803
804	646416	519718464	28·3549	9·2986	·001244	804
805	648025	521660125	28·3725	9·3025	·001242	805
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
806	649636	523606616	28·3901	9·3063	·001241	806
807	651249	525557943	28·4077	9·3102	·001239	807
808	652864	527514112	28·4253	9·3140	·001238	808
809	654481	529475129	28·4429	9·3179	·001236	809
810	656100	531441000	28·4605	9·3217	·001235	810
811	657721	533411731	28·4781	9·3255	·001233	811
812	659344	535387328	28·4956	9·3294	·001232	812
813	660969	537367797	28·5132	9·3332	·001230	813
814	662596	539353144	28·5307	9·3370	·001229	814
815	664225	541343375	28·5482	9·3408	·001227	815
816	665856	543338496	28·5657	9·3447	·001225	816
817	667489	545338513	28·5832	9·3485	·001224	817
818	669124	547343432	28·6007	9·3523	·001222	818
819	670761	549353259	28·6182	9·3561	·001221	819
820	672400	551368000	28·6356	9·3599	·001220	820
821	674041	553387661	28·6531	9·3637	·001218	821
822	675684	555412248	28·6705	9·3675	·001217	822
823	677329	557441767	28·6880	9·3713	·001215	823
824	678976	559476224	28·7054	9·3751	·001214	824
825	680625	561515625	28·7228	9·3789	·001212	825
826	682276	563559976	28·7402	9·3827	·001211	826
827	683929	565609283	28·7576	9·3865	·001209	827
828	685584	567663552	28·7750	9·3902	·001208	828
829	687241	569722789	28·7924	9·3940	·001206	829
830	688900	571787000	28·8097	9·3978	·001205	830
831	690561	573856191	28·8271	9·4016	·001203	831
832	692224	575930368	28·8444	9·4053	·001202	832
833	693889	578009537	28·8617	9·4091	·001200	833
834	695556	580093704	28·8791	9·4129	·001199	834
835	697225	582182875	28·8964	9·4166	·001198	835
836	698896	584277056	28·9137	9·4204	·001196	836
837	700569	586376253	28·9310	9·4241	·001195	837
838	702244	588480472	28·9482	9·4279	·001193	838
839	703921	590589719	28·9655	9·4316	·001192	839
840	705600	592704000	28·9828	9·4354	·001190	840
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
841	707281	594823321	29	9·4391	·001189	841
842	708964	596947688	29·0172	9·4429	·001188	842
843	710649	599077107	29 0345	9·4466	·001186	843
844	712336	601211584	29·0517	9·4503	·001185	844
845	714025	603351125	29·0689	9 4541	·001183	845
846	715716	605495736	29·0861	9·4578	·001182	846
847	717409	607645423	29 1033	9·4615	·001181	847
848	719104	609800192	29·1204	9·4652	·001179	848
849	720801	611960049	29·1376	9·4690	·001178	849
850	722500	614125000	29 1548	9·4727	001176	850
851	724201	616295051	29 1719	9·4764	001175	851
852	725904	618470208	29·1890	9·4801	001174	852
853	727609	620650477	29 2062	9·4838	·001172	853
854	729316	622835864	29·2233	9·4875	001171	854
855	731025	625026375	29·2404	9·4912	·001170	855
856	732736	627222016	29·2575	9·4949	·001168	856
857	734449	629422793	29 2746	9·4986	·001167	857
858	736164	631628712	29·2916	9·5023	·001166	858
859	737881	633839779	29·3087	9·5060	·001164	859
860	739600	636056000	29·3258	9·5097	·001163	860
861	741321	638277381	29·3428	9·5134	·001161	861
862	743044	640503928	29·3598	9·5171	·001160	862
863	744769	642735647	29·3769	9·5207	·001159	863
864	746496	644972544	29·3939	9·5244	·001157	864
865	748225	647214625	29·4109	9·5281	·001156	865
866	749956	649461896	29·4279	9·5317	·001155	866
867	751689	651714363	29·4449	9·5354	·001153	867
868	753424	653972032	29·4618	9·5391	001152	868
869	755161	656234909	29·4788	9·5427	·001151	869
870	756900	658503000	29·4958	9·5464	·001149	870
871	758641	660776311	29·5127	9·5501	·001148	871
872	760384	663054848	29·5296	9·5537	·001147	872
873	762129	665338617	29·5466	9·5574	·001145	873
874	763876	667627624	29·5635	9·5610	·001144	874
875	765625	669921875	29·5804	9·5647	·001143	875
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
876	767376	672221376	29 5973	9 5683	001142	876
877	769129	674526133	29·6142	9·5719	·001140	877
878	770884	676836152	29 6311	9·5756	·001139	878
879	772641	679151439	29·6479	9 5792	·001138	879
880	774400	681472000	29·6648	9·5828	·001136	880
881	776161	683797841	29·6816	9 5865	·001135	881
882	777924	686128968	29 6985	9·5901	·001134	882
883	779689	688465387	29·7153	9·5937	·001133	883
884	781456	690807104	29·7321	9·5973	·001131	884
885	783225	693154125	29·7489	9·6010	·001130	885
886	784996	695506456	29·7658	9·6046	·001129	886
887	786769	697864103	29 7825	9·6082	·001127	887
888	788544	700227072	29·7993	9·6118	·001126	888
889	790321	702595369	29·8161	9·6154	·001125	889
890	792100	704969000	29·8329	9 6190	·001124	890
891	793881	707347971	29·8496	9·6226	·001122	891
892	795664	709732288	29 8664	9·6262	·001121	892
893	797449	712121957	29·8831	9·6298	·001120	893
894	799236	714516984	29·8998	9·6334	001119	894
895	801025	716917375	29·9166	9·6370	·001117	895
896	802816	719323136	29·9333	9·6406	·001116	896
897	804609	721734273	29·9500	9·6442	·001115	897
898	806404	724150792	29·9666	9 6477	·001114	898
899	808201	726572699	29·9833	9·6513	·001112	899
900	810000	729000000	30	9·6549	001111	900
901	811801	731432701	30·0167	9·6585	·001110	901
902	813604	733870808	30 0333	9 6620	001109	902
903	815409	736314327	30 0500	9·6656	·001107	903
904	817216	738763264	30 0666	9·6692	·001106	904
905	819025	741217625	30·0832	9 6727	001105	905
906	820836	743677416	30·0998	9·6763	·001104	906
907	822649	746142643	30·1164	9·6799	·001103	907
908	824464	748613312	30·1330	9·6834	·001101	908
909	826281	751089429	30·1496	9·6870	·001100	909
910	828100	753571000	30·1662	9·6905	·001099	910
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
911	829921	756058031	30·1828	9 6941	·001098	911
912	831744	758550528	30·1993	9·6976	·001096	912
913	833569	761048497	30·2159	9·7012	·001095	913
914	835396	763551944	30·2324	9·7047	·001094	914
915	837225	766060875	30·2490	9·7082	·001093	915
916	839056	768575296	30·2655	9 7118	·001092	916
917	840889	771095213	30·2820	9·7153	·001091	917
918	842724	773620632	30·2985	9·7188	·001089	918
919	844561	776151559	30·3150	9·7224	·001088	919
920	846400	778688000	30·3315	9·7259	·001087	920
921	848241	781229961	30·3480	9·7294	001086	921
922	850084	783777448	30 3645	9·7329	·001085	922
923	851929	786330467	30·3809	9·7364	·001083	923
924	853776	788889024	30·3974	9·7400	·001082	924
925	855625	791453125	30·4138	9·7435	·001081	925
926	857476	794022776	30·4302	9·7470	·001080	926
927	859329	796597983	30·4467	9·7505	·001079	927
928	861184	799178752	30·4631	9·7540	·001078	928
929	863041	801765089	30·4795	9·7575	·001076	929
930	864900	804357000	30·4959	9·7610	·001075	930
931	866761	806954491	30·5123	9·7645	·001074	931
932	868624	809557568	30·5287	9·7680	·001073	932
933	870489	812166237	30·5450	9·7715	·001072	933
934	872356	814780504	30 5614	9·7750	·001071	934
935	874225	817400375	30·5778	9·7785	·001070	935
936	876096	820025856	30·5941	9·7819	·001068	936
937	877969	822656953	30·6105	9·7854	·001067	937
938	879844	825293672	30·6268	9·7889	·001066	938
939	881721	827936019	30·6431	9·7924	·001065	939
940	883600	830584000	30·6594	9·7959	·001064	940
941	885481	833237621	30·6757	9·7993	·001063	941
942	887364	835896888	30·6920	9·8028	·001062	942
943	889249	838561807	30·7083	9·8063	·001060	943
944	891136	841232384	30·7246	9·8097	·001059	944
945	893025	843908625	30·7409	9·8132	·001058	945
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*continued*)

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
946	894916	846590536	30·7571	9·8167	·001057	946
947	896809	849278123	30·7734	9·8201	·001056	947
948	898704	851971392	30·7896	9·8236	·001055	948
949	900601	854670349	30·8058	9·8270	·001054	949
950	902500	857375000	30·8221	9·8305	·001053	950
951	904401	860085351	30·8383	9·8339	·001052	951
952	906304	862801408	30·8545	9·8374	·001050	952
953	908209	865523177	30·8707	9·8408	·001049	953
954	910116	868250664	30·8869	9·8443	·001048	954
955	912025	870983875	30·9031	9·8477	·001047	955
956	913936	873722816	30·9192	9·8511	·001046	956
957	915849	876467493	30·9354	9·8546	·001045	957
958	917764	879217912	30·9516	9·8580	·001044	958
959	919681	881974079	30·9677	9·8614	·001043	959
960	921600	884736000	30·9839	9·8648	·001042	960
961	923521	887503681	31	9·8683	·001041	961
962	925444	890277128	31·0161	9·8717	·001040	962
963	927369	893056347	31·0322	9·8751	·001038	963
964	929296	895841344	31·0483	9·8785	·001037	964
965	931225	898632125	31·0644	9·8819	·001036	965
966	933156	901428696	31·0805	9·8854	·001035	966
967	935089	904231063	31·0966	9·8888	·001034	967
968	937024	907039232	31·1127	9·8922	·001033	968
969	938961	909853209	31·1288	9·8956	·001032	969
970	940900	912673000	31·1448	9·8990	·001031	970
971	942841	915498611	31·1609	9·9024	·001030	971
972	944784	918330048	31·1769	9·9058	·001029	972
973	946729	921167317	31·1929	9·9092	·001028	973
974	948676	924010424	31·2090	9·9126	·001027	974
975	950625	926859375	31·2250	9·9160	·001026	975
976	952576	929714176	31·2410	9·9194	·001025	976
977	954529	932574833	31·2570	9·9227	·001024	977
978	956484	935441352	31·2730	9·9261	·001022	978
979	958441	938313739	31·2890	9·9295	·001021	979
980	960400	941192000	31·3050	9·9329	·001020	980
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Numbers (*concluded*).

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
981	962361	944076141	31·3209	9·9363	·001019	981
982	964324	946966168	31·3369	9·9396	·001018	982
983	966289	949862087	31·3528	9·9430	·001017	983
984	968256	952763904	31·3688	9·9464	·001016	984
985	970225	955671625	31·3847	9·9497	·001015	985
986	972196	958585256	31·4006	9·9531	·001014	986
987	974169	961504803	31·4166	9·9565	·001013	987
988	976144	964430272	31·4325	9·9598	·001012	988
989	978121	967361669	31·4484	9·9632	·001011	989
990	980100	970299000	31·4643	9·9666	·001010	990
991	982081	973242271	31·4802	9·9699	·001009	991
992	984064	976191488	31·4960	9·9733	·001008	992
993	986049	979146657	31·5119	9·9766	·001007	993
994	988036	982107784	31·5278	9·9800	·001006	994
995	990025	985074875	31·5436	9·9833	·001005	995
996	992016	988047936	31·5595	9·9866	·001004	996
997	994009	991026973	31·5753	9·9900	·001003	997
998	996004	994011992	31·5911	9·9933	·001002	998
999	998001	997002999	31·6070	9·9967	·001001	999
1000	1000000	1000000000	31·6228	10	·001000	1000
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Squares, Cubes, Square Roots, Cube Roots, and Reciprocals of Fractions.

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n
$\frac{1}{32}$	·00098	·00003	·17678	·31498	32·00000	$\frac{1}{32}$
$\frac{1}{16}$	·00391	00024	·25000	·39685	16·00000	$\frac{1}{16}$
$\frac{3}{32}$	·00879	00082	·30619	·45428	10·66667	$\frac{3}{32}$
$\frac{1}{8}$	·01562	·00195	·35355	·50000	8 00000	$\frac{1}{8}$
$\frac{5}{32}$	·02441	·00381	39528	·53861	6·40000	$\frac{5}{32}$
$\frac{3}{16}$	·03516	00659	·43301	·57236	5·33333	$\frac{3}{16}$
$\frac{7}{32}$	04785	·01047	·46771	·60254	4 57143	$\frac{7}{32}$
$\frac{1}{4}$	·06250	·01562	·50000	·62996	4·00000	$\frac{1}{4}$
$\frac{9}{32}$	07910	·02225	·53033	·65519	3·55556	$\frac{9}{32}$
$\frac{5}{16}$	·09766	·03052	·55902	67860	3·20000	$\frac{5}{16}$
$\frac{11}{32}$	·11816	·04062	·58630	·70051	2·90909	$\frac{11}{32}$
$\frac{3}{8}$	·14062	·05273	61237	·72112	2 66667	$\frac{3}{8}$
$\frac{13}{32}$	·16504	·06705	·63738	·74062	2 46154	$\frac{13}{32}$
$\frac{7}{16}$	·19141	·08374	·66144	·75915	2·28571	$\frac{7}{16}$
$\frac{15}{32}$	·21973	10300	·68165	·77681	2 13333	$\frac{15}{32}$
$\frac{1}{2}$	25000	·12500	·70711	·79370	2·00000	$\frac{1}{2}$
$\frac{17}{32}$	·28223	·14993	·72887	·80990	1·88235	$\frac{17}{32}$
$\frac{9}{16}$	·31641	·17798	·75000	·82548	1·77778	$\frac{9}{16}$
$\frac{19}{32}$	·35254	·20932	·77055	·84049	1·68421	$\frac{19}{32}$
$\frac{5}{8}$	·39062	·24414	·79057	·85499	1·60000	$\frac{5}{8}$
$\frac{21}{32}$	·43066	·28262	·81009	86901	1·52381	$\frac{21}{32}$
$\frac{11}{16}$	·47266	·32495	·82916	·88259	1·45455	$\frac{11}{16}$
$\frac{23}{32}$	51660	·37131	·84779	·89576	1·39130	$\frac{23}{32}$
$\frac{3}{4}$	·56250	·42187	·86603	·90856	1·33333	$\frac{3}{4}$
$\frac{25}{32}$	·61035	·47684	·88388	·92101	1·28000	$\frac{25}{32}$
$\frac{13}{16}$	·66016	·53638	·90139	·93313	1·23077	$\frac{13}{16}$
$\frac{27}{32}$	·71191	·60068	·91856	·94494	1·18519	$\frac{27}{32}$
$\frac{7}{8}$	·76562	·66992	·93541	·95647	1·14286	$\frac{7}{8}$
$\frac{29}{32}$	·82129	·74429	·95197	·96772	1·10345	$\frac{29}{32}$
$\frac{15}{16}$	·87891	·82397	·96825	·97872	1·06667	$\frac{15}{16}$
$\frac{31}{32}$	·93848	·90915	·98425	·98947	1·03226	$\frac{31}{32}$
n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1}{n}$	n

Tables of Aliquot Parts.

One quantity is said to be an aliquot part of another when the first is contained in the second an exact number of times; thus, 1s. 8d. is an aliquot part of £1, because £1 = 12 times 1s. 8d., or 1s. 8d. = $\frac{1}{12}$ of £1

Money

10s = $\frac{1}{2}$ of £1.	3s. 4d. = $\frac{1}{3}$ of 10s.	1s. 8d. = $\frac{1}{3}$ of 5s.
6s 8d = $\frac{1}{3}$,,	2s. 6d. = $\frac{1}{4}$,,	1s. 3d. = $\frac{1}{4}$,,
5s. = $\frac{1}{4}$,,	1s. 8d = $\frac{1}{6}$,,	10d. = $\frac{1}{6}$,,
4s. = $\frac{1}{5}$,,	1s. 3d. = $\frac{1}{8}$,,	7½d. = $\frac{1}{8}$,,
3s. 4d = $\frac{1}{6}$,,	10d. = $\frac{1}{12}$,,	1s 4d = $\frac{1}{3}$ of 4s.
2s. 6d = $\frac{1}{8}$,,	9d. = $\frac{1}{10}$ of 7s 6d.	8d. = $\frac{1}{6}$,,
2s. = $\frac{1}{10}$,,	1s 8d. = $\frac{1}{4}$ of 6s. 8d.	10d. = $\frac{1}{4}$ of 3s. 4d.
1s 8d = $\frac{1}{12}$,,	1s. 4d. = $\frac{1}{5}$,,	8d. = $\frac{1}{5}$,,
1s. 4d. = $\frac{1}{15}$,,	10d. = $\frac{1}{8}$,,	10d. = $\frac{1}{3}$ of 2s. 6d.
1s. 3d. = $\frac{1}{16}$,,	8d. = $\frac{1}{10}$,,	1½d. = $\frac{1}{8}$ of 1s.

Length.

440 yds. = $\frac{1}{4}$ of 1 ml.	9 in. = $\frac{1}{4}$ of 1 yard.	4 in. = $\frac{1}{3}$ of 1 foot.
352 yds = $\frac{1}{5}$,,	6 in. = $\frac{1}{6}$,,	3 in = $\frac{1}{4}$,,
220 yds. = $\frac{1}{8}$,,	4½ in. = $\frac{1}{8}$,,	2 in. = $\frac{1}{6}$,,
176 yds. = $\frac{1}{10}$,,	4 in. = $\frac{1}{9}$,,	1½ in. = $\frac{1}{8}$,,

Avoirdupois Weight.

1120 lbs. = $\frac{1}{2}$ of 1 ton.	280 lbs. = $\frac{1}{8}$ of 1 ton.	16 lbs. = $\frac{1}{7}$ of 1 cwt.
560 lbs. = $\frac{1}{4}$,,	224 lbs. = $\frac{1}{10}$,,	14 lbs. = $\frac{1}{8}$,,
448 lbs. = $\frac{1}{5}$,,	56 lbs. = $\frac{1}{2}$ of 1 cwt.	8 lbs. = $\frac{1}{14}$,,
320 lbs. = $\frac{1}{7}$,,	28 lbs. = $\frac{1}{4}$,,	7 lbs. = $\frac{1}{16}$,,

Decimal Equivalents of Vulgar Fractions.

Vulgar Fractions	Decimal Equivalents	Vulgar Fractions	Decimal Equivalents.	Vulgar Fractions	Decimal Equivalents.
1/64	·015625	11/32	·34375	43/64	·671875
1/32	·03125	23/64	·359375	11/16	·6875
3/64	·046875	3/8	·375	45/64	·703125
1/16	·0625	25/64	·390625	23/32	·71875
5/64	·078125	13/32	·40625	47/64	·734375
3/32	·09375	27/64	·421875	3/4	·75
7/64	·109375	7/16	·4375	49/64	·765625
1/8	·125	29/64	·453125	25/32	·78125
9/64	·140625	15/32	·46875	51/64	·796875
5/32	·15625	31/64	·484375	13/16	·8125
11/64	·171875	1/2	·5	53/64	·828125
3/16	·1875	33/64	·515625	27/32	·84375
13/64	·203125	17/32	·53125	55/64	·859375
7/32	·21875	35/64	·546875	7/8	·875
15/64	·234375	9/16	·5625	57/64	·890625
1/4	·25	37/64	·578125	29/32	·90625
17/64	·265625	19/32	·59375	59/64	·921875
9/32	·28125	39/64	·609375	15/16	·9375
19/64	·296875	5/8	·625	61/64	·953125
5/16	·3125	41/64	·640625	31/32	·96875
21/64	·328125	21/32	·65625	63/64	·984375

Inches and Fractions of an Inch expressed as Decimals of a Foot.

Inches	·0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0	.	·01042	·02083	·03125	·04167	·05208	·0625	·07292
1	·08333	·09375	·10417	·11458	·125	·13542	·14583	·15625
2	·16667	·17708	·1875	·19792	·20833	·21875	·22917	·23958
3	·25	·26042	·27083	·28125	·29167	·30208	·3125	·32292
4	·33333	·34375	·35417	·36458	·375	·38542	·39583	·40625
5	·41667	·42708	·4375	·44792	·45833	·46875	·47917	·48958
6	·5	·51042	·52083	·53125	·54167	·55208	·5625	·57292
7	·58333	·59375	·60417	·61458	·625	·63542	·64583	·65625
8	·66667	·67708	·6875	·69792	·70833	·71875	·72917	·73958
9	·75	·76042	·77083	·78125	·79167	·80208	·8125	·82292
10	·83333	·84375	·85417	·86458	·875	·88542	·89583	·90625
11	·91667	·92708	·9375	·94792	·95833	·96875	·97917	·98958

1/16 inch = ·005208 foot; 1/32 inch = ·002604 foot; 1/64 inch = ·001302 foot.

Useful Functions of π.

The Greek letter π denotes the ratio of the circumference of a circle to its diameter.

	N	2N	3N	4N	5N	6N	7N	8N	9N
π =	3·14159265	6·283185	9·424778	12·566371	15·707963	18·849556	21·991149	25·132741	28·274334
$\frac{\pi}{2}$ =	1·57079633	3·141593	4·712389	6·283185	7·853982	9·424778	10·995574	12·566371	14·137167
$\frac{\pi}{3}$ =	1·04719755	2·094395	3·141593	4·188790	5·235988	6·283185	7·330383	8·377580	9·424778
$\frac{\pi}{4}$ =	·78539816	1·570796	2·356194	3·141593	3·926991	4·712389	5·497787	6·283185	7·068583
$\frac{\pi}{6}$ =	·52359878	1·047198	1·570796	2·094395	2·617994	3·141593	3·665191	4·188790	4·712389
$\frac{\pi}{7}$ =	·44879895	·897598	1·346397	1·795196	2·243995	2·692794	3·141593	3·590392	4·039191
$\frac{\pi}{16}$ =	·19634954	·392699	·589049	·785398	·981748	1·178097	1·374447	1·570796	1·767146
$\frac{\pi}{24}$ =	·13089969	·261799	·392699	·523599	·654498	·785398	·916298	1·047198	1·178097
$\frac{\pi}{32}$ =	·09817477	·196350	·294524	·392699	·490874	·589049	·687223	·785398	·883573
$\frac{\pi}{180}$ =	·01745329	·034907	·052360	·069813	·087266	·104720	·122173	·139626	·157080
	N	2N	3N	4N	5N	6N	7N	8N	9N

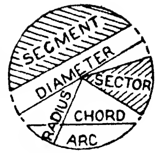
Useful Functions of π (*continued*).

The Greek letter π denotes the ratio of the circumference of a circle to its diameter.

N	2N	3N	4N	5N	6N	7N	8N	9N
π^2 = 9·86960440	19·739209	29·608813	39·478418	49·348022	59 217626	69 087231	78 956835	88·826440
π^3 = 31·00627668	62·012553	93 018830	124·025107	155·031383	186 037660	217 043937	248 050213	279·056490
$\frac{1}{\pi}$ = 31830989	·636620	·954930	1 273240	1 591549	1 909859	2 228169	2 546479	2 864789
$\frac{1}{\pi^2}$ = ·10132118	·202642	·303964	·405285	·506606	·607927	709248	·810569	·911891
$\frac{1}{\pi^3}$ = ·03225153	·064503	·096755	·129006	161258	·193509	225761	258012	290264
$\sqrt{\pi}$ = 1·77245385	3·544908	5·317362	7·089815	8 862269	10 634723	12·407177	14·179631	15·952085
$\sqrt[3]{\pi}$ = 1·46459189	2·929184	4·393776	5·858368	7 322959	8 787551	10 252143	11 716735	13 181327
$\frac{1}{\sqrt{\pi}}$ = ·56418958	1·128379	1·692569	2·256758	2 820948	3 385137	3 949327	4·513517	5·077706
$\frac{1}{\sqrt[3]{\pi}}$ = ·68278406	1·365568	2·048352	2·731136	3 413920	4 096704	4 779488	5·462272	6·145057
Log. π = ·49714987	·994300	1·491450	1·988599	2 485749	2·982899	3 480049	3 977199	4·474349
N	2N	3N	4N	5N	6N	7N	8N	9N

NOTES ON GEOMETRY.

A *circle* is a plane figure contained by one line, which is called the *circumference*, and is such, that all straight lines drawn from a certain point within the figure to the circumference are equal to one another; and this point is called the *centre* of the circle.

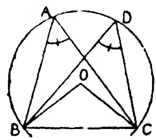


In a segment $BADC$ of a circle the angle BOC at the centre is double of the angle BAC at the circumference.

Angles BAC and BDC in the same segment of a circle are equal to one another.

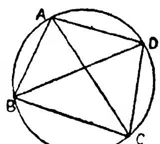
The angle in a semicircle is a right angle.

Similar segments of circles are those which contain equal angles.

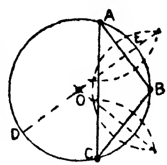


In a quadrilateral $ABCD$ inscribed in a circle the opposite angles ABC and ADC are together equal to two right angles. Also the opposite angles BAD and BCD are together equal to two right angles.

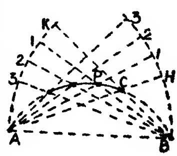
Also $\overline{AC}\times\overline{BD}=\overline{AB}\times\overline{CD}+\overline{AD}\times\overline{BC}$.



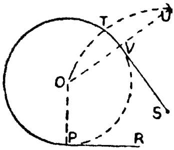
A diameter DE which bisects a chord AB is at right angles to that chord. Hence the construction for drawing a circle through three points, A, B, and C. Bisect the straight lines AB and BC by straight lines at right angles to them. These bisecting lines will intersect at the centre O of the circle required.



To describe an arc of a circle through three points, A, B, and C, when the centre of the circle is inaccessible. With centres A and B describe arcs BH and AK. Join AC and BC, and produce these lines to meet the arcs at H and K. Mark off short equal arcs $H1$, $K1$. The intersection P of the lines $A1$, $B1$, is another point on the arc required. In like manner other points may be found. A fair curve drawn through these points is the arc required.

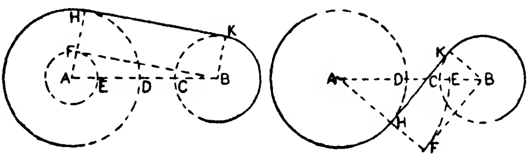


If PR is a tangent to the circle PTV, and P is the point of contact, PR is at right angles to the radius or diameter through P.



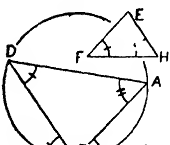
To draw a tangent to the circle PTV from a point S outside the circle. With centre S describe an arc OTU to pass through O, the centre of the circle. With centre O and a radius equal to the diameter of the circle describe an arc to cut the arc OTU at U. Draw OU, cutting the circle at V. SV is the tangent required and V is the point of contact.

To draw a tangent to two given circles. Make $DE = BC$ Draw a circle, with centre A and radius AE. Draw BF a tangent to this circle, F being the point of contact. Draw the line AF,



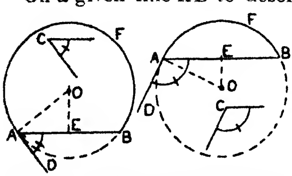
meeting the given circle, whose centre is A, at H. Draw BK parallel to AH. HK is a tangent to both the given circles.

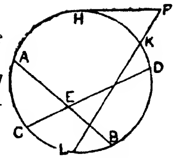
The angle ABC between the chord AB and the tangent BC to the circle at the point B is equal to the angle ADB in the alternate segment.



To inscribe in the circle ABD a triangle equiangular to the triangle EFH. Draw the tangent KBC. Draw AB, making the angle ABC = angle F. Draw DB, making the angle DBK = angle H. BDA is the triangle required.

On a given line AB to describe a segment of a circle which shall contain an angle equal to a given angle C. Draw AD, making the angle DAB = angle C. Draw AO at right angles to AD. Bisect AB at E. Draw EO at right angles to AB, meeting AO at O. O is the centre, and OA is the radius of the circle, of which the segment AFB shall contain an angle = angle C.

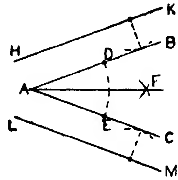




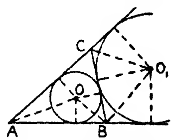
AB and CD are any two chords of a circle intersecting at E. $AE \times BE = CE \times DE$.

FH is a tangent to the circle $ACBD$, H being the point of contact. FKL is a line cutting the circle at K and L. $\overline{FH}^2 = \overline{FK} \times \overline{FL}$.

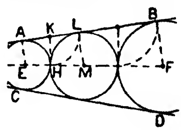
To bisect the angle between two straight lines. (1) The lines AB, AC, intersect within the paper. With centre A describe an arc DE. With centres D and E describe arcs, with equal radii, to intersect at F. The line AF bisects the angle BAC. (2) The lines HK, LM, do not intersect within the paper. Draw AB and AC parallel to HK and LM respectively, and at equal distances from them, so that the point A is accessible. The line AF, which bisects the angle BAC, is the line required.



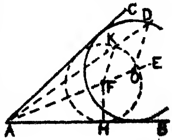
To draw the inscribed and escribed circles of a triangle ABC. The inscribed circle is the one which touches each of the three sides. An escribed circle touches one side and the other two produced. There are three escribed circles to a triangle. When a circle touches each of two straight lines its centre lies on the line bisecting the angle between them. O is the centre of the inscribed circle, and O_1 is the centre of one of the escribed circles.

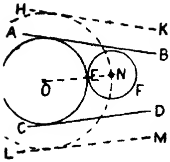


To draw a series of circles to touch one another and two lines AB, CD. Draw EF, bisecting the angle between AB and CD. Let E be the centre of one circle: its radius is EA, the perpendicular on AB. Draw HK perpendicular to EF. Make $KL = KH$. Draw LM perpendicular to AB. M is the centre of the next circle.

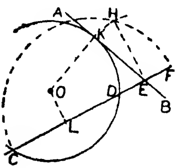


To draw a circle to pass through the point D and touch the lines AB and AC. Draw AE, bisecting the angle BAC. Take any point F in AE. Draw FH perpendicular to AB. With F as centre and FH as radius describe a circle to cut the line AD at K. Draw DO parallel to KF, meeting AE at O. O is the centre, and OD the radius of the circle required.

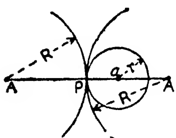




To draw a circle to touch two lines AB, CD, and a circle EF. Draw HK and LM parallel to AB and CD respectively, and at distances from them equal to the radius EN of the circle EF. Draw a circle to pass through N, and touch the lines HK, LM. O the centre of this circle, is the centre of the circle required.

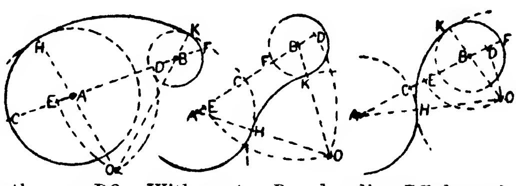


To draw a circle to touch the line AB and pass through the points C and D. Draw CD, and produce it to cut AB at E. Make $EF = ED$. On CF describe a semicircle. Draw EH perpendicular to CF to meet the semicircle at H. Make $EK = EH$. Draw KO perpendicular to AB. Draw LO, bisecting CD at right angles to meet KO at O. O is the centre of the circle required



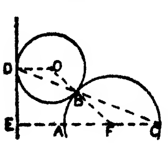
If a circle whose centre is A and radius $= R$ touches another circle whose centre is a and radius $= r$, the point of contact P lies on the line Aa or on that line produced, and the distance Aa between their centres is equal to $R + r$ or $R - r$.

To draw a circle of given radius to touch two given circles. A and B are the centres of the given circles. Make CD and FE each equal to the given radius. With centre A and radius AD

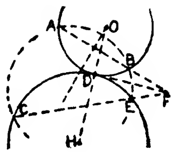


draw the arc DO. With centre B and radius BE draw the arc EO, cutting the former arc at O. Join O with A and B, and produce these lines if necessary. O is the centre of the required circle, and H and K are the points of contact.

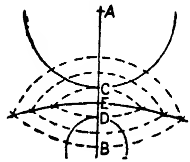
To draw a circle to touch a circle ABC and a straight line DE at the point D. Through F, the centre of the circle ABC, draw FE perpendicular to DE, and produce it to meet the circle at C. Draw DO perpendicular to DE. Join CD, cutting the circle at B. Join FB, and produce it to meet DO at O. O is the centre of the circle required.



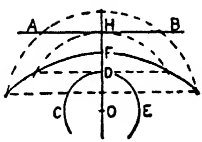
To draw a circle to pass through the points A and B, and touch the circle CDE. Draw a circle $CABE$ through A and B, cutting the circle CDE at C and E. Join CE, and produce it to meet AB produced at F. Draw FD, touching the given circle at D. Join H, the centre of the circle CDE, with D, and produce it to meet the line bisecting AB at right angles at O. O is the centre of the circle required.



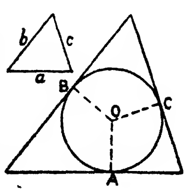
To draw the locus of the centre of a circle which touches two given circles. A and B are the centres of the given circles. Draw AB, cutting the circles at C and D. Bisect CD at E. Mark off from E, above and below it, on AB a number of equal divisions. With centres A and B describe arcs through these divisions, as shown. A fair curve through the intersections of the arcs is the locus required. The curve is an hyperbola.



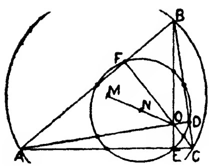
To draw the locus of the centre of a circle which touches a line AB and a circle CDE. Through O, the centre of the circle, draw OH perpendicular to AB. Bisect DH at F. Mark off from F, above and below it, on OH a number of equal divisions. With centre O draw arcs through the divisions above F to meet parallels to AB through the divisions below F, as shown. A fair curve through the intersections of the arcs and parallels is the locus required. The curve is a parabola.



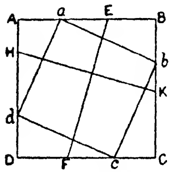
To draw a triangle equiangular to a given triangle, and circumscribing a given circle. From O, the centre of the circle, draw the radii OA, OB, and OC at right angles to the sides a, b, and c of the given triangle respectively. The sides of the required triangle will be tangents to the circle at A, B, and C.



The nine points circle. ABC is a triangle. D, E, and F are the feet of the perpendiculars on the sides from the opposite angles. O is the point of intersection of these perpendiculars. The circle through D, E, and F also passes through the middle



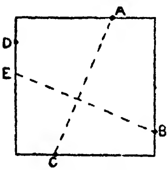
points of the sides and through the middle points of OA, OB, and OC. This circle is known as the nine points circle. If M is the centre of the circumscribing circle of the triangle, and N is the centre of the nine points circle, N is the middle point of OM. The radius of the nine points circle is equal half the radius of the circumscribing circle



$ABCD$ is a square.

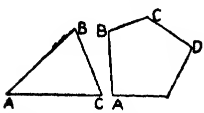
If $Aa=Bb=Cc=Dd$, then $abcd$ is a square.

Lines EF and HK, which are perpendicular to one another and are terminated by the sides of the square, are equal to one another.



To draw a square to pass through four given points, A, B, C, and D Join AC. Draw BE at right angles to AC, and equal to it. Join DE, and produce it both ways. Lines through A and C perpendicular to DE, and a line through B parallel to DE, will complete the square required.

In any triangle, ABC the three angles are together equal to two right angles, *i.e.* $A+B+C=180°$.

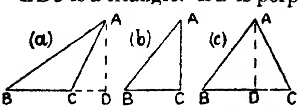


In any polygon, $ABCD$ ... the sum of the interior angles together with four right angles is equal to twice as many right angles as the figure has sides, *i.e.*

$$A+B+C+D+\ldots.+360°=n\times 180°,$$

where n denotes the number of sides

ABC is a triangle. AD is perpendicular to BC or BC produced.

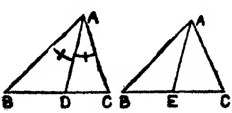


(*a*) $\overline{AB}^2=\overline{AC}^2+\overline{BC}^2+2\overline{BC}\times\overline{CD}$.

(*b*) Angle $C=90°$.

$\overline{AB}^2=\overline{AC}^2+\overline{BC}^2$.

(*c*) $\overline{AB}^2=\overline{AC}^2+BC^2-2BC\times CD$.

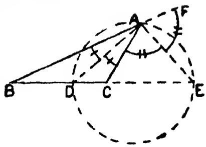


ABC is a triangle. AD bisects the angle BAC.

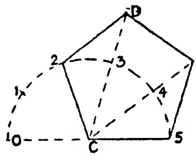
$\overline{AB}\times\overline{AC}=\overline{BD}\times\overline{DC}+AD^2$.

$BE=EC$.

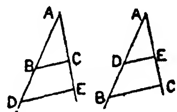
$\overline{AB}^2+\overline{AC}^2=2\overline{BE}^2+2\overline{AE}^2$.



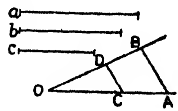
ABC is a triangle. AD bisects the angle BAC.
AE bisects the angle CAF between CA and BA produced.
$AB:AC::BD:CD$, and $AB:AC::BE:CE$.
Angle $DAE = 90°$. If B and C are fixed points, and the ratio of AB to AC is constant, the locus of A is a circle described on DE as diameter.



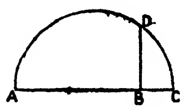
To draw a regular polygon on a given line $C5$. With centre C and radius $C5$ describe the semicircle 035, and divide it into as many equal parts as there are sides in the polygon to be drawn. Join C with 2, 3, etc. With centre 2 and radius $2C$ draw an arc to cut $C3$ produced at D. With centre D and the same radius draw an arc to cut $C4$ produced at E, and so on.



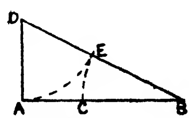
AB and AC are two lines intersecting at A and making any angle with one another. DE is parallel to BC.
$AB:AC::AD:AE$.



To find a fourth proportional to three lines a, b, and c. Draw OA and OB, making any angle with one another. Make $OA = a$, $OB = b$, and $OC = c$. Draw CD parallel to AB. Then $a:b::c:OD$.

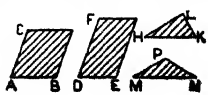


To find a mean proportional to AB and BC. AB and BC are placed in the same straight line. On AC describe a semicircle. Draw BD perpendicular to AC to meet the semicircle at D. Then $AB:BD::BD:BC$, or $BD^2 = \overline{AB} \times \overline{BC}$.



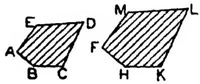
To divide a line AB in medial section. Draw AD perpendicular to AB and equal to half AB. Join BD. Make $DE = DA$, and $BC = BE$. Then $AB : BC :: BC : AC$, or $\overline{BC}^2 = \overline{AB} \times \overline{AC}$.

CB and FE are parallelograms. Angle A = angle D. The parallelograms are equal in area if $\overline{AB} \times \overline{AC} = \overline{DE} \times \overline{DF}$.

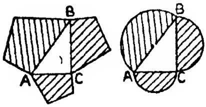


LHK and PMN are triangles. Angle H = angle M. The triangles are equal in area if $\overline{HK} \times \overline{HL} = \overline{MN} \times \overline{MP}$.

Similar figures are those which have their several angles equal, each to each, and the sides about the equal angles proportionals. If AD and FL are similar figures, then angle A = angle F, angle B = angle H, angle C = angle K, and so on. Also $AB:AE::FH:FM$, $BC:BA::HK:HF$, $CD:CB::KL:KH$, and so on.

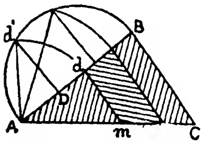


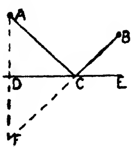
The areas of similar figures are to one another as the squares on their homologous or corresponding sides; thus, area of figure AD: area of figure $FL::\overline{AB}^2:\overline{FH}^2$.



If similar figures be described with corresponding sides on the three sides of a right-angled triangle ABC, then figure on hypotenuse AB = figure on BC + figure on AC The above is true also for circles with the sides of the triangle as diameters, also for similar segments of circles.

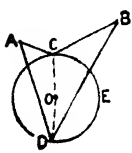
To divide a triangle ABC into a given number of equal parts by lines parallel to one side, BC. On AB describe a semicircle. Divide AB into the given number of equal parts. Let D be one of the points of division. Draw Dd' perpendicular to AB to meet the semicircle at d'. Make $Ad = Ad'$. Draw dm parallel to BC. dm is one of the required lines of division; the others are obtained in a similar manner.





A and B are given points. DE is a given line. C is a point on DE. $AC+BC$ is a minimum when angle ACD = angle BCE.

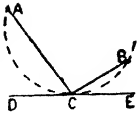
To find C when $AC+BC$ is a minimum. Draw AD perpendicular to DE, and produce it. Make $DF=DA$. Join BF. C is where BF cuts DE.



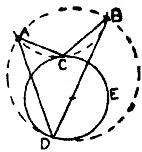
A and B are given points. CDE is a given circle whose centre is O. $AC+BC$ is a minimum when angle ACO = angle BCO. $AD+BD$ is a maximum when angle ADO = angle BDO.

$\overline{AC}^2+\overline{BC}^2$ is a minimum, and $\overline{AD}^2+\overline{BD}^2$ is a maximum when D, O, and C are in a straight line bisecting AB.

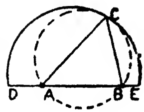
A and B are given points. DE is a given line. C is a point on DE. The angle ACB is a maximum when the circle through A, B, and C touches the line DE.



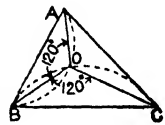
A and B are given points. CDE is a given circle. The angle ACB is a maximum when the circle through A, B, and C touches the circle CDE externally. The angle ADB is a minimum when the circle through A, B, and D is touched internally by the circle CDE.



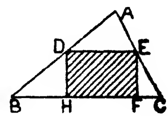
A and B are points on the chord DE. C is a point on the arc DCE of a circle. The angle ACB is a maximum when the circle through A, B, and C touches the arc DCE.



ABC is a triangle. To find a point O such that $AO+BO+CO$ shall be a maximum. On AB and BC describe segments of circles containing angles of 120°. The point O required is at the intersection of the arcs of these segments.

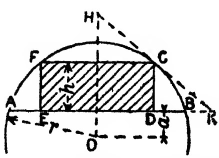


The area of the rectangle $DEFH$ inscribed in the triangle ABC is a maximum when DE bisects the sides AB and AC. The maximum area of the rectangle is half the area of the triangle in which it is inscribed.

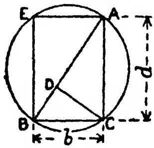


To inscribe in a segment $AFCB$ of a circle a rectangle $CDEF$ of maximum area. Make $h=\dfrac{\sqrt{8r^2+a^2}-3a}{4}$.

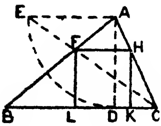
The construction may be verified by drawing the tangent HCK to meet AB produced at K, and OH the perpendicular from the centre O to AB at H. HC should be equal to CK.



In a circle to inscribe a rectangle such that bd^2 shall be a maximum. Draw a diameter AB. Make $BD = \frac{1}{3} AB$. Draw DC perpendicular to AB to meet the circle at C. $ACBE$ is the rectangle required.

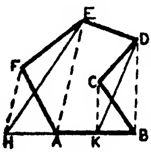


Of all the rectangular beams which can be cut out of a cylinder of diameter AB, the one having the section $ACBE$ has the greatest resistance to bending.

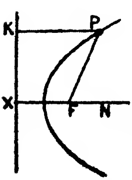


To inscribe a square in a given triangle ABC Draw AD perpendicular to BC, and AE parallel to BC. Make $AE = AD$. Join CE, meeting AB at F. Draw FH parallel to BC, and HK and FL perpendicular to BC. $FHKL$ is the square required.

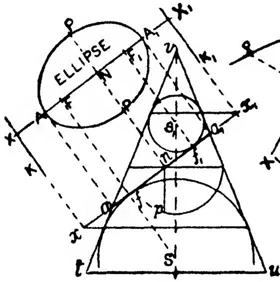
To reduce any plane rectilineal figure $ABCDEF$ to one having the same area but fewer sides Join AE Draw FH parallel to AE, meeting AB or AB produced at H. Join HE. The figure $BCDEH$ has the same area as the original figure. Repeating the above construction the figure $KDEH$ is obtained, which has the same area as the original figure. By the continued application of the above construction any plane rectilineal figure may be reduced to a triangle having the same area.

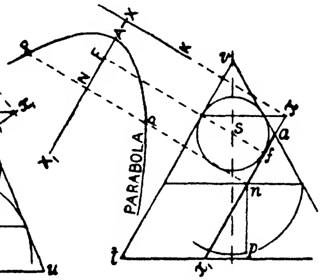


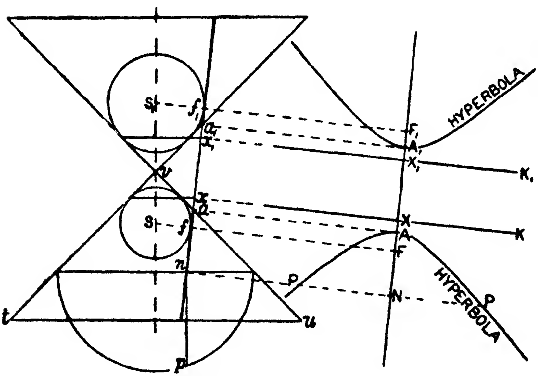
The Conic Sections.—KX is a fixed line, and F is a fixed point. P is a point which moves in the plane containing F and KX in such a manner that its distance from F bears a constant ratio to its perpendicular distance PK from KX. The curve traced out by P is either an *ellipse*, an *hyperbola*, or a *parabola*, according as FP is less than, greater than, or equal to PK. These curves are called *conic sections* or *conics*, because they may be obtained by taking plane sections of a cone. KX is called the *directrix*, F the *focus*, and the line XFN at right angles to KX the *axis* of the conic. The ellipse and hyperbola have two directrices and two foci. The parabola has one directrix and one focus. The constant ratio of FP to PK is called the ***eccentricity*** **of the conic.**



The figures below show how the ellipse, hyperbola, and parabola are obtained by taking sections of the cone. F, F_1 are the foci, and KX, K_1X_1 are the directrices. The foci are the points where spheres inscribed in the cone touch the plane of section. Any point P on the curve is obtained by making NP equal to np.

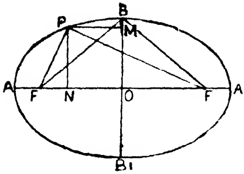






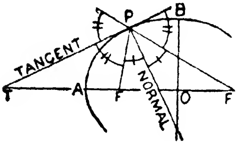
For the ellipse, the plane of section xx_1 cuts vt and vu below the vertex v. For the hyperbola, xx_1 cuts vu below, and vt above the vertex v. For the parabola, xx_1 is parallel to vt.

The Ellipse.—In the ellipse the curve cuts the axis at two points, A and A_1. The line AA_1 is called the *major axis*. A line



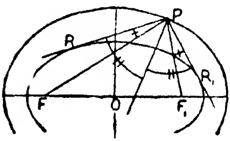
BB_1 bisecting AA_1 at right angles and terminated by the curve is called the *minor axis*. P is any point on the curve. PN is perpendicular to AA_1, and PM is perpendicular to BB_1. $PF + PF_1 = AA_1$. $BF = BF_1 = AO$.

$$PN^2 : AN \times A_1N :: BO^2 : AO^2.$$
$$PM^2 : BM \times B_1M : AO^2 : BO^2.$$



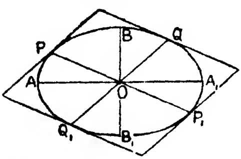
The tangent at P is equally inclined to the focal distances PF, PF_1, so also is the normal.

Ellipses which have the same foci are said to be *confocal*.

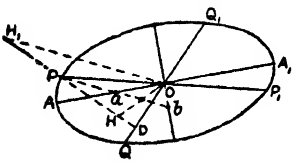


Tangents PR and PR_1 to an ellipse from any point P are equally inclined to PF and PF_1 respectively. If P is on a confocal ellipse, PR and PR_1 are also equally inclined to the normal at P. If angle $RPR_1 = 90°$, the locus of P is a circle with centre O.

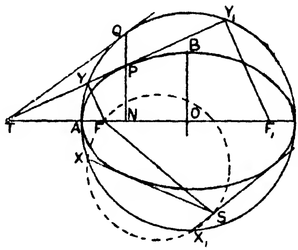
A *diameter* of an ellipse is a line passing through the centre and terminated by the curve. Tangents at the extremities of a diameter are parallel. A diameter bisects all chords parallel to the tangents at its extremities. A diameter QQ_1 parallel to the tangents at P and P_1 is said to be *conjugate* to the diameter PP_1. $\overline{OP}^2 + \overline{OQ}^2 = \overline{OA}^2 + \overline{OB}^2$. The parallelogram formed by the tangents at the extremities of conjugate diameters is equal in area to the rectangle contained by the major and minor axes.



Given, conjugate diameters PP_1 and QQ_1, to find the axes. Draw PD perpendicular to OQ. Make $PH = PH_1 = OQ$. Major axis AOA_1 bisects angle H_1OH. Join P to middle point of OH, to cut major axis at a and minor axis at b. Pb = length of semi-major axis. Pa = length of semi-minor axis.

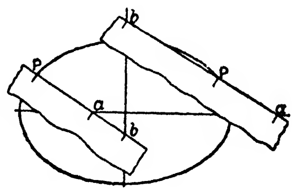


The circle described on the major axis as diameter is called the *auxiliary circle.* The feet of the perpendiculars from the foci on any tangent to the ellipse lie on the auxiliary circle. $BO^2 = FY \times F_1Y_1$ If QPN is perpendicular to the major axis, then $PN : QN :: BO : AO$. The tangent to the ellipse at P, and the tangent to the auxiliary circle at Q, meet at a point T on the major axis produced.

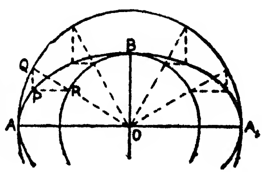


To draw tangents to the ellipse from the point S. On SF or SF_1 as diameter describe a circle to cut the auxiliary circle at X and X_1. SX and SX_1 are the directions of the tangents required.

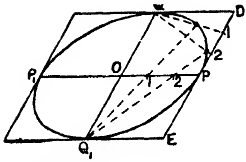
The axes of an ellipse being given, the best practical method of drawing the curve is that involving the application of the trammel. On the straight edge of a strip of paper mark off Pb equal to the semi-major axis, and Pa equal to the semi-minor axis. Place the strip so that a is on the major and b on the minor axis. Mark the point P on the drawing paper. P is a point on the curve. By moving the strip round, any number of points may be determined. a and b may be on the same or on opposite sides of P. If the axes are nearly equal, it is better to place P between a and b.



Another method of drawing the curve, the axes being given. Draw circles with centre O and radii OA and OB. Draw the radial line ORQ. Draw QP parallel to OB and RP parallel to OA. P is a point on the curve. In like manner any number of points may be determined.

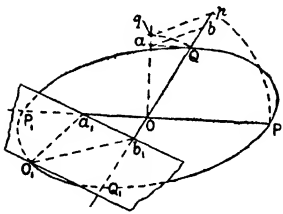


Given the conjugate diameters PP_1, QQ_1, to draw the ellipse. At P and P_1 draw parallels to QQ_1. At Q and Q_1 draw parallels to PP_1. Divide OP into any number of equal parts. Divide PD into the same number of equal parts. Draw lines from Q to the

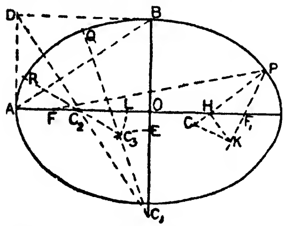


points of division on PD, and lines from Q_1 to the points of division on OP. The intersections of the former and latter lines, taken as shown, are points on the portion PQ of the curve. Points on the other portions of the curve are determined in a similar manner.

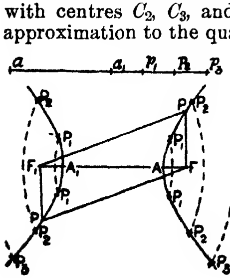
Another method of drawing the ellipse from conjugate diameters is that due to Professor Minchin, and is similar to the trammel method. Draw Oq perpendicular to OP. Draw Qa parallel to OP. Make $Oq = OQ$, and $Op = OP$. Draw ab parallel to qp. Draw the triangle abO on a strip of paper, as shown at $a_1b_1O_1$. If a_1 is placed on the diameter PP_1, and b_1 on the diameter QQ_1, then O_1 will be a point on the curve.



Centre of curvature. P is any point on the ellipse. Draw PC bisecting the angle FPF_1. Draw HK perpendicular to PC. Draw KC perpendicular to PK. The point C is the centre of curvature at the point P, *i.e.* it is the centre of the circle which most nearly coincides with the ellipse at P. Draw BD parallel to OA and AD parallel to OB. Draw DC_2C_1 perpendicular to AB. C_1 is the centre of curvature at B, and C_2 is the centre of curvature at A. Make BE a mean proportional to AO and BO. Make $AL = BE$. Draw the arc EC_3 with centre C_1 and the arc LC_3 with centre C_2. Arcs of circles AR, RQ, and QB, with centres C_2, C_3, and C_1 respectively, will give a very close approximation to the quadrant AB of the true ellipse.



The Hyperbola.—The hyperbola consists of two similar branches, which cut the axis at A and A_1. F and F_1 are the foci. If any point P be taken on the curve, then the difference between the focal distances PF and PF_1 is equal to AA_1. To construct the curve. Draw any line ap_3. Make $aa_1 = AA_1$. Take any number of points p_1, p_2, p_3 on a_1p_3. With centres F and F_1 describe arcs with radii $= ap_1$, and with centres F_1 and

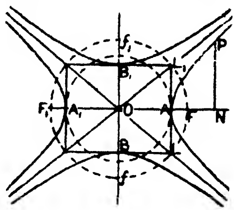


F and radii $= a_1p_1$ describe arcs to cut the former at $P_1P_1P_1P_1$, these are points on the hyperbola. In like manner any number of points may be determined.

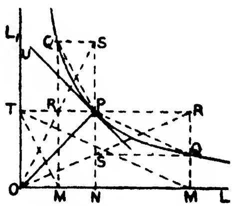
The tangent at P is equally inclined to the focal distances FP and F_1P.

The construction for the centre of curvature is the same as that for the ellipse.

O, the middle point of AA_1, is the centre of the hyperbola. The circle described on AA_1 as diameter is the auxiliary circle. Let the circle with centre O and radius OF meet the tangent at A at L and L_1. Draw LB and L_1B_1 parallel to AA_1 to meet a line through O at right angles to AA_1 at B and B_1. AA_1 is the transverse axis, and BB_1 is the conjugate axis. Let the circle through F cut BB_1 produced at f and f_1. An hyperbola with foci f and f_1 and BB_1 as axis is conjugate to the hyperbola with F and F_1 as foci and AA_1 as axis. Lines OL and OL_1 produced both ways are the ***asymptotes*** of the hyperbola.



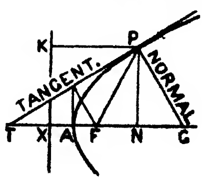
When the transverse and conjugate axes are equal, the asymptotes are at right angles to one another, and the hyperbola is said to be rectangular or equilateral. Given the asymptotes OL and OL_1 of a rectangular hyperbola and a point P on the curve: to draw the curve. Draw any line MR parallel to OL_1 and PN parallel to OL_1. Through P draw TPR parallel to OL. Join OR, meeting PN or PN produced at S. Draw SQ parallel to OL, meeting MR at Q. Q is another point on the curve. Or thus: through P draw PQ parallel to TM to meet MR at Q. These constructions depend on the property of the rectangular hyperbola, viz., $\overline{PN} \times \overline{ON} = \overline{QM} \times \overline{OM}$.



PU is a tangent at P. The angle UPT = angle OPT = angle PON.

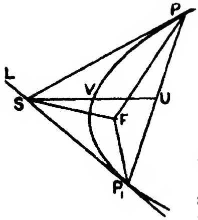
The Parabola.—F is the focus, and KX the directrix. P is any point on the curve. $FP = PK$. The tangent at P is equally inclined to FP and the axis TG.

NG, the subnormal, is constant, and equal to FX. The foot of the perpendicular from F on any tangent lies on the tangent at A. $AT = AN$.

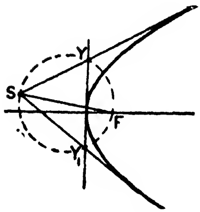


$\overline{PN}^2 = 4\overline{AF} \cdot AN$. Tangents at the ends of a focal chord intersect at right angles on the directrix.

A *diameter* of a parabola is a line parallel to the axis. S, the point of intersection of two tangents, is equidistant from the diameters through the points of contact. A diameter bisects all chords parallel to the tangent at the point where the diameter cuts the curve.

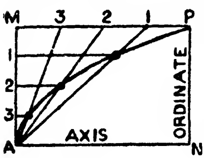


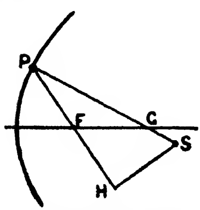
The angles subtended at the focus F by tangents SP and SP_1 are equal to one another and to the angle LSP. The diameter SVU through the point of intersection of the tangents SP and SP_1 bisects the chord of contact PP_1. Also $SV = VU$.



To draw tangents from a point S. Join S to the focus F. On SF as diameter describe a circle to cut the tangent at the vertex in Y and Y_1. SY and SY_1 are the tangents required.

Given the vertex A, the axis AN, and an ordinate PN to draw the curve. Complete the rectangle MN. Divide PM and AM into any, the same, number of equal parts. Join the points of division on PM with A. Lines through the points of division on MA parallel to AN to meet the former lines as shown determine points on the curve.

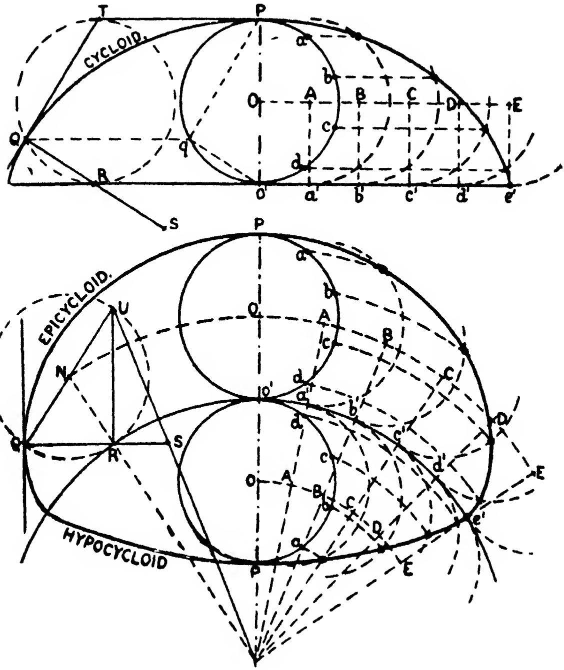




Centre of curvature.—Join PF and produce it, making $FH = FP$. Draw HS perpendicular to PH to meet the normal PG at S. S is the centre, and SP the radius of curvature of the parabola at P.

Cycloidal Curves.—If a circle be made to roll along a line and remain in the same plane, a point on the circumference of the rolling circle will describe a *cycloidal curve.* The line along which the circle rolls is called a *directing line* or *director.* If the director is a straight line, the curve is called a *cycloid*. If the director is a circle, the curve described is called an *epicycloid* or a *hypocycloid*, according as the generating circle rolls on the outside or inside of the directing circle.

The constructions for drawing the cycloid, epicycloid, and hypocycloid are shown below.



Make the straight line or arc $o'e'$ equal to half the circumference of the rolling circle. Divide $o'e'$ into any number of equal parts, and divide the semicircle Pco' into the same number of equal parts. Draw the normals $a'A$, $b'B$, etc., to meet a line or arc OE parallel to or concentric with $o'e'$ at the points A, B, etc. With centres A, B, etc., describe arcs of circles to touch $o'e'$. Draw through the points a, b, c, etc., lines parallel to $o'e'$, or arcs concentric with $o'e'$ to meet the arcs whose centres are at A, B, C, etc.

The intersections of these determine points on the curve required.

The hypocycloid becomes a straight line when the diameter of the rolling circle is equal to the radius of the directing circle.

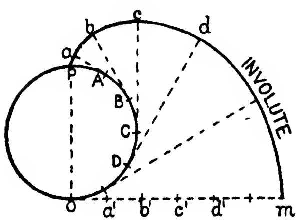
The *normal* at any point Q of the curve passes through R, the point of contact of the rolling circle, and the directing line, when the former passes through Q.

Referring to the cycloid; if Qq be drawn parallel to the directing line, then the normal QR at Q will be parallel to the chord qo', and the tangent QT will be parallel to qP. Again, if PT be drawn parallel to the directing line to meet the tangent QT at T, then the length of the arc PQ of the cycloid will be equal to twice the length of QT, or twice the length of the chord Pq Hence the total length of the cycloid is equal to four times the diameter of the rolling circle.

The *centre of curvature* S of the curve at Q is in the normal QR produced. In the cycloid $RS = QR$. For the epicycloid the construction for finding S is as follows.—Draw the diameter QNU of the rolling circle. Join U with the centre of the directing circle, cutting QR produced at S.

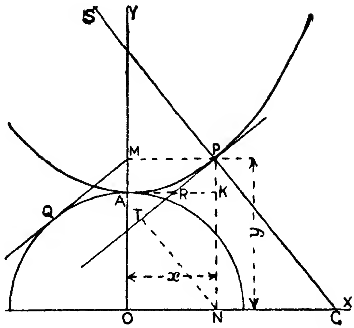
Involute of a Circle.—If a flexible line be wound round a circle, and the part which is off the circle be kept straight, any point in it will describe a curve called the *involute* of the circle. The involute of a circle is what an epicycloid becomes when the generating circle is of infinite diameter, *i.e.* when the generating circle becomes a straight line.

To draw the involute to a given circle OCP, draw the tangent Om, and make Om equal to half the circumference of the circle. Divide the semicircle OCP into any number of equal parts at the points A, B, C, etc., and divide Om into the same number of equal parts at the points a', b', c', etc. At the points A, B, C, etc., draw tangents to the circle, and make Aa, Bb, Cc, etc., equal to Oa', Ob', Oc', etc., respectively. $Pabc$ . . . . m is a portion of the involute which may be extended to any length.



The *normal* to the involute at any point is the tangent to the circle from that point, and the *centre of curvature* at any point is the point of contact of the tangent from that point to the circle.

The Catenary.—The catenary is the curve in which a perfectly flexible and uniform cord hangs when suspended from two points.



A is the lowest point of the curve.

The curve is symmetrical about a vertical axis OAY. OX is a horizontal axis.

$OA = c$. P is any point on the curve, and its coordinates are x and y.

The equation to the curve is, $y = \frac{c}{2}\left(e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right)$ where e is the base of the Naperian system of logarithms.

The table below will facilitate the construction of the curve.

The tangent and normal at P.—PM is parallel to OX. MQ is a tangent to the circle whose centre is O and radius OA. PT, the tangent to the curve at P, is parallel to MQ. SPG, the normal at P, is perpendicular to PT.

If $SP = PG$, then S is the *centre of curvature* of the curve at P.

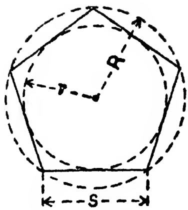
The following are geometrical properties of the curve. If NT is perpendicular to PT, then $NT = OA$, and the length PT = length of the arc AP. Length of arc $AP = \sqrt{y^2 - c^2}$. Area of the figure $OAPN$ = twice the area of the triangle $PTN = OA \times$ length of arc AP. If ARK is parallel to OX, then $RT = RK$.

If s = length of arc AP, then $s = \frac{c}{2}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right)$, and $x = c$ hyp. log. $\frac{y+s}{c}$.

$\frac{x}{c}$	$\frac{y}{c}$	$\frac{x}{c}$	$\frac{y}{c}$	$\frac{x}{c}$	$\frac{y}{c}$	$\frac{x}{c}$	$\frac{y}{c}$
0·0	1·000	1·0	1 543	2·0	3·762	3·0	10·07
0·1	1·005	1·1	1·669	2·1	4·144	3·2	12·29
0·2	1·020	1·2	1·811	2·2	4·568	3·4	15·00
0·3	1·045	1·3	1·971	2·3	5·037	3·6	18·31
0·4	1·081	1·4	2·151	2·4	5·557	3·8	22·36
0·5	1·128	1·5	2·352	2·5	6·132	4·0	27·31
0·6	1·185	1·6	2·577	2·6	6·769	4·5	45·01
0·7	1·255	1·7	2·828	2·7	7·473	5·0	74·21
0·8	1·337	1·8	3·108	2·8	8·253	5·5	122·35
0·9	1·433	1·9	3·418	2·9	9·115	6·0	201·72

Proportions of Regular Polygons.

n = number of sides.
s = length of each side.
R = radius of circumscribing circle.
r = radius of inscribed circle.
A = area of polygon



$\frac{R}{s} = \frac{1}{2} \text{cosec.} \frac{180}{n}$ $\qquad \frac{s}{R} = 2 \sin \frac{180}{n}.$

$\frac{r}{s} = \frac{1}{2} \cot. \frac{180}{n}.$ $\qquad \frac{s}{r} = 2 \tan. \frac{180}{n}.$

$\frac{A}{s^2} = \frac{n}{4} \cot. \frac{180}{n}.$ $\qquad \frac{A}{R^2} = \frac{n}{2} \sin. \frac{360}{n}.$ $\qquad \frac{A}{r^2} = n \tan. \frac{180}{n}.$

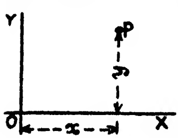
$\frac{s^2}{A} = \frac{4}{n} \tan \frac{180}{n}.$ $\qquad \frac{R^2}{A} = \frac{2}{n} \text{cosec.} \frac{360}{n}.$ $\qquad \frac{r^2}{A} = \frac{1}{n} \cot. \frac{180}{n}.$

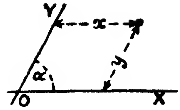
Angle subtended at centre by one side in degrees $= \frac{360}{n}.$

Angle between adjacent sides, in degrees $= 180 - \frac{360}{n}.$

n	$\frac{R}{s}$	$\frac{s}{R}$	$\frac{r}{s}$	$\frac{s}{r}$	$\frac{A}{s^2}$	$\frac{A}{R^2}$	$\frac{A}{r^2}$
3	·57735	1·73205	28868	3·16410	·43301	1·29904	5 19615
4	·70711	1·41421	·50000	2 00000	1·00000	2·00000	4·00000
5	·85065	1·17557	68819	1·45308	1·72048	2·37764	3·63271
6	1·00000	1·00000	·86603	1 15470	2·59808	2·59808	3·46410
7	1·15238	·86777	1·03826	·96315	3·63391	2 73641	3·37102
8	1·30656	·76537	1·20711	·82843	4·82843	2 82843	3 31371
9	1·46190	·68404	1·37374	72794	6·18182	2·89254	3·27573
10	1·61803	·61803	1·53884	·64984	7·69421	2·93893	3·24920
11	1·77473	·56347	1·70284	·58725	9·36564	2·97352	3·22989
12	1·93185	·51764	1·86603	·53590	11·19615	3·00000	3·21539

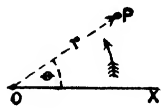
Rectangular Co-ordinates of a Point—The axes OX and OY are at right angles to one another. The position of a point P in the plane of the axes is fixed by its distances x and y from the axes. x is called the *abscissa*, and y the *ordinate* of the point P. Together, x and y are called the *co-ordinates* of P. Distances to the right of OY are positive (+), and those to the left are negative (−). Distances above OX are positive, and those below are negative.





Oblique Co-ordinates of a Point.—The axes OX and OY make any angle α with one another. The co-ordinates x and y are measured parallel to OX and OY respectively.

Polar Co-ordinates of a Point.—OX is a fixed line called the *initial line.* O is a fixed point called the *pole.* $OP=r$ is the *radius vector* of the point P. Angle $POX=\theta$ is the *vectorial angle.* r and θ are the polar co ordinates of the point P. r is always positive. θ is positive when measured from OX in the direction of the arrow, and negative when measured from OX in the opposite direction.



In what follows the co-ordinates are rectangular.

Distance between two Points P and Q.—Co-ordinates of $P=x_1$ and y_1.

Co-ordinates of $Q=x_2$ and y_2.

$$PQ^2=(x_1-x_2)^2+(y_1-y_2)^2.$$

Co-ordinates of a Point R dividing a Straight Line PQ in the ratio of n_1 to n_2.—Co-ordinates of $P=x_1$ and y_1.

Co-ordinates of $Q=x_2$ and y_2.

Co-ordinates of $R=x$ and y.

$$x=\frac{n_1x_2+n_2x_1}{n_1+n_2}. \qquad y=\frac{n_1y_2+n_2y_1}{n_1+n_2}.$$

If R is the middle point of PQ, then $n_1=n_2$ and

$$x=\frac{x_1+x_2}{2}. \qquad y=\frac{y_1+y_2}{2}.$$

Area of a Triangle.—Co-ordinates of angular points, x_1y_1, x_2y_2, and x_3y_3. Area of triangle $=\frac{1}{2}\{y_1(x_2-x_3)+y_2(x_3-x_1)+y_3(x_1-x_2)\}$.

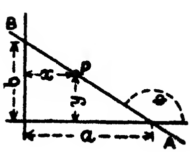
Equations to Lines, Straight or Curved.—An equation which expresses the relation between the co-ordinates of any point on a line is called the equation to that line.

Equation to a Straight Line.—x and y are the co-ordinates of any point P in the straight line AB.

$y=mx+b$. Where $m=\tan.\ \theta$.

Another form of the equation is,

$$\frac{x}{a}+\frac{y}{b}=1.$$



Equation to a Straight Line passing through two given Points P and Q.—Co-ordinates of $P=x_1$ and y_1.

Co-ordinates of $Q=x_2$ and y_2

x and y denote the co-ordinates of any point in the line passing through P and Q

$$\text{Equation to line is, } y-y_1=\frac{y_1-y_2}{x_1-x_2}(x-x_1).$$

Given the Equations to two Straight Lines to find the Co-ordinates of their Point of Intersection.—Given equations, $y=m_1x+b_1$ and $y=m_2x+b_2$. Treating these as simultaneous equations, and solving for x and y,

$$x=\frac{b_1-b_2}{m_2-m_1},\text{ and } y=\frac{b_1m_2-b_2m_1}{m_2-m_1}.$$

These are the co-ordinates requiied.

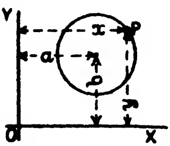
Equation to a Straight Line passing through a given Point x_1y_1, and perpendicular to a given Line $y=mx+b$.

$$\text{Equation required is, } y-y_1=-\frac{1}{m}(x-x_1).$$

The Perpendicular Distance of a given Point x_1y_1 from a given Line $y=mx+b$.

$$\text{Distance}=\frac{y_1-mx_1-b}{\sqrt{(1+m^2)}}.$$

Equation to a Circle.—a and b are the co-ordinates of the centre of the circle. x and y are the co-ordinates of any point P on the circumference. r=radius of circle. The equation to the circle is



$$(x-a)^2+(y-b)^2=r^2.$$

If O the origin be taken at the centre of the circle, then $a=0$, and $b=0$, and the equation to the circle becomes $x^2+y^2=r^2$.

The general form of the equation to a circle is $x^2+y^2+Ax+By+C=0$.

This may be written in the form,

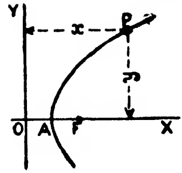
$$\left(x+\frac{A}{2}\right)^2+\left(y+\frac{B}{2}\right)^2=\frac{A^2+B^2-4C}{4},$$

which shows that the radius of the circle is $\frac{1}{2}\sqrt{A^2+B^2-4C}$, and that the co-ordinates of the centre are, $-\frac{A}{2}$ and, $-\frac{B}{2}$.

Equation to a Parabola.—F is the focus. $AO = AF = a$.

$$y^2 = 4a(x - a).$$

If the origin O is at A, then $y^2 = 4ax$.



Equation to an Ellipse.—F is a focus, and OK a directrix.
Major axis $= 2a$ Minor axis $= 2b$.

$OF = p$. $\frac{PF}{PK} =$ eccentricity of curve $= e$.

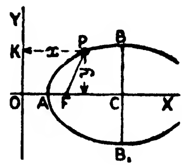
$$y^2 + (x - p)^2 = e^2x^2.$$

If origin O is at A, then

$$y^2 = (1 - e^2)(2ax - x^2).$$

If origin O is at C, then

$$y^2 = (1 - e^2)(a^2 - x^2), \text{ or } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$



Equation to an Hyperbola.—F is a focus, and OK a directrix.
Transverse axis, $AA_1 = 2a$.
Conjugate axis, $BB_1 = 2b$.

$OF = p$. $\frac{PF}{PK} =$ eccentricity of curve $= e$.

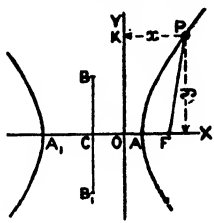
$$y^2 + (x - p)^2 = e^2x^2.$$

If origin O is at A, then

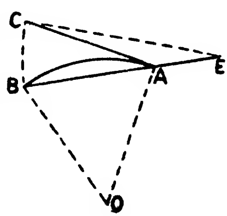
$$y^2 = (e^2 - 1)(2ax + x^2).$$

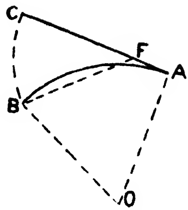
If origin O is at C, then

$$y^2 = (e^2 - 1)(x^2 - a^2), \text{ or } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$



Rankine's Rules for Lengths of Circular Arcs.—AB is an arc of a circle whose centre is O. AC is a tangent to the arc at A. Produce the chord BA to E, making $AE =$ half the chord AB. With centre E and radius EB describe the arc BC, cutting the tangent AC at C. AC is approximately the length of the arc AB.





Make $AF = \frac{1}{4}AC$. With centre F and radius FC describe the arc CB, cutting the arc AB at B. The length of the arc AB is approximately the length of AC.

The errors in the above constructions increase as the angle AOB increases. When the angle AOB is 30°, the straight line AC is shorter than the arc AB by about $\frac{1}{14400}$ of the length of the arc AB. When the angle AOB is 60° the straight line AC is shorter than the arc AB by about $\frac{1}{900}$ of the length of the arc AB.

The Helix or Screw Curve.—If a point moves on the surface of a circular cylinder so that its motion round the axis and its motion parallel to the axis are uniform, the path of the point is a helix. The pitch of the helix is the distance traversed by the point in the direction of the axis while it makes one complete revolution round the axis. From the definition of the helix it follows that if the point makes, say, 1-12th of a revolution it will advance along the cylinder a distance equal to 1-12th of the pitch. Hence the construction for drawing the helix shown in Fig. (a).

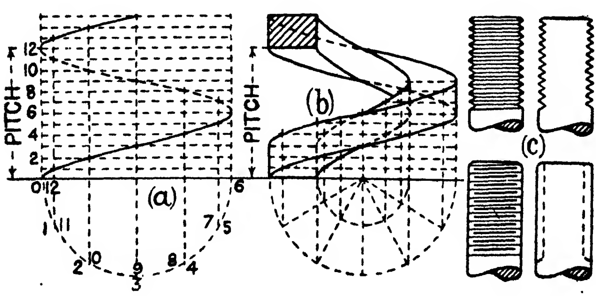
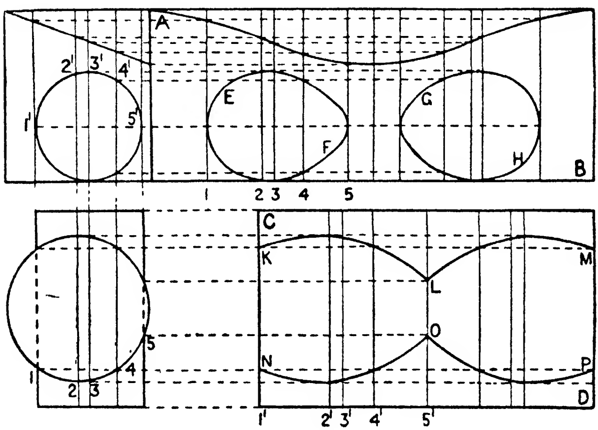


Fig. (b) shows the application of the construction of Fig. (a) to the drawing of a spiral spring of rectangular section.

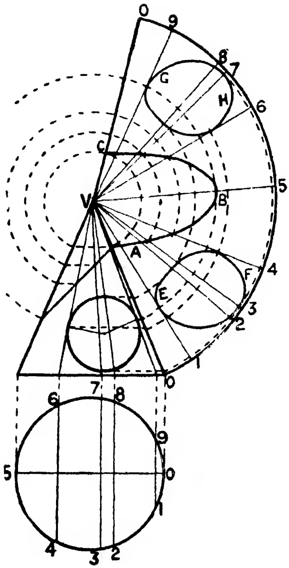
On working drawings screw threads are represented in various conventional ways. Fig (*c*) shows four different ways of representing a triangular screw thread on a bolt.

Development of the Surface of a Cylinder.—If the surface of a cylinder, whose ends are at right angles to its axis, be laid out flat, its shape will be a rectangle whose length is equal to the circumference of the cylinder, and whose width is equal to the length of the cylinder. The Fig. below shows two cylinders intersecting, one being vertical and the other horizontal.



AB is the development of the surface of the vertical cylinder and *CD* is the development of the surface of the horizontal cylinder. The curves *EF* and *GH* are the developments of inter sections of the horizontal with the vertical cylinder. The curves *KLM* and *NOP* are the developments of the intersections of the vertical with the horizontal cylinder. The distance from, say, 4 to 5 on *AB* is equal to the length of the arc 45 on the plan of the vertical cylinder. The distance from, say, 4′ to 5′ on *CD* is equal to the length of the arc 4′5′ on the elevation of the horizontal cylinder.

The upper curve on *AB* is the development of the end of the surface of the vertical cylinder when the latter is bevelled off, as shown by the sloping straight line on the elevation.



Development of the Surface of a Cone.—If the surface of a right circular cone be laid out flat, a sector of a circle is obtained whose radius is equal to the length of the slant side of the cone and whose arc has a length equal to the circumference of the base of the cone. The annexed Fig. shows a right cone, with its axis vertical, intersected by a horizontal cylinder. The sector OVO is the development of the surface of the cone. The curves EF and GH are the developments of the intersections of the cylinder with the cone. The length of, say, the arc 34 on the development is equal to the length of the arc 34 on the plan. The curve ABC is the development of the upper end of a frustum of the cone.

GRAPHIC ARITHMETIC.

Representation of Numbers by Lines (Fig. 1).—If the line AB be taken to represent the number *one*, then a line CD three times the length of AB will represent the number *three*. Again, if AB represents the number one, the number represented by the line EF will be the number of times that EF contains AB. In the above examples AB is called the *unit*.

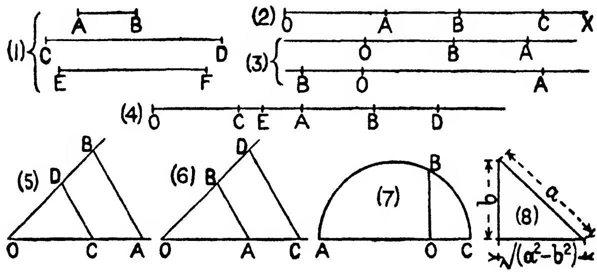
Ordinary drawing scales may be used in marking off lengths for numbers. For example, if the unit is a length equal to $\frac{3}{8}$ inch, a scale of $\frac{3}{8}$ inch to a foot would be suitable for measuring off lengths to represent the numbers.

Addition (Fig. 2).—To add a series of numbers together. Take a line OX of indefinite length. Fix upon the unit, that is, decide what length shall represent the number *one*. Make OA = the first number, AB = the second number, BC = the third number, and so on. From O to the last letter will be the answer.

Subtraction (Fig. 3).—From one number to subtract another.

OA (measured to the *right* of O) = first number. AB (measured to the left of A) = second number. OB = answer. If OB is to the *right* of O the answer is *positive* (+), but if OB is to the *left* of O the answer is *negative* (-).

Addition and Subtraction Combined (Fig. 4).—To find the result of $a+b-c+d-e$. Make $OA=a$, $AB=b$, $BC=c$, $CD=d$, and $DE=e$, then $OE=a+b-c+d-e$. Note that in adding the lengths are measured to the right, and in subtracting they are measured to the left.



In Figs. (5) and (6) the angle AOB may have any magnitude, but for convenience it should not be less than 30° or more than 90°. The lines AB and CD are parallel.

Proportion (Figs. 5 and 6).—$OA : OB :: OC : OD$.

To find a fourth proportional to three given numbers. Make OA = first number, OB = second number, and OC = third number. Join AB, and draw CD parallel to AB. OD = fourth proportional to the given numbers.

To find a mean proportional to two given numbers (Fig. 7). Make OA = one number and OC = the other number. On AC describe a semicircle and draw OB at right angles to AC, then $OA : OB :: OB : OC$, or $OB^2 = OA \times OC$.

Multiplication (Figs. 5 and 6).—$OA : OB :: OC : OD$, therefore

$OA \times OD = OB \times OC$. Make OB = the unit, then,

$OA \times OD = 1 \times OC = OC$.

To multiply one number by another. Make OA = one number, OD = the other number, and OB = the unit. Join AB, and draw DC parallel to BA. Answer = OC.

Division (Figs. 5 and 6).—$OA : OB :: OC : OD$, therefore,

$\frac{OA}{OB} = \frac{OC}{OD}$. Make OB = the unit, then $\frac{OC}{OD} = \frac{OA}{1} = OA$.

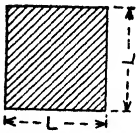
To divide one number by another. Make OC = the dividend, OD = the divisor, and OB = the unit. Join CD, and draw BA parallel to DC Answer $= OA$

Square Root (Fig. 7)—*To find the square root of a number.* Make OA = the number and OC' = the unit. On AC' describe a semicircle, and draw OB at right angles to AC', then $OB = \sqrt{OA}$.

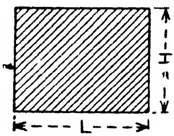
To find $\sqrt{(a^2 - b^2)}$ (Fig 8). Draw a right-angled triangle, having the hypotenuse $= a$, and perpendicular $= b$, then the base $= \sqrt{(a^2 - b^2)}$

Areas of Plane Figures.

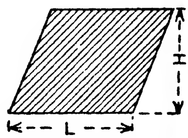
A = area of figure.



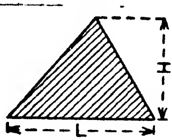
Square.
$A = L \times L = L^2.$



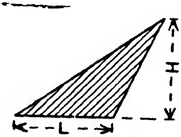
Rectangle
$A = L \times H.$



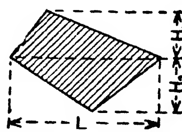
Parallelogram.
$A = L \times H.$



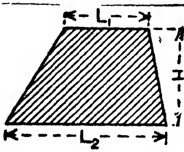
Triangle.
$A = \frac{1}{2} L \times H$



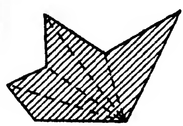
Triangle.
$A = \frac{1}{2} L \times H.$



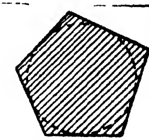
Quadrilateral.
$A = \frac{1}{2} L \times (H_1 + H_2).$



Trapezoid.
$A = \frac{1}{2}(L_1 + L_2) \times H.$



Any Rectilineal Figure.
Divide figure into triangles. A = sum of areas of triangles.



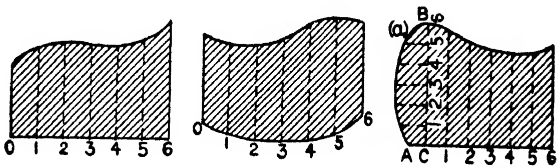
Any Rectilineal Figure Circumscribing a Circle.
$A = \frac{1}{2}$ sum of sides × radius of circle.

Areas of Plane Figures (*continued*).

Simpson's Rule.

To find the area of a plane figure the boundary line of which is curved, or partly curved.

Divide the figure into an *even* number of parallel strips of equal width by ordinates 0, 1, 2, 3, etc. There will be an *odd* number of ordinates. Approximate area of figure is found as



follows :—Add together the first ordinate (0), the last ordinate (6), four times the sum of the alternate ordinates 1, 3, etc., and twice the sum of the remaining ordinates; multiply the result by one-third of the common distance between two adjacent ordinates.

The above rule does not give good results if any considerable part of the curve makes an acute angle with the ordinates as at (*a*). In such a case divide the figure into two parts by the ordinate *BC*, and use ordinates at right angles to *BC* to find area of *ABC*.

The rule gives a more exact result the larger the number of ordinates.

Areas of Circles.

D = diameter of circle. A = area of circle.

$$A = \frac{\pi}{4}D^2 = \cdot 785398D^2. \qquad D = \frac{2\sqrt{A}}{\sqrt{\pi}} = 1\cdot 128379\sqrt{A}.$$

If the diameter is in *inches*, the area is in *square inches*. If the diameter is in *feet*, the area is in *square feet*.

The use of the tables of areas of circles may be extended by applying the following rule :—If the diameter be multiplied or divided by any number, the area must be multiplied or divided by the *square* of that number. Thus, Diameter $= D$. Area $= A$.

$$\text{Diameter} = nD. \quad \text{Area} = n^2A.$$

$$\text{Diameter} = \frac{D}{n}. \quad \text{Area} = \frac{A}{n^2}.$$

Areas of Small Circles.

(DIAMETERS ADVANCING BY 64THS.)

Diam.	Area.	Diam.	Area.	Diam.	Area.	Diam.	Area.
$\frac{1}{64}$	·00019	$\frac{17}{64}$	·05542	$\frac{33}{64}$	·20881	$\frac{49}{64}$	·46039
$\frac{1}{32}$	·00077	$\frac{9}{32}$	·06213	$\frac{17}{32}$	·22166	$\frac{25}{32}$	·47937
$\frac{3}{64}$	·00173	$\frac{19}{64}$	·06922	$\frac{35}{64}$	·23489	$\frac{51}{64}$	·49874
$\frac{1}{16}$	·00307	$\frac{5}{16}$	·07670	$\frac{9}{16}$	·24850	$\frac{13}{16}$	·51849
$\frac{5}{64}$	·00479	$\frac{21}{64}$	·08456	$\frac{37}{64}$	·26250	$\frac{53}{64}$	·53862
$\frac{3}{32}$	·00690	$\frac{11}{32}$	·09281	$\frac{19}{32}$	·27688	$\frac{27}{32}$	·55914
$\frac{7}{64}$	·00940	$\frac{23}{64}$	·10143	$\frac{39}{64}$	·29165	$\frac{55}{64}$	·58004
$\frac{1}{8}$	·01227	$\frac{3}{8}$	·11045	$\frac{5}{8}$	·30680	$\frac{7}{8}$	·60132
$\frac{9}{64}$	·01553	$\frac{25}{64}$	·11984	$\frac{41}{64}$	·32233	$\frac{57}{64}$	·62299
$\frac{5}{32}$	·01917	$\frac{13}{32}$	·12962	$\frac{21}{32}$	·33824	$\frac{29}{32}$	·64504
$\frac{11}{64}$	·02320	$\frac{27}{64}$	·13978	$\frac{43}{64}$	·35454	$\frac{59}{64}$	·66747
$\frac{3}{16}$	·02761	$\frac{7}{16}$	·15033	$\frac{11}{16}$	·37122	$\frac{15}{16}$	·69029
$\frac{13}{64}$	·03241	$\frac{29}{64}$	·16126	$\frac{45}{64}$	·38829	$\frac{61}{64}$	·71349
$\frac{7}{32}$	·03758	$\frac{15}{32}$	·17257	$\frac{23}{32}$	·40574	$\frac{31}{32}$	·73708
$\frac{15}{64}$	·04314	$\frac{31}{64}$	·18427	$\frac{47}{64}$	·42357	$\frac{63}{64}$	·76105
$\frac{1}{4}$	·04909	$\frac{1}{2}$	·19635	$\frac{3}{4}$	·44179	1	·78540

Areas of Circles.

(DIAMETERS ADVANCING BY 32NDS.)

Diam.	0	1	2	3	4	5	Diam.
		·78540	3·1416	7·0686	12·566	19·635	
1/32	·00077	·83525	3·2405	7·2166	12·763	19·881	1/32
1/16	·00307	·88664	3·3410	7·3662	12·962	20·129	1/16
3/32	·00690	·93956	3·4430	7·5173	13·162	20·378	3/32
1/8	01227	·99402	3·5466	7·6699	13·364	20·629	1/8
5/32	·01917	1·0500	3·6516	7·8241	13·567	20·881	5/32
3/16	·02761	1·1075	3·7583	7·9798	13·772	21·135	3/16
7/32	·03758	1·1666	3·8664	8·1370	13·978	21·391	7/32
1/4	·04909	1·2272	3·9761	8·2958	14·186	21·648	1/4
9/32	06213	1·2893	4·0873	8·4561	14·396	21·906	9/32
5/16	·07670	1·3530	4·2000	8·6179	14·607	22·166	5/16
11/32	·09281	1·4182	4·3143	8·7813	14·819	22·428	11/32
3/8	·11045	1·4849	4·4301	8·9462	15·033	22·691	3/8
13/32	·12962	1·5532	4·5475	9·1126	15·249	22·955	13/32
7/16	·15033	1·6230	4·6664	9·2806	15·466	23·221	7/16
15/32	·17257	1·6943	4·7868	9·4501	15·684	23·489	15/32
1/2	·19635	1·7671	4·9087	9·6211	15·904	23·758	1/2
17/32	·22166	1·8415	5·0322	9·7937	16·126	24·029	17/32
9/16	·24850	1·9175	5·1572	9·9678	16·349	24·301	9/16
19/32	·27688	1·9949	5·2838	10·143	16·574	24·575	19/32
5/8	·30680	2·0739	5·4119	10·321	16·800	24·850	5/8
21/32	·33824	2·1545	5·5415	10·499	17·028	25·127	21/32
11/16	·37122	2·2365	5·6727	10·680	17·257	25·406	11/16
23/32	·40574	2·3201	5·8053	10·861	17·488	25·686	23/32
3/4	·44179	2·4053	5·9396	11·045	17·721	25·967	3/4
25/32	·47937	2·4920	6·0753	11·230	17·954	26·250	25/32
13/16	·51849	2·5802	6·2126	11·416	18·190	26·535	13/16
27/32	·55914	2·6699	6·3514	11·604	18·427	26·821	27/32
7/8	·60132	2·7612	6·4918	11·793	18·665	27·109	7/8
29/32	·64504	2·8540	6·6337	11·984	18·906	27·398	29/32
15/16	·69029	2·9483	6·7771	12·177	19·147	27·688	15/16
31/32	·73708	3·0442	6·9221	12·371	19·390	27·981	31/32
Diam.	0	1	2	3	4	5	Diam.

Areas of Circles.

(Diameters advancing by Eighths.)

D.	·0	⅛	¼	⅜	½	⅝	¾	⅞
0	.	·0123	·0491	·1104	·1963	·3068	·4418	·6013
1	·7854	·9940	1·2272	1·4849	1·7671	2·0739	2·4053	2·7612
2	3·1416	3·5466	3·9761	4·4301	4·9087	5·4119	5·9396	6·4918
3	7·0686	7·6699	8·2958	8·9462	9·6211	10·321	11·045	11·793
4	12·566	13·364	14·186	15·033	15·904	16·800	17·721	18·665
5	19·635	20·629	21·648	22·691	23·758	24·850	25·967	27·109
6	28·274	29·465	30·680	31·919	33·183	34·472	35·785	37·122
7	38·485	39·871	41·282	42·718	44·179	45·664	47·173	48·707
8	50·265	51·849	53·456	55·088	56·745	58·426	60·132	61·862
9	63·617	65·397	67·201	69·029	70·882	72·760	74·662	76·589
10	78·540	80·516	82·516	84·541	86·590	88·664	90·763	92 886
11	95·033	97·205	99·402	101·62	103·87	106·14	108·43	110·75
12	113·10	115·47	117·86	120·28	122·72	125·19	127·68	130·19
13	132·73	135·30	137·89	140·50	143·14	145·80	148·49	151·20
14	153·94	156·70	159·48	162·30	165·13	167·99	170·87	173·78
15	176·71	179·67	182·65	185·66	188·69	191·75	194·83	197·93
16	201·06	204·22	207·39	210·60	213·82	217·08	220·35	223·65
17	226·98	230·33	233·71	237·10	240·53	243·98	247·45	250·95
18	254·47	258·02	261·59	265·18	268·80	272·45	276·12	279·81
19	283·53	287·27	291·04	294·83	298·65	302·49	306·35	310·24
20	314·16	318·10	322·06	326·05	330·06	334·10	338·16	342·25
21	346·36	350·50	354·66	358·84	363·05	367·28	371·54	375·83
22	380·13	384·46	388·82	393·20	397·61	402·04	406·49	410·97
23	415·48	420·00	424·56	429·13	433·74	438·36	443·01	447·69
24	452·39	457·11	461·86	466·64	471·44	476·26	481·11	485·98
25	490·87	495·79	500·74	505·71	510·71	515·72	520·77	525·84
26	530·93	536·05	541·19	546·35	551·55	556·76	562·00	567·27
27	572·56	577·87	583·21	588·57	593·96	599·37	604·81	610·27
28	615·75	621·26	626·80	632·36	637·94	643·55	649·18	654·84
29	660·52	666·23	671·96	677·71	683·49	689·30	695·13	700·98
30	706·86	712·76	718·69	724·64	730·62	736·62	742·64	748·69
31	754·77	760·87	766·99	773·14	779·31	785·51	791·73	797·98
32	804·25	810·54	816·86	823·21	829·58	835·97	842·39	848·83
33	855·30	861·79	868·31	874·85	881·41	888·00	894·62	901·26
34	907·92	914·61	921·32	928·06	934·82	941·61	948·42	955·25
35	962·11	969·00	975·91	982·84	989·80	996·78	1003·8	1010·8
D.	·0	⅛	¼	⅜	½	⅝	¾	⅞

Areas of Circles.

(Diameters advancing by Eighths.)

D.	·0	⅛	¼	⅜	½	⅝	¾	⅞
36	1017·9	1025·0	1032·1	1039·2	1046·3	1053·5	1060·7	1068·0
37	1075·2	1082·5	1089·8	1097·1	1104·5	1111·8	1119·2	1126·7
38	1134·1	1141·6	1149·1	1156·6	1164·2	1171·7	1179·3	1186·9
39	1194·6	1202·3	1210·0	1217·7	1225·4	1233·2	1241·0	1248·8
40	1256·6	1264·5	1272·4	1280·3	1288·2	1296·2	1304·2	1312·2
41	1320·3	1328·3	1336·4	1344·5	1352·7	1360·8	1369·0	1377·2
42	1385·4	1393·7	1402·0	1410·3	1418·6	1427·0	1435·4	1443·8
43	1452·2	1460·7	1469·1	1477·6	1486·2	1494·7	1503·3	1511·9
44	1520·5	1529·2	1537·9	1546·6	1555·3	1564·0	1572·8	1581·6
45	1590·4	1599·3	1608·2	1617·0	1626·0	1634·9	1643·9	1652·9
46	1661·9	1670·9	1680·0	1689·1	1698·2	1707·4	1716·5	1725·7
47	1734·9	1744·2	1753·5	1762·7	1772·1	1781·4	1790·8	1800·1
48	1809·6	1819·0	1828·5	1837·9	1847·5	1857·0	1866·5	1876·1
49	1885·7	1895·4	1905·0	1914·7	1924·4	1934·2	1943·9	1953·7
50	1963·5	1973·3	1983·2	1993·1	2003·0	2012·9	2022·8	2032·8
51	2042·8	2052·8	2062·9	2073·0	2083·1	2093·2	2103·3	2113·5
52	2123·7	2133·9	2144·2	2154·5	2164·8	2175·1	2185·4	2195·8
53	2206·2	2216·6	2227·0	2237·5	2248·0	2258·5	2269·1	2279·6
54	2290·2	2300·8	2311·5	2322·1	2332·8	2343·5	2354·3	2365·0
55	2375·8	2386·6	2397·5	2408·3	2419·2	2430·1	2441·1	2452·0
56	2463·0	2474·0	2485·0	2496·1	2507·2	2518·3	2529·4	2540·6
57	2551·8	2563·0	2574·2	2585·4	2596·7	2608·0	2619·4	2630·7
58	2642·1	2653·5	2664·9	2676·4	2687·8	2699·3	2710·9	2722·4
59	2734·0	2745·6	2757·2	2768·8	2780·5	2792·2	2803·9	2815·7
60	2827·4	2839·2	2851·0	2862·9	2874·8	2886·6	2898·6	2910·5
61	2922·5	2934·5	2946·5	2958·5	2970·6	2982·7	2994·8	3006·9
62	3019·1	3031·3	3043·5	3055·7	3068·0	3080·3	3092·6	3104·9
63	3117·2	3129·6	3142·0	3154·5	3166·9	3179·4	3191·9	3204·4
64	3217·0	3229·6	3242·2	3254·8	3267·5	3280·1	3292·8	3305·6
65	3318·3	3331·1	3343·9	3356·7	3369·6	3382·4	3395·3	3408·2
66	3421·2	3434·2	3447·2	3460·2	3473·2	3486·3	3499·4	3512·5
67	3525·7	3538·8	3552·0	3565·2	3578·5	3591·7	3605·0	3618·3
68	3631·7	3645·0	3658·4	3671·8	3685·3	3698·7	3712·2	3725·7
69	3739·3	3752·8	3766·4	3780·0	3793·7	3807·3	3821·0	3834·7
70	3848·5	3862·2	3876·0	3889·8	3903·6	3917·5	3931·4	3945·3
D.	·0	⅛	¼	⅜	½	⅝	¾	⅞

Areas of Circles.

(DIAMETERS ADVANCING BY EIGHTHS.)

D.	·0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
71	3959·2	3973·1	3987·1	4001·1	4015·2	4029·2	4043·3	4057·4
72	4071·5	4085·7	4099·8	4114·0	4128·2	4142·5	4156·8	4171·1
73	4185·4	4199·7	4214·1	4228·5	4242·9	4257·4	4271·8	4286·3
74	4300·8	4315·4	4329·9	4341·5	4359·2	4373·8	4388·5	4403·2
75	4417·9	4432·6	4447·4	4462·2	4477·0	4491·8	4506·7	4521·5
76	4536·5	4551·4	4566·4	4581·3	4596·3	4611·4	4626·4	4641·5
77	4656·6	4671·8	4686·9	4702·1	4717·3	4732·5	4747·8	4763·1
78	4778·4	4793·7	4809·0	4824 4	4839·8	4855·2	4870·7	4886·2
79	4901·7	4917·2	4932·7	4948·3	4963·9	4979·5	4995·2	5010·9
80	5026·5	5042 3	5058·0	5073·8	5089·6	5105·4	5121·2	5137·1
81	5153·0	5168·9	5181·9	5200·8	5216·8	5232·8	5248·9	5264·9
82	5281·0	5297·1	5313·3	5329·4	5345·6	5361·8	5378·1	5394·3
83	5410·6	5426·9	5143·3	5459·6	5476·0	5492·4	5508·8	5525·3
84	5541·8	5558·3	5574·8	5591·4	5607·9	5624·5	5641·2	5657·8
85	5674·5	5691·2	5707·9	5724·7	5741·5	5758·3	5775·1	5791·9
86	5808·8	5825·7	5842·6	5859·6	5876·5	5893·5	5910·6	5927·6
87	5944·7	5961·8	5978·9	5996·0	6013·2	6030·4	6047·6	6064·9
88	6082·1	6099·4	6116·7	6134·1	6151·4	6168·8	6186·2	6203·7
89	6221·1	6238·6	6256·1	6273·7	6291·2	6308·8	6326·4	6344·1
90	6361·7	6379·4	6397·1	6414·9	6432·6	6450·4	6468·2	6486·0
91	6503·9	6521·8	6539·7	6557 6	6575·6	6593·5	6611·5	6629·6
92	6647·6	6665·7	6683·8	6701·9	6720·1	6738·2	6756·4	6774·7
93	6792·9	6811·2	6829·5	6847·8	6866·1	6884·5	6902·9	6921·3
94	6939·8	6958·2	6976·7	6995·3	7013·8	7032·4	7051·0	7069·6
95	7088·2	7106·9	7125·6	7144·3	7163·0	7181·8	7200·6	7219·4
96	7238·2	7257·1	7276·0	7294·9	7313·8	7332·8	7351·8	7370·8
97	7389·8	7408·9	7428·0	7447·1	7466·2	7485·3	7504·5	7523·7
98	7543·0	7562·2	7581·5	7600·8	7620·1	7639·5	7658·9	7678·3
99	7697·7	7717·1	7736·6	7756·1	7775·6	7795·2	7814·8	7834·4
100	7854·0	7873·6	7893·3	7913·0	7932·7	7952·5	7972·2	7992·0
101	8011·8	8031·7	8051·6	8071·4	8091·4	8111·3	8131·3	8151·3
102	8171·3	8191·3	8211·4	8231·5	8251·6	8271·7	8291·9	8312·1
103	8332·3	8352·5	8372·8	8393·1	8413·4	8433·7	8454·1	8474·5
104	8494·9	8515·3	8535·8	8556·2	8576·7	8597·3	8617·8	8638·4
105	8659·0	8679·6	8700·3	8721·0	8741·7	8762·4	8783·2	8803·9
D.	·0	1/8	1/4	3/8	1/2	5/8	3/4	7/8

Areas of Circles. (Diameters advancing by Tenths.)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
1	·785398	·950332	1·13097	1·32732	1·53938	1·76715	2·01062	2·26980	2 54469	2·83529	1
2	3·14159	3·46361	3·80133	4·15476	4·52389	4·90874	5·30929	5·72555	6·15752	6·60520	2
3	7·06858	7·54768	8·04248	8·55298	9·07920	9·62113	10·1788	10·7521	11·3411	11·9459	3
4	12·5664	13·2025	13·8544	14·5220	15·2053	15·9043	16·6190	17·3494	18·0956	18·8574	4
5	19·6350	20·4282	21·2372	22·0618	22·9022	23·7583	24·6301	25·5176	26·4208	27·3397	5
6	28·2743	29·2247	30·1907	31·1724	32·1699	33·1831	34·2119	35·2565	36·3168	37·3928	6
7	38·4845	39·5919	40·7150	41·8539	43·0084	44·1786	45·3646	46·5663	47·7836	49·0167	7
8	50·2655	51·5300	52·8102	54·1061	55·4177	56·7450	58·0880	59 4468	60·8212	62·2114	8
9	63·6172	65·0388	66·4761	67·9291	69·3978	70·8822	72·3823	73·8981	75·4296	76·9769	9
10	78·5398	80·1184	81·7128	83·3229	84·9486	86·5901	88·2473	89·9202	91·6088	93·3131	10
11	95·0332	96·7689	98·5203	100·287	102·070	103·869	105·683	107·513	109·359	111·220	11
12	113·097	114·990	116·899	118·823	120·763	122·718	124 690	126·677	128·680	130·698	12
13	132·732	134·782	136·848	138·929	141·026	143·139	145·267	147 411	149·571	151·747	13
14	153·938	156·145	158·368	160·606	162·860	165·130	167·415	169·717	172·034	174·366	14
15	176·715	179·079	181·458	183·854	186·265	188·692	191·134	193·593	196·067	198·556	15
16	201·062	203 583	206·120	208·672	211·241	213·825	216·424	219·040	221·671	224·318	16
17	226·980	229·658	232·352	235·062	237·787	240·528	243·285	246·057	248·846	251·649	17
18	254·469	257 304	260·155	263·022	265·904	268·802	271·716	274·646	277·591	280·552	18
19	283·529	286·521	289·529	292·553	295·592	298·648	301·719	304·805	307·907	311·025	19
20	314·159	317·309	320·474	323·655	326·851	330·064	333·292	336·535	339·795	343·070	20
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Areas of Circles. (Diameters advancing by Tenths.)

Diam.	0	·1	·2	·3	·4	·5	·6	7	·8	·9	Diam.
21	346·361	349·667	352·989	356·327	359·681	363·050	366·435	369·836	373·253	376·685	**21**
22	380·133	383·596	387·076	390·571	394·081	397·608	401·150	404·708	408·281	411·871	**22**
23	415·476	419·096	422·733	426·385	430·053	433·736	437·435	441·150	444·881	448·627	**23**
24	452·389	456·167	459·961	463·770	467·595	471·435	475·292	479·164	483·051	486·955	**24**
25	490·874	494·809	498·759	502·726	506·707	510·705	514·719	518·748	522·792	526·853	**25**
26	530·929	535·021	539·129	543·252	547·391	551·546	555·716	559·902	564·104	568·322	**26**
27	572·555	576·804	581·069	585·349	589·646	593·957	598·285	602·628	606·987	611·362	**27**
28	615·752	620·158	624·580	629·018	633·471	637·940	642·424	616·925	651·441	655·972	**28**
29	660·520	665·083	669·662	674·256	678·867	683·493	688·134	692·792	697·465	702·154	**29**
30	706·858	711·579	716·314	721·066	725·834	730·617	735·415	740·230	745·060	749·906	**30**
31	754·768	759·645	764·538	769·447	774·371	779·311	784·267	789·239	794·226	799·229	**31**
32	804·248	809·282	814·332	819·398	824·479	829·577	834·690	839·818	844·963	850·123	**32**
33	855·298	860·490	865·697	870·920	876·159	881·413	886·683	891·969	897·270	902·587	**33**
34	907·920	913·269	918·633	924·013	929·409	934·820	940·247	945·690	951·148	956·623	**34**
35	962·113	967·618	973·140	978·677	984·229	989·798	995·382	1000·98	1006·60	1012·23	**35**
36	1017·88	1023·54	1029·22	1034·91	1040·62	1046·35	1052·09	1057·84	1063·62	1069·41	**36**
37	1075·21	1081·03	1086·87	1092·72	1098·58	1104·47	1110·36	1116·28	1122·21	1128·15	**37**
38	1134·11	1140·09	1146·08	1152·09	1158·12	1164·16	1170·21	1176·28	1182·37	1188·47	**38**
39	1194·59	1200·72	1206·87	1213·04	1219·22	1225·42	1231·63	1237·86	1244·10	1250·36	**39**
40	1256·64	1262·93	1269·23	1275·56	1281·90	1288·25	1294·62	1301·00	1307·41	1313·82	**40**
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Areas of Circles. (Diameters advancing by Tenths.)

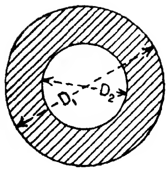
Diam.	·0	·1	·2	·3	·4	5	·6	·7	·8	·9	Diam.
41	1320·25	1326·70	1333·17	1339 65	1346·14	1352·65	1359·18	1365·72	1372·28	1378·85	**41**
42	1385·44	1392·05	1398·67	1405·30	1411·96	1418·63	1425·31	1432·01	1438·72	1445·45	**42**
43	1452·20	1458·96	1465·74	1472·53	1479·34	1486·17	1493·01	1499·87	1506·74	1513·63	**43**
44	1520·53	1527·45	1534·39	1541·34	1548·30	1555·28	1562·28	1569·30	1576·33	1583·37	**44**
45	1590·43	1597·51	1604·60	1611·71	1618·83	1625·97	1633·13	1640·30	1647·48	1654·68	**45**
46	1661·90	1669·14	1676·38	1683·65	1690·93	1698·23	1705·54	1712·87	1720·21	1727·57	**46**
47	1734·94	1742·33	1749·74	1757·16	1764·60	1772·05	1779·52	1787·01	1794·51	1802·03	**47**
48	1809·56	1817·10	1824·67	1832·25	1839·84	1847·45	1855·08	1862·72	1870·38	1878·05	**48**
49	1885·74	1893·45	1901·17	1908·90	1916·65	1924·42	1932·21	1940·00	1947·82	1955·65	**49**
50	1963·50	1971·36	1979·23	1987 13	1995·04	2002·96	2010·90	2018·86	2026·83	2034·82	**50**
51	2042 82	2050·84	2058·87	2066·92	2074·99	2083 07	2091·17	2099·28	2107·41	2115·56	**51**
52	2123·72	2131·89	2140·08	2148·29	2156·51	2164 75	2173·01	2181·28	2189·56	2197·87	**52**
53	2206·18	2214·52	2222·86	2231·23	2239·61	2248 01	2256·42	2264·84	2273·29	2281·75	**53**
54	2290·22	2298·71	2307·22	2315·74	2324·28	2332·83	2341·40	2349·98	2358·58	2367·20	**54**
55	2375·83	2384·48	2393·14	2401·82	2410·51	2419·22	2427·95	2436·69	2445·45	2454·22	**55**
56	2463·01	2471·81	2480·63	2489·47	2498·32	2507·19	2516·07	2524·97	2533·88	2542·81	**56**
57	2551·76	2560·72	2569·70	2578·69	2587·70	2596·72	2605·76	2614·82	2623·89	2632·98	**57**
58	2642·08	2651·20	2660·33	2669·48	2678·65	2687·83	2697·03	2706·24	2715·47	2724·71	**58**
59	2733·97	2743 25	2752·54	2761·84	2771 17	2780·51	2789·86	2799·23	2808·61	2818·02	**59**
60	2827·43	2836·87	2846·31	2855·78	2865 26	2874·75	2884·26	2893·79	2903·33	2912·89	**60**
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Areas of Circles. (Diameters advancing by Tenths)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
61	2922·47	2932·06	2941·66	2951·28	2960·92	2970·57	2980·24	2989·92	2999·62	3009·34	**61**
62	3019·07	3028·82	3038·58	3048·36	3058·15	3067·96	3077·79	3087·63	3097·48	3107·36	**62**
63	3117·24	3127·15	3137·07	3147·00	3156·96	3166·92	3176·90	3186·90	3196·92	3206·95	**63**
64	3216·99	3227·05	3237·13	3247·22	3257·33	3267·45	3277·59	3287·75	3297·92	3308·11	**64**
65	3318·31	3328·53	3338·76	3349·01	3359·27	3369·55	3379·85	3390·16	3400·49	3410·84	**65**
66	3421·19	3431·57	3441·96	3452·37	3462·79	3473·23	3483·68	3494·15	3504·64	3515·14	**66**
67	3525·65	3536·18	3546·73	3557·30	3567·88	3578·47	3589·08	3599·71	3610·35	3621·01	**67**
68	3631·68	3642·37	3653·08	3663·80	3674·53	3685·28	3696·05	3706·84	3717·64	3728·45	**68**
69	3739·28	3750·13	3760·99	3771·87	3782·76	3793·67	3804·59	3815·54	3826·49	3837·46	**69**
70	3848·45	3859·45	3870·47	3881·51	3892·56	3903·63	3914·71	3925·81	3936·92	3948·05	**70**
71	3959·19	3970·35	3981·53	3992·72	4003·93	4015·15	4026·39	4037·65	4048·92	4060·20	**71**
72	4071·50	4082·82	4094·16	4105·50	4116·87	4128·25	4139·65	4151·06	4162·48	4173·93	**72**
73	4185·39	4196·86	4208·35	4219·86	4231·38	4242·92	4254·47	4266·04	4277·62	4289·22	**73**
74	4300·84	4312·47	4324·12	4335·78	4347·46	4359·16	4370·87	4382·59	4394·33	4406·09	**74**
75	4417·86	4429·65	4441·46	4453·28	4465·11	4476·97	4488·83	4500·72	4512·62	4524·53	**75**
76	4536·46	4548·41	4560·37	4572·34	4584·34	4596·35	4608·37	4620·41	4632·47	4644·54	**76**
77	4656·63	4668·73	4680·85	4692·98	4705·13	4717·30	4729·48	4741·68	4753·89	4766·12	**77**
78	4778·36	4790·62	4802·90	4815·19	4827·50	4839·82	4852·16	4864·51	4876·88	4889·27	**78**
79	4901·67	4914·09	4926·52	4938·97	4951·43	4963·91	4976·41	4988·92	5001·45	5013·99	**79**
80	5026·55	5039·12	5051·71	5064·32	5076·94	5089·58	5102·23	5114·90	5127·58	5140·28	**80**
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Areas of Circles. (Diameters advancing by Tenths.)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
81	5153·00	5165·73	5178 48	5191·24	5204 02	5216·81	5229·62	5242·45	5255·29	5268·14	81
82	5281·02	5293·91	5306 81	5319·73	5332·67	5345·62	5358·58	5371·57	5384·56	5397·58	82
83	5410·61	5423·65	5436·71	5449·79	5462·88	5475·99	5489·12	5502·26	5515·41	5528·58	83
84	5541·77	5554·97	5568·19	5581·42	5594·67	5607·94	5621·22	5634·52	5647·83	5661·16	84
85	5674·50	5687·86	5701·24	5714·63	5728·03	5741·46	5754·89	5768 35	5781·82	5795·30	85
86	5808·80	5822·32	5835·85	5849 40	5862·97	5876·55	5890·14	5903·75	5917·38	5931·02	86
87	5944·68	5958·35	5972·04	5985·75	5999·47	6013 20	6026·96	6040·72	6054·51	6068·31	87
88	6082·12	6095·95	6109·80	6123·66	6137·54	6151·43	6165·34	6179·27	6193·21	6207·17	88
89	6221·14	6235·13	6249·13	6263·15	6277·18	6291·24	6305·30	6319·38	6333·48	6347·60	89
90	6361·72	6375·87	6390·03	6404·21	6418·40	6432·61	6446·83	6461·07	6475·32	6489 60	90
91	6503·88	6518·18	6532·50	6546·84	6561·18	6575·55	6589 93	6604·33	6618·74	6633·17	91
92	6647·61	6662·07	6676·54	6691·03	6705·54	6720·06	6734·60	6749·15	6763 72	6778·31	92
93	6792·91	6807·52	6822·16	6836·80	6851·47	6866·15	6880·84	6895·55	6910·28	6925·02	93
94	6939·78	6954·55	6969·34	6984·14	6998·97	7013·80	7028·65	7043·52	7058·40	7073·30	94
95	7088·22	7103·15	7118·09	7133·06	7148·03	7163·03	7178 04	7193·06	7208·10	7223·16	95
96	7238·23	7253·32	7268·42	7283·54	7298·67	7313·82	7328·99	7344·17	7359·37	7374 58	96
97	7389·81	7405·06	7420·32	7435·59	7450·88	7466·19	7481·51	7496 85	7512·21	7527·58	97
98	7542·96	7558·37	7573·78	7589·22	7604·66	7620·13	7635·61	7651·10	7666 62	7682·14	98
99	7697·69	7713·25	7728·82	7744·41	7760·02	7775·64	7791·27	7806·93	7822·60	7838·28	99
100	7853·98	7869·70	7885·43	7901·17	7916·94	7932·72	7948·51	7964·32	7980·15	7995·99	100
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

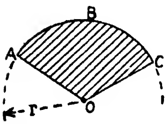


Area of a Flat Circular Ring.—Area of ring = area of larger circle − area of smaller circle $= \frac{\pi}{4}(D_1^2 - D_2^2) = A$

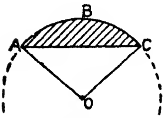
$$= \frac{\pi}{4}(D_1 + D_2)(D_1 - D_2)$$

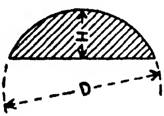
$$D_2 = \sqrt{D_1^2 - \frac{4}{\pi}A}. \quad D_1 = \sqrt{D_2^2 + \frac{4}{\pi}A}.$$

The above formulæ are true whether the circles are concentric or not, provided the smaller is entirely within the larger.



Area of a Sector of a Circle.—Area of sector = ½ length of arc ABC × radius r. If n = number of degrees in angle AOC, then area of sector $= \frac{n}{360} \times \pi r^2$.





Area of a Segment of a Circle.—Area of segment ACB = area of sector $AOCB$ − area of triangle ACO.

D = diameter of circle.

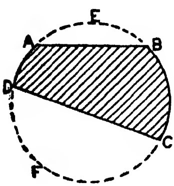
H = height of segment.

Area of segment $= D^2 \times M$.

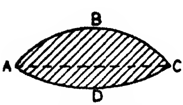
Values of M corresponding to various values of $\frac{H}{D}$ are given in the table on pages 181 to 183.

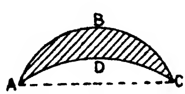
If L = length of chord, then $D = H + \frac{L^2}{4H}$,

$L = 2\sqrt{DH - H^2}$, and $H = \frac{D - \sqrt{D^2 - L^2}}{2}$.



Area of the Portion of a Circle between two Chords.—Area of figure $ABCD$ = area of circle − the sum of the areas of the segments AEB, CFD.





Area of a Figure contained by two Arcs of Circles.—Area of figure = sum or difference of areas of the segments ABC, ADC.

Areas of Segments of a Circle.

D=diameter of circle. H=Height of segment.

Area of segment $= D^2 \times M$. The following table gives values of M corresponding to various values of $\frac{H}{D}$.

$\frac{H}{D}$	M	$\frac{H}{D}$	M	$\frac{H}{D}$	M	$\frac{H}{D}$	M
·001	·000042	·040	·010538	·079	·028894	·118	·052090
·002	·000119	·041	·010932	·080	·029435	·119	·052737
·003	·000219	042	·011331	·081	·029979	·120	·053385
·004	·000337	·043	·011734	·082	·030526	·121	·054037
·005	·000471	·044	·012142	·083	·031077	·122	·054690
·006	·000619	·045	·012555	·084	·031630	·123	·055346
007	·000779	·046	·012971	085	·032186	·124	·056004
·008	·000952	·047	·013393	·086	·032746	·125	056664
·009	·001135	·048	·013818	·087	·033308	·126	057327
·010	·001329	·049	·014248	·088	·033873	·127	057991
011	·001533	·050	·014681	·089	·034441	·128	058658
·012	·001746	·051	·015119	·090	·035012	·129	059328
·013	·001969	·052	·015561	091	·035586	·130	·059999
·014	·002199	·053	·016008	·092	036162	·131	060673
015	·002438	·054	·016458	·093	·036742	132	·061349
·016	·002685	·055	·016912	·094	·037324	·133	·062027
·017	·002940	·056	·017369	·095	037909	·134	·062707
·018	·003202	·057	·017831	·096	·038497	135	·063389
·019	·003472	·058	·018297	·097	·039087	·136	·064074
·020	·003749	·059	·018766	098	·039681	·137	·064761
·021	·004032	·060	·019239	·099	·040277	138	·065449
·022	·004322	·061	·019716	·100	·040875	·139	·066140
·023	·004619	·062	·020197	·101	·041477	140	·066833
·024	·004922	·063	·020681	·102	·042081	·141	·067528
·025	·005231	·064	·021168	·103	·042687	·142	·068225
·026	·005546	·065	·021660	·104	·043296	·143	·068924
·027	·005867	·066	·022155	·105	·043908	·144	·069626
·028	·006194	·067	·022653	·106	·044523	·145	·070329
·029	·006527	·068	·023155	·107	·045140	·146	·071034
·030	·006866	·069	·023660	·108	·045759	·147	·071741
·031	·007209	·070	·024168	·109	·046381	·148	·072450
·032	·007559	·071	·024680	·110	·047006	·149	·073162
·033	·007913	·072	·025196	·111	·047633	·150	·073875
·034	·008273	·073	·025714	·112	·048262	·151	·074590
·035	·008638	·074	·026236	·113	·048894	·152	·075307
·036	·009008	·075	·026761	·114	·049529	·153	·076026
·037	·009383	·076	·027290	·115	·050165	·154	·076747
·038	·009763	·077	·027821	·116	·050805	·155	·077470
·039	·010148	·078	·028356	·117	·051446	·156	·078194

Areas of Segments of a Circle (*continued*).

$\frac{H}{D}$	M	$\frac{H}{D}$	M	$\frac{H}{D}$	M	$\frac{H}{D}$	M
·157	·078921	·200	·111824	·243	·147513	·286	·185425
·158	·079650	·201	·112625	·244	·148371	·287	·186329
·159	·080380	·202	·113427	·245	·149231	·288	·187235
·160	·081112	·203	·114231	·246	·150091	·289	·188141
·161	·081847	·204	·115036	·247	·150953	·290	·189048
·162	·082582	·205	·115842	·248	·151816	·291	·189956
·163	·083320	·206	·116651	·249	·152681	·292	·190865
·164	·084060	·207	·117460	·250	·153546	·293	·191774
·165	084801	·208	·118271	·251	·154413	·294	·192685
·166	·085545	·209	·119083	·252	·155281	·295	·193597
·167	·086290	·210	·119898	253	·156149	·296	·194509
·168	·087037	·211	·120713	254	·157019	·297	·195423
·169	·087785	·212	·121530	·255	·157891	·298	·196337
·170	·088536	·213	122348	·256	·158763	·299	·197252
·171	·089288	·214	·123167	·257	·159636	·300	·198168
·172	·090042	·215	·123988	·258	·160511	·301	·199085
·173	·090797	·216	124811	259	·161386	·302	200003
·174	·091555	·217	·125634	·260	162263	·303	·200922
·175	·092314	·218	·126459	·261	·163141	·304	·201841
·176	·093074	·219	·127286	·262	·164020	·305	·202762
·177	·093837	·220	·128114	·263	·164900	·306	·203683
·178	·094601	·221	·128913	·264	·165781	·307	·204605
·179	·095367	·222	·129773	·265	·166663	·308	·205528
·180	·096135	·223	·130605	·266	·167546	·309	·206452
·181	·096904	·224	·131438	·267	·168431	·310	·207376
·182	·097675	·225	·132273	·268	·169316	·311	·208302
·183	·098447	·226	·133109	·269	·170202	·312	·209228
·184	·099221	·227	·133946	·270	·171090	·313	·210155
·185	·099997	·228	·134784	·271	·171978	·314	·211083
·186	·100774	·229	·135624	·272	·172868	·315	·212011
·187	·101553	·230	·136465	·273	·173758	·316	·212941
·188	·102334	·231	·137307	·274	·174650	·317	·213871
·189	·103116	·232	·138151	·275	·175542	·318	·214802
·190	·103900	·233	·138996	·276	·176436	·319	·215734
·191	·104686	·234	·139842	·277	·177330	·320	·216666
·192	·105472	·235	·140689	·278	·178226	·321	·217600
·193	·106261	·236	·141538	·279	·179122	·322	·218534
·194	·107051	·237	·142388	·280	·180020	·323	·219469
·195	·107843	·238	·143239	·281	·180918	·324	·220404
·196	·108636	·239	·144091	·282	·181818	·325	·221341
·197	·109431	·240	·144945	·283	·182718	·326	·222278
·198	·110227	·241	·145800	·284	·183619	·327	·223216
·199	·111025	·242	·146655	·285	·184522	·328	·224154

Areas of Segments of a Circle (*concluded*).

H/*D*	*M*	*H*/*D*	*M*	*H*/*D*	*M*	*H*/*D*	*M*
·329	·225094	·372	·266111	·415	308110	·458	·350749
·330	·226034	·373	·267078	416	·309096	·459	·351745
·331	·226974	·374	·268046	·417	·310082	·460	·352742
·332	·227916	·375	·269014	·418	·311068	·461	·353739
·333	·228858	·376	·269982	·419	·312055	·462	·354736
·334	·229801	·377	·270951	·420	·313042	·463	·355733
·335	·230745	·378	·271921	·421	·314029	·464	·356730
·336	·231689	·379	·272891	·422	·315017	·465	·357728
·337	·232634	·380	·273861	·423	·316005	·466	·358725
·338	·233580	·381	·274832	·424	·316993	·467	·359723
·339	·234526	·382	·275804	·425	·317981	·468	·360721
·340	·235473	·383	·276776	·426	·318970	·469	·361719
·341	·236421	·384	·277748	·427	·319959	·470	·362717
·342	·237369	·385	·278721	·428	·320949	·471	·363715
·343	·238319	·386	·279695	·429	·321938	·472	·364714
·344	·239268	·387	·280669	·430	·322928	·473	·365712
·345	·240219	·388	·281643	·431	·323919	·474	·366711
·346	·241170	·389	·282618	·432	·324909	·475	·367710
·347	·242122	·390	·283593	·433	·325900	·476	·368708
·348	·243074	·391	·284569	·434	·326891	·477	·369707
·349	·244027	·392	·285545	·435	·327883	·478	·370706
·350	·244980	·393	·286521	·436	·328874	·479	·371705
·351	·245935	·394	·287499	·437	·329866	·480	·372704
·352	·246890	·395	·288476	·438	·330858	·481	·373704
·353	·247845	·396	·289454	·439	·331851	·482	·374703
·354	·248801	·397	·290432	·440	·332843	·483	·375702
·355	·249758	·398	·291411	·441	·333836	·484	·376702
·356	·250715	·399	·292390	·442	·334829	·485	377701
·357	·251673	·400	·293370	·443	·335823	·486	·378701
·358	·252632	·401	·294350	·444	·336816	·487	·379701
·359	·253591	·402	·295330	·445	·337810	·488	·380700
·360	·254551	·403	·296311	·446	·338804	·489	·381700
·361	·255511	·404	·297292	·447	·339799	·490	382700
·362	·256472	·405	·298274	·448	·340793	·491	·383700
·363	·257433	·406	·299256	·449	·341788	·492	·384699
·364	·258395	407	·300238	·450	·342783	·493	·385699
·365	·259358	·408	·301221	·451	·343778	·494	·386699
·366	·260321	·409	·302204	·452	·344773	·495	·387699
·367	·261285	·410	·303187	·453	·345768	·496	·388699
·368	·262249	·411	·304171	·454	·346764	·497	·389699
·369	·263214	·412	·305156	·455	·347760	·498	·390699
·370	·264179	·413	·306140	·456	·348756	·499	·391699
·371	·265145	·414	·307125	·457	·349752	·500	·392699

Circumferences of Circles.

D=diameter of circle. C=circumference of circle.

$$C=\pi D=3{\cdot}141593D.$$

$$D=\frac{C}{\pi}={\cdot}31831C.$$

The use of the tables of circumferences of circles may be extended by applying the following rule :—If the diameter be multiplied or divided by any number, the circumference must be multiplied or divided by the same number.

Thus, Diameter $=D$. Circumference $=C$.
Diameter $=nD$. Circumference $=nC$.
Diameter $=\frac{D}{n}$. Circumference $=\frac{C}{n}$.

Circumferences of Small Circles.

(Diameters advancing by 64ths.)

Diam	Circum.	Diam	Circum.	Diam.	Circum	Diam.	Circum.
1/64	·04909	17/64	·83449	33/64	1·6199	49/64	2·4053
1/32	·09817	9/32	·88357	17/32	1·6690	25/32	2·4544
3/64	·14726	19/64	·93266	35/64	1·7181	51/64	2·5035
1/16	·19635	5/16	·98175	9/16	1·7671	13/16	2·5525
5/64	·24544	21/64	1·0308	37/64	1·8162	53/64	2·6016
3/32	·29452	11/32	1·0799	19/32	1·8653	27/32	2·6507
7/64	·34361	23/64	1·1290	39/64	1·9144	55/64	2 6998
1/8	·39270	3/8	1·1781	5/8	1·9635	7/8	2·7489
9/64	·44179	25/64	1·2272	41/64	2·0126	57/64	2·7980
5/32	·49087	13/32	1·2763	21/32	2·0617	29/32	2·8471
11/64	·53996	27/64	1·3254	43/64	2·1108	59/64	2·8962
3/16	·58905	7/16	1·3744	11/16	2·1598	15/16	2·9452
13/64	·63814	29/64	1·4235	45/64	2·2089	61/64	2·9943
7/32	·68722	15/32	1·4726	23/32	2·2580	31/32	3·0434
15/64	·73631	31/64	1·5217	47/64	2·3071	63/64	3·0925
1/4	·78540	1/2	1·5708	3/4	2·3562	1	3·1416

Circumferences of Circles.

(DIAMETERS ADVANCING BY 32NDS.)

Diam.	0	1	2	3	4	5	Diam.
		3·1416	6·2832	9·4248	12·566	15·708	
1/32	·09817	3·2398	6·3814	9·5230	12·665	15·806	1/32
1/16	·19635	3·3379	6·4795	9·6211	12·763	15·904	1/16
3/32	·29452	3·4361	6·5777	9·7193	12·861	16·002	3/32
1/8	·39270	3·5343	6·6759	9·8175	12·959	16·101	1/8
5/32	·49087	3·6325	6·7741	9·9157	13·057	16·199	5/32
3/16	·58905	3·7306	6·8722	10·014	13·155	16·297	3/16
7/32	·68722	3·8288	6·9704	10·112	13·254	16·395	7/32
1/4	·78540	3·9270	7·0686	10·210	13·352	16·493	1/4
9/32	·88357	4·0252	7·1668	10·308	13·450	16·592	9/32
5/16	·98175	4·1233	7·2649	10·407	13·548	16·690	5/16
11/32	1·0799	4·2215	7·3631	10·505	13·646	16·788	11/32
3/8	1·1781	4·3197	7·4613	10·603	13·744	16·886	3/8
13/32	1·2763	4·4179	7·5595	10·701	13·843	16·984	13/32
7/16	1·3744	4·5160	7·6576	10·799	13·941	17·082	7/16
15/32	1·4726	4·6142	7·7558	10·897	14·039	17·181	15/32
1/2	1·5708	4·7124	7·8540	10·996	14·137	17·279	1/2
17/32	1·6690	4·8106	7·9522	11·094	14·235	17·377	17/32
9/16	1·7671	4·9087	8·0503	11·192	14·334	17·475	9/16
19/32	1·8653	5·0069	8·1485	11·290	14·432	17·573	19/32
5/8	1·9635	5·1051	8·2467	11·388	14·530	17·671	5/8
21/32	2·0617	5·2033	8·3449	11·486	14·628	17·770	21/32
11/16	2·1598	5·3014	8·4430	11·585	14·726	17·868	11/16
23/32	2·2580	5·3996	8·5412	11·683	14·824	17·966	23/32
3/4	2·3562	5·4978	8·6394	11·781	14·923	18·064	3/4
25/32	2·4544	5·5960	8·7376	11·879	15·021	18·162	25/32
13/16	2·5525	5·6941	8·8357	11·977	15·119	18·261	13/16
27/32	2·6507	5·7923	8·9339	12·075	15·217	18·359	27/32
7/8	2·7489	5·8905	9·0321	12·174	15·315	18·457	7/8
29/32	2·8471	5·9887	9·1303	12·272	15·413	18·555	29/32
15/16	2·9452	6·0868	9·2284	12·370	15·512	18·653	15/16
31/32	3·0434	6·1850	9·3266	12·468	15·610	18·751	31/32
Diam.	0	1	2	3	4	5	Diam.

Circumferences of Circles.

(DIAMETERS ADVANCING BY EIGHTHS.)

D.	·0	⅛	¼	⅜	½	⅝	¾	⅞
0	..	·3927	·7854	1·1781	1·5708	1·9635	2·3562	2·7489
1	3·1416	3·5343	3·9270	4·3197	4·7124	5·1051	5·4978	5·8905
2	6·2832	6·6759	7·0686	7·4613	7·8540	8·2467	8·6394	9·0321
3	9·4248	9·8175	10·210	10·603	10·996	11·388	11·781	12·174
4	12·566	12·959	13·352	13·744	14·137	14·530	14·923	15·315
5	15·708	16·101	16·493	16·886	17·279	17·671	18·064	18·457
6	18·850	19·242	19·635	20·028	20·420	20·813	21·206	21·598
7	21·991	22·384	22·777	23·169	23·562	23·955	24·347	24·740
8	25·133	25·525	25·918	26·311	26·704	27·096	27·489	27·882
9	28·274	28·667	29·060	29·452	29·845	30·238	30·631	31·023
10	31·416	31·809	32·201	32·594	32·987	33·379	33·772	34·165
11	34·558	34·950	35·343	35·736	36·128	36·521	36·914	37·306
12	37·699	38·092	38·485	38·877	39·270	39·663	40·055	40·448
13	40·841	41·233	41·626	42·019	42·412	42·804	43·197	43·590
14	43·982	44·375	44·768	45·160	45·553	45·946	46·338	46·731
15	47·124	47·517	47·909	48·302	48·695	49·087	49·480	49·873
16	50·265	50·658	51·051	51·444	51·836	52·229	52·622	53·014
17	53·407	53·800	54·192	54·585	54·978	55·371	55·763	56·156
18	56·549	56·941	57·334	57·727	58·119	58·512	58·905	59·298
19	59·690	60·083	60·476	60·868	61·261	61·654	62·046	62·439
20	62·832	63·225	63·617	64·010	64·403	64·795	65·188	65·581
21	65·973	66·366	66·759	67·152	67·544	67·937	68·330	68·722
22	69·115	69·508	69·900	70·293	70·686	71·079	71·471	71·864
23	72·257	72·649	73·012	73·435	73·827	74·220	74·613	75·006
24	75·398	75·791	76·184	76·576	76·969	77·362	77·754	78·147
25	78·540	78·933	79·325	79·718	80·111	80·503	80·896	81·289
26	81·681	82·074	82·467	82·860	83·252	83·645	84·038	84·430
27	84·823	85·216	85·608	86·001	86·394	86·787	87·179	87·572
28	87·965	88·357	88·750	89·143	89·535	89·928	90·321	90·713
29	91·106	91·499	91·892	92·284	92·677	93·070	93·462	93·855
30	94·248	94·640	95·033	95·426	95·819	96·211	96·604	96·997
31	97·389	97·782	98·175	98·567	98·960	99·353	99·746	100·14
32	100·53	100·92	101·32	101·71	102·10	102·49	102·89	103·28
33	103·67	104·07	104·46	104·85	105·24	105·64	106·03	106·42
34	106·81	107·21	107·60	107·99	108·38	108·78	109·17	109·56
35	109·96	110·35	110·74	111·13	111·53	111·92	112·31	112·70
D.	·0	⅛	¼	⅜	½	⅝	¾	⅞

Circumferences of Circles.

(Diameters advancing by Eighths.)

D.	·0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
36	113·10	113·49	113·88	114·28	114·67	115·06	115·45	115·85
37	116·24	116·63	117·02	117·42	117·81	118·20	118·60	118·99
38	119·38	119·77	120·17	120·56	120·95	121·34	121·74	122·13
39	122·52	122·91	123·31	123·70	124 09	124·49	124·88	125·27
40	125·66	126·06	126·45	126·84	127·23	127·63	128·02	128·41
41	128·81	129·20	129·59	129·98	130·38	130·77	131·16	131·55
42	131·95	132·34	132·73	133·12	133 52	133·91	134·30	134·70
43	135·09	135·48	135·87	136·27	136·66	137·05	137·44	137·84
44	138·23	138·62	139·02	139·41	139·80	140·19	140·59	140·98
45	141·37	141·76	142·16	142·55	142·94	143·34	143·73	144·12
46	144·51	144·91	145·30	145·69	146·08	146·48	146·87	147·26
47	147·65	148·05	148·44	148·83	149·23	149·62	150·01	150·40
48	150·80	151·19	151·58	151·97	152·37	152·76	153·15	153·55
49	153 94	154·33	154·72	155 12	155·51	155·90	156·29	156·69
50	157·08	157 47	157·86	158 26	158·65	159·04	159·44	159·83
51	160·22	160·61	161·01	161·40	161·79	162·18	162·58	162·97
52	163·36	163·76	164·15	164·54	164·93	165·33	165·72	166·11
53	166·50	166·90	167·29	167·68	168·08	168·47	168·86	169·25
54	169·65	170·04	170 43	170·82	171·22	171·61	172·00	172·39
55	172·79	173·18	173·57	173·97	174·36	174·75	175·14	175·54
56	175·93	176·32	176·71	177·11	177·50	177·89	178·29	178·68
57	179·07	179·46	179·86	180·25	180·64	181·03	181·43	181·82
58	182·21	182·61	183·00	183·39	183·78	184·18	184·57	184·96
59	185·35	185·75	186·14	186·53	186·92	187·32	187·71	188·10
60	188·50	188·89	189 28	189 67	190·07	190·46	190·85	191·24
61	191·64	192·03	192·42	192·82	193·21	193·60	193·99	194·39
62	194·78	195·17	195·56	195·96	196·35	196·74	197·13	197·53
63	197·92	198·31	198·71	199·10	199·49	199·88	200 28	200·67
64	201·06	201·45	201·85	202·24	202 63	203·03	203·42	203·81
65	204·20	204·60	204·99	205·38	205·77	206·17	206·56	206·95
66	207·35	207·74	208·13	208·52	208·92	209·31	209·70	210·09
67	210·49	210·88	211·27	211·66	212·06	212·45	212·84	213·24
68	213·63	214·02	214·41	214·81	215·20	215·59	215·98	216·38
69	216·77	217·16	217·56	217·95	218·34	218·73	219·13	219·52
70	219·91	220·30	220·70	221·09	221·48	221·87	222·27	222·66
D.	·0	1/8	1/4	3/8	1/2	5/8	3/4	7/8

Circumferences of Circles.

(DIAMETERS ADVANCING BY EIGHTHS.)

D.	·0	⅛	¼	⅜	½	⅝	¾	⅞
71	223·05	223·45	223·84	224·23	224·62	225·02	225·41	225·80
72	226·19	226·59	226·98	227·37	227·77	228·16	228·55	228·94
73	229·34	229·73	230·12	230·51	230·91	231·30	231·69	232·09
74	232·48	232·87	233·26	233·66	234·05	234·44	234·83	235·23
75	235·62	236·01	236·40	236·80	237·19	237·58	237·98	238·37
76	238·76	239·15	239·55	239·94	240·33	240·72	241·12	241·51
77	241·90	242·30	242·69	243·08	243·47	243·87	244·26	244·65
78	245·04	245·44	245·83	246·22	246·62	247·01	247·40	247·79
79	248·19	248·58	248·97	249·36	249·76	250·15	250·54	250·93
80	251·33	251·72	252·11	252·51	252·90	253·29	253·68	254·08
81	254·47	254·86	255·25	255·65	256·04	256·43	256·83	257·22
82	257·61	258·00	258·40	258·79	259·18	259·57	259·97	260·36
83	260·75	261·14	261·54	261·93	262·32	262·72	263·11	263·50
84	263·89	264·29	264·68	265·07	265·46	265·86	266·25	266·64
85	267·04	267·43	267·82	268·21	268·61	269·00	269·39	269·78
86	270·18	270·57	270·96	271·36	271·75	272·14	272·53	272·93
87	273·32	273·71	274·10	274·50	274·89	275·28	275·67	276·07
88	276·46	276·85	277·25	277·64	278·03	278·42	278·82	279·21
89	279·60	279·99	280·39	280·78	281·17	281·57	281·96	282·35
90	282·74	283·14	283·53	283·92	284·31	284·71	285·10	285·49
91	285·88	286·28	286·67	287·06	287·46	287·85	288·24	288·63
92	289·03	289·42	289·81	290·20	290·60	290·99	291·38	291·78
93	292·17	292·56	292·95	293·35	293·74	294·13	294·52	294·92
94	295·31	295·70	296·10	296·49	296·88	297·27	297·67	298·06
95	298·45	298·84	299·24	299·63	300·02	300·41	300·81	301·20
96	301·59	301·99	302·38	302·77	303·16	303·56	303·95	304·34
97	304·73	305·13	305·52	305·91	306·31	306·70	307·09	307·48
98	307·88	308·27	308·66	309·05	309·45	309·84	310·23	310·62
99	311·02	311·41	311·80	312·20	312·59	312·98	313·37	313·77
100	314·16	314·55	314·94	315·34	315·73	316·12	316·52	316·91
101	317·30	317·69	318·09	318·48	318·87	319·26	319·66	320·05
102	320·44	320·84	321·23	321·62	322·01	322·41	322·80	323·19
103	323·58	323·98	324·37	324·76	325·15	325·55	325·94	326·33
104	326·73	327·12	327·51	327·90	328·30	328·69	329·08	329·47
105	329·87	330·26	330·65	331·04	331·44	331·83	332·22	332·62
D.	·0	⅛	¼	⅜	½	⅝	¾	⅞

Circumferences of Circles. (Diameters advancing by Tenths.)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
1	3·14159	3·45575	3 76991	4·08407	4·39823	4·71239	5·02655	5·34071	5·65487	5·96903	**1**
2	6·28319	6·59735	6·91150	7·22566	7·53982	7·85398	8·16814	8·48230	8·79646	9·11062	**2**
3	9·42478	9·73894	10·0531	10·3673	10·6814	10·9956	11·3097	11·6239	11·9381	12·2522	**3**
4	12·5664	12·8805	13·1947	13·5088	13 8230	14·1372	14 4513	14 7655	15·0796	15·3938	**4**
5	15·7080	16·0221	16·3363	16·6504	16·9646	17·2788	17·5929	17·9071	18·2212	18·5354	**5**
6	18·8496	19·1637	19·4779	19·7920	20·1062	20 4204	20·7345	21·0487	21·3628	21·6770	**6**
7	21·9912	22·3053	22·6195	22·9336	23 2478	23·5619	23·8761	24·1903	24·5044	24·8186	**7**
8	25·1327	25·4469	25·7611	26 0752	26·3894	26·7035	27·0177	27·3319	27·6460	27·9602	**8**
9	28·2743	28·5885	28·9027	29·2168	29·5310	29·8451	30·1593	30·4735	30·7876	31·1018	**9**
10	31·4159	31·7301	32·0442	32·3584	32·6726	32·9867	33·3009	33·6150	33·9292	34·2434	**10**
11	34·5575	34·8717	35·1858	35·5000	35·8142	36 1283	36·4425	36·7566	37·0708	37·3850	**11**
12	37·6991	38·0133	38·3274	38·6416	38·9558	39·2699	39 5841	39·8982	40·2124	40·5265	**12**
13	40·8407	41·1549	41·4690	41 7832	42·0973	42·4115	42·7257	43·0398	43 3540	43·6681	**13**
14	43·9823	44·2965	44·6106	44·9248	45·2389	45 5531	45 8673	46·1814	46·4956	46·8097	**14**
15	47·1239	47·4381	47·7522	48·0664	48·3805	48·6947	49·0089	49·3230	49·6372	49·9513	**15**
16	50·2655	50·5796	50·8938	51·2080	51·5221	51·8363	52·1504	52·4646	52·7788	53·0929	**16**
17	53·4071	53·7212	54·0354	54·3496	54·6637	54·9779	55·2920	55 6062	55·9204	56·2345	**17**
18	56·5487	56·8628	57·1770	57·4912	57·8053	58·1195	58·4336	58·7478	59·0619	59·3761	**18**
19	59·6903	60 0044	60·3186	60 6327	60·9169	61 2611	61·5752	61·8894	62·2035	62 5177	**19**
20	62·8319	63·1460	63·4602	63·7743	64·0885	64·4027	64·7168	65·0310	65·3451	65·6593	**20**
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Circumferences of Circles. (Diameters advancing by Tenths.)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
21	65·9735	66·2876	66·6018	66·9159	67·2301	67·5442	67·8584	68·1726	68 4867	68·8009	21
22	69·1150	69·4292	69·7434	70·0575	70·3717	70·6858	71·0000	71·3142	71·6283	71·9425	22
23	72·2566	72·5708	72·8850	73 1991	73·5133	73·8274	74 1416	74·4558	74·7699	75·0841	23
24	75·3982	75·7124	76·0266	76·3407	76 6549	76·9690	77·2832	77·5973	77·9115	78 2257	24
25	78·5398	78·8540	79·1681	79·4823	79·7965	80·1106	80·4248	80·7389	81·0531	81 3673	25
26	81·6814	81·9956	82·3097	82·6239	82·9381	83·2522	83·5664	83·8805	84·1947	84·5089	26
27	84·8230	85·1372	85·4513	85 7655	86·0796	86·3938	86·7080	87·0221	87 3363	87·6504	27
28	87·9646	88·2788	88·5929	88·9071	89·2212	89 5354	89·8496	90 1637	90·4779	90·7920	28
29	91·1062	91·4204	91·7345	92·0487	92·3628	92·6770	92·9912	93 3053	93·6195	93·9336	29
30	94·2478	94 5619	94·8761	95·1903	95·5044	95·8186	96 1327	96·4469	96·7611	97·0752	30
31	97·3894	97·7035	98 0177	98·3319	98·6460	98·9602	99·2743	99·5885	99·9027	100·217	31
32	100·531	100·845	101·159	101·473	101·788	102·102	102·416	102 730	103·044	103·358	32
33	103·673	103·987	104·301	104·615	104·929	105·243	105 558	105·872	106·186	106 500	33
34	106·814	107·128	107·442	107·757	108 071	108 385	108·699	109·013	109·327	109·642	34
35	109·956	110·270	110·584	110·898	111·212	111·527	111·841	112·155	112·469	112·783	35
36	113·097	113·412	113·726	114·040	114·354	114·668	114·982	115·296	115·611	115·925	36
37	116·239	116·553	116·867	117·181	117·496	117·810	118·124	118·438	118·752	119 066	37
38	119·381	119·695	120·009	120·323	120·637	120·951	121·265	121·580	121·894	122·208	38
39	122·522	122·836	123·150	123·465	123·779	124·093	124·407	124·721	125·035	125·350	39
40	125·664	125·978	126·292	126·606	126·920	127·235	127·549	127·863	128·177	128·491	40
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Circumferences of Circles. (Diameters advancing by Tenths.)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
41	128·805	129·119	129·434	129·748	130·062	130·376	130·690	131·004	131 319	131·633	**41**
42	131·947	132·261	132·575	132·889	133·204	133·518	133·832	134·146	134·460	134·774	**42**
43	135·088	135·403	135·717	136·031	136·345	136·659	136·973	137·288	137·602	137·916	**43**
44	138·230	138·544	138·858	139 173	139·487	139·801	140·115	140·429	140·743	141·058	**44**
45	141·372	141·686	142·000	142·314	142 628	142·942	143·257	143·571	143·885	144·199	**45**
46	144·513	144·827	145·142	145·456	145·770	146·084	146·398	146·712	147·027	147·341	**46**
47	147·655	147·969	148·283	148·597	148·912	149·226	149·540	149·854	150·168	150·482	**47**
48	150·796	151·111	151·425	151·739	152·053	152·367	152·681	152·996	153·310	153·624	**48**
49	153·938	154·252	154·566	154·881	155 195	155·509	155·823	156·137	156·451	156·765	**49**
50	157·080	157 394	157·708	158·022	158·336	158·650	158·965	159·279	159·593	159·907	**50**
51	160·221	160·535	160·850	161·164	161·478	161·792	162·106	162·420	162 735	163·049	**51**
52	163·363	163·677	163·991	164·305	164·619	164·934	165·248	165·562	165·876	166·190	**52**
53	166·504	166·819	167·133	167·447	167·761	168·075	168·389	168·704	169 018	169 332	**53**
54	169·646	169·960	170·274	170·588	170·903	171·217	171·531	171·845	172·159	172 473	**54**
55	172·788	173·102	173·416	173·730	174·044	174·358	174·673	174·987	175·301	175 615	**55**
56	175·929	176·243	176·558	176·872	177·186	177·500	177·814	178 128	178·442	178·757	**56**
57	179·071	179·385	179·699	180·013	180·327	180 642	180·956	181 270	181·584	181 898	**57**
58	182·212	182·527	182·841	183·155	183·469	183·783	184·097	184·412	184·726	185·040	**58**
59	185·354	185·668	185·982	186·296	186·611	186·925	187·239	187·553	187·867	188 181	**59**
60	188·496	188·810	189·124	189·438	189·752	190·066	190·381	190 695	191·009	191·323	**60**
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Circumferences of Circles. (Diameters advancing by Tenths.)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
61	191·637	191·951	192·265	192·580	192·894	193·208	193·522	193·836	194·150	194·465	61
62	194·779	195·093	195·407	195·721	196·035	196·350	196·664	196·978	197·292	197·606	62
63	197·920	198·235	198·549	198·863	199·177	199·491	199·805	200·119	200·434	200·748	63
64	201·062	201·376	201·690	202·004	202·319	202·633	202·947	203·261	203·575	203·889	64
65	204·204	204·518	204·832	205·146	205·460	205·774	206·089	206·403	206·717	207·031	65
66	207·345	207·659	207·973	208·288	208·602	208·916	209·230	209·544	209·858	210·173	66
67	210·487	210·801	211·115	211·429	211·743	212·058	212·372	212·686	213·000	213·314	67
68	213·628	213·942	214·257	214·571	214·885	215·199	215·513	215·827	216·142	216·456	68
69	216·770	217·084	217·398	217·712	218·027	218·341	218·655	218·969	219·283	219·597	69
70	219·912	220·226	220·540	220·854	221·168	221·482	221·796	222·111	222·425	222·739	70
71	223·053	223·367	223·681	223·996	224·310	224·624	224·938	225·252	225·566	225·881	71
72	226·195	226·509	226·823	227·137	227·451	227·765	228·080	228·394	228·708	229·022	72
73	229·336	229·650	229·965	230·279	230·593	230·907	231·221	231·535	231·850	232·164	73
74	232·478	232·792	233·106	233·420	233·735	234·049	234·363	234·677	234·991	235·305	74
75	235·619	235·934	236·248	236·562	236·876	237·190	237·504	237·819	238·133	238·447	75
76	238·761	239·075	239·389	239·704	240·018	240·332	240·646	240·960	241·274	241·589	76
77	241·903	242·217	242·531	242·845	243·159	243·473	243·788	244·102	244·416	244·730	77
78	245·044	245·358	245·673	245·987	246·301	246·615	246·929	247·243	247·558	247·872	78
79	248·186	248·500	248·814	249·128	249·442	249·757	250·071	250·385	250·699	251·013	79
80	251·327	251·642	251·956	252·270	252·584	252·898	253·212	253·527	253·841	254·155	80
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Circumferences of Circles. (Diameters advancing by Tenths.)

Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.
81	254·469	254·783	255·097	255·412	255·726	256·040	256·354	256·668	256·982	257·296	**81**
82	257·611	257·925	258·239	258·553	258·867	259·181	259·496	259·810	260·124	260·438	**82**
83	260·752	261·066	261·381	261·695	262·009	262·323	262·637	262·951	263·265	263·580	**83**
84	263·894	264·208	264·522	264·836	265·150	265·465	265·779	266·093	266·407	266·721	**84**
85	267·035	267·350	267·664	267·978	268·292	268·606	268·920	269·235	269·549	269·863	**85**
86	270·177	270·491	270·805	271·119	271·434	271·748	272·062	272·376	272·690	273·004	**86**
87	273·319	273·633	273·947	274·261	274·575	274·889	275·204	275·518	275·832	276·146	**87**
88	276·460	276·774	277·089	277·403	277·717	278·031	278·345	278·659	278·973	279·288	**88**
89	279·602	279·916	280·230	280·544	280·858	281·173	281·487	281·801	282·115	282·429	**89**
90	282·743	283·058	283·372	283·686	284·000	284·314	284·628	284·942	285·257	285·571	**90**
91	285·885	286·199	286·513	286·827	287·142	287·456	287·770	288·084	288·398	288·712	**91**
92	289·027	289·341	289·655	289·969	290·283	290·597	290·912	291·226	291·540	291·854	**92**
93	292·168	292·482	292·796	293·111	293·425	293·739	294·053	294·367	294·681	294·996	**93**
94	295·310	295·624	295·938	296·252	296·566	296·881	297·195	297·509	297·823	298·137	**94**
95	298·451	298·765	299·080	299·394	299·708	300·022	300·336	300·650	300·965	301·279	**95**
96	301·593	301·907	302·221	302·535	302·850	303·164	303·478	303·792	304·106	304·420	**96**
97	304·735	305·049	305·363	305·677	305·991	306·305	306·619	306·934	307·248	307·562	**97**
98	307·876	308·190	308·504	308·819	309·133	309·447	309·761	310·075	310·389	310·704	**98**
99	311·018	311·332	311·646	311·960	312·274	312·589	312·903	313·217	313·531	313·845	**99**
100	314·159	314·473	314·788	315·102	315·416	315·730	316·044	316·358	316·673	316·987	**100**
Diam.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diam.

Surfaces of Tubes or Cylinders.

D = diameter of tube or cylinder in inches.
L = length of tube or cylinder in inches.
A = area of surface of tube or cylinder in square feet.

$$A = \frac{3{\cdot}1416DL}{144}.$$

The following example illustrates the use of the table given on pages 195 to 199.

The diameter of a tube is $1\frac{7}{8}$ inches, and its length is 11 feet $7\frac{5}{8}$ inches: to find the area of the surface of the tube in square feet

Area of surface of tube 11 ft. long and $1\frac{7}{8}$ in. diam.	= 5·4000	sq. ft.
,, ,, ,, 7 in. ,, ,, ,,	= ·2863	,,
,, ,, ,, $\frac{5}{8}$ in. ,, ,, ,,	= ·0256	,,
,, ,, ,, 11 ft. $7\frac{5}{8}$ in. ,, ,, ,,	= 5·7119	,,

Surfaces of Tubes or Cylinders in Square Feet.

Diam. in Inches	Length in Inches.							Diam in Inches.
	1/8	1/4	3/8	1/2	5/8	3/4	7/8	
1/2	·0014	·0027	·0041	·0055	·0068	·0082	·0095	1/2
9/16	·0015	·0031	·0046	·0061	·0077	·0092	·0107	9/16
5/8	·0017	·0034	·0051	·0068	·0085	·0102	·0119	5/8
11/16	·0019	·0037	·0056	·0075	·0094	·0112	·0131	11/16
3/4	·0020	·0041	·0061	·0082	·0102	·0123	·0143	3/4
13/16	0022	·0044	·0066	·0089	·0111	·0133	·0155	13/16
7/8	·0024	·0048	·0072	·0095	·0119	·0143	·0167	7/8
15/16	·0026	·0051	·0077	·0102	·0128	·0153	·0179	15/16
1	·0027	·0055	·0082	·0109	·0136	·0164	·0191	1
1 1/8	·0031	·0061	·0092	·0123	·0153	·0184	·0215	1 1/8
1 1/4	·0034	·0068	·0102	·0136	·0170	·0205	·0239	1 1/4
1 3/8	·0037	·0075	·0112	·0150	·0187	·0225	·0262	1 3/8
1 1/2	·0041	·0082	·0123	·0164	·0205	·0245	·0286	1 1/2
1 5/8	·0044	·0089	·0133	·0177	·0222	·0266	·0310	1 5/8
1 3/4	·0048	·0095	·0143	·0191	·0239	·0286	·0334	1 3/4
1 7/8	·0051	·0102	·0153	·0205	·0256	·0307	·0358	1 7/8
2	·0055	·0109	·0164	·0218	·0273	·0327	·0382	2
2 1/4	·0061	0123	·0184	·0245	·0307	·0368	·0430	2 1/4
2 1/2	·0068	·0136	·0205	·0273	·0341	·0409	·0477	2 1/2
2 3/4	·0075	·0150	·0225	·0300	·0375	·0450	·0525	2 3/4
3	·0082	·0164	·0245	·0327	·0409	·0491	·0573	3
3 1/4	·0089	·0177	·0266	·0355	·0443	·0532	·0620	3 1/4
3 1/2	·0095	·0191	·0286	·0382	·0477	·0573	·0668	3 1/2
3 3/4	·0102	·0205	·0307	·0409	·0511	·0614	·0716	3 3/4
4	·0109	·0218	·0327	·0436	·0545	·0654	·0764	4
4 1/4	·0116	·0232	·0348	·0464	·0580	·0695	·0811	4 1/4
4 1/2	·0123	·0245	·0368	·0491	·0614	·0736	·0859	4 1/2
4 3/4	·0130	·0259	·0389	·0518	·0648	·0777	·0907	4 3/4
5	·0136	·0273	·0409	·0545	·0682	·0818	·0954	5
5 1/2	·0150	·0300	·0450	·0600	·0750	·0900	·1050	5 1/2
6	·0164	·0327	·0491	·0654	·0818	·0982	·1145	6
7	·0191	·0382	·0573	·0764	·0954	·1145	·1336	7
8	·0218	·0436	·0654	·0873	·1091	·1309	·1527	8
9	·0245	·0491	·0736	·0982	·1227	·1473	·1718	9
10	·0273	·0545	·0818	·1091	·1364	·1636	·1909	10

Surfaces of Tubes or Cylinders in Square Feet.

Diam. in Inches	Length in Inches. 1	2	3	4	5	6	Diam. in Inches.
½	·0109	·0218	·0327	·0436	·0545	·0654	½
9/16	·0123	·0245	·0368	·0491	·0614	·0736	9/16
⅝	·0136	·0273	·0409	·0545	·0682	·0818	⅝
11/16	·0150	·0300	·0450	·0600	·0750	·0900	11/16
¾	·0164	·0327	·0491	·0654	·0818	·0982	¾
13/16	·0177	·0355	·0532	·0709	·0886	·1064	13/16
⅞	·0191	·0382	·0573	·0764	·0954	·1145	⅞
15/16	·0205	·0409	·0614	·0818	·1023	·1227	15/16
1	·0218	·0436	·0654	·0873	·1091	·1309	1
1⅛	·0245	·0491	·0736	·0982	·1227	·1473	1⅛
1¼	·0273	·0545	·0818	·1091	·1364	·1636	1¼
1⅜	·0300	·0600	·0900	·1200	·1500	·1800	1⅜
1½	·0327	·0654	·0982	·1309	·1636	·1963	1½
1⅝	·0355	·0709	·1064	·1418	·1773	·2127	1⅝
1¾	·0382	·0764	·1145	·1527	·1909	·2291	1¾
1⅞	·0409	·0818	·1227	·1636	·2045	·2454	1⅞
2	·0436	·0873	·1309	·1745	·2182	·2618	2
2¼	·0491	·0982	·1473	·1963	·2454	·2945	2¼
2½	·0545	·1091	·1636	·2182	·2727	·3272	2½
2¾	·0600	·1200	·1800	·2400	·3000	·3600	2¾
3	·0654	·1309	·1963	·2618	·3272	·3927	3
3¼	·0709	·1418	·2127	·2836	·3545	·4254	3¼
3½	·0764	·1527	·2291	·3054	·3818	·4581	3½
3¾	·0818	·1636	·2454	·3272	·4091	·4909	3¾
4	·0873	·1745	·2618	·3491	·4363	·5236	4
4¼	·0927	·1854	·2782	·3709	·4636	·5563	4¼
4½	·0982	·1963	·2945	·3927	·4909	·5890	4½
4¾	·1036	·2073	·3109	·4145	·5181	·6218	4¾
5	·1091	·2182	·3272	·4363	·5454	·6545	5
5½	·1200	·2400	·3600	·4800	·6000	·7199	5½
6	·1309	·2618	·3927	·5236	·6545	·7854	6
7	·1527	·3054	·4581	·6109	·7636	·9163	7
8	·1745	·3491	·5236	·6981	·8727	1·047	8
9	·1963	·3927	·5890	·7854	·9817	1·178	9
10	·2182	·4363	·6545	·8727	1·091	1·309	10

Surfaces of Tubes or Cylinders in Square Feet.

Diam. in Inches.	Length in Inches. 7	8	9	10	11	12	Diam. in Inches
½	·0764	·0873	·0982	·1091	·1200	·1309	½
9/16	·0859	·0982	·1104	·1227	·1350	·1473	9/16
⅝	·0954	·1091	·1227	·1364	·1500	·1636	⅝
11/16	·1050	·1200	·1350	·1500	·1650	·1800	11/16
¾	·1145	·1309	·1473	·1636	·1800	·1963	¾
13/16	·1241	·1418	·1595	·1773	·1950	·2127	13/16
⅞	·1336	·1527	·1718	·1909	·2100	·2291	⅞
15/16	·1432	·1636	·1841	·2045	·2250	·2454	15/16
1	·1527	·1745	·1963	·2182	·2400	·2618	1
1⅛	·1718	·1963	·2209	·2454	·2700	·2945	1⅛
1¼	·1909	·2182	·2454	·2727	·3000	·3272	1¼
1⅜	·2100	·2100	·2700	·3000	3300	·3600	1⅜
1½	·2291	2618	·2945	·3272	·3600	·3927	1½
1⅝	·2482	·2836	·3191	·3545	3900	·4254	1⅝
1¾	·2673	·3054	·3436	·3818	4200	·4581	1¾
1⅞	·2863	·3272	·3682	·4091	·4500	·4909	1⅞
2	·3054	·3491	·3927	·4363	·4800	·5236	2
2¼	·3436	·3927	·4418	·4909	·5400	·5890	2¼
2½	·3818	·4363	·4909	·5454	·6000	·6545	2½
2¾	·4200	·4800	·5400	·6000	·6600	·7199	2¾
3	·4581	·5236	·5890	·6545	·7199	·7854	3
3¼	·4963	·5672	·6381	·7090	·7799	·8508	3¼
3½	·5345	·6109	6872	·7636	·8399	·9163	3½
3¾	·5727	·6545	·7363	·8181	·8999	·9817	3¾
4	·6109	·6981	·7854	·8727	·9599	1·047	4
4¼	·6490	·7418	·8345	·9272	1·020	1·113	4¼
4½	·6872	·7854	·8836	·9817	1·080	1 178	4½
4¾	·7254	·8290	·9327	1·036	1·140	1·244	4¾
5	·7636	·8727	·9817	1·091	1·200	1·309	5
5½	·8399	·9599	1·080	1·200	1·320	1·440	5½
6	·9163	1·047	1·178	1·309	1·440	1·571	6
7	1·069	1·222	1·374	1·527	1·680	1·833	7
8	1·222	1·396	1·571	1·745	1·920	2·094	8
9	1·374	1·571	1·767	1·963	2·160	2·356	9
10	1·527	1·745	1·963	2·182	2·400	2·618	10

Surfaces of Tubes or Cylinders in Square Feet.

Diam. in Inches	Length in Feet 2	3	4	5	6	7	8	Diam. in Inches.
1/2	2618	·3927	·5236	·6545	·7854	·9163	1·047	1/2
9/16	·2945	·4418	·5890	·7363	·8836	1·031	1·178	9/16
5/8	·3272	·4909	·6545	·8181	·9817	1·145	1 309	5/8
11/16	·3600	·5400	·7199	·8999	1·080	1·260	1·440	11/16
3/4	·3927	·5890	·7854	·9817	1·178	1·374	1·571	3/4
13/16	·4254	·6381	·8508	1·064	1·276	1·489	1·702	13/16
7/8	·4581	·6872	·9163	1·145	1·374	1·604	1·833	7/8
15/16	·4909	·7363	·9817	1·227	1 473	1·718	1 963	15/16
1	·5236	·7854	1·047	1·309	1·571	1 833	2·094	1
1 1/8	·5890	·8836	1·178	1·473	1·767	2 062	2·356	1 1/8
1 1/4	·6545	·9817	1·309	1·636	1·963	2·291	2·618	1 1/4
1 3/8	·7199	1·080	1·440	1·800	2·160	2·520	2·880	1 3/8
1 1/2	·7854	1·178	1·571	1·963	2·356	2·749	3·142	1 1/2
1 5/8	·8508	1·276	1·702	2·127	2·553	2·978	3·403	1 5/8
1 3/4	·9163	1·374	1·833	2·291	2·749	3·207	3·665	1 3/4
1 7/8	·9817	1·473	1·963	2 454	2·945	3·436	3·927	1 7/8
2	1·047	1·571	2·094	2·618	3·142	3·665	4·189	2
2 1/4	1·178	1·767	2·356	2·945	3·534	4 123	4·712	2 1/4
2 1/2	1·309	1·963	2·618	3·272	3·927	4·581	5·236	2 1/2
2 3/4	1·440	2·160	2·880	3·600	4·320	5·040	5·760	2 3/4
3	1·571	2·356	3·142	3·927	4·712	5·498	6·283	3
3 1/4	1·702	2·553	3·403	4·254	5·105	5·956	6·807	3 1/4
3 1/2	1·833	2·749	3·665	4·581	5·498	6·414	7·330	3 1/2
3 3/4	1·963	2·945	3·927	4·909	5·890	6·872	7·854	3 3/4
4	2·094	3·142	4·189	5·236	6·283	7·330	8·378	4
4 1/4	2·225	3·338	4·451	5·563	6·676	7·789	8·901	4 1/4
4 1/2	2·356	3·534	4·712	5·890	7·069	8·247	9·425	4 1/2
4 3/4	2·487	3·731	4·974	6·218	7·461	8 705	9·948	4 3/4
5	2·618	3·927	5·236	6·545	7·854	9·163	10·47	5
5 1/2	2·880	4·320	5·760	7·199	8·639	10·08	11·52	5 1/2
6	3·142	4·712	6·283	7·854	9·425	11·00	12·57	6
7	3·665	5·498	7·330	9·163	11·00	12·83	14·66	7
8	4·189	6·283	8·378	10·47	12·57	14·66	16·76	8
9	4·712	7·069	9·425	11·78	14·14	16·49	18·85	9
10	5·236	7·854	10·47	13·09	15·71	18·33	20·94	10

Surfaces of Tubes or Cylinders in Square Feet.

Diam. in Inches	Length in Feet 9	10	11	12	13	14	15	Diam. in Inches.
$\frac{1}{2}$	1·178	1·309	1·440	1·571	1·702	1·833	1·963	$\frac{1}{2}$
$\frac{9}{16}$	1·325	1·473	1·620	1·767	1·914	2·062	2·209	$\frac{9}{16}$
$\frac{5}{8}$	1·473	1·636	1·800	1·963	2·127	2·291	2·454	$\frac{5}{8}$
$\frac{11}{16}$	1·620	1·800	1·980	2·160	2·340	2·520	2·700	$\frac{11}{16}$
$\frac{3}{4}$	1·767	1·963	2·160	2·356	2·553	2·749	2·945	$\frac{3}{4}$
$\frac{13}{16}$	1·914	2·127	2·340	2·553	2·765	2·978	3·191	$\frac{13}{16}$
$\frac{7}{8}$	2·062	2·291	2·520	2·749	2·978	3·207	3·436	$\frac{7}{8}$
$\frac{15}{16}$	2·209	2·454	2·700	2·945	3·191	3·436	3·682	$\frac{15}{16}$
1	2·356	2·618	2·880	3·142	3·403	3·665	3·927	1
$1\frac{1}{8}$	2·651	2·945	3·240	3·534	3·829	4·123	4·418	$1\frac{1}{8}$
$1\frac{1}{4}$	2·945	3·272	3·600	3·927	4·254	4·581	4·909	$1\frac{1}{4}$
$1\frac{3}{8}$	3·240	3·600	3·960	4·320	4·680	5·040	5·400	$1\frac{3}{8}$
$1\frac{1}{2}$	3·534	3·927	4·320	4·712	5·105	5·498	5·890	$1\frac{1}{2}$
$1\frac{5}{8}$	3·829	4·254	4·680	5·105	5·531	5·956	6·381	$1\frac{5}{8}$
$1\frac{3}{4}$	4·123	4·581	5·040	5·498	5·956	6·414	6·872	$1\frac{3}{4}$
$1\frac{7}{8}$	4·418	4·909	5·400	5·890	6·381	6·872	7·363	$1\frac{7}{8}$
2	4·712	5·236	5·760	6·283	6·807	7·330	7·854	2
$2\frac{1}{4}$	5·301	5·890	6·480	7·069	7·658	8·247	8·836	$2\frac{1}{4}$
$2\frac{1}{2}$	5·890	6·545	7·199	7·854	8·508	9·163	9·817	$2\frac{1}{2}$
$2\frac{3}{4}$	6·480	7·199	7·919	8·639	9·359	10·08	10·80	$2\frac{3}{4}$
3	7·069	7·854	8·639	9·425	10·21	11·00	11·78	3
$3\frac{1}{4}$	7·658	8·508	9·359	10·21	11·06	11·91	12·76	$3\frac{1}{4}$
$3\frac{1}{2}$	8·247	9·163	10·08	11·00	11·91	12·83	13·74	$3\frac{1}{2}$
$3\frac{3}{4}$	8·836	9·817	10·80	11·78	12·76	13·74	14·73	$3\frac{3}{4}$
4	9·425	10·47	11·52	12·57	13·61	14·66	15·71	4
$4\frac{1}{4}$	10·01	11·13	12·24	13·35	14·46	15·58	16·69	$4\frac{1}{4}$
$4\frac{1}{2}$	10·60	11·78	12·96	14·14	15·32	16·49	17·67	$4\frac{1}{2}$
$4\frac{3}{4}$	11·19	12·44	13·68	14·92	16·17	17·41	18·65	$4\frac{3}{4}$
5	11·78	13·09	14·40	15·71	17·02	18·33	19·63	5
$5\frac{1}{2}$	12·96	14·40	15·84	17·28	18·72	20·16	21·60	$5\frac{1}{2}$
6	14·14	15·71	17·28	18·85	20·42	21·99	23·56	6
7	16·49	18·33	20·16	21·99	23·82	25·66	27·49	7
8	18·85	20·94	23·04	25·13	27·23	29·32	31·42	8
9	21·21	23·56	25·92	28·27	30·63	32·99	35·34	9
10	23·56	26·18	28·80	31·42	34·03	36·65	39·27	10

Capacity of Pipes per Foot of Length, or Discharging Capacity of Pumps per Foot Travel of Piston, in Imperial Gallons.

D = diameter of pipe or pump barrel in inches.
C = capacity per foot length in imperial gallons* = ·03397296D^2.

DIAMETERS ADVANCING BY EIGHTHS.

D	·0	1/8	1/4	3/8	1/2	5/8	3/4	7/8
0		·0005	·0021	·0048	·0085	·0133	·0191	·0260
1	·0340	·0430	·0531	·0642	·0764	·0897	·1040	·1194
2	·1359	·1534	·1720	·1916	·2123	·2341	·2569	·2808
3	·3058	·3318	·3588	·3870	·4162	·4464	·4777	·5101
4	·5436	·5781	·6136	·6503	·6880	·7267	·7665	·8074
5	·8493	·8923	·9364	·9815	1·028	1·075	1·123	1·173
6	1·223	1·275	1·327	1·381	1·435	1·491	1·548	1·606
7	1·665	1·725	1·786	1·848	1·911	1·975	2·041	2·107
8	2·174	2·243	2·312	2·383	2·455	2·527	2·601	2·676
9	2·752	2·829	2·907	2·986	3·066	3·147	3·230	3·313

DIAMETERS ADVANCING BY QUARTERS.

D	·0	1/4	1/2	3/4
10	3·397	3·569	3·746	3·926
11	4·111	4·300	4·493	4·690
12	4·892	5·098	5·308	5·523
13	5·741	5·964	6·192	6·423
14	6·659	6·899	7·143	7·391
15	7·644	7·901	8·162	8·427
16	8·697	8·971	9·249	9·532
17	9·818	10·11	10·40	10·70
18	11·01	11·32	11·63	11·94
19	12·26	12·59	12·92	13·25
20	13·59	13·93	14·28	14·63
21	14·98	15·34	15·70	16·07
22	16·44	16·82	17·20	17·58
23	17·97	18·36	18·76	19·16
24	19·57	19·98	20·39	20·81
25	21·23	21·66	22·09	22·53
26	22·97	23·41	23·86	24·31
27	24·77	25·23	25·69	26·16
28	26·63	27·11	27·59	28·08
29	28·57	29·07	29·56	30·07

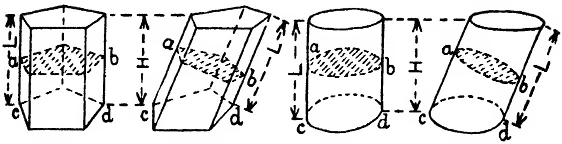
DIAMETERS ADVANCING BY INCHES.

