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ELECTRICAL ENGINEERING TEXTS

ELECTRICAL MEASUREMENTS

BY

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PREFACE TO THE SECOND EDITION

It is to be regretted that Professor Laws did not live to see the publication of this second edition of his book. He had, however, finished the manuscript and it is presented here essentially as he left it. It has been a pleasure to the members of the Electrical Engineering Department Staff of the Massachusetts Institute of Technology to carry out the minor but necessary work on proofs to enable the publication of this valuable work.

We feel that the aims of the second edition are essentially those of the first, namely, to supply a compendium of information on the subject of electrical measurements and, at the same time, a textbook suitable for use by undergraduate students in technical and other schools.

The manuscript has been brought thoroughly up to date in the fields which it covers and yet has retained sufficient of the background material where this is necessary to make evident the course of development of the art. Many references have been added, which make the book particularly useful to those interested in tracing particular subjects or methods in detail.

As a reference book on its chosen subject it is still without rival in attention to detail, in comprehensiveness of treatment, and in completeness.

Professor H. E. Clifford of Harvard University has continued his interest and aid in the second edition as in the first, and to him thanks are due.

RALPH D. BENNETT,
GEORGE B. HOADLEY.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
December, 1937.

PREFACE TO THE FIRST EDITION

In this book it is intended to give a general treatment of the subject of electrical measurements, special emphasis being placed on those matters which are important to the student of electrical engineering.

In preparing a book of this character one has to consider, not only the mature reader who may desire a compendium of methods together with certain practical suggestions, but the student who is beginning the study of Electrical Engineering and who should acquire early in his course a sound knowledge of the process of electrical measurement. This knowledge is fundamental not only to much of the work which the student is required to do in the dynamo laboratory as a matter of engineering training, but to an adequate understanding of electrical testing as it is encountered in the practice of the electrical engineering profession. In the preparation of the text this second class of readers has been particularly in mind.

It is assumed that those who use this text have had courses in physics, the theory of electricity, and in mathematics, such as are given to third year students in technical schools of the first rank.

The choice of material and the method of treatment have been determined by the author's experience in directing for many years the work of the laboratory for Technical Electrical Measurements at the Massachusetts Institute of Technology, and the book is intended as a text for the guidance of students working in such a laboratory as well as a general reference book on the subject.

It is expected that those using the book for purposes of instruction will select such portions as are best suited to their purposes, for more material is presented than can be utilized in both the class room and the laboratory in the time which can properly be allotted to this particular phase of electrical engineering instruction. In this connection it is suggested that interest and discussion are stimulated if the laboratory work is so arranged that the various members of the class while engaged with the same general topic carry out the experimental work by alternative methods.

For the use of those who desire a more detailed discussion of the various methods, references to certain important papers are appended to each chapter. While no attempt had been made to form a bibliography of the subject of electrical measurements it is thought that these references should be of value in directing the students' attention to the original sources of information and thus assisting him to a broad view of this particular part of his professional training.

The various commercial instruments described in the text have been selected simply as giving good illustrations of the application of the particular principles under discussion. No attempt has been made to discuss instruments made by different makers which differ merely in minor details.

The author wishes to thank Professor H. E. Clifford, Gordon McKay Professor of Electrical Engineering at Harvard University, for his continued interest in the work and for the many and valuable suggestions which he has made during its preparation.

F. A. Laws.

Massachusetts Institute of Technology,
July 5, 1917.

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ELECTRICAL MEASUREMENTS

CHAPTER I

THE MEASUREMENT OF CURRENT

Classes of Instruments.—Electrical measuring instruments may be divided into two classes:

1. Absolute instruments.
2. Secondary instruments.

An absolute instrument is one so designed and built that it gives results expressed in the absolute, or c.g.s., system of units. This implies that the numbers of turns in the coils and all the dimensions that are electrically important have been determined, so that the factor that connects the force or the turning moment acting on the movable member, and the numerical value of the quantity under measurement, can be calculated.

A secondary instrument is one so constructed that the relation between its indications and the quantity under measurement must be established experimentally; that is, the instrument must be calibrated.

Examples of absolute instruments are the tangent galvanometer and the Rayleigh current balance (see page 75). The ordinary portable ammeters, voltmeters, and wattmeters are examples of secondary instruments.

Absolute instruments are not adapted for general use and are rarely employed outside such establishments as the Bureau of Standards, the National Physical Laboratory, or the Reichsanstalt. Their particular field of usefulness is in the determination of the fundamental electrical constants, for example, the international ampere.

GALVANOMETERS

The Tangent Galvanometer.—This is an absolute instrument. It consists essentially of a circular coil of insulated wire, having a radius that is large compared with the dimensions of its cross section, together with a small magnetic needle which is so suspended at the center of the coil that it can move about a vertical axis. The needle is provided with a pointer which moves over a scale graduated in degrees. The coil is so placed that its plane is vertical and in the magnetic meridian. When no current is flowing, the pointer stands at zero, for the needle is then controlled by the horizontal component of the local magnetic field H.

On the passage of a current the coil sets up a magnetic field, which at the needle is perpendicular to the plane of the coil and of magnitude

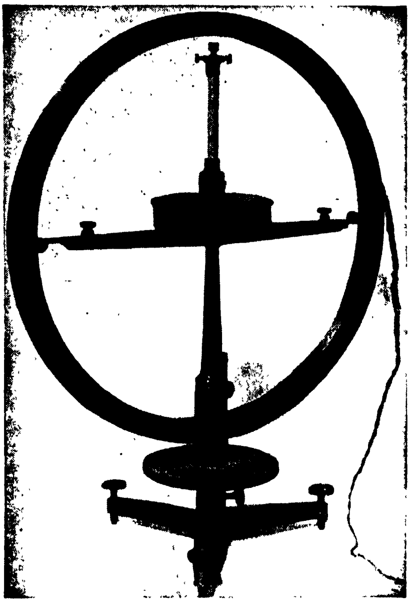


FIG. 1.—Tangent galvanometer.

$$F = \frac{2\pi n}{r} I,$$

where n is the number of turns in the coil, r the mean radius of the coil, and I the current in absolute units. The needle will turn through an angle θ and take up a position along the resultant of F and H. Then

$$\tan \theta = \frac{F}{H} = \frac{2\pi n}{Hr} I$$

or

$$I = \frac{H}{2\pi n/r} \tan \theta. \quad \text{in absolute units} \quad (1)$$

$$I = \frac{10H}{2\pi n/r} \tan \theta. \quad \text{in amperes} \quad (1a)$$

The quantity $2\pi n/r$ depends on the dimensions of the instrument and is the strength of field at the center of the coil due to unit current. It is frequently called the "galvanometer constant" of the coil and denoted by G.

In this elementary demonstration it has been assumed that:

1. The coil is perfectly circular.

2. The mean radius of the windings correctly represents the effective radius.

3. The needle is exactly at the center of the coil, and the field acting on a finite needle is the same as that at the mathematical center of the coil.

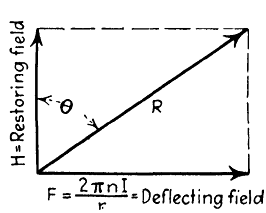


FIG. 2.—Fields at needle of tangent galvanometer.

4. The needle is in a uniform field, and consequently, as it deflects, its poles do not swing into a field of strength differing from that at its zero position.

5. The plane of the coil is in the magnetic meridian and truly vertical.

6. The factor H, or the horizontal intensity of the local magnetic field, has been determined at the place occupied by the instrument and is constant.

7. Only magnetic forces act on the needle; that is, there is no friction and no torsional rigidity in the suspension.

In a careful study of the instrument it would be necessary to discuss each of these items and to determine the numerical effect on the measured value of the current of the unavoidable departures from the assumed conditions.

In absolute electrical measurements this part of the work often calls for mathematical ability of a high order.

The Helmholtz Galvanometer.—In this instrument the needle is suspended on the axis midway between two equal coils whose distance apart is equal to their radius.

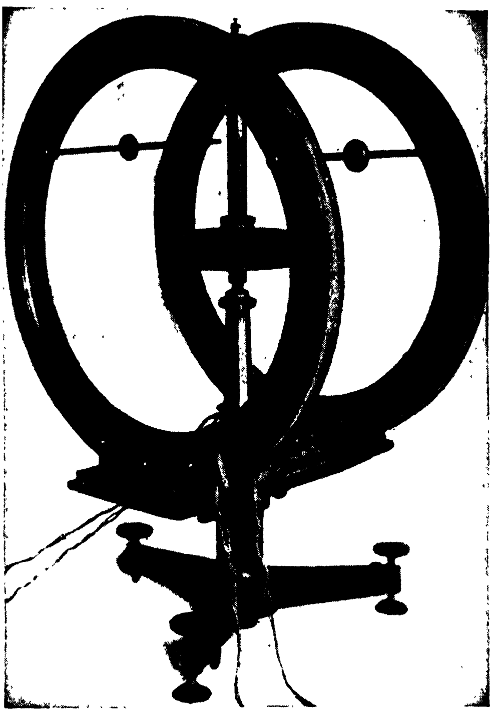


Fig. 3.—Helmholtz tangent galvanometer.

The reason for this construction is that it renders the field in which the needle swings very uniform, so that the correction for the finite length of the needle is much reduced.

This arrangement of coils is frequently employed in other instruments where a uniform magnetic field is desired.

These absolute galvanometers depend for their directive force on the horizontal component of the local field at the place where they are used. This quantity is subject to great and erratic variations due to the proximity of electric cars, feeders, and structural iron work, and, as it enters as a direct factor, present-day conditions have rendered these instruments practically useless.

Any of these absolute galvanometers will be reduced to secondary forms, if, in the attempt to gain sensitiveness, the coils are brought close to the needle.

The Thomson or Kelvin Galvanometer.[1]*—Figure 4 shows, without the magnetic shields, a simple and effective instrument designed and constructed by Hill and Downing;[1] also a more elaborate instrument of the Paschen type made by the Cambridge Instrument Company and embodying the suggestions of Hill and Downing. In Fig. 4*B* the magnetic shields are shown at 1 and 2; 3 is a dustproof protective case; and the two rods projecting from its top serve to adjust the control magnets. The characteristics of the Kelvin galvanometer as now constructed are the use of a strong and exceedingly light astatic needle system carrying a very small and perfect mirror; the division of the winding between two sets of removable coils, one set acting on the upper, and the other on the lower, member of the astatic system; the suspension of the needle system by a long and very fine quartz fiber; and the application of magnetic shields within which are small control magnets adjustable from without, by which the fields at the upper and lower members of the astatic system may be varied. The zero reading and sensitivity are thus controlled.

The Coil Structure.—In order to attain the maximum sensitivity, the windings must be so disposed that they produce the maximum field at the needle. The wire is divided between two sets of coils, and an astatic needle system employed; by this means the wire, as a whole, is brought nearer the needle. An additional advantage is that the over-all galvanometer resistance may be changed to suit the work in hand by connecting the four coils in series, series-parallel, or parallel. In the instrument of Fig. 4*B* the coils are wound on elliptical cores.

When the instrument is properly set up, the distance between the coil faces should be great enough to allow the needle system to rotate freely, so that the suspension fiber may untwist when the controlling field is reduced to practically zero. This is essential for a good law of deflections.

Galvanometer Constant.—The magnetic effect of a coil of fixed dimensions traversed by a current is proportional to the ampere-turns. The strength of the deflecting field at the needle, when a unit current flows in the coils, is called the "galvanometer constant" and is denoted by G.

$$G = kAn'. \qquad (2)$$

Here k is a constant depending on the size of the coil and the shape of

* Numbers refer to references at end of chapter.

its cross section. A is the area of the cross section, and n' is the number of turns of wire per unit area. Consequently,

$$\text{Deflecting field at needle} = kAn'I = k \text{ ampere-turns} = GI. \qquad (3)$$

It is natural to wind galvanometer coils with a rectangular cross section,

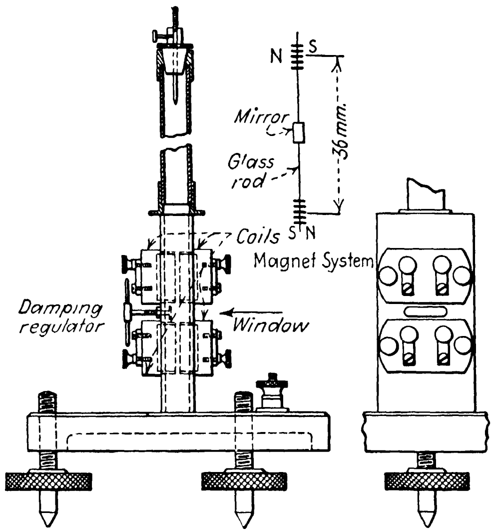


FIG. 4A.—Left, part sectional elevation; right, view looking in direction of arrow.

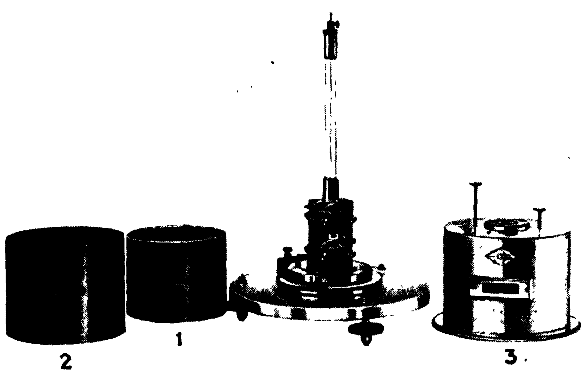


FIG. 4B.—Kelvin galvanometer.

but this general relation (3) applies to coils of any shape, provided they have fixed dimensions.

Best Form of Cross Section.—Theoretically, the rectangular form of cross section is not the best. With a definite length of wire of a certain size, the volume of the coil and its resistance are fixed. The question is

How shall this wire be arranged so that it will produce the maximum effect at the needle?

The effect at the needle of a unit length of wire, bent into an arc of a circle and carrying unit current, is $F = \frac{\sin \alpha}{r^2}$, where r and α are the polar coordinates of the traces of the wire on a plane including the axis. The origin of coordinates being the axis of the coil and the coil center, call $\frac{\sin \alpha}{r^2}$ the efficacy of a unit length of wire and denote it by e_v. Then every unit length of wire whose traces are on the curve $r^2 = \frac{1}{e_v} \sin \alpha$, where e_v has a definite value, will produce the same effect at the center of the coil. Suppose that the wire has been so wound that the boundary of the cross section of the coil is given by $r^2 = \frac{1}{e_v} \sin \alpha$, where e_v has a particular numerical value. If an attempt is then made to alter the form of the cross section by changing the position of a portion of the wire, the field at the needle will be reduced, for the only change possible is that to a region of less efficacy. Consequently, the equation is that of the boundary of the best form of cross section. The boundary is symmetrical about the maximum radius vector, the value of which is $1/\sqrt{e_v}$.

Similarly, to arrange a given number of turns to give the maximum value of G, their traces must be included within the curve

$$r = \left(\frac{2\pi}{e_c}\right) \sin^2 \alpha.$$

The gain from using the best form of cross section is small.

Graded Coils.—As the inner turns of the coil of a Kelvin galvanometer are very near the needle, they are much more effective than those in the outer portion of the coil, which, while they add much to the resistance of the instrument, contribute comparatively little to the galvanometer constant. This suggests that with a coil of a definite resistance, it might be best to concentrate the resistance in the turns near the needle, winding this part of the coil with a finer wire than that used for the outer portion. Maxwell has shown in his "Treatise on Electricity and Magnetism"* (Art. 719) that the diameter of the wire should increase with the diameter of the layer of which it forms a part, the exact law depending on the relation between the diameters of the covered and the bare wire.

As it is not possible to wind the coil with a wire having a cross section which is a function of its distance from the end of the wire, it is customary to wind the coil in three or four sections, each of a single size of wire.

* 3d ed.

In a very sensitive galvanometer having a resistance of 25 ohms, the three sections were wound with Nos. 40, 34, and 26 Brown and Sharpe gage. The thickness of the insulation was 0.002 cm. The galvanometer constant with the instrument so wound was about 33 per cent greater than if a No. 26 wire had been used for the entire coil, this being the size that would be most advantageous if the coil were uniformly wound.* From this it may be inferred that one cannot expect radical improvements in sensitivity to result from modifications of the coil structure.

Relation between the Galvanometer Constant and the Resistance of a Galvanometer.—Suppose, first, that the thickness of the insulation is zero and that the diameter of the wire is B_1. Then, using (2),

$$G = kAn' = K_0 n' = \frac{K_0}{B_1^2},$$

where k and K_0 are constants.

The resistance per unit volume of the coil, w, will be

$$w = \frac{K_1 n'}{B_1^2} = \frac{K_1}{B_1^4},$$

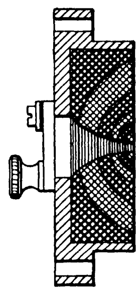


FIG. 5.—Cross section of graded coil.

and the galvanometer resistance, if V is the volume of the coil, will be

$$R_G = Vw = \frac{K_2}{B_1^4}.$$

Therefore

$$G = K_3\sqrt{R_G}. \tag{4}$$

So if the thickness of the insulation is zero, the galvanometer constant is proportional to the square root of the galvanometer resistance.

Suppose the bobbin to be filled with an insulated wire, the diameter outside the insulation, designated by C, being the same as that of the bare wire just considered. Let the diameter of the wire itself be B. As the number of turns has remained the same, G is not changed, but the resistance will be increased in the ratio C^2/B^2; call this ratio y^2; then, to keep G the same if the *new* value of R_G is used,

$$G = K_3\frac{\sqrt{R_G}}{y}. \tag{5}$$

As the deflecting moment acting on the needle, due to the current I, is proportional to GI_G, the deflection will be

$$D = K_4 G I_G = K_5\frac{\sqrt{R_G}}{y} I_G. \tag{6}$$

* The construction of a graded coil of a definite resistance and having three sections is discussed by C. G. Abbot in *Ann. Astrophys. Observatory*, vol. 1, p. 244, 1900, where all the necessary formulae are given.

The deflecting moment in a moving-coil galvanometer is also proportional to the ampere-turns of a coil of fixed dimensions; so (6) applies to that instrument when R_G is the coil resistance.

Best Galvanometer Resistance.—It is well known that in the practice of most methods of electrical testing, the precision obtainable depends on the proper adjustment of the galvanometer resistance to the work in hand.

Let the galvanometer be inserted in a simple circuit containing an e.m.f. E and a resistance R external to the instrument. The deflection will be given by

$$D = K_5 I_G \frac{\sqrt{R_G}}{y} = \frac{K_5 E}{R + R_G}\left(\frac{\sqrt{R_G}}{y}\right).$$

If it is assumed that y is the same for all sizes of wire, the deflection will be a maximum when

$$R_G = R, \tag{7}$$

whence the common statement that the galvanometer should have a resistance equal to that of the rest of the circuit. It will be seen from Table 1 that y is, in fact, variable. Therefore,

$$D = \frac{K_6 E}{R + Vw}\left(\frac{\sqrt{w}}{y}\right), \tag{8}$$

where V is the volume of the coil; w, the resistance per unit volume of the winding; and y, the corresponding value of the ratio of the diameter of the covered wire to the diameter of the bare wire. Table I gives the values of w and y as determined from data furnished by the Belden Company for two sorts of covering.

TABLE I.—DATA FOR INSULATED COPPER WIRE

	Double silk covered				Silk enamel covered				
No. A.W.G.	Outside diam., in.	Ohm per cu. in.	Lb. per cu. in.	y	Outside diam., in.	Ohm per cu. in.	Lb. per cu. in.	y	No. A.W.G
30	0.0140	43.75	0.141	1.40	0.0131	50.56	0.162	1.31	30
31	0.0129	65.08	0.134	1.46	0.0119	77.60	0.157	1.33	31
32	0.0120	95.40	0.126	1.51	0.0109	116.8	0.151	1.37	32
33	0.0111	140.5	0.119	1.57	0.0099	179.1	0.146	1.40	33
34	0.0103	205.8	0.111	1.63	0.0091	265.0	0.137	1.44	34
35	0.0096	297.3	0.102	1.71	0.0084	396.7	0.131	1.50	35
36	0.0090	429.0	0.095	1.80	0.0076	597.3	0.126	1.52	36
37	0.0085	613.5	0.087	1.91	0.0071	887.5	0.120	1.60	37
38	0.0080	875.8	0.081	2.02	0.0065	1294.0	0.112	1.64	38
39	0.0075	1235.0	0.075	2.12	0.0060	1927.0	0.107	1.70	39
40	0.0071	1736.0	0.071	2.26	0.0056	2791.0	0.100	1.78	40

The "best galvanometer resistance," that is, the one giving the largest deflection, is found by substituting appropriate values from the table in (8) and plotting the results as in Fig. 6, where the volume of the coil is assumed to be 1 cu. in. Study of Table I shows that for silk enamel wires, Nos. 30 to 40 inclusive,

$$\frac{\sqrt{w}}{y} = 1.07w^{.42} \qquad \text{approx.}$$

Consequently, for this make of wire and range of gage numbers,

$$D = \frac{K_6'Ew^{.42}}{R + Vw} = \frac{K_6'Ew^n}{R + Vw}.$$

This expression is a maximum when

$$R_G = Vw = R\left(\frac{n}{1 - n}\right) = R\left(\frac{0.42}{0.58}\right) = 0.72R, \qquad \text{if} \qquad n = 0.42.$$

If R is 200 ohms, as assumed in plotting Fig. 6,

$$R_G = 144 \text{ ohms}.$$

Reference to the figure shows that in this case, if silk enamel wire is used, the galvanometer may have any resistance between 40 and 575 ohms and give a deflection 80 per cent of the maximum obtainable. It is seen that the ordinary rule for galvanometer resistance is not one to be slavishly followed.

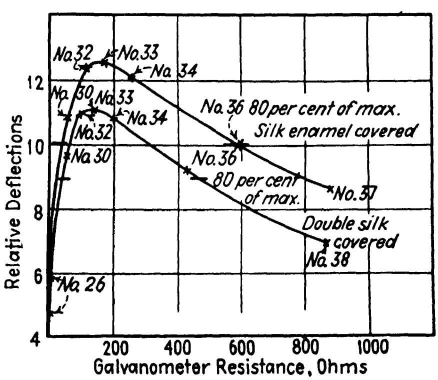


FIG. 6.—Illustrating effect of galvanometer resistance on deflection.

If a moving-coil galvanometer is to be selected from a particular line of instruments alike in all respects except coil resistance, one can, with perhaps a slight sacrifice of sensitivity, select one which will be critically damped (see page 28) and therefore quick working when inserted in the circuit.[1]

The Needle System.—There are two reasons for employing an astatic system:

1. To minimize the effects of local field variations, for it is clear that if the upper and lower members are in the same plane and exactly alike, they will be oppositely affected by variations in a uniform local field, the net effect on the system being zero. This ideally perfect system is never exactly realized.

2. To bring the windings nearer the movable magnets and thus increase the sensitivity.

A high *normal sensitivity*, or *factor of merit*, is attained only by the careful design and construction of the needle system, which, while being magnetically strong, must have a very small moment of inertia. Referring to the instrument shown in Fig. 4A, cobalt steel is used to obtain high magnetic strength. The magnets are 0.9 by 0.25 by 0.1 mm. They are mounted by minute drops of shellac on a straight glass rod 0.12 mm. in diameter. The magnets are so strong that they cause an appreciable electromagnetic damping if the circuit resistance is low. The mirror is very small—1 mm. wide, 3 mm. high, 0.08 mm. thick—and as perfect optically as is obtainable. The use of a long, thin mirror reduces the moment of inertia and contributes to the normal sensitivity. The mass of the complete system is about 4.5 mg. The whole system is constructed with exceeding care so that the axis of the glass rod coincides with that of the suspension fiber, and the center of gravity lies in the axis. *Close attention to symmetry minimizes the effects of mechanical disturbances.* In both the instruments of Fig. 4 the air damping may be controlled by the *damping regulator*, clearly shown in Fig. 4A.

It is seen that in the instrument of Fig. 4A the labor of construction has been concentrated on the all-important needle system, where it gives its return in a high normal sensitivity.

The Broca Galvanometer.[2]—The needle system of this instrument consists of two long, parallel, vertically placed magnets with consequent poles at their centers, the N pole of one being opposite the S pole of the other. Such a system is heavy and has an unduly great moment of inertia.

Magnetic Shields.[3]—In order that a galvanometer may be effectively protected from variations of the local field by means of iron shields, it must be of small size so that the volume of the space to be shielded is reduced to a minimum. The shields may be either spherical or cylindrical; for mechanical reasons the latter form is to be preferred.

The shielding ratio, that is, the ratio of the strength of the external local field to the corresponding field within the inner shield, is greatly affected by the arrangement of the iron. If the iron is all concentrated in a single cylindrical shield, having an outside radius five times the inner radius, the shielding ratio will be about 98 per cent of that for a shield of infinite thickness. If a single shield of permeability 202 is used, the maximum possible shielding ratio is about 50. This shows the futility of trying to protect a galvanometer thoroughly by the use of a single shield of great weight.

If a given weight of iron is used in several concentric shields, with air spaces between them, its effectiveness is vastly increased.

Given the innermost and outermost radii of a system of three shields, the shielding ratio is a maximum when the radii of the shells are in geometrical progression. The shielding ratio is practically proportional to the permeability, which depends upon the degree of magnetization of the iron. The fields against the variations of which a galvanometer must be shielded are very weak; and the fields at the inner shields, still weaker. Therefore the shield material should have a high initial differential permeability; that is, $\Delta B/\Delta H$ should be high for small values of H. For "Mumetal," $\Delta B/\Delta H$ is about 6,000 for low values of H.

Permalloy and similar materials suggest themselves. A. V. Hill[3] employs a Mumetal shield on the galvanometer shown in Fig. 4*A* and obtains a shielding ratio of about 1,000 by use of a composite multiple-layer shield of thin Mumetal sheets. The requisite nonmagnetic spacing is obtained by the use of sheet copper between the layers of Mumetal. The ends of the cylinder are closed by composite sheets of Mumetal and copper, two plates of Mumetal 1/16 in. separated by 1/4 in. of copper.

DATA OF MUMETAL SHIELD (HILL)

Thickness of Mumetal	0.006 in.
Thickness of copper	0.010 in.
Outside diameter	4.0 in.
Height	4.0 in.
Number of layers	20
Total weight of shield	5½ lb.
Weight of Mumetal	2 lb.

The shield was annealed at 900°C. in an atmosphere free of oxygen.

These high-permeability alloys, of which Mumetal is an example, are very susceptible to mechanical distortion; consequently, to obtain the highest permeability, the shields must be given the proper heat treatment after all the mechanical work upon them has been completed. The finished shield must not be exposed to mechanical shocks or to strong magnetic fields. In the instrument shown in Fig. 4*B* the very susceptible Mumetal shield 1 is protected from strong fields by an outer mild steel shield 2. If by chance the shields become magnetized, they must be demagnetized; otherwise the control magnets may not give a sufficiently strong directive field.

E. F. Nichols and S. R. Williams have given the data for concentric cylindrical shields of cast-silicon steel and soft, wrought-iron pipe.[3] With three concentric shields of the steel, weighing 80 lb., they obtained a shielding ratio of above 4,000. With five concentric shields of the iron, weighing 54 lb., they obtained a shielding ratio of over 2,700.

Suspensions.—Quartz fibers[4] are used as suspensions in all high-grade instruments.

Intrinsically, the fibers are very stiff, but this is compensated for by their great strength which permits the use of exceedingly fine threads. A fiber 0.0014 cm. in diameter breaks under a weight of about 10 g. and may be used to carry 5 g.; finer threads break at even higher stresses per unit area.

It is found that with quartz fibers the twist produced by a given turning moment is accurately proportional to the moment and independent of the previous history of the thread; this very important property allows quartz-fiber suspensions to be used in many sorts of instruments where a delicate torsional control is desired. It is important that the suspension fibers be long, of the smallest practicable diameter, and free from initial torsion. Fibers 200 mm. long and about 0.001 mm. in diameter are suitable for delicate instruments. If the torsional rigidity of the fiber is high, the total restoring moment acting on the suspended system is increased, and the sensitivity correspondingly decreased, the deflection remaining proportional to the current. If, in addition, there be initial torsion, and considerable sensitivity is required, it will be found that the deflection is no longer proportional to the current. The sensitivity will decrease as the deflection increases in one direction and will increase as the deflection increases in the opposite direction.

The Sensitivity of Reflecting Galvanometers.—The sensitivity of a Kelvin galvanometer depends on the arrangement and winding of the coils, on the construction of the suspended magnetic system, on the strength and position of the neutralizing magnets which produce the controlling field, on the torsional constant of the suspension fiber, and on its initial torsion. The initial torsion must be removed when the galvanometer is set up, as its presence necessitates a strong controlling field to bring the needle to its zero position and modifies its law of deflection.

Current Sensitivity. Microampere Sensitivity.—Quantitatively, the current sensitivity of a galvanometer is the deflection, as read from the scale, per unit current. This simple statement is not sufficiently definite, so in order to obtain results that admit of comparisons being made between different instruments, some convention must be adopted as to the conditions under which the sensitivity is to be measured.

It will be assumed that one of the mirror and scale methods of reading is used. If the current I_G is stated in amperes; the scale deflection D, in millimeters; and the scale distance L, in meters (1,000 scale divisions), then $S_I = D/LI_G$ is the deflection in millimeters that would be produced by 1 amp. if the scale had been at a meter's distance. S_I will be a very large number; consequently, for convenience in writing, the microampere (10^{-6} amp.) is frequently used as the unit of current. This gives the microampere sensitivity.

Relation between Time of Vibration and Current Sensitivity.—By common consent the sensitivity is measured when the suspended system has a stated time of vibration.

The connection of the sensitivity with the time of vibration of the needle system will be seen from the following. If the damping due to air friction is neglected, the time of vibration T_0 of a suspended magnetic system, having a moment of inertia P and a magnetic moment M, when placed in a field of strength H, will be

$$T_0 = 2\pi\sqrt{\frac{P}{MH}}.$$

The current through the instrument is given nearly enough by

$$I_G = \frac{10H}{G}\frac{D}{2{,}000L}.$$

The deflection is

$$D = \frac{200LI_GG}{H}.$$

Therefore

$$S_I = \frac{D}{I_GL} = \frac{200G}{H} = \frac{KMG}{P}T_0^2 = \frac{K'M\sqrt{R_G}T_0^2}{P}. \tag{9}$$

Therefore, with any given magnetic system, the sensitivity is proportional to the square of the time of vibration. If T_0 is doubled by changing H, the sensitivity will be increased fourfold, for to double the time of vibration the directive force must be reduced to one-fourth of its previous value, so the deflection due to the same value of the current is quadrupled.

If the needle system is very light the damping due to air friction may be great enough to cause a considerable departure from the relation (9).

In comparing instruments, the sensitivity must be reduced to the value that it would have if the movable system had some definite time of vibration; 10 sec. for a complete swing is that commonly taken.

Normal Current Sensitivity.—It is unfair to compare instruments that have very different resistances and different times of vibration. It is desirable to reduce the sensitivities to the values that they would have with the galvanometers wound to a standard resistance—1 ohm—with a wire having an insulation of zero thickness and with a needle system whose time of vibration is 10 sec. From the previous discussion the normal current sensitivity is

$$S_N = \frac{D}{I_GL}\frac{10^2}{T^2}\frac{y}{\sqrt{R_G}}. \tag{10}$$

Voltage Sensitivity. Microvoltage Sensitivity.—The voltage sensitivity under any given conditions is the deflection per unit voltage and is consequently the current sensitivity divided by the resistance. When dealing with moving-coil galvanometers (page 21), the condition should be imposed that the resistance of the circuit shall be such that the galvanometer is critically damped, since critical damping results in a great saving of time. Consequently, the resistance used in computing the voltage sensitivity should be that of the *entire circuit* when critical damping exists and not that of the instrument alone. This should be kept in mind when examining instrument-makers' catalogues. In all cases both the resistance of the galvanometer and the necessary external damping resistance should be known.

Megohm Sensitivity.—The megohm sensitivity is the number of megohms that must be inserted in a circuit containing an e.m.f. of 1 volt in order to obtain a galvanometer deflection of 1 mm. at a meter's scale distance. This is the same, numerically, as the microampere sensitivity, referred to on page 12.

THE MOTION OF THE SUSPENDED SYSTEM OF A GALVANOMETER

Suppose that when the movable system of a galvanometer is at rest, the circuit is suddenly closed so that a current flows through the instrument. Experience shows that in some cases the movable system arrives at its final deflected position by a series of oscillations of diminishing amplitude; in other cases, by a steady increase of the deflection without oscillations.

It is desirable to derive the equations that will represent the motion of the system sufficiently well for practical purposes, for the general considerations thus introduced are of importance in dealing with both current and ballistic galvanometers. An important special case is that of the critically damped instrument of the moving-coil type, as ordinarily used, and also when the period has been so reduced that the instrument has become an oscillograph capable of following a complex wave in its variations.

The Equation of Motion.—1. The angular deflection of a reflecting galvanometer is always small, so the deflecting moment at any instant may be taken as proportional to the instantaneous value of the current and represented by Ci, where C is a constant depending on the construction of the instrument.

2. For small deflections the restoring moment will be proportional to the angle through which the system has been turned; that is, it will be equal to $\tau\theta$, where θ is the angle of deflection and τ is the restoring moment for unit angular deflection.

3. The system as it moves is retarded by air friction, etc., and in some cases by induced currents. It is customary to *assume* that this retarding moment is proportional to the angular velocity of the system and is therefore represented by $k\frac{d\theta}{dt}$ where k is the coefficient of damping.*

* This law of damping was introduced by Gauss and W. Weber in their study of the behavior of the vibrating magnets used in their magnetic measurements at Göttingen, 1836–1837. With air damping, in order that this law may be reasonably well ful-

Let P be the moment of inertia of the movable system. The total moment acting to change the angular velocity of a body rotating about a fixed axis is the product of the moment of inertia and the angular acceleration. On equating this product to the sum of the turning moments acting on the system,

$$P\frac{d^2\theta}{dt^2} = Ci - \tau\theta - k\frac{d\theta}{dt}.$$

Consequently, the motion takes place according to the equation

$$P\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + \tau\theta = Ci. \qquad (11)$$

The right-hand member of Eq. (11) may be a function of t, a constant, or zero according to the conditions of the problem. In this section i will be taken as constant (circuit closed) or zero (circuit opened).

The mathematical form of Eq. (11) and the following discussion should be compared with those for the displacement of electricity in a circuit containing resistance, inductance, and capacitance in series.

The deflection of a galvanometer may be oscillatory or nonoscillatory, and its ultimate value will be most quickly attained if the conditions are such that the motion is just becoming nonoscillatory. This occurs when the relation $k^2/\tau = 4P$ is on the point of being fulfilled. The instrument is then said to be critically damped.

When $k^2/\tau > 4P$, the motion of the suspended system will be nonoscillatory; and when $k^2/\tau < 4P$, it will be oscillatory. When the instrument is not critically damped, the solution of (11) is*

$$\theta = C_1\epsilon^{m_1t} + C_2\epsilon^{m_2t} + \left(\frac{C}{P(m_1 - m_2)}\right)\left(\epsilon^{m_1t}\int\epsilon^{-m_1t}idt - \epsilon^{m_2t}\int\epsilon^{-m_2t}idt\right). \qquad (12)$$

m_1 and m_2 are the roots of the equation

$$Pm^2 + km + \tau = 0,$$

filled, the damping must be slight, the amplitude of the vibration small, and the restoring moment due to the suspension large. That this law is not absolutely exact is apparent, for, according to it, the movable system when once set in vibration would continue to swing for an infinite time with a constantly decreasing amplitude. But it is a matter of common experience that the system comes to rest in a comparatively short time. However, the results obtained by Gauss's theory are in close enough agreement with the observed facts to warrant its use.

* See Cohen, "Differential Equations," p. 105; Campbell, "Differential Equations," p. 56.

or

$$m_1 = -\frac{k}{2P} + \sqrt{\frac{k^2}{4P^2} - \frac{\tau}{P}}. \tag{13}$$

$$m_2 = -\frac{k}{2P} - \sqrt{\frac{k^2}{4P^2} - \frac{\tau}{P}}. \tag{14}$$

The constants C_1 and C_2 must be determined to fit the conditions of the particular case under consideration. When a current of a definite strength is sent through the galvanometer,

$$i = I, \qquad \text{a constant.}$$

At $t = 0$ both the deflection and the angular velocity are zero, or

$$t = 0 \qquad \theta = 0$$

$$t = 0 \qquad \frac{d\theta}{dt} = 0.$$

Imposing the first of these conditions on (12) gives

$$0 = C_1 + C_2 + \frac{CI}{\tau};$$

but $CI/\tau = \theta_F$, the final value of the deflection.

$$\left.\frac{d\theta}{dt}\right]_{t=0} = C_1 m_1 + C_2 m_2 = 0.$$

Therefore

$$C_1 = -\frac{\theta_F m_2}{m_2 - m_1}, \qquad \text{and} \qquad C_2 = \frac{\theta_F m_1}{m_2 - m_1}.$$

The value of θ which fulfills the conditions is therefore

$$\theta = \theta_F\left(-\frac{m_2}{m_2 - m_1}\epsilon^{m_1 t} + \frac{m_1}{m_2 - m_1}\epsilon^{m_2 t}\right) + \theta_F. \tag{15}$$

If, when the needle is at rest in its deflected position, the circuit is broken, the deflecting moment due to the current becomes zero; so

$$P\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + \tau\theta = 0.$$

Therefore

$$\theta = C_1\epsilon^{m_1 t} + C_2\epsilon^{m_2 t}.$$

When

$$t = 0 \qquad \theta = \theta_F$$

and

$$t = 0 \qquad \frac{d\theta}{dt} = 0$$

Therefore

$$\theta_F = C_1 + C_2,$$

and

$$m_1C_1 + m_2C_2 = 0.$$

Therefore

$$\theta = \theta_F\left(\frac{m_2}{m_2 - m_1}\epsilon^{m_1t} - \frac{m_1}{m_2 - m_1}\epsilon^{m_2t}\right). \tag{16}$$

Nonoscillatory Deflection.—If $k^2 > 4\tau P$, both m_1 and m_2 are real and negative, and (15) is the equation of a curve which gradually rises toward the value θ_F; in this case, the galvanometer is said to be overdamped. When the circuit is broken, the needle returns to zero according to equation (16). Equations (15) and (16) are plotted in Fig. 7, where the values $\tau = 0.2$, $P = 0.2$, and $k = 1.0$ are assumed.

Oscillatory Deflection.—When $k^2 < 4\tau P$, m_1 and m_2 are complex; that is

$$m_1 = -a + jb,$$
$$m_2 = -a - jb,$$

where

$$a = \frac{k}{2P} \qquad \text{and} \qquad b = \sqrt{\frac{\tau}{P} - \frac{k^2}{4P^2}} = \sqrt{\frac{\tau}{P} - a^2}.$$

In this case, the final deflection is attained by a series of oscillations. If the values of m_1 and m_2 are substituted in (12), and the resulting equation simplified by the use of the exponential values of the sine and cosine and the relation

$$A \sin\beta + B\cos\beta = \sqrt{A^2 + B^2}\sin\left(\beta + \tan^{-1}\frac{B}{A}\right),$$

it will be found that

$$\theta = \theta_F - \theta_F\epsilon^{-at}\left[\sqrt{\frac{a^2 + b^2}{b^2}}\sin\left(bt + \tan^{-1}\frac{b}{a}\right)\right]. \tag{17}$$

The variable part of (17) represents an oscillation of constantly decreasing magnitude and having a period

$$T = \frac{2\pi}{b} = \frac{2\pi}{\sqrt{\frac{\tau}{P} - \frac{k^2}{4P^2}}}. \tag{18}$$

The time of a complete swing when no damping is present is

$$T_0 = 2\pi\sqrt{\frac{P}{\tau}}.$$

So

$$\theta = \theta_F - \theta_F\frac{T}{T_0}\epsilon^{-\frac{kt}{2P}}\sin\left(\frac{2\pi}{T}t + \tan^{-1}\frac{4\pi P}{kT}\right). \tag{19}$$

The elongations, or maximum and minimum values of the deflection, occur when

$$t = 0, \qquad t = \frac{T}{2}, \qquad t = T, \cdot \cdot \cdot , \qquad t = n\frac{T}{2}.$$

For the first swing, θ_1,

$$\sin\left(\frac{2\pi t}{T} + \tan^{-1}\frac{4\pi P}{kT}\right) = -\frac{T_0}{T}. \tag{19a}$$

Therefore

$$\frac{\theta_1}{\theta_F} = 1 + \epsilon^{-\frac{kT}{4P}},$$

$\epsilon^{-\frac{kT}{4P}}$ is the fractional first overswing. Using the results on page 17, and setting $B_0 = k/2\sqrt{P\tau}$ (page 20), the fractional overswing is

$$\epsilon^{-\pi B_0 \frac{T}{T_0}} = \epsilon^{-\frac{\pi B_0}{\sqrt{1-B_0^2}}}$$

If, when the needle is at rest in its deflected position, the circuit is broken, the return to zero is by a series of oscillations according to the equation

$$\theta = \theta_F \frac{T}{T_0}\epsilon^{-\frac{kt}{2P}} \sin\left(\frac{2\pi}{T}t + \tan^{-1}\frac{4\pi P}{kT}\right). \tag{20}$$

Equations (19) and (20) are plotted in Fig. 7, where the values $\tau = 0.2$, $P = 0.2$ and $k = 0.1$ are assumed.

Logarithmic Decrement.—Let $\lambda = kT/4P$; then (19) and (20) become

$$\theta = \theta_F - \theta_F \frac{T}{T_0}\epsilon^{-\left(\frac{2\lambda}{T}\right)t} \sin\left(\frac{2\pi t}{T} + \tan^{-1}\frac{\pi}{\lambda}\right), \tag{19b}$$

and

$$\theta = \theta_F \frac{T}{T_0}\epsilon^{-\left(\frac{2\lambda}{T}\right)t} \sin\left(\frac{2\pi t}{T} + \tan^{-1}\frac{\pi}{\lambda}\right) \tag{20a}$$

The utility of this substitution lies in the fact that λ is much more easily determined than its components k and P.

λ is called the *Napierian logarithmic decrement;* it is a quantity of importance in the theory of damped vibrations.

The first elongation after the circuit is broken occurs when $t = T/2$. Substituting this value in (20a), and using (22), gives

$$\theta_1 = \theta_F \epsilon^{-\lambda} \cos \pi.$$

The nth elongation, when $t = n\frac{T}{2}$, is

$$\theta_n = \theta_F \epsilon^{-n\lambda} \cos n\pi.$$

Therefore

$$\frac{\theta_1}{\theta_n} = \epsilon^{\lambda(n-1)},$$

and

$$\lambda = \frac{1}{n-1} \log_\epsilon \frac{\theta_1}{\theta_n} \qquad (21)$$

The method of determining λ is obvious: To obtain the necessary data one has only to set the movable system in motion; read an elongation,

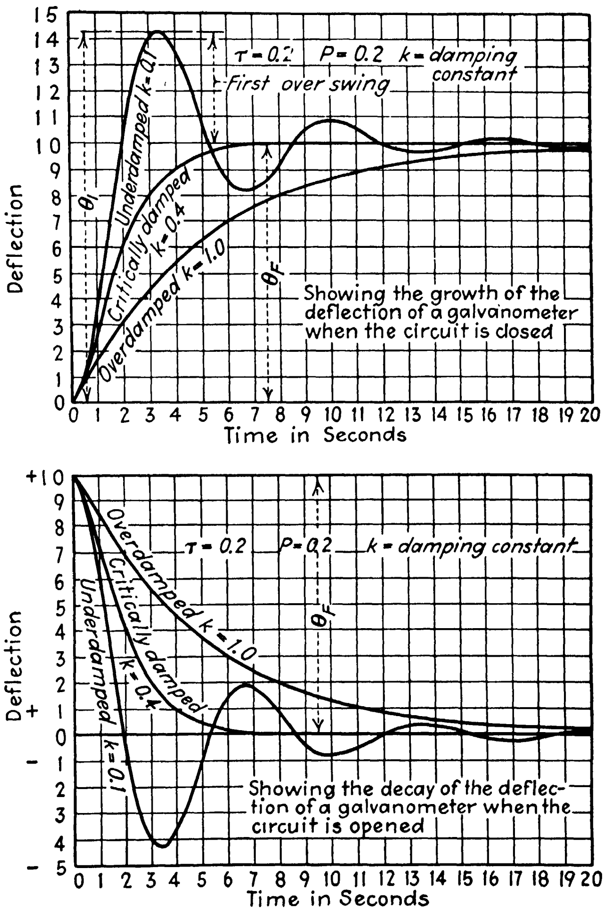


FIG. 7.—Illustrating the motion of a galvanometer needle.

which will be called θ_1; and after a counted number of elongations read θ_n.

Experiment shows that with air damping λ is slightly affected by the amplitude of vibration but not enough to give rise to practical difficulties.

Influence of Damping on the Time of Vibration.—From the foregoing,

$$T = \frac{2\pi}{\sqrt{\frac{\tau}{P} - \frac{k^2}{4P^2}}} = \frac{2\pi}{\sqrt{\frac{4\pi^2}{T_0^2} - \frac{4\lambda^2}{T^2}}}.$$

Therefore

$$\frac{T}{T_0} = \sqrt{\frac{\pi^2 + \lambda^2}{\pi^2}}.$$

Also

$$T = \frac{1}{\frac{1}{2\pi}\sqrt{\frac{\tau}{P}}\sqrt{1 - \frac{k^2}{4\tau P}}}. \tag{22}$$

Denote the ratio of the actual damping coefficient to that required for critical damping (page 18) by B_0, $B_0 = k/k_c = k/2\sqrt{\tau P}$, and the frequency of oscillation by f; then

$$\frac{T}{T_0} = \frac{f_0}{f} = \frac{1}{\sqrt{1 - B_0^2}}, \qquad f = f_0\sqrt{1 - B_0^2}. \tag{22a}$$

It is seen that λ must be large before it greatly affects the time of vibration. For $T/T_0 = 1.01$; $\theta_1/\theta_2 = 1.56$; $\lambda = 0.495$.

Critical Damping—A galvanometer is critically damped when the motion of the needle is just becoming nonoscillatory. In this case, when the current I is constant and $k^2 = 4\tau P$, the solution of (11) becomes

$$\theta = \theta_F + \epsilon^{-\frac{kt}{2P}}(C_1 + C_2 t). \tag{23}$$

To determine C_1 and C_2,

$$t = 0 \qquad \theta = 0$$

$$t = 0 \qquad \frac{d\theta}{dt} = 0.$$

Therefore

$$C_1 = -\theta_F.$$

$$\left.\frac{d\theta}{dt}\right]_{t=0} = -\frac{kC_1}{2P} + C_2 = 0;$$

therefore

$$C_2 = -\frac{\theta_F k}{2P}.$$

So

$$\theta = \theta_F - \theta_F \epsilon^{-\frac{kt}{2P}}\left(1 + \frac{k}{2P}t\right). \tag{24}$$

For this case

$$\frac{k}{2P} = \frac{2\pi}{T_0}$$

and

$$\theta = \theta_F - \theta_F \epsilon^{-\left(\frac{2\pi}{T_0}\right)t}\left(1 + \frac{2\pi}{T_0}t\right). \tag{25}$$

When the circuit is broken, the needle returns to zero in accordance with the equation

$$\theta = \theta_F \epsilon^{-\left(\frac{2\pi}{T_0}\right)t}\left(1 + \frac{2\pi}{T_0}t\right). \tag{26}$$

Figure 7 shows the character of the motion in this case for $\tau = 0.2$ and $P = 0.2$.

Using the preceding equations, the time required for the deflection to approach within a given percentage of its ultimate value may be calculated. The results for a particular case are shown in Fig. 8; it is seen that the reading is most quickly obtained when the instrument is very

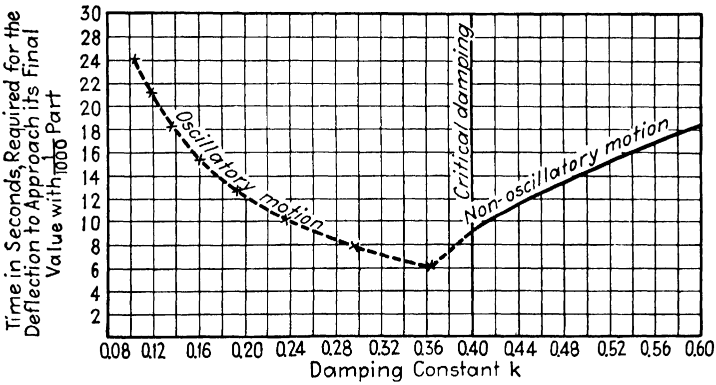


FIG. 8.—Showing the effect of damping on the time required by a galvanometer to attain its deflection.

nearly critically damped. The deflection of a critically damped galvanometer is within $\frac{1}{10}$ per cent of its ultimate value at a time that is approximately equal to $1.5T_0$, and within 1 per cent at a time which is approximately equal to T_0.

Moving-coil, or the D'Arsonval, Galvanometer.[5]—William Thomson used the suspended-coil principle in his siphon recorder (1870), and its application to galvanometers was later suggested by Maxwell in his "Treatise on Electricity and Magnetism."

The name D'Arsonval is frequently applied to galvanometers of this class, attention having been recalled to them by Deprez and D'Arsonval in 1882.

The great practical advantage of the moving-coil instrument lies in its freedom from the effects of stray fields and the ease with which a long and uniform scale may be attained. These are the features that have caused it to be adopted, in a modified form, for direct-current ammeters and voltmeters.

The essential features of a moving-coil galvanometer are shown in Fig. 9. The field is furnished by a strong permanent magnet, and the movable coil swings in the air gaps between the poles of the magnet and a fixed iron core. The coil is hung by a fine wire suspension which also serves as a lead, while below the coil the current is taken out by a loosely coiled metallic spiral. A proper torsion head for adjusting the coil

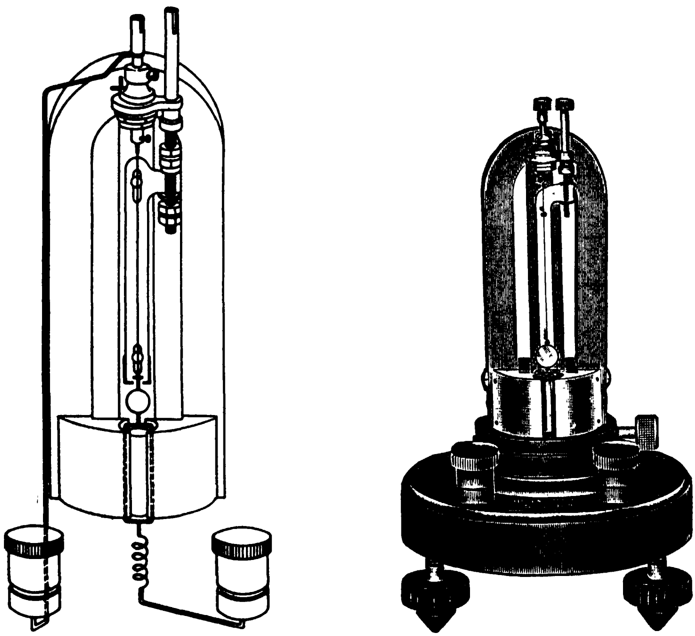


FIG. 9.—Moving-coil galvanometer.

vertically and setting the zero reading is provided. The entire coil system is mounted in a removable frame so that the coils may be readily changed.

Suppose that the coil is rectangular, the total length of active wire is l, and the half breadth of the coil is b and that the field H in which the coil is placed is uniform. The turning moment due to the current I is

$$M = IHlb \cos \theta,$$

where θ is the angle between the lines of force and the plane of the coil. If the motion of the coil be resisted by a spring, the latter will be twisted until the restoring moment due to it is equal to the deflecting moment due to the current. If the zero position of the plane of the coil is in the direction of the lines of force, the restoring moment will be $\tau\theta$, where τ is the restoring moment for unit angular deflection (a constant). Thus, at equilibrium

$$\tau\theta = IHlb \cos \theta.$$

If there are n turns,

$$I = \frac{\tau}{nHlb}\left(\frac{\theta}{\cos \theta}\right).$$

The moving-coil galvanometer is a secondary instrument; that is, the relation between the deflection and the current is always determined by calibration: but the formula serves to direct attention to certain quantities which are involved in its action.

The Magnet Structure.—It is seen that if there were no modifying conditions, the current sensitivity would increase proportionally to H. This indicates that the magnet should be very strong. However, strength is not the only thing to be considered, for it is well known that so-called permanent magnets gradually lose their strength; that is, they "age." The aging depends upon the quality of the steel, the design of the magnetic circuit, and the temperature variations and mechanical jarring to which the magnet is subjected. Any deterioration will influence the sensitivity of the instrument, so in this and in many other cases—for instance, in the magnets used in direct-current ammeters and voltmeters and in watt-hour meters—it is necessary to resort to artificial aging. As pointed out by Strouhal and Barus, this may be done by the proper heat treatment at moderate temperatures. Their procedure was, after the magnet had been hardened, to heat it in a steam bath at 100°C. for 20 or 30 hr., then magnetize it strongly, and afterward heat it again in the steam bath for 4 or 5 hr. In addition, some makers resort to a partial demagnetization. The final result is that while the strength of the magnet is reduced, the remaining magnetization is very permanent.

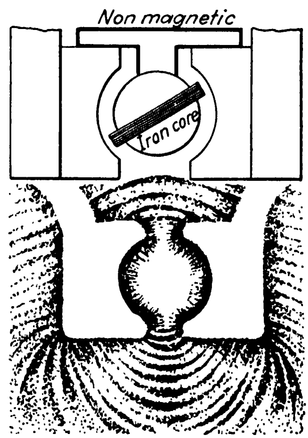


FIG. 10.—Pole pieces for producing a radial field and the resultant field.

The temperature coefficient of magnets such as are used in galvanometers and in direct-current ammeters and voltmeters is about −0.025 per cent per degree centigrade rise of temperature; it varies with the magnet. Frequently a magnetic shunt is used so that H can be adjusted and the instrument made aperiodic when used in circuits having different resistances.

The field in the air gap is usually 700 to 1,000 c.g.s. units. Special instruments having only a few turns and an iron core, and consequently a very narrow air gap, may have fields of 2,000 c.g.s. units. It is not usual to employ special alloy steels for the magnets unless there are space limitations.

The Coil Structure.—Practically, it is found that the increase in current sensitivity may not be proportional to the increase in H, for, as first pointed out by Ayrton and Mather, the coil itself may be slightly magnetic owing to the presence of minute amounts of iron as an impurity in the metal or insulation or which have been worked into the insulation during the process of winding. Consequently, in very strong parallel fields the attraction between the magnet and the induced poles on the coil may be great enough to reduce the sensitivity materially. The magnetic action of the coil also gives rise to indefiniteness of the zero reading, which will be displaced in the direction of the just previous deflection.

These facts being appreciated, the utmost care is now taken to use truly nonmagnetic insulated wire on the movable coil. The difficulties would be reduced to zero if a perfectly uniform *radial* field could be employed. Obviously induced poles, if they existed, would not then affect the restoring moment.

The use of a radial field was introduced in the Weston direct-current ammeters and voltmeters in 1888.

When a coil moves in a truly radial field, the deflecting moment is given by

$$M = 2Hhb(n'AI),$$

where h is the height of the coil, b the half breadth, n' the number of turns of wire per unit area and A the area of the cross section. The deflecting moment is proportional to the ampere-turns, as in the needle galvanometer. As the instruments are constructed, the field is only approximately radial, so this law is not exactly followed.

The moving parts should have a very small moment of inertia, as this reduces the time required for a reading. Consequently in modern instruments, such as the Moll galvanometer (Kipp and Zonen, makers), the coil is long, narrow, and self-supporting, no form or bobbin being used.

The mirror should be very small and light, to reduce the moment of inertia, should be perfect optically, and should be mounted without distortion so that the image on the scale will be sharp, allowing small deflections to be read with accuracy. The irreducible minimum for the moment of inertia of the coil structure is set by that of the mirror. Zernike makes the moment of inertia of the mirror the starting point in design and deduces that for a high voltage sensitivity with critical damping, the moment of inertia of the coil should be small compared with that of the mirror.[6]

The coil structure must be perfectly balanced. This is exceedingly important, as it renders the instrument practically immune to the usual mechanical disturbances. If a taut suspension is used, and the balance is not perfect, a change of level changes the period, sensitivity, and critical damping resistance.

In specially constructed modern high-sensitivity, short-period galvanometers the internal iron cores are frequently omitted, the requisite field strengths being obtained by using narrow air gaps. Magnetic shunts are employed to vary the sensitivities. The coils may be about 17 × 7 mm.; a mass as low as 20 mg. for a coil of 20 turns and a moment of inertia of less than 4.0×10^{-4} g.cm.^{2} are obtainable by expert construction. The wire for the coils of such instruments must be guaranteed free from iron. It is taken directly from the metal spools on which it is purchased, and the surface cleaned with benzol. It is then heated slowly to a dull-red heat by the passage of a current, then cooled to about 100°C., and, while still hot, thinly coated with dilute, uncontaminated bakelite varnish, which is baked on, care being taken that the temperature is not so high that the varnish changes color. Several coats are thus applied. Silk insulation is avoided on account of its iron content. The coils are self-supporting, the turns being cemented together with bakelite varnish. The mirrors employed are about 4.5 × 3.0 × 0.1 mm. and as perfect optically as obtainable. The mass is about 6 mg., and the moment of inertia about 4.5×10^{-5} g.cm.^{2}. With such light movable systems the effect of air damping is appreciable. The results obtained by Downing are shown in Table II.

TABLE II.—PERTAINING TO A MOVING-COIL GALVANOMETER BY DOWNING

Period, sec.	Resistance, ohms	External damping resistance, ohm	Sensitivity $S_I = D/LI_G$
3	50	0	0.53×10^9
3	50	100	0.91×10^9
3	50	500	1.9×10^9

It is not possible to obtain such results with coils constructed in the dust-laden air of the city.

An effective arrangement for clamping the movable system during transportation is necessary.

Suspensions.—The materials commonly used for the suspension wires in commercial instruments are 14 and 24 carat gold. Where thermoelectric e.m.fs. must be avoided, copper is used. Gold is much more resistant to atmospheric corrosion than the phosphor bronze formerly employed.

The sensitivity will be increased by diminishing the torsion constant τ which is proportional to the fourth power of the diameter of the suspension wire. Of course, with a given coil, the stress per unit area on the suspension is inversely as the square of the diameter.

To increase the sensitivity without increasing the unit stress on the suspension, Ayrton and Perry suggested that a flat strip be substituted for the round wire. A strip having a breadth of about ten times its thickness has approximately one-fifth the torsional rigidity of a round wire of the same length and sectional area. Such suspensions are very commonly used in commercial moving-coil galvanometers. Northrup has suggested the employment of a cable of very fine wires as a means of supporting heavy coils. For a cable capable of supporting a given weight, the torsional rigidity decreases in proportion as the number of strands is increased. This form of suspension is frequently used in portable galvanometers.

The use of the taut suspension employed in the original D'Arsonval instrument is not advisable unless the coil is balanced with exceeding care by the makers or there is a special provision for balancing, for it is difficult to attach the suspension wires so that their axes pass through the center of gravity of the coil when the suspension is drawn taut. If this condition is not fulfilled, the center of gravity is coerced into taking up an abnormal position, and the weight of the coil will cause a turning moment which will vary with the tightness of the wire and the level of the instrument. For these reasons, in many sensitive instruments the coil is allowed to hang free, and the electrical connection at the bottom made by a loose spiral which may have a very small torsional rigidity. This procedure also decreases the stress in the upper wire so that it may be made smaller. However, in special cases where the instrument is to be subjected to great changes of level, as on shipboard, the taut wire must be employed. In the Sullivan marine galvanometer and other similar instruments, the center of gravity of the moving coil can be adjusted to its proper position by bending bits of lead wire which project from the coil frame or by adjusting two sets of screws which project at right angles through and perpendicular to the shank supporting the coil.

To gain the highest sensitivities it is necessary to employ restoring moments much below those furnished by suspension strips of the smallest practicable cross sections. In the Zernike galvanometer (Kipp and Zonen, makers), a fine quartz fiber is used to support the coil, and the current is led in and out by exceedingly fine gold ribbons which hang loosely along the sides of the quartz suspension. They furnish only about 10 per cent of the total restoring torque, so the effect of semi-permanent set is practically nil. It is essential that there be no tension on the ribbons; otherwise they exercise considerable torsional control.

For this type of suspension, τ may be of the order of magnitude of 0.005 c.g.s. units.

W. R. Coley (1924) proposed a different solution for the difficulty.[5] Ordinarily, to annul the effect of magnetic impurities in the coil, the field is made radial. However, in the high-sensitivity galvanometers made by the Leeds and Northrup Company according to Coley's suggestion the pole pieces may be adjusted radially with respect to the internal iron core by very finely pitched screws, so that the field is distorted and no longer radial. This distorted field acts on the magnetic impurities of the coil or, as also suggested by Coley, on very small amounts of a magnetic material which have been intentionally attached to the coil, in such a manner that a torque is set up, opposed to that of the suspension. Consequently the net control is much reduced without decreasing the cross section of the suspension. The sensitivity can thus be increased five or ten times.

This fundamental idea is used by Moll in his galvanometer of high voltage sensitivity. A small piece of paramagnetic material is attached to the movable system at some distance from the coil. It moves in a field that can be adjusted so that a couple proportional to the angle of deflection acts to annul the restoring torque of the suspension.

All connections about the suspended system of a moving-coil galvanometer should be soldered, as otherwise extraneous resistances may be introduced by loose contacts or corrosion. In addition, abnormally high thermal e.m.fs. may be introduced into the circuit. Rosin should be used as a flux, as it is difficult to remove the last traces of acid, which would seriously corrode delicate wires. For the same reason, so-called noncorrosive soldering liquids should be avoided.

Effect of Changes of Temperature; Parasitic E.m.fs.—If the instrument is to be used as an unshunted current galvanometer, a change of room temperature will alter the calibration. Experiments show that for phosphor-bronze strip, the elasticity decreases about 0.05 per cent per degree rise of temperature. The strength of the magnet also diminishes with an increase of temperature, the change being 0.01 or 0.02 per cent per degree. It varies with different magnets. The tendency of these effects is toward compensation, but in general their relative magnitudes will not be such as to eliminate the error.

If the instrument is shunted, the multiplying power of the shunt will depend, to a certain extent, upon temperature conditions, for the galvanometer is wound with copper, and the shunt is most probably of a material having zero temperature coefficient.

When the instrument is used as a potential galvanometer or millivoltmeter, the change in resistance introduces an error. If a series resistance is employed, and it is made partly of copper and partly of manganin in the proper proportion, its net temperature coefficient may be made such that the instrument is almost exactly compensated for variations of room temperature.

It should be remembered that when the temperature varies very rapidly, there will be a time lag in the change of resistance and in the change of the strength of the magnet.

In using an instrument of high voltage sensitivity, the effect of sporadic temperature variations on the circuit as a whole must not be lost sight of. They continually set up minute parasitic e.m.fs. to which the coil responds, thus rendering the zero and deflection readings indefinite, when the highest precision is desired. The entire circuit should be as free as possible from junctions of dissimilar metals and should be shielded from drafts and from other sources of temperature variations.

Magnetic Damping.—To prevent loss of time when using any galvanometer it is necessary that it be properly damped. Immediately the coil begins to move, an e.m.f. is set up which sends a current through the circuit; by Lenz's rule the direction of this current is such as to check or damp the motion. If the total circuit resistance be high, little effect will be produced. The instrument will be underdamped, and the observer will lose time waiting for the deflection to come to its final value. Critical damping may be obtained by shunting but at the expense of the ultimate deflection. It may also be obtained by attaching a closed loop of wire of the proper resistance to the coil or by winding the coil on a very light metal bobbin, as is done in direct-current ammeters and voltmeters. The last two expedients increase the moment of inertia of the movable parts.

If the total circuit resistance is low, critical damping may be obtained by the insertion of the proper additional resistance in the circuit. This will reduce the ultimate deflection. The sacrifice is justified by the increased speed with which the readings may be taken (see page 21). In many modern instruments, adjustable magnetic shunts are used to bring about critical damping by altering the strength of the field in which the movable coil swings. This allows the promptness of response to be maintained even if the resistance of the circuit is altered.

When it is necessary that the coil be perfectly free to move from and be brought back quickly to its zero position, a short-circuiting key, which must be free from thermoelectromotive forces, may be placed across the terminals of the galvanometer. The motion of the coil is promptly checked by depressing the key.

The damped D'Arsonval instrument is very useful as a ballistic galvanometer on account of its quick return to the zero reading.

The Critically Damped Moving-coil Galvanometer.[5]—The field in which the coil moves will be assumed to be radial.

SYMBOLS USED IN THE DISCUSSION

H = strength of field.
l = total length of active wire.
b = one-half the breadth of movable coil.
lb = A, sum of the areas of all the turns.
P = moment of inertia of entire moving system.
τ = torsion constant of suspension, restoring moment for unit angular deflection.
θ = deflection at any instant.
θ_F = final value of θ when a constant current I_F flows in coil, or initial value of θ when circuit is broken.
i = current in coil at any instant.
I_F = final value of current after coil has come to rest in its deflected position.
T_0 = time of an undamped vibration of movable system.
R = total resistance of circuit, including galvanometer.
L = total inductance of circuit.
R_G = resistance of coil of galvanometer.
E = e.m.f. of battery.
E_B = back e.m.f. generated by movement of coil.
C = Hlb, turning moment which acts on coil when it carries unit current.
S_I = current sensitivity.
S_V = S_I/R, voltage sensitivity.

The turning moment acting on the coil at any instant is $iHlb = iC$, while the restoring moment is $\tau\theta$. After the coil has come to rest,

$$\tau\theta_F = I_F C,$$

or

$$I_F = \frac{\tau\theta_F}{C}. \tag{27}$$

An important use of the moving-coil instrument is in a closed circuit of moderate resistance, as occurs with the Dieselhorst or the Brooks potentiometer or when the instrument is used in connection with thermoelectric junctions. In these cases, the resistance of the circuit is constant or nearly so.

Consider the instrument to form a part of a galvanic circuit which includes a source of e.m.f. and a resistance external to the galvanometer. When the circuit is closed, the coil will begin to move; and as it swings in the field H, a back e.m.f. will be set up; its magnitude will be

$$E_B = Hlb\frac{d\theta}{dt} = C\frac{d\theta}{dt}.$$

Consequently the current at any instant will be

$$i = \frac{E - L\frac{di}{dt} - C\frac{d\theta}{dt}}{R}. \tag{28}$$

Let k' be the damping coefficient when the galvanometer circuit is open. This damping may be due to currents induced in damping loops attached to tbe movable coil or in a metal frame upon which the coil is wound and, to a slight extent, to the air damping. It follows that

$$P\frac{d^2\theta}{dt^2} = \left(\frac{E - L\frac{di}{dt} - C\frac{d\theta}{dt}}{R}\right)C - k'\frac{d\theta}{dt} - \tau\theta,$$

or

$$P\frac{d^2\theta}{dt^2} + \left(\frac{C^2}{R} + k'\right)\frac{d\theta}{dt} + \tau\theta = \frac{EC}{R} - \frac{CL}{R}\frac{di}{dt}. \tag{29}$$

The term containing di/dt may be neglected, so

$$P\frac{d^2\theta}{dt^2} + \left(\frac{C^2}{R} + k'\right)\frac{d\theta}{dt} + \tau\theta = \frac{EC}{R} = I_F C = \tau\theta_F. \tag{29a}$$

Here $\left(\frac{C^2}{R} + k'\right)$ is the damping constant and replaces k in the equation on page 15.

The case of special importance is when the constants of the circuit are such that the instrument is critically damped. Then

$$\frac{C^2}{R} = 2\sqrt{\tau P} - k' = \frac{4\pi P}{T_0} - k'. \tag{30}$$

Current and Voltage Sensitivity.—If it is assumed that the damping on open circuit is negligible compared with that arising from the electromagnetic action of the coil,

$$C^2 = 2R\sqrt{\tau P} = (Hbl)^2. \tag{30a}$$

$$C = \sqrt{\frac{\tau_0 \tau R}{\pi}} = 2\sqrt{\frac{\pi P R}{T_0}} = Hbl.$$

Using θ in radians and I_F in absolute units,

$$S_I' = \frac{\theta_F}{I_F} = \frac{C}{\tau}.$$

$$S_F' = \frac{C}{\tau} = \frac{1}{\sqrt{\pi}}\sqrt{\frac{T_0 R}{\tau}} = \frac{T_0^{3/2}}{2\pi^{3/2}}\sqrt{\frac{R}{P}} = \sqrt{2}\sqrt{\frac{R}{\tau}}\left(\frac{P}{\tau}\right)^{1/4} = \frac{4R^2P}{C^3} = \frac{2R}{C}\sqrt{\frac{P}{\tau}}.$$

R is the total resistance of the circuit necessary for critical damping. Using the microampere sensitivity (page 12), if R is expressed in ohms and the reading in millimeters, on a scale at a meter's distance, these results become

$$S_I = 2 \times 10^{-4}\frac{C}{\tau} = 3.57\sqrt{\frac{T_0 R}{\tau}} = 0.57 T_0^{3/2}\sqrt{\frac{R}{P}} = 8.9\sqrt{\frac{R}{\tau}}\left(\frac{P}{\tau}\right)^{1/4}$$

$$= 8 \times 10^{14}\frac{R^2P}{C^3} = \frac{4R10^6}{C}\sqrt{\frac{P}{\tau}}, \tag{31}$$

for the angle of deflection measured on the scale is twice that turned through by its coil, and the deflections are always small. The voltage sensitivity or the deflection per microvolt *applied to the circuit* is

$$S_V = \frac{S_I}{R},$$

so

$$S_V = 2 \times 10^{-4}\frac{C}{R\tau} = 3.56\sqrt{\frac{T_0}{\tau R}} = 0.57T_0^{3/2}\sqrt{\frac{1}{PR}} = 8.9\sqrt{\frac{1}{\tau R}}\left(\frac{P}{\tau}\right)^{1/4}$$

$$= 8 \times 10^{14}\frac{RP}{C^3}\frac{4 \times 10^5}{C}\sqrt{\frac{P}{\tau}}. \quad (32)$$

To illustrate the utility of (31) and (32), suppose that an instrument, already constructed, is to be used in a circuit having a fixed resistance and that the conditions are such that it is critically damped but not sufficiently sensitive for the work in hand. There are two factors which may be altered to increase the sensitivity without using a new coil: the torsional control τ and the field strength H. If the field strength is increased, the ultimate deflection due to any current will be increased in the same proportion, but the instrument will no longer be critically damped; it will become sluggish in its action, so that the time that must elapse before the reading can be taken is unduly increased. If τ is decreased, the same is true, so *two* changes are necessary if the critical damping is to be preserved.

Suppose that the restoring moment of the suspension is reduced to one-sixteenth of its original value. Then by (31) and (32) both S_I and S_V will be increased eightfold, for T_0^3 is increased to sixty-four times its former value, and both S_I and S_V are proportional to $\sqrt{T_0^3}$. In detail, by (30*a*) if τ is reduced to one-sixteenth of its former value, in order to maintain critical damping C must be halved. As the coil is not to be altered, the quantity $C = Hbl$ must be halved by halving the strength of field H. If the field is halved and the restoring moment reduced to one-sixteenth of its first value, the sensitivity will become eight times its initial value.

The modification will render the instrument less prompt in its action, for the time of an undamped vibration T_0 will be increased to four times its original value. Thus, although the deflection has been increased and critical damping maintained, the working characteristics have been seriously impaired, for it is essential that the working speed be as great as practicable, not only to economize time but to render possible observations in circuits when the conditions are changing or drifting. It is apparent that a single galvanometer cannot successfully be put to all sorts of uses and that in important work it should be designed to fit the circuit in which it is to be used.

Field Required to Produce Critical Damping.—From (30),

$$H'^2 = \frac{R}{b^2l^2}[2\sqrt{\tau P} - k'] \qquad 2\sqrt{\tau P} = \frac{T_0\tau}{\pi}$$

$$= \frac{R}{b^2l^2}\left[\frac{T_0\tau}{\pi} - k'\right].$$

Expressing R in ohms,

$$H = 17{,}900\frac{\sqrt{R}}{bl}\sqrt{T_0\tau - \pi k'}.$$

Neglecting k', the *total* flux through the coil required for critical damping is

$$H'bl = 112{,}000\sqrt{\frac{PR}{T_0}} = 64{,}000\frac{RT_0}{S_I}. \qquad (33)$$

Air damping has an appreciable effect on the periods of the small, exceedingly light, movable system used in modern high-sensitivity, short-period instruments.

Adjustment of the Circuit Resistance.—If an instrument is overdamped and it is amply sensitive, it may be desirable to insert enough extra resistance in the circuit to bring about critical damping. The ultimate deflection will be reduced in the same proportion as the resistance is increased but with the compensating advantage that the reading is more promptly obtained, and fluctuating circuit conditions more readily followed.

If the instrument is underdamped, the resistance of the circuit to the damping current is too high. It may be reduced by shunting the galvanometer.

If R_C is the total critical damping resistance, R_K the resistance that must be placed in series with the galvanometer to accomplish critical damping, R' the resistance of the apparatus to which the galvanometer is to be connected, and R_G and R_S the resistances of the galvanometer and the shunt, then

$$R_K = \frac{R'R_S}{R' + R_S},$$

and the resistance for critical damping is

$$R_C = R_G + R_K.$$

Therefore

$$R_C(R' + R_S) = R_G(R' + R_S) + R'R_S,$$

$$I_G = \frac{ER_S}{R_G(R' + R_S) + R'R_S} = \frac{ER_S}{R_C(R' + R_S)} = \frac{E}{R_C}\left(\frac{R_K}{R'}\right).$$

The deflection is

$$D = S_I I_G = \frac{ES_I}{R_C}\left(\frac{R_K}{R'}\right) = ES_V\left(\frac{R_K}{R'}\right). \tag{34}$$

It is seen that the voltage sensitivity is reduced in the ratio (R_K/R') (see page 13).