D	C	D	C
30	30·58	50	84·93
31	32·65	51	88·36
32	34·79	52	91·86
33	37·00	53	95·43
34	39·27	54	99·07
35	41·62	55	102·8
36	44·03	56	106·5
37	46·51	57	110·4
38	49·06	58	114·3
39	51·67	59	118·3
40	54·36	60	122·3
41	57·11	61	126·4
42	59·93	62	130·6
43	62·82	63	134·8
44	65·77	64	139·2
45	68·80	65	143·5
46	71·89	66	148·0
47	75·05	67	152·5
48	78·27	68	157·1
49	81·57	69	161·7

* The gallon is taken as 277·420 cubic inches (see p. 2).

Volumes and Surfaces of Solids.

V = volume of solid. A = area of surface of solid.



Right Prism. *Oblique Prism.* *Right Cylinder.* *Oblique Cylinder.*

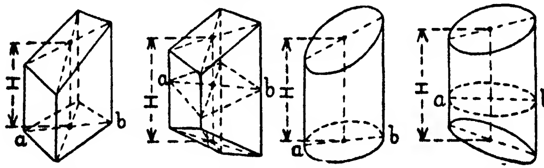
For each of the above solids,

V = area of base cd × perpendicular height H

= area of section ab perpendicular to sides × length L.

A = perimeter of section ab × length L + sum of areas of the two ends.

In a right prism or right cylinder the normal section ab is equal in all respects to the base cd.



Frustum of Prism. *Frustum of Prism* *Frustum of Cylinder.* *Frustum of Cylinder.*

The following rules apply to the frustum of a prism whose normal section is either a triangle, square, rectangle, parallelogram, or any regular polygon, and to the frustum of a cylinder whose normal section is a circle :—

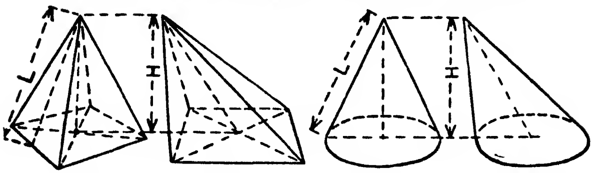
V = area of normal section ab × length of axis H.

A = perimeter of normal section ab × length of axis H + sum of areas of the two ends.

The axis mentioned above is the line joining the centres of gravity of the ends. This line is parallel to the parallel edges of the solid, and its length H is the mean of the lengths of the parallel edges, *i.e.* H = sum of lengths of parallel edges ÷ number of parallel edges.

If the prism whose frustum is under consideration be such that the above rules do not apply, it may be divided into a series of frustums of triangular prisms whose volumes may be determined separately by the rule above, and then added together for the whole volume of the solid. The area of the surface may be obtained by adding together the areas of the separate faces.

Volumes and Surfaces of Solids (*continued*).



Right Pyramid. *Oblique Pyramid* *Right Cone.* *Oblique Cone.*

For each of the above solids,

V = area of base × $\frac{1}{3}$ perpendicular height H

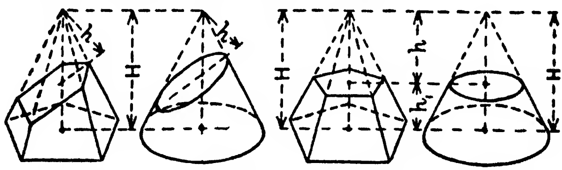
For the right pyramid when the base circumscribes a circle, and for the right circular cone,

A = perimeter of base × $\frac{1}{2}$ length of slant side L + area of base.

For the right circular cone whose base has a radius r,

$$L = \sqrt{r^2 + H^2}, \text{ and } A = \pi r(L + r) = \pi r(r + \sqrt{r^2 + H^2}).$$

In a right pyramid L is measured from the vertex in a direction at right angles to one side of the base.



Frustum of Pyramid. *Frustum of Cone.* *Frustum of Pyramid* *Frustum of Cone.*

The volume of a frustum of a pyramid or cone, *i.e.* the volume of the portion of the pyramid or cone lying between the base and a plane section of the solid, as shown above, is equal to the difference between the volume of the solid whose height is H and the volume of the solid whose height is h, the base of the first solid being the base of the original solid, and the base of the second solid being the section mentioned above.

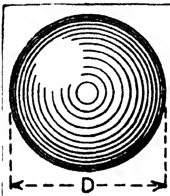
A = area of slant surface of solid of height H − area of slant surface of solid of height h + area of lower end + area of upper end.

If the ends of the frustum are at right angles to the axis, and therefore parallel to one another, then

V = (sum of areas of two ends + $\sqrt{\text{product of areas of two ends}}$) × $\frac{1}{3}$ height of frustum.

If a_1 = area of one end, a_2 = area of other end, and h_1 = height of frustum, then $V = (a_1 + a_2 + \sqrt{a_1 a_2}) \times \frac{1}{3} h_1$.

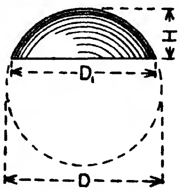
Volumes and Surfaces of Solids (*concluded*).



Sphere.

$$V = \frac{\pi}{6} D^3$$

$$A = \pi D^2.$$

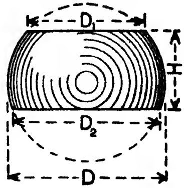


Segment of a Sphere.

$$V = \frac{\pi H}{6}\left(\frac{3}{4}D_1^2 + H^2\right)$$

$$= \frac{\pi H^2}{6}(3D - 2H).$$

$$A = \pi DH + \frac{\pi}{4}D_1^2.$$

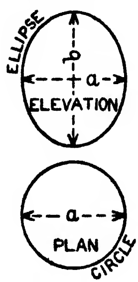


Zone of a Sphere.

$$V = \frac{\pi H}{6}\left(\frac{3}{4}D_1^2 + \frac{3}{4}D_2^2 + H^2\right)$$

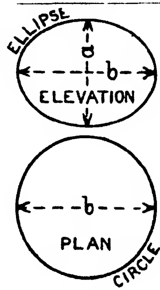
$$A = \pi DH + \frac{\pi}{4}(D_1^2 + D_2^2).$$

If a cone, sphere, and cylinder have the same diameter, and the cone and cylinder each have a height equal to their diameter; then, if the volume of the cone be denoted by 1, the volume of the sphere will be denoted by 2, and the volume of the cylinder by 3. Also, the area of the surface of the sphere will be equal to the area of the *curved* surface of the cylinder. Again, the area of the curved surface of a zone or segment of the sphere will be equal to that of a zone of the cylinder of the same height.



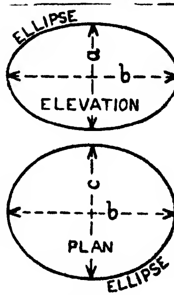
Prolate Spheroid.

$$V = \frac{\pi}{6}a^2 b.$$



Oblate Spheroid.

$$V = \frac{\pi}{6}ab^2.$$



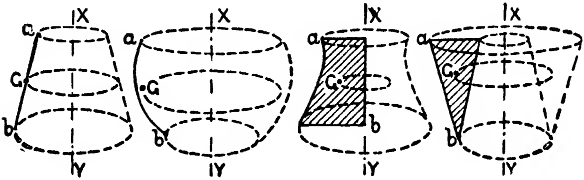
Ellipsoid.

$$V = \frac{\pi}{6}abc.$$

The volume of a spheroid or ellipsoid is equal to two-thirds of the volume of the circumscribing cylinder.

The Theorems of Guldinus.

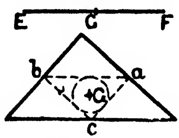
(1.) If a line *ab*, straight or curved, whose centre of gravity is *G*, be made to revolve about an axis *XY* in its plane, the area of the curved surface traced out by *ab* is equal to the length of *ab* multiplied by the length of the path of *G*.



(2.) If a plane figure *ab*, whose centre of gravity is *G*, be made to revolve about an axis *XY* in its plane, but not cutting the figure, the volume of the solid swept out by the figure is equal to the area of the figure *ab* multiplied by the length of the path of *G*.

Centre of Gravity.

The centre of gravity is indicated by the letter *G*.

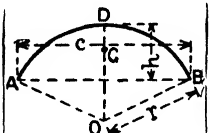


Straight Line *EF*.

G is the middle point of *EF*.

Sides of a Triangle

a, *b*, and *c* are the middle points of the sides. The centre of gravity of the three sides is at the centre of the circle inscribed in the triangle *abc*.



Arc of a Circle.

l = length of arc *ADB*.

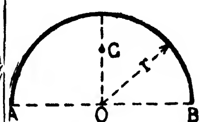
OG is perpendicular to *AB*.

$$OG = \frac{cr}{l} = \frac{c(c^2 + 4h^2)}{8hl}.$$

n = number of degrees in angle *AOB*.

$$l = \frac{\pi r n}{180}.$$

$$OG = \frac{180}{n} \times \frac{c}{\pi}.$$

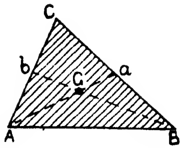


Arc of a Circle = Half Circumference of Circle.

OG is perpendicular to *AB*.

$$OG = \frac{2r}{\pi} = \cdot 6366r.$$

Centre of Gravity (*continued*).

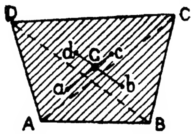


Triangle

$Ab = bC$. $Ba = aC$.
G is at the intersection of Aa and Bb

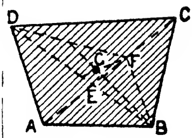
$$aG = \frac{1}{3} Aa.$$

$$bG = \frac{1}{3} Bb$$



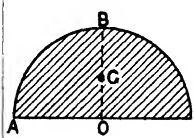
Quadrilateral.

a, b, c, and d are the centres of gravity of the triangles ABD, BCA, CDB, and DAC respectively. G is at the intersection of ac and bd.



Quadrilateral
(*another method*).

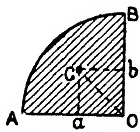
Make $CF = AE$
G, the centre of gravity of the triangle BDF, is also the centre of gravity of the quadrilateral $ABCD$.



Semicircle.

OA = radius = r.
OB is perpendicular to OA.

$$OG = \frac{4r}{3\pi} = \cdot 4244r.$$

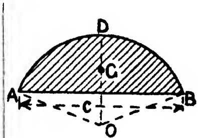


Quadrant of a Circle.

$OA = OB$ = radius = r.
Angle $AOB = 90°$.
Ga is perpendicular to OA.
Gb is perpendicular to OB.

$$Ob = Oa = \frac{4r}{3\pi} = \cdot 4244r$$

$$OG = \frac{4r\sqrt{2}}{3\pi} = \cdot 6002r.$$



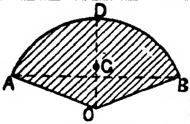
Segment of a Circle.

O is the centre of the circle.
OD is perpendicular to AB.
a = area of segment.

$$OG = \frac{c^3}{12a}.$$

The centre of gravity of a regular plane figure, such as an equilateral triangle, a square, a rectangle, a parallelogram, or any regular polygon, or a circle, or an ellipse, is at the *geometrical centre* of the figure.

Centre of Gravity (*continued*).



Sector of a Circle.

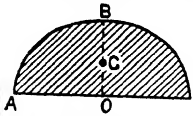
Chord $AB=c$.

Length of arc $ADB=l$.

Radius OA or $OB=r$.

OD is perpendicular to AB.

$OG=\frac{2cr}{3l}$.

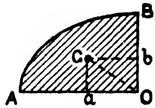


Half of an Ellipse

OA is semi major axis.

OB is semi-minor axis $=b$.

$OG=\frac{4b}{3\pi}=\cdot4244b$.



Quarter of an Ellipse

OA is semi-major axis.

OB is semi-minor axis.

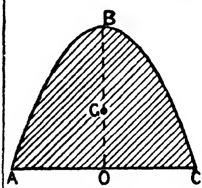
Ga is perpendicular to OA.

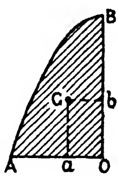
Gb is perpendicular to OB.

$Oa=\frac{4OA}{3\pi}=\cdot4244\times\overline{OA}$.

$Ob=\frac{4\overline{OB}}{3\pi}=\cdot4244\times\overline{OB}$.

Parabola and Semi-Parabola





B is the vertex and OB the axis.

AOC is perpendicular to OB.

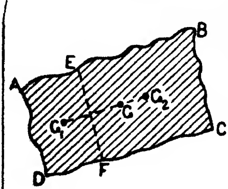
Ga is perpendicular to OA.

Gb is perpendicular to OB.

For parabola, $OG=\frac{2}{5}\overline{OB}$.

$Ob=\frac{2}{5}\overline{OB}$. $Oa=\frac{3}{8}\overline{OA}$.

$ABCD$ is any plane figure. G is the centre of gravity of the whole figure. G_1 is the centre of gravity of the part $AEFD$. G_2 is the centre of gravity of the remaining part $BEFC$.



a=area of part $AEFD$. b=area of part $BEFC$.

G_1GG_2 is a straight line.

$a\times G_1G=b\times G_2G$.

$(a+b)\times\overline{GG_1}=b\times\overline{G_2G_1}$.

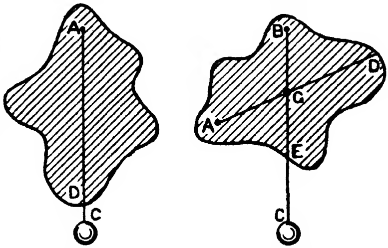
$(a+b)\times\overline{GG_2}=a\times\overline{G_1G_2}$.

Hence if the centres of gravity of the whole figure and one part be known, the centre of gravity of the remaining part may be found. Also if the centres of gravity of the parts be known, the centre of gravity of the whole may be found. The areas a and b are supposed to be known in each case.

Centre of Gravity (*continued*).

Any Plane Figure, by Experiment.

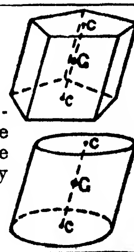
Cut the figure out of a piece of cardboard of uniform thickness. Make a pin-hole at any point A near the edge. Suspend the figure in a vertical plane on a pin through A. To the pin attach a thread carrying a weight C at its lower end. Draw a line AD on the figure in the direction of the thread. Repeat the experiment,



suspending the figure from another point B. The intersection of the lines AD and BE is the centre of gravity of the figure.

This method gives excellent results, and is specially useful for irregular figures.

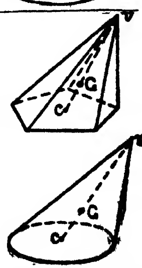
Prism or Cylinder.



Right or oblique, regular or irregular.

Centre of gravity of *surface*, excluding or including *both* ends.* G is at the centre of the line joining C and c, the centres of gravity of the parallel ends. This is also the centre of gravity of the *solid*.

Pyramid or Cone.



Right or oblique, regular or irregular.

Centre of gravity of *surface*, excluding the base.* G is in the line joining the vertex v with c, the centre of gravity of the base. $cG = \frac{1}{3}\ cv$.

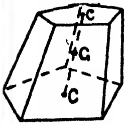
Centre of gravity of *solid* is in same line cv. $cG = \frac{1}{4}\ cv$.

* If in the case of the prism or cylinder it is required to include *one end*, or if in the pyramid or cone it is required to include the *base*, or if in the frustum of a pyramid or cone it is required to include *one end* or *both ends*, apply the rule at the bottom of page 206.

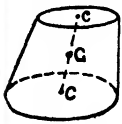
Centre of Gravity (*continued*)

Frustum of Pyramid or Cone.

Right or oblique, regular or irregular, ends parallel.
M = perimeter or circumference of lower end.



m = perimeter or circumference of upper end.
S = length of one side of lower end
s = length of corresponding side of upper end.
R = radius of lower end if circular.
r = radius of upper end if circular.
A = area of lower end.
a = area of upper end.



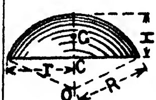
Centre of gravity of *surface*, excluding both ends.* G is in the line joining C and c, the centres of gravity of the ends.

$$CG = \frac{Cc}{3} \times \frac{M+2m}{M+m} = \frac{Cc}{3} \times \frac{S+2s}{S+s} = \frac{Cc}{3} \times \frac{R+2r}{R+r}.$$

Centre of gravity of *solid*. G is in same line Cc.

$$CG = \frac{Cc}{4} \times \frac{S^2+3s^2+2Ss}{S^2+s^2+Ss} = \frac{Cc}{4} \times \frac{R^2+3r^2+2Rr}{R^2+r^2+Rr} = \frac{Cc}{4} \times \frac{A+3a+2\sqrt{Aa}}{A+a+\sqrt{Aa}}.$$

Segment of a Sphere.



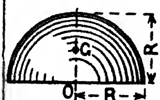
Centre of gravity of *curved surface.**

$$CG = \frac{1}{2}H.$$

Centre of gravity of *solid*. $OG = \frac{3}{4} \times \frac{(2R-H)^2}{3R-H}.$

$$CG = \frac{H}{2} \times \frac{2r^2+H^2}{3r^2+H^2} = \frac{H}{4} \times \frac{4R-H}{3R-H}.$$

Hemisphere.



Centre of gravity of *curved surface.**

$$OG = \frac{1}{2}R.$$

Centre of gravity of *solid*. $OG = \frac{3}{8}R.$

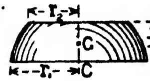
The centre of gravity of a regular solid, such as a cube, a right circular cylinder, a sphere, a spheroid, or ellipsoid, is the *geometrical centre* of the solid.

* See footnotes on pages 207 and 209.

Centre of Gravity (*continued*).

Zone of a Sphere.

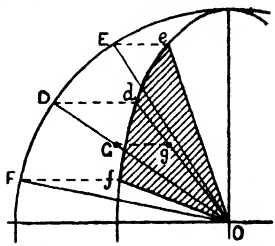
Centre of gravity of *curved surface.** $CG = \frac{1}{2}H.$



Centre of gravity of *solid.*

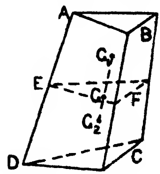
$$CG = \frac{H}{2} \times \frac{2r_1^2 + 4r_2^2 + H^2}{3r_1^2 + 3r_2^2 + H^2}.$$

To Determine the Centre of Gravity by "Projection."



If one figure is a parallel projection † of another, the centre of gravity of the one is the corresponding projection of the centre of gravity of the other. For example, an ellipse is a parallel projection of a circle, the diameter of the circle being equal to the major axis of the ellipse. g, the centre of gravity of a sector oef of an ellipse, is the corresponding projection of G, the centre of gravity of the corresponding sector OEF of the circle.

* If it is required to include the *base* of the segment of a sphere, or if it is required to include *one end* or *both ends* of a zone of a sphere, apply the rule given below.



S = area of the surface of any solid $ABCD$.
S_1 = area of a part $EABF$ of the surface of the solid.
S_2 = area of the remaining part $FCDE$ of the surface.
G is the centre of gravity of whole surface of solid.
G_1 is the centre of gravity of the part $EABF$ of the surface.
G_2 is the centre of gravity of the remaining part $FCDE$ of the surface.
G_1, G, and G_2 are in a straight line, and $S_1 \times G_1G = S_2 \times G_2G$. Also, $S_1 \times G_1G_2 = S \times GG_2$, and $S_2 \times G_2G_1 = S \times GG_1$.
V = volume of any solid $ABCD$.
V_1 = volume of a part $ABFE$ of the solid.
V_2 = volume of the remaining part $EFCD$.
G is the centre of gravity of the whole solid.
G_1 is the centre of gravity of the part $ABFE$.
G_2 is the centre of gravity of the remaining part $EFCD$.
G, G_1, and G_2 are in a straight line, and $V_1 \times G_1G = V_2 \times G_2G$ Also, $V_1 \times G_1G_2 = V \times GG_2$, and $V_2 \times G_2G_1 = V \times GG_1$.

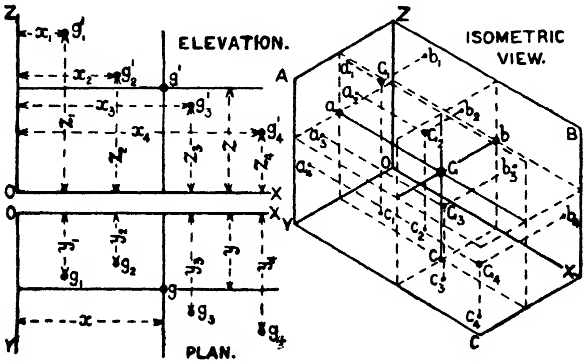
Hence if the areas of the parts of a surface of a solid, or the volumes of the parts of a solid, be known, and also the positions of their centres of gravity, the position of the centre of gravity of the whole surface, or the whole solid, may be determined. Likewise, the position of the centre of gravity of one part may be determined if the positions of the centres of gravity of the whole and the other part be known.

† In parallel projection the projectors are parallel to one another, but may be either inclined or perpendicular to the plane of projection.

Centre of Gravity (*concluded*).

To Determine the Centre of Gravity of any Structure, such as a Complete Machine.

Select three planes of reference, $OYAZ$, $OXBZ$, and $OXCY$, perpendicular to one another. Let G_1, G_2, G_3, etc., be the centres of gravity of the various parts of the structure. w_1, w_2, w_3, etc. = the weights of the parts whose centres of gravity are G_1, G_2, G_3, etc., respectively. x_1, x_2, x_3, etc. = distances of G_1, G_2, G_3, etc., from the plane $OYAZ$. y_1, y_2, y_3, etc. = distances



of G_1, G_2, G_3, etc., from the plane $OXBZ$. z_1, z_2, z_3, etc. = distances of G_1, G_2, G_3, etc., from the plane $OXCY$. G is the centre of gravity of the complete structure. x, y, and z = distances of G from the planes $OYAZ$, $OXBZ$, and $OXCY$ respectively. w = total weight of structure = $w_1 + w_2 + w_3 +$ etc.

$$x = \frac{w_1x_1 + w_2x_2 + w_3x_3 + \text{etc.}}{w}.$$

$$y = \frac{w_1y_1 + w_2y_2 + w_3y_3 + \text{etc.}}{w}.$$

$$z = \frac{w_1z_1 + w_2z_2 + w_3z_3 + \text{etc.}}{w}.$$

If the points G_1, G_2, G_3, etc., are not all on the same side of any one of the planes of reference, then if distances measured on one side are positive, those on the other side must be taken as negative.

Moments of Inertia.

Moment of Inertia.—Let w_1, w_2, w_3, etc., be the weights of the material particles making up a rigid body, and let their distances from a given axis be respectively r_1, r_2, r_3, etc., then,

Moment of inertia of body about given axis $= I$.

$$I = w_1 r_1^2 + w_2 r_2^2 + w_3 r_3^2 + \text{etc.}$$

Again, let a_1, a_2, a_3, etc., be the areas of the small parts making up the surface of a given figure, and let the distances of these small parts from a given axis be respectively r_1, r_2, r_3, etc., then,

Moment of inertia of surface about given axis $= I$.

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \text{etc.}$$

If I_1, I_2, I_3, etc., be the moments of inertia of the parts of a solid or surface about a given axis, then,

Moment of inertia of whole solid or whole surface $= I$.

$$I = I_1 + I_2 + I_3 + \text{etc.}$$

Radius of Gyration.—Let $W =$ total weight of rigid body mentioned above.

Let $A =$ total area of surface mentioned above;
and let k be such a length that,

$$Wk^2 = I = w_1 r_1^2 + w_2 r_2^2 + w_3 r_3^2 + \text{etc., or,}$$

$$Ak^2 = I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \text{etc.,}$$

then k is called the radius of gyration of the body or surface.

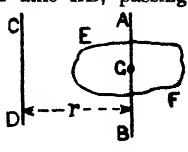
Moments of Inertia about Parallel Axes.—Let $I =$ moment of inertia of a body or surface EF about an axis AB, passing through its centre of gravity G.

$I_1 =$ moment of inertia of same body or surface about an axis CD, parallel to AB, and at a distance r from it.

$W =$ weight of body. $A =$ area of surface.

$I_1 = I + Wr^2$ for the body.

$I_1 = I + Ar^2$ for the surface.



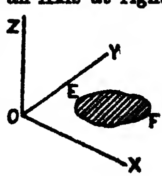
Moment of Inertia of a Plane Figure about an Axis at right angles to its Plane.—OX and OY are axes at right angles to one another, and in the plane of the figure EF. OZ is an axis at right angles to the plane of the figure EF.

$I_x =$ moment of inertia of EF about axis OX.

$I_y =$ moment of inertia of EF about axis OY.

$I_z =$ moment of inertia of EF about axis OZ.

$$I_z = I_x + I_y.$$

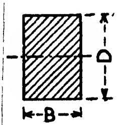


Areas, Moments of Inertia, and Moduli of Various Sections.

A = area. I = moment of inertia. Z = modulus $= \frac{I}{y}$.

The axis of moments is the neutral axis of the section, and passes through the centre of gravity of the section. y = distance of axis of moments from the top or bottom of the section. Where no value is given for y, it is equal to half the total depth of the section. Where the section is not symmetrical about the neutral axis, there are two values for the modulus, $Z_1 = I \div y_1$, and $Z_2 = I \div y_2$.

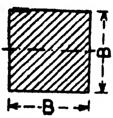
Radius of gyration $= k = \sqrt{\frac{I}{A}}$.



$A = BD.$

$I = \frac{1}{12} BD^3.$

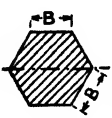
$Z = \frac{1}{6} BD^2.$



$A = B^2.$

$I = \frac{1}{12} B^4.$

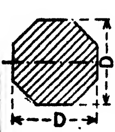
$Z = \frac{1}{6} B^3.$



$A = \frac{3\sqrt{3}}{2} B^2 = 2{\cdot}598 B^2.$

$I = \frac{5\sqrt{3}}{16} B^4 = {\cdot}5413 B^4.$

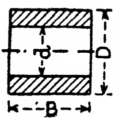
$Z = \frac{5}{8} B^3.$



$A = 2(\sqrt{2} - 1) D^2.$
$= {\cdot}8284 D^2.$

$I = \frac{1}{12}(4\sqrt{2} - 5) D^4.$
$= {\cdot}0547 D^4.$

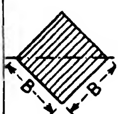
$Z = \frac{1}{6}(4\sqrt{2} - 5) D^3.$
$= {\cdot}1094 D^3.$



$A = B(D - d).$

$I = \frac{B}{12}(D^3 - d^3).$

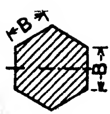
$Z = \frac{B(D^3 - d^3)}{6D}.$



$A = B^2.$

$I = \frac{1}{12} B^4.$

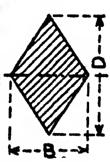
$Z = \frac{\sqrt{2}}{12} B^3 = {\cdot}118 B^3.$



$A = \frac{3\sqrt{3}}{2} B^2.$

$I = \frac{5\sqrt{3}}{16} B^4.$

$Z = \frac{5\sqrt{3}}{16} B^3.$

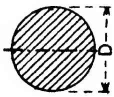


$A = \frac{1}{2} BD.$

$I = \frac{BD^3}{48}.$

$Z = \frac{BD^2}{24}.$

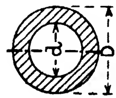
Areas, Moments of Inertia, and Moduli of Various Sections (*continued*).



$A = \frac{\pi}{4}D^2 = .7854D^2.$

$I = \frac{\pi}{64}D^4 = .0491D^4.$

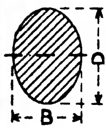
$Z = \frac{\pi}{32}D^3 = .0982D^3.$



$A = \frac{\pi}{4}(D^2 - d^2).$

$I = \frac{\pi}{64}(D^4 - d^4).$

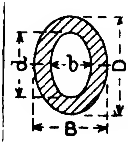
$Z = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)$



$A = \frac{\pi}{4}BD = .7854BD.$

$I = \frac{\pi}{64}BD^3 = .0491BD^3.$

$Z = \frac{\pi}{32}BD^2 = .0982BD^2.$



$A = \frac{\pi}{4}(BD - bd).$

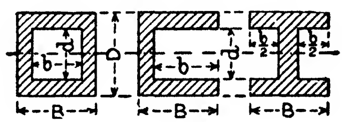
$I = \frac{\pi}{64}(BD^3 - bd^3).$

$Z = \frac{\pi}{32}\left(\frac{BD^3 - bd^3}{D}\right)$

$A = BD - bd.$

$I = \frac{1}{12}(BD^3 - bd^3).$

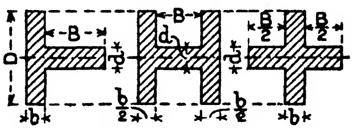
$Z = \frac{BD^3 - bd^3}{6D}.$



$A = bD + Bd.$

$I = \frac{1}{12}(bD^3 + Bd^3).$

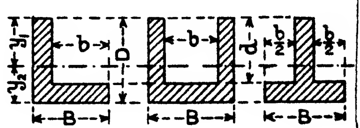
$Z = \frac{bD^3 + Bd^3}{6D}.$



$A = BD - bd.$

$y_1 = \frac{BD^2 - bd^2}{2(BD - bd)}.$

$y_2 = \frac{BD^2 - 2bdD + bd^2}{2(BD - bd)}.$

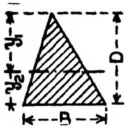


$$I = \frac{(BD^2 - bd^2)^2 - 4BDbd(D - d)^2}{12(BD - bd)}.$$

$$Z_1 = \frac{I}{y_1} = \frac{(BD^2 - bd^2)^2 - 4BDbd(D - d)^2}{6(BD^2 - bd^2)}.$$

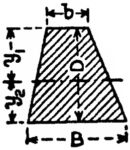
$$Z_2 = \frac{I}{y_2} = \frac{(BD^2 - bd^2)^2 - 4BDbd(D - d)^2}{6(BD^2 - 2bdD + bd^2)}.$$

Areas, Moments of Inertia, and Moduli of Various Sections (*continued*).



$$A=\frac{1}{2}BD. \qquad y_1=\frac{2}{3}D. \qquad y_2=\frac{1}{3}D.$$

$$I=\frac{BD^3}{36}. \qquad Z_1=\frac{I}{y_1}=\frac{BD^2}{24}. \qquad Z_2=\frac{I}{y_2}=\frac{BD^2}{12}.$$



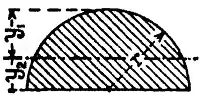
$$A=\frac{1}{2}D(B+b). \qquad y_1=\frac{D(2B+b)}{3(B+b)}.$$

$$y_2=\frac{D(B+2b)}{3(B+b)}. \qquad I=\frac{(B^2+4Bb+b^2)D^3}{36(B+b)}.$$

$$Z_1=\frac{I}{y_1}=\frac{(B^2+4Bb+b^2)D^2}{12(2B+b)}.$$

$$Z_2=\frac{I}{y_2}=\frac{(B^2+4Bb+b^2)D^2}{12(B+2b)}.$$

$$A=\frac{\pi r^2}{2}=1{\cdot}5708r^2.$$

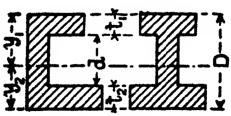


$$y_1=\left(1-\frac{4}{3\pi}\right)r=\cdot5756r. \quad y_2=\frac{4r}{3\pi}=\cdot4244r.$$

$$I=\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)r^4=\cdot1098r^4.$$

$$Z_1=\frac{I}{y_1}=\cdot1907r^3. \quad Z_2=\frac{I}{y_2}=\cdot2586r^3.$$

a_1 = area of top flange.
a_2 = area of bottom flange.
a = area of web.



$$y_1=\frac{a_2(2D-t_2)+a_1t_1+a(d+2t_1)}{2(a_1+a_2+a)}.$$

$$y_2=\frac{a_1(2D-t_1)+a_2t_2+a(d+2t_2)}{2(a_1+a_2+a)}.$$

$$I=\frac{a_1t_1^2+a_2t_2^2+ad^2}{12}+\frac{a_1a_2(D+d)^2+a_1a(t_1+d)^2+a_2a(t_2+d)^2}{4(a_1+a_2+a)}.$$

$$Z_1=\frac{I}{y_1}. \qquad Z_2=\frac{I}{y_2}.$$

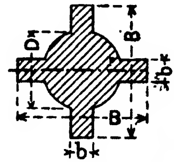
In actual practice it is often sufficiently accurate to take $Z_1=a_1h$, and $Z_2=a_2h$, where h is the perpendicular distance between the centres of the flanges.

Areas, Moments of Inertia, and Moduli of Various Sections (*concluded*).

$$A=\frac{\pi}{4}D^2+2b(B-D).$$

$$I=\frac{1}{12}\left\{\frac{3\pi}{16}D^4+b(B^3-D^3)+b^3(B-D)\right\}.$$

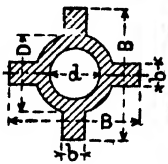
$$Z=\frac{1}{6B}\left\{\frac{3\pi}{16}D^4+b(B^3-D^3)+b^3(B-D)\right\}.$$



$$A=\frac{\pi}{4}(D^2-d^2)+2b(B-D).$$

$$I=\frac{1}{12}\left\{\frac{3\pi}{16}(D^4-d^4)+b(B^3-D^3)+b^3(B-D)\right\}.$$

$$Z=\frac{1}{6B}\left\{\frac{3\pi}{16}(D^4-d^4)+b(B^3-D^3)+b^3(B-D)\right\}.$$



Radii of Gyration of Various Solids.

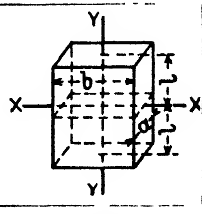
k_x = radius of gyration of body about axis XX.
k_y = radius of gyration of body about axis YY.
The axes XX and YY pass through the centre of gravity of the solid in each case

Rectangular Prism.

Sides of rectangle, a and b.
Length of solid, $2l$.

$$k_x^2=\frac{a^2}{12}+\frac{l^2}{3}.$$

$$k_y^2=\frac{a^2+b^2}{12}.$$

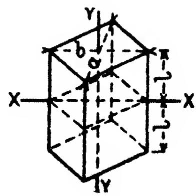


Rhombic Prism.

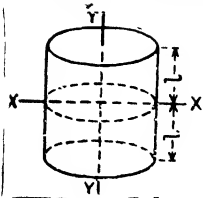
Diagonals, a and b.
Length of solid, $2l$.

$$k_x^2=\frac{a^2}{24}+\frac{l^2}{3}.$$

$$k_y^2=\frac{a^2+b^2}{24}.$$



Radii of Gyration of Various Solids (*continued*).

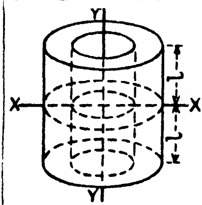


Solid Circular Cylinder.

Radius, r.
Length, $2l$.

$$k_x^2 = \frac{r^2}{4} + \frac{l^2}{3}.$$

$$k_y^2 = \frac{r^2}{2}.$$

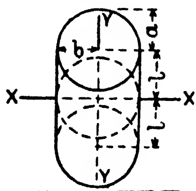


Hollow Circular Cylinder.

External radius, r_1.
Internal radius, r_2 Length, $2l$.

$$k_x^2 = \frac{r_1^2 + r_2^2}{4} + \frac{l^2}{3}.$$

$$k_y^2 = \frac{r_1^2 + r_2^2}{2}.$$

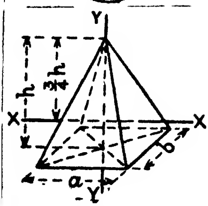


Elliptic Cylinder.

Semi-axes of ellipse, a and b.
Length of cylinder, $2l$.

$$k_x^2 = \frac{a^2}{4} + \frac{l^2}{3}.$$

$$k_y^2 = \frac{a^2 + b^2}{4}.$$

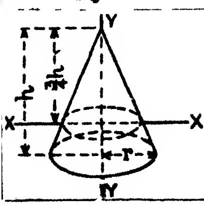


Rectangular Pyramid.

Sides of rectangle, a and b.
Height of solid, h.

$$k_x^2 = \frac{3}{80}h^2 + \frac{b^2}{20}.$$

$$k_y^2 = \frac{a^2 + b^2}{20}.$$



Circular Cone.

Radius of base, r.
Height, h.

$$k_x^2 = \frac{3}{20}\left(r^2 + \frac{h^2}{4}\right).$$

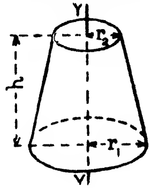
$$k_y^2 = \frac{3}{10}r^2.$$

Radii of Gyration of Various Solids (*concluded*).

Frustum of Circular Cone.

Radii of ends, r_1 and r_2.

$$k_y^2=\frac{3}{10}\left(\frac{r_1^5-r_2^5}{r_1^3-r_2^3}\right).$$



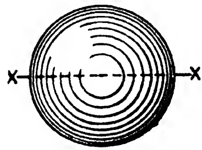
Solid Sphere.

Radius, r. $k_x^2=\frac{2}{5}r^2$.

Hollow Sphere

External radius, r_1 Internal radius, r_2

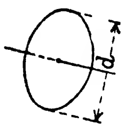
$$k_x^2=\frac{2}{5}\left(\frac{r_1^5-r_2^5}{r_1^3-r_2^3}\right).$$



Polar Moment of Inertia

I_o=moment of inertia of plane figure about an axis at right angles to its plane and passing through its geometrical centre or centre of gravity.

k_o=corresponding radius of gyration.

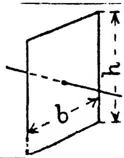


Circle.

Diameter$=d$.

$$I_o=\frac{\pi d^4}{32}.$$

$$k_o^2=\frac{d^2}{8}.$$

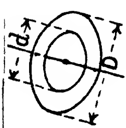


Rectangle.

Sides$=b$ and h.

$$I_o=\frac{bh(b^2+h^2)}{12}.$$

$$k_o^2=\frac{b^2+h^2}{12}.$$

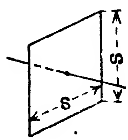


Hollow Circle.

Diameters $=D$ and d.

$$I_o=\frac{\pi(D^4-d^4)}{32}.$$

$$k_o^2=\frac{D^2+d^2}{8}.$$

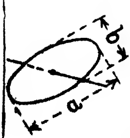


Square.

Sides$=s$.

$$I_o=\frac{s^4}{6}.$$

$$k_o^2=\frac{s^2}{6}.$$



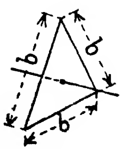
Ellipse.

Major axis$=a$.

Minor axis$=b$.

$$I_o=\frac{\pi ab(a^2+b^2)}{64}.$$

$$k_o^2=\frac{a^2+b^2}{16}.$$



Equilateral Triangle.

Sides$=b$.

$$I_o=\frac{b^4}{16\sqrt{3}}.$$

$$k_o^2=\frac{b^2}{12}.$$

STATICS—The Equilibrium of Forces.

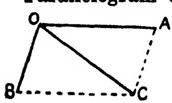
Force is that which moves or tends to move a body, or which changes or tends to change the motion of a body.

The **Unit of Force** used by engineers is the attraction which the earth exerts in the latitude of London upon a certain piece of platinum kept in the Exchequer Office. This unit is called a **pound**, or the "Imperial standard pound avoirdupois." Multiples and submultiples of the pound are also used as units of force.

The single force which would produce the same effect as a number of forces acting together is called the **Resultant** of these forces, and the forces are called the **Components** of their resultant.

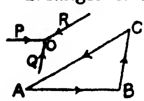
The resultant of a number of forces acting in a straight line is equal to the algebraical sum of the forces. (If forces acting in one direction along a line are positive, those acting in the opposite direction are negative.)

Parallelogram of Forces.—If the straight lines *OA* and *OB*



represent in magnitude and direction two forces acting at the point *O*, the diagonal *OC* of the parallelogram *OACB* will represent their resultant in magnitude and direction. Conversely, if a parallelogram *OACB* be described on *OC* as diagonal, *OA* and *OB* will represent components of the force represented by *OC*.

Triangle of Forces.—If three forces, *P*, *Q*, and *R*, acting at a



point *O*, can be represented in magnitude and direction by the sides of a triangle *ABC* taken in order, the forces are in equilibrium. Conversely, if three forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the sides of a triangle taken in order.

If three forces acting on a body are in equilibrium, their lines of action must meet at a point, or be parallel.

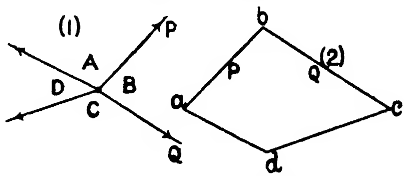
Polygon of Forces.—If any number of forces acting at a point can be represented in magnitude and direction by the sides of a polygon taken in order, the forces are in equilibrium. Conversely, if any number of forces acting at a point are in equilibrium, they can be represented in magnitude and direction by the sides of a polygon taken in order.

The above statements are true whether the forces act in the same plane or not, provided they all act at the same point.

Forces are considered to act at the same point when their lines of action intersect at the same point.

If any number of forces acting in the same plane, but not at the same point, are in equilibrium, they can be represented in magnitude and direction by the sides of a polygon taken in order. The converse of this is, however, not necessarily true.

Lettering of Forces- Bow's Notation.—The diagram (1) shows the lines of action of a number of forces which act at a point on a rigid body, and which are in equilibrium. The diagram (2) is the corresponding polygon of forces. In one system of lettering each force is denoted by a single letter, as



P. In *Bow's notation* each force is denoted by two letters, and they are placed on opposite sides of the line of action of the force in diagram (1), and at the angular points of the polygon in diagram (2). In Bow's notation the force *P* is referred to as the force *AB*. In like manner the force *Q* is referred to as the force *BC*. Bow's notation is of great value in *graphic statics*.

The **Moment of a Force** about a point is the product of the magnitude of the force and the perpendicular distance of its line of action from the point.

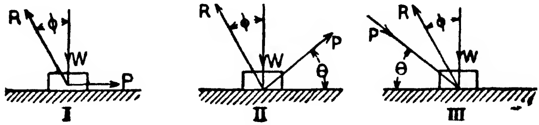
The moment of a force about an *axis* which is at right angles to the line of action of the force is the product of the magnitude of the force and the perpendicular distance of its line of action from the axis. If the axis is not at right angles to the line of action of the force, then the moment of the force about the axis is the product of the rectangular component of the force at right angles to the axis, and in a plane parallel to the axis, and its perpendicular distance from the axis.

Principle of Moments.—When a number of forces acting on a rigid body are in equilibrium, then, the moments of all the forces about any given axis being taken, the sum of the moments of those forces which tend to turn the body in one direction about the axis is equal to the sum of the moments of those forces which tend to turn the body in the opposite direction about the same axis.

Couples.—A *couple* consists of two equal parallel forces acting in opposite directions. The *arm* of a couple is the perpendicular distance between the two parallel forces. The *moment* of a couple is the product of the magnitude of either of the equal forces and the arm of the couple. A couple causes or tends to cause a body to *rotate*.

Two couples will balance one another when (1) they are in the same plane or in parallel planes, (2) they have equal moments, and (3) their directions of rotation are opposite.

Force required to Move a Body on a Horizontal Plane against the Resistance of Friction.



W = weight of body, or force pressing the body on the plane.
R = reaction of the plane on the body.
The direction of R makes an angle with the normal to the plane equal to ϕ, the "limiting angle of resistance."
μ = coefficient of friction = tan ϕ. (For values of μ see p. 239.)
P = force necessary to move the body against the resistance of friction

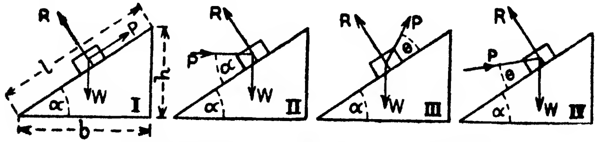
I. $P = W \tan \phi = \mu W$. $\qquad R = \dfrac{W}{\cos \phi}$.

II. $P = W \dfrac{\sin \phi}{\cos(\theta - \phi)}$ = *minimum* when $\theta = \phi$. $\qquad R = W \dfrac{\cos \theta}{\cos(\theta - \phi)}$.

III. $P = W \dfrac{\sin \phi}{\cos(\theta + \phi)}$ = *maximum* when $\theta = 90 - \phi$. $\qquad R = W \dfrac{\cos \theta}{\cos(\theta + \phi)}$.

The Inclined Plane.

α = inclination of plane. $\qquad h$ = height of plane.
l = length of incline. $\qquad b$ = length of base.
P = force necessary to move the body along the plane.
W = weight of body. $\qquad R$ = reaction of plane on body.
The direction of R makes an angle with the normal to the plane equal to ϕ, the "limiting angle of resistance." When friction is neglected the direction of R is perpendicular to the plane.
μ = coefficient of friction = tan ϕ.
θ = acute angle between the direction of P and the plane.



Friction neglected. Motion up the plane.

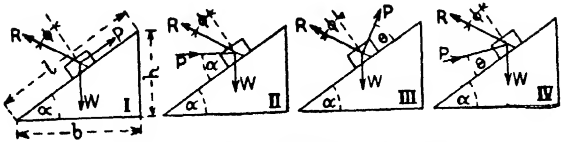
I. $P = W \sin \alpha = W \dfrac{h}{l}$. $\qquad R = W \cos \alpha = W \dfrac{b}{l}$.

The Inclined Plane (*continued*)

II $P = W \tan \alpha = W\dfrac{h}{b}$ $\qquad R = \dfrac{W}{\cos \alpha} = W\dfrac{l}{b}$

III $P = W\dfrac{\sin \alpha}{\cos \theta}$ $\qquad R = W\dfrac{\cos(\alpha + \theta)}{\cos \theta}$

IV $P = W\dfrac{\sin \alpha}{\cos \theta}$ $\qquad R = W\dfrac{\cos(\alpha - \theta)}{\cos \theta}$.



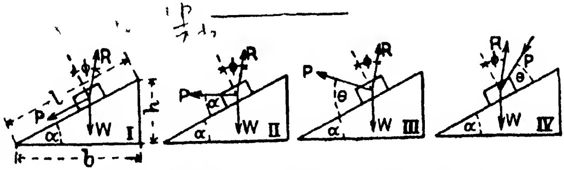
Friction considered Motion up the plane

I $P = W\dfrac{\sin(\alpha + \phi)}{\cos \phi} = W\dfrac{h + b\mu}{l}$ $\qquad R = W\dfrac{\cos \alpha}{\cos \phi} = W\dfrac{b}{l}\sqrt{1 + \mu^2}$.

II $P = W \tan(\alpha + \phi)$ $\qquad R = \dfrac{W}{\cos(\alpha + \phi)}$

III $P = W\dfrac{\sin(\alpha + \phi)}{\cos(\theta - \phi)}$ $\qquad R = W\dfrac{\cos(\alpha + \theta)}{\cos(\theta - \phi)}$.

IV $P = W\dfrac{\sin(\alpha + \phi)}{\cos(\theta + \phi)}$ $\qquad R = W\dfrac{\cos(\alpha - \theta)}{\cos(\theta + \phi)}$



Friction considered Motion down the plane.

I $P = W\dfrac{\sin(\phi - \alpha)}{\cos \phi} = W\dfrac{b\mu - h}{l}$ $\qquad R = W\dfrac{\cos \alpha}{\cos \phi} = W\dfrac{b}{l}\sqrt{1 + \mu^2}$.

II. $P = W \tan(\phi - \alpha)$. $\qquad R = \dfrac{W}{\cos(\phi - \alpha)}$.

The Inclined Plane (*continued*).

III. $P = W\dfrac{\sin(\phi - \alpha)}{\cos(\phi - \theta)}$. $\qquad R = W\dfrac{\cos(\theta - \alpha)}{\cos(\theta - \phi)}$.

IV. $P = W\dfrac{\sin(\phi - \alpha)}{\cos(\phi + \theta)}$. $\qquad R = W\dfrac{\cos(\theta + \alpha)}{\cos(\theta + \phi)}$.

The Wedge, Key, or Cotter.—The diagrams show the wedge as used to separate two pieces A and B which move within suitable guides.

P = force required to drive the wedge.

Q, Q = forces pushing the pieces A and B against the wedge.

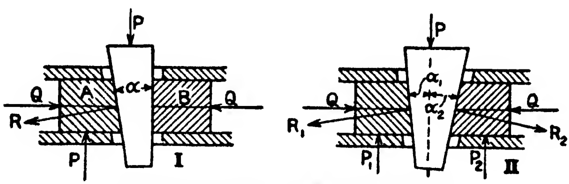
R_1, R_2 = reactions of wedge on pieces A and B.

P_1, P_2 = reactions of the guides on the pieces A and B.

α_1, α_2, or α = angles which acting faces of wedge make with the direction of the force P

ϕ = limiting angle of resistance.

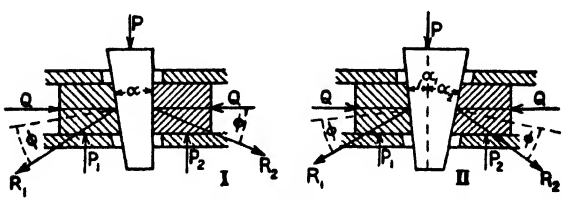
μ = coefficient of friction = $\tan \phi$.



Friction neglected. Driving wedge in.

I. $P = Q \tan \alpha$.

II. $P = P_1 + P_2 = Q(\tan \alpha_1 + \tan \alpha_2)$.

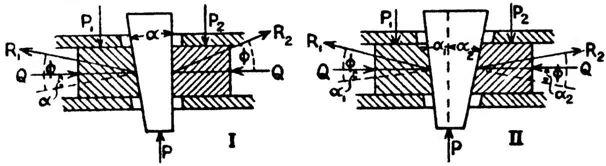


Friction of the wedge considered. Driving wedge in.

I. $P = P_1 + P_2 = Q\{\tan(\alpha + \phi) + \tan \phi\}$.

II. $P = P_1 + P_2 = Q\{\tan(\alpha_1 + \phi) + \tan(\alpha_2 + \phi)\}$.

The Wedge, Key, or Cotter (*continued*).



Friction of the wedge considered. *Driving wedge out.*

I. $P = P_1 + P_2 = Q\{\tan(\phi - \alpha) + \tan\phi\}$.
II. $P = P_1 + P_2 = Q\{\tan(\phi - \alpha_1) + \tan(\phi - \alpha_2)\}$.

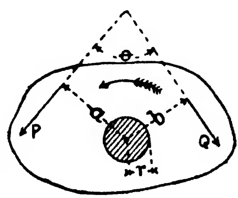
If the friction is just sufficient to prevent the wedge coming out under the action of the forces Q, Q, then $P=0$, and $\alpha = 2\phi$, and $\alpha_1 + \alpha_2 = 2\phi$.

In the above, the friction between the guides and the pieces A and B is neglected.

Equilibrium of a Body on an Axle.

P = force acting on the body at a perpendicular distance a from the centre of the axle.
Q – force acting on the body at a perpendicular distance b from the centre of the axle.

r = radius of the axle.
μ = coefficient of friction = $\tan\phi$.
ϕ = limiting angle of resistance.
θ = angle between the directions of P and Q.



If motion is about to take place in the direction of the arrow, *i.e.* if P is on the point of overcoming Q and the resistance of friction, then the equation of equilibrium is,

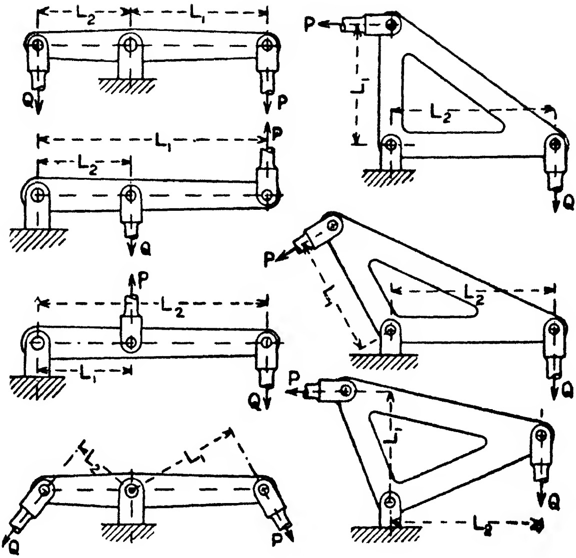
$$Pa = Qb + r\sin\phi\sqrt{P^2 + Q^2 + 2PQ\cos\theta}.$$

If $\theta = 0$, *i.e.* if the directions of P and Q are parallel, $\cos\theta = 1$, and $Pa = Qb + r\sin\phi(P+Q)$;

hence, $P = \dfrac{Q(b + r\sin\phi)}{a - r\sin\phi}$. $\sin\phi = \dfrac{\mu}{\sqrt{1+\mu^2}}$.

If ϕ is a small angle, $\sin\phi$ is nearly equal to μ.

Levers



$$P \times L_1 = Q \times L_2. \qquad P = \frac{Q \times L_2}{L_1}. \qquad Q = \frac{P \times L_1}{L_2}.$$

L_1 and L_2 are the perpendicular distances of the lines of action of P and Q respectively from the fulcrum.

Pressure on fulcrum $= F =$ resultant of P and Q.

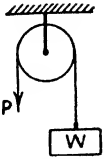
When P and Q are parallel and in the same direction, $F = P + Q$.

When P and Q are parallel and in opposite directions, $F = P - Q$, or $Q - P$.

When the lines of action of P and Q are inclined to one another, F is determined by the parallelogram of forces, and is equal to the resultant of P and Q. The line of action of F passes through the intersection of the lines of action of P and Q and through the fulcrum. When the lines of action of P and Q are at right angles to one another, $F = \sqrt{P^2 + Q^2}$.

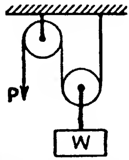
Simple Machines

Friction neglected



Fixed Pulley.

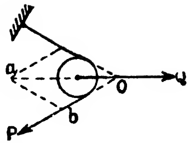
$P = W.$



Movable Pulley.

Parts of rope parallel

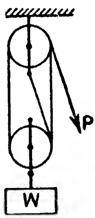
$P = \frac{1}{2} W.$



Movable Pulley.

Parts of rope inclined.

By parallelogram of forces, $Oa = Q$, $Ob = P$.

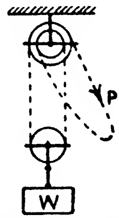


Ordinary Block and Tackle

n = number of sheaves in lower block.

$P = \frac{W}{2n}$ when one end of rope is attached to upper block.

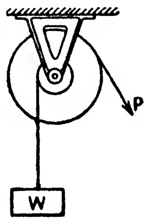
$P = \frac{W}{2n+1}$ when one end of rope is attached to lower block.



Differential Pulley-Block.

R and r = radii of two parts of upper pulley.

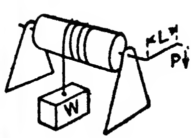
$\frac{P}{W} = \frac{R - r}{2R}.$



Wheel and Axle.

R = radius of wheel.
r = radius of axle or barrel.

$P \times R = W \times r.$



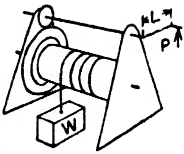
Windlass.

L = length of crank arm.
r = radius of barrel.

$P \times L = W \times r.$

Simple Machines (*continued*).

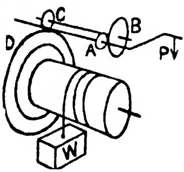
Single Purchase Crab.



L = length of crank arm
r = radius of barrel
n_1 = number of teeth in pinion.
n_2 = number of teeth in wheel.

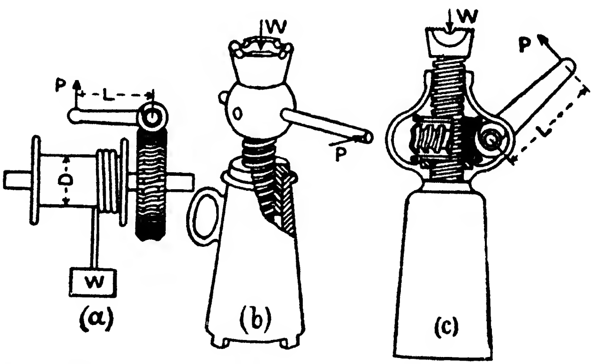
$$\frac{P}{W}=\frac{n_1}{n_2}\times\frac{r}{L}$$

Double Purchase Crab.



L = length of crank arm
r = radius of barrel.
n_1 = number of teeth in pinion A.
n_2 = number of teeth in wheel B
n_3 = number of teeth in pinion C
n_4 = number of teeth in wheel D.

$$\frac{P}{W}=\frac{n_1}{n_2}\times\frac{n_3}{n_4}\times\frac{r}{L}.$$



Endless Screw, or Worm and Worm Wheel.

n = number of teeth in worm wheel

A single threaded worm makes n revolutions for one revolution of the worm wheel.

A double threaded worm makes $\frac{n}{2}$ revolutions for one revolution of the worm wheel.

Endless Screw, or Worm and Worm Wheel (*continued*).

In the machine shown at (*a*), friction being neglected,

$$\frac{P}{W}=\frac{D}{2Ln} \text{ for a single threaded worm.}$$

$$\frac{P}{W}=\frac{D}{Ln} \text{ for a double threaded worm.}$$

Taking friction into account,

$$\frac{P}{W}=\frac{3D}{2Ln}, \text{ approximately, for a single threaded worm.}$$

$$\frac{P}{W}=\frac{2D}{Ln}, \text{ approximately, for a double threaded worm.}$$

Simple Screw Jack (*b*).

p = pitch of screw.

L = length of lever, measured from axis of screw to point where power P is applied.

Neglecting friction, $\frac{P}{W}=\frac{p}{2\pi L}$.

Taking friction into account, $\frac{P}{W}=\frac{7p}{2\pi L}$ to $\frac{3.5p}{2\pi L}$.

Screw Jack, with Worm and Worm Wheel (*c*).

p = pitch of upright screw. n = number of teeth in worm wheel.

Neglecting friction,

$$\frac{P}{W}=\frac{p}{2\pi Ln} \text{ for a single threaded worm.}$$

$$\frac{P}{W}=\frac{p}{\pi Ln} \text{ for a double threaded worm.}$$

Taking friction into account,

$$\frac{P}{W}=\frac{15p}{2\pi Ln}, \text{ approximately, for a single threaded worm.}$$

$$\frac{P}{W}=\frac{10p}{\pi Ln}, \text{ approximately, for a double threaded worm.}$$

Fluid Pressure.

The direction of the pressure of a fluid on any surface with which it is in contact is, at any point, at right angles to the surface at that point.

If a fluid completely fills a closed vessel, and if a pressure be applied to the fluid at any part of its surface, that pressure will be transmitted equally in all directions through the fluid to every part of the surface of the vessel.

The Pressure of a Liquid due to its Weight.—The intensity of the pressure at any point of a liquid due to its weight is directly proportional to the depth of the point below the upper surface of the liquid.

The total pressure on any surface immersed in a liquid is equal to the weight of a right prism of the liquid whose base is equal in area to the given surface, and whose height is equal to the depth of the centre of gravity of that surface below the upper surface of the liquid.

Let A = area of surface in square feet.

H = depth of centre of gravity of surface below upper surface of liquid, in feet.

w = weight of a cubic foot of liquid in lbs.

Total pressure on surface in lbs. $= AHw$.

The **resultant of the pressure** of a liquid on the wetted surface of a rigid body is the single force which, acting at a point on that surface, would produce the same effect on the body as the liquid pressure on it. In the case of a plane surface, the magnitude of the resultant pressure is equal to the total pressure on it

The **centre of pressure** of a surface subjected to fluid pressure is the point on it at which the resultant pressure acts.

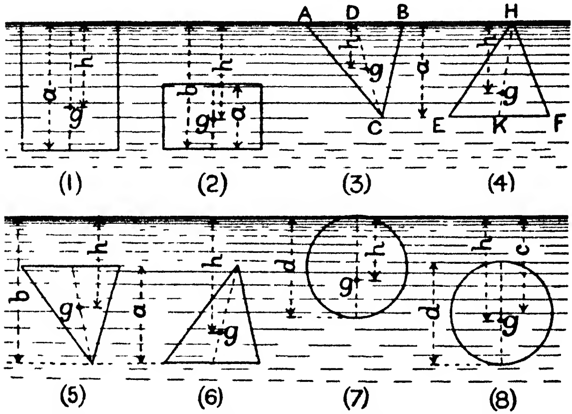
The following are the cases of most frequent occurrence in practice. g is the centre of pressure in each case:—

(1.) A rectangle or parallelogram with its highest side on the surface of the liquid. g is in the line which bisects the horizontal sides of the rectangle or parallelogram. $h = \frac{2}{3}a$.

(2.) A rectangle or parallelogram with its highest side below and parallel to the surface of the liquid. g is in the line which bisects the horizontal sides of the rectangle or parallelogram.

$$h = \frac{2}{3}\left(\frac{3b^2 - 3ab + a^2}{2b - a}\right) = \frac{2}{3}\left(\frac{b^2 + bc + c^2}{b + c}\right), \text{ where } c = b - a.$$

(3.) A triangle with its base on the surface of the liquid. g is in the line joining the vertex C with the middle point D of the base AB. $Dg = \frac{1}{2}DC$. $h = \frac{1}{2}a$, (4.) A triangle with its base parallel to and its vertex on the surface of the liquid. g is in the line joining the vertex with the middle point K of the base EF. $Hg=\frac{3}{4}HK$. $h=\frac{3}{4}a$.



(5.) A triangle with its base below and parallel to the surface of the liquid, the vertex being below the base. g is in the line joining the vertex with the middle point of the base.

$$h=\frac{3a^2-8ab+6b^2}{6b-4a}=\frac{b^2+2bc+3c^2}{2b+4c}, \text{ where } c=b-a.$$

(6.) A triangle with its base parallel to the surface of the liquid, the vertex being above the base and below the surface of the liquid. g is in the line joining the vertex with the middle point of the base. $h=\frac{a^2-4ab+6b^2}{6b-2a}$.

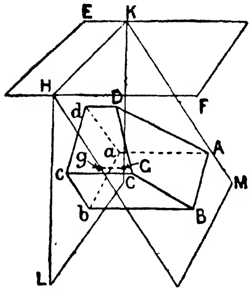
(7.) A circle with its highest point on the surface of the liquid. $h=\frac{5}{8}d$.

(8.) A circle entirely immersed, its centre being at a distance c from the surface of the liquid. $h=c+\frac{d^2}{16c}$.

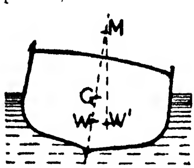
(9.) A semicircle of diameter d, with the diameter on the surface of the liquid. $h=\frac{3\pi d}{32}=\cdot 2945d$.

Note.—When the area of a surface is small compared with its depth below the surface of the liquid, its centre of pressure nearly coincides with its centre of gravity.

A general method of determining the centre of pressure of a plane surface *abcd* immersed in a liquid is as follows:—Produce the plane *KL* of the surface to meet the surface of the liquid in the line *HK*. Take a plane *HM* containing the line *HK* and inclined to the plane *KL*. On *abcd* as base construct a prism whose parallel edges *aA*, *bB*, *cC*, and *dD* meet the plane *HM* at *A*, *B*, *C*, and *D* respectively. Determine *G* the centre of gravity of the solid *abcdDCBA*. A line through *G* parallel to the parallel edges of the prism and meeting the plane of the figure *abcd* at *g* determines the centre of pressure of *abcd*.



Floating Bodies.—When a body floats in a liquid the weight of the body is equal to the weight of the liquid which it displaces, and the straight line which joins the centre of gravity of the body with the centre of gravity of the displaced liquid is vertical.



The annexed figure shows a floating body slightly displaced from its position of equilibrium. *WG* is the line which joins the centres of gravity of the displaced liquid and the body when the latter is in its position of equilibrium. *W'* is the new position of the centre of gravity of the displaced liquid. A vertical line through *W'* meets the line *WG* at *M*. The point *M* is called the **metacentre of the floating body**. The equilibrium of the floating body is more stable the higher the point *M* is above *G* the centre of gravity of the body, and the equilibrium is unstable when *M* is below *G*.

A floating body which is in stable equilibrium will tend to right itself after being slightly displaced from its position of equilibrium, while a floating body which is in unstable equilibrium will tend to become still further displaced after it has been turned slightly from its position of equilibrium.

KINEMATICS—The Science of Motion.

Motion is change of position. **Velocity** is rate of motion or rate of change of position. Velocity is either *uniform* or *variable*, and is generally expressed either in *feet per second*, *feet per minute*, or *miles per hour*. When velocity is variable the rate at which it changes is called *acceleration* if the velocity is increasing, and *retardation* if it is diminishing. Retardation may be regarded as *negative* acceleration. Acceleration may be uniform or variable.

Notation.—The following symbols are used in this section in dealing with the motion of a particle or small body :—

v_1 = initial velocity.

v = velocity after an interval of time, t.

f = uniform acceleration.

s = space described or distance moved in time, t.

(If s is measured in *feet* and t in *seconds*, then v_1 and v are measured in *feet per second* and f in *feet per second per second.*)

For Uniform Velocity—

$$s = v_1 t = vt.$$

For Uniform Acceleration—

$$v = v_1 + ft. \qquad \text{If } v_1 = o, \text{ then } v = ft.$$

$$s = v_1 t + \frac{1}{2}ft^2. \qquad \text{If } v_1 = o, \text{ then } s = \frac{1}{2}ft^2.$$

$$v^2 = v_1^2 + 2fs. \qquad \text{If } v_1 = o, \text{ then } v^2 = 2fs.$$

For Falling Bodies, under the free action of gravity the acceleration is denoted by g, which varies from a minimum of 32·088 feet per second per second at the equator to a maximum of 32·258 feet per second per second at the poles. At the latitude of London g = 32·191 feet per second per second. The above values of g are only strictly true at the level of the sea. In all engineering calculations it is sufficiently accurate to take g = 32·2. The above formulæ applied to falling bodies become—

$$v = v_1 + gt. \qquad \text{If } v_1 = o, \text{ then } v = gt.$$

$$s = v_1 t + \frac{1}{2}gt^2. \qquad \text{If } v_1 = o, \text{ then } s = \frac{1}{2}gt^2.$$

$$v^2 = v_1^2 + 2gs. \qquad \text{If } v_1 = o, \text{ then } v^2 = 2gs.$$

It must be remembered that these formulæ for falling bodies are only strictly true when the body falls freely in a *vacuum*. The air has a retarding effect, which is greater the greater the surface presented by the body to the air compared with its weight. The retarding effect of the air also depends greatly on the *shape* of the body.

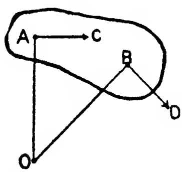
Velocities may be *compounded*, or *resolved into components* in exactly the same way as forces (see p. 218).

Angular Velocity.—If a point P moves in a plane, and OX is a fixed line in that plane, then the rate at which the angle POX changes is called the *angular velocity* of P about O.

Motion in a Circle.—If a point P moves along the circumference of a circle of radius r with a linear velocity v, and if the angular velocity of the point about O the centre of the circle be denoted by ω (omega), then $\omega=\frac{v}{r}$, and $v=\omega r$. (ω is the circular measure of the angle described by OP in a unit of time.)

If P makes n revolutions in a unit of time, and the angular velocity is uniform, then $v=2\pi rn$, and $n=\frac{v}{2\pi r}=\frac{\omega}{2\pi}$.

Instantaneous Axis.—Let A and B be two points in a rigid body, and at a given instant let A be moving in the direction AC, and let B be moving in the direction BD at the same instant. Draw AO perpendicular to AC and BO perpendicular to BD, and let these perpendiculars meet at O. An axis through O at right angles to the plane AOB is called the *instantaneous axis* of the body for its given position. At any instant a body may be considered to be rotating about its instantaneous axis at that instant.



The point O is called the *instantaneous centre*.

It follows that if V_1 is the velocity of A in the direction AC, and V_2 is the velocity of B in the direction BD, then $\frac{V_1}{V_2}=\frac{AO}{BO}$.

KINETICS.

The science which considers the relation of force to motion is called *Kinetics*.

The **Mass** of a body is the quantity of matter which it contains. The **unit of mass** used by engineers is the standard pound.

The **Momentum** of a body is the product of its mass and velocity. If W=the weight of a body in standard pounds, and v= its velocity, then its momentum= Wv.

If a force of P lbs. acts, in the direction of motion, on a mass weighing W lbs., the acceleration f produced, is given by the formula, $f = \frac{P}{W}g$. Having determined f, the space moved through and the velocity acquired in a given time are determined by the formulæ on p. 231.

Newton's Laws of Motion.—LAW I.—Every body continues in its state of rest, or of uniform motion in a straight line, except in so far as it may be compelled by impressed forces to change that state.

LAW II.—Change of motion, or change of momentum, is proportional to the impressed force, and takes place in the direction of the straight line in which the force acts.

LAW III.—To every action there is always an equal and opposite reaction; or, the mutual actions of any two bodies are always equal and oppositely directed.

Work.—When a force acting on a body causes that body to move, the force is said to do *work*. The **Unit of Work** generally used by engineers is the *foot-pound*, and is the work done by a force of one pound acting through a distance of one foot. The other units sometimes used are the *inch-pound* and the *foot-ton*. An *inch-pound* is the work done by a force of one pound acting through a distance of one inch. A *foot-ton* is the work done by a force of one ton acting through a distance of one foot.

The work done by a force is found by multiplying the magnitude of the force by the distance through which it acts.

The **Work done by a Variable Force** is equal to the *average* magnitude of the force multiplied by the distance through which it acts.

The **Work done in Raising a System of Weights** is equal to the sum of the weights multiplied by the distance through which their centre of gravity is raised.

Rate of doing Work—Horse-Power.—The working power of an agent is measured by the amount of work which it can do in a unit of time. A working agent is said to be of one *horse-power* when it can do 33,000 foot-pounds of work in one minute, or 550 foot-pounds in one second.

The horse-power of any working agent is obtained by dividing the number of foot-pounds of work which it does in one minute by 33,000.

The **Approximate Working Power of Men and Beasts** is given in the following table:—

Kind of Work.	Duration of labour in hours per day.	Force exerted in lbs	Work done per minute in foot-pounds.
Man raising his own weight, as in walking up a stair or ladder	8	140	4,300
Man pushing or pulling horizontally, as at an oar or capstan	8	25–60	3,200
Man turning a crank or winch handle	8	18	2,700
Man raising weights with rope and single pulley	6	40	1,800
Man raising weights by hand	6	44	1,500
Man carrying load on his back up stairs and returning unloaded	6	140	1,100
Man shovelling earth to a height of 5 feet	10	6	470
Average draught horse drawing a load	8	120	26,000
Ox drawing a load	8	120	17,000
Mule drawing a load	8	60	12,000
Ass drawing a load	8	30	6,000

The force exerted will depend on the speed. If the speed is increased, the force must be diminished; and if the speed is diminished, the force exerted may be increased within certain limits.

The **work done in moving a body up an inclined plane** is equal to the work done in drawing the body along the base of the plane against the resistance of friction *plus* the work done in raising the body against the resistance of gravity through a distance equal to the height of the plane.

The **work done in moving a body down an inclined plane** is equal to the work done in drawing the body along the base of the plane against the resistance of friction *minus* the work done in raising the body against the resistance of gravity through a distance equal to the height of the plane.

Energy is capacity for performing work.

Potential Energy is the energy which a body possesses by virtue of its position in relation to other bodies, or to the positions of its molecules.

Kinetic Energy is the energy which a body possesses by virtue of its motion. If W is the weight of a body in lbs., and v its velocity in feet per second, then its kinetic energy is $\frac{Wv^2}{2g}$ foot-pounds.

Energy of a Rotating Body.—Let W=weight of body in lbs., k=radius of gyration of body, about axis of rotation, in feet, n=number of revolutions per second, v=linear velocity of a point, at a distance r feet from the axis, in feet per second,

$$\text{Energy in foot-pounds} = \frac{2\pi^2 n^2 W k^2}{g} = \frac{W k^2 v^2}{2 g r^2}.$$

($\pi^2=9{\cdot}87$ nearly, and $g=32{\cdot}2$.)

Centrifugal Force.—When a particle of weight W lbs. moves in a circle of radius R feet, with a uniform linear velocity of v feet per second, it must be pulled or pushed towards the centre of the circle by a force F whose magnitude in lbs. $=\frac{Wv^2}{gR}$. This force F is called the *centripetal* or *centrifugal* force, according as the force impelling the particle towards the centre, or its reaction, is considered.

For a body of considerable size R is the distance of its centre of gravity from the axis of rotation, and v is the velocity of the centre of gravity.

Centrifugal Tension in a Thin Revolving Hoop.—The hoop is supposed to revolve about its geometrical axis. The centrifugal forces of the particles of the hoop act outwards in radial directions, and, like the pressure of a fluid inside a pipe, cause a tension at each cross section of the hoop.

R=radius of hoop in feet.
V=linear velocity of hoop in feet per second.
n=number of revolutions of hoop per second.
w=weight of one cubic inch of material of hoop, in lbs.
g=accelerating effect of gravity=32·2.
s=centrifugal tension in hoop, in lbs. per square inch of cross section.

$$s = \frac{12 w V^2}{g} = \frac{48 w \pi^2 R^2 n^2}{g}.$$

The centrifugal tension in the rim of a revolving wheel or pulley may be determined, with sufficient accuracy for practical purposes, by using the above formula, R being the mean radius of the rim.

Centrifugal Tension in a Thin Revolving Rod.—The rod is supposed to revolve about an axis at right angles to the rod, and passing through one end.

l=length of rod in feet.
s=centrifugal tension in rod at a distance x feet from the axis.
n, w, and g are the same as in the preceding article.

$$s=\frac{24w\pi^2n^2(l^2-x^2)}{g}.$$

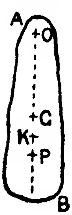
A Simple Pendulum consists of a heavy particle attached to one end of a fine thread, the other end of the thread being attached to a fixed point. When the particle is displaced from its lowest position and left to swing under the action of gravity, the time of an oscillation is found to be independent of the length of the arc described, provided the arc is small compared with the length of the thread. The length of the thread is the length of the pendulum An oscillation is a movement of the particle, or bob of the pendulum, from one end of the arc in which it swings to the other end and back.

t=time of one oscillation in seconds.
l=length of pendulum in inches.

$$t=2\pi\sqrt{\frac{l}{g}}, \qquad l=\frac{gt^2}{4\pi^2}, \qquad g=\frac{4\pi^2l}{t^2}.$$

If $t=2$ seconds, $l=39{\cdot}1393$ inches, and $g=32{\cdot}191\times12$ inches per second per second, at the latitude of London.

A Compound Pendulum is a body of any size or shape which is made to oscillate about a fixed axis.



AB represents a compound pendulum which oscillates about a fixed axis or centre of suspension O. G is the centre of gravity of AB. $OG=R$. P is a point in OG produced such that $OP=l$ the length of a simple pendulum which has the same time of oscillation as the compound pendulum AB. K is a point in OP such that $OK=k$ the radius of gyration of AB about O.

t=time of one oscillation (in seconds) of the compound pendulum about O.

$$t=2\pi\sqrt{\frac{l}{g}}=2\pi\sqrt{\frac{k^2}{gR}} \text{ and } l=\frac{k^2}{R}.$$

The point P is called the **centre of oscillation.** If P be made the centre of suspension, O will become the centre of oscillation, hence *the centres of oscillation and suspension are interchangeable.*

The point P is also the **centre of percussion**, and is the point of the body at which if a blow be received no jar will be felt at the axis of suspension.

The **Ballistic Pendulum** is used to determine the momentum of a projectile, or the impulse of a blow. It consists of a heavy block suspended by rods from an axis. The block may be of wood, or it may be a box filled with moist clay. The projectile is fired into the block, and the pendulum swings forward through an angle which is measured by a light pointer, which moves forward with the pendulum but does not return with it.

w = weight of projectile in lbs.

v = velocity of projectile in feet per second.

r = perpendicular distance of the axis of the pendulum from the line of flight of the projectile in feet.

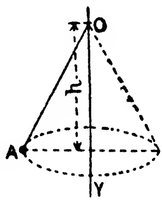
W = joint weight of pendulum and projectile in lbs.

l = length in feet of simple pendulum equivalent to compound pendulum of weight W.

R = distance of axis of pendulum from common centre of gravity of pendulum and projectile in feet.

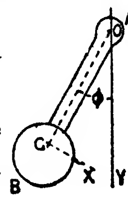
ϕ = angle through which pendulum is moved.

$$v = \frac{2WR \sin\frac{\phi}{2}\sqrt{gl}}{wr}$$



Revolving Simple Pendulum.—A is a small body revolving about a vertical axis OY, to which it is connected by a fine thread or slender rod AO. AO describes the surface of a right circular cone of height h feet. n = number of revolutions of A in one second. t = time of one revolution in seconds.

$$t = 2\pi\sqrt{\frac{h}{g}}. \quad h = \frac{gt^2}{4\pi^2}. \quad n = \frac{1}{2\pi}\sqrt{\frac{g}{h}}. \quad h = \frac{g}{4\pi^2 n^2}.$$



Revolving Compound Pendulum.—AB is the compound pendulum, O the point of suspension, OY the axis of rotation, and G the centre of gravity of AB.

k_1 = radius of gyration of AB about the axis GO.

k_2 = radius of gyration of AB about an axis GX at right angles to GO.

ϕ = angle GOY. $R = GO$.

n = number of revolutions of pendulum in one second.

h = height of equivalent revolving simple pendulum.

$$h = \frac{g}{4\pi^2 n^2} = \frac{R^2 + k_2^2 - k_1^2}{R}\cos\phi.$$

Motion of a Gun and Projectile.—W = weight of gun (including the carriage), w = weight of projectile, V = initial velocity of recoil of the gun, v = initial velocity of projectile. $WV = wv$, or **the momentum of the gun and carriage = momentum of the projectile.**

Collision or Impact of Bodies.—The following formulæ apply to cases of direct impact, *i.e.* to cases where the common normal to the surfaces of the bodies at their point of contact is the line of motion of the bodies :—

W = weight of body A. w = weight of body B.
V = velocity of A before impact.
v = velocity of B before impact.
V' = velocity of A after impact.
v' = velocity of B after impact.
e = coefficient of restitution.

The following are some values of e :—Glass and glass, or ivory and ivory, $e = \cdot 94$; cork and cork, $e = \cdot 65$; iron and iron, $e = \cdot 66$; brass and brass, $e = \cdot 36$; lead and lead, $e = \cdot 2$; lead and iron, $e = \cdot 13$.

For perfectly elastic bodies, $e = 1$; for inelastic bodies, $e = o$.

CASE I. The bodies A and B move in the same direction, and A overtakes B.

$$V' = V - (1+e)\frac{w(V-v)}{W+w}. \qquad v' = v + (1+e)\frac{W(V-v)}{W+w}$$

CASE II. The bodies A and B move in opposite directions.

$$V' = V - (1+e)\frac{w(V+v)}{W+w}. \qquad v' = (1+e)\frac{W(V+v)}{W+w} - v$$

CASE III. One body B is at rest before impact ($v = o$).

$$V' = V - (1+e)\frac{wV}{W+w}. \qquad v' = (1+e)\frac{WV}{W+w}.$$

CASE IV. One body B is rigidly connected to the earth, and therefore becomes a part of the latter, hence w is infinite compared with W. ($v = o$, and $v' = o$.) $V' = -eV$.

Note.—If a velocity is positive (+) when a body is moving in one direction, it will be negative (−) when the body moves in the opposite direction.

The total kinetic energy before impact $= \frac{WV^2}{2g} + \frac{wv^2}{2g}$, and the total kinetic energy after impact $= \frac{WV'^2}{2g} + \frac{wv'^2}{2g}$.

The difference between these two totals is the amount of energy converted into vibration and heat.

The following two theorems form the foundation of the theory of impact :—

(1) $\frac{\text{The relative velocity after impact}}{\text{The relative velocity before impact}} = \frac{V' - v'}{V - v} = -e.$

(2) **Sum of momenta after impact = sum of momenta before impact, or $WV' + wv' = WV + wv$.**

FRICTION.

Work done in Overcoming Friction.

P = resultant load on bearing surface in lbs.
μ = coefficient of friction
N = number of revolutions per minute.
U = work done per minute on friction in foot-pounds.

$$\text{Horse-power} = \frac{U}{33000}.$$

Body sliding on a flat surface $U = \mu P V$,
where V = velocity of sliding in feet per minute.

Cylindrical journal. $U = \frac{\pi}{12}\mu P d N$,

where d = diameter of journal in inches.

Footstep or pivot with flat end. $U = \frac{\pi}{18}\mu P d N$,

where d = diameter of pivot in inches.

Footstep or pivot with conical end.

$$U = \frac{\pi}{18}\mu P d N \operatorname{cosec} a = \frac{\pi}{18}\mu P N \sqrt{d^2 + 4h^2},$$

where d = diameter of pivot in inches.
a = semi-vertical angle of cone.
h = height of cone in inches.

Collar bearing, or square threaded screw.

$$U = \frac{\pi}{18}\mu P N \frac{d_1^2 + d_1 d_2 + d_2^2}{d_1 + d_2} = \frac{\pi}{24}\mu P N(d_1 + d_2), \text{ nearly},$$

where d_1 = greater diameter, and d_2 = smaller diameter in inches.

Coefficients of Sliding Friction of Solids.

For small and moderate pressures and low speeds.

Wood on wood, dry	·25 to ·5
" " soaped	·1 " ·2
" " greased	·02 " ·1
Metal on wood, dry	2 " ·6
Metal on metal, dry	·15 " ·3
" " oiled intermittently	·07 " ·08
" " " continuously	·04 " ·06
Leather on wood, dry	·3 " ·5
Leather on metal, dry	·56
" " wet	·36
" " greased	·23
" " oiled	·15
Hemp ropes on metal, dry	·2 to ·34
" " " greased	·15

The foregoing values of the coefficient of friction must be taken as approximate only. The results of experiments on friction are very discordant. It has been found that the coefficient of friction depends on the material of the sliding bodies, the state of their surfaces as regards smoothness, the intensity of the pressure between the surfaces, the velocity of sliding, the nature and quantity of the lubricant, and the manner in which it is applied, and also on the temperature.

The friction at starting from rest or *statical friction* is greater than the friction of motion, and depends on the hardness of the bodies and the length of time during which they have been in contact.

The so-called **laws of friction** are—

(1.) The force of friction is directly proportional to the pressure between the surfaces in contact.

(2.) The force of friction is independent of the extent of the surfaces in contact.

(3.) The force of friction is independent of the velocity of sliding.

The above "laws" are approximately true when the intensity of the pressure between the surfaces is small, and when the speed of sliding is low.

Friction of Journal Bearings.—The results of Mr. Beauchamp Tower's experiments * showed, according to Unwin,† that the coefficient of friction is approximately proportional to the square root of the velocity, and inversely proportional to the intensity of the pressure when the journal runs in an oil bath. Thus, $\mu = c\frac{\sqrt{v}}{p}$, where μ is the coefficient of friction, v the velocity of the surface of the journal in feet per second, and p the intensity of the pressure in lbs. per square inch of projected area of the bearing.

The following values of c may be used for the lubricants mentioned:—Olive oil, ·289; lard oil, ·281; mineral grease, ·431; sperm oil, ·194; rape oil, ·212; mineral oil, ·276.

For syphon lubrication, $\mu = \frac{c'}{p}$, where $c' = 2·02$ for rape oil.

For pad lubrication, μ is approximately constant, and = ·01 for rape oil.

The following results were obtained by Mr. Tower with a steel journal, 4 inches in diameter and 6 inches long, at a speed of 150 revolutions per minute, or 157 feet per minute. The "brass" was of gun-metal, and embraced nearly one-half of the circum-

* *Proceedings of the Institution of Mechanical Engineers*, 1883 and 1885.
† Unwin's "Machine Design," Part I. p. 184.

ference of the journal, and was placed on the top. The lubricant used was rape oil :—

Method of Lubrication.	Actual Load in Lbs per Square Inch.	Coefficient of Friction.
Oil bath	263	·00139
Syphon lubricator . . .	252	·00980
Pad under journal . . .	272	·00900

With the same journal Mr. Tower obtained the following results at a speed of 20 revolutions, or 21 feet per minute in a bath of mineral oil :—

Nominal load in lbs. per sq. inch	443	333	211	89
Coefficient of friction . . .	·00132	·00168	·00247	·0044

Chord of arc of contact of brass, 3·9 inches. Temperature, 90° Fahr.

The nominal load per square inch is the total load divided by the product of the diameter and length of the journal. Actual load per square inch = nominal load per square inch × 4 ÷ 3·9.

Mr. Tower's experiments on friction, at different temperatures, indicate a very great diminution in the friction as the temperature rises. Thus, in the case of lard oil, taking a speed of 450 revolutions per minute, the coefficient of friction at a temperature of 120° was only one-third of what it was at a temperature of 60° Fahr.

The following figures show the comparative friction with various lubricants tried by Mr. Tower, under as nearly as possible the same conditions. Temperature, 90° Fahr. Lubrication by oil bath :—

Sperm oil . .	·484	Lard oil . .	·652
Rape oil . . .	·512	Olive oil . .	·654
Mineral oil . .	·623	Mineral grease .	1·048

These figures are the means of the actual frictional resistances at the surface of the journal (4 inches diameter) in lbs. per square inch of bearing, at a speed of 300 revolutions per minute (314 feet per minute), with all nominal loads from 100 to 310 lbs per square inch. They also represent the relative thickness or body of the various oils, and also in their order, though perhaps not exactly in their numerical proportions, their relative weight-carrying power. Thus sperm oil, which has the highest lubricating power, has the least weight-carrying power; and though the best oil for light loads, would be inferior to the thicker oils if heavy pressures or high temperatures were to be encountered.

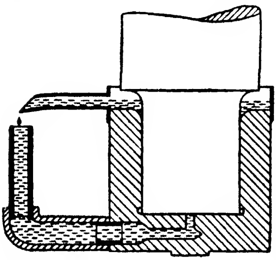
Previous to the publication of the results of Mr. Tower's experiments it was usual to take much higher values for the coefficient of journal friction. Morin gave the coefficient of friction for a lubricated journal bearing at from ·045 to ·08.

For mill shafting Mr. S. Webber * gave $\mu = \cdot 066$ for ordinary oiling, and $\mu = \cdot 044$ for continuous oiling.

For railway axles Kirchweger gave $\mu = \cdot 009$ to ·014 when the journals were lubricated with oil.

Bokelberg and Welkner obtained values of μ ranging from ·003 for small pressures and low velocities to ·013 for great pressures and high velocities.

Friction of Pivot or Footstep Bearings.—The table below gives the results of the experiments on the friction of a pivot bearing carried out for the Institution of Mechanical Engineers by the Research Committee on Friction.† The pivot experimented with was of steel 3 inches in diameter, and flat ended. The bearing was of manganese bronze. The bearing is shown in the adjoining illustration. The oil was introduced, as shown, through a central hole, and distributed over the bearing by a single diametrical groove, terminating at each end within $\frac{1}{16}$ inch of the circumference of the bearing. It was found that the oil circulated automatically, the pivot and bearing acting like a centrifugal pump.



Load in Lbs. per Sq. Inch.	Revolutions per Minute.				
	50	128	194	290	353
	Coefficients of Friction.				
20	·0196	·0080	·0102	·0178	·0167
40	·0147	·0054	·0061	·0107	·0096
60	·0167	·0053	·0051	·0078	·0073
80	·0181	·0063	·0045	·0064	·0063
100	·0219	·0077	·0044	·0056	·0057
120	·0221	·0083	·0052	·0048	·0053
140	...	·0093	·0062	·0046	·0053
160	...	·0113	·0068	·0044	·0054

* *Journal of the Franklin Institute*, 1874.
† *Proceedings of the Institution of Mechanical Engineers*, 1891.

The coefficients of friction in the foregoing table were calculated from the observed frictional moments, on the assumption that the mean leverage of the friction was two-thirds of the radius of the pivot, which would be correct if the pressure on the bearing was uniformly distributed.

Friction of Collar Bearings.—In the third report of the Research Committee of the Institution of Mechanical Engineers on Friction, the experiments of Mr. Beauchamp Tower on the friction of a collar bearing are described * The results showed that the coefficient of friction in this type of bearing is practically independent of the speed. The following table gives the mean values of the coefficient of friction (μ) obtained with different intensities of pressure (p) on the bearing ring, in lbs. per square inch.

p	15	30	45	60	67·5	75	82·5
μ	·0542	·0463	·0369	·0361	·0355	·0349	·0336

It was found in the above experiments that the greatest load which the bearing would carry was 75 lbs. per square inch at the highest speed, and 90 lbs. per square inch at the lowest speed.

The friction of a collar bearing is much greater than that of a cylindrical bearing, because of the greater difficulty of properly lubricating the former.

Friction of Cotton Mill Engines and Gearing.—The following table contains a summary of results obtained by Mr. Alfred Saxon, and given by him in a paper read to the Manchester Association of Engineers in October 1892:—

		Spur Gearing	Rope Gearing.	Belt Gearing.
Number of examples		10	8	2
Friction of engine and gearing in per cent. of full load I.H.P. of engine . .	maximum	35·6	32·8	28·2
	minimum	18·17	23·3	27·0
	mean .	25·96	29·6	27·6

Friction of Steam Engines.—Experiments have shown that the amount of power consumed in overcoming the friction of the mechanism of a steam engine is very nearly the same for all loads. According to Professor Thurston, the friction of steam engines varies from about 4 lbs. per square inch on the piston in small engines (25 to 50 horse-power), down to 1 lb. per square inch in very large marine engines. This gives power consumed

* *Proceedings of the Institution of Mechanical Engineers*, 1888.

in friction, ranging from 16 per cent. to 3 per cent. The crank shaft bearings are responsible for the largest part of the work lost in friction, in some cases as much as one-half.

Mechanical Efficiency of Machines.—

P = force acting at the driving point = the effort.

W = force acting at the working point = the resistance.

r = velocity ratio of machine.

$= \frac{\text{displacement of driving point}}{\text{displacement of working point}}$.

M = mechanical advantage of machine $= \frac{W}{P}$.

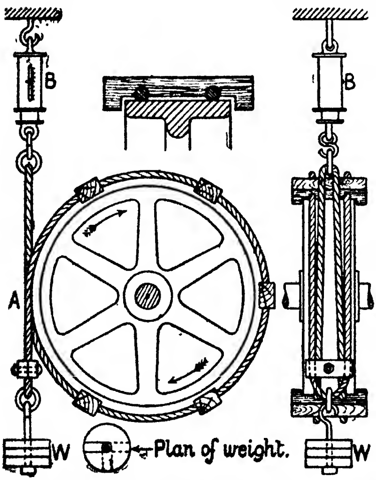
E = mechanical efficiency of machine $= \frac{\text{mechanical advantage}}{\text{velocity ratio}}$.

When friction is neglected $P = \frac{W}{r}$ and $E = \frac{M}{r} = 1$.

When friction is taken into account and the machine has a straight line law, $P = mW + k$, where m and k are constants determined from experiments with the machine, then

$$E = \frac{M}{r} = \frac{W}{r(mW + k)}.$$

Friction Brake Dynamometer.—The effective horse-power of an engine is best determined by absorbing all the power at the crank shaft by means of a friction brake. The simplest and most reliable form of friction brake dynamometer is the rope brake, shown here. One, two, or more lengths of rope are passed once round the rim of the fly-wheel or the rim of a pulley fixed on the crank shaft. The different lengths of rope are kept in position by blocks of wood, as shown, the blocks being laced to the rope. The upper ends of the several lengths of rope are united and attached



to a suspended spring balance B, while the other ends are united and attached to the weight W. (*Note.*—The length of hanging rope from A to the weight should be greater than is shown in the illustration.)

W = hanging weight (including portion of rope, etc., hanging from A) in lbs.

w = tension registered by spring balance, less the weight of rope, etc., between A and the balance, in lbs.

R = effective radius of wheel = radius of wheel + radius of rope, in feet.

N = number of revolutions of shaft per minute.

$B.H.P.$ = effective, actual, or brake horse-power.

$$B.H.P. = \frac{2\pi RN}{33000}(W - w) = \cdot0001904\ RN(W - w).$$

Where the amount of power to be absorbed is large, the rim of the brake wheel is cooled by water. In that case the rim is of a channel section, and the water is held in it by centrifugal force.

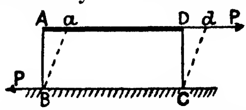
STRENGTH AND STIFFNESS OF MATERIALS.

Load, Strain, and Stress.

The **load** on a piece of construction is the combination of external forces acting on it. A **dead load** is one that remains constant. A **live load** is one that varies continually.

Strain is the change of form produced in a piece by the action of the load. *Tensile* and *compressive strains* are measured by the *ratio* of the increase or decrease in length to the length of the unloaded piece.

If a rectangular piece $ABCD$ is distorted by a load P so that it assumes the shape $aBCd$, the piece is subjected to *shearing*, and the *shearing strain* is measured by the *ratio* of Aa to AB, that is, by the tangent of the angle ABa.



When a bar is lengthened or shortened by a load, it at the same time becomes narrower or wider. The ratio of this strain in the direction of the width to the simultaneous strain in the direction of the length is called **Poisson's Ratio**. The following are values of Poisson's ratio :—

Copper	·38	Steel	·31
Brass	·33	Wrought-iron	·28
Glass	·33	Cast-iron	·27

Stress is the combination of internal forces which are called into play in the material of a structure to resist the tendency of the load to produce strain. Stress is usually measured either in lbs. per square inch or in tons per square inch.

Moduli or Coefficients of Elasticity.

Modulus of direct elasticity $= E = \dfrac{\text{tensile or compressive stress}}{\text{tensile or compressive strain}}$.

If S be the strain produced in a bar, whose cross section is A square inches, by a direct load of W lbs., then

$$S = \frac{W}{EA},\ E = \frac{W}{SA},\ W = SEA, \text{ and } A = \frac{W}{SE}.$$

E is usually determined from experiments on the deflection of beams (see p. 251).

Modulus of transverse elasticity $= C = \dfrac{\text{shearing stress}}{\text{shearing strain}}$.

C is best determined from experiments on the torsion of shafts (see p. 427)

Average Values of E in Lbs per Square Inch.

Material	E	Material	E.
Aluminium (commercially pure) cast	12,000,000	Leather	25,000
Aluminium (commercially pure) sheet	13,000,000	Phosphor-bronze	14,000,000
Aluminium (commercially pure) wire	19,000,000	Steel, mild	30,000,000
Aluminium-bronze (9 alumin. to 1 copper)	18,000,000	Steel, tempered	36,000,000
		Wood—	
Brass, cast	9,000,000	Ash	1,600,000
Brass, rolled	12,500,000	Beech	1,300,000
Brass, wire	14,200,000	Box	1,800,000
Copper, cast	12,000,000	Larch	1,200,000
Copper, rolled	15,000,000	Lignum-vitæ	1,000,000
Copper, wire	17,000,000	Mahogany	1,400,000
Delta-metal, cast	12,000,000	Oak	1,500,000
Delta-metal, rolled	13,000,000	Pine, white	1,000,000
Glass	8,000,000	Pine, yellow	1,600,000
Gun-metal	13,500,000	Pine, red	1,800,000
Iron, cast	17,000,000	Pine, pitch	1,900,000
Iron, wrought, bars	29,000,000	Teak	2,000,000
Iron, wrought, plates	26,000,000		

Average Values of C in Lbs. per Square Inch.

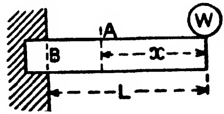
Material.	C.	Material.	C.
Brass wire	5,300,000	Steel, cast, untempered	12,000,000
Copper, rolled	5,600,000	Steel, cast, tempered	14,000,000
Iron, cast	6,300,000	Steel plates	13,000,000
Iron, wrought, bars	10,500,000	Steel, forged	13,000,000
Iron, wrought, plates	14,000,000	Wood, Ash and elm	76,000
Phosphor-bronze	5,300,000	Wood, Oak	82,000

Examples of Bending Moments and Shearing Loads on Beams.

M = bending moment. F = vertical shearing force.

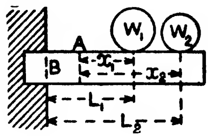
One Concentrated Load W.

M at $A = Wx$ F at $A = W$.
M greatest at B, and $= WL$.
F uniform throughout.



Two Concentrated Loads W_1, W_2

M at $A = W_1x_1 + W_2x_2$.
F at $A = W_1 + W_2$.
M greatest at B, and $= W_1L_1 + W_2L_2$.
F uniform throughout length L_1



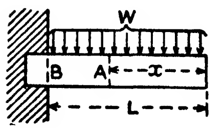
Uniform Load W, or w per unit of length of beam.

$$M \text{ at } A = \frac{wx^2}{2} = \frac{Wx^2}{2L}.$$

$$F \text{ at } A = wx = \frac{Wx}{L}.$$

$$M \text{ greatest at } B, \text{ and} = \frac{wL^2}{2} = \frac{WL}{2}.$$

F greatest at B, and $= wL = W$.



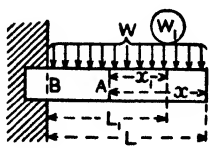
Uniform Load W, or w per unit of length and a Concentrated Load W_1.

$$M \text{ at } A = \frac{wx^2}{2} + W_1x_1.$$

F at $A = wx + W_1$.

$$M \text{ greatest at } B, \text{ and} = \frac{WL}{2} + W_1L_1.$$

F greatest at B, and $= W + W_1$.

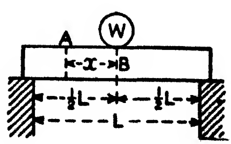


One Concentrated Load W at middle of beam.

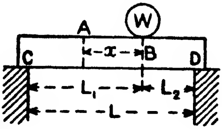
$$M \text{ at } A = \frac{W}{2}\left(\frac{L}{2} - x\right). \quad F \text{ at } A = \frac{W}{2}.$$

$$M \text{ greatest at } B, \text{ and} = \frac{WL}{4}.$$

F uniform throughout.



Examples of Bending Moments (*continued*)



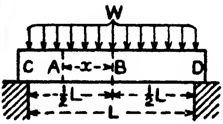
One Concentrated Load W placed anywhere

M at $A = \frac{WL_2}{L}(L_1 - x)$

M greatest at B, and $= \frac{WL_1L_2}{L}$

F at points between C and $B = \frac{WL_2}{L}$

F at points between D and $B = \frac{WL_1}{L}$.

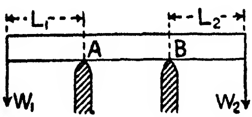


Uniform Load W, or w per unit of length.

M at $A = \frac{w}{2}\left(\frac{L^2}{4} - x^2\right)$ F at $A = wx$.

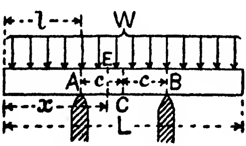
M greatest at B, and $= \frac{wL^2}{8} = \frac{WL}{8}$.

F greatest at C and D, and $= \frac{wL}{2} = \frac{W}{2}$.



Beam supported at two points, and loaded at the ends.

At all points between A and B, $M = W_1L_1 = W_2L_2$.

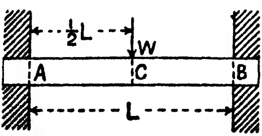


Beam supported at two points, and loaded uniformly.

At any point E between A and B,

$$M = \frac{Wx}{2}\left(\frac{x}{L} + \frac{l}{x} - 1\right).$$

At A, $M = \frac{Wl^2}{2L}$. At C, $M = \frac{WL}{2}\left(\frac{l}{L} - \frac{1}{4}\right)$



Beam fixed at the ends, and loaded at the centre.

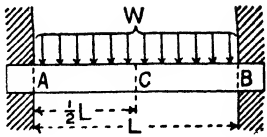
M is a maximum at A, B, and C, and $= \frac{WL}{8}$ at these points.

Examples of Bending Moments (*concluded*).

Beam fixed at the ends, and loaded uniformly.

M is a maximum at A and B, and $=\frac{WL}{12}$ at these points.

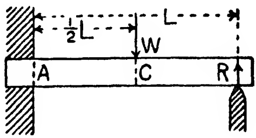
M at $C=\frac{WL}{24}$.



Beam fixed at one end, supported at the other, and loaded at the centre.

Reaction $R=\frac{5}{16}W$.

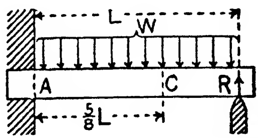
M is a maximum at A, and $=\frac{3WL}{16}$ at that point. M at $C=\frac{5WL}{32}$.



Beam fixed at one end, supported at the other, and loaded uniformly.

Reaction $R=\frac{3}{8}W$.

M is a maximum at A, and $=\frac{WL}{8}$ at that point. M at $C=\frac{9WL}{128}$.



Moment of Resistance of a Beam to Bending.—The neutral axis of a transverse section of a beam passes through the centre of gravity of the section, and is at right angles to the plane in which the beam bends.

The stress on one side of the neutral axis is tensile, and on the other side compressive. The stress varies uniformly from zero at the neutral axis to a maximum at that part of the section which is farthest from the neutral axis.

Let f denote the maximum stress, then the moment of resistance of the beam to bending at the section under consideration is denoted by fz, where z is a quantity, called the *modulus of the section*, depending on the form of the section.

Where the section is not symmetrical about its neutral axis there are two values of z, one, z_1, corresponding to the part on one side of the neutral axis, and the other, z_2, corresponding to the part on the other side. If f_1 is the greatest stress on the part of the section to which z_1 refers, and f_2 is the greatest stress on the side to which z_2 refers, then the moment of resistance is equal to f_1z_1 or f_2z_2. If f_1 is tensile stress, f_2 is compressive stress, and *vice versa.*

For values of z, z_1, and z_2 corresponding to various forms of sections, see pp. 212 215.

The above remarks only apply to cases where the beam is not strained beyond the elastic limit.

Safe Working Strength of a Beam.—If the bending moment at any section of a beam be denoted by M, the safe working strength is given by the following equations :—

$M=fz$ for symmetrical sections,
$M=f_1z_1$ or f_2z_2 for unsymmetrical sections,
where f, f_1, and f_2 are the safe working stresses.

Where the moment of resistance has two values, the smaller must be taken.

Breaking Strength of a Beam.—The relation between the bending moment and the moment of resistance just given, viz. $M=fz$, only applies when the beam is not strained beyond the elastic limit; and if f denotes the breaking stress determined by direct experiment on a piece in tension or compression, the above equation does not give the breaking strength of the beam when it is broken by bending.

Let M' denote the bending moment which actually breaks the beam, and let $M=fz$ where f is the breaking stress determined by direct experiment on a piece in tension or compression, then the ratio $\frac{M'}{M}$ depends on the form of the section, and also on the material of the beam.

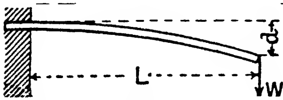
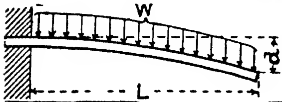
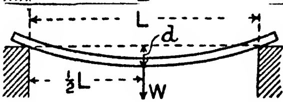
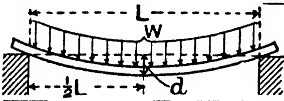
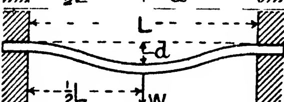
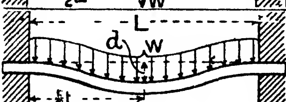
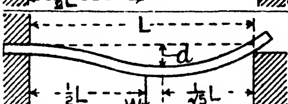
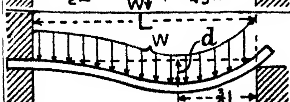
$\frac{M'}{M}$ is greatest for a circular section. For flanged beams M' may be taken equal to M.

The following values of $\frac{M'}{M}$ for rectangular sections are given by Mr. Lineham in his "Text-Book of Mechanical Engineering":—Fir, ·52 to ·94; oak, ·7 to 1; pitch pine, ·8 to 2·2; cast-iron, 1·57 to 2·3; wrought-iron, 1·21; forged steel, 1·47 to 1·6; gun-metal, 1 to 1·9.

Beams of Uniform Strength.—Economy of material is obtained when each section of a beam is so proportioned that its moment of resistance to bending is equal to the bending moment at the section, and the beam is then said to be of uniform strength so far as resistance to bending is concerned. Care must, however, be taken to see that each section is sufficient to resist the shearing load at the section.

Deflection of Beams of Uniform Section —

W = load on beam in lbs
L = effective length of beam in inches
I = moment of inertia of cross section of beam (see pp 212–215).
d = deflection of beam in inches
E = modulus of elasticity (see p 216)

Manner of Supporting and Loading		d
	Fixed at one end, and loaded at the other	$\frac{WL^3}{3EI}$
	Fixed at one end, and loaded uniformly	$\frac{WL^3}{8EI}$
	Supported at the ends, and loaded at the centre	$\frac{WL^3}{48EI}$
	Supported at the ends, and loaded uniformly	$\frac{5WL^3}{384EI}$
	Fixed at the ends, and loaded at the centre	$\frac{WL^3}{192EI}$
	Fixed at the ends, and loaded uniformly	$\frac{WL^3}{384EI}$
	Fixed at one end, supported at the other, and loaded at the centre	$\frac{WL^3}{107EI}$
	Fixed at one end, supported at the other, and loaded uniformly.	$\frac{WL^3}{187EI}$

Combined Straining Actions.

Tension or Thrust combined with Bending.—

A = area of section at ab.

f_1 = stress at side a of section ab.

f_2 = stress at side b of section ab.

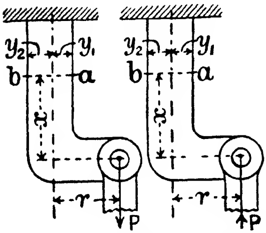
y_1 = distance of centre of gravity of section ab from side a.

y_2 = distance of centre of gravity of section ab from side b.

r = perpendicular distance of point of application of load P from neutral axis. (Neutral axis passes through centre of gravity of section ab).

I = moment of inertia of section ab about an axis perpendicular to the plane of the paper and passing through the centre of gravity of the section

Z_1 and Z_2 = moduli of section ab. $Z_1 = \frac{I}{y_1}$, and $Z_2 = \frac{I}{y_2}$.



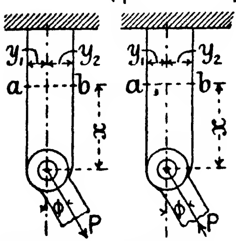
CASE I.

$$f_1 = \frac{P}{A} + \frac{Pr}{Z_1}$$

$$f_2 = \frac{P}{A} - \frac{Pr}{Z_2}$$

$$P = \frac{AZ_1f_1}{Z_1 + Ar} \text{ or } \frac{AZ_2f_2}{Z_2 - Ar}.$$

f_1, f_2, and P are independent of x.



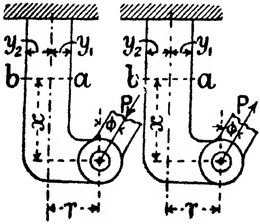
CASE II.

$$f_1 = \frac{P\cos\phi}{A} + \frac{Px\sin\phi}{Z_1}$$

$$f_2 = \frac{P\cos\phi}{A} - \frac{Px\sin\phi}{Z_2}$$

$$P = \frac{AZ_1f_1}{Z_1\cos\phi + Ax\sin\phi}$$

$$= \frac{AZ_2f_2}{Z_2\cos\phi - Ax\sin\phi}$$



CASE III.

$$f_1 = \frac{P\cos\phi}{A} + \frac{Pr\cos\phi + Px\sin\phi}{Z_1}.$$

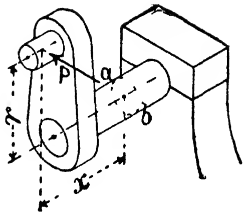
$$f_2 = \frac{P\cos\phi}{A} - \frac{Pr\cos\phi + Px\sin\phi}{Z_2}.$$

$$P = \frac{AZ_1f_1}{Z_1\cos\phi + Ar\cos\phi + Ax\sin\phi}.$$

$$= \frac{AZ_2f_2}{Z_2\cos\phi - Ar\cos\phi - Ax\sin\phi}.$$

Twisting combined with Bending.—

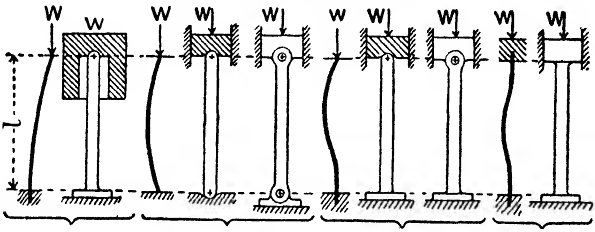
T = twisting moment at section ab.
B = bending moment at section ab.
$T = Pr$. $B = Px$.
T_e = equivalent twisting moment at ab.
B_e = equivalent bending moment at ab.



$$T_e = B + \sqrt{B^2 + T^2}.$$
$$= P(x + \sqrt{x^2 + r^2}).$$
$$B_e = \frac{1}{2}B + \frac{1}{2}\sqrt{B^2 + T^2}.$$
$$= \frac{1}{2}P(x + \sqrt{x^2 + r^2}).$$

The piece subjected to twisting *and* bending is designed to resist the equivalent twisting moment *or* the equivalent bending moment.

Strength of Long Columns.



I. II. III. IV.

Fig. I. Column fixed at one end and free at the other.

Fig. II. Column with both ends free, but guided in the direction of the load.

Fig. III. Column with one end fixed, the other free and guided in the direction of the load.

Fig. IV. Column with both ends fixed in direction.

In each Fig. the thick line diagram shows the manner in which the column bends when loaded.

l = length of column in inches.

A = area of cross section in square inches.

I = least moment of inertia of cross section about an axis passing through its centre of gravity. (This axis will be at right angles to the plane in which the column will most easily bend.)

k = least radius of gyration of cross section $= \sqrt{\frac{I}{A}}$.

E = modulus of elasticity of the material in lbs. per square inch (see p. 246).

n = factor of safety = 5 for wrought-iron or steel, 6 for cast-iron, and 10 for wood.

W = working load in lbs.

Euler's Formulæ.—

Column fixed at one end and free at the other (Fig. I.),

$$W=\frac{\pi^2}{4}\times\frac{EI}{nl^2}=\frac{\pi^2}{4}\times\frac{Ek^2A}{nl^2}. \qquad \left(\frac{\pi^2}{4}=2\cdot4674.\right)$$

Column with both ends free, but guided in the direction of the load (Fig. II.),

$$W=\pi^2\frac{EI}{nl^2}=\pi^2\frac{Ek^2A}{nl^2}. \qquad (\pi^2=9\cdot8696.)$$

Column with one end fixed, the other free and guided in the direction of the load (Fig. III.),

$$W=2\pi^2\frac{EI}{nl^2}=2\pi^2\frac{Ek^2A}{nl^2}. \qquad (2\pi^2=19\cdot7392.)$$

Column with both ends fixed in direction (Fig. IV.),

$$W=4\pi^2\frac{EI}{nl^2}=4\pi^2\frac{Ek^2A}{nl^2}. \qquad (4\pi^2=39\cdot4784.)$$

In each case W must not exceed Af, where f is the safe crushing stress in lbs. per square inch.

Gordon's Formulæ as modified by Rankine.—

Column with both ends free, but guided in the direction of the load (Fig. II.),

$$nW=\frac{Ac_1}{1+\frac{4l^2}{c_2k^2}}.$$

Column with one end fixed, the other free and guided in the direction of the load (Fig. III.),

$$nW=\frac{Ac_1}{1+\frac{16l^2}{9c_2k^2}}.$$

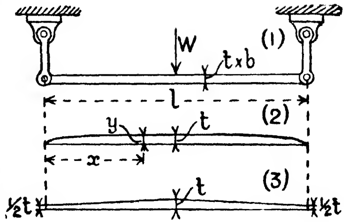
Column with both ends fixed in direction (Fig. IV.),

$$nW=\frac{Ac_1}{1+\frac{l^2}{c_2k^2}}.$$

For wrought-iron or mild steel .	$c_1=36{,}000$,	$c_2=36{,}000$
For cast-iron	$c_1=80{,}000$,	$c_2=6{,}400$
For dry timber, strong kinds .	$c_1=7{,}200$,	$c_2=3{,}000$

Springs.

Deflection of Flat Springs.—



t = thickness in inches.
b = breadth in inches.
l = length in inches.
W = load in lbs.
E = modulus of elasticity in lbs. per square inch.
δ = deflection in inches.

(1.) *Spring of uniform cross section.* $\delta=\frac{Wl^3}{4Ebt^3}$.

(2.) *Spring of uniform strength and uniform breadth.* $\delta=\frac{Wl^3}{2Ebt^3}$, where t is the thickness at the centre.

For uniform strength the thickness at a point which is at a distance x from the end nearest to that point is $y=t\sqrt{\frac{2x}{l}}$.

The bottom being straight, the outline of the top is two parabolas, with their vertices at the ends.

It will be observed that the deflection of (2) is double the deflection of (1) for the same stress in the material.

The form (2) may be approximated to by tapering the spring uniformly from the centre to the ends, as shown at (3), the thickness at the ends being half the thickness at the centre.

If f is the maximum tensile or compressive stress in the spring corresponding to a deflection δ, then

$$\delta=\frac{fl^2}{6Et}, \text{ and } f=\frac{6Et\delta}{l^2} \text{ for spring (1).}$$

$$\delta=\frac{fl^2}{3Et}, \text{ and } f=\frac{3Et\delta}{l^2} \text{ for spring (2).}$$

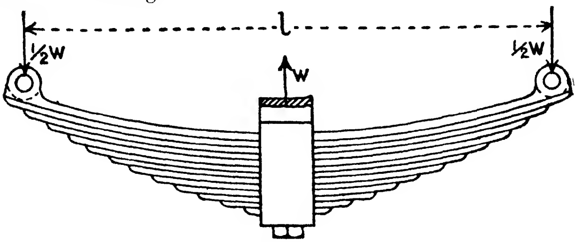
For steel, E varies from 30,000,000 to 40,000,000.

Locomotive Bearing Springs.—

l = span of spring in inches.
b = breadth of plates in inches.
t_1 and t_2 = thickness of plates in inches.
n_1 = number of plates of thickness t_1.
n_2 = ,, ,, ,, t_2.
E = modulus of direct elasticity of material in lbs. per sq. in.
W = working load on spring in tons.

$$W=\frac{cb(n_1t_1^2+n_2t_2^2)}{l},$$

where c is a multiplier which is generally between 18 and 26, and has an average value of 23.



If the spring has n plates all of the same thickness t, then

$$W = \frac{cbnt^2}{l}.$$

The following table gives particulars of a number of locomotive bearing-springs from actual practice :—

l	b	Number and Thickness of Plates	W	l	b	Number and Thickness of Plates	W
Ins	Ins	In.	Tons	Ins.	Ins	In.	Tons
32	4½	16 ⅜	6·4	42	5	{ 2 ½ ; 16 ⅜ }	6·9
36	4	{ 1 ½ ; 17 ⅜ }	5·7	42	5	11 ½	7·0
36	4½	{ 1 ½ ; 21 5/16 }	7·3	42	5	11 ½	7·3
36	4	14 ½	8·9	42	5	13 ½	7·5
36½	4	14 ½	7·5	42	5	{ 1 ½ ; 18 ⅜ }	7·3
39	5	{ 1 ½ ; 9 ⅜ }	4·5	45	4	16 ½	8·9
39	5	{ 1 ½ ; 14 ⅜ }	6·4	48	5	12 ½	7·5
42	4½	13 ½	6·4	48	5	12 ½	7·7
42	5	12 ½	6·6	48	5	12 ½	7·9
				48	6	12 ½	8·0
				54	5	14 ½	8·0

The deflection of locomotive bearing-springs may be found approximately from the formula,

$$\text{Deflection in inches per ton of load} = \frac{l^3}{kb(n_1t_1^3 + n_2t_2^3)},$$

where k varies from 40,000 to 48,000.

The theory of the deflection of beams gives the following formula,

$$\delta = \frac{840Wl^3}{Eb(n_1t_1^3 + n_2t_2^3)},$$

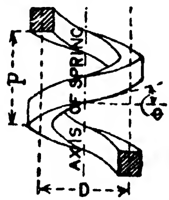
neglecting the friction between the plates. Comparing this formula with the preceding one, $k=\frac{E}{840}$. If $E=30,240,000$, $k=36,000$.

If f=maximum stress in lbs. per square inch (the maximum stress is the same in each plate), then

$$f=\frac{3360\,Wl}{b(n_1t_1^2+n_2t_2^2)}.$$

If W is the safe working load in tons, then substituting from the formula already given for W, $f=3360c$, which gives $f=77,280$ when 23, the average value of c is taken.

Helical Springs.—D=diameter of cylindrical surface passing through the centres of coils, in inches



n=number of effective coils in spring.
p=pitch of helix, in inches.
θ=angle of inclination of helix.

$$\cos\theta=\frac{\pi D}{\sqrt{\pi^2D^2+p^2}}.\qquad \sin\theta=\frac{p}{\sqrt{\pi^2D^2+p^2}}.$$

l=length of wire forming effective coils of spring, in inches.

$$l=n\sqrt{\pi^2D^2+p^2}.$$

I=moment of inertia of a transverse section of the wire about an axis in the plane of the section, and passing through its centre in a direction parallel to axis of spring.

I_0=moment of inertia of a transverse section of the wire about an axis at right angles to the section, and passing through its centre. (Polar moment of inertia.)

A=area of transverse section of wire, in square inches.

K=constant depending on form of section of wire.
$=4\pi^2$ for circular and elliptical sections $(=39\ 478)$.
$=42{\cdot}66$ for a square section.
$=42$ for a rectangular section when the longer side is not greater than $3\frac{1}{3}$ times the shorter side.

W=load on spring in lbs., acting along axis of spring, and not greater than greatest safe load.

W_1=greatest safe steady load, in lbs.

f_1=greatest shearing stress produced by load W_1, in lbs. per square inch.

C=modulus of transverse elasticity, in lbs. per square inch.

E=modulus of direct elasticity.

δ=deflection of spring (extension or compression), in inches.

The greatest safe suddenly applied load is half of the greatest safe steady load.

The principal strain on the spring is torsional, but there is also a bending strain and a small transverse shearing strain.

The third of these may always be neglected, and in most cases the second also.

The following formulæ give the deflection when the torsional and bending strains are considered :—

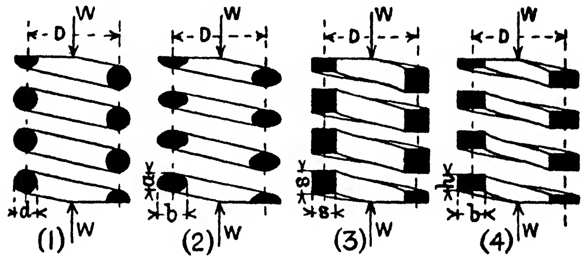
$$\delta = \frac{WlD^2}{4}\left(\frac{KI_o \cos^2\theta}{CA^4} + \frac{\sin^2\theta}{EI}\right).$$

$$= \frac{WnD^2}{4\sqrt{\pi^2D^2+p^2}}\left(\frac{KI_o\pi^2D^2}{CA^4} + \frac{p^2}{EI}\right).$$

If p is less than $\frac{1}{4}D$, that is, if θ is less than 5°, the bending action may be neglected, and the following simpler formulæ may be used :—

$$\delta = \frac{WlD^2KI_o}{4CA^4} = \frac{Wn\pi D^3KI_o}{4CA^4}$$

The latter formula is used in obtaining the results given below.



(1) *For coils of round section*

$$\delta = \frac{8WnD^3}{Cd^4}. \qquad W_1 = \frac{\pi d^3 f_1}{8D},$$

(2) *For coils of elliptical section*

$$\delta = \frac{4WnD^3(a^2+b^2)}{Ca^3b^3}. \qquad W_1 = \frac{\pi ba^2 f_1}{8D},$$

where b is the major axis of the ellipse.

(3) *For coils of square section*

$$\delta = \frac{5{\cdot}584\,WnD^3}{Cs^4}. \qquad W_1 = \frac{{\cdot}416 s^3 f_1}{D}.$$

(4) *For coils of rectangular section*

$$\delta = \frac{2{\cdot}75\,WnD^3(b^2+h^2)}{Cb^3h^3}. \qquad W_1 = \frac{b^2h^2f_1}{(1{\cdot}6b + {\cdot}8h)D}$$

where b is not greater than $3\frac{1}{4}\,h$, and h is less than b.

If h is small compared with b, then

$$\delta = \frac{2\cdot36\,WnD^3}{Cbh^3}, \qquad \text{and } W_1 = \frac{2bh^2f_1}{3D}.$$

The greatest stress f_1 is at the points on the boundary line of the section which are nearest to the centre of the section.

Mr. Wilson Hartnell found from his experiments on springs such as are used for governors and safety-valves that C varied from 13,000,000 for $\frac{1}{4}$-inch wire to 11,000,000 for $\frac{3}{8}$-inch wire, and he gives the safe stress f_1 as 60,000 to 70,000 for $\frac{3}{8}$-inch wire, and 50,000 for $\frac{1}{2}$-inch wire.

For springs made of wire less than $\frac{3}{8}$-inch in diameter Mr. Hartnell gives the following rules :—

$$W_1 = \frac{24{,}000d^3}{D}, \text{ and } \delta = \frac{nD^3W}{1{,}440{,}000d^4}.$$

The Board of Trade rules for safety-valve springs are,

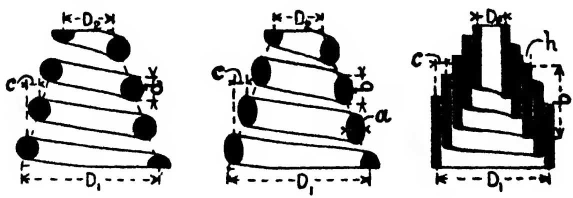
$$W_1 = \frac{8000d^3}{D} \text{ for round steel,}$$

and

$$W_1 = \frac{11{,}000d^3}{D} \text{ for square steel.}$$

These rules correspond to $f_1 = 20{,}371$ for round steel, and $f_1 = 26{,}442$ for square steel.

Conical Spiral Springs.—For notation used, see p. 257 and the illustrations below.



For coils of circular section

$$\delta = \frac{W(D_1^4 - D_2^4)}{Ccd^4}. \qquad W_1 = \frac{\pi d^3 f_1}{8D_1}.$$

For coils of elliptical section

$$\delta = \frac{W(a^2+b^2)(D_1^4 - D_2^4)}{2Cca^3b^3}. \qquad W_1 = \frac{\pi ba^2 f_1}{8D_1},$$

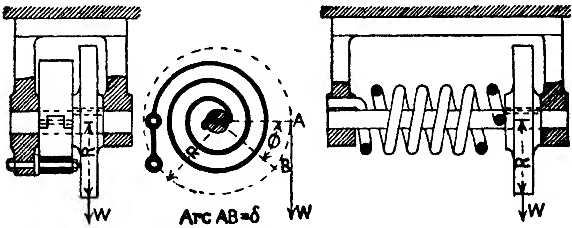
where b is the major axis of the ellipse.

For coils of rectangular section

$$\delta = \frac{3\pi W(D_1^4 - D_2^4)}{32Ccbh^3}. \qquad W_1 = \frac{2bh^2f_1}{3D_1}.$$

The greatest stress f_1 is at the points of the boundary line of the section which are nearest to the centre of the section.

Spiral Springs in Torsion.



One end of the spring is fixed, and the other end is attached to the axle which is turned by the force W, which acts at a distance R from the axis of axle.

ϕ = angle turned through by axle under the action of W, in circular measure.

δ = distance moved by W while the axle turns through the angle ϕ.

Z = moment of resistance of section of spring to bending, see p. 212.

f = greatest tensile or compressive stress in material of spring due to bending moment.

For the rest of the notation employed, see p. 257.

$$\delta = \frac{WR^2l}{EI} = R\phi. \qquad f = \frac{WR}{Z}.$$

Straight Bars in Torsion as Springs.—

ϕ = angle of twist of bar by load W, in circular measure.

δ = distance moved by W while the bar twists through the angle ϕ.

For the rest of the notation employed, see p. 257 and the figures on the opposite page.

General formulæ

$$WR = \frac{A^4C\phi}{KI_0l} = \frac{A^4C\delta}{KI_0lR}.$$

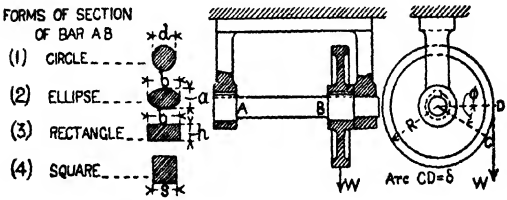
$$\delta = \frac{WR^2KI_0l}{A^4C} = R\phi.$$

(1) *For bar of circular section*

$$\delta=\frac{32\,WR^2l}{\pi d^4C}. \qquad W_1=\frac{\pi d^3f_1}{16R}.$$

(2) *For bar of elliptical section*

$$\delta=\frac{16\,WR^2l(a^2+b^2)}{\pi a^3b^3C}. \qquad W_1=\frac{\pi ba^2f_1}{16R}.$$



(3) *For bar of rectangular section*

$$\delta=\frac{3{\cdot}5\,WR^2(b^2+h^2)l}{b^3h^3C}. \qquad W_1=\frac{b^2h^2f_1}{2R(1{\cdot}6b+{\cdot}8h)},$$

where b is not greater than $3\frac{1}{2}\,h$, and h is less than b. If h is small compared with b, then

$$\delta=\frac{3\,WR^2l}{bh^3C}, \qquad \text{and } W_1=\frac{bh^2f_1}{3R}.$$

(4) *For bar of square section*

$$\delta=\frac{7{\cdot}11\,WR^2l}{s^4C}. \qquad W_1=\frac{{\cdot}208s^3f_1}{R}.$$

Note on the Deflection of Springs.—In consequence of the uncertainty of the values of the moduli of elasticity, the deflection of springs for given loads on them can only be determined approximately by the formulæ beforehand. If it is necessary to know the exact deflections, as in the cases of springs for dynamometers, spring balances, etc., then, after the springs are constructed, they should be carefully tested with known weights and the deflections observed.

Elastic Energy of Springs.—

U=work stored up in spring, in inch lbs

U_1=work stored up in 1 cubic inch of spring, in inch lbs.

U_2=work stored up in 1 lb. weight of spring, in inch lbs.

V=volume of spring, in cubic inches.

w=weight of 1 cubic inch of spring, in lbs.

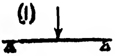
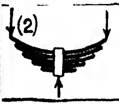
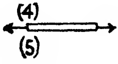
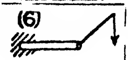
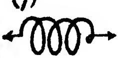
W=load on spring, in lbs.

f=greatest stress due to load W, in lbs. per square inch.

δ = distance moved by W, in inches, when spring is deflected.
E = modulus of direct elasticity of material of spring, in lbs. per square inch.
C = modulus of transverse elasticity, in lbs per square inch.

$$U = \frac{W\delta}{2}. \qquad U_1 = \frac{U}{V}. \qquad U_2 = \frac{U}{Vw}.$$

The values of U_1 and U_2 in the table below are for tempered steel under the greatest safe steady load; f being taken at 80,000, E at 32,000,000, C at 12,800,000, and w at ·284.

Kind of Spring, and Manner of Loading		Form of Section of Spring	U	U_1	U_2
(1) 	Straight bar supported at the ends, and loaded at the centre.	Circle or ellipse	$\frac{Vf^2}{24E}$	8·3	29·3
		Square or rectangle.	$\frac{Vf^2}{18E}$	11·1	39·1
(2) 	Compound plate spring.	Rectangle.	$\frac{Vf^2}{6 \cdot 3E}$ (approx.)	31·7	111·8
(3) 	Helical spring in torsion.	Circle or ellipse.	$\frac{Vf^2}{8E}$	25	88
		Square or rectangle	$\frac{Vf^2}{6E}$	33·3	117·4
(4) (5) 	Straight bar in tension or compression.	Any form.	$\frac{Vf^2}{2E}$	100	352·1
(6) 	Straight bar in torsion, or helical spring in tension or compression.	Circle.	$\frac{Vf^2}{4C}$	125	440·1
(7) 		Square.	$\frac{Vf^2}{6 \cdot 5C}$	76·9	270·9

In (1), (2), and (3) the material of the spring is subjected to bending, in (4) to tension, in (5) to compression, and in (6) and (7) to twisting.

CHAINS.

(From data suggested by E. Baylie & Co. Ltd., Stourbridge.)

Classification.—Welded chain can be classified under two main headings: *lifting chain* and *cable chain*. Lifting chain is made in *short link*, *long link*, and *block* or *pitched chain*. Cable chain is made in *stud link* and *short* or *open link* and is used to attach a ship to its mooring.

Short link lifting chain is normal for cranes without seated wheels and for sling chains. It is made from wrought iron of 21 to 24 tons/sq. in. ultimate strength. It can be of any length and is generally sold by weight.

Chain size is designated by the nominal diameter of the iron; the actual diameter is normally slightly oversize. Exact size chain can be obtained.

Short Link Wrought-iron Crane Chain.*

Nominal Size of Chain.	Weight per Fathom.	Proof Load.	Test Load.				Elongation at Test Load on 36 In.	
			Standard Quality.		Special Quality.		Standard Quality.	Special Quality.
In.	Lb.	Tons.	Tons.	Cwt.	Tons.	Cwt.	In.	In.
5/16	7½	1⅛	2	19	3	3	5	6½
3/8	10¼	1⅝	4	3	4	8	5¼	6¾
7/16	13½	2¼	5	10	5	17	5¼	6¾
½	17½	3	7	2	7	10	5¼	6¾
9/16	22½	3¾	8	17	9	8	5½	7
⅝	27	4⅝	10	16	11	10	5½	7
11/16	32	5⅝	13	0	13	16	5½	7
¾	37	6¾	15	7	16	6	5½	7
13/16	43	8	17	18	19	0	5½	7
⅞	51	9⅛	20	13	21	18	5½	7
15/16	57	10½	23	12	25	1	5½	7
1	66	12	26	14	28	8	5½	7
1 1/16	73	13½	30	1	31	18	5½	7
1⅛	82	15⅛	33	12	35	14	5½	7
1 3/16	89	17	37	6	39	13	5¼	6¾
1¼	98	18¾	41	5	43	16	5¼	6¾
1 5/16	107	20⅝	45	7	48	4	5¼	6¾
1⅜	118	22⅝	49	14	52	16	5	6½
1 7/16	129	24¾	54	4	57	12	5	6½
1½	140	27	58	18	62	12	5	6½

* Extracted from B.S. 394 : 1944.

Short Link Wrought-iron Crane Chain (*continued*).—The outside dimensions of a link shall be not more than $4\frac{1}{2}$ by $3\frac{1}{4}$ times the actual diameter of the iron used. The actual diameter may be up to $\frac{1}{32}$ in. greater than the nominal diameter.

Normally the *safe working load* should not exceed half the proof load, and for hazardous conditions it should be less than half.

The whole of a short link chain is subjected to the proof load, which is approximately $12d^2$ tons, where d is the nominal diameter of the iron in inches. The test load is applied to selected 36-inch samples which have been marked accurately after subjection to the proof load. Test loads produce a stress of 16 tons/sq. in. in standard quality chain and 17 tons/sq. in. in special quality chain, calculated on iron $\frac{1}{32}$ in. larger than the nominal size and considering the sum of the cross-sectional areas of both sides of a link. The elongation is measured after removal of the test load. The samples are finally tested to destruction. Hot bend, cold bend, and nicked fracture tests are also made.

Pitched or Calibrated Wrought-iron Load Chain for Hand-operated Pulley Blocks.*

Nominal Size of Chain.	Proof Load.	Safe Working Load.	Unpolished. Test Load.	Unpolished. Min. Elong. on 36 In. at T. Ld.	Polished. Test Load.	Polished. Min. Elong. on 36 In. at T. Ld.
In.	T. C. Q.	T. C. Q.	T. C. Q.	In.	T. C. Q.	In.
$\frac{7}{32}$	.. 8 1	.. 5 2	1 2 2	$3\frac{1}{2}$	1 1 0	$2\frac{1}{2}$
$\frac{1}{4}$	. 11 1	.. 7 2	1 9 2	$3\frac{1}{2}$	1 7 2	$2\frac{1}{2}$
$\frac{9}{32}$	.. 13 2	.. 9 0	1 17 2	$3\frac{1}{2}$	1 14 3	$2\frac{1}{2}$
$\frac{5}{16}$	.. 17 0	.. 11 1	2 6 0	$3\frac{1}{2}$	2 3 0	$2\frac{1}{2}$
$\frac{11}{32}$	1 1 0	.. 14 0	2 15 2	$3\frac{1}{2}$	2 12 0	$2\frac{1}{2}$
$\frac{3}{8}$	1 4 2	.. 16 1	3 6 0	$3\frac{3}{4}$	3 1 3	$2\frac{3}{4}$
$\frac{13}{32}$	1 8 2	.. 19 0	3 18 0	4	3 12 2	3
$\frac{7}{16}$	1 13 3	1 2 2	4 10 0	4	4 4 1	3
$\frac{15}{32}$	1 19 2	1 6 1	5 3 2	4	4 16 1	3
$\frac{1}{2}$	2 5 0	1 10 0	5 18 0	4	5 10 1	3
$\frac{9}{16}$	3 0 0	2 0 0	7 9 0	4	6 19 1	3
$\frac{5}{8}$	3 15 0	2 10 0	9 4 0	4	8 11 3	3
$\frac{11}{16}$	4 10 0	3 0 0	11 2 2	4	10 7 3	3
$\frac{3}{4}$	5 12 2	3 15 0	13 5 0	4	12 7 1	3
$\frac{13}{16}$	6 15 0	4 10 0	15 11 0	4	14 10 1	3
$\frac{7}{8}$	7 10 0	5 0 0	18 1 0	4	16 16 3	3
$\frac{15}{16}$	9 7 2	6 5 0	20 14 0	4	19 6 2	3
1	11 5 0	7 10 0	23 11 0	4	21 19 3	3

* Extracted from B.S. 465 : 1932.

Pitched or Calibrated Wrought-iron Load Chain for Hand-operated Pulley Blocks (*continued*).—This chain is used exclusively with pocketed sheaves and, although commonly termed "block chain," is used for other purposes than with chain pulley blocks. Actual diameter of iron in the links shall be not greater than, and not more than 5 per cent. less than, the nominal diameter. Overall length of a link shall be not more than 6 times, and width not more than 3¼ times, the nominal diameter.

Short Link and Pitched Steel Chain, Electrically Welded Mild Steel.*—The diameter of the material shall not vary by more than plus 0 003 in. or minus 0 002 in. from the sizes in col. 1 of table.

Size of Chain.		Weight † per Fathom.		Proof Load.			Safe Working Load.			Test Load. Standard Quality.		Test Load. Special Quality.		Min. Elong. on 36 In.
S.W.G.	In.	Lb.	Oz	T.	C.	Q.	T.	C.	Q.	T.	C.	T.	C.	In.
6	0·192	2	1	..	8	0	..	4	0	1	1	1	6	4
5	0 212	2	5	..	10	0	..	5	0	1	5	1	11	4
4	0 232	3	0	..	12	0	..	6	0	1	10	1	18	4
3	0 252	3	6	..	15	0	..	7	2	1	16	2	4	4
2	0·276	4	3	..	18	0	..	9	0	2	3	2	13	4
1	0·300	5	1	1	0	0	..	10	0	2	11	3	2	4
	5/16	5	5	1	2	2	..	11	1	2	15	3	7	4
	11/32	6	7	1	8	0	..	14	0	3	7	4	2	5
	3/8	8	0	1	12	2	..	16	1	4	0	4	17	5
	13/32	9	4	1	18	0	..	19	0	4	13	5	14	5
	7/16	11	0	2	5	0	1	2	2	5	8	6	13	5
	15/32	12	5	2	12	2	1	6	1	6	4	7	12	5
	1/2	14	2	3	0	0	1	10	0	7	1	8	13	5
	17/32	16	0	3	7	2	1	14	0	8	0	9	15	5

The outside dimensions of the links, in terms of the diameter of the material, shall be not more than 5 by 3½ for *short link chain* and not more than 6 by 3½ for *pitched* or *calibrated chain.*

Chain Slings.—Nominal length is measured from inside the ring or link at one end to inside the link or hook at the other end. In two-, three-, or four-leg slings, the legs should be equal in length within ¼ in. The load lifted must be reduced as the angle between each leg and the vertical is increased. If P is the total safe load when all legs are vertical and θ is the angle between each leg and the vertical when they are inclined, then, assuming equal load distribution, the safe load is $P \cos \theta$. Examples are given for two-leg slings in the following table:—

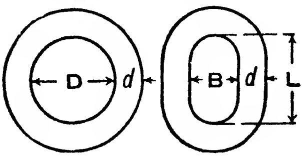
* Extracted from B.S. 590 : 1935.

† These weights do not apply to pitched or calibrated chain.

Safe Working Loads for Two-leg Slings.

Size of Sling Chain.	Angle between each Leg and the Vertical.									
	0°		15°		30°		45°		60°	
In.	Tons	Cwt.	Tons.	Cwt.	Tons.	Cwt.	Tons.	Cwt.	Tons.	Cwt.
$\frac{5}{16}$	1	2	1	2	..	19	..	16	..	11
$\frac{1}{2}$	3	0	2	18	2	12	2	2	1	10
1	12	0	11	12	10	8	8	10	6	0
$1\frac{1}{2}$	27	0	26	1	23	8	19	2	13	10

Rings and Alternative Links for Chain Slings.*—The proof load for a single sling ring, or alternative link, is equal to that of the sling chain, and the proof loads for two-, three-, and four-leg sling rings, or alternative links, are respectively two, three, and four times that of the sling chain. The safe working load should not exceed half the proof load.



Ring. *Alternative Link.*

Rings.—Dimensions (see fig.) in terms of d_c, the size of the short link chain in the sling, are tabulated.

Sling.	d	D	d	D	d	D	d	D
Single .	$1{\cdot}79d_c$	$6{\cdot}5d_c$	$1{\cdot}91d_c$	$8d_c$	$1{\cdot}98d_c$	$9d_c$	$2{\cdot}05d_c$	$10d_c$
Two-leg .	..	..	$2{\cdot}42d_c$	$8d_c$	$2{\cdot}51d_c$	$9d_c$	$2{\cdot}60d_c$	$10d_c$
Three-leg	..	..	..	..	$2{\cdot}89d_c$	$9d_c$	$2{\cdot}98d_c$	$10d_c$
Four-leg	..	..	..	..	..	..	$3{\cdot}29d_c$	$10d_c$

Proof Load $= 14{\cdot}8d^3/(D + 0{\cdot}3d)$ tons, approx., where d and D are in inch units and the ratio D/d is between 2 and 7.

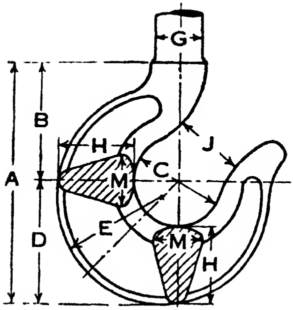
Links Alternative to Rings.—Dimensions (see fig.) in terms of d_c.

Sling.	d	L	B	Sling.	d	L	B
Single .	$1{\cdot}60d_c$	$6{\cdot}5d_c$	$3{\cdot}9d_c$	Three-leg	$2{\cdot}60d_c$	$9d_c$	$5{\cdot}4d_c$
Two-leg	$2{\cdot}18d_c$	$8{\cdot}0d_c$	$4{\cdot}8d_c$	Four-leg	$3{\cdot}06d_c$	$10d_c$	$7{\cdot}0d_c$

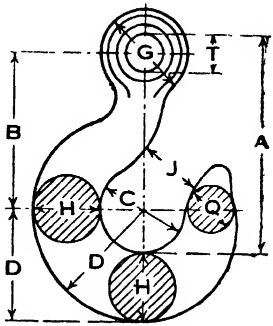
For non-standard links geometrically similar to the above standards $d = K\sqrt{\textit{Proof load}}$ inches, approx., where K is 0·46, 0·445, 0·435, and 0·44 for single, two-leg, three-leg, and four-leg slings respectively, and the proof load is in tons.

* Extracted from B.S. 781 : 1938. Wrought-iron Chain Slings and Rings, Links Alternative to Rings, Egg Links and Intermediate Links.

Crane Hooks.*—A few dimensions are tabulated below; others will be found in B.S. 482 : 1945.



Shank Hook, Trapezoidal Section.



Eye Hook, Circular Section.

	Trapezoidal Sections.				Circular Sections.	
	Shank Hooks.		Eye Hooks.		Shank Hooks.	Eye Hooks.
C	$1{\cdot}5\sqrt{W}$	$1{\cdot}84\sqrt{W}$	$4{\cdot}5d_c = 1{\cdot}84\sqrt{W}$	C	$1{\cdot}5\sqrt{W}$	$1{\cdot}5\sqrt{W}$
A	$2{\cdot}75C$	$2{\cdot}46C$	$10{\cdot}10d_c$	A	$2{\cdot}67C$	$2{\cdot}47C$
B	$1{\cdot}31C$	$1{\cdot}17C$	$7{\cdot}00d_c$	B	$1{\cdot}35C$	$1{\cdot}75C$
D	$1{\cdot}44C$	$1{\cdot}29C$	$5{\cdot}83d_c$	D	$1{\cdot}32C$	$1{\cdot}32C$
E	$1{\cdot}25C$	$1{\cdot}13C$	$5{\cdot}0d_c$	..	..	..
G	$0{\cdot}55C$	$0{\cdot}45C$	$4{\cdot}0d_c$	G	$0{\cdot}55C$	$1{\cdot}00C$
H	$0{\cdot}93C$	$0{\cdot}78C$	$3{\cdot}5d_c$	H	$0{\cdot}82C$	$0{\cdot}82C$
J	$0{\cdot}75C$	$0{\cdot}75C$	$3{\cdot}4d_c$	J	$0{\cdot}77C$	$0{\cdot}79C$
M	$0{\cdot}60C$	$0{\cdot}51C$	$2{\cdot}3d_c$	Q	$0{\cdot}54C$	$0{\cdot}54C$
T	..	..	$1{\cdot}5d_c$	T	..	$0{\cdot}44C$

W = safe working load in tons. C is in inches. d_c = nominal diameter in inches of appropriate chain iron. *Proof load* $= 2W$, up to $W = 50$ tons. Over this value a modified proof load may be necessary.

Second column under trapezoidal sections, shank hooks, is the alternative for hooks of increased internal diameter for small loads.

* Extracted from B.S. 482 : 1945. Wrought Iron and Mild Steel Hooks for Cranes, Chains, Slings and General Engineering Purposes, Excluding Building Operations.

ROPES.

Construction of Hempen Ropes.—*Yarns* are made by spinning or twisting the fibres together with a right-handed twist. A *strand* is made by twisting yarns together left-handed. A *hawser-laid* rope or *hawser* is made by twisting three strands together right-handed. A *shroud-laid* rope has four strands twisted round a core. A *cable-laid* rope or *cable* consists of three hawsers twisted together left-handed.

The size of a rope is its circumference or *girth*. The size of a rope which is used for transmitting power is, however, generally taken as the diameter.

Weight and Strength of Hempen Ropes.—

C = girth of rope in inches.
W = weight of rope per fathom (6 feet) in lbs.
S = breaking strength of rope in tons.

$$W = \cdot16C^2 \text{ to } \cdot20C^2.$$
$$S = \cdot18C^2 \text{ to } \cdot54C^2.$$

The following simple rules for the weight and strength of hempen ropes may be used in making rough calculations :—

$$W = \frac{C^2}{5}, \qquad S = \frac{C^2}{3}.$$

Tarred ropes have only about three-fourths of the strength of untarred ropes of the same material. Tarred ropes, however, retain their original strength longer than white ropes, and they are more impervious to water.

The *working load* is generally from one-sixth to one-tenth of the breaking load, except for ropes used for transmitting power, when the working load is generally much less (see p. 483).

British Standard Manila Ropes *—3-Strand, Plain or Hawser Laid.

Circumference		Weight per 120 Fathoms			Breaking Load					
					Grade 1		Grade 2		Grade 3	
In	Mm	Cwt	Qr	Lb	Tons	Cwt	Tons	Cwt	Tons	Cwt
⅞	22	..	..	17	..	7¼		6½		5¾
1	25	..	..	26	..	10¾	..	9½	..	8¼
1¼	32	..	1	6	..	14	..	12½	..	11
1⅜	35	..	1	15	..	17¾	..	15¾	..	13¾
1½	38	..	1	24	1	1		18¾	..	16½
1⅝	41	..	2	4	1	4¾	1	2	..	19¼
1¾	44	..	2	13	1	8½	1	5¼	1	2
2	51	..	3	10	2	0	1	15½	1	11
2¼	57	1	0	0	2	8	2	2½	1	17
2½	64	1	1	6	3	3½	2	16½	2	9½
2¾	70	1	2	4	3	15½	3	7	2	18½
3	76	1	3	10	4	10	4	0	3	10
3¼	83	2	0	16	5	4½	4	13	4	1½
3½	89	2	2	3	5	19½	5	6½	4	13
3¾	95	2	3	8	6	16½	6	1½	5	6
4	102	3	1	5	7	13½	6	16½	5	19½
4¼	108	3	2	20	8	11	7	12	6	13
4½	114	4	0	15	9	11½	8	10	7	9
4¾	121	4	2	11	10	11	9	7½	8	4
5	127	5	0	15	11	13	10	7	9	11
5½	140	6	0	23	13	19	12	8	10	17
6	152	7	1	12	16	9	14	12	12	15
6½	165	8	2	18	19	3	17	0	14	17
7	178	10	0	4	22	1	19	12	17	3
8	203	13	0	12	28	7	25	4	22	1
9	229	16	2	6	35	9	31	10	27	11
10	254	20	1	24	43	9	38	12	33	15
11	279	24	3	0	52	4	46	8	40	12
12	305	29	1	20	61	18	55	0	48	2
13	330	34	2	7	72	5	64	4	56	3
14	356	40	0	9	83	5	74	0	64	15
15	381	45	3	26	95	13	85	0	74	7
16	406	52	1	10	108	11	96	10	84	9
17	432	59	0	10	122	1	108	10	94	19
18	457	66	0	24	136	14	121	10	106	6

Weights include an allowance of approximately 2½ per cent. for wrappers. 1 fathom = 6 feet. 1 ton = 2240 lb.

* Extracted from B.S. 431 : 1946.

Steel Wire Ropes.
Minimum Diameters of Barrels and Sheaves.

(*British Ropes Ltd., Charlton, S.E. 7.*)

Rope Circumference (Inches).	Diameters of Barrels and Sheaves (Inches). Constructions.				
	6×12	6×19	6×24	6×37	6×61
1	6	7	6	5¼	..
1¼	7½	9	7½	6½	..
1½	9	11	9	8	..
1¾	10½	13	10½	9	..
2	12	15	12	10½	..
2¼	13½	16½	13½	12	..
2½	15	18	15	13	10½
2¾	16½	20	16½	14½	11½
3	18	22	18	16	12½
3¼	20	24	20	17	13½
3½	21	25½	21	18½	14½
3¾	22½	27½	22½	20	15½
4	24	29	24	21	16½
4¼	26	31	26	22½	17½
4½	27	33	27	23½	18½
4¾	28½	35	28½	25	19½
5	30	36½	30	26	20½
5¼	32	38½	32	27½	22
5½	33	40½	33	29	23
5¾	35	42	35	30	24
6	36½	44	36½	31½	25

To convert inches into millimetres multiply by 25·4.

The above table should only be taken as a guide, as hard and fast figures cannot be stated, but depend on local conditions. The recommendations cover speeds up to 120 feet per minute.

Descriptions of Steel Wire Ropes.

Tensile Breaking Strength.	Trade Description.
Tons per Sq. In.	
80–90	Best patent steel.
90–100	Special improved patent steel.
100–110	Best plough steel.
110–120	Special improved plough steel.
115–125	Extra special improved plough steel.

British Standard Steel Wire Ropes.*
5 × 6, 5 × 7, 6 × 6, and 6 × 7 Constructions.

Circumference.	Weight.†		Actual Breaking Load.				
	Per 100 Ft.	Per Metre.	80–90 Tons/ Sq. In.	90–100 Tons/ Sq. In.	100–110 Tons/ Sq. In.	110–120 Tons/ Sq. In.	115–125 Tons/ Sq. In.
Inches.	Lbs.	Kg.	Tons.	Tons.	Tons.	Tons.	Tons.
$\frac{3}{4}$	9	·134	1·9	2·1	2·4	2·5	2·6
$\frac{7}{8}$	12	·178	2·5	2·8	3·2	3·4	3·6
1	19	·283	3·3	3·8	4·2	4·5	4·7
$1\frac{1}{8}$	24	·357	4·2	4·6	5·1	5·5	5·8
$1\frac{1}{4}$	27	·402	5·0	5·6	6·2	6·7	7·1
$1\frac{3}{8}$	34	·506	6·2	6·9	7·7	8·4	8·8
$1\frac{1}{2}$	40	·595	7·3	8·1	9·0	9·9	10·3
$1\frac{5}{8}$	47	·699	8·4	9·4	10·5	11·4	11·9
$1\frac{3}{4}$	54	·804	10·0	11·1	12·3	13·5	14·1
$1\frac{7}{8}$	60	·893	11·3	12·7	14·0	15·3	15·9
2	72	1·07	13·0	14·6	16·2	17·7	18·5
$2\frac{1}{8}$	80	1·19	14·6	16·3	18·0	19·7	20·6
$2\frac{1}{4}$	87	1·29	16·2	18·1	20·0	21·9	22·8
$2\frac{3}{8}$	97	1·44	18·3	20·4	22·6	24·8	25·8
$2\frac{1}{2}$	107	1·59	20·1	22·5	24·8	27·2	28·4
$2\frac{5}{8}$	120	1·79	22·5	25·1	27·7	30·3	31·6
$2\frac{3}{4}$	132	1·96	24·9	27·8	30·7	33·7	35·1
$2\frac{7}{8}$	140	2·08	26·5	29·6	32·7	35·8	37·4
3	154	2·29	29·1	32·6	36·0	39·4	41·2
$3\frac{1}{8}$	168	2·50	31·3	35·0	38·7	42·4	44·3
$3\frac{1}{4}$	184	2·74	34·3	38·2	42·3	46·4	48·3
$3\frac{3}{8}$	196	2·92	36·7	41·0	45·3	49·6	51·8
$3\frac{1}{2}$	217	3·23	39·8	44·5	49·3	53·9	56·3
$3\frac{5}{8}$	232	3·45	43·1	48·1	53·2	58·3	60·9
$3\frac{3}{4}$	247	3·68	45·8	51·2	56·7	62·0	64·7
$3\frac{7}{8}$	262	3·90	48·6	54·3	60·1	65·8	68·7
4	275	4·09	51·5	57·6	63·7	69·7	72·7
$4\frac{1}{8}$	297	4·42	55·3	61·7	68·2	74·8	78·0
$4\frac{1}{4}$	308	4·58	58·3	65·2	72·1	78·9	82·4
$4\frac{3}{8}$	336	5·00	61·5	68·7	75·9	83·2	86·8
$4\frac{1}{2}$	350	5·21	65·5	73·2	81·0	88·7	92·5
$4\frac{5}{8}$	364	5·42	68·9	77·0	85·1	93·2	97·3
$4\frac{3}{4}$	392	5·83	73·1	81·7	90·4	99·0	103·3
$4\frac{7}{8}$	406	6·04	76·6	85·7	94·8	103·7	108·3
5	420	6·25	80·3	89·8	99·2	108·6	113·4

* Extracted from B.S. 330 : 1941. † If wire main core add $\frac{1}{9}$th.
Certain restrictions made in October 1941 are not indicated above.

British Standard Steel Wire Crane Ropes.*
6 × 19 Construction.

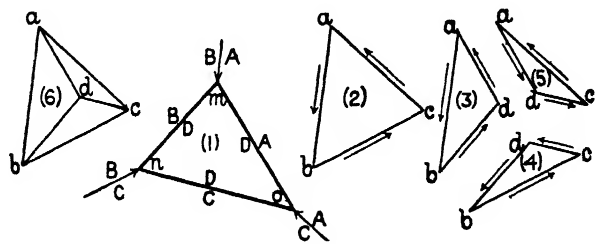
Circumference.	Approx. Weight.			Actual Breaking Load.			
	Per Fathom.	Per 100 Ft.	Per Metre.	80–90 Tons/ Sq. In.	90–100 Tons/ Sq. In.	100–110 Tons/ Sq. In.	110–120 Tons/ Sq. In.
In.	Lb.	Lb.	Kg.	Tons.	Tons.	Tons.	Tons.
1	1·08	18	0·268	2·8	3·0	3·4	3·7
1⅛	1·26	21	0·312	3·3	3·7	4·1	4·4
1¼	1·50	25	0·372	4·3	4·7	5·2	5·8
1⅜	1·80	30	0·446	4·9	5·5	6·1	6·6
1½	2·16	36	0·536	6·0	6·7	7·4	8·2
1⅝	2·58	43	0·640	7·2	8·1	9·0	9·9
1¾	3·00	50	0·744	8·1	9·1	10·1	11·0
2	3·96	66	0·982	11·1	12·4	13·7	15·0
2⅛	4·44	74	1·10	12·1	13·6	15·0	16·5
2¼	5·04	84	1·25	13·9	15·6	17·2	18·8
2⅜	5·52	92	1·37	15·7	17·6	19·4	21·3
2½	6·12	102	1·52	17·0	19·1	21·1	23·1
2¾	7·38	123	1·83	20·5	22·9	25·3	27·7
3	9·24	154	2·29	25·8	28·9	31·9	34·9
3⅛	10·08	168	2·50	27·5	30·7	33·9	37·2
3¼	11·04	184	2·74	30·0	33·6	37·1	40·6
3⅜	11·76	196	2·92	32·7	36·6	40·4	44·3
3½	13·02	217	3·23	35·5	39·6	43·8	48·0
3¾	14·82	247	3·68	40·4	45·1	49·9	54·6
3⅞	15·72	262	3·90	43·5	48·6	53·7	58·8
4	16·50	275	4·09	45·6	50·9	56·3	61·7
4¼	18·48	308	4·58	51·1	57·1	63·1	69·1
4⅜	20·16	336	5·00	54·6	61·0	67·4	73·8
4½	21·00	350	5·21	58·1	65·0	71·8	78·7
4¾	23·52	392	5·83	64·3	71·9	79·5	87·1
5	25·20	420	6·25	70·9	79·2	87·6	95·9
5⅛	26·88	448	6·67	75·0	83·7	92·6	101·4
5¼	28·56	476	7·09	79·1	88·5	97·7	107·1
5½	31·38	523	7·79	86·4	96·5	106·7	116·8
5¾	34·44	574	8·54	93·9	104·9	116·0	127·0
6	37·56	626	9·32	103·3	115·5	127·7	139·8
6¼	40·32	672	10·00	111·6	124·7	137·8	150·9
6½	43·68	728	10·84	120·1	134·3	148·4	162·6

* Extracted from B.S. 302 : 1938. Certain restrictions made in November 1941 are not indicated above.

STRESS DIAGRAMS FOR BRACED STRUCTURES.

The assumptions which are usually made in determining the stresses in the members of a braced structure are: (1) that the members are connected together at their ends by pin joints; (2) that the various loads are placed at the joints. If a bar carries a load uniformly distributed over its length, this load is divided into two equal parts, and one part is placed at each end of the bar. If a bar carries a load concentrated at a point between its ends, this load is divided into two parts, which are to one another as the distances of the load from the ends of the bar; these parts are then placed one at each end of the bar, the greater part being at that end of the bar which is nearest to the original load.

The General Method of Drawing Stress Diagrams for Braced Structures is illustrated by the diagrams below, which show the method applied to a simple triangular frame loaded at the



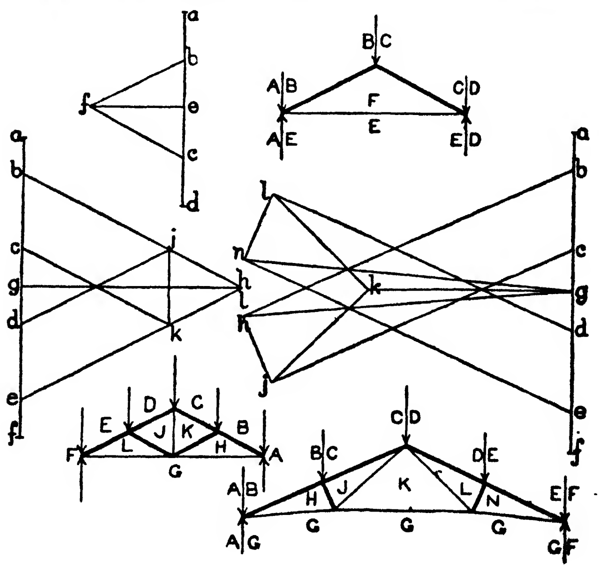
corners. *mno* Fig. (1) is the frame, and *A B*, *BC*, and *CA* are the loads acting at its corners. Fig. (2) is the polygon of forces for the loads, and may be called the load diagram or load polygon. If the external forces acting on the frame are parallel, the load polygon becomes a straight line.

Figs. (3), (4), and (5) are the polygons of forces acting at the points *m*, *n*, and *o* respectively. Fig. (6) shows the polygons of Figs. (2), (3), (4), and (5) combined in one diagram, which is called the stress diagram or force diagram for the loaded frame shown in Fig. (1). The complete stress diagram Fig. (6) may be drawn without first drawing the separate diagrams, but Figs. (2), (3), (4), and (5) are drawn here to make the construction of Fig. (6) quite clear.

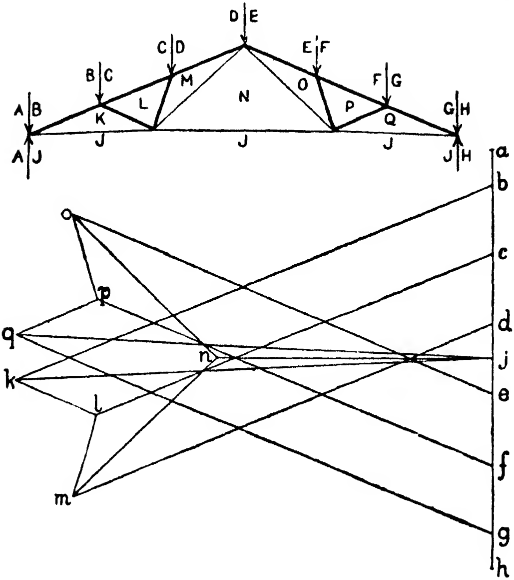
To determine whether a particular bar in a structure is in tension or compression.—Examine the polygon of forces for one end of the bar. If this polygon shows that the force in the bar acts towards the end which is being considered, then the bar is in compression; and if the force acts in the opposite direction, the bar is in tension.

It is usual to show bars which are in compression by thick lines on the frame diagram, while those which are in tension are indicated by thin lines.

Examples of Stress Diagrams for Roof Trusses.



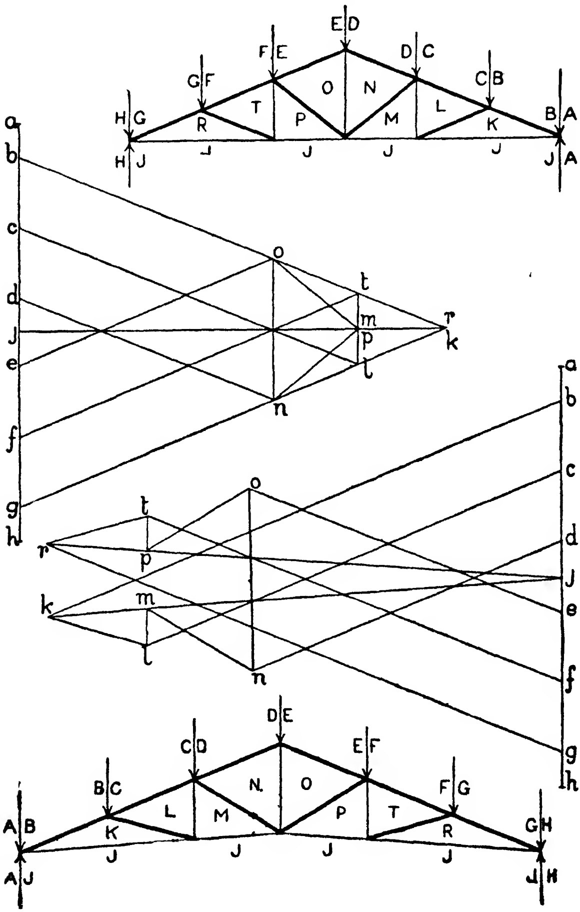
Examples of Stress Diagrams for Roof Trusses.



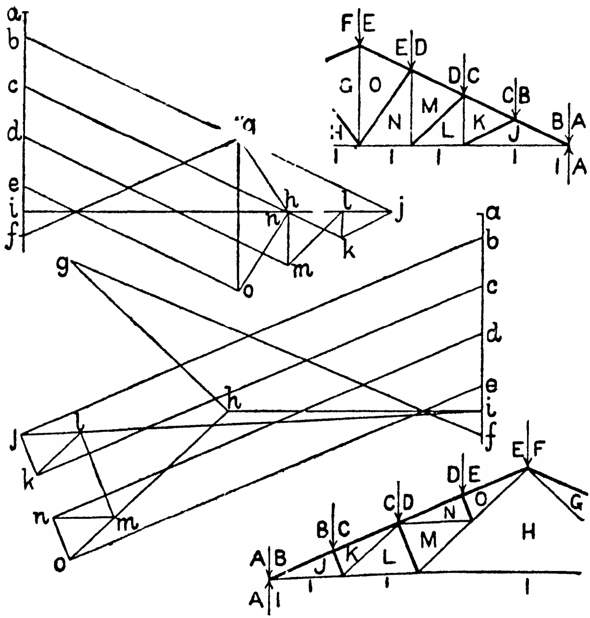
The above stress diagram may be drawn as follows:—First draw *ah*, the line of loads. $ab = AB$, $bc = BC$, etc. The loads being symmetrical about the centre line of the truss, the reactions *HJ* and *JA*, at the points of support, must be equal, therefore *j* is at the middle point of *ah*. Now draw the polygon of forces *abkja* for the point *ABKJA*. Next add the lines to complete the polygon *bclkb*, which is the polygon of forces for the point *BCLKB*. Proceeding next to the point *CDMLC*, the polygon *cdmlc* is completed. The next point to be considered is the point *LMNJKL*, and the polygon *lmnjkl* is completed. This completes the stress diagram for one half of the truss, and the diagram for the other half may be drawn in the same way.

When a truss is loaded symmetrically, it is only necessary to determine the stresses in the bars of one half of the truss, because the stresses in the corresponding bars of the other half will be the same.

Examples of Stress Diagrams for Roof Trusses.



Examples of Stress Diagrams for Roof Trusses.



In the two examples above the trusses are supposed to be symmetrical and symmetrically loaded, and only a little more than one half of the truss is considered in each case.

Approximate Weight of Roof Trusses.

The following formula for the approximate weight of wrought-iron roof trusses is given by Merriman in his "Roofs and Bridges":—

$$w=\frac{3}{4}al\left(1+\frac{1}{10}l\right).$$

w = total weight of one truss in lbs.
l = span in feet.
a = distance between adjacent trusses.

Approximate Weight of Roof Coverings.

In lbs. per square foot of roof surface.

Felt and asphalt	1 to 3	Wood shingles	2 to 3
Corrugated iron	2 to $3\frac{1}{2}$	Tiles	10 to 25
Sheet iron	$1\frac{1}{2}$ to $2\frac{1}{2}$	Slates	6 to 10
Sheet lead	5 to 8	Purlins, wood	1 to 3
Sheet zinc	1 to 2	Purlins, iron	2 to 4
Wood boards 1 inch thick	3 to 4	Rafters	$1\frac{1}{2}$ to 3

Allowance for Weight of Snow on Roofs.

In England it is usual to allow 6 lbs. per square foot of area covered for the weight of snow.

In the United States the allowance varies from 10 lbs. to 30 lbs. per square foot of area covered.

Inclinations of Roofs.

Pitch of roof = rise ÷ span.

$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$
Inclination of roof to horizontal in degrees.							
18·4	21·8	26·6	33·7	45	53·1	56·3	58

The minimum slope of a roof depends on the nature of the covering, and is about 4° for sheet lead, sheet zinc, or corrugated iron. For slates or tiles the minimum slope is from 22° to 27°.

Pressure of Wind.

The pressure of the wind on a convex cylindrical surface, the direction of the wind being at right angles to the axis of the cylinder, is about half the pressure on a plane rectangular surface whose length and width equal the length and diameter respectively of the cylinder.

V = velocity of wind in miles per hour.

P = pressure of wind, in lbs. per square foot, on a plane surface at right angles to the direction of the wind.

$$P=\frac{V^2}{200}.$$

The above formula also gives the resistance to the motion of a plane surface through the air, the surface being at right angles to the direction of motion.

Velocity of Wind.			Pressure in Lbs. per Square Foot.	Description of Wind.
Miles per Hour.	Feet per Minute.	Feet per Second.		
5	440	7·33	·125	Gentle breeze.
10	880	14·67	·5	Moderate breeze.
15	1320	22	1·125	
20	1760	29·33	2	Strong breeze.
30	2640	44	4·5	High wind.
40	3520	58·67	8	Heavy gale.
50	4400	73·33	12·5	Storm.
60	5280	88	18	Violent storm.
70	6160	102·67	24·5	
80	7040	117·33	32	Hurricane.
90	7920	132	40·5	
100	8800	146·67	50	

Normal Pressure of Wind on an Oblique Plane Surface.

P = pressure of wind on a plane at right angles to the direction of the wind, in lbs. per square foot.

p = normal pressure of wind on a plane surface inclined to the direction of the wind at an angle θ degrees, in lbs. per square foot.

$$\frac{p}{P} = (\sin\theta)^{1.84\cos\theta - 1}\quad \text{(Hutton's formula.)}$$

$$\log\frac{p}{P} = (1{\cdot}84\cos\theta - 1)\log\sin\theta.$$

Angle θ in Degrees.	$\frac{p}{P}$	Values of p when $P=$			
		30	40	50	56
5	·131	3·93	5·24	6·55	7·34
10	·241	7·23	9·64	12·05	13·50
15	·350	10·50	14·00	17·50	19·60
20	·457	13·71	18·28	22·85	25·59
25	·563	16·89	22·52	28·15	31·53
30	·663	19·89	26·52	33·15	37·13
35	·754	22·62	30·16	37·70	42·22
40	·834	25·02	33·36	41·70	46·70
45	·901	27·03	36·04	45·05	50·46
50	·952	28·56	38·08	47·60	53·31
60	1·012	30·36	40·48	50·60	56·67
70	1·023	30·69	40·92	51·15	57·29
80	1·010	30·30	40·40	50·50	56·56

When $\theta = 90°$. $p = P$.

The following formulæ are sometimes preferred to Hutton's :—

$$\frac{p}{P}=\frac{2\sin\theta}{1+\sin^2\theta}.\quad\text{(Duchemin's formula.)}$$

$$\frac{p}{P}=\frac{(4+\pi)\sin\theta}{4+\pi\sin\theta}.\quad\text{(Rayleigh and Gerlach's formula.)}$$

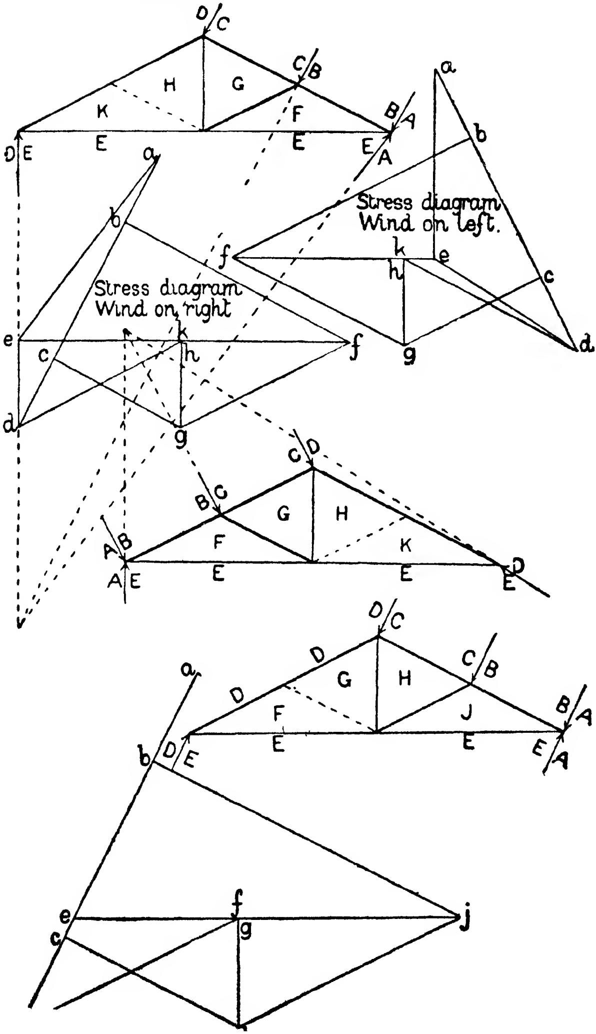
Stresses Due to Wind Pressure on a Roof Truss.

The direction of the wind may be assumed to be horizontal, and its pressure on a plane at right angles to its direction may be taken at 50 lbs. per square foot. The inclination of the roof being known, the normal pressure of the wind on it may be determined by one of the formulæ in the preceding article. The wind is assumed to act on one side of the roof only at one time. The total load due to the wind pressure is divided up into parts, which are placed at the joints, as explained on p. 273, for a distributed load.

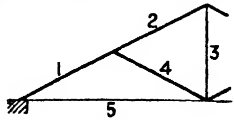
The upper figures on page 281 show the stress diagrams for a roof truss (1) when the wind acts on the right-hand side, and (2) when the wind acts on the left-hand side. In this example the truss is supposed to be fixed at the right-hand end, and supported on friction rollers, or attached to the top of a long column at the left-hand end. The reaction at the left-hand end must therefore be vertical, and the line of the reaction at the other end must pass through the point where the resultant of the wind pressure cuts the line of the reaction at the left-hand end.

The directions of the reactions having been fixed, the load polygon *abcde* can be drawn, and upon this the stress diagram is built, as in previous examples. It will be noticed that the wind pressure causes no stress in the bar *HK*.

In the example shown by the lower figures on p. 281 the truss is supposed to be fixed at both ends, and the reactions are assumed to be parallel to the normal wind pressure. The magnitudes of the reactions are determined by the rules for parallel forces, or by the principle of moments. In this case it is only necessary to draw the stress diagram for the wind on one side, because the diagram for the wind on the other side will be similar to the first, but reversed.