Possible Adjustments.—An instrument not specifically designed for a given piece of work may sometimes be made more effective by special adjustment. The things that may be varied are:

1. The total resistance of the circuit. This may be adjusted by resistances in series or in shunt with the galvanometer, as necessary.

2. The field strength. This may be decreased by using a magnetic shunt or increased by using a second set of magnets in parallel with the original one.

3. The torsion constant τ. The suspension may be changed.

4. The damping, by the use of an auxiliary damping loop.

5. The moment of inertia P. This may be increased by placing weights on the movable element.

As the relation $(C^2/R)^2 = 4P\tau$, which is necessary for critical damping, is to be preserved, *two* changes will be required, the second to compensate for the effect of the first on the damping.

For example, suppose that it is desirable to increase the total resistance R of the circuit N times. If this is done, the instrument will become underdamped, and the voltage sensitivity will be reduced N times. To restore the damping, a damping loop may be added. This will increase T_0 somewhat, as a result of the increased moment of inertia. As indicated by the equations, other compensating changes are possible.

The general principles governing the design of critically damped moving-coil galvanometers have been discussed by Wenner[6]. Zernicke,[6] Hill,[6] and Downing[6] have discussed in particular the construction of sensitive short-period instruments.

Theoretical Limit of Sensitivity of Electromagnetically Damped Galvanometers.[7]—Ising, who has studied the effect of the Brownian movements on instruments having delicately controlled movable systems, assumes, when dealing with the critically damped moving-coil galvanometer, that in order to differentiate with certainty between disturbances of the movable system due to the current and those due to the Brownian movements, the deflection due to the current should be four times the root mean square of the irregular deflections due to the Brownian movements and deduces that the least detectable current is then

$$\delta I = 4.48 \times 10^{-10}\sqrt{\frac{1}{T_0 R}}\ \text{amp.}$$

and that the least detectable voltage is

$$\delta V = 4.48 \times 10^{-10}\sqrt{\frac{R}{T_0}} \text{ volts.}$$

R is the total critical damping resistance in ohms. These results give the order of magnitude of the least detectable current and voltage and imply that there is a limit to the gain that can be realized by the use of any device that multiplies the deflection of the spot of light on the scale.

The Thermoelectric Relay.[8]—This device, suggested by W. H. Wilson but practically developed by Moll and Kipp and Zonen, may be employed to amplify small deflections of a sensitive galvanometer if conditions are

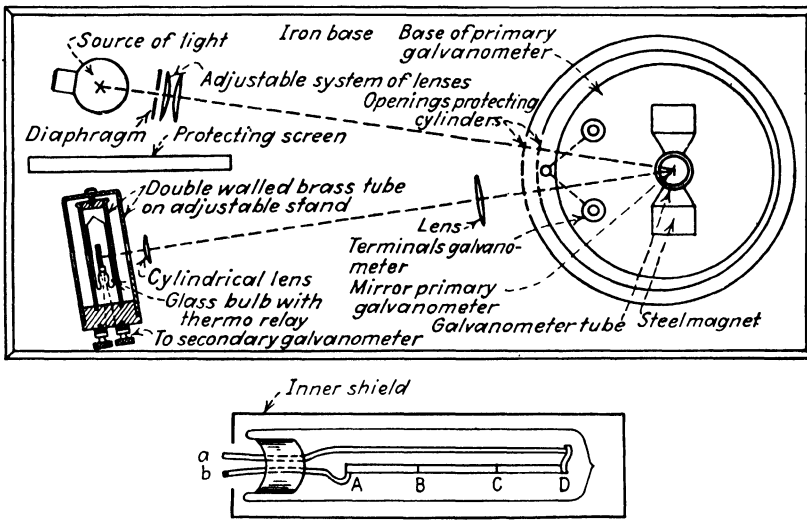


FIG. 11.—Thermoelectric relay.

such that there is no disturbance of the instrument from mechanical jarring. This start is essential.

Referring to Fig. 11, AB and CD are very thin constantan strips. BC is a very thin manganin strip. All are about 0.001 mm. in thickness. To conserve the heat and protect the junctions from accidental disturbances, this thermoelectric structure is contained in a shielded evacuated bulb, which is placed in a horizontal position. The current to be measured is led through a sensitive galvanometer which, when no current flows, throws a strong beam of light on the middle of BC. The junctions B and C are thus equally heated. A second galvanometer, which serves as the indicator, is connected between a and b. If the first instrument is slightly deflected, the spot of light moves along BC, the junctions are unequally heated, and a current flows through the indicating instrument, which, if the original deflection is not large, is proportional to the current being measured. As the heat capacity of the strips is very small, the time of response is only about 2.5 sec.

If the amplification is above fiftyfold, there is no gain in precision from using the relay with a very sensitive galvanometer, for the unavoidable effects of the Brownian movements begin to appear. That this is so is shown if the record of the zero line and a sustained deflection is taken on a moving photographic film. Both records are so irregular and in such haphazard fashion that it is impossible to measure the deflection on the film with precision.

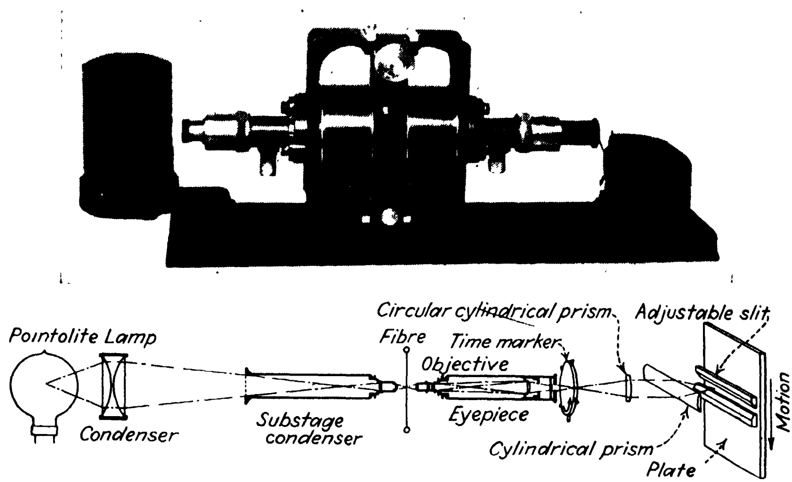


FIG. 12.—Einthoven string galvanometer.

Einthoven String Galvanometer.[9]—In this instrument, the movable element is a delicate, silvered or gilded quartz or glass fiber 5 to 15 cm. long and from 0.001 to 0.0025 mm. in diameter or, in some cases, an exceedingly fine wire. The fiber, which is usually gilded by cathode sputtering, is placed in the narrow air gap of a strong electromagnet which is worked near saturation. It is so mounted that the tension on it can be varied by a micrometer adjustment.

When a current is sent through the fiber, it moves across the magnetic field. The motion may be observed through a microscope provided with a micrometer eyepiece, but frequently an intense light source is used, and the magnified shadow of the string projected on a screen. A magnification of 500 to 1,000 diameters is employed. Permanent records may be obtained photographically on a moving plate or film. A narrow slit is placed immediately in front of the photographic surface and behind a cylindrical lens, so that the image of the fiber appears on the sensitized surface as a shadow, which prevents the exposure of the part of the surface on which it happens to fall. The time scale on the record is given by interrupting the light by a spoked wheel (seen at the right in Fig. 12) rotated at a known speed. The current scale is given by a series of lines ruled on the back of the cylindrical lens; the record then appears on a system of rectangular coordinates, and difficulties due to the stretching of the film are avoided. The modern String Galvanometer has been treated theoretically and also practically by Horatio B. Williams.[9]

The width of the air gap in the instrument shown in Fig. 12 is 0.5 mm. The use of so narrow a gap, although it introduces difficulties, renders it possible to construct an instrument of high sensitivity which is of moderate size and weight.

The magnet is provided with 1,000 magnetizing turns. The energy loss is about 10 watts. The design is such that the m.m.f. is concentrated on the air gap, in which a flux density of about 24,000 gausses is obtained. The small energy loss and the ample provision for dissipating the heat are important in minimizing the effects due to the expansion of the magnet, the string, and the string support and to air currents.

It is essential that the construction be of the utmost rigidity and the design such that there will be no mechanical deformation when the magnet is energized, for in some instruments the tractive force is something like half a ton. Leveling screws are to be avoided. Referring to the instrument shown in Fig. 12, the strings are mounted in removable holders provided with guides so that they may be safely removed from the galvanometer and when replaced will again be in focus and at substantially the same zero position. By the use of carefully constructed jigs the strings can always be mounted in the holders in exactly the same position in respect to the guides. This allows a string to be quickly and safely replaced by one better suited to the work in hand. A precision adjustment allows the string support to be moved and accurately centered in the air gap. The string tension can be altered by another precision adjustment. These adjustments must operate smoothly without backlash and without possibility of deforming the support and throwing the string out of focus or altering the zero reading. On account of magnetostriction effects, the string supports should not be attached directly to the pole pieces but carried by a strong nonmagnetic plate firmly bolted in position. The housing which covers the string support is draft- and dustproof, and great care must be taken that no magnetic dust adhere to the strings or pole pieces. The strings must initially be free from kinks and bends and before and after being mounted in the instrument must be thoroughly protected and manipulated with the utmost care; otherwise they may pick up magnetic particles or have the conducting coating broken, thus becoming useless.

The microscope is rigidly mounted in the pole pieces of the field magnet, so that there is no possibility of displacement with respect to the string due to vibration of the support on which the instrument is placed. Mechanical perfection in all details is of the utmost importance; otherwise the performance of the instrument and convenience of manipulation will fall far short of those desired.

A good string galvanometer should give a deflection proportional to the current up to a deflection of about 8 cm. on each side of the zero, the magnification being 1,000 diameters. In many cases, proportionality of the deflection to the current is of the utmost importance. Some designs of commercial instruments are defective in this respect. Proportionality of deflection is obtained by making and adjusting the field poles with great care. The string must move across a uniform field; consequently the pole faces must be parallel planes and *must so remain when the magnet is energized.*

The period of the string is very short; it depends on the length, diameter, and material of the string and the tension upon it. An instrument having a commercially obtainable gold-coated string 0.003 mm. in diameter and 13.5 cm. in length (resistance 3,500 ohms) gave the following results when the field strength was 20,000 gausses and the magnification 920 diameters:*

Sensitivity amp./cm.	Time to Complete Deflections sec.
10^{-7}	0.015
10^{-8}	0.15
10^{-9}	1.5
10^{-10}	15.

The simple relation between deflection time and sensitivity is to be noted. The last result was obtained with a very slack string, probably under no tension except its own weight. The high sensitivity is coupled with extreme slowness of action. It is customary to adjust the instrument so that the damping for the fundamental period of the string is critical. With this adjustment the odd partials are not critically damped.

The advantages of this form of galvanometer are extreme quickness of action, stability of zero reading, freedom from the effects of stray fields, and immunity from the effects of mechanical vibrations. It should be remembered that the electromagnet sets up a strong stray field.

In a high-resistance circuit the damping is by air friction; but if the resistance is low and shunts are employed, electromagnetic damping is also present. Einthoven has shown[9] that if a high-resistance instrument is placed in series with an adjustable resistance and in parallel with a condenser, and the whole combination shunted around another resistance, it is possible to adjust the combination so that the galvanometer is dead beat. The instrument thus becomes a low-period oscillograph, suitable

* Communicated by H. B. Williams.

for recording phenomena whose cycle is completed in a few tenths of a second.

Butterfield[10] has examined theoretically and practically the application of the Irwin resonating shunt to the string galvanometer, taking into account the back e.m.f. due to the motion of the string.

The theory of the moving-coil galvanometer is not applicable to this instrument. In the moving-coil galvanometer, all parts of the movable system have the same angular velocity, and the motion is in accordance with the solution of an ordinary differential equation. In the string galvanometer, the deflection of a point on the string depends on the time elapsed since the application of the current and on the distance of the point from the end of the string, and the motion is given by the solution of a partial differential equation, which is considered in detail in the Williams paper, which is of the utmost importance to any prospective user of the string galvanometer.

Anyone who is to employ this instrument should carefully study the peculiarities of the motion of the string under various forms of impulse.

The string galvanometer was devised and has largely been developed by those interested in physiological and medical research; under the name of the electrocardiograph it is an instrument of importance to diagnosticians. When supplied with a copper or silver string, it may be used as a vibration galvanometer (see also page 465). Instruments, provided with several metal strings mounted in one optical field, are coming into use as indicators for geophysical seismographs and as oscillographs in communication research.

The Duddell Thermogalvanometer.—The essential features of this instrument, which is based on the radiomicrometer of C. Vernon Boys, are indicated in Fig. 13.

A single loop of silver wire having a high conductivity is suspended by a quartz fiber in the strong magnetic field due to a permanent magnet. At the lower end of the loop there is a bismuth-antimony thermocouple, and beneath this and as near as possible without touching it, is a straight heater wire 3 or 4 mm. long through which the current to be measured is sent. On the passage of the current the lower thermal junction is warmed by radiation and convection; consequently a direct current, due to the thermoelectric action, flows around the loop, which is thus deflected. The inductance of the heater is very small; the instrument is consequently adapted for the measurement of alternating currents of high periodicity, the deflection, which is read by the mirror and scale method, being practically proportional to the mean square value of the current through the heater. The damping is due to currents induced in the loop as it moves in the field, and the electrical constants are such that critical damping is attained. The period of the instrument is 3 or 4 sec.

As with any thermoelectric device, constancy of zero reading depends on uniformity of temperature. Sudden fluctuations in room temperature should be avoided. Slow variations which give time for the temperatures of the hot and cold junctions to equalize are not nearly so important. To assist in maintaining constant temperature conditions, the working parts of the instrument are enclosed in a heavy gunmetal case, the front of which *E* may be removed when it is necessary to inspect or adjust the instrument. A double cover, not shown, is used to protect the instrument

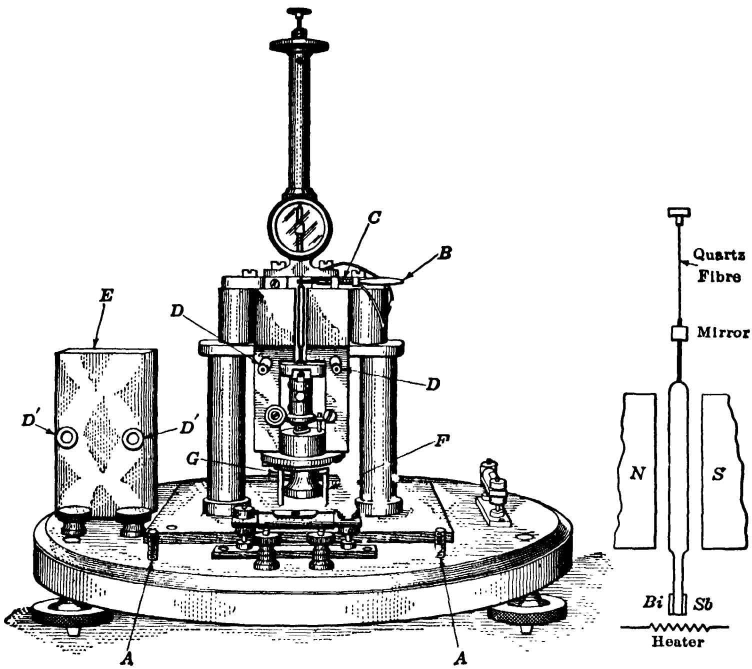


FIG. 13.—Duddell thermogalvanometer.

against external temperature effects. The zero should be read after each observation.

Interchangeable heaters are used. They are of various resistances depending on the sensitivity required. Those having a resistance below 40 ohms are made of wire, while those above this value consist of a deposit of platinum on quartz, made into the form of a grid.

The instrument may be calibrated with direct currents and then used on alternating-current circuits. It is more sensitive than the electrodynamometer, not subject to errors due to inductance or capacitance, and at high frequencies does not disturb the circuit conditions so much as the dynamometer.

The sensitivity may be controlled to a certain extent by adjusting the proximity of the heater to the hot junction. This is done by turning the ebonite milled head *F*. Great care is necessary in manipulating the instrument not to injure the delicate loop or the thermocouple.

The same principle is applied in the Duddell thermoammeter.

The sensitivity attained, as given by the makers of the instruments, the Cambridge Instrument Company, is shown in the following table.

TABLE III.—SCALE DISTANCE 1 M.

Approximate resistance of heater, ohms	1,000	100	10	4	1
Current to give 10 mm. deflection, microamp	20	60	200	300	600

POINTS TO BE CONSIDERED WHEN SELECTING A GALVANOMETER

A galvanometer must be selected with special reference to the work to be done, for no instrument is equally useful under all sorts of conditions. Experience is the best guide, but among the points to be considered are the following.

Sensitivity.—The sensitivity should be sufficient for the work in hand so that measurements may be made without undue fatigue and loss of time. On the other hand, a much higher sensitivity is not an advantage, for it means a more delicate instrument and therefore one more liable to injury and more difficult to manipulate. Also, high sensitivity may mean an unduly long period of vibration and a consequent loss of time in making measurements. Sensitivity, though important, should not be the only thing considered in estimating the utility of a galvanometer.

Sensitivity here means usable sensitivity. A galvanometer may give a large deflection per microampere but have so unstable a zero reading that the useful sensitivity is much lower than that inferred from the normal sensitivity.

Period.—The time of vibration of the movable system should be short, so that the instrument will respond quickly to the current.

Damping.—*When the instrument is in use,* the system should, if possible, be critically damped. This will economize time. In many cases the damping is not an inherent property of the galvanometer but depends both on the instrument and on the circuit to which it is attached (see page 32). With the moving-coil galvanometer and the needle galvanometer with cobalt-steel magnets, the total critical damping resistance should be known.

Promptness of Response.—It is very important that the *final* reading be quickly attained, for in many cases the circuit conditions are continually changing, and unless the galvanometer responds promptly so that the deflection is directly proportional to the current instant by instant, it will be impossible to interpret its readings properly. This implies a short period and critical damping.

Resistance.—The resistance should be appropriate for the measurement in hand so that the maximum sensibility of method may be obtained.

Freedom from Effects of Mechanical Disturbances.—Great care should be exercised in making and mounting the movable system so that symmetry about the axis of rotation is attained; this contributes much to the stability of the system when it is subjected to mechanical disturbances. Choice of location for the instrument and the method of setting up are important.

Freedom from Stray-field Effects.—It is essential that the indications be uninfluenced by the unavoidable variations of the local field.

Definiteness of Zero Reading.—The zero reading should be definite, and the deflections should come promptly to their final values with no viscous action of the suspension.

Law of Deflection.—Throughout its useful range the deflection, as read from the scale, should be proportional to the current.

Visibility of Suspended Parts.—When the instrument is set up and ready for use, it should be possible to see the movable parts and to satisfy oneself that the clearances are properly adjusted.

Accessibility for Repairs.—It should be possible to take out easily the entire movable system with its suspension.

Temperature Effects.—The effect of temperature on the sensitivity should be small, and inequalities of temperature should not set up thermoelectric currents in the galvanometer circuit.

Optical System.—The definition obtained by the optical system used in reading the deflections should be so perfect that readings to the limit of accuracy of the instrument may be obtained without undue fatigue. The mirror should be of the smallest practicable dimensions.

THE JULIUS DEVICE FOR ELIMINATING THE EFFECTS OF MECHANICAL DISTURBANCES ON GALVANOMETERS[11]

The object to be attained is the suspension of the movable system of the instrument from a point that is practically stationary.

The Julius suspension, as developed at the Bell Telephone Laboratories,[11] is shown in Fig. 14. t_1, t_2, t_3 are triangular metal frames which are interconnected by the coiled springs *Sp*; screen-door springs 15 in. long, which stretch to about 18 in. when in use, are satisfactory. Before the instrument is placed on the lower platform *p*, the springs are tightly wound with friction tape (lap, one-half the width of the tape) over their entire lengths. The frame t_1 is firmly clamped in position so that it cannot tilt or turn with reference to the building structure. There must be no swinging of the device. The frame carrying the galvanometer is suspended from t_3 by three wires provided with turnbuckles for adjustment. The frame consists of a heavy lead baseplate p, in which are bolted three vertical rods r_1, r_2, r_3; a light metal ring secures the upper ends of the rods. W_1, W_2, W_3 are lead weights which can be adjusted vertically. The frame is supported from the wires through three brackets C_1, C_2, C_3, the three points of attachment being in a plane through the point of suspension of the movable system of the galvanometer. The weights, W_1, W_2, W_3 are adjusted vertically until the center of oscillation of the frame and instrument coincides with the point of support of the movable system. Complete shielding from drafts is essential.

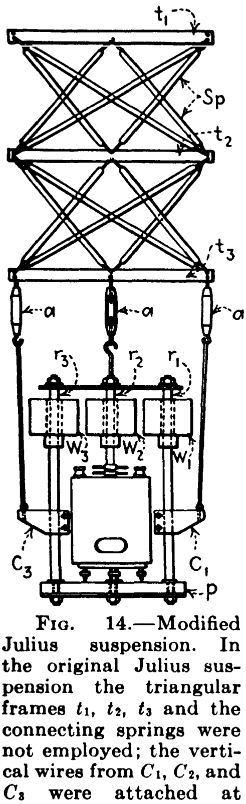


FIG. 14.—Modified Julius suspension. In the original Julius suspension the triangular frames t_1, t_2, t_3 and the connecting springs were not employed; the vertical wires from C_1, C_2, and C_3 were attached at their upper ends to a spider carried by the building structure. (*Bell Telephone Laboratories.*)

SHUNTS

In using galvanometers, it is often found either that the instruments are too sensitive or that their carrying capacities are insufficient. In such cases, shunts placed between the terminals of the galvanometer and acting as by-passes for the current are employed (see Fig. 15).

When zero methods are used, shunts are resorted to for the purpose of protecting the galvanometers during preliminary adjustments. Much time is thus saved, for the violence of the deflection and consequently the time necessary for the needle to come to rest are reduced. A familiar example of the use of shunts to extend the range of galvanometers is found in direct-current, moving-coil ammeters.

Where it is desired to compute the total current in a circuit from the indication of a shunted galvanometer, an exact knowledge of the resistance of both shunt and galvanometer at the time of use is necessary. Attention must be given to possible sources of error, such as defective contacts and changes of resistance due to temperature.

Let I = line current.
I_G = galvanometer current.
I_S = shunt current.
R_G = resistance of galvanometer.
S = resistance of shunt.

Then

$$I = I_S + I_G = I_G\left(\frac{R_G + S}{S}\right).$$

$\left(\frac{R_G + S}{S}\right)$ is called the multiplying power of the shunt. The ordinary arrangement of a shunt box for use with a reflecting galvanometer is shown in Fig. 15. By changing the position of the plug, definite portions, usually one-tenth, one-hundredth, or one-thousandth of the total current, can be sent through the galvanometer, which may be short-circuited by placing the plug in the last hole to the right.

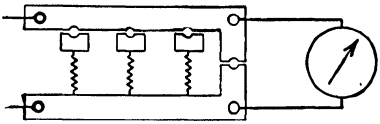


FIG. 15.—Diagram for ordinary shunt box.

With an ordinary shunt, the resistance through which the current set up by the motion of the movable system of the galvanometer flows will depend on the multiplying power that is used. Consequently, the damping cannot be kept constant. This effect is present with movingneedle instruments, especially if cobalt-steel magnets are used. It becomes very important with moving-coil galvanometers, for it is highly desirable that critical damping be obtained and preserved for all values of the multiplying power. Again, when a ballistic galvanometer is shunted, the first throw is dependent on the amount of damping present; and if it varies, the first throws obtained with different multiplying powers cannot be compared unless a correction for damping has been applied.

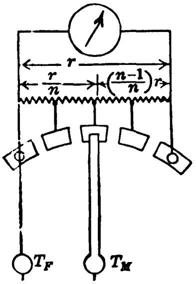


FIG. 16.—Diagram for universal shunt box.

Ayrton-Mather Universal Shunt.[12]—This form of shunt, designed for open-circuit work or for use in circuits of high resistance, is shown diagrammatically in Fig. 16. A high resistance, of r ohms, is permanently connected across the galvanometer terminals, one of which, T_F, is permanently connected to the external circuit. By the proper arrangements, the other lead from the external circuit, T_M, may be connected at will to points on r which are commonly distant r, $r/10$, $r/100$, etc., from the fixed terminal. Referring to the figure, it will be seen that the line current is given by

$$I = I_G \frac{R_G + \left(\frac{n-1}{n}\right)r + \frac{r}{n}}{r/n} = I_G \frac{(R_G + r)n}{r}.$$

For any particular galvanometer and shunt box, $\left(\frac{R_G + r}{r}\right)$ is constant. This factor is the multiplying power of the shunt when the movable terminal is at the extreme right of r.

It is seen that the *relative* multiplying power is n. The values of n depend on the locations of the taps by which the movable terminal T_M is connected to r. They are independent of the relative magnitudes of the resistances of the galvanometer and the shunt. The box is graduated in terms of the relative multiplying powers; consequently, it may be used with any galvanometer; hence the name universal. However, though the multiplying powers are not affected, the same shunt box cannot be applied indiscriminately to all galvanometers, and satisfactory results attained. For the maximum current through the instrument is $I_G = I\frac{r}{R_G + r}$. Therefore, in order that practically the full sensitivity of the galvanometer may be realized, r must be much larger than R_G; if it is nine times the galvanometer resistance, 90 per cent of the sensitivity may be realized. Again, if r is too small, and a moving-coil galvanometer is employed, the instrument will be overdamped and, therefore, sluggish in its action.

The distinct advantage of the universal shunt box is that when it is used in *open-circuit work*, any damping due to currents set up by the motion of either the needle or the movable coil of the galvanometer is constant. This is especially important when capacitances are being compared by means of the ballistic galvanometer.

Also, when a universal shunt is used with a moving-coil galvanometer in a circuit of very high resistance, it is possible, by properly choosing r, to render the galvanometer dead beat for all values of the multiplying power. Such a case arises when insulation resistances are being measured.

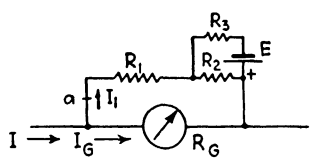


FIG. 17.—Shunt for increments of current.

Shunts for Increments of Current.—If it is desired to measure the change of a current from some initial value, the connections of Fig. 17 may be used.

The current I is fixed by conditions external to the shunted ammeter. E is a cell of constant e.m.f.

If the circuit is opened at a, the voltage that appears across the break is

$$V_a = IR_G + \frac{ER_2}{R_2 + R_3}.$$

By Thévenin's theorem (page 387),

$$I_1 = \frac{IR_G + \frac{ER_2}{R_2 + R_3}}{R_G + R_1 + \frac{R_2R_3}{R_2 + R_3}}.$$

The galvanometer current is

$$I_G = I - I_1 = \frac{I\left(R_1 + \frac{R_2R_3}{R_2 + R_3}\right) - \frac{ER_2}{R_2 + R_3}}{R_G + \left(R_1 + \frac{R_2R_3}{R_2 + R_3}\right)}.$$

When the datum current is flowing, the network is adjusted until $I_G = 0$. Therefore

$$I\left(R_1 + \frac{R_2R_3}{R_2 + R_3}\right) = \frac{ER_2}{R_2 + R_3}.$$

If the current then changes to $I + \Delta I$,

$$\Delta I = \Delta I_G\left(\frac{R_G}{R_1 + \dfrac{R_2R_3}{R_2 + R_3}} + 1\right).$$

$R_1 + \dfrac{R_2R_3}{R_2 + R_3}$ is the total resistance of the shunt which for changes of current obeys the ordinary law.

Networks to Reduce Voltage Sensitivity and Provide Constant Damping.[13]—If a universal shunt is used with a galvanometer in a closed circuit of low resistance, the damping is no longer constant and is critical only when its shunt is so set that the net resistance, measured between the terminals to which the galvanometer is to be attached, is equal to the necessary external critical damping resistance. Consequently, all readings cannot be taken with equal facility. If a ballistic galvanometer is used, the elongations cannot be corrected by use of the multiplying power of the shunt.

When a galvanometer is used to measure potential difference, it is sometimes desirable to reduce the voltage sensitivity. A simple series, or shunt, resistance will do this but will alter the damping which should be kept near its critical value. If the resistance of the circuit to which the galvanometer is applied is less than the proper external damping resistance, there is only one series resistance that can be used; if it is greater, there is only one shunt resistance, so that there is no possibility of obtaining any required reduction of sensitivity and at the same time maintaining critical damping. However, combinations of series and shunt resistances may be so arranged that the voltage sensitivity may be reduced in any desired degree, and constant damping maintained.

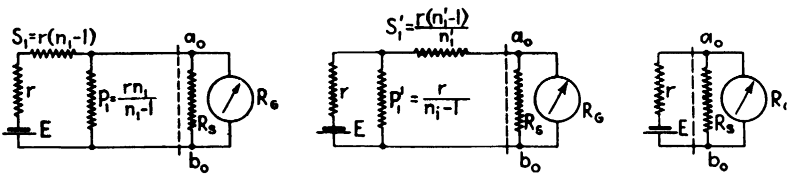
It will be assumed that a moving-coil galvanometer is used and that the damping is due entirely to the induced currents set up by the motion of the coil.

Let E = e.m.f. of source.
r = resistance of source.
r' = resistance of source plus added resistance.
L = inductance of source.
I_G = galvanometer current.
R_G = galvanometer resistance.
R'_G = resistance of shunted galvanometer.

R_S = resistance of shunt.
m = multiplying power of shunt applied to galvanometer in preliminary adjustment.
$1/n$ = fractional reduction of sensitivity from the usable value.
S = series resistance used in the adjustments.
P = parallel resistance used in the adjustments.

If the resistance of the source of e.m.f. is greater than the required external damping resistance, the desired degree of damping must first be established by permanently shunting the galvanometer. This is the preliminary adjustment. When it has been made, the galvanometer has its maximum usable voltage sensitivity under the imposed damping conditions.

If the resistance of the source of e.m.f. is less than the required external damping resistance, it must be increased until the desired degree of



A. Parallel resistance at galvanometer
$$S_v = \frac{S_{v\ max.}}{n_1}$$

B. Parallel resistance at source
$$S_v = \frac{S_{v\ max.}}{n_1'}$$

C. Preliminary adjustment with resistance of source greater than critical damping resistance.

FIG. 18.—Reduction of voltage sensitivity maintaining constant damping.

damping is attained. This is the preliminary adjustment. When it has been made, the galvanometer has its maximum usable voltage sensitivity under the imposed damping conditions. Damping that is very nearly critical is highly desirable.

The problem is to reduce the voltage sensitivity to $1/n_1$ of its maximum usable value and keep the desired degree of damping. The deflection can be cut down by increasing the resistance of the circuit between the source and the galvanometer, and the desired degree of damping obtained by a suitable parallel resistance applied either at the source or at the galvanometer, as indicated in Fig. 18.

Considering the case where the resistance of the source is greater than the required external damping resistance, the shunt is applied as in Fig. 18*C*. The maximum usable galvanometer current to which the maximum usable deflection is proportional is

$$I_{G\ max.} = \left(\frac{E}{r + R_G'}\right)\frac{1}{m}.$$

It will be noted that the damping resistance to the left of a_0b_0 is r; and hence whatever the arrangement of resistances adopted to decrease the deflection, this relation must be maintained; that is, referring to Fig. 18A,

$$\frac{(r + S_1)P_1}{r + S_1 + P_1} = r. \tag{35}$$

If the circuit is opened at a_0, the Thévenin voltage which appears across the break is

$$V_0 = \frac{EP_1}{r + S_1 + P_1} = E\frac{r}{r + S_1}. \tag{36}$$

The current through the galvanometer is

$$I_G = \left(\frac{Er}{r + S_1}\right)\left(\frac{1}{R'_G + r}\right)\frac{1}{m}.$$

Therefore

$$\frac{I_{G\ \text{max.}}}{I_G} = \frac{r + S_1}{r} = n_1, \tag{37}$$

or for the first reduction of sensitivity,

$$S_1 = r(n_1 - 1). \tag{38}$$

Inserting this value in (35) gives

$$P_1 = \frac{rn_1}{n_1 - 1}. \tag{39}$$

With the arrangement of Fig. 18C, it is obvious that when the circuit is opened at a_0, the voltage that appears across the break is E. With the arrangement of Fig. 18A, it is, by (36) and (37), E/n_1. It is seen that the characteristics of the added network are that irrespective of any resistances to the right of a_0b_0 the voltage across the break is always reduced in the ratio $1/n$, while the damping resistance to the left of a_0b_0 is maintained at the value r.

If it is desired to reduce still further the voltage sensitivity in the ratio $1/n_2$, another mesh having the same characteristics may be added, for instance, one in which

$$S_2 = r(n_2 - 1). \tag{40}$$

$$P_2 = \frac{rn_2}{n_2 - 1}. \tag{41}$$

The second added network has the same characteristics as the first, that is, the voltage that would appear across a break at a_1 is E/n_2, and the resistance to the left of a_1b_1 is equal to r. The second added network replaces the original source; consequently the voltage that would appear at a_0 if the circuit were opened would be E/n_1n_2, and the voltage sensitivity is reduced in the ratio $1/n_1n_2$, and so on for additional meshes. If the parallel resistance is applied at the source (Fig. 18B), the resistance to the left of a_0b_0 must always be equal to r; consequently

$$S_1' + \frac{rP_1'}{r + P_1'} = r. \tag{42}$$

If the circuit were opened at a_0, the Thévenin voltage which would appear across the break is

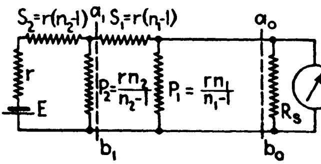


FIG. 19.—Second reduction of voltage sensitivity $s_v = \frac{S_{v\ max.}}{n_1n_2}$

$$V_0 = E\left(\frac{P_1'}{r + P_1'}\right) = E\left(\frac{r - S_1'}{r}\right), \tag{43}$$

and the galvanometer current is

$$I_G = E\left(\frac{r - S_1'}{r}\right)\left(\frac{1}{R_G' + r}\right)\frac{1}{m}.$$

The ratio of $I_{G\ max.}$ to I_G is

$$\frac{I_{G\ max.}}{I_G} = \frac{r}{r - S_1'} = n_1'. \tag{44}$$

Therefore for the first reduction of voltage sensitivity,

$$S_1' = \frac{r(n_1' - 1)}{n_1'}. \tag{45}$$

Using (42),

$$P_1' = \frac{r}{n_1' - 1}. \tag{46}$$

By (43) and (44), the Thévenin voltage is

$$V_0 = \frac{E}{n_1'};$$

the resistance to the left of a_0b_0 is equal to r, and the voltage sensitivity is reduced in the ratio $1/n_1'$. Further reductions of sensitivity are made by adding networks having similar characteristics. If one mesh is added, the reduction of voltage sensitivity is in the ratio $1/n_1$; if two are added, it is $1/n_1'n_2$; etc.

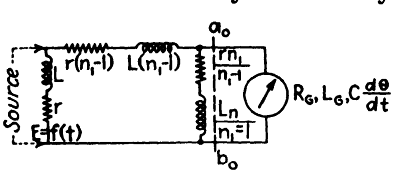


FIG. 20.—Inductive source; reduction of voltage sensitivity.

In general, the e.m.f., instead of being constant, may be any function of time $E = f(t)$, and the source may have an inductance L as well as a resistance r. The various members of the added network must now contain inductance as well as resistance, and the time constant of each member must be equal to that of the source L/r as indicated in Fig. 20.

When a network proportioned as shown in Fig. 20 is added, and the circuit opened at a_0, the current that flows from the source is

$$i = \left(\frac{n_1 - 1}{n_1^2 r}\right)f(t) - \frac{L}{r}\left(\frac{di}{dt}\right), \tag{47}$$

and the voltage that appears across the break is

$$V_{a_0} = \left(\frac{rn_1}{n_1 - 1}\right)i + \left(\frac{Ln_1}{n_1 - 1}\right)\left(\frac{di}{dt}\right). \tag{48}$$

Substituting the value of i from (47) gives

$$V_{a_0} = \frac{f(t)}{n_1}. \tag{48a}$$

If a moving-coil galvanometer forms part of a circuit having constant parameters, and the e.m.f. in the circuit is reduced from $f(t)$ to $f(t)/n_1$, then at every instant the deflection due to the second e.m.f. is $1/n_1$ of that due to the first. The two paths which are in parallel at the left of a_0b_0 have equal time constants. The equivalent resistance at the left of a_0b_0 is r, and the equivalent inductance is L. That is, the resistance and inductance are the same as if the source were connected directly to the galvanometer. The effective e.m.f. is $f(t)/n_1$, so the voltage sensitivity has been reduced in the ratio $1/n_1$. If another section is added, it will also have equivalent constants r and L and pass on to the following sections a voltage $f(t)/n_2$, so the total reduction in voltage sensitivity will be in the ratio $1/n_1n_2$, and the method holds for any e.m.f., impulsive or alternating.

AMMETERS

An ammeter, in distinction from a galvanometer, is an instrument so constructed that the current strength in amperes can be read directly.

Before referring to various designs of these instruments, it will be well to refer to certain considerations which have influenced the development of indicating electrical instruments, especially for direct-current work.

1. The resistance of all current-measuring instruments should be very low, while that of all instruments for measuring voltages should be as high as practicable, the reason in both cases being that the disturbance of the circuit conditions by the insertion of the instrument must be reduced to a minimum. Another way of stating the same thing is that the energy dissipated in the instrument must be a minimum.

2. The construction must be such that the instrument will maintain its reliability. The relative positions of the parts must be maintained in spite of rough handling, and the strength of all magnets used must be insured by proper aging.

3. There must be no "set" of the controlling springs due to standing under load, and no indefiniteness of the zero reading due to magnetic impurities in the movable coils.

4. The indications of the instrument must be independent of stray fields. The importance of this in industrial testing cannot be overemphasized.

5. The indications must be independent of room temperature, and no errors must result from the heating due to the passage of the current.

6. All shunts must be free from errors due to thermoelectromotive forces.

7. Ammeter shunts must be so constructed that they will not be injured by abnormal currents of short duration, and ample provision must be made for dissipating the heat due to continuous operation.

8. There must be no effects due to the retentiveness of any soft-iron parts, for this causes the indications of the direct-current instruments to depend on their previous history.

9. Pivot friction must be reduced to a minimum, and the moving system properly balanced.

10. The damping should be just short of critical; fluctuation of load will then be promptly followed, and the very slight overswing gives the user assurance that abnormal friction is not present.

11. The graduation should be convenient. This reduces the liability to mistakes in readings that have to be taken hurriedly.

Moving-coil Ammeters.—In direct-current instruments, the fulfillment of the conditions stated above is most readily obtained by employing the moving-coil principle.

The first thoroughly practical instrument of this class was designed by Edward Weston in 1888. Since that date, the moving-coil ammeter has become the standard instrument for high-grade measurements and is made, with variation of detail, by many instrument makers.

The instrument is essentially a shunted moving-coil galvanometer, so designed that it fulfills the requirements of portability and general reliability.

The magnet, which is of the horseshoe type, is made of tungsten or cobalt steel and is artificially aged. Carefully shaped soft-iron pole pieces are attached—in some cases welded—to the magnet so that the space between them is cylindrical. In this space is placed a soft-iron cylinder supported from a nonmagnetic yoke attached to the pole pieces. The air gap may be about 0.05 in. wide; consequently, the coil moves in a radial field (see page 23). The movable coil, of copper, is wound on an aluminum frame which also serves as a damping device to make the instrument dead beat. The movable system is provided with steel pivots[14] which turn in two jeweled (sapphire) bearings which are carried by nonmagnetic yokes attached to the pole pieces in such a manner that the coil is truly centered. Some makers assemble the iron core, its support, and the entire movable system in a single structure, which can readily be removed for inspection or repairs. The controlling force is given by two flat spiral springs, one above and one below the coil; they are made of nonmagnetic material and also serve as leads to the movable coil. The inner ends of the springs are attached to nonmagnetic collars which form the terminals of the coil, the outer ends to the extremities of two insulated crossarms which can be moved coaxially with the coil, to adjust the zero.

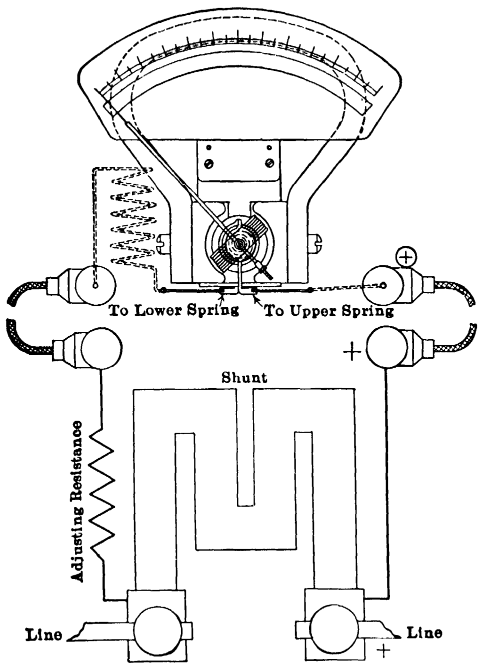


FIG. 21.—Diagram for Weston moving-coil ammeter.

This adjustment may be made without opening the case. The pointer is an aluminum-alloy tube flattened at the index end. The moving parts are balanced by adjustable counterweights on three short arms which project at right angles to the axis of rotation and are in the plane of motion of the pointer; adjustment is made by moving the weights along the arms. Parallax is eliminated by the use of a mirror beneath the pointer.

The graduation of the scale is practically uniform, but no particular law of deflection is assumed. The principal points are determined by comparison with a standard instrument, and the subdivision is done by a dividing engine.

A small resistance coil is included in the movable-coil circuit, and the final adjustment is made by altering its resistance.

For self-contained portable milliammeters and ammeters, the present practice of the Weston Instrument Company is to use a drop of approximately 50 millivolts at full-scale deflection. In external shunts of the precision type, the drop is approximately 100 millivolts, and in the switchboard shunts it is exactly 50 millivolts.

In self-contained instruments the shunts are mounted in the base which, if the current capacity is considerable, is properly ventilated. For large currents, the best procedure is to separate the millivoltmeter and the shunt and mount them in different cases; this allows the shunt to be designed so that the heat is readily dissipated. An external-shunt

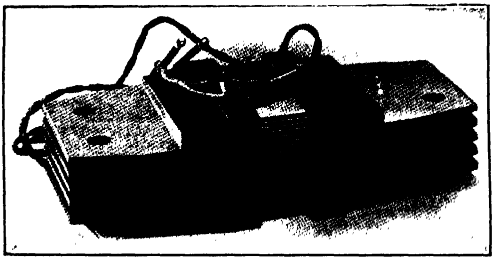


FIG. 22.—Switchboard shunt.

instrument gives a flexible arrangement very convenient for general testing, for shunts of different ranges may be used with the same millivoltmeter.

In using any form of shunt and millivoltmeter, it is necessary to calibrate and to use the instrument with the same set of leads connecting the shunt and the millivoltmeter and to avoid all extraneous resistances in the leads due to imperfect contacts at the terminals.

The moving-coil principle is now universally employed in the best makes of direct-current instruments. Separate millivoltmeters and shunts are universally used in direct-current switchboard work. The shunts are put at any convenient point in the busbars, and small leads are run to the indicating part which is on the front of the switchboard. This greatly simplifies the construction of the board and reduces expense.

Figure 22 shows a switchboard shunt. It will be noted that the resistance strips are very short and are soldered into massive terminal blocks which can be interleaved with the busbar. The heat is thus disposed of by conduction as well as by convection.

Reference should be had to the introduction to the chapter on the "Calibration of Instruments" where various errors found in commercial ammeters, voltmeters, etc., are discussed.

Rectifier Instruments.[15]—The moving-coil milliammeter may be combined with the copper oxide (Rectox) rectifier, as indicated in Fig. 23, to form an instrument capable of dealing with alternating currents of audiofrequencies, above the usual range of electrodynamic, soft-iron, or thermal instruments. In doing this, accuracy is sacrificed in favor of small power consumption, ruggedness of construction, and ability to stand transient overloads.

In rectifier instruments, the direct-current milliammeter registers the *average* value of the current; however, if only sinusoidal currents are dealt with, the scale may be graduated to read r.m.s. values. If the wave contains harmonics, the form factor is no longer 1.11, and the graduation is incorrect, the amount of the error depending on the form of the particu-

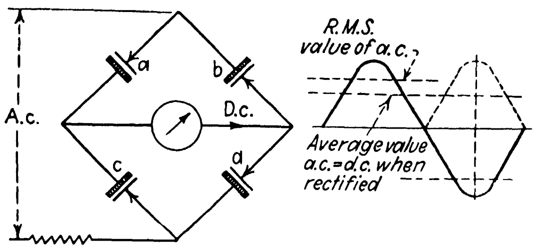


FIG. 23.—Showing principle of "Rectox" rectifier instruments. *a, b, c, d,* copper oxide rectifiers. (*Westinghouse Electric and Manufacturing Company.*)

lar wave being used. This error is most important, is inherent in all instruments of the rectifier type, and limits their use to sinusoidal currents. In addition, the copper oxide rectifier has peculiarities of its own, for its conductivity is not truly unilateral. Referring to Fig. 24, the "forward resistance," that is, the resistance with the indicator short-circuited, is comparatively low (a few hundred ohms), but the "leakage resistance," that is, the resistance with the indicator circuit open, is not infinity, as it would be in an ideal rectifier. It is considered that the "rectifying ratio," that is, leakage resistance/forward resistance, should be above 25. It is usually in the neighborhood of 50 to 60.

The temperature coefficients of both the forward and the leakage resistances are negative, the latter being two or three times the former. A rise of temperature thus decreases the rectifying ratio and at the same time decreases the total resistance. For voltmeters these two effects tend toward compensation, which may be made within the commercial limits of tolerance by using for the necessary series resistances the proper combinations of wires having high- and low-temperature coefficients. On account of temperature effects, it is best to use Rectox instruments in the temperature range 15° to 30°C.

Both the forward and leakage resistances vary in a nonlinear manner with the current density. The forward resistance decreases very rapidly at first, then more slowly with an increase in density, while the leakage resistance increases rapidly at first, then decreases somewhat as the current density is increased. A maximum current density of 65 milliamp. per square centimeter is allowable. The resistance of a rectifier instrument thus depends on the current flowing through it, being a minimum at full-scale deflection and perhaps twice the minimum at 0.3 full scale.

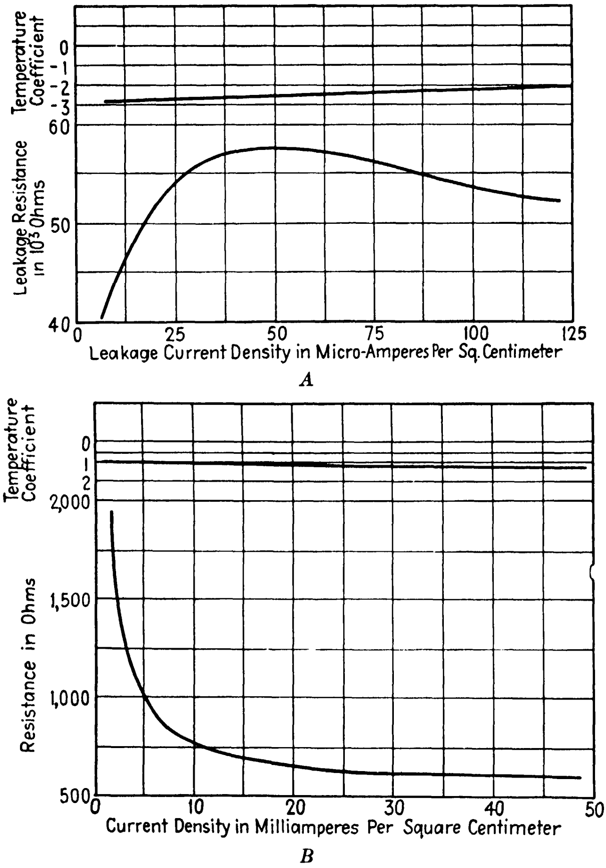


FIG. 24.—Resistance-current density characteristics of copper oxide rectifier. *A*, leakage resistance at 25° C. *B*, forward resistance at 25° C.

Consequently, these instruments put a varying load on the circuit in which they are used and may cause changes of wave form. Whether the effects of the resistance variations are serious or not depends on the resistance of the circuit, in the case of a milliammeter, and, in the case of a voltmeter, on its range; that is, on the magnitude of the series resistance. The preferred range of Rectox voltmeters is from 3 or 4 volts up, the resistance being 1,000 ohms per volt, a result attained without undue sacrifice in the temperature coefficient. Milliammeters are available from 0.5 to 20 milliamp. The resistance of the rectifier used in a 500-microamp. instrument might have a value of a little over 700 ohms at full scale and something over 1,500 ohms at 0.3 full-scale deflection. Instruments of this type have a frequency error due to the capacitance of the rectifier which measured at 1,000 cycles per second is of the order of 0.009 μf. The indications decrease from 0.5 to 1 per cent for each increase of 1,000 cycles per second; the error thus becomes large within the audio range, especially in low-range voltmeters; it can be compensated for a limited band of frequencies by shunting a portion of the series resistance by a condenser.

The possibility of permanent changes in the rectifying elements themselves should not be lost sight of. It is seen that rectifier instruments should be used with a full knowledge of the peculiarities of the particular instrument employed and of the constants of the circuit in which it is inserted.

Thermal Instruments.[16]—The rise of temperature, the increase in resistance, and the elongation of a fine wire due to the passage of a current have all been utilized for measurement purposes. Ammeters and voltmeters which function by causing the elongation of the wire to move a pointer over the scale have decided faults. The zero is indefinite and requires frequent adjustment; the consumption of energy is large; the ultimate deflection is attained only slowly; variations of room temperature affect the indications. For these reasons the expansion type of instrument has been supplanted by the thermoelectric type.

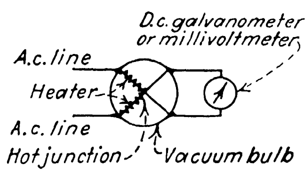


FIG. 25.—Elements of vacuum thermocouple.

The most successful measuring devices depending on thermal action are those in which the rise of temperature of a slender conductor is registered by a thermoelectric junction in intimate contact with it. The junction may be soldered or welded directly to the conductor, or it may be separated from it by very thin insulation. For small currents, the arrangement shown in Fig. 25 is used.

By enclosing the heater and the hot junction in a vacuum bulb, the sensitivity and reliability are much improved since convection currents in the air about the junction are avoided. The Western Electric Company employs a carbon filament for the heater. Iron and "advance" metal are used for the junction. The leads from the thermocouple are carried to a direct-current galvanometer or sometimes to a potentiometer. For small values of current, the voltage due to the couple is approximately proportional to the square of the current through the heater. Owing to radiation, when the current is increased, the curve departs from the parabolic form, becoming almost straight, and at still higher currents begins to drop.

DATA FOR A PARTICULAR WESTERN ELECTRIC VACUUM THERMOCOUPLE

Resistance of heater	300 ohm max.
Resistance of couple	12 ohm ± 10 per cent
E.m.f.	5 millivolts with 2.5 to 3.1 milliamp.
Maximum allowable current	6 milliamp.

The vacuum thermocouple of this type is particularly adapted to the measurement of small alternating currents of the order of a few milliamperes. As the heater is practically nonreactive, there being no coils, the instrument is adapted to audiofrequency work and is used instead of a sensitive electrodynamometer, as it disturbs the circuit conditions much

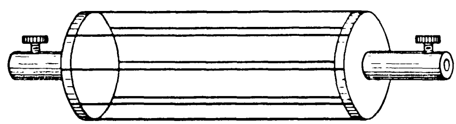


FIG. 26.—Arrangement of parallel wires for avoiding distribution errors in a high-frequency ammeter.

less. At very high frequencies, errors due to skin effect and distributed capacitance begin to appear.

The Parallel-wire Ammeter.[17]—In this instrument, which was formerly much employed in high-frequency measurement, the conductor is a group of several straight wires of the same length and diameter and, in the ideal instrument, so fine that changes of resistance due to skin effect are negligible. They are parallel in direction and equally spaced around the circumferences of two circular terminal blocks, as indicated in Fig. 26. Connection to the circuit is made at the centers of the terminal blocks, which have negligible impedances. In a perfect instrument, all the wires will have the same resistance, and a direct or low-frequency current will divide equally among them, all inductance effects being negligible. At very high frequencies, the distribution of current between the wires is determined by the self- and mutual inductances rather than by the resistances. If the wires are arranged as indicated in Fig. 26, the mutual inductance effects on each and every wire are the same; and as all the self-inductances are identical a high-frequency current will divide equally among the wires.

If all the wires were arranged in one plane, as was formerly done, grave mutual and self-induction errors would arise. Dellinger[17] cites the case of a seven-wire instrument, the indicator of which was operated by one of the wires somewhat distant from the others. At 100,000 cycles per second it gave readings 10 per cent too high: at 750,000 cycles per second the readings were 46 per cent too high.

It is essential that the wires be of uniform resistance. Lack of uniformity may be due to variations in hardness or to small variations of cross section. These things do not affect the inductances, so the wires may carry equal currents at high frequencies, but very different currents at low frequencies where the resistance effects predominate. It is best to utilize the whole heat production rather than that in one wire for actuating the indicator, for this minimizes the errors due to any difference that may exist between the current distribution at low and high frequencies. This is readily accomplished if the indicator is actuated by thermocouples. Difficulties may also arise from capacitance effects which shunt a portion of the current entering the instrument away from the wires. In order to avoid alteration of circuit conditions, the impedance of the ammeter at high frequencies should be low.

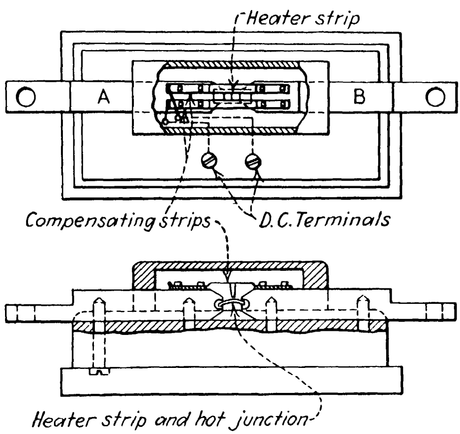


FIG. 27.—Weston thermal ammeter.

Weston Thermal Ammeters.[16]—The Weston Instrument Company has developed a portable thermoammeter for high-frequency measurements, the heater element of which is shown diagrammatically in Fig. 27.

In Fig. 27, the heater strip is very short so that there is little connection loss, and of small mass, so that there is little heat capacity. It is soldered to the copper terminals *A* and *B*, which have so large a mass and radiating surface that their temperatures are but little influenced by the heat conducted away from the strip. The protective case minimizes the loss by convection so that the larger part of the heat is dissipated by conduction to the terminals.

The hot junction is welded or hard soldered to the heater strip midway between the terminals. Any thermoelectric device necessarily involves a cold junction. As it is desired that the instrument function solely by virtue of the rise of temperature of the strip, due to the current, and as the temperature of the terminals is but slightly affected by the current, the arrangement should be such that the indicator is actuated by the difference of temperature between the mid-point of the strip and the terminals. However, the two terminals may not be exactly at the same temperature, and a small amount of heat is actually dissipated by convection and radiation, so a compensating feature is introduced. The terminals are thermally but not electrically connected by two copper compensating strips. The insulation between the terminals and the strips is very thin mica. The strips are so proportioned that they are influenced by variations of the air temperature and inequalities of the temperatures of the terminals to the same extent and at the same rate as is the heater strip. The cold junction is soldered to these compensating strips, to which the leads to the direct-current millivoltmeter are attached.

The theory underlying the action of the instrument may be illustrated by the simplified case where all the heat generated in the strips is assumed to flow to the terminals by conduction, there being no radiation or convection losses.

Let T = difference of temperature between mid-point of strip and terminals.
V = difference of potential between terminals.
w = amount of heat generated in unit length of strip.
k = thermal conductivity of strip material.
ρ = resistivity of strip material.
a = cross section of strip.
l = half length of strip.
x = distance of a point on the strip from the mid-point.
t = temperature of strip at x.
T_1 = temperature of terminal.
T_2 = temperature of mid-point of strip.

The heat flows symmetrically to the two terminals by conduction. The heat generated in a length x of the strip is wx; this heat flows to the terminal through a length dx of the strip; consequently, by the law of heat flow,

$$wx = -ak\frac{dt}{dx};$$

therefore

$$t = -\frac{w}{ak}\int xdx = -\frac{wx^2}{2ak} + C;$$

when $x = l$, $t = T_1$.
Therefore

$$T_1 = -\frac{wl^2}{2ak} + C \qquad \text{or} \qquad C = T_1 + \frac{wl^2}{2ak};$$

therefore

$$t = T_1 + \frac{w}{2ak}(l^2 - x^2).$$

At the mid-point of the strip, $x = 0$; then

$$T_2 = T_1 + \frac{wl^2}{2ak} \quad \text{or} \quad T_2 - T_1 = \frac{wl^2}{2ak},$$

as $w = TV/2l$ and $V = 2l\rho I/a$,

$$T_2 - T_1 = \frac{V^2}{8\rho k}. \tag{49}$$

Therefore the temperature difference of the mid-point and the terminals is proportional to the square of the voltage drop or to the square of the current. A complete treatment of this instrument has been given by its inventor W. N. Goodwin, Jr.[16]

The full-load drop between terminals is 150 millivolts; the overload capacity is 50 per cent. For the higher ranges, 60 to 100 amp., two similar strips in parallel are used, the hot junction being attached to one of them. The thermoelectric element is not affected by changes of room temperature or by the length of time that the instrument is in circuit. On account of the small mass of heater strip, the deflection is promptly attained. The damping is that of the direct-current millivoltmeter. There is no zero shift, and the calibration is not subject to the uncertainties encountered with the expansion type of instrument. The inductance is practically nil, and the resistance low; alteration of circuit conditions is reduced to a minimum. There are no unsymmetrical shunts to give trouble at high frequencies.

All instruments of the heater type must be used with care, for their overload capacity is much smaller than for instruments of the soft-iron or electrodynamometer types.

SOFT-IRON INSTRUMENTS

The first ammeters and voltmeters used for commercial measurements on electric light and power circuits were of the soft-iron type, instruments in which an iron core is drawn into a solenoid, in opposition to the action of either gravity or a controlling spring.

As the induced pole reverses in sign with reversal of the current, a soft-iron instrument deflects in but one direction, irrespective of the direction of the current through it.

The Westinghouse Electric and Manufacturing Company developed this type of instrument for use with their then recently introduced alternating-current system in 1888.

The law connecting the current and the pull of a solenoid on an iron core depends on the degree of saturation of the iron. Suppose the core to remain fixed in position. If it is but weakly magnetized by the current, the strength of the induced pole will be roughly proportional to the current. This pole reacts with the field of the solenoid which is proportional to the current; so the attraction is approximately proportional to the square of the current. If the magnetizing field is so strong that the core is "saturated," any further increase of the current alters the strength of the induced pole but little, and the attraction is approximately proportional to the current.

Thus, as the current is gradually increased from zero to a high value, the law of the pull changes from that of the square to that of the first power of the current, and the pull is also modified by the change in the position of the iron core due to the yielding of the spring or gravity control. The net result is that the scales of direct-current soft-iron instruments are not uniformly divided.

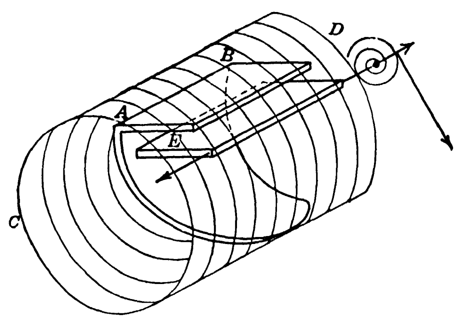


FIG. 28.—Illustrating principle of magnetic-vane instruments.

The quality of the iron is important. In order that the indications may be independent of the previous magnetic history of the iron, it should be as free from hysteresis as possible. For direct-current work, soft-iron instruments are now employed where cheapness and robustness of construction are essential and a moderate accuracy will suffice.

In alternating-current work, this construction is employed in ammeters where its use avoids the necessity of taking large currents into the movable parts of the instrument.

The principle involved in the magnetic-vane instruments is sufficiently illustrated by Fig. 28.

As shown in the figure, *E* is a soft-iron vane fixed to the spindle which carries the pointer and the inner end of the controlling spring.

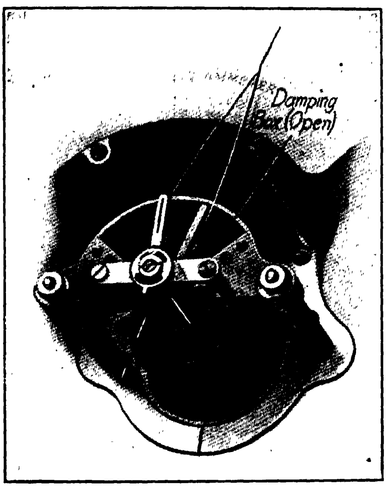
On the passage of the current through the coil *CD*, in which the foregoing arrangement is inserted, the iron is magnetized, the like poles repel each other, and the needle is moved over the scale. An air damper is usually added.

There are many variations on this fundamental design, which is used for both ammeters and voltmeters.

In the Weston soft-iron instruments, the arrangement is as indicated by Fig. 29.

A thin piece of soft iron *abc* of the form shown is bent to conform to a cylinder; *de* is another thin piece of iron of rectangular form so bent that it is coaxial with *abc*. It is rigidly attached to the spindle. The coil has a large opening at the center in which this arrangement is placed with the spindle coinciding with the axis.

On the passage of the current the neighboring edges of the iron are similarly magnetized, the like poles repel each other, and the index is forced over the scale. This construction is employed in both ammeters and voltmeters intended for use with alternating currents.



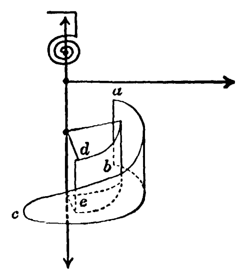


Fig. 29.—Weston soft-iron instrument.

The General Electric Company has increased the length of scale of soft-iron instruments by adopting the inclined-coil principle. Referring to Fig. 30, the current flows through the coil which is inclined at an angle of about 45 deg. to the spindle which carries the iron vanes, the pointer, and the aluminum damping sector that moves between the poles of the permanent magnets. When no current is passing, the plane of the vanes makes a slight angle with that of the coil; on the passage of the current, the vanes tend to place themselves along the lines of force, that is, perpendicular to the plane of the coil.

By adopting the inclined-coil arrangement, a long scale is obtained, for in order to turn the plane of the soft iron from a position coincident with the plane of the coil to one perpendicular to it, it is necessary to turn the pivot, and therefore the pointer, through 180 deg.; the actual working range of deflection is about 100 deg. These instruments have laminated magnetic shields, one of which has been cut away to show the working parts more clearly.

Effect on Circuit of Introduction of Current Measuring Device.[18]—All galvanometers and ammeters possess resistance and consequently when inserted in a circuit alter the very thing that they are intended to measure. In almost all cases, the effect is negligible. If it is not, and the circuit conditions are constant, the magnitude of the current before the instrument was introduced may be determined by taking two readings: the

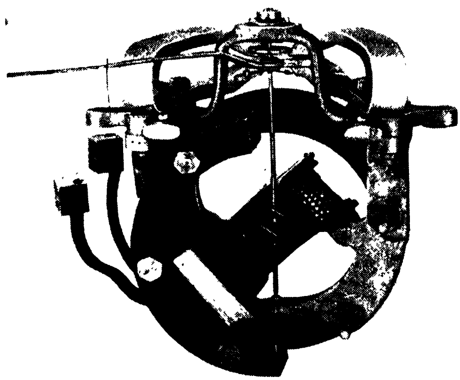


FIG. 30.—Element for G-E type P-3 ammeter.

first with the ammeter as usual, the second with an additional resistance equal to the resistance of the ammeter placed in series with it.[18]

Let R = total resistance of circuit before introduction of the ammeter or galvanometer.

R_G = resistance of ammeter or galvanometer.

E = e.m.f. applied to the circuit.

The desired current is $I = E/R$.

The first reading gives

$$I_1 = \frac{E}{R + R_G}.$$

The second gives

$$I_2 = \frac{E}{R + 2R_G}.$$

Combining the two equations gives the required current

$$I = \frac{E}{R} = \frac{I_1 I_2}{2I_2 - I_1}. \qquad (50)$$

THE ELECTRODYNAMOMETER AND THE CURRENT BALANCE

The electrodynamometer is distinguished from the moving-coil galvanometer by having the movable member suspended not in the field of a permanent magnet but in that due to a system of fixed coils which are traversed by the current.

Electrodynamometers may be either absolute or secondary instruments. The coils may be arranged in various ways, but in an absolute instrument the arrangement and proportions must be such that the constant of the instrument may be accurately calculated from the measured dimensions. The Helmholtz arrangement is sometimes adopted for both the fixed and the movable members, but in the instrument shown in Fig. 31, which was especially designed for absolute measurements, the coils are wound on cylinders.[19]

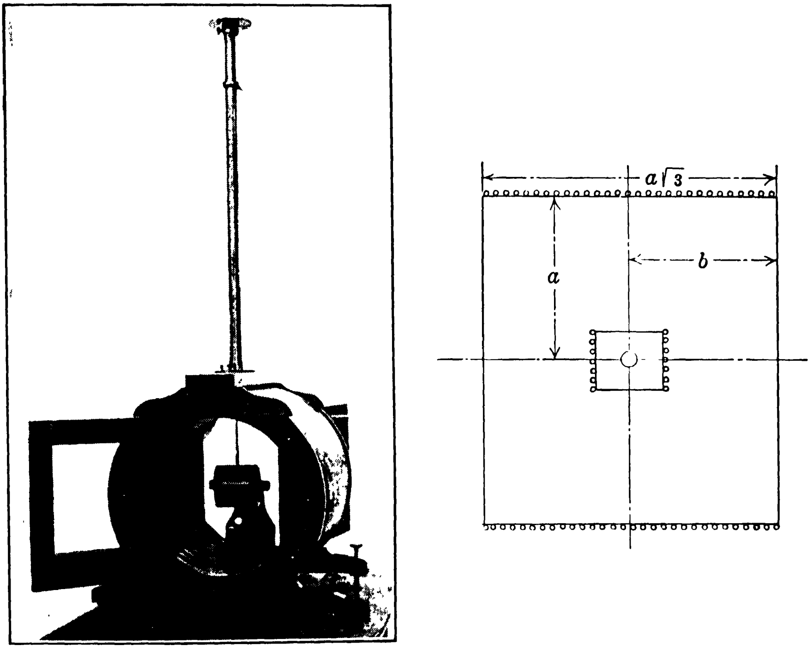


Fig. 31.—Absolute electrodynamometer.

The fixed coil of N_F turns consists of a single layer of insulated wire wound on a plaster of Paris or marble cylinder, which is very accurately turned.

The movable coil of N_M turns is wound in a single layer on a carefully ground porcelain cylinder. The coils are placed concentrically. The diameter of the movable coil is about one-fifth that of the fixed coil.

The movable member is suspended by a torsion wire, and the current is taken to it by two mercury cups, the leads being as far as possible concentric, to avoid disturbing effects.

A torsion head is used in reading, so that any effect due to the angular displacement of the axes of the coils from the perpendicular position is avoided. The suspension wire is the most troublesome feature of the instrument, for its torsional properties must be so definite that they can be determined to a high degree of accuracy. A well-aged phosphor-bronze wire is the most satisfactory.

The force, in any given direction, that acts on a circuit carrying a current I_M, when it is placed in a magnetic field, is equal to the product of the current and the space rate of change of flux through the circuit when the circuit is displaced in the given direction. If the flux is due to a second circuit, its value will be $I_F m$, where I_F is the current in the second circuit and m is the coefficient of mutual induction of the two circuits. Consequently, the force in the direction x is

$$F = I_F I_M \frac{dm}{dx}. \tag{51}$$

Similarly, the turning moment acting between the coils of an electro-dynamometer and tending to change the angle θ between their axes is given by

$$M = I_F I_M \frac{dm}{d\theta}. \tag{52}$$

To use these relations, it is necessary to have expressions for m in terms of the numbers of turns in the coils, their dimensions, and their distance apart; in general, m must be expressed in the form of a series, examples of which may be found in Maxwell's "Treatise on Electricity and Magnetism" and Gray's "Absolute Electrical Measurements."

In accordance with the preceding, if a plane circuit of area A, which is traversed by a current I_M, is suspended in a uniform magnetic field of strength H', it will experience a turning moment $-M = AI_M H' \sin \theta$, where θ is the angle between the perpendicular to the coil and the direction of the field H'. This simple relation cannot be used in connection with the electrodynamometer except as a first approximation, for the field due to the fixed coil is not uniform throughout the space occupied by the movable coil.

In electrodynamometers when used with direct currents, H' is the sum of the field due to the current circulating in the fixed coils, which form a part of the instrument, and the local field.

An *approximate* expression for the turning moment acting on the movable coil, assuming it to be a plane circuit, may be obtained.

Referring to Fig. 31, the field at the point O, the center of the fixed coil, may be obtained as follows: Let the number of turns per centimeter of length of the coil be n, and let x be the axial distance of a turn from the center. If I_F is the current in the wire, a belt of winding δx cm. long will produce at the center a field whose component along the axis is

$$\delta H = \frac{2\pi a n \delta x I_F}{(a^2 + x^2)} \cdot \frac{a}{\sqrt{a^2 + x^2}} = 2\pi a^2 n I_F \frac{\delta x}{(a^2 + x^2)^{3/2}}.$$

The effect of the whole coil will be

$$H = 2\pi a^2 n I_F \int_{-b}^{+b} \frac{dx}{(a^2 + x^2)^{3/2}} = 2\pi n I_F \frac{x}{\sqrt{a^2 + x^2}}\Bigg]_{-b}^{+b} = \frac{4\pi n I_F b}{\sqrt{a^2 + b^2}}.$$

If N_F is the total number of turns,

$$H = \frac{2\pi N_F I_F}{\sqrt{a^2 + b^2}}.$$

If the movable coil of N_M turns has a radius r, and the field in which it is placed is uniform and of strength H, the mutual inductance between the fixed and movable coils is

$$m = H\pi r^2 N_M \cos\theta = \frac{2\pi^2 r^2 N_M N_F \cos\theta}{\sqrt{a^2 + b^2}},$$

where θ is the angle between the axes of the coils. The turning moment is

$$M = I_F I_M \frac{dm}{d\theta} = \frac{2\pi^2 r^2 N_M I_M N_F I_F \sin\theta}{\sqrt{a^2 + b^2}},$$

and when the axes are perpendicular, this becomes

$$M = \frac{2\pi^2 r^2 N_M I_M N_F I_F}{\sqrt{a^2 + b^2}}.$$

As implied above, the turning moment due to the mutual action of the two coils cannot be exactly calculated in this simple manner, for, to be exact, m must be expressed in the form of a series. However, in this particular case, as pointed out by A. Gray,[19] if the ratio of the radius of each coil to its length is $1/\sqrt{3}$, all the terms in the series between the first and seventh drop out, and all but the first term are so very small that they may be considered as corrections, to be calculated if the accuracy of the work demands it.

The result of careful analysis shows that the turning moment due to the mutual action of the two coils, if they are concentrically placed with their axes perpendicular, is given, to a very high degree of approximation, by

$$M = \frac{2\pi^2 r^2 N_M I_M N_F I_F}{\sqrt{a^2 + b^2}}$$

or, if the two coils are in series, by

$$M = \frac{2\pi^2 r^2 N_F N_M I^2}{\sqrt{a^2 + b^2}}.$$

Of course the field in which the movable coil is placed is not uniform, but with coils proportioned as stated above, the instrument acts as if a plane circuit having the net area of the movable coil were suspended in a uniform field of strength

$$H = \frac{2\pi N_F I_F}{\sqrt{a^2 + b^2}}.$$

Secondary Electrodynamometer. Siemens Dynamometer.—A form of secondary electrodynamometer formerly in common use is shown in Fig. 32. The fixed coils are firmly supported from a wooden pillar which carries at its top a torsion head provided with a pointer which can be moved over a uniformly graduated circle.

The wooden frame effectually prevents any errors which might be introduced in alternating-current work by currents induced in the supports for the coils.

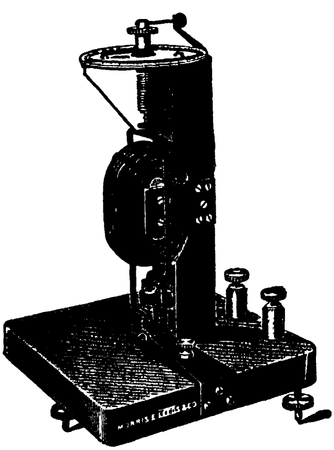


FIG. 32.—Siemens electrodynamometer.

The movable coil hangs freely from a pivot which rests in a jewel carried by a stirrup attached to the graduated plate. The torsion head is connected to the movable system by a loosely coiled spiral spring. A pointer, which normally stands at zero, is attached to the movable coil; on the passage of the current, this pointer deflects against a stop and is brought back to its original position by turning the torsion head. The amount of twist that it is necessary to give the spring in order to return the coil to its zero position is read from the graduated circle. If the spring is perfect, the moment exercised by it will be proportional to this angle of twist. As springs cannot, in general, be relied upon throughout the whole range of twist, the instrument should be calibrated at a number of points, and a calibration curve drawn.

The current is led into the movable coil by two stout wires which dip into mercury cups.

Setting up the Siemens Electrodynamometer.—The instrument must be leveled, and the same relative position of the coils maintained during calibration and subsequent use. Though the instrument is to be used with alternating currents, it is convenient to employ direct currents in the calibration. If this is done, it is desirable to place the instrument so that the local field will have no influence. This may be accomplished by turning the dynamometer in azimuth until a position is found where the strongest current that is to be used produces no deflection when sent through the movable coil alone.