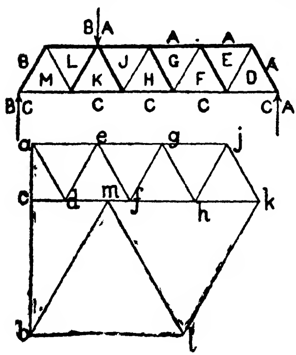

Stress diagram
Wind on left.
Stress diagram
Wind on right

Maximum Stresses.—The maximum stress in any member of a truss is obtained by adding the greatest stress due to the wind pressure to the stress due to the dead load. (The dead load includes the weight of the truss, the weight of the roof covering, and an allowance for the weight of snow.) For convenience the stresses are tabulated, as in the following example (+ denotes compression, and − denotes tension):—



Member	Dead Load W.	Wind on Right P.	Wind on Left. Q	Maximum Stress.	
1	+6760	+3230	+4570	$+W+Q$	+11330
2	+4370	+3230	+2600	$+W+P$	+ 7600
3	−1880	−1420	−1420	$-W-P$	− 3300
4	+2190	none	+3300	$+W+Q$	+ 5490
5	−5940	−2200	−5190	$-W-Q$	−11130

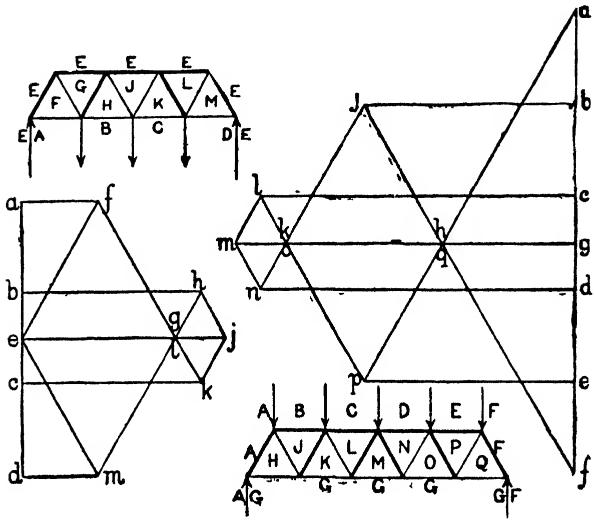
Examples of Stress Diagrams for Bridge Trusses.



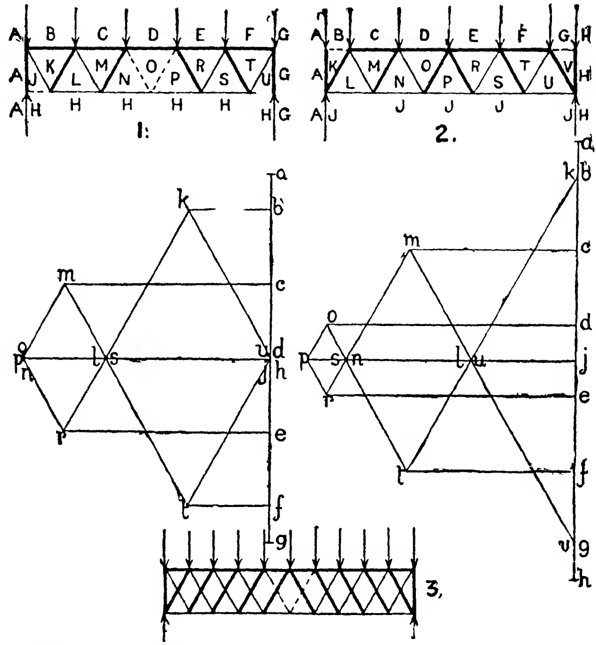
The adjacent figure shows the stress diagram for a *Warren girder* or *half-lattice girder* carrying a single load AB concentrated at one of the joints in the top boom.

The reactions BC and CA (equal to bc and ca respectively) are first determined either by taking moments, or by the graphic method explained on p. 286. The stress diagram is then built up on bc and ca, starting with the forces acting at one of the points of support.

The left-hand figure below shows the stress diagram for a Warren girder when loaded equally at the intermediate joints of the bottom boom, and the right-hand figure shows the stress diagram for the same kind of girder when loaded equally at the joints of the top boom.



Lattice Girders.—If two elementary braced girders (Figs. 1 and 2, p. 284) be placed side by side and joined together, a *lattice girder* (Fig. 3) is obtained. The stresses in the bars of the elementary girders are determined by means of the stress diagrams shown. When the elementary girders are put together to form the compound girder, if any bar of the one coincides with a bar of the other, the stress in the compound bar so formed is equal to the algebraical sum of the stresses in the bars of the elementary girders of which it is made up, and the stresses in those bars of the elementary girders which remain distinct in the compound girder will be unaltered. A lattice girder which is made up of more than two elementary girders is treated in a similar manner.



Note.—Bars shown by dotted lines are not subjected to stress with the assumed manner of loading.

Approximate Weight of Bridges.

The following formula is given in Unwin's "Iron Bridges and Roofs":—

$$w = \frac{Wlr}{Cs - lr}.$$

W = total external distributed weight in tons (exclusive of girder).

w = weight of girder itself in tons.

l = clear span in feet.

s = average stress in tons per square inch of the gross section of the booms, at the centre, usually 4.

r = ratio of span to depth.

C = a coefficient depending on the description of girder.

Values of C in Different Bridges.

Conway, tubular . .	1700	Cannon Street, box girder	1540
Britannia ,, . .	1461	,, ,, plate girder	1598
Torksey ,, . .	1197	Charing Cross, lattice .	1880
Crumlin, Warren . .	1820	Lough Ken, bowstring .	1490

For small plate girders, 30 feet to 60 feet span $C=1500$.

For *Highway Bridges*, Merriman, in his "Roofs and Bridges," gives the following formula :—

$$w=140+12b+0{\cdot}2bl-0{\cdot}4l.$$

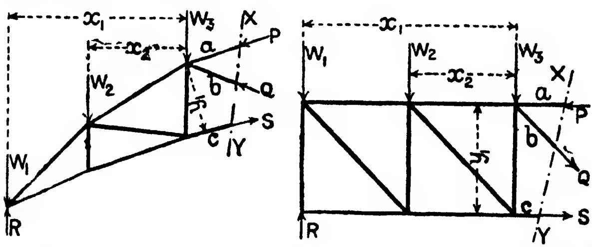
w = weight of bridge in lbs. per linear foot.
l = span in feet.
b = width in feet.

For *Railway Bridges* of less than 100 feet span, Merriman gives the formulæ :—

$$w=\ \ 560+\ \ 5{\cdot}6l, \text{ for single track.}$$
$$w=1070+10{\cdot}7l, \text{ for double track.}$$

w = total dead load of bridge in lbs. per linear foot.
l = span in feet.

Method of Sections.—Suppose that a braced structure is cut transversely into two parts by a plane XY, the plane cutting three bars a, b, and c. Next suppose that one part is removed, and that external forces P, Q, and S are applied to the bars a, b, and c respectively, so as to keep the remaining part of the structure in equilibrium.



Taking moments about the point of intersection of the bars a and b,

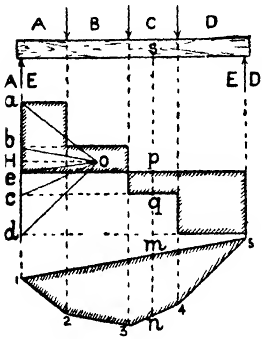
$$Sy_1 \quad Rx_1 - W_1x_1 - W_2x_2,$$

since the external forces are in equilibrium, hence the stress in the bar c is determined.

Then, in like manner, by taking moments about another point in the bar b, the force P is determined. Lastly, taking in like manner moments about a third point, the force Q is determined.

Shearing Force and Bending Moment Diagrams.

The illustration shows a horizontal beam carrying three vertical loads *AB*, *BC*, and *CD*, the reactions at the supports being *DE* and *EA*.



Shearing Force Diagram.—Draw *ad*, the line of loads. Through the points *a*, *b*, *c*, *d*, and *e* draw horizontals across the spaces lettered *A*, *B*, *C*, *D*, and *E* respectively. The shearing force at any section *S* is equal to *pq*, measured with the force scale.

Bending Moment Diagram.—Select a point *o*. Join *oa*, *ob*, *oc*, and *od*. Across the space *A* draw 12 parallel to *oa*, across the space *B* draw 23 parallel to *ob*, across the space *C* draw 34 parallel to *oc*, and across the space *D* draw 45 parallel to *od*. The closing line 15 across the space *E* will be parallel to *oe*. (*Note.*—This construction may be used for finding the point *e*, and therefore the magnitudes of the reactions *DE* and *EA*.) The bending moment at any section *S* is equal to *mn* (measured with the force scale) multiplied by the perpendicular *oH* on the line of loads (measured with the linear scale). Hence *mn*, measured with a suitable scale, measures the bending moment at *S*.

PROPERTIES OF ROLLED STEEL SECTIONS.

The tables on pp. 288–321 are reproduced with the kind permission of Messrs. Dorman, Long & Co., Ltd., Middlesbrough, from their *Handbook for Constructional Engineers*, and with the consent of the British Standards Institution.

Dimensions and Properties

Complete tables are given of dimensions and properties of the various sections illustrated, dimensions being in inches and properties in inch units.

The areas and properties have been carefully calculated on correct profiles and full sections without holing. In the case of beams, however, the net moment of inertia is also given. All fillets, rounded corners, taper of flanges, etc., have been taken into consideration. In the tables of properties of channels the sections marked with an asterisk are obtained by lifting the rolls. In the tables of unequal and equal angles the sections printed in Roman figures are the standards with true profiles, and the sections printed in italics are obtained by adjusting the rolls.

Least Radii of Gyration.

The least radii of gyration have been determined for all sections, and will be found in the tables, the values being given in inches.

In sections such as beams, channels, tees, and equal angles, which have an axis of symmetry, this radius is either about that axis or one at right angles to it. In the case of unequal angles, having no axis of symmetry, the position of the axis, about which the radius is least, has been calculated and is given in the tables; this axis is shown in the diagrams as "Minor Axis" and is the "Major Axis" of the "Ellipse of Inertia," or axis V–V in the case of unequal angles.

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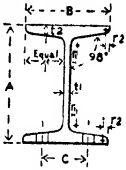

BEAMS

Dimensions and Properties

Reference Mark	Size A × B ins.	Weight per foot lbs.	Standard Thicknesses Web t_1	Standard Thicknesses Flange t_2	Radii Root r_1	Radii Toe r_2	Depth between Fillets D	Centres of holes C	Dia. of rivet or bolt
BSB 140	24 × 7½	95	·57	1·011	·73	·36	20·22	4½	⅞
" 139	22 × 7	75	·50	·834	·69	·34	18·68	4	"
" 138	20 × 7½	89	·60	1·010	·73	·36	16·22	4½	"
" 137	20 × 6½	65	·45	·820	·65	·32	16·81	3¾	"
" 136	18 × 8	80	·50	·950	·77	·38	14·23	4¾	"
" 135	18 × 7	75	·55	·928	·69	·34	14·50	4	"
" 134	18 × 6	55	·42	·757	·61	·30	15·03	3½	¾
" 133	16 × 8	75	·48	·938	·77	·38	12·26	4¾	⅞
" 132	16 × 6	62	·55	·847	·61	·30	12·86	3½	¾
" 131	16 × 6	50	·40	·726	·61	·30	13·09	3½	"
" 130	15 × 6	45	·38	·655	·61	·30	12·23	3½	"
" 129	15 × 5	42	·42	·647	·53	·26	12·46	2¾	"
" 128	14 × 8	70	·46	·920	·77	·38	10·29	4¾	⅞
" 127	14 × 6	57	·50	·873	·61	·30	10·80	3½	¾
" 126	14 × 6	46	·40	·698	·61	·30	11·14	3½	"
" 125	13 × 5	35	·35	·604	·53	·26	10·54	2¾	"
" 124	12 × 8	65	·43	·904	·77	·38	8·32	4¾	⅞
" 123	12 × 6	54	·50	·883	·61	·30	8·78	3½	¾
" 122	12 × 6	44	·40	·717	·61	·30	9·12	3½	"
" 121	12 × 5	32	·35	·550	·53	·26	9·66	2¾	"

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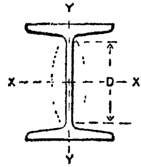
BEAMS

Dimensions and Properties

Note.—One hole is deducted from each flange in calculating the Nett Moment of Inertia about X-X.



Size A × B ins.	Area sq. ins.	Moments of Inertia			Radii of Gyration		Section Moduli	
		About X-X		About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
		Max.	Nett					
24 × 7½	27·94	2533·04	2290	62·54	9·52	1·50	211·09	16·68
22 × 7	22·06	1676·80	1505·	41·07	8·72	1·36	152·44	11·73
20 × 7½	26·19	1672·85	1507·	62·54	7·99	1·55	167·29	16·68
20 × 6½	19·12	1226·17	1088·	32·56	8·01	1·31	122·62	10·02
18 × 8	23·53	1292·07	1167·	69·43	7·41	1·72	143·56	17·36
18 × 7	22·09	1151·18	1026·	46·56	7·22	1·45	127·91	13·30
18 × 6	16·18	841·76	752·	23·64	7·21	1·21	93·53	7·88
16 × 8	22·06	973·91	878	68·30	6·64	1·76	121·74	17·08
16 × 6	18·21	725·05	647·	27·14	6·31	1·22	90·63	9·05
16 × 6	14·71	618·09	551·	22·47	6·48	1·24	77·26	7·49
15 × 6	13·24	491·91	439·	19·87	6·10	1·23	65·59	6·62
15 × 5	12·36	428·49	375·	11·81	5·89	·93	57·13	4·72
14 × 8	20·59	705·58	634·	66·67	5·85	1·80	100·80	16·67
14 × 6	16·78	533·34	473·	27·94	5·64	1·29	76·19	9·31
14 × 6	13·59	442·57	394·	21·45	5·71	1·26	63·22	7·15
13 × 5	10·30	283·51	246·	10·82	5·25	1·03	43·62	4·33
12 × 8	19·12	487·77	437·	65·18	5·05	1·85	81·30	16·30
12 × 6	15·89	375·77	332·	28·28	4·86	1·33	62·63	9·43
12 × 6	13·00	316·76	281·	22·12	4·94	1·30	52·79	7·37
12 × 5	9·45	221·07	192·	9·69	4·84	1·01	36·84	3·88

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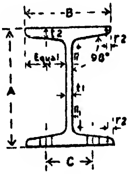

BEAMS

Dimensions and Properties

Reference Mark	Size A × B ins.	Weight per foot lbs	Standard Thicknesses: Web t_1	Standard Thicknesses: Flange t_2	Radii: Root r_1	Radii: Toe r_2	Depth between Fillets D	Centres of holes C	Dia. of rivet or bolt
BSB 120	10 × 8	55	40	783	77	·38	6·56	4½	⅞
" 119	10 × 6	40	·36	·709	·61	·30	7·13	3½	¾
" 118	10 × 5	30	·36	·552	·53	·26	7·64	2¾	¾
" 117	10 × 4½	25	30	505	49	·24	7·84	2½	¾
" 116	9 × 7	50	·40	825	69	·34	5·69	4	⅞
" 115	9 × 4	21	·30	·457	45	·22	7·04	2¼	⅝
" 114	8 × 6	35	35	648	·61	·30	5·25	3½	¾
" 113	8 × 5	28	·35	·575	·53	·26	5·60	2¾	¾
" 112	8 × 4	18	·28	·398	·45	·22	6·16	2¼	⅝
" 111	7 × 4	16	·25	·387	·45	·22	5·18	2¼	⅝
" 110	6 × 5	25	·41	·520	·53	·26	3·72	2¾	¾
" 109	6 × 4½	20	·37	·431	·49	·24	4·00	2½	¾
" 108	6 × 3	12	·23	·377	·37	·18	4·41	1½	½
" 107	5 × 4½	20	·29	·513	·49	·24	2·83	2½	¾
" 106	5 × 3	11	·22	·376	·37	·18	3·40	1½	½
" 105	4¾ × 1¾	6·5	·18	·325	·27	·13	3·52	¾	⅜
" 104	4 × 3	10	·24	·347	·37	·18	2·47	1½	½
" 103	4 × 1¾	5	·17	·239	·27	·13	2·94	¾	⅜
" 102	3 × 3	8·5	·20	·332	·37	·18	1·50	1½	½
" 101	3 × 1½	4	·16	·249	·25	·12	1·97	¾	⅜

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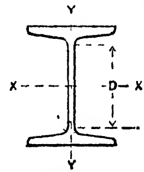
BEAMS

Dimensions and Properties



Note.—One hole is deducted from each flange in calculating the Nett Moment of Inertia about X-X.

Size A × B ins.	Area sq. ins	Moments of Inertia			Radii of Gyration		Section Moduli	
		About X-X		About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
		Max.	Nett					
10 × 8	16 18	288 69	259·	54·74	4·22	1 84	57·74	13·69
10 × 6	11 77	204·80	181·	21·76	4·17	1·36	40·96	7·25
10 × 5	8 85	146·23	127·	9·73	4·06	1·05	29 25	3·89
10 × 4½	7·35	122 34		6·49	4·08	·94	24 47	2 88
9 × 7	14·71	208·13		40·17	3·76	1·65	46·25	11·48
9 × 4	6·18	81·13		4·15	3·62	·82	18·03	2·07
8 × 6	10 30	115·06		19·54	3·34	1·38	28·76	6·51
8 × 5	8·28	89·69		10 19	3·29	1·11	22·42	4·08
8 × 4	5·30	55·63		3·51	3·24	·81	13·91	1·75
7 × 4	4·75	39 51		3·37	2 89	·84	11·29	1·69
6 × 5	7·37	43·69		9·10	2·44	1·11	14·56	3·64
6 × 4½	5·89	34·71		5·40	2·43	·96	11·57	2·40
6 × 3	3·53	20·99		1·46	2·44	·64	7·00	·97
5 × 4½	5·88	25 03		6·59	2·06	1·06	10·01	2·93
5 × 3	3·26	13·68		1·45	2·05	·67	5·47	·97
4¾ × 1¾	1·91	6·73		·26	1·88	·37	2·83	·30
4 × 3	2·94	7·79		1·33	1·63	~~·67~~	~~2·89~~	·88
4 × 1¾	1·47	3·66		·19	1·58	·36	1·83	·21
3 × 3	2·52	3·81		1·25	1·23	·70	2·54	·83
3 × 1½	1·18	1·66		·13	1·19	·33	1·11	·17

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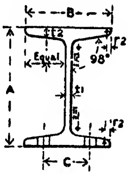


SPECIAL BEAMS

Dimensions and Properties

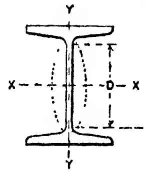
Reference Mark	Size A × B ins.	Weight per foot lbs.	Standard Thicknesses: Web t_1	Standard Thicknesses: Flange t_2	Radii: Root r_1	Radii: Toe r_2	Depth between Fillets D	Centres of holes C	Dia. of rivet or bolt
BSB 30	24 × 7½	100	·60	1·07	70	·35	20·16	4½	⅞
BSB 26	15 × 6	59·	·50	·880	·60	·30	11·82	3½	¾
DLB 25a	15 × 5*	39·5	·40	·59	·52	·26	—	2¾	¾
NBSB 12	14 × 5½	40·	·37	·627	·57	·28	11·39	3¼	¾
DLB 20a	12 × 5	39·	·44	664	·54	·27	9·41	2¾	¾
NBSB 10	12 × 5	30·	·33	·507	·53	·26	9·74	2¾	¾
BSB 19	10 × 8	70·	·60	·970	·70	·35	6·32	4¾	⅞
BSB 18	10 × 6	42·	·40	·736	50	·25	7·27	3½	¾
BSB 16	9 × 7	58·	·55	·924	·65	·325	5·58	4	⅞
NBSB 6	7 × 3½	15·	·25	·398	·41	·20	5·26	2	½
BSB 7	5 × 4½	18·	·29	·448	·39	·195	3·12	2½	¾
NBSB 4	5 × 2½	9·	·20	·347	·33	·16	3·57	1⅜	⅜
BSB 4	4 × 3	9·5	·22	·336	·32	·16	2·58	1¼	½

DORMAN, LONG & CO., LIMITED

SPECIAL BEAMS

Dimensions and Properties

Note.—One hole is deducted from each flange in calculating the Nett Moments of Inertia.



Size A × B ins.	Area sq. ins.	Moments of Inertia			Radii of Gyration		Section Moduli	
		About X-X		About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
		Max.	Nett					
24 × 7½	29·4	2654·	2397·	66·92	9·50	1·50	221·1	17·84
15 × 6	17·35	628·9	559·	28·22	6·02	1·27	83·85	9·406
15 × 5	11·62	399·03	—	10·60	5·86	·955	53·2	4·242
14 × 5½	11·77	377·1	333·	14·79	5·66	1·12	53·87	5·377
12 × 5	11·47	260·9	226·	12·16	4·77	1·03	43·48	4·86
12 × 5	8·827	206·9	180·	8·77	4·84	·997	34·49	3·508
10 × 8	20·6	344·9	309·	71·67	4·09	1·86	68·98	17·91
10 × 6	12·35	211·5	186·	22·95	4·13	1·36	42·3	7·65
9 × 7	17·06	229·5	—	46·3	3·66	1·64	51·0	13·22
7 × 3½	4·416	35·90	—	2·408	2·85	·738	10·26	1·376
5 × 4½	5·29	22·69	—	5·664	2·07	1·03	9·076	2·517
5 × 2½	2·647	10·91	—	·789	2·03	·546	4·364	·631
4 × 3	2·794	7·52	—	1·281	1·64	·677		

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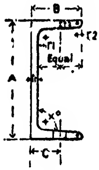


CHANNELS

Dimensions and Properties

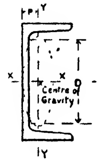
Reference Mark	Size A × B inches	Weight per foot lbs.	Standard Thicknesses Web t_1	Standard Thicknesses Flange t_2	Angle of Flange X	Radii Root r_1	Radii Toe r_2	Depth between fillet D	Back Mark for Holes C	Dia. of Rivet or Bolt
BSC 120a	17×4	51·28	·60*	·68	95°	·60	·30	14·24	2¼	⅞
〃 120	17×4	44·34	·48	·68	〃	·60	·30	14·23	〃	〃
〃 119a	15×4	42·49	·53*	·62	〃	·60	·30	12·36	〃	〃
〃 119	15×4	36·37	·41	·62	〃	·60	·30	12·35	〃	〃
〃 118a	13×4	38·92	·53*	·62	〃	·60	·30	10·36	〃	〃
〃 118	13×4	33 18	·40	·62	〃	·60	·30	10·35	〃	〃
〃 117a	12×4	36·63	·53*	·60	〃	·60	·30	9·40	〃	〃
〃 117	12×4	31·33	·40	·60	〃	·60	·30	9·39	〃	〃
〃 116a	12×3½	30·45	·48*	·50	〃	·54	·27	9·75	2	〃
〃 116	12×3½	26·37	·38	·50	〃	·54	·27	9·74	〃	〃
〃 115a	11×3½	30·52	·48*	·58	〃	·54	·27	8·59	〃	〃
〃 115	11×3½	26·78	·38	·58	〃	·54	·27	8·58	〃	〃
〃 114a	10×3½	28·54	·48*	·56	〃	·54	·27	7·63	〃	〃
〃 114	10×3½	24·46	·36	·56	〃	·54	·27	7·62	〃	〃
〃 113a	10×3	21·33	·38*	·45	〃	·48	·24	7·99	1¾	¾
〃 113	10×3	19·28	·32	·45	〃	·48	·24	7·99	〃	〃
〃 112b	9×3½	25·63	·45*	·54	〃	·54	·27	6·66	2	⅞
〃 112a	9×3½	23·49	·38*	·54	〃	·54	·27	6·66	〃	〃
〃 112	9×3½	22·27	·34	·54	〃	·54	·27	6·65	〃	〃
〃 111a	9×3	19·91	·38*	·44	〃	·48	·24	7·01	1¾	¾
〃 111	9×3	17·46	·30	·44	〃	·48	·24	7·00	〃	〃

* Web thickness obtained by raising the rolls.

DORMAN, LONG & CO., LIMITED

CHANNELS

Dimensions and Properties



Size A × B inches	Area square inches	Dimension P	Moments of Inertia		Radii of Gyration		Section Moduli	
			About X-X	About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
17×4	15·08	·91	569·31	16·96	6·14	1·06	66·98	5·28
17×4	13·04	·92	520·18	15·26	6·32	1·08	61·20	4·96
15×4	12·50	·94	382·85	14·97	5·54	1·09	51·05	4·71
15×4	10·70	·97	349·10	13·34	5·71	1·12	46·55	4·40
13×4	11·45	1·01	270·66	14·51	4·86	1·13	41·64	4·64
13×4	9·76	1·04	246·86	12·76	5·03	1·14	37·98	4·31
12×4	10·77	1·02	218·81	13·80	4·51	1·13	36·47	4·44
12×4	9·21	1·06	200·09	12·12	4·66	1·15	33·35	4·12
12×3½	8·96	·81	174·13	7·96	4·41	·94	29·02	2·86
12×3½	7·76	·83	159·73	7·15	4·54	·96	26·62	2·68
11×3½	8·98	·91	152·96	8·86	4·13	·99	27·81	3·30
11×3½	7·88	·93	141·87	7·93	4·24	1·00	25·80	3·09
10×3½	8·39	·94	119·52	8·50	3·77	1·01	23·90	3·17
10×3½	7·19	·97	109·52	7·42	3·90	1·02	21·90	2·93
10×3	6·27	·73	87·66	4·31	3·74	·83	17·53	1·85
10×3	5·67	·74	82·66	3·98	3·82	·84	16·53	1·76
9×3½	7·54	·97	89·30	7·86	3·44	1·02	19·84	2·98
9×3½	6·91	·99	85·05	7·26	3·51	1·03	18·90	2·85
9×3½	6·55	1·00	82·62	6·90	3·55	1·03	18·36	2·76
9×3	5·86	·76	67·38	4·18	3·39	·85	14·97	1·80
9×3	5·14	·78	62·52	3·75	3·49	·86	13·89	1·69

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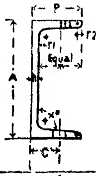

CHANNELS

Dimensions and Properties

Reference Mark	Size A × B inches	Weight per foot lbs.	Standard Thicknesses Web t_1	Standard Thicknesses Flange t_2	Angle of Flange X	Radii Root r_1	Radii Toe r_2	Depth between fillet D	Back Mark for Holes C	Dia. of Rivet or Bolt
BSC 110a	8×3½	23·20	·43*	·52	95	·54	·27	5·70	2	⅞
" 110	8×3½	20·21	·32	·52	"	·54	·27	5·69	2	⅞
" 109a	8×3	18·68	·38*	·44	"	·48	·24	6·01	1¾	¾
" 109	8×3	15·96	·28	·44	"	·48	·24	6·00	1¾	¾
" 108a	7×3½	20·18	·38*	·50	"	·54	·27	4·74	2	⅞
" 108	7×3½	18·28	·30	·50	"	·54	·27	4·73	2	⅞
" 107a	7×3	17·07	·38*	·42	"	·48	·24	5·05	1¾	¾
" 107	7×3	14·22	·26	·42	"	·48	·24	5·04	1¾	¾
" 106a	6×3½	18·52	·38*	·48	"	·54	·27	3·78	2	⅞
" 106	6×3½	16·48	·28	·48	"	·54	·27	3·77	2	⅞
" 105a	6×3	17·53	·43*	·48	"	·48	·24	3·94	1¾	¾
" 105	6×3	16·51	·38	·48	"	·48	·24	3·93	1¾	¾
" 104a	6×3	13·64	·31*	·38	"	·48	·24	4·13	1¾	¾
" 104	6×3	12·41	·25	·38	"	·48	·24	4·12	1¾	¾
" 103a	5×2½	11·24	·31*	·38	"	·42	·21	3·28	1⅜	¾
" 103	5×2½	10·22	·25	·38	"	·42	·21	3·27	1⅜	¾
" 102a	4×2	7·91	·30*	·31	"	·36	·18	2·57	1⅛	⅝
" 102	4×2	7·09	·24	·31	"	·36	·18	2·57	1⅛	⅝
" 101a	3×1½	5·11	·25*	·28	"	·30	·15	1·78	⅞	½
" 101	3×1½	4·60	·20	·28	"	·30	·15	1·78	⅞	½

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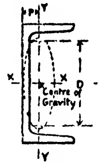
CHANNELS

Dimensions and Properties



Size A × B inches	Area square inches	Dimension P	Moments of Inertia		Radii of Gyration		Section Moduli	
			About X-X	About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
8 × 3½	6·82	1·01	65·27	7·30	3·09	1·03	16·32	2·81
8 × 3½	5·94	1·05	60·57	6·37	3·19	1·04	15·14	2·60
8 × 3	5·49	·81	50·99	4·11	3·05	·87	12·75	1·79
8 × 3	4 69	·83	46·72	3·58	3·16	·87	11·68	1·65
7 × 3½	5·94	1·07	45·12	6·48	2·76	1·05	12·89	2·58
7 × 3½	5·38	1·09	42·83	5·83	2·82	1·04	12·24	2·42
7 × 3	5·02	·84	36·18	3·87	2·68	·88	10·34	1·70
7 × 3	4·18	·88	32·75	3·26	2·80	·88	9·36	1·53
6 × 3½	5·45	1·11	30·68	6·05	2·37	1·05	10·23	2·43
6 × 3½	4·85	1·14	28·88	5·29	2·44	1·05	9·63	2·25
6 × 3	5·16	·90	27·18	3·95	2·30	·86	9·06	1·84
6 × 3	4·86	·91	26·28	3·70	2·33	·87	8·76	1·77
6 × 3	4·01	·87	22·35	3·10	2·36	·88	7·45	1·42
6 × 3	3·65	·89	21·27	2·83	2·41	·88	7·09	1·34
5 × 2½	3·31	·76	12·50	1·82	1·94	·74	5·00	1·01
5 × 2½	3·01	·77	11·87	1·64	1·99	·74	4·75	·95
4 × 2	2·33	·59	5·38	·79	1·52	·58	2·69	·54
4 × 2	2·09	·60	5·06	·70	1·56	·58	2·53	·50
3 × 1½	1·50	·48	1·94	·30	1·14	·44	1·29	·28
3 × 1½	1·35	·48	1·82	·26	1·16	·44	1·22	·26

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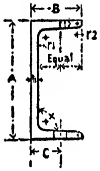


SPECIAL CHANNELS

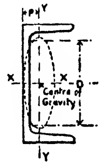
Dimensions and Properties

Reference Mark	Size A × B inches	Weight per foot lbs.	Standard Thicknesses		Angle of Flange X	Radii		Depth between fillet D	Back Mark for Holes C	Dia. of Rivet or Bolt
			Web t_1	Flange t_2		Root r_1	Toe r_2			
BSC 27	15 × 4	41·94	·53	·63	92°	·63	·44	12·40	2¼	⅞
NBSC 14	12 × 3½	25·25	·35	·50	95	·54	·27	9·74	2	⅞
BSC 22	11 × 3½	29·82	·48	·58	92	·58	·40	8·61	2	⅞
BSC 21	10 × 4	30·16	·48	·58	〃	·58	·40	7·60	2¼	⅞
DLC 21a	10 × 4	18·86	·31	·31	〃	·60	·20	8·09	2¼	⅞
BSC 12	8 × 3	19·30	·38	·50	〃	·50	·35	5·94	1¾	¾
DLC 9a	7 × 2⅛	9·75	·23	·33	〃	·33	·23	5·64	1⅛	¾
BSC 6	6 × 3	14·49	·31	·44	〃	·44	·30	4·18	1¾	¾
DLC 5a	5⅛ × 2⅞	16·08	·44	·50	〃	·50	·35	3·07	1½	¾
BSC 4	5 × 2½	10·98	·31	·38	〃	·38	·26	3·43	1⅜	¾
Special	4½ × 1½	13·48	·63	·63	95	·44	·13	2·36	⅞	½
DLC 3a	4 × 3	14·20	·38	·50	92	·50	·35	1·94	1¾	¾
DLC 3b	4 × 3	11·89	·38	·38	〃	·38	·26	2·41	1¾	¾
Special	4 × 2½	10·66	·38	·38	95	·38	·22	2·36	1⅜	¾
BSC 3	4 × 2	7·96	·25	·38	92	·38	·26	2·45	1⅛	⅝
BSC 2	3½ × 2	6·75	·25	·31	〃	·31	·22	2·22	1⅛	⅝
BSC 1	3 × 1½	5·27	·25	·31	〃	·31	·22	1·74	⅞	½
DLC 2a	2½ × 1	4·14	·31	·31	〃	·25	·20	1·37	11/16	⅜

DORMAN, LONG & CO., LIMITED

SPECIAL CHANNELS

Dimensions and Properties



Size A × B inches	Area square inches	Dimension P	Moments of Inertia		Radii of Gyration		Section Moduli	
			About X-X	About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
15 × 4	12·33	·94	377·00	14·55	5·53	1·09	50·27	4·75
12 × 3½	7·43	·85	156·39	7·07	4·59	·98	26·07	2·67
11 × 3½	8·77	·90	148·61	8·42	4·12	·98	27·02	3·23
10 × 4	8·87	1·10	130·72	12·02	3·84	1·16	26·14	4·15
10 × 4	5·55	·93	82·58	7·14	3·86	1·13	16·52	2·32
8 × 3	5·68	·84	53·43	4·33	3·07	·87	13 36	2·01
7 × 2⅛	2·86	·55	20·48	1·07	2·67	·61	5·85	·68
6 × 3	4·26	·94	24·01	3·50	2·37	·91	8·00	1·70
5⅛ × 2⅞	4·73	·92	18·13	3·39	1·96	·85	7·08	1·73
5 × 2½	3·23	·76	12·13	1·77	1·94	·74	4·85	1·02
4½ × 1½	3·96	·48	9·04	·55	1·51	·37	4·02	·54
4 × 3	4·18	1·08	10·15	3·43	1·56	·91	5·08	1·79
4 × 3	3·50	·99	8·54	2·84	1·56	·90	4·27	1·41
4 × 2½	3 14	·77	7·34	1·78	1·53	·75	3·67	1·03
4 × 2	2·34	·66	5·71	·84	1·56	·60	2·86	·63
3½ × 2	1·99	·65	3·70	·71	1·37	·60	2·12	·53
3 × 1½	1·55	·48	1·99	·30	1·14	·44	1·33	·29
2½ × 1	1·22	·33	·93	·09	·87	·26	·74	·13

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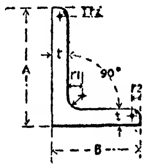


UNEQUAL ANGLES

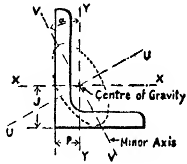
Dimensions and Properties

Reference Mark	Size and Thickness A × B × t	Area sq. ins.	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Dimensions J	Dimensions P
BSUA 123	9×4×⅞	10·61	36·07	·51	·36	3·43	·95
" 123	9×4×13/16	9·90	33·67	"	"	3·41	·92
" 123	9×4×¾	9·19	31·24	"	"	3·38	·90
" 123	9×4×11/16	8·47	28·78	"	"	3·36	·87
" 123	9×4×⅝	7·73	26·30	"	"	3·33	·85
" 123	9×4×9/16	7·00	23·79	"	"	3·30	·82
" 123	9×4×½	6·25	21·25	"	"	3·27	·80
BSUA 122	8×6×⅞	11·48	39·05	·54	·38	2·59	1·60
" 122	8×6×13/16	10·72	36·43	"	"	2·57	1·57
" 122	8×6×¾	9·94	33·79	"	"	2·54	1·55
" 122	8×6×11/16	9·15	31·12	"	"	2·52	1·52
" 122	8×6×⅝	8·36	28·42	"	"	2·49	1·50
" 122	8×6×9/16	7·56	25·70	"	"	2·47	1·47
" 122	8×6×½	6·75	22·95	"	"	2·44	1·45
BSUA 121	8×4×¾	8·44	28·69	·43	·34	2·93	·94
" 121	8×4×11/16	7·78	26·44	"	"	2·91	·92
" 121	8×4×⅝	7·11	24·17	"	"	2·88	·90
" 121	8×4×9/16	6·43	21·87	"	"	2·86	·87
" 121	8×4×½	5·75	19·55	"	"	2·83	·85
BSUA 120	8×3½×⅝	6·80	23·11	·47	·33	3·00	·77
" 120	8×3½×9/16	6·15	20·92	"	"	2·97	·74
" 120	8×3½×½	5·50	18·70	"	"	2·95	·72
" 120	8×3½×7/16	4·84	16·46	"	"	2·92	·69
" 120	8×3½×⅜	4·17	14·19	"	"	2·89	·66
BSUA 119	7×4×¾	7·69	26·14	·45	·32	2·49	1·00
" 119	7×4×11/16	7·09	24·10	"	"	2·47	·98
" 119	7×4×⅝	6·48	22·05	"	"	2·44	·95
" 119	7×4×9/16	5·87	19·96	"	"	2·42	·93
" 119	7×4×½	5·25	17·85	"	"	2·39	·90

DORMAN, LONG & CO., LIMITED

UNEQUAL ANGLES

Dimensions and Properties



Size and Thickness A × B × t	Moment of Inertia About				Radii of Gyration About				Minimum Section Moduli About		Tan α
	X-X	Y-Y	U-U	V-V	X-X	Y-Y	U-U	V-V	X-X	Y-Y	
9 × 4 × 7/8	86·13	10·60	89·48	7·25	2·85	1·00	2·90	·83	15·47	3·47	·21
9 × 4 × 13/16	80·86	9·99	84·06	6·79	2·86	1·00	2·91	·83	14·46	3·25	·21
9 × 4 × 3/4	75·45	9·37	78·49	6·33	2·87	1·01	2·92	·83	13·43	3·02	·21
9 × 4 × 11/16	69·91	8·73	72·78	5·86	2·87	1·02	2·93	·83	12·39	2·79	·21
9 × 4 × 5/8	64·23	8·06	69·91	5·38	2·88	1·02	2·94	·83	11·33	2·56	·21
9 × 4 × 9/16	58·42	7·37	60·88	4·90	2·89	1·03	2·95	·84	10·25	2·32	·21
9 × 4 × 1/2	52·46	6·65	54·70	4·41	2·90	1·03	2·96	·84	9·16	2·08	·22
8 × 6 × 7/8	71·49	34·29	87·16	18·63	2·49	1·73	2·75	1·27	13·22	7·79	·54
8 × 6 × 13/16	67·10	32·24	81·90	17·44	2·50	1·73	2·76	1·28	12·35	7·28	·55
8 × 6 × 3/4	62·60	30·14	76·49	16·24	2·51	1·74	2·77	1·28	11·47	6·77	·55
8 × 6 × 11/16	57·99	27·97	70·93	15·03	2·52	1·75	2·78	1·28	10·58	6·25	·55
8 × 6 × 5/8	53·27	25·74	65·21	13·80	2·52	1·75	2·79	1·28	9·67	5·72	·55
8 × 6 × 9/16	48·43	23·44	59·33	12·54	2·53	1·76	2·80	1·29	8·75	5·18	·55
8 × 6 × 1/2	43·47	21·08	53·28	11·26	2·54	1·77	2·81	1·29	7·82	4·63	·55
8 × 4 × 3/4	54·36	9·14	57·52	5·98	2·54	1·04	2·61	·84	10·73	2·99	·26
8 × 4 × 11/16	50·42	8·52	53·40	5·54	2·55	1·05	2·62	·84	9·90	2·77	·26
8 × 4 × 5/8	46·37	7·87	49·15	5·10	2·55	1·05	2·63	·85	9·06	2·54	·26
8 × 4 × 9/16	42·22	7·20	44·78	4·64	2·56	1·06	2·64	·85	8·21	2·30	·26
8 × 4 × 1/2	37·95	6·50	40·28	4·18	2·57	1·06	2·65	·85	7·34	2·06	·26
8 × 3½ × 5/8	44·24	5·28	45·95	3·57	2·55	·88	2·60	·72	8·85	1·93	·21
8 × 3½ × 9/16	40·30	4·84	41·88	3·25	2·56	·89	2·61	·73	8·02	1·75	·21
8 × 3½ × 1/2	36·24	4·38	37·69	2·93	2·57	·89	2·62	·73	7·17	1·57	·21
8 × 3½ × 7/16	32·08	3·89	33·38	2·59	2·57	·90	2·63	·73	6·31	1·39	·21
8 × 3½ × 3/8	27·79	3·39	28·93	2·25	2·58	·90	2·63	·73	5·44	1·20	·21
7 × 4 × 3/4	37·42	8·86	40·72	5·57	2·21	1·07	2·30	·85	8·30	2·95	·32
7 × 4 × 11/16	34·75	8·26	37·86	5·16	2·21	1·08	2·31	·85	7·67	2·73	·32
7 × 4 × 5/8	32·00	7·64	34·90	4·74	2·22	1·09	2·32	·86	7·02	2·51	·33
7 × 4 × 9/16	29·17	7·00	31·85	4·32	2·23	1·09	2·33	·86	6·37	2·28	·33
7 × 4 × 1/2	26·26	6·33	28·69	3·89	2·24	1·10	2·34	·86	5·70	2·04	·33

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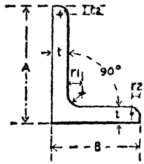

UNEQUAL ANGLES

Dimensions and Properties

Reference Mark	Size and Thickness A × B t	Area sq. ins.	Weight per foot lbs.	Radii		Dimensions	
				Root r_1	Toe r_2	J	P
BSUA 118	$7 \times 3\frac{1}{2} \times \frac{5}{8}$	6·17	20·99	·41	·31	2·55	·81
″ 118	$7 \times 3\frac{1}{2} \times \frac{9}{16}$	5·59	19·01	″	″	2·53	·79
″ 118	$7 \times 3\frac{1}{2} \times \frac{1}{2}$	5·00	17·00	″	″	2·50	·76
″ 118	$7 \times 3\frac{1}{2} \times \frac{7}{16}$	4·40	14·97	″	″	2·47	·74
″ 118	$7 \times 3\frac{1}{2} \times \frac{3}{8}$	3·80	12·91	″	″	2·44	·71
BSUA 117	$6 \times 4 \times \frac{3}{4}$	6·94	23·59	·42	·29	2·06	1·07
″ 117	$6 \times 4 \times \frac{11}{16}$	6·40	21·77	″	″	2·04	1·05
″ 117	$6 \times 4 \times \frac{5}{8}$	5·86	19·93	″	″	2·02	1·02
″ 117	$6 \times 4 \times \frac{9}{16}$	5·31	18·06	″	″	1·99	1·00
″ 117	$6 \times 4 \times \frac{1}{2}$	4·75	16·16	″	″	1·97	·97
″ 117	$6 \times 4 \times \frac{7}{16}$	4·19	14·23	″	″	1·94	·95
″ 117	$6 \times 4 \times \frac{3}{8}$	3·61	12·28	″	″	1·91	·92
BSUA 116	$6 \times 3\frac{1}{2} \times \frac{5}{8}$	5·55	18·86	·41	·29	2·11	·87
″ 116	$6 \times 3\frac{1}{2} \times \frac{9}{16}$	5·03	17·09	″	″	2·09	·85
″ 116	$6 \times 3\frac{1}{2} \times \frac{1}{2}$	4·50	15·30	″	″	2·06	·82
″ 116	$6 \times 3\frac{1}{2} \times \frac{7}{16}$	3·96	13·48	″	″	2·04	·80
″ 116	$6 \times 3\frac{1}{2} \times \frac{3}{8}$	3·42	11·63	″	″	2·01	·77
″ 116	$6 \times 3\frac{1}{2} \times \frac{5}{16}$	2·87	9·76	″	″	1·98	·75
BSUA 115	$6 \times 3 \times \frac{5}{8}$	5·24	17·80	·39	·27	2·22	·73
″ 115	$6 \times 3 \times \frac{9}{16}$	4·75	16·14	″	″	2·20	·71
″ 115	$6 \times 3 \times \frac{1}{2}$	4·25	14·45	″	″	2·17	·68
″ 115	$6 \times 3 \times \frac{7}{16}$	3·75	12·74	″	″	2·15	·66
″ 115	$6 \times 3 \times \frac{3}{8}$	3·24	11·00	″	″	2·12	·63
″ 115	$6 \times 3 \times \frac{5}{16}$	2·72	9·24	″	″	2·09	·61
BSUA 114	$5 \times 4 \times \frac{5}{8}$	5·24	17·80	·39	·27	1·61	1·11
″ 114	$5 \times 4 \times \frac{9}{16}$	4·75	16·14	″	″	1·58	1·09
″ 114	$5 \times 4 \times \frac{1}{2}$	4·25	14·45	″	″	1·56	1·06
″ 114	$5 \times 4 \times \frac{7}{16}$	3·75	12·74	″	″	1·53	1·04
″ 114	$5 \times 4 \times \frac{3}{8}$	3·24	11·00	″	″	1·51	1·01

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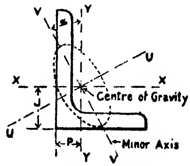
UNEQUAL ANGLES

Dimensions and Properties



Size and Thickness A × B × t	Moment of Inertia About				Radii of Gyration About				Minimum Section Moduli About		Tan α
	X-X	Y-Y	U-U	V-V	X-X	Y-Y	U-U	V-V	X-X	Y-Y	
7 × 3½ × ⅝	30·53	5·14	32·31	3·35	2·22	·91	2·29	·74	6·86	1·91	·26
7 × 3½ × 9/16	27·84	4·71	29·50	3·06	2·23	·92	2·30	·74	6·22	1·74	·26
7 × 3½ × ½	25·07	4·27	26·59	2·75	2·24	·92	2·31	·74	5·57	1·56	·26
7 × 3½ × 7/16	22·22	3·80	23·58	2·44	2·25	·93	2·31	·74	4·91	1·38	·26
7 × 3½ × ⅜	19·28	3·31	20·47	2·12	2·25	·93	2·32	·75	4·23	1·19	·26
6 × 4 × ¾	24·26	8·53	27·75	5·04	1·87	1·11	2·00	·85	6·16	2·91	·43
6 × 4 × 11/16	22·57	7·96	25·86	4·67	1·88	1·12	2·01	·85	5·70	2·70	·43
6 × 4 × ⅝	20·82	7·37	23·90	4·29	1·88	1·12	2·02	·86	5·23	2·48	·43
6 × 4 × 9/16	19·01	6·76	21·85	3·91	1·89	1·13	2·03	·86	4·74	2·25	·43
6 × 4 × ½	17·14	6·11	19·73	3·52	1·90	1·13	2·04	·86	4·25	2·02	·44
6 × 4 × 7/16	15·21	5·44	17·53	3·13	1·91	1·14	2·05	·86	3·75	1·78	·44
6 × 4 × ⅜	13·21	4·74	15·24	2·72	1·91	1·15	2·05	·87	3·23	1·54	·44
6 × 3½ × ⅝	19·85	4·96	21·73	3·08	1·89	·95	1·98	·75	5·11	1·89	·33
6 × 3½ × 9/16	18·13	4·55	19·87	2·81	1·90	·95	1·99	·75	4·63	1·72	·34
6 × 3½ × ½	16·36	4·13	17·95	2·53	1·91	·96	2·00	·75	4·15	1·54	·34
6 × 3½ × 7/16	14·52	3·68	15·59	2·25	1·91	·96	2·01	·75	3·66	1·36	·34
6 × 3½ × ⅜	12·62	3·21	13·87	1·96	1·92	·97	2·01	·76	3·16	1·18	·34
6 × 3½ × 5/16	10·65	2·72	11·72	1·65	1·93	·97	2·02	·76	2·65	·99	·34
6 × 3 × ⅝	18·80	3·14	19·86	2·07	1·89	·77	1·95	·63	4·98	1·38	·25
6 × 3 × 9/16	17·18	2·88	18·18	1·89	1·90	·78	1·96	·63	4·52	1·26	·26
6 × 3 × ½	15·51	2·62	16·43	1·70	1·91	·78	1·97	·63	4·05	1·13	·26
6 × 3 × 7/16	13·78	2·34	14·61	1·51	1·92	·79	1·97	·64	3·58	1·00	·26
6 × 3 × ⅜	11·99	2·05	12·72	1·32	1·93	·80	1·98	·64	3·09	·87	·26
6 × 3 × 5/16	10·13	1·74	10·76	1·12	1·93	·80	1·99	·64	2·59	·73	·26
5 × 4 × ⅝	12·44	7·02	15·79	3·67	1·54	1·16	1·74	·84	3·67	2·43	·62
5 × 4 × 9/16	11·39	6·44	14·48	3·34	1·55	1·16	1·75	·84	3·33	2·21	·62
5 × 4 × ½	10·29	5·83	13·11	3·01	1·56	1·17	1·76	·84	2·99	1·98	·62
5 × 4 × 7/16	9·15	5·19	11·68	2·67	1·56	1·18	1·77	·84	2·64	1·75	·62
5 × 4 × ⅜	7·97	4·53	10·18	2·32	1·57	1·18	1·77	·85	2·28	1·52	·63

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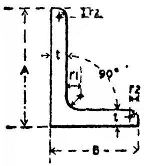


UNEQUAL ANGLES

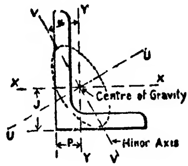
Dimensions and Properties

Reference Mark	Size and Thickness A × B t	Area sq. ins	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Dimensions J	Dimensions P
BSUA 113	$5 \times 3\frac{1}{2} \times \frac{5}{8}$	4·92	16·74	·38	·26	1·69	·94
" 113	$5 \times 3\frac{1}{2} \times \frac{9}{16}$	4·47	15·19	"	"	1·66	·92
" 113	$5 \times 3\frac{1}{2} \times \frac{1}{2}$	4·00	13·61	"	"	1·64	·90
" 113	$5 \times 3\frac{1}{2} \times \frac{7}{16}$	3·53	12·00	"	"	1·62	·87
" 113	$5 \times 3\frac{1}{2} \times \frac{3}{8}$	3·05	10·37	"	"	1·59	·85
" 113	$5 \times 3\frac{1}{2} \times \frac{5}{16}$	2·56	8·71	"	"	1·56	·82
BSUA 112	$5 \times 3 \times \frac{9}{16}$	4·18	14·23	·36	·25	1·76	·77
" 112	$5 \times 3 \times \frac{1}{2}$	3·75	12·75	"	"	1·73	·74
" 112	$5 \times 3 \times \frac{7}{16}$	3·31	11·25	"	"	1·71	·72
" 112	$5 \times 3 \times \frac{3}{8}$	2·86	9·73	"	"	1·68	·69
" 112	$5 \times 3 \times \frac{5}{16}$	2·40	8·17	"	"	1·66	·67
BSUA 111	$4\frac{1}{2} \times 3 \times \frac{9}{16}$	3·90	13·27	·35	·24	1·55	·80
" 111	$4\frac{1}{2} \times 3 \times \frac{1}{2}$	3·50	11·91	"	"	1·52	·78
" 111	$4\frac{1}{2} \times 3 \times \frac{7}{16}$	3·09	10·51	"	"	1·50	·75
" 111	$4\frac{1}{2} \times 3 \times \frac{3}{8}$	2·67	9·09	"	"	1·47	·73
" 111	$4\frac{1}{2} \times 3 \times \frac{5}{16}$	2·25	7·64	"	"	1·44	·70
BSUA 110	$4 \times 3\frac{1}{2} \times \frac{5}{8}$	4·30	14·61	·35	·24	1·29	1·04
" 110	$4 \times 3\frac{1}{2} \times \frac{9}{16}$	3·90	13·27	"	"	1·26	1·01
" 110	$4 \times 3\frac{1}{2} \times \frac{1}{2}$	3·50	11·91	"	"	1·24	·99
" 110	$4 \times 3\frac{1}{2} \times \frac{7}{16}$	3·09	10·51	"	"	1·21	·97
" 110	$4 \times 3\frac{1}{2} \times \frac{3}{8}$	2·67	9·09	"	"	1·19	·94
" 110	$4 \times 3\frac{1}{2} \times \frac{5}{16}$	2·25	7·64	"	"	1·16	·92
BSUA 109	$4 \times 3 \times \frac{9}{16}$	3·62	12·31	·33	·23	1·34	·84
" 109	$4 \times 3 \times \frac{1}{2}$	3·25	11·05	"	"	1·32	·82
" 109	$4 \times 3 \times \frac{7}{16}$	2·87	9·76	"	"	1·29	·80
" 109	$4 \times 3 \times \frac{3}{8}$	2·49	8·45	"	"	1·27	·77
" 109	$4 \times 3 \times \frac{5}{16}$	2·09	7·11	"	"	1·24	·75
BSUA 108	$4 \times 2\frac{1}{2} \times \frac{7}{16}$	2·65	9·02	·32	·22	1·38	·64
" 108	$4 \times 2\frac{1}{2} \times \frac{3}{8}$	2·30	7·81	"	"	1·36	·61
" 108	$4 \times 2\frac{1}{2} \times \frac{5}{16}$	1·93	6·58	"	"	1·33	·59
" 108	$4 \times 2\frac{1}{2} \times \frac{1}{4}$	1·56	5·32	"	"	1·30	·56

DORMAN, LONG & CO., LIMITED

UNEQUAL ANGLES

Dimensions and Properties



Size and Thickness A × B × t	Moment of Inertia About				Radii of Gyration About				Minimum Section Moduli About		Tan α
	X-X	Y-Y	U-U	V-V	X-X	Y-Y	U-U	V-V	X-X	Y-Y	
5 × 3½ × 5/8	11·89	4·74	13·91	2·72	1·55	·98	1·68	·74	3·59	1·86	·47
5 × 3½ × 9/16	10·88	4·36	12·76	2·48	1·56	·98	1·69	·75	3·26	1·69	·47
5 × 3½ × ½	9·84	3·96	11·56	2·24	1·57	·99	1·70	·75	2·93	1·52	·48
5 × 3½ × 7/16	8·76	3·53	10·30	1·99	1·58	1·00	1·71	·75	2·59	1·34	·48
5 × 3½ × 3/8	7·63	3·09	8·99	1·73	1·58	1·01	1·72	·75	2·24	1·16	·48
5 × 3½ × 5/16	6·46	2·62	7·61	1·47	1·59	1·01	1·72	·76	1·88	·98	·48
5 × 3 × 9/16	10·31	2·76	11·37	1·71	1·57	·81	1·65	·64	3·18	1·24	·35
5 × 3 × ½	9·33	2·51	10·31	1·54	1·58	·82	1·66	·64	2·86	1·11	·35
5 × 3 × 7/16	8·31	2·25	9·19	1·37	1·58	·82	1·67	·64	2·53	·99	·36
5 × 3 × 3/8	7·25	1·97	8·03	1·19	1·59	·83	1·68	·65	2·18	·85	·36
5 × 3 × 5/16	6·14	1·68	6·80	1·01	1·60	·84	1·68	·65	1·84	·72	·36
4½ × 3 × 9/16	7·66	2·69	8·76	1·59	1·40	·83	1·50	·64	2·59	1·22	·43
4½ × 3 × ½	6·94	2·45	7·96	1·43	1·41	·84	1·51	·64	2·33	1·10	·43
4½ × 3 × 7/16	6·19	2·19	7·11	1·27	1·42	·84	1·52	·64	2·06	·98	·43
4½ × 3 × 3/8	5·41	1·92	6·22	1·11	1·42	·85	1·53	·64	1·79	·85	·44
4½ × 3 × 5/16	4·59	1·64	5·28	·94	1·43	·85	1·53	·65	1·50	·71	·44
4 × 3½ × 5/8	6·28	4·45	8·55	2·19	1·21	1·02	1·41	·71	2·31	1·81	·74
4 × 3½ × 9/16	5·77	4·09	7·87	1·99	1·22	1·02	1·42	·71	2·11	1·65	·75
4 × 3½ × ½	5·24	3·72	7·16	1·79	1·22	1·03	1·43	·72	1·90	1·48	·75
4 × 3½ × 7/16	4·68	3·32	6·41	1·59	1·23	1·04	1·44	·72	1·68	1·31	·75
4 × 3½ × 3/8	4·09	2·91	5·61	1·39	1·24	1·04	1·45	·72	1·45	1·14	·75
4 × 3½ × 5/16	3·47	2·47	4·77	1·18	1·24	1·05	1·46	·72	1·22	·96	·75
4 × 3 × 9/16	5·48	2·60	6·64	1·44	1·23	·85	1·35	·63	2·06	1·21	·54
4 × 3 × ½	4·97	2·37	6·04	1·30	1·24	·85	1·36	·63	1·85	1·09	·54
4 × 3 × 7/16	4·44	2·13	5·41	1·16	1·24	·86	1·37	·63	1·64	·96	·54
4 × 3 × 3/8	3·89	1·87	4·75	1·01	1·25	·87	1·38	·64	1·42	·84	·55
4 × 3 × 5/16	3·30	1·59	4·04	·86	1·26	·87	1·39	·64	1·20	·71	·55
4 × 2½ × 7/16	4·18	1·24	4·67	·75	1·25	·68	1·33	·53	1·59	·67	·38
4 × 2½ × 3/8	3·66	1·10	4·10	·66	1·26	·69	1·34	·53	1·38	·58	·38
4 × 2½ × 5/16	3·11	·94	3·49	·56	1·27	·70	1·34	·54	1·17	·49	·39
4 × 2½ × ¼	2·54	·77	2·85	·46	1·27	·70	1·35	·54	·94	·40	·39

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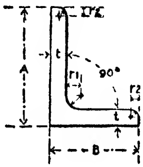


UNEQUAL ANGLES

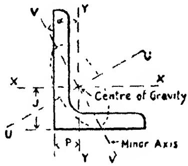
Dimensions and Properties

Reference Mark	Size and Thickness A × B × t	Area sq. ins.	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Dimensions J	Dimensions P
BSUA 107	3½ × 3 × 9/16	3·31	11·36	·32	·22	1·14	·89
" 107	3½ × 3 × ½	3·00	10·20	"	"	1·12	·87
" 107	3½ × 3 × 7/16	2·65	9·02	"	"	1·09	·84
" 107	3½ × 3 × ⅜	2·30	7·81	"	"	1·07	·82
" 107	3½ × 3 × 5/16	1·93	6·58	"	"	1·04	·79
" 107	3½ × 3 × ¼	1·56	5·32	"	"	1·01	·77
BSUA 106	3½ × 2½ × 7/16	2·43	8·28	·30	·21	1·17	·68
" 106	3½ × 2½ × ⅜	2·11	7·17	"	"	1·15	·65
" 106	3½ × 2½ × 5/16	1·78	6·04	"	"	1·12	·63
" 106	3½ × 2½ × ¼	1·44	4·89	"	"	1·09	·60
BSUA 105	3 × 2½ × 7/16	2·22	7·53	·29	·20	·97	·72
" 105	3 × 2½ × ⅜	1·92	6·54	"	"	·94	·70
" 105	3 × 2½ × 5/16	1·62	5·51	"	"	·92	·67
" 105	3 × 2½ × ¼	1·31	4·47	"	"	·89	·65
BSUA 104	3 × 2 × 7/16	2·00	6·79	·27	·19	1·05	·56
" 104	3 × 2 × ⅜	1·73	5·90	"	"	1·03	·53
" 104	3 × 2 × 5/16	1·46	4·98	"	"	1·00	·51
" 104	3 × 2 × ¼	1·19	4·04	"	"	·98	·48
" 104	3 × 2 × 3/16	·90	3·07	"	"	·95	·46
BSUA 103	2½ × 2 × ⅜	1·55	5·26	·26	·18	·82	·57
" 103	2½ × 2 × 5/16	1·31	4·45	"	"	·80	·55
" 103	2½ × 2 × ¼	1·06	3·61	"	"	·77	·53
" 103	2½ × 2 × 3/16	·81	2·75	"	"	·75	·50
BSUA 102	2½ × 1½ × 5/16	1·15	3·92	·24	·17	·89	·39
" 102	2½ × 1½ × ¼	·94	3·19	"	"	·86	·37
" 102	2½ × 1½ × 3/16	·71	2·43	"	"	·83	·34
BSUA 101	2 × 1½ × 5/16	1·00	3·39	·23	·16	·68	·43
" 101	2 × 1½ × ¼	·81	2·76	"	"	·65	·41
" 101	2 × 1½ × 3/16	·62	2·11	"	"	·63	·38

DORMAN, LONG & CO., LIMITED

UNEQUAL ANGLES

Dimensions and Properties



Size and Thickness A × B × t	Moment of Inertia About X-X	Moment of Inertia About Y-Y	Moment of Inertia About U-U	Moment of Inertia About V-V	Radii of Gyration About X-X	Radii of Gyration About Y-Y	Radii of Gyration About U-U	Radii of Gyration About V-V	Minimum Section Moduli About X-X	Minimum Section Moduli About Y-Y	Tan α
3½ × 3 × 9/16	*3·74*	*2·51*	*4·98*	*1·26*	*1·06*	*·87*	*1·22*	*·61*	*1·58*	*1·19*	*·71*
3½ × 3 × ½	3·40	2·28	4·55	1·13	1·06	·87	1·23	·61	1·43	1·07	·71
3½ × 3 × 7/16	*3·04*	*2·05*	*4·08*	*1·01*	*1·07*	*·88*	*1·24*	*·62*	*1·26*	*·95*	*·72*
3½ × 3 × 3/8	2·67	1·80	3·59	·88	1·08	·88	1·25	·62	1·10	·83	·72
3½ × 3 × 5/16	2·27	1·54	3·06	·75	1·08	·89	1·26	·62	·92	·70	·72
3½ × 3 × ¼	*1·86*	*1·26*	*2·50*	*·61*	*1·09*	*·90*	*1·27*	*·62*	*·75*	*·56*	*·72*
3½ × 2½ × 7/16	*2·86*	*1·20*	*3·38*	*·69*	*1·08*	*·70*	*1·18*	*·53*	*1·23*	*·66*	*·49*
3½ × 2½ × 3/8	2·51	1·06	2·98	·59	1·09	·71	1·19	·53	1·07	·57	·49
3½ × 2½ × 5/16	2·14	·91	2·55	·51	1·10	·71	1·20	·53	·90	·48	·50
3½ × 2½ × ¼	1·75	·74	2·08	·41	1·10	·72	1·20	·54	·73	·39	·50
3 × 2½ × 7/16	*1·84*	*1·15*	*2·41*	*·59*	*·91*	*·72*	*1·04*	*·52*	*·91*	*·65*	*·67*
3 × 2½ × 3/8	1·62	1·02	2·13	·51	·92	·73	1·05	·52	·79	·56	·67
3 × 2½ × 5/16	1·39	·87	1·82	·44	·93	·73	1·06	·52	·67	·48	·68
3 × 2½ × ¼	1·14	·72	1·50	·36	·93	·74	1·07	·52	·54	·39	·68
3 × 2 × 7/16	*1·71*	*·59*	*1·94*	*·36*	*·92*	*·55*	*·99*	*·42*	*·88*	*·41*	*·42*
3 × 2 × 3/8	1·50	·53	1·72	·31	·93	·55	1·00	·42	·76	·36	·42
3 × 2 × 5/16	1·29	·45	1·48	·26	·94	·56	1·00	·43	·65	·30	·43
3 × 2 × ¼	1·06	·38	1·22	·22	·94	·56	1·01	·43	·52	·25	·43
3 × 2 × 3/16	*·81*	*·29*	*·94*	*·17*	*·95*	*·57*	*1·02*	*·43*	*·40*	*·19*	*·43*
2½ × 2 × 3/8	*·89*	*·50*	*1·13*	*·27*	*·76*	*·57*	*·85*	*·41*	*·53*	*·35*	*·61*
2½ × 2 × 5/16	·77	·43	·98	·23	·77	·57	·86	·42	·45	·30	·62
2½ × 2 × ¼	·63	·36	·81	·19	·77	·58	·87	·42	·37	·24	·62
2½ × 2 × 3/16	·49	·28	·62	·14	·78	·58	·88	·42	·28	·18	·62
2½ × 1½ × 5/16	*·70*	*·18*	*·77*	*·11*	*·78*	*·40*	*·82*	*·32*	*·43*	*·17*	*·35*
2½ × 1½ × ¼	·58	·15	·64	·09	·78	·40	·82	·32	·35	·14	·35
2½ × 1½ × 3/16	·45	·12	·49	·07	·79	·41	·83	·32	·27	·10	·35
2 × 1½ × 5/16	*·37*	*·17*	*·44*	*·10*	*·61*	*·42*	*·67*	*·31*	*·28*	*·16*	*·53*
2 × 1½ × ¼	·31	·15	·37	·08	·61	·42	68	·31	·23	·13	·54
2 × 1½ × 3/16	·24	·11	·29	·06	·62	·43	·68	·32	·17	·10	·54

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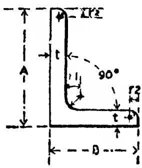


SPECIAL UNEQUAL ANGLES

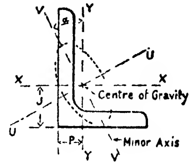
Dimensions and Properties

Reference Mark	Size and Thickness A×B×t	Area sq. ins.	Weight per foot lbs.	Radii		Dimensions	
				Root r_1	Toe r_2	J	P
BSUA 24	6½×4½×¾	7·69	26·13	·45	·325	2·18	1·19
" 24	6½×4½×⅝	6·48	22·04	"	"	2·13	1·14
" 24	6½×4½×½	5·25	17·84	"	"	2·08	1·09
" 24	6½×4½×⅜	3·98	13·54	"	"	2·03	1·04
BSUA 22	6½×3½×¾	6·94	23·59	·425	·30	2·38	·89
" 22	6½×3½×⅝	5·86	19·92	"	"	2·33	·84
" 22	6½×3½×½	4·75	16·15	"	"	2·28	·79
" 22	6½×3½×⅜	3·61	12·27	"	"	2·22	·74
BSUA 19	5½×3½×⅝	5·24	17·80	·40	·275	1·90	·91
" 19	5½×3½×½	4·25	14·46	"	"	1·85	·86
" *19*	*5½×3½×⅜*	*3·24*	*11·00*	"	"	*1·80*	*·81*
" 19	5½×3½×5/16	2·72	9·24	"	"	1·77	·78
BSUA 18	5½×3 ×⅝	4·93	16·75	·375	·25	2·00	·76
" 18	5½×3 ×½	4·00	13·61	"	"	1·95	·71
" 18	5½×3 ×⅜	3·05	10·37	"	"	1·90	·66
" 18	5½×3 ×5/16	2·56	8·71	"	"	1·87	·64
BSUA 14	4½×3½×⅝	4·61	15·67	·35	·25	1·48	·99
" 14	4½×3½×½	3·75	12·75	"	"	1·44	·94
" 14	4½×3½×⅜	2·86	9·72	"	"	1·39	·89
" 14	4½×3½×5/16	2·40	8·17	"	"	1·36	·87
Special	*1¾×1½×¼*	*·69*	*2·34*	*·21*	*·15*	*·59*	*·36*
"	1¾×1¼×3/16	·53	1·79	"	"	·57	·32

DORMAN, LONG & CO., LIMITED

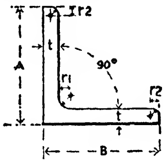
SPECIAL UNEQUAL ANGLES

Dimensions and Properties



Size and Thickness A×B×t	Moment of Inertia About				Radii of Gyration About				Minimum Section Moduli About		Tan ∝
	X-X	Y-Y	U-U	V-V	X-X	Y-Y	U-U	V-V	X-X	Y-Y	
6½×4½×¾	31·66	12·32	36·90	7·07	2·03	1·27	2·19	·96	7·33	3·72	·46
6½×4½×⅝	27·09	10 60	31·67	6·02	2 04	1·28	2·21	·96	6·20	3·16	·47
6½×4½×½	22·24	8·75	26·06	4·93	2·06	1·29	2·23	·97	5·03	2·57	·47
6½×4½×⅜	17·08	6·76	20·04	3·79	2·07	1·30	2·24	·98	3·82	1·95	·47
6½×3½×¾	28·96	5·84	31·01	3·79	2·04	·92	2 11	·74	7·03	2·24	·29
6½×3½×⅝	24·83	5·06	26·66	3·23	2·06	·93	2·13	·74	5·95	1·90	·29
6½×3½×½	20·43	4·20	21·98	2·65	2·07	·94	2·15	·75	4 84	1·55	·30
6½×3½×⅜	15·73	3·27	16·95	2·05	2·09	·95	2·17	·75	3·68	1·18	·30
5½×3½×⅝	15·55	4·86	17·49	2·92	1·72	·96	1·83	·75	4·32	1·87	·39
5½×3½×½	12·84	4·05	14·49	2·40	1·74	·98	1·85	·75	3·52	1·53	·40
5½×3½×⅜	*9·93*	*3·15*	*11·23*	*1 85*	*1·75*	*·99*	*1·86*	*·76*	*2·68*	*1.17*	*·40*
5½×3½×5/16	8·39	2·68	9·50	1·57	1·76	·99	1·87	·76	2·25	·98	·40
5½×3 ×⅝	14·74	3·08	15·83	1·98	1·73	·79	1·79	·63	4·21	1·37	·29
5½×3 ×½	12·19	2·57	13·14	1·63	1·75	·80	1·81	·64	3·41	1·12	·30
5½×3 ×⅜	9·45	2·02	10·2[illegible]	1·26	1·76	·81	1·83	·64	2 62	·86	·30
5½×3 ×5/16	8·00	1·72	8·64	1·07	1·77	82	1·84	·65	2·20	·73	·31
4½×3½×⅝	8·81	4·61	10·93	2·48	1·38	1·00	1·54	·73	2·92	1·83	·58
4½×3½×½	7·31	3·84	9·12	2·04	1·40	1·01	1·56	·74	2·39	1·50	·59
4½×3½×⅜	5·69	3·01	7·12	1·58	1·41	1·03	1·58	·74	1·83	1·15	·59
4½×3½×5/16	4·82	2·55	6·04	1·33	1·42	1·03	1·59	·75	1·53	·97	·59
1¾×1¼×¼	*·20*	*·08*	*0·23*	*·05*	*·53*	*·34*	*·58*	*·26*	*·17*	*·09*	*·48*
1¾×1¼×3/16	·15	·06	0·18	·04	·54	·35	·59	·26	·13	·07	·49

DORMAN, LONG & CO., LIMITED



EQUAL ANGLES

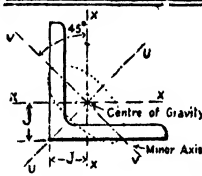
Dimensions and Properties

Reference Mark	Size and Thickness $A \times B \times t$	Area sq. inches	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Dimension J
BSEA 115	$8\times8\times1$	15·00	51·01	·60	·42	2·35
" 115	$8\times8\times\frac{15}{16}$	14·12	48·02	"	"	2·33
" 115	$8\times8\times\frac{7}{8}$	13·24	45·00	"	"	2·30
" 115	$8\times8\times\frac{13}{16}$	12·34	41·96	"	"	2·28
" 115	$8\times8\times\frac{3}{4}$	11·44	38·89	"	"	2·25
" 115	$8\times8\times\frac{11}{16}$	10·53	35·80	"	"	2·23
" 115	$8\times8\times\frac{5}{8}$	9·61	32·68	"	"	2·20
BSEA 114	$7\times7\times1$	13·00	44·20	·54	·38	2·10
" 114	$7\times7\times\frac{15}{16}$	12·25	41·64	"	"	2·08
" 114	$7\times7\times\frac{7}{8}$	11·48	39·05	"	"	2·06
" 114	$7\times7\times\frac{13}{16}$	10·72	36·43	"	"	2·03
" 114	$7\times7\times\frac{3}{4}$	9·94	33·79	"	"	2·01
" 114	$7\times7\times\frac{11}{16}$	9·15	31·12	"	"	1·98
" 114	$7\times7\times\frac{5}{8}$	8·36	28·42	"	"	1·96
" 114	$7\times7\times\frac{9}{16}$	7·56	25·70	"	"	1·93
" 114	$7\times7\times\frac{1}{2}$	6·75	22·95	"	"	1·91
BSEA 113	$6\times6\times\frac{7}{8}$	9·73	31·10	·48	·34	1·81
" 113	$6\times6\times\frac{13}{16}$	9·09	30·90	"	"	1·78
" 113	$6\times6\times\frac{3}{4}$	8·44	28·69	"	"	1·76
" 113	$6\times6\times\frac{11}{16}$	7·78	26·44	"	"	1·74
" 113	$6\times6\times\frac{5}{8}$	7·11	24·17	"	"	1·71
" 113	$6\times6\times\frac{9}{16}$	6·43	21·87	"	"	1·69
" 113	$6\times6\times\frac{1}{2}$	5·75	19·55	"	"	1·66
" 113	$6\times6\times\frac{7}{16}$	5·06	17·20	"	"	1·64
" 113	$6\times6\times\frac{3}{8}$	4·36	14·82	"	"	1·61
BSEA 112	$5\times5\times\frac{3}{4}$	6·94	23·59	·42	·29	1·51
" 112	$5\times5\times\frac{11}{16}$	6·40	21·77	"	"	1·19
" 112	$5\times5\times\frac{5}{8}$	5·86	19·93	"	"	1·47
" 112	$5\times5\times\frac{9}{16}$	5·31	18·06	"	"	1·44
" 112	$5\times5\times\frac{1}{2}$	4·75	16·16	"	"	1·42
" 112	$5\times5\times\frac{7}{16}$	4·19	14·23	"	"	1·39
" 112	$5\times5\times\frac{3}{8}$	3·61	12·28	"	"	1·37

DORMAN, LONG & CO., LIMITED

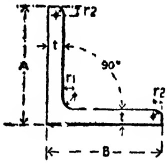
EQUAL ANGLES

Diménsions and Properties



Size and Thickness	Moment of Inertia			Radii of Gyration			Minimum Section Modulus
	About X-X	About U-U	About V-V	About X-X	About U-U	About V-V	About X-X
8×8×1	87·85	139·40	36·31	2·42	3·05	1·56	15·55
8×8×15/16	83·20	132·12	34·28	2·43	3·06	1·56	14·66
8×8×7/8	78·44	124·65	32·23	2·43	3·07	1·56	13·77
8×8×13/16	73·57	116·98	30·16	2·44	3·08	1·56	12·86
8×8×3/4	68·58	109·11	28·06	2·45	3·09	1·57	11·94
8×8×11/16	63·48	101·03	25·93	2·46	3·10	1·57	11·00
8×8×5/8	58·26	92·75	23·78	2·46	3·11	1·57	10·05
7×7×1	57·16	90·97	23·96	2·10	2·65	1·36	11·73
7×7×15/16	54·50	86·37	22·62	2·11	2·66	1·36	11·07
7×7×7/8	51·15	81·64	21·27	2·12	2·67	1·36	10·41
7×7×13/16	48·33	76·75	19·91	2·12	2·68	1·36	9·73
7×7×3/4	45·12	71·72	18·53	2·13	2·69	1·37	9·04
7×7×11/16	41·83	66·53	17·14	2·14	2·70	1·37	8·34
7×7×5/8	38·45	61·19	15·72	2·14	2·71	1·37	7·63
7×7×9/16	34·98	55·68	14·29	2·15	2·71	1·37	6·90
7×7×1/2	31·42	50·02	12·82	2·16	2·72	1·38	6·17
6×6×7/8	31·51	49·86	13·16	1·80	2·26	1·16	7·51
6×6×13/16	29·65	46·99	12·31	1·81	2·27	1·16	7·03
6×6×3/4	27·74	44·01	11·47	1·81	2·28	1·17	6·54
6×6×11/16	25·77	40·92	10·61	1·82	2·29	1·17	6·04
6×6×5/8	23·73	37·73	9·74	1·83	2·30	1·17	5·54
6×6×9/16	21·64	34·42	8·86	1·83	2·31	1·17	5·02
6×6×1/2	19·48	30·99	7·96	1·84	2·32	1·18	4·49
6×6×7/16	17·25	27·45	7·04	1·85	2·33	1·18	3·95
6×6×3/8	14·95	23·79	6·11	1·85	2·34	1·18	3·40
5×5×3/4	15·51	24·57	6·50	1·50	1·88	·97	4·46
5×5×11/16	14·47	22·93	6·02	1·50	1·89	·97	4·12
5×5×5/8	13·37	21·21	5·53	1·51	1·90	·97	3·78
5×5×9/16	12·22	19·42	5·03	1·52	1·91	·97	3·44
5×5×1/2	11·04	17·55	4·53	1·52	1·92	·98	3·08
5×5×7/16	9·81	15·60	4·01	1·53	1·93	·98	2·72
5×5×3/8	8·53	13·57	3·49	1·54	1·94	·98	2·35

DORMAN, LONG & CO., LIMITED



EQUAL ANGLES

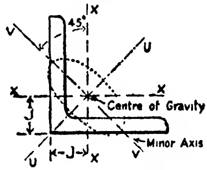
Dimensions and Properties

Reference Mark	Size and Thickness A × B × t	Area sq. inches	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Dimension J
BSEA 111	4½ × 4½ × ¾	6·19	21·04	·39	·27	1·39
" 111	4½ × 4½ × 11/16	5·72	19·44	"	"	1·37
" 111	4½ × 4½ × ⅝	5·24	17·80	"	"	1·34
" 111	4½ × 4½ × 9/16	4·75	16·14	"	"	1·32
" 111	4½ × 4½ × ½	4·25	14·45	"	"	1·29
" 111	4½ × 4½ × 7/16	3·75	12·74	"	"	1·27
" 111	4½ × 4½ × ⅜	3·24	11·00	"	"	1·24
" 111	4½ × 4½ × 5/16	2·72	9·24	"	"	1·22
BSEA 110	4 × 4 × ¾	5·44	18·49	·36	·25	1·26
" 110	4 × 4 × 11/16	5·03	17·10	"	"	1·24
" 110	4 × 4 × ⅝	4·61	15·68	"	"	1·22
" 110	4 × 4 × 9/16	4·18	14·23	"	"	1·20
" 110	4 × 4 × ½	3·75	12·75	"	"	1·17
" 110	4 × 4 × 7/16	3·31	11·25	"	"	1·15
" 110	4 × 4 × ⅜	2·86	9·73	"	"	1·12
" 110	4 × 4 × 5/16	2·40	8·17	"	"	1·10
BSEA 109	3½ × 3½ × ⅝	3·99	13·55	·33	·23	1·09
" 109	3½ × 3½ × 9/16	3·62	12·31	"	"	1·07
" 109	3½ × 3½ × ½	3·25	11·05	"	"	1·05
" 109	3½ × 3½ × 7/16	2·87	9·76	"	"	1·02
" 109	3½ × 3½ × ⅜	2·49	8·45	"	"	1·00
" 109	3½ × 3½ × 5/16	2·09	7·11	"	"	·97
" 109	3½ × 3½ × ¼	1·69	5·74	"	"	·95
BSEA 108	3 × 3 × 9/16	3·06	10·40	·30	·21	·95
" 108	3 × 3 × ½	2·75	9·35	"	"	·92
" 108	3 × 3 × 7/16	2·43	8·28	"	"	·90
" 108	3 × 3 × ⅜	2·11	7·17	"	"	·88
" 108	3 × 3 × 5/16	1·78	6·04	"	"	·86
" 108	3 × 3 × ¼	1·44	4·89	"	"	·83

DORMAN, LONG & CO., LIMITED

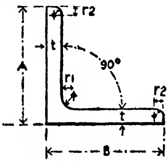
EQUAL ANGLES

Dimensions and Properties



Size and Thickness	Moment of Inertia			Radii of Gyration			Minimum Section Modulus
	About X-X	About U-U	About V-V	About X-X	About U-U	About V-V	About X-X
4½×4½×¾	11·08	17·47	4·68	1·34	1·68	·87	3·56
4½×4½×11/16	10·34	16·34	4·33	1·34	1·69	·87	3·30
4½×4½×⅝	9·56	15·15	3·98	1·35	1·70	·87	3·03
4½×4½×9/16	8·76	13·90	3·62	1·36	1·71	·87	2·75
4½×4½×½	7·92	12·59	3·26	1·37	1·72	·88	2·47
4½×4½×7/16	7·05	11·22	2·89	1·37	1·73	·88	2·18
4½×4½×⅜	6·15	9·78	2·52	1·38	1·74	·88	1·89
4½×4½×5/16	5·21	8·27	2·14	1·38	1·75	·89	1·59
4 ×4 ×¾	7·57	11·89	3·25	1·18	1·48	·77	2·77
4 ×4 ×11/16	7·08	11·15	3·00	1·19	1·49	·77	2·57
4 ×4 ×⅝	6·56	10·37	2·76	1·19	1·50	·77	2·36
4 ×4 ×9/16	6·02	9·54	2·51	1·20	1·51	·77	2·15
4 ×4 ×½	5·46	8·66	2·26	1·21	1·52	·78	1·93
4 ×4 ×7/16	4·87	7·74	2·00	1·21	1·53	·78	1·71
4 ×4 ×⅜	4·26	6·77	1·75	1·22	1·54	·78	1·48
4 ×4 ×5/16	3·61	5·74	1·48	1·23	1·55	·78	1·24
3½×3½×⅝	4·27	6·72	1·82	1·03	1·30	·68	1·77
3½×3½×9/16	3·93	6·20	1·65	1·04	1·31	·68	1·62
3½×3½×½	3·57	5·65	1·49	1·05	1·32	·68	1·46
3½×3½×7/16	3·20	5·07	1·32	1·05	1·33	·68	1·29
3½×3½×⅜	2·80	4·45	1·15	1·06	1·34	·68	1·12
3½×3½×5/16	2·38	3·79	·98	1·07	1·35	·68	·94
3½×3½×¼	1·94	3·09	·80	1·07	1·35	·69	·76
3 ×3 ×9/16	2·39	3·75	1·02	·88	1·11	·58	1·16
3 ×3 ×½	2·18	3·44	·92	·89	1·12	·58	1·05
3 ×3 ×7/16	1·96	3·09	·82	·90	1·13	·58	·93
3 ×3 ×⅜	1·72	2·73	·71	·90	1·14	·58	·81
3 ×3 ×5/16	1·47	2·33	·60	·91	1·16	·58	·68
3 ×3 ×¼	1·20	1·91	·49	·91	1·15	·59	·55

DORMAN, LONG & CO., LIMITED



EQUAL ANGLES

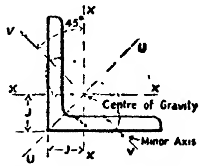
Dimensions and Properties

Reference Mark		Size and Thickness A × B × t	Area sq. inches	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Dimension J
BSEA	107	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{1}{2}$	2·25	7·65	·27	·19	·80
"	107	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{7}{16}$	2·00	6·79	"	"	·78
"	107	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{3}{8}$	1·73	5·90	"	"	·75
"	107	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{5}{16}$	1·46	4·98	"	"	·73
"	107	$2\frac{1}{2}\times2\frac{1}{2}\times\frac{1}{4}$	1·19	4·04	"	"	·70
BSEA	106	$2\frac{1}{4}\times2\frac{1}{4}\times\frac{3}{8}$	1·55	5·26	·26	·18	·69
"	106	$2\frac{1}{4}\times2\frac{1}{4}\times\frac{5}{16}$	1·31	4·45	"	"	·67
"	106	$2\frac{1}{4}\times2\frac{1}{4}\times\frac{1}{4}$	1·06	3·61	"	"	·64
"	106	$2\frac{1}{4}\times2\frac{1}{4}\times\frac{3}{16}$	·81	2·75	"	"	·62
BSEA	105	$2\times2\times\frac{3}{8}$	1·30	4·62	·24	·17	·63
"	105	$2\times2\times\frac{5}{16}$	1·15	3·92	"	"	·61
"	105	$2\times2\times\frac{1}{4}$	·94	3·19	"	"	·58
"	105	$2\times2\times\frac{3}{16}$	·71	2·43	"	"	·56
BSEA	104	$1\frac{3}{4}\times1\frac{3}{4}\times\frac{5}{16}$	1·00	3·39	·23	·16	·54
"	104	$1\frac{3}{4}\times1\frac{3}{4}\times\frac{1}{4}$	·81	2·76	"	"	·52
"	104	$1\frac{3}{4}\times1\frac{3}{4}\times\frac{3}{16}$	·62	2·11	"	"	·49
BSEA	103	$1\frac{1}{2}\times1\frac{1}{2}\times\frac{5}{16}$	·84	2·85	·21	·15	·48
"	103	$1\frac{1}{2}\times1\frac{1}{2}\times\frac{1}{4}$	·69	2·34	"	"	·46
"	103	$1\frac{1}{2}\times1\frac{1}{2}\times\frac{3}{16}$	·53	1·79	"	"	·43
BSEA	102	$1\frac{1}{4}\times1\frac{1}{4}\times\frac{1}{4}$	·56	1·91	·20	·14	·40
"	102	$1\frac{1}{4}\times1\frac{1}{4}\times\frac{3}{16}$	·43	1·47	"	"	·37
"	102	$1\frac{1}{4}\times1\frac{1}{4}\times\frac{1}{8}$	·30	1·01	"	"	·34

DORMAN, LONG & CO., LIMITED

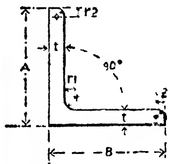
EQUAL ANGLES

Dimensions and Properties



Size and Thickness	Moment of Inertia About X-X	Moment of Inertia About U-U	Moment of Inertia About V-V	Radii of Gyration About X-X	Radii of Gyration About U-U	Radii of Gyration About V-V	Minimum Section Modulus About X-X
2½ × 2½ × ½	1·21	1·89	·52	73	·92	·48	·71
2½ × 2½ × 7/16	1·09	1·71	·46	·74	·93	·48	·63
2½ × 2½ × 3/8	·96	1·52	40	·74	·94	·48	·55
2½ × 2½ × 5/16	·83	1·31	34	·75	·95	·48	·47
2½ × 2½ × ¼	·68	1·08	·28	76	·95	·49	·38
2¼ × 2¼ × 3/8	·69	1·08	·29	·67	84	·43	·44
2¼ × 2¼ × 5/16	·59	·94	·25	67	·85	·43	·37
2¼ × 2¼ × ¼	·49	·77	·20	·68	·85	·44	·30
2¼ × 2¼ × 3/16	·38	·60	·16	·68	·86	·44	·23
2 × 2 × 3/8	·47	·74	·20	·59	·74	·38	·34
2 × 2 × 5/16	·40	64	17	59	75	·38	·29
2 × 2 × ¼	·34	·53	14	·60	·75	·39	·24
2 × 2 × 3/16	26	·41	11	·60	76	·39	·18
1¾ × 1¾ × 5/16	·26	·41	11	51	·64	·34	·22
1¾ × 1¾ × ¼	·22	·35	·09	·52	·65	·34	·18
1¾ × 1¾ × 3/16	·17	·27	·07	·52	·66	·34	·14
1½ × 1½ × 5/16	·16	·25	·07	44	·55	·29	·16
1½ × 1½ × ¼	·13	·21	·06	·44	55	·29	·13
1½ × 1½ × 3/16	·10	·17	·04	·45	·56	29	·10
1¼ × 1¼ × ¼	·07	·12	·03	·36	·45	·24	·09
1¼ × 1¼ × 3/16	·06	·09	·02	·37	·46	·24	·07
1¼ × 1¼ × 1/8	·04	·06	·02	·37	·47	·24	·05

DORMAN, LONG & CO., LIMITED



SPECIAL EQUAL ANGLES

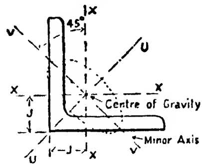
Dimensions and Properties

Reference Mark	Size and Thickness A × B × t	Area sq. inches	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Dimension J
DLEA 20	12×12×1¼	28·44	98·69	·84	·59	3·43
" 20	*12×12×1*	*23·00*	*78·21*	"	"	*3·34*
" 20	12×12×⅞	20·24	68·80	"	"	3·29
NBSEA 9	2¾×2¾×½	2·50	8·50	·29	·20	·86
" 9	2¾×2¾×⅜	1·92	6·54	"	"	·81
" 9	2¾×2¾×¼	1·31	4·46	"	"	·76

DORMAN, LONG & CO., LIMITED

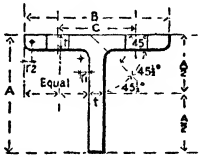
SPECIAL EQUAL ANGLES

Dimensions and Properties



Size and Thickness	Moment of Inertia About X-X	Moment of Inertia About U-U	Moment of Inertia About V-V	Radii of Gyration About X-X	Radii of Gyration About U-U	Radii of Gyration About V-V	Minimum Section Modulus About X-X
12×12×1¼	381·43	606·48	156·38	3·66	4·62	2·34	44·53
12×12×1	*313·36*	*498·88*	*127·84*	*3·69*	*4·66*	*2·36*	*36·17*
12×12×⅞	277·71	442·23	113·19	3·70	4·67	2·36	31·87
2¾×2¾×½	1·64	2·59	·70	·81	1·02	·53	·87
2¾×2¾×⅜	1·30	2·07	·54	·82	1·07	·53	·67
2¾×2¾×¼	·92	1·46	·38	·84	1·05	·54	·46

DORMAN, LONG & CO., LIMITED



TEES

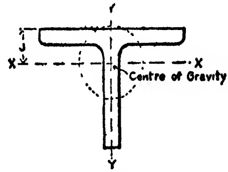
Dimensions and Properties

Reference Mark	Size and Thickness B × A × t	Area square inches	Weight per foot lbs	Radii Root r_1	Radii Toe r_2	Centres of Holes C	Dia. of Rivet or Bolt	Dimension J
BST 119	6 × 6 × ⅝	7·13	24·23	·48	·34	3½	¾	1·69
" 118	6 × 6 × ½	5·77	19·62	·48	·34	3½	"	1·63
" 117	6 × 4 × ⅝	5·88	19·99	·42	·29	3½	"	1·02
" 116	6 × 4 × ½	4·77	16·22	·42	·29	3½	"	·97
" 115	6 × 3 × ½	4·27	14·52	·39	·27	3½	"	·68
" 114	6 × 3 × ⅜	3·26	11·08	·39	·27	3½	"	·63
" 113	5 × 4 × ½	4·27	14·50	·39	·27	2¾	"	1·05
" 112	5 × 4 × ⅜	3·25	11·06	·39	·27	2¾	"	1·00
" 111	5 × 3 × ½	3·77	12·80	·36	·25	2¾	"	·74
" 110	5 × 3 × ⅜	2·88	9·79	·36	·25	2¾	"	·69
" 109	4 × 4 × ½	3·76	12·79	·36	·25	2¼	⅝	1·16
" 108	4 × 4 × ⅜	2·87	9·77	·36	·25	2¼	"	1·10
" 107	4 × 3 × ½	3·26	11·09	·33	·23	2¼	"	·82
" 106	4 × 3 × ⅜	2·50	8·49	·33	·23	2¼	"	·77
" 105	3 × 3 × ⅜	2·12	7·20	·30	·21	1½	½	·87
" 104	2½ × 2½ × ⅜	1·74	5·92	·27	·19	1⅜	⅜	·75
" 103	2½ × 2½ × ¼	1·20	4·07	·27	·19	1⅜	"	·70
" 102	2 × 2 × ¼	·94	3·21	·24	·17	1⅛	¼	·58
" 101	1½ × 1½ × ¼	·69	2·36	·21	·15	¾	"	·46

DORMAN, LONG & CO., LIMITED

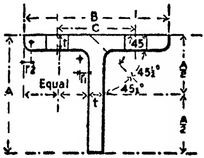
TEES

Dimensions and Properties



Size and Thickness B×A×t	Moments of Inertia		Radii of Gyration		Section Moduli	
	About X-X	About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
6 ×6 ×⅝	23·31	10 87	1·81	1·23	5·40	3·62
6 ×6 ×½	19·04	8·56	1·82	1·22	4·36	2·85
6 ×4 ×⅝	7·33	10·93	1·12	1·36	2·46	3·64
6 ×4 ×½	6 07	8·64	1·13	1·35	2·00	2·88
6 ×3 ×½	2·63	8·67	·78	1·42	1·14	2·89
6 ×3 ×⅜	2·06	6·40	·80	1·40	·87	2·13
5 ×4 ×½	5·77	5·02	1·16	1·09	1·96	2·01
5 ×4 ×⅜	4·47	3·70	1·17	1·07	1·49	1·48
5 ×3 ×½	2·51	5·04	·82	1·16	1·11	2·01
5 ×3 ×⅜	1·97	3·72	·83	1·14	·85	1·49
4 ×4 ×½	5·40	2·59	1·20	·83	1·90	1·30
4 ×4 ×⅜	4·19	1·90	1·21	·81	1·45	·95
4 ×3 ×½	2·37	2·60	·85	·89	1·08	1·30
4 ×3 ×⅜	1·86	1·91	·86	·87	·83	·96
3 ×3 ×⅜	1·71	·81	·90	·62	·80	·54
2½×2½×⅜	·96	·47	·74	·52	·55	·38
2½×2½×¼	·68	·30	·75	·50	·38	·24
2 ×2 ×¼	·34	·16	·60	·41	·24	·16
1½×1½×¼	·14	·07	·44	·31	·13	·09

DORMAN, LONG & CO., LIMITED

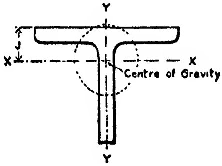


SPECIAL TEES

Dimensions and Properties

Reference Mark	Size and Thickness B×A×t	Area square inches	Weight per foot lbs.	Radii Root r_1	Radii Toe r_2	Centres of Holes C	Dia. of Rivet or Bolt	Dimension J
BST 21	6 ×4 ×3/8	3 63	12·36	·43	·30	3½	3/4	·92
" 13	3½×3½×½	3·26	11·08	·33	·23	2	½	1·04
" 13	3½×3½×3/8	2·50	8·49	·33	·23	2	½	·99
" 11	3 ×3 ×½	2·76	9·38	·30	·20	1½	½	·92
NBST 5	3 ×3 ×5/16	1·79	6·07	·30	·21	1½	½	·84
BST 10	3 ×2½×½	2·51	8·52	·28	·20	1½	½	74
" 10	3 ×2½×3/8	1·93	6·56	·28	·20	1½	½	·70
NBST 4	2½×2½×5/16	1·47	5·00	·27	·19	1 3/8	3/8	·72
BST 7	2¼×2¼×3/8	1·55	5·28	·25	·18	1¼	3/8	·69
" 7	2¼×2¼×¼	1·07	3·64	·25	·18	1¼	3/8	·64
NBST 3	2 ×2 ×3/8	1·37	4·64	·24	·17	1⅛	¼	·63
" 3	2 ×2 ×5/16	1·16	3·94	24	·17	1⅛	¼	·60
DLT 6a	2 ×1½×3/8	1·18	4·01	·23	·15	1⅛	¼	·46
" 6a	2 ×1½×¼	·82	2·79	·23	15	1⅛	¼	·41
BST 4	1¾×1¾×5/16	1·00	3·40	23	·15	7/8	¼	·54
" 4	1¾×1¾×¼	·82	2·79	·23	·15	7/8	¼	·52
" 4	1¾×1¾×3/16	·63	2·14	·23	·15	7/8	¼	49
" 5	1½×2 ×5/16	1·00	3·41	·23	·15	¾	¼	·67
" 5	1½×2 ×¼	·82	2·79	·23	·15	¾	¼	·65
NBST 2	1½×1½×3/16	·53	1·81	·21	·15	¾	¼	·43

DORMAN, LONG & CO., LIMITED



SPECIAL TEES

Dimensions and Properties

Size and Thickness B × A × t	Moments of Inertia		Radii of Gyration		Section Moduli	
	About X-X	About Y-Y	About X-X	About Y-Y	About X-X	About Y-Y
6 × 4 × ⅜	4·70	6·34	1·14	1·32	1·52	2·11
3½ × 3½ × ½	3·54	1·75	1·04	·73	1·44	1·00
3½ × 3½ × ⅜	2·77	1·28	1·05	·72	1·10	·73
3 × 3 × ½	2·17	1·12	·89	·64	1·04	·74
3 × 3 × 5/16	1·46	·67	·90	·61	·67	·44
3 × 2½ × ½	1·28	1·11	·71	·67	·73	·74
3 × 2½ × ⅜	1·02	·81	·73	·65	·56	·54
2½ × 2½ × 5/16	·82	·39	·75	·51	·46	·31
2¼ × 2¼ × ⅜	·69	·35	·66	·47	·44	·31
2¼ × 2¼ × ¼	·49	·22	·68	·46	·30	·20
2 × 2 × ⅜	·47	·25	·59	·43	·34	·25
2 × 2 × 5/16	·41	·20	·59	·42	·29	·20
2 × 1½ × ⅜	·20	·25	·41	·46	·19	·25
2 × 1½ × ¼	·15	·16	·43	·44	·14	·16
1¾ × 1¾ × 5/16	·27	·14	·52	·37	·22	·16
1¾ × 1¾ × ¼	·22	·11	·52	·36	·18	·12
1¾ × 1¾ × 3/16	·17	·08	·52	·35	·14	·09
1½ × 2 × 5/16	·37	·09	·61	·30	·28	·12
1½ × 2 × ¼	·31	·07	·61	·29	·23	·09
1½ × 1½ × 3/16	·11	·05	·45	·30	·10	·07

Weldless Steel Tubes—Moments of Inertia.

Calculated from data supplied by Accles & Pollock, Birmingham.

From p. 213, $I = \frac{\pi}{64}(D^4 - d^4)$ for bending. Values of I in inch^4.

Thickness.		External Diameter in Inches.							
I.S.W.G.	Inch.	½	⅝	¾	⅞	1	1⅛	1¼	1⅜
26	·018	·0008	·0016	·0028	·0045	·0067	·0096	·0132	·0177
25	·020	·0009	·0017	·0031	·0049	·0074	·0106	·0146	·0195
24	·022	·0009	·0019	·0033	·0054	·0081	·0116	·0160	·0214
23	·024	·0010	·0020	·0036	·0058	·0088	·0126	·0174	·0232
22	·028	·0012	·0023	·0041	·0067	·0101	·0145	·0201	·0269
21	·032	·0013	·0026	·0047	·0075	·0114	·0164	·0227	·0305
20	·036	·0014	·0029	·0052	·0084	·0127	·0183	·0253	·0340
19	·040	·0015	·0032	·0056	·0092	·0139	·0201	·0279	·0374
18	·048	·0018	·0036	·0066	·0107	·0163	·0236	·0328	·0441
17	·056	·0020	·0041	·0074	·0121	·0186	·0269	·0375	·0506
..	$\frac{1}{16}$	·0021	·0044	·0080	·0132	·0203	·0295	·0412	·0556
16	·064	·0021	·0045	·0082	·0135	·0207	·0301	·0420	·0568
15	·072	·0023	·0049	·0089	·0148	·0227	·0332	·0464	·0627
14	·080	·0024	·0052	·0096	·0159	·0246	·0361	·0506	·0685
13	·092	·0026	·0056	·0105	·0176	·0273	·0401	·0565	·0767
..	$\frac{3}{32}$	·0026	·0057	·0106	·0178	·0277	·0407	·0573	·0778
12	·104	·0027	·0060	·0113	·0191	·0298	·0439	·0620	·0844
11	·116	·0028	·0063	·0120	·0204	·0320	·0474	·0671	·0917
..	$\frac{1}{8}$	·0029	·0065	·0125	·0213	·0336	·0499	·0708	·0968
10	·128	·0029	·0066	·0126	·0216	·0340	·0506	·0719	·0985
9	·144	·0030	·0069	·0133	·0229	·0365	·0545	·0778	·1069
..	$\frac{5}{32}$	·0030	·0070	·0137	·0239	·0381	·0572	·0819	·1129
8	·160	·0030	·0071	·0139	·0241	·0386	·0580	·0831	·1147
7	·176	·0030	·0072	·0143	·0251	·0404	·0611	·0879	·1217
..	$\frac{3}{16}$	·0031	·0073	·0146	·0257	·0416	·0631	·0911	·1264
6	·192	·0031	·0073	·0147	·0259	·0420	·0638	·0922	·1281
5	·212	·0031	·0074	·0150	·0267	·0437	·0668	·0970	·1353
..	$\frac{7}{32}$	..	·0074	·0151	·0270	·0442	·0677	·0984	·1375
4	·232	..	·0075	·0152	·0274	·0450	·0693	·1011	·1417
..	$\frac{1}{4}$	..	·0075	·0153	·0278	·0460	·0711	·1043	·1467

Weldless Steel Tubes—Moments of Inertia (*continued*).

Thickness.		External Diameter in Inches.							
I.S.W.G.	Inch.	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2½
26	·018	·0230	·0293	·0367	·0453	·0550	·0661	·0786	·1081
25	·020	·0255	·0325	·0407	·0501	·0610	·0733	·0871	·1198
24	·022	·0279	·0356	·0446	·0550	·0669	·0804	·0956	·1315
23	·024	·0303	·0387	·0485	·0598	·0727	·0874	·1040	·1431
22	·028	·0351	·0448	·0562	·0693	·0843	·1014	·1206	·1661
21	·032	·0398	·0508	·0637	·0787	·0958	·1152	·1371	·1889
20	·036	·0444	·0567	·0712	·0880	·1071	·1289	·1535	·2115
19	·040	·0489	·0626	·0786	·0971	·1183	·1424	·1696	·2339
18	·048	·0578	·0740	·0930	·1150	·1403	·1690	·2014	·2780
17	·056	·0663	·0850	·1070	·1325	·1617	·1949	·2324	·3212
..	$\frac{1}{16}$	·0730	·0938	·1181	·1463	·1787	·2155	·2571	·3557
16	·064	·0746	·0958	·1206	·1495	·1826	·2202	·2628	·3636
15	·072	·0825	·1061	·1338	·1660	·2029	·2450	·2924	·4051
14	·080	·0902	·1162	·1467	·1821	·2227	·2691	·3215	·4457
13	·092	·1013	·1306	·1652	·2053	·2515	·3042	·3637	·5052
..	$\frac{3}{32}$	·1028	·1327	·1678	·2086	·2556	·3092	·3698	·5137
12	·104	·1117	·1444	·1829	·2276	·2792	·3380	·4046	·5628
11	·116	·1216	·1575	·1997	·2490	·3058	·3706	·4440	·6187
..	$\frac{1}{8}$	·1287	·1668	·2119	·2644	·3250	·3942	·4727	·6594
10	·128	·1309	·1699	·2158	·2694	·3313	·4020	·4820	·6728
9	·144	·1426	·1854	·2361	·2953	·3637	·4419	·5307	·7423
..	$\frac{5}{32}$	·1509	·1966	·2508	·3141	·3873	·4712	·5663	·7935
8	·160	·1533	·1999	·2551	·3197	·3944	·4799	·5770	·8088
7	·176	·1632	·2134	·2729	·3426	·4233	·5159	·6210	·8725
..	$\frac{3}{16}$	·1699	·2224	·2849	·3582	·4431	·5405	·6514	·9165
6	·192	·1724	·2259	·2895	·3641	·4506	·5499	·6629	·9334
5	·212	·1827	·2402	·3086	·3891	·4826	·5900	·7123	1·0057
..	$\frac{7}{32}$	·1859	·2447	·3147	·3971	·4928	·6029	·7283	1·0292
4	·232	·1920	·2531	·3261	·4121	·5122	·6273	·7586	1·0740
..	$\frac{1}{4}$	·1994	·2637	·3405	·4312	·5369	·6587	·7977	1·1321

Atomic Weights, 1937.

(The Chemical Society, London.)

Element.	Symbol.	At. No.	Atomic Weight.	Element.	Symbol.	At. No.	Atomic Weight.
Aluminium	Al	13	26·97	Neon	Ne	10	20·183
Antimony	Sb	51	121·76	Nickel	Ni	28	58·69
Argon	A	18	39·944	Niobium (Columbium)	Nb (Cb)	41	92·91
Arsenic	As	33	74·91	Nitrogen	N	7	14·008
Barium	Ba	56	137·36	Osmium	Os	76	191·5
Beryllium	Be	4	9·02	Oxygen	O	8	16·0000
Bismuth	Bi	83	209·00	Palladium	Pd	46	106·7
Boron	B	5	10·82	Phosphorus	P	15	31·02
Bromine	Br	35	79·916	Platinum	Pt	78	195·23
Cadmium	Cd	48	112·41	Potassium	K	19	39·096
Cæsium	Cs	55	132·91	Praseodymium	Pr	59	140·92
Calcium	Ca	20	40·08	Protoactinium	Pa	91	231
Carbon	C	6	12·01	Radium	Ra	88	226·05
Cerium	Ce	58	140·13	Radon	Rn	86	222
Chlorine	Cl	17	35·457	Rhenium	Re	75	186·31
Chromium	Cr	24	52·01	Rhodium	Rh	45	102·91
Cobalt	Co	27	58·94	Rubidium	Rb	37	85·48
Copper	Cu	29	63·57	Ruthenium	Ru	44	101·7
Dysprosium	Dy	66	162·46	Samarium	Sm	62	150·43
Erbium	Er	68	167·64	Scandium	Sc	21	45·10
Europium	Eu	63	152·0	Selenium	Se	34	78·96
Fluorine	F	9	19·00	Silicon	Si	14	28·06
Gadolinium	Gd	64	156·9	Silver	Ag	47	107·880
Gallium	Ga	31	69·72	Sodium	Na	11	22·997
Germanium	Ge	32	72·60	Strontium	Sr	38	87·63
Gold	Au	79	197·2	Sulphur	S	16	32·06
Hafnium	Hf	72	178·6	Tantalum	Ta	73	180·88
Helium	He	2	4·002	Tellurium	Te	52	127·61
Holmium	Ho	67	163·5	Terbium	Tb	65	159·2
Hydrogen	H	1	1·0078	Thallium	Tl	81	204·39
Indium	In	49	114·76	Thorium	Th	90	232·12
Iodine	I	53	126·92	Thulium	Tm	69	169·4
Iridium	Ir	77	193·1	Tin	Sn	50	118·70
Iron	Fe	26	55·84	Titanium	Ti	22	47·90
Krypton	Kr	36	83·7	Tungsten	W	74	184·0
Lanthanum	La	57	138·92	Uranium	U	92	238·07
Lead	Pb	82	207·21	Vanadium	V	23	50·95
Lithium	Li	3	6·940	Xenon	Xe	54	131·3
Lutecium	Lu	71	175·0	Ytterbium	Yb	70	173·04
Magnesium	Mg	12	24·32	Yttrium	Y	39	88·92
Manganese	Mn	25	54·93	Zinc	Zn	30	65·38
Mercury	Hg	80	200·61	Zirconium	Zr	40	91·22
Molybdenum	Mo	42	96·0				
Neodymium	Nd	60	144·27				

Specific Gravity.—The specific gravity of a substance (solid or liquid) is the ratio of the weight of a given volume of the substance to the weight of an equal volume of pure water. The specific gravity of a gas is generally given in terms of air or hydrogen.

Weight of Materials. Metals and Alloys.

Material.		Specific Gravity.	Weight in Lbs of one Cubic Ft	Weight in Lbs of one Cubic In.	Cubic In. in One Lb.
Aluminium, cast		2·569	160	·093	10·80
,, wrought		2·681	167	·097	10·35
,, bronze		7·787	485	·281	3·56
Antimony		6 712	418	·242	4·13
Arsenic		5·748	358	·207	4·83
Bismuth		9 827	612	·354	2·82
Brass, cast	from	7·868	490	·284	3·53
	to	8·430	525	·304	3·29
	average	8·109	505	·292	3·42
,, Muntz-metal		8·221	512	·296	3·37
,, naval (rolled)		8·510	530	·307	3·26
,, sheet		8·462	527	·305	3·28
,, wire		8·558	533	·308	3·24
Bronze (gun-metal)	from	8·478	528	·306	3·27
	to	8·863	552	·319	3·13
	average	8·735	544	·315	3·18
Copper, cast		8·622	537	·311	3·22
,, hammered		8·927	556	·322	3·11
,, sheet		8·815	549	·318	3·15
,, wire		8·895	554	·321	3·12
Gold (pure)		19·316	1203	·696	1·44
,, standard 22 carat fine (Gold 11, copper 1)		17·502	1090	·631	1·59
Iron, cast	from	6·904	430	·249	4·02
	to	7·386	460	·266	3·76
	average	7·209	449	·260	3·85
Iron, wrought	from	7·547	470	·272	3·68
	to	7·803	486	·281	3·56
	average	7·707	480	·278	3·60
Lead, cast		11·368	708	·410	2·44
,, sheet		11·432	712	·412	2·43
Manganese		8·012	499	·289	3·46
Nickel, cast		8·285	516	·299	3·35
,, rolled		8·687	541	·313	3·19
Platinum		21·516	1340	·775	1·29
Silver		10·517	655	·379	2·64
Steel	from	7·820	487	·282	3·55
	to	7·916	493	·285	3·51
	average	7·868	490*	·284	3·53
Tin		7·418	462	·267	3·74
White metal (Babbitt's)		7·322	456	·264	3·79
Zinc, cast		6·872	428	·248	4·04
,, sheet		7 209	449	·260	3·85

* A widely accepted value is 489·6 lb. per cu. ft.

Weight of Materials.

Woods (Dry).

Material.	Weight in Lbs. of One Cub. Foot.	Material.	Weight in Lbs. of One Cub. Foot.
Ash	43–53	Larch	31–37
Beech	43–53	Lignum-vitæ	83
Birch	40–46	Mahogany, Honduras	35
Boxwood	57–83	,, Spanish	53
Cork	15	Oak, American red	54
Ebony	70–83	,, English	48–58
Elm	34–45	Pine, red	30–44
Fir, spruce	30–44	,, white	27–34
Greenheart	70	,, yellow	29–41
Hornbeam	47	Teak	41–55

Stones, Earths, etc.

Material.	Weight in Lbs. of One Cub. Foot.	Material.	Weight in Lbs. of One Cub. Foot.
Asphaltum	64–112	Grindstone	134
Brick, common	100–125	Lime, quick	52
,, fire	137–150	Limestones and marbles	150–179
Cement, Portland	80–90	Mortar, hardened	88–118
Clay	120	Mud, dry and close	80–110
Concrete	120–140	,, wet and fluid	104–120
Earth	77–120	Sand, dry	88–110
Glass, crown	156	,, wet	118–129
,, flint	187	Sandstone	130–170
,, plate	169	Victoria stone (crushed granite, Portland cement, silica)	144
Granite	164–175		
Gravel	90–125		

Miscellaneous Substances.

Material.	Weight in Lbs. of One Cub. Foot.	Material.	Weight in Lbs. of One Cub. Foot.
Bone	119	Ivory	117
Grain, barley	40	Lard	59·2
,, oats	33	Leather	60
,, wheat	50	Rosin	69
Guttapercha	61	Sulphur	125
Ice	57·4	Tallow	58
Indiarubber	58	Wax	60·5

Specific Gravity and Bulk of Coal.

Anthracite coal has a specific gravity varying from 1·3 to 1·8, and a ton stowed in the ordinary way occupies from 40 to 45 cubic feet.

Bituminous coal has a specific gravity varying from 1·2 to 1·5, and the bulk of one ton is from 43 to 48 cubic feet.

Relative Weights of Metals.

Metal.	Weight of One Cubic Inch.	Relative Weights. Aluminium =1.	Cast-iron =1.	Wrought-iron =1	Steel =1	Brass =1.	Copper =1.
Aluminium	·097	**1·000**	·373	·349	·342	·318	·305
Cast-iron	·260	2·680	**1·000**	·935	·915	·852	·818
Wrought-iron	·278	2·866	1·069	**1·000**	·979	·911	·874
Steel	·284	2·928	1·092	1·022	**1·000**	·931	·893
Brass	·305	3·144	1·173	1·097	1·074	**1·000**	·959
Copper	·318	3·278	1·223	1·144	1·120	1·043	**1·000**
Lead	·412	4·247	1·585	1·482	1·451	1·351	1·296

Weight of Square Wrought-iron Bars in Lbs. per Foot of Length.

S = side of square in inches.
Weight of bar per foot of length, in lbs. = 3·336 S^2.
The weight of a cubic inch of wrought-iron is taken as ·278 lb.

S	·0	⅛	¼	⅜	½	⅝	¾	⅞
0	..	·0521	·2085	·4691	·8340	1·303	1·876	2·554
1	3·336	4·222	5·212	6·307	7·506	8·809	10·22	11·73
2	13·34	15·06	16·89	18·82	20·85	22·99	25·23	27·57
3	30·02	32·58	35·24	38·00	40·87	43·84	46·91	50·09
4	53·38	56·76	60·26	63·85	67·55	71·36	75·27	79·28
5	83·40	87·62	91·95	96·38	100·9	105·6	110·3	115·1
6	120·1	125·2	130·3	135·6	140·9	146·4	152·0	157·7
7	163·5	169·4	175·3	181·4	187·6	194·0	200·4	206·9
8	213·5	220·2	227·1	234·0	241·0	248·2	255·4	262·8
9	270·2	277·8	285·4	293·2	301·1	309·0	317·1	325·3

Weight of Square Steel Bars in Lbs. per Foot of Length.

S = side of square in inches
Weight of bar per foot of length, in lbs. = 3·408 S^2.
The weight of a cubic inch of steel is taken as ·284 lb.

S	·0	⅛	¼	⅜	½	⅝	¾	⅞
0	...	·0532	·2130	·4792	·8520	1·331	1·917	2 609
1	3·408	4·313	5·325	6·443	7·668	8·999	10·44	11·98
2	13·63	15·39	17·25	19·22	21·30	23·48	25·77	28·17
3	30·67	33·28	36 00	38·82	41·75	44·78	47·92	51·17
4	54·53	57·99	61·56	65·23	69·01	72·90	76·89	80·99
5	85·20	89·51	93·93	98·46	103·1	107·8	112 7	117·6
6	122 7	127·9	133·1	138·5	144·0	149·6	155·3	161·1
7	167·0	173·0	179·1	185·4	191·7	198·1	204·7	211·3
8	218·1	225·0	232·0	239·0	246·2	253·5	260·9	268·4
9	276·0	283·8	291·6	299·5	307·6	315·7	324·0	332·3

Weight of Round Wrought-iron Bars in Lbs. per Foot of Length.

D = diameter of bar in inches.
Weight of bar per foot of length, in lbs. = 2·62 D^2.
The weight of a cubic inch of wrought-iron is taken as ·278 lb.

D	·0	⅛	¼	⅜	½	⅝	¾	⅞
0		·0409	·1638	·3684	·6550	1·023	1·474	2·006
1	2·620	3·316	4·094	4·953	5·895	6·918	8·024	9·211
2	10·48	11·83	13·26	14·78	16·37	18·05	19 81	21·66
3	23·58	25·59	27·67	29·84	32 09	34·43	36·84	39·34
4	41·92	44·58	47·32	50·15	53·05	56·04	59·11	62·27
5	65·50	68·82	72·21	75·69	79·25	82·90	86·62	90·43
6	94·32	98·29	102·3	106·5	110·7	115·0	119·4	123 8
7	128·4	133·0	137·7	142·5	147·4	152·3	157·4	162·5
8	167·7	173 0	178·3	183·8	189·3	194·9	200·6	206 4
9	212·2	218·2	224·2	230·3	236·5	242·7	249·1	255·5
10	262·0	268·6	275·3	282·0	288·9	295 8	302·8	309·9
11	317·0	324·3	331·6	339·0	346·5	354·1	361·7	369·5
12	377·3	385·2	393·2	401·2	409·4	417·6	425·9	434·3
13	442·8	451·3	460·0	468 7	477·5	486·4	495·3	504·4
14	513·5	522·7	532·0	541·4	550·9	560·4	570·0	579·7

Weight of Round Steel Bars in Lbs. per Foot of Length.

D = diameter of bar in inches.
Weight of bar per foot of length, in lbs. = 2·6766 D^2.
The weight of a cubic inch of steel is taken as ·284 lb.

D	·0	⅛	¼	⅜	½	⅝	¾	⅞
0		·0418	·1673	·3764	·6692	1·046	1·506	2·049
1	2·677	3·387	4·182	5·060	6·022	7·068	8·197	9·410
2	10·71	12·09	13·55	15·10	16·73	18·44	20·24	22·12
3	24·09	26·14	28·27	30·49	32·79	35 17	37·64	40·19
4	42·83	45·54	48·35	51·23	54 20	57·25	60·39	63·61
5	66·91	70·30	73·77	77·33	80·97	84·69	88·50	92·38
6	96·36	100·4	104·6	108·8	113·1	117·5	122·0	126·5
7	131·2	135·9	140·7	145·6	150·6	155·6	160·8	166·0
8	171·3	176·7	182·2	187·7	193·4	199·1	204·9	210·8
9	216·8	222·9	229·0	235·2	241·6	248·0	254·4	261·0
10	267·7	274·4	281·2	288·1	295·1	302·2	309·3	316·5
11	323·9	331·3	338·8	346·3	354·0	361·7	369·5	377·4
12	385·4	393·5	401·7	409·9	418·2	426·6	435·1	443·7
13	452·3	461·1	469·9	478·8	487·8	496·9	506·0	515·3
14	524·6	534·0	543·5	553·1	562·8	572·5	582·3	592·2

Weight of Flat Wrought-iron Bars in Lbs. per Foot of Length.

Weight per foot of length = width × thickness × 3·336.
The width and thickness are both in inches.
The weight of a cubic inch of wrought-iron is taken as ·278 lb.

Width in Ins.	Thickness in Fractions of an Inch.								
	1/16	1/8	1/4	3/8	1/2	5/8	3/4	7/8	1
½	·104	·208	·417	·625	·834	1·042	1·251	1·459	1·668
⅝	·130	·261	·521	·782	1·042	1·303	1·564	1·824	2·085
¾	·156	·313	·625	·938	1·251	1·564	1·876	2·189	2·502
⅞	·182	·365	·730	1·095	1·459	1·824	2·189	2·554	2·919
1	·208	·417	·834	1·251	1·668	2·085	2·502	2·919	3·336
1⅛	·235	·469	·938	1·407	1·876	2·346	2·815	3·284	3·753
1¼	·261	·521	1·042	1·564	2·085	2·606	3·127	3·649	4·170
1⅜	·287	·573	1·147	1·720	2·293	2·867	3·440	4·014	4·587
1½	·313	·625	1·251	1·876	2·502	3·127	3·753	4·378	5·004
1⅝	·339	·678	1·355	2·033	2·710	3·388	4·066	4·743	5·421
1¾	·365	·730	1·459	2·189	2·919	3·649	4·378	5·108	5·838
1⅞	·391	·782	1·564	2·346	3·127	3·909	4·691	5·473	6·255
2	·417	·834	1·668	2·502	3·336	4·170	5·004	5·838	6·672
2¼	·469	·938	1·876	2·815	3·753	4·691	5·629	6·568	7·506
2½	·521	1·042	2·085	3·127	4·170	5·212	6·255	7·297	8·340
2¾	·573	1·147	2·293	3·440	4·587	5·734	6·880	8·027	9·174
3	·625	1·251	2·502	3·753	5·004	6·255	7·506	8·757	10·008
3¼	·678	1·355	2·710	4·066	5·421	6·776	8·131	9·487	10·842
3½	·730	1·459	2·919	4·378	5·838	7·297	8·757	10·216	11·676
3¾	·782	1·564	3·127	4·691	6·255	7·819	9·382	10·946	12·510
4	·834	1·668	3·336	5·004	6·672	8·340	10·008	11·676	13·344
4¼	·886	1·772	3·544	5·317	7·089	8·861	10·633	12·406	14·178
4½	·938	1·876	3·753	5·629	7·506	9·382	11·259	13·135	15·012
4¾	·990	1·981	3·961	5·942	7·923	9·904	11·884	13·865	15·846
5	1·042	2·085	4·170	6·255	8·340	10·425	12·510	14·595	16·680
5½	1·147	2·293	4·587	6·880	9·174	11·467	13·761	16·054	18·348
6	1·251	2·502	5·004	7·506	10·008	12·510	15·012	17·514	20·016
6½	1·355	2·710	5·421	8·131	10·842	13·552	16·263	18·973	21·684
7	1·459	2·919	5·838	8·757	11·676	14·595	17·514	20·433	23·352
7½	1·564	3·127	6·255	9·382	12·510	15·637	18·765	21·892	25·020
8	1·668	3·336	6·672	10·008	13·344	16·680	20·016	23·352	26·688
9	1·876	3·753	7·506	11·259	15·012	18·765	22·518	26·271	30·024
10	2·085	4·170	8·340	12·510	16·680	20·850	25·020	29·190	33·360
11	2·293	4·587	9·174	13·761	18·348	22·935	27·522	32·109	36·696
12	2·502	5·004	10·008	15·012	20·016	25·020	30·024	35·028	40·032

Weight of Flat Steel Bars in Lbs. per Foot of Length.

Weight per foot of length, in lbs. = width × thickness × 3·408.
The width and thickness are both in inches.
The weight of a cubic inch of steel is taken as ·284 lb.

Width in Ins.	Thickness in Fractions of an Inch.								
	1/16	1/8	1/4	3/8	1/2	5/8	3/4	7/8	1
½	·106	·213	·426	·639	·852	1·065	1·278	1·491	1·704
⅝	·133	·266	·532	·799	1·065	1·331	1·597	1·864	2·130
¾	·160	·319	·639	·958	1·278	1·597	1·917	2·236	2·556
⅞	·186	·373	·745	1·118	1·491	1·864	2·236	2·609	2·982
1	·213	·426	·852	1·278	1·704	2·130	2·556	2·982	3·408
1⅛	·240	·479	·958	1·438	1·917	2·396	2·875	3·355	3·834
1¼	·266	·532	1·065	1·597	2·130	2·662	3·195	3·727	4·260
1⅜	·293	·586	1·171	1·757	2·343	2·929	3·514	4·100	4·686
1½	·319	·639	1·278	1·917	2·556	3·195	3·834	4·473	5·112
1⅝	·346	·692	1·384	2·077	2·769	3·461	4·153	4·846	5·538
1¾	·373	·745	1·491	2·236	2·982	3·727	4·473	5·218	5·964
1⅞	·399	·799	1·597	2·396	3·195	3·994	4·792	5·591	6·390
2	·426	·852	1·704	2·556	3·408	4·260	5·112	5·964	6·816
2¼	·479	·958	1·917	2·875	3·834	4·792	5·751	6·709	7·668
2½	·532	1·065	2·130	3·195	4·260	5·325	6·390	7·455	8·520
2¾	·586	1·171	2·343	3·514	4·686	5·857	7·029	8·200	9·372
3	·639	1·278	2·556	3·834	5·112	6·390	7·668	8·946	10·224
3¼	·692	1·384	2·769	4·153	5·538	6·922	8·307	9·691	11·076
3½	·745	1·491	2·982	4·473	5·964	7·455	8·946	10·437	11·928
3¾	·799	1·597	3·195	4·792	6·390	7·987	9·585	11·182	12·780
4	·852	1·704	3·408	5·112	6·816	8·520	10·224	11·928	13·632
4¼	·905	1·810	3·621	5·431	7·242	9·052	10·863	12·673	14·484
4½	·958	1·917	3·834	5·751	7·668	9·585	11·502	13·419	15·336
4¾	1·012	2·023	4·047	6·070	8·094	10·117	12·141	14·164	16·188
5	1·065	2·130	4·260	6·390	8·520	10·650	12·780	14·910	17·040
5½	1·171	2·343	4·686	7·029	9·372	11·715	14·058	16·401	18·744
6	1·278	2·556	5·112	7·668	10·224	12·780	15·336	17·892	20·448
6½	1·384	2·769	5·538	8·307	11·076	13·845	16·614	19·383	22·152
7	1·491	2·982	5·964	8·946	11·928	14·910	17·892	20·874	23·856
7½	1·597	3·195	6·390	9·585	12·780	15·975	19·170	22·365	25·560
8	1·704	3·408	6·816	10·224	13·632	17·040	20·448	23·856	27·264
9	1·917	3·834	7·668	11·502	15·336	19·170	23·004	26·838	30·672
10	2·130	4·260	8·520	12·780	17·040	21·300	25·560	29·820	34·080
11	2·343	4·686	9·372	14·058	18·744	23·430	28·116	32·802	37·488
12	2·556	5·112	10·224	15·336	20·448	25·560	30·672	35·784	40·896

Weight of Sheets or Plates of Various Metals in Lbs. per Square Foot.

Thicknesses advancing by 32nds of an Inch.

Weight of sheet per square foot, in lbs. = thickness in inches × C. $C = 40{\cdot}032$ for wrought-iron, $= 40{\cdot}896$ for steel, $= 45{\cdot}76$ for copper, $= 43{\cdot}92$ for brass, $= 59{\cdot}328$ for lead, $= 37{\cdot}44$ for zinc.

Thickness.	Wrought-iron	Steel	Copper	Brass.	Lead	Zinc.
1/32	1·25	1·28	1·43	1·37	1·85	1·17
1/16	2·50	2·56	2·86	2·74	3·71	2·34
3/32	3·75	3·83	4·29	4·12	5·56	3·51
1/8	5 00	5·11	5·72	5·49	7·42	4·68
5/32	6·25	6·39	7·15	6·86	9·27	5·85
3/16	7·51	7·67	8·58	8·23	11·12	7·02
7/32	8·76	8·95	10·01	9·61	12·98	8·19
1/4	10·01	10·22	11·44	10·98	14·83	9·36
9/32	11·26	11·50	12·87	12·35	16·69	10·53
5/16	12·51	12·78	14·30	13·72	18·54	11·70
11/32	13·76	14·06	15 73	15·10	20·39	12·87
3/8	15·01	15·34	17·16	16·47	22·25	14·04
13/32	16·26	16·61	18·59	17·84	24·10	15·21
7/16	17·51	17·89	20·02	19·21	25·96	16·38
15/32	18·76	19·17	21·45	20·59	27·81	17·55
1/2	20·02	20 45	22·88	21·96	29·66	18·72
17/32	21·27	21·73	24·31	23·33	31·52	19·89
9/16	22·52	23·00	25·74	24·70	33·37	21·06
19/32	23·77	24 28	27·17	26·08	35·23	22·23
5/8	25·02	25·56	28·60	27·45	37·08	23·40
21/32	26·27	26·84	30·03	28 82	38·93	24·57
11/16	27·52	28·12	31·46	30·19	40·79	25·74
23/32	28·77	29 39	32·89	31 57	42·64	26·91
3/4	30·02	30·67	34·32	32·94	44·50	28·08
25/32	31·27	31·95	35·75	34·31	46·35	29·25
13/16	32·53	33·23	37·18	35·68	48·20	30·42
27/32	33·78	34·51	38·61	37·06	50·06	31·59
7/8	35·03	35·78	40·04	38·43	51·91	32·76
29/32	36·28	37·06	41·47	39·80	53·77	33·93
15/16	37·53	38·34	42·90	41·17	55·62	35·10
31/32	38·78	39·62	44·33	42·55	57·47	36·27
1	40·03	40·90	45·76	43·92	59·33	37·44

Weight of Sheets or Plates of Various Metals in Lbs. per Square Foot.

Thicknesses by Imperial Standard Wire Gauge.

Thickness. I.S.W.G	Thickness. Inch.	Wrought-iron.	Steel.	Copper.	Brass.	Lead.	Zinc.
7/0	·500	20·02	20·45	22·88	21·96	29·66	18·72
6/0	·464	18·57	18·98	21·23	20·38	27·53	17·37
5/0	·432	17·29	17·67	19·77	18·97	25·63	16·17
4/0	·400	16·01	16·36	18·30	17·57	23·73	14·98
3/0	·372	14·89	15·21	17·02	16·34	22·07	13·93
2/0	·348	13·93	14·23	15·92	15·28	20·65	13·03
0	·324	12·97	13·25	14·83	14·23	19·22	12·13
1	·300	12·01	12·27	13·73	13·18	17·80	11·23
2	·276	11·05	11·29	12·63	12·12	16·37	10·33
3	·252	10·09	10·31	11·53	11·07	14·95	9·43
4	·232	9·29	9·49	10·62	10·19	13·76	8·69
5	·212	8·49	8·67	9·70	9·31	12·58	7·94
6	·192	7·69	7·85	8·79	8·43	11·39	7·19
7	·176	7·05	7·20	8·05	7·73	10·44	6·59
8	·160	6·41	6·54	7·32	7·03	9·49	5·99
9	·144	5·76	5·89	6·59	6·32	8·54	5·39
10	·128	5·12	5·23	5·86	5·62	7·59	4·79
11	·116	4·64	4·74	5·31	5·09	6·88	4·34
12	·104	4·16	4·25	4·76	4·57	6·17	3·89
13	·092	3·68	3·76	4·21	4·04	5·46	3·44
14	·080	3·20	3·27	3·66	3·51	4·75	3·00
15	·072	2·88	2·94	3·29	3·16	4·27	2·70
16	·064	2·56	2·62	2·93	2·81	3·80	2·40
17	·056	2·24	2·29	2·56	2·46	3·32	2·10
18	·048	1·92	1·96	2·20	2·11	2·85	1·80
19	·040	1·60	1·64	1·83	1·76	2·37	1·50
20	·036	1·44	1·47	1·65	1·58	2·14	1·35
21	·032	1·28	1·31	1·46	1·41	1·90	1·20
22	·028	1·12	1·15	1·28	1·23	1·66	1·05
23	·024	·96	·98	1·10	1·05	1·42	·90
24	·022	·88	·90	1·01	·97	1·31	·82
25	·020	·80	·82	·92	·88	1·19	·75
26	·018	·72	·74	·82	·79	1·07	·67
27	·0164	·66	·67	·75	·72	·97	·61
28	·0148	·59	·61	·68	·65	·88	·55
29	·0136	·54	·56	·62	·60	·81	·51

Weight of Nuts and Bolt Heads.

d = diameter of bolt in inches.

Width of nut or bolt head across the flats $= 1\frac{1}{2}d + \frac{1}{8}$ inch.

Height of nut = height of bolt head = d.

The weight in lbs. is given approximately by the following formulæ:—

Weight of hexagonal nut $= (\cdot 37d + \cdot 09)d^2$.
Weight of square nut $= (\cdot 45d + \cdot 11)d^2$.
Weight of hexagonal bolt head $= (\cdot 55d + \cdot 09)d^2$.
Weight of square bolt head $= (\cdot 63d + \cdot 11)d^2$

The nuts are supposed to be screwed.

The above weights are for wrought-iron or steel nuts. For brass nuts add about 10 per cent.

The following table of weights of nuts and bolt heads has been calculated by means of the above formulæ:—

d	Nuts.		Bolt Heads.		d	Nuts.		Bolt Heads.	
	Hex.	Sq.	Hex	Sq		Hex.	Sq.	Hex.	Sq.
$\frac{1}{2}$	·07	·08	·09	·11	2	3·3	4·0	4·8	5·5
$\frac{3}{4}$	·21	·25	·28	·33	$2\frac{1}{4}$	4·7	5·7	6·7	7·7
1	·46	·56	·64	·74	$2\frac{1}{2}$	6·3	7·7	9·2	10·5
$1\frac{1}{4}$	·86	1·05	1·21	1·40	$2\frac{3}{4}$	8·4	10·2	12·1	13·9
$1\frac{1}{2}$	1·45	1·77	2·06	2·37	3	10·8	13 1	15·7	18·0
$1\frac{3}{4}$	2·26	2·75	3·22	3·71	$3\frac{1}{4}$	13·7	16·6	19·8	22·8

Weight of Tubes, Pipes, or Cylinders.

D = external diameter, in inches.
d = internal diameter, in inches

t = thickness, in inches $= \frac{1}{2}(D - d)$.

w = weight, in lbs., of one cubic inch of material.

$w = \cdot 26$ for cast-iron.	$w = \cdot 308$ for brass.
$= \cdot 278$ for wrought-iron.	$= \cdot 321$ for copper.
$= \cdot 284$ for steel.	$= \cdot 412$ for lead.

W = weight, in lbs., of one lineal foot of tube, pipe, or cylinder.

$W = 12\pi wt(D - t) = 12\pi wt(d + t)$.
$= 9\cdot 8t(D - t) = 9\cdot 8t(d + t)$ for cast-iron.
$= 10\cdot 48t(D - t) = 10\cdot 48t(d + t)$ for wrought-iron.
$= 10\cdot 7t(D - t) = 10\cdot 7t(d + t)$ for steel. See also p. 336.
$= 11\cdot 6t(D - t) = 11\cdot 6t(d + t)$ for brass.
$= 12\cdot 1t(D - t) = 12\cdot 1t(d + t)$ for copper.
$= 15\cdot 5t(D - t) = 15\cdot 5t(d + t)$ for lead.

The tables on pp. 334–341 have been calculated by means of the above formulæ.

Weight of Wrought-iron Tubes in Lbs. per Foot of Length.

For formula see p. 333.

Thickness		External Diameter in Inches.							
I.S W.G.	Ins	½	⅝	¾	⅞	1	1¼	1½	1¾
18	·018	·227	·290	·353	·416	·479	·605	·730	·856
17	·056	·261	·334	·407	·481	·554	·701	·847	·994
16	·064	·292	·376	·460	·544	·628	·795	·963	1·131
15	·072	·323	·417	·512	·606	·700	·889	1·078	1·266
14	·080	·352	·457	·562	·667	·771	·981	1·191	1·400
13	·092	·392	·514	·634	·755	·875	1·116	1·358	1·599
12	·104	·432	·568	·704	·840	·977	1·249	1·522	1·794
11	·116	·467	·619	·771	·923	1·075	1·379	1·683	1·986
10	·128	·499	·667	·834	1·002	1·170	1·505	1·840	2·176
9	·144	·537	·726	·915	1·103	1·292	1·669	2·016	2·424

Thickness		External Diameter in Inches.							
I.S W.G.	Ins	2	2¼	2½	2¾	3	3¼	3½	3¾
13	·092	1·840	2·081	2·322	2·563	2·804	3·045	3·286	3·527
12	·104	2·066	2·339	2·611	2·884	3·156	3·429	3·701	3·974
11	·116	2·290	2·594	2·898	3·202	3·506	3·810	4·114	4·418
10	·128	2·511	2·847	3·182	3·517	3·853	4·188	4·523	4·859
9	·144	2·801	3·178	3·555	3·933	4·310	4·687	5·065	5·442
8	·160	3·085	3·505	3·924	4·343	4·762	5·181	5·601	6·020
7	·176	3·364	3·825	4·287	4·748	5·209	5·670	6·131	6·592
6	·192	3·638	4·141	4·644	5·147	5·650	6·153	6·656	7·159
5	·212	3·973	4·528	5·083	5·639	6·194	6·750	7·305	7·861
4	·232	4·299	4·906	5·514	6·122	6·730	7·338	7·946	8·554

Thickness		External Diameter in Inches.							
I.S W.G.	Ins	4	4½	5	5½	6	6½	7	7½
10	·128	5·19	5·86	6·54	7·21	7·88	8·55	9·22	9·89
9	·144	5·82	6·57	7·33	8·08	8·84	9·59	10·35	11·10
8	·160	6·44	7·28	8·12	8·95	9·79	10·63	11·47	12·31
7	·176	7·05	7·98	8·90	9·82	10·74	11·66	12·59	13·51
6	·192	7·66	8·67	9·67	10·68	11·69	12·69	13·70	14·70
5	·212	8·42	9·53	10·64	11·75	12·86	13·97	15·08	16·19
4	·232	9·16	10·38	11·59	12·81	14·02	15·24	16·46	17·67
3	·252	9·90	11·22	12·54	13·86	15·18	16·50	17·82	19·14
2	·276	10·77	12·22	13·66	15·11	16·56	18·00	19·45	20·90
1	·300	11·63	13·20	14·78	16·35	17·92	19·49	21·06	22·64

Weight of Wrought-iron Tubes in Lbs. per Foot of Length.

For formula see p. 333.

Thickness.		Internal Diameter in Inches.							
I S W G	Ins.	½	⅝	¾	⅞	1	1¼	1½	1¾
18	·048	·276	·339	·401	·464	·527	·653	·779	·904
17	·056	·326	·400	·473	·546	·620	·766	·913	1·060
16	·064	·378	·462	·546	·630	·714	·881	1·049	1·217
15	·072	·432	·526	·620	·715	·809	·998	1·186	1·375
14	·080	·486	·591	·696	·801	·905	1·115	1·325	1·534
13	·092	·571	·691	·812	·932	1·053	1·294	1·535	1·776
12	·104	·658	·795	·931	1·067	1·203	1·476	1·748	2·021
11	·116	·749	·901	1·053	1·205	1·357	1·661	1·965	2·268
10	·128	·842	1·010	1·178	1·345	1·513	1·849	2·184	2·519
9	·144	·972	1·161	1·349	1·538	1·726	2·104	2·481	2·858
		Internal Diameter in Inches							
		2	2¼	2½	2¾	3	3¼	3½	3¾
13	·092	2·017	2·258	2·499	2·740	2·981	3·222	3·463	3·704
12	·104	2·293	2·566	2·838	3·111	3·383	3·656	3·928	4·201
11	·116	2·572	2·876	3·180	3·484	3·788	4·092	4·396	4·700
10	·128	2·855	3·190	3·525	3·861	4·196	4·531	4·867	5·202
9	·144	3·236	3·613	3·990	4·367	4·745	5·122	5·499	5·877
8	·160	3·622	4·041	4·460	4·879	5·299	5·718	6·137	6·556
7	·176	4·014	4·475	4·936	5·397	5·858	6·319	6·780	7·241
6	·192	4·411	4·914	5·417	5·920	6·423	6·926	7·429	7·932
5	·212	4·915	5·470	6·025	6·581	7·136	7·692	8·247	8·803
4	·232	5·427	6·035	6·642	7·250	7·858	8·466	9·074	9·682
		Internal Diameter in Inches.							
		4	4½	5	5½	6	6½	7	7½
10	·128	5·54	6·21	6·88	7·55	8·22	8·89	9·56	10·23
9	·144	6·25	7·01	7·76	8·52	9·27	10·03	10·78	11·54
8	·160	6·98	7·81	8·65	9·49	10·33	11·17	12·01	12·84
7	·176	7·70	8·62	9·55	10·47	11·39	12·31	13·24	14·16
6	·192	8·43	9·44	10·45	11·45	12·46	13·47	14·47	15·48
5	·212	9·36	10·47	11·58	12·69	13·80	14·91	16·02	17·13
4	·232	10·29	11·51	12·72	13·94	15·15	16·37	17·58	18·80
3	·252	11·23	12·55	13·87	15·19	16·51	17·83	19·15	20·47
2	·276	12·37	13·81	15·26	16·71	18·15	19·60	21·05	22·49
1	·300	13·52	15·09	16·66	18·24	19·81	21·38	22·95	24·52

Weight of Steel Tubes in Lb. per Foot of Length.

For formula see p. 333. If wt. of steel is taken as 489·6 lb./cu. ft., $W = 10{\cdot}6814t(D - t)$, and tabulated wts. are high by 1·74 lb. in 1000 lb.

Thickness I.S.W.G.	Thickness Ins	½	⅝	¾	⅞	1	1¼	1½	1¾
18	·048	·232	·296	·361	·425	·489	·617	·746	·874
17	·056	·266	·341	·416	·491	·566	·715	·865	1·015
16	·064	·299	·384	·470	·555	·641	·812	·983	1·155
15	·072	·330	·426	·522	·619	·715	·908	1·100	1·293
14	·080	·360	·467	·574	·681	·788	1·002	1·216	1·430
13	·092	·402	·525	·648	·771	·894	1·140	1·386	1·632
12	·104	·441	·580	·719	·858	·997	1·275	1·553	1·832
11	·116	·477	·632	·787	·942	1·097	1·408	1·718	2·028
10	·128	·509	·681	·852	1·023	1·194	1·537	1·879	2·221
9	·144	·549	·741	·934	1·126	1·319	1·704	2·089	2·475

External Diameter in Inches

Thickness I.S.W.G.	Thickness Ins	2	2¼	2½	2¾	3	3¼	3½	3¾
13	·092	1·878	2·124	2·370	2·617	2·863	3·109	3·355	3·601
12	·104	2·110	2·388	2·666	2·944	3·223	3·501	3·779	4·057
11	·116	2·338	2·649	2·959	3·269	3·580	3·890	4·200	4·511
10	·128	2·564	2·906	3·249	3·591	3·933	4·276	4·618	4·961
9	·144	2·860	3·245	3·630	4·015	4·401	4·786	5·171	5·556
8	·160	3·150	3·578	4·006	4·434	4·862	5·290	5·718	6·146
7	·176	3·435	3·906	4·377	4·847	5·318	5·789	6·260	6·731
6	·192	3·714	4·228	4·742	5·255	5·769	6·282	6·796	7·310
5	·212	4·056	4·623	5·190	5·757	6·324	6·891	7·458	8·026
4	·232	4·389	5·009	5·630	6·251	6·871	7·492	8·112	8·733

External Diameter in Inches.

Thickness I.S.W.G.	Thickness Ins	4	4½	5	5½	6	6½	7	7½
10	·128	5·30	5·99	6·67	7·36	8·04	8·73	9·41	10·10
9	·144	5·94	6·71	7·48	8·25	9·02	9·79	10·56	11·33
8	·160	6·57	7·43	8·29	9·14	10·00	10·85	11·71	12·57
7	·176	7·20	8·14	9·08	10·03	10·97	11·91	12·85	13·79
6	·192	7·82	8·85	9·88	10·90	11·93	12·96	13·99	15·01
5	·212	8·59	9·73	10·86	12·00	13·13	14·26	15·40	16·53
4	·232	9·35	10·59	11·84	13·08	14·32	15·56	16·80	18·04
3	·252	10·11	11·45	12·80	14·15	15·50	16·85	18·20	19·54
2	·276	11·00	12·47	13·95	15·43	16·90	18·38	19·86	21·33
1	·300	11·88	13·48	15·09	16·69	18·30	19·90	21·51	23·11

Weight of Steel Tubes in Lb. per Foot of Length.

For formula see p. 333. For note on weight of steel and tabulated weights see p. 336.

Thickness.		Internal Diameter in Inches.							
I.S.W.G	Ins	½	⅝	¾	⅞	1	1¼	1½	1¾
18	·048	·281	·346	·410	·474	·538	·667	·795	·923
17	·056	·333	·408	·483	·558	·633	·783	·932	1·082
16	·064	·386	·472	·557	·643	·729	·900	1·071	1·242
15	·072	·441	·537	·633	·730	·826	1·018	1·211	1·404
14	·080	·496	·603	·710	·817	·924	1·138	1·352	1·566
13	·092	·583	·706	·829	·952	1·075	1·321	1·567	1·813
12	·104	·672	·811	·950	1·089	1·229	1·507	1·785	2·063
11	·116	·765	·920	1·075	1·230	1·385	1·695	2·006	2·316
10	·128	·860	1·031	1·203	1·374	1·545	1·887	2·230	2·572
9	·144	·992	1·185	1·377	1·570	1·763	2·148	2·533	2·918
		Internal Diameter in Inches.							
		2	2¼	2½	2¾	3	3¼	3½	3¾
13	·092	2·059	2·305	2·552	2·798	3·044	3·290	3·536	3·782
12	·104	2·341	2·620	2·898	3·176	3·454	3·732	4·011	4·289
11	·116	2·626	2·937	3·247	3·557	3·868	4·178	4·488	4·798
10	·128	2·915	3·257	3·599	3·942	4·284	4·627	4·969	5·311
9	·144	3·303	3·689	4·074	4·459	4·844	5·229	5·615	6·000
8	·160	3·698	4·126	4·554	4·982	5·410	5·838	6·266	6·694
7	·176	4·098	4·569	5·039	5·510	5·981	6·452	6·923	7·393
6	·192	4·503	5·017	5·530	6·044	6·558	7·071	7·585	8·098
5	·212	5·018	5·585	6·152	6·719	7·286	7·853	8·420	8·987
4	·232	5·541	6·161	6·782	7·403	8·023	8·644	9·264	9·885
		Internal Diameter in Inches.							
		4	4½	5	5½	6	6½	7	7½
10	·128	5·65	6·34	7·02	7·71	8·39	9·08	9·76	10·45
9	·144	6·39	7·16	7·93	8·70	9·47	10·24	11·01	11·78
8	·160	7·12	7·98	8·83	9·69	10·55	11·40	12·26	13·11
7	·176	7·86	8·81	9·75	10·69	11·63	12·57	13·51	14·46
6	·192	8·61	9·64	10·67	11·69	12·72	13·75	14·78	15·80
5	·212	9·55	10·69	11·82	12·96	14·09	15·23	16·36	17·49
4	·232	10·51	11·75	12·99	14·23	15·47	16·71	17·95	19·19
3	·252	11·47	12·81	14·16	15·51	16·86	18·21	19·55	20·90
2	·276	12·63	14·10	15·58	17·06	18·53	20·01	21·49	22·96
1	·300	13·80	15·41	17·01	18·62	20·22	21·83	23·43	25·04

Weight of Copper Tubes in Lbs. per Foot of Length.

For formula see p. 333.

Thickness. I.S.W.G.	Thickness. Ins	External Diameter in Inches. ½	⅝	¾	⅞	1	1¼	1½	1¾
18	·048	·263	·335	·408	·480	·553	·698	·843	·989
17	·056	·301	·386	·470	·555	·640	·809	·978	1·148
16	·064	·338	·434	·531	·628	·725	·918	1·112	1·306
15	·072	·373	·482	·591	·700	·808	1·026	1·244	1·462
14	·080	·407	·528	·649	·770	·891	1·133	1·375	1·617
13	·092	·451	·593	·732	·872	1·011	1·289	1·567	1·846
12	·104	·498	·656	·813	·970	1·128	1·442	1·757	2·071
11	·116	·539	·714	·890	1·065	1·241	1·592	1·943	2·293
10	·128	·576	·770	·963	1·157	1·351	1·738	2·125	2·512
9	·144	·620	·838	1·056	1·274	1·491	1·927	2·363	2·798

I.S.W.G.	Ins	External Diameter in Inches. 2	2¼	2½	2¾	3	3¼	3½	3¾
13	·092	2·124	2·402	2·681	2·959	3·237	3·515	3·794	4·072
12	·104	2·386	2·700	3·015	3·330	3·644	3·959	4·274	4·588
11	·116	2·644	2·995	3·346	3·697	4·048	4·399	4·750	5·101
10	·128	2·899	3·287	3·674	4·061	4·448	4·835	5·223	5·610
9	·144	3·234	3·669	4·105	4·541	4·976	5·412	5·847	6·283
8	·160	3·562	4·046	4·530	5·014	5·498	5·982	6·466	6·950
7	·176	3·884	4·417	4·949	5·482	6·014	6·546	7·079	7·611
6	·192	4·200	4·781	5·362	5·943	6·524	7·104	7·685	8·266
5	·212	4·587	5·228	5·869	6·510	7·152	7·793	8·434	9·076
4	·232	4·963	5·665	6·367	7·069	7·770	8·472	9·174	9·876

I.S.W.G.	Ins	External Diameter in Inches. 4	4½	5	5½	6	6½	7	7½
10	·128	6·00	6·77	7·55	8·32	9·09	9·87	10·64	11·42
9	·144	6·72	7·59	8·46	9·33	10·20	11·07	11·95	12·82
8	·160	7·43	8·40	9·37	10·34	11·31	12·27	13·24	14·21
7	·176	8·14	9·21	10·27	11·34	12·40	13·47	14·53	15·60
6	·192	8·85	10·01	11·17	12·33	13·49	14·65	15·82	16·98
5	·212	9·72	11·00	12·28	13·56	14·85	16·13	17·41	18·70
4	·232	10·58	11·98	13·38	14·79	16·19	17·60	19·00	20·40
3	·252	11·43	12·95	14·48	16·00	17·53	19·05	20·58	22·10
2	·276	12·44	14·11	15·78	17·45	19·12	20·79	22·46	24·13
1	·300	13·43	15·25	17·06	18·88	20·69	22·51	24·32	26·14

Weight of Copper Tubes in Lbs. per Foot of Length.

For formula see p. 333.

Thickness.		Internal Diameter in Inches							
I S.W.G.	Ins.	½	⅝	¾	⅞	1	1¼	1½	1¾
18	·048	·318	·391	·463	·536	·609	·754	·899	1·044
17	·056	·377	·461	·516	·631	·716	·885	1·054	1·224
16	·064	·437	·534	·630	·727	·824	1·018	1·211	1·405
15	·072	·498	·607	·716	·825	·934	1·152	1·370	1·587
14	·080	·561	·682	·803	·924	1·045	1·287	1·529	1·771
13	·092	·659	·798	·937	1·076	1·216	1·494	1·772	2·051
12	·104	·760	·917	1·075	1·232	1·389	1·704	2·018	2·333
11	·116	·865	1·040	1·216	1·391	1·566	1·917	2·268	2·619
10	·128	·973	1·166	1·360	1·553	1·747	2·134	2·521	2·909
9	·144	1·122	1·340	1·558	1·776	1·993	2·429	2·865	3·300
		Internal Diameter in Inches.							
		2	2¼	2½	2¾	3	3¼	3½	3¾
13	·092	2·329	2·607	2·885	3·164	3·442	3·720	4·00	4·28
12	·104	2·648	2·962	3·277	3·591	3·906	4·221	4·54	4·85
11	·116	2·970	3·321	3·672	4·023	4·374	4·725	5·08	5·43
10	·128	3·296	3·683	4·070	4·457	4·845	5·232	5·62	6·01
9	·144	3·736	4·171	4·607	5·043	5·478	5·914	6·35	6·78
8	·160	4·182	4·666	5·150	5·634	6·118	6·602	7·09	7·57
7	·176	4·634	5·166	5·699	6·231	6·764	7·296	7·83	8·36
6	·192	5·092	5·673	6·254	6·835	7·416	7·996	8·58	9·16
5	·212	5·674	6·316	6·957	7·598	8·239	8·881	9·52	10·16
4	·232	6·266	6·967	7·669	8·371	9·073	9·775	10·48	11·18
		Internal Diameter in Inches.							
		4	4½	5	5½	6	6½	7	7½
10	·128	6·39	7·17	7·94	8·72	9·49	10·27	11·04	11·81
9	·144	7·22	8·09	8·96	9·83	10·71	11·58	12·45	13·32
8	·160	8·05	9·02	9·99	10·96	11·93	12·89	13·86	14·83
7	·176	8·89	9·96	11·02	12·09	13·15	14·22	15·28	16·35
6	·192	9·74	10·90	12·06	13·22	14·39	15·55	16·71	17·87
5	·212	10·80	12·09	13·37	14·65	15·94	17·22	18·50	19·78
4	·232	11·88	13·28	14·69	16·09	17·49	18·90	20·30	21·71
3	·252	12·97	14·49	16·01	17·54	19·06	20·59	22·11	23·64
2	·276	14·28	15·95	17·62	19·29	20·96	22·63	24·30	25·97
1	·300	15·61	17·42	19·24	21·05	22·87	24·68	26·50	28·31

Weight of Cast-iron Pipes in Lbs. per Foot of Length.

For formula see p. 333.

Inside Diam. Inches.	Thickness in Inches								
	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4	1 3/8
1	5·05	7·35	9·95	12·86	16 08	19·60	23·43	27·56	32·00
1½	6·89	9·80	13·02	16·54	20·37	24·50	28·94	33·69	38·74
2	8·73	12·25	16·08	20·21	24·65	29·40	34·45	39 81	45·48
2½	10·57	14·70	19·14	23·89	28·94	34·30	39·97	45·94	52 22
3	12·40	17·15	22·20	27·56	33·23	39·20	45·48	52·06	58·95
3½	14·24	19·60	25·27	31·24	37·52	44·10	50·99	58·19	65·69
4	16 08	22·05	28·33	34·91	41·80	49·00	56·50	64·31	72·43
4½	17·92	24·50	31 39	38·59	46·09	53·90	62·02	70·44	79·17
5	19·75	26·95	34·45	42·26	50·38	58 80	67·53	76·56	85·90
5½	21·59	29·40	37·52	15·94	54·67	63·70	73·04	82·69	92·64
6	23·43	31·85	40·58	49·61	58·95	68·60	78·55	88·8	99·4
6½	25·27	34·30	43·64	53·29	63·24	73·50	84 07	94·9	106·1
7	27·10	36·75	46·70	56·96	67·53	78·40	89·58	101·1	112·9
7½	28·94	39·20	49·77	60·64	71·82	83·30	95·09	107·2	119·6
8	30·78	41·65	52·83	64·31	76·10	88·20	100·6	113·3	126·3
8½	32·62	44·10	55·89	67·99	80·39	93·1	106·1	119·4	133·1
9	34·45	46·55	58·95	71·66	84·68	98·0	111·6	125·6	139·8
9½	36·29	49·00	62·02	75·34	88 97	102·9	117·1	131·7	146·5
10	38·13	51·45	65·08	79·01	93·25	107·8	122·7	137·8	153·3
10½	39·97	53·90	68·14	82·69	97·54	112·7	128·2	143·9	160·0
11	41·80	56·35	71·20	86·4	101·8	117·6	133·7	150·1	166·8
11½	43·64	58·80	74·27	90·0	106·1	122·5	139·2	156·2	173·5
12	45·48	61·25	77·33	93·7	110·4	127·4	144·7	162·3	180·2
13	49·15	66·15	83·45	101·1	119·0	137·2	155·7	174·6	193·7
14	52·83	71·05	89·58	108·4	127·6	147·0	166·8	186·8	207·2
15	56·50	75·95	95·7	115·8	136·1	156·8	177·8	199·1	220·7
16	60·18	80·85	101·8	123·1	144·7	166·6	188·8	211·3	234·1
17	63·85	85·75	108·0	130·5	153·3	176·4	199·8	223·6	247·6
18	67·53	90·65	114·1	137·8	161·9	186·2	210·9	235·8	261·1
19	71·20	95·55	120·2	145·2	170·4	196·0	221·9	248·1	274·6
20	74·88	100·5	126·3	152·5	179·0	205·8	232·9	260·3	288·0
21	78·55	105·4	132·5	159·9	187·6	215·6	243·9	272·6	301·5
22	82·23	110·3	138·6	167·2	196·2	225·4	255·0	284·8	315·0
23	85·90	115·2	144·7	174·6	204·7	235·2	266·0	297·1	328·5
24	89·58	120·1	150·8	181·9	213·3	245·0	277·0	309·3	341·9

Weight of Brass Tubes in Lbs. per Foot of Length.

For formula see page 333.

Thickness.		External Diameter in Inches.							
I S W G	Ins.	½	⅝	¾	⅞	1	1¼	1½	1¾
20	·036	·194	·246	·298	·350	·403	·507	·611	·716
19	·040	·213	·271	·329	·387	·445	·561	·677	·793
18	·048	·252	·321	·391	·460	·530	·669	·808	·948
17	·056	·288	·370	·451	·532	·613	·776	·938	1·100
16	·064	·324	·416	·509	·602	·695	·880	1·066	1·252
15	·072	·357	·462	·566	·671	·775	·984	1·193	1·401
14	·080	·390	·506	·622	·738	·854	1·086	1·318	1·550
13	·092	·435	·569	·702	·836	·969	1·236	1·503	1·769
12	·104	·478	·629	·779	·930	1·081	1·383	1·684	1·986
11	·116	·517	·685	·853	1·021	1·190	1·526	1·862	2·199
10	·128	·552	·738	·924	1·109	1·295	1·666	2·037	2·408
9	·144	·595	·803	1·012	1·221	1·430	1·847	2·265	2·683

Weight and Volume of Water at Various Temperatures.*

Temperature in Degrees. Cent.	Fahr.	Relative Volume.	Relative Density.	Weight in Lbs of One Cubic Foot.	Weight in Lbs of One Gallon.
0	32	1·00013	·99987	62·3512	10·0101
4	39·2	1·00000	1·00000	62·3593	10·0114
10	50	1·00027	·99973	62·3425	10·0087
16·67	62	1·00114	·99886	62·2882	10·0000
20	68	1·00177	·99823	62·2489	9·9937
30	86	1·00435	·99567	62·0893	9·9681
40	104	1·00782	·99224	61·8754	9·9337
50	122	1·01207	·98807	61·6153	9·8920
60	140	1·01705	·98324	61·3141	9 8436
70	158	1·02269	·97781	60·9755	9·7893
80	176	1·02899	·97183	60·6026	9·7294
90	194	1·03590	·96534	60·1979	9·6644
100	212	1·04343	·95838	59·7639	9·5947

Relative densities are the same as densities in grammes per millilitre. Relative volumes are the reciprocals of relative densities.

It is important to realize that, whereas a litre contains 1 kilogramme of pure water at 4° C. and 760 mm., a gallon only *appears* to contain 10 lbs. of water at 62° F. and with the barometer at 30 inches. The weighing of the litre is adjusted to vacuum conditions, and the *apparent* weight in air at 4° C. and 760 mm. would be less than 1 kilogramme. In the case of the gallon the *apparent* weight in air, at 62° F. and 30 inches, is 10 lbs. and the weight in a vacuum would be more than 10 lbs.

Taking the weight of a cubic foot of pure water at 62° F. as 62·2882 lbs., the following results are obtained:—

1 ton of pure	water at	62° F.	contains	35·9619 cubic feet.
1 pound	,,	,,	,,	27·7420 cubic inches.
1 ton	,,	,,	,,	224 gallons.
1 cubic foot	,,	,,	,,	6·22882 gallons.
1 gallon	,,	,,	,,	277·420 cubic inches.
1 cubic inch	,,	,,	weighs	·0360464 pound.

* The water is free from air and at atmospheric pressure.

Pressures Corresponding to given Heads of Water.

Water at maximum density. Temperature 39·2° F.
H = head in feet. *P* = pressure in lbs. per sq. inch = ·433 *H*.

H	*P*	*H*	*P*	*H*	*P*	*H*	*P*	*H*	*P*
1	·433	21	9·09	41	17·75	61	26·41	81	35·07
2	866	22	9·53	42	18·19	62	26·85	82	35·51
3	1·299	23	9·96	43	18·62	63	27·28	83	35·94
4	1·732	24	10·39	44	19·05	64	27·71	84	36·37
5	2·165	25	10·82	45	19·48	65	28·14	85	36·80
6	2·598	26	11·26	46	19·92	66	28·58	86	37·24
7	3·031	27	11·69	47	20·35	67	29·01	87	37·67
8	3·464	28	12·12	48	20·78	68	29·14	88	38·10
9	3·897	29	12·56	49	21·22	69	29·88	89	38·54
10	4·330	30	12 99	50	21·65	70	30·31	90	38·97
11	4·763	31	13·42	51	22·08	71	30·74	91	39·40
12	5·196	32	13·86	52	22·52	72	31·18	92	39·84
13	5·629	33	1 29	53	22·95	73	31·61	93	40·27
14	6·062	34	14·72	54	23·38	74	32·04	94	40·70
15	6·495	35	15·15	55	23·81	75	32·47	95	41·13
16	6·928	36	15·59	56	24·25	76	32·91	96	41·57
17	7·361	37	16·02	57	24·68	77	33·34	97	42·00
18	7 794	38	16·45	58	25·11	78	33·77	98	42·43
19	8·227	39	16·89	59	25·55	79	34·21	99	42·87
20	8·660	40	17·32	60	25·98	80	34·64	100	43·30

Pressures Corresponding to given Heads of Water.

Water at maximum density. Temperature 39·2° F.
h = head in inches. *P* = pressure in lbs. per sq. inch = ·03608 *h*.

h.	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
0	...	·004	·007	·011	·014	·018	·022	·025	·029	·032
1	·036	·040	·043	·047	·051	·054	·058	·061	·065	·069
2	·072	·076	·079	·083	·087	·090	·094	·097	·101	·105
3	·108	·112	·115	·119	·123	·126	·130	·133	·137	·141
4	·144	·148	·152	·155	·159	·162	·166	·170	·173	·177
5	·180	·184	·188	·191	·195	·198	·202	·206	·209	·213
6	·216	·220	·224	·227	·231	·235	·238	·242	·245	·249
7	·253	·256	·260	·263	·267	·271	·274	·278	·281	·285
8	·289	·292	·296	·299	·303	·307	·310	·314	·318	·321
9	·325	·328	·332	·336	·339	·343	·346	·350	·354	·357
10	·361	·364	·368	·372	·375	·379	·382	·386	·390	·393
11	·397	·400	·404	·408	·411	·415	·419	·422	·426	·429

Weight of One Cubic Inch of Mercury at Various Temperatures.

Temperature in Degrees Cent.	Temperature in Degrees Fahr.	Weight of One Cubic Inch in Pounds	Temperature in Degrees Cent.	Temperature in Degrees Fahr.	Weight of One Cubic Inch in Pounds
0	32	·49056	120	248	·48007
10	50	·48968	130	266	·47920
20	68	·48880	140	284	·47833
30	86	·48792	150	302	·47746
40	104	·48705	160	320	·47660
50	122	·48617	170	338	·47573
60	140	·48530	180	356	·47487
70	158	·48442	190	374	·47400
80	176	·48355	200	392	·47314
90	194	·48268	210	410	·47228
100	212	·48181	220	428	·47141
110	230	·48094	230	446	·47055

Pressure Corresponding to a given Column of Mercury.

h = height of column in inches.

P = pressure in lbs. per square inch corresponding to the column of height h.

t = temperature of mercury.

w = weight of one cubic inch of mercury at temperature t.

$$P = wh.$$

Pressures, in Lbs. per Square Inch, Corresponding to Various Heights of the Barometer, at Various Temperatures.

Height in Inches	Temperature in Degrees Fahrenheit. 32°.	50°.	68°.	86°.	104°.
28·2	13·834	13·809	13·784	13·759	13·735
·4	13·932	13·907	13 882	13·857	13·832
·6	14·030	14·005	13·980	13·955	13·930
·8	14·128	14·103	14·077	14·052	14·027
29·0	14·226	14·201	14·175	14·150	14·124
·2	14·324	14·299	14·273	14·247	14·222
·4	14·422	14·397	14·371	14·345	14·319
·6	14·521	14·495	14·468	14·442	14·417
·8	14·619	14·592	14·566	14·540	14·514
30·0	14·717	14·690	14·664	14·638	14·611
·2	14·815	14·788	14·762	14·735	14·709
·4	14·913	14·886	14·860	14·833	14·806
·6	15·011	14·984	14·957	14·930	14·904

Weight of Acids and Acid Solutions.

Temperature 15° C. = 59° F.

Acid or Acid Solution.	Sp. Gravity. Water at 39·2° F. = 1	Weight of One Cubic Foot in Lbs.	Weight of One Gallon in Lbs.
Hydrochloric acid (HCl)—			
40 per cent. acid . . .	1·200	74·82	12·013
30 " " . . .	1·152	71·82	11·533
20 " " . . .	1·100	68·58	11·012
10 " " . . .	1·050	65·47	10·512
Nitric acid (HNO_3)—			
100 per cent. acid . . .	1·530	95·39	15·317
80 " " . . .	1·460	91·03	14·616
60 " " . . .	1·374	85·67	13·755
40 " " . . .	1·251	78·00	12·524
20 " " . . .	1·120	69·83	11·212
10 " " . . .	1·060	66·09	10·612
Sulphuric acid (H_2SO_4)—			
100 per cent. acid . . .	1·842	114·85	18·440
80 " " . . .	1·733	108·05	17·349
60 " , . . .	1·503	93·71	15·047
40 " " . . .	1·306	81·43	13·074
20 " " . . .	1·145	71·39	11·463
10 " " . . .	1·069	66·65	10·702

Weight of Oils.

Temperature 15° C. = 59° F.

Name of Oil.	Class of Oil.	Sp. Gravity. Water at 39·2° F. = 1.	Weight of One Cubic Foot in Lbs.	Weight of One Gallon in Lbs.
Castor . . .	Vegetable	·970	60·48	9·71
Colza. . . .	Vegetable	·914	56·99	9·15
Cotton-seed .	Vegetable	·925	57·67	9·26
Linseed . . .	Vegetable	·935	58·30	9·36
Naphtha . .	Mineral	·848	52·87	8·49
Neat's-foot . .	Animal	·914	56·99	9·15
Olive	Vegetable	·915	57·05	9·16
Petroleum . .	Mineral	·878	54·74	8·79
Rape-seed . .	Vegetable	·915	57·05	9·16
Sperm . . .	Animal	·880	54·87	8·81
Turpentine . .	Vegetable	·870	54·24	8·71
Whale . . .	Animal	·925	57·67	9·26

Specific Gravity, Weight, and Volume of Gases.

Pressure = 1 atmosphere (14·7 lbs. per square inch).
Temperature = 32° Fahr. unless stated to be otherwise.

Gas.	Specific Gravity. Air at 32° F. = 1.	Weight of One Cub. Foot in Lbs.	Volume of One Lb. in Cubic Feet.
Air, dry and pure, at 32° F.	1·00000	·080728	12·387
,, ,, 62° F.	0·94263	·076097	13·141
Carbonic acid (CO_2)	1·52901	·123434	8·101
Carbonic oxide (CO)	0·96780	·078129	12·799
Coal gas (16 candle)	0·370	·029869	33·480
,, (19 ,,)	0·425	·034309	29·147
,, (20 ,,)	0·455	·036731	27·225
,, (36 ,,) from Boghead Cannel	0·750	·060546	16·516
Cyanogen (C_2N_2)	1·80640	·145827	6·857
Hydrogen	0 06926	·005591	178·859
Marsh gas (CH_4)	0·55900	·045127	22·160
Nitrogen	0·97137	·078417	12·752
Olefiant gas (C_2H_4)	0·97840	·078984	12·661
Oxygen	1·10563	·089255	11·204
Steam, at 212° F.	0·47034	·037970	26·337
Sulphurous acid (SO_2)	2·21126	·178511	5·602

Weight in Lbs. of a Cubic Foot of Air containing a Standard Amount of Carbonic Acid.

(Board of Trade, Standards Department.)

Condition of Air.	Temperature in Degrees Fahr.		
	32°	62°	80°
Dry air	·08098	·07632	·07377
Ordinary air $\left(\text{saturation} = \frac{2}{3}\right)$	·08093	·07596	·07313
Moist air (saturation = 1)	·08080	·07578	·07281

The standard amount of carbonic acid mentioned above is 6 volumes of carbonic acid to 10,000 volumes of air.

NOTES ON MATERIALS.

(*Revised by A. E. W. Smith, B.Sc., Ph.D.*)

Ferrous Alloys.

When iron ore is dug from the earth and smelted in the blast furnace, the metal produced is a crude iron called *pig-iron*. This is very impure, only about 90–95 per cent. iron, the foreign elements present being chiefly carbon, silicon, manganese, sulphur, and phosphorus. This material is the starting-point for making every kind of engineering iron and steel. The cheapest way of dealing with it is simply to remelt and cast it into shapes, and in this form it is called *cast-iron*.

Cast-iron is an admirable material for foundry work, because it melts much more easily than does steel, and flows well in the mould. It is cleaner and more uniform than pig-iron, but nevertheless still contains most of the impurities, notably 2½–4½ per cent. of carbon. A small part of this carbon (up to 0·9 per cent.) is in a complex combined form which strengthens the iron, but the rest may be there either as free carbon, in the form of graphite flakes, which are weak and soft, or as chemically combined carbon, in which case it is present as the compound iron carbide, which is hard and brittle. In the former case the cast-iron is *grey* and has a dark fracture; whereas in the latter it is *white*, so called from its brighter fracture; and *mottled* irons occur as a cross between these two, *i.e.* they are partly grey and partly white. It is the ratio of combined carbon to free carbon which determines the quality and hardness of the iron; the greater the quantity of combined carbon, the harder and whiter the iron.

Grey irons are relatively soft, are machinable, and may wear well owing to the self-lubricating action of the graphite flakes; but they tend when very grey to be weak and porous. White irons are very hard and may be stronger than the grey variety; but they are also brittle and practically unmachinable. Within these limitations, however, each kind gives excellent service if correctly used. Control of the degree of greyness is largely attained by the chemical composition of the iron.

Silicon may be present up to 4 per cent., and it has the effect of decomposing the iron carbide to liberate graphite, and therefore the greyness increases with the silicon content. This is shown by the typical analyses of pig-irons in Table 1, where the softest irons are seen to contain the most silicon, and the hard, white irons contain the least. White irons of low silicon content do not make good castings, as the metal is too viscous to flow well, and is also liable to produce blowholes in the casting.

The presence of *phosphorus* in cast-iron makes it more fusible, and causes it to remain longer in the fluid state when poured;

Table 1.*

Typical Analyses of some (East Coast) Pig-irons, showing Relation of Silicon to Combined Carbon.

	Combined Carbon.	Graphitic Carbon.	Silicon.	Sulphur.	Phosphorus.	Manganese.
No. 1	0·30	3·73	2·50	0·02	0·05	1·00
,, 2	0·45	3·53	2·25	0·03	0·05	1·00
,, 3	0·56	3·18	2·00	0·04	0·05	1·00
,, 4	1·00	2·75	1·50	0·10	0·05	1·00
,, 5	1·55	2·45	1·00	0·20	0·05	0·75
Mottled	2·05	1·50	0·75	0·25	0·05	0·50
White	3·15	trace	0·65	0·30	0·05	0·50

but it also induces brittleness. For this reason the phosphorus content is rarely as high as 1 per cent. This type of iron is useful, however, for castings of thin section or of intricate design, and for many light engineering castings.

Sulphur tends to make cast-iron hard and brittle, and is generally regarded as detrimental; it should be kept well below 0·1 per cent. for most foundry purposes.

Manganese also tends in a different way to whiten and harden a cast-iron, and is therefore often kept below 0·75 per cent. But it helps to exert a controlling influence over the harmful effect of sulphur, and for any particular purpose these two impurities should be considered in conjunction.

The grey irons have a higher melting-point than the white irons, but they are more fluid in the molten state. They also expand during solidification and therefore take a good impression of the mould, a property not possessed by the white varieties. After cast-iron has solidified it contracts in cooling, about one-eighth of an inch per foot of length. Allowance for this contraction must be made in patterns for the foundry. The shrinkage is less in the softer than in the harder irons.

A part of a casting may be *chilled* by having the corresponding part of the mould lined with cast-iron, protected by a thin coating of loam. The chilled (*i.e.* more rapidly cooled) portion is found to contain more combined carbon, and is therefore harder. The hard skin produced in this way may be up to 1 inch thick; for instance, the wearing surface of a chilled iron roll.

Castings of hard white iron may be converted into *malleable* castings by embedding them in powdered *hæmatite* (an oxide of

* Gregory, *Metallurgy*, 1932, Blackie & Son, Ltd.

iron) and keeping them at a bright red-heat for several days, the time depending on their size. They are then cooled very slowly. By this process most of the iron carbide is decomposed and a large part of the carbon oxidised away, this variety of malleable iron being called *white-heart.* A similar kind of treatment, but without the surrounding oxide, leaves all the carbon embedded in the iron, collected into graphite "nodules," and this is called *black-heart* malleable from its darker fracture.

An idea of the strength of ordinary cast-irons is obtained from the following figures, and from the fact that the B.S.S., No. 321, 1928, called for a tensile strength of only 11 tons per square inch (on the 1·2 inch-diameter bar).

	Maximum Stress. Tons/Sq. In.	Crushing Strength. Tons/Sq. In.	Transverse Tests. 36-in. × 2-in. × 1-in. Bars.
Average .	8–11	40	28 cwt.
Very good .	14–16	50	30–40 ,,
Poor . .	5	30	18 ,,

In recent years, however, really outstanding improvements have been made in cast-iron, and this material can no longer be regarded as the Cinderella of metals. Not only in composition, but especially in melting practice in the cupola and in casting procedure, far greater control has been created by research, resulting in irons of much finer texture and greater uniformity. A great many irons have been patented. The general improvement is indeed so great that new specifications have been required, and the B.S.S., No. 786, 1938, demands in these high-duty irons a tensile strength as follows:—

	Grade I.	Grade II.	Grade III.	
1·2 inch-diameter bar	14	17	20	tons/sq. in.

It should be remembered that these are minimum figures, and indeed a strength of 30 tons per square inch is by no means unusual in practice. With these high-duty irons it is often worth while to employ heat-treatment (as with steels) for further improvement, as in applications such as cylinder liners, cams, etc.

Alloy additions also are now frequently made to cast-irons, the commonest being nickel, chromium, and silicon. These may produce greater wear-resistance, or strength, as in Grades II. and III. above, or resistance to *growth.* Ordinary cast-irons, when repeatedly heated and cooled (as in travelling fire-grates), are gradually altered in their internal structure, and actually increase in length in the process. This growth causes buckling.

The new non-magnetic alloy irons have almost completely overcome this phenomenon and can be heated with safety. Table 2 gives the properties of some of these irons.

Table 2.*

Physical Properties of Nickel Alloy Austenitic Cast-irons.