The Law of the Electrodynamometer.—The law of the electrodynamometer is dependent on the method of reading. Two cases will be considered:

1. When the movable coil is always brought back to its initial position by the use of a torsion head, as in the Siemens instrument.

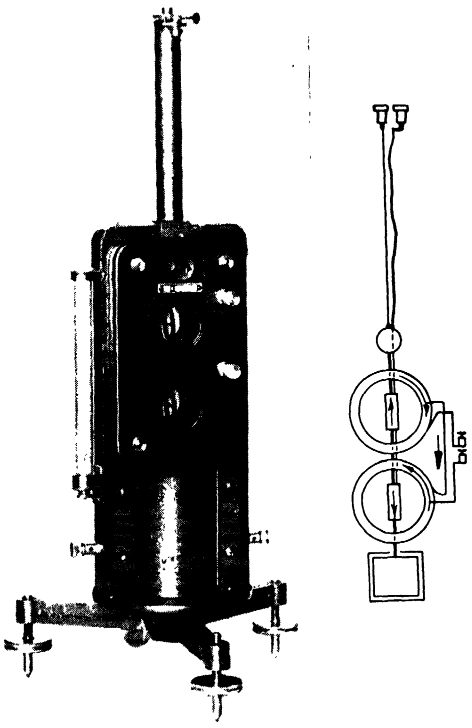


FIG. 33.—Astatic electrodynamometer.

2. When the movable coil is allowed to deflect, as in an ordinary reflecting galvanometer.

1. When a simple electrodynamometer, set up without regard to the local field, is used with direct currents, a part of the turning moment is due to the action of that field on the movable system. This will be proportional to the current through the movable coil. The field due to the fixed coil is proportional to the current through it, so the total turning

moment will be $M = K_1 I_F I_M \pm K_2 H I_M$. If the coils are in series, the moment corresponding to a current I will be

$$M = K_1 I^2 \pm K_2 H I.$$

K_1 is a factor that depends on the dimensions and the number of turns of both coils and on the angle between their axes. K_2 depends on the dimensions and number of turns of the movable coil and on the position of the coil in the local field, H.

It is seen that with direct currents, the local field effect will greatly complicate the action of the simple instrument. It will not be present when alternating currents are used, but, in that case, alternating stray fields introduce errors, so recourse is had to astatic instruments (Fig. 33). The movable system consists of two coils, identical in all respects and attached to a rigid stem. They are so connected that they experience turning moments in opposite directions. Consequently, in a *uniform* field, either direct or alternating, the system as a whole experiences no turning moment. There are two sets of fixed coils, so connected that they both tend to turn the movable system in the same direction. The use of the astatic principle in reflecting electrodynamic wattmeters increases their reliability and greatly facilitates calibration. If the astaticism is not perfect, the slight residual effect may be eliminated by taking reversed readings.

If an alternating or regularly pulsating current is employed, the turning moment passes through a cycle of values with each complete period. As the natural time of vibration of the movable system is much greater than the period of the current, the system will take up a position dependent upon the average turning moment; that is, the twist in the controlling spring will be given by

$$\theta = \frac{K_1}{\tau}\frac{1}{T}\int_0^T i_F i_M dt. \tag{53}$$

The subscripts F and M refer to the fixed and movable coils, respectively; T is the time of a complete cycle; hence, the deflection when a torsion head is used is proportional to the mean product of the currents in the two coils. Therefore, if the coils are in series, the deflection is proportional to the mean square of the current or to the square of the effective value.

The currents in the two coils may differ in wave form and in time phase as well as in magnitude. For instance, expressing both i_F and i_M in the form of a series, if the waves are nonsinusoidal,

$$i_F = A_1 \sin \omega t + A_2 \sin (2\omega t - \varphi_2) + A_3 \sin (3\omega t - \varphi_3) + \cdots$$
$$i_M = B_1 \sin (\omega t - \varphi_1') + B_2 \sin (2\omega t - \varphi_2') + B_3 \sin (3\omega t - \varphi_3') + \cdots.$$

It is well known that the mean product of two sine curves which differ in periodicity is zero and that the mean product of two sine curves of the same periodicity is $I_1 I_2/2 \cos \alpha$, where I_1 and I_2 are the maximum values and α is the angle of phase difference. Consequently,

$$\tau\theta = K_1 \frac{1}{T}\int_0^T i_F i_M dt = K_1\left[\frac{A_1B_1}{2}\cos \varphi_1' + \frac{A_2B_2}{2}\cos(\varphi_2 - \varphi_2') + \frac{A_3B_3}{2}\cos(\varphi_3 - \varphi_3') + \cdots\right]. \quad (54)$$

If the fixed and movable coils are in series, as they are when currents are measured, $A_n = B_n = I_n$, and $\cos(\varphi_n - \varphi_n') = 1$, so

$$\tau\theta = K_1 \frac{1}{T}\int_0^T i^2 dt = K_1\left(\frac{I_1^2}{2} + \frac{I_2^2}{2} + \frac{I_3^2}{2} + \cdots\right) = K_1 I^2. \quad (55)$$

I_1, I_2, I_3, etc., are the maximum values of the various components. $I_1^2/2$ is the square of the effective value of the fundamental; $I_2^2/2$ is the square of the effective value of the second harmonic, etc.; and I^2 is the square of the effective value of the current.

2. When the movable coil is allowed to deflect, the factor K_1 becomes a variable depending on the angle between the axes of the fixed and movable coils. If the field due to the fixed coil is uniform, or, what practically amounts to the same thing, if the movable coil is very much smaller than the fixed coil, K_1 is proportional to the cosine of the angle of displacement of the axes of the coils from a perpendicular position. However, Lord Rayleigh* called attention to the fact that the mutual inductance of two concentric circles, the ratio of whose radii is

$$\sqrt{\frac{0.3}{1}} = 0.548,$$

is, over a considerable range, very nearly proportional to the angular displacement of the axes from the perpendicular position. This is illustrated by Fig. 34 in which relative values of the mutual inductance are plotted against the angular displacements of the axes of the coils from the perpendicular position.

From the figure it is seen that, over a wide range, $dm/d\theta$ is practically constant. As the turning moment due to the mutual action of two coils is $I_F I_M \frac{dm}{d\theta}$, the deflection of an electrodynamometer with coils of small cross section thus proportioned, neglecting the action of local fields, should be very nearly proportional to the square of the current. Also, if

* The Inductance and Resistance of Compound Conductors, *Phil. Mag.*, December, 1886, p. 470.

the coils of a deflectional wattmeter (page 315) are thus proportioned, the scale should be sensibly uniform throughout the working range of the instrument. Advantage is taken of this suggestion by instrument makers, who, by experiment, design coils of large cross section, which give practically uniform scales in wattmeters.

In instruments of the reflecting type, the zero position of the movable system should be that in which there is no mutual induction between the

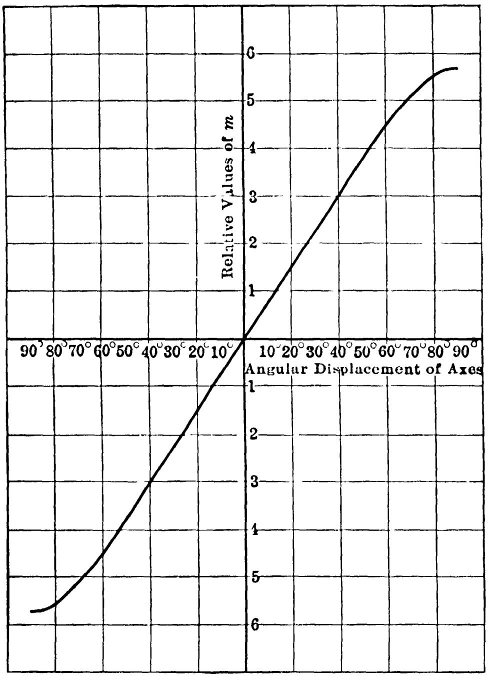


FIG. 34.—Showing relation between the mutual inductance of two circles and the angular displacement of their axes from the perpendicular position when the ratio of the radii is 0.548.

fixed and moving coils. To make this adjustment, an alternating current may be sent through the fixed coil, the circuit of the movable coil being closed through a telephone. The movable system is turned by the torsion head until the telephone is silent.

In the instrument shown in Fig. 33, the damping is obtained by means of mica vanes, which move in the closed chamber at the base of the instrument. An astatic instrument can be set up without regard to local fields and calibrated with direct currents. The curves so obtained are immediately applicable to either direct- or alternating-current measurements.

The Irwin Astatic Electrodynamometer.—The Irwin astatic instrument is intermediate between the electrodynamometer and the current balance. The fixed turns are in the form of two coaxial circular coils of the same diameter, through which the current circulates in opposite directions, thus producing between the faces of the coils a field that is directed radially outward. The movable member consists of two semi-circular coils mounted on a thin disk of mica, the directions of the currents being as indicated. The straight sides are as nearly as possible in the axis of rotation; the curved sides move in the field between the two fixed coils, being attracted by one and repelled by the other. This construction brings the two movable coils very near together, which is advantageous, as it reduces any effect that may be due to nonuniformity of the local or stray field.

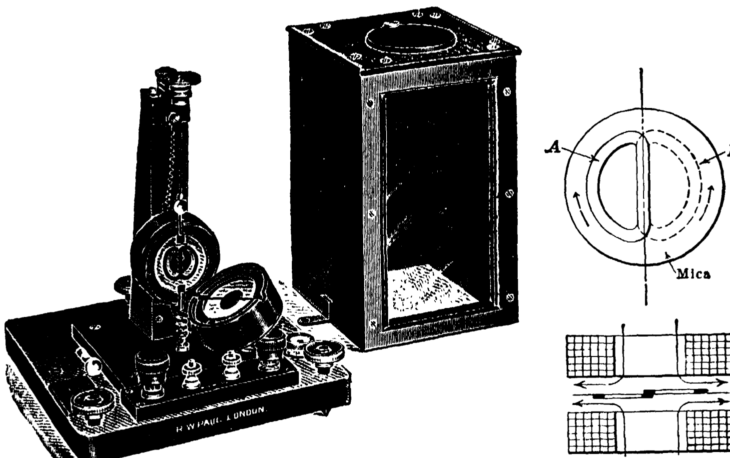


FIG. 35.—Irwin astatic electrodynamometer.

Rewinding an Electrodynamometer to Obtain a Given Performance.—As the deflection of an electrodynamometer when used with alternating currents is proportional to the square of the current, the range of the instrument is limited. It is desirable, therefore, to have the fixed coil subdivided by taps so that the number of turns may be varied.

It is sometimes necessary to alter the sensitivity of secondary dynamometers so that definite deflections may be obtained with stated currents. If the corresponding dimensions of the coils of the original and the rewound instruments are the same, the calculation of the proper number of turns is simply a matter of proportion, provided one has data concerning the number of turns on the coils and the performance of the original instrument. If the instrument is properly set up,

$$\tau\theta = KFMI^2. \qquad (56)$$

F and M are the numbers of turns on the fixed and movable coils, respectively, and K is a constant depending upon the geometry of the coil system. This may be determined from a knowledge of the torsion constant of the spring, the number of fixed and moving turns on the original instrument, and the deflection corresponding to a given current. After K has been found, the value of FM, the required product of the fixed and moving turns for the new winding, is readily determined.

Ballistic Electrodynamometer.—For a discussion of the ballistic use of the electrodynamometer, see the paper by G. F. C. Searle, Ballistic Measurement of Hysteresis, *Jour. Inst. Elec. Eng.*, vol. 49, 1902, pp. 100, 219; also the paper by M. E. Rice and B. McCollum, Testing Iron by the Ballistic Electrodynamometer, *Phys. Rev.*, vol. 29, 1909, p. 132.

Electrodynamometers for Heavy Currents.—For several reasons, it is difficult to apply the ordinary electrodynamometer to the measurement of very large currents. If the coils are used in series, trouble is experienced in getting the current into and out of the movable member. Where any considerable current is to be carried, the necessary flexible connections are made by means of mercury cups, and no dynamometer in which they are employed can be considered a portable instrument, as that term is now generally understood. There are also the structural difficulties due to the necessity of supporting a heavy movable coil on pivots that are practically free from friction and are sufficiently strong so that the instrument will stand handling. These considerations preclude the use of the electrodynamometer with its coils in series as an alternating-current ammeter and are the reasons for the survival of soft-iron ammeters as alternating-current instruments.

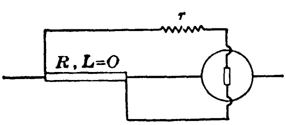


FIG. 36.—Wattmeter method for measuring large alternating currents.

Again, it is essential that the field at the movable coil be the same with both direct and alternating currents, for direct currents will be used in calibrating, and alternating currents in the subsequent use of the instrument. This equality of fields may be attained either by arranging the metal of the coil so that the current distribution will be the same in both cases or by adopting an arrangement which from its symmetry is such that the change in distribution in passing from direct to alternating current does not affect the field in which the movable coil swings.

It is also necessary that there be no error due to eddy currents induced in the mass of the coils or in the frames by which they are supported. For this reason, in instruments of moderate capacity, recourse is had to stranding the conductors. This must be very carefully done, so that all the strands will have the same effective inductance and resistance and the same increase of resistance due to heating.

Wattmeter Method for Measuring Large Alternating Currents.—To obviate the necessity of taking a large current into the movable coil, recourse has been had to a wattmeter method, as indicated in Fig. 36.

The current coil of the wattmeter and a known resistance R are inserted in the circuit; the connections are such that the I^2R loss in the constant resistance R is measured, and thus the current is determined.

Agnew Tubular Electrodynamometer.[20]—This instrument is designed primarily for measuring very large alternating currents, up to 5,000 amp.,

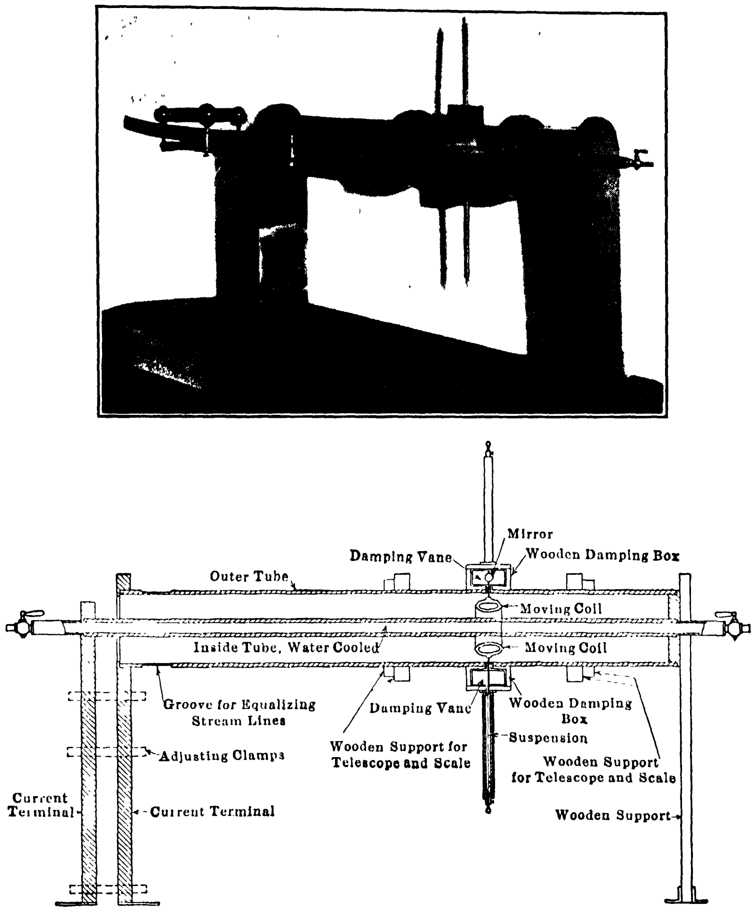


Fig. 37.—Agnew tubular electrodynamometer.

by the wattmeter method. It can, of course, be used as an ordinary wattmeter for power measurements. Its distinctive feature is the means taken to avoid errors due to the skin effect in the very massive conductors that must be used for the current coil.

As seen from Fig. 37, the current "coil" is made in the form of two coaxial tubes. When they are traversed by the current, a strong field will exist in the space between them, while the field external to the tubes will be *nil*. Stray-field effects due to the heavy current in the instrument are thus avoided.

The movable system consists of two rigidly connected coils, one above, the other below, the central tube. The working position of the movable system is approximately 90 deg. from that shown in Fig. 37. As the movement takes place in a strong field, it is essential that the movable system be entirely free from magnetic impurities. If this is not the case, the zeros with the current on and with the current off will not coincide.

The suspension strip is of phosphor-bronze, and air damping is provided. Diaphragms between the tubes are necessary to prevent disturbance of the movable system by air currents.

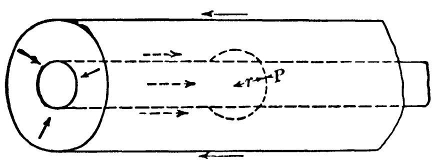


FIG. 38.—Pertaining to the Agnew tubular electrodynamometer.

Theory of the Tubular Electrodynamometer.—Suppose that two coaxial circular tubes are arranged as shown in Fig. 38, the directions of the current being as indicated. Consider any point P in the space between the tubes and distant r from the axis. A direct current will distribute itself uniformly over the cross section of the tubes. The work done in taking a unit pole around the indicated path is

$$2\pi rH = 4\pi I,$$

where I is the current encircled by the path, that is, the current that flows in the central tube; and H is the field strength at any and all points in the path. Therefore,

$$H = \frac{2I}{r}.$$

It is obvious that this relation will hold as long as a symmetrical arrangement of the current around the central axis is preserved. It is thus evident that the generating lines of the surfaces of the coaxial tubes need not be straight but may have any form whatsoever. Symmetry is the all-important thing. It is also apparent that any symmetrical redistribution of the current in the conductors will not alter H.

If an alternating current be substituted for a direct current, the distribution of the current over the cross section of the tubes will be altered, owing to the skin effect, but from the symmetry of the conductors the new distribution of the current will be symmetrical about the axis, and therefore the field due to the changed distribution of the current is the same as before.

In the actual instrument, owing to the manner of taking the current into the large outside tube, the natural distribution will not be quite uniform. For this reason, a groove about 5 cm. wide is turned eccentrically in the tube. By filing, the groove may be so adjusted that the streamlines are uniformly distributed.

It is essential that the symmetrical distribution of currents about the axis be maintained; therefore, any springing of the slender inner tube must be avoided. A test for distribution errors may be made by the method described on page 324.

The principal dimensions of the instrument, as used at the Bureau of Standards, are

Outer tube, length, 101 cm.; radii, 6.41 and 7.07 cm.

Inner tube, length, 125 cm.; radii, 0.50 and 1.27 cm.

Current capacity, air-cooled, 1,200 amp., water-cooled, 5,000 amp.

Field at center of movable coils at full load, 300 gausses.

Movable coils, 116 turns of 0.2-mm. silver wire; diameters, 2.5 and 5.0 cm.; weight of each coil, 7.3 gm.; total resistance of movable-coil circuit, 14.3 ohms; current capacity, 0.06 amp.; inductance, 1.4 millihenrys.

Sensitiveness, 100-cm. deflection at 86-cm. scale distance requires 100 amp. in tubes and 0.06 amp. in movable coils.

The Current Balance.—In this class of instruments, the current is measured by weighing, with a gravity balance, the pull exerted by a coil upon another placed in a parallel plane, their axes being coincident.

An absolute current balance of this sort was used by Lord Rayleigh and Mrs. Sidgwick (1884) in their determination of the electrochemical equivalent of silver. Since that time, the instrument has been brought to a very high degree of perfection, particularly through the work of Ayrton, Mather, and Smith at the National Physical Laboratory in England and of Rosa, Dorsey, Miller, and Curtis at the Bureau of Standards at Washington, D. C.[21]

Rayleigh and, following him, Rosa, Dorsey, Miller, and Curtis used a balance with two equal fixed coils, the smaller movable coil being placed midway between them, all three coils being coaxial.

The Rayleigh current balance is not intended for general use as a current-measuring device, but for the absolute measurement of currents in special investigations, such as are necessary in the determination of electrical standards, it is of great service and is generally considered to be the most accurate device that has been developed for the purpose. Its advantages are:

1. The constant of the instrument depends principally upon the ratio of the effective radii of the coils. This number can be determined

experimentally to a high degree of accuracy, by a method originally due to Bosscha; the difficulties met with in determining the mean radii of multiple-layer coils from mechanical measurements are thus avoided. This is the peculiar advantage of the Rayleigh form of balance.

2. The measurements are independent of the local field and its variations.

3. The determination of torsion constants is entirely avoided.

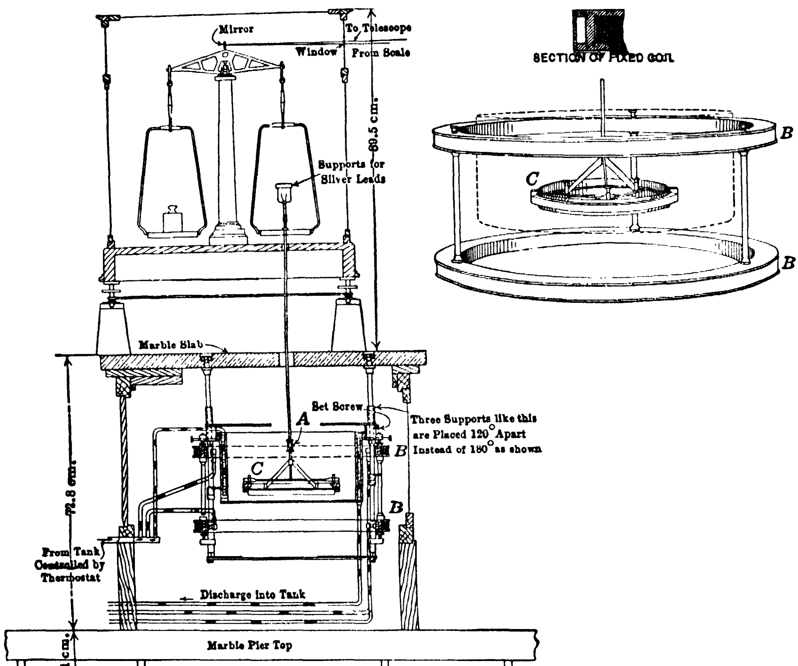


FIG. 39.—Absolute current balance used at Bureau of Standards by Rosa, Dorsey, and Miller.

4. Analysis shows that when the distance between the fixed and movable coils is equal to one-half the radius of the larger coil, slight inaccuracies in the placing of the movable coil produce very small errors in the calculated constant of the instrument.

Figure 39 shows the current balance used by Rosa, Dorsey, and Miller, and later by Curtis, together with an enlarged view of the coil system. The two fixed coils have a radius of 50 cm. and are placed 25 cm. apart. They are wound on brass frames, the material for which was carefully selected, for even good brass is slightly magnetic. Enameled wire was used. The movable coil, 25 cm. in diameter, is hung from a precision balance, and the force is determined by the change in weight necessary to restore the balance to equilibrium when the current in the fixed coil is reversed. This change was 6 g. To prevent disturbance of the balance by air currents set up by the heating of the coils, the fixed coils are water-cooled, and the movable coil is hung in a water-jacketed chamber which is kept at a constant temperature.

When the balance is used in the manner indicated,

$$I^2K = \frac{Mg}{2}.$$

M is the change in the weights, corrected for buoyancy of the air, which is necessary to restore the balance to equilibrium when the current in the fixed coil is reversed; and g is the acceleration due to gravity.

The factor K depends principally on the ratio of the radii of the fixed and movable coils. It is equal to dm/dx, where m is the mutual inductance of the coils and dx refers to an axial displacement of the movable coil. m can be calculated, in the form of a series, from the numbers of turns and the dimensions of the coils.

Secondary Current Balances. The Kelvin Balance.—In the Kelvin balance (Fig. 40), which was formerly in common use as a secondary standard for alternating-current measurements, four fixed and two movable coils are connected in series. The movable coils are carried at the ends of the balance arm which turns about a horizontal axis. In place of knife-edges, the beam is suspended by a large number of filaments of copper wire, forming a sort of stranded ribbon. The electrical turning moment acting on the movable system is balanced by a sliding weight which can be moved along a uniformly graduated bar. The weight may be placed in a definite position on a carriage which can be manipulated by cords acting through a self-releasing pendant which allows the beam to swing freely when the cords are relaxed. The weight is in two portions; one half is placed on the carriage, and one half in a definite position in a V-shaped trough at the right-hand end of the beam. The current is given by $I = K2\sqrt{R}$, where K is a constant depending on the weights and R is the displacement of the carriage as read from the scale. The range of the instrument is altered by changing the weights. It is obvious that the instrument is adapted for use only in a laboratory where the circuit conditions can be carefully controlled; consequently it has been superseded by more convenient arrangements. The Westinghouse Company has developed a line of laboratory standards having the Kelvin arrangement of coils but with a convenient spring control and torsion head substituted for the gravity control.

Alternating- and Direct-current Comparators.[22] **Silsbee Comparator.**—Silsbee has described an alternating- and direct-current comparator, developed from the original ideas of H. B. Brooks, in which the currents flow in two electrically separate but thoroughly interwound coil systems. Each winding taken alone forms a complete electrodynamometer, the alternating- and direct-current movable coils being of necessity mounted on the same spindle. The connections are such that the direct- and alternating-current torques on the movable system are in opposite directions and can therefore be balanced. To avoid taking any considerable current into the movable coils, the wattmeter method of measurement (page 72) is adopted for the current comparator. The direct current through the movable system is made adjustable in small

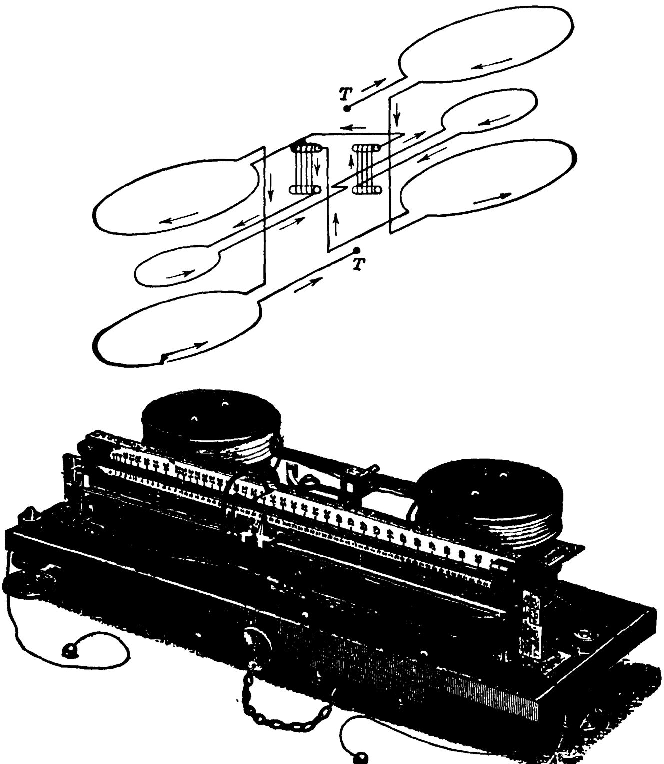


Fig. 40.—Kelvin current balance.

steps, so it is practicable to balance approximately the two torques while the direct current in the fixed coils is maintained at a constant value—1 amp.—in the instrument being described. The small difference of the two torques gives rise to a deflection which is read from a calibrated scale. Though the difference between the torques is small, either is large enough to give many times the full-scale deflection if it acted alone.

Referring to Fig. 41, the entire alternating current flows through the fixed coils 3 and 3′; the movable-coil current is derived from the 1-ohm shunt R_Q; therefore $i_1 = i_3\left(\frac{R_Q}{R_Q + R_P + R_1}\right)$, nearly enough.

For the alternating-current member, the electrodynamic factor is G_{13}, and the turning moment is

$$M_{a\text{-}c.} = G_{13}\frac{1}{T}\int_0^T i_1 i_3 dt = G_{13}\left(\frac{R_Q}{R_Q + R_P + R_1}\right)\frac{1}{T}\int_0^T i_3^2 dt =$$

$$G_{13}\left(\frac{R_Q}{R_Q + R_P + R_1}\right)I_{a\text{-}c.}^2,$$

$$G_{13} = \frac{dm_{13}}{d\theta} \times 10^7,$$

where m_{13} is the mutual inductance, in henrys, of coils 1 and 3, and θ is the angle between their axes; similarly for the other electrodynamic constants.

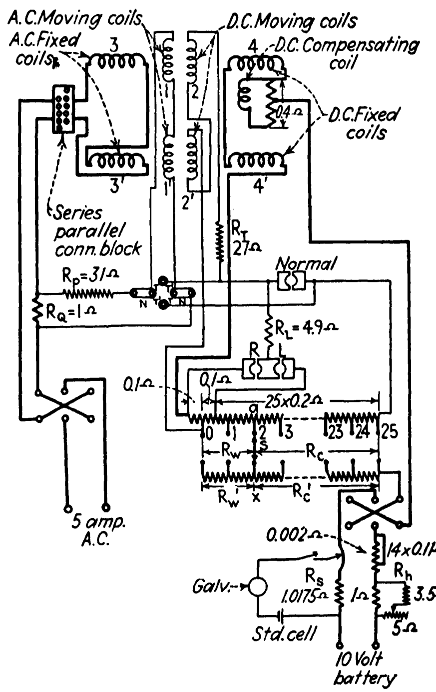


FIG. 41.—Silsbee comparator.

The direct current I_4, which is kept at 1 amp. by aid of the standard cell arrangement and the adjustable rheostat Rh, flows through the fixed coils, then divides, one part flowing through R_W, the other through the astatically arranged movable coils and the resistances R_T and R_C. The two parts, which unite at a, then flow through the sliding switch S and the coils R_C' back to the source.

The resistance $R_W + R_C$ consists of 25 equal coils. The resistors composing $R'_W + R'_C$ are so proportioned that when the galvanometer is balanced, the current I_4 is always 1 amp. and is unaffected by changes in the setting of S.

The movable-coil current is $I_2 = \frac{I_4 R_W}{R_T + R_2 + R_C + R_W}$; therefore the direct-current torque is $M_{\text{d-c.}} = \frac{G_{24} I_4{}^2 R_W}{R_2 + R_T + R_C + R_W}$. The net torque is $M_{\text{a-c.}} - M_{\text{d-c.}}$ If τ is the torsion constant of the suspension and θ is the angular deflection, $\theta = \frac{M_{\text{a-c.}} - M_{\text{d-c.}}}{\tau}$. If the length of the pointer is L scale units and the scale reading is D,

$$\frac{D}{L} = \frac{M_{\text{a-c.}} - M_{\text{d-c.}}}{\tau}.$$

Substituting the values of $M_{\text{a-c.}}$ and $M_{\text{d-c.}}$ gives

$$I_{\text{a-c.}}{}^2 = \left(\frac{G_{24}}{G_{23}}\right)\left[\frac{(R_Q + R_P + R_1)I_4{}^2}{R_Q(R_2 + R_T + R_C + R_W)}\right]R_W + \frac{\tau}{G_{13}L}\left(\frac{R_Q + R_P + R_1}{R_Q}\right)D, \tag{57}$$

or

$$I_{\text{a-c.}} = K_W R_W + K_D D, \tag{58}$$

when K_W and K_D are factors depending on the construction of the comparator.

The indication is seen to consist of two parts: the larger, given by the setting of the slider S, is proportional to R provided (G_{24}/G_{13}) is independent of the deflection of the movable coil. The smaller is proportional to the reading D, provided, G_{13} is constant. This is analogous to the setting of a deflection potentiometer where the major part of the reading is given by the dial switch and the remaining portion, which may be slightly varying, by the deflection of the galvanometer.

The fixed alternating-current coils consist of 1,000 turns of wire in five sections which are connected in parallel at the connection block, giving 200 net turns for a 5-amp. instrument. One half the turns are on the upper, and one half on the lower, pair of coils. The fixed direct-current coils consist of 1,000 turns in series similarly distributed. The alternating- and direct-current windings are thoroughly mixed. The direct-current compensating coils are used to adjust for any differences in the scale laws of the alternating- and direct-current windings. Their axes are horizontal, and perpendicular to that of the fixed coils, within which they are placed. Their effect may be varied by adjusting the 0.4-ohm shunt. All of the movable coils are rigidly attached to the same bakelite spindle which is 21 cm. long and 0.3 cm. in diameter. Air damping is employed. A thin aluminum vane, 12 by 4.5 cm., swings with very small clearances in a damping box. Each movable coil has an outside diameter of about 3.5 cm. and contains 75 turns of No. 28 silk-enamel wire. A taut suspension of "phonoelectric" strip, about 17 cm. long and 0.056 by 0.006 cm. in cross section, is employed. A tension of perhaps 50 to 100 g. is exerted on the lower end of the suspension by a helical spring. This serves to locate the movable system with respect to the fixed coils. The deflection is read by a compact mirror and scale arrangement.

The direction of the alternating fields from the fixed coils is such that they induce in the direct-current windings, by reason of mutual inductance, e.m.fs. that are equal and oppositely directed, and are therefore balanced. The instrument contains two self-checking features. The deflection constant may be checked by removing the plug from NORMAL and inserting it in either R or L, the a-c. circuits being open. A deflection of 50 divisions either to the right or to the left should result.

To check the ratio (G_{24}/G_{13}), all the alternating fixed turns are put in series at the series-parallel connection block, thus making G_{13} and G_{23} five times their normal value. The alternating- and direct-current circuits are connected in series, so that the same current I_4 flows through both circuits. The alternating-current movable coils are disconnected from the shunt by removing the links N and connected in series with the direct-current movable coils by means of the links at TT, the plug being removed from NORMAL. The two movable windings are thus placed in series around the main dial. The current in them is

$$I_2 = \left(\frac{R_W}{R_2 + R_T + R_1 + R_C + R_W}\right)I_4. \tag{59}$$

With the links at TT,

$$M = I_2I_4(5G_{13} - G_{24} + G_{14} + 5G_{23}) = \frac{\tau D}{L}. \tag{60}$$

If the current in 1 is reversed in respect to that in 2 by placing the links at $T'T'$; and the current in 3 and 3′ reversed by the alternating-current reversing switch, I_2 and I_4 remaining the same,

$$M' = I_2I_4(5G_{13} - G_{24} - G_{14} - 5G_{23}) = \frac{\tau D'}{L}. \tag{61}$$

From (57) and (58),

$$\frac{G_{24}}{G_{13}} = 5\left[1 - \left(\frac{\tau}{L}\right)\left(\frac{D + D'}{10G_{13}I_2I_4}\right)\right]. \tag{62}$$

From (56) and (57),

$$\frac{\tau}{L} = K_DG_{13}\left(\frac{R_Q}{R_Q + R_P + R_1}\right).$$

Therefore

$$\frac{G_{24}}{G_{13}} = 5\left[1 - \left(\frac{K_D R_Q(R_2 + R_T + R_1 + R_C + R_W)}{10 I_4^2 R_W(R_Q + R_P + R_1)}\right)(D + D')\right]. \quad (63)$$

MEASUREMENT OF CURRENTS IN PERMANENTLY CLOSED CIRCUITS[23]

Occasionally it is necessary to measure the current in a conductor that cannot be broken to allow the introduction of an ammeter or shunt. For example, such cases occur in the investigation of the electrolytic deterioration of underground pipes for water or gas, due, for instance, to the stray currents caused by the use of a ground return, or imperfect bonding, in a traction system. The damage occurs where the current leaves the pipe and may cause such a menace to health and property that large expendi-

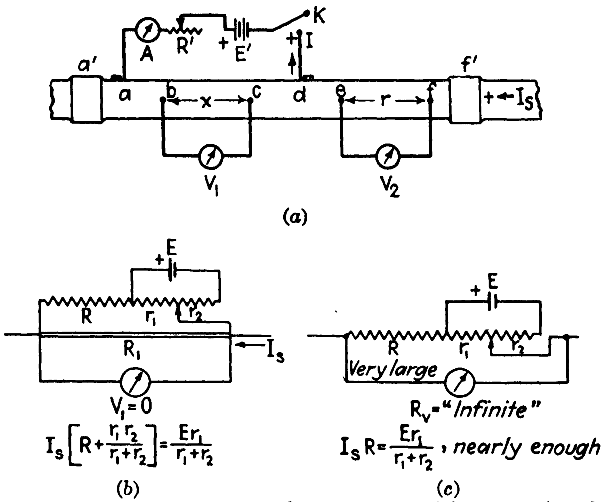


FIG. 42.—Connections for measuring a direct current without opening the circuit.

tures of time and money are justifiable in locating the source of the trouble and in its elimination.

In this or similar cases, if the resistance between two potential points on the pipe or other conductor can be determined, the current may be measured by using this resistance as a shunt, a millivoltmeter being used to determine the potential difference between the points. The problem thus resolves itself into the determination of the resistance of a portion of a conductor which may be carrying a current and which must be measured *in situ* without being opened.

Suppose that, as a preliminary to measuring the stray current in, for instance, a water main, it is necessary to determine the resistance between two plugs that have been screwed into the pipe to serve as potential terminals.