	Typical Composition. Per Cent.					Tensile Strength. Tons/ Sq. In.	Transverse Stress. Tons/ Sq. In.	Brinell Hardness Number.	Machinability.
	C	Si	Ni	Cr					
"Nomag"	2·8	1·5	11	..	Mn 6	14–17	..	200–220	Good
"Ni-Resist"	2·8	1·5	14	2	Cu 6	12–16	..	180–220	Good
"Nicrosilal"—									
A. Hard	1·9	5·0	18	5	..	16–18	36–40	320–350	Difficult
B. Soft	1·9	5·0	18	2	..	12–14	26–30	110–130	Good
Unalloyed engineering cast-iron	3·2	1·8	..	..	..	12–16	24–30	180–210	Possibly difficult at corners

The properties of other alloy irons, such as may be used for cast crankshafts, are shown in Table 3, with a nickel-chromium alloy steel for comparison.

Wrought-iron.†—Wrought-iron is nearly pure iron, and is made from pig-iron. Although pig-iron can be cast into shapes and has moderate strength, it cannot be forged or bent; but it is made forgeable or malleable by removing the impurities picked up in the blast furnace.

Wrought-, or "puddled," iron is made in a shallow hearth furnace at a high temperature (1200°–1400° C.) by melting down certain varieties of pig containing the most suitable amounts of impurities, and then oxidising this metal by a blast of air, together with iron ore or scale. Some iron is lost as oxide, but the impurities are almost all removed by preferential oxidation. The removal of the impurities raises the melting-point, so that during the refining the iron solidifies to a pasty mass, which is raked and worked into balls by the puddlers. The ball, whilst still white-hot, is taken to a hammer forge and beaten, to squeeze out most of the still fluid slag. The ball is rolled down to plates or billets, which are then packed into piles (piling) or bent back (doubling) and re-rolled, so that the final product is laminated but perfectly welded together. Good wrought-iron is thus nearly pure iron in which the original droplets of slag are rolled out into fibres; hence wrought-iron breaks with a fibrous

* "Nickel Cast-iron for Engineers"—Publication No. B 27.—The Bureau of Information on Nickel.

† Notes summarised partly from *Wrought-Iron and Malleable Cast-Iron*, by R. C. Tucker.

Table 3.*

Physical Properties of Crankshaft Cast-irons.

Property.	Alloy Steel (a).	Copper-Chromium Iron (b).	Inoculated Cast-iron.	Chromium-Molybdenum Iron.	Nickel-Chromium Iron.
Composition, per cent.—					
Carbon (total)	0·32	1·56	2·75	3·28	3·36
„ (graphitic)	..	..	..	2·47	2·39
Silicon	0·23	1·16	1·59	2·19	1·22
Manganese	0·88	0·44	0·88	0·95	0·92
Sulphur	0·04	0·05	0·07	0·10	0·11
Phosphorus	0·03	0·07	0·08	0·17	0·12
Nickel	2·42	trace	..	..	1·87
Chromium	0·49	0·46	..	0·42	0·47
Molybdenum	0·38	..	0·29	0·95	..
Copper	0·13	1·75	..	..	..
Brinell hardness number	260	260	245	280	265
Izod impact, ft.-lbs.	24·8	1·2	1·5	0·8	0·4
Tensile properties—					
Ultimate strength, tons/sq. inch.	52·3	32·3	23·3	20·8	18·8
Relative ult. strength, per cent.	100	62	45	40	36
Elongation, per cent.	10·0	0·2	0·4	0·8	0·8
Modulus, lbs./sq. in. $\times 10^{-6}$	29·3	26·8	21·7	20·1	18·7
Fatigue properties (c)—					
Fatigue limit, tons/sq. in. (cycles of reversed bending stress).	±26·6	±18·2	±12·9	±10·8	±10·6
Ratio to tensile strength	0·51	0·56	0·55	0·52	0·56
Relative values of ratio, per cent.	100	111	109	102	111

(a) Steel billet annealed at 860° C. (1575° F.); test-pieces cut and oil-quenched from 850° C. (1560° F.), and tempered at 650° C. (1200° F.).

(b) Heated to 900° C. (1650° F.) for 20 minutes, air-cooled to 650° C. (1200° F.), re-heated to 760° C. (1400° F.) for 60 minutes, furnace-cooled to 540° C. (1000° F.), and thence in air.

(c) Using the N.P.L. "combined stress fatigue testing machine."

fracture. The amount of slag (ferrous silicate) is very small, but it is a characteristic feature of this metal, and makes it slightly stronger in its length than across the fibre.

The pig-iron used is called "forge iron," and is intermediate between the hard mottled irons and the grey soft irons generally used for making cast-iron castings. The carbon and silicon contents must be such that the process of puddling just eliminates

* H. J. Gough and H. V. Pollard, "Properties of Some Materials for Cast Crankshafts," *Proc. I.A.E.*, vol. xxxi. (1937), p. 821.

them without too much burning of the iron itself. Moreover, since phosphorus and sulphur are both deleterious, these elements must not be over specified amounts. In wrought-iron excess silicon, not removed, gives hard and brittle metal. Wrought-iron should contain less than 0·35 per cent. silicon. Phosphorus, similarly, should be less than 0·25 per cent. Sulphur makes the wrought-iron brittle at welding temperatures (red short), and should be less than 0·03 per cent.

Many cheap varieties of "wrought-iron" have been made by enclosing mild steel bars, containing considerable quantities of manganese, in the presence of slag, between wrought-iron plates and hot-rolling the whole mass to give a coherent "wrought-iron" plate. The material is, however, by no means as satisfactory, partly due to its lack of ductility, and has probably been responsible for the supposed deterioration in the quality of genuine wrought-iron. Consequently, material should not be regarded as good wrought-iron if it contains manganese in excess of 0·15 per cent.

The excellence of wrought-iron depends on its easy and reliable weldability, its good toughness and ductility. Typical tests show a tensile strength of 22–24 tons per sq. inch and 10–25 per cent. elongation. Wrought-iron should pull out or extend if it is overloaded, and thus give warning before failure occurs. It is the best material for chains and hooks for heavy duty. It is better in ductility than even the best black-heart malleable iron, as a comparison of the properties of the malleable irons shows.

	White-heart.	Black-heart.	Wrought-iron.	Mild Steel.
Structure.	Iron + Pearlite.	Iron + Temper Carbon + some Pearlite.	Iron + Slag Filaments.	Iron + Pearlite.
Tensile strength, tons/sq. inch.	19–29	16–26	23	25–35
Elongation, per cent.	6–2	15–4½	26	35–30
Reduction in area, per cent.	6–2	15–4½	40	65–60

When it is exposed to corrosive conditions it wears away uniformly, whereas steel corrodes badly at some points more than others, and this causes pitting. The actual loss is about the same, but the wrought-iron will last longer. In use, if wrought-iron is hammered cold, it becomes hard and brittle, so that chains and hooks exposed to shocks should be periodically annealed at a bright heat, to recrystallise the metal that has been hardened by the "cold work."

Wrought-iron, by its laborious method of manufacture, is very expensive, and has been ousted by the cheaper mild steel from many of its original applications. Its peculiar properties, however, give it a place which substitutes cannot fill. In fact, a new process has been developed by an American company for "synthesising" wrought-iron from an artificial mixture of molten "dead mild" steel and fluid silicate slag. This material is quite as good as genuine puddled iron, and is cheaper to make, and it meets the best American specifications for wrought-iron for naval requirements.

Steel.—About four-fifths of the pig-iron produced goes to the manufacture of steels. Pig-iron is refined to steel by various methods, all having the common feature that the impurities are oxidised away in the molten condition and liquid steel is left, providing the complete elimination of the slag, a contrast with wrought-iron. Further, after the iron is thus purified, controlled quantities of useful carbon, etc., are added to give the desired strength and hardness. Thus steel is really an alloy of purified iron and carbon, with which the iron combines. In good steel the amount of the sulphur and phosphorus impurities left should not exceed 0·05 per cent.

The first steel was made by strongly heating wrought-iron while surrounded with carbon, whereby *cementation* occurred and the iron absorbed some of the carbon. Several of these slabs were then welded together and the composite block reduced to any desired size by hot forging, giving what is called *shear steel*, a material which is very little made now. The cemented bars may also be melted in crucibles, adding more ingredients if necessary, and cast into ingots, called *cast steel*. These may be rolled hot into small sections, also called cast steel, but as they are mainly used for cutting tools, *tool steel* is a better name, as many other grades of steel are used for casting purposes nowadays. It is a high-grade product, clean, and uniform in carbon content.

Modern structural steels, however, are made on a much larger scale, in either the Bessemer converter or the open-hearth (Siemens-Martin) furnace. Each of these may use either an acid or basic refractory lining for the furnace, according to the type of pig being melted, and this gives the so-called *acid* and *basic* steels. Basic linings are becoming the more popular, as the by-product slag is useful for fertilising, but acid open-hearth steel has been regarded as the best grade of them all.

Bessemer steel is made by pouring melted pig-iron into a vessel called a converter, through which a blast of air is then blown. By this means the carbon and other impurities are burnt away and comparatively pure iron remains. To this may be added the necessary quantities of carbon, silicon, and manganese, in various forms, and the molten steel cast into ingots or into prepared moulds in the foundry.

Open-hearth steel is made by melting pig-iron, together with up to 50 per cent. of steel scrap, in the shallow hearth of a reverberatory furnace fired by producer gas, and the impurities are oxidised away as before, but from the surface of the bath, which therefore takes much longer. The composition of the steel is adjusted as before, and the metal is cast into large ingots for forging.

In addition to these methods there are the two newer electrical melting furnaces, of the arc and the high frequency induction types, used mainly for obtaining the very high temperatures necessary for making steels of high alloy content, *e.g.* stainless, high-speed, etc. The high-frequency furnace is now ten years old industrially, has almost superseded the older crucible melting plant, and gives far better control over the cleanliness and thorough mixing of the alloy steels.

Strength of Steel.—Pure iron is a soft and ductile metal. As the carbon content gradually increases so that the iron becomes first *mild steel*, then *medium carbon steel*, and finally *high carbon steel*, its tensile strength gradually increases also, reaching a maximum of about 65 tons per sq. inch at 0·9 per cent. carbon. The hardness goes up continuously with the percentage of combined carbon, whilst the percentage elongation falls steadily, as the following figures for high-grade normalised steels show:—

Carbon. Per Cent.	Condition.	Approximate Physical Properties.			
		Tons/Sq. In.		Elongation. Per Cent.	Brinell Hardness Number.
		Yield.	Ultimate.		
Pure iron	Annealed	13	20	40	96
0·15	Normalised at 880° C.	15	24	35	120
0·25	,, ,, 850° C.	18	30	32	145
0·35	,, ,, 830° C.	23	35	28	170
0·45	,, ,, 830° C.	30	42	25	195
0·55	,, ,, 820° C.	32	47	23	220

With higher carbon contents approaching 1 per cent., the steel is not only hard but has become almost devoid of elongation, *i.e.* is brittle. Structural steels are therefore limited to the lower carbon varieties, and typical uses of the various grades are shown in Table 4.

Table 4.

Typical Applications of Carbon Steels.

	Carbon. Per Cent.	Approximate Tensile Strength. Tons/Sq. In.	Use.
Mild steels (good for welding)	Below 0·1	20–25	Tinplate, galvanised iron.
	·10– ·18	24–28	Boiler plates, ships' plates, general, case-hardening for gear wheels, cams (hard case, tough core).
	·15– ·25	26–32	Structural for buildings, bridges, general engineering purposes, crank axles, shafting.
	·25– ·35	32–38	Turbine rotor shafts, hydraulic cylinders, rams, spindles, marine shafts.
Medium carbon	·35– ·45	35–45	Railway and tramway axles, rails, turbine discs, crank-pins, connecting-rods.
	·45– ·55	45–55	Steel mill rolls, gear wheels, rifle barrels, gun parts, shells.
High carbon (tool steels)	·55– ·65	50–60	Wheel tyres, die blocks, gears, mandrels.
	·65– ·75	60–70	Crusher rolls, hammers, general tools.
	·75– ·85	60–70	Ball mill parts, hand chisels, scissors.
	·85–1·0	60–70	Taps, drills, dies, ball races, wood tools.
	1·0 –1·2	55–65	Tools, drills, razors, wire dies, pens.

It is not only the carbon content, however, that determines the strength of a steel, because by adopting various heat-treatments any one steel can be put into a number of different conditions, each showing its own combination of strength, toughness, etc. For instance, if a carbon steel be heated to a good red-heat (suitable to its composition) and quenched in water, it will become very hard, probably brittle, and may be cracked or distorted. Less drastic cooling may be obtained by oil-quenching, with correspondingly less severe results. Slow cooling in the furnace, or in still air, on the other hand, will soften the material. Gentle heating after quenching can be used to remove the brittleness and make the metal tough while still being very hard; this is called *tempering* (or in America, "drawing"). Some typical contrasting properties are shown in Tables 5 and 6.

Table 5.*

Effect of Various Heat-treatments on Carbon Steels.

Carbon. Per Cent.	Heat-treatment.	B.H.	Y.P.	M.S.	E.%.	R.%	Izod.
0·06	norm. at 920° C.	121	19	26	41	71	32
	w.q. at 920° C.	167	..	35	30	67	51
	o.q. at 920° C.	137	..	30	35	69	90
0·20	norm. at 900° C.	167	24	36	34	60	82
	w.q. at 900° C.,	241	..	52	18	46	33
	and t. at: 300° C.	223	..	48	20	52	33
	400° C.	..	..	49	20	52	45
	500° C.	217	35	46	23	57	61
	600° C.	212	34	44	25	63	79
0·45	norm. at 870° C.	192	27	44	27	54	31
	w.q. at 870° C.,	321	..	67	12	28	14
	and t. at: 300° C.	311	..	65	15	38	16
	400° C.	302	46	64	17	47	19
	500° C.	277	42	59	21	55	34
	600° C.	235	36	52	25	62	48
	700° C.	207	31	45	28	67	59
0·55	ann. at 700° C.	..	24	35	30	..	..
	norm. at 820° C.	..	32	48	25	..	..
	o.q. at 850° C., and t. at 400° C.	..	75	80	8	..	..

B.H. = Brinell hardness.
Y.P. = yield point, tons/sq. inch.
M.S. = maximum stress, tons/sq. inch.
E.% = elongation, per cent.
R.% = reduction in area, per cent.
Izod = energy absorbed in ft.-lbs.

norm. = normalised.
ann. = annealed.
w.q. = water-quenched.
o.q. = oil-quenched.
t. = tempered.

* Mainly from Automobile Steel Research Report of the I.A.E., 1920.

Table 6.*

Effect of Heat-treatment on some Alloy Steels.

Composition. Per Cent.	Heat-treatment.	B.H.	Y.P.	M.S.	E.%.	R.%.	Izod.
	Case-hardening.						
C 0·15 Ni 2·6	norm. at 880° C.	143	24	32	37	69	95
	w.q. at 880° C., and t. at 600° C.	179	30	38	31	75	115
C 0·07 Ni 5·1	norm. at 860° C.	143	26	31	39	72	104
	w.q. at 860° C., and t. at 600° C.	163	29	34	36	77	114
	o.q. at 860° C., and t. at 600° C.	159	28	32	38	77	116
	3½ Per Cent. Nickel.						
C 0·37 Ni 3·65	norm. at 860° C.	196	27	44	28	56	36
	w.q. at 860° C.,	544	..	124	9	25	4
	and t. at: 400° C.	340	67	75	16	56	8
	500° C.	285	54	61	21	61	52
	600° C.	248	45	53	25	63	72
	650° C.	228	39	48	28	65	81
	o.q. at 860° C.,	495	..	116	10	..	6
	and t. at: 500° C.	293	54	63	20	60	50
	650° C.	235	40	49	27	66	82
	1½ Per Cent. Nickel-chrome.						
Ni 1·52 Cr 0·80 C 0·28	norm. at 850° C.	174	29	40	33	65	74
	w.q. at 850° C., and t. at: 500° C.	302	56	64	18	58	50
	650° C.	229	40	47	27	69	93
	o.q. at 850° C.,	302	..	64	16	34	17
	and t. at: 400° C.	291	..	60	17	51	32
	500° C.	262	44	54	21	62	70
	600° C.	223	37	46	27	68	90
	650° C.	212	35	44	29	70	93
	3 Per Cent. Nickel-chrome.						
Ni 3·27 Cr 0·82 C 0·31	norm. at 820° C.	341	..	76	15	40	6
	o.q. at 820° C.,	495	..	116	14	37	11
	and t. at: 400° C.	388	75	85	16	54	8
	500° C.	331	65	71	19	56	28
	600° C.	285	54	60	22	62	59
	650° C.	269	48	57	24	65	74
	Air-hardening.						
Ni 3·70 Cr 1·42 C 0·32	air-hardened from 850° C.	477	..	115	13	40	18
	„ from 820° C.,	477	..	114	11	39	17
	and t. at: 400° C.	418	80	90	14	52	14
	500° C.	351	67	74	18	56	34
	650° C.	262	46	56	25	67	83
	o.q. at 820° C., and t. at: 400° C.	418	87	91	13	52	17
	500° C.	364	71	78	16	56	37
	650° C.	262	47	57	24	67	86

* Mainly from Automobile Steel Research Report of the I.A.E., 1920.

The effects of heat-treatment become more pronounced with the higher carbon contents, as is seen with the 0·55 per cent. carbon steel, whose tensile strength of only 35 tons per sq. inch in the fully softened condition may be raised to 80 tons per sq. inch by suitable treatment. Another method of increasing the tensile strength is by cold-working, such as rolling or drawing through a die; and hence great strength may be induced in wires, such as piano wire, by selected combinations of drawing and heat-treatment, *e.g.*

Spring Wire. 0·7 per Cent. Carbon.	Yield Point. Tons/Sq. In.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent. on 10 In.	Reduction in Area. Per Cent.
Hard-drawn .	62	103	2	30
Patented . .	44·5	69	7	40
Annealed . .	24·5	49	10	55
Oil-quenched and tempered .	84·5	106	3	40

Case-hardening is an application of the cementation process of making steel. If a mild steel article, after it is machined and finished, be heated in contact with substances rich in carbon, the iron at the surface will be converted into steel, which may be hardened by quenching in water, so as to produce a wear-resisting surface. In addition to solid carburising mixtures, molten cyanides are now also used, or the articles may be heated in a gas mixture which contains carbon monoxide. In all cases the carbon content of the surface reaches 0·9 to 1·0 per cent., and is therefore capable of developing full hardness in the later heat-treatment, while at the same time the lower carbon centre is put into a tough condition to resist shocks.

A newer kind of hard case is made by allowing active nitrogen, derived from ammonia gas, to penetrate the surface of the steel. This is called *nitriding*, and was formerly applicable only to certain patented alloy steels, but it is now finding wider use. It is also applicable to certain cast-irons. The case is much harder, equivalent to 900–1000 Brinell, thinner and more resistant than the carbon case, and no after-treatment is necessary. The process is advantageous in many ways but is somewhat more costly. Fig. 1* shows results produced by different hardening processes.

Alloy Steels.—The addition of small amounts of other elements to carbon steels is now very commonly used to produce materials of greater all-round strength than can conveniently be obtained with plain carbon steels; *e.g.* 1½, 3, or 5 per cent. of nickel, with

* "Nickel Alloys in Aeronautical Engineering"—Publication No. H 27.—The Bureau of Information on Nickel.

or without ½ or 1 per cent. of chromium, are quite usual in engineering steels. Often a combination of desirable properties is attainable with low alloy steels which is not possible with carbon steel. These steels respond to heat-treatment in a similar way to the carbon steels (each composition requiring its own peculiar temperatures), but in general the alloy additions ease the rate at which the internal alterations occur, so that hardening may be produced by oil-cooling or even air-cooling instead of

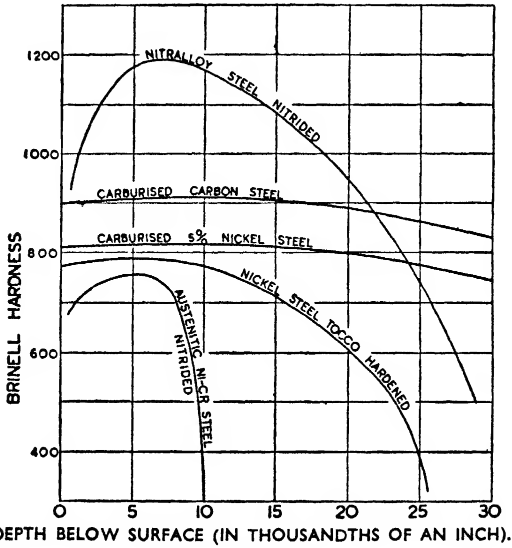


FIG. 1.

water-quenching. Thus with suitable amounts of nickel and chromium, or other elements, there are the *oil-hardening* and the *air-hardening* types of steel. Furthermore, this more gentle rate of internal change eliminates the distortion and cracking so liable to occur with plain steels, and gives more uniform hardness throughout thicker sections. Similarly, where ordinary case-hardening steels do not provide a strong enough core, low nickel additions may be used to give greater strength. Table 6 gives the properties of typical steels of these kinds and the effects of some heat-treatments on them, while Table 7 gives a fuller list of commercial steels. In general, nickel may be

Table 7.—Range of Structural Electric Steels Suitable for Automobiles, Aero Engines, and Aircraft.

(Samuel Fox & Co., Ltd., associated with The United Steel Companies Ltd., Sheffield.)

Class of Steel.	British Standard Specifications.	Heat-treatment, °C. A, O, W = Cool in Air, Oil, or Water, respectively.		Maximum Stress. Tons/Sq. In.	Yield Point. Tons/Sq. In.	Elongation. Per Cent. on 2 In.	Reduction of Area. Per Cent.	Izod Impact. Ft.-lbs.
Carbon. Case-hardening	2 S.14, 5005/101	Refine 900/W	Harden 780/W	32/45	..	32/22	65/50	100/50
3 per cent. Ni. Case-hardening	3 S.15, 5005/103	„ 860/O	„ 770/W	45/60	..	32/20	65/45	80/45
3 per cent. Ni. C.H. (Single quench)	.. ..	.. ..	„ 780/O	34/44	..	35/25	65/55	100/60
5 per cent. Ni. C.H. (Low limit)	S.67 ..	Refine 850/O	„ 760/O	40/60	..	33/20	65/50	90/50
5 per cent. Ni. C.H. (High limit)	S.90, 5005/104	„ 850/O	„ 760/O	65/85	..	22/13	55/40	60/30
Ni, Cr. Case-hardening	.. ..	„ 850/O	„ 780/O	50/65	..	27/20	65/45	75/50
3 per cent. Ni, Cr. Case-hardening	.. ..	„ 840/O	„ 800/O	65/80	..	22/15	60/45	60/30
Ni, Cr, Mo. Case-hardening	S.82 ..	„ 860/O	„ 760/O	85/100	..	18/13	60/40	30 min.
1 per cent. Ni. Oil-hardening	2 S.76, 5005/204	Harden 850/O	Temper 550/650/O	40/55	28/38	30/20	65/45	95/50
3 per cent. Ni. Oil-hardening	.. 5005/401	„ 830/O	„ 550/650/O	45/60	35/45	27/20	68/55	95/50
3½ per cent. Ni. Oil-hardening	S.69, 5005/402	„ 830/O	„ 550/650/O	55/70	45/55	25/16	65/45	70/35
1½ per cent. Ni, Cr. Oil-hardening	.. 5005/503	{ „ 820/O	„ 200/A	115/130	..	17/12	45/35	15 min.
		„ 820/O	„ 550/650/O	55/70	45/60	25/17	65/50	80/50
3 per cent. Ni, Cr. Oil-hardening	3 S.11, 5005/501	„ 830/O	„ 550/650/O	55/70	45/60	26/20	65/50	80/45
Ni, Cr. Air-hardening	2 S.28, 5005/502	„ 820/A	„ 200/A	100/115	..	16/12	45/30	20 min.
Cr, Mo. Oil-hardening	.. ..	„ 850/O	„ 550/680/A	50/65	40/53	25/18	65/55	75/45
3 per cent. Ni, Cr, Mo. Oil-hardening	S.65 ..	„ 830/O	„ 550/650/A	65/75	53/62	23/18	65/55	75/50
1½ per cent. Ni, Cr, Mo. Oil-hardening	.. ..	{ „ 830/O	„ 200/A	120/135	..	17/12	48/35	15 min.
		„ 830/O	„ 550/650/A	60/75	52/65	25/17	65/50	70/45
3 per cent. Ni, Cr, Mo. Oil-hardening	S.81 ..	„ 830/O	„ 550/650/A	65/80	53/66	22/17	65/50	60/40
2½ per cent. Ni, Cr, Mo. Oil-hardening	.. ..	„ 820/O	„ 500/650/A	55/80	47/65	25/15	70/45	80/35
Ni, Cr, Mo. Air-hardening	2 S.28 ..	„ 820/A	„ 200/A	105/120	..	16/12	45/30	20 min.
Nitralloy steel	D.T.D.87 ..	„ 900/O	„ 600/700/O	55/65	42/55	22/17	55/45	50/35
3 per cent. Cr, Mo. Nitriding	D.T.D.317 ..	„ 900/O	„ 600/700/A	45/60	35/48	26/20	70/60	90/60
3 per cent. Cr, Mo. Nitriding	D.T.D.306 ..	„ 900/O	„ 600/675/A	60/75	48/60	23/17	65/50	80/50
"Valmax 3/8" Si, Cr, Mo.	.. ..	„ 1050/A	„ 800/860/A	50/65	40/52	35/20	65/45	22/12
Austenitic valve	D.T.D.49 B ..	Soften 1050/W	.. ..	45/65	..	40/25	60/35	50/25
Austenitic valve	D.T.D.282 ..	„ 1050/W	.. ..	45/65	..	40/25	60/35	50/25

regarded as a strengthener, whereas chromium, in small amounts, is primarily a hardener. Like most alloying additions, they both also give a finer grain to the steel. Other small additions may be employed for many reasons: to assist in the melting and casting procedure, to ensure soundness and a cleaner steel, to improve resistance to shock or promote greater elastic properties. For instance, up to ½ per cent. molybdenum markedly improves fatigue properties, amongst other benefits, and a like quantity of vanadium will often be put into spring steels, while copper is found to generate better weather-resistance.

Since the properties of steels can be altered by heat-treating, it follows that they cannot maintain their full strength if used under conditions where they get hot, as in steam plants. In the range 350°–500° C. (665°–930° F.) carbon steels lose almost all their tensile strength, but some alloy steels are much less affected. Notable among these resistant steels are some combinations of molybdenum and chromium; molybdenum also counteracts the embrittlement to which many other steels are liable when heated for long periods in the 350°–500° C. range. "Durehete" is a steel of this type, and the effect of high-temperature tempering on bars (1⅛ in. dia.) which have been oil-hardened from 840°–860° C. is as follows:—*

Tempering Temperature, °C.	Maximum Stress. Tons/Sq. in.	Yield Point. Tons/Sq. In.	Elongation. Per Cent. on 2 In.	Reduction of Area. Per Cent.	Izod Impact. Ft.-lbs.
500	90·5	84·0	11·5	24	27
600	74·3	69·3	17·5	52	47
650	64·8	58·0	20·5	55	63
700	54·1	47·7	23·0	59	70

These figures are not maintained, of course, if the steel be held at the temperature for long periods, but gradually fall away. The stress which the metal *will* stand is a more useful figure, and on the Barr-Bardgett creep testing machine it is possible to furnish results approximating closely to *safe working stresses*, that is, stresses which will produce negligible initial deformation, and permanence of dimensions for long life. As seen from the table † on p. 362, the molybdenum steels are far superior to carbon steel; but the higher chromium contents are not so effective as the lower. Work on the choice of the best creep-resisting steels is still going on.

* Samuel Fox & Co., Ltd., associated with The United Steel Companies Ltd.

† *Alloy Metals Review*, June 1938. Review of paper on "Special Steels in Engineering and Shipbuilding," by Dr T. Swindon.

Type of Steel.	Condition.	Tested at °C.	Tested at °F.	Safe Working Stress. Tons/Sq. In.
Mild steel, 0·12 per cent. C.	Normalised, 910° C.	500	(930)	1·1
½ per cent. Mo.	Annealed, 910° C.	500	(930)	2·5
½ per cent. Mo, 1 per cent. Cr.	Normalised, 910° C.	510	(950)	3·5
½ per cent. Mo, 6 per cent. Cr.	Air-cooled, 950° C. ,, , 750° C.	500	(930)	1·6

To obtain other specific properties, larger additions of special elements are employed, usually with one element predominating, and these produce steels which behave quite differently from ordinary steels, so that their heat-treatment must always follow the instructions supplied by their particular maker. For instance, stainless steels of the hardenable cutlery variety contain 13 per cent. of chromium, and the non-magnetic, austenitic varieties, including the "Staybrite" range, have 18 per cent. chromium and 8 per cent. nickel; high-speed steels contain around 19 per cent. tungsten, and permanent magnet steels may have up to 35 per cent. cobalt, though these latter are being rivalled by the newer nickel-aluminium magnet steels. Manganese in proportions of 12–13 per cent. has long been used to provide exceptional resistance to wear, as in rail and tramway crossings; for heat-resistance, chromium and silicon additions are made, as shown in Table 8.

Steels for exhaust valves supply an example of the special application of alloying elements, because in this field the conditions are severe and the metal must possess a combination of properties which it has been hard to obtain. The table on p. 364 * shows the hot and cold strengths of various steels which have been used for valves, and it is seen that the high nickel-chromium steels are vastly superior. This type of steel, being austenitic, is not susceptible to hardening (and consequent embrittlement) on overheating, as may be the case with steels used previously. In other words, it has that important attribute, permanence of properties after many re-heatings. The nickel content varies from 8 to 30 per cent., and the chromium from 12 to 25 per cent. Silicon up to 2 per cent. has been found to improve still further the resistance to oxidation, and the presence of tungsten up to 4 per cent. improves the hot strength. Thus at the present time the heat-resisting nickel-chromium steels containing silicon

* *Nickel Bulletin*, February 1937.

Table 8.*

Typical Heat-resisting Steels manufactured by Messrs. Hadfields, Ltd., Sheffield.

Approximate Analysis.					Treatment.	Mechanical Properties.										Coeff. of Expansion 10^{-6} per °C. at 20°.
						Room Temperature.						800° C.				
C	Cr	Ni	W	Si		Y.P. Tons per Sq. Inch.	M.S. Tons per Sq. Inch.	El. Per Cent.	R.A. Per Cent.	Brinell Hardness No.	Izod Ft.-lbs.	M.S. Tons per Sq. Inch.	El. Per Cent.	R.A. Per Cent.	Creep Stress 10^{-6} per Hour at 1000 Hours (40 Days).	
0·40	13·5	13·5	2·5	1·5	Forged	37	60	37	42	260	50	24	32	60	1·0	15·1
0·30	20	7	4·0	1·5	,,	38	61	34	41	260	35	25	36	60	1·2	13·3
0·25	20	7	..	1·5	,,	30	56	42	46	190	50	23	38	61	0·5	15·7
0·45	14	28	4·0	1·5	,,	32	50	30	45	200	35	21	32	43	0·55	14·1
0·43	10	37	..	0·25	,,	28	48	30	50	200	50	17	44	61	..	7·3
0·25	25	18	..	2·0	,,	33	50	29	46	195	50	20	33	48	0·5	14·3
0·50	30	..	..	1·5	,,	25	45	18	30	215	5	Same order as for carbon steel.			0·28	8·7
0·50	12	60	..	1·5	..	25	51	33	47	185	50	23	43	53	0·48	11·0

* "Heat-resisting Steels"—Publication No. F 4.—The Bureau of Information on Nickel.

and tungsten provide a material which is not only the strongest available, but also the one with the best resistance to oxidation.

Steel.	Maximum Stress. Tons/Sq. In.	Izod Impact. Ft.-lbs.	Maximum Stress. Tons/Sq. In.	
	At 20° C.	At 20° C.	At 750° C.	At 850° C.
3 per cent. nickel-chromium	56·5	66	11·2	6·9
High-speed steel (19 per cent. tungsten).	61	5	16·8	7·7
Stainless steel (13 per cent. chromium).	48·3	50	8·0	5·6
Chromium steel (10 per cent. chromium).	51	28	13·0	7·2
Silicon - chromium - tungsten (2½/8/2).	63	9	17·8	7·2
Cobalt-chromium (4/13) .	58	4	13·6	5·8
High nickel-chromium .	68	55	28·6	19·4

The subject of alloy steels is far too large to be dealt with in a few lines, and further reference should be made to fuller accounts.* It may be mentioned, however, that alloy steels should *not* be welded without a full knowledge of the material, as they may not respond well to this treatment. Special grades of some steels, *e.g.* stainless, are available for welding purposes.

Non-ferrous Alloys.

Copper.—Pure copper is a soft and ductile metal, and it has its greatest electrical conductivity in the pure state. Conductivity copper for cables is therefore over 99·9 per cent. copper. This material in the annealed condition has a tensile strength of only about 14 tons per sq. inch, with a very low yield and high elongation. By sufficient cold-working the tensile strength may be nearly doubled, but in this condition the metal is naturally more sensitive to local overworking, *e.g.* bending. It may be softened by annealing above 350° C. The electrical and thermal conductivity of copper are both reduced by alloying, but this loss may be more than balanced by improvement in other properties. The reduction caused by silicon is not so serious as with most other additions, and the presence of up to 0·2 per cent. silicon is acceptable in the copper for telegraph and telephone wires on account of the marked improvement, about 50 per cent., which

* See Simons and Gregory, *The Structure of Steel*, 1938, Blackie & Son, Ltd., for a simple account of alloy steels.

it produces in the tensile strength. The addition of ½ per cent. of arsenic reduces the conductivity by more than 50 per cent., but slightly increases tensile strength, resistance to fatigue, and helps the metal to retain its strength at elevated temperatures and to resist oxidation. Hence, this arsenical copper is used for fireboxes, boiler tubes, stay bolts, rivets, etc. The oxygen content, usually between 0·05 and 0·1 per cent., must be carefully controlled in these alloys if good mechanical properties are to be obtained. It should be possible to hot-forge a bar until it is flat, cool it in water, and then double it back and flatten it on itself without any cracks appearing at the bend.

Alloys of Copper.—The mechanical weakness of copper is overcome by alloying, and the alloys may be endowed with a wide range of properties by varying their composition and the mechanical and heat treatment to which they are subjected. Hence, they rank next to steel in importance to the engineer.

Copper can dissolve a certain amount of most elements and still retain its own simple structure, but above a certain limit the structure becomes duplex. Thus in all copper alloy series there is a division between those which behave like copper and can be cold-worked, and those more complicated alloys, harder and stronger, which can only be worked hot. Alloys of copper and zinc are *brasses*; those of copper and tin are *bronzes*.

Brasses.—Up to about 36 per cent. zinc added to copper provides a long range of ductile alloys, whose tensile strength gradually increases with zinc content, and which will withstand a very great amount of deformation before cracking. The brass called *70/30, i.e.* 70 per cent. copper and 30 per cent. zinc, or *cartridge brass*, is typical of this series, and it is eminently suitable for rolling, pressing, and spinning operations. When it becomes hardened with work it may be annealed at 550°–600° C. under carefully controlled conditions, whereby it becomes softened again. The properties of this alloy are:

Material.	Yield Point. Tons/Sq. In.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.	Brinell Hardness Number.
Cartridge brass:				
Chill castings .	6	16	60–70	60
Hard-rolled sheet	> 20	30–40	10–15	150–200
Annealed sheet .	6	20–23	65–70	60

Fig. 2 * shows the effect of cold work, as represented by percentage reduction in thickness, on the mechanical properties of 70/30 brass, and the curves are similar for all alloys of this

* D. P. C. Neave, "Copper and its Alloys in Automobile Design," *Proc. I.A.E.*, vol. xxxi. p. 624.

type. No metal should be put into service in too highly a stressed condition. This is particularly so in the case of brasses, which are liable to develop spontaneous "season cracking" during

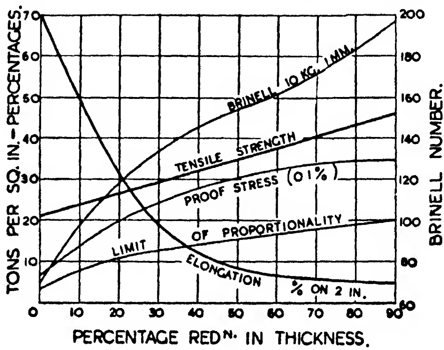


FIG. 2.

service; but this may be obviated by heating for ½ hour at 270°–280° C., a treatment which relieves the internal stress (the cause of the cracking) without losing strength or hardness. Table 9 shows the effect of increasing temperatures on drawn brasses.

Table 9.*

Effect of Temperature on Brasses.

Admiralty Brass Tube.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.
As drawn	41·85	9
Reheated to 150° C. . .	41·9	8
,, ,, 250° C. . .	41·78	10
,, ,, 270° C. . .	36·64	16
,, ,, 280° C. . .	36·62	18
,, ,, 290° C. . .	35·47	19
,, ,, 300° C. . .	27·48	48
,, ,, 350° C. . .	26·57	57
,, ,, 450° C. . .	24·41	63
,, ,, 550° C. . .	21·63	74
,, ,, 650° C. . .	20·81	79

* Aitchison and Barclay, *Engineering Non-ferrous Metals and Alloys*, Oxford University Press.

Table 9* (*continued*).

Naval Brass.	Method of Cooling.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.
Annealed for 30 min. at—			
450° C.	Quenched	29·18	20
475° C.	,,	28·74	31
500° C.	,,	27·70	47
550° C.	Slowly	27·86	31
550° C.	Quenched	28·40	36
600° C.	Slowly	24·02	39
600° C.	Quenched	24·44	57
700° C.	Slowly	25·10	41
700° C.	Quenched	25·56	51

The ductility may be seriously impaired by certain impurities, and for this reason the specification for cartridge brass places the limit at 0·05 per cent. for lead and at 0·006 per cent. for bismuth.

With as much as 40 per cent. zinc, the solution limit mentioned on p. 365 is passed. This alloy is harder and stronger, and can

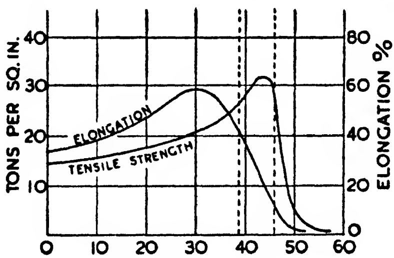


FIG. 3.

only be hot-worked. Fig. 3 shows the sudden increase in tensile strength which occurs with zinc contents in this region, and *Muntz metal* or *60/40 brass* is typical of this kind of brass. It will be noticed that with still higher zinc contents both the

* Aitchison and Barclay, *Engineering Non-ferrous Metals and Alloys*, Oxford University Press.

tensile and elongation figures drop very suddenly, and these alloys are therefore not of engineering use. In order to take advantage of this peak in the curve, the alloys in this range are made very carefully with accurate compositior.

Typical properties of Muntz metal are:

Material.	Proof Stress. Tons/Sq. In.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.	Brinell Hardness Number.
Muntz metal:				
Hot-rolled and cold-drawn.	14·8	25·8	48·5	116
Extruded and cold-drawn.	21·6	28·5	31·0	126
Extruded and rolled.	11·1	27·6	33·5	116

Many other alloying elements are added to both the 70/30 and 60/40 kinds of brass, for various reasons, and the varieties and trade names are legion. For instance, 1 per cent. tin assists resistance to sea-water corrosion, and when added to the 70/30 alloy gives *Admiralty brass*, or to 60/40 gives *Naval brass*. The latter is widely used for marine and engineering castings. Similarly, up to 2 per cent. of aluminium is often added to condenser-tube brass to resist corrosion.

Free-machining, or leaded, brasses are of the 60/40 type, containing up to 3 per cent. of lead, and are particularly suitable for hot extrusion into lengths of rod to be machined later on automatic lathes, for such products as screws, nuts, and small instrument parts. The lead provides ease of machining, but at the sacrifice of other mechanical properties.

Extruded leaded brass (58 per cent. copper, 2 per cent. lead) has a yield point of 12 and maximum stress of 26·5 tons per sq. inch, with an elongation of 28 per cent.

The 60/40 brasses are frequently strengthened by additions of small percentages of such elements as manganese, aluminium, iron, tin, nickel, and silicon, to a total of about 4 per cent., and are then called *high-tensile brasses*. Those containing manganese, though really manganese brasses, are usually incorrectly termed *manganese-bronzes*. A typical manganese-bronze * in extruded or hot-rolled form has a tensile strength of about 35 tons per sq. inch, with 22 per cent. elongation and proof stress (0·15 per cent.)

* D. P. C. Neave, "Copper and its Alloys in Automobile Design," *Proc. I.A.E.*, vol. xxxi. p. 624.

of 17 tons per sq. inch. Manganese-bronze can be hot-stamped and extruded easily, and is used for bushes, pinions, etc. It is also used in the cast condition, especially for propellers. Manganese is beneficial first of all as a deoxidant, giving clean and sound castings, but to obtain its maximum benefit about 1 per cent. excess manganese should be left in the final metal. Aluminium in conjunction with manganese gives a greater improvement, and nickel additions give a still higher combination of physical properties which make the brass especially resistant to erosive action. This is particularly necessary in marine propellers, and these elements are seen to be present in turbadium-bronze (Table 10), which was used for the propellers of the *Queen Mary*.

Delta metal is another excellent brass of the 60/40 type. Its chief alloying ingredient is iron, up to 3 per cent., but nowadays it may also contain small amounts of manganese or other elements. It develops a strength of over 30 tons per sq. inch after being extruded or hot-worked above 550° C., and is also considerably resistant to corrosion, so that it may be used to replace mild steel under conditions where corrosion is also a consideration. Those alloys which contain small amounts of several added elements, particularly manganese, aluminium, iron, silicon, and nickel, seem to possess resistance to corrosion far superior to that of the straight brasses. *Tungum* is a high copper alloy of this type, which can actually be used for certain acid pans.

The presence of nickel in brasses and in bronzes is useful in giving an all-round improvement in properties, and in larger proportions it is especially beneficial in maintaining the strength at elevated temperatures. Tables 11 and 12 demonstrate these points.

Bronzes.—Tin is more powerful than zinc as an alloying element in copper, 1 per cent. tin being roughly equivalent to 3 per cent. zinc. Up to 10 per cent. tin the bronzes may be cold-worked, but are stiffer and stronger than the corresponding brasses, as well as being darker in colour, more expensive, and somewhat more resistant to corrosion. Coinage bronze, 95 per cent. copper, 5 per cent. tin, is an example. To assist deoxidation and ensure casting sound ingots, 1–2 per cent. zinc may be added to these tin alloys. *Admiralty gun-metal,* for instance, has 88 per cent. copper, 10 per cent. tin, and 2 per cent. zinc, and is generally known as 88/10/2. With tin contents much over 10 per cent., the alloys are harder and must be hot-worked.

Gun-metal is used chiefly for castings, and has a yield point of about 11 and a maximum stress of 17 tons per sq. inch, with 12–13 per cent. elongation on 2 inches. In the cast state it may be used for bearings, but if annealed and quenched from 700° C., the tensile strength will be raised and the elongation improved, but the bearing properties impaired.

Table 10.*

Mechanical Properties of Sand-cast High-tensile Brass.

Alloy.	Composition. Per Cent.							Yield Point. Tons/Sq. In.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent. on 2 Inches.	Brinell Hardness Number.
	Cu	Zn	Al	Ni	Fe	Mn	Sn				
Manganese-bronze	57·2	41·0	0·3	..	0·7	0·8	..	15·0	32·0	25·0	129
	56·97	37·76	2·02	..	1·6	1·6	..	..	39·0	21·0	..
	†50·0	44·0	..	2·0	1·0	1·75	0·5	17·0	38·0	18·0	154
	56·0	36·0	1·0	3·0	1·0	2·0	..	17·8	35·7	20·0	166
	64·0	24·0	5·0	..	2·0	3·0	..	20·1	40·2	25·0	208
High-tensile brass	60·0	34·0	3·0	3·0	..	..	..	16·6	36·7	19·0	125
	60·0	31·45	4·1	3·0	1·45	..	..	23·4	44·5	12·0	159

* "Modern Non-ferrous Engineering Castings"—Publication No. D 11.—The Bureau of Information on Nickel.

† Turbadium-bronze.

Table 11.*

Effect of Nickel on Strength of Bronzes.

Alloy.	Composition. Per Cent.					Mechanical Properties.			
	Cu	Sn	Zn	Pb	Ni	Yield Point. Tons/Sq. In.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent. on 2 Inches.	Brinell Hardness Number.
Gear bronze . . .	87·5	11·5	..	..	..	10·7	18·5	15·5	83
	87·0	11·5	..	..	1·5	12·6	24·6	19·8	95
Gun-metal . . .	88·0	8·5	3·5	..	..	9·9	19·3	19·8	78
	88·0	8·0	3·5	..	0·5	10·4	21·1	26·0	77
Gun-metal (red brass) .	84·0	5·0	5·0	6·0	..	6·3	11·3	11·3	52
	82·5	5·0	5·0	6·0	1·5	8·3	15·3	20·3	61
Leaded bronze . .	78·5	10·0	..	11·5	..	8·5	11·1	10·0	58
	78·0	10·0	..	11·5	0·5	9·3	13·1	7·5	64
High leaded bronze .	67·5	6·0	1·5	25·0	..	7·4	12·3	18·8	56
	70·0	7·0	1·5	21·0	0·5	7·9	12·8	16·8	58

Note.—The above bronzes were de-oxidised with phosphorus and contain a residual percentage of from 0·02 to 0·04 of this element.

* N. B. Pilling and T. F. Kihlgren, "Some Effects of Nickel on Bronze Foundry Mixtures," *Trans. Amer. Found. Assn.*, 1931, vol. ii. p. 93.

Table 12.*

Properties of Non-ferrous Alloys at Elevated Temperatures.

Alloy.	Composition.								Room Temp.		205° C. (400° F.).		315° C. (600° F.).		427° C. (800° F.).	
	Cu	Zn	Al	Ni	Fe	Mn	Sn	Pb	Max. Stress. Tons/Sq. In.	Elongation. Per Cent.	Max. Stress. Tons/Sq. In.	Elongation. Per Cent.	Max. Stress. Tons/Sq. In.	Elongation. Per Cent.	Max. Stress. Tons/Sq. In.	Elongation. Per Cent.
Manganese brass (extruded).	55·1	41·89	0·07	0·28	0·84	0·36	0·77	0·52	40	21	24	35	15	50	5	60
Gun-metal (cast) .	86·28	5·10	..	0·23	0·06	..	5·99	2·33	15	20	..	..	11	16	6	2
Gun-metal (cast) .	88·0	2·0	..	..	..	..	10·0	..	17	17	15	15	12	4	8	2
Bronze (cast) .	86·52	1·29	..	0·09	0·01	..	11·86	0·17	19	17	18	15	13	4	9	2
Bearing bronze (cast).	79·50	free	..	..	..	..	10·25	10·15	15	21	..	..	8	3	..	..
Bearing bronze (cast).	70·30	..	..	..	..	..	4·90	24·70	10	14	9	12	6	5	..	..
Low nickel bronze (cast).	82·0	4·0	..	3·5	..	..	10·00	..	16	25	14	23	12	15	11	8
High nickel bronze (cast).	32·55	..	0·32	53·8	0·48	..	12·72	..	32	2	..	..	..	..	30	2

* "Modern Non-ferrous Engineering Castings"—Publication No. D 11.—The Bureau of Information on Nickel.

Phosphorus is added to bronzes first as a deoxidant, and then any excess phosphorus left in the bronze improves the tensile strength and gives added resistance to corrosion by sea-water. With low alloy contents, up to 6 per cent. tin and 0·3 per cent. phosphorus, the alloys may be wrought into sheet and wire, with a tensile strength of 24 tons per sq. inch and 13 per cent. elongation.

Cast *phosphor-bronzes* for bearings, etc., contain 10–13 per cent. tin and 0·5–1·0 per cent. phosphorus. These may also be used for gears. Their maximum stress should be $\nless$ 10 tons per sq. inch, and the elongation per cent. $\nless$ 1·5, $\ngtr$ 4. The structure of most bearing alloys consists of particles of a hard constituent, which actually resists the wear, dispersed through a matrix of a softer but tough material which beds down well. The phosphorus serves to build up the hard particles, and hence there must be the correct amount present. It is highly necessary that these particles should be not only of the right size but correctly distributed, and the casting technique with bearing materials is as important as the composition.

Aluminium Bronzes.—These are really aluminium-copper alloys and have no tin content. With 7–8 per cent. aluminium the alloys are fairly ductile, and possess a beautiful golden colour. The more useful engineering alloys have 10–12 per cent. aluminium, possibly with small amounts of other elements, and are remarkable for their strength and toughness. They respond to heat-treatment rather like steels, and also maintain their strength quite well as working temperatures rise in service. The 10 per cent. aluminium alloy has a strength of 33 tons per sq. inch at 300° C. They are used for die-castings, pump-rods, etc., and even in place of steel for "non-sparking" chisels. The properties of the alloys are:

Material.	Yield Point. Tons/ Sq. In.	Maximum Stress. Tons/ Sq. In.	Elongation. Per Cent.	Brinell Hardness Number.
Aluminium bronze:				
7 per cent. aluminium—				
Hard-rolled . .	38·7	39·8	17·5	195
30 min. at 650° C. and air-cooled.	7·0	27·5	71	75·5
10 per cent. aluminium—				
Chill-cast.	13·4	36·2	22·5	..
Water-quenched from 900° C.	19·6	43·2	1·5	..
Quenched at 900° C. and tempered at 650° C.	17·6	38·6	24·0	..

Silicon has a very powerful effect on copper, small percentages producing very tough alloys, which have also a fair resistance to corrosion. These may or may not have tin in them, but are all called *silicon bronzes.* They are being developed especially in America, and they are likely to be of wider importance in future, as the silicon can be used to replace the more expensive tin.

Nickel and copper will blend together in all proportions, giving a long and useful series of alloys. Nickel soon removes the copper colour, and *cupro-nickel* with 20 per cent. nickel is almost white. Higher up the series the colour becomes quite steel-like. Increasing nickel content increases the toughness of the alloys, their electrical resistance, corrosion resistance, and price. All the alloys can be worked without complications to form sheet, wire, and tubes. They are also suitable for hot-forging and stamping. The following table * shows a few of the uses of typical alloys:—

Alloy.	Nickel. Per Cent.	Copper. Per Cent.	Special Uses.
"Cupro-nickel" as commonly known	10	90	Locomotive stay rods.
	15	85	Coinage alloy.
	20	80	Bullet envelopes, drop forgings, solid drawn tubes.
	25	75	Coinage alloy, automobile radiator sheets (not plated).
"Constantan" .	40	60	Electrical resistance and thermo-electric work.
"Monel metal" .	65/70	30/35	General engineering.

The strength * of alloys of this series is:

Nickel. Per Cent.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.
5	18·0	50
10	21·5	30
15	20	35/40
20	23	40/45
25	23·5	39/42
40	30	45
65/70	40	40

Monel metal is a rather special alloy because it is made, not by melting down the pure metal ingredients together, but by smelting the mixed nickel-copper ore direct. It contains approximately 2/3rds nickel and 1/3rd copper, together with essential

* Aitchison and Barclay, *Engineering Non-ferrous Metals and Alloys*, Oxford University Press.

small quantities of iron and manganese, etc. It is resistant to many chemicals, oils, salts, etc., as well as to the atmosphere, sea-water, and to superheated steam, and this, combined with its malleability and toughness, enables it to be used for a very wide range of engineering applications. All the nickel-copper alloys keep their strength reasonably well at high temperatures, but Monel is better than the others, having a tensile strength at 300° C. of over 30 tons per sq. inch.

Where nickel and zinc are added together to copper within certain wide limits, there is produced a range of cheaper, softer alloys, pale yellow or silver white, which are really a cross between 70/30 brass and cupro-nickel. During the War their name of *German silvers* was changed to *nickel silvers*, but, as they contain no silver, the more correct modern name is *nickel brasses*.

The nickel content varies from	5–35	per cent.
,, zinc ,, ,, ,,	35–20	,,
,, copper ,, ,, ,,	60–45	,,

All these alloys are of similar constitution and can be worked and rolled. Their strength varies from 15–25 tons per sq. inch, and the elongation from 45–25 per cent. They do not readily oxidise or corrode, and are used as the basis for electro-plated silver goods, and for innumerable automobile, railway, and shipping fittings, as well as for electrical resistance wires.

Light Alloys.—The specific gravities of the common commercial metals are as follows:—

Magnesium	1·75	Tin	7·3	Copper	8·8
Aluminium	2·68	Steel	7·8	Nickel	8·9
Cast-iron	7·2	Brass	8·4	Lead	11·3
Zinc	7·2	Bronze	8·6		

Magnesium and aluminium are therefore the only common metals available for providing the reduction in weight so keenly sought in aeroplane and automobile construction.

Aluminium and its Alloys.—Aluminium conducts electricity and heat well. Its specific conductivities are about 60 per cent. those of copper, but weight for weight it is better. It is mechanically a weak metal, however, its tensile strength being only 6–7 tons per sq. inch. Therefore when making cables of aluminium a steel wire is incorporated to provide strength. As in the case of conductivity copper (p. 364), it is possible to improve the tensile strength without undue loss of conductivity by making small alloy additions, *e.g.* ½ per cent. magnesium, ½ per cent. silicon, and ½ per cent. manganese. Such an alloy may be strengthened also by heat-treating, and is more resistant to corrosion.

Commercially pure aluminium always contains traces of iron and silicon from its mode of preparation, but good quality metal is about 99·8 per cent. aluminium. It is largely used for sheets, wire, and tubes, and its mechanical properties are:

	Tensile Strength. Tons/Sq. In.	Elongation. Per Cent.
Cast . . .	6–7	3
Cold-worked . .	12	..
Annealed 350°–400° C.	6–7	40

Even in the worked and annealed condition, however, it is not strong enough for structural purposes, and alloys have been developed for castings, also for producing greater strength, and for resistance to sea-water corrosion, these representing three fields where pure aluminium falls short.

Casting alloys contain copper, and/or zinc. The alloys with zinc alone are more fluid than the copper alloys, but are hot-short. Therefore copper is added with the zinc. All these alloys have high contraction on setting, but alloys with about 12 per cent. silicon flow well and have only a small contraction, and are therefore useful for thin section castings. The strength of typical casting alloys is shown in Table 13. The 12 per cent. copper (L.8) alloys have found considerable application for pistons.

Alloys containing copper are capable of being heat-treated by the process known as "age-hardening" to improve their strength. This is more effective when some magnesium and silicon are also present, and the process was first developed with *duralumin*. This has approximately 4 per cent. copper, and ½ per cent. each of magnesium, silicon, iron, and manganese. The treatment takes place in two stages: first, heating to a relatively high temperature (about 500° C.) to make the alloys homogeneous, and quenching in water to retain this condition. This is called "solution" or "homogenising," and leaves the metal in a soft condition. Second, on keeping the alloys at room temperature for several days, or somewhat above room temperature for some hours, an internal change gradually proceeds which brings the alloy to its maximum strength. This is called "ageing." It is inconvenient to have the metal changing while it is being handled, and this has been overcome in two ways: either the metal, after solution treatment, may be kept at a low temperature in a refrigerator until required, as practised with duralumin rivets, or different alloy compositions may be used so that the change will only occur when warmed. Duralumin is used mainly in the forged condition, and the surprising improvement effected by ageing is shown by the following figures:—

Condition.	Y.P. Tons/ Sq. In.	M.S. Tons/ Sq. In.	Elongation. Per Cent.	Reduction in Area. Per Cent.	Brinell Hardness No.
Air-cooled from 500° C.	7	18	21	41	65
Quenched in water from 500° C.	7	17	20	41	63
Quenched and aged	15	26	20	35	98

Y alloy is similar to duralumin, but having 4 per cent. copper, 1·5 per cent. magnesium, and 2 per cent. nickel. It is used largely for castings, but may also be forged, and retains its strength above 200° C. rather better than duralumin does. Table 13 shows its properties when cast and heat-treated. Forging plus heat-treating produces a maximum stress of about 22 tons per sq. inch, with an elongation of 15 per cent.

Table 13.*

Aluminium Casting Alloys.

Common Designation.	Nominal Composition (Remainder Aluminium). Per Cent.	Treatment.	Minimum Tensile Strength. Tons/ Sq. In.		Proof Stress (0·1 per Cent.). Tons/ Sq. In.		Elongation. Per Cent. on 2 In.		B.S. Specification No.
			Sand Cast.	Chill Cast.†	Sand Cast.†	Chill Cast.†	Sand Cast.	Chill Cast.†	
Y alloy	Cu 3·5–4·5 Ni 1·8–2·3 Mg 1·2–1·7	As cast	10	12	8·5	8·5	..	..	703 (2 L.24)
		Heat-treated ‡	14	18	13	14	..	2	704 (L.35)
4 L.11	Cu 6·0–8·0 Sn 1 (optional)	As cast	7·5	9	3·5	3·5	1·5	3	361 (4 L.11)
3 L.8	Cu 11–13	As cast	7	9	4·5	4·5	..	..	362 (3 L.8)
3 L.5	Zn 12·5–14·5 Cu 2·5–3·0	As cast	9	11	3·5	3·5	2	3	363 (3 L.5)
Alpax Wilmil BA/40 D	Si 10·0–13·0	As cast	10·5	13	3·5	4·5	5	8	702 (L.33)

* The British Aluminium Co., Ltd.

† The values given in these columns are not specification requirements, but may be regarded as reasonable minima.

‡ Six hours at 500°–520° C. and quenched. Aged at room temperature for five days, or boiling water two hours.

A large number of age-hardening alloys have been produced by Messrs. Rolls-Royce, Ltd., and Messrs. High Duty Alloys, Ltd., and are known as the *Hiduminium RR* alloys. These may be regarded as superior alloys of the duralumin type, and they give rather greater strengths which are retained over a wider working range of temperature. They contain iron and a small amount of titanium for refining the grain. The heat-treated alloys as a class have the strength of steel, although their specific gravity is less than 3, and they are likely to find very much wider application for purely structural purposes in the future. The latest alloy, RR 77, provides the highest strength to weight ratio of any available light alloy.

The Mechanical Properties of RR Alloys.

	Proof Stress (0·1 per Cent.). Tons/Sq. In.	Yield Point. Tons/Sq. In.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.	Reduction in Area. Per Cent.	Brinell Hardness No.
RR 50, as cast	7	8	14–15	7–10	12	72
,, heat-treated	12–14	14	16	4–8	10	80
RR 53, as cast	12	13	14	3	4	80
,, heat-treated	22	23	23–25	1	1·5	132–152
RR 56, heat-treated	21–23	22–24	27–32	15–10	14–20	125–148
RR 59, heat-treated	19·2	21	24	8	17·8	127
RR 77, annealed	4–8	..	12–14	20–14	..	45–65
Solution-treated and naturally aged.	18–21	..	29–32	21–16	..	130–140
Solution-treated and artificially aged.	28–33	..	33–38	16–10	..	160–180

It must be remembered that the melting-point of aluminium is only 660° C., and therefore the whole range of temperatures within which its alloys may be heat-treated and used is much narrower than with the steels. This means, first, that the treatments are extremely delicate, and it is foolish for anyone to attempt them who has not a full knowledge of the alloy concerned, combined with the necessary accurately controlled apparatus. Secondly, the alloys cannot be expected to remain stable at any high temperatures. One of the most popular alloys is RR 56, which is used in the form of tubes, channels, and sheets in much constructional work, *e.g.* Imperial Airways liners, London Transport buses, crane booms and fire brigade escapes. The effect of temperature on this heat-treated alloy is shown by the following figures:—*

Temperature of Test, °C.	Maximum Stress. Tons/Sq. In.	Brinell Hardness at Test Temperature.	Brinell Hardness after Cooling.
Normal	29·0	138	..
100	27·0	130	138
150	25·5	120	138
200	24·0	107	138
250	20·0	80	120
300	16·75	37	85
350	8·50	16	66

The effect of temperature on other alloys is demonstrated by the table † below, which shows the softening developed when the load is maintained.

Creep Brinell Tests—Six hours, load 250 kilos at 100° C. and 250° C.

Alloy.	Heat-treatment.	Original Brinell Hardness (Cold).	Brinell Hardness after 6 Hours. 100° C. Loaded.	Brinell Hardness after 6 Hours. 250° C. Loaded.
RR 59 . . .	Solution and precipitation.	138	121	67
Y alloy . .	" "	118	104	62
RR 53 . . .	" "	142	124	69
RR 50 . . .	Precipitation only.	72	69	53
8 per cent. copper .	Solution and precipitation.	93	82	48
12 per cent. zinc, 2 " copper	Air-aged 7 days.	96	48	23
12 " silicon .	" "	58	51	26

All the aluminium alloys have relatively high thermal conductivity (0·39 C.G.S. units average, as compared with 0·11 for steel and 0·052 for cast-iron), and this is very useful in cast pistons and forged connecting-rods. But the expansion of the alloys is also rather high, and this must be allowed for in the design. The

* "RR 56 Aluminium Alloy—Properties, Heat-treatment, and Uses"—Publication No. D 8.—The Bureau of Information on Nickel.

† "Nickel Alloys in Aeronautical Engineering"—Publication No. H 27.—The Bureau of Information on Nickel.

coefficients are $22–27 \times 10^{-6}$ per °C., as compared with $11–15 \times 10^{-6}$ for steel and $8–14 \times 10^{-6}$ for cast-iron.

The alloys normally do not resist corrosion well, and many processes are developed which satisfactorily treat the surface to produce a protective *anodised* film. Also pure aluminium is better than its alloys against atmospheric corrosion, and the pure metal may be welded and rolled on to the surfaces of an alloy sheet, *e.g.* duralumin, giving what is effectively three-ply metal, thus providing strength plus better corrosion resistance. These are termed "clad" alloys.

For sea-water corrosion the compositions contain either around 12 per cent. silicon, which have already been mentioned, giving low expansion alloys of the Alpax type; or magnesium, up to 10 per cent., with other special additions. These alloys are strong and resistant, but do not normally respond to ageing treatment.

The alloys MG 7 and RR 66 are of this class, and the casting alloy "Birmabright" has manganese in addition to the magnesium. The corresponding German type of alloy is the *Seewasser* alloy, which contains small amounts of antimony.

Magnesium Alloys.—These alloys, being lighter than aluminium, are coming rapidly to the fore, and are likely to be extended in use considerably as new methods of producing magnesium metal are just now being exploited. The fire hazard in casting and machining this material is not now considered at all serious if reasonable precautions are taken. The alloys—commonly known by the German name *Elektron*—usually contain aluminium, zinc, and manganese. Some of them may be heat-treated in a manner similar to aluminium alloys. Typical casting alloys are as follows:—*

Composition. Per Cent. Maximum.			Application.	D.T.D. Specification.
Al	Zn	Mn		
4·5	3·5	0·4	Sand castings	59A
8·0	1·5	0·4	Sand castings	59A
3·25	1·25	0·4	Sand and gravity die castings	136A
10·0	1·5	0·4	Sand, gravity, and pressure die castings	136A

The 4·5 per cent. aluminium alloy has a tensile strength of 16 tons per sq. inch with 7 per cent. elongation, and the 8·0 per cent. aluminium alloy may be heat-treated to give 25 tons per sq. inch with 10 per cent. elongation. Magnesium alloys are quite commonly used for cylinder heads, crank-cases, etc.

* *Magnesium and its Alloys*, H.M. Stationery Office, 1937.

The average properties of typical wrought alloys are as follows:—*

Approximate Composition. Per Cent.	Condition.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.
Al 6·5 Zn 1·5 Mn 0·5	Extruded .	19	12
Al 9·0 Zn 0·5 Mn 0·3	Forged . .	22	12
Mn 2·5 max.	Sheet annealed. Extruded rod .	15 16	7 6

The last-named alloy with 2·5 per cent. manganese (D.T.D. 118) is obtainable in sheet form and is useful for fuel tanks. It may be welded better than the other alloys, and possesses satisfactory strength and ductility. "Leaded" fuels, however, may cause corrosion.

Silver and cadmium have been added to magnesium alloys to produce greater strength. An alloy with 8·5 per cent. aluminium and 2·5 per cent. silver may have the following properties:—†

	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.
Annealed at 350° C. for 1 hour . .	22	7·2
Annealed at 410° C. for 2 hours and quenched.	21·8	8·4
Annealed at 410° C. for 2 hours and aged at 175° C. for 2 days.	27·1	4·8
Annealed at 410° C. for 2 hours and aged at 175° C. for 3 days.	28·7	4·8

Magnesium alloys unfortunately lose strength at high temperatures even more rapidly than aluminium alloys. Attempts have been made to find alloys which will keep more of their

* *Magnesium and its Alloys*, H.M. Stationery Office, 1937.

† J. L. Haughton, "Developments in Magnesium Alloys," *Proc. Inter. Assn. for Testing Materials*, London, 1937.

strength up to as high as 300° C. for internal combustion engine parts. One such attempt contains 10 per cent. cerium and about 1·5 per cent. each of cobalt and manganese; its hot strength is as follows:—

Temperature of Test, °C.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.
20	20	0
100	16	2·4
200	13	2·0
300	7·6	152·0

This alloy, though considerably weaker than other alloys at room temperature, is better at higher temperatures. The silver-bearing alloy mentioned above has a strength at 300° C. of only 4·7 tons per sq. inch and an elongation of 60 per cent.

As a protection against atmospheric corrosion, all magnesium alloys need surface treatment similar to that of the aluminium alloys. Treatment in chromate solutions gives the well-known golden finish, and in nitric acid solutions a black finish.

White-metals.—White-metal alloys for bearings * are either tin base or lead base, the former type covering the range of Babbitt metals. The tin base are in most ways superior to the lead base, but are more costly.

The hardening constituents necessary for the wear-resistance are provided by the presence of antimony, up to 15 per cent., and of copper, usually limited to 4 per cent. In the *tin-base* alloys it is desired to produce two well-defined compounds, which will be embedded in the softer tin-rich matrix: (*a*) A compound of tin and copper which crystallises from the molten alloy in the form of needles and forms a sort of network; (*b*) a hard compound of tin and antimony which separates in the form of characteristic cuboids. The melting procedure, casting temperature, etc., must be carefully controlled so that these two compounds are correctly arranged or the bearing will not wear uniformly. For cheaper bearings lead may be substituted for part of the tin. *Lead-base* alloys contain anything over 50 per cent. lead, and these are also hardened with antimony and copper. Where loads are not excessive and speeds are slow they are exceptionally good. Table 14 gives the properties of a wide range of white-metals.

Lead-base alloys have also been made with other alloy additions; *e.g.* the German *Bahnmetall* † (railway axle-box bearing metal) is lead base with 0·7 per cent. calcium, 0·6 per cent. sodium, and 0·04 per cent. lithium.

* Notes taken partly from M. Melhuish, "White-metal and Bronze Bearings from the Manufacturers' Point of View," *Proc. I.A.E.*, vol. xxx.

† H. N. Bassett, *Bearing Metals and Alloys*, 1937, Edward Arnold & Co.

Table 14.*—Properties of some White-metal Bearing Alloys.

Composition. Per Cent.				Tensile Properties.		Compression Test Values. Tons/Sq. In.		Brinell Hardness Number.	Remarks.
Tin.	Antimony.	Copper.	Lead.	Maximum Stress. Tons/Sq. In.	Elongation. Per Cent.	Yield Point.†	Load.‡		
93·0	3·5	3·5	..	5·12	11·6	3·6	14·7	24·9	Combines maximum toughness with strength—probably ideal for big end bearings.
86·0	10·5	3·5	..	6·65	7·1	4·4	17·2	33·3	Harder than the above but still tough—very useful for general work for main bearings.
83·0	10·5	2·5	4·0	5·60	nil	4·3	17·6	34·5	Lead reduces elongation to nil, yet alloy still has excellent shock-resisting properties.
80·0	11·0	3·0	6·0	5·70	nil	4·6	17·5	32·1	Very useful for heavy loads with high speeds: Diesel engines, turbines, rolling-mills, locomotives.
60·0	10·0	1·5	28·5	5·04	nil	4·0	12·9	27·1	For I.C. engines of all types, steam-engines, dynamos, locomotives.
40	10·0	1·5	48·5	4·58	nil	3·7	11·3	21·8	Generally useful for heavy pressure and medium speed, or medium pressure and high speed: automobile engines, railway and tramway bearings.
20	15·0	1·5	63·5	5·48	nil	4·0	12·2	31·3	Also useful for medium pressure and speed, or light pressure and high speed.
78·0	11·0	11·0	..	6·36	nil	4·6	17·9	37·0	"Plastic metal"—has a long range of solidification, and used extensively by millwrights and marine engineers for repair work; hard and durable and self-tinning.
5·0	15·0	..	80·0	4·69	2·8	3·6	13·4	24·9	"Magnolia metal" type—capable of carrying load and continuing duty under increased temperatures to a greater extent than many tin-base bearing metals in similar circumstances.

* Gregory, *Metallurgy*, 1932, Blackie & Son, Ltd.
† Load in tons/sq. inch which produced 0·001 inch permanent deformation—original length of test-piece 0·564 inch.
‡ Load required to compress piece to half-length.

Other Bearing Alloys.—With the continued increase in bearing loads and speeds there is a constant search for new mixtures that will serve at higher temperatures than the lower melting-point white-metals. *Cadmium* alloys, with small amounts of silver or nickel, have been found satisfactory in some fields. In the nickel alloy * the Brinell hardness of the hard constituent is 260 and of the matrix 55, whilst the melting-point is 310° C. as against 230° C. for the tin-base alloys. The cadmium alloy also withstands greater crushing loads than the white-metals, but it has its own difficulties, because cadmium is a very volatile metal which may introduce casting troubles.

Lead added to copper alloys gives a plasticity which can be utilised in bearings. Lead helps a bronze to resist wear, but does not increase its coefficient of friction. Alloys of this type, called *lead-bronzes*, contain 5–10 per cent. tin, 8–10 per cent. lead, and up to 0·5 per cent. phosphorus for deoxidation.

Higher lead contents, around 30 per cent., are also used with copper, and the alloys are called *plastic bronzes* even though they may contain no tin. Lead and copper do not alloy together but remain as two separate metals, *i.e.* the mixture is a mechanical one with the lead dispersed throughout the copper. Hence it is difficult to prevent segregation of the lead occurring, and the bearings must be specially cast as a thin lining on a steel shell. One satisfactory composition † used on crankshaft and other driving-shaft bearings is:

Copper	69–74 per cent.	Iron	0–·25 per cent.
Lead	26–31 ,,	Nickel	0–·1 ,,
Silver	1·5 per cent. max.	Tin	0–·15 ,,

Tin may be added up to 5 per cent., and this forms genuine bronze with the copper, but this harder material is likely to wear the shaft away unless this is made of a harder steel, such as nitralloy. These alloys will stand heavier loads and higher temperatures, up to 650° C., than the white-metal bearings.

A new alloy recently introduced claims to have overcome the two difficulties associated with other copper-base bearing metals, namely, that they are available only in the cast form and that they have low ductility. This is a *wrought chromium-bronze* alloy, which was produced in Messrs. Stone's foundry in 1936. It can be prepared in cast or wrought forms, in both of which the hard constituent is very stable, due to its chromium content, and this enables the bearings to retain their characteristics even after heating as high as 850° C. The following characteristics are claimed as compared with phosphor-bronze:—

* Imperial Smelting Corporation.

† "Aircraft Engine Materials," *S.A.E. Journal*, April 1937. See also "Bearing Materials," *Automotive Industries*, March 1938, p. 412.

Properties at Ordinary Temperatures.	Chromium-bronze.	Chill-cast Phosphor-bronze.
0·1 per cent. proof stress (tension), tons/sq. in.	10– 14	10–12
Ultimate tensile stress, tons/sq. in.	23– 27	18–20
Elongation on 4 $\sqrt{}$Area, per cent. .	30– 40	1–5
Brinell hardness number . .	90–130	90–120
Izod value, ft.-lbs. . . .	62– 65 (unbroken)	2–5
0·1 per cent. proof stress (compression), tons/sq. in.	10– 12	10–12

Pressure Die-castings.—During the last fifteen years the casting of metal under pressure has been very extensively developed, and is now widely used to produce sound and strong castings of such accurate dimensions that they need no final machining to size. The surface also is smooth and good. This is a great saving, and innumerable small engineering mass-production parts are cast in this way.

Special varieties of aluminium and magnesium alloys, and even of brass, have been developed for this work, but the most generally used alloys of all are those of a *zinc base*. Two of the most prominent * have the following alloy additions: (*a*) 4 per cent. aluminium, 2·7 per cent. copper, 0·1–0·3 per cent. magnesium; (*b*) 4 per cent. aluminium, 0·05 per cent. magnesium. The first is more generally used in this country, as it is harder and stronger and rather more easily die-cast than the copper-free alloy.

The mechanical properties of two of the zinc-base "Mazak" alloys are given below, and it will be seen that they have tensile and shock-resisting properties very considerably superior to cast-iron and other similar materials.

Mechanical Properties of Pressure-cast Mazak Alloys.†

Properties.	Mazak No. 3.	Mazak No. 5.
Tensile strength (tons/sq. in.) .	18	20
Elongation, per cent. (on 2 in.) .	4·5	3
Impact strength, ft.-lbs. (¼ in. sq. unnotched bar).	20	17·5
Compressive strength (tons/sq.in.)	27	39
Brinell hardness number . .	62	73

* A. C. Street, "An A B C of Die-casting," *Metal Industry*, 1937.
† National Alloys Ltd. (Imperial Smelting Corporation).

Wood.—The principal woods used by mechanical engineers are *ash*, *beech*, *boxwood*, *elm*, *fir*, *hornbeam*, *lignum-vitæ*, *mahogany*, *oak*, *pine*, and *teak*.

Ash, a straight-grained, tough, and elastic wood, is largely used where sudden shocks have to be resisted, as in the handles of tools, shafts of carriages, and the framing and other portions of agricultural machinery, when such are not made of metal. This wood is very durable if it is protected from the weather.

Beech takes a smooth surface, and is very compact in its grain. It is largely used for joiners' tools.

Boxwood is very hard and heavy, and takes a very smooth surface. It is of a bright yellow colour, and is used for sheaves of pulley-blocks, bearings in machinery, small rollers, etc.

Elm is valuable on account of its durability when constantly wet, and is used for piles, floats of paddle-wheels, etc.

Fir and *pine* are largely used for various purposes, because they are cheap, easy to work, and possess considerable strength. White or yellow pine is much used for pattern-making.

Mahogany is a durable, strong, straight-grained wood, and is less liable to crack or twist in seasoning than almost any other wood. It is used to a considerable extent by pattern-makers for light patterns.

Lignum-vitæ is a very hard wood of very high specific gravity, being one and one-third times the weight of the same volume of water. It is very valuable for bearings of machinery which are under water. It is also used for sheaves of pulley-blocks, and for other purposes where great hardness and strength are required.

Oak is one of the strongest and most durable of woods. It is tough and straight-grained, and is durable in either a wet or dry situation. It is good for framing or for bearings of machinery. English oak is considered the best.

Teak is also a very strong and durable wood, possessing considerable toughness. It is also valuable for many purposes, on account of the small amount of shrinkage which takes place with seasoning. The oil which this wood contains prevents the rusting of bolts or other iron parts which may be used in framing it.

Strength of Bridge and Trestle Timbers.

***From** a Report of a Committee of the American Association of Railway Superintendents of Bridges and Buildings, October* 1895.

Kind of Timber.	Tension.		Compression.			Transverse Rupture.		Shearing.	
			With Grain						
	With Grain.	Across Grain.	End Bearing.	Columns under 15 Diameters.	Across Grain.	Extreme Fibre Stress.	Modulus of Elasticity	With Grain.	Across Grain.
	Average Ultimate Breaking Stresses in Pounds per Square inch.								
White oak	10,000	2000	7000	4500	2000	6000	1,100,000	800	4000
White pine	7,000	500	5500	3500	800	4000	1,000,000	400	2000
Southern long leaf or Georgia yellow pine	12,000	600	8000	5000	1400	7000	1,700,000	600	5000
Douglas, Oregon, and Washington fir or pine — Yellow fir	12,000	...	8000	6000	1200	6500	1,400,000	600	...
Douglas, Oregon, and Washington fir or pine — Red fir	10,000	...	...	...	...	5000	...	...	...
Northern or short-leaf yellow pine	9,000	500	6000	4000	1000	6000	1,200,000	400	4000
Red pine	9,000	500	6000	4000	800	5000	1,200,000	...	...
Norway pine	8,000	...	6000	4000	800	4000	1,200,000	...	...
Canadian (Ottawa) white pine	10,000	...	...	5000	...	...	...	350	...
Canadian (Ontario) red pine	10,000	...	...	5000		5000	1,400,000	400	...
Spruce and eastern fir	8,000	500	6000	4000	700	4000	1,200,000	400	3000
Hemlock	6,000	...	...	4000	600	3500	900,000	350	2500
Cypress	6,000	...	6000	4000	700	5000	900,000	...	...
Cedar	8,000	...	6000	4000	700	5000	700,000	...	1500
Chestnut	9,000	...	...	5000	900	5000	1,000,000	600	1500
California redwood	7,000	...	...	4000	800	4500	700,000	400	...
California spruce	...	...	...	4000	...	5000	1,200,000	...	...
Factors of safety recommended	10	10	5	5	4	6	2	4	4

TABLES OF STRENGTH OF MATERIALS.

Ultimate Tensile Strength.

Approximate average values in tons per square inch.

Cast-iron	11	Phosphor-bronze—	
Wrought-iron—		Wrought	24
Along the grain	21	Cast	12
Across ,,	19	Manganese-bronze—	
Steel—		Extruded	25
Castings	35	Tin—	
Forgings	40	Cast	1·5
Mild steel, normalised	30	Zinc—	
Copper—		Cast	1·5
Cast	10	Wrought	7·0
Wrought	20	"Monel" metal—	
Annealed	14	Annealed	30
Brass (70/30)—		"Staybrite" steel—	
Cast	16	Annealed	40
Wrought	35	Duralumin	26
Annealed	22	Aluminium alloy—	
Muntz metal—		RR 77	38
Cast	18	MG 7	29
Extruded	25	Magnesium alloy (Elektron)	20
Naval brass—		Wood (along the fibres)—	
Annealed	24	Ash	7
Delta metal—		Beech	5·4
Extruded	30	Elm	5·8
Bronze gun-metal (90/10)	17	Fir and pine	5·1
Aluminium-bronze—		Lignum-vitæ	5
Chill-cast	36	Mahogany	7
Quenched and tempered	38	Oak	7
		Teak	7

Ultimate Tensile Strength of Wire.

In tons per square inch.

Material of Wire.	Unannealed.	Annealed.
Wrought-iron	30–45	20–30
Mild steel	40–80	27–55
Crucible cast-steel . . .	90–150	...
Copper	26–30	15–20
Phosphor-bronze . . .	45–75	23–30
Delta-metal	45–62	...
Silicon-bronze	28–50	...

The tenacity of wire per square inch is greater the smaller the diameter of the wire, especially when it is not annealed.

Breaking Strength of Wires at Low Temperature.*

Diameter of wires, 0·098 inch.

Material of Wires.	Breaking Load in Lbs. 15° C. = 59° F.	–182° C. = –295·6° F.
Steel (soft)	420	700
Iron	320	670
Copper	200	300
Brass	310	440
German silver . . .	470	600
Gold	255	340
Silver	330	420

Wires that were cooled to the temperature of - 182° C., and allowed to regain the ordinary temperature, were in no way changed as regards their tenacity.

Ordinary Working Stresses (for a Steady or Dead Load).

In lbs. per square inch.

Tension.

Cast-iron . . .	4,000	Gun-metal . . .	6,000
Wrought-iron—		Phosphor-bronze .	10,000
Bars or forgings.	14,000	Manganese-bronze	12,000
Plates, along the grain	14,000	Brass . . .	4,000
„ across „	12,000	Muntz-metal . .	7,000
Steel—		Naval brass . . .	8,000
Castings or forgings	20,000	Delta-metal, cast.	10,000
Mild steel . . .	18,000	„ rolled	13,000
Copper, cast . . .	4,000	Wood . . .	2,000
„ forged . .	6,000	Leather . .	800
„ sheet . .	5,000		

* From a paper by Professor Dewar, read before the Royal Institution in 1896, on the "Scientific Uses of Liquid Air."

Ordinary Working Stresses (for a Steady or Dead Load)—*continued.*

Compression.

Cast-iron . . .	16,000	Copper	6,000
Wrought-iron . .	14,000	Gun-metal . . .	6,000
Steel—		Phosphor-bronze .	10,000
Castings or forgings .	20,000	Brass	3,000
Mild steel . . .	18,000	Wood	1,500

Shearing.

Cast-iron . . .	3,000	Copper, rolled . .	2,500
Wrought-iron . .	11,000	Wood, across the grain	500
Mild steel . .	11,000	,, along ,, .	100

For a live load which produces a stress always of the same kind or in the same direction, the working stress may be taken at two-thirds of the working stress for a steady or dead load.

For a live load which produces equal stresses in opposite directions, the working stress may be taken at one-third of the working stress for a steady or dead load.

Strength to Weight Ratios.

Material.	Tensile Strength. Tons/Sq. In.	Average Specific Gravity.	Strength to Weight Ratio.
Steel piano wire, drawn very fine .	160	7·84	20·4
RR 77 aluminium alloy, heat-treated	38	2·8	13·57
Alloy steel, heat-treated . . .	100	7·85	12·74
RR 56 aluminium alloy, heat-treated	32	2·75	12·40
Aluminium alloy MG 7 . .	29	2·6	11·20
Magnesium alloy (Elektron) . .	20	1·86	10·76
Stainless steel, 18 : 8, hard-rolled .	85	7·93	10·72
Duralumin, forged and aged . .	26	2·8	9·30
Alloy steel, heat-treated . . .	55	7·85	7·01
"Monel" metal, rolled . . .	50	8·86	5·64
Mild steel, normalised . . .	30	7·85	3·82
Aluminium, hard-rolled . . .	10·5	2·71	3·95
,, annealed . . .	5·5	2·71	2·04
Wrought-iron	21	7·80	2·67
Brass, hard-rolled	32	8·4	3·81
,, annealed	18	8·4	2·14
Spruce for aircraft	4·4	0·435	10·1

Brinell Hardness Testing.*

The Brinell hardness number is the quotient of the applied load divided by the spherical area of the impression, and is given by

$$H=\frac{P}{\frac{1}{2}\pi D\{D-(D^2-d^2)^{\frac{1}{2}}\}}=\frac{P}{D^2}\left\{\frac{2/\pi}{1-\{1-(d/D)^2\}^{\frac{1}{2}}}\right\},$$

where P = load in kilogrammes, D = diameter of ball in millimetres, d = diameter of impression in millimetres, H = Brinell hardness number.

The value of d is to be the average of two readings at right angles.

Standard Balls and Loads.

Diameter of Ball.	Load.			
	$\frac{P}{D^2}=1.$	$\frac{P}{D^2}=5.$	$\frac{P}{D^2}=10.$	$\frac{P}{D^2}=30.$
Mm.	Kg.	Kg.	Kg.	Kg.
1	1	5	10	30
2	4	20	40	120
5	25	125	250	750
10	100	500	1000	3000

The same Brinell hardness number is given by tests on the same uniform material with balls of different diameters when the same value of P/D^2 is used.

The centre of the impression shall be not less than $2{\cdot}5d$ from any edge of specimen.

The thickness of specimen shall be at least ten times the depth of impression as given by: Depth in mm. $=P/\pi DH$. Lower values may be permitted in some instances.

For guidance in specifying an appropriate value for P/D^2, approximate values for representative materials are as follows: Steels, cast-iron, 30; copper alloys, aluminium alloys, 10; copper, aluminium, 5; lead, tin, and their alloys, 1.

The load shall be applied slowly and progressively to the specimen in a direction normal to the surface. The full load to be maintained for 15 seconds.

The approximate tensile strength of steel, in tons per sq. inch, may be found by multiplying the Brinell hardness numbers by 0·22.

A table showing values of H corresponding to various values of d is given on p. 392 for the case where $P/D^2=30$. This and similar tables for other values of P/D^2 are to be found in B.S. 240 : 1937.

* Extracted mainly from B.S. 240 : 1937.

Brinell Hardness Numbers.

Diameter of ball = 10 mm. Load = 3000 kg.

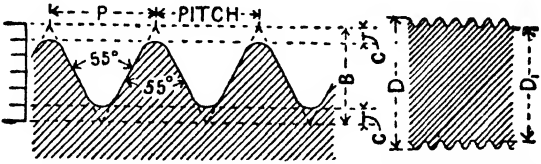
$$\frac{P}{D^2} = 30.$$

Diameter of Impression. Mm.		·01	·02	·03	·04	·05	·06	·07	·08	·09
2·00	945	936	926	917	908	899	890	882	873	865
2·10	856	848	840	832	824	817	809	802	794	787
2·20	780	772	765	758	752	745	738	732	725	719
2·30	712	706	700	694	688	682	676	670	665	659
2·40	653	648	643	637	632	627	621	616	611	606
2·50	601	597	592	587	582	578	573	569	564	560
2·60	555	551	547	543	538	534	530	526	522	518
2·70	514	510	507	503	499	495	492	488	485	481
2·80	477	474	471	467	464	461	457	454	451	448
2·90	444	441	438	435	432	429	426	423	420	417
3·00	415	412	409	406	404	401	398	395	393	390
3·10	388	385	383	380	378	375	373	370	368	366
3·20	363	361	359	356	354	352	350	347	345	343
3·30	341	339	337	335	333	331	329	326	325	323
3·40	321	319	317	315	313	311	309	307	306	304
3·50	302	300	298	297	295	293	292	290	288	286
3·60	285	283	282	280	278	277	275	274	272	271
3·70	269	268	266	265	263	262	260	259	257	256
3·80	255	253	252	250	249	248	246	245	244	242
3·90	241	240	239	237	236	235	234	232	231	230
4·00	229	228	226	225	224	223	222	221	219	218
4·10	217	216	215	214	213	212	211	210	209	208
4·20	207	205	204	203	202	201	200	199	198	198
4·30	197	196	195	194	193	192	191	190	189	188
4·40	187	186	185	185	184	183	182	181	180	179
4·50	179	178	177	176	175	174	174	173	172	171
4·60	170	170	169	168	167	167	166	165	164	164
4·70	163	162	161	161	160	159	158	158	157	156
4·80	156	155	154	154	153	152	152	151	150	150
4·90	149	148	148	147	146	146	145	144	144	143
5·00	143	142	141	141	140	140	139	138	138	137
5·10	137	136	135	135	134	134	133	133	132	132
5·20	131	130	130	129	129	128	128	127	127	126
5·30	126	125	125	124	124	123	123	122	122	121
5·40	121	120	120	119	119	118	118	117	117	116
5·50	116	115	115	114	114	114	113	113	112	112
5·60	111	111	110	110	110	109	109	108	108	107
5·70	107	107	106	106	105	105	105	104	104	103
5·80	103	103	102	102	101	101	101	100	99·9	99·5
5·90	99·2	98·8	98·4	98·0	97·7	97·3	96·9	96·6	96·2	95·9
6·00	95·5	..	..	..	..	..	..	..	..	..

Note.—This table is correct for a ball of 1 mm. diameter and a load of 30 kilogrammes if the decimal point in the value (diameter of impression) is moved one place to the left.

SCREWS, BOLTS, AND NUTS.

British Standard Whitworth (B.S.Whit.) Screw Threads.—The angle of the **V** is 55°, and an amount, C, equal to one-sixth of the total depth, B, is rounded off at the top and bottom, as shown.



D = major diameter = diameter of screw over thread, in inches.
D_1 = minor diameter = diameter at bottom of thread, in inches.
P = pitch of screw thread, in inches.
N = number of threads per inch.

$$D_1 = D - \frac{1 \cdot 280654}{N}. \qquad P = \frac{1}{N}. \qquad N = \frac{1}{P}.$$

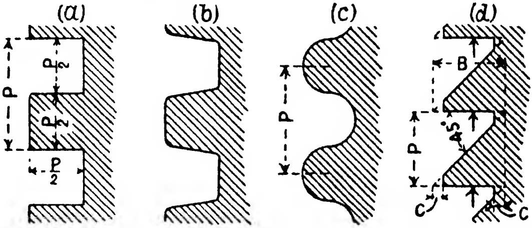
*British Standard Whitworth (B.S.Whit.) Screw Threads.**

Nominal Diam.	Threads per Inch.	Minor Diam.	Area at Bottom of Thread.	Nominal Diam.	Threads per Inch.	Minor Diam.	Area at Bottom of Thread.
In.		In.	Sq. In.	In.		In.	Sq. In.
⅛	40	0·0930	0·0068	1½	6	1·2866	1·300
3/16	24	0·1341	0·0141	1¾	5	1·4938	1·753
¼	20	0·1860	0·0272	2	4·5	1·7154	2·311
5/16	18	0·2413	0·0457	2¼	4	1·9298	2·925
⅜	16	0·2950	0·0683	2½	4	2·1798	3·732
7/16	14	0·3461	0·0941	2¾	3 5	2·3840	4·464
½	12	0·3932	0·1214	3	3·5	2·6340	5·449
9/16	12	0·4557	0·1631	3¼	3·25	2·8560	6·406
⅝	11	0·5086	0·2032	3½	3·25	3·1060	7·577
11/16 †	11	0·5711	0·2562	3¾	3	3·3232	8·674
¾	10	0·6220	0·3039	4	3	3·5732	10·03
⅞	9	0·7328	0·4218	4½	2·875	4·0546	12·91
1	8	0·8400	0·5542	5	2·75	4·5344	16·15
1⅛	7	0·9420	0·6969	5½	2·625	5·0122	19·73
1¼	7	1·0670	0·8942	6	2·5	5·4878	23·65

* Extracted from B.S. 84 : 1940.
† To be dispensed with wherever possible.

In the **Square Screw Thread** (*a*) the thickness and depth of the thread are each equal to half the pitch. The number of threads per inch for a square-threaded screw is usually half the number for a triangular-threaded screw of the same diameter.

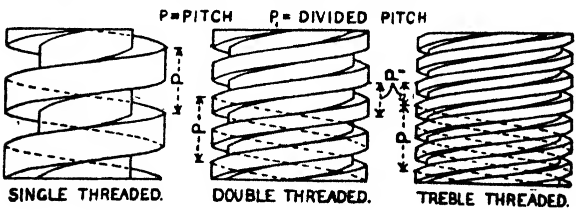
The **leading screw of a lathe** is frequently made, as shown at (*b*), to permit the nut, which is divided into two parts, to readily disengage or engage with the screw.



The edges of the square thread are often more or less rounded, so that they shall not be so easily injured; in the **Knuckle Thread** (*c*) this rounding is carried to excess.

The **Buttress Thread** (*d*) is designed to combine the smaller friction of the square thread with the greater strength of the triangular thread. The load on the thread should act as shown by the arrows. Angle of section of thread, 45°. The points and roots of the thread may be cut off straight or rounded. The amount *C* cut off may be from one-eighth to one-sixth of the total depth *B*.

Multiple-Threaded Screws.—When the pitch of a screw is required to be larger than usual for a given diameter, the loss of strength in the rod upon which it is cut may be diminished by making the screw multiple-threaded; the several threads



being parallel to one another, and of the same pitch. **The total working surface of a screw, for a nut of a given length, is increased by making the screw multiple-threaded. The divided pitch (P_1) of a multiple-threaded screw is the pitch (P) divided by the number of threads.**

British Association (B.A.) Screw Threads.—The British Standards Institution recommends the B.A. form of thread for screws below $\frac{1}{4}$ inch diameter. An exception is the $\frac{7}{32}$ inch B.S.F. screw, which is slightly smaller than No. 0 B.A. and should be used in preference to it.

The thread is V-shaped, the angle of the V being $47\frac{1}{2}°$, with the top and bottom rounded off to two-elevenths of the pitch.

Each size of screw has a distinguishing number (n), which is connected with the pitch (P) by the formula, $P=(\cdot 9)^n$, and the diameter (D) is given by the formula, $D=6P^{\frac{6}{5}}$, the dimensions being in millimetres. The standard dimensions are taken as the results of the above formulæ expressed correctly to two significant figures. The following table gives the standard dimensions in millimetres, and their approximate equivalents in inches:—

Dimensions of British Association (B.A.) Screw Threads.

Number.	Dimensions in Millimetres.		Dimensions in Inches.		Threads per Inch.
	Diameter.	Pitch.	Diameter.	Pitch.	
0	6·0	1·00	·236	·0394	25·4
1	5·3	·90	·209	·0354	28·2
2	4·7	·81	·185	·0319	31·4
3	4·1	·73	·161	·0287	34·8
4	3·6	·66	·142	·0260	38·5
5	3·2	·59	·126	·0232	43·0
6	2·8	·53	·110	·0209	47·9
7	2·5	·48	·098	·0189	52·9
8	2·2	·43	·087	·0169	59·1
9	1·9	·39	·075	·0154	65·1
10	1·7	·35	·067	·0138	72·6
11	1·5	·31	·059	·0122	81·9
12	1·3	·28	·051	·0110	90·7
13	1·2	·25	·047	·0098	101·0
14	1·0	·23	·039	·0091	110·0
15	·90	·21	·035	·0083	121·0
16	·79	·19	·031	·0075	134·0
17	·70	·17	·028	·0067	149·0
18	·62	·15	·024	·0059	169·0
19	·54	·14	·021	·0055	181·0
20	·48	·12	·019	·0047	212·0
21	·42	·11	·017	·0043	231·0
22	·37	·10	·015	·0039	259·0
23	·33	·09	·013	·0035	285·0
24	·29	·08	·011	·0031	317·0
25	·25	·07	·010	·0028	353·0

British Standard Fine (B.S. Fine) Screw Threads.*

Nominal Diam.	Threads per Inch.	Pitch.	Depth of Thread.	Major Diam.	Effective Diam.	Minor Diam.	Area at Bottom of Thread.
In.		In.	In.	In.	In.	In.	Sq. In.
3/16	32	0·03125	0·0200	0·1875	0·1675	0·1475	0·0171
7/32	28	0·03571	0·0229	0·2188	0·1959	0·1730	0·0235
1/4	26	0·03846	0·0246	0·2500	0·2254	0·2008	0·0317
9/32	26	0·03846	0·0246	0·2812	0·2566	0·2320	0·0423
5/16	22	0·04545	0·0291	0·3125	0·2834	0·2543	0·0508
3/8	20	0·05000	0·0320	0·3750	0·3430	0·3110	0·0760
7/16	18	0·05556	0·0356	0·4375	0·4019	0·3663	0·1054
1/2	16	0·06250	0·0400	0·5000	0·4600	0·4200	0·1385
9/16	16	0·06250	0·0400	0·5625	0·5225	0·4825	0·1828
5/8	14	0·07143	0·0457	0·6250	0·5793	0·5336	0·2236
11/16	14	0·07143	0·0457	0·6875	0·6418	0·5961	0·2791
3/4	12	0·08333	0·0534	0·7500	0·6966	0·6432	0·3249
13/16	12	0·08333	0·0534	0·8125	0·7591	0·7057	0·3911
7/8	11	0·09091	0·0582	0·8750	0·8168	0·7586	0·4520
1	10	0·10000	0·0640	1·0000	0·9360	0·8720	0·5972
1 1/8	9	0·11111	0·0711	1·1250	1·0539	0·9828	0·7586
1 1/4	9	0·11111	0·0711	1·2500	1·1789	1·1078	0·9639
1 3/8	8	0·12500	0·0800	1·3750	1·2950	1·2150	1·159
1 1/2	8	0·12500	0·0800	1·5000	1·4200	1·3400	1·410
1 5/8	8	0·12500	0·0800	1·6250	1·5450	1·4650	1·686
1 3/4	7	0·14286	0·0915	1·7500	1·6585	1·5670	1·928
2	7	0·14286	0·0915	2·0000	1·9085	1·8170	2·593
2 1/4	6	0·16667	0·1067	2·2500	2·1433	2·0366	3·258
2 1/2	6	0·16667	0·1067	2·5000	2·3933	2·2866	4·106
2 3/4	6	0·16667	0·1067	2·7500	2·6433	2·5366	5·054
3	5	0·20000	0·1281	3·0000	2·8719	2·7438	5·913
3 1/4	5	0·20000	0·1281	3·2500	3·1219	2·9938	7·039
3 1/2	4·5	0·22222	0·1423	3·5000	3·3577	3·2154	8·120
3 3/4	4·5	0·22222	0·1423	3·7500	3·6077	3·4654	9·432
4	4·5	0·22222	0·1423	4·0000	3·8577	3·7154	10·84
4 1/4	4	0·25000	0·1601	4·2500	4·0899	3·9298	12·13

NOTE.—It is recommended that for larger diameters in this series four threads per inch be used.

* Extracted from B.S. 84 : 1940.

British Standard Pipe (B.S. Pipe) Screw Threads.*

B.S.P. Size (Nom. Bore of Tube).	Outside Diameter of Black Tube.		Threads per Inch.	Diameters at Gauge Plane (Basic).			Gauge Length.
	Max.	Min.		Major (Gauge Diam.).	Effective.	Minor.	
In.	In.	In.		In.	In.	In.	In.
⅛	0·412	0·387	28	0·383	0·3601	0·3372	0·1563
¼	0·550	0·525	19	0 518	0·4843	0 4506	0·2367
⅜	0·688	0·663	19	0·656	0·6223	0·5886	0·2500
½	0·859	0·834	14	0·825	0·7793	0·7336	0·3214
¾	1·075	1·050	14	1·041	0·9953	0·9496	0·3750
1	1·351	1·320	11	1·309	1·2508	1·1926	0·4091
1¼	1·692	1·661	11	1·650	1·5918	1·5336	0·5000
1½	1·924	1·893	11	1·882	1·8238	1·7656	0·5000
2	2·403	2·358	11	2·347	2·2888	2·2306	0·6250
2½	3·021	2·971	11	2·960	2·9018	2·8436	0·6875
3	3 526	3·471	11	3·460	3·4018	3·3436	0·8125
3½	4·021	3·961	11	3·950	3·8918	3·8336	0·8750
4	4·526	4·461	11	4·450	4·3918	4·3336	1·0000
5	5·536	5·461	11	5·450	5·3918	5·3336	1·1250
6	6·541	6·461	11	6·450	6·3918	6·3336	1·1250
7	7·575	7·463	10	7·450	7·3860	7·3220	1·3750
8	8·585	8·463	10	8·450	8·3860	8·3220	1·5000
9	9·595	9·463	10	9·450	9·3860	9·3220	1·5000
10	10·605	10·463	10	10·450	10·3860	10·3220	1·6250
11	11·615	11·465	8	11·450	11·3700	11·2900	1·6250
12	12·625	12·465	8	12·450	12·3700	12·2900	1·6250

NOTE.—*Tubes 7 in. and upwards.* The ends shall be specially sized prior to screwing, in order to ensure ample thickness below the root of the thread. This condition shall be complied with for the screwing of cut tube at site.

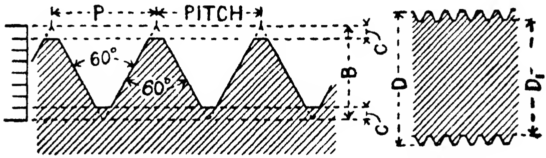
Gauge diameter is the basic major diameter of the thread, whether external or internal, parallel or taper. For taper threads the gauge diameter is theoretically located at the face of the internal screw (coupling) or at a distance equal to the basic gauge length from the small end of the external screw (pipe-end).

Gauge plane is the plane in which the gauge diameter is located.

Gauge length is the distance of the gauge plane on an external taper screw (pipe-end) from the small end of the screw.

* Extracted from B.S. 21 : 1938.

Sellers Screws.*—The *Sellers* screw thread is the standard form of triangular thread used by American engineers. The angle of the V is 60°, and an amount, *C*, equal to one-eighth of the total depth, *B*, is cut off at the top and bottom, parallel to the axis of the screw, as shown.



D=diameter of screw over the thread, in inches.
D_1=diameter of screw at bottom of thread, in inches.
P=pitch of screw thread, in inches.
N=number of threads per inch.

$$D_1 = D - \frac{1{\cdot}29904}{N}. \qquad P = \frac{1}{N}. \qquad N = \frac{1}{P}.$$

Dimensions of Sellers Screws.

Diam. of Screw. D	Number of Threads per Inch. N	Diam. at Bottom of Thread. D_1	Area at Bottom of Thread. A	Diam. of Screw. D	Number of Threads per Inch. N	Diam. at Bottom of Thread. D_1	Area at Bottom of Thread. A
Inches.	Threads.	Inches.	Sq. In.	Inches.	Threads.	Inches.	Sq. In.
¼	20	·185	·0269	2	4½	1·711	2·3001
5/16	18	·240	·0454	2¼	4½	1·961	3·0212
⅜	16	·294	·0678	2½	4	2·175	3·7161
7/16	14	·345	·0933	2¾	4	2·425	4·6194
½	13	·400	·1257	3	3½	2·629	5·4276
9/16	12	·454	·1620	3¼	3½	2·879	6·5090
⅝	11	·507	·2018	3½	3¼	3·100	7·5491
¾	10	·620	·3020	3¾	3	3·317	8·6413
⅞	9	·731	·4193	4	3	3·567	9·9930
1	8	·838	·5510	4¼	2⅞	3·798	11·3304
1⅛	7	·939	·6931	4½	2¾	4·028	12·7404
1¼	7	1·064	·8898	4¾	2⅝	4·255	14·2203
1⅜	6	1·158	1·0541	5	2½	4·480	15·7661
1½	6	1·283	1·2938	5¼	2½	4·730	17·5746
1⅝	5½	1·389	1·5148	5½	2⅜	4·953	19·2676
1¾	5	1·490	1·7441	5¾	2⅜	5·203	21·2617
1⅞	5	1·615	2·0490	6	2¼	5·423	23·0943

* Still in use, but the American Standard screw threads are now as given on pp. 399, 400.

American Standard Screw Threads.—The form of thread for all American Standard threads is known as the American National Form, and is the same as that previously known as the United States Standard (or Sellers).

Coarse Thread Series.

Size No. or Diam.	Threads per Inch.	Major * Diam.	Minor * Diam.	Size Diam.	Threads per Inch.	Major Diam.	Minor Diam.
		Inches.	Inches.			Inches.	Inches.
1	64	0·0730	0·0527	7/8	9	0·8750	0·7307
2	56	0·0860	0·0628	1	8	1·0000	0·8376
3	48	0·0990	0·0719	1 1/8	7	1·1250	0·9394
4	40	0·1120	0·0795	1 1/4	7	1·2500	1·0644
5	40	0·1250	0·0925	1 3/8	6	1·3750	1·1585
6	32	0·1380	0·0974	1 1/2	6	1·5000	1·2835
8	32	0·1640	0·1234	1 3/4	5	1·7500	1·4902
10	24	0·1900	0·1359	2	4 1/2	2·0000	1·7113
12	24	0·2160	0·1619	2 1/4	4 1/2	2·2500	1·9613
1/4	20	0·2500	0·1850	2 1/2	4	2·5000	2·1752
5/16	18	0·3125	0·2403	2 3/4	4	2·7500	2·4252
3/8	16	0·3750	0·2938	3	4	3·0000	2·6752
7/16	14	0·4375	0·3447	3 1/4	4	3·2500	2·9252
1/2	13	0·5000	0·4001	3 1/2	4	3·5000	3·1752
9/16	12	0·5625	0·4542	3 3/4	4	3·7500	3·4252
5/8	11	0·6250	0·5069	4	4	4·0000	3·6752
3/4	10	0·7500	0·6201				

* *Major diameter* is largest diameter; *minor diameter* is smallest diameter.

American Standard Screw Threads (*continued*).

Fine Thread Series.

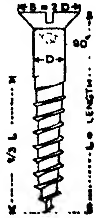
Size No. or Diam.	Threads per Inch.	Major Diam.	Minor Diam.	Size Diam.	Threads per Inch.	Major Diam.	Minor Diam.
		Inches.	Inches.			Inches.	Inches.
0	80	0·0600	0·0438	3/8	24	0·3750	0·3209
1	72	0·0730	0·0550	7/16	20	0·4375	0·3725
2	64	0·0860	0·0657	1/2	20	0·5000	0·4350
3	56	0·0990	0·0758	9/16	18	0·5625	0·4903
4	48	0·1120	0·0849	5/8	18	0·6250	0·5528
5	44	0·1250	0·0955	3/4	16	0·7500	0·6688
6	40	0·1380	0·1055	7/8	14	0·8750	0·7822
8	36	0·1640	0·1279	1	14	1·0000	0·9072
10	32	0·1900	0·1494	1 1/8	12	1·1250	1·0167
12	28	0·2160	0·1696	1 1/4	12	1·2500	1·1417
1/4	28	0·2500	0·2036	1 3/8	12	1·3750	1·2667
5/16	24	0·3125	0·2584	1 1/2	12	1·5000	1·3917

Wood Screws.

Mild Steel—Countersunk Heads and Gimlet Points.

(Guest, Keen & Nettlefolds, Ltd., Birmingham.)

Screw Gauge.	Equivalent in Parts of an Inch.	Length, Inches.		Screw Gauge.	Equivalent in Parts of an Inch.	Length, Inches.	
		Shortest Screw.	Longest Screw.			Shortest Screw.	Longest Screw.
4/0	·054	1/8	3/8	12	·220	1/2	6
3/0	·057	1/8	1/2	13	·234	5/8	6
2/0	·060	1/8	3/4	14	·248	5/8	6
0	·063	1/8	3/4	15	·262	3/4	6
1	·066	1/8	1	16	·276	3/4	6
2	·080	1/8	1½	18	·304	1	7
3	·094	3/16	2	20	·332	1	7
4	·108	1/4	2½	22	·360	1	7
5	·122	1/4	3	24	·388	1¼	7
6	·136	1/4	3½	26	·416	1¼	7
7	·150	3/8	3½	28	·444	1½	7
8	·164	3/8	4	30	·472	1½	7
9	·178	3/8	4½	32	·500	2	7
10	·192	1/2	5	36	·556	3	7
11	·206	1/2	5½	40	·612	3	7

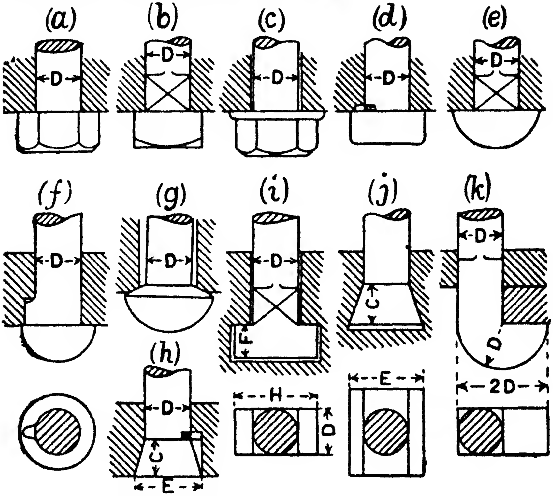


Longer or shorter lengths than those given in the table can be made to suit special requirements.

The principal dimensions of a countersunk head wood screw are as shown in the figure. Diameter (B) of head is twice diameter (D) of screw. Angle of countersunk head is 45°. Length of threaded portion is two-thirds the length of the screw.

Wood screws of various types are obtainable in mild steel, brass, "staybrite" stainless steel, copper, gun-metal, aluminium alloy, nickel silver, etc.

Forms and Proportions of Bolt-heads.—



D = diameter of bolt in inches.

(*a*) *Hexagonal head.* (*b*) *Square head.* (*c*) *Hexagonal head*, with *collar* or *flange* formed on it to increase its bearing surface. (*d*) *Cylindrical* or *cheese head.* (*e*) and (*f*) *Spherical heads.* (*g*) *Spherical head*, with *spherical bearing surface* to permit of bolt canting over slightly. (*h*) *Conical* or *counter-sunk head.* (*i*) T-*head.* (*j*) *Wedge-shaped head.* (*k*) *Hook-bolt head.*

The usual **proportions** for the above forms of bolt-heads are as follows:—

Hexagonal head.—

Width across flats $= 1\frac{1}{2}D + \frac{1}{8}$ inch.

Width across angles = width across flats × 1·155.

Height = from $\frac{3}{4}D$ to D.

For table of dimensions of Whitworth standard hexagonal bolt-heads, see p. 407.

Square head.—

Width across flats $= 1\frac{1}{2}D + \frac{1}{8}$ inch.

Width across angles = width across flats × 1·414.

Height = from $\frac{3}{4}D$ to D.

Cheese head.—

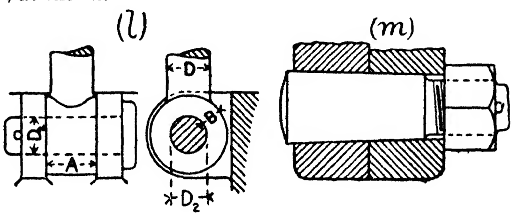
Diameter = from $1{\cdot}3D$ to $1{\cdot}5D$.

Height = from $\cdot5D$ to $\cdot8D$.

Spherical head.—
Diameter $=1{\cdot}5D$. Height $=\cdot75D$.

Other forms.—
$C=\cdot75D$. $E=1{\cdot}5D$. $F=$ from $\cdot75D$ to D.
$H=1\frac{1}{2}D+\frac{1}{8}$ inch.

A bolt may be prevented from rotating by making a portion of it next the head square, as shown at (*b*), the square portion fitting into a square hole. A pin, screwed or driven into the bolt, as shown at (*d*) and (*h*), or a snug formed on the bolt, as shown at (*f*), serves the same purpose; the pin or snug fitting into a recess, as shown.



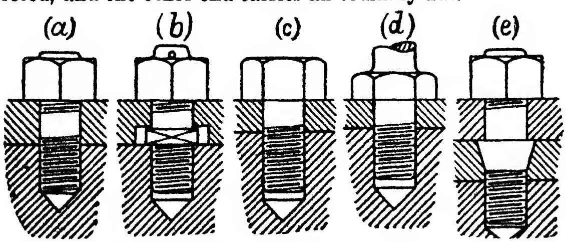
(*l*) *Eye-bolt head.*—$A=$ from D to $1{\cdot}2D$. $B=\frac{D^2}{2A}$.

$D_2=$ from $\cdot8D$ to D when the pin is supported at both sides of the bolt.

$D_2=1{\cdot}2D$ when the pin is overhung.

(*m*) *Taper bolt.*—This is easier made a tight fit in the hole, and is easier to withdraw, than a parallel bolt. Usual taper, three-eighths of an inch, on the diameter, per foot of length.

Stud and Tap Bolts.—A *Stud* or *Stud-bolt* (*a*) is screwed at both ends. One end is screwed into one of the pieces to be connected, and the other end carries an ordinary nut.

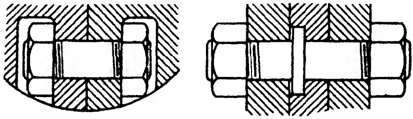


At (*b*) is shown a stud with a *collar*, which may be circular or square.

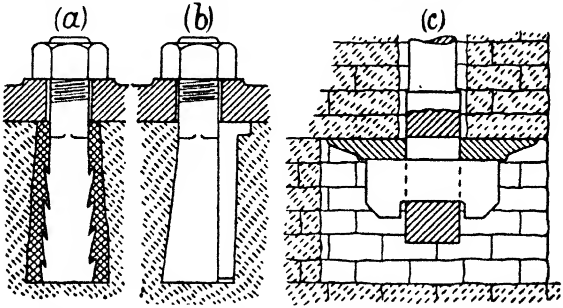
A *Tap-bolt* (*c*) is a bolt with a head, and is screwed into one of the pieces to be connected while the head presses on the other piece.

Tap and stud bolts combined are shown at (*d*) and (*e*).

Bolts with Nuts at each End.—



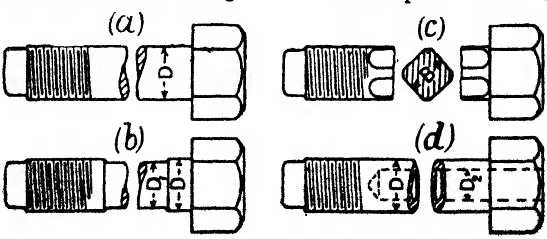
Foundation Bolts.—(*a*) Bolt with jagged head. The head is wider at the bottom than at the top, and is let into a tapered hole in a large stone. The space between the head of the bolt and the sides of the hole is filled with molten lead or molten sulphur.



The form shown at (*b*) is an improvement on (*a*), in that it can easily be removed on taking out the key.

The form shown at (*c*) has a cotter or gib which presses on a cast-iron plate. The area of the surface of the plate pressing on the foundation, multiplied by the safe compressive stress of the material of the foundation, should be equal to the safe load on the bolt.

Bolts of Uniform Strength.—The weakest part of the bolt (*a*)



is the screwed portion. A bolt is better able to resist suddenly applied loads when the area of its cross section is uniform throughout its length. The unscrewed portion may be forged or turned down, as shown at (*b*), but a portion at each end should be left of the full diameter, so that the bolt may not shake in the hole through which it passes. D_1, the diameter of the reduced part, should be equal to the diameter of the screwed part at the bottom of the thread.

The unscrewed portion may have flats forged or cut on it, as shown at (*c*), so that the cross section is a square with the corners rounded off. For bolts above half-inch in diameter, B, the breadth across the flats is given very approximately by the formula,

$$B = 1{\cdot}414D - \sqrt{D^2 - A},$$

where A is the area of the section of the bolt at the bottom of the screw thread.

The cross section of the unscrewed portion may be reduced by drilling a hole from the head to near where the screw ends, as shown at (*d*). The diameter of the hole is given by the formula,

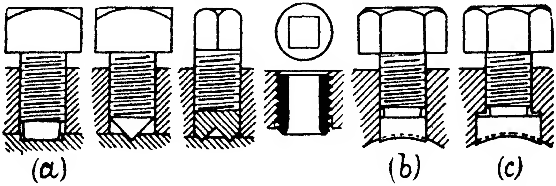
$$D_2 = \sqrt{D^2 - D_1^2},$$

where D_1 is the diameter of the bolt at the bottom of the screw thread.

The following table gives the values of B and D_2 for various sizes of bolts, with *Whitworth screw threads* and *Sellers screw threads* :—

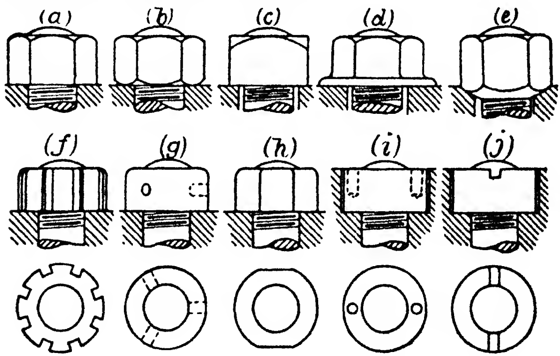
For Bolts with Whitworth Screw Threads.	D	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼
	B	·75	·95	1·15	1·33	1·53	1·72	1·95	2·13	2·36	2·56
	D_2	·54	·65	·77	·91	1·03	1·16	1·22	1·37	1·44	1·55
	D	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾
	B	2·79	2·98	3·21	3·42	3·65	3·86	4·10	4·30	4·53	4·74
	D_2	1·61	1·74	1·80	1·89	1·95	2·05	2·11	2·21	2·26	2·37
For Bolts with Sellers Screw Threads.	D	1	1¼	1½	1¾	2	2¼	2½	2¾	3	3¼
	B	·74	·95	1·14	1·33	1·52	1·75	1·94	2·17	2·35	2·58
	D_2	·55	·66	·78	·92	1·03	1·10	1·23	1·30	1·44	1·51
	D	3½	3¾	4	4¼	4½	4¾	5	5¼	5½	5¾
	B	2·78	2·97	3·21	3·42	3·62	3·83	4·03	4·26	4·46	4·70
	D_2	1·62	1·75	1·81	1·91	2·01	2·11	2·22	2·28	2·39	2·45

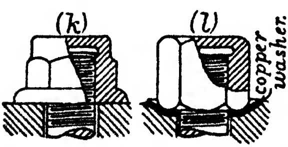
Set-Screws.—A set-screw is a screw or bolt which presses on a piece so as to prevent the rotation or sliding of that piece. Set-screws are generally made of steel, and their points should be hardened. In cases where the piece to be locked is not secured permanently in one position, the point of the set-screw



should press on the bottom of a shallow groove in the piece, as shown at (*a*), so that the bur raised by the point of the set-screw shall not interfere with the rotation or sliding of the piece when the latter has to be shifted. The piece to be locked may be protected from the damaging action of the set-screw by a metal pad, as shown at (*b*) and (*c*).

Forms and Proportions of Nuts.—





D=diameter of screw, over the threads, in inches.

(*a*) *Hexagonal nut,* chamfered on top. (*b*) *Hexagonal nut,* chamfered on top and bottom. (*c*) *Square nut.* (*d*) *Flanged nut.* (*e*) Nut with *spherical bearing surface,* which permits of bolt canting over slightly. (*f*), (*g*), (*h*), (*i*), and (*j*) *Circular nuts.*

(*k*) and (*l*) *Cap nuts.* These being closed at their outer ends, the leakage of a liquid or gas past the screw threads is prevented.

The usual **proportions** for nuts are as follows :—

Hexagonal nut.—

Width across flats $= 1\frac{1}{2}D + \frac{1}{8}$ inch. Height $= D$.

Width across angles = width across flats × 1·155.

Square nut.—

Width across flats $= 1\frac{1}{2}D + \frac{1}{8}$ inch. Height $= D$.

Width across angles = width across flats × 1·414.

Circular nuts.—

Diameter = from $1\frac{1}{2}D + \frac{1}{8}$ inch to $1\frac{3}{4}D$. Height $= D$.

Dimensions of Whitworth Standard Hexagonal Nuts and Bolt-heads.

Diameter of Bolt.	Width of Nut or Bolt-head across Flats.	Height of Bolt-head.	Diameter of Bolt.	Width of Nut or Bolt-head across Flats.	Height of Bolt-head.
$\frac{1}{8}$	·338	·109	$1\frac{1}{4}$	2·048	1·094
$\frac{3}{16}$	·448	·164	$1\frac{3}{8}$	2·215	1·203
$\frac{1}{4}$	·525	·219	$1\frac{1}{2}$	2·413	1·312
$\frac{5}{16}$	·601	·273	$1\frac{5}{8}$	2·576	1·422
$\frac{3}{8}$	·709	·328	$1\frac{3}{4}$	2·758	1·531
$\frac{7}{16}$	·820	·383	$1\frac{7}{8}$	3·018	1·641
$\frac{1}{2}$	·919	·437	2	3·149	1·750
$\frac{9}{16}$	1·011	·492	$2\frac{1}{8}$	3·337	1·859
$\frac{5}{8}$	1·101	·547	$2\frac{1}{4}$	3·546	1·969
$\frac{11}{16}$	1·201	·601	$2\frac{3}{8}$	3·750	2·078
$\frac{3}{4}$	1·301	·656	$2\frac{1}{2}$	3·894	2·187
$\frac{13}{16}$	1·390	·711	$2\frac{5}{8}$	4·049	2·297
$\frac{7}{8}$	1·479	·766	$2\frac{3}{4}$	4·181	2·406
$\frac{15}{16}$	1·574	·820	$2\frac{7}{8}$	4·346	2·516
1	1·670	·875	3	4·531	2·625
$1\frac{1}{8}$	1·860	·984			

The height of the nut is in each case equal to the diameter of the bolt.

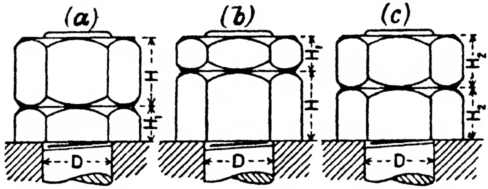
The proportions of "**United States Standard**" square and hexagonal nuts and bolt-heads are as follows :—

Width across flats $= 1\frac{1}{2}D + \frac{1}{8}$ inch.

Height of nut $= D$. Height of bolt-head $= \frac{3}{4}D + \frac{1}{16}$ inch.

Common Lock Nut.—The lower nut is first screwed up tight. The upper nut is next screwed *almost* as tight as it can be made against the lower nut. The upper nut is then held with one spanner, while the lower is screwed back tight against the upper by another spanner. Or, instead of proceeding as above, the lower nut may be first screwed up *almost* as tight as it can be made, and then, while the lower nut is held by one spanner, the upper is screwed as tight as possible against the lower by another spanner. In any case the nuts are wedged tight against one another on the screw, and *the outer nut carries the load.*

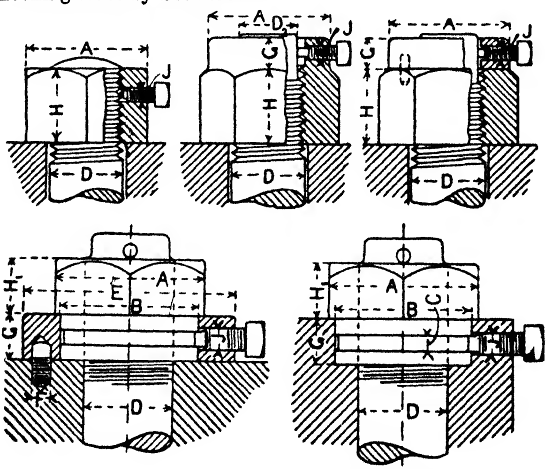
Hence the thicker of the two nuts should be on the outside, as shown at (*a*). In practice the thin nut is often on the outside, as shown at (*b*), for the reason that ordinary spanners are generally too thick to act on the thin nut when placed under



the other. Sometimes a compromise is made by having each nut of a thickness equal to three-fourths of the thickness of an ordinary nut, as shown at (*c*).

$H = D \qquad H_1 = \frac{1}{2}D \qquad H_2 = \frac{3}{4}D.$

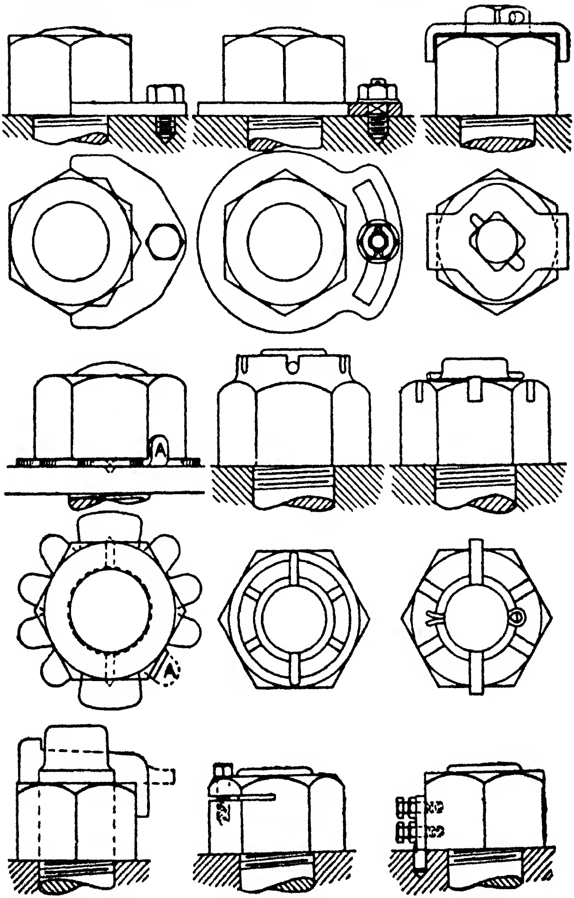
Locking Nuts by Set-screws —



D = diameter of bolt.
$A = 1\frac{1}{2}D + \frac{1}{8}$ inch.
$B = 1\frac{1}{2}D - \frac{1}{16}$ inch.
C = diameter of set-screw at bottom of thread $+ \frac{1}{16}$ inch.
Depth of groove $= \frac{1}{8}J$.

$E = 1\frac{7}{8}D + \frac{1}{8}$ inch.
$F = \frac{1}{8}D + \frac{1}{16}$ inch.
$G = 2J$.
$H = D$.
H_1 = from $\frac{3}{4}D - \frac{1}{4}$ in. to $D - \frac{1}{4}$ in.
$J = \frac{1}{8}D + \frac{1}{8}$ inch.

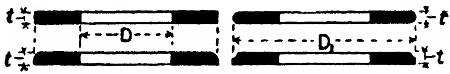
Miscellaneous Locking Arrangements for Nuts.—

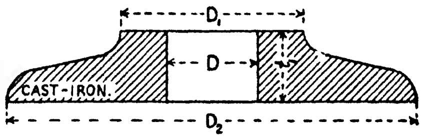


Split Pins for Bolts.—Frequently the end of a bolt projects a little beyond the nut, and this portion has the screw thread turned off, and a hole drilled across it to receive a split pin, which prevents the nut coming off.

Diam. of bolt in inches	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Diam. of pin No. I.S.W.G.	14	12	10	8	6	4	2	1
Diam. of bolt in inches	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{3}{4}$	4
Diam. of pin in inches	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{1}{2}$

Washers.—Ordinary washers are usually made of wrought-iron.
D_1 = diameter of nut across the angles × $1\frac{1}{8}$.
t = from $\cdot 1D$ to $\cdot 2D$.





For small punched or stamped washers $t = 14BWG = \cdot 083$ inch.
For *cast-iron washers* for wood D_2 = from $3D$ to $6D$, $t_1 = \frac{1}{8}D_2$, and D_1 = diameter of nut across angles × $1\frac{1}{8}$.

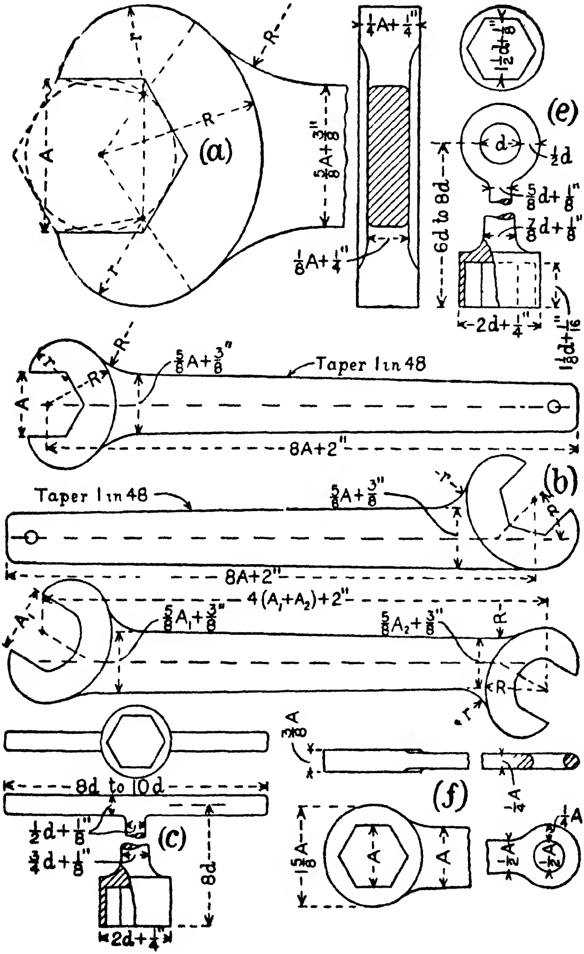
Spanners.—The best spanners are made of wrought-iron or forged steel, and the jaws are case hardened. Cheap spanners are made of malleable cast-iron.

The forms and proportions of ordinary spanners are shown on the next page. d = diameter of screw in inches. A = width of nut across the flats $= 1\frac{1}{2}d + \frac{1}{8}$ inch. The graphic construction shown at (a) gives good proportions for the head of an open spanner. The angle a between the centre line of the head and the axis of the arm varies from 0° to 45°.

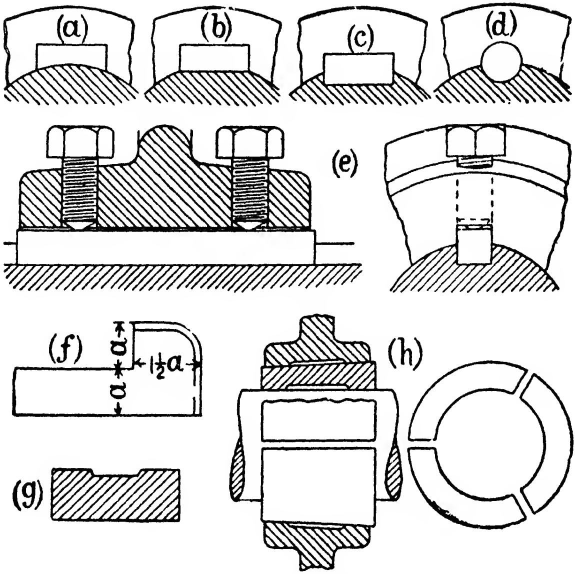
If a spanner is used as a hammer, which is frequently the case, it is better to form it as shown at (b), where the angle $a = 45°$.

A box spanner is shown at (c). For large nuts the outer end of the box spanner has an eye to receive a lever, as shown at (e).

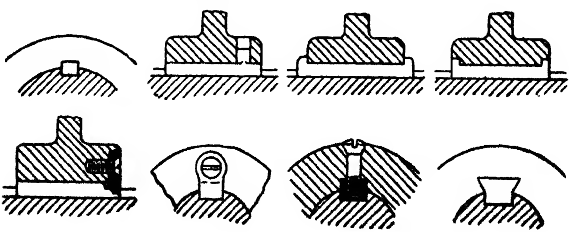
For very large nuts which require to be screwed up very tight, the closed spanner shown at (f) is used. The outer end has an eye, through which a rope may be passed at which several men may pull at the same time.


(a)
(b)
(c)
(e)
(f)
Taper 1 in 48
Taper 1 in 48
8A+2"
8A+2"
4(A1+A2)+2"
8d to 10d
8d
6d to 8d

KEYS.



(*a*) Saddle key. (*b*) Flat key or key on a flat. (*c*) Sunk key. (*d*) Round or pin key. (*e*) Sunk key secured by set-screws. (*f*) Key with gib-head. (*g*) Section of a large key showing a shallow groove to diminish the surface to be filed in fitting the key into its key-way. (*h*) Cone keys, generally made of cast-iron. These are made in one piece, and after being bored and turned the piece is divided into three parts, as shown.



The illustrations at the bottom of the opposite page show various forms of "feather" or "sliding" keys.

Proportions of Keys.—

B = breadth of key in inches.
$L = nD$ = length of key in inches.
t = mean thickness of key in inches.
D = diameter of shaft in inches.
T = twisting moment, in inch lbs., which is transmitted by the piece which is keyed to the shaft.

$B = \frac{T}{5500DL}$ for a steel key.

$B = \frac{T}{4500DL}$ for a wrought-iron key.

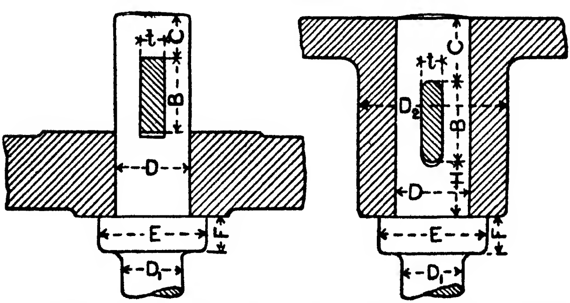
If the key is to be capable of transmitting the full power of the shaft, then,

$B = \frac{\cdot 32D^2}{L} = \frac{\cdot 32D}{n}$ for a steel key and wrought-iron shaft.

$B = \frac{\cdot 39D^2}{L} = \frac{\cdot 39D}{n}$ when the key and shaft are both of wrought-iron or both of steel.

An empirical rule often used is, $B = \frac{1}{4}D + \frac{1}{8}$ inch
$t = \frac{3}{8}B + \frac{1}{8}$ inch.
For sliding or feather keys, $t = B$.
The taper of keys varies from $\frac{1}{8}$ inch to $\frac{3}{16}$ inch per foot of length, or 1 in 96 to 1 in 64.
Sliding or feather keys have no taper.

COTTERS.



$\left.\begin{array}{l} B = 1\frac{1}{4}D \\ t = \frac{1}{4}D \end{array}\right\}$ when the bar and cotter are both made of wrought-iron or both of steel.

$\left.\begin{array}{l} B = D \\ t = \frac{1}{4}D \end{array}\right\}$ when the bar is made of wrought-iron and the cotter of steel.

$C = \frac{1}{2}D$ to D. $D_1 = \cdot 82D$. $D = 1\cdot 22D_1$.
$E = 1\frac{1}{2}D$. $F = \frac{1}{3}D$ to $\frac{1}{2}D$. $D_2 = 2D$.

The taper of cotters is usually from 1 in 24 to 1 in 48. If the taper is greater than 1 in 24, special means should be adopted to lock the cotter to prevent it from slackening back.

PIPES AND PIPE JOINTS.

Strength of Pipes and Cylinders Subjected to Internal Pressure.—

D=external diameter, d=internal diameter, and t=thickness, in inches. p=excess of internal over external pressure, in lb. per sq. in. f=allowable working tensile stress, in lb. per sq. in. e=efficiency of longitudinal joint.

Case I.—Thin pipes, cylinders, or cylindrical boiler shells.

$$p = \frac{2tfe}{d}, \qquad t = \frac{pd}{2fe},$$

where $e = 1$ when there is no longitudinal joint. For efficiencies of riveted joints, see section on riveted joints.

In practice *minimum* pipe thicknesses are calculated from the formula

$$t = \frac{pD}{2fe} + c.$$

Examples of values of f, e and c are given below, but see B.S. specifications for full information.*

Cast-iron steam or water pipes. $f = 3000$. $e = 1$. $c = 0\cdot 015D + 0\cdot 25$.

Cast-steel pipes. Up to 550° F., $f = 10{,}000$; over 600° F. up to 650° F., $f = 8000$; over 800° F. up to 850° F., $f = 6300$; over 875° F. up to 900° F., $f = 4400$. $e = 1$. $c = 0\cdot 015D + 0\cdot 25$.

Cold drawn weldless steel pipes. Up to 550° F., $f = 11{,}200$; over 600° F. up to 650° F., $f = 8900$; over 800° F. up to 850° F., $f = 7000$; over 875° F. up to 900° F., $f = 4880$. $e = 1$. $c = 0\cdot 09$.

Butt welded steel pipes. Up to 500° F., $f = 10{,}000$. $e = 0\cdot 9$ for values of t up to $\frac{7}{8}$ in., $0\cdot 85$ for values of t over $\frac{7}{8}$ in. up to $1\frac{1}{8}$ in., $0\cdot 80$ for values of t over $1\frac{1}{8}$ in. $c = 0\cdot 09$.

Solid drawn copper pipes. $f = 2500$ for steam, 3000 for feed. $e = 1$. $c = 0\cdot 03$.

* B.S. 806 : 1942. Ferrous Pipes and Piping Installations.
B.S. 1306 : 1946. Non-ferrous Pipes and Piping Installations.

Case II.—Thick pipes or cylinders.

Lamé's formula. $\dfrac{p}{f}=\dfrac{D^2-d^2}{D^2+d^2}$ and $t=\dfrac{d}{2}\left\{\sqrt{\left(\dfrac{f+p}{f-p}\right)}-1\right\}.$

Steel Tubes (*Stewarts & Lloyds, Ltd.*).—The principal classes are welded and weldless. *Welded* is made from strip formed into shape and then either butt welded or lap welded. There is no ridge, the finish being smooth and continuous outside and inside. *Weldless* is made from a billet and the tube may be finished by hot rolling or cold drawing, the latter producing a slightly smoother surface with slightly less tolerances.

Steels used are Bessemer steel and open hearth low tensile steel with 22 to 30 tons/sq. in. ult. tensile strength and high tensile steel with 35 to 41 tons/sq. in. ult. tensile strength. Tubes may be untreated, oiled, painted, or coated with coal tar or bitumen.

Steel Tubes Suitable for Screwing to B.S. 21 Pipe Threads.*

Tubes all hydraulically tested to 700 lb./sq. in.

Three thicknesses, designated Class A, B, and C, for each size.

Nominal Bore.	Approx. Outside Diameter.	Thicknesses.			Ordinary Sockets.	
		Class A.	Class B.	Class C.	Approx. Outside Diameter.	Minimum Length.
In.	In.	In.	In.	In.	In.	In.
$\frac{1}{8}$	$\frac{13}{32}$	0·072	0·080	0·104	$\frac{19}{32}$	$\frac{3}{4}$
$\frac{1}{4}$	$\frac{17}{32}$	0·072	0·080	0·104	$\frac{3}{4}$	1
$\frac{3}{8}$	$\frac{11}{16}$	0·072	0·092	0·116	$\frac{29}{32}$	$1\frac{1}{8}$
$\frac{1}{2}$	$\frac{27}{32}$	0·080	0·104	0·128	$1\frac{3}{32}$	$1\frac{1}{2}$
$\frac{3}{4}$	$1\frac{1}{16}$	0·092	0·116	0·144	$1\frac{11}{32}$	$1\frac{5}{8}$
1	$1\frac{11}{32}$	0·104	0·128	0·160	$1\frac{21}{32}$	$1\frac{7}{8}$
$1\frac{1}{4}$	$1\frac{11}{16}$	0·104	0·144	0·176	$2\frac{1}{32}$	$2\frac{1}{8}$
$1\frac{1}{2}$	$1\frac{29}{32}$	0·116	0·160	0·192	$2\frac{9}{32}$	$2\frac{1}{4}$
2	$2\frac{3}{8}$	0·116	0·160	0·192	$2\frac{25}{32}$	$2\frac{1}{2}$
$2\frac{1}{2}$	3	0·128	0·176	0·212	$3\frac{7}{16}$	$2\frac{3}{4}$
3	$3\frac{1}{2}$	0·128	0·176	0·212	4	3
$3\frac{1}{2}$	4	0·144	0·176	0·212	$4\frac{1}{2}$	$3\frac{1}{4}$
4	$4\frac{1}{2}$	0·144	0·176	0·212	$5\frac{1}{16}$	$3\frac{1}{2}$
5	$5\frac{1}{2}$	..	0·176	0·212	$6\frac{1}{8}$	$3\frac{3}{4}$
6	$6\frac{1}{2}$	..	0·176	0·212	$7\frac{1}{4}$	$3\frac{3}{4}$

* Extracted from B.S. 1387 : 1947.

Weights of Steel Tubes Suitable for Screwing to B.S. 21 Pipe Threads.*

Nominal Bore.	Weights per Foot of Black Tube.					
	Plain End.			Screwed and Socketed.†		
	Class A.	Class B.	Class C.	Class A.	Class B.	Class C.
In.	Lb.	Lb.	Lb.	Lb.	Lb.	Lb.
1/8	0·244	0·273	0·328	0·247	0·276	0·331
1/4	0·348	0·391	0·482	0·351	0·393	0·484
3/8	0·455	0·573	0·693	0·459	0·577	0·696
1/2	0·643	0·825	0·982	0·650	0·831	0·987
3/4	0·941	1·173	1·413	0·951	1·182	1·421
1	1·349	1·651	2·009	1·364	1·664	2·020
1¼	1·729	2·357	2·821	1·751	2·375	2·836
1½	2·201	2·988	3·520	2·233	3·015	3·544
2	2·779	3·795	4·488	2·827	3·836	4·524
2½	3·893	5·301	6·304	3·979	5·375	6·370
3	4·577	6·246	7·442	4·700	6·354	7·540
3½	5·878	7·172	8·557	6·006	7·288	8·660
4	6·647	8·117	9·695	6·831	8·286	9·847
5	..	10·006	11·971	..	10·262	12·205
6	..	11·891	14·241	..	12·301	14·625

* Extracted from B.S. 1387 : 1947.
† Based on a length (measured from end of tube to end of socket) of: 15 ft. for 1/8 in. nominal bore; 19 ft. for 1/4 in. to 6 in. nominal bore inclusive.

Tubes should be described by the number of the specification (B.S. 1387) and the class of tube.

The term *tube* denotes a straight tube of uniform bore and is synonymous with the term *pipe*.

The term *socket* denotes the screwed coupling utilised in jointing the tubes and is synonymous with the term *coupler*.

Lengths of Steel Tubes Suitable for Screwing to B.S. 21 Pipe Threads.*

Lengths.	Size 1/8 In.	Sizes 1/4 In. to 6 In.
Random	10 to 17 ft.	15 to 23 ft.
Exact	All sizes within +1/4 in. −0 in. of specified length	

* Extracted from B.S. 1387 : 1947.

For orders of over 500 ft. of one size of tube ½ in. to 6 in. nominal bore some short random lengths are permitted. See B.S. 1387 for details.

The length of a screwed and socketed "random length" tube is the overall length when one socket has been screwed on.

The length of a screwed and socketed "exact length" tube is the length of the tube exclusive of the socket.

Steel Tubes for Gas, Water, Air, and Sewage.*

(*Stewarts & Lloyds, Ltd.*)

Sizes 2 in. to 72 in. Four thicknesses, designated Class A, B, C, and D, for each size. Welded or weldless up to 16 in., welded only for larger sizes.

"Steel" Sizes.

Nominal Bore.	Outside Diameter.	Class A.			Class B.		
		Thickness.	Weight per Ft.	Test Head.	Thickness.	Weight per Ft.	Test Head.
In.	In.	In.	Lb.	Ft.	In.	Lb.	Ft.
2	2⅜	0·104	2·52	2300	0·116	2·80	2300
2½	3	0·116	3·57	2300	0·128	3·93	2300
3	3½	0·116	4·19	2300	0·144	5·16	2300
4	4½	0·128	5·98	1900	0·144	6·70	2200
5	5½	0·144	8·24	1800	0·160	9·13	2000
6	6½	0·144	9·78	1500	0·176	11·9	1800
7	7½	0·176	13·8	1600	0·192	15·0	1700
8	8½	0·176	15·6	1400	0·192	17·0	1500
9	9½	0·192	19·1	1300	0·212	21·0	1500
10	10½	0·192	21·1	1200	0·212	23·3	1300
12	12½	0·192	25·2	1000	0·212	27·8	1100
14	14½	0·192	29·3	750	0·219	33·4	900
15	15½	0·192	31·4	700	0·219	35·7	800
16	16½	0·192	33·4	650	0·219	38·0	750
18	18½	0·192	37·5	550	0·219	43·7	700
20	20½	0·219	47·4	600	0·25	54·1	700
21	21½	0·219	49·7	600	0·25	56·7	700
22	22½	0·25	59·4	650	0·281	66·8	750
24	24½	0·25	64·8	600	0·313	80·7	800

* In accordance with requirements of B.S. 534.

Steel Tubes for Gas, Water, Air, and Sewage.*

(Stewarts & Lloyds, Ltd.)

" Steel " Sizes (*continued*).

Nominal Bore.	Outside Diameter.	Class C.			Class D.		
		Thickness.	Weight per Ft.	Test Head.	Thickness.	Weight per Ft.	Test Head.
In.	In.	In.	Lb.	Ft.	In.	Lb.	Ft.
2	2⅜	0·128	3·07	2300	0·144	3·43	2300
2½	3	0·144	4·39	2300	0·160	4·85	2300
3	3½	0·176	6·25	2300	0·192	6·78	2300
4	4½	0·176	8·13	2300	0·192	8·84	2300
5	5½	0·176	10·0	2200	0·192	10·9	2300
6	6½	0·192	12·9	2000	0·212	14·2	2300
7	7½	0·212	16·5	1900	¼	19·4	2300
8	8½	0·212	18·8	1700	¼	22·0	2000
9	9½	¼	24·7	1800	9/32	27·7	2000
10	10½	¼	27·4	1600	9/32	30·7	1800
12	12½	¼	32·7	1300	9/32	36·7	1500
14	14½	¼	38·1	1100	9/32	42·7	1200
15	15½	¼	40·7	1000	9/32	45·7	1100
16	16½	¼	43·4	900	9/32	48·7	1000
18	18½	¼	48·7	800	5/16	60·7	1000
20	20½	9/32	60·7	850	5/16	67·4	950
21	21½	9/32	63·7	800	5/16	70·7	900
22	22½	5/16	74·1	850	⅜	88·6	1000
24	24½	11/32	88·7	900	⅜	96·6	950

* In accordance with requirements of B.S. 534.

For "cast-iron" sizes see p. 419.

Lengths of " Steel " and " Cast-iron " Sizes.

Lengths.	Sizes 16 In. and under.	Sizes over 16 In.
Random	16 to 25 ft.	18 to 25 ft.
Average	Not less than 20 ft.	About 25 ft.

Steel Tubes for Gas, Water, Air, and Sewage.*

(Stewarts & Lloyds, Ltd.)

"Cast-iron" Sizes.

Interchangeable with Cast-iron Pipes to B.S. 78.

Nominal Bore.	Outside Diameter.	Class A.			Class B.		
		Thickness.	Weight per Ft.	Test Head.	Thickness.	Weight per Ft.	Test Head.
In.	In.	In.	Lb.	Ft.	In.	Lb.	Ft.
3	3·76	0·116	4·51	2100	0·144	5·56	2300
4	4·80	0·128	6·39	1800	0·144	7·16	2000
5	5·90	0·144	8·85	1600	0·160	9·80	1800
6	6·98	0·160	11·7	1500	0·176	12·8	1700
7	8·06	0·176	14·8	1500	0·192	16·1	1600
8	9·14	0·176	16·9	1300	0·192	18·4	1400
9	10·20	0·192	20·5	1200	0·212	22·6	1400
10	11·26	0·192	22·7	1100	0·212	25·0	1200
12	13·14	0·192	26·6	950	0·212	29·3	1000

"Cast-iron" Sizes.—*Interchangeable with C.I. Pipes (continued).*

Nominal Bore.	Outside Diameter.	Class C.			Class D.		
		Thickness.	Weight per Ft.	Test Head.	Thickness.	Weight per Ft.	Test Head.
In.	In.	In.	Lb.	Ft.	In.	Lb.	Ft.
3	3·76	0·176	6·74	2300	0·192	7·32	2300
4	4·80	0·176	8·69	2300	0·192	9·45	2300
5	5·90	0·176	10·8	2000	0·192	11·7	2200
6	6·98	0·192	13·9	1900	0·212	15·3	2100
7	8·06	0·212	17·8	1800	0·250	20·9	2100
8	9·14	0·212	20·2	1600	0·250	23·7	1900
9	10·20	0·250	26·6	1600	0·281	29·8	1900
10	11·26	0·250	29·4	1500	0·281	33·0	1700
12	13·60	0·250	35·6	1200	0·281	40·0	1400

* In accordance with requirements of B.S. 534.

Joints for Steel Tubes conveying Gas, Water, Air, and Sewage.

(*Stewarts & Lloyds, Ltd.*)

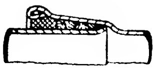


Fig. 1.

Spigot and Socket Joint (Fig. 1) for lead and yarn. Sizes 2 in. to 72 in. Maximum working water pressure 350 lb./sq. in. up to 12 in.; 300 lb./sq. in. for larger sizes.

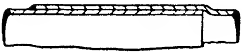


Fig. 2.

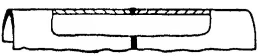


Fig. 3.

Sleeve Welded Joint (Fig. 2) and *Butt Welded Joint* (Fig. 3). Sizes 2 in. to 72 in. Not usually adopted for water services; specially suitable for gas and air mains at usual working pressures.

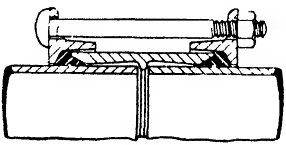


Fig. 4.

Johnson Coupling (Fig. 4). Sizes 3 in. to 72 in. Used on tubes with plain ends. Maximum working water pressure 350 lb./sq. in.

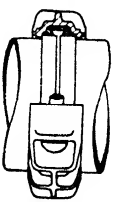


Fig. 5.

Victaulic Joint (Fig. 5). For tubes up to 12 in., but can be supplied on larger sizes. Pressures up to about 1500 lb./sq. in.

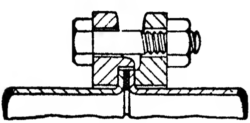


Fig. 6.

Stewarts Loose Flange Joint (Fig. 6) and *Albion Loose Flange Joint* (Fig. 7). Sizes 2 in. and larger. Standardised up to 10 in.

for working water pressures up to 300 lb./sq. in., but larger sizes are obtainable.

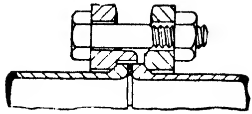


FIG. 7.

Flanges Welded On (Fig. 8). For all diameters and thicknesses of pipes.

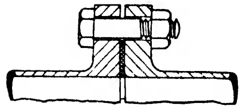


FIG. 8.

Flanges Screwed and Expanded On (Fig. 9). Usually limited to sizes up to 12 in. and to pipes of thicknesses suitable for screwing.

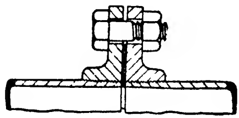


FIG. 9.

SHAFTING.

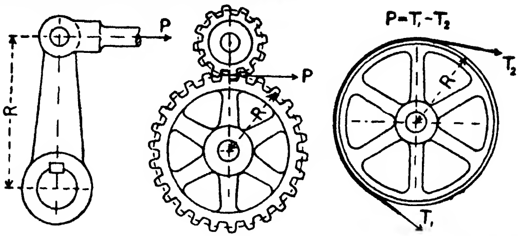
Shafts for transmitting power are usually made of mild steel. They are generally circular in cross-section, but are sometimes square. Large shafts are frequently made hollow. Ordinary mill shafting is made in lengths usually not exceeding 20 feet, and the standard diameters advance by quarters of an inch.

Shafts are subjected to *twisting*, and generally also to *bending*.

An *axle* is a shaft which is subjected to bending only.

The parts of a shaft or axle which rest on the bearings or supports are called *journals*, *pivots*, or *collars*. In journals the supporting pressure is at right angles to the axis of the shaft, while in pivots and collars the pressure is parallel to that axis.

Transmission of Power by Shafts.—



P = force in *lbs.* acting at a perpendicular distance of R *inches* from the centre of the shaft.
T = twisting moment on the shaft in *inch-lbs.* $= PR$.
N = speed of shaft in revolutions per minute.
H = horse-power which is being transmitted.

$$H = \frac{2\pi TN}{12 \times 33000} = \frac{TN}{63025} = \cdot 00001587 TN.$$

$$T = 63025 \frac{H}{N}. \qquad N = 63025 \frac{H}{T}.$$

Resistance of a Shaft to Twisting.—When a shaft is subjected to twisting, the stress induced is a shearing stress, which varies uniformly from nothing at the centre to a maximum at the circumference.

D = diameter of shaft in inches.
f = maximum shearing stress in lbs. per square inch.
T = twisting moment on shaft in inch-lbs.

Moment of resistance of shaft to twisting $= \frac{\pi}{16} D^3 f$.

$$T = \frac{\pi}{16} D^3 f = \cdot 19635 D^3 f.$$

$$D = \sqrt[3]{\frac{16T}{\pi f}} = 1 \cdot 72 \sqrt[3]{\frac{T}{f}}.$$

The formula $T = \frac{\pi}{16} D^3 f$ shows that the strength of a shaft to resist twisting is directly proportional to the cube of its diameter. Hence if T_1 is the twisting moment on a shaft of diameter D_1, and if T_2 is the twisting moment on a shaft of diameter D_2 when both shafts are strained to the same extent, then,

$$\frac{T_1}{T_2} = \frac{D_1^3}{D_2^3} = \left(\frac{D_1}{D_2}\right)^3, \text{ and } \frac{D_1}{D_2} = \sqrt[3]{\frac{T_1}{T_2}}.$$

Values of $\frac{\pi}{16}D^3$.

D	·0	⅛	¼	⅜	½	⅝	¾	⅞
1	·1963	·2796	·3835	·5104	·6627	·8425	1·052	1·294
2	1·571	1·884	2·237	2·630	3·068	3·552	4·083	4·666
3	5·301	5·992	6·740	7·548	8·418	9·353	10·35	11·42
4	12·57	13·78	15·07	16·44	17·89	19·43	21·04	22·75
5	24·54	26·43	28·41	30·49	32·67	34·95	37·33	39·82
6	42·41	45·12	47·94	50·87	53·92	57·09	60·39	63·80
7	67·35	71·02	74·82	78·76	82·83	87·05	91·40	95·89
8	100·5	105·3	110·3	115·3	120·6	126·0	131·5	137·3
9	143·1	149·2	155·4	161·8	168·3	175·1	182·0	189·1
10	196·3	203·8	211·4	219·3	227·3	235·5	243·9	252·5
11	261·3	270·4	279·6	289·0	298·6	308·5	318·5	328·8
12	339·3	350·0	360·9	372·1	383·5	395·1	407·0	419·1
13	431·4	443·9	456·7	469·8	483·1	496·6	510·4	524·5
14	538·8	553·3	568·2	583·2	598·6	614·2	630·1	646·3
15	662·7	679·4	696·4	713·6	731·2	749·0	767·1	785·5
16	804·2	823·2	842·5	862·1	882·0	902·2	922·7	943·5
17	964·7	986·1	1008	1030	1052	1075	1098	1121
18	1145	1169	1193	1218	1243	1269	1294	1320
19	1347	1374	1401	1428	1456	1484	1513	1542
20	1571	1600	1630	1661	1692	1723	1754	1786
D	·0	⅛	¼	⅜	½	⅝	¾	⅞

The safe twisting moment for a shaft of a given diameter is found by multiplying the corresponding number in the above table by the safe stress.

To find the proper diameter of shaft for a given twisting moment proceed as follows :—Divide the given twisting moment by the safe stress, and find the number in the above table which is nearest to the quotient ; the diameter corresponding to this number is the diameter required.

The moment of resistance of a **square shaft** to twisting is $\cdot 208 s^3 f$, where s is the length of the side of the square in inches.

To diminish the bending action on a shaft the power should be taken off as near to a bearing as possible. Couplings should be placed near to a bearing for the same reason.

Safe or Working Stress in Shafting (*Unwin*).

	Steel	Wrought-iron.	Cast-iron.
Stress changing little during work and not reversing . . .	13,500	9,000	3,600
Part of the stress reversing at each revolution	9,000	6,000	2,400
Stress constantly changing between equal and opposite values	4,500	3,000	1,200

Strength of Shafts.

Calculated by the formula $T = \frac{\pi}{16} D^3 f.$

Diameter of Shaft, D. Inches.	Twisting Moment, in Inch-lbs., when the Stress f in Lbs. per Square Inch is— 3,000	4,500	6,000	9,000	13,500
1	589	884	1,178	1,767	2,651
1¼	1,150	1,726	2,301	3,451	5,177
1½	1,988	2,982	3,976	5,964	8,946
1¾	3,157	4,735	6,314	9,471	14,206
2	4,712	7,069	9,425	14,137	21,206
2¼	6,710	10,064	13,419	20,129	30,193
2½	9,204	13,806	18,408	27,612	41,417
2¾	12,250	18,376	24,501	36,751	55,127
3	15,904	23,856	31,809	47,713	71,569
3¼	20,221	30,331	40,442	60,663	90,994
3½	25,255	37,883	50,511	75,766	113,650
3¾	31,063	46,595	62,126	93,189	139,784
4	37,699	56,549	75,398	113,097	169,646
4½	53,677	80,516	107,354	161,031	241,547
5	73,631	110,447	147,262	220,893	331,340
5½	98,003	147,004	196,006	294,009	441,013
6	127,234	190,852	254,469	381,703	572,555
6½	161,767	242,651	323,535	485,302	727,953
7	202,044	303,065	404,087	606,131	909,196
7½	248,505	372,757	497,010	745,514	1,118,272
8	301,593	452,389	603,186	904,778	1,357,168
9	429,416	644,125	858,833	1,288,249	1,932,374
10	589,048	883,573	1,178,097	1,767,145	2,650,718
11	784,024	1,176,035	1,568,047	2,352,071	3,528,106

Horse-power of Shafts.

D = diameter of shaft in inches.
f = maximum shearing stress in lbs. per square inch.
T = twisting moment on shaft in inch-lbs.
N = speed of shaft in revolutions per minute.
H = horse-power transmitted.

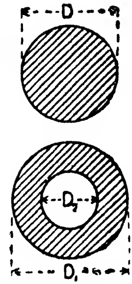
$$\frac{H}{N}=\frac{T}{63025} \qquad T=\frac{\pi}{16}D^3f.$$

Diameter of Shaft in Inches.		Values of $\frac{H}{N}$ when the Stress f in Lbs. per Square Inch is—				
		3,000	4,500	6,000	9,000	13,500
1		·0093	·0140	·0187	·0280	·0420
	$1\frac{1}{4}$	·0183	·0274	·0365	·0548	·0821
$1\frac{1}{2}$		·0315	·0473	·0631	·0946	·1419
	$1\frac{3}{4}$	·0501	·0752	·1003	·1504	·2256
2		·0748	·1122	·1495	·2243	·3365
	$2\frac{1}{4}$	·1065	·1597	·2129	·3194	·4790
$2\frac{1}{2}$		·1460	·2190	·2921	·4381	·6571
	$2\frac{3}{4}$	·1944	·2916	·3887	·5831	·8747
3		·2523	·3785	·5047	·7570	1·136
	$3\frac{1}{4}$	·3208	·4813	·6417	·9625	1·444
$3\frac{1}{2}$		·4007	·6011	·8014	1·202	1·803
	$3\frac{3}{4}$	·4929	·7393	·9857	1·479	2·218
4		·5982	·8972	1·196	1·794	2·692
	$4\frac{1}{2}$	·8517	1·278	1·703	2·555	3·833
5		1·168	1·752	2·337	3·505	5·257
	$5\frac{1}{2}$	1·555	2·332	3·110	4·665	6·997
6		2·019	3·028	4·038	6·057	9·085
	$6\frac{1}{2}$	2·567	3·850	5·133	7·700	11·55
7		3·206	4·809	6·412	9·617	14·43
	$7\frac{1}{2}$	3·943	5·914	7·886	11·83	17·74
8		4·785	7·178	9·571	14·36	21·53
	9	6·813	10·22	13·63	20·44	30·66
10		9·346	14·02	18·69	28·04	42·06
	11	12·440	18·66	24·88	37·32	55·98

Span between Bearings of Shafts.—D = diameter of shaft in inches. S = span between bearings in feet. $S=C\sqrt[3]{D^2}$, where C is equal to from 5 to 6 for shafts which carry their own weight only, and from 4·5 to 5 for shafts carrying an ordinary number of pulleys or wheels.

Hollow Shafts.—

D_1 = external diameter of hollow shaft.
$D_2 = cD_1$ = internal diameter of hollow shaft.
D = diameter of solid shaft of same strength as hollow shaft.
T = twisting moment on shaft in inch-lbs.
f = maximum shearing stress in lbs. per square inch.



Moment of resistance of hollow shaft

$$=\frac{\pi}{16}\left(\frac{D_1^4 - D_2^4}{D_1}\right)f.$$

Moment of resistance of solid shaft $=\frac{\pi}{16}D^3 f.$

$$\frac{D_1^4 - D_2^4}{D_1} = D^3. \qquad D_1 = D\sqrt[3]{\left(\frac{1}{1-c^4}\right)} = c_1 D.$$

$$T=\frac{\pi}{16}\left(\frac{D_1^4 - D_2^4}{D_1}\right)f = \frac{\pi}{16}D_1^3(1-c^4)f.$$

$$D_1 = \sqrt[3]{\left(\frac{16T}{\pi(1-c^4)f}\right)} = 1\cdot72c_1\sqrt[3]{\frac{T}{f}}.$$

c	c_1	c	c_1	c	c_1
·25	1·0013	·45	1·0141	·65	1·0677
·3	1·0027	·5	1·0218	·7	1·0958
·35	1·0051	·55	1·0325	·75	1·1352
·4	1·0087	·6	1·0474	·8	1·1920

Resistance of a Shaft to Combined Twisting and Bending.—

T = twisting moment on shaft.
B = bending moment on shaft.
T_1 = *equivalent twisting moment.*

$$T_1 = B + \sqrt{(B^2+T^2)}.$$

The equivalent twisting moment must be used in determining the diameter of the shaft.

If $p=\frac{B}{T}$, then $T_1 = T(p+\sqrt{p^2+1})$.

If $q=(p+\sqrt{p^2+1})$, then $T_1 = qT$.
If D = diameter of shaft for twisting moment T,
D_1 = diameter of shaft for twisting moment T_1,
then $D_1 = D\sqrt[3]{q}$.

The following table gives the values of q and $\sqrt[3]{q}$ for various values of p.—

p	q	$\sqrt[3]{q}$	p	q	$\sqrt[3]{q}$	p	q	$\sqrt[3]{q}$
·1	1·105	1·034	1·1	2·587	1·373	2·1	4·426	1·642
·2	1·220	1·068	1·2	2·762	1·403	2·2	4·617	1·665
·3	1·344	1·104	1·3	2·940	1 433	2·3	4·808	1·688
·4	1·477	1·139	1·4	3·120	1·461	2 4	5·000	1·710
·5	1·618	1·174	1·5	3·303	1·489	2·5	5·193	1·732
·6	1·766	1·209	1·6	3·487	1·516	2·6	5·386	1·753
·7	1·921	1·243	1·7	3·672	1·543	2·7	5·579	1·774
·8	2·081	1·277	1·8	3·859	1·569	2·8	5·773	1·794
·9	2·245	1·309	1·9	4·047	1·594	2·9	5·968	1·814
1·0	2·414	1·342	2·0	4·236	1·618	3·0	6·162	1·833

Angle of Torsion.—

D = diameter of solid shaft in inches.

D_1 = external diameter of hollow shaft in inches.

D_2 = internal diameter of hollow shaft in inches.

T = twisting moment on shaft in inch-lbs.

L = length of shaft in inches.

θ = angle moved through by one end of shaft in advance of the other end, in circular measure.

n = same angle in degrees.

C = modulus or coefficient of transverse elasticity of the material of the shaft (see p. 246).

f = greatest shearing stress in lbs. per square inch.

For solid shafts,

$$\theta = \frac{32TL}{\pi CD^4} = 10{\cdot}186\frac{TL}{CD^4} = \frac{2fL}{CD}.$$

$$n = \frac{180 \times 32TL}{\pi^2 CD^4} = 583\ 61\frac{TL}{CD^4} = 114{\cdot}59\frac{fL}{CD}.$$

For hollow shafts,

$$\theta = 10{\cdot}186\frac{TL}{C(D_1^4 - D_2^4)} = \frac{2fL}{CD_1}.$$

$$n = 583{\cdot}61\frac{TL}{C(D_1^4 - D_2^4)} = 114{\cdot}59\frac{fL}{CD_1}$$

C = 9,000,000 to 11,000,000 lbs. per square inch for wrought-iron.

C = 11,000,000 to 13,000,000 lbs. per square inch for steel.

Shafts of Ships (***Board of Trade and Lloyd's Rules***). ***Turbine-driven Shafting.***—

$$d = \sqrt[3]{\frac{\text{S.H.P.}}{R}} \times F.$$

d is diameter of the intermediate shaft in inches.
S.H.P. is maximum designed shaft horse-power.
R is number of revolutions per minute at that power.
$F=64$ for ocean-going and home-trade vessels.
$F=58$ for vessels trading on estuaries, rivers, lochs, or lakes.

Wheel shafts of geared turbine-driven installations shall be not less than $1{\cdot}05d$ in diameter; but where there is only one pinion gearing into the wheel, or where there are two pinions set to subtend an angle at the centre of the shaft of less than 120°, the diameter of the wheel shaft at the wheel and the adjacent journals shall be not less than $1{\cdot}1d$. Abaft the journals the shaft may be tapered to $1{\cdot}05d$.

Shafting of Steam Reciprocating Engines.—

$$d=\sqrt[3]{\frac{D^2\times S\times p}{f(r+2)}}.$$

d is diameter of the intermediate shaft in inches.
D is diameter in inches of the low-pressure cylinder, or the equivalent diameter where two or more low-pressure cylinders are used.
S is stroke of piston in inches.
p is working pressure in the boiler in lbs. per sq. inch.
r is ratio of the swept volume of the low-pressure cylinder or cylinders to that of the high-pressure cylinder or cylinders.
f is a coefficient given in the following table:—

Compound, Triple, or Quadruple Expansion Reciprocating Engines.	Values of f.	
	Ocean-going and Home-trade Vessels.	Vessels Trading on Estuaries, Rivers, Lochs, and Lakes.
2 cranks at 90° . .	1900	2100
2 cranks at 180° . .	1350	1500
3 cranks at 120° . .	2150	2400
4 cranks balanced .	2150	2400
4 cranks at 90° . .	2100	2300

The diameter of the crank shaft is not to be less than $1{\cdot}05d$.

Crank webs of built shafts should have dimensions not less than the following:—

$$h=0{\cdot}625d_c. \qquad t=\sqrt{\frac{0{\cdot}12d_c^3}{h}}.$$

h is thickness, in inches, of the web measured parallel to the axis.
t is thickness, in inches, of metal around the eyeholes, measured radially.
d_c is diameter of the crank shaft in inches.

Crank webs should be securely shrunk on the body-pieces and crank pins, or forced on by hydraulic pressure. One or two keys or cylindrical dowels should be fitted at the junctions of the body-pieces and webs.

Thrust Shafts.—The diameter at the collars of thrust shafts transmitting torque should not be less than $1{\cdot}05d$. Thrust shafts may be tapered down outside the collars to the diameter d required for the intermediate shaft.

Tube Shafts (shafts passing through stern tubes but not carrying the screw propellers).—Diameter should not be less than $1{\cdot}05d$, and any part of the shaft within the tube which may be exposed to sea-water should not be less than $1{\cdot}075d$.

Tail or Screw Shafts (shafts carrying the screw propellers).—

$$d_t = d + \frac{P}{K}.$$

d_t is diameter of tail shaft in inches or mm.
d is diameter of intermediate shaft in inches or mm.
P is diameter of screw propeller in inches or mm.
$K = 144$ when a continuous liner is fitted.
$K = 100$ when a continuous liner is not fitted.

Tail shafts which run in stern tubes may have the end forward of the stern gland tapered down to a diameter at the coupling flange equal to $1{\cdot}05d$.

Bronze Liners on Shafts.—The thickness of liners fitted on tail shafts or on tube shafts, in way of the bushes, should not be less than

$$t = \frac{d_t + 9{\cdot}25}{32} \quad \text{or} \quad t_m = \frac{d_t + 235}{32}.$$

t is thickness of liner in inches, t_m is thickness of liner in mm.
d_t is diameter, in inches or mm., required for the tail or tube shaft within the liner.

Thickness of a continuous liner at the part between the bushes should be $\frac{3}{4}t$ or $\frac{3}{4}t_m$. Liners must be carefully shrunk on or be forced on to the shafts by hydraulic pressure, and are not to be secured by pins.

The length of the bearing in the stern bush next to the propeller should be not less than four times the diameter required for the shaft within the liner.

SHAFT COUPLINGS.

Coupling Flanges and Bolts (*Board of Trade and Lloyd's Rules*).—

$$\left.\begin{array}{l}\text{Diameter of coupling bolts,}\\ \text{in inches or mm.}\end{array}\right\} = \sqrt{\frac{d^3}{3{\cdot}5 \times n \times r}}.$$

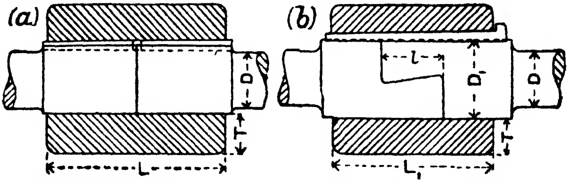
d is diameter of intermediate shaft, in inches or mm.
n is number of bolts in the coupling.
r is radius of pitch circle of the bolts, in inches or mm.

The thickness of the coupling flanges at the pitch circle of the bolt holes is not to be less than the diameter of the coupling bolts at the face of the coupling. The thickness of the screw shaft coupling flange is not to be less than 0·25 of the diameter of the intermediate shaft.

The radius of curvature at the fillet where the flange starts from the shaft is not to be less than 0·125 times the diameter of the shaft adjacent to the flange.

When couplings are separate from the shafts, provision is to be made to resist the astern pull.

Box or Muff Couplings.—(*a*) *Ordinary muff coupling*, in which the shafts butt against one another. The box or muff is made of cast-iron, and is secured to the shafts by sunk keys.

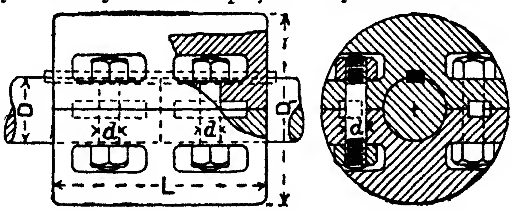


(*b*) *Fairbairn's half-lap coupling.*—The box is made of cast-iron, and is secured to the shafts by a saddle key.

D = diameter of shaft in inches. $T = \cdot 4D + \cdot 5$ inch.
$D_1 = 1\cdot 375D + \cdot 25$ inch. $L = 2\cdot 5D + 2$ inches.
$l = \cdot 875D + \cdot 125$ inch. $L_1 = 2\cdot 25D + \cdot 75$ inch.
Slope of lap 1 in 12. For proportions of keys see p. 413.

D	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6
D_1	$2\frac{5}{16}$	3	$3\frac{11}{16}$	$4\frac{3}{8}$	$5\frac{1}{16}$	$5\frac{3}{4}$	$6\frac{7}{16}$	$7\frac{1}{8}$	...	...
l	$1\frac{7}{16}$	$1\frac{7}{8}$	$2\frac{5}{16}$	$2\frac{3}{4}$	$3\frac{3}{16}$	$3\frac{5}{8}$	$4\frac{1}{16}$	$4\frac{1}{2}$	...	...
T	$1\frac{1}{8}$	$1\frac{5}{16}$	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{15}{16}$	$2\frac{1}{8}$	$2\frac{5}{16}$	$2\frac{1}{2}$	$2\frac{3}{4}$	$2\frac{15}{16}$
L	$5\frac{3}{4}$	7	$8\frac{1}{4}$	$9\frac{1}{2}$	$10\frac{3}{4}$	12	$13\frac{1}{4}$	$14\frac{1}{2}$	$15\frac{3}{4}$	17
L_1	$4\frac{1}{8}$	$5\frac{1}{4}$	$6\frac{3}{8}$	$7\frac{1}{2}$	$8\frac{5}{8}$	$9\frac{3}{4}$	$10\frac{7}{8}$	12	...	...

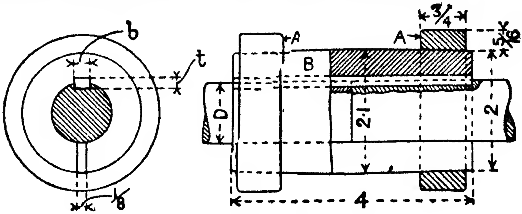
Split Muff Couplings.—The box or muff gripes the shaft firmly. The keys have no taper, and they fit on the sides only.



D	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4
D_1	$4\frac{1}{2}$	$5\frac{1}{4}$	$5\frac{3}{4}$	6	$6\frac{1}{2}$	$7\frac{1}{4}$	$7\frac{7}{8}$	$9\frac{3}{4}$	$10\frac{1}{2}$
L	6	7	8	9	10	11	12	14	16
d	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$
N	4	4	4	4	4	4	4	6	6

N = number of bolts.

Another design of split muff coupling is shown below.

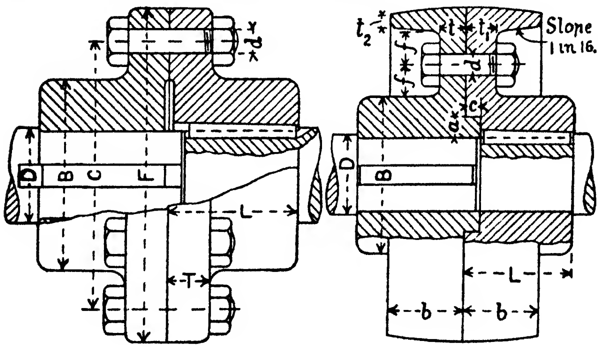


Unit for proportions $= D$.