The pipe is supposed to be traversed by stray currents of unknown strength. The necessary connections are shown in Fig. 42*a*. At V_1 and V_2 are two millivoltmeters, with, perhaps, 10-millivolt scales; they are connected to potential points at *b*, *c* and *e*, *f*. These points are obtained by drilling into the pipe and firmly inserting brass plugs to which the leads may be soldered. It is essential that the instruments be calibrated with the leads that are to be used in the test. At E' is a storage battery capable of yielding enough current to give a good reading on V_1 (100 or 200 amp. for a 15-in. iron pipe); at *A* is the ammeter by which the current from E' is measured; and *K* is the switch by which the current is controlled.

The spacing of the points *a*, *b*, *c*, *d* is important, for the four points *b*, *c*, *e*, *f* should be on four equipotential planes through the pipe. Therefore the distance between *a* and *b* and between *c* and *d* should be great enough so that the current from E' may spread out, and the streamlines become uniformly distributed before the points *c* and *b* are reached; the lengths of *ab* and *cd* should be about twice the diameter of the pipe, the points *a* and *d* being on the top, and *b* and *c* on the side.

The positive direction of the currents may be assumed as indicated; it is essential that such an assumption be made at the start; otherwise confusion may arise, and the results are of no value.

The procedure is as follows: Observe, if necessary, the temperature of the conductor. With *K* open, *simultaneous* readings of the millivoltmeters are taken. Denote them by V_1 and V_2; then

$$\frac{x}{r} = \frac{V_1}{V_2}.$$

K is then closed, and *simultaneous* readings of the ammeter and of the two millivoltmeters are taken. Call the readings I, V'_1, V'_2, and denote the stray current in the pipe at the instant of reading by I_s. Then the current across x, at that instant, will be

$$I_x = -I + I_s;$$

also

$$I_x x = V'_1,$$
$$I_s r = V'_2.$$

From these

$$I_x = \frac{V'_1}{x} = -I + I_s$$

$$V'_1 = -Ix + I_s x = -Ix + V'_2\frac{x}{r}.$$

Therefore

$$x = -\frac{V'_1}{I} + \frac{V'_2 V_1}{I V_2}$$

and

$$r = -\frac{V_2 V_1'}{V_1 I} + \frac{V_2'}{I}.$$

The test should be repeated with the battery reversed; I is then $-$. Throughout, care must be taken as to the algebraic signs of all the deflections.

The final result is independent of the stray current in the pipe, I_s; its elimination is possible, even though it be varying rapidly, because all three instruments are read simultaneously. The periods and the damping of the three instruments must be such that they keep pace with one another when the current changes. It is frequently desirable to obtain records extending over a considerable time, in which case the resistance of a section of pipe may be determined as above and a registering millivoltmeter used.

If the current I is so adjusted that the reading V_1' becomes zero,

$$I = I_s$$

also

$$r = V_2'/I$$

and the reading of the ammeter gives the strength of the stray current which then flows.

When $V_1 = 0$, the current I_s is not exactly the same as the original stray current, although in most cases the difference is negligible. The resistance from a to d has been virtually removed from the circuit.

The resistance r may be a joint which it is inadvisable to open, as, for instance, that between a neutral bus connection and a water pipe.

In cases where the current is not sufficient for the measurement, it may be temporarily increased and controlled by a second battery and an adjustable rheostat connected between a' and f'.

Figure 42*b* shows an application that is convenient when dealing with small currents.

To measure a current of the order of a few microamperes in a high-resistance circuit, the modification shown in Fig. 42*c* is useful. In this case, R is the regular resistance of the circuit and may be many megohms. The voltmeter resistance should be "infinite."

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CHAPTER II

THE BALLISTIC GALVANOMETER

In the comparison of the electrostatic capacities of condensers and cables, and in the examination of the magnetic properties of iron, an instrument is necessary that will measure the quantity of electricity displaced in a circuit by a transient current. For this purpose, the ballistic galvanometer is employed; and to avoid disturbances due to local magnetic fields, an instrument of the D'Arsonval type is now generally used.

For reading the instrument, a telescope and a uniformly divided circular scale, having its center at the axis of the movable system, should be employed; with this arrangement the deflection as read from the scale is directly proportional to the angle turned through by the movable system.

Observations are made as follows: The movable system is brought to rest in its proper zero position; the discharge is then passed through the instrument, giving an impulse to the movable system, which slowly deflects; the reading is taken at the first turning point, or elongation, just as the movable system is about to begin its swing back toward zero. If this maximum angular deflection of the coil from its original position is called θ_1, and Q is the quantity of electricity in the discharge, then when a D'Arsonval instrument is employed,

$$Q = K'\theta_1.$$

For any particular instrument used in a definite manner, K' is a constant. As will be shown later, its value depends on the current sensitivity of the instrument, on the time of vibration of the movable system, on the amount of damping, and on the manner in which the discharge is sent through the galvanometer.

The ballistic instrument differs from the ordinary current galvanometer in one essential particular, in that the moment of inertia of the movable system is made very large compared with the restoring moment due to the suspension strip. In other words, the instrument is one having a long time of vibration. This is necessary in order that the response to the impulse caused by the passage of the transient current may be rendered so sluggish that the entire quantity due to the discharge of the condenser or the change of magnetic flux may have time to pass through the instrument before the system has deflected appreciably. In the ordinary discussion of the ballistic galvanometer, this is assumed to be true; but in certain cases, the assumption is not tenable.

Some of the devices for obtaining a large moment of inertia are indicated in Fig. 43.

No. 1 is for the Kelvin galvanometer, while the others are for the D'Arsonval type. In 1 and 2, the crossbar is of aluminum with a screw thread on it; the little nonmagnetic weights are thus made adjustable. In 3, the weights may be removed from the pans, and others substituted as desired. Instruments of very long period may have the moment of inertia increased as shown in Fig. 44. The rim of the disk, seen just below the movable coil, is made of brass; the web, of aluminum.

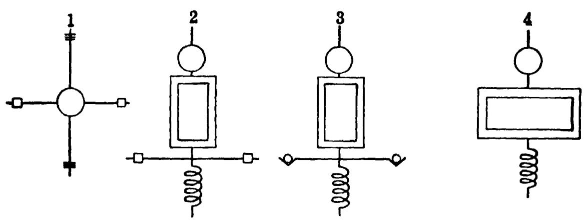


FIG. 43.—Suspended systems for ballistic galvanometer.

The time of vibration which it is necessary to give the movable system in order to obtain accurate results depends entirely on the use to which the instrument is to be put. In comparing condensers, when the resistance of the circuit is low, the discharge is practically instantaneous, and an instrument with a period of about 20 sec. is convenient; so long a period is not necessary in this case for the fulfillment of the assumption that the entire discharge has passed before the movable system has been deflected appreciably, but it renders the reading of the instrument much easier. For magnetic work with the usual small-sized specimens, such a period would be adequate; but for investigation of the behavior of massive electromagnets, an instrument with so short a period would be of no value whatsoever, since in this case the change of flux is very slow. When a solid iron core is tested, as much as 30 sec. may elapse before the change in flux is practically complete; for such work, a galvanometer with a period as great as 600 sec. is sometimes employed.

If the motion of the movable system results from a series of impulses given as the coil swings from its zero position, or if it is due to a prolonged discharge, the magnitude of the deflection will be affected by an amount that will depend on the manner in which the galvanometer current varies.

It is frequently stated that one of the essential characteristics of a ballistic galvanometer is absence of damping, or, as damping must of necessity be present to a certain degree, that it must be reduced to a minimum. This does not mean, however, that a damped instrument cannot be used ballistically; in fact, a critically damped ballistic galvanometer is frequently most convenient, being a great timesaver. The damping should be electromagnetic; the law governing it is then definite

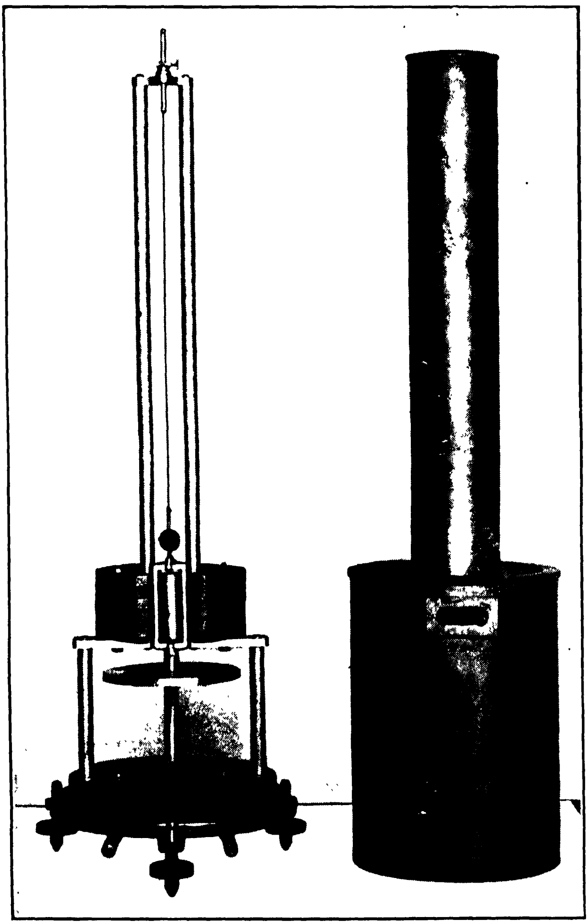


FIG. 44.—Long-period ballistic galvanometer.

and capable of a simple mathematical expression. If air damping is present, the proviso that it be small is a safe one, for its law is not exactly known. In the analytical theory, it is assumed to be the same as for electromagnetic damping.

Checking Devices.—If there is little damping, it is necessary in order to economize time to have some form of checking device by which the moving system may be brought promptly to rest. For general purposes, that shown in Fig. 45 is convenient.

By a little practice, the motion of the magnet and the manipulation of the key may be so timed that the zero is promptly attained. A key in series with a resistance placed across the galvanometer terminals is frequently convenient; the resistance should be of such a value that the instrument may be critically damped. Thermoelectric currents are frequently present and somewhat complicate the action of these devices.

Precautions in Reading.[1]—Trouble is likely to be experienced with galvanometers of the D'Arsonval form, due to changes in direction of the very weak magnetism of the supposedly nonmagnetic coil, and also possibly to "set" in the suspension. Both of these must be reduced to a minimum in the manufacture of the instrument, the first by the use of a radial field and extreme care in the preparation of the materials used and in the winding of the coil. The set may be minimized by the proper choice of suspension strip and by care in mounting it.

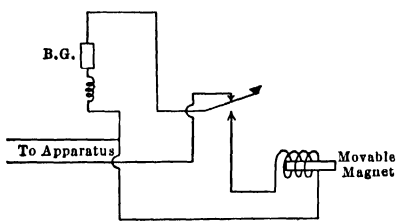


FIG. 45.—Checking device for ballistic galvanometer.

When readings are made, they should all be taken toward the same end of the scale, and the coil should not be allowed to swing very much beyond zero on its return; proper damping or use of the checking device will insure this. Before taking any readings, a deflection in the proper direction and as large as any that are to be used should be given the system; after this there will be no appreciable change of zero. This precaution should be taken each time that the instrument is used.

If it is necessary to take a reading when the coil is not absolutely at rest but swinging so that the amplitude as read on the scale is only a small fraction of a centimeter, the impulse should be given to the system when the swing is at its maximum, and the elongation θ should be calculated from the true mechanical zero, not from the scale reading when the discharge was passed. This applies when the swinging on either side of the mechanical zero is not more than about 3 per cent of the first elongation θ_1.

Thermoelectromotive forces in any part of the circuit are troublesome; those arising in the galvanometer itself should be minimized by shielding from drafts or anything else that could cause irregularities of temperature. In the best instruments the binding posts, connections to the movable coil and the coil itself, are all of copper. In specially designed instruments, the current is not taken in through the suspension, which may be of steel, but through spiral connections made of very thin copper strip. This strip may be made by rolling out a fine wire, about No. 40. The spirals may be made so delicate that they contribute practically nothing to the restoring moment.

The Calibration of a Ballistic Galvanometer.—It will be shown that the quantity of electricity which is instantaneously discharged through a ballistic galvanometer is given by

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(1 + \frac{\lambda}{2}\right)\theta_1',$$

where T = time of a complete swing.
θ = steady deflection caused by a steady current of strength I_G.
θ_1' = first elongation, or throw.
λ = logarithmic decrement, or natural logarithm of the ratio of two successive elongations.

In the following discussion, if the displacement of electricity is "instantaneous," the corresponding value of θ_1 will be primed; if the displacement is not instantaneous, the prime will be omitted.

With any definite arrangement of the apparatus,

$$Q = K'\theta_1'.$$

A determination of the time of vibration and current sensitivity, together with λ, enables the constant of the instrument to be calculated. For most purposes, however, it is preferable to calibrate by discharging a known quantity of electricity through the galvanometer and reading the corresponding deflection θ_1'. If λ at calibration differs from its value during the subsequent work, it must be determined and allowed for.

To obtain a definite quantity of electricity, an earth inductor, a mutual inductance, or a standard condenser charged to a known voltage may be employed.

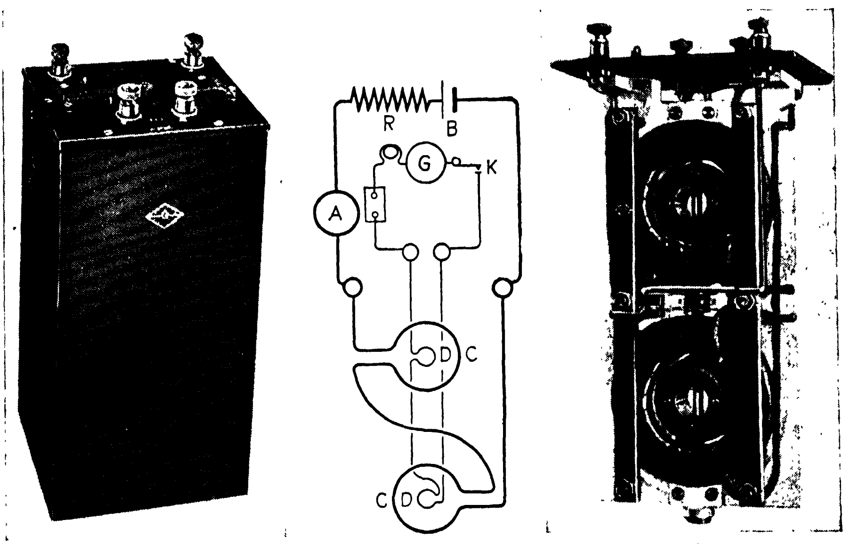
The earth inductor is a large coil of many turns mounted on a vertical or horizontal axis so that it can be quickly turned through 90 or 180 deg. The total area of the turns is known. The coil is included in the galvanometer circuit. If the plane of the coil is originally in the magnetic meridian, and the rotation is through 90 deg. about a vertical axis, the quantity of electricity displaced in the circuit is $Q = AH/r$. A is the total area of the turns; H, the horizontal intensity of the local field; and r, the resistance of the galvanometer circuit. Owing to erratic variations of the local field, this method of calibration has ceased to be of importance.

Duddell has developed the idea embodied in the earth inductor so that an instrument of practical value has resulted. In his magnetic standard, two movable coils arranged astatically are used in series with the galvanometer, and the local field is replaced by the fields due to two oppositely wound fixed coils, one acting on each movable coil. On releasing a catch, the movable system is rotated through 180 deg. by a spring, thus cutting the lines due to the fixed coils.

This is a secondary instrument. The number of lines cut by the movable coils is

$$n = KI,$$

where I is the current in the fixed coils; and K, an experimentally determined constant the value of which is furnished by the instrument maker.



(a) (b) (c)

FIG. 46.—Duddell inductor.

It is seen that the number of lines cut may be varied by altering the current through the fixed coils.

When a mutual inductance is employed, the galvanometer is placed in series with the secondary winding. The primary circuit may be arranged so that a measured value of the current may be suddenly *reversed*. In this case,

$$Q = \frac{2mI}{r}, \tag{1}$$

where m is the mutual inductance; I, the primary current; and r, the resistance of the secondary or galvanometer circuit.

A common form of mutual inductance used for this purpose consists of a long, straight primary coil of one layer wound on a nonmagnetic core and a short secondary coil wound outside the primary. The arrangement, commonly called a "solenoidal inductor," is indicated in Fig. 47.

Let l be the length of the primary coil, a its radius, n_P the number of primary turns per unit length, N_S the *total* number of secondary turns

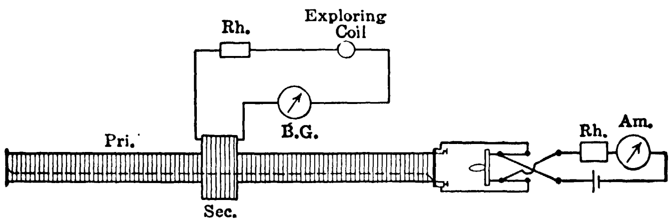


FIG. 47.—Solenoidal inductor.

and A the cross section of the primary. Then if l is many times a the mutual inductance is approximately

$$m = 4\pi n_P N_S A,$$

and

$$Q = \frac{8\pi n_P N_S A I}{r},$$

all quantities being in the c.g.s. system,
or

$$Q = \frac{8\pi n_P N_S A I}{r 10^9}, \qquad (2)$$

Q, I, and r in the practical system.

Ordinarily, when this method of calibration is employed, the instrument is to be used in a closed circuit. The damping will therefore be dependent on r, and the constant of the galvanometer becomes a function of the resistance of the circuit. For this reason, it is common in magnetic work to arrange the apparatus so that the secondary of the calibrating solenoid is continuously kept in circuit with the galvanometer and the exploring coil. A substitution method is customarily used. First, a known change of flux is produced in the circuit by the mutual inductance, and the change of flux per scale division of the galvanometer thus determined. After this, an observation of θ_1', corresponding to any change of flux in the specimen, enables the change in linkages to be calculated. If r is altered, recalibration is necessary.

Concerning the use of standard condensers, see page 367.

Theory of the Undamped Ballistic Galvanometer.—A D'Arsonval galvanometer with a uniform radial field will be assumed. With such an instrument, when it is traversed by a steady current of strength I_G,

$$I_G C = \tau\theta.$$

C is the coil constant or factor which when multiplied by the galvanometer current gives the turning moment acting on the movable system. Its value depends on the strength of field, the length of active wire, and the breadth of the coil. τ is the torsion constant of the suspension, or the restoring moment per unit angular deflection; $\tau\theta$ is then the restoring moment due to twisting the strip through an angle θ.

It will be necessary to recall that when a body having a moment of inertia P is rotating about a fixed axis with an angular velocity $d\theta/dt$, its kinetic energy is given by $E = \frac{1}{2}P(d\theta/dt)^2$; that when a body so rotating has its angular velocity changed, the moment of the forces producing the change is $M = P(d^2\theta/dt^2)$, where $d^2\theta/dt^2$ is the angular acceleration.

Suppose the coil to be at rest in its zero position and a transient current whose intensity at any instant is i_G to be sent through the instrument. Its electromagnetic action gives rise to a force which lasts for the very short time during which the current flows. This imparts a certain amount of energy to the movable system, which swings to its extreme deflection in opposition to the restoring force due to the suspension. At any instant, the total energy of the system is in part kinetic and in part the potential energy stored in the twisted suspension. When the coil swings through its zero position, all the energy is kinetic, while at the end of the swing it is all potential. These two amounts of energy must be equal, for by supposition there is no damping and therefore no dissipation of energy as the coil swings.

The turning moment acting on the coil at any instant is

$$i_G C - \tau\theta.$$

Therefore

$$P\frac{d^2\theta}{dt^2} = i_G C - \tau\theta.$$

It is assumed that the time occupied by the passage of the current is so short that the coil has not moved appreciably from its zero position; in other words, during the discharge, $\tau\theta$ is zero. Therefore

$$C\int i_G dt = P\int \frac{d^2\theta}{dt^2}dt.$$

Therefore, if Q is the total quantity in the discharge,

$$CQ = P\left(\frac{d\theta}{dt}\right)_{\theta=0}.$$

$\left(\frac{d\theta}{dt}\right)_{\theta=0}$ is the angular velocity at the zero position of the movable system, that is, at the time when all the energy is kinetic. The energy imparted to the system is then

$$E = \frac{1}{2}P\left(\frac{d\theta}{dt}\right)^2_{\theta=0} = \frac{1}{2}\frac{C^2Q^2}{P}.$$

The coil deflects, and this amount of energy is expended in twisting the suspension through an angle θ'_1. The coil then swings back through its zero position and continues to oscillate.

The work done in twisting the suspension through the angle θ'_1 is

$$W = \tau\int_0^{\theta'_1}\theta\, d\theta = \frac{\tau}{2}(\theta'_1)^2.$$

As, by supposition, there is no dissipation of energy during the swing from $\theta = 0$ to $\theta = \theta'_1$,

$$\frac{C^2Q^2}{P} = \tau(\theta'_1)^2.$$

Therefore

$$Q = \left(\frac{\sqrt{P\tau}}{C}\right)\theta'_1.$$

θ'_1 is the first throw, or elongation. The quantities P, τ, and C are not easily determined, and the formula may be put in a more useful shape by introducing the time of vibration of the movable system considered as a torsion pendulum. If T_0 is the time of vibration when there is no damping,

$$T_0 = 2\pi\sqrt{\frac{P}{\tau}}.$$

Hence

$$Q = \left(\frac{T_0}{2\pi}\right)\left(\frac{\tau}{C}\right)\theta'_1.$$

There still remains the factor τ/C. This may be evaluated as follows: If a current of constant intensity I_G is sent through the instrument, a deflection of constant magnitude θ will result, and

$$\frac{\tau}{C} = \frac{I_G}{\theta}.$$

Therefore

$$Q = \left(\frac{T_0}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\theta'_1. \quad (3)$$

Obviously, the ratio of the ballistic to the current sensitivity is

$$\frac{2\pi}{T_0}.$$

For the description of a ballistic galvanometer specially designed to have very small damping and an interesting application of the method of successive impulses,* see a paper by W. B. Ellwood.[12]

Formula for the Kelvin Galvanometer.—With a Kelvin instrument, the work done in turning the suspended system through an angle θ'_1 is $MH(1 - \cos\theta'_1) = 2MH\sin^2\frac{\theta'_1}{2}$, where M is the magnetic moment of the movable system; and H, the strength of the controlling field.

The moment of the force due to the current in the coils is at any instant $GiM\cos\theta$, where G is the galvanometer constant or strength of field at the needle, due to a unit current in the coils. The time of vibration of the needle system considered as a magnetic pendulum is

$$T_0 = 2\pi\sqrt{\frac{P}{MH}}.$$

Therefore, in this case, the expression for Q is

$$Q = \frac{T_0 H}{\pi G}\sin\frac{\theta'_1}{2}.$$

Theory of the Damped Ballistic Galvanometer.[2]—In the practical case, a certain amount of damping is always present. It may be due to:

1. Induced currents set up in the metallic parts of the movable system by their motion through the field of the instrument.

2. Modification of the current through the instrument by the e.m.f. induced in the movable coil by its motion.

3. Air friction.

4. Internal friction in the suspension wire.

In the D'Arsonval type of instrument, 4 is entirely negligible, and 3 is small.

As the ballistic galvanometer is ordinarily used, the time during which the current flows is very short. But cases arise where the displacement of electricity through the instrument is not instantaneous, and such cases must be included in the general discussion.[3]

The ballistic galvanometer is commonly employed:

a. In the determination of magnetic fluxes.

b. In the comparison of capacities.

In case *a*, the instrument is used in a closed circuit, consisting of the galvanometer and of the exploring coil wound around the specimen.

In any case where the instrument is used in a closed circuit of resistance r, the equation governing the motion is (see page 30)

$$P\frac{d^2\theta}{dt^2} + \left(k' + \frac{C^2}{r}\right)\frac{d\theta}{dt} + \tau\theta = \frac{C}{r}\left(e - L\frac{di}{dt}\right); \quad (4)$$

* Maxwell, "Electricity and Magnetism," 3d ed., p. 390.

k' is the damping coefficient when the instrument is on open circuit, and C^2/r is the damping coefficient due to the e.m.f. set up in the main circuit by the motion of the coil. In this discussion, let

$$k = k' + \frac{C^2}{r};$$

e is the instantaneous value of the e.m.f. impressed on the circuit; it is a function of t and usually becomes zero in a comparatively short time. $-L\frac{di}{dt}$ is the back e.m.f. due to the inductance of the galvanometer circuit. In the following discussion, it will be assumed to be negligible.

If the conditions are such that the current through the instrument is not modified by the e.m.f. due to the motion of the coil, then, in (4), the term C^2/r is absent, and the second member becomes Ci. The process of solution for $\int_0^t idt$ is the same as that employed below for $\int_0^t edt$. Assuming that $L\frac{di}{dt}$ is negligible, the solution of (4) is*

$$\theta = C_1\epsilon^{m_1t} + C_2\epsilon^{m_2t} + \frac{C}{rP(m_1 - m_2)}\left(\epsilon^{m_1t}\int e\epsilon^{-m_1t}dt - \epsilon^{m_2t}\int e\epsilon^{-m_2t}dt\right). \quad (5)$$

The values of m_1 and m_2 are given on page 16. When the e.m.f. is first applied to the circuit, the movable system is supposed to be at rest in its zero position; that is, when

$$t = 0; \qquad \theta = 0, \qquad \frac{d\theta}{dt} = 0.$$

Since e is a function of t, these conditions are imposed if

$$C_1 = -\frac{C}{rP(m_1 - m_2)}\left(\int e\epsilon^{-m_1t}dt\right)_{t=0} \quad (6)$$

and

$$C_2 = \frac{C}{rP(m_1 - m_2)}\left(\int e\epsilon^{-m_2t}dt\right)_{t=0} \quad (7)$$

With these values of C_1 and C_2 substituted in (5), the deflection at any definite time t is

$$\theta = \frac{C}{rP(m_1 - m_2)}\left(\epsilon^{m_1t}\int_0^t e\epsilon^{-m_1t}dt - \epsilon^{m_2t}\int_0^t e\epsilon^{-m_2t}dt\right). \quad (8)$$

Equations (5) and (8) apply in all cases. A difficulty is encountered in using them, since in comparatively few instances is it possible to express e as an algebraic function of t. This precludes the taking of the integrals by purely analytical methods.

* Cohen. "Differential Equations," p. 103.

In the preliminary study of a proposed investigation, if it is found that the displacement of electricity through the ballistic galvanometer will not be "instantaneous," it is necessary to inquire how much the first elongation will be influenced by the manner in which e varies. For (see page 92) the galvanometer is calibrated by methods in which e is applied to the circuit "instantaneously."

To obtain the necessary data, the logarithmic decrement and the time of vibration must be found, and a preliminary test made which will experimentally determine the curve connecting e and t. This curve must be one that fairly represents the conditions that will exist in the subsequent work and is to be used as described below. If the computations show that the first elongation will be greatly influenced by the manner in which the discharge is sent through the galvanometer, it will be necessary to modify the instrument by giving it a longer period of vibration.

Suppose that the experimentally determined graph connecting e and t shows that e has become sensibly equal to zero before the galvanometer deflection has reached its first elongation. If the time that elapses before e becomes sensibly equal to zero is denoted by t', the values of the integrals in (8) taken for times greater than t' are practically constant. In the case where the galvanometer is overdamped, that is, where m_1 and m_2 are real, let M and N be two quantities defined as follows:

$$M = \frac{\int_0^{t \gtreqless t'} e\epsilon^{-m_1 t}dt}{\int_0^{t \gtreqless t'} edt}, \qquad \text{a constant.} \tag{9}$$

$$N = \frac{\int_0^{t \gtreqless t'} e\epsilon^{-m_2 t}dt}{\int_0^{t \gtreqless t'} edt}, \qquad \text{a constant.} \tag{10}$$

Then

$$\left(\theta\right)_{t \gtreqless t'} = \frac{C}{rP(m_1 - m_2)}\left(\int_0^{t \gtreqless t'} edt\right)(M\epsilon^{m_1 t} - N\epsilon^{m_2 t})_{t \gtreqless t'}. \tag{11}$$

θ_1, which is the observed reading of the instrument, occurs at a time t_1, when $d\theta/dt = 0$. Neglecting the constant coefficient in (11),

$$\frac{d\theta}{dt} = Mm_1\epsilon^{m_1 t} - Nm_2\epsilon^{m_2 t} = 0,$$

or

$$\epsilon^{(m_1 - m_2)t} = \frac{Nm_2}{Mm_1}. \tag{11a}$$

Therefore

$$t_1 = \frac{1}{m_1 - m_2} \log_\epsilon \frac{Nm_2}{Mm_1}.$$

Substituting this value of t_1 in (11) gives, for the first elongation,

$$\theta_1 = \frac{C}{rP(m_1 - m_2)}\left(\int^{t \gtreqless t'} e dt\right)\left\{M\left(\frac{Nm_2}{Mm_1}\right)^{\frac{m_1}{m_1 - m_2}} - N\left(\frac{Nm_2}{Mm_1}\right)^{\frac{m_2}{m_1 - m_2}}\right\},$$

or

$$\theta_1 = \frac{C}{rP(m_1 - m_2)}\left(\int_0^{t \gtreqless t'} e dt\right)\left\{\left(\frac{m_2}{m_1}\right)^{\frac{m_1}{m_1 - m_2}} - \left(\frac{m_2}{m_1}\right)^{\frac{m_2}{m_1 - m_2}}\right\} N^{\frac{m_1}{m_1 - m_2}} M^{\frac{m_2}{m_2 - m_1}}. \quad (12)$$

The quantity in the braces { } depends only on m_1 and m_2 and is therefore independent of the manner in which e varies. If the displacement of electricity through the galvanometer is "instantaneous," that is, if e goes through its cycle of values "instantaneously," then, by (9) and (10),

$$M = 1, \qquad N = 1.$$

Therefore when the motion of the movable system is nonperiodic, the ratio of the actual elongation to that which would have occurred had the same *integral* change in e been made instantaneously is

$$\frac{\theta_1}{\theta_1'} = N^{\frac{m_1}{m_1 - m_2}} M^{\frac{m_2}{m_2 - m_1}}. \quad (13)$$

It is seen that (13) gives a measure of the error produced in the deflection by the prolongation of the discharge through the galvanometer.

To obtain the numerical values of M and N, the ordinates of the graph connecting e and t are multiplied by $\epsilon^{-m_1 t}$ and $\epsilon^{-m_2 t}$, and the two curves thus obtained are plotted to the original scale. All three curves are then integrated by a planimeter, giving the necessary data for calculating both M and N.

When the motion of the movable system is periodic, that is, when $k^2/4P^2 < \tau/P$, m_1 and m_2 are complex.

$$m_1 = -a + jb, \qquad m_2 = -a - jb,$$

where

$$a = \frac{k}{2P} = \frac{2\lambda}{T} \qquad \text{and} \qquad b = \frac{2\pi}{T}.$$

These values of m_1 and m_2 when substituted in (8) give

$$\vartheta = \frac{C}{rP} \cdot \frac{\epsilon^{-at}}{2jb}\left(\epsilon^{jbt}\int_0^t e\epsilon^{(a-j)bt}dt - e^{-jbt}\int_0^t e\epsilon^{(a+jb)t}dt\right); \quad (14)$$

but

$$\epsilon^{jbt} = \cos bt + j \sin bt,$$

and

$$\epsilon^{-jbt} = \cos bt - j \sin bt.$$

Hence

$$\theta = \frac{C\epsilon^{-at}}{rPb}\left[(\sin bt)\int_0^t e\epsilon^{at} \cos btdt - (\cos bt)\int_0^t e\epsilon^{at} \sin btdt.\right] \quad (15)$$

The method of dealing with this equation, when e falls sensibly to zero before the first elongation is reached, is similar to that just employed in the nonperiodic case (see page 99).

Let R and S be defined as follows:

$$R = \frac{\int_0^{t \geqq t'} e\epsilon^{at} \cos btdt}{\int_0^{t \geqq t'} edt}, \qquad \text{a constant.} \quad (16)$$

$$S = \frac{\int_0^{t \geqq t'} e\epsilon^{at} \sin btdt}{\int_0^{t \geqq t'} edt}, \qquad \text{a constant.} \quad (17)$$

Then, when $t \geqq t'$,

$$\theta = \frac{C\epsilon^{-at}}{rPb}\left(\int_0^{t \geqq t'} edt\right)(R \sin bt - S \cos bt). \quad (18)$$

To find θ_1, determine t_1 by placing $d\theta/dt = 0$, and substitute the result in (18). Neglecting the constant coefficient,

$$\frac{d\theta}{dt} = \epsilon^{-at}[(-aR + bS) \sin bt + (aS + bR) \cos bt] = 0.$$

Therefore

$$\frac{\sin bt_1}{\cos bt_1} = \tan bt_1 = \frac{aS + bR}{aR - bS} = \frac{\lambda S + \pi R}{\lambda R - \pi S}.$$

Substituting t_1 in (18) gives

$$\theta_1 = \frac{CT}{2rP}\epsilon^{-\frac{\lambda}{\pi}\tan^{-1}\frac{\lambda S + \pi R}{\lambda R - \pi S}}\left(\int_0^{t \geqq t'} edt\right)\frac{\sqrt{R^2 + S^2}}{\sqrt{\pi^2 + \lambda^2}}. \quad (19)$$

If the same integral change in e had been made instantaneously, then, by (16) and (17),

$$R = 1, \qquad S = 0,$$

and the deflection would have been

$$\theta_1' = \frac{CT}{2rP}\epsilon^{-\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}}\left(\int_0^{t \geqq t'} edt\right)\frac{1}{\sqrt{\pi^2 + \lambda^2}}. \quad (20)$$

It follows in this case that

$$\frac{\theta_1}{\theta_1'} = \frac{\sqrt{R^2 + S^2}}{\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}}}. \quad (21)$$

This quantity is a measure of the error produced in the deflection by the prolongation of the discharge through the galvanometer.

Equations (19) and (20) are more conveniently expressed if C and P are replaced by quantities that are more easily determined. Since

$$T = \frac{2\pi}{\sqrt{\frac{\tau}{P} - \frac{4\lambda^2}{T^2}}}$$

and

$$\tau\theta = I_G C,$$

substituting in (19) and solving for

$$\left(\int_0^{t \overline{\gtrless} t'} edt\right)$$

gives

$$\left(\int_0^{t \overline{\gtrless} t'} edt\right) = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(\frac{\pi r}{\sqrt{\pi^2 + \lambda^2}}\right)\frac{\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\lambda S + \pi R}{\lambda R - \pi S}}}{\sqrt{R^2 + S^2}}\theta_1. \tag{22}$$

Discussion. When the Displacement of Electricity is Instantaneous.—The circuit in which e acts contains the inductance of the galvanometer. If e goes through its cycle instantaneously, that is, in a time so short that the cycle is over before the coil of the galvanometer moves appreciably, then, as the current is zero both at the start and at the finish,

$$r\int_0^t idt = \int_0^{t \overline{\gtrless} t'} edt = rQ.$$

In the case of an instantaneous displacement of electricity through the instrument, the quantity Q is given by

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left\{\frac{\pi}{\sqrt{\pi^2 + \lambda^2}}\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}}\right\}\theta_1'. \tag{23}$$

This is the formula commonly used for the ballistic galvanometer. The term involving λ gives the correction for damping.

The quantity in the braces { } may be expanded by Maclaurin's theorem, giving

$$\frac{\pi}{\sqrt{\pi^2 + \lambda^2}}\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}} = 1 + 0.5\lambda - 0.026\lambda^2 - 0.055\lambda^2 \cdots$$

and if λ is small,

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(1 + \frac{\lambda}{2}\right)\theta_1'. \tag{24}$$

For a secondary instrument, assuming that T is not sensibly affected by any change in damping that is likely to be encountered,

$$Q = K\left(1 + \frac{\lambda}{2}\right)\theta_1', \tag{25}$$

where K is a constant.