AA are wrought-iron or steel rings shrunk on, or driven on to the cast-iron box B. Sometimes the key is dispensed with. The key has no taper, and it fits on the sides only.

$b = \frac{3}{16}D + \frac{1}{4}$ inch. $t = \frac{1}{8}D + \frac{1}{8}$ inch.

Cast-iron Flange and Pulley Couplings.—



n = number of bolts = from $\frac{2}{3}D + 2$ to $D + 2$.

$B = 1{\cdot}8D + {\cdot}8$.

$L = 1{\cdot}2D + {\cdot}8$.

$d = \dfrac{{\cdot}423D}{\sqrt{n}} + {\cdot}3$.

$C = B + 3{\cdot}2d$.

$F = B + 6d$.

$T = {\cdot}35D + {\cdot}35$.

$a = {\cdot}2D + {\cdot}2$.

$b = {\cdot}5D + 1$, but not less than ${\cdot}3D + 1{\cdot}3d + {\cdot}3$.

$c = {\cdot}1D + {\cdot}1$.

$f = 1{\cdot}5d$.

$t = {\cdot}25D + {\cdot}25$.

$t_1 = {\cdot}3D + {\cdot}3$.

$t_2 = {\cdot}1D + {\cdot}2$.

For proportions of keys see p. 413.

Dimensions of Cast-iron Flange and Pulley Couplings.

D	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6
B	$3\frac{1}{2}$	$4\frac{3}{8}$	$5\frac{5}{16}$	$6\frac{1}{4}$	$7\frac{1}{8}$	8	$8\frac{7}{8}$	$9\frac{13}{16}$	$10\frac{3}{4}$	$11\frac{5}{8}$
L	$2\frac{5}{8}$	$3\frac{1}{4}$	$3\frac{11}{16}$	$4\frac{3}{8}$	5	$5\frac{5}{8}$	$6\frac{1}{4}$	$6\frac{13}{16}$	$7\frac{3}{8}$	8
n	3	4	4	4	4	6	6	6	6	6
d	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	1	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$1\frac{3}{8}$
C	$5\frac{1}{2}$	$6\frac{3}{4}$	$8\frac{1}{8}$	$9\frac{1}{2}$	$10\frac{5}{16}$	$11\frac{1}{4}$	$12\frac{1}{2}$	$13\frac{13}{16}$	$14\frac{3}{4}$	16
F	$7\frac{1}{4}$	$8\frac{7}{8}$	$10\frac{9}{16}$	$12\frac{1}{4}$	$13\frac{1}{8}$	14	$15\frac{5}{8}$	$17\frac{5}{16}$	$18\frac{1}{4}$	$19\frac{7}{8}$
T	$\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$1\frac{7}{16}$	$1\frac{5}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{8}$	$2\frac{5}{16}$	$2\frac{1}{2}$
a	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{5}{16}$	$1\frac{3}{8}$
b	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{3}{4}$	4
c	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{11}{16}$
f	$\frac{15}{16}$	$1\frac{1}{8}$	$1\frac{5}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{11}{16}$	$1\frac{7}{8}$	$1\frac{7}{8}$	$2\frac{1}{16}$
t	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{3}{4}$
t_1	$\frac{3}{4}$	$\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{3}{16}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{11}{16}$	2	$2\frac{1}{8}$
t_2	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$

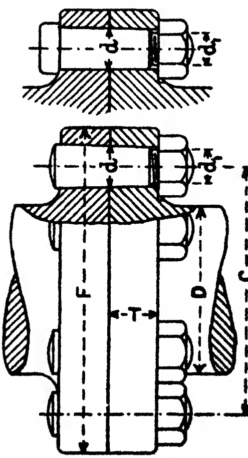
All the dimensions in the above table are in inches.

The outside of the section of the rim of the pulley coupling may be either straight or curved; if curved, the radius of the curve may be from $6b$ to $10b$.

Marine or Solid Flange Coupling.—If the bolts are made of the same material as the shaft, which is the usual practice, then the bolts and shaft will have the same strength when $2nd^2C = D^3$ for solid shafts, or

$$2nd^2C = \frac{D_1^4 - D_2^4}{D_1} \text{ for hollow shafts,}$$

where n = number of bolts, d = diameter of bolts, C = diameter of bolt circle, D = diameter of solid shaft, D_1 = external diameter of hollow shaft, and D_2 = internal diameter of hollow shaft.



In practice C varies from $D + 1\frac{1}{2}d$ to $D + 2\frac{1}{2}d$, and is on the average equal to $D + 2d$. Hence, for solid shafts, $2nd^2(D + 2d) = D^3$, and for hollow shafts

$$2nd^2(D_1 + 2d) = \frac{D_1^4 - D_2^4}{D_1}.$$

The following table gives the values of d corresponding to suitable values of n for solid shafts from 3 inches to 24 inches in diameter —

D	n	d	n	d	D	n	d	n	d
3	3	$\frac{15}{16}$	4	$\frac{7}{8}$	14	6	$3\frac{5}{16}$	8	$2\frac{15}{16}$
4	3	$1\frac{1}{4}$	4	$1\frac{1}{8}$	15	6	$3\frac{9}{16}$	8	$3\frac{1}{8}$
5	4	$1\frac{7}{16}$	6	$1\frac{3}{16}$	16	6	$3\frac{11}{16}$	8	$3\frac{3}{8}$
6	4	$1\frac{11}{16}$	6	$1\frac{7}{16}$	17	6	$4\frac{1}{16}$	8	$3\frac{9}{16}$
7	4	2	6	$1\frac{11}{16}$	18	6	$4\frac{5}{16}$	8	$3\frac{3}{4}$
8	4	$2\frac{1}{4}$	6	$1\frac{7}{8}$	19	8	4	9	$3\frac{13}{16}$
9	6	$2\frac{1}{8}$	8	$1\frac{7}{8}$	20	8	$4\frac{3}{16}$	9	4
10	6	$2\frac{3}{8}$	8	$2\frac{1}{8}$	21	8	$4\frac{3}{8}$	9	$4\frac{3}{16}$
11	6	$2\frac{5}{8}$	8	$2\frac{5}{16}$	22	8	$4\frac{5}{8}$	9	$4\frac{3}{8}$
12	6	$2\frac{7}{8}$	8	$2\frac{1}{2}$	23	8	$4\frac{13}{16}$	9	$4\frac{9}{16}$
13	6	$3\frac{1}{8}$	8	$2\frac{3}{4}$	24	8	$5\frac{1}{16}$	9	$4\frac{13}{16}$

The bolts may either be parallel or tapered. In tapered bolts the heads are often dispensed with.

Taper of bolts $= \frac{3}{8}$ inch (on the diameter) per foot of length.

Diameter of screwed part of bolt $= d_1 = \dfrac{7d + 1 \text{ inch}}{8}$.

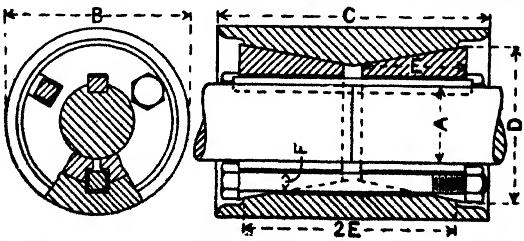
Thickness of nut $= \frac{3}{4}d_1$ to $\frac{7}{8}d_1$.

Thickness of flange $= T = \cdot 27D + 2$ inch for solid shafts.

$$= \cdot 29\sqrt[3]{\frac{D_1^4 - D_2^4}{D_1}} \text{ for hollow shafts.}$$

Diameter of flange $= F = C + 1\cdot 9d = D + 3\cdot 9d$.

Sellers' Cone Coupling.—This coupling consists of a cast-iron muff, bored out to a double conical form, and two split cast-iron conical sleeves, which are pulled together by bolts, as shown.

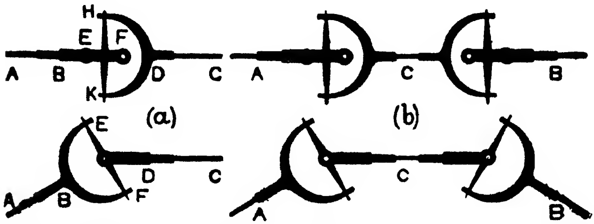


The friction between the sleeves and the shaft may be quite sufficient to prevent slipping, but as an additional security a side-fitting sunk key without taper is generally added.

A	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	3	$3\frac{1}{2}$	4	5
B	$4\frac{1}{2}$	$5\frac{1}{4}$	$6\frac{1}{8}$	$6\frac{1}{2}$	$7\frac{1}{4}$	$7\frac{5}{8}$	$8\frac{1}{2}$	$9\frac{3}{4}$	11	$12\frac{7}{8}$
C	6	7	8	$8\frac{3}{4}$	$9\frac{3}{4}$	$10\frac{3}{4}$	$11\frac{3}{4}$	$13\frac{1}{2}$	$15\frac{1}{4}$	$18\frac{3}{4}$
D	$3\frac{1}{2}$	4	$4\frac{3}{4}$	5	$5\frac{3}{4}$	6	$6\frac{3}{4}$	$7\frac{3}{4}$	$8\frac{3}{4}$	$10\frac{1}{4}$
E	$2\frac{1}{4}$	$2\frac{5}{8}$	3	$3\frac{3}{8}$	$3\frac{3}{4}$	$4\frac{1}{8}$	$4\frac{1}{2}$	$5\frac{1}{4}$	6	$7\frac{1}{2}$
F	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$

Taper of cones 3 inches on the diameter per foot of length.
All the dimensions in the above table are in inches.

Hooke's Joint or Universal Coupling.—This form of coupling is used to connect two shafts whose axes intersect, and it has the advantage that the angle between the shafts may be varied while they are in motion.

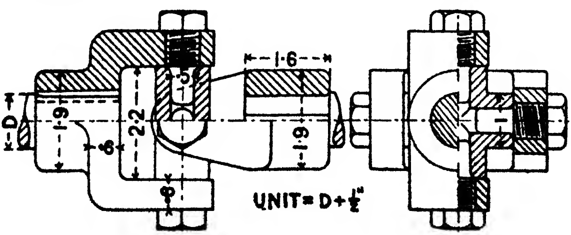


The diagram (*a*) shows the main features of the coupling. *AB* and *CD*, the shafts to be coupled, are forked at their ends. The forks carry between them a cross *EKFH*, the arms of which are at right angles to one another. The arms of the cross are jointed to the forks so that they may turn freely about their axes.

The angular velocities of the shafts *AB* and *CD* are unequal except at every quarter of a revolution, and this inequality is greater the greater the acute angle between the shafts.

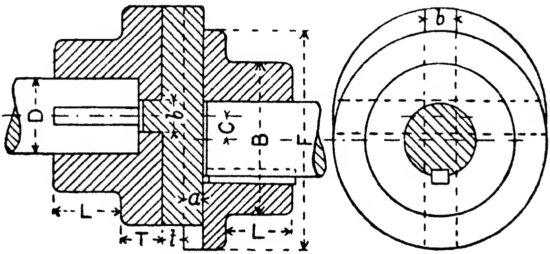
By using a double joint, as shown at (*b*), the shafts *A* and *B* have the same angular velocities, provided they make equal angles with the intermediate shaft *C*, and are in the same plane with it.

The illustration below shows the details of the construction of one form of Hooke's joint, the parts being made of wrought-iron or steel.



Oldham's Coupling.—This coupling is used for connecting shafts whose axes are parallel and at a comparatively short distance from one another. The grooved discs are secured to the shafts, and a third disc is placed between them. This third disc has two cross bars, one on each face, and at right angles to one another. These cross bars fit into the grooves on the other discs. All the discs are made of cast-iron.

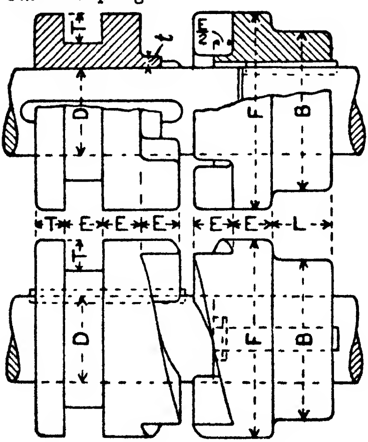
The angular velocities of the two shafts and the intermediate disc are equal to one another at every instant.



D = diameter of shaft in inches.
C = distance between axes of shafts in inches.

$B = 1{\cdot}8D + {\cdot}8$ inch. $F = 3D + C.$ $L = {\cdot}75D + {\cdot}5$ inch.
$b = {\cdot}4D + {\cdot}15C.$ $a = t = {\cdot}25D + {\cdot}1C.$ $T = {\cdot}6D + {\cdot}25C.$

Claw Couplings.—

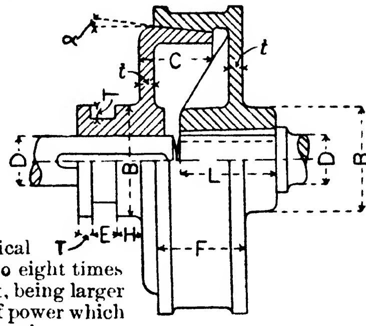


$B = 1{\cdot}7D + 1$ inch.
$E = {\cdot}4D + {\cdot}4$ inch.
$F = 2D + 2$ inches.
$L = {\cdot}6D + {\cdot}6$ inch.
$T = {\cdot}3D + {\cdot}3$ inch.
$t = {\cdot}1D + {\cdot}2$ inch.

The dimensions are in inches.

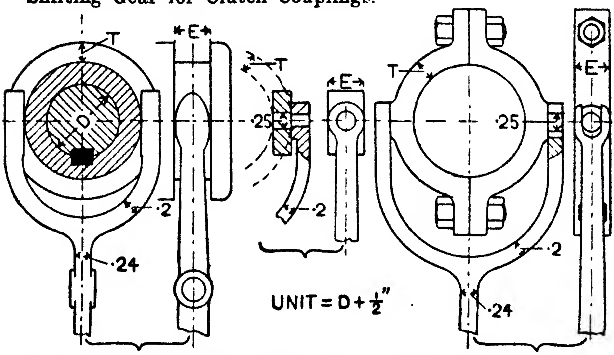
Conical Friction Coupling.—

$B = 2D + 1$ inch.
$B_1 = 2D + \cdot 5$ inch.
$C = 1 \cdot 5D$.
$E = \cdot 4D + \cdot 4$ inch.
$F = 1 \cdot 8D$.
$H = \cdot 5D$.
$L = 2D$.
$T = \cdot 3D + \cdot 3$ inch.
$t = \cdot 23D + \cdot 2$ inch.
$a =$ from 4 to 10 degrees.



Mean diameter of conical part may be from four to eight times the diameter of the shaft, being larger the greater the amount of power which the coupling has to transmit.

Shifting Gear for Clutch Couplings.



The dimensions E and T are made to suit the groove on the coupling.

BEARINGS FOR SHAFTS.

Area of a Bearing.—The area of a bearing is the area of its projection on a plane at right angles to the direction of the load on the bearing.

Let $a =$ area of bearing.

For a cylindrical journal bearing of diameter d and length l, $a = dl$.

For a pivot bearing of diameter d, $a = \cdot 7854d^2$.

For a collar bearing having n collars of outside diameter D_1 and inside diameter D_2, $a = \cdot 7854(D_1^2 - D_2^2)n$, when the bearing surface is all round the collars. When horse-shoe collars, such as are shown on p. 449, are used, the total area of the bearing is approximately $\pi(D_1 - D_2)(\cdot 78 D_1 + \cdot 17 D_2)$.

Intensity of Pressure on Bearings.—

R = load on bearing in lbs.

a = area of bearing in square inches.

p = intensity of pressure on bearing in lbs. per square inch.

$p = \frac{R}{a}$, and $R = pa$.

p varies greatly in different cases. It is generally smaller the greater the speed; and, in cases where the load is intermittent or changes from one side of the bearing to the other during each revolution, p may be greater than in cases where the load is a steady load.

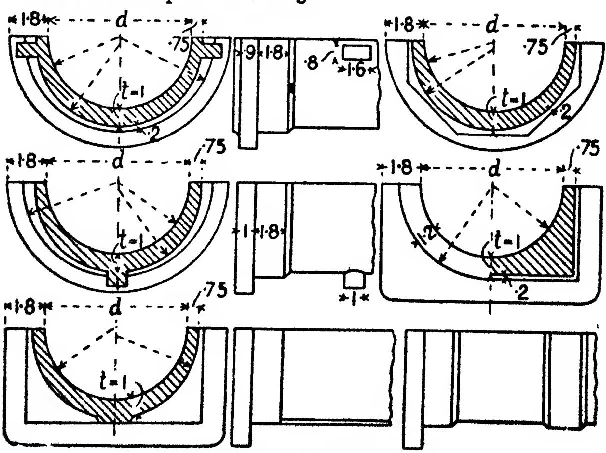
For the main journal bearings of steam-engines, the maximum value of p is 600 for slow and 400 for fast going engines; but where space will permit, it is desirable to make the bearings of such a length that p is from 200 to 300.

For railway axles p varies from 160 to 300. According to the late Mr. Joseph Tomlinson, p should not exceed 280 for railway axles.

For the thrust bearings of propeller shafts, p = 40 to 70.

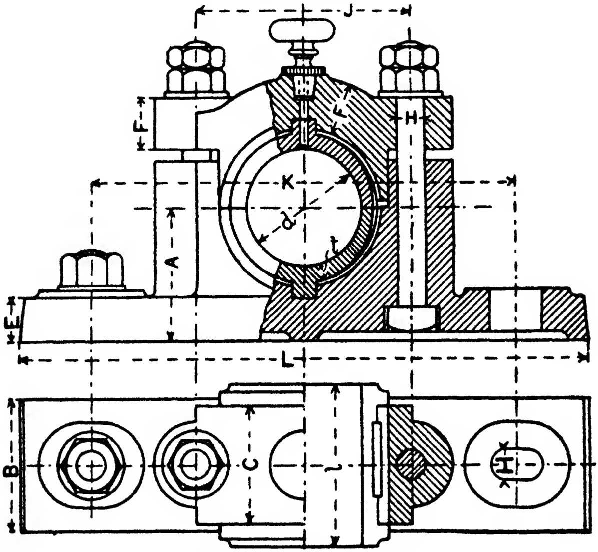
For pivot bearings which have to run continuously at moderate and high speeds, p should not exceed 250.

Brasses or Steps for Bearings.—



Unit = t = ·09d + ·15 **inch.**

Ordinary Plummer or Pillow Block.



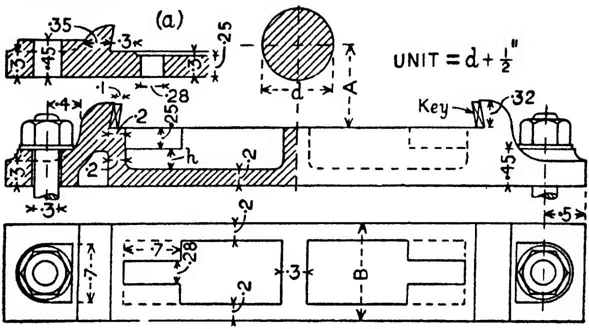
d = diameter of bearing in inches.

$A = 1{\cdot}05d + {\cdot}5$ inch.
$B = {\cdot}8l$.
$C = {\cdot}7l$.
$E = {\cdot}3d + {\cdot}3$ inch.
$F = {\cdot}3d + {\cdot}4$ inch.

$H = {\cdot}25d + {\cdot}25$ inch.
$J = 1{\cdot}6d + 1{\cdot}5$ inches.
$K = 2{\cdot}7d + 4{\cdot}2$ inches.
$L = 3{\cdot}6d + 5$ inches.
$t = {\cdot}09d + {\cdot}15$ inch.

l varies considerably. One rule is, $l = d + 1$ inch. Another rule is, $l = 1{\cdot}5d$.

Sole Plates.—

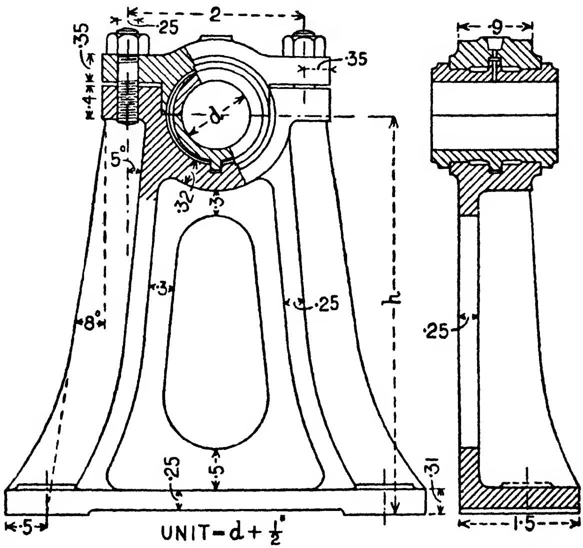


Unit for proportions $= d + \frac{1}{2}$ inch.
$d =$ diameter of bearing in inches.

The width B and the length between the joggles must be made to suit the particular pillow block or pedestal to be supported. The distance h may be made to suit the height to which it is required to raise the pillow block, but it must not be less than the thickness of the heads of the bolts used to secure the block to the sole plate. The height of the sole plate may be reduced by adopting the design shown at (a).

Standard and Bearing.—The height h is made to suit the height of the shaft from the floor to which the standard is attached.

For the proportions of the brasses see p. 438.

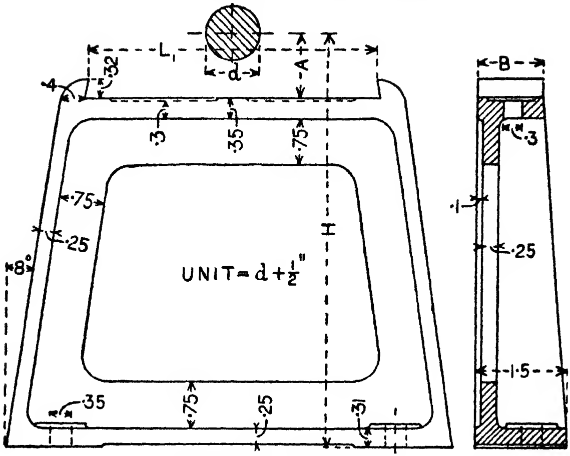


The standard may be bolted directly to the flooi, but preferably it should be mounted on a sole plate such as is shown at (*a*) on p. 440; this will permit more readily of accurate lateral adjustment.

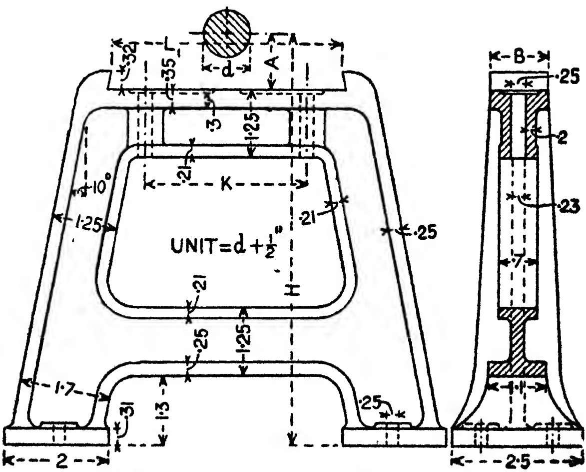
Standards for Pillow Blocks.—The height of the standard must be made to suit the height H of the shaft and the height A of the pillow block. The length L_1 between the joggles and the width B must be made to suit the particular pillow block to be supported. For proportions of pillow blocks see p 439.

The second illustration on the opposite page shows a heavier design than the preceding one As before, the height of the standard depends on H and A. The dimensions B, L_1, and K are made to suit the pillow block.

Unit for proportions $= d + \frac{1}{2}$ inch.
d = diameter of bearing in inches.

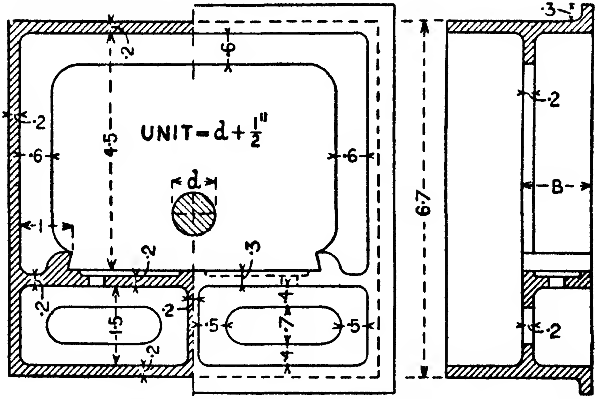


Standard for Pillow Block.



Standard for Pillow Block (Heavier Design).

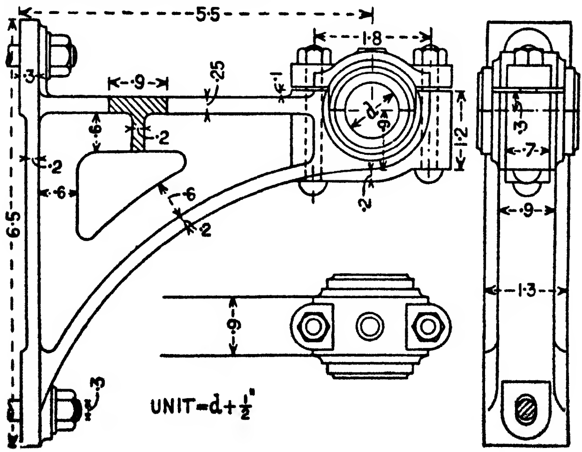
Wall Box for Pillow Block.—



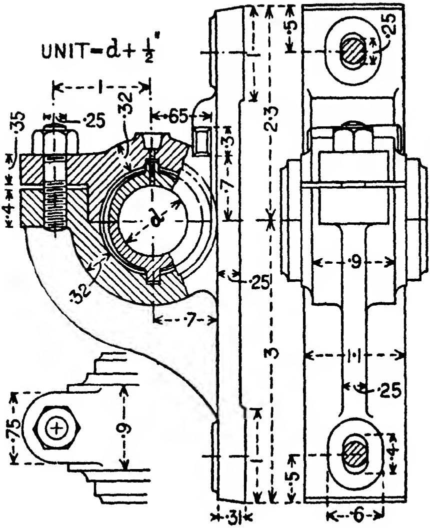
d=diameter of bearing of pillow block in inches.

The length between the joggles and the width B must be made to suit the particular pillow block which has to be supported.

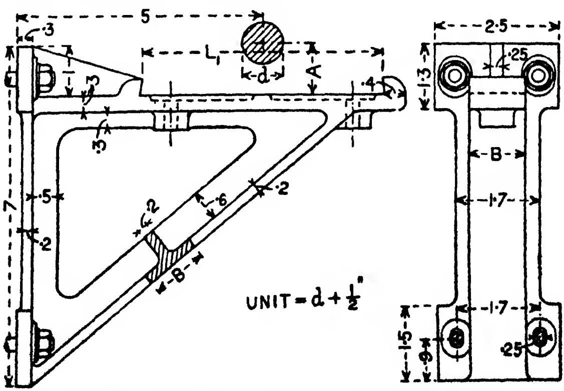
Wall Bracket and Bearing.—



Pillar Bracket.—This design of bracket has a minimum amount of overhang, and is suitable for supporting a horizontal shaft from a pillar or column where there is no wall in the way of the wheels or pulleys on the shaft. It may also be used as a wall bracket in cases where the shaft does not carry wheels or pulleys which would be interfered with by the wall. For the proportions of the brasses see p. 438.



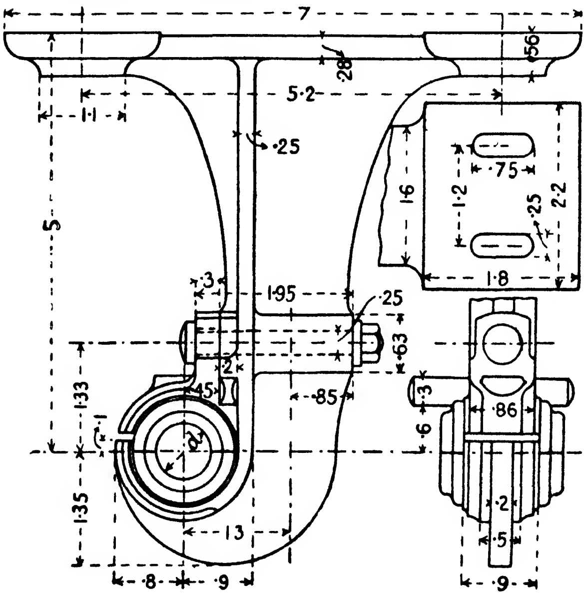
Wall Bracket for Pillow Block.—



The dimensions marked B and L_1 must be made to suit the particular pillow block which the bracket is designed to carry.

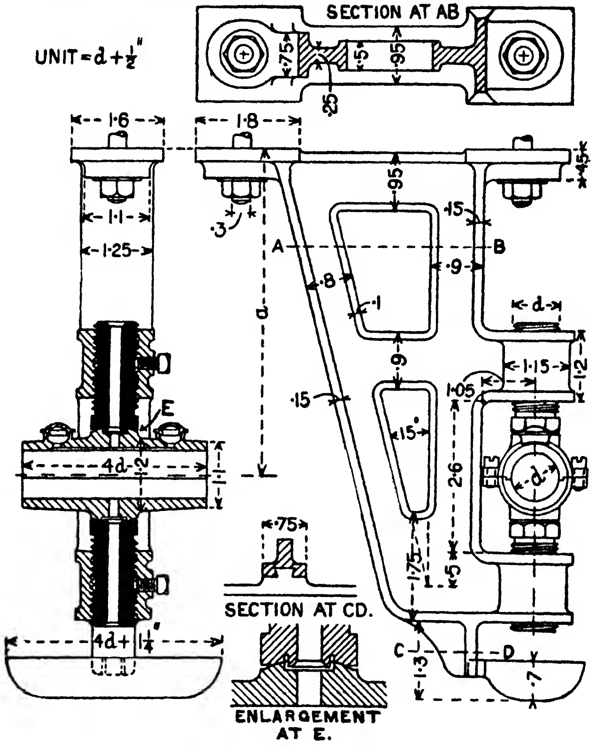
so also must the distance between the holes for the bolts which secure the block to the bracket. For proportions of pillow blocks see p. 439.

"J" Hanger with Bearing.—



Unit for proportions $= 1{\cdot}15d + {\cdot}4$ inch.
d = diameter of bearing in inches.

Hanger with Adjustable Swivel Bearing.—

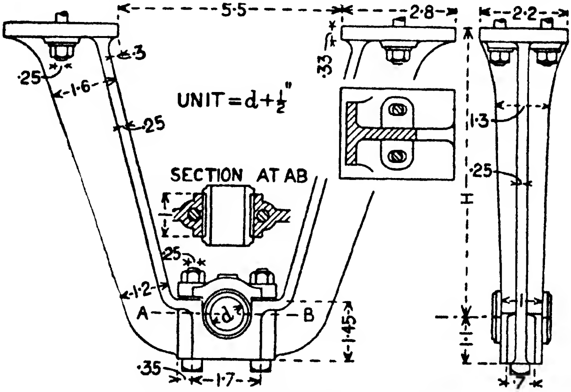


Unit for proportions $= d + \frac{1}{2}$ inch.
d = diameter of bearing in inches.

With the exception of the bolts and set screws, all the parts are made of cast-iron.

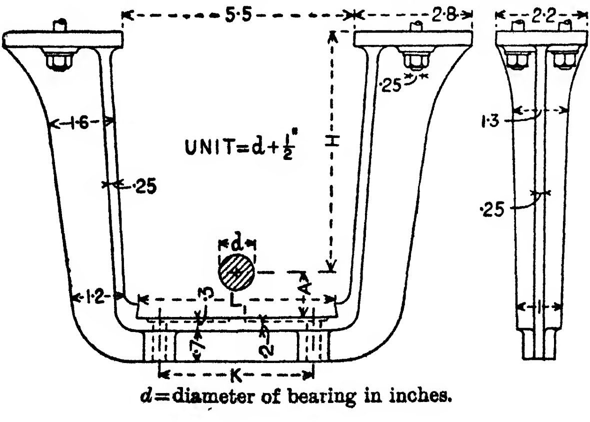
The distance a may be varied within wide limits, but it usually lies between $6d$ and $12d$.

Sling Hanger with Bearing.—The drop H usually varies in different cases from five to nine times the unit. On the average it is about seven. For the proportions of the brasses see p. 438.



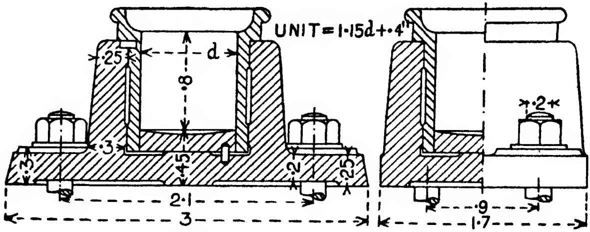
d=diameter of bearing in inches.

Sling Hanger for a Pillow Block.—The dimensions K, L_1, and A depend on the particular pillow block to be carried by the hanger. For proportions of pillow blocks see p. 439. Usually H=from four to eight times the unit, average about six.



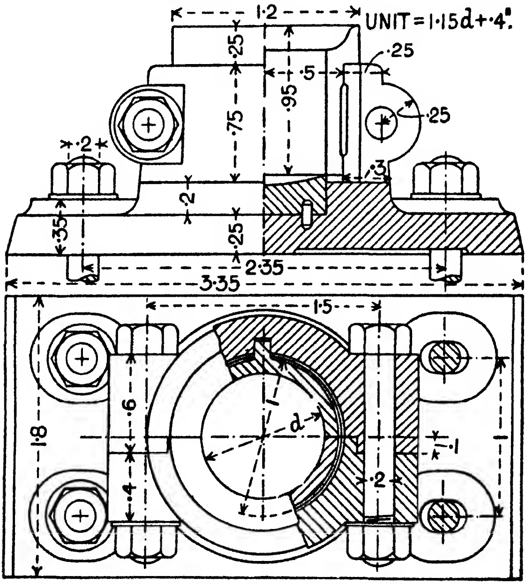
d=diameter of bearing in inches.

Ordinary Footstep Bearing.—The footstep end of the shaft should be of steel, and it may be quite flat or slightly convex.



Unit for proportions $=1{\cdot}15d+{\cdot}4$ inch.
$d=$ diameter of bearing in inches

Divided Footstep Bearing—



Unit for proportions $=1{\cdot}15d+{\cdot}4$ inch.
$d=$ diameter of bearing in inches.

Thrust of a Screw Propeller.—

H = indicated horse-power of engines.

K = speed of ship in knots.

F = speed of ship in feet per minute.

R = thrust of propeller, or total load on thrust bearing in lbs.

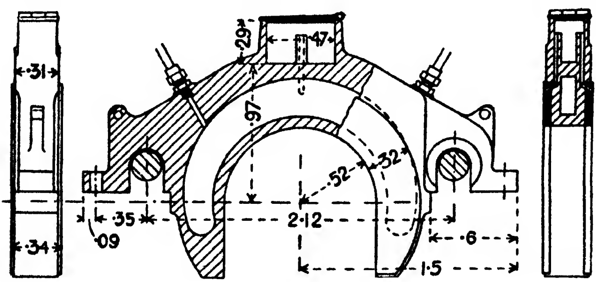
E = mechanical efficiency of engines and propeller, or ratio of power used to propel the ship to the indicated horse-power.

$$R \times F = H \times 33000 \times E. \qquad R = \frac{H \times 33000 \times E}{F} = \frac{H \times 33000 \times E}{K \times 101{\cdot}33}.$$

E may be taken equal to ·7.

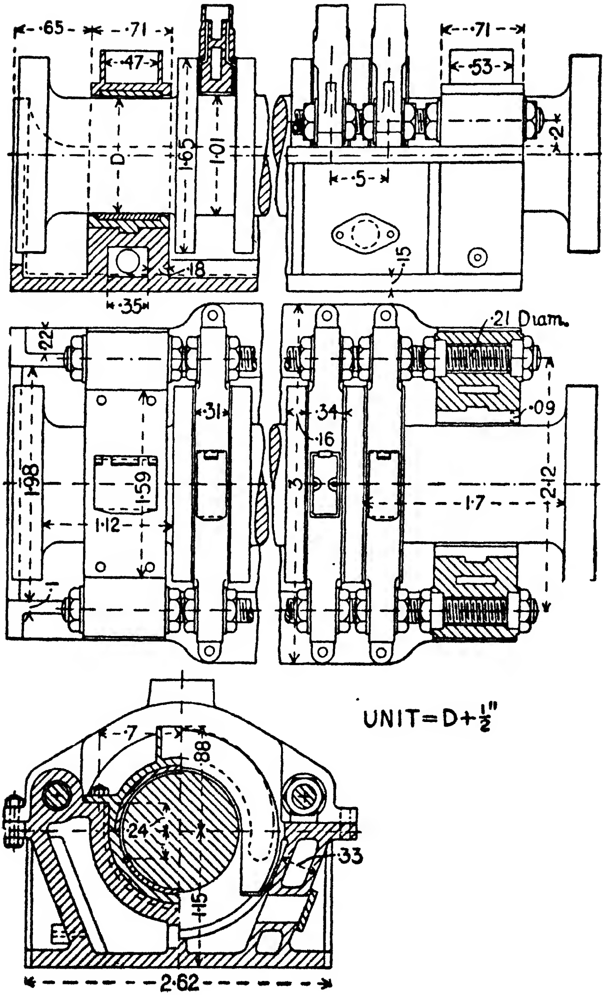
Thrust Bearings for Screw Propeller Shafts.*—Small shafts up to 8 inches diameter have often only one thrust collar, but sometimes comparatively small shafts have several thrust collars. In the latter case the bearing may have a brass bush in halves containing grooves to receive the collars on the shaft, or the bearing may contain a number of rings in halves which fit between the collars on the shaft. The general practice now, especially with large shafts, is to place between the collars cast-iron or cast-steel horse-shoe shaped pieces faced with brass or white metal. For steel shafts the horse-shoe pieces should be faced with white metal.

The illustrations below and on the following page show a design for a thrust bearing by Messrs. David Rollo & Sons,



Liverpool. The proportions marked on the illustrations are in terms of the unit $D + \frac{1}{2}$ inch, where D is the diameter of the shaft in inches. These proportions have been deduced from the dimensions of a bearing of this design for a shaft $16\frac{1}{2}$ inches in diameter, given in *Engineering*, vol. lvi. p. 206. The shoes are of cast-steel, and are hollow, water for cooling purposes circulating through them. The sides of the block are double, and wacer is

* For Michell thrust bearings see p. 467.


·65
·71
·47
D
1·65
1·01
·18
·35
·71
·53
·2
·5
·15
·22
·21 Diam.
·09
·31
·34
·16
1·98
1·59
1·12
3
1·7
2·12
·1
UNIT = D + ½"
·7
·88
·24
·33
1·15
2·62

passed through these also. The longitudinal screws for adjusting and holding the shoes are of manganese-bronze.

For the area of the bearing surface of thrust collars, and for the working pressure allowed on them, see pp. 437 and 438.

The number of collars is generally about $\frac{D}{2}-2$, where D is the diameter of the shaft in inches.

Mr. G. R. Bate gives the following rule* for the surface of thrust collars :—

$$S=\frac{217}{KP}, \text{ where}$$

S=surface of thrust collars in square inches per indicated horse-power.

K=speed of vessel in knots.

P=pressure on thrust collars in lbs. per square inch.

In ordinary practice P varies from 50 to 60 in naval work, and 40 to 50 in mercantile steamers. Where white metal is fitted these loads may be safely increased by 25 per cent.

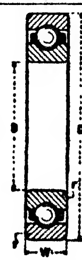
* *The Practical Engineer*, vol. x. p. 344.

Hoffmann Ball Bearings.*

The Hoffmann Manufacturing Co. Ltd., Chelmsford.

Deep Groove Ball Journals—Light Type.

Dimensions in Inches.				Approx. Weight, Lbs.	Working Load, Lbs. R.P.M.				
B	*D*	*W*	*r*		100	300	600	1000	1500
1/2	1 5/16	3/8	1/32	·08	390	270	220	180	160
5/8	1 9/16	7/16	1/32	·14	510	350	280	240	210
3/4	1 7/8	9/16	1/16	·25	800	560	440	370	330
7/8	2	9/16	1/16	·32	860	600	470	400	350
1	2 1/4	5/8	1/16	·41	890	620	490	410	360
1 1/8	2 1/2	5/8	1/16	·53	930	640	510	430	370
1 1/4	2 3/4	11/16	1/16	·7	1100	780	620	520	450
1 3/8	3	11/16	1/16	·83	1350	930	730	620	540
1 1/2	3 1/4	3/4	3/32	1·01	1720	1200	950	800	700
1 5/8	3 1/2	3/4	3/32	1·19	1800	1250	990	840	730
1 3/4	3 3/4	13/16	3/32	1·48	2250	1550	1250	1050	910
1 7/8	4	13/16	3/32	1·72	2350	1650	1300	1100	960
2	4	13/16	3/32	1·77	2700	1850	1450	1250	1100
2 1/4	4 1/2	7/8	3/32	2·4	3350	2350	1850		
2 1/2	5	15/16	3/32	3·28	4100	2850	2250		
2 3/4	5 1/4	15/16	3/32	3·46	4500	3100	2450		
3	5 3/4	1 1/16	3/32	4·74	4850	3350	2650		
3 1/4	6	1 1/16	3/32	5·06	5100	3550	2800		
3 1/2	6 1/2	1 1/8	1/8	6·28	6000	4150	3300		
3 3/4	6 3/4	1 1/8	1/8	6·53	6500	4500	3600		
4	7 1/4	1 1/4	1/8	8·5	7600	5000	4150		
4 1/4	7 1/2	1 1/4	1/8	8·76	8100	5600	4450		
4 1/2	8	1 5/16	1/8	10·57	8600	5900	4700		

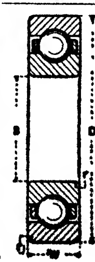


Load Tables.—The load tables are based on the manufacturers' experience, and may be used without reservation in many cases where conditions are normal. For shock loads and dead weights roller bearings are preferable, and a factor of safety of 1·5 to 2 should be allowed. For continuous running, 24 hours a day, a factor of safety of 1·2 to 1·5 is advised.

* The tables give a selection from the ranges of bearings available. The manufacturers should be consulted for further particulars of ball and roller bearings

Hoffmann Ball Bearings.
Deep Groove Ball Journals—Medium Type.

Dimensions in Inches.				Approx. Weight, Lbs.	Working Load, Lbs.				
					R.P.M.				
B	*D*	*W*	*r*		100	300	600	1000	1500
1/4	1 5/16	9/16	1/16	·15	510	350	280	240	210
3/8	1 1/2	9/16	1/16	·22	640	450	350	300	260
1/2	1 5/8	5/8	1/16	·24	790	550	430	370	320
5/8	1 13/16	5/8	1/16	·29	820	570	450	380	330
3/4	2	11/16	1/16	·38	990	690	550	460	400
7/8	2 1/4	11/16	1/16	·49	1050	720	570	480	420
1	2 1/2	3/4	3/32	·61	1600	1100	890	750	650
1 1/8	2 13/16	13/16	3/32	·85	2100	1450	1150	980	860
1 1/4	3 1/8	7/8	3/32	1·12	2700	1850	1470	1250	1090
1 3/8	3 1/2	7/8	3/32	1·62	2800	1930	1530	1300	1130
1 1/2	3 3/4	15/16	3/32	2·01	3100	2150	1700	1450	1250
1 5/8	4	15/16	3/32	2·31	3450	2400	1900	1600	1400
1 3/4	4 1/4	1 1/16	3/32	2·92	3800	2600	2100	1750	1550
1 7/8	4 1/2	1 1/16	3/32	3·26	4550	3150	2500	2100	1850
2	4 1/2	1 1/16	3/32	3·18	4550	3150	2500	2100	1850
2 1/4	5	1 1/4	1/8	4·52	5300	3700	2950	2450	2150
2 1/2	5 1/2	1 1/4	1/8	5·49	6700	4700	3700	3100	2700
2 3/4	6 1/4	1 3/8	1/8	8·02	8800	6100	4850	4100	3550
3	7	1 9/16	5/32	11·54	11000	7600	6100	5100	4500
3 1/4	7 1/2	1 9/16	5/32	13·33	11000	7600	6100	5100	4500
3 3/8	7 1/2	1 9/16	5/32	13·02	11000	7600	6100	5100	4500
3 1/2	8 1/8	1 3/4	5/32	17·64	13500	9400	7500	6300	5500
3 3/4	8 1/4	1 3/4	5/32	17·54	13500	9400	7500		
4	8 1/2	1 3/4	5/32	18·29	14500	9900	7900		
4 1/4	8 3/4	1 3/4	5/32	19·12	14500	9900	7900		
4 1/2	9 3/8	2	3/16	25	15600	10800	8600		
4 3/4	10	2	3/16	29·25	17000	12000	9300		
5	10	2	3/16	28	17000	12000	9300		
5 1/2	11	2	3/16	34·25	19500	13500	11000		
6	12	2 1/4	3/16	45·5	25800	17900	14200		
6 1/2	13	2 1/2	3/16	60·25	27000	18800	14900		
7	13 1/2	2 1/2	3/16	63	29000	20000			



Hoffmann Single Thrust Ball Bearings—Medium Type.

Fig. 1. Flat seating.
Fig. 2. Spherical seating, with housing.
Fig. 3. Spherical seating, with seating ring.

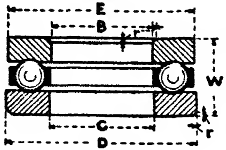


Fig. 1.

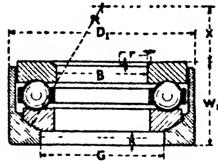


Fig. 2.

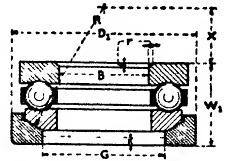


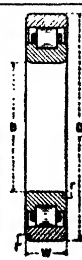
Fig. 3.

Dimensions in Inches.												Working Load, Lbs.					
												R.P.M.					
B	*C*	*D*	D_1	*E*	*G*	*W*	W_1	*K*	*R*	*X*	*r*	*	100	300	600	1000	2000
3/4	13/16	1 9/16	1 3/4	1 1/2	1	23/32	27/32	1/16	1 3/16	3/8	1/32	2490	900	520	370	280	200
7/8	15/16	1 13/16	2	1 3/4	1 3/16	3/4	29/32	3/32	1 3/8	1/2	1/32	3750	1280	740	520	400	280
1	1 1/8	2	2 1/8	1 7/8	1 5/16	3/4	29/32	3/32	1 5/8	3/4	1/16	3750	1280	740	520	400	280
1 1/8	1 1/4	2 1/4	2 3/8	2 1/8	1 1/2	7/8	1 1/16	3/32	1 5/8	9/16	1/16	2 1/4 tons	1710	980	690	540	380
1 1/4	1 3/8	2 1/2	2 5/8	2 3/8	1 11/16	1	1 3/16	1/8	2	13/16	1/16	3 ,,	2180	1260	890	690	480
1 1/2	1 5/8	2 7/8	3	2 3/4	2	1 1/8	1 3/8	1/8	2 3/16	13/16	1/16	3 3/4 ,,	2650	1530	1080	830	590
1 3/4	1 7/8	3 5/16	3 1/2	3 3/16	2 5/16	1 1/4	1 1/2	1/8	2 9/16	1 1/32	1/16	4 1/2 ,,	3130	1810	1280	990	700
2	2 1/8	3 11/16	3 7/8	3 9/16	2 5/8	1 3/8	1 11/16	5/32	2 3/4	1 1/32	3/32	5 1/2 ,,	3650	2110	1490	1150	810
2 1/4	2 3/8	4 1/4	4 1/2	4 1/8	3	1 11/16	2	3/16	3 9/16	1 9/16	3/32	7 1/2 ,,	4860	2800	1980	1530	1080
2 1/2	2 5/8	4 3/4	5 1/8	4 5/8	3 3/8	2	2 5/16	7/32	4	1 21/32	3/32	10 ,,	6090	3510	2480	1920	
2 3/4	2 7/8	5	5 3/8	4 7/8	3 5/8	2	2 5/16	7/32	4	1 9/16	3/32	10 ,,	6090	3510	2480	1920	
3	3 1/8	5 5/8	6	5 1/2	4	2 1/4	2 5/8	1/4	4 1/2	1 25/32	1/8	12 1/2 ,,	7480	4320	3060	2370	

* The load given in the first column is only suitable where an occasional revolution is required, such as the pivot of a crane or a crane hook.

Hoffmann Roller Bearings.
Rigid Roller Journals—Light Type.

Dimensions in Inches.				Approx. Weight, Lbs.	Working Load, Lbs.				
					R.P.M.				
B	*D*	*W*	*r*		100	300	600	1000	1500
1/2	1 5/16	3/8	1/32	·1	800	610	510	450	410
5/8	1 9/16	7/16	1/32	·17	890	680	570	500	450
3/4	1 7/8	9/16	1/16	·3	1500	1150	970	860	770
7/8	2	9/16	1/16	·32	1600	1200	1000	890	810
1	2 1/4	5/8	1/16	·46	1650	1250	1050	930	840
1 1/8	2 1/2	5/8	1/16	·57	1800	1350	1150	1000	920
1 1/4	2 3/4	11/16	1/16	·74	2750	2050	1750	1550	1400
1 3/8	3	11/16	1/16	·9	2950	2250	1900	1650	1500
1 1/2	3 1/4	3/4	3/32	1·1	3950	3000	2500	2200	2000
1 5/8	3 1/2	3/4	3/32	1·34	4100	3100	2600	2300	2100
1 3/4	3 3/4	13/16	3/32	1·58	4250	3250	2700	2400	2150
1 7/8	4	13/16	3/32	1·87	4400	3350	2800	2450	2250
2	4	13/16	3/32	1·75	4400	3350	2800	2450	2250
2 1/4	4 1/2	7/8	3/32	2·45	6000	4550	3850	3400	3050
2 1/2	5	15/16	3/32	3·2	7500	5750	4800	4250	3850
2 3/4	5 1/4	15/16	3/32	3·4	7800	5900	5000	4400	4000
3	5 3/4	1 1/16	3/32	4·62	9900	7500	6300	5600	5100
3 1/4	6	1 1/16	3/32	4·84	10200	7800	6500	5700	5200
3 1/2	6 1/2	1 1/8	1/8	6·04	12500	9600	8100	7100	6400
3 3/4	6 3/4	1 1/8	1/8	6·37	13000	10000	8400	7400	6700
4	7 1/4	1 1/4	1/8	8·14	15500	11500	9800	8600	7800
4 1/4	7 1/2	1 1/4	1/8	8·51	16500	12500	10500	9200	8400
4 1/2	8	1 5/16	1/8	10·36	19000	14500	12000		
4 3/4	8 1/4	1 5/16	1/8	10·82	20000	15000	12500		
5	9	1 3/8	1/8	14·19	23000	17500	14500		
5 1/2	9 1/2	1 3/8	1/8	15·14	23500	18000	15000		
6	10 1/2	1 9/16	5/32	21·19	28500	21500	18000		
6 1/2	11	1 9/16	5/32	22·57	29000	22000	18500		
7	12	1 3/4	5/32	30·25	37000	28000	23500		
7 1/2	12 1/2	1 3/4	5/32	32	38000	29000	24500		
8	13	1 3/4	5/32	34·5	39000	29500	25000		
8 1/2	14	2	5/16	46·25	49500	37500	32000		



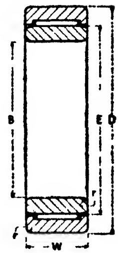
Hoffmann Roller Bearings.
Rigid Roller Journals—Medium Type.

Dimensions in Inches.				Approx. Weight, Lbs.	Working Load, Lbs.				
					R.P.M.				
B	*D*	*W*	*r*		100	300	600	1000	1500
1/4	1 5/16	9/16	1/16	·14	1100	840	700	620	560
3/8	1 1/2	9/16	1/16	·21	1250	970	810	710	640
1/2	1 5/8	5/8	1/16	·26	1250	970	810	710	640
5/8	1 13/16	5/8	1/16	·32	1450	1100	920	810	730
3/4	2	11/16	1/16	·41	2300	1750	1450	1300	1150
7/8	2 1/4	11/16	1/16	·53	2400	1850	1550	1350	1200
1	2 1/2	3/4	3/32	·67	3300	2500	2100	1840	1660
1 1/8	2 13/16	13/16	3/32	·98	3400	2600	2200	1920	1740
1 1/4	3 1/8	7/8	3/32	1·23	4400	3350	2800	2450	2250
1 3/8	3 1/2	7/8	3/32	1·67	4900	3700	3150	2750	2500
1 1/2	3 3/4	15/16	3/32	1·97	6400	4900	4100	3600	3250
1 5/8	4	15/16	3/32	2·23	6400	4900	4100	3600	3250
1 3/4	4 1/4	1 1/16	3/32	2·79	8100	6200	5200	4550	4150
1 7/8	4 1/2	1 1/16	3/32	3·16	8500	6500	5400	4800	4350
2	4 1/2	1 1/16	3/32	3·04	8500	6500	5400	4800	4350
2 1/4	5	1 1/4	1/8	4·27	10000	7600	6400	5600	5100
2 1/2	5 1/2	1 1/4	1/8	5·3	11500	8700	7400	6500	5800
2 3/4	6 1/4	1 3/8	1/8	7·79	17000	12500	10500	9400	8500
3	7	1 9/16	5/32	11·14	22000	16500	14000	12500	11000
3 1/4	7 1/2	1 9/16	5/32	12·97	23000	17500	14500	13000	11500
3 3/8	7 1/2	1 9/16	5/32	12·65	23000	17500	14500	13000	11500
3 1/2	8 1/8	1 3/4	5/32	17·04	28500	21500	18000	16000	14500
3 3/4	8 1/4	1 3/4	5/32	17·07	28500	21500	18000		
4	8 1/2	1 3/4	5/32	17·66	29500	22500	19000		
4 1/4	8 3/4	1 3/4	5/32	18·42	29500	22500	19000		
4 1/2	9 3/8	2	3/16	24·5	36000	27000	23000		
4 3/4	10	2	3/16	28	39000	29500	25000		
5	10	2	3/16	27	39000	29500	25000		
5 1/2	11	2	3/16	33·75	40000	30500	25500		
6	12	2 1/4	3/16	44·5	47500	36000	30500		
6 1/2	13	2 1/2	3/16	58·25	57000	43500	36500		
7	13 1/2	2 1/2	3/16	62	62000	47500	40000		

Hoffmann Needle Roller Bearings.

Complete Bearings.

Dimensions in mm.				
B	*D*	*W*	*E*	*r*
12	30	20	18·4	1
15	35	20	22·3	1
17	37	20	24·7	1
20	42	20	28·7	1
25	47	22	33·5	1
30	52	22	38·2	1
35	58	22	44	1
40	65	22	49·7	1·5
45	72	22	55·4	1·5
50	80	28	62·1	2
55	85	28	68·8	2
60	90	28	72·6	2
65	95	28	78·3	2
70	100	28	83·1	2
75	110	32	90·8	2
80	115	32	95·5	2
85	120	32	101·2	2
90	125	32	105	2
95	130	32	110·8	2
100	135	32	115·5	2
110	150	40	127	3
120	160	40	137	3
130	180	52	151·5	3
140	190	52	161·7	3
150	200	52	171·9	3



The bearing consists of one plain cylindrical race and one lipped race, having between them a sufficient number of rollers of almost needle-like proportions to fill practically the whole of the space between the two. It is useful for oscillating motion or where loads and speeds fluctuate continuously.

These bearings can be supplied, as specials, with rollers retained in the outer race, but this entails extra width.

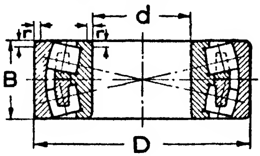
Needle Rollers.

Diameter .	mm.	2	2	2·5	2·5	2·5	2·5
Length .	mm.	9·8	15·8	7·8	9·8	13·8	15·8
Diameter .	mm.	3	3	3	3	3·5	4
Length .	mm.	11·8	15·8	19·8	23·8	29·8	39·8

SKF Spherical Roller Bearings.

The Skefko Ball Bearing Co. Ltd., Luton.

Medium Type, Self-aligning Radial Roller Bearings.



Millimetres.				Weight. Lbs.	Basic Load at 1–15 R.P.M. Lbs.	Max. Catalogue Speed. R.P.M.
d	*D*	*B*	*r* approx.			
40	90	33	2·5	2·27	18000	5000
45	100	36	2·5	3·08	22000	5000
50	110	40	3	4·20	31000	5000
55	120	43	3	5·28	36500	5000
60	130	46	3·5	6·61	45000	5000
65	140	48	3·5	7·92	48500	5000
70	150	51	3·5	9·59	64000	5000
75	160	55	3·5	11·88	66000	3000
80	170	58	3·5	14·04	77000	3000
85	180	60	4	16·28	86000	3000
90	190	64	4	19·40	100000	3000
95	200	67	4	22·66	110000	3000
100	215	73	4	28·65	130000	1500
110	240	80	4	40·00	160000	1500
120	260	86	4	48·70	195000	1500
130	280	93	5	63·00	220000	1500
140	300	102	5	78·46	240000	750
150	320	108	5	93·70	275000	750
160	340	114	5	113	295000	750
170	360	120	5	132	340000	750
180	380	126	5	155	375000	750
190	400	132	6	179	405000	500
200	420	138	6	207	440000	500
220	460	145	6	269	520000	500
240	500	155	6	340	610000	300
260	540	165	8	423	700000	300
280	580	175	8	516	790000	300

Permissible loads when the bearings are at rest are about two-thirds of the tabulated loads for 1–15 r.p.m. The permissible load P at a speed of n r.p.m. may be found approximately from the formula $P=K_{1-15}/f$, where K_{1-15} is the tabulated load and f is the speed factor. Values of f are given for a number of speeds.

R.P.M.	f	R.P.M.	f	R.P.M.	f	R.P.M.	f
30	1·26	100	1·88	300	2·71	1500	4·62
50	1·49	150	2·16	500	3·21	3000	5·83
75	1·71	200	2·37	750	3·68	5000	6·92

The tabulated loads are based on a life of 500 hours. For other lengths of life, a life factor s is used. Some values of s are given below.

Life-Hours.	500	4000	13000	30000	60000
s	1	2	3	4	5

In practice P is known, and a load near the value given by $K_{1-15}=sfP$ is then found in the table, and so the right size of bearing is selected.

When radial and thrust loads act simultaneously on a radial bearing, $P=R+yA$, where P is the equivalent radial load, R is the actual radial load, A is the actual thrust load, and y is a coefficient depending on the type of bearing. For the bearing illustrated on p. 458, $y=2$, provided the actual thrust load is small compared with the radial load. The value of y is 2·3 when the thrust load is 50 per cent. of the radial load, 2·5 when the thrust load is equal to the radial load, and 2·7 when the load is purely thrust.

Notes on a Few Types of Skefko Bearings.*

Deep groove, single row ball bearing (Fig. 1).—Deep ball tracks give full support to the balls, even under thrust load. No filling slot. Light but strong cage, suitable for relatively high speeds. Thrust load taken in either direction, alone or in combination with radial load. For thrust duty some slackness is necessary in the bearing, so that, when thrust is applied, a line through the contact points between the balls and tracks will make an angle with the axis of rotation. Axial and radial play can be eliminated by providing initial thrust between two bearings. Misalignment up to about $\frac{1}{2}°$ can be dealt with.

Double row, self-aligning ball bearing (Fig. 2).—There are two tracks of normal form in the inner ring, but the outer ring track is spherical, the radius of curvature having its centre at the

* The manufacturers should be consulted for full particulars of their ball and roller bearings.

geometrical centre of the bearing. This bearing is particularly valuable where misalignment is expected. It is also supplied with taper bores and adapter sleeves for secure fixing on a shaft without shoulders. In addition to radial load, thrust in either direction may be taken, but the single row type has a greater thrust capacity. A wide double row series is made, and this takes a fairly high thrust.

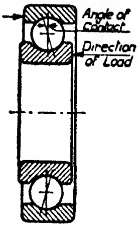


FIG. 1.

FIG. 2.

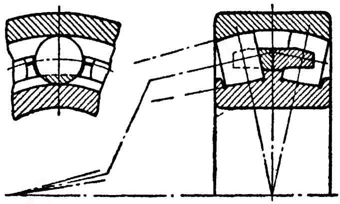


FIG. 3.

Double row, spherical roller bearing (Fig. 3).—This bearing is intended for heavy loads, as in rolling mills, railway axle-boxes, etc. The rollers are barrel-shaped and make line contact with the inner ring. The outer ring has a spherical race-way, common to both rows of rollers. The axes of the rollers converge to a point on the main axis of the bearing. The bearing is self-contained, and can accept both radial and thrust loads in either direction.

FIG. 4.

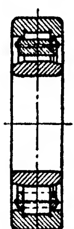


FIG. 5.

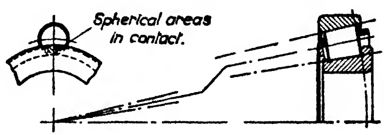


FIG. 6.

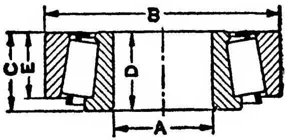
Cylindrical roller bearing.—The two main types have either a flanged inner ring (Fig. 4) or a flanged outer ring (Fig. 5). The flanged inner ring is the more common. All types are made in light, medium, and heavy series.

Tapered roller bearing (Fig. 6).—The bearing takes thrust in one direction as well as radial load. The large ends of the rollers are spherical in form, and the inner ring flange against which they bear is also a sphere of the same radius; this prevents the rollers from skewing and reduces the frictional resistance.

Timken Tapered Roller Bearings.*

British Timken Ltd., Birmingham.

Single Row Bearings—Light Type.



Dimensions in Inches.					Capacity at 500 R.P.M. with 500 Hours Life.	
A	*B*	*C*	*D*	*E*	Radial Lbs.	Thrust Lbs.
$\frac{1}{2}$	$1\frac{1}{2}$	·5245	·550	$\frac{7}{16}$	780	420
$\frac{5}{8}$	$1\frac{5}{8}$	$\frac{9}{16}$	·578	$\frac{7}{16}$	840	515
$\frac{3}{4}$	1·850	·566	·562	$\frac{7}{16}$	995	695
$\frac{7}{8}$	2	$\frac{19}{32}$	·557	$\frac{1}{2}$	1045	825
1	2	$\frac{11}{16}$	·709	$\frac{17}{32}$	1410	900
$1\frac{1}{8}$	$2\frac{1}{2}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{5}{8}$	1920	1320
$1\frac{1}{4}$	2·717	$\frac{25}{32}$	·771	$\frac{5}{8}$	1960	1450
$1\frac{3}{8}$	2·717	$\frac{25}{32}$	·771	$\frac{5}{8}$	1960	1450
$1\frac{1}{2}$	$3\frac{5}{32}$	·8268	·8244	$\frac{5}{8}$	2280	1790
$1\frac{3}{4}$	3·3465	·748	·7545	$\frac{5}{8}$	2370	2070
2	$3\frac{13}{16}$	·875	·875	$\frac{3}{4}$	3220	2135
$2\frac{1}{4}$	$3\frac{7}{8}$	·8268	·889	·7018	3355	2325
$2\frac{1}{2}$	4·333	·866	·866	·6786	3475	2645

* The tables give a selection from the ranges of bearings which are generally available. A few other types are shown in the Figs. on pp. 464 and 465. This firm, who also make ball and parallel roller bearings, should be consulted for further particulars.

Timken Tapered Roller Bearings.

Single Row Bearings—Medium Type.

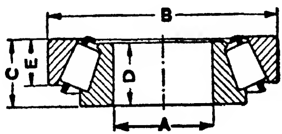
Dimensions in Inches.					Capacity at 500 R.P.M. with 500 Hours Life.	
A	*B*	*C*	*D*	*E*	Radial Lbs.	Thrust Lbs.
$\frac{5}{8}$	$1\frac{3}{4}$	$\frac{13}{16}$	·888	$\frac{9}{16}$	1330	635
$\frac{3}{4}$	1·938	$\frac{29}{32}$	·848	$\frac{11}{16}$	1440	755
$\frac{7}{8}$	2·240	$\frac{49}{64}$	·781	$\frac{5}{8}$	1735	1045
1	$2\frac{1}{2}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{5}{8}$	1920	1320
$1\frac{1}{8}$	2·8346	·748	·745	$\frac{5}{8}$	2075	1465
$1\frac{1}{4}$	2·615	1	·973	$\frac{13}{16}$	2945	1580
$1\frac{1}{2}$	3	$\frac{15}{16}$	1·010	$\frac{3}{4}$	3100	1845
$1\frac{3}{4}$	$3\frac{7}{16}$	$1\frac{7}{16}$	1·466	$\frac{15}{16}$	4230	2540
2	3·6719	$1\frac{3}{16}$	1·193	$\frac{15}{16}$	4620	3065
$2\frac{1}{4}$	$4\frac{1}{4}$	$1\frac{3}{32}$	1·162	$\frac{7}{8}$	4900	3220
$2\frac{1}{2}$	$4\frac{7}{16}$	$1\frac{3}{16}$	1·183	$\frac{15}{16}$	5155	4060
$2\frac{3}{4}$	4·7244	1·1417	1·142	·923	5510	4155
3	$5\frac{3}{8}$	$1\frac{3}{16}$	$1\frac{11}{64}$	$\frac{7}{8}$	6075	5285
$3\frac{1}{4}$	5·5118	$1\frac{7}{16}$	1·4212	$1\frac{1}{8}$	7655	6060
$3\frac{1}{2}$	6	$1\frac{9}{16}$	1·430	$1\frac{3}{16}$	8450	7320
$3\frac{3}{4}$	$6\frac{3}{8}$	$1\frac{7}{16}$	1·422	$1\frac{1}{32}$	8685	8080
4	$6\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{3}{16}$	10400	9610
$4\frac{1}{2}$	$7\frac{1}{2}$	$1\frac{7}{8}$	$1\frac{15}{16}$	$1\frac{3}{8}$	14490	11805
5	$9\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{15}{16}$	22910	16595

Timken Tapered Roller Bearings.
Single Row Bearings—Heavy Type.

Dimensions in Inches.					Capacity at 500 R.P.M. with 500 Hours Life.	
A	*B*	*C*	*D*	*E*	Radial Lbs.	Thrust Lbs.
3/4	2·240	49/64	·781	5/8	1735	1045
7/8	2·240	15/16	·975	13/16	2475	1125
1	2·615	15/16	·976	3/4	2790	1390
1 1/8	2 3/4	15/16	·973	3/4	2945	1580
1 1/4	2·8594	1 3/16	1·1811	15/16	3690	2160
1 3/8	3 5/32	1 5/32	1·1965	15/16	4030	2190
1 1/2	3 7/16	1 3/16	1·216	15/16	4230	2400
1 5/8	3·4844	1 1/16	1·145	7/8	4250	2450
1 3/4	3·6719	1 3/16	1·193	15/16	4620	3065
1 7/8	4	1 3/8	1·420	1 1/16	6205	3465
2	4 3/8	1 1/2	1·455	1 3/16	6510	3785
2 1/4	4 7/8	1 1/2	1·444	1 3/16	7105	4830
2 1/2	5	1 7/16	1·444	1 1/8	7395	5270
2 3/4	5 3/8	1 5/8	1 5/8	1 1/4	9060	6435
3	5·596	1 3/4	1·815	1 5/16	9420	7130
3 1/4	5·909	1 3/4	1·838	1 7/16	12040	7680
3 1/2	6 3/8	1 7/8	1·9	1 1/2	12550	8390
3 3/4	7 1/8	1 7/8	1 7/8	1 1/2	13040	9850
4	7 1/2	2 1/4	2·265	1 3/4	17850	11705
4 1/2	8 3/8	2 5/8	2 5/8	2 1/8	21020	13420
5	10 3/16	3 1/16	3 1/4	2 1/4	28435	17915

Timken Tapered Roller Bearings.

Single Row Bearings—High Thrust Type.



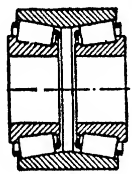
Dimensions in Inches.					Capacity at 500 R.P.M. with 500 Hours Life.	
A	*B*	*C*	*D*	*E*	Radial Lbs.	Thrust Lbs.
1	$2\frac{9}{16}$	$\frac{7}{8}$	·845	$\frac{5}{8}$	1930	2805
$1\frac{1}{8}$	2·8594	$\frac{31}{32}$	·955	$\frac{11}{16}$	2635	3120
$1\frac{1}{4}$	$3\frac{1}{8}$	1	·9478	$\frac{11}{16}$	2935	3845
$1\frac{1}{2}$	3·4844	1	·933	$\frac{11}{16}$	3030	4635
$1\frac{3}{4}$	$3\frac{7}{8}$	$1\frac{7}{32}$	1·114	$\frac{13}{16}$	3680	5345
2	$4\frac{7}{8}$	$1\frac{7}{16}$	1·291	1	5765	8330
$2\frac{1}{2}$	5·513	1·437	1·3085	·926	6855	11635
3	7	$2\frac{3}{16}$	2	$1\frac{3}{8}$	11200	16765
4	7·874	$2\frac{5}{64}$	$1\frac{15}{16}$	$1\frac{3}{8}$	15435	19135
$4\frac{1}{2}$	9	$2\frac{1}{8}$	1·946	$1\frac{1}{2}$	15780	22780
5	$12\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{1}{4}$	33235	47670

Other types of tapered roller bearings are shown in the illustrations on p. 465.

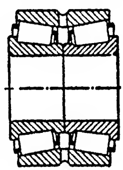
Speed Factors.—For bearing rating at any speed, multiply factor by the rating at 500 r.p.m. Factors not in the table may be obtained approximately by drawing a graph of the given values.

R.P.M.	Factor.	R.P.M.	Factor.	R.P.M.	Factor.	R.P.M.	Factor.
100	1·621	500	1·000	1500	0·719	3000	0·584
200	1·317	750	0·885	1750	0·687	3500	0·558
300	1·166	1000	0·812	2000	0·660	4000	0·536
400	1·069	1250	0·760	2500	0·617	5000	0·501

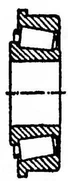
Service Factors.—In selecting a tapered roller bearing a service factor must be taken into account. This is a product of a factor appropriate to the application and one giving a selected number of life-hours. This factor varies from 1·75 for household appliances to 7 for paper mill machinery. A good average value is 3·5.



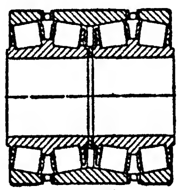
TYPE DO



TYPE NA



TYPE SF



TYPE QO

Selection of Bearings.—The tabulated radial and thrust ratings are to be considered separately. Calculate the required load on the bearing, then—

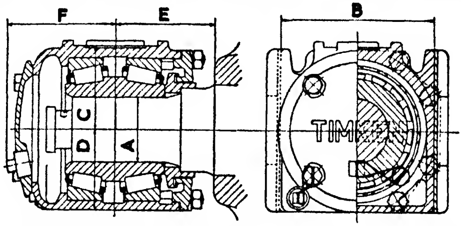
$$\text{Required rating at 500 r.p.m.} = \frac{\text{Calculated load} \times \text{Service factor}}{\text{Speed factor}}.$$

When radial and thrust loads are carried simultaneously, calculate the equivalent radial load from the formula—

$$\text{Equivalent radial load} = 0{\cdot}66 \times \text{Radial load} + k \times \text{Thrust load},$$

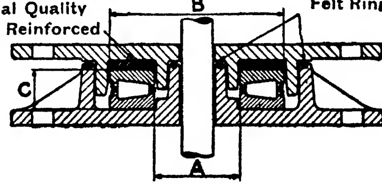
where $k = \dfrac{\text{Radial rating}}{\text{Thrust rating}}$. For a first trial take $k = 1{\cdot}5$.

Timken Railway Axle-boxes for Outside Journals.



Leading Dimensions—Inches.					Weight, Lbs.	Radial Rating at 500 R.P.M., Lbs.
A	*B*	*C = D*	*E*	*F*		
2·875	7·000	3·625	4·750	4·625	70	4850
3·000	7·250	3·750	5·000	4·875	80	5140
3·125	7·500	3·875	5·250	5·125	95	7720
3·750	8·375	4·375	5·500	5·375	120	8700
4·000	9·250	4·750	6·375	6·250	160	11300
4·750	10·250	5·250	6·500	6·375	190	12720
4·875	10·625	5·500	6·750	6·625	225	14960
5·000	11·250	5·875	7·000	6·875	260	18640
5·375	12·000	6·375	7·125	7·000	300	26000
6·000	13·500	6·875	7·250	7·125	350	30600
7·000	14·500	7·500	7·625	7·500	400	32360

Timken Bogie Centre Pivot Bearings.



The design incorporates a rubber pad of oil-resisting and high-loading qualities. This pad prevents uneven loading of the bearing and absorbs vibration and noise.

Dimensions—In.			Load, Lbs.	Weight, Lbs.	Dimensions—In.			Load, Lbs.	Weight, Lbs.
A	*B*	*C*			*A*	*B*	*C*		
4	7 5/8	1 3/4	23100	15	6 7/8	14 1/8	3 1/4	83650	99
4 7/8	8 1/8	1 1/4	26600	17	7	14 1/4	3 1/2	88550	104
4	8 1/2	1 13/16	30800	20	8	16 1/8	3 5/8	114450	153
5	10 1/2	2 5/16	46830	39	7	17	4	131950	190
6 1/8	12 1/2	3 1/8	59150	78	9	19	4 1/4	154000	230
6	12 1/4	2 3/4	65800	64					

MICHELL BEARINGS.

Michell Bearings Ltd., Newcastle-on-Tyne.

The Michell bearing makes use of the principle that bearing loads should be carried completely by the lubricating oil, instead of the oil acting merely to reduce friction between metallic surfaces. The bearing takes its name from the inventor, Mr A. G. M. Michell.

In all Michell bearings there are two elements: the shaft (either journal or thrust collar), and the six or eight bearing parts which are pivoted and known as *pads*. These two elements never come into contact, being forcibly separated and kept apart by automatically generated tapered oil films drawn from the normal oil supply. There is no metallic friction, no wear, and no renewal of parts so long as good clean oil is present in sufficient volume to carry the load.

Figs. 1 and 2 illustrate the action in a Michell thrust block

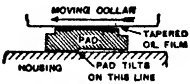


FIG. 1.

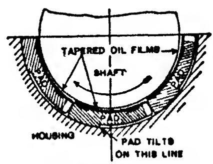


FIG. 2.

and a Michell journal bearing respectively. The tapered pressure oil film, or wedge of lubricant, is self-generated by the motion of the shaft or collar and is not dependent on any extraneous pressure from an oil pump. The pads are so designed that they tilt and float on their own oil films, and they have white metal faces, as white metal is less liable to damage from foreign matter which may be present in the oil.

In the thrust bearing, as the thrust collar revolves in its oil bath, the oil adhering to its surface is carried round and lifts every pad at its leading edge to admit the tapered oil film. Thus every pad generates a tapered pressure oil film of a thickness appropriate to the load, the speed, and the viscosity of the lubricating medium. For maximum efficiency, the pivot is "off-set" from the centre of the circumferential width of the pad, and this off-set is right-handed or left-handed to suit the direction of rotation. When necessary, the pads can be pivoted centrally to suit both directions of rotation, with a slight reduction in efficiency.

The Michell thrust bearing is a simple single-collar unit capable of carrying at least twenty times the load per square inch of a flat multi-collar thrust bearing. Thrust shafts may be disposed at any angle and standardised designs are available.

The Michell journal bearing usually has six pads surrounding the shaft journal. Each pad is free to tilt, and is prevented from cross-winding by suitable flanges. Oil is automatically introduced between each pair of pads from an annulus in the housing.

Load-carrying Capacity. — The working load depends on several factors, mainly diameter, length, peripheral speed, and viscosity. The capacity increases with the speed, and loads of several thousand pounds per square inch have been sustained in prolonged tests. For bearings which do not start under full load, up to 500 lbs. per square inch may be used, but 350 lbs. per square inch is taken as the limit when starting under full load. However, it is to be noted that bearings will work satisfactorily with considerable overloads.

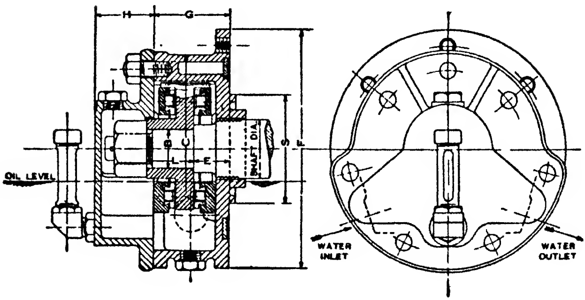
The formula for the thrust of a propeller is given on p. 449.

Friction. — Experiments with a Michell bearing loaded to 560 lbs. per square inch gave a coefficient of friction $\mu = 0{\cdot}0020$, and the calculated figure was 0·0022. The coefficient of friction of a good ordinary bearing is $\mu = 0{\cdot}036$, *about eighteen times as much.*

Lubricating Oil.—Any good quality pure mineral oil is recommended. In average cases a mineral oil having the following approximate viscosity at a temperature of 140° F. (60° C.) is suitable: 0·2 absolute viscosity in C.G.S. units, corresponding to 92·5 Redwood seconds, or 111 Saybolt seconds, or 3·3 Engler number. Forced lubrication is never required.

Dimensions.—Drawings and sizes of typical bearings are shown on pp. 469 to 472.

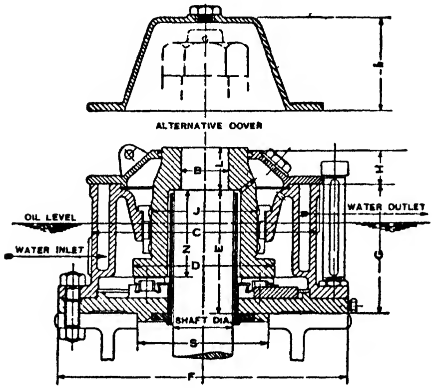
Michell Horizontal Thrust Bearings.



Shaft Diameter. Inches.	Thrust Surface. Sq. In.	Inches.							
		F	*C**	*G*	*H*	*S*	*B*	*L*	*E*
7/8 to 1 1/8	2 1/2	5 1/2	4	1·722	1 1/4	1 3/4	3/4	1	13/16
1 1/8 to 1 3/8	3 1/2	6 5/8	4 7/8	1·975	1 1/2	2 1/2	7/8	1 1/4	15/16
1 3/8 to 1 5/8	4 3/4	7 5/8	5 3/4	2·35	1 3/4	3	1	1 3/8	1 1/8
1 5/8 to 2	6 1/4	8 1/2	6 1/2	2·789	2	3 3/4	1 1/4	1 1/2	1 3/8
2 to 2 1/2	9	10	7 3/4	3·193	2 1/2	4 1/2	1 5/8	2	1 1/2
2 1/2 to 3	14	11 1/8	8 7/8	3·757	3	5 1/2	2	2 1/4	1 3/4
3 to 3 1/2	19	13 1/8	10 1/2	4·442	3 1/2	6 1/2	2 3/8	2 1/2	2 1/8
3 1/2 to 4	26	14 7/8	11 7/8	5·01	4	7 1/2	2 3/4	3	2 3/8
4 to 5	40	17 7/8	14 1/2	6·07	4 1/2	9	3 1/2	3 1/2	2 7/8
5 to 6	55	20 3/4	17 1/4	7·012	5	11	4 1/2	4	3 1/4
6 to 7	80	23 1/4	19 3/4	7·891	6	13	5 1/2	4 1/2	3 5/8

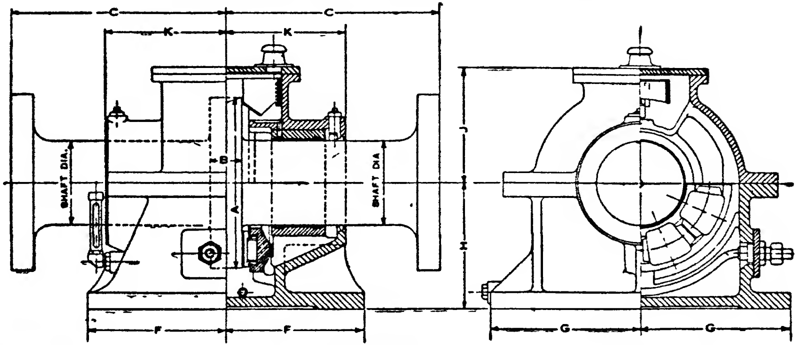
* The dimension *C* is the outside diameter of the casing over the water jacket which extends only about 270°.

Michell Vertical Thrust and Journal Bearings.



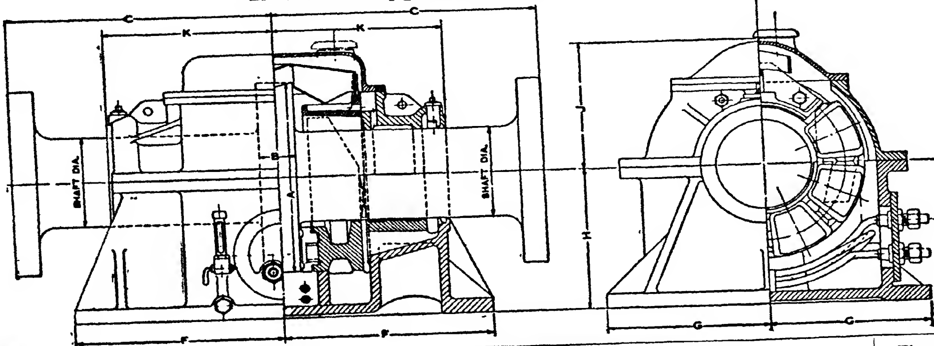
Shaft Diam. Inches.	Thrust Surface. Sq. In.	Inches.											
		F	*C*	*G*	*H*	*h*	*S*	*B*	*L*	*E*	*J*	*D*	*N*
5/8 to 7/8	2 1/4	6	4	2 5/8	9/16	1 13/16	3 1/8	5/8	5/8	2 5/8	1 3/4	2 1/8	1 3/4
7/8 to 1 1/8	3 1/2	6 7/8	4 3/4	2 15/16	11/16	2 1/8	3 3/4	7/8	3/4	2 15/16	2 1/4	3	2
1 1/8 to 1 3/8	4 3/4	8 1/4	6 3/8	3 1/4	7/8	2 1/8	4	1	1	3 3/4	2 5/8	3 1/2	2 1/4
1 3/8 to 1 5/8	6 1/4	9 3/8	6 7/8	4 3/8	7/8	3	4 3/8	1 1/4	1 1/8	4 1/4	3	4	3
1 5/8 to 2 1/8	9	10 3/4	8	4 3/4	1 1/8	3 1/2	5	1 3/8	1 3/8	4 3/8	3 3/4	5	3 1/4
2 1/8 to 2 1/2	14	12 1/4	9 1/2	5 3/8	1 3/8	4	5 1/8	2	1 3/4	5 1/8	4 1/8	6	3 5/8
2 1/2 to 3	19	14 1/8	11	6 1/8	1 1/2	4 3/8	6 1/8	2 3/8	2	5 3/4	5 1/4	7	4
3 to 3 1/2	26	15 3/8	12 1/4	7	1 7/8	5 1/4	7 1/8	2 3/4	2 1/8	6 5/8	6	8	4 3/8
3 1/2 to 4 1/2	40	18 1/8	15	8 3/8	2 1/4	6 1/8	9	3 1/8	3	7 7/8	7 1/8	10	5 3/8
4 1/2 to 5 1/2	55	21 3/4	18	9 3/4	2 3/4	8	11	4 1/8	3 3/4	9 1/4	9	12	6 3/8
5 1/2 to 6 1/2	80	24	20 1/4	11 1/8	3 1/8	9 3/4	13	5 1/8	4 1/8	10 3/8	10 1/4	14	7 1/4

Michell S-Type Marine Thrust Block.



Diameter of Shaft. Inches.	Shaft Dimensions. Inches.			Block Dimensions. Inches.					Thrust Surface. Sq. In.
	A	B	C	F	G	H	J	K	
1⅜ to 1¾	4	¾	8	4¼	5	3¾	3¼	4	2·8
2 to 2¼	5	1	9	4¾	5⅝	4¼	3⅝	4½	4·4
2⅜ to 2¾	6	1⅛	10¼	5⅛	6¼	5¼	4¼	5¼	6·6
3 to 3¼	7	1¼	11¼	6	7	6	5⅜	5¾	9·2
3⅜ to 3¾	8	1⅜	12½	6¾	7¾	7	5⅞	6½	12·0
4 to 4¾	10	1⅝	15½	8	9	8	7	7½	18·6
5 to 6	12	2¼	18	9¼	10¼	9	8⅜	8¾	27·0
6 to 7	14½	2½	19	11	12	10½	9½	9½	36·2

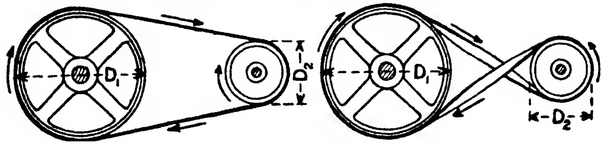
Michell B-Type Marine Thrust Block.



Diameter of Shaft. Inches.	Shaft Dimensions. Inches.			Block Dimensions. Inches.					Thrust Surface. Sq. In.
	A	*B*	*C*	*F*	*G*	*H*	*J*	*K*	
6 to 7½	14½	2½	20½	13	11¼	10½	10⅞	11½	45½
7 to 8½	17	3	25	17½	13½	12½	11¼	14	80½
8½ to 10	20	3½	28	19	15¼	14½	12⅞	15¾	111
10 to 12	23½	4	34	24	18	16¾	15¼	19½	145
12 to 14	27½	4½	40	27½	20½	18¾	17	23	195
14 to 16	33	5	44	30	24	24	20	25	290
16 to 18	36	5½	50	33½	26½	25	26¼	29	360
18 to 20	41	6	56	36½	29	28½	29¼	31¾	475

BELT GEARING.

Transmission of Motion by Bands.—

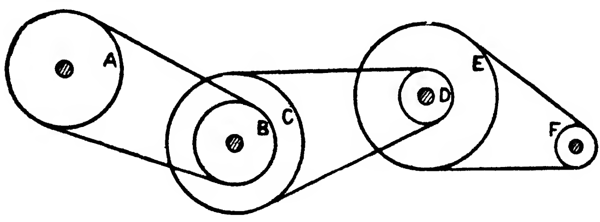


D_1 = diameter of driving pulley, or driver, in feet.
D_2 = diameter of following pulley, or follower, in feet.
N_1 = speed of driver in revolutions per minute.
N_2 = speed of follower in revolutions per minute.
V = speed of band in feet per minute.

$$V = 3{\cdot}1416 D_1 N_1 = 3{\cdot}1416 D_2 N_2.$$

$$D_1 N_1 = D_2 N_2. \qquad \frac{N_2}{N_1} = \frac{D_1}{D_2}.$$

The *effective* diameter of a pulley carrying a band is equal to the nominal diameter plus the thickness of the band. The above formulæ are only strictly true when D_1 and D_2 are the *effective* diameters of the pulleys.

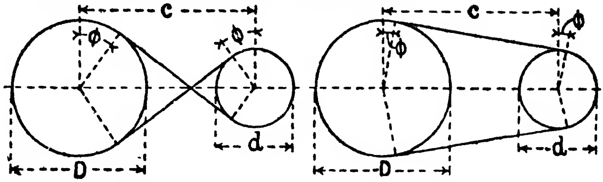


When motion is transmitted from a pulley A to a pulley F, through a number of intermediate pulleys B, C, D, E, of which B and C are fixed to one shaft and D and E fixed to another, then,

$$\frac{N_6}{N_1} = \frac{D_1}{D_2} \times \frac{D_3}{D_4} \times \frac{D_5}{D_6},$$

where D_1, D_2, D_3, D_4, D_5, and D_6 are the diameters of the pulleys A, B, C, D, E, and F respectively, and N_1 and N_6 are the speeds of A and F respectively. To be exact, the diameters taken should be the effective diameters, as explained above.

Length of a Belt connecting Two Pulleys.—



Crossed band *Open band.*

The length (L) may be obtained by measurement from a scale drawing, or by calculation, as follows:—

First calculate sin ϕ from

$$\sin\phi=\frac{D+d}{2c}\text{ for a crossed band,}$$

$$\text{or }\sin\phi=\frac{D-d}{2c}\text{ for an open band.}$$

Next find from a table of sines the angle ϕ in degrees, and then find cos ϕ from the table of cosines.

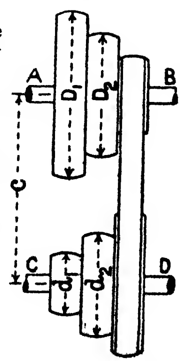
If n is the number of degrees in the angle, its circular measure is $\frac{\pi n}{180}$; let this be denoted by ϕ, then,

$$L=\left(\frac{\pi}{2}+\phi\right)(D+d)+2c\cos\phi\text{ for a crossed band,}$$

$$L=\frac{\pi}{2}(D+d)+\phi(D-d)+2c\cos\phi\text{ for an open band,}$$

$$=\frac{\pi}{2}(D+d)+\frac{(D-d)^2}{4c}+2c\text{ nearly, when }\phi\text{ is a small angle.}$$

Stepped Pulleys.—Stepped pulleys must be designed so that the same belt will be equally tight on each pair of opposite steps.



N=speed of driving shaft AB in revolutions per minute.

n_1, n_2, etc.=speeds at which shaft CD may be required to run, in revolutions per minute.

D_1, D_2, etc.=diameters of steps of pulley on AB.

d_1, d_2, etc.=diameters of steps of pulley on CD.

$$\frac{d_1}{D_1}=\frac{N}{n_1},\quad\frac{d_2}{D_2}=\frac{N}{n_2},\text{ etc.}$$

Assume the diameter D_1, then $d_1 = \frac{ND_1}{n_1}$.

For a *crossed band*, $D_2 + d_2 = D_1 + d_1$,

$$\text{hence } D_2 = \frac{n_2(D_1 + d_1)}{N + n_2}, \text{ and } d_2 = \frac{N(D_1 + d_1)}{N + n_2} = \frac{ND_2}{n_2}.$$

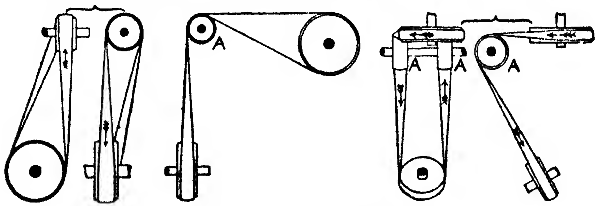
For an *open band*, determine D_2 and d_2 as above for a crossed band, and use their difference $D_2 - d_2 = x$ in the following formulæ to determine more approximate values of D_2 and d_2:—

$$D_2 = \frac{n_2}{N + n_2}\left\{ D_1 + d_1 + \frac{(D_1 - d_1)^2 - x^2}{2\pi c} \right\}.$$

$$d_2 = \frac{N}{N + n_2}\left\{ D_1 + d_1 + \frac{(D_1 - d_1)^2 - x^2}{2\pi c} \right\} = \frac{ND_2}{n_2}.$$

The equations for a crossed band are correct, but those for an open band are not quite exact; they are, however, sufficiently approximate for all practical purposes.

Pulleys on Shafts which are not Parallel. Guide Pulleys —



The pulleys marked A are guide pulleys.

The important rule to be observed in arranging the positions of the pulleys is this, the point at which the band leaves one pulley must be in the central plane of the next pulley.

Speed of Belts.—For main driving belts the lineal velocity should be from 3000 to 4000 feet per minute. In America a speed as high as 6000 feet per minute has been attained in a main driving belt, but such a high velocity is unusual.

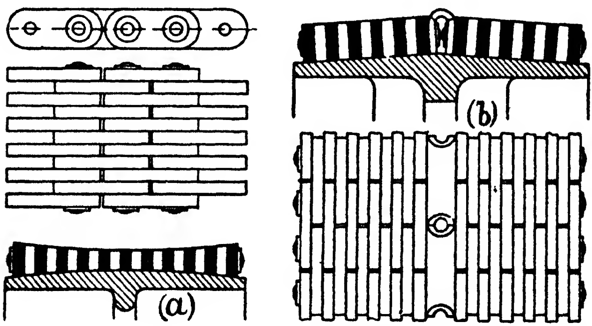
Strength of Leather Belts.—The tenacity of the leather used for belting may be taken at from 3000 to 5000 lbs. per square inch. The strength of a riveted joint is about half the strength of the solid leather, and the strength of a laced joint is only about one-third of the strength of the solid leather.

The working stress is usually taken at from one-fourth to one-third of the ultimate strength of the weakest part of the belt. It will be found in practice that the working stress is generally from 200 to 350 lbs. per square inch of solid belt section.

The following table gives the working strength per inch of width for belts of different thicknesses :—

Thickness of Belt in Inches.		$^3/_{16}$	$^7/_{32}$	$^1/_4$	$^9/_{32}$	$^5/_{16}$
Safe tension in lbs. per inch of width when the safe stress is	200	37	44	50	56	62
	250	47	55	62	70	78
	300	56	66	75	84	94
	350	66	77	87	98	109

Leather Link or Chain Belting.—The links are made of leather, and are connected by wrought-iron or steel pins. The belt may be made of any width, and it works freely on pulleys of small diameter It works well at high speeds.

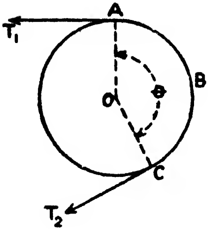


When a link belt of considerable width works on a curved pulley rim, the section of the belt should be made to suit the curvature of the rim, as shown at (*a*), or it should be hinged longitudinally in the centre, as shown at (*b*). In the latter case it is best to make the rim of the pulley of a double conical form, so that the section consists of two straight portions instead of the usual curve.

b = breadth of links in inches.
t = thickness of links in inches.
d = diameter of pins in inches.
n = number of links in extreme width of belt.
f = safe working stress on leather in lbs. per square inch.

Safe load on tight side of belt $=\frac{n}{2}(b-d)tf$ when n is an even number, and $=\frac{n-1}{2}(b-d)tf$ when n is an odd number.

Friction of a Band on a Pulley.—



T_1 = tension on "tight" or driving portion of band.

T_2 = tension on "slack" portion of band

θ = angle of contact AOC between band and pulley, *in circular measure*

$= \frac{\text{arc } ABC}{\text{radius } OA}$.

n = angle AOC *in degrees.*

μ = coefficient of friction between band and pulley.

e = base of Napierian system of logarithms.

When the band is on the point of slipping, $\frac{T_1}{T_2} = e^{\mu\theta}$.

$$\log \frac{T_1}{T_2} = \mu\theta \log e = \cdot 4343\mu\theta.$$
$$= \cdot 00758\mu n.$$

The following table gives values of the ratio $\frac{T_1}{T_2}$ for various values of μ and θ when the band is on the point of slipping :—

Angle of Contact (AOC). Circular Measure (θ).	Degrees (n).	Ratio of Tensions $\frac{T_1}{T_2}$ when the Band is on the Point of Slipping. $\mu = \cdot 2$	$\mu = \cdot 3$	$\mu = \cdot 4$	$\mu = \cdot 5$
·3491	20	1·072	1·110	1·150	1·191
·6981	40	1·150	1·233	1·322	1·418
1·0472	60	1·233	1·369	1·520	1·688
1·3963	80	1·322	1·520	1·748	2·010
1·7453	100	1·418	1·688	2·010	2·393
2·0944	120	1·520	1·874	2·311	2·850
2·4435	140	1·630	2·081	2·658	3·393
2·7925	160	1·748	2·311	3·056	4·040
3·1416	180	1·874	2·566	3·514	4·811
3·4907	200	2·010	2·850	4·040	5·728
3·8397	220	2·155	3·164	4·646	6·820
4·1888	240	2·311	3·514	5·342	8·121
4·7124	270	2·566	4·111	6·586	10·551
5·2360	300	2·850	4·811	8·121	13·709

For a leather belt on an iron pulley, μ may be taken at ·3 or ·4; but if the pulley and belt are greasy, μ will not exceed ·2.

Slip of Belts.—Professor Lanza found by careful experiments that under ordinary working conditions the speed of slip between bands and pulleys was from 3 to 12 feet per minute, and that the coefficient of friction had then a mean value of ·27.

Driving Force in a Belt connecting Two Pulleys.—

T_1 = tension on tight or driving portion of belt in lbs.
T_2 = tension on slack portion of belt in lbs.
P = driving force in lbs.
$P = T_1 - T_2$.

If P is given, then $T_1 = \frac{Px}{x-1}$, and $T_2 = \frac{P}{x-1}$, where $x = \frac{T_1}{T_2} = e^{\mu\theta}$.

In many cases in practice T_1 may be taken equal to $2T_2$, then $P = \frac{1}{2}T_1$, and $T_1 = 2P$.

Centrifugal Tension in Belts.—The centrifugal force of a belt as it rotates with a pulley causes an additional tension in it. The amount of this centrifugal tension is $\frac{wv^2}{g}$ in lbs. per square inch of belt section, where w = weight of a portion of belt one foot long, and one square inch in section = ·4 for leather, and v = velocity of belt in feet per second.

The total tension on the tight side of the belt is therefore

$$T_1 + \frac{wbtv^2}{g} = T_1 + \frac{btv^2}{80} \text{ for leather belts,}$$

where b = width, and t = thickness of belt in inches.

At a speed of 3000 feet per minute the centrifugal force increases the stress in the belt by about 31 lbs. per square inch. At 6000 feet per minute the increase would be 125 lbs. per square inch with a leather belt.

Power Transmitted by Belts, Centrifugal Tension Neglected.—

V = velocity of belt in feet per minute.
P = driving force in lbs.
H = horse-power transmitted.

$$H = \frac{VP}{33000}.$$

If the belt passes over a pulley of diameter D (in feet) which makes N revolutions per minute, then,

$$H = \frac{\pi DNP}{33000}.$$

Assuming $T_1 = 2P$, and taking the safe working stress on the belt at 300 lbs. per square inch, then $P = 150bt$, where b = width, and t = thickness of belt in inches.

For rough calculations the following rules may be used :—

$$H = \frac{B\,V}{800} \text{ for single leather belts.}$$

$$H = \frac{B\,V}{450} \text{ for double leather belts.}$$

Power Transmitted by Belts, allowing for Centrifugal Tension.—

b = width of belt in inches.
t = thickness of belt in inches.
w = weight of belt per foot of length and one square inch section, in lbs.
v = velocity of belt in feet per second.
V = velocity of belt in feet per minute.
F = centrifugal tension on belt, in lbs.
T_1 = tension on driving side of belt, exclusive of centrifugal tension, in lbs.
T_2 = tension on slack side of belt, exclusive of centrifugal tension, in lbs.
T = total tension on driving side of belt, in lbs.
P = driving force, or driving tension, in lbs.
H = horse-power transmitted by belt.

$$P = T_1 - T_2. \qquad T_1 = T - F.$$

$$F = \frac{wbtv^2}{32{\cdot}2}. \qquad H = \frac{Pv}{550} = \frac{PV}{33000}.$$

For leather belts with laced joints, $T = 300bt$, $w = {\cdot}4$, and $T_1 = 2T_2$. Using these values, the following formulæ are obtained :—

$$P = \frac{T_1}{2} = \frac{T - F}{2} = \frac{(24150 - v^2)bt}{161}.$$

$$H = \frac{Pv}{550} = \frac{(24150 - v^2)btv}{88550}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

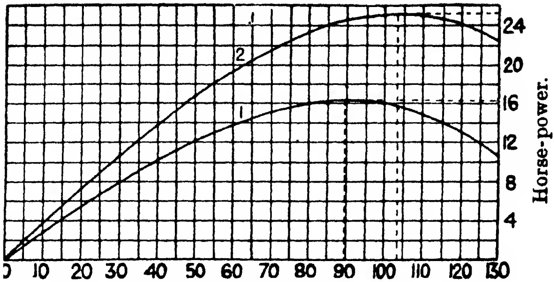
The horse-power calculated by the above formula will be a maximum when the velocity of the belt is 89·7 feet per second, or 5382 feet per minute.

For leather belts with riveted joints T may be taken at 400 lbs. per square inch, and the formula for the horse-power then becomes

$$H = \frac{(32200 - v^2)btv}{88550}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which will become a maximum when the velocity is 103·6 feet per second, or 6216 feet per minute.

The diagram at the top of the next page shows the horse-power of leather belts per square inch of section for various speeds of belt, (1) when T = 300 lbs. per square inch, (2) when T = 400 lbs. per square inch.



Velocity in feet per second.

The following table has been calculated by the formula (1) on the preceding page :—

Horse-power of Leather Belts per Inch of Width.

t=Thick- of Belt in Inches	v=Velocity of Belt in Feet per Second.						
	30	40	50	60	70	80	90
	V=Velocity of Belt in Feet per Minute.						
	1800	2400	3000	3600	4200	4800	5400
$\frac{3}{16}$	1·48	1·91	2·29	2·61	2·85	3·01	3·06
$\frac{7}{32}$	1·72	2·23	2·67	3·05	3·33	3·51	3·57
$\frac{1}{4}$	1·97	2·55	3·06	3·48	3·80	4·01	4·08
$\frac{9}{32}$	2·22	2·86	3·44	3·92	4·28	4·51	4·59
$\frac{5}{16}$	2·46	3·18	3·82	4·35	4·76	5·01	5·10
$\frac{3}{8}$	2·95	3·82	4·58	5·22	5·71	6·01	6·12
$\frac{7}{16}$	3·45	4·46	5·35	6·09	6·66	7·02	7·14
$\frac{1}{2}$	3·94	5·09	6·11	6·96	7·61	8·02	8·16

Thickness and Width of Belts of One Square Inch Section.—

t=thickness in inches. b=width in inches.

$t=\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$
b=5·33	4·57	4·00	3·56	3·20	2·67	2·29	2·00

Rims of Pulleys.—The rim of a pulley for a belt which has to be shifted to different parts in the width of the rim should be straight on the outside of its cross section. Wrought-iron rims are nearly always straight.

When the rims are curved, the radius of curvature may be from 3 to 5 times the breadth of the rim. Various authorities give the rise at the centre of a curved rim at from one-tenth to one-ninety-sixth of the breadth of the rim.

In the Van den Kerkove form of rim section a portion at the centre of the width is straight and parallel to the axis of the pulley, and the parts on each side of this slope slightly. In one example of this form the rim of a belt fly-wheel, $18\frac{1}{2}$ inches broad, was straight in the middle for about one-fifth of the width, and the outside portions sloped straight down to ·18 inch below the crown. The wheel was about 16 feet $4\frac{1}{2}$ inches in diameter. The pulley driven from the wheel was about 4 feet 11 inches in diameter, and the slope on the outer portions of the section of the rim was about half that on the wheel.*

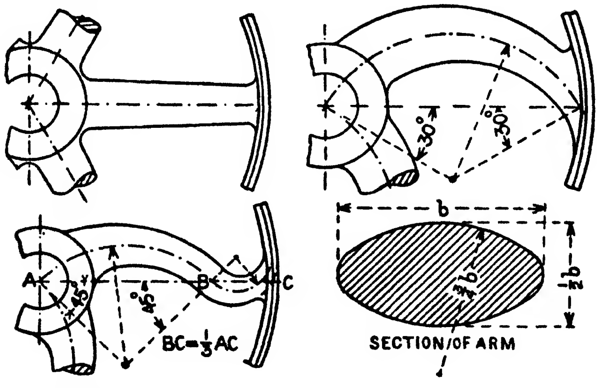
For cast-iron rims the thickness at the edge may be,

$$\frac{D}{200}+\frac{1}{8}\text{ inch for single belts,}$$

$$\text{and }\frac{D}{200}+\frac{1}{4}\text{ inch for double belts,}$$

where D is the diameter of the pulley in inches

Arms of Cast-iron Pulleys.—



R = radius of pulley in inches.
b = breadth of arm (measured at centre of pulley) in inches.
m = number of arms in pulley.
f = working stress in lbs. per square inch.
P = driving force in lbs. at the circumference of the pulley, that is, the difference between the tensions in the tight and slack sides of the belt.

* *Proceedings of the Institution of Mechanical Engineers*, 1895, p. 648.

The bending moment on each arm is approximately $\frac{PR}{m}$.

The moment of resistance of the oval-shaped arm to bending is approximately $\cdot 05b^3f$,

$$\text{therefore } \frac{PR}{m} = \cdot 05b^3f, \text{ and } b = \sqrt[3]{\frac{20PR}{mf}}$$

$$\text{taking } f = 2000,\ b = \sqrt[3]{\frac{PR}{100m}}.$$

The breadth and thickness of the arms at the rim may be two-thirds of the breadth and thickness respectively at the centre.

If P cannot be conveniently determined, it may, for the purpose of designing the arms of the pulley, be taken equal to $50B$ for single belts and $100B$ for double belts, where B is the width of the belt in inches; then

$$b = \sqrt[3]{\frac{BR}{2m}} \text{ for single belts,}$$

$$\text{and } b = \sqrt[3]{\frac{BR}{m}} \text{ for double belts.}$$

Bosses of Pulleys.—

D = diameter of pulley in inches.
B = breadth of rim in inches.
d = diameter of shaft in inches.
t = thickness of boss in inches.

$$t = \cdot 14\sqrt[3]{BD} + \tfrac{1}{4} \text{ inch for single belts.}$$
$$t = \cdot 18\sqrt[3]{BD} + \tfrac{1}{4} \text{ inch for double belts}$$

The above rules are given by Unwin.
Box gives the following rule in his "Mill Gearing":—

$$t = \frac{D}{96} + \frac{d}{8} + \tfrac{5}{8} \text{ inch.}$$

Length of boss $= \tfrac{2}{3}B$ to B.

The above rules apply to pulleys keyed to their shafts.

For loose pulleys the bosses should be longer, and they need not be so thick. It is a good plan to line the boss of a loose pulley with a brass bush.

Centrifugal Tension in Wheel and Pulley Rims.—As the rim rotates each portion tends to fly outwards, and produces a tension in the rim, and if the speed is high enough the rim will burst.

v = velocity of rim in feet per second.
w = weight of one cubic inch of material of rim, in lbs.
g = accelerating effect of gravity = 32·2 feet per sec. per sec.
f = stress in rim due to centrifugal force, in lbs. per sq inch.

$$f=\frac{12wv^2}{g}.$$

The stress f is independent of the area of the section of the rim.

A rim speed of 100 feet per second, or 6000 feet per minute, produces a centrifugal tension in the rim = 970 lbs. per square inch for cast-iron, 1040 lbs. per square inch for wrought-iron, and 1060 lbs. per square inch for steel.

For toothed wheels the centrifugal tension is greater than that given by the above formula, because the teeth add to the centrifugal force without increasing the area of the section of the rim.

ROPE GEARING.

Ropes.—The material used for the ropes in the rope gearing of mills is either hemp or cotton. Formerly hemp was much preferred, but cotton is now the material most used. The ropes are white or untarred, and are usually "hawser laid," that is, they consist of three strands twisted together. Four-strand ropes are also often used.

The ropes most commonly used are from $1\frac{1}{2}$ inches to 2 inches in diameter. Of late years ropes of $1\frac{5}{8}$ inches or $1\frac{3}{4}$ inches in diameter have been largely used.

The weight of the ropes when dry is given approximately by the formula $W = \cdot 3D^2$, where W is the weight per foot of length in lbs., and D the diameter in inches.

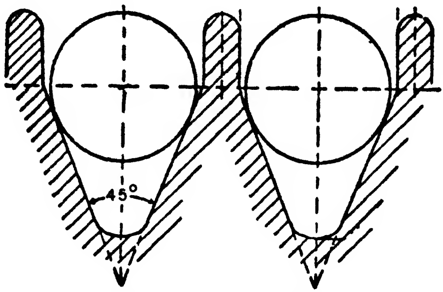
The breaking strength of the ropes is from 7000 to 12,000 lbs. per square inch, but the working load is seldom greater than one-thirty-sixth of the breaking load, and it is often as small as one-sixtieth. A small working load on a rope ensures great durability. A good rope should run well for ten years.

The splice should be made with great care, and should have a length not less than sixty times the diameter of the rope.

The speed of the ropes varies in practice from 3000 to 6000 feet per minute. The advantage of running at a higher speed than 5000 feet per minute is very doubtful.

Cotton rope provides a constant speed drive; no allowance need be made for slip, and heavy loads can be transmitted with little or no tension in the slack side of the drive.

Pulleys.—The pulleys are made of cast-iron, and the rims are grooved, as shown below. The angle of the grooves is generally 45°.



The following proportions are for grooves having an angle of 45°:—

D = diameter of rope in inches.	Width $= 1\frac{1}{12}D$.
Pitch $= 1\frac{3}{8}D$.	Mid-feather $= \frac{1}{4}D$.
Depth $= 1\frac{1}{2}D$.	Bottom radius $= \frac{1}{4}D$.

The diameter of a rope pulley, measured to the centre of the rope, should not be less than that given by the following rule:—

$$\text{Minimum diameter of pulley} = (10D + 16)D,$$

where D is the diameter of the rope.

The line joining the centres of two pulleys connected by ropes should not be inclined at more than 45° to the horizontal.

Whenever possible the driving side of the rope is placed on the bottom and the slack side on the top, in order to increase the arc of contact between the rope and the pulleys.

Friction of Rope on Pulley.—If μ = coefficient of friction between the rope and a cylindrical pulley, and μ_1 = coefficient of friction between the rope and a wedge-shaped groove, the angle of the groove being ϕ, then $\mu_1 = \mu$ cosecant $\frac{\phi}{2}$.

For hemp or cotton ropes on iron pulleys, μ varies from ·15 to ·35, and, therefore, for grooves with the usual angle, 45°, μ_1 varies from ·4 to ·9. The value μ_1 = ·4 is for greasy pulleys. For ropes and pulleys in ordinary working condition μ_1 may be taken at ·7, the angle of the grooves being 45°.

T_1 = tension on driving side of rope.
T_2 = tension on slack side of rope.
θ = angle of contact between rope and pulley in circular measure.
n = number of degrees in angle of contact.
μ_1 = coefficient of friction between rope and pulley.

Then as for belt gearing (see p. 477),

$$\log \frac{T_1}{T_2} = \cdot 4343 \mu_1 \theta = \cdot 00758 \mu_1 n.$$

Angle of Contact in Degrees. n	Ratio of Tensions $\frac{T_1}{T_2}$ when the Rope is on the Point of Slipping.					
	$\mu_1 = \cdot 4$	$\mu_1 = \cdot 5$	$\mu_1 = \cdot 6$	$\mu_1 = \cdot 7$	$\mu_1 = \cdot 8$	$\mu_1 = \cdot 9$
60	1·52	1·69	1·87	2·08	2·31	2·57
90	1·87	2·19	2·57	3·00	3·51	4·11
120	2·31	2·85	3·51	4 33	5·31	6·59
150	2·85	3·70	4·81	6·25	8·12	10·55
180	3·51	4·81	6·59	9 02	12·35	16·90
210	4·33	6·25	9·02	13·01	18·77	27·08

Centrifugal Tension in Ropes.—

W = weight of rope per foot of length in lbs.
v = velocity of rope in feet per second.
F = centrifugal tension in lbs.

$$F = \frac{Wv^2}{g} = \frac{Wv^2}{32 \cdot 2}.$$

Power Transmitted by Ropes.—

D = diameter of rope in inches.
W = weight of rope per foot of length in lbs.
v = velocity of rope in feet per second.
V = velocity of rope in feet per minute.
F = centrifugal tension on rope in lbs.
T_1 = tension on driving side of rope, exclusive of centrifugal tension, in lbs.
T_2 = tension on slack side of rope, exclusive of centrifugal tension, in lbs.
T = total tension on driving side of rope in lbs.
t = total tension on slack side of rope in lbs.
P = driving force, or driving tension, in lbs.
H = horse-power transmitted by one rope.

$$T = T_1 + F. \qquad t = T_2 + F. \qquad P = T_1 - T_2.$$

$$F = \frac{Wv^2}{32 \cdot 2}. \qquad H = \frac{Pv}{550} = \frac{PV}{33000}.$$

From numerous examples in actual practice, it may be taken that $T=195D^2$, $T_1=1{\cdot}3P$, $T_2={\cdot}3P$, and $W={\cdot}3D^2$. Substituting these values in the above formulæ, the following practical rules are obtained:—

$$P=\frac{T-F}{1{\cdot}3}=\frac{(62790-3v^2)D^2}{418{\cdot}6}.$$

$$H=\frac{Pv}{550}=\frac{(62790-3v^2)D^2v}{230230}.$$

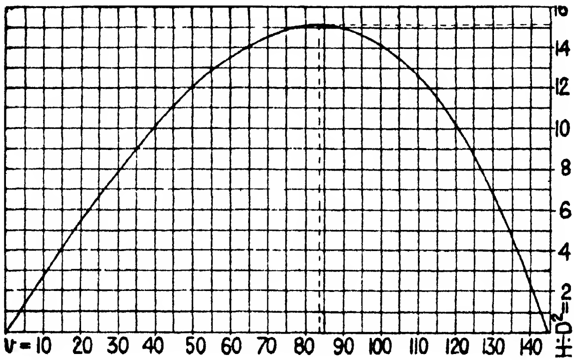
The horse-power will be a maximum when the speed of the rope is 83·5 feet per second, or 5010 feet per minute. Above that speed the power transmitted diminishes, until at a speed of 8680 feet per minute it is zero.

The following table has been calculated by the above rules:—

D	W	T	$v=50$ $V=3000$			$v=60$ $V=3600$			$v=70$ $V=4200$		
			F	P	H	F	P	H	F	P	H
1	·30	195	23	132	12·0	34	124	13·5	46	115	14·6
$1\frac{1}{8}$	·38	247	29	167	15·2	42	157	17·1	58	145	18·5
$1\frac{1}{4}$	·47	305	36	206	18·8	52	194	21·2	71	180	22·8
$1\frac{3}{8}$	·57	369	44	250	22·7	63	235	25·6	86	217	27·6
$1\frac{1}{2}$	·68	439	52	297	27·0	75	279	30·5	103	258	32·9
$1\frac{5}{8}$	·79	515	61	349	31·7	89	328	35·8	121	303	38·6
$1\frac{3}{4}$	·92	597	71	404	36·8	103	380	41·5	140	352	44·8
$1\frac{7}{8}$	1·05	686	82	464	42·2	118	437	47·6	160	404	51·4
2	1·20	780	93	528	48·0	134	497	54·2	183	459	58·5

D	W	T	$v=80$ $V=4800$			$v=90$ $V=5400$			$v=100$ $V=6000$		
			F	P	H	F	P	H	F	P	H
1	·30	195	60	104	15·1	75	92	15·0	93	78	14·2
$1\frac{1}{8}$	·38	247	75	132	19·2	96	116	19·0	118	99	18·0
$1\frac{1}{4}$	·47	305	93	163	23·7	118	144	23·5	146	122	22·3
$1\frac{3}{8}$	·57	369	113	197	28·6	143	174	28·4	176	148	26·9
$1\frac{1}{2}$	·68	439	134	234	34·1	170	207	33·8	210	176	32·0
$1\frac{5}{8}$	·79	515	157	275	40·0	199	243	39·7	246	207	37·6
$1\frac{3}{4}$	·92	597	183	319	46·4	231	282	46·1	285	240	43·6
$1\frac{7}{8}$	1·05	686	210	366	53·2	265	323	52·9	328	275	50·1
2	1·20	780	239	417	60·6	302	368	60·2	373	313	56·9

Best Speed for Ropes.—Values of $H \div D^2$ (or the horse-power of a rope one inch in diameter), for various values of v, are plotted in the diagram below. This diagram shows that the



horse-power increases very slowly as the speed for maximum horse-power (83·5 feet per second) is approached. For example, as the speed increases from 75 to 83·5 feet per second, the horse-power is only increased about $1\frac{1}{2}$ per cent., and this is without taking into account the additional loss of power due to the increased friction of the ropes in passing through the air. It would seem, therefore, that there is little, if any, advantage in running the ropes at more than 75 feet per second, or 4500 feet per minute. Many engineers are of the opinion that the most efficient speed is 4800 feet per minute.

Deflection or Sag of Ropes.—The deflection of the rope corresponding to a given tension is found by the formulæ on p. 490.

TELODYNAMIC TRANSMISSION.

This is the name given to the system of transmitting power to long distances by means of pulleys and wire ropes.

The ropes are made of iron or steel wires. A rope has from six to ten strands twisted on a central hempen core. Each strand has usually a hempen core surrounded by from six to twelve wires. The cores are saturated with tar or boiled oil free from acid.

The wires have a diameter varying from ·02 to ·088 inch, and the diameter of the ropes varies from ·36 to 1·28 inch.

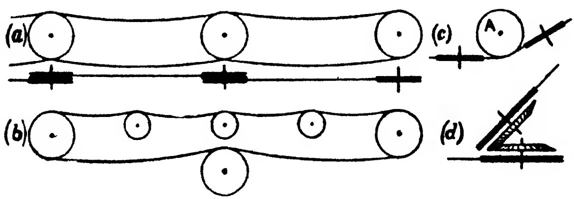
If D = diameter of rope, d = diameter of wires, and n = number of wires in rope, then from a number of examples it has been found that $\frac{D}{d}$ varies from $\frac{n}{6}+3$ to $\frac{n}{7}+2$.

The weight of the rope, in lbs. per foot of length, is about $3{\cdot}3d^2n$.

The speed of the rope varies from 3000 to 6000 feet per minute, the average being about 4000.

The pulleys are of large diameter (6 to 18 feet), and each carries one rope, unless it is a relay pulley, when it carries two.

The distance between the pulleys varies from 80 to 500 feet. When it is required to transmit power to a greater distance than 500 feet (the distance may amount to several miles) it is done either by relays, as shown at (a), or by introducing guide pulleys, as shown at (b), so that the rope is supported at intervals not exceeding 500 feet. The guide pulleys under the tight or driving side have a diameter equal to that of the driving pulley, but those under the slack side may be of half that diameter.



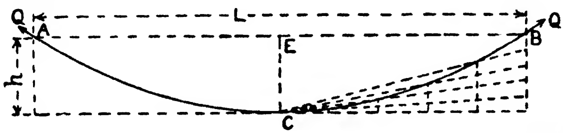
Each pulley is fixed on a shaft, which is supported on bearings at each end. These bearings are carried by masonry, iron, or wood-framed piers, which are sufficiently high to prevent the rope touching the ground.

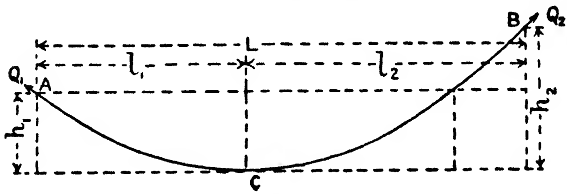
The direction of the rope may be changed by means of a horizontal guide pulley A, as shown at (c), but bevel wheels, as shown at (d), are generally used.

If T_1 = tension on tight side, T_2 = tension on slack side, P = driving force, V = velocity of rope in feet per minute, and H = horse-power transmitted, then $H = \frac{PV}{33000}$.

In practice $T_1 = 2T_2$, and therefore $P = T_1 - T_2 = T_2$.

Tensions in a Suspended Flexible Rope.—The curve in which a flexible rope ACB hangs when suspended from points A and B is a catenary, but for questions on rope gearing it is sufficiently accurate to substitute for this curve a parabola. If the points A and B are at the same level (horizontal transmission), the vertex C of the parabola is vertically under the middle point E of AB. The construction for drawing the parabola is given on p. 156





If the points A and B are at different levels (inclined transmission), the position of the vertex C is given by the following equations :—

$$l_1 = \frac{L\sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}, \text{ and } l_2 = \frac{L\sqrt{h_2}}{\sqrt{h_1} + \sqrt{h_2}}.$$

The tension in the rope due to its weight varies from a minimum at its lowest point to a maximum at its highest point. Let w = weight of rope in lbs. per foot of length, and let all distances be measured in feet.

T = tension, in lbs., at lowest point C.

$$T = \frac{wL^2}{8h} \text{ for horizontal transmission.}$$

$$T = \frac{wl_1^2}{2h_1} = \frac{wl_2^2}{2h_2} \text{ for inclined transmission.}$$

When a rope or chain hangs in a catenary curve, the difference

between the tensions at any two points in the curve is equal to the weight of a portion of the rope or chain whose length is equal to the difference between the levels of the two points. Hence, neglecting the difference between the catenary and the parabola,

$$Q - T = wh. \qquad Q = T + wh = \frac{wL^2}{8h} + wh.$$

$$Q_1 - T = wh_1. \qquad Q_1 = T + wh_1 = \frac{wl_1^2}{2h_1} + wh_1.$$

$$Q_2 - T = wh_2. \qquad Q_2 = T + wh_2 = \frac{wl_2^2}{2h_2} + wh_2.$$

Also, $Q_2 - Q_1 = w(h_2 - h_1)$.

$$h = \frac{Q}{2w} - \sqrt{\frac{Q^2}{4w^2} - \frac{L^2}{8}}.$$

$$h_1 = \frac{Q_1}{2w} - \sqrt{\frac{Q_1^2}{4w^2} - \frac{l_1^2}{2}}. \qquad h_2 = \frac{Q_2}{2w} - \sqrt{\frac{Q_2^2}{4w^2} - \frac{l_2^2}{2}}.$$

Stresses in a Wire Rope when Transmitting Power.—

f_w = stress, in lbs. per square inch, due to weight of rope per foot of length, the distance between the pulleys and the depth of the curve in which the rope hangs.

f_b = stress, in lbs. per square inch, due to the bending of the rope on the pulley rim.

f_c = stress, in lbs. per square inch, due to centrifugal force.

$$f_w = \frac{4Q}{\pi d^2 n} = \frac{w(L^2 + 8h^2)}{2\pi d^2 nh} \text{ for horizontal transmission,}$$

$$= \frac{4Q_2}{\pi d^2 n} = \frac{2w(l_2^2 + 2h_2^2)}{\pi d^2 n h_2} \text{ for inclined transmission.}$$

If $w = 3{\cdot}3d^2n$, then

$$f_w = \frac{{\cdot}525(L^2 + 8h^2)}{h} \text{ for horizontal transmission,}$$

$$= \frac{2{\cdot}1(l_2^2 + 2h_2^2)}{h_2} \text{ for inclined transmission.}$$

$f_b = \dfrac{Ed}{2R}$, where R is the radius of the pulley in inches, and E the modulus of elasticity = 29,000,000 for wrought-iron wire, and 30,000,000 for steel wire.

$f_c = \dfrac{4wv^2}{\pi d^2 ng} = {\cdot}13v^2$, where v is the velocity of the rope in feet per second.

$f_w + f_b + f_c$ should not exceed 25,600 lbs. per square inch.

Horse-power Transmitted by a Wire Rope.—To find the horse-power which a wire rope will transmit, first determine f_b and f_c by the formulæ in the preceding article, then

$$f_w = 25600 - (f_b + f_c), \quad \text{and horse-power} = H = \frac{\pi d^2 n f_w V}{264000},$$

where d=diameter of wires, n=number of wires in rope, and V=speed of rope in feet per minute.

For rough calculations the following formula may be used:—

$$H = \frac{d^2 n V}{7}.$$

Efficiency of Telodynamic Transmission.—According to the results of experiments by Ziegler, the efficiency of one relay is ·962, and if there are altogether m stations, the efficiency of the whole=e= $\sqrt{\cdot962^m}$.

m=	2	3	4	5	6	7	8
e=	·962	·944	·925	·908	89	·873	·856

Horse-power Transmitted for each Wire of a Wire Rope.—

Diameter of Wire. I.S.W.G.	Diameter of Wire. Inch.	Suitable Diameter of Pulley. Ft.	In.	Speed of Rope in Feet per Minute. 3000	3600	4200	4800	5400	6000
				Horse-power Transmitted for each Wire.					
25	·020	4	0	·182	·216	·249	·280	·309	·337
24	·022	4	3	·220	·262	·301	·339	·374	·408
23	·024	4	9	·262	·311	·358	·403	·446	·485
22	·028	5	6	·357	·424	·488	·549	·607	·660
21	·032	6	3	·467	·554	·638	·717	·793	·863
20	·036	7	3	·591	·701	·807	·908	1·00	1·09
19	·040	8	0	·730	·866	·997	1·12	1·23	1·34
18	·048	9	6	1·05	1·24	1·43	1·61	1·78	1·94
17	·056	11	3	1·43	1·69	1·95	2·19	2·42	2·64
16	·064	12	9	1·86	2·21	2·55	2·87	3·17	3·45
15	·072	14	3	2·36	2·80	3·23	3·63	4·01	4·36
14	·080	16	0	2·92	3·46	3·98	4·48	4·95	5·39
13	·092	18	3	3·86	4·58	5·27	5·93	6·55	7·13
Stress Due to Weight and Curve of Rope.									
Tight side . . .				12775	12632	12463	12268	12047	11800
Slack side . . .				6387	6316	6231	6134	6023	5900

The horse-power transmitted by the whole rope is obtained by multiplying the horse-power transmitted by one wire by the total number of wires in the rope.

The foregoing table has been calculated as follows :—

Stress due to bending $= f_b = E \times \frac{d}{2R} = 30000000 \times \frac{1}{2400} = 12500.$

Stress due to centrifugal force $= f_c = \cdot 13v^2.$

Total stress $= 25600.$

Stress on tight side due to weight of wire and the curve in which it hangs $= f_u = 25600 - 12500 - \cdot 13v^2 = 13100 - \cdot 13v^2.$

Load on tight side due to weight and curve of wire $= T_1 = \frac{\pi}{4} d^2 f_w.$

Load on slack side due to weight and curve of wire $= T_2 = \frac{1}{2} T_1.$

Driving force $= P = T_1 - T_2 = \frac{\pi}{8} d^2 f_w.$

Horse-power $= \frac{PV}{33000} = \frac{d^2 f_u V}{84000} = \frac{d^2 f_u v}{1400}.$

Deflection of Ropes in Telodynamic Transmission.—The formula, given on p. 490, for the deflection of the rope is—

$$h = \frac{Q}{2w} - \sqrt{\frac{Q^2}{4w^2} - \frac{L^2}{8}},$$

$$\text{but } Q = \frac{\pi}{4} d^2 n f_u, \text{ and } w = 3 \cdot 3 d^2 n,$$

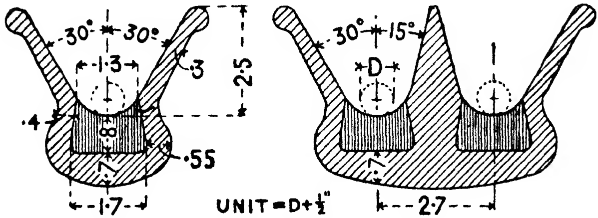
$$\text{hence } h = \frac{f_w}{8 \cdot 4} - \sqrt{\left(\frac{f_w}{8 \cdot 4}\right)^2 - \frac{L^2}{8}}.$$

$$h \text{ is nearly equal to } \frac{\cdot 5252 L^2}{f_w}.$$

The following table gives values of h for the tight and slack sides of the rope for the stresses corresponding to speeds of 3000 and 6000 feet per minute given in the table on the preceding page :—

Stress. (f_w)	Span in Feet (L)						
	100	150	200	250	300	350	400
	Deflection in Feet (h).						
{ 12775	·41	·93	1·64	2·57	3·70	5·04	6·58
{ 6387	·82	1·85	3·30	5·16	7·44	10·14	13·27
{ 11800	·45	1·00	1·78	2·78	4·02	5·47	7·14
{ 5900	·89	2·00	3·57	5·58	8·06	10·98	14·38

Pulleys for Telodynamic Transmission.—These are usually from 6 to 18 feet in diameter, or from 2000 to 2800 times the diameter of the wires forming the rope. They may be made entirely of cast-iron, or they may have wrought-iron arms and cast-iron rims and bosses. The form of the rim of a pulley carrying one rope is shown at (*a*), and the form of the rim of a pulley for two



(*a*) (*b*)

ropes at a relay station is shown at (*b*). The dovetailed recess at the bottom of the groove is generally filled with leather, but wood, guttapercha, tarred oakum, and tarred jute yarn are also used. When leather is used for filling the recess, it may be cut into pieces from the hide or from scrap by means of a die to the shape of the recess, and then placed in on edge.

Weight of Pulleys for Telodynamic Transmission.—According to M. Achard,* the weights of the most ordinary sizes of pulleys employed, including their shafts, are on an average as given in the adjoining table.

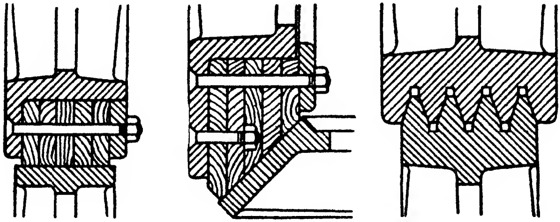
Diameter.		Single-Groove Pulley.	Double-Groove Pulley.
Ft.	In.	Lbs.	Lbs.
7	0	798	1164
12	4	2425	4078
14	9	5180	6988
18	0	6232	8267

FRICTION GEARING.

The driving power of friction wheels is increased if one of a pair is faced with wood, as shown at (*a*) for cylindrical, and at (*b*) for bevel wheels. The grain of the wood should lie in a tangential direction to the working surface. The different layers of wood are nailed together and further secured by glue or white lead. Maple is the best wood for this purpose. Instead of wood, leather or paper may be used, put on in layers, the edges of the layers forming the working surface. The wheel which

* *Proceedings of the Institution of Mechanical Engineers*, 1880.

is faced with wood, leather, or paper should be the driver, the follower being made of cast-iron.

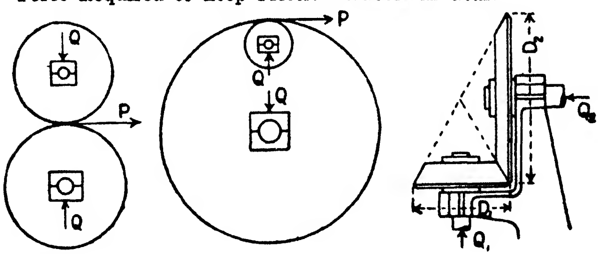


(a) (b) (c)

In Robertson's friction gearing the wheels are made of cast-iron, and they are grooved, as shown at (c). The angle between the sides of the grooves is usually 40°. When first introduced, the inventor recommended a pitch of $\frac{1}{8}$ inch to $\frac{3}{4}$ inch, but a much larger pitch has been used in some cases.

Friction gearing is most efficient when the force to be transmitted is comparatively small and the speed high.

Force Required to keep Friction Wheels in Gear.—



P = tangential driving force.
μ = coefficient of friction between the surfaces in contact.
= $\frac{1}{6}$ for metal on metal, and $\frac{1}{4}$ for wood on metal.
Q = force pressing wheels together.

For cylindrical wheels, $Q=\frac{P}{\mu}$, and $P=\mu Q$.

For Robertson's friction gearing, $Q=3P$ when the angle of the grooves is 40°.

For bevel wheels,

$$Q_1=\frac{PD_1}{\mu\sqrt{D_1^2+D_2^2}},\text{ and } Q_2=\frac{PD_2}{\mu\sqrt{D_1^2+D_2^2}}$$

Power Transmitted by Friction Gearing.—

V = velocity of surfaces in contact, in feet per minute.
P = tangential driving force, in lbs.
H = horse-power transmitted.

$$H=\frac{PV}{33000}.$$

When the driver is faced with soft maple, P may be taken at about 30 lbs. per inch width of face, and at about 15 to 20 lbs. per inch width of face when the driver is faced with basswood or pine.

TOOTHED GEARING.

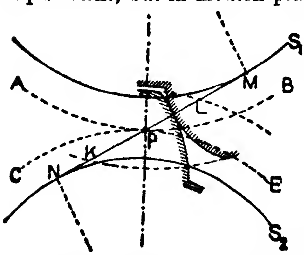
(Revised by David Brown & Sons (Hudd.), Ltd., Huddersfield.)

Gears are used to transmit motion positively from one shaft to another.

Types.—(1) Spur or helical gears for parallel shafts.
(2) Straight bevel or spiral bevel gears for shafts whose axes intersect.
(3) Spiral gears, hypoid gears, or worm gears for shafts whose axes are not parallel and do not intersect.

Gears for Parallel Shafts.

Shape of Tooth.—The essential requirement is that the ratio of angular velocities of the gears is the same in all phases of engagement. This is satisfied if the common normal to the tooth surfaces at any point of contact passes through a fixed point. There are several curves which meet the fundamental requirement, but in modern practice the involute tooth form is almost universal.



Let *AB* and *CE* be the pitch circles, and *P* the pitch point. Through *P* draw the straight line *MPN*, and draw circles S_1 and S_2 concentric with the pitch circles to touch *MPN*. Teeth whose curves are involutes of the circles S_1 and S_2 will work correctly together. The line *LK*, part of *MN*, is the path of contact, that is, the line which contains the point where the two involutes touch one another.

If the centres of the gears be pushed further apart or closer together, the wheels will still work correctly together. This is a special property of involute teeth, and is a valuable one in cases where the distance between the axes of two wheels cannot be kept constant.

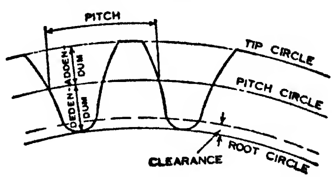
The involute is the curve traced by the end of a taut string unwound from a circular disc which represents the *base circle*, S_1 or S_2. The curvature at any point on an involute gear tooth depends on the distance of the point from the base circle. A rack is a part of a gear of infinite diameter; the teeth of an involute rack have infinite radius of curvature, *i.e.* they are straight-sided. The angle between the side of the tooth and a line perpendicular to the length of the rack is called the *pressure angle* (ψ).

Such rack teeth describe involute gear teeth on the plane of any pitch circle of radius r rolled along a straight line parallel to the length of the rack. The radius of the base circle is $r \cos \psi$.

Should any part of the rack tooth come within the base circle, part of the gear tooth profile is ineffective. Should any part of the rack tooth come within a distance $r \cos^2 \psi$ of the centre of the circle, the gear tooth is *undercut*. The rolling circle is the *pitch circle*.

The process of cutting gear teeth by *rolling* the blank with a cutter in the form of a rack (or other gear) or its equivalent is called *generation*. Nearly all modern gear-cutting machines operate on the generating principle, and with them it is unnecessary, so far as gear production is concerned, to draw the desired tooth shape. It may be done, however, by rolling a sector of the required pitch radius along a straight edge to which is fixed a template in the shape of the rack tooth and tracing the form of the rack tooth (in numerous positions) on a card attached to the sector. A flexible metal strip attached to sector and straight edge ensures rolling without slip.

Proportions of Teeth.—The *nominal* or *circular pitch* (p) of a gear is the distance between similar flanks of adjacent teeth measured along the *pitch circle*.



The *addendum* is the radial length of tooth lying outside the pitch circle, the *dedendum* is the radial length lying inside the pitch circle.

For spur and helical gears the addendum (a) and dedendum (b), corrected to prevent undercutting, are given by

$$a_p = \frac{p}{\pi}\left[1 + 0{\cdot}4\left(1 - \frac{t}{T}\right)\right], \qquad a_w = \frac{p}{\pi}\left[1 - 0{\cdot}4\left(1 - \frac{t}{T}\right)\right];$$

$$b_p = 0{\cdot}716p - a_p, \qquad b_w = 0{\cdot}716p - a_w,$$

where the suffix p refers to the *pinion* (the gear with the smaller number of teeth) and w to the mating gear or *wheel.*

t = number of teeth in pinion.

T = number of teeth in wheel.

The old Brown and Sharpe standards are given by

$$a_p = a_w = \frac{p}{\pi} \quad \text{and} \quad b_p = b_w = \frac{1{\cdot}157p}{\pi}.$$

The British Standard pressure angle is 20°, although an older standard (14½°) is frequently employed because of existing cutter stocks.

The *diametral pitch* (P) is the number of teeth divided by the pitch circle diameter (D) in inches.

Since $tp = \pi D$ and $P = t/D$, therefore $P = \pi/p$.

Integral values of P up to 8 are standardised, together with 1¼, 1½, 1¾, 2¼, 2½, 2¾, 3½, 10, 12, 14, 16, 20, 24.

The *module* is the pitch circle diameter, in inches or millimetres, divided by the number of teeth, and the units intended should be specified.

Relation between Number and Pitch of Teeth and Diameter of Pitch Circle.

D = diameter of pitch circle. p = pitch (circular) of teeth.
t = number of teeth in wheel. $\pi = 3{\cdot}14159265$.

$tp = \pi D.$ $t = \frac{\pi}{p} D.$ $p = \frac{\pi}{t} D.$ $D = \frac{p}{\pi} t.$

The table below will simplify the use of the above formulæ—

p	$\frac{\pi}{p}$	$\frac{p}{\pi}$	p	$\frac{\pi}{p}$	$\frac{p}{\pi}$
1/2	6·2832	·1592	3	1·0472	·9549
9/16	5·5851	·1790	3 1/8	1·0053	·9947
5/8	5·0265	·1989	3 1/4	·9666	1·0345
11/16	4·5696	·2188	3 3/8	·9308	1·0743
3/4	4·1888	·2387	3 1/2	·8976	1·1141
13/16	3·8666	·2586	3 5/8	·8666	1·1539
7/8	3·5904	·2785	3 3/4	·8378	1·1937
15/16	3·3510	·2984	3 7/8	·8107	1·2335
1	3·1416	·3183	4	·7854	1·2732
1 1/16	2·9568	·3382	4 1/8	·7616	1·3130
1 1/8	2·7925	·3581	4 1/4	·7392	1·3528
1 3/16	2·6456	·3780	4 3/8	·7181	1·3926
1 1/4	2·5133	·3979	4 1/2	·6981	1·4324
1 5/16	2·3936	·4178	4 5/8	·6793	1·4722
1 3/8	2·2848	·4377	4 3/4	·6614	1·5120
1 7/16	2·1855	·4576	4 7/8	·6444	1·5518
1 1/2	2·0944	·4775	5	·6283	1·5916
1 9/16	2·0106	·4974	5 1/4	·5984	1·6711
1 5/8	1·9333	·5173	5 1/2	·5712	1·7507
1 11/16	1·8617	·5371	5 3/4	·5464	1·8303
1 3/4	1·7952	·5570	6	·5236	1·9099
1 13/16	1·7333	·5769	6 1/4	·5027	1·9894
1 7/8	1·6755	·5968	6 1/2	·4833	2·0690
1 15/16	1·6215	·6167	6 3/4	·4654	2·1486
2	1·5708	·6366	7	·4488	2·2282
2 1/8	1·4784	·6764	7 1/4	·4333	2·3077
2 1/4	1·3963	·7162	7 1/2	·4189	2·3873
2 3/8	1·3228	·7560	7 3/4	·4054	2·4669
2 1/2	1·2566	·7958	8	·3927	2·5465
2 5/8	1·1968	·8356	8 1/2	·3696	2·7056
2 3/4	1·1424	·8754	9	·3491	2·8648
2 7/8	1·0927	·9151	9 1/2	·3307	3·0239

Helical Teeth.—Smoother action in gears connecting parallel shafts may be secured by setting the teeth at an angle to the axes of the shafts. Such gears are called *single helical gears.* The teeth are in the form of helices which are right-handed on one gear and left-handed on the other. The ratio of the leads of the helices must be the ratio of the numbers of teeth.

Single helical gears whose helix angles are not equal and opposite may be meshed together with shafts making an angle equal to the sum of the helix angles (reckoning right hand as positive and left hand as negative). Such gears are known as *spiral gears,* except in the special case where the shafts are parallel, when they are described as *helical gears.*

A rack which meshes with an involute helical gear has straight-sided teeth. A section of the rack on a plane perpendicular to the axis of the gear shows a rack form whose pressure angle is the *transverse pressure angle* of the gear. A section on a plane perpendicular to the direction of the rack teeth shows a rack form whose pressure angle is the *normal pressure angle* of the gear.

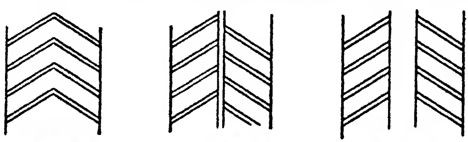
If L = lead of helix,
r = radius of pitch circle,
r_0 = radius of base circle,
σ = helix angle at pitch circle,
ψ_t = transverse pressure angle,
ψ_n = normal pressure angle,

$$\cos \psi_t = r/r_0, \qquad \tan \sigma = \frac{2\pi r}{L},$$

$$\tan \psi_n = \tan \psi_t \cos \sigma.$$

In British Standard practice, $\psi_n = 20°$ and $\sigma = 30°$.

Double Helical Gears (Range of ratio—unity to 12).—By making each of the gears connecting parallel shafts in the form of two helical gears of equal lead and opposite hand, the total end thrust on each shaft is zero. This assumes that one of the shafts is mounted with some axial *float* so that it may adjust



CONTINUOUS STAGGERED IN-LINE

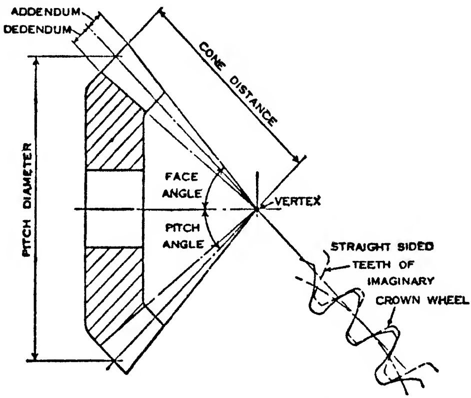
itself to the axial position which gives equal tooth load on each helix.

The teeth of such gears may be continuous, but if not, they may be either *staggered* or *in-line.* The most accurate process (hobbing) for producing the teeth demands a gap between the helices to provide clearance for the hob. Hobbed double helical gears are successfully used at pitch circle speeds up to 15,000 feet per minute.

Triple Helical Teeth.—Occasional use of triple helical teeth is due to imaginary advantages. These teeth cannot be produced with the accuracy practicable in double helical gears.

Gears for Non-parallel Shafts.

Bevel Gears (Range of ratio—unity to 8).—The pitch surfaces of bevel gears are portions of cones having the intersection of the shaft axes as a common vertex. The tooth elements (pitch, addendum, dedendum) diminish as the vertex is approached. The nominal elements are those applying to the outermost section of the tooth.



If corresponding points on different sections of a tooth lie on a straight line passing through the point of intersection of the shaft axes, the gear is a *straight tooth bevel* gear.

If they lie on a line or curve inclined to straight lines passing through the vertex, the gear is a *single helical bevel* or *spiral bevel* gear.

Gears of these types have the advantage of quieter operation. The Gleason process, in which the teeth are approximately circular arcs, is the most widely used method of producing spiral bevel gears.

Double helical bevel gears have been used, but they cannot be produced with any very high degree of accuracy.

A bevel gear of 90° pitch angle is known as a *crown wheel.*

A pair of bevel gears may be regarded as meshing simultaneously with an imaginary crown wheel whose teeth are straight-sided.

Hypoid Gears (Range of ratio—unity to 6).—This is a modification of the spiral bevel gear, the axes of the mating gears being perpendicular but not intersecting. The *offset* or distance between the axes is usually limited to about one-tenth of the diameter of the wheel. Production of hypoid gears demands highly specialised plant and experience.

The hypoid type of gear was primarily developed for automobile rear axles, the chief advantage being that it permits of a lower propeller shaft line than does the normal spiral bevel drive. In industrial service the hypoid gear is also occasionally advantageous in that it permits the two shafts to be extended past each other, a condition which is of course impossible with spiral bevel gears.

Worm Gears (Range of ratio—unity to 100).—This type of gear is used for shafts whose axes do not intersect; in nearly every case they are perpendicular to each other. The worm gear was originally used only for high reduction ratios, but the useful range extends down to unity.

The worm is nearly always the smaller member, and is usually *parallel*, *i.e.* of uniform external diameter, although *hollow-faced* or *hour-glass* worms are still used. The mating gear, the *worm wheel*, generally has teeth curved to match the circular cross-section of the worm.

One form of worm thread used is straight-sided on a section containing the axis of the worm, but the British Standard worm thread is involute in a section perpendicular to the axis of the worm, the normal pressure angle being 20°.

The worm wheel teeth are of complicated form, and they are *generated* by a cutter (or *hob*) which corresponds in dimensions to the mating worm.

The worm is essentially a helical gear with helix angle usually greater than 45°. The complement of the helix angle is called the *lead angle*.

The teeth of a worm are described as *threads*, and the reduction ratio is $\dfrac{\text{Number of teeth in worm wheel}}{\text{Number of threads in worm}}$.

Highest efficiency requires a lead angle in the neighbourhood of 45°, but this is impracticable in high-ratio worm gears because it would require an excessively slender worm.

The amount of relative sliding of the contacting surfaces is comparatively large in worm gears, and it is therefore important to minimise the coefficient of friction between them. The best combination of load capacity and low friction is obtained by making the worm from case-hardened steel and precision-grinding the threads, the worm wheel being of phosphor-bronze. To reduce cost, large worm wheels are often made in the form of a bronze ring fixed to a cast-iron centre.

Velocity Ratio or Reduction Ratio.—

$$\left(\begin{matrix}\text{Reduction ratio } R \text{ of}\\ \text{train of gears}\end{matrix}\right)=\frac{\text{Revolutions of driving gear}}{\text{Corresponding revolutions of driven gear}}.$$

If axes of gears are fixed,

$$R=\frac{\text{Product of numbers of teeth in driven gears}}{\text{Product of numbers of teeth in driving gears}}.$$

In *epicyclic* gear trains not all the axes are fixed. Let gears be A, B, C, etc. Assuming axes to be fixed in position, gear A is imagined to turn through angle a. Let b, c, d, etc., be corresponding angles of rotation of B, C, D, etc.

Now, if in the actual assembly gear P is fixed, relative angular movements are

$$a-p, \quad b-p, \quad c-p, \quad \text{etc.},$$

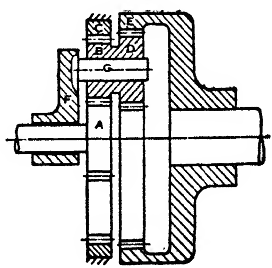
and reduction ratio between any two members, say C and E, is

$$\frac{c-p}{e-p}.$$

Example of Epicyclic Gear Train.—The member F rotates and carries a spindle G on which the double gear BD rotates freely. The internal gear C is fixed. Rotation of A causes rotation of F and of all the other gears except C.

To find the reduction ratios of the train, assume F to be fixed and C free. Then if A rotates through an angle a, and the letters A, B, C, D, E denote the numbers of teeth in the respective gears, the angular movements are those shown in the second column of the table below.

A	a	$a\left(1+\frac{A}{C}\right)$
BD	$-\frac{A}{B}a$	$a\left(\frac{A}{C}-\frac{A}{B}\right)$
C	$-\frac{A}{C}a$	0
E	$-\frac{A}{B}\frac{D}{E}a$	$a\left(\frac{A}{C}-\frac{A}{B}\frac{D}{E}\right)$
F	0	$a\frac{A}{C}$



Since C is stationary in the actual assembly, $-\frac{A}{C}a$ is subtracted from each of the quantities in the second column, giving those in the third column. Hence the reduction ratio between (say) A and E is $\left(1+\frac{A}{C}\right)\Big/\left(\frac{A}{C}-\frac{A}{B}\frac{D}{E}\right)$.

Load Capacity of Gears.

The safe load for a pair of gears is limited either by surface pressure on the teeth or by bending stress near the roots of the teeth. For a given effective driving force per unit face, the surface pressure is determined largely by the diameter of the gear, and is almost independent of pitch. The bending stress depends largely on pitch, and is only slightly affected by changes in diameter.

Permissible stress depends on frequency of application (*i.e.* rotational speed) and length of life expected. In worm gears the permissible stress is also controlled by rubbing speed.

Spur and Helical Gears.—Permissible load (lbs.) per inch width of face on basis of surface pressure $= VMA\frac{d^{0\cdot 8}}{1+r}$, . (1)

where V = speed factor (Table 1).
M = material factor (Table 2).
A = type factor (Table 3).
d = pitch diameter (in.) of pinion.
$r = \dfrac{\text{pitch diameter of pinion}}{\text{pitch diameter of wheel}}$.

Formula (1) must be worked out for both pinion and wheel, with appropriate values of V and M.

Permissible load (lbs.) per inch width of face on basis of bending stress $= VNBFm$, ... (2)

where V = speed factor (Table 1).
N = material factor (Table 2).
B = type factor (Table 4).
F = strength factor (Table 5).
$m = \dfrac{\text{pitch diameter}}{\text{number of teeth}}$.

Formula (2) must be worked out for both pinion and wheel, with appropriate values of V, N, and F.

Bevel Gears.—The load capacities are determined by the above formulæ with these modifications. In (1), write $d\sqrt{(1+r^2)}$ instead of d and r^2 instead of r. In finding F from Table 5, use as the number of teeth in the pinion the actual number multiplied by $\sqrt{(1+r^2)}$, and for the wheel multiply the number of teeth by $\sqrt{\left(1+\frac{1}{r^2}\right)}$.

$$\text{Take } m=\frac{\text{Diameter at mid-point of face width}}{\text{Number of teeth}}.$$

The permissible load thus calculated is taken as acting at the mid-point of the face width.

Worm Gears.—Permissible load (lbs.) per inch width of wheel face $=GHD^{0\cdot8}$ . . . . (3)

on basis of surface pressure,

where D = pitch diameter of worm wheel (in.).
G = surface stress factor (from Table 6).
H = load application factor (from Table 7).

Those material-combinations, for which no value of G is given in Table 6, are not recommended.

Permissible load (lbs.) per inch width of wheel face on basis of tooth breakage $=13\,VMp_n$ . . . . (4)

where V = speed factor (Table 1).
M = breaking stress factor (Table 6).
p_n = normal pitch.

The values of V and M are to be taken for both worm and worm wheel, and the lower of the two results is to be used.

Table 1.

Speed Factors V.

R.P.M.	V.	R.P.M.	V.
0	420	300	158
1	345	400	150
5	300	500	144
10	275	750	132
20	250	1000	125
40	223	1250	118
60	209	1500	113
80	201	2000	105
100	194	2500	99
150	179	3000	93
200	170	4000	85
250	165	5000	79

Table 2.
Material Factors M and N.

	Tensile Strength (tons per sq. in.).	Brinell No.	*M*.	*N*.
3½% Ni case-hardened steel .	40	550	6·7	2·7
3½% Ni. 1% Cr steel . .	55	240	2·0	2·0
0·55% carbon steel . .	45	200	1·3	1·5
0·4% carbon steel . .	35	145	1·0	1·3
Phosphor-bronze . .	15	80	0·55	0·55
Cast-iron	12	180	0·75	0·4

Table 3.
Type Factors A.

Type of Gear.	Parallel Shaft.		Bevel.	
Helix Angle.	Normal Pressure Angle.		Normal Pressure Angle.	
	20°	14½°	20°	14½°
0°	0·75	0·68	0·56	0·5
22½°	0·87	0·8	0·66	0·6
30°	1·0	0·9	0·75	0·68
45°	1·0	0·9	0·75	0·68

Table 4.
Type Factors B.

Type of Gear.	Parallel Shaft.		Bevel.	
Helix Angle.	Normal Pressure Angle.		Normal Pressure Angle.	
	20°	14½°	20°	14½°
0°	31	28	29	23·5
22½°	28·5	25·7	26·7	21·5
30°	26	23·5	24·5	19·5
45°	17·5	15·5	16·5	13·0

Table 5.
Strength Factors F.

Number of Teeth in Gear.	Number of Teeth in Mating Gear.									
	14	17	20	25	30	40	50	75	100	400
14	0·86	0·90	0·94	1·02	1·10	1·17	1·20	1·24	1·26	1·4
17	0·88	0·96	1·00	1·07	1·15	1·22	1·27	1·31	1·35	1·44
20	0·90	1·0	1·08	1·16	1·22	1·28	1·33	1·38	1·40	1·48
25	1·0	1·06	1·16	1·24	1·28	1·34	1·39	1·44	1·46	1·52
30	1·1	1·16	1·25	1·30	1·35	1·40	1·45	1·50	1·52	1·60
40	1·2	1·25	1·30	1·36	1·4	1·44	1·50	1·54	1·57	1·70
50	1·25	1·30	1·35	1·40	1·44	1·48	1·54	1·57	1·60	1·73
75	1·3	1·35	1·40	1·43	1·47	1·50	1·56	1·60	1·64	1·75
100	1·35	1·39	1·43	1·46	1·49	1·52	1·57	1·62	1·68	1·77
400	1·4	1·42	1·46	1·48	1·50	1·54	1·58	1·64	1·70	1·8

Table 6.

Breaking Stress Factors M and Surface Stress Factors G for Worm Gears.

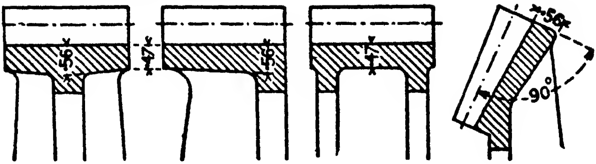
Material.	M	G when working with			
		Case-hardened Steel.	0·55% Carbon Steel.	Phosphor-bronze.	Cast-iron.
Case-hardened steel	6·7	34	..	115	69
0·55% carbon steel	1·3	..	..	34	19
Phosphor-bronze .	0·55	27	15·5	..	13·5
Cast-iron . .	0·75	13	10	15·5	10

Table 7.

Load Application Factors H for Worm Gears.

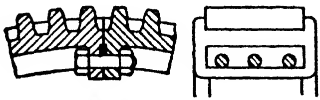
R.P.M.	Rubbing Speed (Ft. per Min.).										
	0	5	10	25	50	100	250	500	750	1000	1500
0	20	18	15	13·6	12·8	11·8	10·4	9·5	8·8	8·4	7·8
5	18	14·5	13·6	12·6	11·6	10·6	9·6	8·6	8·0	7·5	7·0
10	15	13	12·0	11·4	10·6	9·8	8·6	7·6	7·2	6·8	6·4
25	12	11·2	10·3	9·8	9·0	8·1	7·3	6·6	6·2	5·9	5·6
50	11	10·6	9·6	9·2	8·4	7·6	6·8	6·2	5·8	5·5	5·1
100	10	9·2	8·6	7·8	7·2	6·8	6·0	5·4	5·0	4·7	4·4
250	8	7·4	6·7	6·1	5·7	5·5	4·9	4·4	4·1	3·8	3·5
500	6·5	6·1	5·8	5·1	4·7	4·5	4·0	3·6	3·4	3·2	2·9
750	5·5	5·3	5·0	4·4	4·0	3·9	3·5	3·1	2·95	2·8	2·6
1000	5·3	4·8	4·5	4·1	3·8	3·5	3·1	2·8	2·6	2·5	2·3
1500	4·7	4·4	4·1	3·7	3·4	3·1	2·7	2·5	2·4	2·2	2·0
2000	4·2	3·8	3·9	2·2	3·0	2·8	2·5	2·2	2·1	2·0	1·8

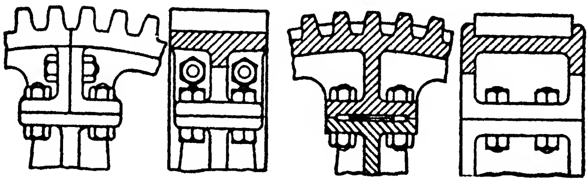
Rims of Gear Wheels.—



The proportions marked on the above illustrations are in terms of the pitch p of the teeth.

The adjacent illustrations show examples of the rims of gear wheels which are made in segments.

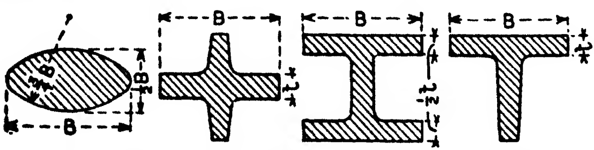




Arms of Gear Wheels.—

P = driving force at pitch line in lbs.
R = radius of pitch circle in inches.
p = pitch of teeth in inches.
b = breadth of teeth in inches.
m = number of arms in wheel.
f = working stress in lbs. per square inch.

The bending moment on each arm is approximately $\frac{PR}{m}$, and this must be equated to the moment of resistance of the arm to bending.



For the oval section shown above the moment of resistance to bending may be taken equal to $\cdot 05B^3f$.

For the other sections shown it will be sufficiently accurate to take their moment of resistance to bending equal to $\frac{1}{6}B^2tf$.

For the oval section $\frac{PR}{m} = \cdot05B^3f$, and $B = \sqrt[3]{\frac{20PR}{mf}}$.

For the other sections $\frac{PR}{m} = \frac{1}{6}B^2tf$, and $B = \sqrt{\frac{6PR}{mtf}}$.

Putting $P = \frac{bpf}{17\cdot5}$, $t = \cdot48p$ in the ┼ and ┬ sections, $\frac{t}{2} = \cdot48p$ in the I section, and taking the stress in the arms equal to five-sevenths of the stress in the teeth, to allow for the initial straining actions due to unequal contraction in the mould, and also for the possible unequal distribution of the bending action on the arms, we get,

$$B = \sqrt[3]{\frac{1\cdot6bpR}{m}} \text{ for the oval section,}$$

$$B = \sqrt{\frac{bR}{m}} \text{ for the ┼ and ┬ sections, and}$$

$$B = \sqrt{\frac{bR}{2m}} \text{ for the I section.}$$

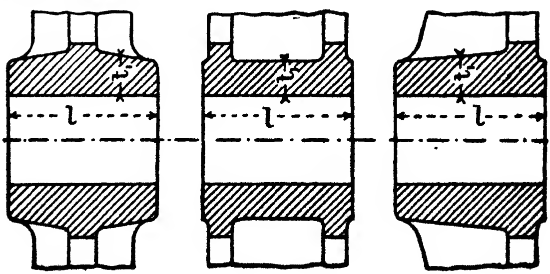
In each case B is measured at the centre of the wheel, supposing the arm to be produced to that point.

The breadth measured at the pitch line may be $B - \cdot04R$.

The mean thickness of the ribs or web may be $\cdot4p$.

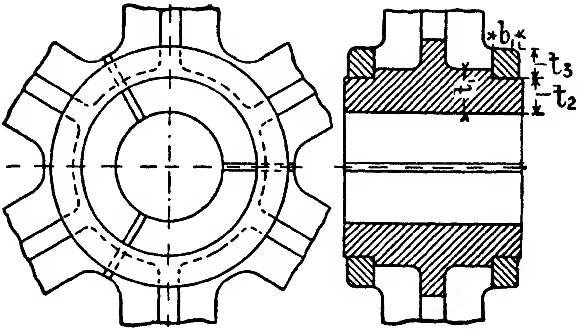
The number of arms is approximately $m = \frac{D}{36} + 4$, where D is the diameter of the pitch circle in inches. m is generally an *even* number.

Wheel Bosses.—



Thickness of boss $= t_1 = \frac{\sqrt[3]{bpR}}{3}$.

Another rule is, $t_1 = \cdot 8p + \cdot 02R$.
Length of boss $= l = b$ to $1 \cdot 4b$.



Steel or wrought-iron hoops shrunk on to the bosses of large wheels, as shown above, may have the following proportions:—

$$t_3 = \frac{2}{3}t_1; \quad b_1 = \frac{7}{8}t_3.$$

Thickness of boss under the rings $= t_2 = \frac{3}{4}t_1$.

CRANKS.

Overhung Crank Pins.—

d = diameter of pin in inches.
l = length of journal in inches.
P = load on pin in lbs.
p = pressure on journal in lbs. per square inch of "projected area." (Projected area of journal $= dl$.)
f = greatest tensile stress due to bending in lbs. per square inch.

$$d = \frac{1 \cdot 5\sqrt{P}}{\sqrt[4]{pf}} = K\sqrt{P}. \qquad \frac{l}{d} = \frac{4}{9}\sqrt{\frac{f}{p}} = C.$$

The following table gives values of K and C for various values of p and f:—

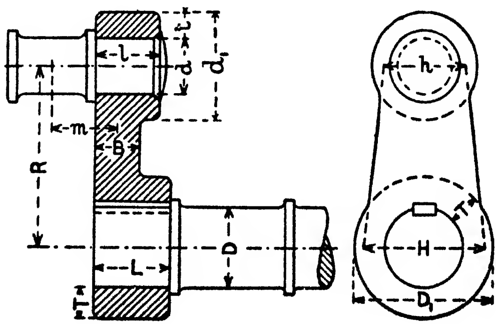
p	$f=6500$		$f=8000$		$f=9500$		$f=11,000$		$f=12,500$	
	K	C	K	C	K	C	K	C	K	C
200	·044	2·53	·042	2·81	·040	3·06	·039	3·30	·038	3·51
300	·040	2·07	·038	2·30	·037	2·50	·035	2·69	·034	2·87
400	·037	1·79	·035	1·99	·034	2·17	·033	2·33	·032	2·48
500	·035	1·60	·034	1·78	·032	1·94	·031	2·08	·030	2·22
600	·034	1·46	·032	1·62	·031	1·77	·030	1·90	·029	2·03
700	·032	1·35	·031	1·50	·030	1·64	·029	1·76	·028	1·88
800	·031	1·27	·030	1·41	·029	1·53	·028	1·65	·027	1·76
900	·030	1·19	·029	1·32	·028	1·44	·027	1·55	·026	1·66

For small high-speed engines, $p=200$.
For large low-speed engines, $p=900$.
For wrought-iron, $f=6500$ to 9500.
For steel, $f=8000$ to $12,500$.

If it is desired to make the length of the crank pin less than that given by the foregoing rules, the diameter must be increased so as to satisfy the equation $pdl=P$.

If $l=nd$, then $d=\sqrt{\dfrac{P}{np}}$. The crank pin proportioned by this rule will have an excess of strength when n is less than $\dfrac{4}{9}\sqrt{\dfrac{f}{p}}$.

Overhung Cranks.—The following rules are based on the assumption that the crank is made of wrought-iron or steel, and that the material of the shaft is the same as that of the crank.



$L = CD$. C varies from ·7 to 1·1. Average value of C is ·9.
$T = KD$. K varies with C as follows:—

C = ·7	·8	·9	1·0	1·1.
K = ·44	·40	·37	·34	·31.

$l = cd$. c varies from ·9 to 1·4. Average value of c is 1·2.
$t = kd$. k varies with c as follows:—

c = ·9	1	1·1	1·2	1·3	1·4.
k = ·62	·5	·41	·35	·30	·26.

$D_1 = D + 2T$. $d_1 = d + 2t$.
$H = \cdot 7D_1$ to D_1. $h = \cdot 7d_1$ to d_1.

To determine B. First assume $B = \frac{1 \cdot 18 D^3}{H^2}$, and find m, the distance of the centre of the crank pin journal from the centre line of the crank web. Let P = load on crank pin at right angles to crank arm. This force produces a bending moment on the arm $= M_b = PR$, and also a twisting moment $= M_t = Pm$. Combining these, the equivalent bending moment is

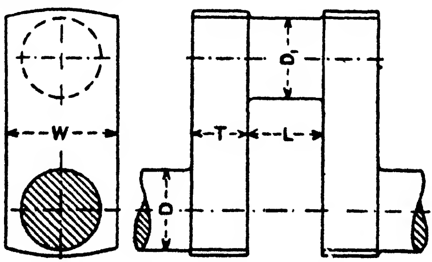
$$M_e = \frac{1}{2} M_b + \frac{1}{2} \sqrt{M_b^2 + M_t^2}.$$

Then $B = \frac{6 M_e}{H^2 f}$, where f is the safe tensile stress.

Width of key $= \frac{1}{4} D$ for large cranks, and $\frac{1}{4} D + \frac{1}{8}$ for small cranks.

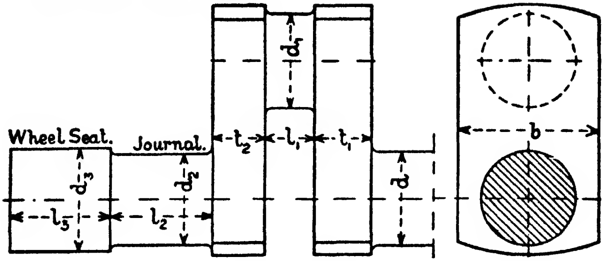
Thickness of key = ·4 to ·7 of its width.

Forged Crank Shafts.—



$T = \cdot 6D$ to $\cdot 8D$. Average value, $T = \cdot 7D$.
$TW^2 = \cdot 8D^3$ to $1 \cdot 06 D^3$. Average value, $TW^2 = \cdot 9D^3$.
If $T = \cdot 7D$, and $TW^2 = \cdot 9D^3$, then $W = 1 \cdot 134 D$.
D_1 generally $= D$, but sometimes as large as $1 \cdot 05 D$.
$L = D$ to $1 \cdot 3D$.

Locomotive Crank Axles.—



Examples from Actual Practice.

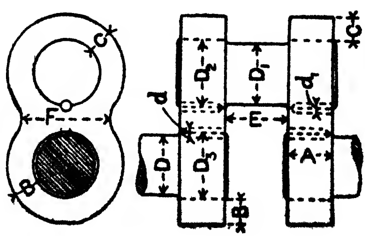
D=diameter of cylinders. L=stroke of pistons.

D	L	d	d_1	d_2	d_3	l_1	l_2	l_3	t_1	t_2	b
13	20	5¼	6¼	5¾	5¾	4	8	6⅛	4½	4⅛	8
16	22	7	7¼	7	8½	4½	9	6¾	4⅜	4⅜	10
16½	20	6½	6½	6½	7½	4	8½	5⅞	4¼	4	10
17	24	6⅜	7	6¾	8	4	9	7	4	4	11
17½	24	6¾	7	6¼	8	4	7½	6¾	4½	4	12
17½	26	7	7¾	7½	9	4	7½	7½	4½	4¼	12
17½	26	7	7½	7½	8½	4	9	7 1/32	4¼	4¼	13
18	24	7	8	7½	9	4½	9	8½	4¾	4¼	12
18	25	7	7¾	7¼	8½	4	9	7	4⅛	4⅛	14
18	26	7½	7½	7½	8½	4	8½	7⅛	4¼	4	13
18	26	8	8½	8	8¾	4¼	7½	8	4½	4⅜	13
18¼	26	7	7½	7	8½	4	7	6⅞	4¼	3¾	12
18¼	26	7¾	8¼	8	8¼	4	8⅛	7 1/16	5	5	11
19	26	7½	8¼	8¼	8¼	4	8½	8¾	5¼	4⅞	12
20	24	7½	8	8	9	5	10	6¾	4¾	4¾	13

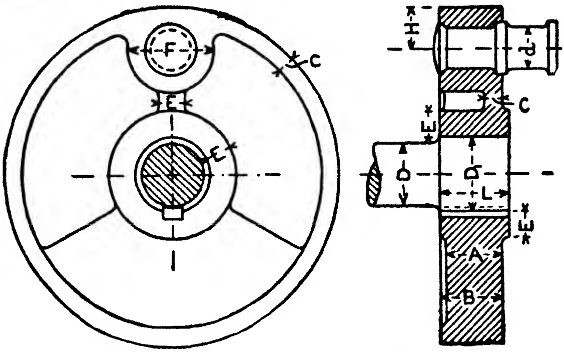
All the dimensions in the above table are in inches.

Built-up Crank Shafts.—

$A=\cdot 75D.$
$B=\cdot 42D.$
$C=\cdot 4D.$
$D_1=D$ to $1\cdot 03D.$
$D_2=1\cdot 02D_1.$
$D_3=1\cdot 06D.$
$d=\cdot 12D$ to $\cdot 2D.$
$d_1=\cdot 75d$ to d
$E=\cdot 95D$ to $1\cdot 12D.$
$F=1\cdot 3D.$

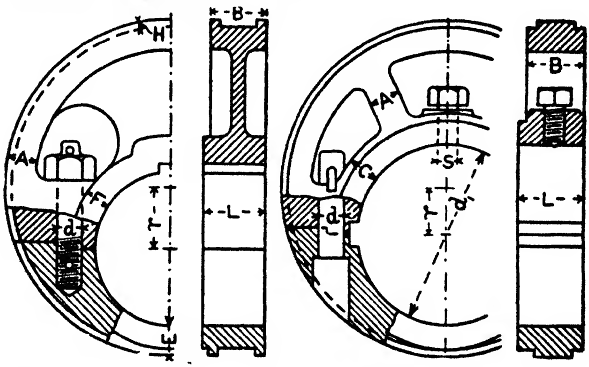


Cast-iron Crank Discs.—



$A = \cdot 9B.$	$E = \cdot 4D.$
$B = \cdot 7D$ to $D.$	$F = 2d.$
$C = \cdot 25D.$	$H = d$ to $1 \cdot 5d.$
$D_1 = D$ to $1 \cdot 2D.$	$L = B$ to $1 \cdot 25B.$

Eccentric Sheaves or Pulleys.—



The breadth B is first determined from the force required to work the valve which the eccentric has to drive.

Let l = length of slide valve in inches.

b = breadth of slide valve in inches.

p = steam pressure on back of valve in lbs. per square inch.

$B = c\sqrt{lbp}.$

For locomotive engines c varies from ·017 to ·02, and has an average value of ·0185.

For marine engines $c = ·01$.

B may also be determined as follows:—

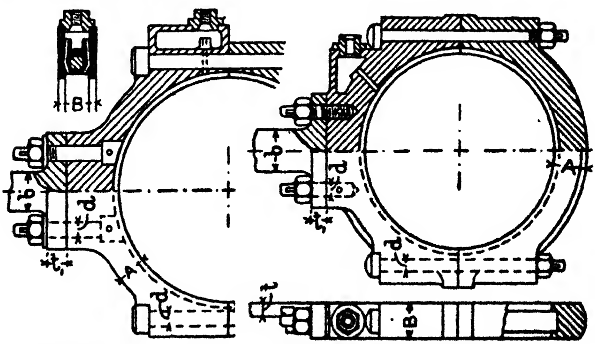
D = diameter of steam cylinder in inches.
p = initial steam pressure in lbs. per square inch.
$B = ·0132D\sqrt{p}$ for locomotive engines.
$B = ·009D\sqrt{p}$ to $·015D\sqrt{p}$ for marine engines.

For stationary engines the breadth of the eccentric should not be less than that given for locomotive or marine engines, and if space will permit, B may be made from $·013D\sqrt{p}$ to $·03D\sqrt{p}$.

$A = ·55B$.
$d = ·4B$.
$E = ·4B$ (minimum for wrought-iron or steel).
$E = ·5B$ (minimum for cast-iron).
$C = ·65B$.
$S = ·32B$ for two set-screws.
$S = ·4B$ for one set-screw.
$F = ·4B$.
$H = ·13B$.
$L = B$ to $1·6B$.

Thickness of key $= ·2B$ to $·3B$.
Breadth of key = 1 to 2 times its thickness, generally $1\frac{1}{2}$ times.
Breadth of cotters in bolts $= d$ to $1\frac{1}{4}d$.
Thickness of cotters in bolts $= ·25d$.
Diameter of sheave $= d_1 + 2r + 2E$, where r is the radius or eccentricity of the eccentric.

Eccentric Straps.—Eccentric straps may be made of cast-iron, brass, malleable cast-iron, wrought-iron, or steel. When made of cast-iron or brass no liner is necessary, but when made of the other materials a brass or white metal liner is required.



B = breadth of strap = breadth of eccentric sheave.

Thickness of strap if of cast-iron $= A = \cdot 7B$ to $\cdot 9B$.
Thickness of strap if of wrought-iron or steel $= A = \cdot 5B$ to $\cdot 6B$.
Thickness of strap if of brass or malleable cast-iron $= A = \cdot 6B$ to $\cdot 8B$.
Diameter of bolts or studs $= d = \cdot 4B$ to $\cdot 5B$.
Thickness of palm on end of eccentric rod $= t_1 = \cdot 45B$.
Breadth of eccentric rod (if rectangular) at strap end $= b = 1 \cdot 3B$ to $1 \cdot 5B$.
Thickness of eccentric rod (if rectangular) $= t = \cdot 4B$.
Diameter of eccentric rod (if round) at strap end $= \cdot 9B$ to B.

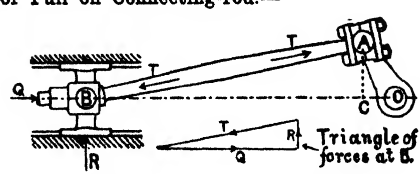
CONNECTING-RODS.

D = diameter of piston in inches.
L = length of stroke of piston in inches.
l = length of connecting-rod in inches, measured from the centre of the crosshead pin or gudgeon to the centre of the crank pin.
d = diameter of rod in inches, at the middle of its length, if of circular section.
b = breadth or thickness of rod in inches if of rectangular section.
h = depth of rod in inches, at the middle of its length, if of rectangular section (h being greater than b).

$$r = \frac{l}{d} \text{ or } \frac{l}{b}. \qquad n = \frac{l}{L}. \qquad m = \frac{b}{h}.$$

p = maximum effective pressure of steam on piston in lbs. per square inch.
Q = effective load on piston in lbs.
T = thrust or pull on connecting-rod in lbs.
V = velocity of crank pin in feet per second.

Thrust or Pull on Connecting-rod.—



$$Q = \cdot 7854 D^2 p. \qquad T = \frac{AB}{BC} \times Q.$$

A is the centre of the crank pin, B is the centre of the cross-head pin or gudgeon, O is the centre of the crank shaft, and C is the foot of the perpendicular from A on BO.

The greatest possible value of T is when AO is at right angles to BO, then,

$$T=\frac{AB}{BO}\times Q=\frac{lQ}{\sqrt{l^2-\frac{L^2}{4}}}=\frac{2nQ}{\sqrt{4n^2-1}}=cQ.$$

$n=1{\cdot}75$	2	2·5	2·75	3
$c=1{\cdot}044$	1·033	1·021	1·017	1·014.

For stationary engines, $n=2{\cdot}5$ to 3, but it is sometimes as small as 2, and sometimes as large as 3·75.

For ordinary marine engines, $n=2$, but it is sometimes as large as 2·5.

For the engines of war-ships, n is sometimes as small as 1·75.

For locomotives, $n=3$, but it is sometimes as small as 2·7, and in some exceptional cases as large as 4·8.

For portable engines, $n=3{\cdot}5$.

Strength of Connecting-rods.—*Round Rods.*—The following empirical formula is based on numerous examples from actual practice :—

$$d=\frac{\sqrt{300+r^2}\sqrt{T}}{1000}=c_1\sqrt{T}.$$

Putting $T=1{\cdot}025Q=1{\cdot}025\times{\cdot}7854D^2p$, then,

$$d=\frac{\sqrt{300+r^2}}{1115}D\sqrt{p}=c_2D\sqrt{p}.$$

The material of the rod is supposed to be wrought-iron or mild steel.

$r=$ 8	10	12	14	16	18	20
$c_1=$ ·019	·020	·021	·022	·024	·025	·026
$c_2=$ ·017	·018	·019	·020	·021	·022	·023

Messrs. Seaton and Rounthwaite, in their pocket-book of marine engineering, recommend an empirical formula, of which the following is a slight modification :—

$d={\cdot}0418\sqrt{l}\sqrt[4]{Q}$ for mercantile engines.

$d={\cdot}0371\sqrt{l}\sqrt[4]{Q}$ for naval engines.

The stress in a connecting-rod due to the thrust or pull is,

$$\frac{T}{{\cdot}7854d^2};$$

and the stress due to the bending action caused by the inertia of the rod is approximately,

$$\frac{{\cdot}103\,V^2l^2}{Ld}.$$

The sum of these two stresses should not exceed 6000 for wrought-iron, and 7000 for mild steel.

If, as is generally the case, the crank-pin end of the connecting-rod is much heavier than the crosshead end, the rod is made with a straight taper; the diameter at the crosshead end being not less than $\cdot 9d$. Sometimes a rod with a straight taper has a diameter at the crosshead end equal to $\cdot 95d$, but generally it is about $\cdot 92d$.

When the two ends of the rod are equally heavy and the rod is comparatively short, it is usually made parallel. Long rods with equally heavy ends are generally made barrel-shaped, and the diameter of each end is then from $\frac{3}{4}d$ to $\frac{7}{8}d$.

Sometimes the rod is tapered from the crosshead end to the middle, and is parallel for the remainder of its length.

The smallest diameter of the rod should not be less than $\cdot 0146\sqrt{T}$ or $\cdot 013D\sqrt{p}$ for wrought-iron, and not less than $\cdot 0135\sqrt{T}$ or $\cdot 012D\sqrt{p}$ for mild steel.

Rectangular Rods.—The following empirical formula is based on examples of locomotive connecting-rods:—

$$h = \frac{\sqrt{5800 + r^2}\sqrt{T}}{6500\sqrt{m}} = \frac{c_3\sqrt{T}}{\sqrt{m}}.$$

Putting $T = 1{\cdot}014Q = 1{\cdot}014 \times {\cdot}7854D^2p$, then,

$$h = \frac{\sqrt{5800 + r^2}}{7280\sqrt{m}}D\sqrt{p} = \frac{c_4D\sqrt{p}}{\sqrt{m}}.$$

$r=$	35	40	45	50	55
$c_3=$	·0129	·0132	·0136	·0140	·0145
$c_4=$	·0115	·0118	·0121	·0125	·0129

In practice m varies from ·42 to ·67, an average value being ·5.

The rod is either uniform in depth or it has a straight taper. In the latter case the depth at the crosshead end is from $\frac{3}{4}h$ to $\frac{7}{8}h$. The thickness b is nearly always uniform throughout the length of the rod.

The stress in the rod due to the thrust or pull is $\frac{T}{bh}$; and the stress due to the bending action * caused by the inertia of the rod is approximately $\frac{\cdot 079\,V^2l^2}{Lh}$.

The sum of these two stresses should not exceed 6000 for wrought-iron, and 7000 for mild steel.

Coupling-rods.—

l = length of coupling-rod in inches, measured between centres of crank pins.

R = radius of cranks in inches.

V = velocity of crank pins in feet per second.

A = area of cross section of rod in square inches.

* The bending moment is greatest at about $\cdot 42l$ from the crank pin.

Z = modulus of section of rod (see p. 212).
T = thrust or pull on rod in lbs.

The greatest bending moment due to the inertia of the rod is at the middle of its length, and is $= \cdot 013\,\frac{V^2 l^2}{R}$.

The stress due to the greatest bending moment is $= \cdot 013\,\frac{V^2 l^2}{RZ}$.

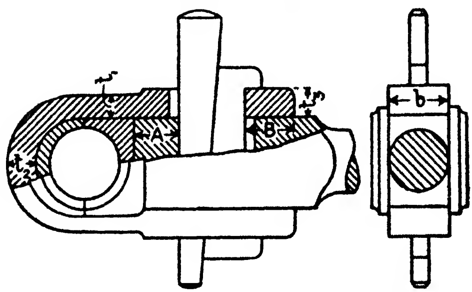
The stress due to the thrust or pull is $= \frac{T}{A}$.

The sum of these two stresses should not exceed 6000 for wrought-iron, and 7000 for mild steel.

Locomotive coupling-rods are usually of uniform section throughout, and are either of rectangular or I section.

CONNECTING-ROD ENDS.

Common Strap End.—T = maximum pull on the connecting-rod. f = safe tensile stress for material of strap in lbs. per square inch = 6000 for wrought-iron, and 7000 for steel.



$t_1 = \frac{T}{2bf}$. $t_2 = 1{\cdot}17t_1$ to $1{\cdot}5t_1$. $t_3 = 1{\cdot}2t_1$ to $1{\cdot}5t_1$.

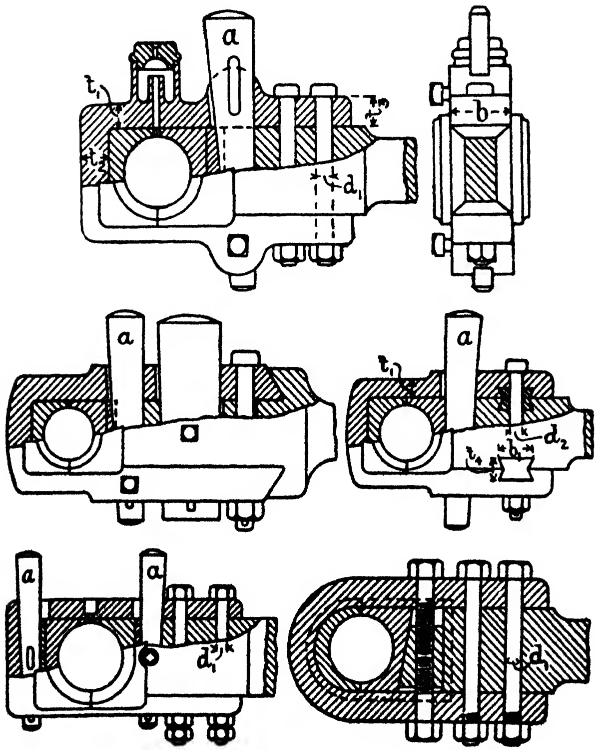
t_3 should be such that the area of the cross section at the cotter hole is not less than bt_1.

The area of the cross section of the gib and cotter combined should not be less than bt_1.

The thickness of the cotter is generally $\cdot 25b$, then t_3 should not be less than $1{\cdot}33t_1$.

A may equal $2t_1$, and B may equal $2{\cdot}5t_1$.

Fixed Strap End.—The dimensions t_1, t_2, and t_3 are determined as for a common strap end.



$d_1 = \cdot 009\sqrt{T}$ for wrought-iron $= \cdot 008\sqrt{T}$ for steel.

$d_2 = \cdot 7t_1$. $b_1 = 1 \cdot 9t_1$. $t_4 = 1 \cdot 3t_1$.

The thickness of the cotters may be $\cdot 25b$.

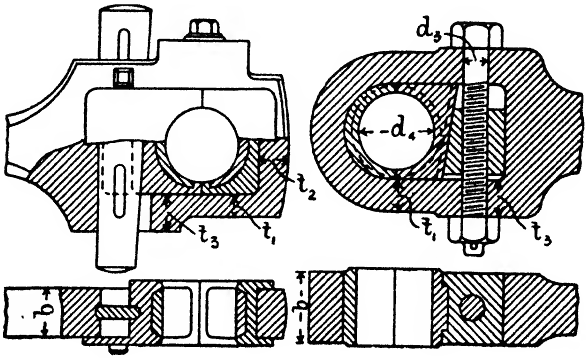
The thickness of the steel bearing piece sometimes introduced to distribute the pressure of the cotter on the brass may be equal to the thickness of the cotter.

The cotters marked *a* are subjected to compression only.

Solid or Box End.—t_1, t_2, and t_3 are found as for a common strap end. The cotter is in compression only.

$d_3 = \cdot 2d_4$ to $\cdot 3d_4$.

The slope of the block B next the brass equals 1 in 4 to 1 in 7, generally 1 in 6.



Brasses for Connecting-rod Ends.—

t = thickness of brass or step at bottom in inches.

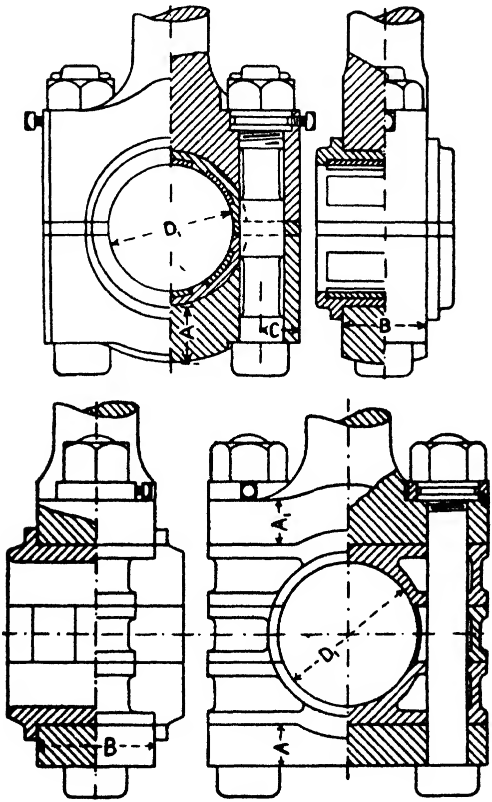
d = diameter of bearing in inches.

$t = \frac{1}{8}d + \frac{1}{4}$ inch for bearings up to 8 inches in diameter.

$t = \frac{1}{16}d + \frac{3}{4}$ inch for bearings above 8 inches in diameter.

The thickness at the sides is generally from $\cdot 5t$ to t, but is sometimes as small as $\cdot 2t$.

Marine Type of Connecting-rod End.—This is largely used for stationary as well as for marine engines.



Diameter of bolts at bottom of screw thread $= d_1 = \cdot 8\sqrt{\frac{T}{f}}$.

For bolts 2 inches in diameter and upwards $f = 5000$ for wrought-iron, and 7000 for steel. For smaller diameters f must be diminished. For bolts under 1 inch in diameter f should not exceed 3000 for wrought-iron, and 4200 for steel.

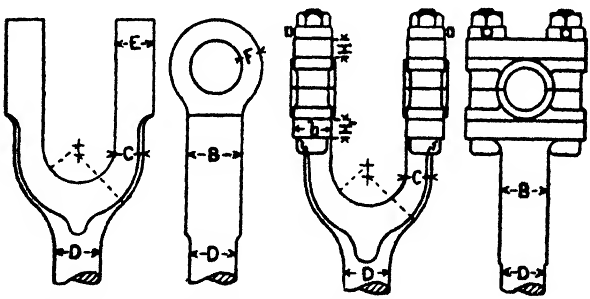
The bolts are generally made of mild steel, and the nuts of wrought-iron.

The breadth B of the rod end is generally a little greater than the diameter of the adjacent part of the rod. B should not be less than $1{\cdot}73d + {\cdot}15$ inch, where d is the diameter of the bolt over the screw thread in inches.

$$A = 1{\cdot}086 d_1 \sqrt{\frac{D_1}{B}}. \qquad A_1 = A \text{ to } 1{\cdot}2A.$$

C varies from ${\cdot}9d$ to d, but should not be less than half the minimum value of B, given above.

Forked Ends.—a = area of cross section of adjacent part of rod $= {\cdot}7854D^2$. The total area of the section of the jaws is usually from $1{\cdot}25a$ to $1{\cdot}8a$, and in the case of the solid forked end the total area of the section through the eyes is usually from $1{\cdot}4a$ to $1{\cdot}9a$. The following proportions make the total section through the jaws about $1{\cdot}4a$, and the total section through the eyes about $1{\cdot}5a$:—



$B = D$ to $1{\cdot}2D$, average value $= 1{\cdot}1D$.
$C = {\cdot}5D$ when $B = 1{\cdot}1D$.
$E = C$ to $1{\cdot}35C$, average value $= 1{\cdot}2C = {\cdot}6D$ when $C = {\cdot}5D$.
$F = {\cdot}5D$ when $E = {\cdot}6D$.

For the design shown to the right B and C are generally greater than for the design shown to the left.

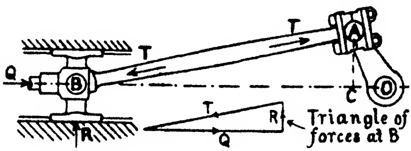
Diameter of each of the four bolts at the bottom of the screw thread $= d_1 = {\cdot}564\sqrt{\frac{T}{f}}$, where f has a value equal to 90 per cent. of the value given in the preceding article for two bolts.

d = diameter of bolts over the screw thread in inches.
$H = d$. $H_1 = d$ to $1{\cdot}2d$.
b should not be less than $1{\cdot}73d + {\cdot}15$ inch.

CROSS-HEADS.

Rubbing Surface of Slide Block.-

D = diameter of piston in inches.
L = length of stroke of piston in inches.
l = length of connecting-rod in inches, measured from the centre of the cross-head pin or gudgeon to the centre of the crank pin.
$n = l \div L$.
p = maximum effective pressure of steam on piston in lbs. per square inch.
Q = effective load on piston in lbs.
R = reaction of the guide bar on the slide block in lbs.
q = pressure on the rubbing surface of the slide block in lbs. per square inch.
a = area of rubbing surface of slide block in square inches.
S = speed of piston in feet per minute.



$Q = \cdot 7854 D^2 p.$ $\qquad R = \frac{AC}{BC} \times Q.$

A is the centre of the crank pin, B is the centre of the cross-head pin or gudgeon, O is the centre of the crank shaft, and C is the foot of the perpendicular from A on BO.

The maximum value of R is when AO is at right angles to BO, then

$$R = \frac{AO}{BO} \times Q = \frac{LQ}{2\sqrt{l^2 - \frac{L^2}{4}}} = \frac{Q}{\sqrt{4n^2 - 1}} = kQ.$$

$n =$	1·75	2	2·5	2·75	3
$k =$	·298	·258	·204	·185	·169

$$a = \frac{R}{q}.$$

For stationary engines, $q = \frac{12000}{S}$, but should not be less than 30 or more than 60.

For locomotive engines, $q = \frac{45000}{S}$, but should not be less than 40 or more than 60.

For marine engines, $q = \frac{50000}{S}$, **but should not be less than 50 or more than 80.**

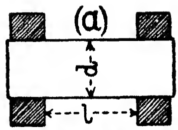
Cross-head Pin or Gudgeon.–

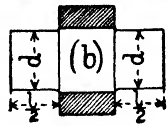
D = diameter of piston in inches.

P = maximum effective pressure of steam on piston in lbs. per square inch.

d = diameter of pin journal in inches.

l = length of pin journal in inches when supported at both ends, as shown in Fig. (*a*).





$\frac{1}{2}l$ = length of pin journals in inches when the pin is fixed at the middle, as shown in Fig. (*b*).

$n = l \div d$, and therefore $l = nd$.

p = pressure on projected area of journal $(d \times l)$ in lbs. per square inch.

f = maximum safe stress in lbs. per square inch.

The pin will have sufficient bearing area when

$$d = \frac{\cdot 9D\sqrt{P}}{\sqrt{np}} = k\frac{D\sqrt{P}}{100}, \text{ where } k = \frac{90}{\sqrt{np}}.$$

The pin will have sufficient strength when

$$d = \frac{D\sqrt{Pn}}{\sqrt{f}} = k_1\frac{D\sqrt{P}}{100}, \text{ where } k_1 = \frac{100\sqrt{n}}{\sqrt{f}}.$$

The following table gives values of k and k_1 corresponding to various values of n and p or f:—

n	Values of k when p =				Values of k_1 when f =			
	800	1000	1200	1400	4000	5000	6000	7000
·9	3·35	3·00	2·74	2·54	1·50	1·34	1·22	1·13
1·0	3·18	2·85	2·60	2·41	1·58	1·41	1·29	1·20
1·1	3·03	2·71	2·48	2·29	1·66	1·48	1·35	1·25
1·2	2·90	2·60	2·37	2·20	1·73	1·55	1·41	1·31
1·3	2·79	2·50	2·28	2·11	1·80	1·61	1·47	1·36
1·4	2·69	2·41	2·20	2·03	1·87	1·67	1·53	1·41
1·5	2·60	2·32	2·12	1·96	1·94	1·73	1·58	1·46
1·6	2·52	2·25	2·05	1·90	2·00	1·79	1·63	1·51
1·7	2·44	2·18	1·99	1·84	2·06	1·84	1·68	1·56
1·8	2·37	2·12	1·94	1·79	2·12	1·90	1·73	1·60
1·9	2·31	2·06	1·88	1·74	2·18	1·95	1·78	1·65
2·0	2·25	2·01	1·84	1·70	2·24	2·00	1·83	1·69

In practice n generally varies from 1 to 1·3 for pins supported at the ends, as in Fig. (*a*), and from 1·3 to 2 for pins overhung, as in Fig. (*b*).

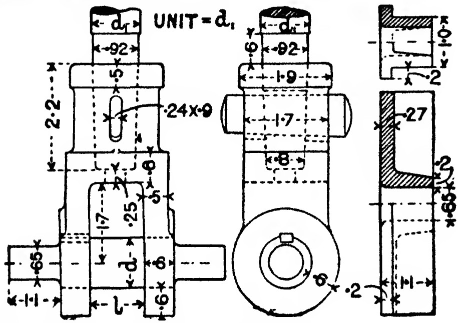
p does not generally exceed 1200.

f may be taken at 5000 for wrought-iron, and 7000 for steel.

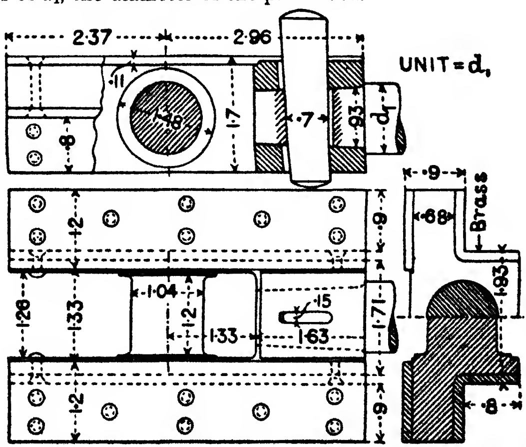
Wrought-iron or mild steel pins should be case hardened and afterwards ground true.

If when the pin has sufficient bearing surface it is found to have an excess of strength, it may be lightened by drilling a hole through it of a diameter $d_1 = \sqrt[4]{\left\{ d^4 - d\left(\frac{D\sqrt{Pn}}{\sqrt{f}}\right)^3 \right\}}$.

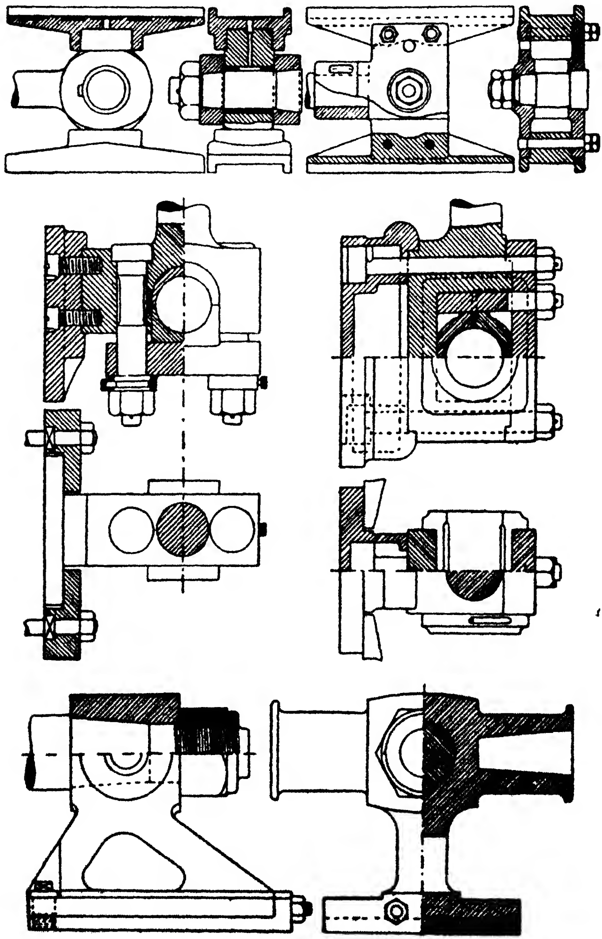
Types of Cross-heads.—



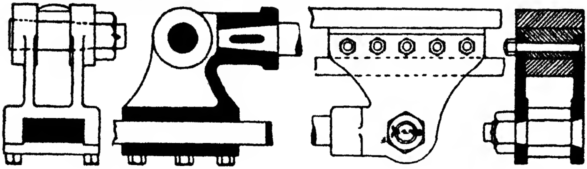
Wrought-iron or steel cross-head, with cast-iron or brass slide blocks. Approximate proportions are marked on the Figs., in terms of d_1, the diameter of the piston-rod.

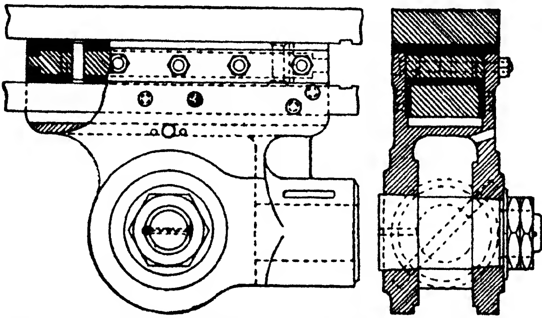


Cast-steel cross-head faced with brass. The cross-head pin is cast in one piece with the cross-head. The brass facings are connected to the cross-head with brass rivets. This type is chiefly used in America on locomotives and high-speed engines.



Types of Cross-heads (*continued*).—





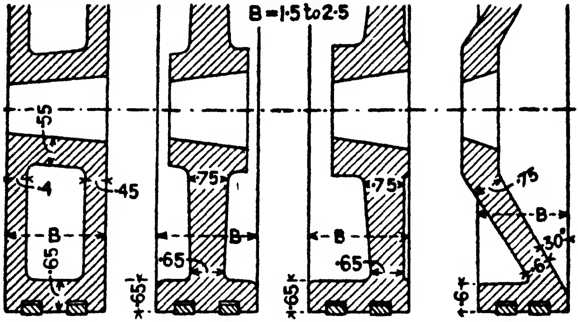
When slide blocks are made of wrought-iron or steel, their sliding surfaces should be faced with a softer metal. In American practice it is common to tin the sliding surfaces of wrought-iron or steel slide blocks to a depth of $\frac{1}{16}$ inch.

Guide Bars for Cross-heads.—R, the maximum load on the guide bar, cannot exceed $\frac{Q}{\sqrt{4n^2-1}}$, where Q is the greatest effective load on the piston, and n is the ratio of the length of the connecting-rod to the stroke of the piston. If R is assumed to act at the middle of the length of the bar, then the greatest bending moment on the bar is $\frac{RS}{4}$, where S is the distance between the supports of the bar. If the load R is carried by two bars, then the greatest bending moment on each is $\frac{RS}{8}$. The bending moment must be equated to the moment of resistance of the bar, as explained on p. 250. The stress f may be taken at 3000 lbs. per square inch for cast-iron, and 6000 for wrought-iron or steel.

Cast-iron bars are usually of ⊥ section, while wrought-iron bars are of rectangular section.

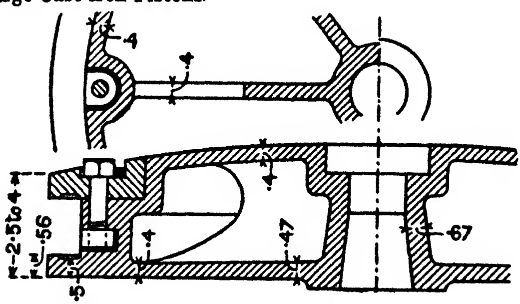
PISTONS.

Small and Medium Sized Pistons.—



Unit for proportions $= \frac{D\sqrt{P}}{100}$, where D = diameter of piston in inches, and P = initial steam pressure in lbs. per square inch.

Large Cast-iron Pistons.—

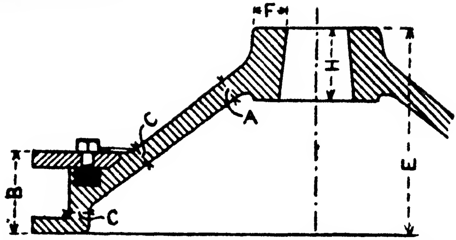


Unit for proportions $= \frac{D\sqrt{P}}{100}$, where D = diameter of piston in inches, and P = initial steam pressure in lbs. per square inch.

The junk ring bolts are made of wrought-iron or steel, and the nuts of brass. Diameter of junk ring bolts $= \frac{\cdot 28 D\sqrt{P}}{100} + \frac{1}{4}$ inch. Pitch of junk ring bolts = 7 to 10 times their diameter.

Number of internal ribs about $\frac{D}{10} + 2$.

Cast-steel Pistons.—

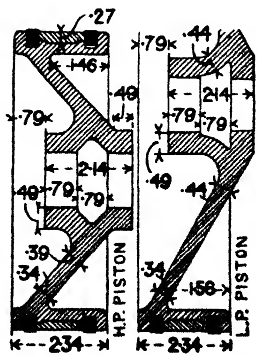


The following rules are based on examples from triple expansion marine engines:—

Unit for proportions $= \frac{D\sqrt{P}}{100}$, where D = diameter of *high pressure piston* in inches, and P = boiler pressure (above the atmosphere) in lbs. per square inch.

$A = \cdot48$ for h.-p. piston.
$A = \cdot54$ for i.-p. piston.
$A = \cdot64$ for l.-p. piston.
$B = 1\cdot8$ to $3\cdot1$, average $2\cdot2$.
$C = \cdot33$ for h.-p. piston.
$C = \cdot34$ for i.-p. piston.
$C = \cdot38$ for l.-p. piston.
$F = \cdot74$.
$E = 3\cdot8$ to $5\cdot4$, average $4\cdot6$, and is generally such as will make the sloping part of the low-pressure piston inclined at about 20°.
$H = 1\cdot5$ to $2\cdot7$, average $1\cdot7$.

The annexed illustrations show the form and proportions of cast-steel pistons introduced by the Rogers Locomotive Company, of Paterson, N.J., for their compound locomotives. The unit for the proportions is that already given, viz., $\frac{D\sqrt{P}}{100}$. The Ramsbottom packing-rings are of cast-iron, ¾ inch square in section. The cutting of the cylinder which sometimes takes place with cast-steel pistons is prevented by a broad cast-iron ring placed between the packing-rings. This ring is split at the top, and sprung on to the piston, to which it is secured by six ½-inch copper rivets.



The packing-rings of pistons which work in cylinders fitted with forged-steel liners should be made of hard bronze.

Piston-rods.—

d = diameter of piston-rod in inches.

l = length of piston-rod in inches.

$r = \frac{l}{d}$.

D = diameter of piston in inches.

p = greatest effective pressure on piston in lbs. per square inch,

$d = cD\sqrt{p}$, where $c = \frac{\sqrt{20250 + 16r^2}}{9000}$.

r =	10	15	20	25	30
c =	·0164	·0171	·0181	·0193	·0207

To calculate d, first assume a value for r, and use the corresponding value of c, to find the approximate value of d. Dividing l by this approximate value of d, a more correct value of r is obtained; and using the corresponding value of c, a sufficiently exact value of d is found.

For the piston-rods of oscillating cylinders, $d = \cdot 022D\sqrt{p}$.

Let d_1 = diameter of the screwed end of the rod at the bottom of the screw thread.

$d_1 = kD\sqrt{p}$.

$k = \cdot 013$ for large screws (d_1 greater than 2 inches) of wrought-iron.

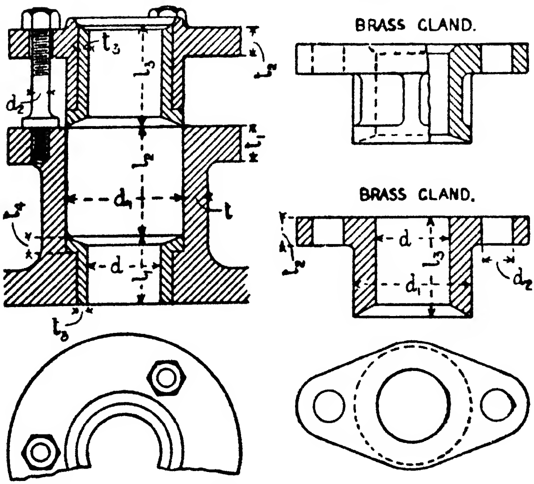
$= \cdot 011$ for large screws (d_1 greater than 2 inches) of steel.

For smaller screws, k is increased until

$k = \cdot 018$ for small screws (d_1 less than $\frac{3}{4}$ inch) of wrought-iron.

$= \cdot 016$ for small screws (d_1 less than $\frac{3}{4}$ inch) of steel.

STUFFING-BOXES.

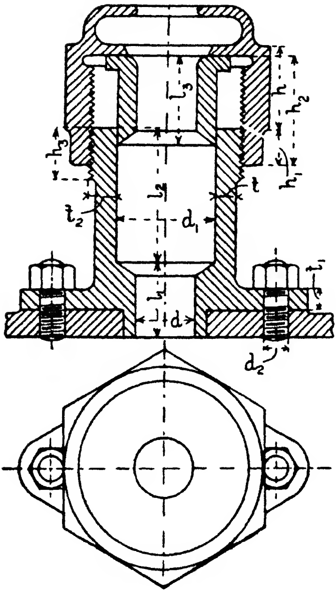


The proportions of stuffing-boxes and their glands vary considerably in actual practice. The following rules are based on a large number of examples:—

d = diameter of rod.	$t = \cdot 1d + \cdot 6$.
$d_1 = 1 \cdot 22d + \cdot 6$.	$t_1 = 1 \cdot 4t = \cdot 14d + \cdot 84$.
$l_1 = \cdot 4d + 1$.	$t_2 = t$.
$l_2 = d + 1$ to $1 \cdot 3d + 1$.	$t_3 = \cdot 04d + \cdot 2$, but not to exceed $\frac{1}{2}$ inch.
$l_3 = \cdot 75 l_2$.	$t_4 = \cdot 1d + \cdot 13$, but not to exceed 1 inch.

d_2 = diameter of bolts $= \cdot 12d + \cdot 5$ when two are used.

For n bolts, $d_2 = \frac{1 \cdot 6}{\sqrt{n}}(\cdot 12d + \cdot 5)$, when n is greater than two.

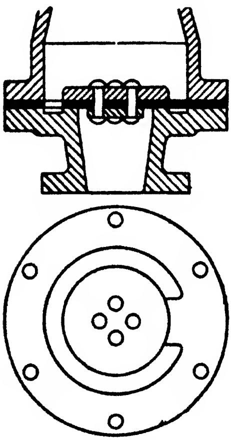


The annexed illustration shows a form of brass stuffing-box and gland suitable for small rods.

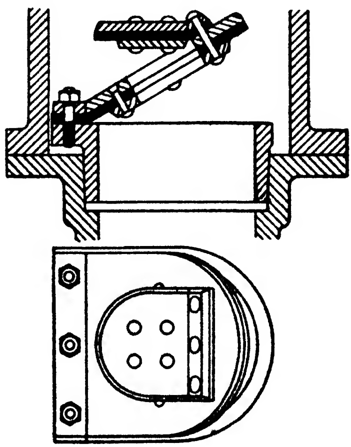
d = diameter of rod.
$d_1 = 1·3d + ·6$.
$d_2 = ·15d + ·5$.
$l_1 = ·4d + 1$
$l_2 = d + 1·5$
$l_3 = ·6d + 1$.
$t = ·1d + ·3$.
$t_1 = ·15d + ·5$
$t_2 = ·13d + ·4$.
$h = ·6d + 1$.
$h_1 = ·14d + ·4$.
$h_2 = ·67d + 1·2$.
$h_3 = ·3d + ·7$.

VALVES.

Flap or Clack Valves.—At one time flap or clack valves were very common, but they are now used to a very limited extent. These valves may be made entirely of brass or other metal, but they are generally made of leather stiffened by metal plates.



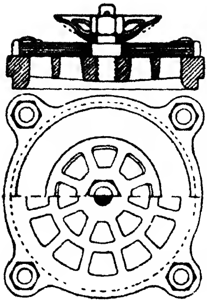
Single Flap Valve.



Double Flap Valve.

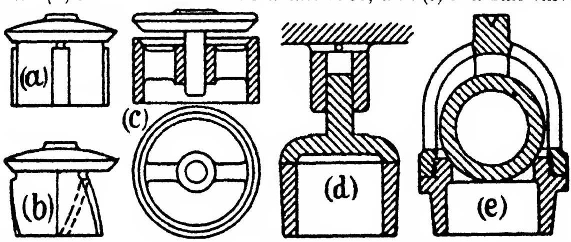
Large flap valves work very much smoother if made double, as shown above, the area of the opening of the inner valve being about one-third of the area of the opening of the main valve. Mr. Henry Teague states * that double flap valves 15 inches in diameter have worked incessantly for five years without changing a leather, and without showing the least sign of leakage, under 350 feet head of water, and without the slightest concussion. For a velocity of 160 feet per minute of the pump piston, Mr. Teague found that the weight of the flap should be about 2 lbs. per square inch. The width of the seat for a flap valve may be from one-eighth to one-twelfth of the diameter of the valve, and the flap should open about 35°.

* *Proceedings of the Institution of Mechanical Engineers*, 1887.



India-rubber Disc Valves.—The thickness of the india-rubber is generally $\frac{3}{8}$ inch to $\frac{1}{2}$ inch for small, and $\frac{5}{8}$ inch to $\frac{7}{8}$ inch for large valves. The area of the seat or grating in contact with the india-rubber should be sufficient to prevent the pressure between them exceeding 40 lbs. per square inch. The perforated guard which limits the lift of the valve may be either conical or spherical. If conical the slant side may slope to the valve seat at an angle of 30°, and if spherical its radius may be equal to three-fourths of the diameter of the india-rubber disc.

Single-beat Direct-lift Valves.—(*a*), (*b*), and (*c*) are conical disc valves. The face of the valve and its seat are parts of the surface of a cone whose slant side usually makes 45° with its axis. (*d*) is a disc valve with a flat face, and (*e*) is a ball valve.



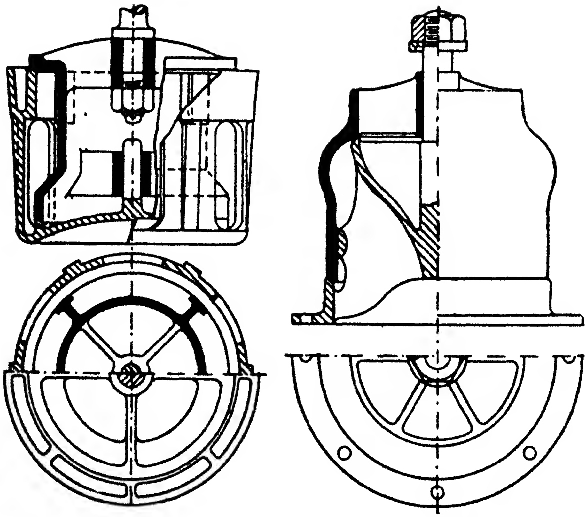
(*a*) and (*b*) are guided in rising and falling by feathers on their under sides, which fit into the cylindrical parts of their seats. The feathers on (*b*) are of a screw form, which enables the fluid to give the valve a rotary motion as it rises, and thus prevents the parts of the valve face from always beating on the same parts of the seat, and secures more uniform wear. (*c*) and (*d*) are guided by a central spindle, as shown.

The lift of a single-beat valve should not exceed one-fourth of its diameter, and when the valve is controlled automatically by the action of the fluid the lift is generally much less than this. When the pressure on the valve is great, the lift should not exceed $\frac{1}{4}$ inch.

The width of the valve seat may be as small as $\frac{1}{32}$ inch, and it is sometimes as much as $\frac{1}{4}$ inch. The narrower the seat the easier is it to make the valve tight, but the area of the seat must be sufficient to prevent the material of the valve and seat from being crushed.

These valves and their seats are generally made of brass.

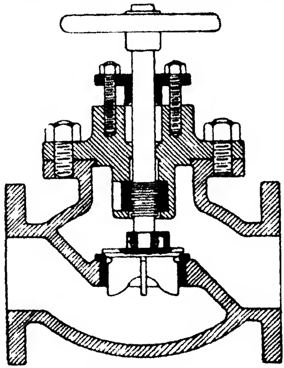
Double-beat Valves.—



These valves and their seatings are made of brass or bronze.

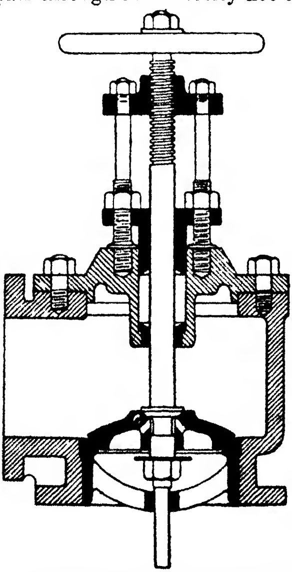
Let D_1 and D_2 be the diameters (in inches) of the larger and smaller seats of a double-beat valve, H the lift (in inches) which gives the maximum opening, and P the effective pressure of the fluid in lbs. per square inch. Then, $H=\frac{D_1^2}{4(D_1+D_2)}$, and the force required to open the valve is $\cdot7854(D_1^2-D_2^2)P$, neglecting the width of the seats and the weight of the valve.

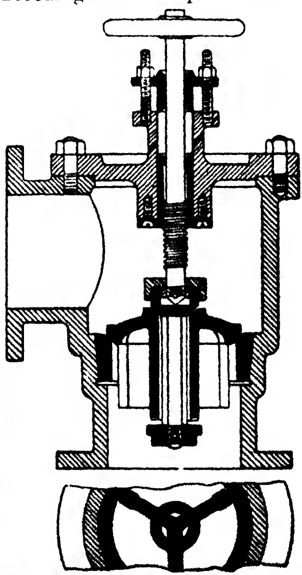
Stop Valves.—Stop valves on steam boilers are generally single-beat valves operated by a hand-wheel on a screwed spindle which passes through a stuffing-box in the valve casing. The valve and its seat are made of brass or gun-metal. The casing may be made of cast-iron, gun-metal, or cast-steel. The nut into which the screw on the spindle works may be at the bottom of the stuffing-box, but in large valves it is generally in a cross-head outside the casing.



The valve is full open when it is raised a distance equal to one quarter of its diameter.

The area of the valve must be sufficient to allow the steam to pass through at a velocity not exceeding 8000 feet per minute.

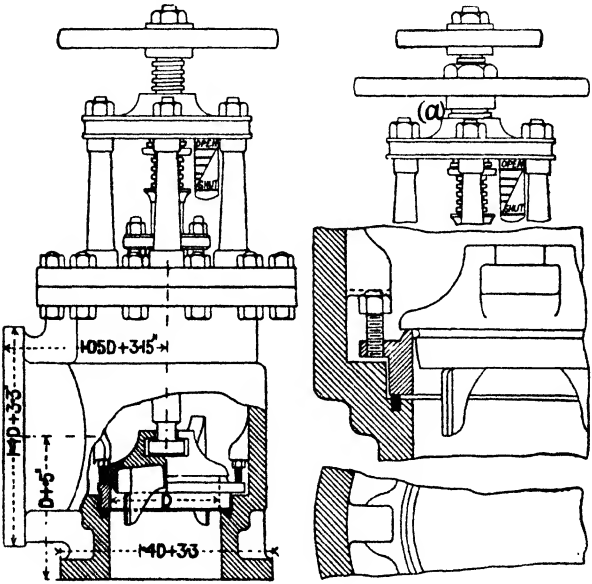




The above illustrations show two forms of stop valve with a

pilot, relief, or by-pass valve in the centre. The relief valve, being small, is easily opened, and when opened it allows the steam to surround the main valve, and thus place it in equilibrium ; the main valve is then easily opened.

Turnbull's is a good design of stop valve, and is illustrated below. The special features of this design are—(1) gradual



opening due to the curved ring on the under side of the valve, (2) free expansion seating, and (3) a seating which is easily removed.

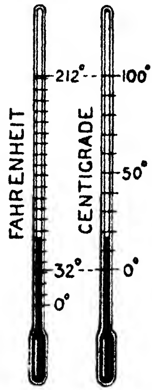
All large stop valves of the single-beat design, without relief valves, should be fitted with a second hand-wheel and differential screw arrangement to press the valve to its seat in closing, and to start it in opening. This arrangement is shown at (*a*) in the illustration of Turnbull's valve. The lower hand-wheel, or power-wheel, is fixed upon a screwed bush, the inside of which fits the screw on the valve spindle, while the outside fits into the screw in the cross-head. The outer screw on the bush being of greater pitch than the inner screw, the motion of the valve for one revolution of the power-wheel will be equal to the difference between the pitches of the two screws; and, as this may be made much smaller than the pitch of the screw on the valve spindle, a much smaller force at the circumference of the wheel will be sufficient to exert a great force on the valve. The smaller wheel on the valve spindle is used for rapidly opening or closing the valve.

HEAT.

Thermometric Scales.—The standard freezing temperature is the temperature of a mixture of water and ice under ordinary atmospheric pressure.

The standard boiling temperature is the temperature of steam from water under a pressure of 14·7 lbs. per square inch, or 29·9 inches of mercury at the standard freezing temperature.

On the *Fahrenheit scale* the freezing point is marked 32°, and on the *centigrade scale* it is marked 0°, or zero. On the Fahrenheit scale the boiling point is marked 212°, and on the centigrade scale it is marked 100°. The zero point on the Fahrenheit scale is 32° below the freezing point. Temperatures below the zero point on either scale are distinguished from those above the zero point by prefixing the sign − (minus) to them. Thus −10° means 10 degrees below zero.



Conversion of Thermometric Scales.—

F = temperature on the Fahrenheit scale.

C = equivalent temperature on the centigrade scale.

$$C=\frac{5}{9}(F-32). \qquad F=\frac{9}{5}C+32.$$

Examples.

(1) To convert 194° Fahr. into degrees cent.

$$C=\frac{5}{9}(194-32)=\frac{5}{9}\times 162=90^\circ.$$

(2) To convert 14° Fahr. into degrees cent.

$$C=\frac{5}{9}(14-32)=\frac{5}{9}\times(-18)=-10^\circ.$$

(3) To convert 20° cent. into degrees Fahr.

$$F=\frac{9}{5}\times 20+32=36+32=68^\circ.$$

Equivalent Temperatures—Centigrade to Fahrenheit.

	- 0° C.	- 10° C.	- 20° C.	- 30° C.	- 40° C.	- 50° C.	- 60° C.	- 70° C.	- 80° C.	- 90° C.
- 200° C.	- 328° F.	- 346° F.	- 364° F.	- 382° F.	- 400° F.	- 418° F.	- 436° F.	- 454° F.	..	..
- 100° C.	- 148	- 166	- 184	- 202	- 220	- 238	- 256	- 274	- 292	- 310
0° C.	+ 32	+ 14	- 4	- 22	- 40	- 58	- 76	- 94	- 112	- 130
	0° C.	**10° C.**	**20° C.**	**30° C.**	**40° C.**	**50° C.**	**60° C.**	**70° C.**	**80° C.**	**90° C.**
0° C.	32° F.	50° F.	68° F.	86° F.	104° F.	122° F.	140° F.	158° F.	176° F.	194° F.
100° C.	212	230	248	266	284	302	320	338	356	374
200° C.	392	410	428	446	464	482	500	518	536	554
300° C.	572	590	608	626	644	662	680	698	716	734
400° C.	752	770	788	806	824	842	860	878	896	914
500° C.	932	950	968	986	1004	1022	1040	1058	1076	1094
600° C.	1112	1130	1148	1166	1184	1202	1220	1238	1256	1274
700° C.	1292	1310	1328	1346	1364	1382	1400	1418	1436	1454
800° C.	1472	1490	1508	1526	1544	1562	1580	1598	1616	1634
900° C.	1652	1670	1688	1706	1724	1742	1760	1778	1796	1814
	0° C.	**100° C.**	**200° C.**	**300° C.**	**400° C.**	**500° C.**	**600° C.**	**700° C.**	**800° C.**	**900° C.**
1000° C.	1832° F.	2012° F.	2192° F.	2372° F.	2552° F.	2732° F.	2912° F.	3092° F.	3272° F.	3452° F.
2000° C.	3632	3812	3992	4172	4352	4532	4712	4892	5072	5252
3000° C.	5432	5612	5792	5972	6152	6332	6512	6692	6872	7052

Differences.

1° C.	2° C.	3° C.	4° C.	5° C.	6° C.	7° C.	8° C.	9° C.
1·8° F.	3·6° F.	5·4° F.	7·2° F.	9·0° F.	10·8° F.	12·6° F.	14·4° F.	16·2° F.

For differences of 10°, 20°, 30° C., etc., multiply the given values by 10.

Examples.—

- 156° C. = - 150° - 6° C.
= - 238° - 10·8° F. = - 248·8° F.

781° C. = 780° + 4° C.
= 1436° + 7·2° F. = 1443·2° F.

1650° C. = 1600° + 50° C.
= 2912° + 90° F. = 3002° F.

Equivalent Temperatures—Fahrenheit to Centigrade.

	-0° F.	-10° F.	-20° F.	-30° F.	-40° F.	-50° F.	-60° F.	-70° F.	-80° F.	-90° F.
-400° F.	-240·0° C.	-245·6° C.	-251·1° C.	-256·7° C.	-262·2° C.	-267·8° C.	..	..	..	..
-300° F.	-184·4	-190·0	-195·6	-201·1	-206·7	-212·2	-217·8	-223·3	-228·9	-234·4
-200° F.	-128·9	-134·4	-140·0	-145·6	-151·1	-156·7	-162·2	-167·8	-173·3	-178·9
-100° F.	- 73·33	- 78·89	- 84·44	- 90·00	- 95·56	-101·1	-106·7	-112·2	-117·8	-123·3
0° F.	- 17·78	- 23·33	- 28·89	- 34·44	- 40·00	- 45·56	- 51·11	- 56·67	- 62·22	- 67·78
	0° F.	**10° F.**	**20° F.**	**30° F.**	**40° F.**	**50° F.**	**60° F.**	**70° F.**	**80° F.**	**90° F.**
0° F.	-17·78° C.	-12·22° C.	- 6·67° C.	- 1·11° C.	4·44° C.	10·00° C.	15·56° C.	21·11° C.	26·67° C.	32·22° C.
100° F.	37·78	43·33	48·89	54·44	60·00	65·56	71·11	76·67	82·22	87·78
200° F.	93·33	98·89	104·4	110·0	115·6	121·1	126·7	132·2	137·8	143·3
300° F.	148·9	154·4	160·0	165·6	171·1	176·7	182·2	187·8	193·3	198·9
400° F.	204·4	210·0	215·6	221·1	226·7	232·2	237·8	243·3	248·9	254·4
500° F.	260·0	265·6	271·1	276·7	282·2	287·8	293·3	298·9	304·4	310·0
600° F.	315·6	321·1	326·7	332·2	337·8	343·3	348·9	354·4	360·0	365·6
700° F.	371·1	376·7	382·2	387·8	393·3	398·9	404·4	410·0	415·6	421·1
800° F.	426·7	432·2	437·8	443·3	448·9	454·4	460·0	465·6	471·1	476·7
900° F.	482·2	487·8	493·3	498·9	504·4	510·0	515·6	521·1	526·7	532·2
1000° F.	537·8	543·3	548·9	554·4	560·0	565·6	571·1	576·7	582·2	587·8
1100° F.	593·3	598·9	604·4	610·0	615·6	621·1	626·7	632·2	637·8	643·3
1200° F.	648·9	654·4	660·0	665·6	671·1	676·7	682·2	687·8	693·3	698·9
1300° F.	704·4	710·0	715·6	721·1	726·7	732·2	737·8	743·3	748·9	754·4
1400° F.	760·0	765·6	771·1	776·7	782·2	787·8	793·3	798·9	804·4	810·0

Equivalent Temperatures—Fahrenheit to Centigrade (*continued*).

	0° F.	10° F.	20° F.	30° F.	40° F.	50° F.	60° F.	70° F.	80° F.	90° F.
1500° F.	815·6° C.	821·1° C.	826·7° C.	832·2° C.	837·8° C.	843·3° C.	848·9° C.	854·4° C.	860·0° C.	865·6° C.
1600° F.	871·1	876·7	882·2	887·8	893·3	898·9	904·4	910·0	915·6	921·1
1700° F.	926·7	932·2	937·8	943·3	948·9	954·4	960·0	965·6	971·1	976·7
1800° F.	982·2	987·8	993·3	998·9	1004·4	1010·0	1015·6	1021·1	1026·7	1032·2
1900° F.	1037·8	1043·3	1048·9	1054·4	1060·0	1065·6	1071·1	1076·7	1082·2	1087·8

	0° F.	100° F.	200° F.	300° F.	400° F.	500° F.	600° F.	700° F.	800° F.	900° F.
2000° F.	1093·3° C.	1148·9° C.	1204·4° C.	1260·0° C.	1315·6° C.	1371·1° C.	1426·7° C.	1482·2° C.	1537·8° C.	1593·3° C.
3000° F.	1648·9	1704·4	1760·0	1815·6	1871·1	1926·7	1982·2	2037·8	2093·3	2148·9
4000° F.	2204·4	2260·0	2315·6	2371·1	2426·7	2482·2	2537·8	2593·3	2648·9	2704·4
5000° F.	2760·0	2815·6	2871·1	2926·7	2982·2	3037·8	3093·3	3148·9	3204·4	3260·0
6000° F.	3315·6	3371·1	3426·7	3482·2	3537·8	3593·3	3648·9	3704·4	3760·0	3815·6

Differences.

1° F.	2° F.	3° F.	4° F.	5° F.	6° F.	7° F.	8° F.	9° F.
0·56° C.	1·11° C.	1·67° C.	2·22° C.	2·78° C.	3·33° C.	3·89° C.	4·44° C.	5·00° C.

For differences of 10°, 20°, 30° F., etc., multiply the given values by 10.

Examples.—

−126° F. = −120° − 6° F.
= −84·44° − 3·33° C. = −87·8° C.

452° F. = 450° + 2° F.
= 232·2° + 1·11° C. = 233·3° C.

2870° F. = 2800° + 70° F.
= 1537·8° + 38·9° C. = 1576·7° C.

The Practical Measurement of Temperature (Abstracted from the *Cambridge Instrument Co., Ltd.*, publication: *Accurate Measurement of Temperature*).—The gas thermometer is of fundamental importance in determining standard temperatures, but it is too elaborate and difficult to be used in the majority of laboratories or workshops. Other means of measuring temperature have therefore to be devised. Any property of a substance which varies with temperature can theoretically form the basis of a temperature-measuring instrument. Many properties have been used, but those generally employed are as follows:—

1. The expansion of liquids.
2. The vapour pressure of liquids.
3. The electrical resistance of metal wires.
4. The thermo-electric potential between two dissimilar wires.

In addition, certain instruments do not measure temperatures directly, but measure the radiation emitted by the hot body, from which value the temperature of the body can be determined. The table on p. 545 classifies the main types of instruments, together with the ranges for which they are suitable.

Mercury-in-glass thermometers are still in general use, and within their range may be read to a high degree of accuracy. Mercury-in-steel and vapour-pressure thermometers are robust, and the dial may be in a convenient position at a distance from the heat-receiving bulb and a number of instruments may be grouped together. Resistance thermometers are robust and accurate, and suitable for centralised readings; they have advantages over the thermo-electric type for measurement of low temperatures. Thermo-electric thermometers are the most generally useful type for temperatures between 400° C. and 1200° C., being simple and reliable to a high degree of accuracy when properly installed and maintained. Radiation and optical pyrometers have a much higher range than the other types.

All temperature problems resolve themselves into dealing with a gas, a liquid, or a solid.

For gas temperatures it is usually only necessary to immerse the sensitive part of the thermometer in the gas and allow it to acquire the temperature. If the gas is not in motion there may be a time lag due to the small thermal capacity of the gas and the large capacity of the thermometer, while if it is in rapid motion the pressure and temperature changes will necessitate a very sensitive thermometer to follow them; in such cases the sensitive bulb should be placed parallel to the gas stream.

Liquids have a greater heat capacity and do not present the same difficulties.

The accurate measurement of the temperature of a solid body

Type.	Description.	Range and Remarks.	
Liquid expansion	Toluol or pentane in glass.	−200° C. to +30° C.	Indicating.
Liquid expansion	Mercury-in-glass.	−30° C. to +500° C.	Indicating.
	Mercury-in-steel.	−40° C. to 600° C. Readings can be transmitted up to distances of 120 feet.	Indicating and recording.
Vapour pressure	Ether or other organic liquid.	−50° C. to 400° C. Readings can be transmitted up to distances of 200 feet.	Indicating and recording.
Vapour pressure	Mercury-in-steel.	350° C. to 800° C. Readings can be transmitted up to distances of 120 feet.	Indicating and recording.
Electric resistance (change in resistance of a platinum wire with temperature).	Mica frame— (in which the wire is wound on mica and has a resistance of 2·5 ω or 25 ω at 0° C.).	−200° C. to +1000° C. Suitable for accurate laboratory work.	Subdivided into two groups of direct deflection or null measurement, can be indicating or recording.
	Steatite spool— (in which the wire is wound on steatite and then glazed, and has a resistance of 100 ω at 0° C.).	−100° C. to 500° C. Particularly suitable for the direct reading of air temperatures in buildings, power plants, ships, etc.	" "
	Copper tube— (in which the wire covered with silk is drawn through a thin-walled copper tube).	−100° C. to 140° C. for the quick measurement of temperature of liquids or for obtaining the mean temperatures of large enclosures.	" "
Thermo-electric	Platinum, platinum-rhodium.	−200° C. to 1500° C.	Subdivided into groups of direct deflection or null measurement, either indicating or recording.
	Base-metal titan wires.	−200° C. to 1200° C.	
	" iron constantan.	−200° C. to 800° C.	
	" copper constantan.	−200° C. to 500° C.	
Radiation (total)	Féry pyrometer.	600° C. upwards. Particularly useful for recording temperatures above 1000° C.	Indicating or recording.
Optical	Wanner type (polarised light).	700° C. upwards without limit.	Indicating.
	Disappearing filament.	700° C. to 4000° C.	Indicating.

is much more difficult. A thermocouple can sometimes be inserted in a small hole drilled into the body, but errors due to conduction along the thermocouple must be considered. In many cases the solid body is situated in a furnace, and unless the temperature is high enough for a radiation pyrometer to be used, it is the temperature of the gas surrounding the body that must be measured. There are now available many special forms of pyrometers designed for measuring the surface temperatures of solids, in cases where the temperatures do not exceed 800° C., and where it is possible to bring the pyrometer into actual contact with the surface.

Quantity of Heat—Unit of Heat.—A *unit of heat* is the quantity of heat required to raise a unit mass of pure water one degree in temperature.

The *British thermal unit of heat* (B.Th.U.) is the 1/180th part of the heat required to raise one pound of water from 32° F. to 212° F. This may be called the pound-degree Fahrenheit unit.

The *pound-degree centigrade* unit is the 1/100th part of the heat required to raise one pound of water from 0° C. to 100° C. This unit is called either the *centigrade heat unit* (C.H.U.) or the *pound-calorie* (lb.-cal.).

In the *gramme-degree* unit the gramme is the unit of mass, and the centigrade scale of temperature is used. This unit is called the *gramme-calorie*, or French unit of heat.

In the *kilogramme-degree* unit the kilogramme is the unit of mass, and the centigrade scale is used. This unit is called the *major calorie* or *kilogramme-calorie*, but frequently it is called simply the calorie.

The First Law of Thermodynamics—The Mechanical Equivalent of Heat.—Heat and mechanical work are mutually convertible, and one unit of heat is equivalent to a definite amount of mechanical work called the mechanical equivalent of heat.

The mechanical equivalent of heat is usually denoted by the letter J in honour of Joule, who did most to determine its numerical value.

If a quantity of work W is converted into H units of heat, then the first law of thermodynamics is expressed by the equation $W=JH$.

The value of J is 778 ft.-lbs. per B.Th.U., or 1400 ft.-lbs. per C.H.U.

Specific Heat of Solids and Liquids.—The specific heat of a substance is the ratio of the amount of heat required to raise its temperature 1° to the amount of heat required to raise the same weight of water 1°.

Substance.	Specific Heat	Substance.	Specific Heat
Aluminium	·214	Glass	·198
Antimony	·051	Stones from	·208
Bismuth	·031	Stones to	·218
Brass	·094	Coal, anthracite	·201
Copper	·095	Coke	·202
„ from 32° to 572° F.	·101	Charcoal	·241
Gold	·032	Sulphur	·203
Iron, cast	·130	Ice	·504
„ wrought	·114	Mercury	·033
Lead	·031	„ from 32° to 572° F.	·035
Nickel	·109	Olive oil	·310
Platinum	·033	Water, near 32° F.	1·0000
„ from 32° to 900° F.	·035	„ from 32° to 104° F.	1·0013
„ „ 32° to 2000° F.	·038	„ „ 32° to 176° F.	1·0035
Silver	·056	„ „ 32° to 248° F.	1·0067
Steel	·117	„ „ 32° to 320° F.	1·0109
Tin	·056	„ „ 32° to 392° F.	1·0160
Zinc	·094	„ „ 32° to 446° F.	1·0204

The specific heats given in the above table are the *mean* specific heats between the temperatures 32° and 212° Fahr., except in the case of ice and where other ranges of temperatures are stated.

The specific heat of a substance increases with the temperature.

The amount of heat required to raise the temperature of a given weight (w) of a substance, whose specific heat is s, through a given number of degrees (t), is equal to wts.

Specific Heat of Gases.—Gases have two specific heats: (1) specific heat at constant pressure, and (2) specific heat at constant volume. The former is greater than the latter, because when the pressure is constant the volume of the gas increases when heat is applied, and this heat has not only to raise the temperature, but it has to do work in expanding the gas against the external pressure.

The following table gives the specific heats of equal weights of various gases at constant pressure:—

Specific Heats of Gases at Constant Pressure.

Air	·2374	Carbonic acid	·2169
Hydrogen	3·4090	Carbonic oxide	·2450
Oxygen	·2175	Ammonia	·5084
Chlorine	·1210	Marsh gas	·5929
Nitrogen	·2438	Olefiant gas	·4040
Steam (gaseous)	·4805	Sulphurous acid	·1554

The simple gases air, hydrogen, oxygen, and nitrogen have the same specific heat *per unit of volume.*

Ratio of the two specific heats.—

$$\frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}} = 1{\cdot}408.$$

Latent Heat of Fusion.—The latent heat of fusion is the amount of heat required to convert a unit weight of a solid into the liquid state without raising its temperature.

Non-metallic Substances.	Melting Point. Fahr.	Latent Heat. B.Th.U.	Metals.	Melting Point. Fahr.	Latent Heat. B.Th.U.
Ice	32°	144	Bismuth . .	516°	23·4
Nitrate of soda	591	113·3	Cadmium. .	610	24·6
Nitrate of potass	642	85·3	Lead . . .	621	9
Phosphorus. .	111	9·1	Mercury . .	−37·8	5·4
Spermaceti . .	120	148	Silver . . .	1733	37·9
Sulphur. . .	239	16·9	Tin . . .	450	25·2
Wax. . . .	149	175	Zinc . . .	784	50·4

M. Person gives the following formula for the latent heat of fusion of non-metallic substances:—

$$l = (s' - s)(t + 256),$$

where l = latent heat of fusion in British thermal units.
s = specific heat of substance in the solid state.
s' = specific heat of substance in the liquid state.
t = temperature of fusion in degrees Fahrenheit.

Latent Heat of Evaporation.—The latent heat of evaporation is the amount of heat required to convert a unit weight of a liquid into the gaseous state without raising its temperature.

Latent Heat of Evaporation of various Liquids under a Pressure of one Atmosphere, or 14·7 Lbs. per Square Inch.

Liquid.	Specific Heat of Liquid.	Boiling Point. Fahr.	Latent Heat. B.Th.U.
Water	1·000	212°	970·7
Alcohol	·624	172·8	369
Bisulphide of carbon . .	·235	115	180
Ether	·516	94·1	162
Oil of turpentine . . .	·463	315	124
Wood spirit	·601	150	475

Specific Gravities and Boiling Points of Salt Water.—

Proportion of Salt in Water.	Specific Gravity.	Boiling Point. Fahr.	Proportion of Salt in Water.	Specific Gravity.	Boiling Point. Fahr.
$\frac{1}{32}$	1·029	213·2	$\frac{7}{32}$	1·203	220·3°
$\frac{2}{32}$	1·058	214·4°	$\frac{8}{32}$	1·232	221·5°
$\frac{3}{32}$	1·087	215·5	$\frac{9}{32}$	1·261	222·7°
$\frac{4}{32}$	1·116	216·7°	$\frac{10}{32}$	1·290	223·8°
$\frac{5}{32}$	1·145	217·9°	$\frac{11}{32}$	1·319	225·0°
$\frac{6}{32}$	1·174	219 1°	$\frac{12}{32}$	1·348	226·1°

The water is supposed to be under ordinary atmospheric pressure, corresponding to 30 inches of mercury in the barometer.

Ordinary sea water usually contains about $\frac{1}{32}$ of its weight of salt.

The freezing point of ordinary sea water is about 27° Fahr.

A saturated solution of salt water freezes at about 4° Fahr., and it contains about $\frac{12}{32}$ of its weight of salt.

Before the introduction of surface condensation the boilers of steam-ships were fed with sea water, and it was then the practice to prevent the proportion of salt in the boiler water from increasing beyond $\frac{3}{32}$. This necessitated blowing off from the boiler $\frac{1}{3}$ of all the water fed into it.

Melting Points or Temperatures of Fusion.—

Solid.	Cent.	Fahr.	Solid.	Cent.	Fahr.
Aluminium .	659	1218	Steel, mild. .	1475	2687
Antimony . .	630	1166	,, hard. .	1420	2588
Bismuth . .	271	520	Tin	232	450
Brass . . .	1030	1886	Zinc . . .	419	787
Bronze. . .	920	1688	Carbonic acid .	– 65	– 85
Copper. . .	1083	1981	Glass . . .	1100	2012
Gold . . .	1063	1945	Mercury . .	– 38·9	– 38·0
Iron, cast, grey	1220	2228	Nitro-glycerine	7·2	45
,, ,, white	1135	2075	Paraffin wax .	54	129
,, pure . .	1530	2786	Sulphur . .	115	239
Lead . . .	327	621	Sulphurous acid	– 76	– 105
Manganese .	1230	2246	Tallow . . .	33·3	92
Platinum . .	1775	3227	Turpentine .	– 10	14
Silver . . .	961	1761	Beeswax . .	65	149

Conduction of Heat.—

Relative Conductivity of Metals.

Silver . . .	100·0	Iron	11·9
Copper . . .	92·0	Steel	11·6
Gold . . .	53·2	Lead	8·5
Aluminium . .	48·0	Platinum . . .	8·4
Zinc . . .	27·0	Antimony . . .	4·0
Brass . . .	23·6	Bismuth . . .	1·8
Tin . . .	14·5	Mercury . . .	1·3

Transmission of Heat through Boiler Plates and Tubes.—

$$q = \frac{(t_1 - t_2)^2}{a} \text{ (Rankine's approximate formula),}$$

where q is the number of British thermal units of heat transmitted through one square foot of plate or tube in one hour, t_1 and t_2 the temperatures of the hotter and cooler sides of the plate respectively in degrees Fahrenheit, and a is a number which lies between 160 and 200.

Radiation and Absorption of Heat.—"The rate of radiation of heat by the hotter of a pair of bodies, and of its absorption by the colder, are increased by darkness and roughness of the surfaces of the bodies, and diminished by smoothness and polish" (*Rankine*)

Coefficients of Linear Expansion of Solids by Heat.—The coefficient of linear expansion of a solid by heat is the ratio of its increase in length for 1° to its length at 0° C. or 32° F.

Coefficients of Linear Expansion at Temperatures between 32° Fahr. and 212° Fahr.

Material.		For 1° Cent	For 1° Fahr.
Aluminium, cast		·0000222	·0000123
Aluminium, rolled		·0000207	·0000115
Antimony		·0000110	·0000061
Bismuth		·0000139	·0000077
Brass		·0000189	0000105
Copper		·0000171	·0000095
Gold		·0000153	·0000085
Iron, cast		·0000108	·0000060
Iron, wrought		·0000117	·0000065
Lead		·0000284	·0000158
Nickel		·0000126	·0000070
Platinum		·0000087	·0000048
Silver		·0000198	·0000110
Steel, untempered		·0000108	·0000060
Steel, tempered		·0000126	·0000070
Tin		·0000207	·0000115
Zinc		·0000288	·0000160
Brick, best stock		·0000055	·0000031
Fire-brick		·0000049	·0000027
Building stones	from	·0000072	·0000040
	to	·0000144	·0000080
Glass		·0000088	·0000049
Porcelain		·0000036	·0000020
Roman cement, dry		·0000144	·0000080
Slate		·0000104	·0000058
Wedgwood ware		·0000088	·0000049

Let a = coefficient of linear expansion.
L = length of solid at 0° C. or 32° F.
L_1 = length of solid at t_1° above 0° C. or 32° F.
L_2 = length of solid at t_2 above 0° C. or 32° F.

$$L_1 = L(1 + at_1). \qquad L_2 = L(1 + at_2).$$

$$L_1 = L_2\left(\frac{1 + at_1}{1 + at_2}\right). \qquad L_2 = L_1\left(\frac{1 + at_2}{1 + at_1}\right).$$

It is, however, generally sufficiently accurate to take

$$L_2 = L_1\{1 + a(t_2 - t_1)\}, \text{ and } L_1 = L_2\{1 - a(t_2 - t_1)\}.$$

That is, the increase or decrease in length is equal to the product of the rise or fall in temperature, the coefficient of expansion, and the original length.

Cubic Expansion of Solids by Heat.—The coefficient of cubic expansion of a solid by heat may, for all practical purposes, be taken as equal to three times the coefficient of linear expansion.

Real and Apparent Expansion of Liquids by Heat.—When heat is applied to a liquid the vessel containing the liquid is also heated, and both will undergo a change of volume. If the expansion of the liquid is determined from the rise of the level of the liquid in the vessel, neglecting the expansion of the vessel, this is called the *apparent expansion* of the liquid

Let E_a = coefficient of apparent expansion of a liquid,
E_v = coefficient of (cubic) expansion of containing vessel,
E_r = coefficient of real expansion of liquid,
then $E_r = E_a + E_v$ (very nearly).

If V is the volume of a vessel, and a the coefficient of *linear* expansion of the material of the vessel, then when its temperature is raised $t°$ its volume will become approximately $V(1 + 3at)$.

Absolute Expansion of Mercury (*Regnault*).—

Temperature (t) in Degrees.		Whole Expansion from 0° C to $t°$	Mean Coefficient of Expansion between 0° C. and $t°$		True Coefficient of Expansion at $t°$.	
Cent.	Fahr.		Cent.	Fahr	Cent.	Fahr.
0	32	...	...	...	·0001791	·0000995
50	122	·009013	·0001803	·0001001	·0001815	·0001008
100	212	·018153	·0001815	·0001008	·0001841	·0001023
150	302	·027419	·0001828	·0001016	·0001866	·0001037
200	392	·036811	·0001841	·0001023	·0001891	·0001051
250	482	·046329	·0001853	·0001030	·0001916	·0001064
300	572	·055973	·0001866	·0001037	·0001941	·0001078
350	662	·065743	·0001878	·0001044	·0001967	·0001093

For absolute expansion of water see p. 342.

Change of Volume and Density of Solids and Liquids by Heat.—

Let A = real coefficient of cubic expansion of solid or liquid.

V_1 = volume at temperature $t_1°$ above 0° C. or 32° F.

V_2 = ,, ,, ,, $t_2°$,, ,,

W_1 = weight ,, ,, $t_1°$,, ,,

W_2 = ,, ,, ,, $t_2°$,, ,,

$$\frac{V_2}{V_1}=\frac{1+At_2}{1+At_1}. \qquad \frac{W_2}{W_1}=\frac{V_1}{V_2}=\frac{1+At_1}{1+At_2}.$$

$$=1+A(t_2-t_1) \text{ nearly.} \qquad =1-A(t_2-t_1) \text{ nearly.}$$

Relations between the Volume, Pressure, and Temperature of a Gas.—The following formulæ are nearly true for the more permanent gases, such as oxygen, nitrogen, hydrogen, and atmospheric air :—

(1) *Temperature constant.*

Let V_1 denote the volume of a given quantity of a gas, and P_1 its pressure. Let the volume V_1 be changed to V_2, then P_1 will change to P_2, and

$$\frac{V_1}{V_2}=\frac{P_2}{P_1}, \text{ or } P_1V_1=P_2V_2 \text{ (\textit{Boyle's law})},$$

(2) *Pressure constant.*

Let V_1 denote the volume of a given quantity of a gas, and t_1 its temperature. Let the temperature t_1 be changed to t_2, then V_1 will change to V_2, and

$$\frac{V_1}{V_2}=\frac{1+at_1}{1+at_2} \text{ (\textit{Charles' or Gay Lusac's law})},$$

where a is the coefficient of expansion of the gas for 1° of temperature. Values of a for different gases are given in the table on the opposite page.

(3) *Volume constant.*

Let P_1 denote the pressure of a given quantity of a gas, and t_1 its temperature. Let the temperature t_1 be changed to t_2, then P_1 will change to P_2, and $\frac{P_1}{P_2}=\frac{1+ct_1}{1+ct_2}$,

where c is the coefficient of change of pressure of the gas for 1° of change of temperature. Values of c for different gases are given in the table on the following page.

If the gas conforms strictly to Boyle's law, then $a=c$.

In the above formulæ the temperatures are in degrees above the freezing temperature of water; hence, if the temperatures t_1 and t_2 are on the Fahrenheit scale, t_1-32 and t_2-32 must be substituted for t_1 and t_2 respectively in the formulæ. The pressures are *absolute* pressures.

The values of a and c given in the following table are those determined by Regnault:—

Gas.	a		c	
	Cent	Fahr	Cent	Fahr
Air	·003670	·002039	·003665	·002036
Nitrogen			·003668	·002038
Hydrogen	·003661	·002034	·003667	·002037
Carbonic oxide	·003669	·002038	·003667	·002037
Carbonic acid	·003710	·002061	·003688	·002049
Nitrous oxide	·003720	·002067	·003676	·002042
Cyanogen	·003877	·002154	·003829	·002127
Sulphurous acid	·003903	·002168	·003845	·002136

In ordinary calculations on the more permanent gases it is usual to take $a=c=\frac{1}{273}$ for temperatures on the centigrade scale, and $=\frac{1}{492}$ for the Fahrenheit scale.

The Air Thermometer—Absolute Temperature.—A thermometer in which the expanding substance is air has the following advantages over the mercurial thermometer:—

It is much more sensitive, since the expansion of air is more than twenty times that of mercury for the same change of temperature.

The variation of volume of air for a given variation of temperature is the same at all temperatures.

The expansion of air is so great compared with that of the vessel containing it, that the expansion of the latter has very little effect on the reading of the air thermometer.

The specific heat of air is the same at all temperatures.

Air requires much less heat to change its temperature than an equal bulk of any liquid.

The air thermometer may be used for very high or very low temperatures.

The great objection to the use of the air thermometer is that it must be used in conjunction with the barometer, and a calculation has to be made before a temperature is determined. The air thermometer is therefore not used for ordinary observations of temperature.

A volume of air equal to 273 cubic inches at 0° C. becomes 373 cubic inches at 100° C., hence on the air thermometer it is convenient to denote the freezing point of water by 273 and the boiling point by 373. The zero on the air thermometer is therefore 273° C. below the freezing point of water. This is called **absolute zero** Temperatures on the air thermometer are called **absolute temperatures**, because they correspond very closely to the absolute scale of temperature which is based on thermodynamic considerations.

The absolute zero by Fahrenheit's scale is 492° F. below the freezing point of water, or 460° F. below the zero on that scale.

Absolute temperature in centigrade degrees is obtained by adding 273 to the ordinary temperature centigrade.

Absolute temperature in Fahrenheit degrees is obtained by adding 460 to the ordinary temperature Fahrenheit.

The numbers 273 and 460 given above may be used for all ordinary purposes; but the former is probably barely large enough, and the latter rather too large.

It has been stated (p. 552) that

$$\frac{V_1}{V_2}=\frac{1+at_1}{1+at_2}=\frac{1+\frac{1}{273}t_1}{1+\frac{1}{273}t_2}=\frac{273+t_1}{273+t_2},$$

using the centigrade scale.

But $273+t_1$ and $273+t_2$ are the absolute temperatures corresponding to the ordinary temperatures t_1 and t_2, and if these absolute temperatures are denoted by T_1 and T_2, then

$$\frac{V_1}{V_2}=\frac{T_1}{T_2} \text{ when the pressure is constant,}$$

$$\text{also } \frac{P_1}{P_2}=\frac{T_1}{T_2} \text{ when the volume is constant,}$$

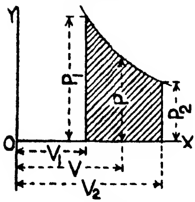
$$\text{and } \frac{P_1V_1}{P_2V_2}=\frac{T_1}{T_2} \text{ when the pressure and volume both vary.}$$

For weight and volume of air and other gases see p. **346.**

Isothermal Expansion or Compression of a Gas.—When a given quantity of a gas has its volume and pressure changed under such conditions that its temperature remains constant, the change is said to take place *isothermally*, and the curve which exhibits the relation between the volume and pressure is called an isothermal curve.

When a gas is compressed slowly in a metal cylinder, the heat produced during the compression is dissipated and the temperature remains nearly constant, so that the compression takes place nearly isothermally.

The isothermal curve for a perfect gas is a rectangular hyperbola, the axes OX and OY being the asymptotes of the curve.



The equation to the curve is PV = constant.

If a volume of gas V_1 having a pressure P_1 be expanded isothermally until its volume is V_2 and its pressure P_2, or if a volume of gas V_2 having a pressure P_2 be compressed isothermally until its volume is V_1 and its pressure P_1, then the work done during the expansion or compression is given by the formula,

$$\text{work done} = P_1 V_1 \log_e \frac{V_2}{V_1} = P_2 V_2 \log_e \frac{V_2}{V_1},$$

$$= mP_1 V_1 \log_{10} \frac{V_2}{V_1} = mP_2 V_2 \log_{10} \frac{V_2}{V_1},$$

where $m = 2{\cdot}302585$.

If the pressures be stated in lbs. per square foot, and the volumes in cubic feet, the work done will be expressed in foot-pounds.

Adiabatic Expansion or Compression of a Gas.—When a given quantity of a gas has its volume and pressure changed under such conditions that no heat enters or leaves the gas during the operation, the change is said to take place *adiabatically*, and the curve which exhibits the relation between the volume and pressure is called an *adiabatic curve.*

In practice the expansion or compression of a gas is approximately adiabatic when the change of volume takes place rapidly, and the change is more approximately adiabatic the better the non-conducting power of the containing vessel.

Let the volume of a given quantity of gas be V_1, its pressure P_1, and its absolute temperature T_1. Now suppose this quantity of gas to change adiabatically so that its volume becomes V_2, its pressure P_2, and its absolute temperature T_2, then,

$P_1 V_1^n = P_2 V_2^n$ which gives the relation between the pressure and volume,

$T_1 V_1^{n-1} = T_2 V_2^{n-1}$ which gives the relation between the temperature and volume,

$\left(\frac{T_1}{T_2}\right)^n = \left(\frac{P_1}{P_2}\right)^{n-1}$ which gives the relation between the temperature and pressure,

where n = ratio of the specific heat at constant pressure to the specific heat at constant volume = 1·408.

The foregoing equations require logarithms for their solution, thus,

$$\log P_1 + n \log V_1 = \log P_2 + n \log V_2$$
$$\log T_1 + (n-1) \log V_1 = \log T_2 + (n-1) \log V_2$$
$$n \log T_1 - n \log T_2 = (n-1) \log P_1 - (n-1) \log P_2.$$

The work done when the volume changes from V_1 to V_2 is given by the formula,

$$\text{work done} = \frac{P_1 V_1 - P_2 V_2}{n-1}.$$

The Critical Temperature of a Gas.—Any gas may be liquefied by pressure, provided its temperature is not higher than a certain temperature called its *critical temperature.* Above the critical temperature the substance can only exist as a gas, however great be the pressure.

If a gas is to be liquefied by pressure, it must therefore have its temperature reduced at least to its critical temperature. The lower the temperature of the gas is, the smaller is the pressure required to liquefy it.

The following table gives (approximately) the critical temperatures of various gases:—

Gas.	Critical Temperature.	
	Cent	Fahr.
	°	°
Carbonic acid	30·9	87·6
Ether	190	374
Carbon bisulphide	272	521·6
Alcohol	234	453·2
Chloroform	260	500
Oxygen	−112	−169·6
Nitrogen	−146	−230·8

COMBUSTION AND FUEL.

Constituents of Fuel.—The combustible constituents of fuel are carbon and hydrogen, and in coal there is generally also a small quantity of sulphur. The incombustible constituents are oxygen and nitrogen, and the various mineral substances which go to form the ash which is left after the fuel is burned.

When a fuel contains oxygen this element is in combination with hydrogen in the form of water, and this water is either combined with other constituents of the fuel, or it is present as moisture, which may be expelled by drying.

Fuels which contain any considerable quantity of water, either combined or free, are objectionable, because when they are burned in a furnace in the ordinary way a large amount of heat passes away with the waste gases as latent heat in the steam formed.

Combustion in Oxygen.—1 lb. of *hydrogen* (H) combines with 8 lbs. of *oxygen* (O) to form 9 lbs. of *water* (H_2O) or *steam.*

1 lb. of *carbon* (C) combines with $1\frac{1}{3}$ lbs. of *oxygen* (O) to form $2\frac{1}{3}$ lbs. of *carbonic oxide* (CO).

1 lb. of *carbon* (C) combines with $2\frac{2}{3}$ lbs. of *oxygen* (O) to form $3\frac{2}{3}$ lbs. of *carbonic acid* (CO_2).

1 lb. *sulphur* (S) combines with 1 lb. of *oxygen* (O) to form 2 lbs. of *sulphurous acid* (SO_2).

Composition of Air.—For all questions on combustion, atmospheric air may be taken as composed of 23 parts by weight of *oxygen* and 77 parts of *nitrogen.*

Weight of air containing 1 lb. of oxygen $=\frac{100}{23}=4{\cdot}348$ lbs.

Hence, weight of air required for the combustion of any fuel equals 4·348 times the weight of oxygen required.

Air required for Combustion of Fuel.—1 lb. of hydrogen requires 8 lbs. of oxygen, or 34·78 lbs. of air, say, 35 lbs.

1 lb. of carbon requires $2\frac{2}{3}$ lbs. of oxygen, or 11·6 lbs. of air, say, 12 lbs., for the complete combustion of the carbon to carbonic acid.

1 lb. of sulphur requires 1 lb. of oxygen, or 4·35 lbs. of air.

In estimating the amount of oxygen or air required for the combustion of fuel, the amount of oxygen already in the fuel must be taken into account.

Let $C=$ weight of carbon in 1 lb. of fuel.
$H=$ weight of hydrogen in lb. of fuel.
$O=$ weight of oxygen in 1 lb of fuel.
$W=$ weight of air required for combustion of 1 lb. of fuel.

Then neglecting the sulphur, which is usually small in amount or entirely absent,

$$W = 12C + 35\left(H - \frac{O}{8}\right).$$

This is the *minimum* amount of air required. In actual practice it is seldom possible to ensure that all the oxygen in the air supplied shall combine with the constituents of the fuel, and a considerable excess of air is usually necessary. The excess of air varies from 50 to 100 per cent., so that the actual weight of air supplied to a furnace is generally from $1\frac{1}{2}W$ to $2W$.

For coal and coke W may be taken as 12 lbs.

12 lbs. of air at 62° Fahr., and under a pressure of one atmosphere, occupies about 158 cubic feet.

The following table gives, approximately, the minimum weight of air required for the combustion of 1 lb. weight of various fuels, and its volume at the normal temperature and under a pressure of one atmosphere (14·7 lbs. per square inch) :—

Fuel.	Weight of Air in Lbs	Vol. of Air at 62° F. in Cubic Feet.
Straw, with 16 per cent. of water	4·4	57·8
Wood, dry	6·1	80·2
Peat, dry	7·6	99·9
Charcoal, from wood, dry	10·8	141·9
Charcoal, from peat, dry	9·7	127·5
Coal: Lignite, air dried	9·3	122 2
Coal: Bituminous, average	11	144·6
Coal: Anthracite, average	11·7	153·7
Coke, average	11	144·6
Petroleum	14·5	190 5

Total Heat of Combustion, or Calorific Value of a Fuel.—The calorific value of a fuel is the number of units of heat produced by the combustion of 1 lb. weight of it. The unit of heat is the amount of heat required to raise 1 lb of water 1° Fahr.

The calorific value of a fuel is independent of the *rate* of combustion.

The calorific values of various fuels are given in the table on p. 560.

The calorific value of a fuel may be determined experimentally by burning a sample of it, of known weight, in a specially constructed calorimeter in which the products of combustion are cooled by passing them through water. The weight of the water and its increase of temperature being known, the total amount of heat given out by the combustion of the sample is determined.

The calorific value of a compound of carbon and hydrogen, as determined by experiment, is generally a little less than the sum of the calorific values of its elements. In the case of olefiant gas (C_2H_4) there is practically no such difference, but with marsh gas (CH_4) the calorific value, as determined by experiment, is 23513, and, by calculation from the calorific values of its elements, the result is 26416, or a difference of 2903 heat units.

In calculating the calorific value of a fuel from its chemical composition it is usual to assume that it is equal to the sum of the calorific values of its elements. In estimating the heating value of the hydrogen, care must be taken to neglect as much of it as is in combination with the oxygen in the fuel.

When the calorific value of hydrogen is determined by experiment, the steam formed gives up its latent heat to the water in the calorimeter; but when the hydrogen is burned in an ordinary furnace, the latent heat of the steam formed is not available, and must therefore be deducted from the total heat of combustion of the hydrogen. 1 lb. of hydrogen combines with 8 lbs. of oxygen to form 9 lbs. of steam whose latent heat $=966 \times 9=8694$. Deducting this from 62032, the total heat of combustion of hydrogen gives 53338 units of heat per lb. of hydrogen.

Let C = weight of carbon in 1 lb. of fuel.
H = weight of hydrogen in 1 lb. of fuel.
S = weight of sulphur in 1 lb. of fuel.
O = weight of oxygen in 1 lb. of fuel.
h = calorific value of 1 lb. of fuel.

$$h=14544C+53338\left(H-\frac{O}{8}\right)+3996S.$$

$$=14544\left\{C+3{\cdot}667\left(H-\frac{O}{8}\right)+{\cdot}275S\right\}.$$

Carbon Value of Fuel.—The carbon value of any fuel is the weight of carbon in lbs. having the same heating value as 1 lb. of the fuel. Carbon value equals calorific value of fuel divided by calorific value of carbon.

Theoretical Evaporative Power of Fuel.—The theoretical evaporative power of fuel is stated in lbs. of water evaporated from and at 212° Fahr., and is obtained by dividing the calorific value of the fuel by 971.

Actual Evaporative Power of Coal in Steam Boilers.—From numerous experiments on steam boilers it appears that the actual evaporative power of coal varies from 50 per cent. to

85 per cent. of the theoretical evaporative power. An average of a considerable number of tests gave the actual evaporative power equal to 70 per cent. of the theoretical evaporative power of the coal.

Calorific Value, Carbon Value, and Evaporative Power of Various Fuels.—

Combustible		Calorific value in British Thermal Units.	Carbon Value	Evaporative Power in Lbs. of Water from and at 212° Fahr.
Carbon, burned to carbonic acid		14544	1·000	15·06
Carbon, burned to carbonic oxide		4451	·306	4·61
Carbonic oxide		4325	·297	4·48
Marsh gas		23513	1·617	24·34
Olefiant gas		21344	1·468	22·10
Hydrogen		62032	4·265	64·22
Hydrogen, deducting latent heat in steam formed		53338	3·667	55·22
Sulphur		3996	·275	4·14
Straw, with 16 per cent. water		5200	·358	5·38
Wood, kiln dried		8000	·550	8·28
Wood, air dried, with 20 per cent. water		5600	·385	5·80
Peat, kiln dried		10000	·688	10·35
Peat, air dried, with 20 per cent. water		6500	·447	6·73
Charcoal from wood, dry		13000	·894	13·46
Charcoal from peat, dry		11600	·798	12·01
Coal: Lignite, air dried		11000	·756	11·39
Coal: Bituminous	from	13000	·894	13·46
	to	15700	1·079	16·25
	average	14100	·969	14·60
Coal: Anthracite	from	14000	·963	14·49
	to	16200	1·114	16·77
	average	15000	1·031	15·53
Coke	from	12000	·825	12·42
	to	13700	·942	14·18
Block fuel	average	15000	1·031	15·53
Petroleum		20000	1·375	20·70
Natural gas (Pennsylvanian)		26000	1·788	26·92

Mean Specific Heat of Products of Combustion.—

Let w_1, w_2, w_3, etc., be the weights of the separate gases and the ash making up the products of the combustion of 1 lb. of fuel, including surplus air.

s_1, s_2, s_3, etc., the specific heats of these gases and ash respectively.

$w = w_1 + w_2 + w_3 +$ etc., the total weight of the products of the combustion of 1 lb. of fuel, including surplus air.

s, the mean specific heat of the products of combustion.

$$s = \frac{s_1 w_1 + s_2 w_2 + s_3 w_3 + \text{etc.}}{w}.$$

The following are the specific heats of gases (under constant pressure) found in the products of combustion of fuel: Air, ·238; oxygen, ·218; nitrogen, ·244; carbonic oxide, ·245; carbonic acid, ·217; steam, ·475; sulphurous acid, ·155.

The specific heat of the ash is about ·2.

Calorific Intensity, or Temperature of Combustion.—

h = total heat of combustion of 1 lb. of fuel.

w = total weight of products of combustion of 1 lb. of fuel, including surplus air.

s = mean specific heat of products of combustion.

t = ideal temperature (in degrees Fahr.) of products of combustion, and the surplus air mixed with them at the instant that the combustion is complete, measured from the initial temperature.

$$t = \frac{h}{sw}.$$

Examples of Calorific Intensity of Fuels.

FUEL. 1 lb. weight of fuel with minimum supply of air for complete combustion, unless otherwise stated.	Weight of Products of Combustion in Lbs. (w)	Mean Specific Heat of Products of Combustion. (s)	Ideal Temperature of Combustion. (t)
Hydrogen	35·8	·302	4933
Carbon	12·6	·236	4891
Marsh gas	18·4	·268	4768
Olefiant gas	15·9	·257	5223
Sulphur	5·35	·211	3540
Wood, dry	7·1	·255	4419
Coal, average	12·0	·247	4892
Coal, average, with 50 per cent. surplus air	17·5	·244	3396
Coal, average, with 100 per cent. surplus air	23·0	·242	2605
Coke	12·0	·236	4590
Petroleum	15·5	·256	5040

Varieties of Fuel.

(Revised by J. W. Farmery, M.A., A.I.C., D.I.C.)

Wood.—Thoroughly dried wood contains about 50 per cent. of carbon, 6 per cent. of hydrogen, 43 per cent. of oxygen, less than 1 per cent. of nitrogen plus sulphur, and a trace of ash. On account of the large amount of oxygen present, less than 1 per cent. of hydrogen is available for combustion, and the heat of combustion of wood is almost entirely due to the carbon it contains.

Ordinary air-dried firewood usually contains about 20 per cent. of moisture, so that its composition per cent. is: carbon, 40; hydrogen, 4·8; oxygen, 34·4; less than 0·8 of nitrogen plus sulphur; a trace of ash; and water, 20.

Newly felled wood contains from 20 to 50 per cent. of moisture.

Peat, or turf, consists of the decomposed remains of vegetable matter, generally mosses and aquatic plants. As found, it usually contains over 90 per cent. of moisture, but after air-drying about 20 per cent.

The mean composition of perfectly dry peat is approximately: carbon, 57; hydrogen, 6; oxygen, 32; nitrogen and sulphur, 1·5; and ash, 3·5 per cent.

The specific gravity of peat in its ordinary state varies from ·3 to 1, and when compressed it varies from ·9 to 1·8.

Coal is the product of vegetable matter which has, during the course of ages, been decomposed and solidified under great pressure.

The principal varieties of coal are as follows:—I. Lignite. II. Bituminous coals, including long-flame non-caking coals, caking coals, and cannel coals. III. Semi-bituminous (non-caking) coals. IV. Anthracitic coals and true anthracite.

The gradual conversion of woody fibre into peat and the different kinds of coal is shown in the following table:—

	Carbon.	Hydrogen.	Oxygen.
Wood	100	12	88
Peat	100	10·5	56
Lignite	100	7·7	52
Bituminous coal . .	100	6	21
Anthracite . . .	100	3	3·5

Lignite or brown coal is intermediate in appearance and properties between peat and true coal. It burns with a very long smoky flame, and it is generally non-caking. After drying in the air lignite contains from 15 to 20 per cent. of moisture. If thoroughly dried in a stove and again exposed to the air it reabsorbs the water which it lost in drying. The composition of lignite varies considerably. A sample of good quality when thoroughly dried contained 57 per cent. of carbon, 5·7 per cent. of hydrogen, 32 per cent. of oxygen, 1·6 per cent. of nitrogen plus sulphur, and 3·7 per cent. of ash. The specific gravity varies from 1·2 to 1·3.

Cannel coal burns with a long flame, and gives off large quantities of smoke. It contains a relatively large amount of disposable hydrogen. Specific gravity 1·27 to 1·32.

Caking bituminous coal softens and swells when heated, and the parts adhere together, forming a pasty mass. It burns with a fairly long flame, and requires careful stoking to avoid smoke. Specific gravity 1·26 to 1·36.

Non-caking or dry bituminous coal burns with a shorter flame than that of the caking coal, and it gives off little or no smoke. Specific gravity 1·28 to 1·42.

Anthracite burns without flame or smoke, and with an intense local heat, but it requires a strong draught for its combustion. It is hard and brittle, and many varieties decrepitate considerably when heated, especially when the heat is applied suddenly. Care has therefore to be taken that the fire is so managed that the small pieces do not fall through between the fire-bars and get lost. Specific gravity 1·35 to 1·7.

Briquette fuel is usually made by mixing coal dust with pitch or some other binding material, the mixture being pressed and formed into hard blocks of rectangular shape. Good briquette fuel contains about 7 per cent. of ash, 8 per cent. of pitch, and 3 per cent. of moisture. Its calorific value is about 14,500 B.Th.U. per lb. One ton of briquette fuel occupies about 37 cubic feet of space.

For full information on the manufacture of briquette fuel see *Minutes of Proceedings of the Institution of Civil Engineers*, vol. cxviii.

Wood charcoal is made by heating wood out of contact with the atmosphere, or with only a limited supply of air, to a temperature not lower than 550° Fahr. The higher the temperature, the blacker and harder is the charcoal produced. The yield of charcoal varies from 15 to 25 per cent. by weight of the wood from which it is produced, the yield being lower the higher the temperature. Dry charcoal contains from 80 to 95 per cent. of carbon, ·5 to 3 per cent. of available hydrogen, and 1 to 5 per cent. of ash, the remainder being nitrogen and combined oxygen and hydrogen. Charcoal which has been exposed to the air usually contains from 5 to 12 per cent. of moisture.

Peat charcoal is prepared from peat in the same manner that wood charcoal is made from wood. Good peat charcoal when perfectly dry contains from 80 to 90 per cent. of carbon, and 10 to 15 per cent. of ash. It is usually extremely friable.

Coke is the solid carbonaceous material left after coal has been heated out of contact with air. High temperature coke is made at 1200°–1300° C., and low temperature coke or semi-coke at 500°–750° C.

The best coke for fuel is prepared from bituminous coals. It is hard, brittle, and porous, of a dark grey colour and slightly metallic lustre.

The yield of coke from bituminous coals is from 50 to 80 per cent. of the weight of the coal. Anthracite yields from 80 to 95 per cent. of powdery residue, of no commercial value.

Good dry coke contains from 85 to 95 per cent. of carbon, ·25 to 2 per cent. of sulphur, and 4 to 12 per cent. of ash. Exposed to the air, it absorbs from 10 to 20 per cent. of moisture.

Petroleum, or natural mineral oil, contains from 82 to 87 per cent. of carbon, 11 to 15 per cent. of hydrogen, and ·1 to 5 per cent. of oxygen.

The following table gives the specific gravity, composition, gross calorific value, and flash point of various oil fuels and, for the purpose of comparison, particulars for average English coal:—

Fuel.	Specific Gravity at 60° F.	Composition. Per Cent.			Gross Calorific Value B.Th.U./Lbs.	Flash Point.
		Carbon.	Hydrogen.	Oxygen plus Sulphur.		
Kerosene . .	0·792	86·4	13·5	0·1	20,050	100° F. (Abel)
Gas oil . .	0·870	86·3	12·7	1·0	19,760	170° F. (P.-M.)
Light fuel oil .	0·895	86·1	12·3	1·6	19,260	175° F. (P.-M.)
Heavy fuel oil	0·950	86·0	11·8	2·2	18,900	232° F. (P.-M.)
Good English coal, mean of 98 samples	1·380	80·0	5·0	8·0+1·25	14,112	..

Natural gas issues from strata at depths from 500 feet to 2000 feet, and reaches the surface at a mean pressure of 150 to 200 lbs. per square inch. When first reached the pressure is very high (1000 lbs. per square inch is not unusual). The density is from ·45 to ·75 (air = 1); calorific value 750 to 1600 B.Th.U. per cubic foot.

Natural gas is important as the source of *natural petrol*, and is used locally to provide heat and light.

STEAM.

Saturated Steam.—Steam at a given temperature is said to be *saturated* when it is of maximum density for that temperature. Steam in contact with water is saturated steam.

Wet or Supersaturated Steam.—Steam which has water (in the form of small drops) suspended in it is called *wet or supersaturated steam*. If wet steam be heated until all the water suspended in it is evaporated, it is said to be dried.

Superheated Steam.—If dry saturated steam be heated when not in contact with water, its temperature is raised and its density is diminished or its pressure is raised. The steam is then said to be superheated.

Dryness Fraction of Steam.—Let W = weight of a given quantity of wet steam, w = weight of water suspended in this steam, then dryness fraction $= \frac{W-w}{W}$.

Under ordinary conditions and good stoking the dryness fraction is about 95 per cent.

Steam Tables.

The table of Properties of Saturated Steam (pp. 566–574) is given in the form in which it was arranged by D. A. Low some years ago, with Centigrade and Fahrenheit values on each page. With the exception of the figures in the column headed w the figures in the tables on pp. 566–582 are taken, by permission of Messrs. Edward Arnold & Co., from the Abridged Callendar Steam Tables, to which reference should be made for fuller information.

On pp. 566–574 the columns to the left of the first thick line and the columns between the thick lines apply to degrees Centigrade and C.H.U. Also, the columns to the right of the second thick line and the columns between the thick lines apply to degrees Fahrenheit and B.Th.U.

Symbols.—

t = saturation temperature.
h = sensible heat of 1 lb. of steam, or the total heat in 1 lb. of water at temperature t.
H = total heat of 1 lb. of steam.
L = latent heat of 1 lb. of steam.
v = volume of 1 lb. of steam, in cubic feet.
w = weight of 1 cubic foot of steam $= 1/v$, in pounds.
p = pressure of steam in lbs. per sq. inch, absolute.
ϕ_w = entropy of 1 lb. of water.
ϕ_s = total entropy of 1 lb. of steam.

Properties of Saturated Steam.

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
26·42	26·45	608·95	582·50	643·0	0·00156	**0·5**	0·0924	2·0367	1048·5	1096·1	47·6	79·6
28·05	28·06	609·67	581·61	587·0	0·00170	**0·55**	0·0978	2·0288	1046·9	1097·4	50·5	82·5
29·57	29·58	610·34	580·76	540·6	0·00185	**0·6**	0·1028	2·0214	1045·4	1098·6	53·2	85·3
30·97	30·98	610·97	579·99	500·8	0·00200	**0·65**	0·1073	2·0143	1044·0	1099·7	55·7	87·8
32·28	32·28	611·55	579·27	466·6	0·00214	**0·7**	0·1117	2·0082	1042·7	1100·8	58·1	90·1
33·52	33·51	612·10	578·59	437·3	0·00229	**0·75**	0·1156	2 0025	1041·5	1101·8	60·3	92·3
34·67	34·66	612·61	577·95	411·7	0·00243	**0·8**	0·1196	1·9970	1040·3	1102·7	62·4	94·4
35·77	35·76	613·09	577·33	388·9	0·00257	**0·85**	0·1230	1·9919	1039·2	1103·5	64·3	96·3
36·80	36·80	613·54	576·74	368·7	0·00271	**0·9**	0·1264	1·9871	1038·1	1104·3	66·2	98·2
37·80	37·79	613·95	576·16	350·5	0·00285	**0·95**	0·1296	1·9826	1037·1	1105·1	68·0	100·0
38·74	38·74	614·34	575·60	334·0	0·00299	**1·0**	0·1326	1·9783	1036·1	1105·8	69·7	101·7
39·64	39·65	614·72	575·07	318·9	0·00314	**1·05**	0·1353	1·9742	1035·2	1106·5	71·3	103·3
40·52	40·52	615·09	574·57	305·2	0·00328	**1·1**	0·1381	1·9702	1034·3	1107·2	72·9	104·9
41·36	41·36	615·45	574·09	292·6	0·00342	**1·15**	0·1407	1·9665	1033·4	1107·8	74·4	106·4
42·17	42·17	615·80	573·63	281·1	0·00356	**1·2**	0·1433	1·9630	1032·5	1108·4	75·9	107·9
42·95	42·94	616·13	573·19	270·4	0·00370	**1·25**	0·1460	1·9596	1031·7	1109·0	77·3	109·4
43·70	43·70	616·45	572·75	260·5	0·00384	**1·3**	0·1484	1·9563	1030·9	1109·6	78·7	110·7
44·43	44·42	616·76	572·34	251·4	0·00398	**1·35**	0·1506	1·9532	1030·2	1110·2	80·0	112·0
45·14	45·12	617·06	571·94	243·0	0·00412	**1·4**	0·1527	1·9501	1029·5	1110·8	81·3	113·3
45·84	45·80	617·34	571·54	235·2	0·00425	**1·45**	0·1548	1·9471	1028·8	1111·3	82·5	114·5

Properties of Saturated Steam *(continued)*.

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
46·49	46·45	617·61	571·16	228·0	0·00439	**1·5**	0·1569	1·9442	1028·1	1111·8	83·7	115·7
47·77	47·73	618·14	570·41	214·3	0·00467	**1·6**	0·1609	1·9387	1026·8	1112·8	86·0	118·0
48·98	48·94	618·65	569·71	202·5	0·00494	**1·7**	0·1646	1·9336	1025·5	1113·7	88·2	120·2
50·13	50·08	619·14	569·06	191·8	0·00521	**1·8**	0·1681	1·9288	1024·4	1114·6	90·2	122·2
51·22	51·16	619·63	568·47	182·3	0·00549	**1·9**	0·1715	1·9243	1023·3	1115·4	92·1	124·2
52·27	52·22	620·11	567·89	173·7	0·00576	**2·0**	0·1749	1·9200	1022·2	1116·2	94·0	126·1
54·24	54·18	620·91	566·73	158·8	0·00630	**2·2**	0·1809	1·9120	1020·2	1117·7	97·5	129·6
56·06	55·98	621·67	565·69	146·4	0·00683	**2·4**	0·1864	1·9047	1018·3	1119·1	100·8	132·9
57·75	57·66	622·39	564·73	135·8	0·00736	**2·6**	0·1916	1·8981	1016·5	1120·3	103·8	135·9
59·34	59·26	623·05	563·79	126·7	0·00789	**2·8**	0·1963	1·8920	1014·8	1121·5	106·7	138·8
60·83	60·78	623·67	562·89	118·7	0·00842	**3·0**	0·2008	1·8869	1013·2	1122·6	109·4	141·5
62·24	62·20	624·28	562·08	111·7	0·00895	**3·2**	0·2050	1·8819	1011·7	1123·7	112·0	144·0
63·58	63·55	624·87	561·32	105·5	0·00948	**3·4**	0·2091	1·8770	1010·3	1124·7	114·4	146·4
64·85	64·82	625·44	560·62	100·0	0·01000	**3·6**	0·2128	1·8722	1009·0	1125·7	116·7	148·7
66·07	66·04	625·98	559·94	95·10	0·01052	**3·8**	0·2164	1·8676	1007·8	1126·7	118·9	150·9
67·23	67·20	626·49	559·29	90·63	0·01103	**4·0**	0·2199	1·8632	1006·7	1127·7	121·0	153·0
68·34	68·31	626·97	558·66	86·58	0·01155	**4·2**	0·2231	1·8590	1005·6	1128·6	123·0	155·0
69·40	69·38	627·42	558·04	82·88	0·01207	**4·4**	0·2262	1·8552	1004·6	1129·5	124·9	156·9
70·43	70·41	627·84	557·43	79·50	0·01258	**4·6**	0·2292	1·8516	1003·6	1130·3	126·7	158·8
71·42	71·40	628·23	556·83	76·39	0·01309	**4·8**	0·2321	1·8482	1002·6	1131·1	128·5	160·6

Properties of Saturated Steam (*continued*).

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
72·38	72·36	628·60	556·24	73·52	0·01360	**5·0**	0·2348	1·8449	1001·6	1131·8	130·2	162·3
73·30	73·28	628·96	555·68	70·88	0·01411	**5·2**	0·2374	1·8417	1000·6	1132·5	131·9	163·9
74·19	74·18	629·31	555·13	68·41	0·01462	**5·4**	0·2401	1·8386	999·6	1133·1	133·5	165·5
75·06	75·05	629·66	554·61	66·12	0·01512	**5·6**	0·2426	1·8356	998·6	1133·7	135·1	167·1
75·90	75·89	630·00	554·11	63·98	0·01563	**5·8**	0·2450	1·8327	997·6	1134·2	136·6	168·6
76·72	76·71	630·33	553·62	61·98	0·01613	**6·0**	0·2473	1·8299	996·6	1134·7	138·1	170·1
78·67	78·68	631·04	552·36	57·50	0·01739	**6·5**	0·2529	1·8236	994·3	1135·9	141·6	173·6
80·49	80·52	631·72	551·20	53·64	0·01864	**7·0**	0·2582	1·8176	992·2	1137·1	144·9	176·9
82·21	82·25	632·39	550·14	50·30	0·01988	**7·5**	0·2631	1·8119	990·3	1138·3	148·0	180·0
83·84	83·89	633·05	549·16	47·35	0·02112	**8·0**	0·2676	1·8065	988·5	1139·5	151·0	182·9
85·38	85·43	633·69	548·26	44·73	0·02236	**8·5**	0·2719	1·8015	986·8	1140·6	153·8	185·7
86·84	86·88	634·30	547·42	42·40	0·02358	**9·0**	0·2762	1·7968	985·2	1141·7	156·5	188·3
88·24	88·28	634·88	546·60	40·31	0·02481	**9·5**	0·2801	1·7925	983·8	1142·7	158·9	190·8
89·58	89·61	635·43	545·82	38·42	0·02603	**10·0**	0·2836	1·7884	982·5	1143·8	161·3	193·2
90·87	90·91	635·94	545·03	36·71	0·02724	**10·5**	0·2872	1·7845	981·1	1144·7	163·6	195·6
92·10	92·15	636·41	544·26	35·14	0·02846	**11·0**	0·2906	1·7807	979·6	1145·5	165·9	197·8
93·29	93·34	636·85	543·51	33·71	0·02966	**11·5**	0·2939	1·7771	978·2	1146·3	168·1	199·9
94·44	94·50	637·25	542·75	32·40	0·03086	**12·0**	0·2970	1·7735	976·9	1147·0	170·1	202·0
95·55	95·62	637·64	542·02	31·17	0·03208	**12·5**	0·3001	1·7703	975·7	1147·7	172·0	204·0
96·62	96·69	638·03	541·34	30·05	0·03328	**13·0**	0·3029	1·7672	974·6	1148·5	173·9	205·9

Properties of Saturated Steam (*continued*).

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
97·65	97·73	638·41	540·68	29·01	0·03447	**13·5**	0·3059	1·7642	973·4	1149·2	175·8	207·8
98·65	98·73	638·79	540·06	28·03	0·03568	**14·0**	0·3086	1·7613	972·2	1149·9	177·7	209·6
99·63	99·71	639·16	539·45	27·12	0·03687	**14·5**	0·3112	1·7585	971·0	1150·5	179·5	211·3
100·00	100·06	639·28	539·22	26·80	0·03731	**14·696**	0·3122	1·7574	970·6	1150·7	180·1	212·0
100·57	100·65	639·5	538·9	26·28	0·03805	**15**	0·3137	1·7556	970·0	1151·2	181·2	213·0
102·40	102·51	640·2	537·7	24·74	0·04042	**16**	0·3187	1·7505	967·9	1152·4	184·5	216·3
104·13	104·27	640·8	536·5	23·38	0·04277	**17**	0·3231	1·7456	965·9	1153·5	187·6	219·5
105·78	105·94	641·4	535·5	22·17	0·04511	**18**	0·3276	1·7411	964·0	1154·6	190·6	222·4
107·36	107·53	642·0	534·5	21·07	0·04746	**19**	0·3319	1·7368	962·2	1155·7	193·5	225·2
108·87	109·05	642·6	533·6	20·09	0·04978	**20**	0·3358	1·7327	960·4	1156·7	196·3	228·0
110·32	110·53	643·1	532·6	19·19	0·05211	**21**	0·3396	1·7287	958·8	1157·7	198·9	230·6
111·71	111·94	643·6	531·7	18·38	0·05441	**22**	0·3433	1·7250	957·2	1158·6	201·4	233·1
113·05	113·30	644·1	530·8	17·63	0·05672	**23**	0·3468	1·7215	955·6	1159·5	203·9	235·5
114·34	114·61	644·6	530·0	16·94	0·05903	**24**	0·3502	1·7181	954·0	1160·3	206·3	237·8
115·59	115·87	645·1	529·2	16·30	0·06135	**25**	0·3534	1·7148	952·5	1161·1	208·6	240·1
116·80	117·11	645·5	528·4	15·72	0·06361	**26**	0·3565	1·7118	951·1	1161·9	210·8	242·2
117·97	118·31	645·9	527·6	15·17	0·06592	**27**	0·3595	1·7089	949·7	1162·6	212·9	244·4
119·11	119·47	646·3	526·8	14·67	0·06817	**28**	0·3625	1·7060	948·3	1163·3	215·0	246·4
120·21	120·58	646·7	526·1	14·19	0·07047	**29**	0·3654	1·7032	947·0	1164·0	217·0	248·4
121·3	121·7	647·1	525·4	13·73	0·07283	**30**	0·3682	1·7004	945·6	1164·6	219·0	250·3

Properties of Saturated Steam (*continued*).

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
122·3	122·8	647·5	524·7	13·31	0·07513	**31**	0·3709	1·6977	944·3	1165·2	220·9	252·2
123·3	123·8	647·9	524·1	12·93	0·07734	**32**	0·3735	1·6952	943·1	1165·8	222·7	254·0
124·3	124·8	648·3	523·5	12·56	0·07962	**33**	0·3760	1·6928	941·9	1166·4	224·5	255·8
125·3	125·8	648·6	522·8	12·21	0·08190	**34**	0·3785	1·6905	940·7	1167·0	226·3	257·6
126·3	126·8	648·9	522·1	11·89	0·08410	**35**	0 3809	1·6883	939·6	1167·6	228·0	259·3
127·2	127·7	649·2	521·5	11·58	0·08636	**36**	0·3833	1·6860	938·5	1168 2	229·7	260·9
128·0	128·6	649·5	520·9	11·29	0·08857	**37**	0·3856	1·6838	937·4	1168·8	231·4	262·5
128·9	129·5	649·8	520·3	11·02	0·09074	**38**	0·3879	1·6817	936·4	1169·4	233·0	264·1
129·8	130·4	650·1	519·7	10·76	0·09294	**39**	0·3901	1·6796	935·4	1170·0	234·6	265·7
130·7	131·2	650·4	519·2	10·50	0·09524	**40**	0·3923	1·6776	934·4	1170·5	236·1	267·2
132·3	132·9	650·9	518·0	10·03	0·09970	**42**	0·3964	1·6737	932·3	1171·4	239·1	270·3
133·9	134·5	651·4	516·9	9·600	0·1042	**44**	0·4003	1·6700	930·3	1172·3	242·0	273·1
135·4	136·0	651·9	515·9	9·209	0·1086	**46**	0·4041	1·6664	928·3	1173·2	244·9	275·8
136·9	137·5	652·3	514·8	8·848	0·1130	**48**	0·4077	1·6630	926·4	1174·0	247·6	278·5
138·3	139·0	652·8	513·8	8·516	0·1174	**50**	0·4112	1·6597	924·6	1174·8	250·2	281·0
139·7	140·4	653·2	512·8	8·208	0·1218	**52**	0·4146	1·6566	922·9	1175·6	252·7	283·5
141·0	141·8	653·6	511·8	7·922	0·1262	**54**	0·4179	1·6536	921·1	1176·3	255·2	285·9
142·3	143·1	654·0	510·9	7·656	0·1306	**56**	0·4211	1·6507	919·4	1177·0	257·6	288·3
143·6	144·4	654·4	510·0	7·407	0·1350	**58**	0·4242	1·6478	917·8	1177·7	259·9	290·5
144·9	145·6	654·8	509·2	7·175	0·1394	**60**	0·4272	1·6450	916·2	1178·4	262·2	292·7

Properties of Saturated Steam *(continued)*.

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
146·1	146·8	655·2	508·4	6·957	0·1437	**62**	0·4302	1·6423	914·6	1179·0	264·4	294·9
147·3	148·0	655·6	507·6	6·752	0·1481	**64**	0·4331	1·6398	913·1	1179·6	266·5	296·9
148·4	149·2	655·9	506·7	6·560	0·1524	**66**	0·4359	1·6374	911·6	1180·2	268·6	299·0
149·5	150·3	656·2	505·9	6·378	0·1568	**68**	0·4386	1·6350	910·1	1180·8	270·7	301·0
150·6	151·5	656·5	505·0	6·206	0·1611	**70**	0·4412	1·6327	908·7	1181·4	272·7	302·9
151·6	152·6	656·8	504·2	6·044	0·1655	**72**	0·4437	1·6304	907·4	1182·0	274·6	304·8
152·6	153·6	657·0	503·4	5·890	0·1698	**74**	0·4462	1·6282	906·0	1182·5	276·5	306·7
153·6	154·7	657·3	502·6	5·743	0·1741	**76**	0·4486	1·6261	904·6	1183·0	278·4	308·5
154·6	155·7	657·5	501·8	5·604	0·1784	**78**	0·4510	1·6240	903·2	1183·5	280·3	310·3
155·6	156·7	657·8	501·1	5·472	0·1827	**80**	0·4533	1·6219	901·9	1184·0	282·1	312·0
156·5	157·7	658·0	500·3	5·346	0·1871	**82**	0·4556	1·6199	900·6	1184·5	283·9	313·7
157·5	158·6	658·2	499·6	5·226	0·1914	**84**	0·4579	1·6180	899·4	1185·0	285·6	315·4
158·4	159·6	658·5	498·9	5·110	0·1957	**86**	0·4601	1·6161	898·1	1185·4	287·3	317·1
159·4	160·5	658·8	498·3	5·000	0·2000	**88**	0·4622	1·6142	896·8	1185·8	289·0	318·7
160·3	161·5	659·1	497·6	4·896	0·2042	**90**	0·4643	1·6124	895·5	1186·2	290·7	320·3
161·2	162·4	659·3	496·9	4·796	0·2085	**92**	0·4664	1·6106	894·3	1186·6	292·3	321·9
162·0	163·3	659·6	496·3	4·699	0·2128	**94**	0·4684	1·6088	893·1	1187·0	293·9	323·3
162·8	164·1	659·8	495·7	4·607	0·2171	**96**	0·4704	1·6071	891·9	1187·4	295·5	324·8
163·6	165·0	660·0	495·0	4·519	0·2213	**98**	0·4723	1·6054	890·8	1187·8	297·0	326·3
164·4	165·8	660·1	494·3	4·434	0·2255	**100**	0·4742	1·6038	889·7	1188·2	298·5	327·8

Properties of Saturated Steam (*continued*).

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
166·4	167·9	660·6	492·7	4·230	0·2364	**105**	0·4789	1·6000	886·9	1189·1	302·2	331·3
168·2	169·8	661·0	491·2	4·046	0·2472	**110**	0·4833	1·5963	884·2	1189·9	305·7	334·8
170·0	171·7	661·5	489·8	3·880	0·2577	**115**	0·4876	1·5927	881·5	1190·7	309·2	338·1
171·8	173·6	661·9	488·3	3·729	0·2682	**120**	0·4918	1·5891	878·9	1191·4	312·5	341·3
173·5	175·4	662·3	486·9	3·587	0·2788	**125**	0·4958	1·5856	876·4	1192·1	315·7	344·4
175·2	177·1	662·7	485·6	3·456	0·2894	**130**	0·4997	1·5823	874·0	1192·8	318·8	347·3
176·8	178·8	663·0	484·2	3·335	0·2999	**135**	0·5035	1·5792	871·5	1193·4	321·9	350·2
178·3	180·5	663·4	482·9	3·222	0·3104	**140**	0·5071	1·5763	869·1	1194·0	324·9	353·0
179·8	182·1	663·7	481·6	3·116	0·3209	**145**	0·5106	1·5733	866·8	1194·6	327·8	355·8
181·3	183·7	664·0	480·3	3·015	0·3317	**150**	0·5140	1·5705	864·5	1195·1	330·6	358·4
182·8	185·2	664·3	479·1	2·922	0·3422	**155**	0·5173	1·5678	862·3	1195·6	333·3	361·0
184·2	186·7	664·6	477·9	2·835	0·3527	**160**	0·5205	1·5652	860·1	1196·1	336·0	363·6
185·5	188·1	664·9	476·8	2·754	0·3631	**165**	0·5236	1·5627	858·0	1196·6	338·6	366·0
186·9	189·5	665·2	475·7	2·677	0·3736	**170**	0·5267	1·5602	855·9	1197·1	341·2	368·4
188·2	190·9	665·4	474·5	2·603	0·3842	**175**	0·5297	1·5578	853·9	1197·6	343·7	370·8
189·5	192·3	665·6	473·3	2·534	0·3946	**180**	0·5326	1·5554	851·9	1198·0	346·1	373·1
190·7	193·6	665·8	472·2	2·468	0·4052	**185**	0·5355	1·5531	849·9	1198·4	348·5	375·3
192·0	194·9	666·0	471·1	2·407	0·4155	**190**	0·5383	1·5509	847·9	1198·8	350·9	377·5
193·1	196·2	666·2	470·0	2·347	0·4261	**195**	0·5410	1·5487	846·0	1199·2	353·2	379·7
194·3	197·5	666·4	468·9	2·290	0·4367	**200**	0·5437	1·5466	844·0	1199·5	355·5	381·8

Properties of Saturated Steam (*continued*).

t° C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_w	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	t° F.
195·5	198·7	666·6	467·9	2·237	0·4470	**205**	0·5463	1·5445	842·0	1199·8	357·8	383·9
196·6	199·9	666·8	466·9	2·185	0·4577	**210**	0·5488	1·5424	840·1	1200·1	360·0	385·9
197·7	201·1	667·0	465·9	2·136	0·4682	**215**	0·5513	1·5404	838·3	1200·4	362·1	387·9
198·8	202·3	667·2	464·9	2·089	0·4787	**220**	0·5537	1·5384	836·5	1200·7	364·2	389·9
199·9	203·5	667·4	463·9	2·043	0·4895	**225**	0·5562	1·5366	834·8	1201·0	366·2	391·8
200·9	204·7	667·6	462·9	1·999	0·5003	**230**	0·5587	1·5347	833·0	1201·3	368·3	393·7
202·0	205·8	667·7	461·9	1·958	0·5107	**235**	0·5611	1·5329	831·2	1201·5	370·3	395·5
203·0	206·9	667·9	461·0	1·918	0·5214	**240**	0·5634	1·5311	829·4	1201·7	372 3	397·4
204·0	208·0	668·1	460·1	1·880	0·5319	**245**	0·5656	1·5293	827·7	1201·9	374·2	399·2
204·9	209·0	668·2	459·2	1·844	0·5423	**250**	0·5677	1·5276	826 0	1202·1	376·1	401·0
206·9	211·1	668·4	457·3	1·775	0·5634	**260**	0·5721	1·5243	822·6	1202 5	379·9	404·4
208·8	213·1	668·6	455·5	1·711	0·5845	**270**	0·5762	1·5211	819·3	1202·9	383·6	407·8
210·7	215·1	668·7	453·6	1·651	0·6057	**280**	0·5803	1·5179	816·1	1203·2	387·1	411·1
212·4	217·1	668·8	451·7	1·595	0·6270	**290**	0·5843	1·5148	812 9	1203·5	390·6	414·2
214·1	219·0	668·9	449·9	1·543	0·6481	**300**	0·5881	1·5117	809·8	1203·8	394·0	417·3
215·8	220·8	669·0	448·2	1·494	0·6693	**310**	0·5918	1·5088	806·8	1204·1	397·3	420·4
217·4	222·6	669·1	446·5	1·448	0·6906	**320**	0·5954	1·5060	803·7	1204·3	400·6	423·3
219·0	224·3	669·2	444·9	1·405	0·7117	**330**	0·5990	1·5032	800·7	1204·5	403·8	426·2
220·6	226·0	669·3	443·3	1·364	0·7331	**340**	0·6024	1·5005	797·8	1204·7	406·9	429·0
222·1	227·7	669·4	441·7	1·326	0·7541	**350**	0·6058	1·4979	795·0	1204·9	409·9	431·7
223·5	229·3	669·4	440·1	1·289	0·7758	**360**	0·6092	1·4953	792·3	1205·1	412·8	434·4
224·9	231·0	669·5	438·5	1·255	0·7968	**370**	0·6124	1·4928	789·5	1205·2	415·7	437·0

Properties of Saturated Steam (*concluded*).

$t°$ C.	h C.H.U.	H C.H.U.	L C.H.U.	v	w	p	ϕ_u	ϕ_s	L B.Th.U.	H B.Th.U.	h B.Th.U.	$t°$ F.
226·4	232·6	669·5	436·9	1·222	0·8183	**380**	0·6155	1·4904	786 7	1205·3	418·6	439·6
227·8	234·3	669·6	435·3	1·191	0·8396	**390**	0·6186	1 4880	784·0	1205·4	421 4	442·1
229·2	235·8	669·6	433·8	1·161	0·8613	**400**	0·6216	1 4857	781·3	1205·5	424·2	444·6
230·5	237·3	669·7	432·4	1·133	0 8826	**410**	0·6245	1 4834	778·6	1205·5	426·9	447·0
231·8	238·8	669·7	430·9	1·106	0 9042	**420**	0 6274	1·4811	775·9	1205·5	429·6	449·4
233·2	240·2	669·7	429·5	1·080	0·9259	**430**	0·6303	1·4789	773·3	1205·5	432·2	451·7
234·5	241·6	669·8	428·2	1·056	0·9470	**440**	0·6331	1·4767	770·8	1205·6	434·8	454·0
235·8	243·0	669·8	426·8	1·032	0·9690	**450**	0·6358	1·4746	768·2	1205·6	437·4	456·3
237·0	244·4	669·8	425·4	1·009	0·9911	**460**	0·6385	1·4725	765·7	1205·6	439·9	458·5
238·2	245·8	669·7	423·9	0·988	1·012	**470**	0·6412	1·4705	763·2	1205·6	442·4	460·7
239·4	247·2	669·7	422·5	0·967	1·034	**480**	0·6438	1·4685	760·7	1205·5	444·8	462·8
240·6	248·5	669·7	421·2	0·947	1·056	**490**	0·6464	1·4665	758·3	1205·5	447·2	464·9
241·7	249·8	669·7	419·9	0·928	1·078	**500**	0·6489	1·4646	755·8	1205·4	449·6	467·0
247·2	256·3	669·4	413·1	0·843	1·186	**550**	0·6614	1·4553	743·9	1204·9	461·0	476·9
252·3	262·1	669·0	406·9	0·770	1·299	**600**	0·6722	1·4466	732·4	1204·2	471·8	486·2
261·7	273·2	667·9	394·7	0·655	1·527	**700**	0·6927	1·4308	710·5	1202·2	491·7	503·1
270·1	283·3	666·4	383·1	0·569	1·757	**800**	0·7110	1·4165	689·7	1199·6	509·9	518·2
284·8	301·4	662·7	361·3	0·446	2·242	**1000**	0·7432	1·3909	650·2	1192·8	542·6	544·6
313·4	339·8	649·5	309·7	0·277	3·610	**1500**	0·8086	1·3360	557·5	1169·1	611·6	596·2
335·4	373·2	631·2	258·0	0·188	5·319	**2000**	0·8620	1·2857	464·3	1136·1	671·8	635·8
353·4	405·8	606·7	200·9	0·132	7·576	**2500**	0·9128	1·2323	361·5	1092·0	730·5	668·1
368·6	446·2	564·7	118·5	0·086	11·63	**3000**	0·9728	1·1580	214·1	1016·4	802·3	695·4

Total Heat of Superheated Steam—C.H.U. per lb.

Abs. Press. Lb./In.²	Saturation.		Degrees of Superheat (Centigrade).						
	t° C.	H	10°	20°	30°	40°	50°	60°	70°
15	100·6	639·5	645·6	649·4	654·3	659·0	663·7	668·5	673·2
20	108·9	642·6	647·8	652·6	657 4	662·2	667·0	671·9	676·7
30	121·3	647·1	652·4	657·2	662 0	666·9	671·9	676·8	681·6
40	130·7	650·4	655·6	660·5	665·4	670·4	675·4	680·3	685·2
50	138·3	652·8	658·0	663·0	668·1	673·1	678·1	683·1	688·0
60	144·9	654·8	660·0	665·2	670·3	675 4	680·4	685·4	690·4
70	150·6	656·5	661·7	667·0	672·2	677·3	682·4	687·4	692·4
80	155·6	657·8	663·2	668·5	673·8	679·0	684·1	689 2	694·3
90	160·3	659·1	664·5	669·9	675·2	680·5	685·7	690 9	696·1
100	164·4	660·1	665·7	671·1	676 5	681·8	687·0	692 2	697·4
120	171·8	661·9	667·6	673·3	678·7	684·1	689·5	694 8	700·0
140	178·3	663·4	669·2	675·0	680·5	686·0	691·5	696 9	702·2
160	184·2	664·6	670·6	676·5	682·1	687·7	693·2	698 6	703·9
180	189·5	665·6	671·8	677·7	683·5	689·1	694·6	700·1	705·5
200	194·3	666·4	672·7	678·7	684·6	690·3	695·9	701·5	707·0
250	204·9	668·2	674·5	680 8	687 0	692·9	698 7	704·4	710·0
300	214·1	668·9	675·6	682 2	688 7	694·8	700·8	706·6	712 4
400	229·2	669·6	677·0	684 1	691·0	697·4	703·7	709 9	715·9
500	241·7	669·7	677·2	685·0	692 0	698 9	705·6	712·0	718·3
600	252·3	669·0	677·3	685·2	692·7	700 0	707·0	713·6	720 1
700	261·7	667·9	677·0	685·2	693·0	700·6	707·7	714·7	721·4
800	270·1	666·4	676·2	684·9	693·1	700·9	708·3	715·5	722·4
1000	284·8	662·7	673·5	683·4	692·5	701·0	709·0	716·7	724·0
2000	335·4	631·2	651·6	667·2	680·7	692·7	703·6	713·6	722·9

Total Heat of Superheated Steam—C.H.U. per lb. *(continued)*.

Abs. Press. Lb./In.²	Degrees of Superheat (Centigrade).							
	80°	90°	100°	120°	140°	160°	180°	200°
15	677·9	682·6	687·2	696·7	706·2	715·7	725·2	734·7
20	681·4	686·1	690·7	700·2	709·8	719·3	728·9	738·5
30	686·4	691·1	695·8	705·4	715·1	724·7	734·4	744·1
40	690·0	694·8	699·5	709·2	718·9	728·6	738·4	748·2
50	692·9	697·8	702·6	712·4	722·2	731·9	741·7	751·6
60	695·3	700·2	705·1	714·9	724·7	734·5	744·4	754·3
70	697·4	702·3	707·3	717·1	727·0	736·9	746·8	756·7
80	699·4	704·4	709·3	719·2	729·1	739·0	748·9	758·9
90	701·2	706·2	711·1	721·0	730·9	740·8	750·7	760·7
100	702·6	707·7	712·7	722·8	732·8	742·7	752·5	762·5
120	705·2	710·3	715·4	725·5	735·6	745·7	755·7	765·7
140	707·4	712·6	717·7	727·9	738·1	748·2	758·3	768·4
160	709·2	714·5	719·8	730·1	740·3	750·5	760·7	770·9
180	710·9	716·3	721·6	732·0	742·3	752·6	762·9	773·1
200	712·4	717·8	723·1	733·5	744·0	754·4	764·8	775·0
250	715·5	721·1	726·6	737·3	747·9	758·4	768·9	779·2
300	718·0	723·6	729·3	740·1	750·9	761·5	772·2	782·7
400	722·0	727·7	733·5	744·7	755·8	766·7	777·5	788·4
500	724·6	730·6	736·6	748·1	759·5	770·7	781·8	792·8
600	726·5	732·8	739·0	750·6	762·2	773·7	785·0	796·3
700	728·0	734·6	741·0	753·0	764·9	776·5	788·0	799·3
800	729·2	735·9	742·4	755·0	767·2	779·2	790·9	802·2
1000	731·1	737·9	744·6	757·4	770·0	782·2	794·1	805·8
2000	731·6	739·8	747·8	762·9	777·1	790·7	803·9	816·6

Total Heat of Superheated Steam—B.Th.U. per lb.

Abs. Press. Lb./In.²	Saturation.		Degrees of Superheat (Fahrenheit).						
	t° F.	H	20°	40°	60°	80°	100°	120°	140°
15	213·0	1151·2	1161·2	1170·9	1180·5	1190·0	1199·5	1208·9	1218·4
20	228·0	1156·7	1167·0	1176·7	1186·3	1195·8	1205·3	1214·8	1224·3
30	250·3	1164·6	1175·1	1184·9	1194·7	1204·4	1214·0	1223·6	1233·2
40	267·2	1170·5	1181·0	1190·9	1200·8	1210·7	1220·6	1230·3	1240·0
50	281·0	1174·8	1185·5	1195·7	1205·7	1215·6	1225·5	1235·3	1245·1
60	292·7	1178·4	1189·3	1199·5	1209·7	1219·8	1229·7	1239·6	1249·5
70	302·9	1181·4	1192·5	1202·8	1213·0	1223·2	1233·3	1243·3	1253·4
80	312·0	1184·0	1195·2	1205·6	1215·9	1226·2	1236·5	1246·7	1256·8
90	320·3	1186·2	1197·5	1208·0	1218·5	1228·9	1239·3	1249·6	1259·8
100	327·8	1188·2	1199·3	1210·2	1220·9	1231·4	1241·8	1252·2	1262·5
120	341·3	1191·4	1202·8	1214·0	1224·9	1235·7	1246·3	1256·8	1267·2
140	353·0	1194·0	1205·8	1217·1	1228·2	1239·1	1249·9	1260·6	1271·2
160	363·6	1196·1	1208·3	1219·7	1231·0	1242·1	1253·1	1263·9	1274·6
180	373·1	1198·0	1210·3	1222·0	1233·5	1244·7	1255·9	1266·8	1277·7
200	381·8	1199·5	1212·1	1224·0	1235·6	1247·0	1258·2	1269·3	1280·2
250	401·0	1202·1	1215·3	1227·6	1239·7	1251·7	1263·4	1274·9	1286·2
300	417·3	1203·8	1217·3	1230·3	1242·9	1255·1	1267·2	1278·9	1290·3
400	444·6	1205·5	1219·9	1234·1	1247·5	1260·5	1273·1	1285·3	1297·2
500	467·0	1205·4	1220·8	1235·7	1250·0	1263·6	1277·0	1289·7	1302·1
600	486·2	1204·2	1220·7	1236·5	1251·7	1266·0	1279·7	1292·7	1305·5
700	503·1	1202·2	1220·0	1236·6	1252·2	1267·0	1281·3	1295·0	1308·1
800	518·2	1199·6	1218·7	1236·1	1252·2	1267·4	1282·1	1296·3	1309·9
1000	544·6	1192·8	1214·5	1233·9	1251·6	1268·4	1283·9	1298·9	1313·2
2000	635·8	1136·1	1176·1	1206·6	1232·9	1256·0	1276·7	1295·6	1313·5

Total Heat of Superheated Steam—B.Th.U. per lb. (*continued*).

Abs. Press. Lb./In.²	Degrees of Superheat (Fahrenheit).							
	160°	180°	200°	240°	280°	320°	360°	400°
15	1227·7	1237·0	1246·2	1265·0	1284·0	1303·1	1322·3	1341·5
20	1233·7	1243·1	1252·5	1271·5	1290·6	1309·8	1329·1	1348·5
30	1242·8	1252·3	1261·8	1281·1	1300·5	1319·9	1339·3	1358·7
40	1249·7	1259·3	1268·9	1288·3	1307·7	1327·2	1346·7	1366·3
50	1254·9	1264·7	1274·4	1293 9	1313·4	1332·9	1352 6	1372·4
60	1259·4	1269·3	1279·1	1298·7	1318·3	1338·0	1357·8	1377·6
70	1263·4	1273·3	1283·1	1302 8	1322·5	1342·3	1362·1	1382·0
80	1266·9	1276·9	1286·8	1306·5	1326·2	1346·0	1365·8	1385·7
90	1270·0	1280·1	1290·1	1309·9	1329·7	1349·5	1369·3	1389·2
100	1272·7	1282·9	1293·0	1313·0	1332·9	1352·7	1372·6	1392·5
120	1277·5	1287·8	1298·0	1318·2	1338·2	1358·1	1378·1	1398·0
140	1281·7	1292·0	1302·3	1322·6	1342·8	1362·9	1383·0	1403·0
160	1285·2	1295·7	1306·0	1326·3	1346·6	1366·9	1387·2	1407·5
180	1288·3	1298·8	1309·2	1329·7	1350·2	1370·7	1391·1	1411·5
200	1291·0	1301·6	1312·1	1332·9	1353·7	1374·3	1394·9	1415·3
250	1297·1	1307·7	1318·5	1340·0	1361·2	1382·0	1402·6	1423·1
300	1301·6	1312·6	1323·4	1345·1	1366 5	1387·6	1408·6	1429·4
400	1308·9	1320·3	1331·6	1353·9	1375·9	1397·5	1418·7	1439·8
500	1314·0	1325·8	1337·5	1360·5	1383·4	1405·3	1427·0	1448·3
600	1317·9	1330·1	1342·1	1365·5	1388·6	1411·2	1433·5	1455·3
700	1320·9	1333·5	1345·9	1369·9	1393·4	1416·3	1438·8	1460·9
800	1323·2	1336·2	1349·0	1373·6	1397·5	1420·9	1443·7	1465·8
1000	1326·8	1340·0	1353·2	1378·5	1402·8	1426·9	1450·2	1473·1
2000	1330·1	1346·0	1361·4	1390·4	1417·9	1444·4	1469·7	1494·0

Entropy of Superheated Steam—C.H.U. per lb./°C.

Abs. Press. Lb./In.²	Saturation		Degrees of Superheat (Centigrade).						
	$t°$ C.	ϕ_s	10°	20°	30°	40°	50°	60°	70°
15	100·6	1·7556	1·7692	1·7811	1·7930	1·8049	1·8166	1·8279	1·8388
20	108·9	1·7327	1·7460	1·7578	1·7698	1·7815	1·7930	1·8040	1·8149
30	121·3	1·7004	1·7133	1·7252	1·7370	1·7484	1·7596	1·7704	1·7809
40	130·7	1·6776	1·6900	1·7019	1·7135	1·7250	1·7358	1·7464	1·7568
50	138·3	1·6597	1·6716	1·6834	1·6950	1·7063	1·7172	1·7277	1·7379
60	144·9	1·6450	1·6570	1·6688	1·6803	1·6916	1·7026	1·7129	1·7233
70	150·6	1·6327	1·6448	1·6566	1·6681	1·6794	1·6905	1·7009	1·7112
80	155·6	1·6219	1·6341	1·6460	1·6575	1·6688	1·6798	1·6902	1·7005
90	160·3	1·6124	1·6248	1·6368	1·6484	1·6597	1·6705	1·6809	1·6912
100	164·4	1·6038	1·6163	1·6285	1·6401	1·6513	1·6621	1·6728	1·6830
120	171·8	1·5891	1·6014	1·6137	1·6253	1·6366	1·6474	1·6579	1·6680
140	178·3	1·5763	1·5886	1·6010	1·6127	1·6241	1·6348	1·6453	1·6554
160	184·2	1·5652	1·5773	1·5898	1·6015	1·6131	1·6240	1·6345	1·6447
180	189·5	1·5554	1·5674	1·5799	1·5917	1·6032	1·6142	1·6247	1·6349
200	194·3	1·5466	1·5585	1·5710	1·5827	1·5942	1·6052	1·6158	1·6261
250	204·9	1·5276	1·5403	1·5531	1·5651	1·5767	1·5879	1·5987	1·6091
300	214·1	1·5117	1·5250	1·5381	1·5503	1·5622	1·5737	1·5846	1·5951
400	229·2	1·4857	1·5002	1·5135	1·5261	1·5382	1·5498	1·5608	1·5714
500	241·7	1·4646	1·4796	1·4934	1·5066	1·5189	1·5310	1·5416	1·5528
600	252·3	1·4466	1·4616	1·4762	1·4899	1·5030	1·5151	1·5264	1·5377
700	261·7	1·4308	1·4469	1·4621	1·4763	1·4894	1·5019	1·5135	1·5248
800	270·1	1·4165	1·4337	1·4494	1·4641	1·4776	1·4904	1·5021	1·5135
1000	284·8	1·3909	1·4100	1·4274	1·4429	1·4572	1·4703	1·4828	1·4948
2000	335·4	1·2857	1·3174	1·3430	1·3645	1·3833	1·4000	1·4148	1·4286

Entropy of Superheated Steam—C.H.U. per lb./°C. *(continued)*.

Abs. Press. Lb./In.²	Degrees of Superheat (Centigrade).							
	80°	90°	100°	120°	140°	160°	180°	200°
15	1·8493	1·8596	1·8694	1·8891	1·9079	1·9261	1·9435	1·9604
20	1·8252	1·8352	1·8451	1·8646	1·8832	1·9011	1 9184	1·9350
30	1·7912	1·8011	1·8109	1·8300	1·8482	1·8658	1·8830	1 8995
40	1·7670	1·7768	1·7865	1·8053	1·8233	1·8408	1·8577	1·8741
50	1·7480	1·7578	1·7675	1·7860	1·8040	1·8215	1·8383	1 8546
60	1·7334	1·7432	1·7528	1·7712	1·7890	1·8063	1·8230	1·8394
70	1·7213	1·7310	1·7404	1·7587	1·7764	1·7936	1·8102	1·8265
80	1·7106	1·7202	1·7297	1·7480	1·7655	1·7826	1·7991	1·8153
90	1·7013	1·7109	1·7203	1·7384	1·7559	1·7730	1·7894	1·8057
100	1·6930	1·7026	1·7119	1·7302	1·7477	1·7645	1·7808	1·7969
120	1·6780	1·6877	1·6971	1·7154	1·7328	1·7496	1·7660	1·7817
140	1·6654	1·6751	1·6846	1·7028	1·7201	1·7369	1·7533	1·7689
160	1·6548	1·6646	1·6738	1·6921	1·7094	1·7261	1·7424	1·7579
180	1·6450	1·6548	1·6643	1·6825	1·7000	1·7165	1·7327	1·7482
200	1·6362	1·6460	1·6555	1·6737	1·6913	1·7080	1·7241	1·7396
250	1·6192	1·6290	1·6385	1·6568	1·6742	1·6909	1·7069	1·7224
300	1·6052	1·6150	1·6247	1·6428	1·6603	1·6770	1·6931	1·7085
400	1·5815	1·5916	1·6015	1·6196	1·6370	1·6537	1·6696	1·6850
500	1·5633	1·5735	1·5834	1·6016	1·6191	1·6358	1·6520	1·6675
600	1·5484	1·5585	1·5683	1·5869	1·6043	1·6212	1·6375	1·6533
700	1·5356	1·5458	1·5578	1·5745	1·5922	1·6093	1·6252	1·6414
800	1·5245	1·5348	1·5450	1·5638	1·5816	1·5997	1·6148	1·6308
1000	1·5058	1·5164	1·5266	1·5458	1·5638	1·5809	1·5972	1·6129
2000	1·4416	1·4536	1·4646	1·4848	1·5041	1·5220	1·5389	1·5548

Entropy of Superheated Steam—B.Th.U. per lb./°F.

Abs. Press. Lb./In.²	Saturation.		Degrees of Superheat (Fahrenheit)						
	t° F.	ϕ_s	20°	40°	60°	80°	100°	120°	140°
15	213·0	1·7556	1·7706	1·7844	1·7969	1·8101	1·8230	1·8353	1·8470
20	228·0	1·7327	1·7475	1·7610	1 7742	1·7867	1·7991	1·8110	1·8227
30	250·3	1·7004	1·7148	1·7281	1·7410	1·7534	1·7655	1·7773	1·7887
40	267·2	1·6776	1·6912	1·7045	1·7172	1·7295	1·7416	1·7531	1·7644
50	280·9	1·6597	1·6729	1·6861	1·6987	1·7109	1·7228	1·7344	1·7457
60	292·7	1·6450	1·6580	1·6713	1·6839	1·6961	1·7084	1·7200	1·7312
70	302·9	1·6327	1·6459	1·6593	1·6719	1·6840	1·6962	1·7078	1·7189
80	312·0	1·6219	1·6353	1·6488	1·6615	1·6736	1·6855	1·6971	1·7082
90	320·3	1·6124	1·6261	1·6396	1·6524	1·6645	1·6763	1·6878	1·6989
100	327·8	1·6038	1·6176	1·6311	1·6440	1·6561	1·6680	1·6795	1 6906
120	341·3	1·5891	1·6031	1·6166	1·6296	1·6417	1 6537	1·6652	1·6762
140	353·0	1·5762	1·5903	1·6038	1·6169	1·6290	1·6411	1·6526	1·6635
160	363·6	1·5652	1·5791	1·5926	1·6058	1·6179	1·6301	1·6416	1·6527
180	373·1	1·5554	1·5690	1·5825	1·5958	1·6080	1·6203	1·6318	1·6429
200	381·8	1·5466	1·5598	1·5735	1·5865	1·5990	1·6111	1·6228	1·6339
250	401·0	1·5276	1·5417	1·5557	1·5690	1·5818	1·5939	1·6057	1·6169
300	417·3	1·5117	1·5264	1·5408	1·5545	1·5675	1·5798	1·5918	1·6031
400	444·6	1·4857	1·5015	1·5164	1·5302	1·5434	1·5560	1·5680	1·5794
500	467·0	1·4646	1·4810	1·4959	1·5107	1·5240	1·5372	1·5594	1·5612
600	486·2	1·4466	1·4632	1·4793	1·4943	1·5084	1·5216	1·5341	1·5459
700	503·1	1·4308	1·4486	1·4654	1·4807	1·4950	1 5085	1·5211	1·5333
800	518·2	1·4165	1·4353	1·4529	1·4677	1·4832	1 4969	1·5099	1·5221
1000	544·6	1·3909	1·4119	1·4307	1·4477	1·4630	1·4772	1·4906	1·5031
2000	635·8	1·2857	1·3206	1·3481	1·3710	1·3908	1·4082	1·4241	1·4387

Entropy of Superheated Steam—B.Th.U. per lb./°F. (*continued*).

Abs. Press. Lb./In.²	Degrees of Superheat (Fahrenheit).							
	160°	180°	200°	240°	280°	320°	360°	400°
15	1·8584	1·8694	1·8802	1·9015	1·9220	1·9415	1·9604	1·9786
20	1·8340	1·8450	1·8556	1·8770	1·8973	1·9166	1·9352	1·9529
30	1·8000	1·8108	1·8213	1·8422	1·8621	1·8812	1·8996	1·9173
40	1·7755	1·7865	1·7969	1·8176	1·8374	1·8563	1·8745	1·8920
50	1·7567	1·7675	1·7778	1·7980	1·8175	1·8363	1·8546	1·8724
60	1·7422	1·7529	1·7628	1·7829	1·8023	1·8210	1·8392	1·8569
70	1·7299	1·7406	1·7503	1·7702	1·7895	1·8080	1·8261	1·8437
80	1·7192	1·7298	1·7397	1·7595	1·7787	1·7970	1·8150	1·8325
90	1·7099	1·7204	1·7302	1·7500	1·7691	1·7874	1·8054	1·8228
100	1·7015	1·7120	1·7220	1·7418	1·7607	1·7789	1·7969	1·8142
120	1·6870	1·6974	1·7073	1·7270	1·7460	1·7639	1·7818	1·7990
140	1·6744	1·6848	1·6947	1·7144	1·7333	1·7512	1·7690	1·7861
160	1·6636	1·6740	1·6840	1·7036	1·7224	1·7402	1·7579	1·7749
180	1·6539	1·6643	1·6743	1·6939	1·7128	1·7305	1·7482	1·7652
200	1·6449	1·6554	1·6655	1·6854	1·7042	1·7220	1·7395	1·7563
250	1·6280	1·6384	1·6485	1·6684	1·6872	1·7050	1·7225	1·7390
300	1·6142	1·6246	1·6349	1·6547	1·6734	1·6914	1·7086	1·7250
400	1·5905	1·6011	1·6114	1·6313	1·6501	1·6679	1·6851	1·7016
500	1·5724	1·5831	1·5935	1·6133	1·6320	1·6502	1·6675	1·6842
600	1·5574	1·5682	1·5785	1·5986	1·6176	1·6358	1·6532	1·6698
700	1·5448	1·5558	1·5664	1·5864	1·6056	1·6236	1·6410	1·6578
800	1·5338	1·5449	1·5557	1·5759	1·5951	1·6133	1·6306	1·6472
1000	1·5151	1·5264	1·5374	1·5579	1·5771	1·5954	1·6129	1·6296
2000	1·4522	1·4648	1·4768	1·4988	1·5191	1·5380	1·5557	1·5720

Equivalent Evaporation from and at 212° Fahr.—For the purpose of comparison it is usual to state the weight of steam produced in a boiler on the assumption that the feed water is supplied at a temperature of 212° Fahr., and that it is evaporated at that temperature.

W = weight of steam produced at any given temperature or pressure.

H = total heat in 1 lb. weight of this steam above that in water at 32°.

t_1 = temperature of feed water.

h_1 = heat in 1 lb. weight of feed water above that in water at 32° (h_1 is nearly equal to $t_1 - 32$).

W_1 = equivalent weight of steam at 212° temperature produced from feed water at 212°.

970·6 = latent heat of steam at 212° temperature.

$$W_1 = \frac{W(H - h_1)}{970 \cdot 6} = \frac{W(H - t_1 + 32)}{970 \cdot 6} \text{ nearly.}$$

The quantity $\frac{H - h_1}{970 \cdot 6}$ or its approximate value $\frac{H - t_1 + 32}{970 \cdot 6}$ is called the **Factor or Equivalent Evaporation**, or simply the **Factor of Evaporation.**

Factors of Equivalent Evaporation.

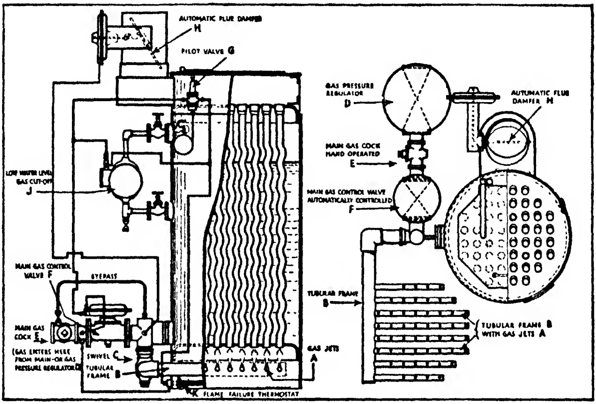
Temp. of Feed Water in Degrees F.	Boiler Pressure (Absolute) in Pounds per Square Inch.								
	40	50	60	70	80	90	100	110	120
35	1·201	1·206	1·210	1·213	1·216	1·218	1·221	1·223	1·225
40	1·196	1·201	1·204	1·208	1·210	1·213	1·215	1·218	1·220
45	1·191	1·195	1·199	1·202	1·205	1·208	1·210	1·212	1·214
50	1·186	1·190	1·194	1·197	1·200	1·203	1·205	1·207	1·209
55	1·181	1·185	1·189	1·192	1·195	1·198	1·200	1·202	1·204
60	1·176	1·180	1·184	1·187	1·190	1·192	1·195	1·197	1·199
65	1·170	1·175	1·179	1·182	1·185	1·187	1·190	1·192	1·194
70	1·165	1·170	1·173	1·177	1·179	1·182	1·184	1·187	1·189
75	1·160	1·164	1·168	1·171	1·174	1·177	1·179	1·181	1·183
80	1·155	1·159	1·163	1·166	1·169	1·172	1·174	1·176	1·178
85	1·150	1·154	1·158	1·161	1·164	1·166	1·169	1·171	1·173
90	1·145	1·149	1·153	1·156	1·159	1·161	1·164	1·166	1·168
95	1·139	1·144	1·147	1·151	1·153	1·156	1·159	1·161	1·163
100	1·134	1·139	1·142	1·145	1·148	1·151	1·153	1·156	1·157
105	1·129	1·133	1·137	1·140	1·143	1·146	1·148	1·150	1·152
110	1·124	1·128	1·132	1·135	1·138	1·141	1·143	1·145	1·147
115	1·119	1·123	1·127	1·130	1·133	1·135	1·138	1·140	1·142
120	1·113	1·118	1·121	1·125	1·127	1·130	1·133	1·135	1·137
125	1·108	1·113	1·116	1·120	1·122	1·125	1·127	1·130	1·131
130	1·103	1·107	1·111	1·114	1·117	1·120	1·122	1·124	1·126
135	1·098	1·102	1·106	1·109	1·112	1·115	1·117	1·119	1·121
140	1·093	1·097	1·101	1·104	1·107	1·109	1·112	1·114	1·116
145	1·087	1·092	1·095	1·099	1·101	1·104	1·106	1·109	1·111
150	1·082	1·086	1·090	1·093	1·096	1·099	1·101	1·103	1·105
155	1·077	1·081	1·085	1·088	1·091	1·094	1·096	1·098	1·100
160	1·072	1·076	1·080	1·083	1·086	1·088	1·091	1·093	1·095
165	1·066	1·071	1·075	1·078	1·081	1·083	1·086	1·088	1·090
170	1·061	1·066	1·069	1·072	1·075	1·078	1·080	1·083	1·084
175	1·056	1·060	1·064	1·067	1·070	1·073	1·075	1·077	1·079
180	1·051	1·055	1·059	1·062	1·065	1·068	1·070	1·072	1·074
185	1·046	1·050	1·054	1·057	1·060	1·062	1·065	1·067	1·069
190	1·040	1·045	1·048	1·052	1·054	1·057	1·060	1·062	1·064
195	1·035	1·040	1·043	1·046	1·049	1·052	1·054	1·057	1·059
200	1·030	1·034	1·038	1·041	1·044	1·047	1·049	1·051	1·053
205	1·025	1·029	1·033	1·036	1·039	1·042	1·044	1·046	1·048

Factors of Equivalent Evaporation.

Temp. of Feed Water in Degrees F.	Boiler Pressure (Absolute) in Pounds per Square Inch.								
	130	140	150	160	170	180	190	200	210
35	1·227	1·229	1·230	1·232	1·233	1·235	1·236	1·238	1·239
40	1·222	1·223	1·225	1·227	1·228	1·230	1·231	1·233	1·234
45	1·216	1·218	1·220	1·222	1·223	1·225	1·226	1·227	1·229
50	1·211	1·213	1·215	1·216	1·218	1·219	1·221	1·222	1·223
55	1·206	1·208	1·210	1·211	1·213	1·214	1·216	1·217	1·218
60	1·201	1·203	1·204	1·206	1·208	1·209	1·210	1·212	1·213
65	1·196	1·197	1·199	1·201	1·202	1·204	1·205	1·207	1·208
70	1·191	1·192	1·194	1·196	1·197	1·199	1·200	1·201	1·203
75	1·185	1·187	1·189	1·191	1·192	1·194	1·195	1·196	1·198
80	1·180	1·182	1·184	1·185	1·187	1·188	1·190	1·191	1·192
85	1·175	1·177	1·179	1·180	1·182	1·183	1·185	1·186	1·187
90	1·170	1·172	1·173	1·175	1·177	1·178	1·179	1·181	1·182
95	1·165	1·166	1·168	1·170	1·171	1·173	1·174	1·176	1·177
100	1·159	1·161	1·163	1·165	1·166	1·168	1·169	1·170	1·172
105	1·154	1·156	1·158	1·159	1·161	1·162	1·164	1·165	1·166
110	1·149	1·151	1·153	1·154	1·156	1·157	1·159	1·160	1·161
115	1·144	1·146	1·147	1·149	1·151	1·152	1·153	1·155	1·156
120	1·139	1·140	1·142	1·144	1·145	1·147	1·148	1·150	1·151
125	1·133	1·135	1·137	1·139	1·140	1·142	1·143	1·144	1·146
130	1·128	1·130	1·132	1·133	1·135	1·136	1·138	1·139	1·141
135	1·123	1·125	1·127	1·128	1·130	1·131	1·133	1·134	1·135
140	1·118	1·120	1·121	1·123	1·125	1·126	1·127	1·129	1·130
145	1·113	1·114	1·116	1·118	1·119	1·121	1·122	1·124	1·125
150	1·107	1·109	1·111	1·113	1·114	1·116	1·117	1·118	1·120
155	1·102	1·104	1·106	1·107	1·109	1·110	1·112	1·113	1·114
160	1·097	1·099	1·100	1·102	1·104	1·105	1·106	1·108	1·109
165	1·092	1·093	1·095	1·097	1·098	1·100	1·101	1·103	1·104
170	1·086	1·088	1·090	1·092	1·093	1·095	1·096	1·097	1·099
175	1·081	1·083	1·085	1·086	1·088	1·089	1·091	1·092	1·093
180	1·076	1·078	1·080	1·081	1·083	1·084	1·086	1·087	1·088
185	1·071	1·073	1·074	1·076	1·078	1·079	1·080	1·082	1·083
190	1·066	1·067	1·069	1·071	1·072	1·074	1·075	1·077	1·078
195	1·060	1·062	1·064	1·066	1·067	1·069	1·070	1·071	1·073
200	1·055	1·057	1·059	1·060	1·062	1·063	1·065	1·066	1·067
205	1·050	1·052	1·054	1·055	1·057	1·058	1·060	1·061	1·062

STEAM BOILERS.

Cochran-Kirke Sinuflo Gas-fired Vertical Boiler.—



Gas is burnt in a number of jets (*A*) fitted to a tubular frame (*B*) arranged on a swivel (*C*), so that the frame can be withdrawn from under the boiler when the jets are lighted or cleaned. The gas jets are for a gas pressure of 2½ inches water gauge, and a gas pressure regulator (*D*) maintains this pressure.

The hot gases passing up the *Sinuflo* tubes impinge upon the sides, and heat is thus conducted to the water surrounding the tubes more efficiently than would be possible with straight tubes. The gas supply to the jets is automatically controlled, it enters at the main gas cock (*E*) and then passes through the main gas control valve (*F*), which is opened or shut by a pilot valve (*G*) as the pressure in the boiler rises or falls. In the case of hot-water boilers the pilot valve is actuated by a rise or fall in temperature.

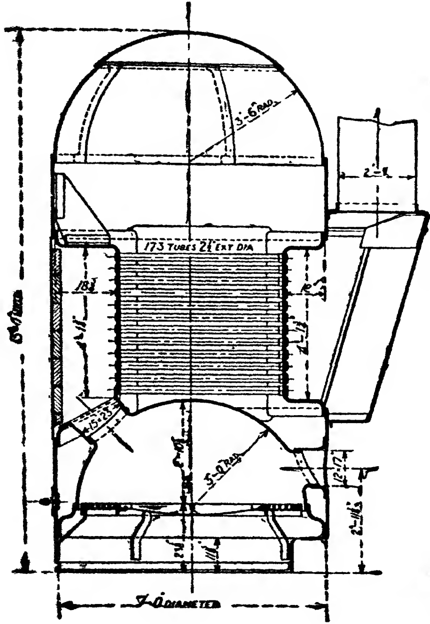
The automatic flue damper (*H*) prevents cold air passing through the boiler when the gas supply is shut off. It opens automatically when the gas is on. Should the level of the water fall below the bottom gauge cock, the low water-level gas cut-off (*J*) reduces the gas supply to a minimum until the water-level is restored, when the gas automatically comes on again. The flame failure thermostat (*K*) is a safety device which cuts off the gas to the burners if the main supply fails temporarily. It closes the main gas control valve (*F*), and ensures its remaining closed until the operator relights the jets.

Standard Sizes.

Pressures 60 to 120 lbs. per sq. inch.

Diam.		Height.		Evaporation. Lbs. of Steam per Hour.		Gas Consumption per Hour.	
				From and at 212° F.	120 lbs./sq. in. from Feed-water at 60° F.	Therms.	Cubic Ft.*
Ft.	In.	Ft.	In.				
1	6	6	3	140	120	1·89	378
1	9	6	6	230	190	3·10	620
2	0	6	6	285	240	3·84	770
2	3	6	9	380	315	5·12	1024
2	6	6	9	500	420	6·75	1350
2	9	6	9	620	515	8·35	1670
3	0	7	0	760	635	10·25	2050
4	0	7	6	1000	830	13·50	2700
4	6	7	6	1330	1110	17·90	3580
5	0	7	9	1665	1390	22·40	4480
5	6	8	0	2130	1770	28·80	5760
6	0	8	3	2700	2250	36·40	7280
6	6	8	9	3140	2610	42·40	8480
7	0	8	9	3750	3120	50·50	10100
7	6	9	0	4300	3580	57·80	11560
8	0	9	3	4800	4000	64·50	12900
8	6	9	6	5600	4670	75·50	15100
9	0	9	6	6450	5370	86·90	17400

* Assuming a calorific value of 500 B.Th.U. per cubic foot.



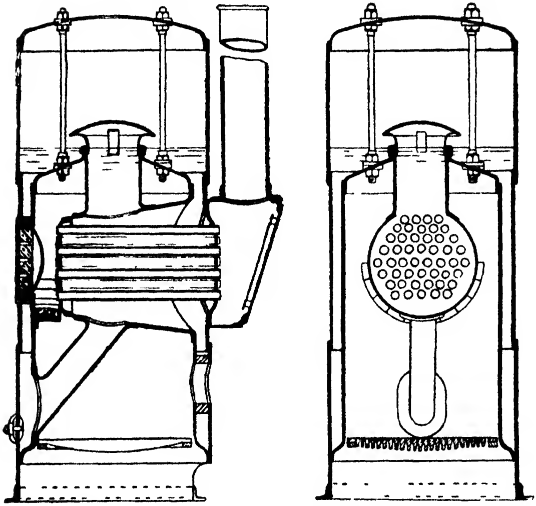
Cochran's Vertical Multitubular Boiler.

Working pressure 100 lbs. per sq. inch.

Diam.		Height.		Heating Surface.	Grate Area.	No. of Tubes. Ex. Diam. 2¼ Inches.	Coal-fired. Evap. from 60° F.	Approx. Weight.
Ft.	In.	Ft.	In.	Sq. Ft.	Sq. Ft.		Lbs. per Hr.	Cwts.
3	0	6	9	50	4·75	23	250	23
3	3	7	6	60	5·75	23	340	25
3	9	8	6	100	7·50	37	540	32
4	0	9	0	110	8·50	42	630	41
4	3	9	6	140	9·25	58	750	45
4	6	10	0	160	9·75	65	820	50
4	9	10	3	190	11·75	74	1030	58
5	0	11	3	220	12·50	84	1150	65
5	3	11	9	250	14·00	92	1320	73
5	6	12	3	300	16·75	102	1620	82
5	9	13	0	350	18·75	123	1880	95
6	0	12	6	350	18·75	110	1880	96
6	0	13	6	350	18·75	110	1880	100
6	0	14	0	400	18·75	136	2000	107
6	6	13	6	450	22·50	143	2360	116
6	6	14	0	450	22·50	143	2360	121
6	6	14	6	500	22·50	158	2480	125
7	0	14	0	500	26·75	143	2750	140
7	0	15	0	600	26·75	173	3020	150
7	6	16	3	750	31·50	224	3650	188
8	0	16	6	850	37·00	224	4320	213
8	6	18	0	1000	41·00	270	4960	258
9	0	19	0	1250	48·00	322	6030	328

The Sharpe-Palmer Vertical Boiler.

(Abbott & Co., Newark-on Trent.)



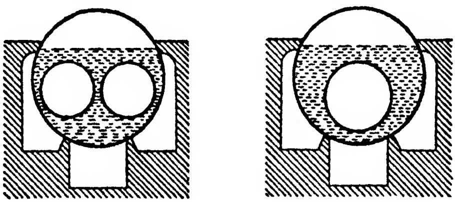
Height of Shell. Ft.	In.	Diameter of Shell Ft.	In.	Height of Fire-box. Ft.	In.	Number of Tubes.	Diameter of Tubes. In.	Thickness of Shell Plates. In.	Thickness of Fire box. In.	Thickness of Crowns. In	Heating Surface Sq Ft.	Grate Surface. Sq. Ft.	Approximate Weight. Cwts
5	0	2	6	3	0	22	$1\frac{1}{2}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{16}$	45	3·0	10
5	6	2	9	3	9	28	$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	55	3·7	14
6	0	3	0	4	0	34	$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	70	4·6	18
7	0	3	3	4	6	38	$1\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	80	5·6	24
8	0	3	6	5	3	28	2	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	100	6·7	29
8	6	3	9	5	9	30	2	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	120	7·9	33
9	0	4	0	6	0	34	2	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{16}$	134	9·0	37
10	0	4	6	6	3	42	$2\frac{1}{2}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{1}{2}$	200	12·0	54
10	0	5	0	6	6	44	$2\frac{1}{2}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{1}{2}$	250	14·7	65
11	6	5	6	7	0	56	$2\frac{1}{2}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{2}$	330	18·3	80
12	0	6	0	7	4	74	$2\frac{1}{2}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	440	22·3	92
13	6	6	6	8	0	108	$2\frac{1}{2}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	560	26·0	123

Computation of Heating Surface of Boiler Tubes.—A formula is given on p. 194, and tables are given on pp. 195–199, for determining the surface of tubes. There is a difference in practice in estimating the heating surface of boiler tubes. The most common practice is to take the external surface of the tube as the heating surface, the length of the tube being measured between the tube-plates. Some engineers, however, prefer to take the internal surface when the furnace gases pass through the tubes. In boiler tubes the external surface is from 10 to 20 per cent. greater than the internal surface, according to the diameter and thickness of the tubes.

Let A = external surface of tube. D = external diameter of tube.
a = internal surface of tube. d = internal diameter of tube.

$$\text{Then } A = \frac{aD}{d}, \text{ and } a = \frac{Ad}{D}.$$

Lancashire and Cornish Boilers.—The Lancashire boiler has two internal cylindrical flues extending the whole length of the boiler, the furnaces being in these flues at their front ends.



Lancashire Boiler. *Cornish Boiler.*

The Cornish boiler has one internal flue extending the whole length of the boiler, the furnace being in this flue at its front end.

The majority of boilers of these types built to-day are of the Lancashire variety.

Lancashire boilers may be subdivided as follows:—

(1) Flat end with gusset stays.
(2) Double dished end of the Continental variety.
(3) Unidish boiler (made by Daniel Adamson & Co., Ltd.), which has a dished end at the front and a flat stayed end at the back.

These boilers are set in brickwork, and care must be taken to ensure that air leakages do not occur between the metal shells and the bricks. Various expansion joints are on the market, and one of those manufactured by Daniel Adamson & Co., Ltd., consists of a partial ring of asbestos which presses against the boiler shell, but is free to move radially in a groove in a metal casing attached to the brickwork.

Approximate Evaporation for Lancashire Boilers. (Hand Fired.)

(*Made by Daniel Adamson & Co. Ltd., Dukinfield.*)

No.	Size.			Size of Flues.			Grate Area in Sq. Ft.	Evaporation. Lbs. per Hour.		Approximate Heating Surface.	
	Length. Ft.	Diam. Ft.	Diam. In.	No.	Diam. Ft.	Diam. In.		Feed at 60° F.	From and at 212 F.	Sq. Ft.	Sq. M.
1	18	6	0	2	2	4	22·5	2580	3100	460	43·0
2	20	6	0	2	2	4	22·5	2860	3430	510	47·5
3	22	6	0	2	2	4	22·5	3250	3900	580	54·0
4	24	6	0	2	2	4	22·5	3530	4230	630	58·5
5	18	6	6	2	2	7	25·0	2880	3460	505	47·0
6	20	6	6	2	2	7	25·0	3190	3830	560	52·0
7	22	6	6	2	2	7	25·0	3530	4230	620	57·5
8	24	6	6	2	2	7	27·5	3850	4620	675	62·5
9	26	6	6	2	2	7	27·5	4190	5050	735	68·5
10	24	7	0	2	2	9	29·5	4290	5150	740	69·0
11	26	7	0	2	2	9	32·0	4640	5570	800	74·5
12	28	7	0	2	2	9	32·0	4980	5980	860	80·0
13	30	7	0	2	2	9	32·0	5370	6450	925	86·0
14	24	7	6	2	3	0	32·5	4800	5750	800	74·5
15	26	7	6	2	3	0	35·5	5220	6280	870	81·0
16	28	7	6	2	3	0	35·5	5640	6760	940	87·5
17	30	7	6	2	3	0	35·5	6000	7200	1000	93·0
18	24	8	0	2	3	3	38·0	5420	6500	860	80·0
19	26	8	0	2	3	3	38·0	5900	7090	935	87·0
20	28	8	0	2	3	3	38·0	6360	7610	1010	94·0
21	30	8	0	2	3	3	38·0	6850	8220	1085	101·0
22	24	8	6	2	3	6	41·0	6070	7300	920	85·5
23	26	8	6	2	3	6	41·0	6600	7900	1000	93·0
24	28	8	6	2	3	6	41·0	7130	8580	1080	100·5
25	30	8	6	2	3	6	41·0	7650	9200	1160	108·0
26	24	9	0	2	3	9	44·0	6760	8100	980	91·0
27	26	9	0	2	3	9	44·0	7400	8900	1070	99·5
28	28	9	0	2	3	9	44·0	7950	9530	1150	107·0
29	30	9	0	2	3	9	44·0	8500	10200	1230	114·5
30	24	9	6	2	4	0	47·0	7630	9200	1060	98·5
31	26	9	6	2	4	0	47·0	8200	9810	1140	106·0
32	28	9	6	2	4	0	47·0	8800	10600	1220	113·5
33	30	9	6	2	4	0	47·0	9450	11350	1310	122·0
35	26	10	0	2	4	0	50·0	9150	11000	1220	113·5
37	30	10	0	2	4	0	50·0	10400	12500	1385	129·0

Note.—The figures in the above table are based on coal having a calorific value of 13,500 B.Th.U. per lb.

Approximate Evaporation for Cornish Boilers. (Hand Fired.)

(Made by Daniel Adamson & Co. Ltd., Dukinfield.)

No.	Size.		Size of Flue.	Grate Area in Sq. Ft.	Evaporation. Lbs. per Hour.		Approximate Heating Surface	
	Length. Ft.	Diam. Ft. In.	Diam. Ft. In.		Feed at 60° F.	From and at 212° F.	Sq. Ft.	Sq. M.
1	18	6 0	3 3	16·0	2150	2580	390	36·0
2	20	6 0	3 3	16·0	2370	2840	430	40·0
3	22	6 0	3 3	16·0	2680	3220	470	43·5
4	24	6 0	3 3	17·5	2830	3400	515	48·0
5	18	6 6	3 6	17·0	2400	2880	420	39·0
6	20	6 6	3 6	17·0	2650	3180	465	43·0
7	22	6 6	3 6	19·0	2910	3500	510	47·5
8	24	6 6	3 6	19·0	3160	3790	555	51·5

Note.—The figures in the above table are based on coal having a calorific value of 13,500 B.Th.U. per lb.

Brickwork Setting for Lancashire Boilers.*—(See illustrations on opposite page.)

Width E of side flues at top = 12 inches.

Width F of bottom flue = half diameter of shell of boiler.

Depth G of bottom flue from bottom of shell not less than 2 feet 6 inches, and is usually made 3 feet.

The dimensions of the seating blocks and side flue covers are shown in the illustrations. The seating block at (1) is for boilers less than 8 feet diameter, and that at (2) is for boilers of 8 feet diameter and over.

Seating blocks, flue covers, and all brickwork in contact with boiler to be set in lime mortar, but no mortar should come in contact with the boiler plates, fireclay being used at these points.

All flues should be faced with firebricks forming a lining 4½ inches thick, suitably bonded into the walls.

The short partition wall (*a*) at the back end is frequently omitted, but this is bad practice, and may lead to severe "humming" and vibration of the boiler.

The boiler shell should slope downwards towards the front end at the rate of about ½ inch in 10 feet, and the floors of the flues should also have this slope.

Width H of downtake at back end = 2 feet 6 inches.

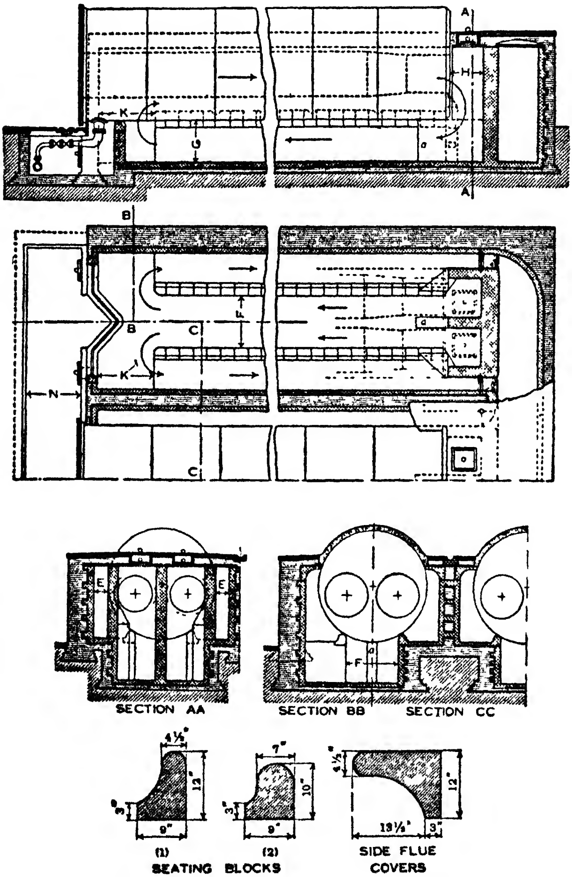
Width K of gap at front end = 3 feet 6 inches.

Width N of blow-out pit = 3 feet 6 inches to 4 feet.

* Revised from information supplied by the National Boiler & General Insurance Co., Ltd., Manchester.

Floor of blow-out pit 6 inches to 9 inches below floor of bottom flue.

In the plan view, the groove round the top of the brickwork of the blow-out recess wall is for asbestos rope.