Another approximation, sometimes used in making the correction for damping, results from assuming λ to be very small, in which case

$$\frac{\pi}{\sqrt{\pi^2 + \lambda^2}}\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}} = \epsilon^{\frac{\lambda}{\pi}\cdot\frac{\pi}{2}} = \epsilon^{\frac{\lambda}{2}} \qquad \text{approx.}$$

But, from the law of damped oscillations (page 18),

$$\frac{\theta_1}{\theta_2} = \epsilon^{\lambda} \qquad \frac{\theta_1}{\theta_3} = \epsilon^{2\lambda}.$$

Consequently

$$\epsilon^{\frac{\lambda}{2}} = \left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{2}} = \left(\frac{\theta_1}{\theta_3}\right)^{\frac{1}{4}},$$

and Q is given approximately by

$$Q = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(\frac{\theta_1}{\theta_2}\right)^{\frac{1}{2}}\theta'_1 = \left(\frac{T}{2\pi}\right)\left(\frac{I_G}{\theta}\right)\left(\frac{\theta_1}{\theta_3}\right)^{\frac{1}{4}}\theta'_1. \tag{26}$$

Discussion. When the Discharge through the Instrument is Prolonged.— If the equation connecting e and t is known, the integrations indicated in (5), (6), and (7) may be performed. Suppose, for example, that

$$e = E_0\epsilon^{-pt}.$$

In this case, which is of practical importance,[4] the total quantity of electricity displaced in this circuit is

$$Q = \frac{1}{r}\int_0^\infty edt = \frac{E_0}{rp}.$$

Substituting the value of e in (5),

$$\theta = \frac{CE_0}{2rPjb}\left(\frac{\epsilon^{m_1t}}{p + m_1} - \frac{\epsilon^{m_2t}}{p + m_2}\right) + \frac{CE_0}{rP}\cdot\frac{\epsilon^{-pt}}{(p + m_2)(p + m_1)}. \tag{27}$$

To find t_1, the time of the first elongation or the turning point, this value of θ is differentiated, and the result placed equal to zero. Combining the equation so formed with (27) gives

$$\theta_1 = \frac{CE_0}{2rPjbp}(\epsilon^{m_1t_1} - \epsilon^{m_2t_1}),$$

or, after the values of m_1 and m_2 are substituted,

$$\theta_1 = \left(\frac{CT}{2\pi P}\right)\left(\frac{E_0}{rp}\right)\epsilon^{-\frac{2\lambda}{T}t_1}\sin\left(\frac{2\pi}{T}\right)t_1. \tag{28}$$

Equating $d\theta/dt$ to zero gives t_1, the time of the first elongation, as a solution of the equation

$$\epsilon^{-\left(p-\frac{2\lambda}{T}\right)t_1} = \cos\left(\frac{2\pi}{T}\right)t_1 + \left(\frac{-p\lambda T + 2\lambda^2 + 2\pi^2}{\pi pT}\right)\sin\left(\frac{2\pi}{T}\right)t_1. \tag{29}$$

The value of t_1 is obtained by successive approximation.

Referring to (20) and (28), it will be seen that the ratio of the deflection to that which would have been obtained had the same quantity been discharged through the galvanometer instantaneously is

$$\frac{\theta_1}{\theta_1'} = \left(\frac{\sqrt{\pi^2 + \lambda^2}}{\pi}\right)\left(\frac{\epsilon^{-\frac{2\lambda}{T}t_1}}{\epsilon^{-\frac{\lambda}{\pi}\tan^{-1}\frac{\pi}{\lambda}}}\right)\sin\left(\frac{2\pi}{T}\right)t_1. \quad (30)$$

This ratio gives a measure of the effect produced on the first elongation by the prolongation of the discharge.[5]

As another example, take one that may be realized by the manipulation of the apparatus used for calibrating a ballistic galvanometer by means of a mutual inductance.

Suppose the primary circuit to be traversed by a current. At a time $t = 0$, the circuit is broken by the reversing switch. This *instantaneously* removes $n/2$ magnetic linkages from the secondary. After an interval of 10 sec., the circuit is made in the reverse direction, and a second *instantaneous* change of $n/2$ linkages is made.

It is desired to know how the deflection so obtained will compare with that which would have been obtained had the change of n linkages been made instantaneously. In this case, using (16) and (17),

$$R = \frac{\int_0^{t \gtrless 10} e\epsilon^{at}\cos bt\,dt}{\int^{t \gtrless 10} edt} = \frac{\frac{n}{2} + \frac{n}{2}\epsilon^{10a}\cos 10b}{n}.$$

$$S = \frac{\int^{t \gtrless 10} e\epsilon^{at}\sin bt\,dt}{\int^{t \gtrless 10} edt} = \frac{0 + \frac{n}{2}\epsilon^{10a}\sin 10b}{n}.$$

Suppose that the following data apply to the galvanometer in question:

Time of a complete vibration $T = 149$ sec.

Ratio of two successive swings $\theta_1/\theta_2 = 1.063$.

Logarithmic decrement $\lambda = \log_\epsilon 1.063 = 0.0611$

$$b = \frac{2\pi}{T} = 0.0422$$

$$a = \frac{b\lambda}{\pi} = \frac{2\lambda}{T} = 0.00082.$$

Consequently,

$$R = \tfrac{1}{2}(1 + \epsilon^{0.0082}\cos 0.422) = 0.9597.$$
$$S = \tfrac{1}{2}(\epsilon^{0.0082}\sin 0.422) = 0.2064.$$
$$\sqrt{R^2 + S^2} = 0.982.$$
$$\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}} = \epsilon^{0.0041} = 1.004.$$

By (21),

$$\frac{\theta_1}{\theta_1'} = \frac{0.982}{1.004} = 0.978.$$

The error is therefore 2.2 per cent. An actual test under the foregoing conditions gave

$$\frac{\theta_1}{\theta_1'} = 0.978.$$

When the ballistic galvanometer is used in series with a test coil for determining magnetic fluxes through iron cores, it is not possible to apply the preceding purely analytical method, for the law connecting time and the e.m.f. induced in the test coil is not known. In this case, the integration must be made with the aid of a planimeter.

The graph connecting e and t may be obtained by an oscillograph, an exploring coil being wound for that purpose on the specimen under test. In general, the oscillograph is not inserted in the galvanometer circuit, for the current in it is modified by the e.m.f. set up by the movable coil. In cases where this e.m.f. is insignificant compared with that due to the change of flux through the circuit, the separate exploring coil is not necessary. Suppose that the e.m.f.-time curve $OABC$, shown in Fig. 48, has been obtained.

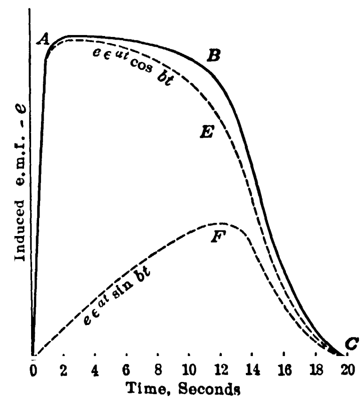


FIG. 48.—Pertaining to effect of prolonged discharge through a ballistic galvanometer.

The curves OEC and OFC are obtained by multiplying the ordinates of $OABC$ by the corresponding values of $\epsilon^{at}\cos bt$ and $\epsilon^{at}\sin bt$.

From the figure, it will be seen that t' is 20 sec. This means that e is practically zero after this time; in reality, a very small e.m.f. may persist for a considerably longer period, but the quantity of electricity displaced by it may be neglected.

$\int_0^{t'} edt$ is obtained from the area under the curve $OABC$. Integrating the curves by a planimeter, and using (16) and (17),

$$R = 0.920,$$
$$S = 0.320.$$

Therefore

$$\sqrt{R^2 + S^2} = 0.973.$$

If $\lambda = 0.0611$,

$$\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}} = \epsilon^{0.0065} = 1.007.$$

By (21),

$$\frac{\theta_1}{\theta_1'} = \frac{\sqrt{R^2 + S^2}}{\epsilon^{\frac{\lambda}{\pi}\tan^{-1}\frac{S}{R}}} = \frac{0.973}{1.007} = 0.966.$$

The error due to the prolongation of the discharge is 3.4 per cent for this particular form of e.m.f. curve.

The Critically Damped Ballistic Galvanometer.—Mathematically, critical damping occurs when the roots of the equation $Pm^2 + km + \tau = 0$ are equal, or when $m_1 = m_2 = -\frac{k}{2P}$ corresponding to

$$\frac{k^2}{4P^2} = \frac{\tau}{P}.$$

In this case, the solution of (4) becomes

$$\theta = (C_1 + C_2 t)\epsilon^{-\frac{kt}{2P}} + \frac{C}{rP}\left(t\epsilon^{-\frac{kt}{2P}}\int e\epsilon^{\frac{kt}{2P}}dt - \epsilon^{-\frac{kt}{2P}}\int te\epsilon^{\frac{kt}{2P}}dt\right). \qquad (31)$$

The movable system is supposed to be at rest in its zero position when e is applied to the circuit, that is, when

$$t = 0 \qquad \theta = 0.$$

$$t = 0 \qquad \frac{d\theta}{dt} = 0.$$

These conditions will be fulfilled if

$$C_1 = \frac{C}{rP}\left(\int te\epsilon^{\frac{kt}{2P}}dt\right)_{t=0}. \qquad (32)$$

$$C_2 = -\frac{C}{rP}\left(\int e\epsilon^{\frac{kt}{2P}}dt\right)_{t=0}. \qquad (33)$$

So

$$\theta = \frac{C}{rP}\epsilon^{-\frac{kt}{2P}}\left(t\int_0^t e\epsilon^{\frac{kt}{2P}}dt - \int_0^t te\epsilon^{\frac{kt}{2P}}dt\right). \qquad (34)$$

If the integral change in e is instantaneous,

$$\theta = \frac{C}{rP}\left(\int_0^t edt\right)t\epsilon^{-\frac{kt}{2P}}. \qquad (35)$$

The first elongation occurs when

$$\frac{d\theta}{dt} = 0 = 1 - \frac{kt}{2P},$$

$$t_1 = \frac{2P}{k} = \sqrt{\frac{P}{\tau}} = \frac{T_0}{2\pi}.$$

Substituting this value of t_1 gives

$$\left(\int^t edt\right) = \frac{\sqrt{\tau P}r\epsilon\theta_1'}{C} = \frac{2\pi}{T_0}\cdot\frac{rP}{C}\epsilon\theta_1' = \left(\frac{T_0}{2\pi}\right)\left(\frac{I_G}{\theta}\right)r\epsilon\theta_1' = K\theta_1', \quad (36)$$

where θ is the deflection due to a steady current I_G.

It is seen that the elongation is proportional to the quantity of electricity in the discharge and that the galvanometer factor is ϵ times that of the undamped instrument. Consequently, the quantity sensitivity is $1/\epsilon$, or 37 per cent that of the undamped instrument. Also, the time necessary for arriving at the elongation θ_1 is $2/\pi$, or 64 per cent that of the undamped instrument.

Equation (36) applies when the ballistic galvanometer is used in a series circuit, the resistance r being such that the galvanometer is critically damped. When the instrumental constants used in Eq. (4) are introduced, (36) becomes

$$\left(\int_0^t edt\right) = \left(\frac{\epsilon C\sqrt{\tau P}}{2\sqrt{\tau P} - k'}\right)\theta_1'. \quad (37)$$

If the resistance of the apparatus to which the galvanometer is attached is so high that the instrument is underdamped, critical damping may be obtained by shunting the galvanometer. In this case, if R is the resistance of the apparatus to which the galvanometer is attached; and R_G, that of the galvanometer,

$$\left(\int_0^t edt\right) = \left(\frac{\epsilon RC\sqrt{\tau P}}{C^2 - 2R_G\sqrt{\tau P} + R_Gk'}\right)\theta_1'. \quad (38)$$

As no time is lost in bringing the movable system to rest, the critically damped ballistic galvanometer is frequently a most convenient instrument.

Effect of Damping Resistance on Sensitivity. Randall Curve.[8]— The quantity sensitivity of a ballistic galvanometer with only electromagnetic damping depends on the resistance of the circuit through which the damping current flows. For an instantaneous discharge, the quantity sensitivity of the undamped instrument is

$$S_0 = \frac{\theta_1'}{Q} = \left(\frac{2\pi}{T_0}\right)\left(\frac{\theta}{I_G}\right). \quad (39)$$

If k is the damping coefficient, in Eq. (4), when the entire resistance of the damping circuit is R; and k_c, its value when the damping is critical, the ratio k_c/k may be designated by n.

$$\frac{k_c}{k} = n = \frac{R}{R_c},$$

where R_c is the critical damping resistance. For critical damping,

$$k_c = 2\sqrt{\tau P}.$$

As the damping coefficient is increased from zero, the motion is in three stages: first oscillatory, then critically damped, and finally overdamped. Considering the first stage and referring to page 17,

$$a = \frac{k}{2P} \qquad b = \sqrt{\frac{\tau}{P} - a^2} \qquad B_0 = \frac{k}{k_c}.$$

Therefore

$$a = \frac{k_c}{2nP} = \frac{1}{n}\sqrt{\frac{\tau}{P}}$$

$$b = \frac{1}{n}\sqrt{\frac{\tau}{P}}\sqrt{n^2 - 1}$$

$$\frac{a}{b} = \frac{1}{\sqrt{n^2 - 1}}$$

also (page 100),

$$\frac{a}{b} = \frac{\lambda}{\pi}.$$

Therefore

$$n = \sqrt{\frac{\pi^2}{\lambda^2} + 1.}$$

Using (20), p. 101, the underdamped sensitivity is

$$S_{UD} = \left(\frac{2\pi}{T_0}\right)\left(\frac{\theta}{I_G}\right)\epsilon^{-\frac{a}{b}\tan^{-1}\frac{b}{a}}.$$

Therefore

$$\frac{S_{UD}}{S_0} = \epsilon^{-\frac{a}{b}\tan^{-1}\frac{b}{a}} = \epsilon^{-\frac{1}{\sqrt{n^2-1}}\tan^{-1}\sqrt{n^2-1}}.$$

If the damping is critical, using (36) (page 107),

$$\frac{S_c}{S_0} = \epsilon^{-1}.$$

If the instrument is overdamped, using the definitions of m_1 and m_2 from page 16,

$$m_1 = -a + \sqrt{a^2 - \frac{\tau}{P}} = -\frac{1}{n}\sqrt{\frac{\tau}{P}}(1 - \sqrt{1 - n^2}),$$

$$m_2 = -a - \sqrt{a^2 - \frac{\tau}{P}} = -\frac{1}{n}\sqrt{\frac{\tau}{P}}(1 + \sqrt{1 - n^2}),$$

and (11) (page 99), the overdamped sensitivity is

$$S_{OD} = \frac{c}{P(m_1 - m_2)}\epsilon^{m_1 t_1}(1 - \epsilon^{(m_2 - m_1)t_1}) = -\frac{c\epsilon^{m_1 t_1}}{Pm_2},$$

as by (11*a*)

$$\epsilon^{(m_2-m_1)t_1} = \frac{m_1}{m_2}.$$

Therefore

$$\frac{S_{OD}}{S_0} = -\sqrt{\frac{T}{P}}\left(\frac{\epsilon^{m_1t_1}}{m_2}\right) = \left(\frac{n}{1+\sqrt{1-n^2}}\right)\epsilon^{-\frac{1-\sqrt{1-n^2}}{2\sqrt{1-n^2}}\log_\epsilon\frac{1+\sqrt{1-n^2}}{1-\sqrt{1-n^2}}}$$

These expressions for relative ballistic sensitiveness are independent of the characteristics of any particular galvanometer except its critical damping resistance, and therefore a table of values of S_n/S_0 and n, or a single curve plotted with n and S_n/S_0 as coordinates, enables one to study the effect of the damping resistance in all cases. This is Randall's universal calibration curve.

TABLE IV.—RELATION BETWEEN $R/R_c = n$ AND RELATIVE BALLISTIC SENSITIVITY S_n/S_0

$R/R_c = n$	S_n/S_0	$R/R_c = n$	S_n/S_0
0.100	0.0494	2.000	0.5463
0.200	0.0964	2.500	0.6029
0.300	0.1403	3.000	0.6471
0.400	0.1810	3.500	0.6825
0.500	0.2186	4.000	0.7115
0.600	0.2533	5.000	0.7561
0.700	0.2853	6.000	0.7888
0.800	0.3150	8.000	0.8335
0.900	0.3424	10.000	0.8626
1.000	0.3679	20.000	0.9267
1.500	0.4713	50.000	0.9695

These values are plotted in Fig. 49.

Figure 50 is a plot of the values of S_0/S_{β_0} and $R_c/R_{\beta_0} = \beta_0$, the reciprocals of the values given in Table IV, when the motion is oscillatory. It will be seen that for this range of values, the maximum deviation from a straight line through $S_0/S_{\beta_0} = 1$ and $S_0/S_{\beta_0} = \epsilon = 2.718$ is about 1.7 per cent, so that it is easy to form a fair estimate of the effects of changes of damping resistance on the sensitivity.[9] To within a maximum divergence of 0.7 per cent

$$\frac{S_0}{S_{\beta_0}} = 1 + 1.602\beta_0 + 0.116\beta^2.$$

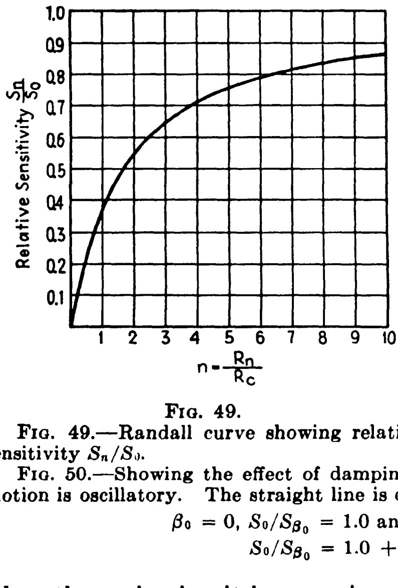
$$\beta_0 = \frac{R_c}{R_{\beta_0}} = -6.90 + \sqrt{47.60 + 8.61\left(\frac{S_0}{S_{\beta_0}} - 1\right)}.$$

The critical damping resistance R_c may be found from the undamped ballistic sensitivity S_0 and the sensitivity S_{β_0} when a damping resistance R_{β_0} is used.

Damping coefficients having effects proportional to the angular velocity of the moving parts are additive, so if the instrument is provided with an auxiliary damping loop, the total coefficient is the sum of the damping coefficient due to the loop and that due to the circulation of the damping current through the windings of the instrument; that is,

$$k = \frac{c^2}{R} + k_1 = \frac{c^2}{nR_c} + k_1.$$

To find k_1 in terms of c and R_c, the logarithmic decrement λ_1 is observed



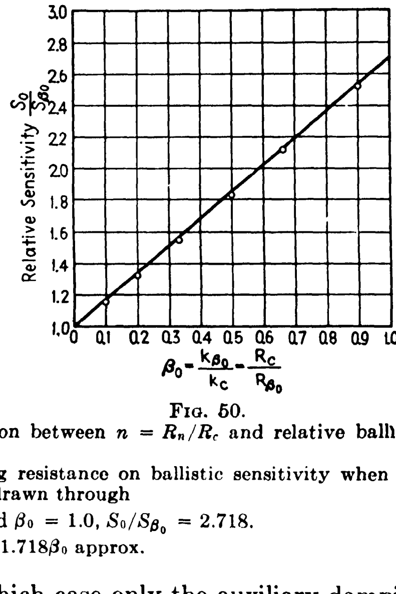


FIG. 49.　　　　FIG. 50.

FIG. 49.—Randall curve showing relation between $n = R_n/R_c$ and relative ballistic sensitivity S_n/S_0.

FIG. 50.—Showing the effect of damping resistance on ballistic sensitivity when the motion is oscillatory. The straight line is drawn through

$\beta_0 = 0$, $S_0/S_{\beta_0} = 1.0$ and $\beta_0 = 1.0$, $S_0/S_{\beta_0} = 2.718$.

$S_0/S_{\beta_0} = 1.0 + 1.718\beta_0$ approx.

when the main circuit is open, in which case only the auxiliary damping is present. Then

$$n_1 = \sqrt{\frac{\pi^2}{\lambda_1^2} + 1} = \frac{R_1}{R_c}.$$

n_1 is the resistance ratio which it would be necessary to employ if the same damping effect were to be obtained by inserting resistance in the main circuit, the damping loop being removed. Therefore

$$k = \frac{c^2}{R_c n} + \frac{c^2}{R_c n_1} = \frac{c^2}{R_c}\left(\frac{n_1 + n}{n n_1}\right).$$

The effective value of n is $N = \dfrac{n n_1}{n_1 + n}$.

To illustrate, a ballistic galvanometer is to be used in a circuit having a resistance of 450 ohms; the undamped sensitivity is 120 mm. per microcoulomb; the critical damping resistance is 300 ohms.

In this case, $n = 450/300 = 1.5$.

Referring to the table, or to the curve, it is seen that the sensitivity of the instrument when used in the 450-ohm circuit will be 0.47 times that of the undamped instrument, or $120 \times 0.47 = 56.5$ mm. per microcoulomb.

If the same instrument is provided with auxiliary damping, and, on open circuit the ratio of two successive elongations is 1.7, then $\lambda_1 = 0.531$, and

$$n_1 = \sqrt{\left(\frac{\pi}{0.531}\right)^2 + 1} = 6;$$

$$N = \frac{1.5 \times 6}{1.5 + 6} = 1.2.$$

From the curve,

$$\frac{S_n}{S_0} = 0.414.$$

Therefore the sensitivity is now $120 \times 0.414 = 49.7$ mm. per microcoulomb.

The Use of Shunts with the Ballistic Galvanometer.—At first sight, it would seem that if a ballistic galvanometer were shunted with a non-inductive resistance, the total quantity of electricity discharged from a condenser would not, on account of the inductance of the galvanometer, divide inversely as the resistance of the galvanometer and of the shunt. However, the quantity does so divide, as will be seen from the following:

Let R_G = galvanometer resistance.
L_G = galvanometer inductance.
i_G = galvanometer current.
Q_G = quantity discharged through galvanometer.
S = shunt resistance.
L_S = shunt inductance.
i_S = shunt current.
Q_S = quantity discharged through shunt.

Then

$$R_G i_G + L_G \frac{di_G}{dt} = S i_S + L_S \frac{di_S}{dt}.$$

Consequently

$$R_G Q_G + L_G \int di_G = S Q_S + L_S \int di_S.$$

Both currents are zero at the beginning and zero at the end of the discharge, so

$$\frac{Q_G}{Q_S} = \frac{S}{R_G}.$$

It is seen that any error caused by the shunt must be due to the variation of the damping when the multiplying power is changed.

A practical difficulty met with when applying shunts of the ordinary sort to a moving-coil ballistic galvanometer is that the range of the instrument cannot be greatly extended before the damping becomes excessive.

In open-circuit work, that is, in measurements upon condensers or cables, the universal shunt (page 43) should be used. The advantage of this arrangement is that the resistance through which the damping current, set up by the motion of the coil, must flow is always the same. Consequently, λ does not vary, even though the multiplying power of the shunt is changed. Another advantage is that the total shunt resistance r may be made so great that the instrument is not overdamped, even though it is heavily shunted.

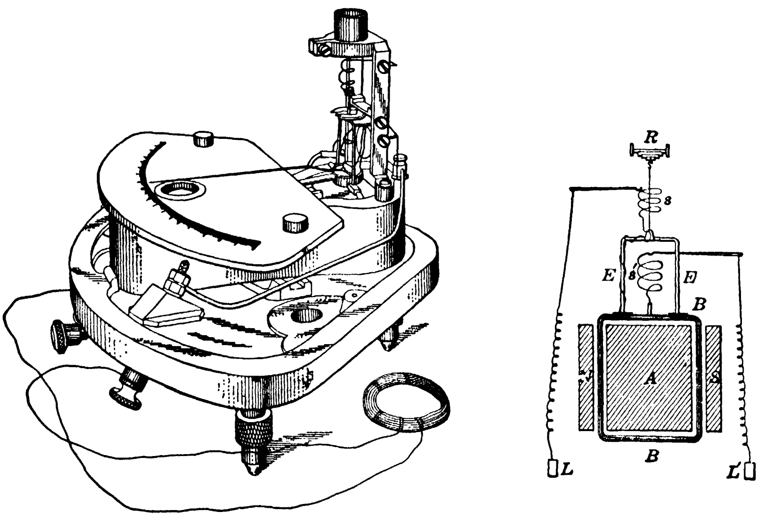


Fig. 51.—Fluxmeter.

Obviously, this form of shunt loses its advantages, if the condenser be replaced by a closed circuit, as an exploring coil for magnetic work, and recourse must be had to combining series and parallel arrangements.[10]

The Fluxmeter.—The fluxmeter,[6] as its name indicates, is designed for measuring the flux in magnetic circuits. This instrument is essentially a moving-coil ballistic galvanometer in which the restoring moment is reduced to a minimum by the removal of the controlling spring. The fluxmeter instrument is used in circuits of low resistance, so that it is greatly overdamped. This is essential to the action of the instrument. In a perfect instrument, the restoring couple would be zero. Practically, it is difficult to reduce the controlling action of the necessary leads to the movable coil to a negligible amount, and usually after the movable system has been deflected it very gradually sinks back toward zero.

Figure 51 shows one form of the instrument and its suspended system. The moving coil is hung by a silk fiber from the spring support R; the current is led to the coil through the delicate spirals s, s', which are of annealed silver and supposed to exercise no controlling effect on the movable system. A mechanical or electrical device is employed by which the system may be quickly reset to its zero position.

The instrument is used in series with an exploring coil and therefore in a closed circuit. When the change of flux through the test coil is completed, the movable system comes to rest in its deflected position, and the change in linkages is given by

$$n = C\theta_1, \tag{40}$$

which is true, irrespective of the manner in which the flux is changed and of the time occupied in making the change. This connection between the deflection and the change of flux linkages may be seen from Eq. (12) (page 100).

When the torsional control τ is indefinitely reduced, m_1 and m_2, which are the roots of the equation $Pm^2 + km + \tau = 0$, approach the values

$$m_1 = 0,$$
$$m_2 = -\frac{k}{P}.$$

Under these conditions (see page 99),

$M = 1,$

$N =$ some definite numerical value,

and the factor $N^{\frac{m_1}{m_1-m_2}} M^{\frac{m_2}{m_2-m_1}}$ in (12), which depends on the manner in which the discharge takes place, always becomes unity. The factor in (12) which is within the braces also approaches unity, so if n is the total change of flux linkages,

$$\theta_1 = \frac{Cn}{rk}.$$

As

$$k = \frac{C^2}{r} + k' \qquad \text{(see page 98)},$$
$$n = \left(C + \frac{k'r}{C}\right)\theta_1.$$

The air damping is very small, so $n = C\theta_1$, approximately. This relation may be proved directly as follows:

Let n = number of linkages of flux with exploring coil.
θ_1 = final value of deflection.
C = coil constant.
ω = angular velocity of movable coil.
(dn/dt) = rate of change of linkages through exploring coil.
L = inductance of exploring-coil circuit.
P = moment of inertia of movable system.
k' = damping constant for air damping.
r = resistance of exploring-coil circuit.
i = current at any instant.
N' = number of turns in exploring coil.
φ = flux through exploring coil.

As soon as the coil begins to move, it sets up a back e.m.f. whose value is ωC.

The current at any instant is

$$i = \frac{\frac{dn}{dt} - C\omega - L\frac{di}{dt}}{r},$$

and

$$P\frac{d\omega}{dt} = Ci - k'\omega = \frac{C}{r}\left(\frac{dn}{dt}\right) - \frac{C^2\omega}{r} - \frac{CL}{r}\frac{di}{dt} - k'\omega.$$

At the beginning and at the end of the deflection, both ω and i are zero, so, on integrating,

$$0 = \frac{Cn}{r} - \left(\frac{C^2}{r} + k'\right)\int \omega dt.$$

Therefore

$$n = \left(C + \frac{k'r}{C}\right)\theta_1.$$

The air damping is small. Very closely, then, the change of flux linkages is given by

$$n = C\theta_1,$$

and

$$\varphi = \frac{C\theta_1}{N'}.$$

That is, the deflection is proportional to the change in the flux threading the exploring coil. In the actual instrument, the scale is calibrated experimentally by use of a mutual inductor. If only electromagnetic damping were present, the sensitivity would be independent of the circuit resistance; it follows, as the air damping is small, that the sensitivity is not greatly changed by alteration of the circuit resistance. A tenfold change in the resistance may make a 25 per cent change in the constant. The advantages of the fluxmeter as just described over the ballistic galvanometer are portability, ease of reading, and independence of the time required for the flux changes, *provided they do not take place too slowly*, and of the manner in which they take place. It is therefore a convenient workshop instrument. As ordinarily constructed, its accuracy is inferior to that of the ballistic galvanometer.[7]

Compensated Fluxmeter.[11]—The defects of the ordinary instrument are due to the small but unavoidable restoring moment arising from the lead-in wires or from the suspension, if an ordinary moving-coil galvanometer be used as a fluxmeter. The suspension causes the coil to drift back very slowly toward its new position, so that independence of the time required for the flux change is more theoretical than real. This defect may be annulled by the arrangement indicated in Fig. 52. A long, slim photoelectric cell is connected across the terminals of the instrument. Light reflected from a mirror attached to the movable system is focused on the cell, the amount of light received being determined by the V-shaped slit B, which may be empirically shaped to allow for nonuniformity in the sensitivity of the tube. The result is that at all usable deflections the restoring moment of the leads is balanced. The circuit associated with B_2 allows the effect of stray light e.m.fs., and thermal e.m.fs. to be compensated.

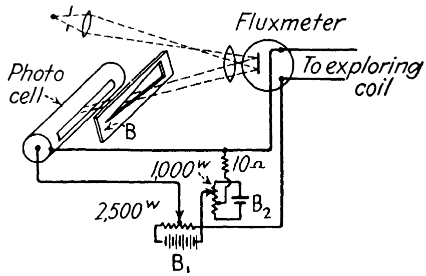


FIG. 52.—Compensation circuit for fluxmeter.

With the compensated fluxmeter, it is not necessary that the resistance of the exploring coil be kept low. A galvanometer having a resistance of 800 ohms, requiring an external resistance of 100,000 ohms for critical damping, gives satisfactory results with a 100-ohm exploring coil. This permits the determination of small fluxes by use of an exploring coil of many turns. The deflection may be recorded photographically on a slowly rotating drum.

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CHAPTER III

RESISTANCE DEVICES

The resistance devices used in electrical measurements may be divided into two groups: resistance boxes and rheostats.

A resistance box is a device by which the resistance of a circuit may be altered by accurately known amounts. It contains an aggregation of coils, each coil having a definite and known resistance, and the construction is such that the coils may be connected in various combinations, so that any required resistance, up to the full capacity of the box, may be obtained.

The term "rheostat" is applied to the devices commonly used for varying the resistance of the circuit where regulation of currents and absorption of power are concerned. The circumstances under which rheostats are used render it unnecessary that the magnitudes of the variations in resistance be known.

Resistance Coils.—Formerly it was the universal practice to wind resistance coils on wooden bobbins; but in the better class of work, these bobbins have been replaced by metal spools (see Fig. 53). A layer of shellacked silk, which is dried out by baking before the coil is wound, serves to insulate thoroughly the wire from the metal spool.

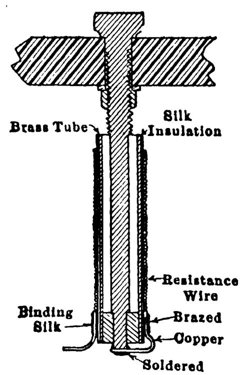


FIG. 53.—Section of resistance coil.

Noninductive windings are always employed. The wire is arranged in a bight before it is wound upon the bobbin, and the two wires are kept side by side in the coil.

If possible, the winding is concentrated in a single layer, for, as all the heat must be dissipated through the surfaces of the coil, one wound several layers deep with a large wire is not superior to one wound with but a single layer of small wire.

After it is wound, the coil is shellacked and then baked for 2 weeks at 120°C.; this frees the entire coil of moisture and alcohol and at the same time anneals the wire. To avoid possible chemical action, the paraffin coating formerly used is now omitted. Some makers coat the coil with ceresin. The coil should be allowed to age several months before it is used, for the resistance rises appreciably after the baking process.

The resistance wires are hard soldered to copper terminals, which, in turn, are soft soldered to the working terminals.

The prime requisite of the resistance material used for winding coils is permanence. In addition, its temperature coefficient should be small; its thermal e.m.f. low when opposed to copper; and to obtain compactness, its resistivity should be high. Alloys rather than the pure metals are used, for they have smaller temperature coefficients and higher resistivities. To settle the all-important question of permanence, prolonged investigation is of course necessary. Up to the present time, the alloy that has most commended itself is that known as manganin.[2] Other alloys are used for certain kinds of work, but manganin has been under critical examination longer than the others, and its properties are more definitely known.

Edward Weston discovered in 1889 that alloys of copper and nickel containing some manganese have very small temperature coefficients and high resistivities. Investigation has shown that the particular alloy known as manganin is, when properly employed, sufficiently permanent for resistance coils and resistance standards.

The composition of manganin is given as 84 per cent copper, 12 per cent manganese, and 4 per cent nickel. Its resistivity at 20°C. is about 44.5 microhms (cm.), and its thermo-e.m.f. when opposed to copper is only 0.000002 volt per degree centigrade. Apparently quite large variations in the composition have little effect on the stability of the alloy.

To insure permanence, this material must be protected by a well-dried coating of shellac.

The curious effect of a rise of temperature on a manganin resistance coil is shown in Fig. 54. The point at which the temperature coefficient changes sign varies with different samples of wire.

It will be noted that the temperature coefficient is very small, the average value between 15 and 20°C. being only 0.0005 per cent. For engineering work, consequently, temperature corrections may be neglected. In work of the highest precision (a few parts in 100,000), temperature corrections must be made, in which case the coefficient applying to the particular coil in hand must be employed.

The possible effects of a rise of temperature on a resistance coil are:

1. If the rise of temperature is small, there will be a temporary change in the resistance, that is, one that disappears when the normal temperature is regained.

2. If the rise of temperature is great, there will be a permanent alteration of the resistance.

3. At a still higher temperature, the insulation will be impaired.

It is evident that the ability of a resistance coil to dissipate the heat due to the passage of the current is most important, and as the precisions that may be obtained with various methods of measurement are proportional to the currents employed, it is advisable, when selecting a resistance box for general use, to choose one having coils of a high watt capacity.

In any case, the safe working current is that which will heat the coil to a temperature just below that at which a permanent alteration, or "set," in the resistance will take place. A coil adjusted to 0.01 per cent, which is expected to maintain this degree of reliability, must be more carefully treated than one certified to 0.05 per cent, which is a degree of adjustment frequently used in the better class of bridges and resistance boxes for general use.

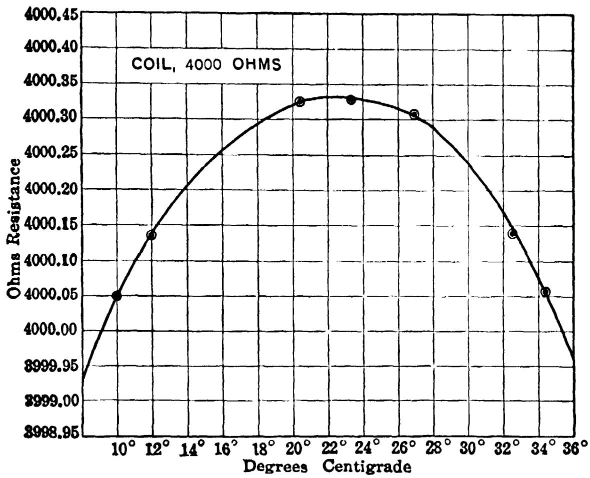


Fig. 54.—Showing effect of temperature on a manganin resistance coil.

No definite statement can be made as to the safe carrying, or watt capacity, of the resistance coils used in boxes for general laboratory work, for it depends on the construction of the coils and of the box in which they are mounted and on the accuracy of the initial adjustment. A temperature rise of approximately 15°C. may be conservatively rated as introducing an error of 0.05 per cent. The power necessary to produce this rise depends on the design of the coil and may be, if a metal bobbin is used, from 0.5 to 1.0 watt. If the bobbin is of poor thermal conductivity, the value is reduced to about one-third or one-half of this. The instrument maker should furnish the purchaser a statement as to the watt capacity of his coils.

Standard Resistances.[1]—Standard resistances are used for two purposes:

1. As standards of reference with which other resistances are compared.

2. As current-carrying standards for use in potentiometer methods.

With the first class of coils, permanence is all-important. The watt capacity is not so important, provided it is great enough to allow comparisons to be made with the desired precision.

Experience has shown that lack of permanence in the finished coil may be due to corrosion, to stresses in the coil caused by sharp bends or by winding the wire on a small spool, to stresses due to the absorption of moisture by the insulating materials, and to the use of soft solder at the terminals which may crack and alter the effective length of the wire. These factors are now generally recognized, and the coils prepared accordingly.

For coils of the second class, a high carrying capacity is absolutely necessary, and one can tolerate a lesser degree of permanence provided comparisons are made from time to time with carefully preserved resistance standards; as a matter of convenience, permanence is highly desirable.

The designs commonly employed for resistance standards are those developed at the Reichsanstalt, Charlottenburg, and at the Bureau of Standards, Washington, D. C. Figure 55 shows a group of these coils.

For standards having a resistance of $\frac{1}{10}$ ohm and greater, the resistance material is used in the form of wire; below $\frac{1}{10}$ ohm, strips are employed.

The coils are kept cool by immersion in oil. Great care must be taken that there are no acids or sulphur in the oil and that it is kept free from water. Otherwise, the resistance material may be attacked, and the accuracy of the standards impaired. The oil should be renewed from time to time.

As a result of careful experiments at the Bureau of Standards, it has been found that coils constructed like that shown at *A* in Fig. 55 are subject to slight variations due to the absorption of moisture by the shellac used in insulating the windings. This swells and stresses the wire. The effect is most noticeable when fine wires are used, that is, in high-resistance coils. It follows the seasonal variations of atmospheric humidity and may be as great as 0.04 per cent in a 1,000-ohm coil.

Immersion of the coil in oil which is freely exposed to the air, though retarding the effect, is not an absolute preventive, for the oil absorbs moisture and imparts it to the shellac. To overcome this source of error, Rosa developed the form of sealed resistance standard[3] shown at *B* (Fig. 55).