Cochran Sinuflo Economic Boiler.—This boiler is of the horizontal multitubular dry-back type with induced draught provided by a fan which is usually mounted with its prime mover on a platform fixed to the top of the boiler shell. No external brick setting is required and the boiler shell is made in two strakes or belts, with the mid-circumferential seam treble-riveted. The primary heating surface consists of one or more horizontal furnaces, above which is arranged a single pass of patent horizontal *Sinuflo* tubes. These tubes, which are similar to those illustrated in the vertical boiler on p. 586, ensure that much of the remaining heat in the gases is transmitted to the surrounding water.

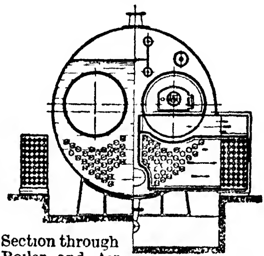
There is a large brick-lined combustion chamber into which brick arches project from the ends of the furnaces; thus the hot gases are compelled to travel round the combustion chamber before entering the Sinuflo tubes, and smokeless combustion is ensured.

The rated outputs listed below are obtained with suitable coal when the boilers are fitted with mechanical stokers of the Coking, Sprinkler, or Underfeed type, usually driven by electric motor or, in the case of a battery of boilers, from overhead shafting.

Satisfactory results may also be obtained with oil fuel, coke-oven gas, or other fuels.

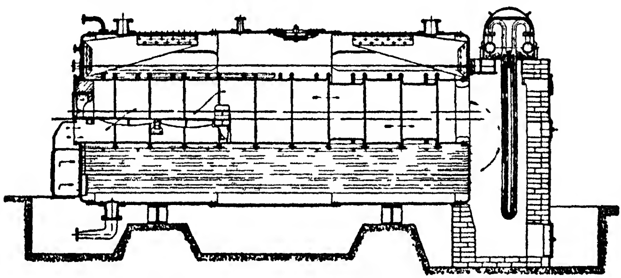
Boiler Diam.		Overall Length.		No. of Flues.	Diam. of Flues.		Maximum Working Pressure. Lbs. per Sq. In.	Evaporation. Lbs. per Hour.	
								From Feed at 60° F.	From and at 212° F.
Ft.	In.	Ft.	In.		Ft.	In.			
6	6	18	7	1	2	9	250	4800	5800
7	0	21	7	1	3	0	250	5700	6800
7	6	21	7	1	3	6	250	6700	8000
8	0	21	7	1	4	0	225	7600	9100
9	0	22	7	1	4	6	200	8500	10200
9	0	22	7	2	2	9	250	10400	12500
9	6	22	7	2	3	0	250	11300	13600
10	6	22	7	2	3	6	250	13300	16000
11	6	22	7	2	4	0	225	15200	18200
11	9	22	7	3	3	0	250	17100	20500
12	6	22	7	3	3	6	250	20000	24000
13	0	22	7	4	3	0	250	22900	27500
13	6	22	7	4	3	3	250	25000	30000

Daniel Adamson's Patent Super Lancashire Boiler.—This plant consists of a cylindrical boiler having furnace tubes similar to a Lancashire boiler but with smoke tubes arranged beneath them. A superheater of the Adamson sectional type is fitted in the downtake, if required. There are no external flues, and the gases from the smoke tubes pass through two air heaters, one at each side, which heat the air supply to the furnaces. Finally, the gases are delivered to a short steel or brick chimney by means of an induced draught fan. The boiler is also provided with forced draught.



Section through Boiler and Air Heater. Section through Front Gas and Air Boxes.

The fuel (coal, oil, coke, timber, etc.) is fed into the furnaces by hand, mechanical stoker, jet or pulveriser, etc. The downtake



Longitudinal Sectional Elevation through Boiler and Superheater.

chamber is constructed of steel and is lined with insulating bricks. No other bricks are required, for the boiler is arranged on cast-iron chairs on a concrete base without brickwork setting, thus avoiding possible losses due to defective brickwork. The connection between the boiler and the downtake chamber is made with a patent flexible air-excluding device.

The makers claim, amongst other things, economy of space, quick steaming, high overall efficiency, balanced draught (no cold air entering furnace whether hand or stoker fired), and practically the entire elimination of smoke.

The boiler is usually made 20 feet long, and there are eight sizes in the diameter, from 8 feet to 12 feet. The working pressures range up to 260 lbs. per sq. inch.

Boiler Trial.

An Abstract of the Report of a Test made by the National Boiler & General Insurance Co., Ltd., of the Adamson Patent Super Lancashire Boiler Unit.

General Description of Unit.

Boiler.

Type and size—Super Lancashire, 8 ft. 6 in. diam., 20 ft. long.

Permissible working pressure	lbs. per sq. in.	260
Total grate area	sq. ft.	38
Heating surface	,,	1427

Superheater.

Heating surface (effective)	,,	285

Air Heaters (2).

Heating surface (effective)	,,	1060

Draught Plant—Mechanical draught.

Fans.

Induced draught fan direct-coupled to small high-speed engine	r.p.m.	390
Forced draught fan direct-coupled to 5 B.H.P. electric motor	,,	765

Mean Observations and Derived Data.

Duration of test	hours	12·117
Temperature of outside air	°F.	58·5
Temperature of air at inlet to forced draught fan	°F.	65·9

Furnace.

Description of fuel	"Manton Cobbles."	
Method of stoking	Hand.	
Rate of firing	lbs. per hour	1078·5
Ultimate analysis of fuel as fired—		
Moisture	per cent.	7·40
Ash	,,	6·12
Carbon	,,	71·35
Hydrogen	,,	4·53
Nitrogen	,,	1·29
Sulphur	,,	0·63
Oxygen, etc. (by difference)	,,	8·68
		100·00

Calorific value of fuel as fired	**Gross.**	**B.Th.U. per lb.**	**12835**
,, ,, ,,	Net.	,, ,,	**12330**

Method and time of clearing and cleaning grates.	Clinker broken up and removed by hand at intervals of 6 hours.	
Residue (ashes and clinker) formed .	lbs. per hour	42·1
Combustible matter in residue . .	. per cent.	12·82
Calorific value (gross) of residue .	B.Th.U. per lb.	1859
Boiler and Superheater.		
Feed-water—Rate of feed . .	. lbs. per hour	8773
Mean temperature of feed-water at boiler .	. °F.	113·1
Steam.		
Pressure in saturated steam space .	lbs. per sq. in.	138·3
Moisture in steam entering superheater	. per cent.	1·05
Temperature of saturation . .	. . °F.	360·0
Steam superheated . . .	lbs. per hour	8078
Pressure at superheater outlet . .	lbs. per sq. in.	137·8
Temperature of superheated steam .	. . °F.	680·4
Air Heaters.		
Temperature of air at entry	°F.	65·9
,, ,, ,, exit	°F.	255·8
Draught Plant.		
Temperature of air supply at inlet to forced draught fan	°F.	65·9
Pressure at air heater—		
Flue gas inlet . . .	inches suction (W.G.)	1·03
,, ,, outlet . . .	,, ,, ,,	2·27
Air inlet . . .	inches pressure (W.G.)	1·14
,, outlet . . .	,, ,, ,,	0·67
Mean power to operate induced draught fan	. B.H.P.	4·48
,, ,, ,, forced ,, ,,	. K.W.	2·11
Total power to operate induced and forced draught fans	. K.W.	5·83
Flue Gases.		
Average temperature in downtake . . .	°F.	1234
,, ,, at inlet to air heaters .	°F.	590·2
,, ,, at outlet from air heaters .	°F.	407·3
Analysis of dry flue gases at air heater outlet—		
Carbon dioxide (by volume) . . .	per cent.	12·9
Oxygen	,,	6·2
Carbon monoxide	,,	0·0
Nitrogen, etc. (by difference) . . .	,,	80·9
		100·0

Conclusions.

Fuel fired per sq. ft. of grate . .	lbs. per hour	28·38
Air used per lb. of fuel	lbs.	13·26
Air theoretically required per lb. of fuel as fired .	,,	9·65
Ratio of air used to air theoretically required .		1·37
Weight of products of combustion per lb. of fuel as fired	lbs.	10·59
Weight of gases per lb. of fuel as fired . .	,,	14·20
Water evaporated per lb. of fuel as fired . .	,,	8·13
Equivalent evaporation from and at 212° F. per lb. of fuel as fired	,,	10·68
Heat absorbed by water in boiler .	B.Th.U. per lb.	1119
Heat absorbed by steam in superheater	,, ,,	170·2
Heat absorbed by forced and induced draught fans	B.Th.U. per hour	109500
Heat absorbed by forced and induced draught fans	per cent. of total value	0·82
Heat transmitted per sq. ft. of boiler heating surface	B.Th.U. per hour	6879
Heat transmitted per sq. ft. of superheater heating surface . . .	B.Th.U. per hour	4824

Air Heaters.

Calculated weight of air heated per lb. of fuel as fired	lbs.	13·26
Heat transmitted per sq. ft. of heating surface of air heaters	B.Th.U. per hour	623
Percentage of total heat imparted to air in air heaters	per cent.	4·96

Draught Plant.

Total power to produce draught . .	K.W.	5·83

Heat Account.

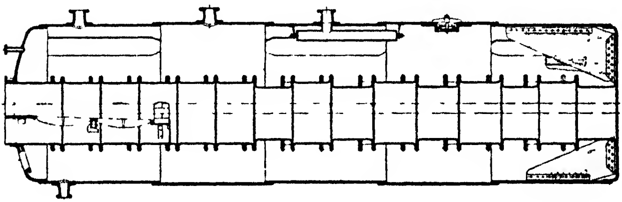
(Per lb. of Fuel as Fired.)

	B.Th.U.	Per cent.
Heat leaving boiler plant in steam . .	10371	84·13
Heat lost in products of combustion . .	909	7·37
Heat lost in excess air	302	2·43
Heat lost in combustible matter in ashes .	101	0·82
Balance of account—Radiation and other unmeasured losses	647	5·25
Total net heat value of 1 lb. of coal as fired	**12330**	**100·00**

Thermal Efficiency.

Gross overall thermal efficiency	84·13
Heat absorbed by induced and forced draught plant	0·82
Net overall thermal efficiency	83·31

Daniel Adamson's "Unidish" Boiler—This boiler is of the Lancashire type, with a dished end at the front and a flat end at the back, as shown in the figure. The makers claim the advantages of both the dished end and flat-end types. The flat back end gives "breathing space" to take up the difference in



expansion of the flues and the shell of the boiler, and makes it more convenient to install a superheater than is the case with a dished back end. The dished front end overcomes any possibility of leakage at gusset angle rivets, as there are none. There are, of course, gusset stays at the back end, but there the temperature conditions are less severe than at the front end.

The boiler is made in various sizes and for working pressures up to 260 lbs. per sq. inch.

The Dry-back or Economic Boiler.—This boiler is of the shell type with one or more internal flues, and, in addition, smoke tubes are provided above the main flue or flues, and it may be either brick-set or self-contained. It is used for hotels, institutions, or small works, or other buildings where space is limited. The heating surface is large, and the steam is generated quickly and with reasonable efficiency. Working pressures from 60 to 260 lbs. per sq. inch are obtainable. Sizes and evaporation rates are given in the tables (p. 600).

Ordinary Marine Boilers.

Furnaces.—Boilers up to 9 feet in diameter may have one furnace in an end. Boilers from 8 feet to 13 feet in diameter may have two furnaces in an end. Boilers from 11 feet 6 inches to 15 feet 6 inches in diameter may have three furnaces in an

Approximate Evaporation for Dry-Back or Economic Boilers.

(Brick-set or Self-contained. Hand Fired.)

(Made by Daniel Adamson & Co. Ltd., Dukinfield.)

No.	Size.				Size of Flues.			Grate Area in Sq. Ft.	Evaporation. Lbs. per Hour.		Approximate Heating Surface.	
	Length.		Diam.		No.	Diam.			Feed at 60° F.	From and at 212° F.		
	Ft.	In.	Ft.	In.		Ft.	In.				Sq. Ft.	Sq. M.
1	9	6	6	0	1	2	10	13·75	2100	2520	420	39·0
2	11	0	6	3	1	2	11	14·7	2425	2910	485	45·0
3	12	6	6	6	1	3	0	16·0	2950	3540	590	55·0
4	12	6	7	0	2	2	4	22·5	3450	4140	690	64·0
5	12	6	7	6	2	2	6	26·5	3975	4770	795	74·0
6	14	0	7	6	2	2	6	29·0	4475	5370	895	83·0
7	14	0	8	0	2	2	9	32·0	5875	7050	1175	109·0
8	14	0	8	6	2	2	10	34·0	6700	8040	1340	124·5
9	15	6	8	9	2	3	0	35·0	7650	9180	1530	142·0
10	15	6	9	0	2	3	1	36·0	8370	10044	1674	155·5
11	15	6	9	6	2	3	3	38·0	9130	10956	1826	169·5
12	15	6	9	9	2	3	5	40·0	10250	12300	2050	190·5
13	16	0	10	0	2	3	7	42·0	11085	13302	2217	206·0
14	16	0	10	6	2	3	9	44·0	12555	15066	2511	233·0
15	16	0	11	0	2	4	0	47·0	13440	16128	2688	250·0

Approximate Evaporation for Dry-Back or Economic Boilers.

(Double Return. Hand Fired.)

(Made by Daniel Adamson & Co. Ltd., Dukinfield.)

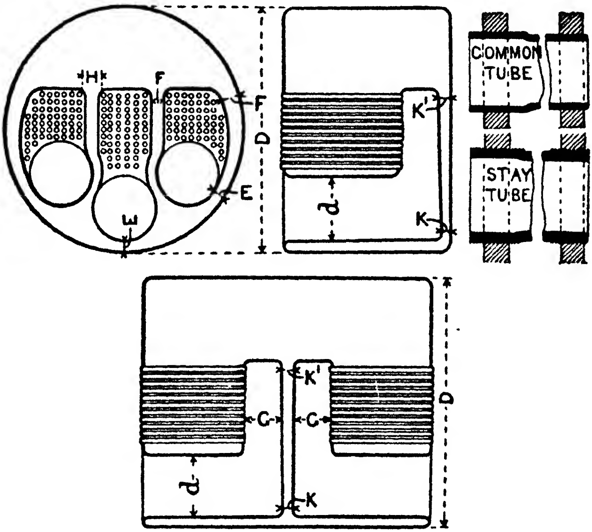
No.	Size.				Size of Flues.			Grate Area in Sq. Ft.	Evaporation. Lbs. per Hour.		Approximate Heating Surface.	
	Length.		Diam.		No.	Diam.			Feed at 60° F.	From and at 212° F.		
	Ft.	In.	Ft.	In.		Ft.	In.				Sq. Ft.	Sq. M.
1	7	6	6	0	1	2	6	9·75	1800	2160	360	33·5
2	8	0	6	6	1	2	9	10·75	2105	2526	421	39·0
3	8	3	7	0	1	3	0	11·75	2580	3096	516	48·0
4	8	6	7	6	1	3	2	14·0	3070	3684	614	57·0
5	9	0	7	9	2	2	4	20·0	3695	4434	739	68·5
6	9	3	8	0	2	2	6	22·0	4330	5196	866	80·5
7	9	6	8	6	2	2	8	23·5	4780	5736	956	89·0
8	9	6	9	0	2	2	10	27·5	6110	7332	1222	113·5
9	9	6	9	6	2	3	0	32·0	6705	8046	1341	124·5
10	9	6	9	9	2	3	2	34·5	7150	8580	1430	133·0
11	10	0	10	0	2	3	3	38·0	8880	10656	1776	165·0
12	10	6	11	0	2	3	5	40·0	10730	12876	2146	199·5
13	10	6	12	0	2	3	9	44·0	13345	16014	2669	248·0

Note.—The figures in the above tables are based on coal having a calorific value of 13,500 B.Th.U. per lb.

end; and boilers above 14 feet 6 inches in diameter may have four furnaces in an end.

D = diameter of shell in inches.
d = diameter of furnace in inches.
$d = \frac{1}{2}D$, when there is one furnace in an end.
$d = \frac{1}{4}D + 5$ inches, when there are two furnaces in an end.
$d = \frac{1}{4}D - 1$ inch, when there are three furnaces in an end.

The above rules need not be strictly adhered to, although they represent average practice.



Combustion Chambers.—The height of the crown of the combustion chamber above the centre of the boiler is usually about one-sixth of the diameter of the shell. The length G of the combustion chamber should not exceed $\frac{7}{8}d$; it is generally about $\frac{3}{4}d$, but in boilers having not more than two furnaces in an end it is sometimes as small as $\frac{1}{2}d$.

Water Spaces.—$E = 4\frac{1}{2}$ inches to 6 inches. $F = 5$ inches to 7 inches.

$K = 5$ inches to 7 inches. K' must be at least equal to K, and is sometimes as much as 12 inches. These are all inside dimensions between the plates.

Tubes.—The tubes are generally of wrought-iron or steel, and they are usually from $2\frac{1}{2}$ inches to $3\frac{1}{2}$ inches in external diameter. They are arranged in horizontal and vertical rows. The pitch, measured from centre to centre, is about $1\frac{3}{8}$ times the external diameter of the tubes. The clear space *H* between the nests of tubes varies from $10\frac{1}{2}$ inches to $12\frac{1}{2}$ inches, the average being about 11 inches.

The common tubes are swelled at their front ends $\frac{1}{16}$ inch larger in diameter, and they are secured to the tube plates by expanding them at their ends with a tube expander.

The stay tubes are screwed into both tube plates. The diameter over the screw thread at the front end is $\frac{1}{8}$ inch larger than the diameter over the thread of the back end. The screws have 10 or 11 threads per inch.

The length of the tubes between the tube plates varies from 5 feet 9 inches to 7 feet 6 inches, the average being about 6 feet 9 inches. For steam pressures up to about 285 lbs. per square inch the thickness of the common tubes may be No. 8 S.W.G. (·16 inch). The thickness of the stay tubes varies from $\frac{3}{16}$ inch to $\frac{3}{8}$ inch at the bottom of the screw thread. About one-third of all the tubes are stay tubes.

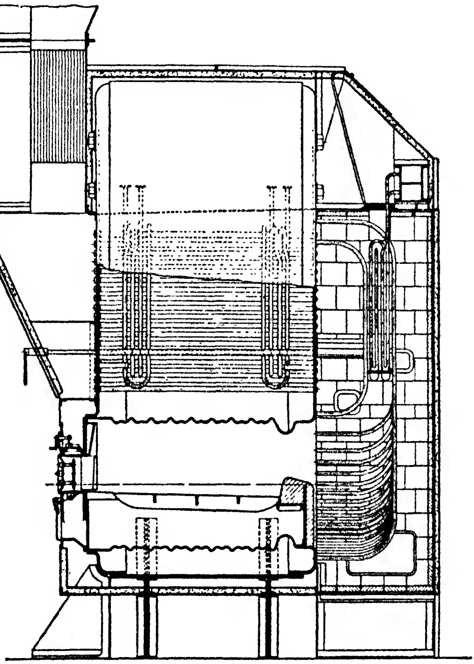
The Howden-Johnson Air-jacketed Improved Scotch Boiler.

The designers claim that while this boiler possesses ample water reserve, it is 30 per cent. lighter and appreciably smaller than the standard Scotch boiler of the same heating surface. It is suited for the burning of powdered coal or oil, and also for coal burning by hand firing or mechanical stokers. The maximum working pressure is 300 lbs. per sq. inch, and the superheater gives temperatures up to 800° F.

The boiler consists of a cylindrical shell in which furnaces and return tubes are fitted. There is a dry-back combustion chamber in which is arranged a series of tubes forming a water wall, and these tubes maintain a rapid circulation in the boiler. The furnaces have bellows ends at the back to give flexibility and are welded to the back plate, and both the front and back plates, where not made in one piece, have welded butt joints instead of riveted lap joints.

The combustion chamber is common to all the furnaces, and the nests of tubes are arranged so that there is one more than the number of furnaces. The vertical lanes between the tube nests are above the furnace crowns, thus permitting easy liberation of the steam bubbles formed on the furnace crowns. The whole boiler and combustion chamber are enclosed in a lagged casing in which the air for combustion circulates from the air heater to the furnace.

There is a combustion chamber type of superheater, and the superheat is controlled by regulating the gas flow with dampers in the combustion chamber baffle. Steam flows through the



Longitudinal Sectional Elevation showing general construction. The water-tubes, superheater, desuperheater, and superheat damper control are shown, also the air-jacket and brick-lined combustion chamber.

superheater under all circumstances, and steam required for auxiliaries is subsequently passed through a desuperheater.

A summary of results of tests on one of these boilers, when burning coal and when burning oil, is given at the top of p. 604.

Forced Draught.—By the use of forced draught the combustion of the fuel is effected with a less quantity of air in excess

Fuel	Coal. Hand-fired.		Mexican Oil.	
Gross calorific value . . B.Th.U./lb.	13580.		18510.	
Rating	Low.	High.	Low.	High.
Steam press. Saturated . lb./sq. in.	270	269	273	268
Superheated lb./sq. in.	265	258	266	248
Steam temp. Superheated . °F.	767	790	743	772
Feed temp. . . . . . °F.	43	42	44	44
Actual evap./sq. ft. H.S. . lb./hr.	4·69	6·89	5·71	8·79
Actual evap./lb. of fuel . . lb.	8·38	8·16	11·60	11·11
Equiv. evap. from and at 212° F./sq. ft. H.S. . lb./hr.	6·75	9·99	8·13	12·67
Equiv. evap. from and at 212° F./lb. of fuel . . . lb.	12·07	11·85	16 54	16·02
Coal per sq. ft. grate area . lb./hr.	23·5	36·0	.	..
Boiler efficiency on gross C.V. per cent.	85·7*	84·2*	86·0*	83·2*

* Apparent discrepancy between efficiency and equivalent evaporation per lb. of fuel from and at 212° F. is due to a small percentage of steam produced being subsequently desuperheated and therefore some heat given back by desuperheater.

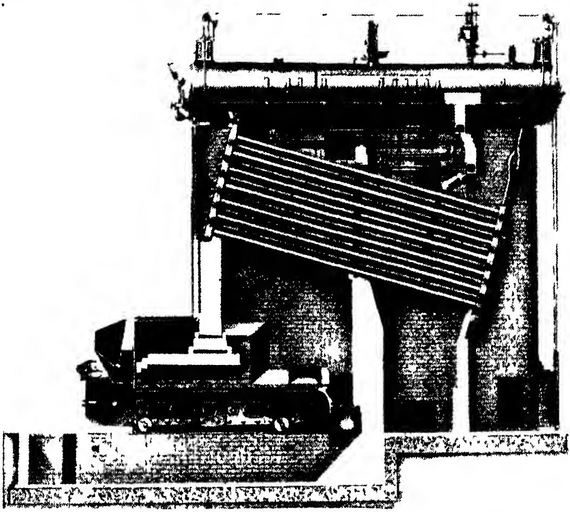
of that theoretically required, hence less heat will be carried away through the chimney by the excess air. Also, a larger quantity of fuel is burned on a given grate in a given time, and therefore a smaller boiler will produce the same quantity of steam. An inferior quality of coal may be used economically when forced draught is used. Another advantage of forced draught is that the draught being produced by a fan is under control, and may be varied to suit the state of the atmosphere and the rate of combustion required.

Forced draught also allows higher gas velocities through the boiler, resulting in greater heat transfer, and hence a higher rating and efficiency. It also permits the use of air preheaters to heat the air for combustion by recovering some of the heat from the waste gases. In cases where high efficiency air heaters of the Howden-Ljungstrom rotary type are used, the resultant gas temperature may be so low (say 230° F.) that the pull of the funnel, due to the temperature difference, is diminished to such an extent that induced draught fans may be necessary. This balanced draught arrangement consisting of both forced and induced draught fans, in conjunction with high efficiency air heaters, is almost universally adopted in important land boiler plants, and is becoming quite a normal feature of high efficiency marine steam installations.

The reduction in the consumption of fuel per I.H.P., due to the use of forced draught, has been shown to be about 15 per cent., while in the steam-producing power of a given boiler there is an increase of from 30 to 50 per cent.

The air pressure in ordinary stationary and marine boilers is from ½ inch to 2 inches of water. In torpedo boats the air pressure ranges up to 5 inches.

Babcock and Wilcox Water-tube Boilers.—



Longitudinal Multiple Drum Sectional Header Boiler with Natural Draught Stoker and Superheater.—This boiler is composed of seamless steel tubes placed in an inclined position, and connected to one another by forged steel headers of serpentine form, so that the tubes are arranged zigzag. The headers are connected by tubes with the steam and water drums, as shown in the above illustration. The rear headers are also connected to a forged steel mud drum at their bottom ends. The tubes are expanded into bored holes in the headers. The handhole openings in the headers opposite the ends of the tubes are closed by internal handhole caps with gaskets and secured by bolts and clamps (all of forged steel). The boiler is suspended, entirely independent of the brickwork, from girders resting on columns.

The products of combustion pass from the furnace up between the tubes into a combustion chamber under the steam and water drums; from thence they pass down between the tubes, then once more up between the tubes, and off to the chimney.

The water circulates from the back to the front in the inclined tubes, up through the front headers into the drums above, then along the drums and down the back tubes into the rear of the inclined tubes again.

The soot is removed from the tubes by soot blowers.

Babcock and Wilcox Water-tube Boilers. Some Standard Sizes of the Longitudinal Drum Sectional Header Type.

Total Heating Surface.		No. of Tubes in		Length of Tubes.	Drums.				Approx. Total Weight (Packed).		Space Occupied (Hand-fired).		
		Width.	Height.		No.	Diam.	Length.				Height to Tube Door.	Length over Brickwork.	
Sq. Ft.	Sq. M.			Ft.		In.	Ft.	In.	Tons.	Kilos.	Ft.	Ft.	In.
121	11	3	4	6	1	30	11	3	$3\frac{3}{4}$	3800	7	9	6
152	14	3	4	8	1	30	13	4	$4\frac{1}{2}$	4600	7	11	6
183	17	3	5	8	1	30	13	4	$4\frac{3}{4}$	4800	7	11	6
221	21	3	5	10	1	30	15	6	5	5100	7	13	6
293	27	4	5	10	1	30	15	6	$5\frac{1}{2}$	5600	7	13	6
343	32	4	5	12	1	30	17	6	$5\frac{3}{4}$	5800	7	16	0
401	37	4	6	12	1	30	18	0	6	6100	7	16	6
460	42	4	6	14	1	30	20	1	$6\frac{1}{2}$	6600	7	19	0
526	49	4	7	14	1	30	20	1	7	7100	7	19	0
593	55	4	8	14	1	30	20	1	$7\frac{1}{2}$	7600	8	19	0
735	68	5	8	14	1	36	20	7	$8\frac{1}{2}$	8600	8	19	0
870	81	6	7	16	1	36	22	6	$10\frac{1}{4}$	10400	8	21	0
983	91	6	8	16	1	36	22	6	$10\frac{3}{4}$	10900	8	21	0
1098	102	6	9	16	1	36	22	6	$11\frac{1}{2}$	11700	8	21	0
1218	113	6	9	18	1	36	24	8	$12\frac{1}{4}$	12400	8	23	0
1265	117	7	8	18	1	36	24	8	$13\frac{1}{4}$	13400	8	23	0
1411	131	7	9	18	1	36	24	8	14	14100	8	23	0
1426	132	7	9	18	1	42	24	11	$14\frac{1}{4}$	14400	8	23	0
1619	150	8	9	18	1	42	24	11	$14\frac{3}{4}$	14900	8	23	0
1741	162	12	7	16	2	36	22	6	$18\frac{1}{2}$	18900	8	21	0
1790	166	8	10	18	1	42	24	11	$16\frac{1}{4}$	16500	8	23	0
1827	170	9	9	18	1	48	25	4	17	17300	8	23	0
1966	182	12	8	16	2	36	22	6	19	19300	8	21	0
2010	186	9	10	18	1	48	25	4	$19\frac{1}{4}$	19600	8	23	0
2197	204	12	9	16	2	36	22	6	$20\frac{1}{4}$	20600	8	21	0
2255	209	10	10	18	1	54	25	7	$20\frac{3}{4}$	21100	8	23	0
2437	226	12	9	18	2	36	24	8	22	22400	8	23	0
2531	235	14	8	18	2	36	24	8	$24\frac{1}{2}$	25000	8	23	0
2690	250	12	10	18	2	36	24	8	23	23400	8	23	0
2823	262	14	9	18	2	36	24	8	$25\frac{1}{4}$	25700	8	23	0

Babcock and Wilcox Water-tube Boilers Some Standard Sizes of the Longitudinal Drum Sectional Header Type (*continued*).

Space Occupied (Hand-fired). Width over Brickwork. Singly.		Battery.		Space Occupied (Stoker-fired). Height to Tube Door.	Length over Brickwork.		Width over Brickwork. Singly.		Battery.		Hand-fired Grate Area.	Style 6 Stoker Size.	Steam Stop Valve Size.
Ft.	In.	Ft.	In.	Ft.	Ft.	In.	Ft.	In.	Ft.	In.	Sq. Ft.	Ft. In. Ft. In.	In.
4	4	7	11	..	..		..		..		5·20	..	2
4	4	7	11	..	..		..		..		6·25	..	2
4	4	7	11	..	..		..		..		7·28	..	2
4	4	7	11	..	..		..		..		8·33	..	2
4	11	9	1	..	..		..		..		10·64	..	2
4	11	9	1	..	..		..		..		10·64	..	2
4	11	9	1	..	..		..		..		11·97	..	2
4	11	9	1	..	..		..		..		13·30	..	2
5	8	10	7	8	19	0	5	8	10	7	16·00	2 0 × 7 0	2
5	8	10	7	8	19	0	5	8	10	7	16·00	2 0 × 8 6	2
6	3	11	0	8	19	0	6	3	11	0	19·50	2 7 × 8 6	2½
6	10	12	2	8	21	0	6	10	12	2	26·85	3 0 × 8 6	2½
6	10	12	2	8	21	0	6	10	12	2	26·85	3 0 × 8 6	2½
6	10	12	2	8	21	0	6	10	12	2	26·85	3 0 × 10 0	2½
6	10	12	2	8	23	0	6	10	12	2	26·85	3 0 × 10 0	3
7	5	13	4	8	23	0	7	5	13	4	30·91	3 8 × 10 0	3
7	5	13	4	8	23	0	7	5	13	4	30·91	4 0 × 10 0	3
7	5	13	4	8	23	0	7	5	13	4	30·91	4 0 × 10 0	3
8	0	14	6	8	23	0	8	0	14	6	34·98	4 0 × 12 0	3½
10	4	19	2	8	21	0	10	4	19	2	51·31	5 0 × 10 0	3½
8	0	14	6	8	23	0	8	0	14	6	34·98	4 0 × 12 0	3½
8	7	15	8	8	23	0	8	7	15	8	39·08	4 5 × 12 0	3½
10	4	19	2	8	21	0	10	4	19	2	51·31	5 6 × 10 0	3½
8	7	15	8	8	23	0	8	7	15	8	39·08	4 5 × 12 0	3½
10	4	19	2	8	21	0	10	4	19	2	51·31	6 0 × 10 0	4
9	2	16	10	9	23	0	9	2	16	10	43·16	5 0 × 12 0	4
10	4	19	2	9	23	0	10	4	19	2	51·31	5 6 × 12 0	4
11	6	21	6	9	23	0	11	6	21	6	59·46	6 0 × 12 0	4
10	4	19	2	9	23	0	10	4	19	2	51·31	6 6 × 12 0	5
11	6	21	6	9	23	0	11	6	21	6	59·46	6 6 × 12 0	5

Babcock and Wilcox Water-tube Boilers. Some Standard Sizes of the Longitudinal Drum Sectional Header Type (*continued*).

Total Heating Surface.		No. of Tubes in		Length of Tubes.	Drums.				Approx. Total Weight (Packed).		Space Occupied (Hand-fired).		
		Width.	Height.		No.	Diam.	Length.				Height to Tube Door.	Length over Brickwork.	
Sq. Ft.	Sq. M.			Ft.		In.	Ft.	In.	Tons.	Kilos.	Ft.	Ft.	In.
2852	265	14	9	18	2	42	24	11	25¾	26200	8	23	0
3140	292	14	10	18	2	42	24	11	26¾	27200	..	..	
3240	301	16	9	18	2	42	24	11	27¼	27700	..	..	
3580	333	16	10	18	2	42	24	11	28½	29000	..	..	
3654	340	18	9	18	2	48	25	4	32	32500	..	..	
4020	374	18	10	18	2	48	25	4	33¾	34300	..	..	
4510	419	20	10	18	2	54	25	8	40½	41200	..	..	
4780	445	18	12	18	2	48	25	4	37½	38100	..	..	
5346	496	20	12	18	2	54	25	8	48	48800	..	..	
5540	515	18	14	18	2	48	25	11	42	42700	..	..	
6182	574	20	14	18	2	54	26	0	48½	49300	..	..	
7135	663	27	12	18	3	48	26	3	63	64000	..	..	
7322	683	24	14	18	3	42	25	10	60	61000	..	..	
8283	770	27	14	18	3	48	26	3	69¾	70900	..	..	
9273	863	30	14	18	3	54	26	4	80¾	82100	..	..	

Pressures.—From 350 to 400 lbs. per square inch represents the usual maximum final steam pressure of these boilers. Where higher pressures are required, the cross drum sectional header boiler must be installed.

Hand-fired Grate Sizes.—In selecting the size of boiler, care must be taken to choose a unit having sufficient grate area to burn the necessary quantity of the particular fuel available. The grate areas tabulated do not allow for any stepping out of the side walls to accommodate a larger area. Where no grate size is given, hand-firing is not recommended.

Stoker Sizes.—The above remarks *re* selection of a boiler also apply here. Stoker sizes may be modified by using special settings, etc. Boiler sizes which have no stoker sizes allocated should receive special consideration, as the largest stoker that can be fitted with a normal setting will not carry the usual evaporation.

Babcock and Wilcox Water-tube Boilers (*continued*).

Space Occupied (Hand-fired). Width over Brickwork. Singly.		Battery.		Space Occupied (Stoker-fired). Height to Tube Door.	Length over Brickwork.		Width over Brickwork. Singly.		Battery.		Hand-fired Grate Area.	Style 6 Stoker Size.	Steam Stop Valve Size.
Ft.	In.	Ft.	In.	Ft.	Ft.	In.	Ft.	In.	Ft.	In.	Sq. Ft.	Ft. In. Ft. In.	In.
11	6	21	6	9	23	0	11	6	21	6	59·46	6 6 × 12 0	5
..		..		9	23	0	11	6	21	6	..	6 0 × 14 0	5
..		..		9	23	0	12	8	23	10	..	7 6 × 12 0	5
..		..		9	23	0	12	8	23	10	..	8 6 × 12 0	5
..		..		9	23	0	13	10	26	2	..	8 6 × 12 0	5
..		..		9	23	0	13	10	26	2	..	8 0 × 14 0	5
..		..		9	23	0	15	0	28	6	..	9 0 × 14 0	6
..		..		11	23	0	13	10	26	2	..	8 6 × 16 0	6
..		..		9	23	0	15	0	28	6	..	$^{5}_{5}$ $^{0}_{0}$ × 14 0	6
..		..		11	23	6	13	10	26	2	..	9 0 × 16 0	6
..		..		..	23	6	..		..		..	..	7
..		..		9	23	0	19	1	..		..	$^{7}_{7}$ $^{0}_{0}$ × 14 0	7
..		..		..	23	6	..		..		..	..	7
..		..		11	23	6	19	1	..		..	$^{7}_{7}$ $^{0}_{0}$ × 16 0	7
..		..		11	23	6	20	10	..		..	$^{8}_{8}$ $^{0}_{0}$ × 16 0	8

Babcock and Wilcox Water-tube Boilers—Cross Drum Types.— The categories of cross drum boilers may be listed briefly as follows: Marine, portable, cross type, and power station boilers.

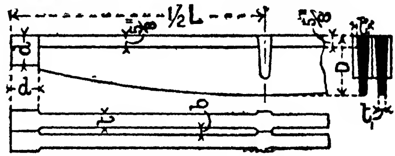
Marine Water-tube Boilers are made in sizes up to evaporative capacities of about 60,000 lbs. of water per hour, and for working pressures up to about 625 lbs. per square inch. They are used on warships, ocean-going liners, tugs, and other craft. An illustration of an oil-fired marine boiler with superheater is shown on p. 611.

Portable Boilers range in evaporative capacity from 500 to 6000 lbs. of water per hour, and for working pressures up to 200 lbs. per square inch. They can be constructed so that no single part weighs more than 280 lbs., thus facilitating transport in difficult country.

Cross Type Boilers range in evaporative capacity from 360 to 8770 lbs. of water per hour, and for working pressures up to 205 lbs. per square inch. The weight of the steam and water drum, which is the heaviest part, does not exceed 8 cwt. in the smaller sizes of these boilers.

Power Station Boilers, known as the C.T.M. type, have been manufactured to give an evaporation of over 1,000,000 lbs. of water per hour, and for working pressures exceeding 1500 lbs. per square inch. For such high pressures, welded drums and seamless forged steel drums are used in place of riveted drums, and the water tubes are reduced in diameter.

Fire-bars.—These are generally made of cast-iron, and are usually of the form shown in the annexed illustration. The following are approximate rules for proportioning common cast-iron fire-bars:—



L = length of fire-bar (usually from 2 feet to 3 feet).

$D = \frac{L}{16} + 1\frac{1}{4}$ inches. $\qquad d = \frac{L}{32} + \frac{3}{4}$ inch.

$t = \frac{3}{4}$ inch to $\frac{7}{8}$ inch. $\qquad t_1 = \frac{1}{2}\,t$.

$b = \frac{3}{8}$ inch to $\frac{5}{8}$ inch, depending on the kind of fuel and the force of the draught.

The area of the air spaces between the fire-bars varies from ·15 to ·4 of the grate area. The average of a large number of examples gave area of air spaces equal to ·28 of grate area.

Gain Due to Heating the Feed-water in Steam Boilers.—

t_1 = temperature of feed-water before heating, in degrees Fahr.

t_2 = temperature of feed-water after heating, in degrees Fahr.

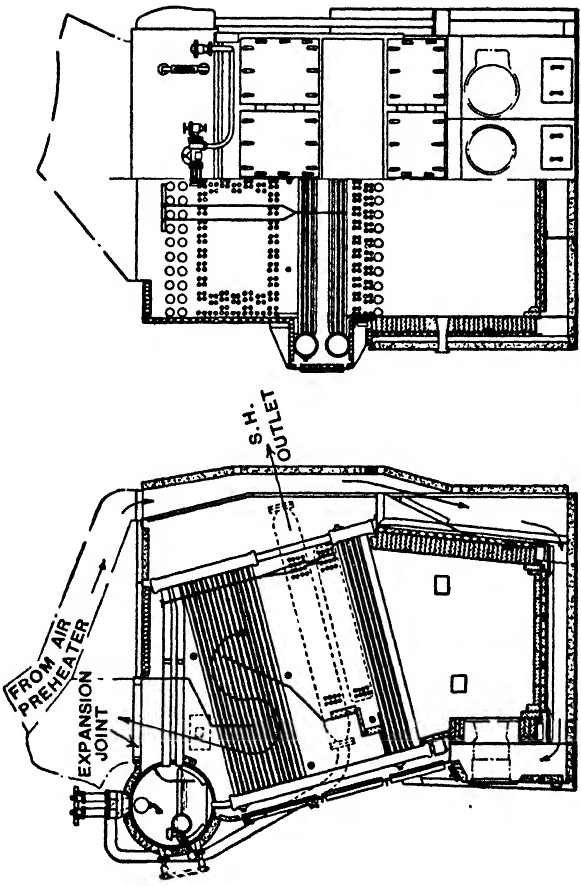
h_1 = heat in feed-water (above that in water at 32° F.) before heating, in British thermal units.

h_2 = heat in feed-water (above that in water at 32° F.) after heating, in British thermal units.

H = total heat in steam at boiler pressure (above that in water at 32° F.), in British thermal units.

$$\text{Gain per cent.} = \frac{100(h_2 - h_1)}{H - h_1} = \frac{100(t_2 - t_1)}{H - t_1 + 32} \text{ very nearly.}$$

It is assumed that the feed-water is heated by waste gases, the heat of which would otherwise be lost.



Babcock and Wilcox Oil-fired Marine Boiler with Superheater.

The following table shows the gain per cent. due to heating the feed-water to various temperatures for various pressures of steam :—

Temp. t_2 in Degrees Fahr.	Absolute Pressure of Boiler Steam in Lbs. per Square Inch					
	45	135	225	45	135	225
	Gain per Cent. due to Heating Feed-water to Temp. t_2.					
	When $t_1 = 60°$ Fahr.			When $t_1 = 100°$ Fahr.		
100	3·52	3·45	3·41	—	—	—
150	7·91	7·75	7·67	4·56	4·46	4·41
200	12·31	12·06	11·93	9·11	8·92	8·82
250	16·70	16·37	16 19	13·67	13·38	15·23

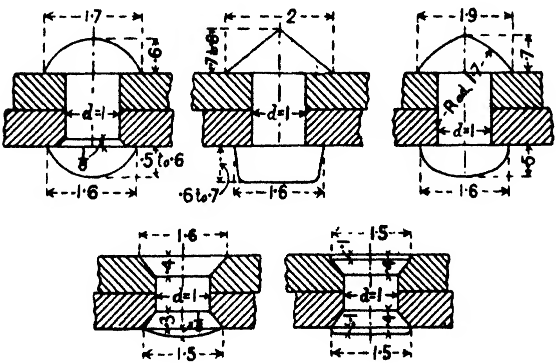
Hydraulic Tests for Steam Boilers.—*Lloyd's Rules.*—In all new boilers working at pressures up to 100 lbs. per square inch the hydraulic test is to be twice the working pressure. For boilers working at pressures greater than 100 lbs. per square inch the hydraulic test pressure is to be 1½ times the working pressure plus 50 lbs. per square inch.

Board of Trade Rules.—All new boilers to be tested by hydraulic pressure to 1½ times the working pressure plus 50 lbs. per square inch. The tests to be made before the boilers are in the vessel and before they are lagged. This latter instruction applies also to evaporators, superheaters, and steam chests, but these should be tested to twice the working pressure, except that when a superheater forms an integral part of a boiler it should be tested to the same pressure as and with the boiler of which it forms a part.

RIVETED JOINTS.

Rivets and Rivet Holes.*— For the proportions marked on the figures below the unit is *d*, the diameter of the rivet.

The length of rivet required to form a rivet head is about $1\frac{1}{4}d$ for conical and snap heads. For countersunk heads the allowance is from $\frac{3}{4}d$ to *d*.



* For particulars of British Standard Rivets, see B.S.S., No. 275, Dimensions of Rivets, and B.S.S., No. 425, Forms and Dimensions of Boiler Rivets.

As the rivet expands when heated, its diameter when cold should be less than that of the hole. In practice, the diameter of the rivet is generally one-sixteenth of an inch less than that of the hole for rivets $\frac{3}{4}$ inch in diameter and upwards.

In all calculations on the strength of riveted joints, and in all rules and tables, the diameter of the hole is taken as the diameter of the rivet.

In boiler work the rivet holes are drilled after the plates have been bent or flanged and put together in their proper places. After drilling the plates are taken asunder, and any burs which have been formed at the edges of the holes are removed.

Notation for Formulæ for Riveted Joints.—All dimensions are in inches, unless otherwise stated.

t = thickness of plates.

t_1 = thickness of butt straps; when inner and outer straps have different thicknesses, t_i = thickness of inner strap, t_o = thickness of outer strap.

d = diameter of rivets, after riveting.

p = greatest pitch of rivets.

n = number of rivets in a width of joint equal to p in lap joints.

= number of rivets on each side of the butt in a width of joint equal to p in butt joints.

l = distance from edge of plate to centre line of nearest row of rivets.

c, c', c_1, and c'_1 = distances between rows of rivets.

f_t = tensile strength of plates in lbs. per square inch.

f_s = shearing strength of rivets in lbs. per square inch.

$\frac{f_s}{f_t} = \frac{23}{28}$, or say 0·8, for steel plates and steel rivets, but varies according to ultimate strength of plate.

C = ratio of shearing strength of rivet in joint to shearing strength of rivet in single shear.

= 1 for lap joints.

= 1·875 for butt joints with double cover straps (Board of Trade and Lloyd's rules).

R_t = resistance of plate to tearing between the rivets of pitch p, expressed as a percentage of the resistance of the solid plate.

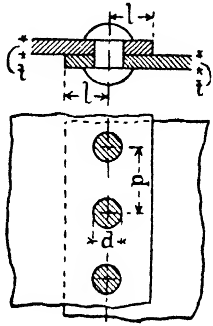
R_s = resistance of rivets of joint to shearing expressed as a percentage of the resistance of the solid plate to tearing.

R_1 = resistance of rivets of outer row to shearing combined with the resistance of the plate between the inner row of rivets to tearing, expressed as a percentage of the resistance of the solid plate to tearing. (For joints in which the pitch of the rivets in the inner row is half the pitch of the rivets in the outer row.)

R = the least of the quantities R_t, R_s, R_1.

E = efficiency of the joint $= \dfrac{R}{100}$.

Single Riveted Lap Joints.—



$$R_t = \frac{100(p-d)}{p}.$$

$$R_s = \frac{100 \times \cdot 7854 d^2}{pt} \times \frac{f_s}{f_t}.$$

If $R_t = R_s$, then $p = \dfrac{\cdot 7854 d^2}{t} \times \dfrac{f_s}{f_t} + d.$

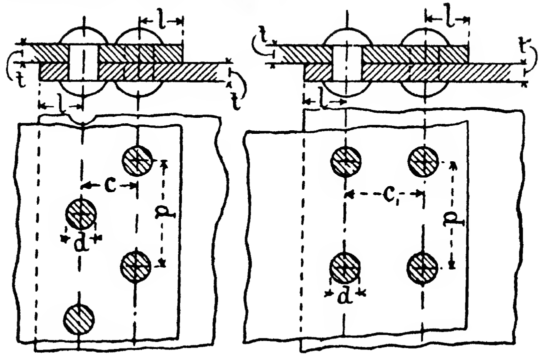
For joints which have to be made steam-tight the pitch of the rivets is generally less than that given by the above formula.

The following table gives proportions of single riveted lap joints suitable for boiler work, when $f_s/f_t = 0{\cdot}8$:—

t	Steel Plates and Steel Rivets.			
	d	p	R_t	R_s
$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{7}{16}$	56·5	54·6
$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{11}{16}$	55·6	55·9
$\frac{7}{16}$	$\frac{7}{8}$	2	56·2	55·0
$\frac{1}{2}$	$\frac{15}{16}$	$2\frac{1}{16}$	54·5	53·6
$\frac{9}{16}$	1	$2\frac{1}{8}$	52·9	52·6
$\frac{5}{8}$	$1\frac{1}{16}$	$2\frac{1}{4}$	52·8	50·4
$\frac{11}{16}$	$1\frac{1}{8}$	$2\frac{3}{8}$	52·6	48·7

$l = 1\frac{1}{2}d.$

Double Riveted Lap Joints.—



$$R_t = \frac{100(p-d)}{p}. \qquad R_s = \frac{100 \times 2 \times \cdot 7854 d^2}{pt} \times \frac{f_s}{f_t}.$$

$$\text{If } R_t = R_s, \text{ then } p = \frac{2 \times \cdot 7854 d^2}{t} \times \frac{f_s}{f_t} + d.$$

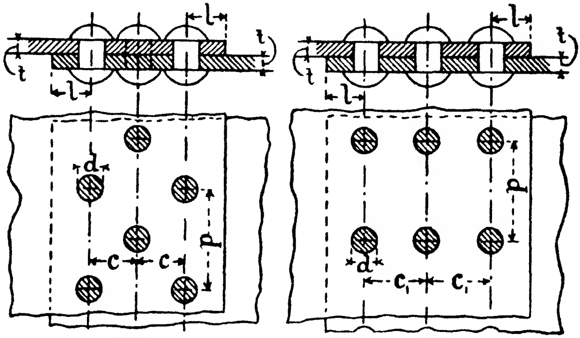
$$c = \tfrac{1}{10}\sqrt{(11p+4d)(p+4d)}. \qquad c_1 = 2d \text{ to } 2d + \tfrac{1}{2} \text{ inch.}$$

$$f_s/f_t = 0{\cdot}8.$$

t	Steel Plates and Steel Rivets.				
	d	p	c	c_1	R
$\frac{5}{16}$	$\frac{11}{16}$	$2\frac{9}{16}$	$1\frac{5}{16}$	$1\frac{7}{8}$	73·2
$\frac{3}{8}$	$\frac{3}{4}$	$2\frac{5}{8}$	$1\frac{3}{8}$	2	71·4
$\frac{7}{16}$	$\frac{13}{16}$	$2\frac{3}{4}$	$1\frac{7}{16}$	$2\frac{1}{8}$	69·0
$\frac{1}{2}$	$\frac{7}{8}$	$2\frac{13}{16}$	$1\frac{1}{2}$	$2\frac{1}{4}$	68·4
$\frac{9}{16}$	$\frac{15}{16}$	$2\frac{7}{8}$	$1\frac{9}{16}$	$2\frac{3}{8}$	67·4
$\frac{5}{8}$	1	3	$1\frac{5}{8}$	$2\frac{1}{2}$	66·7
$\frac{11}{16}$	$1\frac{1}{16}$	$3\frac{1}{8}$	$1\frac{11}{16}$	$2\frac{5}{8}$	66·0
$\frac{3}{4}$	$1\frac{1}{8}$	$3\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{3}{4}$	65·2
$\frac{13}{16}$	$1\frac{3}{16}$	$3\frac{3}{8}$	$1\frac{13}{16}$	$2\frac{7}{8}$	64·6
$\frac{7}{8}$	$1\frac{1}{4}$	$3\frac{1}{2}$	$1\frac{15}{16}$	3	64·1
$\frac{15}{16}$	$1\frac{5}{16}$	$3\frac{5}{8}$	2	$3\frac{1}{8}$	63·7
1	$1\frac{3}{8}$	$3\frac{3}{4}$	$2\frac{1}{8}$	$3\frac{1}{4}$	63·3

In each case R_t is nearly equal to R_s, and the value under R is the smaller of the two. $l = 1\frac{1}{2}d.$

Treble Riveted Lap Joints.—



$$R_t=\frac{100(p-d)}{p}.\qquad R_s=\frac{100\times 3\times \cdot 7854 d^2}{pt}\times\frac{f_s}{f_t}.$$

$$\text{If } R_t=R_s, \text{ then } p=\frac{3\times \cdot 7854 d^2}{t}\times\frac{f_s}{f_t}+d.$$

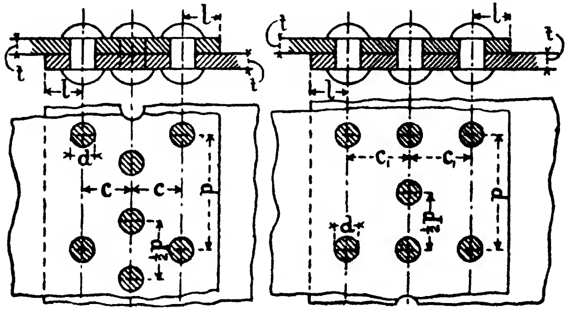
$$c=\tfrac{1}{10}\sqrt{(11p+4d)(p+4d)}.\qquad c_1=2d \text{ to } 2d+\tfrac{1}{8} \text{ inch.}$$

$$f_s/f_t=0\cdot 8.$$

t	Steel Plates and Steel Rivets.				
	d	p	c	c_1	R
$\frac{5}{8}$	$\frac{7}{8}$	$3\frac{1}{8}$	$1\frac{5}{8}$	$2\frac{1}{4}$	72·0
$\frac{11}{16}$	$\frac{15}{16}$	$3\frac{3}{8}$	$1\frac{3}{4}$	$2\frac{3}{8}$	71·4
$\frac{3}{4}$	1	$3\frac{1}{2}$	$1\frac{13}{16}$	$2\frac{1}{2}$	71·4
$\frac{13}{16}$	$1\frac{1}{16}$	$3\frac{11}{16}$	$1\frac{15}{16}$	$2\frac{5}{8}$	71·0
$\frac{7}{8}$	$1\frac{1}{8}$	$3\frac{7}{8}$	2	$2\frac{3}{4}$	70·4
$\frac{15}{16}$	$1\frac{3}{16}$	4	$2\frac{1}{16}$	$2\frac{7}{8}$	70·3
1	$1\frac{1}{4}$	$4\frac{3}{16}$	$2\frac{3}{16}$	3	70·1
$1\frac{1}{16}$	$1\frac{5}{16}$	$4\frac{3}{8}$	$2\frac{1}{4}$	$3\frac{1}{8}$	69·9
$1\frac{1}{8}$	$1\frac{3}{8}$	$4\frac{1}{2}$	$2\frac{3}{8}$	$3\frac{1}{4}$	69·4

In each case R_t is nearly equal to R_s, and the value under R is the smaller of the two. $l=1\frac{1}{2}d.$

Treble Riveted Lap Joints in which the Pitch of the Rivets of the Inner Row is half the Pitch of the Rivets in the Outer Rows.—



$$R_t=\frac{100(p-d)}{p}. \qquad R_s=\frac{100\times 4\times \cdot 7854d^2}{pt}\times\frac{f_s}{f_t}.$$

$$\text{If } R_t=R_s, \text{ then } p=\frac{4\times \cdot 7854d^2}{t}\times\frac{f_s}{f_t}+d.$$

$$c=\sqrt{\left(\tfrac{11}{20}p+d\right)\left(\tfrac{1}{20}p+d\right)}. \quad c_1=2d \text{ to } 2d+\tfrac{1}{2} \text{ inch.} \quad l=1\tfrac{1}{2}d.$$

The resistance of this joint to the shearing of the rivets of an outer row, and the tearing of the plate between the rivets of the inner row, expressed as a percentage of the strength of the solid plate, is $R_1=\frac{100\{\cdot 7854d^2f_s+(p-2d)tf_t\}}{ptf_t}=\frac{R_s}{4}+\frac{100(p-2d)}{p}$.

If $R_1=R_t=R_s$, then $p=\frac{4\times \cdot 7854d^2}{t}\times\frac{f_s}{f_t}+d$, as before, and

$$d=\frac{t}{\cdot 7854}\times\frac{f_t}{f_s}=1\cdot 27t \text{ for iron plates and iron rivets.}$$
$$=1\cdot 59t \text{ for steel plates and steel rivets.}$$

If the diameter of the rivets is greater than that given by the above formula, the strength of the joint must be taken as R_t or R_s; but if the diameter is less, which is generally the case, then the strength of the joint must be taken as R_1.

General Formulæ for Lap Joints.—

$$R_t=\frac{100(p-d)}{p}. \qquad R_s=\frac{100\times n\times \cdot 7854d^2}{pt}\times\frac{f_s}{f_t}.$$

$$\text{If } R_t=R_s, \text{ then } p=\frac{n\times \cdot 7854d^2}{t}\times\frac{f_s}{f_t}+d.$$

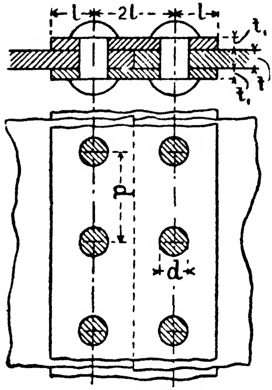
When the pitch of the rivets in the inner rows is half the pitch of the rivets in the outer rows,

$$R_1=\frac{R_s}{n}+\frac{100(p-2d)}{p}.$$

Butt Joints with Single Butt Straps.—A butt joint with a cover strap on one side only is made up of two lap joints, and may therefore be proportioned by the rules for lap joints.

Thickness of butt strap = thickness of plate × $1\frac{1}{8}$.

Single Riveted Butt Joints with Double Butt Straps.—



$$R_t = \frac{100(p-d)}{p}.$$

$$R_s = \frac{100 \times C \times \cdot 7854 d^2}{pt} \times \frac{f_s}{f_t}.$$

If $R_t = R_s$, then

$$p = \frac{C \times \cdot 7854 d^2}{t} \times \frac{f_s}{f_t} + d.$$

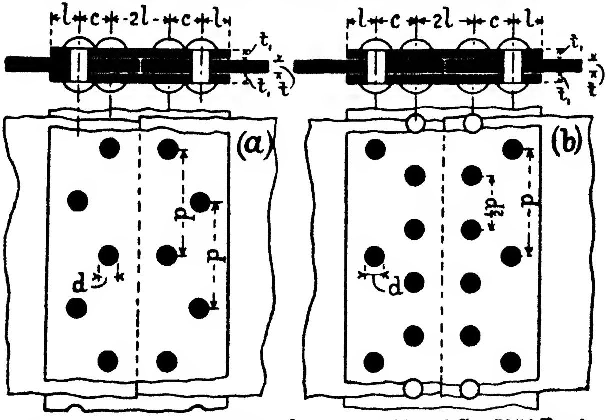
The diameter of the rivets may be—

$d = t + \frac{1}{4}$ inch for iron plates and iron rivets.

$d = t + \frac{5}{16}$ inch for steel plates and steel rivets.

$t_1 = \frac{5}{8}t.$

Double Riveted Butt Joints with Double Butt Straps.—



For Fig. (*a*). $R_t = \frac{100(p-d)}{p}$. $R_s = \frac{100 \times 2C \times \cdot 7854 d^2}{pt} \times \frac{f_s}{f_t}$.

If $R_t = R_s$, then $p = \frac{2C \times \cdot 7854 d^2}{t} \times \frac{f_s}{f_t} + d.$

The diameter of the rivets may be,

$d=t+\frac{3}{16}$ inch for iron plates and iron rivets.
$d=t+\frac{1}{4}$ inch for steel plates and steel rivets.

$c=\frac{1}{10}\sqrt{(11p+4d)(p+4d)}$. $\qquad t_1=\frac{5}{8}t.\qquad l=1\frac{1}{2}d.$

For Fig. (*b*). $\quad R_t=\frac{100(p-d)}{p}. \qquad R_s=\frac{100\times 3C\times\cdot 7854d^2}{pt}\times\frac{f_s}{f_t}.$

$$\text{If } R_t=R_s, \text{ then } p=\frac{3C\times\cdot 7854d^2}{t}\times\frac{f_s}{f_t}+d.$$

The diameter of the rivets may be,

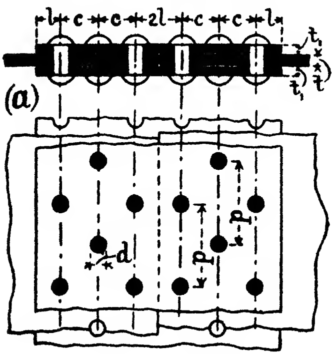
$d=t+\frac{1}{8}$ inch for iron plates and iron rivets.
$d=t+\frac{3}{16}$ inch for steel plates and steel rivets.

$c=\sqrt{(\frac{11}{20}p+d)(\frac{1}{20}p+d)}$. $\qquad t_1=\frac{5t(p-d)}{8(p-2d)}.\qquad l=1\frac{1}{2}d.$

$$R_1=\frac{100\{C\times\cdot 7854d^2f_s+(p-2d)tf_t\}}{ptf_t}=\frac{R_s}{3}+\frac{100(p-2d)}{p}.$$

With rivets of ordinary diameter R_1 will be greater than R_t or R_s.

Treble Riveted Butt Joints with Double Butt Straps.—



For Fig. (*a*).

$$R_t=\frac{100(p-d)}{p}.$$

$$R_s=\frac{100\times 3C\times\cdot 7854d^2}{pt}\times\frac{f_s}{f_t}.$$

If $R_t=R_s$, then

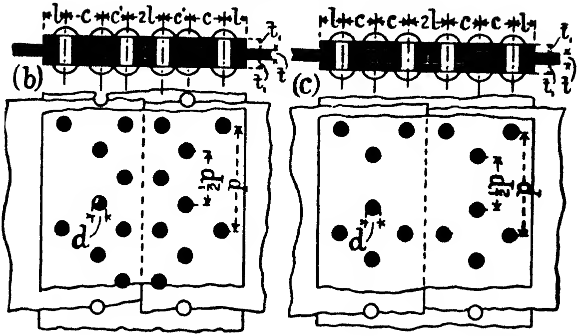
$$p=\frac{3C\times\cdot 7854d^2}{t}\times\frac{f_s}{f_t}+d.$$

$c=\frac{1}{10}\sqrt{(11p+4d)(p+4d)}$.

$t_1=\frac{5}{8}t.\qquad l=1\frac{1}{2}d.$

The diameter of the rivets may be,

$d=t+\frac{1}{16}$ inch for iron plates and iron rivets.
$d=t+\frac{1}{8}$ inch for steel plates and steel rivets.



For Fig. (*b*). $R_t=\dfrac{100(p-d)}{p}$. $R_s=\dfrac{100\times 5C\times \cdot 7854d^2}{pt}\times\dfrac{f_s}{f_t}$.

If $R_t=R_s$, then $p=\dfrac{5C\times \cdot 7854d^2}{t}\times\dfrac{f_s}{f}+d$.

d may be from $t+\frac{1}{8}$ inch for plates $\frac{5}{8}$ inch thick to t for plates 1 inch thick and upwards.

$c=\sqrt{(\tfrac{11}{20}p+d)(\tfrac{1}{20}p+d)}$. $c'=\tfrac{1}{20}\sqrt{(11p+8d)(p+8d)}$.

$t_1=\dfrac{5t(p-d)}{8(p-2d)}$. $l=1\tfrac{1}{2}d$.

$$R_1=\frac{100\{C\times \cdot 7854d^2f_s+(p-2d)tf_t\}}{ptf_t}=\frac{R_s}{5}+\frac{100(p-2d)}{p}.$$

With rivets of ordinary diameter R_1 will be greater than R_t or R_s.

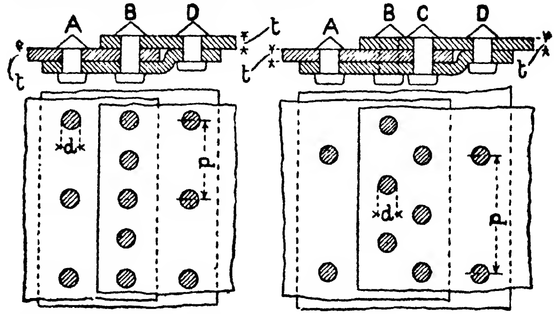
For Fig. (*c*). $R_t=\dfrac{100(p-d)}{p}$. $R_s=\dfrac{100\times 4C\times \cdot 7854d^2}{pt}\times\dfrac{f_s}{f_t}$.

If $R_t=R_s$, then $p=\dfrac{4C\times \cdot 7854d^2}{t}\times\dfrac{f_s}{f_t}+d$.

d may be from $t+\frac{1}{8}$ inch for plates $\frac{5}{8}$ inch thick to t for plates 1 inch thick and upwards.

$c=\sqrt{(\tfrac{11}{20}p+d)(\tfrac{1}{20}p+d)}$. $t_1=\tfrac{5}{8}t$. $l=1\tfrac{1}{2}d$.

Combined Lap and Butt Joints.—



Strength of joint of width $=p$.
Resistance to tearing at A or $D=(p-d)tf_t$ . . . . (1)
Resistance to shearing at A and tearing at B

$$=\frac{\pi}{4}d^2f_s+(p-2d)tf_t \quad (2)$$

Resistance to shearing at B and D (first figure) $=\frac{3\pi}{4}d^2f_s$ (3a)

Resistance to shearing at B, C, and D (second figure)

$$=\frac{5\pi}{4}d^2f_s \quad (3b)$$

From (1), (2), and (3a), $p=4d$, and $d=\frac{4}{\pi}\times\frac{f_t}{f_s}\times t=1{\cdot}6t$ for steel plates and steel rivets, which are the proportions for the first design.

From (1), (2), and (3b), $p=6d$, and $d=\frac{4}{\pi}\times\frac{f_t}{f_s}\times t=1{\cdot}6t$ for steel plates and steel rivets, which are the proportions for the second design.

Efficiency of joint $=\frac{p-d}{p}=\frac{4d-d}{4d}=\frac{3}{4}=75$ per cent. for first design.

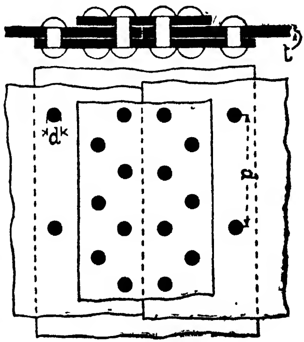
Efficiency of joint $=\frac{p-d}{p}=\frac{6d-d}{6d}=\frac{5}{6}=83{\cdot}3$ per cent. for second design.

The thickness of the cover strap is usually the same as the thickness of the plates.

The above proportions give the maximum efficiency, but in practice they have to be modified for thick plates because the rivets would be too large to be conveniently riveted.

The addition of a cover strap, in the manner shown by the illustrations on the opposite page, is a simple way of increasing very considerably the efficiency of an existing lap joint.

Butt Joint with Cover Straps of Unequal Width.—



Strength of joint of width $=p$.

Resistance to tearing at outer row of rivets $=(p-d)tf_t$. (1)

Resistance to shearing of outer row of rivets and to tearing at second row of rivets $=\frac{\pi}{4}d^2f_s+(p-2d)tf_t$. (2)

Resistance of rivets to shearing $=\frac{9\pi}{4}d^2f_s$ . . . . . (3)

Combining (1), (2), and (3), $p=10d$, and

$d=\frac{4}{\pi}\times\frac{f_t}{f_s}\times t=1{\cdot}6t$ for steel plates and steel rivets.

$$\text{Efficiency of joint}=\frac{p-d}{p}=\frac{10d-d}{10d}=\frac{9}{10}, \text{ or 90 per cent.}$$

The above proportions give the maximum efficiency to the joint, but in practice they would have to be modified. The pitch would in general be too great to ensure a steam-tight joint, and for thick plates the rivets would be too large to be conveniently riveted.

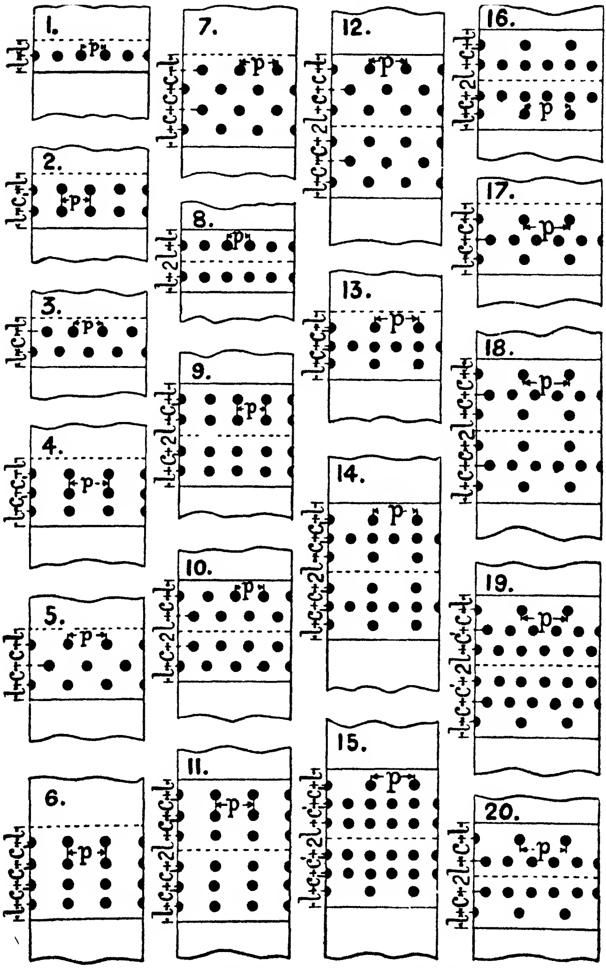
For maximum pitches of rivets allowed by the Board of Trade and Lloyd's rules for joints in steam boilers, see p. 627.

Board of Trade and Lloyd's Rules for Riveted Joints of Cylindrical Boiler Shells.

Joints with Drilled Holes.

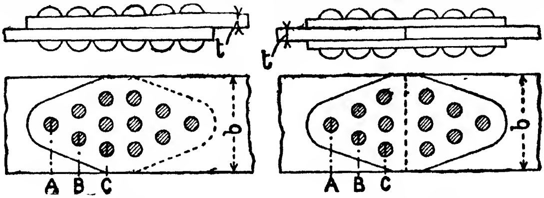
Joints.	Steel Plates, Rivets, and Straps.
1 to 20	$R_t = \frac{100(p-d)}{p}$. $p = \frac{100d}{100-R_t}$. $l = 1\frac{1}{2}d$. $R_s = \frac{100 \times S_2 \times a \times n \times C}{S_1 \times p \times t}$, where $a = 0{\cdot}7854d^2$, S_1 = minimum tensile strength of plate in tons per square inch, S_2 = shearing strength of rivets, which is taken generally to be 23 tons per square inch, and may be 85 per cent. of the minimum tensile strength of the rivet bars. For meanings of other symbols on this page see pp. 614, 615, and illustrations on opposite page.
13 to 20	$R_1 = \frac{100(p-2d)}{p} + \frac{R_s}{n}$. The values which follow are minimum values.
1 to 12, and 14 and 18	For double butt straps, $t_o = \frac{5}{8}t$ and $t_i = t_o + \frac{1}{8}$ inch.
15, 16, 19, and 20	For double butt straps, $t_o = \frac{5t(p-d)}{8(p-2d)}$ and $t_i = t_o + \frac{1}{8}$ inch.
2, 4, 6, 9, and 11	$c_1 = 2d$.
3, 5, 7, 10, and 12	$c = 0{\cdot}33p + 0{\cdot}67d$.
13, 14, 15, and 16	$c_1 = 0{\cdot}33p + 0{\cdot}67d$, or $c_1 = 2d$, whichever is the greater.
17, 18, 19, and 20	$c = 0{\cdot}2p + 1{\cdot}15d$.
15	$c'_1 = 2d$.
19	$c' = 0{\cdot}165p + 0{\cdot}67d$.

Steel plates subject to direct tensile stress may be welded in cases where the weld is covered by a riveted butt strap or straps. All plates which are welded, dished, flanged, or locally heated are to be afterwards efficiently annealed. Butt straps are to be cut from plates and not from rolled strips.



Illustrations of joints referred to on the opposite page.

Riveted Joints for Bars or Narrow Plates in Tension.



Lap Joint.

Resistance to tearing at A
$=(b-d)tf_t$ . . . . (1)

Resistance to shearing at A and tearing at B
$=\frac{\pi}{4}d^2f_s+(b-2d)tf_t$ . . . (2)

Resistance to shearing at A and B and tearing at C
$=\frac{3\pi}{4}d^2f_s+(b-3d)tf_t$ . . . (3)

Resistance of rivets to shearing $=\frac{n\pi}{4}d^2f_s$ . . . . . (4)
where n = total number of rivets in joint.*

From (1) and (2),
$d=\left(\frac{4}{\pi}\right)\left(\frac{f_t}{f_s}\right)t$ . . . . . (5)

$d=1{\cdot}6t$ for steel plates and steel rivets.

From (1), (4), and (5),
$n=\left(\frac{b}{t}\right)\left(\frac{\pi}{4}\right)\left(\frac{f_s}{f_t}\right)-1$ . . (6)

$n={\cdot}63\left(\frac{b}{t}\right)-1$ for steel plates and steel rivets.

Butt Joint.

Resistance to tearing at A
$=(b-d)tf_t$ . . . . (1)

Resistance to shearing at A and tearing at B
$=\frac{2\pi}{4}d^2f_s+(b-2d)tf_t$ . . . (2)

Resistance to shearing at A and B and tearing at C
$=\frac{6\pi}{4}d^2f_s+(b-3d)tf_t$ . . . (3)

Resistance of rivets to shearing $=\frac{n\pi}{4}d^2f_s$ . . . . . (4)
where n = total number of rivets in joint.*

From (1) and (2),
$d=\left(\frac{2}{\pi}\right)\left(\frac{f_t}{f_s}\right)t$ . . . . . (5)

$d={\cdot}8t$ for steel plates and steel rivets.

From (1), (4), and (5),
$n=\pi\left(\frac{b}{t}\right)\left(\frac{f_s}{f_t}\right)-2$ . . . (6)

$n=2{\cdot}5\left(\frac{b}{t}\right)-2$ for steel plates and steel rivets.

A more efficient joint is obtained by making the rivet at A smaller than the rivets at B, and the rivets at B smaller than the rivets at C.

* The total number of rivets must also be $=x(x+1)$, where x is the number of rows of rivets on each side of the centre of the joint. $x=\sqrt{n+\frac{1}{4}}-\frac{1}{2}$. If x comes out a number and a fraction, the next whole number above it may be taken.

The rules deduced on the opposite page for the diameter of the rivets would probably be modified in practice. For thick plates the rule, $d = 1{\cdot}6t$, would give too large a rivet to be conveniently riveted. The other rule, $d = {\cdot}8t$, gives smaller diameters than would be found in practice.

Maximum Pitch of Rivets in Longitudinal Joints (*Board of Trade and Lloyd's Rules*).—

t = thickness of plate in inches.
p = maximum pitch of rivets in inches.
k = constant applicable from the following table:—

Number of rivets in one pitch .	= 1	2	3	4	5
For lap joints .	$k = 1{\cdot}31$	2·62	3·47	4·14	..
For double butt strap joints .	$k = 1{\cdot}75$	3 50	4·63	5·52	6·00

$$p = (k \times t) + 1\tfrac{5}{8}.$$

Circumferential Joints (*Board of Trade and Lloyd's Rules*).—The strength of the seams joining the end plates to the cylindrical shell is not to be less than 42 per cent. of that of the solid shell plate. Where the shell plates exceed $\frac{5}{8}$ inch in thickness the seams connecting the shell plates to the end plates are to be at least double riveted. Where the shell plates exceed $\frac{1}{2}$ inch in thickness the intermediate circumferential seams of double-ended boilers are to be at least double riveted.

The circumferential seam at or near the middle of the length of single-ended boilers is to have a strength of joint not less than 60 per cent. of the solid plate. The inner circumferential seams of double-ended boilers are to have a strength of joint not less than 62 per cent. of the solid plate. In any case there are to be at least three rows of rivets where single-ended boilers have shell plates over $1\frac{3}{8}$ inches in thickness and where double-ended boilers have shell plates over $1\frac{3}{16}$ inches in thickness.

The circumferential seams of the shells of vertical boilers are to have a strength of not less than 42 per cent. of the solid plate. Where these seams are not complete circles, and where the shell plates exceed $\frac{5}{8}$ inch in thickness, the riveting is to be at least double.

Strength of Cylindrical Boiler Shells.—

D = greatest internal diameter of shell in inches.
t = thickness of shell plates in inches.
E = efficiency of riveted joints (longitudinal seams), see p. 615.
f_t = tensile strength of plates in lbs. per square inch.
P = safe working pressure of steam in lbs. per square inch.
F = factor of safety.

$$P = \frac{2tf_tE}{DF}. \qquad t = \frac{PDF}{2f_tE}.$$

Cylindrical Boiler Shells (*Board of Trade and Lloyd's Rules*).—If the thickness of the shell plates does not exceed 1¾ inches,

$$P=\frac{(t-2)\times S\times J}{C\times D}.$$

If the thickness of the shell plates exceeds 1¾ inches, and the longitudinal seams are made with double butt straps,

$$P=\frac{t\times S\times J}{2{\cdot}85\times D}.$$

In the above formulæ

P is working pressure in lbs. per square inch.

t is thickness of the shell plates *in 32nds of an inch.*

S is minimum tensile strength of shell plates in tons per square inch.

J is percentage of strength of longitudinal seams, and its value is either the least of the quantities R_t, R_s, and R_1 (see p. 624), or the least of the quantities R_t and R_s, depending on the type of joint.

C is a coefficient, which is 2·75 where longitudinal seams are made with double butt straps, 2·83 where longitudinal seams are made with lap joints and are treble riveted, 2·9 where they are made with lap joints and are double riveted, and 3·3 where they are made with lap joints and are single riveted.

D is inside diameter of outer strake of plating of cylindrical shell measured in inches.

Superheaters (*Board of Trade Rules*).—

Tubulous Superheaters attached to Cylindrical Boilers.—The headers should be of wrought- or cast-steel.

The minimum thickness of heating tubes is given by

$$t=\frac{P\times d}{75}+5.$$

P is working pressure in lbs. per square inch.

t is thickness of tubes *in 100ths of an inch.*

d is external diameter of tubes in inches.

The tubes should be solid drawn. Superheaters should be tested by hydraulic pressure to double the working pressure.

Superheaters of Water-tube Boilers.—Superheaters forming part of a water-tube boiler should comply with the requirements of boilers of that type as regards drums, headers, construction, and material. Tubes which have only steam within them should be situated in a position shielded from direct radiant heat, and where only hot gases and not flame can impinge upon any part. Working pressure on the tubes should not exceed that obtained from the formula for the upper tubes of water-tube boilers (see p. 639).

The completed superheater should be tested with and to the same hydraulic pressure as the boiler of which it forms part.

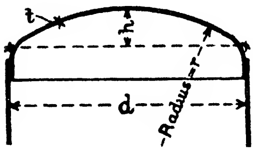
Strength of Spherical Boiler Shells.—A spherical shell has twice the strength of a cylindrical shell of the same diameter and thickness, hence,

$$P=\frac{4tf_tE}{DF}. \qquad t=\frac{PDF}{4f_tE},$$

where the letters have the meanings given on p. 627.

The foregoing formulæ also apply to the hemispherical ends of boiler shells.

Strength of Dished Ends of Boiler Shells.—



$$P=\frac{2tf_t}{rF} \qquad t=\frac{PrF}{2f_t}$$

The dished end will have the same strength as the cylindrical shell of the same thickness when

$$r=\frac{d}{E}$$

In practice r is generally equal to d.

In the above formulæ it is assumed that the dished end is in one piece.

$$r=\frac{d^2+4h^2}{8h}. \qquad h=r-\frac{\sqrt{4r^2-d^2}}{2}.$$

If $r=d$, then $h=\cdot134d$.

Hemispherical Ends and Dished Ends (*Board of Trade and Lloyd's Rules*).—

When the end is a hemisphere without stays,

$$P=\frac{(t-2)\times S\times J}{C\times R}.$$

P is working pressure in lbs. per square inch.

t is thickness of plates *in 32nds of an inch.*

S is minimum tensile strength of plates in tons per square inch.

J is minimum strength of riveted joints per cent. of solid plate.

R is inner radius of curvature in inches.

$C=3\cdot3$ for single riveting, 2·9 for double riveting, 2·83 for treble riveting.

When end is dished outwards to partial spherical form and without stays,

$$P=\frac{15\times S(t-1)}{R}.$$

R is not to exceed diameter of shell. Inside radius of curvature at flange not to be less than four times the thickness of end plate and in no case less than 2·5 inches.

When end has a manhole in it, the thickness of plate is to be increased by $\frac{1}{8}$ inch, and total depth of flange of manhole from the outer surface, in inches, is to be at least

$$\sqrt{T \times w},$$

where T is plate thickness in inches, and w is minor axis of hole in inches.

Plain Furnace Tubes.—

D = outside diameter in inches.

t = thickness in inches.

L = length in feet. (The length is measured between the rings, if the furnace is made with rings.)

P = working pressure of steam in lbs. per square inch.

$$P = \frac{46552 t^{2 \cdot 19}}{D\sqrt{L}}.$$

t =	$\frac{5}{16}$	$\frac{11}{32}$	$\frac{3}{8}$	$\frac{13}{32}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{1}{2}$	$\frac{17}{32}$	$\frac{9}{16}$
$t^{2 \cdot 19}$ =	·078	·096	·117	·139	·164	·190	·219	·250	·284

Board of Trade and Lloyd's Rules for Plain Furnaces.—

The working pressure allowed on plain furnaces or furnaces strengthened by Adamson or other joints, and on the cylindrical bottoms of combustion chambers, is determined by the following formulæ, the lesser pressure obtained being taken :—

$$P = \frac{C(t-1)^2}{(L+24) \times D} \quad \text{or} \quad P = \frac{C_1}{D} \times [10(t-1) - L].$$

P is working pressure in lbs. per square inch.

D is external diameter of furnace or combustion chamber in inches.

t is thickness of furnace plate *in 32nds of an inch.*

L is length, *in inches*, of furnace or combustion chamber bottom between points of substantial support, measured from centres of rivet rows or from the commencement of flange curvature, whichever is applicable.

C = 1450 where the longitudinal seams are welded, and 1300 where they are riveted.

C_1 = 50 where the longitudinal seams are welded, and 45 where they are riveted.

When plain vertical furnaces are tapered, the diameter for calculation purposes shall be the mean of that at the top, and at the bottom where it meets the substantial support from flange or ring. The length for the same purpose shall be the distance from the centre of the row of rivets, connecting the crown to the body of the furnace, to the substantial support at the bottom of the furnace, or to a row of screwed stays connecting the furnace to the shell, provided the pitch of stays at the furnace does not exceed 14 times the thickness of the furnace plate when the stays are riveted at their ends, and 16 times when they are fitted with nuts. The diameter over the threads of such screwed stays must be not less than 2·25 times the thickness of the furnace plate.

No furnace shall exceed $\frac{1}{8}$ inch in thickness.

Corrugated Furnaces (*Board of Trade and Lloyd's Rules*).—

$$P=\frac{C(t-1)}{D}.$$

P is working pressure in lbs. per square inch.

D is external diameter, in inches, measured at the bottom of the corrugations.

t is thickness of furnace plate *in 32nds of an inch*, measured at the bottom of the corrugation or camber.

$C=480$ for the Fox, Morison, Deighton, Purves, and similar furnaces, and is 510 for the Leeds Forge Bulb Suspension furnace.

No furnace shall exceed $\frac{1}{8}$ inch in thickness.

Spherical Furnaces (*Board of Trade and Lloyd's Rules*).—

When furnaces are spherical in form and convex upwards at their tops, and are without support from stays of any kind,

$$P=\frac{275(t-1)}{R}.$$

P is working pressure in lbs. per square inch.

t is thickness of top plate *in 32nds of an inch*.

R is outer radius of curvature of furnace in inches.

Ogee Ring (*Board of Trade and Lloyd's Rules*).—

For the ogee ring which connects the bottom of the furnace to the shell, and sustains the whole load on the furnace vertically,

$$P=\frac{140(t-1)^2}{D\times(D-d)}.$$

P is working pressure in lbs. per square inch.

t is thickness of ogee ring *in 32nds of an inch*.

D is inside diameter of boiler shell in inches.

d is outside diameter, in inches, of lower part of furnace where it joins the ogee ring.

BOILER STAYS.

Direct Stays.—

d = smallest diameter of stay in inches.
A = area of plate supported by one stay in square inches.
P = working pressure of steam in lbs. per square inch.
f = safe stress allowed on stay in lbs. per square inch.

$$d=\sqrt{\frac{AP}{\cdot 7854 f}}=C_1\sqrt{AP}. \qquad A=\frac{\cdot 7854\, d^2 f}{P}=C_2\frac{d^2}{P}.$$

$f=$ 4500	5000	5500	6000	6500	7000	7500	8000	8500	9000
$C_1=$ ·0168	·0160	·0152	·0146	·0140	·0135	·0130	·0126	·0122	·0119
$C_2=$ 3534	3927	4320	4712	5105	5498	5890	6283	6676	7069

For copper stays $f=4000$ to 5000.

Board of Trade and Lloyd's Rules.—Symbols P and A as above, also

d' = diameter of stay *over thread* in inches,
d_1 = diameter of stay at bottom of thread or at smallest unscrewed part,
S = minimum tensile strength of steel in tons per square inch.

For screw stays with threads not coarser than nine threads per inch, made of steel or special wrought iron,

$$P=\frac{8250(d'-0{\cdot}267)^2}{A}$$

but stress must not exceed 9000 lbs. per square inch.

For steel longitudinal stays with threads not coarser than six threads per inch,

$$P=\frac{9500(d'-0{\cdot}340)^2}{A}\times\frac{S}{28}$$

but stress must not exceed 11,000 lbs. per square inch when steel of a minimum tensile strength of 28 tons per square inch is used.

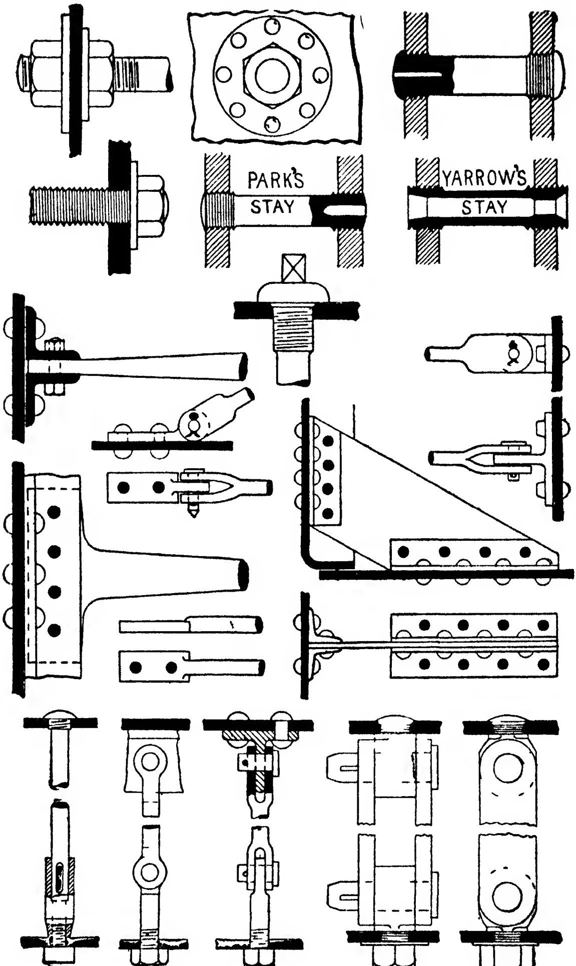
When longitudinal stays have enlarged ends or when the threads are coarser than six threads per inch,

$$P=\frac{9500(d_1-0{\cdot}125)^2}{A}\times\frac{S}{28}.$$

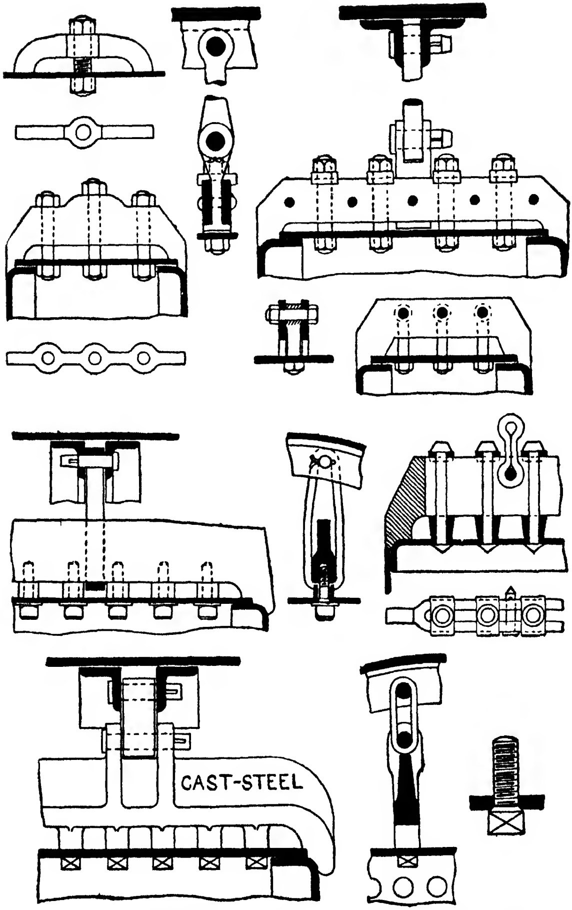
On stay tubes, of wrought-iron or steel, a working stress of 7500 lbs. per square inch of net sectional area at bottom of thread is allowed.

Minimum thickness of stay tubes, measured under threads, is to be $\frac{1}{4}$ inch for marginal stay tubes and $\frac{3}{16}$ inch for other stay tubes. Thickening of ends of stay tubes is to be attained by upsetting and not by any welding process, and the tubes are to be annealed after the upsetting.

Stay tubes are to be expanded by roller expanders and not made tight by caulking only.



Illustrations of direct, diagonal, and gusset stays.



Illustrations of girder stays, mainly from marine and locomotive practice.

Diagonal and Gusset Stays.—The area of the cross section of a diagonal stay is found as follows: Find the area of a direct stay to support the same area of plate, and multiply it by the length of the straight part of the diagonal stay, and divide by the length of its projection on a plane at right angles to the surface supported.

The area of a gusset stay should be in excess of that found by the above rule.

Girders supporting Combustion Chamber Tops (*Board of Trade and Lloyd's Rules*).—

$$P=\frac{C\times d^2\times t}{(L-p)\times D\times L}\times\frac{S}{28}.$$

P is working pressure in lbs. per square inch.

d is depth of girder at centre in inches.

t is effective thickness of girder at centre *in 32nds of an inch.*

L is length in inches, measured between the tube plate and back chamber plate inside, or between tube plates in chambers common to two opposite furnaces.

p is pitch of stays supported by girder, in inches.

D is distance apart of girders, centre to centre, in inches.

S is minimum tensile strength of steel plates forming the girder, in tons per square inch. With forged girders S is taken as 24 for iron and 28 for steel.

$C=\frac{495n}{n+1}$ when n is odd, $C=\frac{495(n+1)}{n+2}$ when n is even.

n is number of stays in a girder.

Plain Smoke Tubes (*Board of Trade and Lloyd's Rules*).—
Wrought Iron or Mild Steel.

Outside Diameter. Inches.	Standard Thicknesses. S.W.G.				Working Pressures. Lbs. per Sq. Inch.			
	A	*B*	*C*	*D*	*A*	*B*	*C*	*D*
2	..	11	10	9	..	155	215	300
2¼	11	10	9	8	140	190	260	315
2½	11	10	9	8	125	175	230	300
2¾	11	10	9	8	110	160	215	275
3	10	9	8	7	140	190	250	300
3¼	10	9	8	7	130	180	230	280
3½	10	9	8	7	120	165	215	260

Flat Plates supported by Stays (*Board of Trade and Lloyd's Rules*).—

$$P = \frac{C(t-1)^2}{a^2+b^2}.$$

P is working pressure in lbs. per square inch.
t is thickness of plate *in 32nds of an inch.*
a is distance apart of rows of stays in inches.
b is pitch of stays in the rows in inches.
C is a coefficient which depends on the method of fixing stays.

Method of Fixing.	C: Plates exposed to Flame.	C: Plates not exposed to Flame.
Stays screwed and ends riveted. Thickness of plate to be at least half diameter of stay measured at bottom of thread.	50	57
Stay tubes screwed and expanded .	..	52
Stay tubes screwed into plate, expanded and fitted with nuts.	..	72
Stays screwed into plate and fitted with nuts on outside.	75	86
Stays passing through plate and fitted with nuts on inside and outside.	84	96
Plate with flange, the inner radius of which is not greater than 2½ times plate thickness. Pitch to be reckoned from commencement of curvature.	96	110

Where stays are irregularly pitched, d^2 is to be used instead of a^2+b^2, d being the diameter of the largest circle which can be drawn through any three points of support without enclosing another point of support. When parts of the plate are supported in different ways, C is to be taken as the mean of the values appropriate for the methods of support at the various points.

Where plates are supported by stays passing through them and are fitted with nuts inside and washers and nuts outside, the diameter of the washers being at least 3·5 times that of the stay, and their thickness at least two-thirds that of the plate,

$$P = \frac{100}{a^2+b^2}[(t-1)^2 + 0{\cdot}15t_w^2],$$

where t_w is thickness, *in 32nds of an inch*, of washers, strips, or doublings, and the other symbols are as before.

Where washers have a diameter of at least two-thirds of the pitch of the stays and a thickness of at least two-thirds of that of the plate and are riveted to the plate in an efficient manner,

$$P=\frac{100}{a^2+b^2}[(t-1)^2+0{\cdot}35t_w^2].$$

Where plate is stiffened by strips at least two-thirds of pitch of stays in breadth and having a thickness at least two-thirds of that of the plate and are riveted to the plate in an efficient manner,

$$P=\frac{100}{a^2+b^2}[(t-1)^2+0{\cdot}55t_w^2].$$

Where the plate is fitted with a doubling plate having a thickness at least two-thirds that of the plate and riveted to it,

$$P=\frac{100}{a^2+b^2}[(t-1)^2+0{\cdot}85t_w^2].$$

If steel of less tensile strength than 26 tons per square inch is used for flat plates, the working pressure allowed is to be correspondingly reduced.

Tube Plates (*Board of Trade and Lloyd's Rules*).—
For the portions of tube plates in the nests of tubes,

$$P=\frac{C(t-1)^2}{p^2}.$$

P is working pressure in lbs. per square inch.

t is thickness of tube plate *in 32nds of an inch.*

p is mean pitch in inches of stay tubes supporting any portions of the plate (being the sum of the four sides of the quadrilateral divided by 4).

$C=38$ when stay tubes are screwed and expanded into tube plates and no nuts are fitted.

$C=49$ when stay tubes are screwed and expanded into tube plates and are fitted with nuts.

For the wide water spaces of tube plates between the nests of tubes and between the wing rows of tubes and the shell,

$$P=\frac{C[(t-1)^2+0{\cdot}55t_w^2]}{a^2+b^2}.$$

t_w is thickness of doubling plate, when fitted, *in 32nds of an inch.*

a is horizontal pitch of stay tubes in inches, measured across the wide water space from centre to centre.

b is vertical pitch of stay tubes in bounding rows, in inches, measured from centre to centre.

$C=52$ when stay tubes are screwed and expanded into the tube plates and no nuts are fitted.
$C=72$ when stay tubes are screwed and expanded into the tube plates and nuts are fitted to each stay tube.
$C=63$ when stay tubes are screwed and expanded into the tube plates and nuts are fitted only to alternate stay tubes.
$C=45$ for each of the foregoing conditions when there are wide spaces in the back tube plate exposed to flame (not in Lloyd's rules).

In cases where tube plates are in compression, the working pressure is obtained from the formula

$$P=875\times\frac{(D-d)\times t}{W\times D}.$$

P is working pressure in lbs. per square inch.
t is thickness of tube plate *in 32nds of an inch.*
D is horizontal pitch of tubes in inches.
d is inside diameter of plain tubes in inches.
W is width of combustion chamber, in inches, from tube plate to back chamber plate.

Tube Plates of Vertical Boilers (*Board of Trade and Lloyd's Rules*).—When vertical boilers have a nest or nests of horizontal tubes so that there is direct tension on the tube plates due to the vertical load on the boiler ends, or to their acting as horizontal ties across the shell, the thickness of the tube plates and the spacing of the tubes are to be such that the section of metal taking the load is sufficient to keep the stress within that allowed on shell plates. Also each alternate tube in the outer vertical rows of tubes is to be a stay tube.

The tube plates between the stay tubes must be designed according to the rule $P=\frac{C(t-1)^2}{p^2}$ as given in the preceding Art., and in addition

$$P=\frac{(t-2)\times S\times(p-d)\times 100}{2{\cdot}9\times D\times p}.$$

P is working pressure in lbs. per square inch.
S is minimum tensile strength of steel plate in tons per square inch.
t is thickness of tube plate *in 32nds of an inch.*
D is twice the radial distance of the centre of the outer row of tube holes from the axis of the shell in inches.
p is the vertical pitch of tubes in inches.
d is the diameter of the tube holes in inches.

Tube Plates of Water-tube Boilers (*Board of Trade and Lloyd's Rules*).—For tube plates forming portions of cylindrical drums of water-tube boilers

$$P=\frac{(t-4)\times S\times(p-d)\times 100}{3\times D\times p}.$$

P is working pressure in lbs. per square inch.

D is internal diameter of drum in inches.

t is thickness of tube plates *in 32nds of an inch.*

S is minimum tensile strength of plate in tons per square inch.

p is pitch of tubes, in inches, on lines parallel with axis of drum.

d is diameter of tube holes in inches.

Tubes of Water-tube Boilers (*Board of Trade and Lloyd's Rules*).—For pressures up to 250 lbs. per square inch, the minimum thickness of tubes is given by

$$t=\frac{P\times d}{F}+7.$$

P is working pressure in lbs. per square inch.

d is external diameter in inches.

t is thickness *in 100ths of an inch.*

$F=55$ for the two rows of tubes next the fire and round the gaps formed in the nests of tubes for the outflow of hot gases from the fire.

$F=75$ for all other tubes, including superheater tubes.

For pressures above 250 lbs. per square inch up to 650 lbs. per square inch, and a designed steam temperature not exceeding 750° Fahr., the minimum thickness of tubes is as follows:—

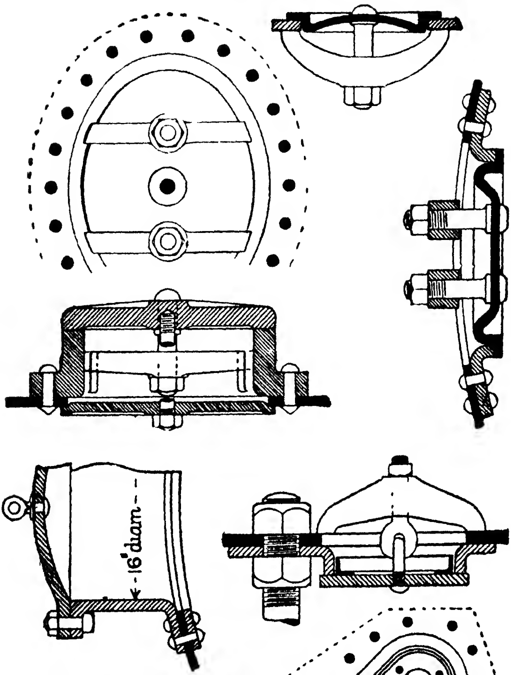
For the two rows of tubes next the fire and round the gaps formed in the nests of tubes for the outflow of hot gases from the fire,

$$t=\frac{d}{200}(P+400)+9.$$

For all other tubes, including superheater tubes,

$$t=\frac{d}{200}(P+400)+6.$$

The maximum thickness of any tube is not to exceed 1 S.W.G. (0·3 inch).



Illustrations of Man-holes.

Man-holes.—The principal man-hole in a boiler should be 16 inches in diameter if round, and 16 inches by 12 inches if oval. The joint for the cover should be a faced joint. The stand-pipe should be wrought steel. The cover for a circular man-hole 16 inches in diameter is generally secured by 16 bolts ⅞ inch or

1 inch in diameter, and the cover is ⅞ inch or 1 inch thick. Manholes in the ends may be from 14 inches by 12 inches to 15 inches by 11 inches. Mud-holes and sight-holes are from 6 inches by 4 inches to 9 inches by 6 inches.

Safety-valves (*Board of Trade and Lloyd's Rules*).—At least two safety-valves are to be fitted to each boiler. They are to be arranged so that the springs and valves are cased in, that the valves cannot be overloaded when steam is up, that they can be lifted by easing gear, and turned round on their seats by hand, and in case of fracture of springs they cannot lift out of their seats. Easing gear is to be arranged to lift all the safety-valves on a boiler together, and is to be workable from some accessible place, free from steam danger.

Vertical boilers having 100 square feet, or more, of total heating surface are to be fitted with two safety-valves each not less than 1·5 inches diameter; those having less than 100 square feet may have one valve not less than 2 inches diameter.

All the safety-valves of each boiler may be fitted in one chest, which is to be separate from any other valve chest and is to be connected direct to the boiler by a strong and stiff neck, the passage through which is to be of not less cross-sectional area than one-half the aggregate area of the safety-valves in the chest. Each safety-valve chest is to be provided with a means by which it can be drained; the drain-pipe is to be led to the bilge or to a tank, clear of the boiler.

The minimum aggregate area of the safety-valves of the ordinary type for saturated steam fitted to each boiler, whether coal fired or oil fired, and whether working under natural, forced or induced draught, is to be found by the following formula:—

$$A=\frac{\text{T.H.S.}\times E}{(p+15)\times 4{\cdot}8}.$$

A is aggregate area of safety-valves in square inches.

T.H.S. is total external surface, in square feet, of the tubes and other parts of the boiler exposed to heat, so as to cause evaporation.

p is working pressure in lbs. per square inch.

E is estimated evaporation in lbs. per square foot of heating surface (T.H.S.) per hour *with a minimum of 6.*

For superheated steam, the aggregate area of the safety-valves is to be

$$A_s=A\times\left(1+\frac{T}{1000}\right).$$

A_s is aggregate area of the safety-valves for superheated steam in square inches.

T is degree of superheat in degrees Fahr.

An approved type of safety-valve of equally good and reliable design may be fitted in lieu of those described in the Rules.

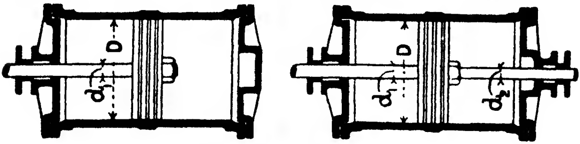
In the case of high lift safety-valves of approved type, the aggregate area of safety-valves, as calculated from either of the above formulæ, may be reduced by not more than 50 per cent.

The waste-steam pipe and the passages leading to it are to have a cross-sectional area not less than 1·1 times the combined areas of the safety-valves given by the formula.

All safety-valves are to be set to the required pressure under steam. During a test of 15 minutes with the stop valves closed and under full firing conditions the accumulation of pressure is not to exceed 10 per cent. of the loaded pressure. During this test no more feed-water should be supplied than is necessary to maintain a safe-working water level.

STEAM-ENGINES.

Indicated Horse-power.—The term "indicated horse-power" denotes the power developed in the cylinder of an engine.



D = diameter of cylinder in inches.
d_1 = diameter of piston-rod in inches.
d_2 = diameter of tail-rod in inches.
A = nominal area of piston in square inches.
= $\cdot 7854 D^2$.
A_1 = effective area of front of piston in square inches.
= $\cdot 7854(D^2 - d_1^2)$.
A_2 = effective area of back of piston in square inches.
= $\cdot 7854(D^2 - d_2^2)$.
P_1 = mean pressure on front of piston during the backward stroke in lbs. per square inch.
p_1 = mean pressure on front of piston during the forward stroke in lbs. per square inch.
P_2 = mean pressure on back of piston during the forward stroke in lbs. per square inch.
p_2 = mean pressure on back of piston during the backward stroke in lbs. per square inch.
L = length of stroke of piston in feet.
N = number of strokes per minute = twice the number of revolutions of the crank shaft per minute.
S = mean speed of piston in feet per minute = LN.
$I.H.P.$ = indicated horse-power.

The mean total effective force on piston during the backward stroke is $A_1P_1 - A_2p_2$.

The mean total effective force on piston during the forward stroke is $A_2P_2 - A_1p_1$.

The mean total effective force on piston during two consecutive strokes is $\dfrac{A_1(P_1 - p_1) + A_2(P_2 - p_2)}{2}$.

$P_1 - p_1$ and $P_2 - p_2$ are the mean heights of the indicator diagrams taken from the front and back ends of the cylinder respectively, these heights being measured by the pressure scale.

Let $P_1 - p_1 = Q_1$, and $P_2 - p_2 = Q_2$, then,

$$I.H.P. = \frac{(A_1Q_1 + A_2Q_2)LN}{2 \times 33000}.$$

If the areas of the piston-rod and tail-rod are neglected, and if $P_1 - p_1 = P_2 - p_2 = P$, then,

$$I.H.P. = \frac{PLAN}{33000} = \frac{\cdot 7854 D^2 PLN}{33000} = \frac{\cdot 7854 D^2 PS}{33000}.$$

The error introduced through neglecting the area of the piston-rod (when there is no tail-rod) is to make the $I.H.P.$ from $\frac{1}{2}$ per cent. to 4 per cent. too large in extreme cases. If there is a tail-rod, and its area is also neglected, the error will be nearly doubled.

If an engine has more than one cylinder, the total $I.H.P.$ of the engine is the sum of the $I.H.P.$'s of the separate cylinders.

$$\text{Diameter of cylinder} = D = \sqrt{\frac{33000 I.H.P.}{\cdot 7854 PS}} = 205 \sqrt{\frac{I.H.P.}{PS}}$$

Mean Piston Speed. The following table gives speeds of pistons to be met with in ordinary steam-engine practice:—

Class of Engine.	Mean Speed of Piston in Feet per Minute.
Ordinary direct-acting pumping engines (non-rotative)	90 to 130
Ordinary horizontal engines	200 to 400
Compound and triple expansion engines .	400 to 800
Ordinary marine engines	400 to 650
Locomotive engines (express) . . .	800 to 1500

For locomotive engines $S = \frac{56 \cdot 02\, ML}{D}$, where S = mean speed of pistons in feet per minute, M = speed of train in miles per hour, L = stroke of pistons in feet, and D = diameter of driving wheels in feet.

✓ **Clearance and Clearance Volume.**—*Clearance* in engine cylinders is the linear distance between the piston and the cylinder cover or cylinder end when that distance is least. The amount of the clearance varies with the size of the engine, being about $\frac{3}{8}$ inch in small engines and $\frac{7}{8}$ inch in large engines. In horizontal engines the clearance is generally the same at both ends of the cylinder, but in vertical engines the clearance at the lower end is usually about one and a half times the clearance at the upper end.

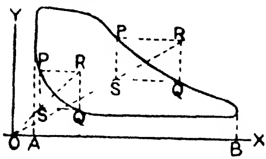
Clearance volume is the volume of the space between the piston and the valve when that space is least—that is, when the piston is at the beginning of its stroke. The clearance volume is generally expressed as a percentage of the volume swept through by the piston in one stroke.

The following table gives values of the clearance volume to be met with in steam-engine practice :—

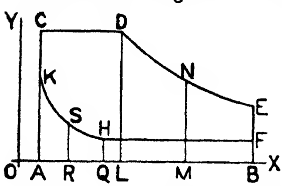
Type of Valve.	Clearance Volume per Cent.
Ordinary slide valve	5 to 13
Piston valve	8 to 15
Slide valve at each end of cylinder	3 to 5
Corliss valves	2 to 3

The clearance volume is best determined by filling the space with a measured quantity of water.

The clearance volume may be determined approximately from an indicator diagram as follows: Select two points P and Q on the expansion curve or on the compression curve. Draw the rectangle $PRQS$, PR being parallel to OX, the line of no pressure. Produce the diagonal RS to meet OX at O. Then, if AB represents the volume swept through by the piston in one stroke, OA will represent the clearance volume. The result will be exact if the curve PQ is an hyperbola.



Theoretical Diagram of Work in a Steam Cylinder.—AB= length of stroke or volume swept through by piston during one stroke.



OA = clearance volume to same scale.

AC = initial steam pressure (absolute) on piston.

CD is the steam admission line, and D the point of cut off.

DNE is the expansion line, which is usually assumed to be a rectangular hyperbola, axes OX and OY.

The exhaust opens and the pressure falls from E to F.

During the return stroke the exhaust pressure is shown by the height of FH above OX. At H the exhaust closes and compression begins, the back pressure rising from H to K. The compression line HSK is assumed to be a rectangular hyperbola, axes OX and OY.

The curves DNE and HSK may be drawn by means of the construction given on p. 155, or points may be obtained by calculation, thus

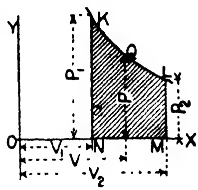
$$MN = \frac{OL \times LD}{OM}, \quad BE = \frac{OL \times LD}{OB}, \quad RS = \frac{OQ \times QH}{OR}, \quad AK = \frac{OQ \times QH}{OA}$$

If the compression curve is required to rise to C, then

$$OQ = \frac{AO \times AC}{QH}.$$

The area of the diagram $CDEFHK$ represents the work done during one stroke of the piston, and its average height represents the mean effective pressure on the piston during one stroke.

The Curve PV^n = constant.—If KQL is a curve such that the co-ordinates P and V of any point Q in it satisfy the condition PV^n = constant, then the area of the figure $KLMN$ bounded by two ordinates, a part of the curve and a part of the axis OX, is given by the formula, area $= \frac{P_1 V_1 - P_2 V_2}{n-1}$. Also, since



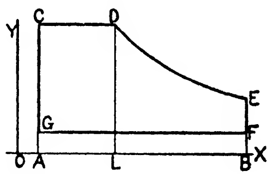
$$P_2 V_2^n = P_1 V_1^n, \quad P_2 = P_1 \left(\frac{V_1}{V_2}\right)^n.$$

For saturated steam $n = \frac{17}{16}$, or more exactly 1·0646.

If $n = 1$, the curve KQL is a rectangular hyperbola and the above formula fails; in this case, area $KLMN = P_1 V_1 \log_e \frac{V_2}{V_1}$.

Theoretical Mean Pressure of Steam used Expansively.—The steam is assumed to expand according to Boyle's law, namely, PV = constant.

As before, AB = length of stroke or volume swept through by piston during one stroke, and OA = clearance volume to same scale.



CASE I. *No Compression or Cushioning.*

$$\text{Area of } DEBL = DL \times OL \log \frac{OB}{OL} = DL(OA + AL) \log \frac{OA + AB}{OA + AL}.$$

Area of $CDEFG$ = area $DEBL$ + area $CDLA$ − area $GFBA$.

$$= DL(OA + AL) \log \frac{OA + AB}{OA + AL} + DL \times AL - BF \times AB.$$

Let $AB = l$, $\quad \frac{OA}{AB} = c$, $\quad \frac{AL}{AB} = r$, $\quad DL = AC = P_1$, $\quad BF = P_2$.

$$\text{Area of } CDEFG = P_1 l(c + r) \log \frac{c+1}{c+r} + P_1 rl - P_2 l.$$

Mean height of $CDEFG$ = area of $CDEFG \div l$.

$$= P_1 \left\{ r + (c + r) \log \frac{c+1}{c+r} \right\} - P_2.$$

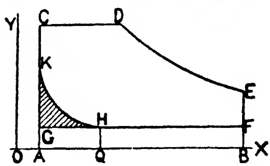
$$= P_1 q_1 - P_2.$$

The logarithms to be used are the Napierian or hyperbolic logarithms.

The following table gives values of q_1 for various values of c and r.

The nominal ratio of expansion, $R = \frac{1}{r}$. The actual ratio of expansion is equal to $\frac{1+c}{r+c}$.

Point of Cut-off, r.	Ratio of Expansion. R.	Clearance Volume, c.							
		·00	·02	·04	·06	·08	·10	·12	·14
		Values of q_1.							
·75	$1\frac{1}{3}$	·966	·967	·967	·968	·969	·969	·970	·970
·7	$1\frac{3}{7}$	·950	·951	·952	·953	·954	·955	·956	·957
·6	$1\frac{2}{3}$	·906	·909	·911	·913	·915	·916	·918	·920
·5	2	·847	·850	·854	·857	·861	·864	·867	·869
·4	$2\frac{1}{2}$	·767	·773	·778	·784	·789	·794	·799	·803
·3	$3\frac{1}{3}$	·661	·671	·680	·689	·697	·705	·712	·719
·25	4	·597	·609	·620	·631	·641	·651	·660	·668
·2	5	·522	·537	·552	·565	·578	·590	·601	·611
·1	10	·330	·357	·381	·403	·423	·441	·458	·474
·05	20	·200	·238	·270	·299	·325	·349	·371	·390



Case II. *With Compression or Cushioning.* — Cushioning reduces the area of the diagram by the amount of the shaded area GHK. $\frac{GH}{AB} = x$, and the other letters have the same meanings as in Case I.

The cushioning diminishes the mean effective pressure by the amount $P_2(x+c)\log\frac{x+c}{c} - P_2x$.

Hence the mean effective pressure in Case II.

$$= P_1\left\{r + (c+r)\log\frac{c+1}{c+r}\right\} - P_2\left\{1 - x + (x+c)\log\frac{x+c}{c}\right\}$$

$$= P_1q_1 - P_2q_2.$$

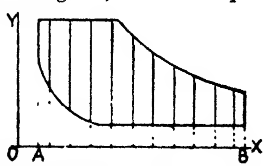
Values of q_1 have already been given, and the following table gives values of q_2:—

Point of Compression x	Clearance Volume, c							
	·01	·02	·04	·06	·08	·10	·12	·14
	Values of q_2.							
·5	2·505	2·194	1·905	1·751	1·649	1·575	1·518	1·473
·4	2·123	1·879	1·655	1·537	1·460	1·405	1·363	1·329
·3	1·765	1·587	1·428	1·345	1·292	1·255	1·226	1·204
·2	1·439	1·328	1·230	1·181	1·151	1·130	1·114	1·102
·1	1·164	1·115	1·075	1·057	1·046	1·039	1·033	1·029

The mean effective pressure on the piston may also be obtained from the theoretical diagram of work, or from the actual diagram drawn by a steam-engine indicator by either of the following methods:—

(a) Find the area of the diagram by means of a planimeter or other area-measuring instrument, and divide this area by the length of the diagram to obtain the mean height of the figure. This mean height multiplied by a factor corresponding to the scale of pressures for the diagram gives the mean effective pressure. Thus, if the mean height of the diagram is 1¼ inches, and the scale of pressures is 1 inch = 32 lbs. per square inch, the mean effective pressure would be $1\frac{1}{4} \times 32 = 40$ lbs. per square inch.

(b) Divide AB, the length of the diagram, into ten equal parts, and through the middle points of these parts draw ordinates across the diagram, as shown. The lengths of the parts of these ordinates intercepted by the diagram being added together, and the sum divided by ten, the result is approximately the mean height of the diagram from which the mean effective pressure is obtained, as already explained.



Mean Pressure referred to Low-pressure Cylinder in Compound and Multiple Stage Expansion Engines.—Neglecting the losses due to condensation and wire-drawing of the steam, the total work done in the cylinders of a compound or multiple stage expansion engine is the same as if the steam was used in the low-pressure cylinder only; the initial pressure and total ratio of expansion being the same in both cases. If P_1 is the absolute initial pressure in the high-pressure cylinder, and P_2 is the absolute terminal pressure in the low-pressure cylinder, then the total ratio of expansion is $P_1 \div P_2$.

If now a diagram of work be drawn for the low-pressure cylinder with the initial pressure equal to P_1, a ratio of expansion equal to $P_1 \div P_2$, and a back pressure equal to that in the low-pressure cylinder, then the mean effective pressure determined from this diagram is the theoretical mean effective pressure referred to the low-pressure cylinder.

The actual mean effective pressure referred to the low-pressure cylinder is determined from the formula,

$$P = \frac{33000 \times I.H.P.}{\cdot 7854 D^2 S}, \text{ where}$$

P = actual mean effective pressure in lbs. per square inch referred to low-pressure cylinder.

$I.H.P.$ = total indicated horse-power of engine.

D = diameter of low-pressure cylinder in inches.

S = mean speed of low-pressure piston in feet per minute.

If P is known, then the diameter of the low-pressure cylinder is given by the formula,

$$D = 205 \sqrt{\frac{I.H.P.}{PS}}.$$

Diagram Factor.—The ratio $\dfrac{\text{actual mean pressure}}{\text{theoretical mean pressure}}$ is called the diagram factor. If, therefore, the diagram factor for a particular case is known, and the theoretical mean pressure be determined, the actual mean pressure is found by multiplying the theoretical mean pressure by the diagram factor for that case.

Examples of Diagram Factors.

Simple engines working expansively . .	·7 to ·9
Compound or two stage expansion engines	·6 to ·8
Triple expansion engines	·6 to ·7

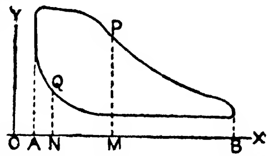
The diagram factor is higher the greater the expansion ratio. It is also higher with jacketed than with unjacketed cylinders. Valves such as those of the Corliss type, which open and close quickly, cause the factor to be higher than simple slide valves driven by eccentrics.

Compound and triple expansion engines of the Woolf type, where there are no receivers, have higher diagram factors than those which require receivers between the cylinders.

In large slow-speed pumping engines with jacketed cylinders the diagram factor is sometimes as high as 1·0.

All the above values of the diagram factor are given on the assumption that in determining the *theoretical* mean pressure clearance and cushioning are neglected.

Determination of Steam Consumption from Indicator Diagram.—Select a point P on the expansion line near to the point of cut-off, and measure the absolute pressure PM at that point. Determine from the table of properties of saturated steam the weight w_1 of a cubic foot of steam having a pressure PM. Select a point Q on the compression line, and measure the absolute pressure QN at that point. Determine the weight w_2 of a cubic foot of steam having a pressure QN.



If AB represents the volume swept through by the piston during one stroke in cubic feet, and OA represents the clearance volume in cubic feet, then weight of steam in cylinder after cut-off $= w_1 \times OM$, and weight of steam in cylinder after exhaust $= w_2 \times ON$. Therefore, weight of steam used in one stroke $= w_1 \times OM - w_2 \times ON$.

If V = volume swept through by piston in one stroke, then weight of steam used in one stroke $= V\left\{w_1 \times \frac{OM}{AB} - w_2 \times \frac{ON}{AB}\right\}$.

The above operations must be repeated on the diagram from the other end of the cylinder. The two results added together gives the weight of steam used in one revolution, and this multiplied by the number of revolutions in a given time gives the weight of steam used in that time.

It is important to note that the consumption of steam determined as above does not include steam which may have condensed in the cylinder before cut-off or in the pipes and passages leading to the cylinder.

Steam Consumption per Indicated Horse-power.—The following table shows approximately the weight of steam consumed per indicated horse-power per hour in various types of engines under ordinary working conditions :—

Type of Engine.	Weight of Steam per I.H.P. per Hour, in Lbs.
Simple non-condensing engines	22 to 40
Simple condensing engines, with steam at 60 lbs. pressure, and fitted with expansion gear	19 to 22
Compound condensing engines, with steam at 60 lbs. pressure	18 to 20
Compound condensing engines, with steam at 100 lbs. pressure	16½ to 18½
Triple expansion condensing engines, with steam at 160 lbs. pressure	13 to 16
Locomotives, simple	24 to 30
Locomotives, compound	22 to 27

Proportions of High-speed Engines.—The following formulæ, given in a paper by Professor John H. Barr, read before the American Society of Mechanical Engineers in 1897, are based on the dimensions of about eighty engines by thirteen different American engine builders. The sizes of the engines ranged from 20 to 240 horse-power. The engines here classed as "high speed" have, generally, a stroke of from one to one and a half diameters, with a rotative speed of 200 to 300 revolutions per minute.

D = diameter of piston.
L = length of stroke.
A = area of piston.
H = indicated horse-power.
P = steam pressure, taken at 100 lbs. per square inch above exhaust as a standard pressure.
V = mean piston speed in feet per minute.
L_1 = length of connecting-rod.
D_1 = diameter of fly-wheel.
N = number of revolutions per minute.
All dimensions in inches and areas in square inches.

Thickness of cylinder walls $\cdot 04D + \cdot 3$ to $\cdot 06D + \cdot 3$, mean $\cdot 05D + \cdot 3$.

Area of ports $\frac{AV}{4500}$ to $\frac{AV}{6500}$, mean $\frac{AV}{5500}$.

Area of steam-pipe $\frac{AV}{5800}$ to $\frac{AV}{7000}$, mean $\frac{AV}{6500}$.

Area of exhaust-pipe $\frac{AV}{2500}$ to $\frac{AV}{5500}$, mean $\frac{AV}{4400}$.

Width of face of piston $\cdot 3D$ to $\cdot 6D$, mean $\cdot 46D$.

Diameter of piston-rod $\cdot 12\sqrt{DL}$ to $\cdot 175\sqrt{DL}$, mean $\cdot 145\sqrt{DL}$.

Mid section of connecting-rod, rectangular, height h, thickness b. b varied from $\cdot 045\sqrt{DL_1}$ to $\cdot 07\sqrt{DL_1}$, mean $\cdot 057\sqrt{DL_1}$. h varied from $2\cdot 2b$ to $4b$, mean $2\cdot 7b$.

Projected area of cross-head pin (diameter × length) $\cdot 06A$ to $\cdot 11A$, mean $\cdot 08A$.

Length of cross-head pin ÷ diameter 1 to 2, mean 1·25.

Projected area of crank pin $\cdot 17A$ to $\cdot 44A$, mean $\cdot 24A$.

Projected area of main journal (or of each of the two journals of a centre crank engine) $\cdot 37A$ to $\cdot 7A$, mean $\cdot 46A$.

Length of main journal ÷ diameter 2 to 3, mean 2·2.

Diameter of main journal $6\cdot 5\sqrt[3]{H \div N}$ to $8\cdot 5\sqrt[3]{H \div N}$, mean $7\cdot 3\sqrt[3]{H \div N}$.

Average piston speed **530 to 660, mean 600 feet per minute.**

Weight of reciprocating parts (piston, piston-rod, cross-head, and one half of connecting-rod) $1,200,000\frac{D^2}{LN^2}$ to $2,300,000\frac{D^2}{LN^2}$, mean $1,860,000\frac{D^2}{LN^2}$ lbs.

Weight of fly-wheel rim (lbs.) $650,000,000,000(H \div D_1^2N^3)$ to $2,000,000,000,000(H \div D_1^2N^3)$, mean $1,200,000,000,000(H \div D_1^2N^3)$.

Mean speed of rims of fly-wheels about 4200 feet per minute.

Weight of engine (including fly-wheel) $100H$ to $135H$, mean $115H$ lbs.

Cylinder Barrels and Cylinder Liners.—

P = maximum steam pressure in cylinder in lbs. per sq. inch.
D = internal diameter of cylinder in inches.
t = thickness of cylinder barrel in inches.
t_1 = thickness of cylinder liner in inches.

The following formulæ agree with the average results given by the rules of a considerable number of authorities:—

$$t = \frac{DP}{3500} + \tfrac{3}{8} \text{ inch.}$$

$$t_1 = \frac{DP}{4000} + \tfrac{1}{4} \text{ inch, for a cast-iron liner.}$$

$$t_1 = \frac{DP}{5000} + \tfrac{3}{8} \text{ inch, for a forged steel liner.}$$

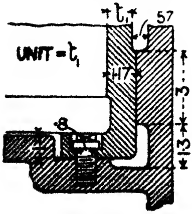
Messrs. Seaton and Rounthwaite give the following rules in their pocket-book for marine engineers:—

$$t = \frac{D(P+50)}{6000} + \cdot 2 \text{ inch, for cylinders fitted with liners.}$$

$$t = \frac{D(P+50)}{6000} + \cdot 4 \text{ inch, for cylinders without liners.}$$

$$t_1 = \cdot 8\left\{\frac{D(P+50)}{6000}\right\} + \cdot 35 \text{ inch, for cast-iron liners.}$$

$$t_1 = \cdot 65\left\{\frac{D(P+50)}{6000}\right\} + \cdot 3 \text{ inch, for forged steel liners.}$$



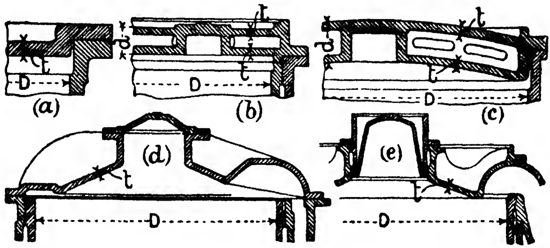
The annexed illustration shows a common method of securing the liner to the cylinder. The liner has an internal flange at one end, which is bolted to the front end of the cylinder.

Cylinder Covers.—

D = diameter of cylinder in inches.

P = greatest steam pressure in cylinder in lbs. per square inch.

t = thickness of metal in inches.



Single dished cast-iron covers, Fig. (*a*).

$$t = \frac{D\sqrt{P}}{400} + \tfrac{1}{2} \text{ inch.}$$

Double cast-iron covers, Figs. (*b*) and (*c*).

$$t = \frac{D\sqrt{P}}{500} + \tfrac{3}{8} \text{ inch.}$$

Depth of cover $= d = 5t$ to $7t$.

Cast-steel covers, single thickness, with ribs, Figs. (*d*) and (*e*).

$$t = \frac{D\sqrt{P}}{500} + \tfrac{1}{4} \text{ inch.}$$

$$\text{Number of radial ribs} = \frac{D}{8} + 4 \text{ (roughly).}$$

Large cylinder covers generally have man-holes, as shown at (*d*) and (*e*); these vary in diameter from 14 inches to 20 inches.

The inside face of a cylinder cover or cylinder end is generally of the same shape as that of the piston, in order that the clearance volume of the cylinder may be as small as possible.

The steam port is generally formed in the cylinder, as shown in Fig. (*e*); but a shorter cylinder and a saving of weight is obtained by forming the port in the cylinder cover, as shown in Fig. (*d*).

Flanges of Cylinder and Cylinder Cover.—Thickness of flange = 1·2 to 1·4 times the thickness of the cylinder barrel.

Distance from centre of bolts or studs to outside of flange not less than $d + \frac{1}{4}$ inch, and not more than $1\frac{1}{2}d$, where d is the diameter of the bolts over the threads, in inches.

Bolts for Cylinder Cover.—

d = nominal diameter of bolts in inches.
d_1 = diameter of bolts at bottom of screw thread in inches.
n = number of bolts.
f = stress on bolts in lbs. per square inch of net section.
D = diameter of cylinder in inches.
P = maximum pressure of steam in cylinder in lbs. per square inch.

$$d_1^2 nf = D^2 P, \text{ and } d_1 = D\sqrt{\frac{P}{nf}}.$$

f should vary with the diameter d, and may be taken equal to $4000d$, but should not exceed 6000.

Bolts of less than $\frac{5}{8}$ inch nominal diameter should not be used for cylinder covers.

In practice d generally varies from $\frac{3}{4}t$ to t, where t is the thickness of the flange.

The pitch of the bolts may be from $4\sqrt{d}$ to $6\sqrt{d}$.

Areas of Steam-pipes, Ports, and Passages.—Let V be the mean velocity of the steam in feet per minute, A the area of the pipe, port, or passage at right angles to the direction of flow, in square inches, D the diameter of the piston in inches, and S its mean speed in feet per minute. Then $A = \dfrac{\cdot 7854 D^2 S}{V}$.

If the pipe, port, or passage be circular, and of a diameter d inches,

$$\text{then } \cdot 7854 d^2 = \frac{\cdot 7854 D^2 S}{V}, \text{ and } d = D\sqrt{\frac{S}{V}}.$$

The following table gives the values of V usually taken in different cases:—

Main steam-pipes	5000 to 8000
Exhaust pipes, ports, and passages	4000 „ 6000
Stop and throttle valves	4000 „ 6000
Steam ports and passages	4000 „ 7000
Steam port opening. Ordinary slide valve	6000 „ 9000
Steam port opening. Quick cut-off valve	9000 „ 12000

Jet Condensers.—The total heat contained in 1 lb. of steam, as it leaves the low-pressure cylinder of a condensing engine, is about 1138 B.T.U. above that contained in 1 lb. of water at 32° F., and the weight of water required to condense this steam is $\dfrac{1138 + 32 - T}{T - t} = \dfrac{1170 - T}{T - t}$, where T is the temperature of the hot-well, and t the temperature of the injection water. T is usually from 100° to 120°.

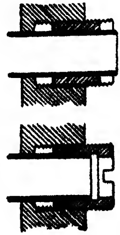
Temperature of Hot-well.	Corresponding Back Pressure in Cylinder.	Temperature of Injection Water, Deg. Fahr.					
		40	50	60	70	80	90
		Ratio of Weight of Injection Water to Weight of Steam.					
Deg. Fahr	Lbs. per Sq. Inch.						
100	0·94	17·8	21·4	26·8	35·7	53·5	107·0
110	1·27	15·1	17·7	21·2	26·5	35·3	53·0
120	1·68	13·1	15·0	17·5	21·0	26·3	35·0
130	2·21	11·6	13·0	14·9	17·3	20·8	26·0
140	2·88	10·3	11·4	12·9	14·7	17·2	20·6

The volume of a jet condenser is usually proportioned according to the volume of the low-pressure cylinder. Many authorities give the volume of a jet condenser as one-quarter to one-half the volume of the low-pressure cylinder, but an examination of the proportions of jet condensers in a number of recent examples of mill-engines showed a ratio of volume of condenser to volume of low-pressure cylinder varying from one-half to one and one-quarter, the average being about three-quarters.

The area of the injection pipe is approximately $\frac{W}{130\sqrt{h}}$, where W = weight of injection water required per minute in lbs., and h = head of water in feet.

Illustrations of jet condensers are shown on pp. 615 and 616 in connection with air-pumps.

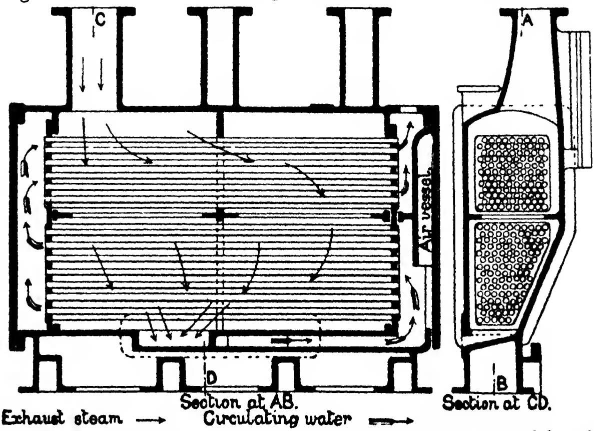
Surface Condensers.—A surface condenser should be used when the cooling water is of such a character as to be injurious to the boiler when used as feed-water. The shell of a surface condenser is generally either rectangular or cylindrical, and may be made of cast-iron, brass, wrought-iron, or steel. The tube plates are made of rolled brass. The tubes are made of brass and are solid drawn, and they are generally tinned outside and inside. They vary in diameter from ½ inch to 1 inch, but generally they are ¾ inch in diameter outside. The thickness of the tubes is from 16 to 19 I.S.W.G. Tubes ¾ inch in diameter are generally No. 18 I.S.W.G. (·048 inch) in thickness.



The tubes are generally secured to the tube plates by screwed glands and stuffing boxes packed with cotton cord or a ring of thick tape. The tubes are placed zigzag, and their pitch, measured from centre to centre, may be from $1{\cdot}5d$ to $1{\cdot}7d$, where d is the external diameter of the tubes.

Thickness of tube plates = diameter of tubes in inches, + ¼ inch.

In the surface condensers of modern triple expansion marine engines the amount of cooling surface is from 1·1 square feet to



1·5 square feet per indicated horse-power. Professor Whitham's rule for the amount of cooling surface is, $S=\frac{WL}{180(T-t)}$, where

S=cooling surface in square feet.
W=weight of steam to be condensed per hour, in lbs.
T=temperature of steam to be condensed.
t=mean temperature of circulating water=arithmetical mean of the initial and final temperatures.
L=latent heat of steam of temperature T.

If T is 135°, and t is about 75°, then $S=\frac{17W}{180}$.

The cooling water in most cases passes through the tubes.

The amount of cooling water required is determined in the same way as for jet condensers, except that it must be noted that the temperature of the cooling water as it leaves the condenser is not the same as that of the condensed steam.

H=total heat in 1 lb. of steam above that contained in 1 lb. of water at 32°.
T_1=temperature of condensed steam.
t=temperature of circulating water as it enters the condenser.
t_1=temperature of circulating water as it leaves the condenser.
W_1=weight of circulating water (in lbs.) required for each lb. of steam condensed.

$$W_1=\frac{H+32-T_1}{t_1-t}.$$

Circulating Pumps.—*Reciprocating Pumps.*—These may be either single or double acting, but the latter are more often used than the former.

D = diameter of barrel in inches.

L = length of stroke in inches.

n = number of working strokes per minute.

w = total weight of circulating water required per minute, in lbs.

Then $nD^2L = 35w$.

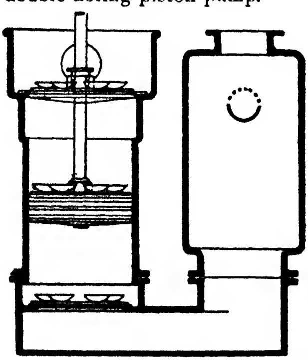
The valves should have a clear waterway sufficient to prevent the velocity of the water exceeding 450 feet per minute, and the pipes should be large enough to prevent the velocity of the water in them exceeding from 500 to 600 feet per minute.

These pumps may be driven by the main engines or by separate and independent engines.

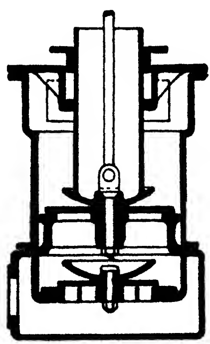
Centrifugal Pumps.—These are always driven by separate and independent engines.

The velocity of the water in the inlet and outlet pipes should not exceed 450 feet per minute, hence the diameter of these pipes should not be less than $\cdot 08\sqrt{w}$, where w is the weight of circulating water required per minute, in lbs. The diameter of the wheel may be from two and a half to three times the diameter of the pipes.

Air-pumps.—The most efficient and most common form of air-pump is the vertical single-acting bucket-pump. When it is desirable that the air-pump should be horizontal, it is usually a double-acting piston-pump.



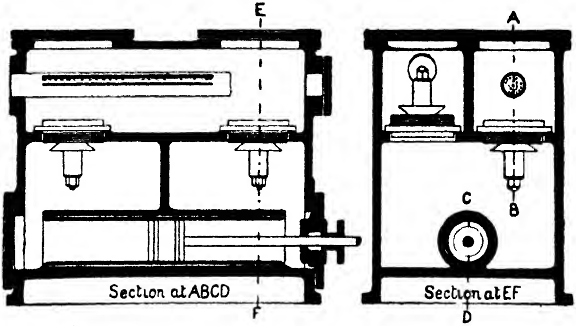
Vertical Single-acting Air-pump and Jet Condenser.



Combined Bucket and Plunder Air-pump.

For a jet-condensing engine the capacity of a vertical single-acting air-pump varies from one-eighth to one-fourth of that of the low-pressure cylinder. The average of twenty-five examples

from recent practice gave the capacity of the air-pump as ·18 of the capacity of the low-pressure cylinder.

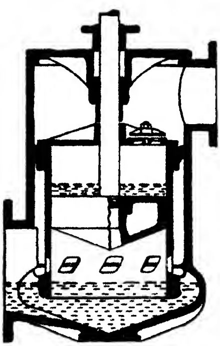


Horizontal Double-acting Air-pump and Jet Condenser.

For a surface-condensing engine the capacity of a vertical single-acting air-pump varies from one-tenth to one-twentieth of that of the low-pressure cylinder. The average of a number of examples from recent practice gave the capacity of the air-pump as one-fifteenth of the capacity of the low-pressure cylinder.

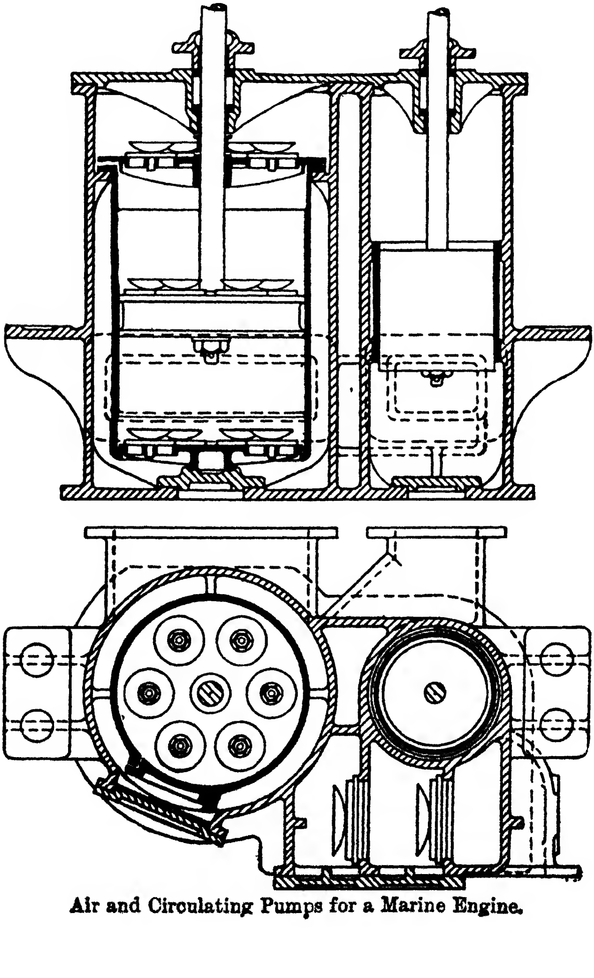
The capacity of a double-acting pump should be from five-eighths to two-thirds of the capacity of a single-acting pump for the same work.

All the above proportions relate to pumps whose buckets or pistons make the same number of strokes per minute as the piston of the low-pressure cylinder. In each case the capacity is taken as the area of the piston or bucket multiplied by the length of stroke



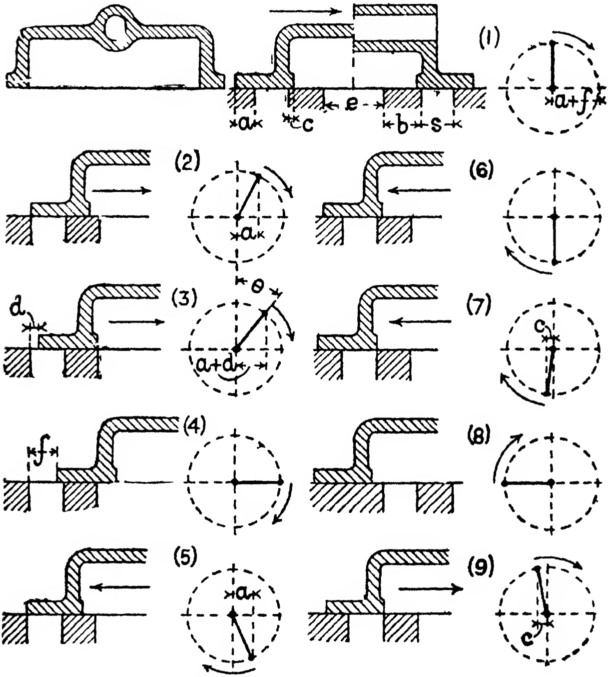
The annexed illustration shows a form of air-pump designed by Mr. Frederick Edwards. This pump is single-acting, and has no foot or bucket valves. A similar design is used by Messrs. Bates and Co., Sowerby Bridge, in their horizontal mill-engines.

Air-pump buckets need not be provided with packing rings. If packing is used, the ordinary rope-packing is the best. On this point Mr. Michael Longridge, in his report to the Engine and Boiler Insurance Company for the year 1895, remarks that the cause of failures of air-pump arrangements is the vicious practice pursued by many makers of fitting the buckets with brass packing-rings, which rapidly wear out and break; and he adds that these rings are utterly useless, a plain bucket without any packing whatever, or a bucket or plunger packed with rope, being as effective as the most elaborate system of metallic rings and springs.



Air and Circulating Pumps for a Marine Engine.

Slide Valve.—The diagrams (1) to (9) below show the relative positions of the slide valve and the eccentric which drives it, as the latter revolves. The effect of the obliquity of the eccentric-rod is neglected.



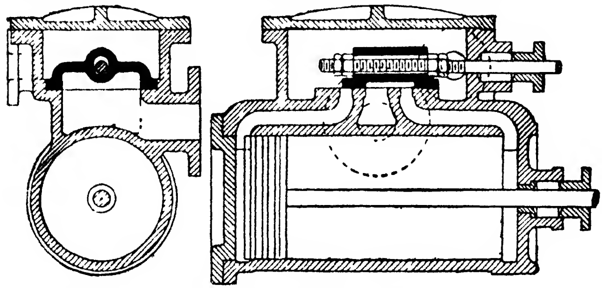
(1) Valve in its middle position.
(2) Valve just about to open to steam.
(3) Valve open to steam by an amount equal to the "lead."
(4) Valve full open to steam, and at one end of its travel.
(5) Steam just cut off.
(6) Valve returned to its middle position.
(7) Valve just about to open to exhaust.
(8) Valve at other end of its travel.
(9) Exhaust just cut off.

a = outside lap. c = inside lap.
d = lead. s = width of steam port.
e = width of exhaust port. f = maximum opening to steam.

r = radius of eccentric. θ = angular advance of eccentric.
b = width of bar between steam and exhaust ports.
Travel of valve $= 2r = 2(a+f)$.

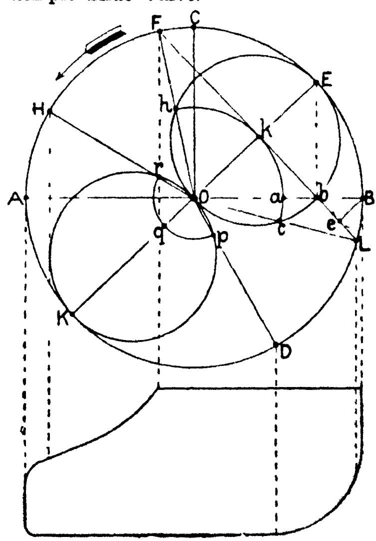
$\sin\theta = \frac{a+d}{r}$. c should not be less than $2r - b$.

The illustration below shows transverse and longitudinal sections of a steam cylinder fitted with the ordinary simple slide valve.



Zeuner's Diagram for a Simple Slide Valve.—

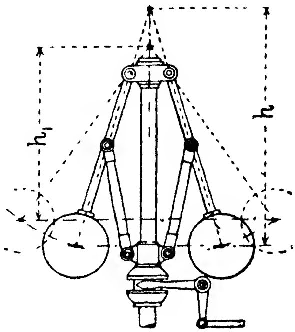
AB = travel of valve.
OA = radius of eccentric.
OC is perpendicular to AB.
Angle COE = angle of advance of eccentric.
EO is produced to K, and circles are described on OE and OK as diameters.
Oa = outside lap of valve.
ab = lead of valve.
Angle $obE = 90°$.
hkc is an arc of a circle, with centre O and radius $= Oa$.
pqr is an arc of a circle, with centre O and radius = inside lap of valve. (If inside lap is negative, points r and p are on circle $Eoch$).



OL, *OF*, *OH*, and *OD* are positions of crank of engine, when steam is admitted, when steam is cut off, when exhaust opens, and when exhaust closes, respectively. The probable indicator diagram is shown at the bottom of the figure, the effect of the obliquity of the connecting-rod being neglected.

The line *FL* touches the arc *hkc* at *k*. *FL* is also perpendicular to *OE*. A perpendicular *Be* on *FL* is equal to *ab* the lead.

Governors.—*The Common Governor.* (See also p. 237.)



N = speed of rotation in revolutions per minute when the height = h.

h = height in feet when the speed = N.

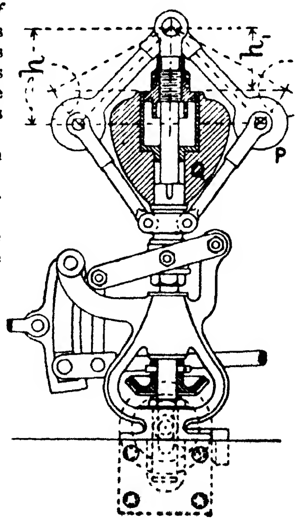
N_1 = speed of rotation in revolutions per minute when the height = h_1.

h_1 = height in feet when the speed = N_1.

$$h=\frac{2936\cdot3}{N^2}. \qquad h_1=\frac{2936\cdot3}{N_1^2}.$$

$$N=\frac{54\cdot19}{\sqrt{h}}. \qquad N_1=\frac{54\cdot19}{\sqrt{h_1}}.$$

The Porter or Loaded Type of Governor.—The illustration shows one form of this type. In this example (Tyrrel and Deed's Patent) there is a dash-pot in the sliding weight to give steadiness to the governor.



P = weight, in lbs., of each pendulum ball.

Q = weight, in lbs., of the central weight.

The proportions of the arms are generally such as to make the rise of Q twice that of P, then

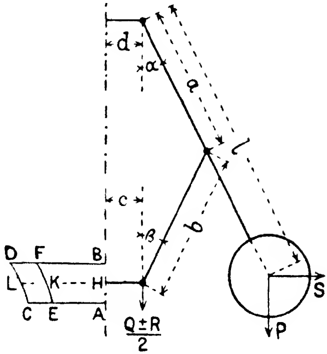
$$h=\left(\frac{P+Q}{P}\right)\frac{2936\cdot3}{N^2}.$$

$$h_1=\left(\frac{P+Q}{P}\right)\frac{2936\cdot3}{N_1^2}.$$

$$N=\frac{54\cdot19}{\sqrt{h}}\sqrt{\frac{P+Q}{P}}.$$

$$N_1=\frac{54\cdot19}{\sqrt{h_1}}\sqrt{\frac{P+Q}{P}}.$$

More exact Formulæ for Pendulum Governors.—



α_1, α_2, and α = minimum, maximum, and normal inclinations of pendulum-rods to vertical.

β_1, β_2, and β = corresponding inclinations of lifting links to vertical.

N_1, N_2, and N = speeds of governor in revolutions per minute corresponding to the inclinations α_1, α_2, and α, when sleeve is being *raised*.

n_1, n_2, and n = corresponding speeds of governor when sleeve is being *lowered*.

P = weight, in lbs., of one ball plus one-half of the weight of the rod to which it is attached.

Q = weight, in lbs., of the sleeve and sliding weight (if there is one) plus the weight of one lifting link.

R = resistance, in lbs., to motion of sleeve due to friction of governor and gear.

Let the governor be in its normal position, and suppose it to be rising.

$$S = \text{centrifugal force} = \frac{(d + l \sin \alpha)PN^2}{2936}.$$

The force $\frac{Q+R}{2}$ causes a tension in the lifting link $= \frac{Q+R}{2 \cos \beta}$.

Considering the forces acting on the pendulum-rod, and taking moments about its point of suspension, the following formula is obtained, the linear dimensions being in feet :—

$$N^2 = \frac{2936}{d + l \sin \alpha} \left\{ \tan \alpha + \frac{(Q+R)a \sin (\alpha + \beta)}{2Pl \cos \alpha \cos \beta} \right\}.$$

The angle β is dependent on the angle α and on the dimensions a, b, c, and d, and it is obtained from the formula,

$$\sin \beta = \frac{d - c + a \sin \alpha}{b}.$$

Generally $c = d$, $a = b$, and $\alpha = \beta$, then,

$$N^2 = \frac{2936 \tan \alpha}{d + l \sin \alpha} \left\{ 1 + \frac{(Q+R)a}{Pl} \right\}.$$

In the porter governor $l=a$.
In the crossed arm governor d is negative.
To find n^2, change R into $-R$, in the above formulæ.
To find N_1^2, change α into α_1, and β into β_1.
To find N_2^2, change α into α_2, and β into β_2.
To find n_1^2, change α into α_1, β into β_1, and R into $-R$.
To find n_2^2, change α into α_2, β into β_2, and R into $-R$.
The diagram $ABDC$ is the speed diagram for the governor. AB = lift of sleeve. $AC=N_1$. $AE=n_1$. $BD=N_2$ $BF=n_2$. $HL=N$. $HK=n$.

The mean normal speed of the governor is $\frac{N+n}{2}$, and this is practically the speed obtained from the formulæ above when R is made $=0$

Steam-engine Fly-wheels.

W = weight of wheel, in tons. R = radius of gyration, in feet.
N = mean speed; N_1 = max. speed; N_2 = min. speed, in r.p.m.
$(N_1-N_2)/N=1/n$ = coefficient of fluctuation of speed.
H = indicated horse-power of engine.
r = ratio of the work to be stored up by fly-wheel, in changing its speed from the minimum to the maximum, to the work done in one revolution.

$$W=\frac{43257Hrn}{R^2N^3}.$$

Approximate Values of r.

For single cylinder condensing engines—

Point of cut-off =	1	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{8}$
r =	·125	·163	·173	·178	·184	·191

For single cylinder non-condensing engines—

Point of cut-off =	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
r =	·125	·16	·186	·209	·232

For two similar engines working on cranks at right angles to each other on the same shaft, the value of r is about one-fourth of its value for one engine.

Approximate Values of n.

Pumps, and shearing and punching machines	20 to 30
Flour-mills	25 ,, 35
Looms, paper-making machines . . .	30 ,, 40
Spinning machinery	50 ,, 100

The diameter of the fly-wheel is usually from three to five times the stroke of the piston.

LOCOMOTIVES.

(By W. J. Reynolds.)

Locomotive design is influenced by the following conditions:—

Nature.—Humidity and dryness affect adhesion of the driving wheels. Impurities in water supply require extra facilities for boiler wash-outs. Lack of water supply calls for extra tank capacity. Type of fuel dictates fire-box design.

Civil Engineering.—British rail gauge is practically standard at 4 ft. 8½ in., but maximum overall width and height govern moving dimensions. Permissible total weight and individual axle load are governed by permanent way and underline bridges. Inclines and their relation to easier grades affect boiler steaming, and radii of curves determine maximum rigid wheel base. Turntables limit the maximum overall length of engine and tender.

Traffic Department.—Account must be taken of load to be hauled, average speed required, and length of run without intermediate stops.

Maximum Availability.—Locomotives must be standardised to work over the maximum number of route miles, and must be capable of a variety of duties to enable them profitably to work home, and they must operate on minimum shed hours and maintenance.

Tractive Force.—The power rating of locomotives, unlike other prime movers (ships, motor vehicles, etc.), is reckoned in terms of tractive effort (T.E.), not horse-power (H.P.).

Assuming the work done in the cylinders and at the draw-bar are equal, then for two, three, and four cylinders respectively—

$$T_D=\frac{D^2SP}{W}, \qquad T_D=\frac{3D^2SP}{2W}, \qquad T_D=\frac{2D^2SP}{W}$$

where T_D = tractive force at draw-bar in lb., neglecting work done in moving engine.
D = diameter of cylinder in inches.
S = stroke of piston in inches.
P = mean effective pressure on pistons in lb. per sq. in.
W = diameter of driving wheels in inches.

Mean effective pressure is usually taken at 85 per cent. boiler pressure at starting, and varies with boiler pressure, steam-chest pressure, percentage of cut-off, and speed of the locomotive. With modern locomotives with long travel valves, large steam passages, and driven with wide-open regulator and early cut-off, the steam-chest pressure and boiler pressure are approximately equal.

Adhesion of Locomotives.—*Adhesion weight* is load on driving wheels (including coupled wheels). *Adhesive force* is the sum of the forces between driving wheels and rails, along the rails, at which slip would begin, and is the maximum possible value of the tractive force. Normally adhesive force is greater than tractive force. *Adhesion* may be stated either as an adhesive force in lb. per ton of load or as the ratio of adhesive force divided by adhesion weight, and this varies from about one-third when rails are clean and sanded to about one-ninth when rails are damp or greasy. When running it is generally assumed to be one-sixth, or at starting, one-fourth. The figures in general use (British practice) for adhesion per ton of load on driving wheels (Molesworth) are as follows:—

Ordinary English weather .	450 lb.	Coef. 0·2
Misty (rails greasy) . .	300 lb.	Coef. 0·13
Frost and snow . . .	200 lb.	Coef. 0·09

Permanent way, underline bridges, etc. determine the number of coupled axles over which the necessary adhesive weight has to be carried.

Let R = weight of rails in lb. per yard.
L = maximum load in tons per axle.
N = number of coupled axles.
T = tractive force in lb.

Then—

$L = \frac{R}{5}$. Maximum load on coupled axles in tons $= LN$.

Maximum adhesive force in tons (Coef. 0·2) $= 0{\cdot}2LN$.

$$T = 2240 \times 0{\cdot}2LN \text{ and } N = \frac{5T}{2240L}.$$

Total weight in excess of the necessary and permissible adhesive weight must rest on carrying axles and wheels.

Weight of Locomotives.—This is a laborious business to calculate by estimating the weight of the various components, but in many instances the weight can be satisfactorily estimated from the known weight of a similar type of locomotive with the necessary adjustments for detail differences. To determine the weight to be carried on each axle, the longitudinal position of the centre of gravity of the total weight carried on the springs must be ascertained. The weight of axles, axle boxes, side rods, wheels, etc. are the dead weights, and the distribution of these can be adjusted.

Resistance of Trains.—*On straight level track* the experiments made by Sir Daniel Gooch were used by D. K. Clark to deduce the following formulæ:—

$$R=\frac{V^2}{240}+6. \qquad R_1=\frac{V^2}{171}+8.$$

V = speed of train in m.p.h.
R = resistance of train (only), lb. per ton.
R_1 = resistance of engine, tender, and train, in lb. per ton.
l = train length in feet.

With modern heavy vehicles the following formulæ are more satisfactory:—

$$R=3{\cdot}5+0{\cdot}1V+0{\cdot}0015V^2. \qquad R=2{\cdot}5+\frac{V^{\frac{5}{3}}}{50{\cdot}8+0{\cdot}0278l}.$$

(*The Engineer*, 10th February 1928.) (Sir J. Aspinall's formula.)

On inclines the resistance due to gravity may be expressed as follows, with + sign signifying ascending and − sign descending:—

$$G=\pm\frac{2240h}{5280}=\pm\frac{14h}{33}$$

where h = rise of incline in feet per mile.
G = resistance in lb. per ton.

On curves.—In American practice it is usual to allow a traction of ½ lb. per ton per degree of curvature to overcome the resistance due to the curve. One degree of curvature corresponds to a radius of 5730 feet—the radius being very nearly inversely proportional to the number of degrees of curvature.

The maximum load in tons that a locomotive can haul may be expressed as—

$$L=\frac{T_D}{R_{RC}+R_G}$$

where T_D = draw-bar tractive effort in lb.
R_{RC} = rolling resistance on straight level line at constant speed in lb. per ton.
R_G = resistance of stock due to gradient in lb. per ton.

Streamlining.—The air resistance of a train is proportional to the square of the train speed and to its "effective surface," which represents the front of the first vehicle as well as all the parts of the train which project beyond.

$$P=0{\cdot}003V^2$$

where P = air resistance in lb. per sq. ft.
V = speed in miles per hour.

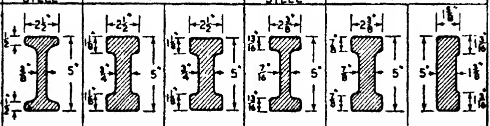
The demand for high-speed trains led to the introduction of streamlined locomotives, and Sir Nigel Gresley stated that with his streamlined Pacifics, hauling the superspeed trains scheduled at average speeds approximating to 70 m.p.h. continuously over 200 to 400 miles, a saving of about 10 per cent. in horse-power and 4 lb. of coal per mile was effected. The question is also bound up with the necessity of lifting smoke and exhaust gases clear of the driver's cab when locomotives are working with an early cut-off. Streamlining effected this satisfactorily.

Boiler Power.—The locomotive boiler must raise quickly and efficiently an adequate volume of steam under all conditions of working, withstand a pressure in excess of that at which it nominally works, and be so designed to transfer the maximum amount of the heat generated in the fire-box to the water and steam.

Boiler power is the ultimate maximum power of any given locomotive, and the tractive effort formula is based on the assumption that the boiler power will make the rated tractive effort effective, and the amount and quality of fuel burned over a given distance is the limitation of that power. The rating is stated as pounds of fuel per mile. There is no high economy standard for this rating as in other forms of steam engine practice, as locomotives work under such varying conditions. From experimental tests made from time to time it may be assumed that 35 to 50 lb. of good coal per mile is a satisfactory average for a modern locomotive using superheated steam, correctly driven, and with a well-designed front end (*i.e.* long travel valves and large free steam passages).

The amount of fuel that can be dealt with is rated in terms of pounds of fuel per hour per sq. ft. of grate, and the amount of heat dealt with is rated in terms of pounds of water evaporated per hour per sq. ft. of total heating surface.

Balancing.—The parts of a locomotive which move relative to the frame fall into two classes: those which revolve and those which reciprocate. The revolving parts can be fully balanced by masses in the wheels, and the degree of balance remains constant at all angular positions during rotation of the wheels, but reciprocating parts can only act in the plane of reciprocation, which for this purpose can be assumed to be horizontal. Neglecting obliquity of the connecting-rod, the reciprocating parts can be fully balanced by revolving balance weights in the wheels. These revolving masses, however, set up hammer blow on the track as the reciprocating forces have no component in the vertical plane, but unbalanced reciprocating masses tend to shake the locomotive, so the whole matter of counterbalancing is a matter of compromise. This position can be met by reducing to a minimum the proportion of reciprocating parts balanced, distributing the balance weights evenly amongst the coupled wheels, and reducing the weight of the reciprocating parts to the minimum by the use of high tensile steels for connecting-rods.

RODS	CONNECTING RODS			COUPLING RODS		
	NICKEL CHROME STEEL	ORDINARY CARBON STEEL		NICKEL CHROME STEEL	ORDINARY CARBON STEEL	
SECTIONS						
	3 CYLDS.	2 CYLDS	2 CYLDS.	3 CYLDS	2 CYLDS.	2 CYLDS.
LENGTH OF ROD	8′–1″	8′–1″	10′–0″	8′–9″	9–0″	6′–10″
WEIGHT PER FT. LBS	16·4	27·7	27·1	17·6	25·3	25·4

British practice is to balance the whole of the revolving and a part of the reciprocating masses. The even torque of 3-cylinder locomotives enables a great reduction to be made in the balance weights, and in the case of the Southern Railway Pacifics the reciprocating parts are entirely unbalanced without ill-effects.

Boiler and Fire-box Design.—The size of the grate is governed by the necessity of keeping the firing at a rate at which the air stream passing through the fuel resting on the fire-bars can ensure complete combustion without unburned fuel being drawn through the tubes and up the chimney. The British computation averages about 60 lb. coal per sq. ft. grate area per hour. About 40 to 45 per cent. of the total heat of combustion is retained by the fire-box heating surface, and the remainder passes through the tubes, of which 15 per cent. escapes into the smoke-box. The ratio of evaporative heating surface to grate area varies from 53 to 67.

The tubes must be so proportioned and the number provided to enable the greatest amount of gas which can be produced by the given grate area to be passed without throttling.

All-steel fire-boxes are universal in America, but British practice is to use copper for the inner fire-box. Higher working pressures and consequent increase in fire-box temperature affect copper, which loses its elasticity and sets up leakage at the tube connections. Steel tubes in copper fire-box plates set up electrolytic action. In the Southern Pacific, using a 280 lb. per sq. in. working pressure, an all-steel fire-box has been designed with a saving in weight of 1½ tons.

Calculations for Heating Surface and Grate Area.—(*a*) *Tubes.*—The *outside* area of both flues and tubes to be taken and calculated on uniform diameters. Length to be reckoned between tube plates.

(*b*) *Superheater Elements.*—The *inside* area to be taken and the length both to and from the smoke-box end of the flue.

(*c*) *Fire-box.*—The outside area of the inner box plates to be taken. Allowance must be made for fire-hole ring, foundation ring, and tube openings. No allowance is to be made for stays.

(*d*) *Smoke-box Tube Plate.*—Not to be included in the area.

(*e*) *Grate Area.*—This is measured between the inner fire-box sides at top face level of the grate.

Stays.—The flat sides of the inner and outer fire-box require to be stayed to prevent distortion. This area in a modern locomotive is about 100 to 120 sq. ft. The pressure load may be as high as 1200 tons, while the crown may have to withstand a downward pressure of as much as 820 tons when the boiler pressure is 250 lb. per sq. in. The pitch of the side fire-box stays varies between 3½ in. and 4 in., that is about 9 stays per sq. ft., and they number anything from 1000 to 2000 in the latest express engines. Direct staying is preferable to the girder type, which complicates washing out. Stays are made of copper, steel, or monel metal, and the maximum stress in any material should not exceed 6000 lb. per sq. in. Experiment has shown that, with copper stays screwed and riveted, the plates show the first sign of bulging at an approximate pressure of 515 lb. per sq. in. with complete failure at 1600 lb. per sq. in. The barrel is stayed with a few longitudinal stays between smoke-box and fire-box front plate.

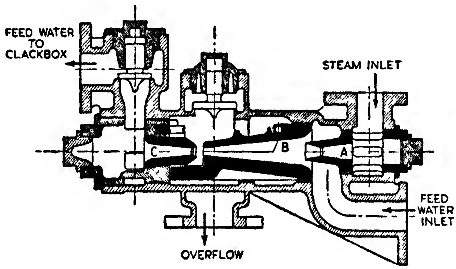
Injectors.—To overcome the pressure within the boiler and feed water into the boiler, the water must acquire kinetic energy and pressure energy.

A locomotive boiler will evaporate 3 to 7 tons of water per hour according to the size, and this must be replaced by the injectors.

Velocity is obtained by a steam supply admitted to a cone, the diameter of which is progressively reduced so that the steam obtains a high velocity. Having attained a high velocity, it is discharged into a conical water space which surrounds the steam nozzle and is condensed. The steam by its velocity has already imparted sufficient impetus to the water and condensed steam to overcome the boiler pressure. The condensation of the steam induces a vacuum sufficient to lift the water supply a reasonable height. The steam attains a velocity of about 1700 ft. per sec. (1160 m.p.h.) according to boiler pressure, and carries forward about 12 times its own weight of water at 131 ft. per sec. (90 m.p.h.), which is sufficient to carry the water into the boiler through the clack valve.

Locomotives are fitted with two types of injector: the live steam type and the exhaust steam type. The former is worked entirely by steam from the boiler, while the latter depends on a supply of exhaust steam taken from the blast pipe together with a small supply of live steam. Normally, when the locomotive is running, the exhaust injector is always on. The live steam combination injector for hot or cold water, at boiler pressure 180 to 225 lb. per sq. in., will deliver 2350 to 2200 gallons per hour (steam pipes $1\frac{1}{2}$ in. diameter and based on an initial feed-water temperature of 60° F.).

The live steam injector augments the initial feed-water temperature by 90° to 100° F., and the average boiler steam consumption is about 10 per cent. of the total boiler evaporation,



The Live Steam Injector.
A = Steam Delivery Nozzle.
B = Combining Cone.
C = Delivery Cone.

but the thermal efficiency of the injector nearly approaches unity. The exhaust steam injector requires a live steam augment of about $2\frac{1}{2}$ per cent. of the total boiler evaporation when delivering against boiler pressures in excess of 150 lb. per sq. in., but in effect it is the simplest form of feed-water heater. The Davies & Metcalfe Type H exhaust injector, which is controlled by the operation of a single external valve, will feed water up to 230° F., but the normal temperature ranges from 190° to 200° F. Water economy due to return of condensate is 10 to 12 per cent., with a similar percentage of fuel economy. To set against these advantages the smoke-box vacuum is reduced by about 5 per cent. The most convenient location for injectors is beneath the footplate on either side of the engine, operated from the cab.

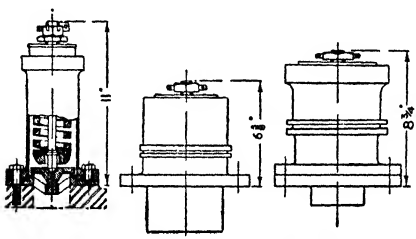
Safety-valves.—Modern practice is to fit " Pop " safety-valves exclusively to locomotives, as these give a large lift rapidly as soon as they open. The discharge area is directly proportional to the diameter of the valve and is taken as πdl, where d = valve diameter and l = lift. In locomotive design d may be varied, but l is regarded as constant. The area of the valve should be directly proportional to the grate area and, owing to a decrease in specific volume when the velocity of the steam increases as its pressure rises, the volume of discharge in a given time varies inversely as the pressure.

The makers of the "Ross" safety-valve recommend the following dimensions for their valves:—

Grate Area. Sq. Ft.	Boiler Pressure. Lb./Sq. In.	No. and Diameter of Valves. In.	Grate Area. Sq. Ft.	Boiler Pressure. Lb./Sq. In.	No. and Diameter of Valves. In.
27	175	$2 \times 2\frac{1}{2}$	42	180	$3 \times 2\frac{1}{2}$
30	170	2×3	44	200	$2 \times 3\frac{1}{2}$
30	200	$2 \times 2\frac{1}{2}$	46	200	$2 \times 3\frac{1}{2}$
35	200	2×3	50	180	3×3
38	180	$2 \times 3\frac{1}{2}$	50	200	2×4
41	175	2×4	56	200	3×3

On the foregoing data $A = \frac{xG}{P}$,

where A = area of "Pop" valves in sq. in.
G = grate area in sq. ft.
P = pressure in lb. per sq. in.
and the average value of x is 81.

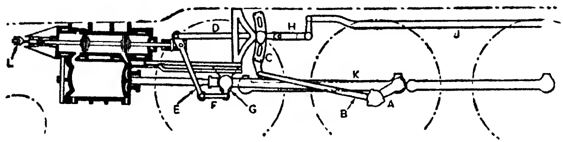


2½" diam. 3" diam. 4" diam.

The Ross valves are made to a standard pattern with an average height of about 11 in., but special designs are available from 6¼ in. to 8¾ in. for use when there are severe height restrictions imposed by large modern boilers.

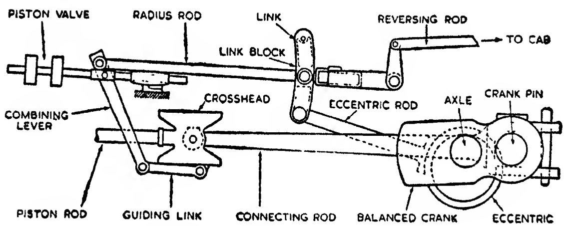
Valve Gears.—Practically all modern locomotives are now fitted with radial gears in place of the former link motions. With radial gears the lead is constant and must be determined in relation to the normal cut-off. Link motions have practically no lead in full gear, but the lead is increased as the engine is notched up.

Radial gears give an appreciable saving in weight, improved steam distribution, and the elimination of eccentrics which have great inertia and excessive friction losses. For inside cylinders the link moves through a small angle, and the link trunnion is a fixed fulcrum, the movement of the link being derived from a single eccentric. The drive is more direct than with link motion, but the trunnions must be of more robust construction to stand up to the total reaction of the valve drive. The phase angle between the main and the return cranks approximates to 90°. Walschaert's valve gear is mainly used in Britain, but is subject to

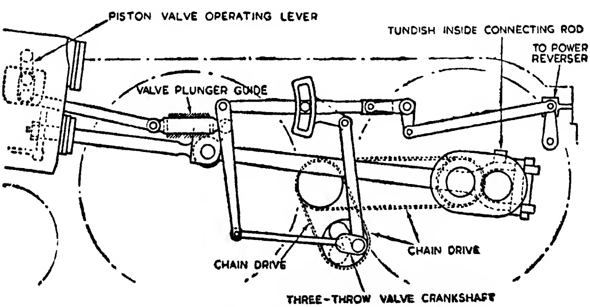


Walschaert Valve Gear with Return Crank.

A = Return Crank. B — Eccentric Rod. C — Radius Link.
D = Radius Rod. E — Combination Lever. F — Union Link.
G = Crosshead Arm. H = Reversing Arm. J = Reversing Rod.
K = Connecting-rod. L = Link for Inside Cylinder Gear.



Walschaert Valve Gear with Eccentric.



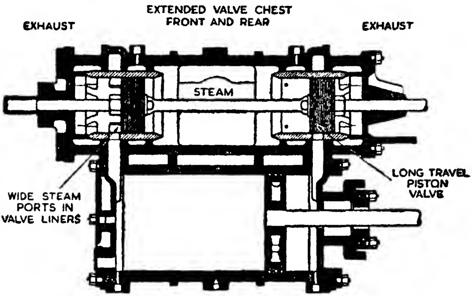
Bulleid Valve Gear with Chain Drive.

variation in design to suit the requirements of different engineers. Churchward (G.W.R.) introduced a single eccentric in place of the return crank—the outside valve being driven through a rocking shaft. Gresley (L.N.E.R.) incorporated his conjugate gear for the inside cylinder of his numerous 3-cylinder engines, which is driven conjointly by the two sets of Walschaert's valve gear through two unequal armed rocking levers. The longer 2 to 1 lever has a fixed fulcrum, and the shorter equal-armed lever has a moving fulcrum.

With Walschaert's valve gear the motion is obtained by two independent movements. From the crosshead, a movement equal to twice the steam lap plus the lead on either side of the central position of the valve is obtained. The remainder of the travel which provides the port opening on each end greater than the lead is derived from a return crank on a crank-pin, or with inside motion, from an eccentric attached to the driving axle.

An entirely new type of valve gear has been designed by Mr. O. V. Bulleid for the Southern Pacifics. This motion is entirely enclosed, and requires no attention between plant overhauls. Each of the three valves is operated by independent motion, and these three sets of gears are operated by a three-throw secondary crank which oscillates its quadrant link by a vertical connecting-rod pinned to an arm extending backward from the link. Simultaneously it reciprocates the foot of the combination lever by a horizontal link pinned to the big-end of the vertical connecting-rod. The upper end of the combination lever is operated by the quadrant link. The valve rods convey the combined motion through a plunger working in a guide to the operating rocker shaft. To revolve the three-throw crank-shaft in phase with the crank-axle a silent rocker chain drive is used. The whole of the motion is enclosed and is lubricated by flood lubrication by two reversible gear pumps, chain-driven from the crank-shaft.

Valves.—The slide-valve has now been almost entirely superseded by the piston type. High superheat and pressures made a new type of valve essential, which could be more efficiently lubricated and at the same time relieved of the high degree of unbalanced pressure on the steam-chest valve face. In addition, the "D" slide-valve required a large amount of power to drive. The piston-valve consists of two pistons attached to one spindle with a distance-piece between, securely held in position by a castellated nut and split-pin. The valve slides in cylindrical liners, with holes cut around the circumference to connect them to the steam ports. In modern valves four Ramsbottom rings are fitted to each of the pistons, which reduce the bearing surfaces and wear. Inside admission valves are usually fitted now. A new method of operating has recently been introduced consisting of driving each pair of piston-valves by a rocker in the exhaust cavity. No valve spindles are used, enabling the glands to be suppressed. The rocker is placed across the cylinders and, when uncoupled, the arm in the exhaust cavity drops clear, allowing the piston-valves to be withdrawn.



A Typical Modern Cylinder and Piston Valve.

The chief point affecting steam flow is length of travel of the piston-valves, which in modern British practice varies from 6 in. to 7 in. Wide steam ports require a long valve travel, and the diameter of the valve liners regulates the number of openings that can be cut in them. The valve chest is extended beyond the ends of the cylinder bore, giving direct passages between valve ports and cylinder. The streamlining of internal surfaces exposed to steam eliminates cavities and ledges, which collect carbon deposits.

Cylinders.—Cylinder castings vary in size and shape to suit the particular design. The materials used are pig and selected irons and steel scrap mixed in the cupola to produce a hard close-grained metal. Cylinders are usually cast separately and bored out to the required diameter, and also bored for piston-valves. Steam and exhaust ports are accurately machined, stud and bolt holes drilled and tapped, studs inserted and covers fitted in position. The piston is turned slightly smaller in diameter than the cylinder bore. An efficient piston should be of sufficient strength to withstand the considerable pressure to which it is subjected, and to prevent leakage of steam from one end of the cylinder to the other, piston rings are fitted. British practice is to fit Ramsbottom piston rings of cast-iron with an outside diameter $\frac{1}{8}$ in. to $\frac{3}{8}$ in. larger than the cylinder bore, a portion being cut away to enable the ends to be pressed together and sprung into the cylinder. Two or more grooves are turned in the piston block into which the rings fit. The tendency of the rings to spring outwards is sufficient to resist the steam pressure. To prevent leakage where the rings have been severed, the gaps are staggered and the rings are secured by pegs inserted in the grooves to prevent them turning.

Pistons are made of cast-iron or steel. A clearance space of $\frac{3}{8}$ in. to $\frac{7}{16}$ in. between the piston head and the inside of the cylinder cover when the limit of the stroke has been reached, compensates for the alteration in position of the rods due to wear, and allows for water due to condensation and for a certain amount of steam which is compressed. This compression has a cushioning effect, bringing the piston gradually to a standstill before the commencement of the next stroke. A strong boss is cast in the centre of the piston, which is secured by nut and pin riveted at both ends to prevent the nut from working back. The rod is maintained steam-tight by gland packings in a stuffing-box. Fibrous packings are now giving place to metallic gland packings, which are of two main types, viz. those which are maintained steam-tight by the end-on pressure exerted by the gland nuts, and those by the action of springs fitted inside the stuffing-boxes. The modern packing consists of three rings of Babbitt metal.

Superheating.—Practically all modern British locomotives are now fitted with superheater equipment. The temperature of steam in contact with water cannot be raised without an increase of pressure, but it may be superheated without a rise in pressure provided it can expand as heat is added. With the locomotive the regulator must be opened and the engine running before the steam can be superheated, because the demand for steam in the cylinder enables steam to expand in the superheater. Saturated boiler steam at 200 lb. per sq. in. has 124 times the volume of an equal weight of water at the same temperature, and is subject to a loss of nearly $\frac{1}{3}$ of its volume, due to condensation between the boiler and the cylinder face. To overcome such serious loss and still further to increase the work available in each 1 lb. of steam, the boiler steam is reheated after it leaves the boiler and before it enters the cylinder steam chest. The effect of superheating the steam as it passes through the elements at practically boiler pressure is to increase its volume, and this volume is approximately proportional to the value of the absolute temperature (F.° + 460°).

Therefore the volume of any weight of superheated steam at, say, 600° F. compared with the volume at, say, 400° F. and at the same pressure is $\frac{600+460}{400+460}$, or approximately $1\frac{1}{4}$ times as great. Thus, comparing the temperature and volume of 1 lb. of saturated steam at 200 lb. per sq. in. with the corresponding values when it is superheated to 700° F. at the same pressure;

	Temp. °F.	Volume. Cu. Ft.	Heat. B.Th.U.
Saturated . . .	382	2·288	1200
Superheated by 318° F.	700	3·376	1373

The additional volume of the superheated steam is nearly 48 per cent., but it requires an increase of 173 B.Th.U., or nearly 15 per cent., to heat it. Most of the added heat is given up by the steam to heat the walls of the cylinder during the admission period, raising the temperature of these walls above that at which boiler-pressure steam will commence to condense.

Assuming 650°–700° F. superheat temperature, economies of 15–25 per cent. fuel and 25–35 per cent. water are achieved. The range of operation of locomotives is increased, and under-boilered engines can have their sphere of usefulness extended.

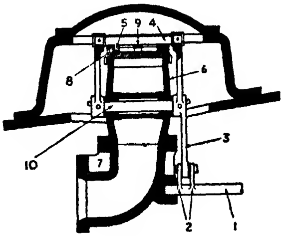
Superheating is regarded as low or high as follows:—

Low . . 10–50 F. of superheat.
High . . above 200° F. of superheat.

Below 200° F. of superheat slide-valves can be retained, but above this value it is necessary to use either balanced slide-valves or piston-valves. The type of lubricant and method of feeding must receive special attention.

Proportions for superheaters are governed by the length and diameter of the boiler barrel, but the cross-section area through the superheater elements must be greater than that of the main steam pipe to allow for volumetric increase and also to allow for flow friction. The ratio varies from 1 : 1 to 5 : 1 of the main steam pipe. The superheater reduces the evaporative heating surface of the boiler from 25 to 30 per cent. The flues occupy 30 to 35 per cent. of the total tube heating surface. The superheater elements should terminate about 1 ft. from the fire-box tube plate. Elements are cold-drawn weldless steel tubes of an internal diameter 1 to 1½ in., with a thickness of 9 S.W.G. The Superheater Co. specification for tensile strength is 20–26 tons per sq. in. Headers are made of cast-iron ¾ in. thick except at the tube face, which is increased to 1 in. thick.

Regulators.—It is essential to use the highest possible steam pressure on the pistons. The position of the regulator valve on the ports of the regulator head determines how far this can be achieved. The valve is governed by the setting and movement of the driver's handle in the sector. The "full open" area through the regulator valve should be equal to the cross-section area of the main steam pipe. The area of a 6½-in. diameter main steam pipe is 33 sq. in.



(1) Regulator Rod.
(2) Short Crank Arm.
(3 and 4) Actuating Levers.
(5) Starting Valve
(6) Regulator Head (Top).
(7) Regulator Head (Bottom).
(8) Main Valve.
(9) Spring.
(10) Fulcrum Pin.

Position of Regulator Handle.	Area of Opening. Sq. In.	Percentage of Opening.
¼ open (from shut)	2 66	8
½ open („ „)	13·84	42
¾ open („ „)	24·40	74
FULL OPEN	33·00	100
¼ shut (from open)	30·05	91
½ shut („ „)	21·93	66
¾ shut („ „)	10 46	32

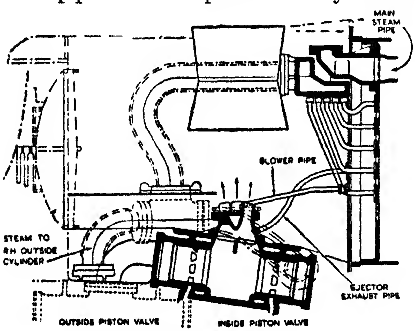
The tabulated values of the area of the regulator valve opening, expressed in sq. in. and as percentages of the steam-pipe area, for various positions of the regulator handle, are typical of a modern design of regulator valve with 6½-in. diameter steam pipe.

Smoke-box.—The smoke-box is constructed of steel wrapper plates ¼ in. to ⅜ in. thick, with an angle iron stiffener 3 in. × 2½ in. × ½ in. section at the front tube-plate and a smoke-box door ⅝ in. thick (dished approximately 4 in.). In modern practice its length varies from 5 ft. 6 in. to 8 ft., and is usually of drumhead formation and not the horseshoe type as in the past. Its cubic capacity should be such that it can maintain a degree of vacuum between exhaust beats and modify their intermittent character and thereby maintain an even draught on the fire. With 3 cylinders the degree of vacuum is lower, but the more rapid beats (6 per rev.) compensate for this. The smoke-box is a reservoir for the combustion gases before expulsion up the chimney, and a receptacle for ash. It houses the blast-pipe with blower ring around the top, petticoat and steam pipes, the superheater header and in some designs the regulator. Its structural function is to resist the lateral movement of the boiler when the locomotive is running.

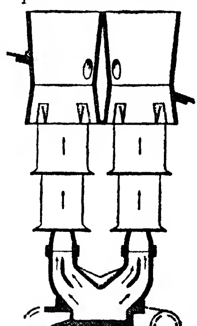
Blast-pipes and Chimneys.—The blast-pipe, taken in conjunction with the chimney, has a most important bearing on the efficiency of the locomotive. High fire-box temperature with a moderate draught through the fire is the ideal to be aimed at in design and dimensions. Modern practice is to fit a petticoat and a low position for the blast nozzle. The area of the opening of the blast-pipe should be of a dimension to avoid back pressure in the cylinder when the exhaust is heavy, but must retain enough energy to carry away the gases when the locomotive is working under easy steam. A free passage to the orifice to reduce eddying is essential. The minimum height of the true cone inside the blast-pipe is 12 in., and branches feeding should be below and directed towards the orifice. To improve the blast various types of blast-pipe tops are in extensive use. The G.W.R. jumper top provides an extra exit for the exhaust when the engine is working heavily.

The multiple jet divides the exhaust into a number of jets; in British practice 5 jets inclined outwards at 1 in 12 are used with $2\frac{5}{8}$-in. diameter nozzles, an increase in area over the normal $5\frac{1}{2}$-in. orifice of 13 per cent. This achieves a reduction of back pressure in the cylinder but entails the fitting of a larger diameter chimney and petticoat 2 ft. 1 in. diameter at choke. A higher smoke-box vacuum, however, is maintained, notwithstanding the lower velocity of the mingled gases of exhaust owing to the larger surface presented by the multiple jets.

The "Kylchap" has much the same effect as the multiple jet, namely breaking up of the exhaust, but consists of branching the blast-pipe with two separate chimneys and two petticoats for each



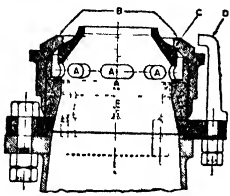
General Arrangement.
Smoke-box of Southern Railway Pacific.



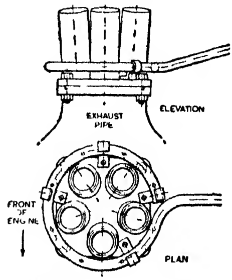
"Kylchap"
Double Blast-pipe
(L.N.E.R.).

of the branches, one above the other and covering the orifice.

The blast-pipe must be co-axial with the chimney, and the orifice should be about 8 in. below the boiler centre line, and the



Jumper Top Blast pipe (G W R.).
A = Extra Exhaust Holes.
B = Ring which lifts.
C = Seat.
D – Stops.



Multiple Jet Blast-pipe (Southern Railway).

area should be such that the mean velocity of the blast will not exceed 1000 ft./sec. The area opening of the chimney should be 8·6 times the area of the blast orifice, and it should have its lowest extremity at a distance from the orifice of about eight orifice diameters.

Lubrication.—Cylinder and valve lubrication may be effected either by sight-feed lubricators or by the pump type of mechanical lubricator. With the latter an atomiser is fitted to ensure that the oil is broken up into small globules and carried forward with the steam as oil haze, and that a constant film of oil is deposited on all working surfaces of the valves and pistons. The sight-feed lubricator effects lubrication by feeding the cylinder oil into the steam in the regulator-box and thence to the cylinder; the oil is displaced through the sight-feed glasses by steam from the boiler condensing, forming a vacuum. The feed passes through a combining valve, where the oil is mixed with steam from the boiler and passes to the smoke-box steam pipes and thence to the cylinders. The mixing valve is operated by moving the regulator-handle, and is so arranged that oil can also be fed to the cylinder when the regulator-valve is shut. Oil reservoirs for big-ends and coupling-rods must be large enough for the longest non-stop runs. (For a mileage of 300 it should be not less than ½ pint for big-ends and ⅓ pint for coupling-rods.) Wire trimmings are more economical in oil than worsted trimmings and pads. The wires should be of hard steel (120 tons per sq. in. tensile, 13 S.W.G. for big-ends and 12 S.W.G. for coupling-rods), and should extend within ¼ in. of the journals. The nipples should be of similar steel and cupped to trap the oil.

Coupled axle-boxes are most satisfactorily lubricated by mechanical lubricators, as oil can be introduced at the point of maximum pressure, preparation of the engine is simplified, the feed is proportional to the speed and is capable of easy adjustment should the box heat. When running, the oil pressure is about 300 lb. per sq. in. Hornfaces absorb about 20 per cent. of the total oil fed to the whole box, the consumption being about 2 oz. per box.

If gravity feed is used, the oil-boxes should be located on the back-plate of the fire-box to maintain the viscosity of the oil and make them accessible for replenishment and examination. Each box should have flexible connections.

Wheels and Tyres.—Modern locomotive wheel centres are now exclusively made of cast-steel, wrought-iron being obsolete. The spokes number 3 per foot of the diameter at the rim. The thickness of the metal of the boss is approximately half the diameter of the axle, and the maximum diameter of the boss is 2 to 2¼ times the diameter of the axle. The rim of the wheel centre is rectangular in section, rounded on the inside, about 2 in. thick and 3¾ in. wide, and is turned to gauge for reception of the tyre. Tyres are 3 in. thick when new, and are turned slightly smaller in diameter on the inside than the wheel centres to allow for shrinking on after heating, the allowance varying from 1/1200 to 1/750 times the diameter. Screwed studs in alternate spaces are now obsolete, and the more usual practice is to leave a flange suitably turned on the inside edge of the tyre and secured to the wheel rim by ¾-in. rivets opposite the centre of each spoke. A recent innovation in wheel centres is known as the B.F.B. disc type, made of cast steel, the web being corrugated with bridge pieces, supporting the tyre opposite each corrugation. The weight of these wheels is 10 per cent. less than the spoked type.

A new type of tyre fastening has been introduced in conjunction with B.F.B. whereby the contact between rim and tyre is much greater. Fixing is effected by heating to 450° F. to expand it sufficiently for the inside lip to pass over the driving-wheel lip. The tyre material used in the Southern Pacifics is nickel-molybdenum-chromium steel, tempered at 600° F.

Springs.—Both laminated and helical springs are used on locomotives. Laminated springs are generally fitted to driving and coupled wheels, the number of plates varying with the conditions demanded. For a modern express locomotive, 15 to 20 plates about 5 in. wide by ½ in. thick is a good average. Helical springs of about 11 in. free length and 6 in. diameter of Timmis section may be fitted to driving wheels. Bogie trucks are fitted with helical springs 10 in. to 12 in. free length, 5 in. to 5¼ in. diameter, Timmis section. Where trailing wheels are fitted, either laminated or helical springs can be used. Independent suspension is usual.

Examples of inverted laminated springs have been included in some modern designs, notably the L.M.S. Pacifics designed by Sir Wm. Stanier.

Brakes.—The energy stored up in a train travelling at 60 m.p.h. (88 ft./sec.), assuming the weight of engine and train to be 300 tons, is

$$\frac{Wv^2}{2g}=\frac{300\times 88^2}{2\times 32{\cdot}2}=36074 \text{ ft.-tons.}$$

The average resistance F to be provided by the brakes to bring the train to rest in, say, 1500 ft., neglecting wind resistance, etc., is,

$$F=\frac{36074}{1500}=24 \text{ tons.}$$

Two main types of brake are in use on locomotives and trains, viz. the Vacuum and the Westinghouse. Both are automatic, that is, their normal state at atmospheric pressure is " brakes on." Modern British practice is to use the Vacuum, but abroad the Westinghouse finds considerable favour, especially in America.

A steam brake is fitted to a number of British locomotives, both in combination with the Vacuum train equipment and separately for engines employed on unbraked goods service.

The principle of the Vacuum automatic brake is the maintenance of a continuous state of exhaustion throughout the system to keep the " brakes off," and any admission of air, intentionally or by accident, results in the application of the brakes. For the purposes of this brake a vacuum of 21 in. for passenger trains and 18 in. for goods trains is used, with minimum values of 18 in. and 16 in. respectively. The system applied to locomotives and trains consists of a continuous pipe throughout the train in conjunction with brake cylinders and vacuum chambers on each vehicle to be braked. The brake cylinder is provided with a piston kept in a state of equilibrium by the exhaustion of space above and below it. Air admitted to the continuous pipe is allowed to exert its pressure on the underside of the piston, but not above, and by this means the piston is raised 3½ to 4 in. and the brakes applied through suitable rigging and blocks.

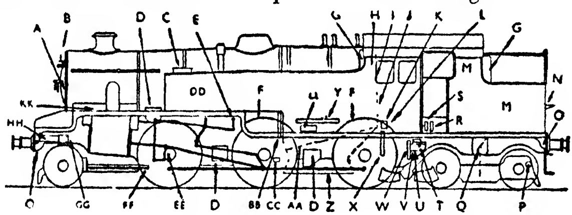
To create the vacuum an ejector is fitted in the locomotive cab, using dry live steam in the form of a jet and exhausting into the smoke-box.

The power employed by the Westinghouse air brake is compressed air which is pumped by a steam pump located on the engine. Air is forced by the pump into a main reservoir, passing through a reducing valve in the driver's brake valve into the train pipe to the triple-valve reservoir on each vehicle. The application of the brake allows air to enter the train pipe, causing the triple valves to move and allowing the compressed air in the reservoirs to pass through these valves into the brake cylinder, forcing the blocks on to the wheels. When the brake is released the reverse action takes place, air flowing from the main reservoir into the train pipe and forcing the triple valve to its former position. This allows air in the brake cylinder to escape through the triple valve into the atmosphere.

The triple valve is so called because it has three functions: (*a*) to charge the auxiliary reservoir, (*b*) to apply the brake, (*c*) to release the brake.

Welding.—Welding was until recently mainly used in repair work, but has now become a standard practice in new construction. An advantage with fabricated units is a saving in weight, and Bulleid has introduced an all-steel welded fire-box in his Pacific design. Fabricated stretchers are now replacing cast steel.

The diagram of a standard L.M.S. 2–6–4 tank engine indicates the extent to which fabricated components are now being introduced.



A = Smoke-box Door, Bar, and Hinged Brackets.
B = Front Lamp Bracket.
C = Filling Hole and Manhole Cover.
D = Sandboxes.
E = Reversing Shaft.
F = Driving and Trailing Splashers.
G = Tank and Air Vents.
H = Pads on Boiler for Studs.
I = Sandbox for Sand Gun.
J = Firedoor Frame.
K = Ashpan Handles and Support.
L = Fire-box Steadying Bracket and Support.
M = Cab and Bunker.
N = Footstep.
O = Frame Gussets.
P = Bogie Frame-end Stay.
Q = Frame Stretcher at Bogie.
R = Frame Stretcher in front of Bunker.
S = Tank Support.
T = Pick-up Shaft.
U = Handbrake Lever Bracket.
V = Water Pick-up Shaft Support.
W = Breeches Pipe Support.
X = Frame Stretcher behind Fire-box.
Y = Expansion Angle.
Z = Ashpan.
AA = Foundation Ring.
BB = Frame Stretcher in front of Fire-box.
CC = Frame Stretcher to Spring Link Brackets.
DD = Side Tanks.
EE = Crosshead Oil-box.
FF = Radial Arm.
GG = Frame Stretcher at Pony Truck Centre.
HH = Front Drag Box to Pony Truck Centre.
KK = Smoke-box Saddle and Frame Stretcher with Exhaust Branches.

INTERNAL COMBUSTION ENGINES.

(By W. Steeds, B.Sc., and R. F. Pattenden, B.Sc.)

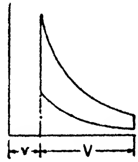
Useful Data.—1 cu. cm. = 0·061 cu. inch. 1 litre = 1·761 pints. 1 h.p.-hour = 1414 C.H.U. = 2545 B.Th.U. 1 C.H.U. = 1400 ft.-lbs. 1 B.Th.U. = 778 ft.-lbs.

An internal combustion (I.C.) engine is one in which ignition, and more or less complete combustion, of the fuel occurs inside the working cylinder. Such engines may be divided into two main groups:

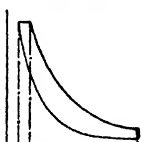
1. Spark-ignition engines.
2. Compression-ignition (C.I.) engines.

In the former, ignition of the fuel is brought about by means of an electric spark, while in the latter, which are commonly called *Diesel* engines, ignition is obtained by raising the temperature of the cylinder contents by compressing them. In engines of the *Hot Bulb* type ignition is assisted by maintaining the hot bulb, or some portion of the cylinder head or piston, at a relatively high temperature.

Thermodynamic Cycles.—Thermodynamically there are several cycles on which, theoretically, I.C. engines may be made to work, the differences between them being in the methods used for adding and abstracting heat from the working fluid, but only two cycles are of practical importance. In the *constant volume cycle* heat is supposed to be added or abstracted while the volume of the working fluid remains constant and the ideal indicator diagram is as shown at (*a*). In what is commonly called the *Diesel cycle* heat is supposed to be added at constant pressure and abstracted at constant volume and the indicator diagram is as at (*b*). In both cycles the compression and expansion curves are adiabatics.



(*a*)



(*b*)

Working Cycles.—Two principal working cycles are used, namely—

(1) The four-stroke cycle.
(2) The two-stroke cycle.

The Four-stroke Cycle.—This is completed in four strokes of the piston as follows. First (outward) stroke, air or air-fuel mixture is drawn into the cylinder through the inlet valve or port. Second (inward) stroke, the charge is compressed into the clearance space in the head of the cylinder. Third (outward) stroke, heat having been added to the charge either by injecting fuel into it or by igniting it, and its temperature and pressure having been thereby raised, it expands and forces the piston outwards; this is the power stroke of the cycle. Fourth (inward) stroke, the products of combustion are expelled from the cylinder by the inward motion of the piston. The four strokes are usually called respectively the induction or suction, compression, firing or power, and exhaust strokes.

The Two-stroke Cycle.—This is used in two ways, namely—

1. With crankcase scavenging.
2. With pump or blower scavenging.

Considering the former, on the outward stroke of the piston the charge in the cylinder, having been ignited when the piston was nearing the inner dead centre, expands and does work on the piston, simultaneously a new charge of either air or air-fuel mixture (which had been drawn into the crankcase by the previous inward stroke of the piston) is compressed in the crankcase, its pressure being raised a few pounds per square inch above atmospheric. Towards the end of the outward stroke of the piston the exhaust port is uncovered, or the exhaust valve is opened, and the products of combustion begin to escape from the cylinder. Almost immediately the transfer port is uncovered, or the transfer valve is opened, and the new charge is transferred from the crankcase to the cylinder and completes the expulsion of the products of combustion. The inward stroke of the piston ensues, the transfer and exhaust ports or valves are closed, the new charge is compressed into the clearance space of the cylinder, and a fresh charge is drawn into the crankcase and the cycle is repeated.

When pump or blower scavenging is used the cycle is essentially the same as the above, but the charge instead of being compressed in the crankcase is compressed in a separate pump or blower. The chief advantages of this method are that a better volumetric efficiency is obtained, and hence a greater power for a given cylinder size, and that certain lubrication difficulties are obviated.

Scavenging and aspiration of the new charge into the cylinder are sometimes obtained by using the wave effects set up in the exhaust pipe on the opening of the exhaust valve or port.

Compression Ratio (r).—

$$r=\frac{v+V}{v},$$

where v = the clearance volume (the volume enclosed in the cylinder when the piston is at T.D.C.),

and V = swept volume of cylinder $=0{\cdot}7854d^2l$ (d = cyl. diameter, l = piston stroke).

Hence $$v=\frac{V}{r-1}.$$

Volumetric Efficiency (η_v).—

$$\eta_v = \frac{\text{Volume of air drawn into the cylinder, reduced to N.T.P.}}{\text{Swept volume of cylinder}}.$$

It varies from nearly 1·0 at low speeds down to 0·7 at high speeds. Occasionally, due to wave effects in the exhaust pipe, it may slightly exceed unity even with natural aspiration.

Power Rating of Engines.—Definitions and Symbols.—

I.H.P. = Indicated horse-power = B.H.P./η_m.

B.H.P. = Brake horse-power = $\eta_m \times$ I.H.P.

η_m = Mechanical efficiency = B.H.P./I.H.P.

p = I.M.E.P. = Indicated mean effective pressure = B.M.E.P./η_m.

B.M.E.P. = Brake mean effective pressure = $\eta_m \times$ I.M.E.P.

T = Mean engine torque in lb.-ft.

N = Revolutions per minute.

n = Number of cylinders.

d = Cylinder diameter in inches.

l = Piston stroke in inches.

V = Swept volume of cylinder = $0{\cdot}7854\ d^2l$ cu. ins.

S = Mean piston speed = $\frac{lN}{6}$ feet per minute.

Then,

$$\begin{aligned}\text{I.H.P.} &= p \times 0{\cdot}7854 \times d^2 \times \frac{l}{12} \times N \times \frac{n}{33000} \times f \\ &= 0{\cdot}000001983\ pd^2lNnf \\ &= pd^2lNnf/504200 \\ &= 0{\cdot}0000119\ pd^2Snf \\ &= pd^2Snf/84033 \\ &= pVNnf/396090\end{aligned}$$

where f is a factor which equals

½ for single-acting four-stroke engines.

1 for single-acting two-stroke or double-acting four-stroke engines.

2 for double-acting two-stroke engines.

Brake Horse-power.—$\text{B.H.P.} = \frac{2\pi NT}{33000}$

$$= 0{\cdot}0001904NT$$
$$= NT/5252.$$

Corrections to B.H.P. for Atmospheric Conditions.—The horse-power an engine develops varies with variations of atmospheric pressure and temperature, and test results are sometimes adjusted to show the horse-power that would have been developed under standard conditions, namely, 760 mm. barometric pressure and 15° C. temperature. Some disagreement exists as to the correct basis on which the adjustment should be made, and the following formulæ are both used:—

$$\text{B.H.P.}_o = \text{B.H.P.}_a \times \frac{P_o}{P_a} \times \frac{T_a}{T_o},$$

and

$$\text{B.H.P.}_o = \text{B.H.P.}_a \times \frac{P_o}{P_a} \times \sqrt{\frac{T_a}{T_o}},$$

where

B.H.P._o = B.H.P. under conditions of pressure and temperature P_o and T_o,

B.H.P._a = B.H.P. under conditions of pressure and temperature P_a and T_a,

these pressures and temperatures being absolute.

$$(T_o^\circ \text{ C.} = 273 + t_o^\circ \text{ C.}, \quad T_o^\circ \text{ F.} = 460 + t_o^\circ \text{ F.})$$

The second formula is the more commonly accepted one. According to Gagg and Farrar (*Trans. S.A.E.*, 1934) it is sufficiently accurate for temperatures between 0° C. and 30° C., but gives erroneous results at higher temperatures.

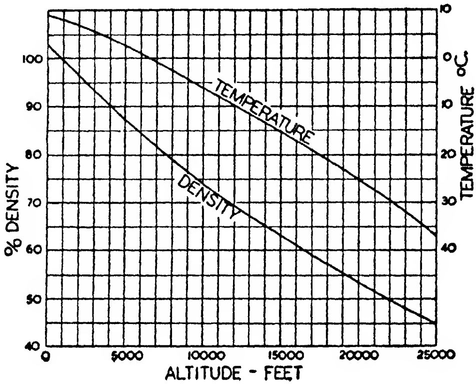
It has been suggested (*Automobile Engineer*, March 1935) that the formula should be used to correct I.H.P. and not B.H.P., since the mechanical efficiency is unaffected by variations of pressure and temperature. This gives

$$\text{B.H.P.}_o = \text{B.H.P.}_a \left[1 - \frac{1}{\eta_m} \left\{ 1 - \frac{P_o}{P_a} \sqrt{\frac{T_a}{T_o}} \right\} \right],$$

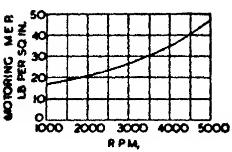
and the mechanical efficiency η_m will usually have to be estimated.

Variations of Atmospheric Pressure and Temperature with Altitude.—The American Bureau of Standards uses the formula $H = 62000 \log_{10} \frac{760}{P}$ to relate the barometer reading P (millimetres) with the altitude H (feet). The standard temperatures and densities used by the British Air Ministry are given in the graphs on p. 688.

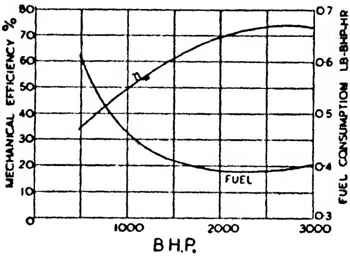
The temperature may be calculated approximately on the assumption that it decreases at the rate of 1° C. per 700 feet up to 15,000 feet, and at 1° C. per 900 feet from 15,000 up to 25,000 feet. Dorand's formula for the temperature is $t°$ C. $=15-0·00198\,H$, the altitude H being in feet.



Mechanical Efficiency (η_m).—This is the ratio, B.H.P./I.H.P. and is determined in low-speed engines by taking indicator cards and thus deriving the I.H.P., while the B.H.P. is obtained either from the brake test or from a shaft dynamometer. In high-speed engines the I.H.P. cannot be obtained directly by means of indicator cards, unless a Farnboro or similar type of indicator is used, and the friction horse-power is generally determined by motoring the engine by means of a swung-field electric motor, the assumption being made that the friction under motoring conditions will be the same as when the engine is running. At full load η_m varies from as low as 70 per cent. up to as high as 90 per cent., but for high-speed petrol engines is usually about 80 per cent. The torque required to motor an engine varies with the speed, and the nature of this variation is shown in the graph, which gives the mean effective pressure corresponding to the motoring torque at various speeds in a six-cylinder engine. The friction horse-power is almost independent of the load, and so the mechanical efficiency falls off as the load decreases, the example shown in the graph on p. 689 being for a Fullagar opposed piston engine running at 200 r.p.m.



For the same reason the mechanical efficiency of a supercharged engine will generally be somewhat higher than that of an unsupercharged engine, and will increase with the amount of boost,



up to a limit. This is illustrated by the results tabulated below, which relate to a Vickers-Armstrong submarine engine of 21-inch bore by 21-inch stroke.

R.P.M.	350	385	400
Intake pressure, lbs. per sq. inch abs.	14·7	18·2	19·7
I.H.P.	3810	5525	6088
B.H.P.	3000	4506	5000
η_m	78·8	81·6	82·2

B.M.E.P in Terms of Torque.—

$$\text{B.M.E.P.} = \eta_m p = \frac{48\pi T}{Vn} \text{ lbs. per sq. inch, for single-acting four-stroke engines}$$

$$= \frac{24\pi T}{Vn} \text{ for single-acting two-stroke engines.}$$

Engine Ratings.—For taxation purposes motor vehicle engines in Britain are rated according to the R.A.C. formula $\text{B.H.P.} = \frac{d^2n}{c}$ where $c = 2·5$ for d in inches and $c = 1613$ for d in millimetres. This formula assumes $S = 1000$ ft./min., $\eta_m = 75$ per cent., I.M.E.P. = 90 lbs. per sq. inch, and these figures are still applicable to low-speed stationary and marine engines, but are largely exceeded in automobile and aero engines, for which limiting values of these factors are $S = 3500$ ft./min., $\eta_m = 90$ per cent., I.M.E.P. = 150 lbs. per sq. inch, it being understood that maximum values of each factor are not obtained simultaneously.

The Society of Motor Manufacturers and Traders formula (which is little used) is B.H.P. = 0·45 $(d - 1·18)(l + d)$, and takes

the stroke l into account. Motor-car manufacturers sometimes designate their cars by the maximum b.h.p. figure, but marine and stationary engines are usually rated at the b.h.p. they will develop continuously.

Aero engines have two, and sometimes three, distinct ratings, namely—(1) take-off power rating; (2) cruising rating; and (3) emergency rating. The first is the maximum power the engine can develop for short periods at ground level. The second is the power the engine can develop for protracted periods at the altitude at which the aeroplane is expected to fly. The third is the power the engine can develop for moderately long periods, such as might be necessary if failure of one engine of a multi-engined machine occurred. The cruising rating is usually about two-thirds of the take-off rating.

Thermal Efficiency (η_t).—

$$\eta_t = \frac{\text{Useful work done in time } T}{\text{Heat supplied in same time } T},$$

both quantities of energy being measured in the same units.

Air Standard Efficiency.—The thermal efficiency of an ideal engine working on the constant volume cycle with air as the working fluid, and assuming the ratio (γ) of the specific heat at constant pressure to that at constant volume to be constant, can be shown to be given by $\eta_t = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$. The value of γ is usually taken as 1·4.

Effect of Mixture Strength.—Tizard and Pye ("Empire Fuels Report," *Proc. I.A.E.*, 1923–24) have shown that for all ordinary hydrocarbon fuels the theoretical efficiency obtainable with the constant volume cycle depends on the mixture strength of the cylinder charge. For chemically correct mixtures they find that $\eta_t = 1 - \left(\frac{1}{r}\right)^{0\cdot258}$, while for 20 per cent. weak mixtures $\eta_t = 1 - \left(\frac{1}{r}\right)^{0\cdot296}$. Values of these efficiencies and of the air standard efficiency for various values of r are given below.

r . .	4.	5.	6.	7.	8.	9.
$1 - \left(\frac{1}{r}\right)^{0\cdot4}$	·4257	·4747	·5116	·5408	·5646	·5848
$1 - \left(\frac{1}{r}\right)^{0\cdot296}$	·3366	·3790	·4116	·4248	·4596	·4782
$1 - \left(\frac{1}{r}\right)^{0\cdot258}$	·3007	·3398	·3702	·3947	·4152	·4314

r . .	10.	12.	14.	16.	18.	20.
$1-\left(\frac{1}{r}\right)^{0\cdot4}$	·6019	·6299	·6520	·6701	·6853	·6983
$1-\left(\frac{1}{r}\right)^{0\cdot296}$	·4942	·5208	·5421	·5599	·5750	·5880
$1-\left(\frac{1}{r}\right)^{0\cdot258}$	·4479	·4733	·4938	·5110	·5256	·5383

Goodenough and Baker (Illinois University) have deduced empirical formulæ for the value of n in the formula $\eta_t=1-\left(\frac{1}{r}\right)^n$, giving the theoretical thermal efficiency obtainable with actual working fluids. For the constant volume cycle with chemically correct or rich mixtures $n=0\cdot524-\frac{24\cdot6}{a}$,

for weak mixtures $n=0\cdot3867-\frac{6\cdot5}{a-35}-\frac{0\cdot043}{r}$,

and for the Diesel cycle $n=0\cdot434-\frac{19\cdot5}{a-r}-\frac{0\cdot7}{r}$,

where a is the ratio (expressed as a percentage) of the air supplied per cycle to the air required for the chemically correct mixture.

The theoretical efficiency of the Diesel cycle, assuming γ to be constant, is given by $\eta_t=1-\left(\frac{1}{r}\right)^{\gamma-1}\left(\frac{1}{\gamma}\cdot\frac{\rho^\gamma-1}{\rho-1}\right)$, where ρ is the *cut-off ratio*, the ratio of the volume enclosed in the cylinder at the end of the constant pressure period to the clearance volume (see Fig. (*b*), p. 684).

Relative Efficiency.—The ratio of the actual thermal efficiency of an engine to the theoretical efficiency of an ideal engine having the same compression ratio and using the same working fluid is called the *relative efficiency*, and is useful as a criterion of the thermodynamic performance of the engine. The ideal engine efficiencies will be those given by Tizard and Pye's formula $\eta_t=1-\left(\frac{1}{r}\right)^{0\cdot296}$ and tabulated above and on p. 690.

In practice relative efficiencies range from about 65 per cent. to 85 per cent.

The Tookey Factor.—This was evolved by W. A. Tookey as a convenient means of comparing I.C. engines of various sizes running at different speeds and operating on a variety of fuels.

It is given by

$$T_m = \text{Tookey factor} = \frac{p}{Q_t},$$

where

p = I.M.E.P. in lbs. per sq. inch,

and

Q_t = "Mixture Strength" in B.Th.U. per cubic foot of *total* cylinder volume,

i.e.

$$Q_t = \frac{\text{Cal. val. of fuel (higher)} \times \text{wt. of fuel per min.}}{(\text{Clearance vol.} + \text{swept vol. of cyl.}) \times n \times N/2},$$

the clearance and swept volumes being in cubic feet.

The Tookey factor is related to the indicated thermal efficiency thus,

$$\text{Indicated thermal efficiency} = \frac{T_m}{5{\cdot}4} \times \left(\frac{r-1}{r}\right).$$

Combustion Factor.—This was introduced by the Admiralty Engines Laboratory, and is defined as B.M.E.P. (lbs. per sq. inch) × fuel consumption (lbs. per b.h.p. hr.).

Heat Balance in I.C. Engines.—Typical figures for this at full load are as follows:—

Heat to b.h.p.	26 to 35 per cent.
Heat to exhaust	40 to 35 per cent.
Heat to jackets	28 to 20 per cent.
Heat to friction h.p.	3 to 5 per cent.
Heat to radiation, etc.	Remainder.

At lower loads the percentages to exhaust, jackets, and friction will of course be greater, but the actual heat flow will be less. The heat to friction h.p. is of course ultimately passed to the cooling water or is dissipated by radiation. A Fullagar opposed piston engine gave the following figures:—

Heat to b.h.p.	36 per cent.
Heat to exhaust	30 per cent.
Heat to cooling water, etc.	28 per cent.
Heat to scavenge pump h.p.	2·5 per cent.
Heat to compressor h.p.	3·5 per cent.

All the above percentages are of the total heat supplied to the engine in the fuel.

Pressures and Temperatures during the Cycle.—*Suction stroke.*—With the throttle closed a vacuum of 20 inches mercury is possible, but with the throttle wide open the pressure during the suction stroke will usually be only 2 to 3 lbs. per sq. inch below atmospheric. The temperature at the end of the suction stroke may be somewhat above atmospheric temperature, but for calculation of compression pressures and temperatures is usually taken as atmospheric. Temperatures lower than atmospheric may be obtained when using fuels with high values of the latent heat (*e.g.* alcohol) without pre-heating.

Compression stroke.—The pressure and temperature at the end of this stroke will depend on those at the beginning and on the compression ratio, the tightness of the piston ring seal, and the heat losses or gains during the stroke. They can be calculated sufficiently accurately for most purposes from the relation $PV^{1\cdot3}=k$. Assuming atmospheric pressure (14·7 lbs. per sq. inch) and a temperature of 15° C. at the commencement, the pressures and temperatures at the end of the stroke, as calculated by the above formula, for various values of the compression ratio r, are given in the table below.

r . . .	4.	5.	6.	7.	8.	10.	12.	14.	16.	18.	20.
p_c lb./in.^{2} . .	89	119	151	184	219	293	371	454	540	629	727
t_c° C. . .	164	194	220	243	264	302	334	363	389	412	434

In some experiments, reported by Day (*Proc. I.Mech.E.*, 1931), on a Diesel engine the first compression when starting from cold followed the law $PV^{1\cdot26}=k$, the second compression $PV^{1\cdot28}=k$, the third $PV^{1\cdot30}=k$, and when settled conditions had been obtained the relationship was $PV^{1\cdot35}=k$.

Compression Ratios in Actual Engines.—Since the maximum thermal efficiency depends on the compression ratio, this is made as high as circumstances permit. The value is limited in spark-ignition engines by the tendency of the fuel to detonate and, except with special fuels, does not exceed 8. It is now rarely less than 4. The size of cylinder affects the tendency to detonate and so the ratios used with small cylinders are usually higher than with large ones. Ricardo gives the following as practical limiting values :—

Cylinder dia., inches .	2–2½	2½–3	3–4	4–5½
Compression ratio .	8	7·5	7	6·5

Another factor which tends to set a limit to the increase in compression ratio is the consequent increase in the maximum pressures attained, which necessitate heavier scantlings throughout the engine.

In compression-ignition engines the compression ratio ranges from as low as 9 in large slow-speed marine engines with air injection up to as high as 20 in small high-speed engines with mechanical injection.

Maximum Pressures.—These depend more on the rate of pressure rise than on the compression ratio. In spark-ignition engines they range from 250 lbs. per sq. inch to 1000 lbs. per sq. inch with normal combustion, but may be higher when detonation or pre-ignition occurs. In C.I. engines they range from about 650 lbs. per sq. inch to 1200 lbs. per sq. inch. Ricardo gives the following as typical of the maximum pressures in petrol engines:—

Comp. ratio .	4	5	6	7	8
Max. press., lb./in.^{2}	360	490	625	770	930

The above values will be increased if supercharging is used. The table below illustrates this for a petrol engine having a compression ratio of 5:—

Intake press., lb./in.^{2} abs.	15	22·5	30	34
Max. press., lb./in.^{2} abs. .	400	630	940	1050
B.M.E.P., lb./in.^{2} . .	99	178	258	297

Maximum Temperatures.—The maximum temperatures attained after the ignition of the charge can be calculated, as shown by Tizard and Pye ("Empire Motor Fuels Report," *Proc. I.A.E.*, 1923–24), allowance being made for dissociation. They vary more with mixture strength than with the compression ratio, being a maximum with mixtures about 20 per cent. richer than the chemically correct strength. The highest value quoted by Tizard and Pye is with benzene as fuel, and is 2701° C., the compression ratio being 5. With the same fuel, but with a compression ratio of 10, the highest temperature is 2760° C. approximately, these being calculated temperatures. The flame temperature as measured by Lloyd-Evans and S. S. Watts (B.A. Report, 1934) for a compression ratio of 5 was 2200° C. For details as to the calculation of maximum temperatures reference should be made to the "Empire Motor Fuels Report" mentioned above, and to a paper by Hershey, Eberhardt, and Hottel in the *Transactions of the Society of Automotive Engineers*, vol. xxxi., 1936.

Detonation.—If a fuel is tested in a variable-compression engine at gradually increasing compression ratios, detonation will, sooner or later, set in, and the characteristic "pinking" sound it produces will become more and more audible. On an indicator diagram detonation is shown by an increased rate of pressure rise on ignition and by somewhat increased maximum pressures. Detonation in a given engine will occur at different ratios with different fuels, but the actual engine design, as regards shape of combustion chamber, position of sparking plugs, etc., also influences the tendency towards detonation. Many attempts have been made to rate fuels according to their detonation tendencies, the earliest attempt being that of Ricardo, who determined the *Highest Useful Compression Ratio* (H.U.C.R.) for a number of fuels. The H.U.C.R. of a fuel is the compression ratio at which the detonation reaches some definite amount. The results obtained differ somewhat with differing designs of engine and methods of measuring the detonation, and for results to be comparable a standard engine and method must be adopted. A large number of authorities have agreed to accept as standard the engine design and testing method developed by the Co-operative Fuels Research Committee in America, which engine is commonly referred to as the C.F.R. engine (see *Trans. Amer. Soc. for Testing Materials*, 1933). The British Air Ministry, however, does not use this engine, and other engines are also used by other bodies. Thus the H.U.C.R. values of fuels will not agree within closer limits than about $\pm$ 0·5, unless they are obtained in exactly similar engines using the same test procedure.

Octane Number of a Fuel.—This is used as a criterion of the liability to detonation of the fuels used in spark-ignition engines, a good fuel in this respect having a high octane number. It is the percentage by volume of iso-octane (C_8H_{18}) in a mixture of octane and heptane (C_7H_{16}) which gives the same detonation characteristics in a C.F.R. engine as does the fuel in question. Octane does not detonate readily, whereas heptane is prone to detonation; and a mixture of the two can be made to match any fuel, thus giving the octane number. Other engines than the C.F.R. engine, and different test procedures, are used by some experimenters; and thus octane numbers, like H.U.C.R. values, are not strictly comparable unless they have been obtained in similar engines by similar methods. The octane number determined in any engine will not usually differ from that obtained with the C.F.R. engine by more than 3 or 4. Although the octane number is a reliable criterion of the tendency of a fuel to detonate when used in ordinary motor-car engines, considerable variations have been found when the fuels have been used in aero engines. The C.F.R. octane number tends to be too low for fuels having a high proportion of aromatics, these fuels performing much better in aero engines than their octane numbers would lead one to expect.

Ignition-lag or Delay Period (C.I. engines).—This is the time that elapses between the introduction of the first particle of fuel into the cylinder and the start of inflammation of the fuel. It depends upon the nature of the fuel, the degree of atomisation and distribution of the fuel spray, the temperature of the cylinder charge, the amount of air swirl, and other factors. In general, a fuel having a long delay period will give a poor performance and the maximum pressures will be higher than with a fuel having a short delay period.

Rating of Diesel Fuels. — Attempts have been and are being made to find some satisfactory method of rating Diesel fuels, much as the octane number rates spark-ignition fuels, but so far no altogether satisfactory method has been developed, although one or two methods are giving quite good results, *e.g.* the *Cetane Number, Aniline-point,* and *Diesel Index.*

Cetane Number.—This is the percentage of cetane ($C_{16}H_{34}$) in a mixture of cetane and alpha-methyl-naphthalene which will give the same ignition characteristics in a standard engine as does the fuel in question. Originally cetene ($C_{16}H_{32}$) was used, but cetane has been found to give more consistent results. A standard engine and testing procedure have not yet been agreed on.

Aniline-point.—This is the lowest temperature at which a mixture of equal parts of a fuel and of aniline will form a clear solution. It is considered by many authorities to be almost as good an index to the ignition qualities of Diesel fuels as is the cetane number. Le Mesurier and Stansfield give the following results correlating the aniline-points and the cetene numbers for a number of fuels:—

Fuel No.	1.	2.	3.	4.	5.	6.	7.	8.
Aniline-point (°C.)	74·0	71·5	68·0	64·5	61·0	56·5	51·5	45·0
Cetene number	70	65	60	55	50	45	40	35

The Diesel Index.—This is defined as

$$\frac{\text{Aniline-point (deg. F.)} \times \text{A.P.I. gravity}}{100},$$

and is considered by some authorities to be the best criterion of the suitability of a fuel for use in a Diesel engine. The A.P.I. (American Petroleum Institute) gravity is measured by means of the A.P.I. hydrometer and the specific gravity is related to it thus:

$$\text{Sp. gr.} = \frac{141{\cdot}5}{131{\cdot}5 + {}^\circ\text{A.P.I.}}.$$

Spontaneous Ignition Temperature (S.I.T.).—Apart from the difficulty of determining this, it has been found to be unsuitable as a criterion of the performance of Diesel fuels except that it can be fairly definitely stated that if the S.I.T. is above 300° C. the fuel will not be suitable.

Some idea of the correlation between cetane numbers, aniline-points, and Diesel indexes may be obtained from the table below.

Fuel No.	1.	2.	3.	4.	5.
Sp. gr., A.P.I.	36·1	34·0	23·1	29·4	45·5
Viscosity (Saybolt universal at 100° F.)	32	33	53	34	32
Aniline-point (°F.)	126·2	119·2	131·5	124·6	152·4
Diesel index	45·6	40·7	30·3	36·6	69·3
Cetane number	29·6	39·7	37·7	51·6	51·1

The fuels were: 1. Straight-run, S. Texas; 2 and 3. Cracked, Mid-continent; 4. Straight-run, Mid-continent; 5. Straight-run, Pennsylvania.

Fuels for I.C. Engines.—A great variety of fuels is available for use in I.C. engines, but the vast majority of engines use fuels consisting of a mixture of various hydrocarbons, the types and proportions of the different hydrocarbons varying with the different sources from which the crude oils are obtained and according to the amount of blending done by the refinery. Hydrocarbons can be divided into: 1. Paraffins, comprising substances whose composition is represented by the formula C_nH_{2n+2}; 2. Olefines and Naphthenes (C_nH_{2n}); 3. Benzenes (Aromatics) (C_nH_{2n-6}). Alcohol and blends of alcohol with petrol are also sometimes used. The chief properties of the fuels used in I.C. engines are given in the tables on pp. 698, 699. The calorific values are seen to vary considerably, but Tizard and Pye have pointed out that for all the liquid hydrocarbons, and for alcohol, the heat liberated by the complete combustion of 1 lb. of air is 720 C.H.U. (1300 B.Th.U.) to an approximation of 5 per cent. For all petrols the value is 716 C.H.U. (1290 B.Th.U.) to an approximation of 1 per cent., and Ricardo has suggested that thermal efficiencies should be calculated from the air consumptions of engines, on the assumption that any fuel supplied in excess of that required for chemically correct combustion is to be debited to inefficiency in carburation or distribution and not to thermal inefficiency. It follows that

$$\eta_t = \frac{\text{B.H.P.}}{\text{lbs. of air per hour} \times C},$$

where C is 1·96 for petrol and 2·02 for ethyl alcohol.

Assuming the value of 720 C.H.U. for the heat of combustion of 1 lb. of air it follows that

$$\text{Swept volume per cylinder} = \frac{1414 \times \text{B.H.P.}}{Nn\eta_t\eta_m\eta_v} \text{ cu. ins.}$$

and this formula may be used to derive the major dimensions of

Properties of Fuels.—I. Liquid.

Fuel.	Specific Gravity.	Latent Heat. C.H.U./Lbs.	Calorific Value. C.H.U./Lbs.		Spontaneous Ignition (in Air). Temperature, °C.	H.U.C.R.	Octane Number.	Air/Fuel. Lbs./Lbs.
			Higher.	Lower.				
Benzene, C_6H_6	·884	96	10090	9700	419	> 15	> 100	13·2
Butane, C_4H_{10}	·58	..	11800	..	..	7	90–100	15·5
Propane, C_3H_8	..	..	12000	..	..	10	125	15·7
Hexane, C_6H_{14}	·68	86	11500	10650	366	5–6	..	15·3
Heptane, C_7H_{16}	·691	75	11560	10710	330	3·75	0	15·15
Methyl alcohol, CH_3OH	·796	270	5241	4762	457	5·2	..	6·45
Ethyl alcohol, C_2H_5OH	·794	213·6	7093	6403	514	> 7·5	..	8·97
Power alcohol mixture, No. 1	·832	138·7	7780	7170	..	9·5–12	..	9·57
Power alcohol mixture, No. 2	·824	69	8800	8190	..	6·25	..	11·29
Benzole	·878	63·7	10025	9600	420	> 7·0	..	13·31
Aviation petrol	·720	70·8	11300	10510	..	..	..	14·9
Petrol, No. 1	·740	68·4	11280	10500	..	5·5–6·0	70–73	14·8
Petrol, No. 3	·745	66	11200	10430	..	5·0	66–70	14·8
Ethyl petrol	·73	70	..	..	..	6–8	80–87	14·8
Kerosene	·793	59·5	11140	10420	..	4·2	..	14·6
Tractor vaporising oil	·780	59·2	11150	10420	..	..	..	14·6
Diesel oil	·87	..	10980	10310	..	..	..	14·4
Light fuel oil	·895	..	10700	10050	..	..	..	14·3
Heavy fuel oil	·949	..	10500	9880	..	..	..	14·0

Properties of Fuels.—II. Gaseous.

Fuel.	Volume per Lb. at N.T.P.	Calorific Value at N.T.P. C.H.U./Cu. Ft.		Air/Fuel.	
		Higher.	Lower.	Cu. Ft./ Cu. Ft.	Lbs./ Lbs.
Hydrogen, H_2	178·0	193	166	2·38	34·3
Carbon monoxide, CO	12·8	190	190	2·38	2·46
Methane, CH_4	22·4	594	543	9·52	17·2
Ethylene, C_2H_4	12·7	931	890	14·28	14·8
Town gas	31	170–360	160–330	5·0	12
Blast-furnace gas	12·6	60	58	0·78	0·8
Coke-oven gas	31	313	280	4·5	11·4
Producer gas	15	80–90	72–82	1·0–1·2	1·2–1·4

a petrol engine to develop a given horse-power at a given speed, suitable values being assumed for the thermal, mechanical, and volumetric efficiencies.

Air/Fuel Ratio.—Finlayson (*Proc. I.A.E.*, 1924) gives the formula $Air/Fuel = 0{\cdot}116\left\{C + 3\left(H - \frac{O}{8}\right)\right\}$, where C, H and O are the percentages, by weight, of carbon, hydrogen and oxygen in the fuel. For hydrocarbons this becomes $0{\cdot}116(C+3H)$. For a hydrocarbon whose formula is C_aH_b the ratio is given by $1\bigg/\left[0{\cdot}0292 + \frac{0{\cdot}2305}{4+b/a}\right]$. These *air/fuel* ratios are by weight.

According to the American Bureau of Standards (Report No. 97), the percentage (by weight) of hydrogen in the hydrocarbon fuels used in petrol and Diesel engines is equal to

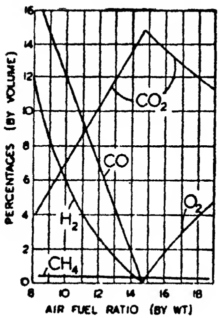
$$26 - (15 \times \text{specific gravity at } 60^\circ \text{ F.}).$$

Maximum power is generally obtained with mixtures about 20 per cent. rich, while maximum economy is obtained with mixtures between 10 per cent. and 20 per cent. weak. In multi-cylinder engines the mixture strength usually has to be richer than in single-cylinder engines because of variations in the distribution to the cylinders.

Petrol Engines on Producer and Town Gas.—The Fuel Research Station at Greenwich have run petrol engines on producer gas made from (*a*) charcoal and (*b*) low temperature coke, the b.h.p. obtained being respectively 90 per cent. and 70 per cent. of that with petrol. The gases have calorific values of 89 and 72 C.H.U. per cubic foot respectively. When a petrol engine is run on town gas, experiments have shown that about 120 cubic feet of gas (C.V. 375 C.H.U. per cubic foot) is equivalent to one gallon of petrol. One gallon of butane was found to be equivalent to about 145 cubic feet of town gas.

Engines Running on Hydrogen.—The *Erren* engine is designed to run on hydrogen and oxygen or hydrogen and air mixtures. With oxygen the hydrogen consumption per b.h.p. per hour is from 30–35 cubic feet and with air from 35–40 cubic feet.

Exhaust Gas Analyses.—The chemical composition of the exhaust gases of engines may be obtained in various ways, and can be used as a means of determining the *air/fuel* ratio of the cylinder charge. In addition to nitrogen the constituents of the exhaust from engines running on hydrocarbon fuels are carbon dioxide, oxygen, carbon monoxide, hydrogen, and methane. The percentages of these gases for various *air/fuel* ratios are shown in the diagram for a fuel whose composition can be assumed equivalent to C_8H_{17}; this is a reasonable assumption for all petrols. According to a report (No. 476) of the National Advisory Committee for Aeronautics (U.S.A.), the ratio of the percentages of hydrogen and carbon monoxide in the exhausts of engines using petrol or Diesel fuels is 0·51. Traces of aldehydes are also often found in the exhaust gases.



Temperature in I.C. Engines.—The hottest part of most engines is the exhaust valve, for which temperatures up to 800° C. have been recorded. Normally exhaust valves run at between 600° C. and 750° C. on full load, the highest temperatures occurring with approximately chemically correct mixture strengths, late ignition, high speeds, and high water-jacket temperatures. In high output aero engines exhaust valves are frequently cooled by making them hollow and partly filling them with a material having a suitable melting-point, sodium being often used. Valve temperatures are increased by having large amounts of uncooled valve guide, the design shown at (*a*), p. 705, being superior to that at (*b*) in this respect. Sparking plugs also run at high temperatures, values of 550° C. to 800° C. having been measured in a petrol engine at 1500 r.p.m. Piston heads are the next hottest spot, cast-iron pistons running at much higher temperatures than aluminium alloy pistons. Temperatures up to 460° C. have been measured at the centre of cast-iron pistons, while in the same engine under the same conditions aluminium alloy pistons reached only 240° C. Cylinder head temperatures in water-cooled engines may reach 200° C. to 250° C., and in air-cooled engines may be as high as 350° C. Aluminium and copper-aluminium alloy heads run at lower temperatures than cast-iron heads in water-cooled engines, and their use in place of cast-iron will usually permit the raising of the compression ratio by about one ratio. Forged aluminium pistons and cylinder heads (air-cooled engines) have been found to be much superior to cast alloys for aero engines, and those parts are now usually made as forgings machined all over.

Horse-power per Litre.—The horse-power per litre figure is a quantity that is useful only to compare engines whose designs are fairly similar. In naturally aspirated engines for small cars, from 20 to 50 b.h.p. per litre may be obtained, and with supercharging 150 b.h.p. per litre has been recorded. The higher values cannot be sustained for long periods, however. In aero engines the b.h.p. per litre calculated on the take-off rating varies between 35 and 50, but in special engines over 70 b.h.p. per litre has been obtained.

Engine Proportions.—*Weights of Engines.* (1) *Automobile.* (*a*) *Cars.*—From 5 to 20 lbs. per b.h.p., the lower values in small sports car engines and the higher in large engines running at comparatively low speeds (2500 r.p.m. max.). Vee-type engines show fairly low weights per b.h.p.

(*b*) *Lorries.*—From 10 to 30 lbs. per b.h.p. Diesel engines are usually about 10 per cent. heavier than petrol engines of similar design by the same maker, but the difference is becoming less and in some cases is already negligible.

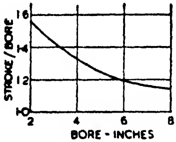
(2) *Aero.*—From 1·8 to 2·5 lbs. per b.h.p. dry and without air-screw. The lower figure is attained in highly developed engines, and the higher in low-powered engines for small aeroplanes.

(3) *Marine.* (*a*) *Medium speed* (500–1500 r.p.m.).—From 20–80 lbs. per b.h.p. for engines designed for marine purposes. Automobile and aero type engines are sometimes used, however, and the weights will then be towards the upper limits of (1) or (2).

(*b*) *Slow speed* (80–300 r.p.m.).—From 150 to 400 lbs. per b.h.p. A figure of 250 lbs. per b.h.p. is common.

Great reductions (up to 40 per cent.) in weight can be effected in medium and high-powered engines running at moderate speeds by the use of welded instead of cast frames.

Stroke/Bore Ratio.—This lies between 0·9 and 1·8, the lower values obtaining in some aero and automobile engines and the higher in stationary and marine engines. Many automobile engines have a ratio between 1·5 and 1·6. Rowell (*Engineering,* 7/3/30) gives the curve shown for the variation of *stroke/bore* ratio with the bore. Present tendency in automobile practice seems to be towards ratios of about 1·3.



Connecting Rod/Crank Ratio.—In some aero engines this is as low as 2·5, but usually it ranges from 3 to 5·5, common values being between 3·7 and 4·5. Multi-cylinder engines generally have a lower value than single-cylinder engines.

Crankpin Diameters.—In petrol engines, with the cylinders in line, these are from $0{\cdot}6D$ to $0{\cdot}7D$ where D is the cylinder diameter, in Vee engines from $0{\cdot}7D$ to $0{\cdot}8D$, and in Diesel engines they are from $0{\cdot}65D$ to $0{\cdot}8D$.

Main Bearings.—These vary from $0{\cdot}5D$ to $0{\cdot}8D$ in line engines, but usual values are between $0{\cdot}6D$ and $0{\cdot}77D$. In Vee engines they are from $0{\cdot}8D$ to $0{\cdot}87D$. In Diesel engines they are from $0{\cdot}6D$ to $0{\cdot}85D$, but values below $0{\cdot}65D$ are not common. Most Diesel engines have a main bearing on each side of every crank throw, but four-cylinder petrol engines have two or three, and six-cylinder engines often have only four main journals.

Gudgeon Pin.—Diameter $=0{\cdot}18D$ to $0{\cdot}4D$, a common value being $0{\cdot}25D$. With pins free to turn in both rod and piston the ratio *Length of little-end bush/Dia. of pin* varies from 1·25 to 2·25, and is usually about 1·6. When the pin is fixed in the rod the ratio *Total length of bearing/Dia. of pin* varies from 1·5 to 2·4, an average value being 1·75. Loads up to 6000 lb./sq. in. of projected area are used with floating pins, but with fixed pins 4000 lb./sq. in. is seldom exceeded.

Piston Length.—In aero engines this may be as low as $0{\cdot}75D$, but in most medium and high-speed engines it is about $1{\cdot}5D$. Lorry Diesel engines may have rather longer pistons, and in stationary and marine engines the piston length may be as much as $2{\cdot}5D$.

Piston Clearances.—The clearances recommended, per inch of diameter, by the makers of Covmo pistons are as follows:—

	Standard Alloy.	Low-expansion Alloy.	Cast-iron.
Solid skirt; water-cooled engines . . .	0·0014	0·0010	0·0008
Solid skirt; air-cooled engines . . .	0·0022	0·0016	0·0011
Solid skirt; two-stroke engines . . .	0·0028	..	..
Split skirt; water-cooled engines . . .	0·0008	..	..

The coefficient of expansion of the standard alloy is 0·000023 per °C., for the low expansion alloy it is 0·000018, and for cast-iron it is 0·000011.

Piston Ring Gaps.—

Water-cooled engines, 0·002 inch per inch diameter.
Air-cooled engines, 0·004 inch per inch diameter.
Racing air-cooled engines, not less than 0·005 inch per inch diameter.

Piston Ring Clearances.—

(1) *On sides.*—Cast-iron pistons, 0·0015 inch.
Aluminium pistons, 0·0025 inch.
(2) *At back.*—0·010 to 0·015 for all sizes up to 6 inches diameter.

Valve Diameters.—These vary from $0{\cdot}35D$ to $0{\cdot}55D$, but in medium and high-speed engines are usually about $0{\cdot}4D$. Inlet and exhaust valves are usually the same in size, but if they are not then the exhaust valve is generally the smaller. Stem diameters are usually between 0·18 and 0·3 of the valve head diameter, an average value being 0·22.

Gas Velocities.—The speed of the gases through the inlet valve ports of car engines at the speed corresponding to maximum torque is round about 130 ft./sec., maximum power being then obtained at a gas speed of from 200–250 ft./sec. In the other parts of the induction system the gas speed at maximum torque is about 80–90 ft./sec., except for the carburettor choke, where it is about 230–300 ft./sec. These gas speeds are calculated from the mean piston speeds and the ratios of cylinder and port areas.

Valve Timings.—*Inlet opening.*—In racing motor-cycle engines this may be as early as 25° before T.D.C., and in stationary engines it may be as late as 5–10° after T.D.C. Common values are from 10° before to 5° after T.D.C.

Inlet closing.—This is usually from 30° to 50° after B.D.C.
Exhaust opening.—This may be from 50° to 40° before B.D.C.
Exhaust closing.—This varies from about T.D.C. to 15°, or sometimes as much as 20°, after T.D.C.

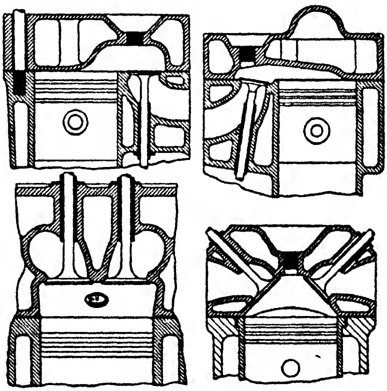
Ignition Timing.—The ignition timing must be adjusted to suit the prevailing conditions of load and speed if the best results are to be obtained, and may be from as little as 8° to as much as 40° before T.D.C. In practically all automobile engines the ignition timing is controlled automatically, partly by a centrifugal control which varies the timing according to engine speed and partly by a vacuum control, which varies the timing according to the intake manifold pressure (advancing the timing with increasing vacuum), and a hand control may also be fitted.

Bearing Materials.—Bronze is still used occasionally in marine engines, but white-metal, lead-bronze, or aluminium alloys are almost universal for big-ends and main bearings. White-metal, which has numerous compositions (a typical one being Sn 87 per cent., Sb 9 per cent., Cu 3 per cent., Pb 1 per cent.), is generally used in bronze or steel shells, but in big-ends of motor-cars is often cast direct in the connecting rod. Clearances of 0·001 to 0·015 inch per inch diameter are used, and oil pressures range from 30–60 lbs. per sq. inch. The thickness of the white-metal lining varies from as little as 0·015 inch upwards, but 0·020 inch is considered preferable as a minimum. For motor-car engines the steel shells are sometimes made very thin (0·030 to 0·040 inch), and are then pressed up from strip. Lead-bronzes are commonly used in Diesel engines, and are becoming common in car engines where the duty is severe. A.E.C. use a composition of Cu 74 per cent., Sn 1 per cent., Pb 25 per cent., and the Ford Motor Co. use Cu 63–68 per cent., Pb 30–35 per cent., Fe 0·5 per cent. max., Ni 1–1·5 per cent. in a thin steel shell pressed to shape. Clearances of 0·002 to 0·005 inch per inch diameter are used. In Diesel engines lead-bronze is often used for the rod half of the big-end bearing, and white-metal for the cap, partly because of cost considerations and partly because white-metal can absorb small abrasive particles better than lead-bronze. The main journals also may have lead-bronze lower halves and white metal upper halves. Alloys containing cadmium and silver have also been used, but not extensively. Various aluminium alloys have been developed for bearings, and are likely to come into extensive use. High Duty Alloys Ltd., in conjunction with Rolls-Royce Ltd., have been responsible for this development in England, and much work has also been done in Germany. Rolls-Royce use these alloys as standard practice in their car engines. They are used without shells and with clearances of about 0·0008 inch per inch diameter. Quarzal, a German alloy containing from 2 to 15 per cent. Cu and other elements in small quantities, is being used in Germany with loads up to 2000 lbs. per sq. inch at speeds up to 33 ft./sec. and with clearances of 0·012 inch per inch diameter. Case-hardened journals are generally used with these alloys, and frequently with the lead-bronze materials. According to Fedden a chrome-molybdenum steel is the best for case-hardening. An alternative to case-hardening is hardening by the Tocco process, this being now fairly common practice in motor-car engines. For details of this process see the *Automobile Engineer* for July 1937.

Checking Bearing Clearances.—This is best done by using an oversize mandrel, but alternatively the bearing may be assembled with copper-foil of suitable thickness inserted, the bearing being machined or scraped until it is just movable when tightened up. In large bearings it is usual to employ lead wire to measure the clearances.

Bearing Temperatures.—The actual temperatures of the main bearings and crankpins of a petrol engine have been measured at the I.A.E. Research Station (*Proc. I.A.E.*, 1938) by means of thermocouples inserted into the white-metal of the main bearing and into the crankpin respectively. Temperatures up to 140° C. were recorded, and when the engine speed and rate of oil flow were kept constant the bearing temperatures varied linearly with the temperature of the oil at the inlet to the main bearing. When the inlet oil temperature and rate of oil flow were constant the bearing temperatures varied linearly with engine speed. The variation of bearing temperature with rate of oil flow followed approximately an hyperbolic law.

Combustion Chambers for Petrol Engines.—A large number of different forms of combustion chamber have been tried for petrol engines, but most modern engines use one of the forms

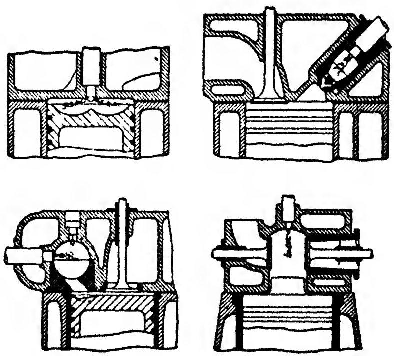


(a) (b) (c) (d)

shown in the illustration. At (*a*) is shown a *turbulent* type in which the last part of the piston stroke, in forcing the charge into the clearance space, produces considerable turbulence in the charge, thus producing rapid inflammation when the charge is ignited. In the type shown at (*b*), which is commonly used, the turbulence is much less than in type (*a*). Both these types are generally used with side-by-side valves but sometimes with the exhaust valve in the flat portion of the cylinder head over the piston. Types (*c*) and (*d*) are for overhead valve engines, the former for ordinary cars and the latter chiefly for racing cars and motor cycles. The position of the sparking plug is of considerable importance, and should be such that the flame travel is from the hottest towards the cooler parts of the combustion chamber, otherwise the liability to detonation will be increased. To obtain good volumetric efficiency the incoming charge should be kept out of contact with very hot spots such as exhaust valves.

Sparking Plugs.—Two sizes are standardised, namely—the 18-mm. and the 14-mm.; the pitch of the thread of the 18-mm. plug is 1·5 mm. and that of the 14-mm. plug is 1·25 mm. The thread form in both cases is the Système International.

Diesel Combustion Chambers.—A great variety of types has been developed for high-speed Diesel engines, but those that have survived fall roughly into three groups. These are shown in the illustration. At (*a*) is shown the *open* type of chamber, the injection being direct into the disc-shaped space between the piston and cylinder head. Overhead valves (not shown) are used, placed vertically in the cylinder head. This



(*a*) (*b*) (*c*) (*d*)

arrangement is also fairly typical of large slow-speed engines. At (*b*) is shown a *pre-combustion* type of chamber, the injection being into the pre-chamber which communicates with the cylinder by the holes shown. The air that is forced into the pre-chamber on the compression stroke is considerably heated on its way through the holes, and this helps to reduce the delay period when the fuel is injected. The design at (*c*) is that developed by Ricardo and A.E.C. and known as the *Comet*; injection is into the air cell, the air in which is given considerable swirl because of the tangential position of the entrance passage from the cylinder. The lower half of the air cell is fairly well heat-insulated and maintains a fairly high temperature. At (*d*) is shown a variation of the open type chamber which is used in comparatively slow-speed engines with success; its chief drawback is that the valve gear is rather difficult to arrange.

The open type chambers usually give easier starting than types (*b*) and (*c*) with which heating plugs generally have to be used, they also show slightly better fuel consumptions. It is claimed that types (*b*) and (*c*) give smoother running than open type chambers.

Injection Systems.—The earliest Diesel engines employed air injection, the fuel, which was fed to the injectors by plunger-type pumps, being forced into the cylinder by a blast of air, at a pressure between 600 and 1000 lbs. per sq. inch, when the injector needle valve was lifted by its cam. The blast air produced good atomisation of the fuel and also increased slightly the amount of oxygen available for combustion. It had, however, a cooling effect. Air injection gives a much closer approach to constant-pressure combustion than does mechanical injection, and the maximum cylinder pressures are not greatly in excess of the compression pressures. The necessity for having a three-stage air compressor makes air injection unsuitable for small engines, and even in large ones mechanical injection is replacing air injection largely because of the reduction in space occupied.

Mechanical or Solid Injection.—The fuel is sprayed into the cylinder by the direct action of a plunger pump, the pressure in the pipe to the injector ranging from 1000 to 4000 lbs. per sq. inch. The system can be used in two ways—(*a*) the *common-rail* system and (*b*) the *jerk-pump* system. In (*a*) the fuel is fed at full injection pressure (about 2500 lbs. per sq. inch) to mechanically or electrically operated injectors. The timing of the beginning of injection is determined by the cam that lifts the injector valve (or closes the electrical contacts), and the amount of fuel injected is determined by the height and duration of the valve lift. The latter is a very critical control, and this system in consequence is difficult to apply to small engines. In (*b*) there is a plunger pump for each cylinder, this plunger being operated by a quick-lift cam. The sudden lift of the plunger produces delivery of the fuel through the injector and determines the timing of the injection, while the quantity of fuel injected is controlled by mechanism embodied in the pump itself and described in the next paragraph.

Fuel Injection Pumps.—Two kinds are in use, variable spill and variable stroke types. In the former, variations in delivery are obtained by varying the point during the plunger stroke at which the suction valve is allowed to close or at which the spill ports are opened. In the second type the delivery is varied by altering the stroke of the pump plunger, this type is used in slow-speed engines, but in medium and high-speed engines the first type is almost universal. Variable delivery is usually controlled by a helical slot formed either in the plunger or the pump barrel, this part being rotated to uncover the spill ports earlier or later in the delivery stroke of the plunger. The maximum delivery of pumps is governed by the plunger diameter and the extent of the delivery stroke during which the suction and spill ports remain closed, this being usually less than half the total plunger stroke. The table below gives the maximum deliveries for C.A.V.-Bosch pumps.

C.A.V.-Bosch Fuel Injection Pumps.

Maximum Useful Output per Stroke of Each Element.

Series.	Plunger Stroke. Mm.	Plunger Dia. Mm.	Output. Cu. Mm.	Output. Cu. In.	Series.	Plunger Stroke. Mm.	Plunger Dia. Mm.	Output. Cu. Mm.	Output. Cu. In.
A	7	4	25	·0015	C	15	10	550	·0336
		5	40	·0024			11	650	·0396
		6	60	·0038			12	800	·0488
B	10	5	65	·0041			13	950	·0518
		6	100	·0061			14	1100	·0670
		6·5	125	·0076			15	1250	·0762
		7	135	·0082			16	1400	·0854
		7·5	160	·0098	D	30	14	2300	·1400
		8	180	·0109			16	3000	·1830
		9	230	·0143			18	4000	·2440
		10	280	·0171			20	5000	·3050

Size of Pump for given Engine.—The best size is usually found by trial. The use of a larger pump than the minimum possible tends to give a shorter delivery period, and this is often an advantage although it usually makes steady idling more difficult to obtain.

Timing Injection Pumps.—For consistent results it is best to time on the closing of the suction port. The pipe connecting the pump cylinder to the injector nozzle is removed and the delivery valve is taken out. On rotation of the pump shaft, fuel will then flow out of the delivery pipe union until the suction port is covered. This point is the commencement of delivery and can be accurately observed. The actual point of injection into the cylinder will be somewhat later, the *injection lag* being governed by the length and diameter of the delivery pipes and on the injection nozzle characteristics.

Pre-heaters.—C.I. engines using swirl chambers, air cells, or pre-combustion chambers usually require heater plugs to enable a start to be made from cold, unless a very high compression ratio is used. Heater plugs are standardised with the same threads as sparking plugs.

Injection Nozzles.—When air injection is employed the injection nozzles are operated by a cam provided on the camshaft, and mechanically operated injection nozzles are sometimes used with mechanical injection, usually when this is arranged on the common rail system. When the jerk-pump system is used the nozzle operation is automatic; two types of nozzle are used—(1) closed and (2) open, the distinction being according to whether the injection pressure is controlled by a spring loaded needle valve or whether a valve is dispensed with. Open nozzles are not much used. Closed nozzles may have an outlet in the form of one or more holes drilled in the nozzle cap; the diameter of these holes may be as small as 0·2 mm., and their *length/dia.* ratio controls the penetration of the spray to some extent. Alternatively the outlet may be in the form of an annular space between a pin or *pintle* on the end of the needle valve and a relatively large hole in the nozzle cap. The pintle has an inverted conical end which can direct the spray into a cone of from 4° to 60° depending on the angle to which the pintle is ground. The ratio *Pump plunger area/Nozzle hole area* controls the maximum pressure attained during injection, and also, to some extent, the duration and degree of atomisation of the spray. Rates of rise and fall of the injection pressure greatly affect "dribble" from the nozzles, a low valve closing rate being usually accompanied by some dribble, which is prejudicial to economy.

The Hesselman Engine.—In this engine air alone is compressed, as in a Diesel engine, but the compression ratio is only from 6 to 9. The fuel is injected during the compression stroke, commencing about 50° before T.D.C. Ignition is by a sparking plug, and is timed to occur about 15° before T.D.C. The engine is started on some volatile fuel, and at part load the air intake is throttled. These engines are being built under licence in several countries, and in America considerable numbers are being built by the Waukesha Company, the sizes ranging from 3¾-inch to 6¼-inch bore, and the horse-powers from 25 to 170. A brake m.e.p. of 104 lbs. per sq. inch is claimed for a production engine of 6¼ inch bore, and fuel consumptions of 0·58 lb. per b.h.p.-hour on full load and 0·52 lb. per b.h.p.-hour on three-quarter load. The engines are claimed to combine the ability to use the cheap fuels of the Diesel engine with the smoothness of running and flexibility of the petrol engine.

Supercharging. — This is a means of obtaining a higher pressure in the cylinder, at the moment of closing of the inlet valve, than would be obtained with normal aspiration, thereby increasing the weight and heat content of the charge and thus the power output. This is done by putting a pump or blower between the atmosphere and the intake of the engine. The pump or blower may be a centrifugal, Roots, eccentric vane, or piston type. The latter is used chiefly in two-stroke and in large slow-speed engines. For motor-cars the other types are all used, but for aero engines the centrifugal type is almost universal, and may be driven either mechanically by gearing from the crankshaft or by an exhaust gas turbine, whose blades are acted on by the exhaust gases as they leave the cylinders. In car engines the blowers are either driven directly off the end of the crankshaft or, more usually, through tooth or chain gearing. In large stationary and marine engines the blower may be a completely separate unit with its own prime mover, or it may be driven direct from the crankshaft or by an exhaust gas turbine. The last is adopted in the *Büchi* system used on large stationary and marine engines, in which the pressure of the exhaust entering the turbine is about 2½ to 3½ lbs. per sq. inch, and the pressure of the charging air entering the engine cylinder is from 3 to 5 lbs. per sq. inch. A valve overlap of about 120° is used, and as much as 20 per cent. of the charging air may pass straight through the cylinders, thus giving excellent scavenging. The arrangement and number of exhaust manifolds must be carefully considered in this system when more than four cylinders are used.

Supercharging is also called *boosting*, and the difference between the intake pressure and atmospheric pressure is the amount of boost. In marine engines supercharging is usually called *pressure charging*. It may be used in two fairly distinct ways—(*a*) to increase the intake pressure at all engine speeds, and thus shift the whole power-speed curve upwards, or (*b*) to increase the manifold pressures at high engine speeds so as to maintain the volumetric efficiency at or about the value it has at low speeds, and thus obviate the falling off in power which occurs with normal aspiration, or at least to bring the peak of the power-speed curve to a higher speed. Similarly in aero engines supercharging may be used to increase the power output at ground level, or, more usually at present, to maintain it at increasing altitudes when it would with normal aspiration fall off because of the reduction in atmospheric pressure and density.

Boosts up to as much as five atmospheres have been used in experimental engines, but in ordinary practice from 2 to 25 lbs. per sq. inch is the range used, and the higher values only in racing engines.

Supercharging increases the maximum and indicated mean effective pressures, these quantities being fairly closely proportional to the absolute intake pressures. Thus

$$\frac{(\text{I.M.E.P.})_a}{(\text{I.M.E.P.})_b}=\frac{P_a}{P_b},$$

where P_a and P_b are the absolute manifold pressures. The mechanical efficiencies of supercharged engines are usually rather higher than those of unblown engines and, for moderate boost (5–10 lbs. per sq. inch), the brake thermal efficiencies are the same in both cases.

Supercharging has the effect of reducing the range of mixture strength over which satisfactory running can be obtained, but it simplifies distribution difficulties in multi-cylinder engines. It also increases the tendency to detonation, partly because it increases the compression pressure and partly because (unless an inter-cooler is used, which is unusual) it raises the temperature at the beginning of the compression stroke and thus at the end also. If an engine, which is on the verge of detonation with a given fuel and normal aspiration, is supercharged, detonation will occur, and to avoid it the compression ratio will have to be lowered or a higher octane number fuel used. The requisite lowering of the compression ratio is given, according to O. Thornycroft (*Proc. I.A.E.*, vol. xxx.), by the relation

$$r_2=r_1\left(\frac{P_2}{P_1}\right)^{0\cdot6\text{ to }0\cdot75},$$

r_2 being the ratio with intake pressure P_2, and r_1 that with pressure P_1, both pressures being absolute. According to McEvoy (*loc. cit.*), the index of the pressure ratio term should be 0·5.

The heat flow in a supercharged engine will be greater, and the flow of cooling water to valve and sparking plug areas must be better, than in an unsupercharged engine. The mean temperature at the end of the power stroke will, however, be somewhat lower in the supercharged than in the unsupercharged engine.

COMPRESSED AIR.

(Revised by R. F. Pattenden, B.Sc.)

Definition of Terms.—*Free Air Delivered* (F.A.D.).—The volume of free air at intake pressure and temperature, expressed in cubic feet per minute, which a compressor will take in, compress and deliver at the stated delivery pressure.

Displacement.—The volume swept by the pistons of a compressor, in cubic feet per minute, under normal running conditions. In a compound compressor this refers only to the first stage.

Volumetric Efficiency (η_v).—The ratio F.A.D. to Displacement, under the given pressure conditions. This is the common definition. More correctly, Volumetric Efficiency is the ratio weight of air taken into the cylinder per cycle to that weight which would fill the swept volume at normal temperature and pressure.

Indicated Air Horse-Power (U_1).—The horse-power necessary to compress air as shown by the indicator diagram of the compressor.

Brake Horse-Power (U).—The horse-power necessary to drive the compressor at the stated working pressure and speed.

Adiabatic Horse-Power.—The theoretical horse-power necessary to compress the F.A.D., assuming adiabatic compression.

Isothermal Horse-Power (U_2).—The theoretical horse-power necessary to compress the F.A.D., assuming isothermal compression.

Compression Efficiency (η_c).—The ratio Isothermal Horse-Power to Indicated Air Horse-Power.

Mechanical Efficiency (η_m).—The ratio Indicated Air Horse-Power to Brake Horse-Power.

Overall Efficiency (η_o).—The ratio Isothermal Horse-Power to Brake Horse-Power.

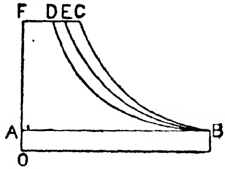
Combined Efficiency (η_T).—The ratio Isothermal Horse-Power to Horse-Power required by the Driving Unit.

Methods of Air Compression.—(1) *Piston Compressors.*—Air is compressed by pistons working in cylinders. The compression may be performed in one or more stages. These compressors are capable economically of a pressure ratio of about 8 per stage, and are therefore suited to high delivery pressures.

(2) *Centrifugal- or Turbo-Compressors.*—Air is compressed by energy imparted to it by rotating vanes. They are always multi-stage machines since the pressure ratio per stage can seldom exceed 1·2. They are thus suited to large volumes at relatively low delivery pressures.

(3) *Rotary Pumps and Blowers.*—Air is compressed either by gear or vane pumps or by single-stage fans. Such blowers are suitable only for low delivery pressures, and are much used for supplying the air blast to foundry cupolas, etc.

Action of an Air Compressor.—During the suction stroke a volume of air AB at, say, atmospheric pressure OA is drawn in.



During the return or compression stroke this air is compressed. If the air neither rejects heat nor receives heat from an external source during compression, it is compressed adiabatically, and the relation between the pressure and volume is represented by the curve BC and by the equation PV^n=a constant, where $n=1{\cdot}4$. When compressed adiabatically the temperature of the air rises.

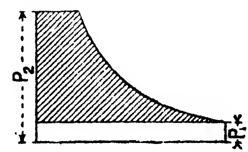
If the air remains at constant temperature during compression, it is compressed isothermally, and the relation between the pressure and volume is represented by the curve BD, and by the equation PV=a constant.

In actual practice the relation between the pressure and volume during compression is represented by a curve BE, which lies between the curves BC and BD, and is expressed by the equation PV^n=a constant, where n lies between 1 and 1·4 (generally 1·3). After compression the air is discharged at the constant pressure OF.

Mean Effective Pressure and Work done in the Cylinder of a Compressor.—

P_1=absolute pressure during suction stroke.
P_2=absolute pressure at end of compression.
P=mean effective pressure.

$$P=P_1\left(\frac{n}{n-1}\right)\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right\}$$



For isothermal compression

$$P=P_1 log_e\left(\frac{P_2}{P_1}\right)$$

A=effective area of piston.
L=length of stroke.

Work done in two strokes or one revolution$=2APL$, if the compressor is double acting, and half this amount if single acting. If A is in square inches, P in lbs. per square inch, and L in feet, the foregoing formula gives the work done in foot-pounds.

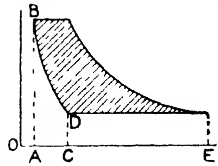
Temperature at End of Compression.—

T_1 and P_1=absolute temperature and absolute pressure respectively at beginning of compression.
T_2 and P_2=absolute temperature and absolute pressure respectively at end of compression.

$$T_2=T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

The compression is assumed to be performed adiabatically.

Effect of Clearance.—At the end of the compression stroke the clearance space OA is full of air at the pressure AB. During the next suction stroke the air in the clearance space expands until its pressure equals CD. The suction valve then opens, the piston being at C, and air is drawn in until the stroke AE is completed. The principal effect of the clearance space is to diminish the amount of air passing through the cylinder. The amount passing through per revolution is equal to $\frac{CE}{AE}\times$ volume swept through by the piston in one stroke for a single acting compressor, and double this for a double acting compressor.



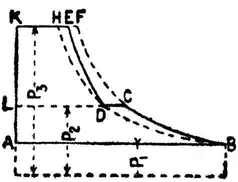
The clearance does not affect the amount of work done in compressing a given quantity of air to a given pressure, but the cylinder must be larger in the ratio of AE to CE.

The clearance volume in a compressor cylinder is generally small (from 1 to 2 per cent. of the volume swept through by the piston in one stroke).

Necessity of Cooling the Air during Compression.—In most cases where air compressors are used, the air is transmitted after compression to some distance before it is used in a motor, and it has time to cool down to the temperature of the surrounding atmosphere, and in consequence it shrinks to the volume which it would have had had it been compressed isothermally; hence any work which is done in the compressor cylinder in addition to that required to compress the air isothermally is lost. The compression curve should therefore be kept as near to the isothermal curve as possible. This is done by cooling the air during compression, either by a water-jacket round the cylinder or by a water spray injected into the cylinder.

Compound Compressors.—A higher efficiency is obtained by compressing the air in two or more stages in separate cylinders, the air being cooled on its way from the larger to the smaller cylinder. The air can be cooled much more effectively in this way, because it can be passed through a long coil of pipe which presents a large surface to the cooling water which surrounds it.

The advantage of compounding is shown by the adjacent diagram. AB represents the volume of the low-pressure cylinder, and BCF is the compression curve, assuming that all the compression is produced in the low-pressure cylinder. When the pressure has been raised from P_1 to P_2 the volume has been reduced from AB to CL. The air is then delivered to the high-pressure cylinder, but on its way it is cooled, as explained above, and its volume is reduced to DL, which is the volume



of the high-pressure cylinder, the point D being on the isothermal BDH. In the high-pressure cylinder the volume is reduced from DL to EK, and the pressure is raised from P_2 to P_3. The area $CDEF$, as compared with the whole area $ABCFK$, represents the saving due to compressing the air in two cylinders instead of in one

The best results are said to be obtained when the proportions are those given by the formulæ below :—

P_1=absolute pressure of air during the suction stroke in the low-pressure cylinder.

P_2=absolute pressure of air delivered from low-pressure cylinder.

P_3=absolute pressure of air delivered from high-pressure cylinder.

V_1=volume of low-pressure cylinder.

V_2=volume of high-pressure cylinder.

$$P_2=\sqrt{P_1P_3}. \qquad \frac{V_2}{V_1}=\sqrt{\frac{P_1}{P_3}}.$$

Available Work in the Air in the Air Main.—

Let

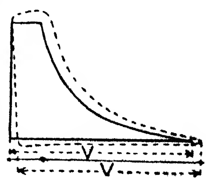
U = brake horse-power of compressor.

U_1 = indicated air horse-power $=\eta_m U$, where η_m = mechanical efficiency, and varies from 0·85 to 0·9.

U_2 = available indicated horse-power of the air delivered into the air main, assuming that the air cools down to atmospheric temperature.

$U_2=\eta_c U_1=\eta_m\eta_c U$, where η_c = compression efficiency and varies from 0·5 to 0·75 for single-stage compressors (in a good example η_c may be taken as 0·7). For a two-stage compressor η_c varies from 0·85 to 0·95 and may be taken as 0·9.

U_2 would be the indicated horse-power of the compressor cylinder if the air were drawn in at atmospheric pressure, compressed isothermally, and delivered at a uniform pressure, as shown by the full line diagram annexed. The probable actual indicator diagram, representing the indicated horse-power U_1, is shown dotted. The suction and delivery lines are outside the suction and delivery lines of the theoretical diagram because of the resistance offered by the valves and passages, and the compression line of the actual diagram is outside the isothermal line because of the heating of the air during compression.



Power Developed in Compressed Air Motor.—The maximum power is obtained when the air expands adiabatically in the cylinder of the motor.

P_2 and V_2=absolute pressure and volume respectively of air taken from air main by the motor in one stroke.

P_3=absolute pressure to which the air is expanded in the motor.

Maximum work done in one stroke$=\frac{n}{n-1}P_2V_2\left\{1-\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}\right\}$

If $n=1{\cdot}41$, then $\frac{n}{n-1}=3{\cdot}44$, and $\frac{n-1}{n}={\cdot}29$.

If before using the air in the motor the air be reheated so that its absolute temperature is raised from T_2 to T_3, then the theoretical maximum work done in one stroke will be increased to

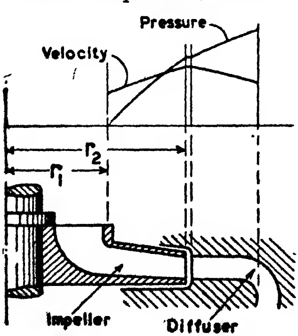
$$\frac{n}{n-1}P_2V_2\left\{1-\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}\right\}\frac{T_3}{T_2}.$$

If P_2 is in lbs. per square foot, and V_2 is in cubic feet, the work done will be expressed in foot-pounds.

Advantage of Reheating the Air.—If the air is reheated before passing into the motor, the available work is increased in the ratio of the absolute temperature after reheating to the absolute temperature before reheating. For example, if the absolute temperature before reheating is 520°, and the absolute temperature after reheating is 760°, then the available work will be $760\div520=1{\cdot}46$ times what it was before reheating. This great increase of work may be obtained by the expenditure of a comparatively small quantity of fuel in a suitable heater.

Besides increasing the work done in the motor, the reheating of the air obviates the inconvenience which attends a low temperature in the exhaust pipe of the motor.

Turbo-Compressors.—As the impeller rotates, it carries air round with it, and this air acquires the velocity of the impeller. Its kinetic energy is therefore increased as it travels from the centre to the edge. Its pressure will also increase, and the figure shows approximately the distribution of pressure and velocity of air in the impeller and diffuser.



If r_1 and r_2 be inner and outer radii of the impeller, and ω its angular velocity, then the tangential velocities of air at inlet and outlet are $u_1=\omega r_1$ and $u_2=\omega r_2$.

Let P_1 and P_2 be inlet and outlet pressures, and v_1 and v_2 velocities of whirl at inlet and outlet, then the pressure rise per stage is $P_2 - P_1 = P_1\left[\left\{\frac{v_2u_2 - v_1u_1}{K_pgT_1} + 1\right\}^{\frac{n-1}{n}} - 1\right]$, where T_1 is the absolute temperature at inlet, and K_p the specific heat of air at constant pressure in foot-pounds per degree.

Temperature at outlet, $T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$.

Work done per lb. of air, $W = \frac{1}{g}(v_2u_2 - v_1u_1)$.

For one stage the pressure ratio does not usually exceed 1·2. Any number of stages can be used, but about 12 is the usual limit. Since the pressure rise is reduced as the inlet temperature increases, cooling is essential on multi-stage compressors.

Blowers.—For very low pressures, *e.g.* 20 inches of water, single-stage centrifugal fans are often used.

If fan outlet is A sq. in., and pressure and velocity in the outlet pipe are P lbs. per sq. in. and v ft. per sec. respectively, then horse-power of fan $= \frac{PAv}{550}$.

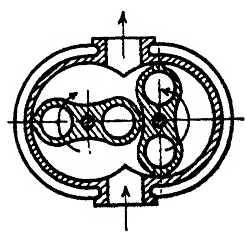
If ρ is density of air at pressure P,

$$P = \frac{v^2\rho}{2g}; \text{ hence horse-power } = \frac{Av^3\rho}{550 \times 2g}, \quad \propto v^3.$$

The efficiency of such fans is usually low.

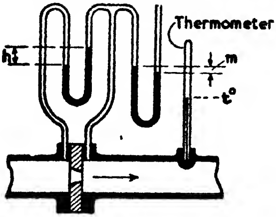
Turbo blowers and compressors have the advantage that their necessarily high speed enables them to be driven directly by steam-turbines or high-speed electric motors.

Roots Blower.—This is a type of gear pump having only two teeth or lobes per rotor. The rotors revolve in opposite directions, and the air trapped between rotor and casing is delivered to the outlet pipe. It will be seen that the volume of each rotor is approximately equal to the volume of air so trapped, so that the air delivered per revolution is $V = \frac{1}{2}\pi d^2 l$, where d is diameter of rotor, and l its length. If the pressure exceeds about 5 lbs. per sq. in. (gauge), the leakage due to the essential working clearances will reduce the output considerably.



Testing of Air Compressors.—For most purposes, the assessment of F.A.D., and brake horse-power necessary to supply this, is the test required by the user.

The F.A.D. is determined by the *British Standard Method*, as laid down in B.S. No. 726—1937. The set-up for testing is shown in the figure. The pressure difference across a calibrated nozzle and that between the downstream side of the nozzle and the atmosphere are measured by water manometers. The pressure at inlet is measured by a mercury barometer and air temperature is observed at inlet and outlet.



At inlet. Atmospheric pressure, in inches of mercury $=P_i$.
Air temperature (° F.) $=t_i$.

At nozzle. Head across nozzle, in inches of water $=h$. Difference in pressure between downstream side of nozzle and atmosphere $=m$ inches of water $=m/13{\cdot}6$ inches of mercury. Absolute pressure on downstream side of nozzle $=m_2$ inches of mercury. $m_2=$ atmospheric pressure $\pm(m/13{\cdot}6)$.
The temperature of air in the pipe line (° F.) $=t$.

Then $\text{F.A.D.}=K\left(\frac{T_i}{P_i}\right)\sqrt{h}\sqrt{\frac{m_2}{T}}$ cubic feet per minute, where $T_i=459{\cdot}6+t_i$ and $T=459{\cdot}6+t$, while the value of K is obtained from the following table *:—

Nozzle. Diam. in Inches.	Suitable Pipe Line. Internal Diam. Inches.	Approx. F.A.D., Cu. Ft./Min.			Constant K.
		$h=0{\cdot}4$ In.	$h=4$ In.	$h=40$ In.	
$\frac{3}{8}$	$\frac{7}{8}$	..	6	18	0·73
$\frac{5}{8}$	$1\frac{1}{2}$	..	16	50	2·03
1	$2\frac{1}{2}$	..	42	130	5·18
$1\frac{1}{2}$	$3\frac{1}{2}$	30	90	300	11·65
$2\frac{1}{2}$	6	80	260	800	32·4
4	10	210	660	2100	82·8
6	15	470	1500	4700	187
10	24	1300	4100	13000	518
15	36	3000	9250	30000	1165

Transmission of Power by Compressed Air.

$P_a=$ pressure in lbs. per square foot . . ⎫
$p_a=$ pressure in lbs. per square inch . . ⎬ of air admitted from the atmosphere to the compressor.
$V_a=$ volume of 1 lb. in cubic feet . . . ⎪
$T_a=$ absolute temperature in degs. Fahr. ⎭

P_1, p_1, V_1, and T_1, the corresponding quantities for air discharged from the compressor into the main.

* Extracted from B.S. No. 726—1937.

P_2, p_2, V_2, and T_2, the corresponding quantities for air arriving at the point of consumption in the main.
U = brake horse-power of compressor.
$U_1 = \eta_m U$ = indicated air horse-power.
$U_2 = \eta_c U_1$ = isothermal horse-power.
U_3 = adiabatic horse-power of motor.

Taking $P_a = 2116 \cdot 8$, $p_a = 14 \cdot 7$, $V_a = 13 \cdot 09$, $T_a = 521°$,
$P_a V_a = 27710$ . . . . (1)

W = weight of air compressed, in lbs., per second, by U horse-power in driving unit.

$$550 \eta_m \eta_c U = 27710 W \log_e \frac{p_1}{p_a}. \qquad W = \frac{\eta_m \eta_c U}{50 \cdot 4 \log_e \frac{p_1}{p_a}} \quad . \quad (2)$$

$$P_1 V_1 = 27710. \qquad p_1 V_1 = 192 \cdot 3 \quad . \quad . \quad (3)$$

v_1 = initial velocity of air in main in feet per second.
d = diameter of main in feet. l = length of main in feet.

$$\frac{\pi}{4} d^2 v_1 = W V_1 = 192 \cdot 3 \frac{W}{p_1}. \qquad d = 15 \cdot 64 \sqrt{\frac{W}{p_1 v_1}} \quad . \quad . \quad (4)$$

$$\frac{p_2}{p_1} = \sqrt{\left\{1 - \frac{v_1^2 l}{74{,}300{,}000 d}\right\}} \quad . \quad . \quad . \quad . \quad (5)$$

If the air is used in the motor without reheating, then,

$$U_3 = \frac{95600 W}{550} \left\{1 - \left(\frac{p_a}{p_2}\right)^{0 \cdot 29}\right\} \quad . \quad . \quad . \quad (6)$$

If the air is reheated to temperature T_3 before admission to the air-motor, then,

$$U_3' = \frac{95600 W}{550} \left(\frac{T_3}{T_a}\right) \left\{1 - \left(\frac{p_a}{p_2}\right)^{0 \cdot 29}\right\} \quad . \quad . \quad (7)$$

Pneumatic Tools.—The following gives a few applications of compressed air for tools used in workshops, together with the air consumption of representative types. The working pressure is usually of the order of 80 lbs. per square inch:—

Type of Tool.	Duty.	Speed of Operation. Revs. or Strokes per Minute.	Air Consumption. Cubic Feet of Free Air per Minute.
Drill . . .	Light	1000–1500	18–20
	Heavy	350–600	30–50
Grinder . .	..	3000–6000	25–45
Hammer, chipping	Extra light	4500	6
	Light	2000–3000	15–18
	Heavy	1500	18
Hammer, riveting	Light	3500	8
	Medium	1500–3000	15–20
	Heavy	1000–1600	26–30
Sand rammers .	..	750–1000	12–22

The Flow of Gas or Steam through Pipes.—In *Engineering*, vol. lxiii. p. 361, Mr. Arthur J. Martin examines the best-known formulæ for the flow of gas or steam through pipes, and finally approves of Unwin's formula, with a slight change in the constant. Mr. Martin then derives from the fundamental formula the other formulæ given below, so that if all the factors but one are known, that one can be determined.

p = loss of pressure in lbs. per square inch.
w = delivery of fluid in lbs. per minute.
Q = delivery of fluid in cubic feet per minute.
v = velocity in feet per second.
d = diameter of pipe in inches.
L = length of pipe in feet.
D = density of fluid in lbs. per cubic foot.
F = loss of pressure in feet of fluid.
H = loss of pressure in inches of water $= 27{\cdot}7p$.
M = loss of pressure in inches of mercury $= 2{\cdot}04p$.

$$p = \frac{w^2 L\left(1+\frac{3\cdot6}{d}\right)}{7000Dd^5} = \frac{Q^2 DL\left(1+\frac{3\cdot6}{d}\right)}{7000d^5} = \frac{v^2 DL\left(1+\frac{3\cdot6}{d}\right)}{65360d}.$$

$$w = \sqrt{\frac{7000pDd^5}{L\left(1+\frac{3\cdot6}{d}\right)}} = QD = \frac{vDd^2}{3\cdot056}.$$

$$Q = \frac{w}{D} = \sqrt{\frac{7000pd^5}{DL\left(1+\frac{3\cdot6}{d}\right)}} = \frac{vd^2}{3\cdot056}.$$

$$v = \frac{3\cdot056w}{Dd^2} = \frac{3\cdot056Q}{d^2} = 256\sqrt{\frac{pd}{DL\left(1+\frac{3\cdot6}{d}\right)}}$$

$$= \sqrt{\frac{454dF}{L\left(1+\frac{3\cdot6}{d}\right)}}.$$

$$d = \sqrt[5]{\frac{\cdot0002w^2L}{pD}} = \sqrt[5]{\frac{\cdot0002Q^2DL}{p}} \text{ approximately}$$

$$L = \frac{7000pDd^5}{\left(1+\frac{3\cdot6}{d}\right)w^2} = \frac{7000pd^5}{\left(1+\frac{3\cdot6}{d}\right)Q^2D}.$$

$$F=\frac{w^2L\left(1+\frac{3\cdot6}{d}\right)}{49D^2d^5}=\frac{Q^2L\left(1+\frac{3\cdot6}{d}\right)}{49d^5}.$$

$$=\frac{v^2L\left(1+\frac{3\cdot6}{d}\right)}{454d}=\frac{144p}{D}.$$

Mr. Martin adds, where the loss of pressure in a pipe is but a small proportion of the original pressure, no great inaccuracy will result from using the density due to the original pressure, or preferably that due to the mean of the original and terminal pressures if both are known. But in long mains, where the loss of pressure is very great, it is better to ascertain the initial velocity by means of the following formula :—

$$u=\sqrt{\left\{\frac{480000d\,(P_1^2-P_2^2)}{\left(1+\frac{3\cdot6}{d}\right)LD_aP_1^2}\right\}}$$

where u = initial velocity in feet per second.
P_1 = absolute initial pressure in lbs. per square inch.
P_2 = absolute terminal pressure in lbs. per square inch.
D_a = density of the gas at atmospheric pressure.
d = diameter of pipe in inches.
L = length of pipe in feet.

The weight of gas delivered can then be ascertained by substituting the value of u thus arrived at for v in the equation,

$$w=\frac{vDd^2}{3\cdot056}.$$

The loss of pressure given by the foregoing formulæ is for a straight pipe.

Mr. Martin concludes by giving the following table, calculated from Hurst's formulæ :—

Length in feet of straight pipe equivalent to a square elbow and to a bend of radius = diameter.

Diam.	Elbow.	Bend.	Diam.	Elbow.	Bend.	Diam.	Elbow.	Bend.
Inches.	Feet.	Feet.	Inches.	Feet.	Feet.	Inches.	Feet.	Feet.
1	1·5	·23	7	32·0	4·8	16	90·1	13·5
2	4·9	·74	8	38·0	5·7	18	104	15·5
3	9·4	1·41	9	44·4	6·7	20	117	17·5
4	14·5	2·2	10	50·7	7·6	22	130	19·6
5	20·0	3·0	12	63·7	9·6	24	144	21·6
6	25·9	3·9	14	76·7	11·5	...	...	...

The loss of pressure at a screw-down valve is about $1\frac{1}{2}$ times that due to a square elbow.

HYDRAULIC TRANSMISSION OF POWER

In **low-pressure systems** the water is stored in reservoirs, and the pressure is obtained by "head" of water, the head being the altitude of the surface of the water in the reservoir above the point at which the power is used. If H=head of water in feet, and p=pressure in lbs. per square inch corresponding to the head H, then, neglecting friction, $p = \cdot 433H$.

Generally H does not exceed 600 feet, which corresponds to a pressure of 260 lbs. per square inch.

In the ordinary water mains of towns the water pressure is generally from 50 to 100 lbs. per square inch.

In **high-pressure systems** with accumulator storage the pressure of the water is obtained by means of pumps, and is usually from 700 to 800 lbs. per square inch. In the systems at work in Manchester and Glasgow, the pressure of the water is 1120 lbs. per square inch.

The **velocity of the water** in the mains does not generally exceed 3 feet per second, but is in some cases as high as 5 feet per second.

The **energy transmitted** by a pipe having a diameter d inches, carrying water at a pressure of p lbs. per square inch at a velocity of v feet per second is, $\frac{\pi}{4}d^2pv$ foot-lbs. per second, and the horse-power is $\frac{\pi d^2pv}{4 \times 550}$.

If p=pressure of water in lbs. per square inch, then,

1 lb. weight of water used produces $2 \cdot 3p$ foot-lbs. of work.
1 gallon of water used produces $23p$ foot-lbs. of work.
1 cubic inch of water used produces $\frac{1}{12}p$ foot-lbs. of work.
1 cubic foot of water used produces $144p$ foot-lbs. of work.

The **pipes** used are made of cast-iron, and for a working pressure of 800 lbs. per square inch they do not exceed $7\frac{1}{2}$ inches in diameter.

The **loss of pressure due to friction** in the pipes, per mile of length, at a velocity of 3 feet per second, is, according to Unwin, about $\frac{107}{d}$ lbs. per square inch, where d is the diameter of the pipe in inches.

The **pumping engines** used are generally of the direct acting triple expansion type. The following are the particulars of the pumping engines at the Glasgow hydraulic power station.* The engines are of the vertical, inverted cylinder, triple expansion type. Diameter of H.P. cylinder, 15 inches. Diameter of I.P.

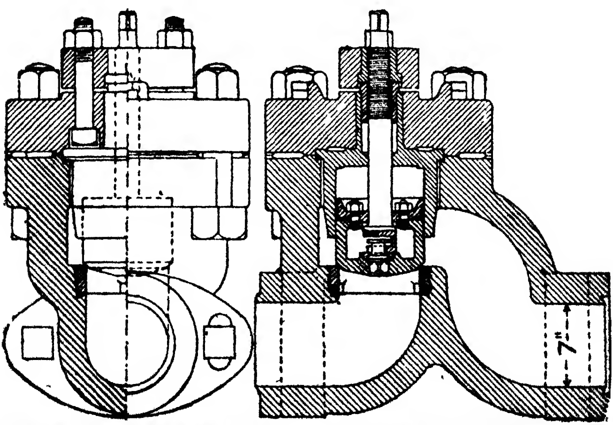
* *Engineering*, vol. lviii. p. 34.

cylinder, 22 inches. Diameter of L.P. cylinder, 36 inches. Stroke of pistons, 2 feet. Each cylinder is steam-jacketed. Depth of pistons, 7¼ inches. Diameter of piston-rods, 3 inches. The L.P. cylinder can either exhaust into the atmosphere or into a surface condenser having 530 square feet of cooling surface. Diameter of crank shaft, 7 inches. Angles between cranks, 120 degrees. Diameter of fly-wheel, 7 feet. Weight of fly-wheel, 2 tons. The crank shaft is below the pumps, and the connecting-rods are made double so as to clear the pump bodies.

The air, circulating, and feed pumps, each having a stroke of 16 inches, are driven from the cross-head of the intermediate cylinder by a rocking lever and links. Diameter of air pump (single acting), 13 inches. Diameter of circulating pump (double acting), 8 inches. Diameter of feed pump (single acting), 2 inches.

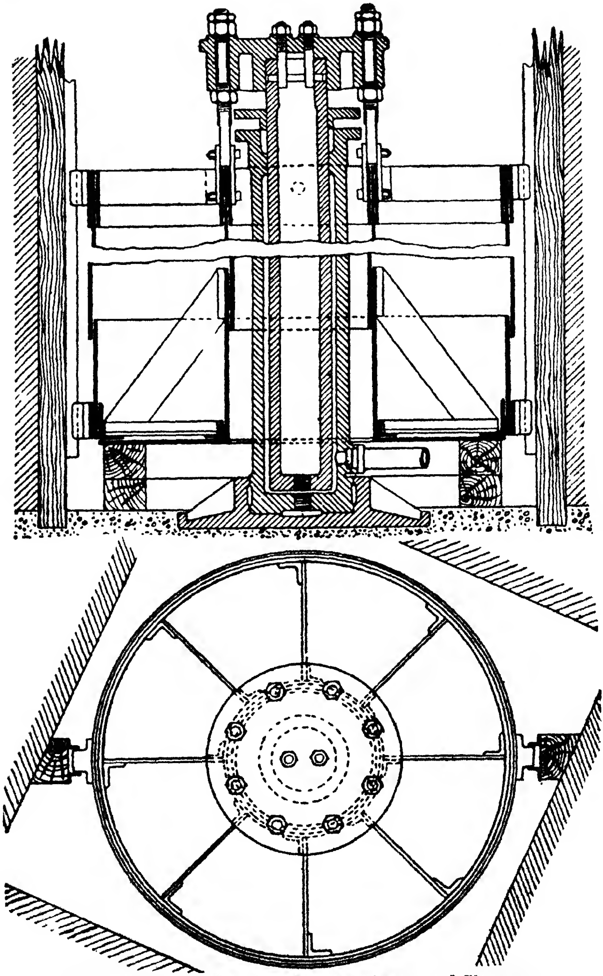
Each steam cylinder drives a single-acting ram pump, the piston-rod and ram being cottered to the same cross-head. The pump bodies are of cast-iron. The rams are of gun-metal, 4½ inches in diameter, with conoidal ends.

The indicated horse-power of each set of engines is 200, and each set is capable of delivering 230 gallons of water per minute against an accumulator pressure of 1120 lbs. per square inch, with a steam pressure of 120 lbs. per square inch, and a piston speed of 240 feet per minute. Weight of feed water, 15 lbs. per I.H.P. per hour.



Transverse Section and Elevation. *Longitudinal Section.*

Hydraulic Stop Valve.



Hydraulic Accumulator used at Manchester and Glasgow.
Diameter of ram, 18 inches; stroke, 23 feet; total load, 127 tons, inclusive of ram and casing.

The **accumulators** at hydraulic power stations have rams, which are generally from 18 to 20 inches in diameter, and the stroke is from 20 to 23 feet. Where two accumulators are used, one is loaded to about 20 lbs. per square inch more than the other, so that one does not rise until the other is at the top of its stroke.

If D = diameter of the ram in inches, h = stroke of ram in feet, W = load on ram in lbs., and p = pressure of water in lbs. per square inch, then, neglecting friction, $W=\frac{\pi}{4}D^2p$, and the number of foot-lbs. of work which may be stored up in the accumulator $= Wh = \frac{\pi}{4}D^2ph$.

The **mechanical efficiency** of the pumping-engines may be taken at from 75 to 80 per cent. The efficiency of an hydraulic press with one ram and one gland or packing leather is about 90 per cent. The efficiency of lifts and cranes fully loaded is about 55 per cent.

The efficiency of a ram with a chain and pulley multiplying gear of good design, and well lubricated, is given approximately by the formula—

$$\text{Efficiency per cent.} = 84 - 2m,$$

where m is the ratio of multiplying power.

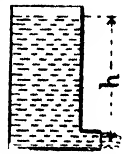
HYDRAULICS.

Pressure due to Head of Water.—

h = head of water in feet.
w = weight of one cubic foot of water in lbs.
p = pressure of water in lbs. per square foot due to head h.

$$p = wh. \qquad h = p \div w.$$

Pressure in lbs. per square inch $= p \div 144$.
For weight of water see p. 342.
For table of pressures of water corresponding to various heads see p. 343.

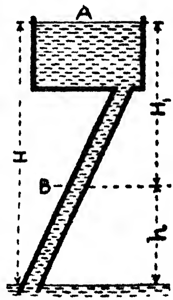


Velocity due to Head of Water.—

h = head of water in feet.
v = velocity of water in feet per second due to head h when there are no resistances.

$$v = \sqrt{2gh}.$$

Energy and Pressure of Water in Motion in a Pipe.—The adjacent illustration shows a tank, open at the top, from which a pipe conveys water to some datum level, say, the level of the sea. H and h are heights in feet above the datum level. Consider a mass of water weighing 1 lb. At A this mass of water has H foot-lbs. of *potential energy*, and it has no velocity and no pressure. At the level B the 1 lb. mass of water has h foot-lbs. of potential energy, and $\frac{v^2}{2g}$ foot-lbs. of *kinetic energy*, where v is the velocity of the water at B in feet per second. If v is less than $\sqrt{2gH_1}$ the mass of water at B will exert a pressure of p lbs. per square foot such that, $\frac{v^2}{2g}+\frac{p}{w}=H_1$, or $p=w\left(H_1-\frac{v^2}{2g}\right)$, where w = weight of 1 cubic foot of water in lbs. H_1 is called the *equivalent head* of water having a velocity v and pressure p. $\frac{p}{w}$ is called the *pressure energy* of the water.



The total energy in the 1 lb. mass of water at B is,

$$\frac{v^2}{2g}+\frac{p}{w}+h=H,$$

which is equal to the energy in the same mass of water at A. It is assumed that there is no loss of energy due to friction of the water in the tank or pipe.

If the sectional area of the pipe varies, the velocity of the water will vary inversely. Thus if the sectional area changes from a_1 to a_2, the velocity will change from v_1 to v_2 such that $v_1 a_1 = v_2 a_2$; also, the pressure will change from p_1 to p_2, and if the potential head or potential energy changes from h_1 to h_2, then,

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + h_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + h_2.$$

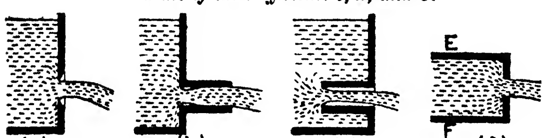
Discharge of Water through Small Orifices.—

H = head of water in feet.
v = velocity of discharge in feet per second, resistances neglected.
v_1 = actual velocity of discharge in feet per second.
c = coefficient of velocity $= v_1 \div v$.
A = area of orifice in square feet.
A_1 = area of smallest section of jet in square feet.
k = coefficient of contraction $= A_1 \div A$.
C = coefficient of discharge $= ck$.
Q = quantity of water discharged per second in cubic feet.

$$v = \sqrt{2gH}. \qquad v_1 = cv = c\sqrt{2gH}.$$

$$Q = v_1 A_1 = ckvA = ckA\sqrt{2gH} = CA\sqrt{2gH}.$$

Values of the coefficients c, k, and C.



(a) (b) (c) (d)

(*a*) **Sharp edged orifices.**
Rectangular. $c = \cdot 97$. $k = \cdot 6$ to $\cdot 63$. $C = \cdot 58$ to $\cdot 61$.
Circular. $c = \cdot 97$. $k = \cdot 64$. $C = \cdot 62$.

(*b*) Square edged orifice and external pipe.
$c = 1$ (nearly). $k = \cdot 815$. $C = \cdot 815$.

(*c*) Square edged orifice and internal pipe.
$c = \cdot 98$. $k = \cdot 5$. $C = \cdot 49$.

(*d*) Rankine gives the following empirical formula for determining k where a pipe EF of sectional area S discharges into the atmosphere through an orifice of sectional area A:—

$$\frac{1}{k} = \sqrt{2\cdot 618 - 1\cdot 618\frac{A^2}{S^2}}.$$

Formulæ for Friction of Water in Pipes.—Of the formulæ given below, Unwin's is probably the most exact, but Darcy's has been most used.

d=internal diameter of pipe in feet.
v=velocity of water in feet per second.
l=length of straight pipe in feet.
h=loss of head in feet, due to friction of water in pipe, or head necessary to overcome friction.

Unwin's formula :—

$$h=\frac{m}{d^x}\times l\times\frac{v^n}{2g}.$$

Values of m, x, and n are given in the following table :—

Kind of Pipe.	m	x	n
Wrought-iron (gas)	·0226	1·21	1·75
New cast-iron	·0215	1·168	1·95
Cleaned cast-iron	·0243	1·168	2·0
Incrusted cast-iron	·044	1·16	2·0

Darcy's formula :—

$$h=z\left\{1+\frac{1}{12d}\right\}\times\frac{4l}{d}\times\frac{v^2}{2g}.$$

z=·005 for new cast-iron pipes.
z=·01 for old incrusted cast-iron pipes.

Weisbach's formula :—

$$h=\frac{2l}{d}\left\{\cdot 0072+\frac{\cdot 0086}{\sqrt{v}}\right\}\times\frac{v^2}{2g}.$$

Weisbach's formula for the loss of head due to a **bend in a pipe** is as follows :—

$$h=\cdot 131+1\cdot 847\left(\frac{d}{2R}\right)^{3\cdot 5}\times\frac{v^2}{2g}\times\frac{a}{180},$$

where R=radius of curvature of centre line of bend in feet, and a=angle, in degrees, subtended by the bend at its centre of curvature. The other symbols have the same meanings as before.

The **loss of energy** due to loss of head is equal to the weight of water delivered in lbs. multiplied by the loss of head in feet.

General Formula for Hydraulic Resistances.—

v_1=actual velocity of water in feet per second at the point where the resistance is encountered.
h=loss of head, in feet, due to resistance.
F=coefficient of resistance.

$$h=F\frac{v_1^2}{2g}.$$

Professor Cotterill in his "Applied Mechanics," fourth edition, p. 486, gives the following values of F:—

Orifice in a thin plate, $F = \cdot 06$.

Square-edged entrance of a pipe, $F = \cdot 5$.

Sudden enlargement of a pipe in the ratio $m:1$, $F = (m-1)^2$, referred to velocity through larger part of pipe.

Bend at right angles in a pipe, radius of bend = three times diameter of pipe, $F = \cdot 14$.

Quick bend at right angles, radius of bend = diameter of pipe, $F = \cdot 3$.

Common cock partially closed, handle turned through 15°, 30°, 45° from position when full open. $F = \cdot 75$, 5·5, and 31 respectively.

Knee in a pipe at right angles, $F = 1$.

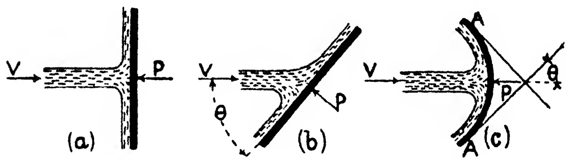
Surface friction of a pipe, the length of which is n times the diameter, $F = 4fn$. For a clean cast-iron pipe d inches in diameter, $4f = \frac{\cdot 036}{\sqrt[6]{d}}$

Impact of Water Jets.—

V = velocity of water before impact in feet per second.

W = weight of water delivered per second in lbs.

P = reaction of plate on jet in lbs.



(a) Plate at right angles to direction of jet.

$$P = \frac{WV}{g}.$$

(b) Plate inclined at an angle θ to direction of jet.

$$P = \frac{WV \sin \theta}{g}.$$

(c) Jet impinges at the centre of a cup.

$$P = \frac{WV(1 + \cos \theta)}{g}.$$

θ is the angle between the tangent at A and the centre line of the jet.

Turbines.

Classification of Turbines.—Considering the action of the water on the vanes, turbines are either *reaction wheels* or *impulse wheels.* In reaction turbines the motive power is obtained mainly from the pressure energy of the water, while in impulse turbines the motive power is obtained from the kinetic energy of the water.

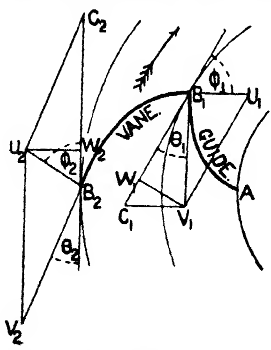
In reaction turbines all the passages in the guide and wheel are filled with water, and the turbine is said to be *drowned.* It follows that in this type of turbine the product, velocity of water across any section of the passage, multiplied by the area of that section, is constant. Reaction turbines are generally used where there is a large quantity of water available under a small or moderate head.

In impulse turbines the buckets are not filled by the water passing through them, and they must be so constructed that the atmosphere has free access to the spaces in the buckets not occupied by the water. Impulse turbines are suitable for moderate or small quantities of water under a great head.

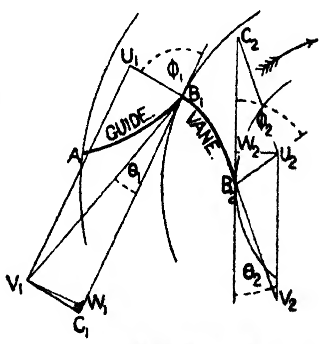
Turbines may also be classed according to the direction in which the water flows through them, into, radial flow, axial or parallel flow, and combined or mixed flow. Radial flow turbines are again divided into two classes, viz., outward flow and inward flow.

The axis of a turbine may be either vertical or horizontal.

Formulæ for Reaction Turbines.—Water enters the fixed guide wheel at A, and is guided by the fixed guide vane AB_1, which delivers the water in the direction V_1B_1 on to the vane or bucket



Outward flow turbine.
Section perpendicular to axis of wheel.



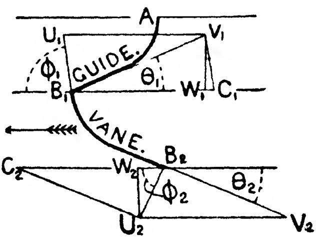
Inward flow turbine.
Section perpendicular to axis of wheel.

B_1B_2 of the revolving wheel at B_1. The water leaves the revolving wheel at B_2 in the direction B_2U_2.

B_1C_1 and B_2C_2 are tangents to the wheel circles at B_1 and B_2 respectively. B_1V_1 is a tangent to the guide vane at B_1. B_1U_1 and B_2V_2 are tangents to the wheel vane at the points B_1 and B_2 respectively. V_1W_1 and U_2W_2 are perpendiculars to B_1C_1 and B_2C_2 respectively.

Note.—A turbine has a considerable number of guides and buckets, but in the illustrations only one of each is shown. The number of guides is generally not the same as the number of buckets.

$v_1 = V_1B_1$ = absolute velocity of outflow of water from guide passages.



Parallel flow turbine.

Section parallel to axis of wheel at mean radius.

$u_2 = B_2U_2$ = absolute velocity of outflow of water from wheel passages.

$c_1 = C_1B_1$ = tangential velocity of wheel at B_1.

$c_2 = B_2C_2$ = tangential velocity of wheel at B_2.

c = tangential velocity of wheel at mean radius r.

$u_1 = U_1B_1$ = relative velocity of inflow of water.

$v_2 = B_2V_2$ = relative velocity of outflow of water.

$w_1 = W_1B_1$ = tangential velocity of water, or velocity of whirl, at entry.

$w_2 = B_2W_2$ = tangential velocity of water, or velocity of whirl, at exit.

r_1 = radius of wheel at B_1 in radial flow turbines.

r_2 = radius of wheel at B_2 in radial flow turbines.

r = mean radius of wheel in parallel flow turbines.

θ_1 = guide angle, or angle of entrance of water.

ϕ_1 = vane angle at entrance.

θ_2 = vane angle at exit.

ϕ_2 = angle of exit of water.

A_1 = area of guide passages at exit, at right angles to direction of flow.

A_2 = area of wheel passages at exit, at right angles to direction of flow.

H = available head of water.

Q = quantity of water available per second in cubic feet.

g = accelerating effect of gravity = 32·2.

E = efficiency of turbine.

$\overline{H\text{-}P}$ = effective or brake horse-power of turbine.
K_1 = coefficient of velocity.
K_2 = coefficient of wheel speed.

All velocities are in feet per second, linear dimensions are in feet, and areas are in square feet.

The following formulæ give the relations between the various velocities:—

$u_1 \sin \phi_1 = v_1 \sin \theta_1.$

$w_1 = v_1 \cos \theta_1.$

$u_1 \sin \phi_1 \cos \theta_1 = w_1 \sin \theta_1.$

$c_1 = w_1 + u_1 \cos \phi_1.$

$c_1 = w_1 (1 + \tan \theta_1 \cot \phi_1).$

$$c_1 = w_1 \frac{\sin(\theta_1 + \phi_1)}{\cos \theta_1 \sin \phi_1}.$$

$$c_1 = v_1 \frac{\sin(\theta_1 + \phi_1)}{\sin \phi_1}.$$

$$\tan \phi_1 = \frac{v_1 \sin \theta_1}{c_1 - v_1 \cos \theta_1}.$$

$u_2 \sin \phi_2 = v_2 \sin \theta_2.$

$w_2 = u_2 \cos \phi.$

$v_2 \sin \theta_2 \cos \phi_2 = w_2 \sin \phi_2.$

$c_2 = w_2 + v_2 \cos \theta_2.$

$$c_2 = w_2 \frac{\sin(\theta_2 + \phi_2)}{\sin \theta_2}.$$

In parallel flow turbines

$c_2 = c_1 = c.$

If $\phi_2 = 90°$, then $w_2 = 0$, and

$c_2 = v_2 \cos \theta_2 = u_2 \cot \theta_2.$

Work done per lb. of water passing through wheel $= \dfrac{c_1 w_1 - c_2 w_2}{g}$.

This will become $\dfrac{c_1 w_1}{g}$ when $w_2 = 0$, which is the case when $\phi_2 = 90°$.

Values of the angles.—The angles θ_1 and θ_2 are assumed in commencing the design of a turbine. In practice θ_1 varies from 15° to 30° in radial outward flow turbines, and from 10° to 25° in radial inward flow and parallel flow turbines. θ_2 varies from 10° to 25°.

Ratio of area A_1 to area A_2.—The ratio $A_1 \div A_2$ is also assumed. In practice $A_1 \div A_2$ varies from ·5 to 1 in radial outward flow turbines, and from ·6 to 1·5 in radial inward flow turbines. In parallel flow turbines $A_1 \div A_2$ is usually about 1.

Ratio of radius r_1 to radius r_2.—The ratio $r_1 \div r_2$ is also assumed. For outward flow $r_1 \div r_2$ varies from ·7 to ·85, and for inward flow from 1·2 to 2.

Efficiency.—The efficiency E of a turbine of good design is usually from ·75 to ·85, and for purposes of design it may be taken at ·8.

Coefficient of velocity.—$K_1 = \sqrt{\dfrac{E}{2\dfrac{A_1}{A_2}\dfrac{r_1}{r_2} \cos \theta_1 \cos \theta_2}}$.

For parallel flow turbines the ratio $\dfrac{r_1}{r_2} = 1$.

Velocity of outflow from guide passages.—$v_1 = K_1 \sqrt{2gH}$.

Coefficient of wheel speed.—

$$K_2 = K_1 \frac{A_1}{A_2} \frac{r_1}{r_2} \cos\theta_2 = K_1 \frac{\sin(\theta_1+\phi_1)}{\sin\phi_1} \text{ for radial flow.}$$

$$K_2 = K_1 \frac{A_1}{A_2} \cos\theta_2 = K_1 \frac{\sin(\theta_1+\phi_1)}{\sin\phi_1} \text{ for parallel flow.}$$

Wheel speed.—Circumferential speed at radius $r_1 = c_1 = K_2\sqrt{2gH}$ in radial flow turbines.

Circumferential speed at mean radius $r = c = K_2\sqrt{2gH}$ in parallel flow turbines.

Revolutions of wheel per minute $= \frac{60K_2\sqrt{2gH}}{2\pi r_1} = \frac{76{\cdot}63K_2\sqrt{H}}{r_1}$ for radial flow turbines. For parallel flow turbines substitute r for r_1.

Area of guide passages.—$A_1 = \frac{Q}{v_1}$.

Effective or brake horse-power.—$\overline{H\text{-}P} = \frac{62{\cdot}3EQH}{550}$.

PUMPS.

Reciprocating Pumps.—

D = diameter of plunger, piston, or bucket, in inches.
d = diameter of piston-rod or bucket-rod, in inches.
L = length of stroke, in inches.
C = discharging capacity, in cubic feet, for two consecutive strokes.
G = discharging capacity, in gallons, for two consecutive strokes.
$G = 6{\cdot}23C$, for water.
H = delivery head, in feet.
h = suction head, in feet.
F = load on pump, or force required to move plunger, piston, or bucket, in lbs. (friction neglected).

Discharging Capacity of Reciprocating Pumps.—

Plunger pumps.—$C = \frac{\pi}{4} \times \frac{D^2L}{1728} = \frac{D^2L}{2200}$.

Double-acting piston pumps.—$C = \frac{(2D^2 - d^2)L}{2200}$.

Single-acting piston pumps.—$C = \frac{D^2L}{2200}$ or $\frac{(D^2 - d^2)L}{2200}$, depending on whether the outer or inner side of the piston is next the liquid.

Single-acting bucket pumps.—$C=\frac{D^2L}{2200}$. In practice this may be considerably exceeded if the bucket is worked rapidly, on account of the inertia of the water preventing the suction valve closing when the bucket commences its return stroke. The flow of water through the pump therefore continues at a diminishing rate during a portion of the return or idle stroke of the bucket.

Combined bucket and plunger pumps.—The discharging capacity for two consecutive strokes is the same as for the bucket without the plunger, viz., $\frac{D^2L}{2200}$, where D is the diameter of the bucket. The presence of the plunger causes a portion of the water raised by the bucket to be delivered during the up stroke and the remainder during the down stroke. If the delivery is to be uniform the area of the plunger should be half that of the bucket, or $d=D\div\sqrt{2}=\cdot707D$, where d=diameter of plunger, and D=diameter of bucket.

Load on a Pump.—

Plunger pump.—

Suction stroke. $F=\frac{\pi}{4}D^2\times\cdot433h=\cdot34D^2h$.

Delivery stroke. $F=\cdot34D^2H$.

Double-acting piston pump.—

Forward stroke. $F=\cdot34\{D^2h+(D^2-d^2)H\}$.

Return stroke. $F=\cdot34\{D^2H+(D^2-d^2)h\}$.

If d the diameter of the piston-rod is neglected, then for each stroke $F=\cdot34D^2(H+h)$.

Single-acting piston pump.—

Suction stroke. $F=\cdot34D^2h$, or $\cdot34(D^2-d^2)h$.

Delivery stroke. $F=\cdot34D^2H$, or $\cdot34(D^2-d^2)H$.

The formulæ to the right are to be used when the water is on the piston-rod side of the piston.

Single-acting bucket pump.—

Up stroke. $F=\cdot34\{D^2h+(D^2-d^2)H\}$.

Down stroke. $F=\cdot34d^2H$.

If d is neglected, then $F=\cdot34D^2(H+h)$ for the up stroke, and $F=0$ for the down stroke.

Combined bucket and plunger pump.—

Up stroke, suction and delivery. $F=\cdot34\{D^2h+(D^2-d^2)H\}$.

Down stroke, delivery only. $F=\cdot34d^2H$.

Where D=diameter of bucket, and d=diameter of plunger.

In *vertical pumps* the weight of the parts raised must be added to F in the foregoing formulæ for the up stroke, and deducted for the down stroke.

Friction in Pipes.—The "loss of head" due to friction in the pipes may be determined by one of the formulæ on p. 728, and if this be added to the actual head in the foregoing formulæ, the force F_1 required to overcome the pressure due to the weight of the water and the friction in the pipes will be found.

Friction of Pumps.—The friction of the moving parts of a pump will increase the force required to work it by from $\frac{1}{10}F_1$ to $\frac{1}{4}F_1$.

Speed of Reciprocating Pumps.—The speed of a pump plunger, piston, or bucket is generally from 50 to 200 feet per minute; speeds of 100 to 150 feet per minute are very common. The speed may be increased as the length of the stroke is increased. A safe rule is, speed in feet per minute $= 45\sqrt[3]{L}$, where L is the length of the stroke in inches.

The air and circulating pumps of marine engines generally run at speeds of 200 to 400 feet per minute.

Diameter of Pipes.—Velocity of Water in Pipes.—The suction and delivery pipes generally have a diameter equal to three-fourths of the diameter of the pump barrel. This makes the velocity of the water in the pipes 1·78 times the speed of the plunger or piston. Sometimes the suction pipe is made larger than the delivery pipe.

Air Vessels.—The delivery of water is more uniform, and the shocks at the beginning of a stroke are reduced by placing an air vessel near to the delivery valve. The volume of the air vessel varies greatly in practice. The volume of the air vessel may be from two to six times the capacity of the pump barrel, sometimes it is as much as ten times.

An air vessel on the suction pipe near the suction valve is also desirable when the suction pipe is long. The capacity of this vessel may be from two to four times the capacity of the pump barrel. The air in this vessel has. of course, a pressure less than that of the atmosphere.

Size of Steam Cylinder for Direct-acting Steam Pump.—

D = diameter of steam cylinder.

d = diameter of pump barrel.

P = mean effective pressure of steam on steam piston.

p = mean effective pressure of water on water piston.

$$\cdot 75D^2P = d^2p. \qquad D = 1\cdot 15d\sqrt{\frac{p}{P}}.$$

This allows 25 per cent. of the work done in the steam cylinder for the friction of the machine. In large modern pumping engines an allowance of 10 per cent. for the friction of the machine would be sufficient.

Centrifugal Pumps.—

d = diameter of suction and delivery pipes in inches.

D = diameter of fan or wheel in inches.

C = water discharged per minute in cubic feet.
H = total height to which water is raised in feet.
v = circumferential speed of wheel in feet per second.
N = number of revolutions of wheel per minute.
V = velocity of water in pipes in feet per minute.
V is generally between 300 and 500 feet per minute.

$$\frac{\cdot 7854 d^2 V}{144} = C. \qquad \therefore d = \frac{13\cdot 54}{\sqrt{V}} \sqrt{C}.$$

If $V = 350$, then $d = \cdot 72 \sqrt{C}$. If $V = 450$, then $d = \cdot 63 \sqrt{C}$.
$D = 2d$ to $3d$.
v varies from $8\sqrt{H}$ in small pumps to $10\sqrt{H}$ in large pumps.

$$N = \frac{229\cdot 2v}{D}.$$

Efficiency of Pumps.—With lifts of 2 or 3 feet the efficiency of a centrifugal pump of moderate size is about 50 per cent. As the lift increases the efficiency increases, and reaches a maximum of about 70 per cent. when the lift is from 15 to 20 feet. Beyond this the efficiency diminishes. With a lift of 50 feet the efficiency is about 50 per cent. The efficiency of large pumps is greater than the efficiency of small ones. Centrifugal pumps are best adapted for low lifts.

The efficiency of reciprocating pumps is small with low lifts. With a lift of 10 feet the efficiency is only from 20 to 30 per cent.* The efficiency increases with the lift. With a lift of 100 feet the efficiency may reach 85 per cent. Reciprocating pumps are therefore best adapted for high lifts.

* Professor Hood of Michigan, who has made many experiments on pumps, states that for the cylinder and valve arrangement alone this is very low for a good modern pump at reasonable speed, and he believes that the maximum efficiency is reached at lifts of 25 to 50 ft. for many pumps. Also for lower lifts and slow speeds 70 to 85 per cent. is not uncommon.

ARDOLOY CUTTING TOOLS.

Alfred Herbert Ltd., Coventry.

Ardoloy is a hard high-speed cutting alloy capable of cutting steel, cast-iron, and non-ferrous metals at very high speeds. Its tensile strength is low, and it can be used for cutting only in the form of tips brazed on to carbon steel shanks. Both tips and standard tipped tools are obtainable.

Standard ardoloy tips are made in three different top rakes: 13° for mild steel; 8° for steel castings, alloy steels, and light cuts in cast-iron; 3½° for manganese steel, chilled iron, gun-metal, phosphor-bronze, and roughing cuts in cast-iron.

Whilst ardoloy will cut at very high speeds and is remarkably durable when used under proper conditions, its cutting edge is rapidly destroyed by vibration.

It is made in six grades:—

Grade 1A (*Brown*).—For light cuts. For fine boring and finish-turning hard cast-iron. For turning chilled-iron rolls and machining aluminium alloys, bakelite, and non-ferrous metals. For tipping reamer and counterboring blades.

Grade 2A (*Red*).—For general purpose work on cast-iron and malleable iron, semi-steel, non-ferrous metals; for tipping milling cutters and dead centres.

Grade 2 (*Blue*).—For general purpose work involving intermittent cutting on cast-iron and non-ferrous metals. Also for planing.

Grade 3 (*Yellow*).—For drawing dies, work guides for machines, such as centreless grinders, and for wood-cutting tools.

Grade S (*Green*).—For general purpose work on steels.

Grade W (Tips painted *White*).—For gauges, steadies, micrometer measuring anvils, faces on fixtures, guides, and other parts subject to wear.

Ardoloy Cutting Speeds.—As cutting speeds vary with conditions under which they are used, the following should be taken only as a general guide:—

[TABLE

ARDOLOY CUTTING SPEEDS.

Material.	Feet per Minute.
Mild steel (28–35 tons tensile)	300–1000
Cast steel (40–90 tons tensile)	80–750
Stainless steel: Bar	100–300
Stainless steel: Castings	60–150
Chrome nickel steel (40–90 tons tensile)	80–750
High-speed steel (annealed)	80–250
Manganese steel (12 per cent. manganese)	10–40
Cast-iron: 200 Brinell	200–700
Close-grained iron	150–400
10 per cent. nickel iron	20–45
Chilled-iron rolls	10–50
High silicon iron	20–70
Pearlite iron	150–400
Malleable iron	100–450
Copper, soft brass	500–1200
Cupro-nickel	350–500
Hard brass, phosphor-bronze, gun-metal	400–1000
Aluminium bronze, Admiralty bronze, manganese bronze	300–750
Aluminium	1000–2000
Aluminium alloys	300–750
Plastics, erinoid, ebonite, hard rubber	500–1000
Porcelain, marble	10–65
Slate	50–100
Glass	30–70

HINTS ON MILLING CUTTERS.

B.S.A. Tools Ltd., Birmingham.

Speed of Cutters.—High-speed cutters, to be efficient, must be run at suitable speeds. It is impossible to specify definite speeds and feeds for various cutters, as much depends upon the circumstances in which they are used. As a guide, to be used only in a general sense, however, the following cutting speeds are suggested:—

Tool steel (well annealed)	45– 50 ft. per min.
0·6–0·8 per cent. carbon steel	60– 65	,,
Mild steel and wrought-iron	75– 90	,,
Cast-iron (about 250 Brinell)	45– 50	,,
Brass (rolled)	200–250	,,
Brass (castings)	100–150	,,

By referring to the tables on pp. 740, 741 the correct speeds, in revolutions per minute, for any cutter can be obtained, based on the above surface speeds.

Lubrication and Cooling.—High-speed steel is more efficient than ordinary carbon steel because it will cut faster, not being so susceptible to tempering or softening resulting from the heat generated by cutting. A milling cutter should be cooled in order to preserve its cutting edges as long as possible.

Rigidity of Machine.—Tests show that a cutter will stand up to its work better on a machine that is rigid than if performing the same work on a less rigid machine. Uneven feeding, due to backlash, causes the edges of the cutter to wear away rapidly and stresses the cutter unnecessarily.

Grinding.—High-speed steel milling cutters should be ground on a fairly soft wheel, and a supply of water may be used with advantage to obviate the heating of the cutter. An efficient cutter-grinding machine soon repays its cost.

Storage.—Careless storage may cause cutting edges to become damaged and chipped. Milling cutters should be inspected and ground before they are stored, in order that they may be in a fit condition for use when next required.

Relation between Circumferential Speed and Revolutions.—

D = diameter of revolving tool or piece of work in feet.

d = diameter of revolving tool or piece of work in inches.

N = number of revolutions per minute of tool or piece of work.

V = circumferential speed in feet per minute of tool or piece of work.

$$\pi DN = V. \qquad N = \frac{1}{\pi} \times \frac{V}{D}. \qquad \pi = 3{\cdot}1416. \qquad \frac{1}{\pi} = {\cdot}3183.$$

$$\frac{\pi}{12} dN = V. \qquad N = \frac{12}{\pi} \times \frac{V}{d}. \qquad \frac{\pi}{12} = {\cdot}2618. \qquad \frac{12}{\pi} = 3{\cdot}82.$$

The tabulated values on the two following pages have been calculated from the formula

$$N = \frac{12}{\pi} \times \frac{V}{d},$$

and give the speeds in revolutions per minute corresponding to various circumferential speeds in feet per minute, for diameters ranging from ¼ inch to 24 inches.

Cutting Speeds*—Ft./Min. and Equiv. R.P.M.

Diam. Inches.	Feet per Minute. 10	15	20	25	30	35	40	45
	Revolutions per Minute.							
1/4	153	229	306	382	458	535	611	688
3/8	102	153	204	255	306	357	407	458
1/2	76·4	115	153	191	229	267	306	344
5/8	61·1	91·7	122	153	183	214	244	275
3/4	50·9	76·4	102	127	153	178	204	229
7/8	43·7	65·5	87·3	109	131	153	175	196
1	38·2	57·3	76·4	95·5	115	134	153	172
1 1/8	34·0	50·9	67·9	84·9	102	119	136	153
1 1/4	30·6	45·8	61·1	76·4	91·7	107	122	138
1 3/8	27·8	41·7	55·6	69·4	83·3	97·2	111	125
1 1/2	25·5	38·2	50·9	63·7	76·4	89·1	102	115
1 3/4	21·8	32·7	43·7	54·6	65·5	76·4	87·3	98·2
2	19·1	28·6	38·2	47·7	57·3	66·8	76·4	85·9
2 1/4	17·0	25·5	34·0	42·4	50·9	59·4	67·9	76·4
2 1/2	15·3	22·9	30·6	38·2	45·8	53·5	61·1	68·8
2 3/4	13·9	20·8	27·8	34·7	41·7	48·6	55·6	62·5
3	12·7	19·1	25·5	31·8	38·2	44·6	50·9	57·3
3 1/4	11·8	17·6	23·5	29·4	35·3	41·1	47·0	52·9
3 1/2	10·9	16·4	21·8	27·3	32·7	38·2	43·7	49·1
3 3/4	10·2	15·3	20·4	25·5	30·6	35·7	40·7	45·8
4	9·5	14·3	19·1	23·9	28·6	33·4	38·2	43·0
4 1/2	8·5	12·7	17·0	21·2	25·5	29·7	34·0	38·2
5	7·6	11·5	15·3	19·1	22·9	26·7	30·6	34·4
5 1/2	6·9	10·4	13·9	17·4	20·8	24·3	27·8	31·3
6	6·4	9·5	12·7	15·9	19·1	22·3	25·5	28·6
6 1/2	5·9	8·8	11·8	14·7	17·6	20·6	23·5	26·4
7	5·5	8·2	10·9	13·6	16·4	19·1	21·8	24·6
7 1/2	5·1	7·6	10·2	12·7	15·3	17·8	20·4	22·9
8	4·8	7·2	9·5	11·9	14·3	16·7	19·1	21·5
8 1/2	4·5	6·7	9·0	11·2	13·5	15·7	18·0	20·2
9	4·2	6·4	8·5	10·6	12·7	14·9	17·0	19·1
10	3·8	5·7	7·6	9·5	11·5	13·4	15·3	17·2
12	3·2	4·8	6·4	8·0	9·5	11·1	12·7	14·3
14	2·7	4·1	5·5	6·8	8·2	9·5	10·9	12·3
16	2·4	3·6	4·8	6·0	7·2	8·4	9·5	10·7
18	2·1	3·2	4·2	5·3	6·4	7·4	8·5	9·5
20	1·9	2·9	3·8	4·8	5·7	6·7	7·6	8·6
22	1·7	2·6	3·5	4·3	5·2	6·1	6·9	7·8
24	1·6	2·4	3·2	4·0	4·8	5·6	6·4	7·2

* The use of the tables may be extended by multiplying by 10.

Cutting Speeds—Ft./Min. and Equiv. R.P.M. (*cont.*).

Diam. Inches.	Feet per Minute.							
	50	60	70	80	90	100	110	120
	Revolutions per Minute.							
$\frac{1}{4}$	764	917	1070	1222	1375	1528	1681	1833
$\frac{3}{8}$	509	611	713	815	916	1019	1120	1222
$\frac{1}{2}$	382	458	535	611	688	764	840	917
$\frac{5}{8}$	306	367	428	489	550	611	672	733
$\frac{3}{4}$	255	306	357	407	458	509	560	611
$\frac{7}{8}$	218	262	306	349	393	437	480	524
1	191	229	267	306	344	382	420	458
$1\frac{1}{8}$	170	204	238	272	306	340	374	407
$1\frac{1}{4}$	153	183	214	244	275	306	336	367
$1\frac{3}{8}$	139	167	194	222	250	278	306	333
$1\frac{1}{2}$	127	153	178	204	229	255	280	305
$1\frac{3}{4}$	109	131	153	175	196	218	240	262
2	95·5	115	134	153	172	191	210	229
$2\frac{1}{4}$	84·9	102	119	136	153	170	187	204
$2\frac{1}{2}$	76·4	91·7	107	122	138	153	168	183
$2\frac{3}{4}$	69·4	83·3	97·2	111	125	139	153	167
3	63·7	76·4	89·1	102	115	127	140	153
$3\frac{1}{4}$	58·8	70·5	82·3	94·0	106	118	129	141
$3\frac{1}{2}$	54·6	65·5	76·4	87·3	98·2	109	120	131
$3\frac{3}{4}$	50·9	61·1	71·3	81·5	91·7	102	112	122
4	47·7	57·3	66·8	76·4	85·9	95·5	105	115
$4\frac{1}{2}$	42·4	50·9	59·4	67·9	76·4	84·9	93·4	102
5	38·2	45·8	53·5	61·1	68·8	76·4	84·0	91·7
$5\frac{1}{2}$	34·7	41·7	48·6	55·6	62·5	69·4	76·4	83·3
6	31·8	38·2	44·6	50·9	57·3	63·7	70·0	76·4
$6\frac{1}{2}$	29·4	35·3	41·1	47·0	52·9	58·8	64·6	70·5
7	27·3	32·7	38·2	43·7	49·1	54·6	60·0	65·5
$7\frac{1}{2}$	25·5	30·6	35·7	40·7	45·8	50·9	56·0	61·1
8	23·9	28·6	33·4	38·2	43·0	47·7	52·5	57·3
$8\frac{1}{2}$	22·5	27·0	31·5	36·0	40·4	44·9	49·4	53·9
9	21·2	25·5	29·7	34·0	38·2	42·4	46·7	50·9
10	19·1	22·9	26·7	30·6	34·4	38·2	42·0	45·8
12	15·9	19·1	22·3	25·5	28·6	31·8	35·0	38·2
14	13·6	16·4	19·1	21·8	24·6	27·3	30·0	32·7
16	11·9	14·3	16·7	19·1	21·5	23·9	26·3	28·6
18	10·6	12·7	14·9	17·0	19·1	21·2	23·3	25·5
20	9·5	11·5	13·4	15·3	17·2	19·1	21·0	22·9
22	8·7	10·4	12·2	13·9	15·6	17·4	19·1	20·8
24	8·0	9·5	11·1	12·7	14·3	15·9	17·5	19·1

GRINDING WHEELS.

Norton Grinding Wheel Co., Ltd.

The abrasives used by the Norton Grinding Wheel Co., Ltd., are known by the trade-marks *Alundum, 19 Alundum, 38 Alundum*, and *Crystolon*. Five distinct bonds are employed—vitrified, silicate, shellac, rubber, and bakelite—and wheels are made in many shapes and sizes, from $\frac{3}{16}$ inch to 60 inches in diameter and from $\frac{1}{64}$ inch to 20 inches thick. Segmental wheels are made as large as 72 inches in diameter and 14 inches thick.

In general, *Alundum* abrasives are used for grinding materials of high tensile strength, and *Crystolon* abrasive is used for those of low tensile strength. Roughly, 50,000 lbs. per sq. inch may be taken as the dividing line between high and low tensile strengths in this instance.

Vitrified bonded wheels are of exceedingly strong bond, and are standard for most grinding operations. *Silicate bonded wheels* have a less harsh grinding action, and are used for work requiring a delicate edge, such as edged tools and cutlery. *Shellac bonded wheels* produce a high degree of finish, and are used in saw sharpening and in the grinding of granite and marble. In some cases they are used for grinding aluminium pistons. *Bakelite bonded wheels*, operating at 9000 surface feet per minute, are used for snagging of steel and malleable castings, and for billet and ingot grinding. *Rubber bonded wheels* are used where thin wheels of great strength are wanted, as in the grinding of grooves.

Grain, Grade, and Structure.—*Grain* size denotes the size of the cutting particles or abrasive. Sizes range from 8 to 600, 8 being the coarsest and 600 the finest. The numbers refer to the meshes per linear inch in the screens through which the various sizes will pass. *Grade* is a term used to denote the hardness of a wheel. It refers to the strength of the bond and not to the hardness of the abrasive. A soft wheel may have a hard abrasive. Norton grades are designated by the letters E to Z, E being the softest wheel and Z the hardest. *Structure* is the arrangement of the constituent parts of a grinding wheel as to the exact degree of its density or abrasive spacing. The various structures are denoted by the numbers 1 to 12, number 1 representing the closest spacing and number 12 the most open.

Grinding Wheel Markings.—The five characteristics which determine the grinding action of Norton wheels are: (1) Abrasive (type), (2) Grain (size of abrasive), (3) Grade (strength of wheel), (4) Structure (grain spacing), and (5) Bond (type).

A typical wheel marking is 3846–M5BE. Here 38, 46, M, 5, and BE refer to the characteristics (1), (2), (3), (4), and (5) respectively.

Determination of Grinding Wheel Specifications.—The following factors must be taken into consideration:—

(1) Material to be ground.
(2) Amount of material to be removed, degree of accuracy, and finish required.
(3) Area of contact.
(4) Type of grinding machine.

A few rules which must be regarded as flexible are given in the following paragraphs:—

Selection of abrasive.—*Alundum* abrasive is suitable for carbon steels, alloy steels, high-speed steel, annealed malleable iron, wrought-iron, tough bronzes, etc. *Crystolon* abrasive is suitable for grey iron, chilled iron, brass and soft bronze, aluminium and copper, marble and other stone, rubber, leather, very hard alloys such as tungsten carbide, etc.

Selection of grain size.—Coarse wheel for fast cutting, except in case of very hard materials where depth of grain penetration is small. Fine grain for fine finish. Coarse grain for soft ductile materials and fine grain for hard and brittle materials.

Selection of grade.—Hard wheels on soft materials and *vice versa.* The smaller the area of contact, the harder the wheel should be. The higher the ratio of work speed to wheel speed, the harder the grade should be, and *vice versa.* Vibration and worn grinding machines usually necessitate using a harder wheel than would be required on a machine in good condition.

Selection of structure.—Soft, tough, and ductile materials require a wheel with a wide spacing of abrasive grains. Hard and brittle materials require a wheel with close spacing of abrasive grains, with the exception of tungsten carbides. Fine finish requires close spacing of particles of fine grain size.

Snagging and other operations, with flexible application of pressure, require wide grain spacing. Surface operations require wide grain spacing. Cylindrical and centreless work, also tool and cutter grinding, are usually done with medium grain spacing. Heavy pressures which tend to destroy the form of shaped wheels require close grain spacing.

Selection of bond.—Vitrified type of bond is generally used, but, in some cases, unusual operating and performance requirements make the selection of other types advantageous.

Thin cutting-off wheels and others subjected to bending require bakelite, shellac, or rubber bonds. Solid wheels over 36 inches diameter require silicate bond.

Vitrified wheels are best for speeds below 6500 surface feet per minute; bakelite, shellac, and rubber wheels are best for higher speeds.

Shellac or rubber bonds are best for high finish where production is not a factor.

Recommended Wheel Speeds in Surface Feet per Minute.

Norton Grinding Wheel Co., Ltd.

Cylindrical grinding	5500– 6500
Internal grinding	2000– 6000
Snagging, offhand grinding (vitrified wheels)	5000– 6000
Snagging (rubber and bakelite wheels) .	7000– 9500*
Surface grinding	4000– 5000
Knife grinding	3500– 4500
Hemming cylinders	2100– 5000†
Wet tool grinding	5000– 6000
Cutlery wheels	4000– 5000
Rubber, shellac, and bakelite cutting-off wheel	9000–16000*

Work Speed.—In general, the faster the work speed the faster the wheel wear. This is not necessarily a drawback, but it must be understood.

Wheel wear is dependent upon the ratio of wheel speed in s.f.p.m. to work speed in s.f.p.m. The higher the ratio, the less work the wheel is required to do in a given time; hence the wheel wears at a slower rate. If the ratio is decreased by increasing the work speed, the wheel will be required to do more work in a given time and will wear faster.

In general, the longer the arc or the larger the area of contact in precision grinding operations, the faster should be the speed of the work, in order that the wheel may cut properly.

In offhand grinding, the rate at which the work is forced against the wheel exerts an influence upon grade selection. The harder the grinding wheel is forced, the harder should be the wheel if abrasive economy is to be expected. This relation also exists in precision grinding of a semi-automatic and high production nature.

* Higher speed recommended only where bearings, protection devices, and machine rigidity are adequate.

† This higher speed is recommended only where suitable bearings are employed.

Grinding Wheel Speeds—Feet per Minute and Equivalent R.P.M.

Wheel Diam. Inches.	Feet per Minute.					
	4000	4500	5000	5500	6000	6500
	Revolutions per Minute.					
1	15279	17189	19099	21008	22918	24828
2	7639	8594	9549	10504	11459	12414
3	5093	5730	6366	7003	7639	8276
4	3820	4297	4775	5252	5730	6207
5	3056	3438	3820	4202	4584	4966
6	2546	2865	3183	3501	3820	4138
7	2183	2456	2728	3001	3274	3547
8	1910	2149	2387	2626	2865	3104
10	1528	1719	1910	2101	2292	2483
12	1273	1432	1592	1751	1910	2069
14	1091	1228	1364	1501	1637	1773
16	955	1074	1194	1313	1432	1552
18	849	955	1061	1167	1273	1379
20	764	859	955	1050	1146	1241
22	694	781	868	955	1042	1129
24	637	716	796	875	955	1035
26	588	661	735	808	881	955
28	546	614	682	750	819	887
30	509	573	637	700	764	828
32	477	537	597	657	716	776
34	449	506	562	618	674	730
36	424	477	531	584	637	690

Grinding Wheel Speeds—Feet per Minute and Equivalent R.P.M. (*continued*).

Wheel Diam. Inches.	Feet per Minute.					
	7000	7500	8000	8500	9000	9500
	Revolutions per Minute.					
1	26738	28648	30558	32468	34377	36287
2	13369	14324	15279	16234	17189	18144
3	8913	9549	10186	10823	11459	12096
4	6685	7162	7639	8117	8594	9072
5	5348	5730	6112	6494	6875	7257
6	4456	4775	5093	5411	5730	6048
7	3820	4093	4365	4638	4911	5184
8	3342	3581	3820	4058	4297	4536
10	2674	2865	3056	3247	3438	3629
12	2228	2387	2546	2706	2865	3024
14	1910	2046	2183	2319	2456	2592
16	1671	1790	1910	2029	2149	2268
18	1485	1592	1698	1804	1910	2016
20	1337	1432	1528	1623	1719	1814
22	1215	1302	1389	1476	1563	1649
24	1114	1194	1273	1353	1432	1512
26	1028	1102	1175	1249	1322	1396
28	955	1023	1091	1160	1228	1296
30	891	955	1019	1082	1146	1210
32	836	895	955	1015	1074	1134
34	786	843	899	955	1011	1067
36	743	796	849	902	955	1008

Twist Drill Sizes.

Drill Gauge.	Inches.		Drill Gauge.	Inches.		Drill Gauge.	Inches.		Drill Gauge.	Inches.	
80		·0135	49		·073	20		·161	*J*		·277
79		·0145	48		·076	19		·166	*K*		·281
..	$\frac{1}{64}$	·015625	..	$\frac{5}{64}$	·078125	18		·1695	..	$\frac{9}{32}$	·28125
78		·016	47		·0785	..	$\frac{11}{64}$	·171875	*L*		·29
77		·018	46		·081	17		·173	*M*		·295
76		·02	45		·082	16		·177	..	$\frac{19}{64}$	·296875
75		·021	44		·086	15		·18	*N*		·302
74		·0225	43		·089	14		·182	..	$\frac{5}{16}$	·3125
73		·024	42		·0935	13		·185	*O*		·316
72		·025	..	$\frac{3}{32}$	·09375	..	$\frac{3}{16}$	·1875	*P*		·323
71		·026	41		·096	12		·189	..	$\frac{21}{64}$	·328125
70		·028	40		·098	11		·191	*Q*		·332
69		·0292	39		·0995	10		·1935	*R*		·339
68		·031	38		·1015	9		·196	..	$\frac{11}{32}$	·34375
..	$\frac{1}{32}$	·03125	37		·104	8		·199	*S*		·348
67		·032	36		·1065	7		·201	*T*		·358
66		·033	..	$\frac{7}{64}$	·109375	..	$\frac{13}{64}$	·203125	..	$\frac{23}{64}$	·359375
65		·035	35		·11	6		·204	*U*		·368
64		·036	34		·111	5		·2055	..	$\frac{3}{8}$	·375
63		·037	33		·113	4		·209	*V*		·377
62		·038	32		·116	3		·213	*W*		·386
61		·039	31		·12	..	$\frac{7}{32}$	·21875	..	$\frac{25}{64}$	·390625
60		·04	..	$\frac{1}{8}$	·125	2		·221	*X*		·397
59		·041	30		·1285	1		·228	*Y*		·404
58		·042	29		·136	*A*		·234	..	$\frac{13}{32}$	·40625
57		·043	28		·1405	..	$\frac{15}{64}$	·234375	*Z*		·413
56		·0465	..	$\frac{9}{64}$	·140625	*B*		·238	..	$\frac{27}{64}$	·421875
..	$\frac{3}{64}$	·046875	27		·144	*C*		·242	..	$\frac{7}{16}$	·4375
55		·052	26		·147	*D*		·246	..	$\frac{29}{64}$	·453125
54		·055	25		·1495	*E*	$\frac{1}{4}$	·25	..	$\frac{15}{32}$	·46875
53		·0595	24		·152	*F*		·257	..	$\frac{31}{64}$	·484375
..	$\frac{1}{16}$	·0625	23		·154	*G*		·261	..	$\frac{1}{2}$	·5
52		·0635	..	$\frac{5}{32}$	·15625	..	$\frac{17}{64}$	·265625	..	$\frac{33}{64}$	·515625
51		·067	22		·157	*H*		·266	..	$\frac{17}{32}$	·53125
50		·07	21		·159	*I*		·272	..	$\frac{35}{64}$	·546875

Above $\frac{35}{64}$ inch, sizes advance by 64ths to 2 inches. Millimetre sizes are also available.

SANDBLASTING.

H. G. Sommerfield, Ltd., London.

Sandblasting is a general term used to describe the projection of abrasive materials on to surfaces, primarily for the purposes of cleaning, etching, or buffing. Sand was formerly the standard abrasive, and hence the designation. Graded flint was also used for heavier work, but now the most popular abrasive consists of graded metallic shot, round or crushed (angular).

Pulverisation occurs with all abrasives, the extent being governed by the material. This gives rise to dust formation, and it is necessary to exhaust the dust-laden atmosphere from the blasting chamber or zone and to collect the dust. Sands and flints generate dust of a siliceous nature, and, owing to the consequent danger of silicosis, the Home Office is influencing employers to substitute metallic abrasives for sands and flints. As a further precaution, it is decreed that operators working inside a sandblast room must wear a helmet fitted with an air line and supplied with a constant supply of fresh air. Induced ventilation must ensure that the atmosphere of any sandblast room, wherein an operator works, shall be changed at least five times per minute, and most manufacturers prefer considerably to exceed this rate of exhaust; purification of the atmosphere may be further assisted by a system of abrasive cleaning.

Apart from the dust which arises through pulverisation of the abrasive, the blasting away of moulding sands and cores and, in some cases, the breaking up of scale or the skin of castings, further complicates the problem. Removal by air separation or other means of such of this material as becomes mixed with the abrasive before the latter is reintroduced into the system is then essential.

The original types of sandblast plant operated through the medium of steam, but contemporary designs favour the use of either compressed air or centrifugal force to impart the high velocity to the stream of abrasive. Compressed air has the advantage of comparative simplicity, and allows of manual control of the abrasive through the medium of a flexible hose and nozzle directed on to the work, and is the best system where large surfaces have to be treated, or, conversely, where the conditions call for a moderate output only. Up to the present, the centrifugal machine is limited to repetition work, as manual control is not practicable and prime cost is heavy.

In addition to the operating medium, there is a great diversity in types of sandblast machines. Room plants, wherein the operator shuts himself inside a chamber with his work and, wearing a helmet and protective clothing, directs the abrasive on to the surfaces to be treated, form the most practical way of dealing with large castings. Tumbling barrels, slowly rotating and partially filled with parts to be treated, having one or more jets projected through the central axis, are admirable for repetition work. In the same way, where great numbers of parts presenting flat surfaces have to be treated, the most practical method is the rotating table, on which the parts are carried into a screened-off blasting zone and then emerge again to be reversed or exchanged for fresh parts as conditions demand. Finally, a popular machine for small works and small components is the cabinet, the operator remaining outside and directing the flow of abrasive through a sight window, and operating the flexible hose through arm-holes.

The chief uses of sandblasting are etching and frosting glass; cleaning castings, both ferrous and non-ferrous; cleaning and "cutting a key" on surfaces preparatory to enamelling or metallising; de-enamelling; and removing scale after heat-treatment.

Sand and shot abrasive is supplied in a wide range of grades (sieve meshes). The following table will assist in the selection of the most suitable abrasive grade and air pressure, which must ultimately be determined by trial, for different purposes:—

Abrasive.	Grade.	Air Pressure. Lbs./Sq.In.	Purpose.
Sharp sand	16 to 32	15 to 25	Cleaning castings.
,,	24 ,, AA	2 ,, 20	Frosting and stencilling glass.
,,	AA or "Flour"	5 ,, 10	Grindery (leather).
Graded flint	10 to 24	15 ,, 30	Cleaning heavy castings; removing cores.
,,	24 ,, 40	15 ,, 30	Cleaning light castings; removing cores.
Metallic shot (round)	10 ,, 20	15 upwards	Dull cleaning heavy ferrous castings.
,,	24 ,, 40	15 to 50	Cleaning light ferrous castings.
,,	24 ,, 40	10 ,, 20	Cleaning or buffing non-ferrous castings.
,,	40 ,, 80	10 ,, 20	Special non-ferrous work where skin must not be penetrated.
Metallic shot (angular)	6 ,, 10	20 ,, 80	Cleaning steel castings.
,,	12 ,, 30	20 ,, 80	Cleaning and cutting key on iron castings.
,,	40 ,, 80	15 ,, 40	Cleaning light non-ferrous castings.
,,	50 ,, 80	5 ,, 10	Glass stencilling.

Table of Theoretical Air Discharge for Nozzles of Different Diameters, Air Velocities, and H.P. of Compressor required.

Air Pressure. Lbs./Sq. In.	Velocity of Discharge. Ft./Sec.	Volume of Free Air (Cu. Ft./Min.) and Compressor H.P.							
		Nozzle Diameter—Inch.							
		¼		⅜		½		⅝	
		Vol.	H.P.	Vol.	H.P.	Vol.	H.P.	Vol.	H.P.
5	552	15·4	·54	34·6	1·2	61·6	2·2	96·5	3·4
10	780	21·8	1·3	49	2·9	87	5·2	136	8·2
20	1030	30·8	2·8	69	6·3	123	11·2	193	17·6
30	1100	40	4·8	90	10·7	161	19·1	252	30
40	1143	49	6·9	110	15·5	196	27·5	307	43
60	1196	67	11·6	151	26·1	268	46·4	420	73
80	1230	85	16·7	191	37·6	340	67	532	105

Note.—In considering the H.P. of driving motors, etc., an adequate margin should be allowed over the theoretical requirement of the compressor.

Given formal requirements, compressor makers generally stipulate the necessary allowance. It is important that oil and water precipitation should be minimised in the sandblast system, otherwise trouble may arise through coagulation of shot. For this reason inter-coolers should be provided in 2-stage compressors, and after-coolers and moisture and oil extractors are sometimes fitted. In cold, moist atmospheres, a gas jet is often fitted beneath pressure chambers to maintain air temperatures and prevent deposition of moisture in the system.

The above table is intended as a practical guide, and is based on more complete figures given by Cox, in respect to air volumes passing through a range of nozzle bores at various pressures, and on theoretical velocities given by Hiscock.

MISCELLANEOUS NOTES AND DATA.

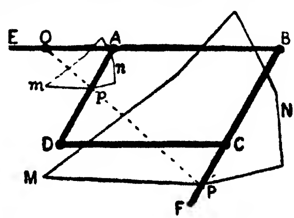
Roman and Ordinary Notation.

I.	1	X.	10	XX.	20	CC.	200
II.	2	XI.	11	XXV.	25	CCC.	300
III.	3	XII.	12	XXX.	30	CCCC.	400
IIII. or IV.	4	XIII.	13	XL.	40	D.	500
		XIV.	14	L.	50	DC.	600
V.	5	XV.	15	LX.	60	DCC.	700
VI.	6	XVI.	16	LXX.	70	DCCC.	800
VII.	7	XVII.	17	LXXX.	80	DCCCC.	900
VIII.	8	XVIII.	18	XC.	90	M.	1000
IX.	9	XIX.	19	C.	100	MDCCCXCVI.	1896

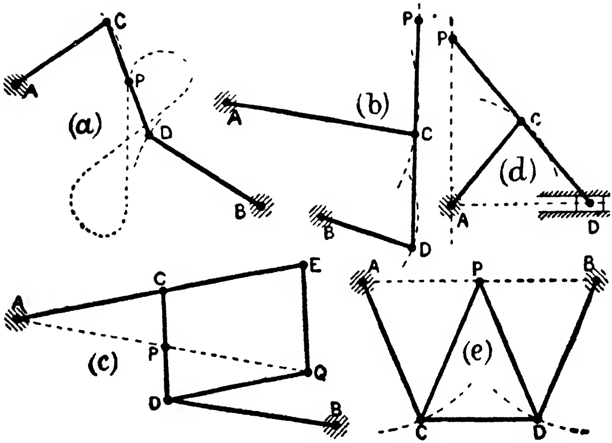
The Greek Alphabet.—As Greek letters are frequently used in mathematical formulæ, they are here given for reference.

Α	α	alpha	Ι	ι	iōta	Ρ	ρ	rho
Β	β	bēta	Κ	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	Τ	τ	tau
Δ	δ	delta	Μ	μ	mu	Υ	υ	upsīlon
Ε	ε	epsīlon	Ν	ν	nu	Φ	φ	phi
Ζ	ζ	zeta	Ξ	ξ	xi	Χ	χ	chi
Η	η	ēta	Ο	ο	omīcron	Ψ	ψ	psi
Θ	θ	thēta	Π	π	pi	Ω	ω	ōmega

The Pantagraph.—Four bars are connected together by pin joints to form a parallelogram $ABCD$. The whole frame can be made to rotate in its plane about a fixed pin which passes through the bar EB at any point O in it. p is a point in the bar AD, and P is a point in the bar FB such that OpP is a straight line. If the frame be moved (O being fixed) so that the point p traces out the figure mn, the point P will describe a figure MN similar to mn, and the linear dimensions of MN will be to the corresponding dimensions of mn as OB is to OA; also, the area of MN will be to the area of mn as the square on OB is to the square on OA.



Parallel Motions.—(*a*) *Watt's Parallel Motion.*—*AC* and *BD* are bars which can oscillate about fixed axes at *A* and *B* respectively, and which are connected by pin joints at *C* and *D* to a link *CD*. A point *P* in *CD* describes a figure of eight when the mechanism receives all the motion of which it is capable. Two portions of this figure are approximately straight lines. The straight portions of the figure described by *P* are longest when the position of *P* is such that *DP* : *CP* : : *AC* : *BD*.



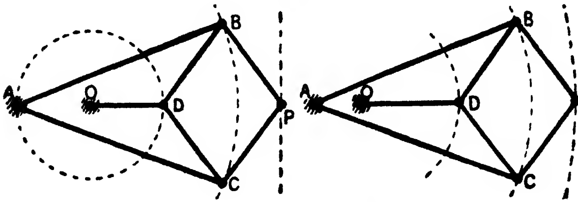
(*b*) The bars *AC* and *BD* are here shown on the same side of the link *CD*. The point which describes the closest approximation to a straight line is the point *P*, in *CD* produced, whose position is such that *DP* : *CP* : : *AC*: *BD*.

(*c*) This shows the application of the pantagraph to the mechanism *ACDB* shown at (*a*) for the purpose of guiding a point *Q* in a line parallel to that described by *P*. The parallelogram *CDQE* is proportioned so that *A*, *P*, and *Q* are in a straight line. *ACE* is one bar.

(*d*) *Scott Russell's Parallel Motion.*—*AC* is a bar which can oscillate about a fixed axis at *A*. *PD* is a bar one end of which is guided in a straight line *AD*. *AC* is connected to *PD* by a pin joint at *C*. *AC*=*PC*=*CD*. *P* moves in a straight line *AP* which is at right angles to *AD*.

(*e*) *Roberts's Parallel Motion.*—*AC* and *BD* are equal bars which can oscillate about fixed axes at *A* and *B* respectively. *CPD* is a frame connected by pin joints at *C* and *D* to the bars *AC* and *BD*. *PC*=*PD*. Part of the path described by the point *P* is an approximation to a straight line.

The Peaucellier Mechanism.—The Peaucellier mechanism consists of seven bars connected by pin joints as shown. The axes of the pins at A and O are fixed. $AB=AC$. $BD=DC=CP=BP$. When the mechanism is moved the point P describes either an exact straight line or an arc of a circle.



If $OD=OA$, the path of P is an exact straight line.

If OD is *greater* than OA, the path of P is an arc of a circle concave towards O, and having a radius given by the formula,

$$R=OD\frac{AB^2-BD^2}{OD^2-OA^2}.$$

If OD is *less* than OA, the path of P is an arc of a circle convex towards O, and having a radius given by the formula

$$R=OD\frac{AB^2-BD^2}{OA^2-OD^2}.$$

The most important form of the Peaucellier mechanism is that for describing an exact straight line. The other forms may be used for describing circular arcs of large radii when the centres of the circles are inaccessible.

Useful Functio
accelerating effect
per second; then

$g=32{\cdot}2.$	$\frac{1}{g}={\cdot}031.$	$\sqrt{g}=5{\cdot}675.$	$\frac{1}{\sqrt{g}}={\cdot}176.$
$2g=64{\cdot}4.$	$\frac{1}{2g}={\cdot}0155.$	$\sqrt{2g}=8{\cdot}025.$	$\frac{1}{\sqrt{2g}}={\cdot}1246.$

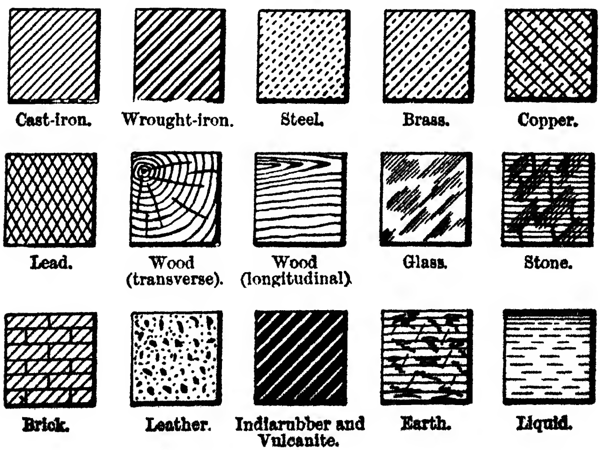
Sizes of Drawing-paper.

	Inches.		Inches.
Demy . . .	20 × 15½	Imperial . .	30 × 22
Medium . .	22 × 17½	Atlas . . .	34 × 26
Royal . . .	24 × 19	Double elephant .	40 × 27
Super royal . .	27½ × 19¼	Antiquarian . .	53 × 31

Colours on Drawings.

Materials.	Colours.
Cast-iron	Payne's grey or neutral tint.
Wrought-iron	Prussian blue.
Steel	Purple (mixture of Prussian blue and crimson lake).
Brass	Gamboge, with a little sienna, or a very little red added.
Copper	A mixture of crimson lake and gamboge, the former colour predominating.
Lead	Light Indian ink, with a very little indigo added.
Brick-work	Crimson lake and burnt sienna.
Firebrick	Yellow and Vandyke brown.
Grey stones	Light sepia or pale Indian ink, with a little Prussian blue added.
Brown freestone	Mixture of pale Indian ink, burnt sienna, and carmine.
Soft woods	For ground work, pale tint of sienna.
Hard woods	For ground work, pale tint of sienna, with a little red added.
	For graining woods, use darker tint, with a greater proportion of red.

Section Lining on Drawings.—On drawings in black and white different materials may be distinguished by differences in "section lining." Examples are given below.



SELECTED BRITISH STANDARD SPECIFICATIONS.*

B.S.

4 : 1932. Channels and Beams for Structural Purposes, Dimensions and Properties of.

4A : 1934. Equal Angles, Unequal Angles, and Tee Bars for Structural Purposes, Dimensions and Properties of.

21 : 1938. Pipe Threads. Part 1. Basic Sizes and Tolerances.

78 : 1938. Cast-Iron Pipes for Water, Gas, and Sewage.

84 : 1940. Screw Threads of Whitworth form.

93 : 1919. Screw Threads, British Association, with Tolerances.

240 : 1937. Brinell Hardness Testing, Method and Tables for.

275 : 1927. Dimensions of Rivets ($\frac{1}{2}$ inch to $1\frac{3}{4}$ inch diameter).

302 : 1938. Round Strand Steel Wire Ropes for Cranes.

330 : 1941. Steel Wire Ropes for Colliery Haulage Purposes.

394 : 1944. Short Link Wrought-Iron Crane Chain.

425 : 1943. Forms and Dimensions of Boiler Rivets (as manufactured) ($\frac{1}{2}$ inch to 2 inch diameter).

431 : 1946. Manila Ropes for General Purposes.

436 : 1940. Machine-Cut Gears. A. Helical and Straight Spur.

465 : 1932. Pitched or Calibrated Wrought-Iron Load Chain for Hand-Operated Pulley Blocks.

482 : 1945. Wrought-Iron and Mild Steel Hooks for Cranes, Chains, Slings and General Engineering Purposes, Excluding Building Operations.

534 : 1934. Steel Spigot and Socket Pipes and Specials for Water, Gas, and Sewage.

545 : 1934. Machine-Cut Gears. B. Bevel (with Helical, Curved, and Straight Teeth).

590 : 1935. Short Link and Pitched Steel Chain, Electrically Welded Mild Steel.

664 : 1936. Cast-Iron Shaft Couplings, Rigid Flanged Type with recessed Bolt-Heads and Nuts.

721 : 1937. Machine-Cut Gears. C. Worm Gearing.

781 : 1938. Wrought-Iron Chain Slings and Rings, Links Alternative to Rings, Egg and Intermediate Links.

806 : 1942. Ferrous Pipes and Piping Installations.

1306 : 1946. Non-ferrous Pipes and Piping Installations.

1387 : 1947. Steel Tubes and Tubulars Suitable for Screwing to B.S. 21 Pipe Threads.

1452 : 1948. Grey Iron Castings.

* British Standards Institution, Sales Dept., 24 Victoria Street, London, S.W.1.

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