The coil itself is prepared as specified by the Reichsanstalt, being insulated with silk and shellac and thoroughly dried by baking; the bobbin is supported from the cover by the thermometer tube. The case is nearly filled with a pure oil which has been carefully freed from moisture. The cover is then screwed into the protective case, and the joint sealed with shellac. Potential terminals are used with coils of 1 ohm or less.

More recently, 1-ohm standards have been constructed at the Bureau,[4] in which bare manganin wire, which has been annealed at 550°C. after winding on a form, is employed. Stresses are thus effectually removed. The coil is sealed in a dry, double-walled container, as shown in Fig. 55*C*. The indications are that this construction results in a remarkably permanent coil. Sealed standards show no seasonal variations in resistance.

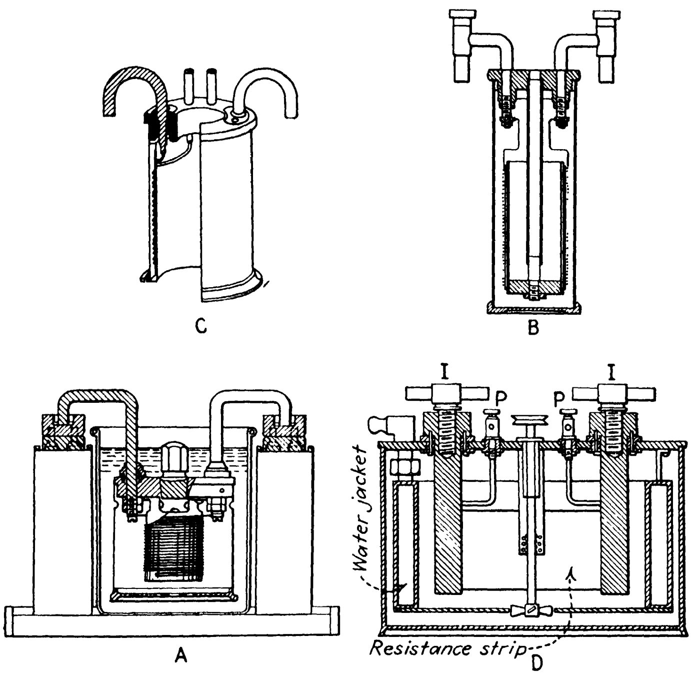


Fig. 55.—Standard resistors.

The same result may be obtained by hermetically sealing a perfectly dried coil in a metal case, all moisture having been removed from the internal surfaces of the case, from the wire, and from the inclosed air. Difficulties from any slight accidental leakage of moist air from the outside may be prevented by the use of a drying agent within the case.

The Bureau of Standards' unit of resistance is the mean obtained from a group of 10 Rosa coils. It has probably not changed, on the average, more than 1 part in 1,000,000 per year since 1910.

Figure 55*D* shows a current-carrying standard. Such resistances are immersed in oil, which is stirred and kept cool by a water jacket through which there is a brisk circulation. It is essential that the strips of resistance material be protected by a coating of shellac and that the oil be kept dry.

Standard low resistances are always designed so that they are four-terminal conductors[5] (see Fig. 55*D*). The current enters and leaves by the heavy "current terminals" *I* and *I*. The value of the resistance is the potential difference between the "potential terminals" *P* and *P* divided by the current. By using this construction errors due to contact resistances at the terminals are avoided.

Resistance Coils for Alternating-current Work.—Though the bifilar winding usually employed in resistance coils reduces the inductance to a minimum, it increases the capacitance effects, since at the terminals of the coil the two wires are separated by only the double thickness of insulation and have full voltage between them. It is apparent that this form of coil is not suitable, in general, for alternating-current work.

With alternating currents, especially at high frequencies, the behavior of a resistance coil will be modified by:

1. The inductance of the winding.
2. The distributed capacitance within the coil.
3. The capacitance to surrounding objects.
4. The behavior of the insulation, which is an imperfect dielectric.
5. The skin effect in the wire.

As the coils are wound with very fine wire of a high-resistivity material, the skin effect is not troublesome unless the frequency is very high. The skin effect in a manganin wire 2 mm. in diameter is about 1 part in 100,000 at 3,000 cycles per second. To reduce the inductance, it is necessary that the net magnetic flux through the coil be brought practically to zero. To do this, there must be, in effect, equal numbers of practically coincident right-handed and left-handed ampere-turns.

To reduce the capacitance effects, the voltage between neighboring wires must be kept as small as practicable, the wires between which there is, of necessity, a considerable voltage being widely separated. This implies that a coil of high resistance should be sectionalized, that is, composed of a number of smaller coils in series.

The over-all reactance of the coil is affected by the dielectric coefficient of the insulation, which should be as near unity as practicable, and by the power losses in the insulation, which should be small. The use of an unimpregnated insulation is indicated, shellac and waxes being excluded. If this is done, the access of moisture must be prevented, for instance by covering the coil with thin sheet mica and then dipping it in melted asphalt compound.[6] The differences observed in performances of apparently identical coils are probably due to the dielectric behavior of the shellac or waxes used to prevent the access of moisture. The use of coils with moisture-proof cover but unimpregnated insulations is not common.

Bifilar-wound coils of low resistance are slightly inductive. However, when the resistance is above 10 ohms, the capacitance begins to neutralize the inductance; and at about 100 ohms, the neutralization is almost complete. Above about 100 ohms, the capacitance effect predominates.

When, in the endeavor to produce a nonreactive coil, both the inductance and the capacitance have been reduced as much as practicable, there still remains a small residual, so that the finished coil will show a very small over-all inductance (measured in microhenrys) or a very small over-all capacitance (measured in micromicrofarads).

The general effect of the small residuals is indicated by assuming as a first approximation that the coil has resistance and a very small inductance and that this combination is shunted by a small capacitance. The impedance operator for such a combination is

$$z = \frac{\left(-\frac{j}{\omega C}\right)(R + j\omega L)}{\left(-\frac{j}{\omega C}\right) + (R + j\omega L)}. \tag{1}$$

Separating the two quadrature components, and noting that both the inductance and the capacitance are small, gives, for the resistance component,

$$R' = \{1 + \omega^2 C(2L - R^2 C)\}R \qquad \text{approx.} \tag{2}$$

and for the inductance component,

$$L' = L - CR^2 \qquad \text{approx.} \tag{3}$$

It is seen that the equivalent inductance may be either positive or negative, as the inductance or the capacitance effect predominates, and that the effective resistance depends on the frequency, the capacitance, and the inductance. Also, if the inductance and capacitance effects are small, they tend to neutralize each other.

It is customary to compare coils on the basis of their time constants t, where $t_L = L/R$ and $t_C = RC$. For a first-class coil, this quantity is of the order of 10^{-8} sec. The phase displacement is given by $\tan \theta = \omega t$. For the coil as given,

$$t = \frac{L'}{R} = \frac{L}{R} - RC. \tag{4}$$

If a coil shows an over-all capacitance, it may be considered as having a negative inductance which gives the same numerical value of t as that observed.

To reduce the net effect of the inductance and capacitance, both of which are made very small individually, a number of different windings have been devised.[7]

In their high-grade coils, Tinsley and Company wind the bulk of the wire so that the resulting coil has a *very slight* residual capacitance. The remainder of the wire is so wound, placed, and mounted that it has a slight inductive effect which can be varied by a screw adjustment until the entire coil is nonreactive. This construction allows the final adjustment to be made after the coil is mounted in its case. If the residual capacitance is very small, the resistance is practically independent of frequency.

The Chaperon winding, the properties of which have been carefully investigated,[7] originally consisted of an even number of layers wound on a tubular metal bobbin. A single layer is wound in the clockwise direction; at the end of the layer the winding is reversed, the second layer being of the same number of turns and wound directly over the first in the counterclockwise direction. This brings the wire back to the starting point, after which the cycle is repeated the required number of times. Each cycle is "noninductive" in the usual sense, and the voltage between contiguous wires is that due to the drop in two layers rather than that due to the drop in the whole coil. Thus capacitance effects are reduced much below those in a bifilar-wound coil. A number of short sections may be wound on a single tube and connected in series, capacitance effects between the sections being reduced by employing an insulating bobbin.

Assuming as an approximation that the formulæ for two parallel wires apply to a resistance coil, Curtis and Grover have deduced the following relations:[7]

Effective resistance, $R' = R[1 + \omega^2 C(\tfrac{1}{3}L - \tfrac{2}{15}CR^2)]$. (5)

Effective inductance, $L' = L - \tfrac{1}{3}CR^2$.

Phase displacement, $\tan^{-1} \dfrac{\omega(L - \tfrac{1}{3}CR^2)}{R}$. (6)

R and L are the total resistance and inductance of the coil, and C is the capacitance between the wires if they are separated at the end of the bight.

It is seen that capacitance and inductance tend to neutralize each other. In coils of low resistance with only one layer of bifilar winding, the inductance will preponderate; but when the resistance of the coil is increased, the effect of capacitance increases more rapidly than that of inductance; consequently, there will be a point where the two effects will be balanced. The balance point is practically independent of the frequency.

From the formulæ, it is evident that the phase displacement and the change of resistance cannot *both* be made zero; but if $L = \tfrac{1}{3}CR^2$, the phase displacement is zero, and the change in resistance negligible.

After having found a construction that gives a practically nonreactive coil, higher resistances may be built up by simply placing coils in series.

The several sections of a high-resistance coil may be wound side by side on the same spool which should be of porcelain or glass. The use of a nonconducting spool avoids troublesome capacitance effects between the sections.

Curtis and Grover[7] recommend the following constructions:

The coils are designed for oil immersion and have approximately 50 sq. cm. surface. With an expenditure of 1 watt per coil, the rise of temperature above the surrounding oil is about 1°, the bobbin being of poor thermal conductivity.

Tenth-ohm Coils.—Only the inductance need be considered. The coil is to be made of manganin strip $\frac{1}{10}$ mm. thick, width about 3 mm., length about 10 cm. The strip is to be folded back on itself at the middle of its length, and the two halves bound together with insulation between them. The effective inductance is about $+0.005$ μh.

One-ohm Coils.—Only the inductance need be considered. The coil is to be of manganin strip $\frac{1}{10}$ mm. thick and 3 mm. wide, folded back on itself at the middle of its length and with proper insulation between the halves. The effective inductance is about $+0.05$ μh.

Ten-ohm Coils.—Only the inductance need be considered. Three 30-ohm coils are used in parallel. They are bifilar wound on spools 2.5 cm. in diameter, a single layer of double-silk-covered manganin wire 0.24 mm. in diameter being employed. The effective inductance is approximately 0.3 μh.

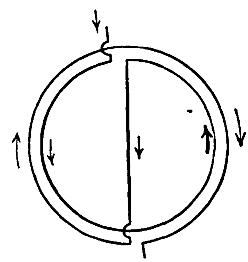


Fig. 56.—Special non-reactive winding for high-resistance coils.

One-hundred-ohm Coils.—The coils are bifilar wound in one section, a single layer of double-silk-covered manganin wire 0.24 mm. in diameter being used. The capacitance effect preponderates, resulting in an effective inductance of -1.6 μh; the phase angle at 2,000 cycles per second is about 41 sec.

One-thousand-ohm Coils.—Five sections, each of 200 ohms, are used in series. Each section consists of a single layer of double-silk-covered manganin wire 0.10 mm. in diameter. The five sections are bifilar wound on a spool 2.5 cm. in diameter, a space of 2 or 3 mm. being left between the sections. The effective inductance is about -16 μh.

Five-thousand-ohm Coils.—Five-thousand-ohm coils may be built up of five of the preceding 1,000-ohm coils in series, or the bifilar winding may be replaced by that illustrated in Fig. 56.

The coil is wound on a porcelain cylinder which is slit along a diameter for about two-thirds of its length. Double-silk-covered manganin wire is used. The wire goes once around the bobbin, then passes through the slit and around the bobbin in the opposite direction back through the slit. This cycle is repeated until the whole coil is wound. The capacitance effect is very small, as there is only a small potential difference between adjacent wires. The effective inductance is about +30 μh. The disadvantages are the difficulty of winding and the introduction of many sharp bends in the wire.

Ten-thousand-ohm Coils.—These are constructed by using in series two of the 5,000-ohm units just described. The effective inductance is about +100 μh.

Shackelton[6] has constructed 1,000-ohm ratio coils for use in an impedance bridge by winding the wire in 10 sections on a glass rod ¾ in. in diameter. Each section of 100 ohms consists of two layers of No. 38 double-silk-covered manganin wire wound in opposite directions and *without shellac or waxes.* The 10 sections are connected in series, and the entire coil protected from moisture. The coil thus constructed has a small, positive inductance. It is protected by an electrostatic shield which increases the capacitance effect, as will be seen from the following table.

TABLE V.—EFFECTIVE INDUCTANCE OF 1,000-OHM SHACKELTON COILS IN MICROHENRYS

Frequency, cycles per second	Before coating with asphalt compound		After coating with asphalt compound		Assembled in shields	
	Coil *A*	Coil *B*	Coil *A*	Coil *B*	Coil *A*	Coil *B*
1,000	7.4	6.9	6.7	6.3	−1.3	−1.0
50,000	7.4	6.9	6.7	6.2	−1.2	−0.8

Ayrton and Mather (1892) introduced a form of winding frequently employed in resistance boxes and bridges for work at frequencies[8] up to about 3,000 cycles per second. A winding of double the required resistance is wound clockwise on an insulating card; a second winding of equal resistance is wound over the first in the counterclockwise direction, and the two connected in parallel.

Obviously, there are as many clockwise as counterclockwise nearly coincident ampere-turns, so the inductance is reduced to a minimum; also, the terminals are widely separated, and the potential difference between adjacent wires so reduced that the capacitance effects are very small. As the wire is sharply bent at the edges of the card, such resistors must be carefully aged to insure permanence.

Micanite cards wound with resistance wire, such as are used for series resistances in the potential circuits of alternating-current instruments, are highly satisfactory for general laboratory purposes, as they have considerable surface. Capacitance effects should be minimized by mounting the cards so that they are at least a centimeter apart. The time constant of one of these cards is 10^{-6} to 10^{-7} sec.

As long ago as 1901, Duddell and Mather introduced resistors woven as indicated at *a* (Fig. 57). It is seen that although the conductors at the terminals are widely separated, the "go" and "return" wires are on opposite sides of the silk warp and consequently are not so nearly coincident as is desirable to insure the minimum inductance. Improvements in the weave have been introduced by the Leeds and Northrup Company, as indicated at *b* and *c* (Fig. 57). The results obtained are shown in Table VI.

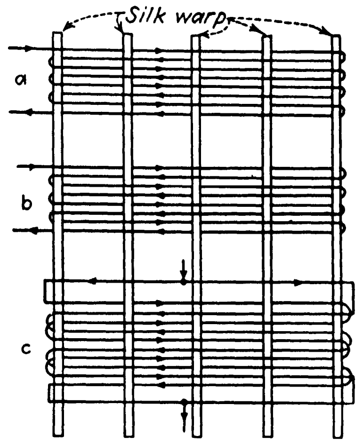


FIG. 57.—Types of woven resistors; *b* and *c* by Leeds and Northrup Company.

Shielded Resistors.[9]—The ideal resistor for alternating-current work would have zero over-all reactances. Practically, this result is not attainable, though so-called "noninductive" windings are employed. There will be a small residual inductance, and there is distributed capacitance within the coil and capacitance to ground and neighboring objects.

Capacitance effects are very troublesome when audio- or high frequencies are employed. With a simple resistance coil, the impedances of

TABLE VI.—CHARACTERISTICS OF INDIVIDUAL WOVEN RESISTANCE COILS

Resistance, ohms	Wire size, Brown and Sharpe gage	Weave	Time constant
20,000	44	*b*	-1.4×10^{-8}
10,000	44	*b*	−0.8
1,000	44	*b*	+0.11
1,000	41	*b*	+0.20
1,000	41	*a*	+0.80
1,000	38	*b*	+0.37
100	41	*b*	+0.36
100	38	*b*	+0.52
100	36	*b*	+0.92
100	39	*c*	+0.33
100	38	*c*	+0.43

the capacitance paths to ground and near-by objects may become so small that an appreciable portion of the current is shunted from the coil so that the entering current at one terminal is not equal to the exit current at the other terminal. Again, if the frequency is low, and the coil forms part of a high-resistance circuit and is at a high potential with respect to its surroundings, an appreciable current will be shunted from the coil in spite of the high impedances of the capacitance paths. Any change of environment alters these disturbing impedances and so changes the effective impedance of the coil. It is evident that the effect of the

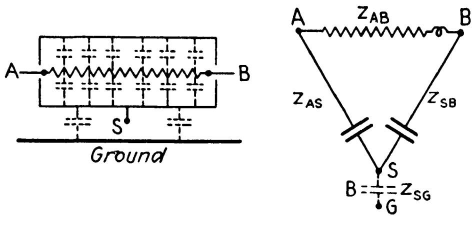


Fig. 58.—Diagrams for shielded resistor.

environment must be made definite. To this end, resistance coils for alternating-current work are surrounded by thin metallic shields which are kept at definite potentials with respect to the coils that they protect by conductively connecting them to predetermined points in the circuit so chosen that the effects of shield to ground capacitances are eliminated.

Figure 58 indicates the construction. The condensers shown dotted

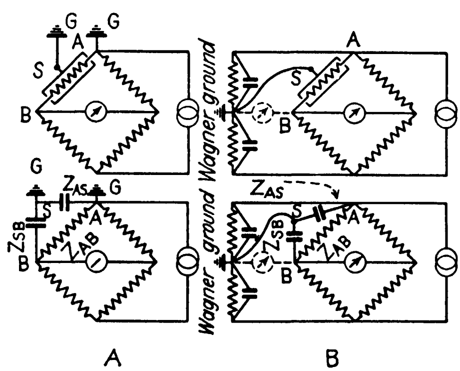


Fig. 59.—Pertaining to shielded resistors.

are intended to indicate capacitance between the coil and the shield and shield and ground.

Evidently the shielded coil is a network with three accessible terminals. It can be represented by the equivalent network shown at Fig. 58*B*, where z_{AB}, z_{AS}, and z_{SB} are the direct impedances between *A* and *B*, *A* and *S*, and *S* and *B*. To illustrate, in an impedance bridge it is usual either to ground one terminal of the source or to ground indirectly one terminal of the detector by using the Wagner earth connection. The two cases are indicated in Fig. 59(A) and (B), where for simplicity only one shielded bridge arm is shown.

If one terminal of the source is grounded, and the shield is also grounded, as in Fig. 59(A), no current will flow through z_{AS}, and the effective impedance of the arm AB will be the parallel impedance of z_{AB} and z_{SB}, or

$$z'_{AB} = \frac{z_{AB}z_{SB}}{z_{AB} + z_{SB}}. \tag{7}$$

If the Wagner ground is used, as indicated in Fig. 59(B), no current will flow through z_{SB}, for S and B are both at ground potential. The impedance z_{AS} shunts one side of the Wagner ground. A current in z_{AS} does not influence the currents in the main bridge, so in this case

$$z'_{AB} = z_{AB} \tag{8}$$

Shielded High Resistances.[10]—When testing potential transformers intended for high voltages, it is necessary to employ very large resistances for the impedance z_1 of Fig. 403. For example, if the voltage is 25,000 to 30,000, and the working current about 0.05 amp., from 500,000 to 600,000 ohms is required. As such a resistor must be composed of many units and must dissipate something like 1¼ kw., it must occupy considerable space. The possibility of large capacitance effects between the units and to ground is apparent.

Suppose the resistor to be unshielded. One end of the resistance wire is connected to ground through a comparatively small impedance. Therefore, beginning at the low-voltage end, the differences in potential between successive portions of the resistance wire and ground become greater and greater until the full primary voltage is reached. Each element of the resistance wire has capacitance to ground and to neighboring objects. It is evident that the current entering at the high-voltage end finds its way to earth both through the wire and through the capacitances to ground and that the exit current at the grounded end of the resistor will differ in magnitude and phase from the entrance current at the high-voltage end.

The magnitude of the current at the grounded end and its phase relation to the applied voltage depend upon the effective impedance, which is very different from the ohmic resistance and depends not only on the resistor itself but on its environment as well. It is seen that it is necessary to have either a truly nonreactive high resistance or one where the reactance effects are reduced to very small and measurable amounts. Consequently, shielded resistors of the construction indicated in Fig. 60 are employed.

Referring to Fig. 60, WO is the working circuit; GO, the guard circuit, which is similar to the working circuit except that it is usually unshielded.

An alternative arrangement is to use in place of *GO* the secondary of the high-voltage supply transformer or, as employed by Weller, an auto-transformer,[10] which is tapped at suitable points.

The working circuit is divided into sections of 20,000 ohms, for example. Each section is surrounded by its own shield, which is kept at a definite potential relative to the section by being connected to the guard circuit at the proper point. By this means, each section is given a definite environment, so that changes of position with respect to the ground and to neighboring objects have no effect—or only a second-order

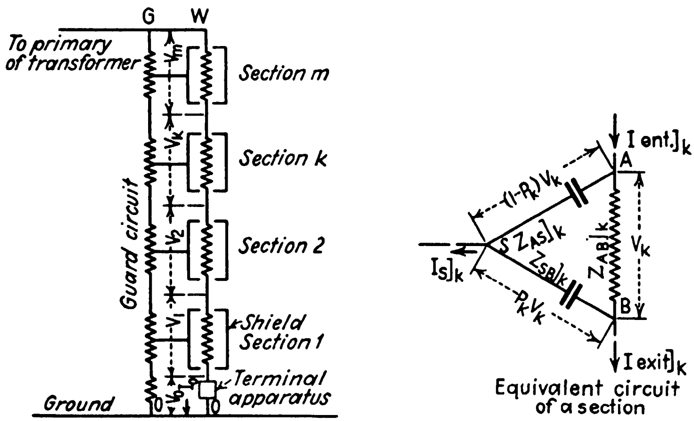


FIG. 60.—Diagrams for shielded high resistances.

effect, for the guard circuit is usually unshielded—on either the magnitude or the phase of the current through the section.

As all parts of a shield must be at the same potential, the potential difference between a coil and its shield must vary from point to point. There is capacitance between the elements of the coil and the shield, with the result that, in general, the exit current at the low-voltage end of a section differs from the entrance current at the high-voltage end; the difference gives rise to a component of current in the tap leading from the shield to the guard circuit.

Each section, for example the *k*th, has two impedances:

1. The exit impedance

$$z_{\text{exit}}]_k = \frac{V_k}{I_{\text{exit}}]_k}.$$

2. The entrance impedance

$$z_{\text{ent.}}]_k = \frac{V_k}{I_{\text{ent.}}]_k}.$$

Using the exit impedance, the voltage drop in a section is

$$V_k = z_{\text{exit}}]_k I_{\text{exit}}]_k.$$

If the current and voltage drop in the terminal apparatus are I_0 and V_0, then, if there are n sections in series,

$$V = V_0 + I_0 z_1 + I_2 z_2 + I_3 z_3 + \cdots + I_n z_n. \tag{9}$$

All the impedances are exit impedances; and all the currents, exit currents. $I_2 z_2$, etc., denotes the product of the current issuing from a section by the exit impedance of that section.

The entrance current of a section, for example, the kth, is the exit current plus the component current I_S which flows to the guard circuit. The entrance current for any section is the exit current of the next section beyond, that is,

$$I_{\text{exit}}]_k + I_S]_k = I_{\text{ent.}}]_k = I_{\text{ex.t}}]_{k+1}.$$

If the ratio of the component guard current to the exit current is denoted by S, then, for Section k,

$$S_k = \frac{I_S]_k}{I_{\text{exit}}]_k} \qquad \text{and} \qquad I_S]_k = S_k I_{\text{exit}}]_k.$$

Therefore

$$I_{\text{exit}}]_{k+1} = I_{\text{exit}}]_k(1 + S_k)$$

If the taps are connected to the guard circuit at the proper points, S_k is always a very small quantity. The exit current of the first section flows through the measuring devices; the exit currents of the various sections are:

$$\begin{aligned}
I_1 &= I_0 &&= I_0 \\
I_2 &= I_0(1 + S_1) &&= I_0(1 + S_1) \\
I_3 &= I_0(1 + S_1)(1 + S_2) &&= I_0(1 + S_1 + S_2) \qquad \text{approx.} \\
I_4 &= I_0(1 + S_1)(1 + S_2)(1 + S_3) &&= I_0(1 + S_1 + S_2 + S_3) \\
&\cdots\cdots\cdots \\
I_n &= I_0(1 + S_1)(1 + S_2)(1 + S_3) \cdots (1 + S_{n-1}) \\
& &&= I_0(1 + S_1 + S_2 + S_3 + \cdots + S_{n-1}).
\end{aligned}$$

The approximations come from neglecting the products of small quantities, as, for example, $S_1 S_2$. The effective impedance is

$$z = \frac{V}{I_0} = \frac{V_0}{I_0} + z_1 + z_2(1 + S_1) + z_3(1 + S_1 + S_2) \cdots z_n(1 + S_1 + S_2 + \cdots + S_{n-1}). \tag{10}$$

To utilize (10), the exit impedance of a section must be found. Each section is a three-terminal network, as indicated in Fig. 60, where z_{AB} is the direct impedance between A and B; z_{AS}, the direct impedance between A and S; and z_{SB}, the direct impedance between S and B.

Referring to Fig. 60, the fall of voltage in a section is V_k; the fall from shield terminal S to terminal B is proportional to V_k. If P_k is the proportionality factor for Section k,

$$V_{SB}]_k = P_k V_k.$$

The subscript k applied to a bracket indicates that the quantities within the brackets are those pertaining to Section k. Similarly, the fall of voltage between terminal A and the shield is

$$V_{AS}]_k = (1 - P_k)V_k.$$

The entrance current for Section k is

$$I_{\text{ent.}}]_k = I_{AB}]_k + I_{AS}]_k = \frac{V_k}{z_{AB}}\bigg]_k + \frac{V_k(1 - P_k)}{z_{AS}}\bigg]_k = V_k\left[\frac{z_{AS} + (1 - P_k)z_{AB}}{z_{AB}z_{AS}}\right]_k$$

The exit current for Section k is

$$I_{\text{exit}}]_k = I_{AB}]_k + I_{SB}]_k = \frac{V_k}{z_{AB}}\bigg]_k + \frac{V_k P_k}{z_{SB}}\bigg]_k = V_k\left[\frac{z_{SB} + P_k z_{AB}}{z_{AB}z_{SB}}\right]_k.$$

The entrance impedance for Section k is

$$z_{\text{ent.}}]_k = \left[\frac{z_{AB}z_{AS}}{z_{AS} + (1 - P_k)z_{AB}}\right]_k. \tag{11}$$

The exit impedance for Section k is

$$z_{\text{exit}}]_k = \left[\frac{z_{AB}z_{SB}}{z_{SB} + P_k z_{AB}}\right]_k. \tag{12}$$

The shield current for Section k is

$$I_S]_k = I_{AS}]_k - I_{SB}]_k = \left[\frac{(1 - P_k)V_k}{z_{AS}} - \frac{P_k V_k}{z_{SB}}\right]_k = V_k\left[\frac{(1 - P_k)z_{SB} - P_k z_{AS}}{z_{AS}z_{SB}}\right]_k.$$

The ratio of the shield current to the exit current for Section k is

$$S_k = \frac{I_S}{I_{\text{exit}}}\bigg]_k = \left[\left(\frac{(1 - P_k)z_{SB} - P_k z_{AS}}{z_{SB} + P_k z_{AB}}\right)\left(\frac{z_{AB}}{z_{AS}}\right)\right]_k. \tag{13}$$

It is natural to make all the sections alike, in which case all the values of z_{AB} will be the same, as will be all the values of z_{AS} and z_{SB}. This, however, does not make all the exit impedances the same, for each depends on the particular value of P pertaining to that section. If the taps of the guard circuit are so located that all values of P are the same, then all the exit impedances will be the same, and all the values of S will be the same. Consequently, for this case, denoting the values common to all the sections by z' and S',

$$z = z_0 + nz' + [1 + 2 + 3 + \cdots + (n - 1)]z'S' = z_0 + \left[n + \frac{n(n-1)}{2}S'\right]z'. \quad (14)$$

If the taps are located so that there is no interchange of current between the shields and the guard circuit, all the values of S are zero, and

$$z = z_0 + nz'. \quad (15)$$

The various direct impedances would be determined by measurements. However, suppose the resistances to be uniformly and noninductively wound on bobbins and the coils to be in the uniform environments of concentric metal shields; then the capacitances will be uniformly distributed along the coils, and the sections may be treated as transmission lines having distributed capacitance but no leakance and no inductance. In this case, the direct impedances can be computed if one end of the conductor is assumed to be grounded, and end effects are neglected.

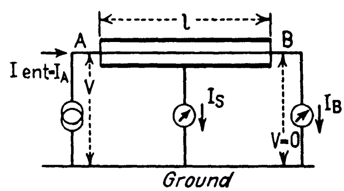


FIG. 61.—Pertaining to direct impedances of resistor with uniformly distributed capacitance.

Referring to the discussion of transmission lines with uniformly distributed constants, for instance that given by Woodruff,* it will be seen that

$$I_A = \frac{V}{z_0 \tanh \theta} \qquad I_B = \frac{V}{z_0 \sinh \theta}.$$

Therefore

$$I_S = I_A - I_B = \frac{V}{z_0}\left[\frac{\sinh \theta - \tanh \theta}{\sinh \theta \tanh \theta}\right],$$

$$z_{AB} = z_0 \sinh \theta,$$

$$z_{AS} = z_{SB} = z_0\left[\frac{\sinh \theta}{\cosh \theta - 1}\right],$$

$$z_{\text{exit}} = \frac{z_{AB}z_{AS}}{z_{SB} + Pz_{AB}} = \frac{z_0 \sinh \theta}{1 + P(\cosh \theta - 1)}. \quad (16)$$

For this case, if R and C are the resistance and capacitance of the whole section, at the frequency to be employed,

$$z_0 = \sqrt{\frac{R}{j\omega C}} \qquad \alpha = \sqrt{\frac{R}{l} \cdot \frac{j\omega C}{l}}$$

$$\theta = \alpha l = \sqrt{jR\omega C}.$$

* "Electric Power Transmission and Distribution," 1st ed., p. 132.

The expansions for sinh and cosh are:[10]

$$\sinh\theta = \theta + \frac{\theta^3}{6} + \frac{\theta^5}{120}\cdots = \sqrt{jR\omega C}\left[1 + j\frac{R\omega C}{6}\cdots\right]$$

$$\cosh\theta = 1 + \frac{\theta^2}{2} + \frac{\theta_1^4}{24}\cdots = 1 + j\frac{R\omega C}{2}\cdots.$$

Consequently,

$$z_{AB} = R\left[1 + j\frac{R\omega C}{6}\cdots\right]$$

$$z_{AS} = 2\left[\frac{1}{j\omega C} + \frac{R}{6}\cdots\right]$$

$$z_{\text{exit}} = \frac{2R\left[2 + \frac{2}{6}jR\omega C - PRj\omega C + \frac{PR^2\omega^2C^2}{6}\cdots\right]}{4 + P^2R^2\omega^2C^2}$$

$$= R\left[1 + \left(\frac{1}{6} - \frac{P}{2}\right)jR\omega C\right] \quad \text{approx.} \tag{17}$$

For this case, $z_{AS} = z_{SB}$. Therefore

$$S_k = \left.\frac{z_{AB}(1 - 2P_k)}{z_{SB} + P_k z_{AB}}\right] = \frac{(1 - 2P_k)jR\omega C(2 - jR\omega CP_k)}{4 + R^2\omega^2C^2P_k^2} \quad \text{approx.}$$

$$S_k = \frac{jR\omega C[2 - k4P_k] + R^2\omega^2C^2P_k[1 - 2P_k]}{4 + R^2\omega^2C^2P_k^2} \quad \text{approx.} \tag{17a}$$

$$S_k = jR\omega C[\tfrac{1}{2} - P_k] \quad \text{approx.} \tag{18}$$

If all the values of P are made $\frac{1}{2}$, all the values of S are zero, and there will be no component of current to the guard circuit, for any section.

$$z_{\text{exit}} = R[1 + jR\omega C(\tfrac{1}{6} - \tfrac{1}{4})]$$

$$= R\left[1 - \frac{jR\omega C}{12}\right] \tag{19}$$

The total exit impedance of n identical sections in series with the terminal apparatus will be

$$z = z_0 + nR\left[1 - \frac{jR\omega C}{12}\right] \tag{20}$$

The exit current from the sections leads the applied section voltage by the angle

$$\tan^{-1}\frac{R\omega C}{12}. \tag{21}$$

This arrangement with taps to the guard circuit at points having the same potentials as the mid-points of the working sections is advantageous, since any convenient number of sections may be used without readjusting the guard circuit.

If a tap is misplaced from its ideal position so that

$$P_k = \frac{1}{2}\left(1 + \frac{a_k}{100}\right),$$

where a_k corresponds to a small percentage change in P_k, then for that section

$$z_{\text{exit}}]_k = R\left[1 - jR\omega C\left(\frac{1}{2} + \frac{a_k}{400}\right)\right]_k. \qquad (22)$$

If a shield voltage is slightly out of phase with its section voltage so that

$$P_k = \tfrac{1}{2}(1 + jb_k),$$

then for that section

$$z_{\text{exit}}]_k = R\left[1 + \frac{R\omega Cb}{4} - j\frac{R\omega C}{12}\right]_k. \qquad (23)$$

When the phase displacement is small, $R\omega Cb/4$ is negligible compared to unity.

If, as suggested by Orlich and Kouwenhoven,[10] the adjustment is such that

$$P_1 = P_n = \tfrac{5}{12}$$

and

$$P_2 = P_3 = P_4 = \cdots = P_{n-1} = \tfrac{1}{2},$$

then

$$z_1 = z_n = R\left[1 - j\frac{R\omega C}{24}\right],$$

and

$$z_2 = z_3 = z_4 = \cdots = z_{(n-1)} = R\left[1 - \frac{jR\omega C}{12}\right] = z',$$

$$S_1 = S_n = jR\omega C\left[\frac{1}{2} - \frac{5}{12}\right] = \frac{jR\omega C}{12},$$

$$S_2 = S_3 = S_4 = \cdots = S_{n-1} = 0.$$

The impedance of the n sections is

$$z = z_1 + z_2(1 + S_1) + z_3(1 + S_1 + 0) + z_4(1 + S_1 + 0) + \cdots + z_n(1 + S_1 + S_{n-1}) \qquad (24)$$

$$= z_1 + z'(n - 2)(1 + S_1) + z_n(1 + S_1),$$

as

$$z_1 = z_n,$$

$$z = z_1(2 + S_1) + z'(n - 2)(1 + S_1).$$

Substituting the values of z_1, z_1', and S_1 gives, neglecting only real second-order terms in ωRC,

$$z = nR. \qquad (25)$$

The resistor is nonreactive and has a resistance sensibly equal to that of the n sections in series.

Low-resistance Four-terminal Resistors or Shunts, for Use in Alternating-current Measurements.[11,12]—In many of the modern methods for alternating-current measurements, for example those for determining the ratios and phase angles of current transformers and for measuring power by the electrostatic wattmeter, "noninductive" shunts are employed. They fall into two groups:

a. Primary standards, the inductances of which can be accurately calculated from their dimensions.

b. Secondary standards, the inductances of which cannot be calculated with accuracy but can be adjusted experimentally to practically zero.

Examples of *a* are a resistor consisting of a loop of measured dimensions formed by two thin, straight, parallel wires, or a resistor consisting of two straight, concentric tubes, as used at the National Bureau of Standards. The first is frequently used with small currents; the second may be given a large carrying capacity by oil immersion and has the additional advantage that the skin effect may be calculated. Group *b* includes low-resistance standards intended for current measurements. Here current-carrying capacity is of paramount importance. Examples are the National Bureau of Standards air-cooled standards and the National Physical Laboratory water-cooled standards.

Generally speaking, the voltage drops in these devices are considerable —perhaps 0.5 volt at full load—and as they are frequently intended for large currents, special means must be provided for dissipating the heat. If air cooling is relied upon, the shunt becomes bulky and expensive unless the air is briskly circulated by a blower, as in the shunts for currents up to 500 amp., designed at the National Bureau of Standards.[12] Frequently the shunt is immersed in oil which is circulated at a rapid rate. In the water-cooled shunts designed at the National Physical Laboratory, the resistor is in the form of a tube, through which water from the city mains is circulated.

If an alternating current of strength I flows through a shunt, and E is the voltage which appears between the potential terminals, then the four-terminal impedance of the shunt is

$$z = \frac{E}{I} = R' + j\omega L.$$

R' is the resistance to alternating current; theoretically, on account of the skin effect, it will differ from the four-terminal resistance determined with direct currents. L is the four-terminal inductance, the quantity that when multiplied by the rate of change of current gives that part of the voltage drop between the potential terminals which is due to variations in the flux through the entire potential circuit, set up by the current. The phase displacement is given by $\tan\theta = \omega L/R'$.

It is necessary that the inductance be reduced to a minimum, for the phase displacement between the current and the potential difference at the shunt terminals must be as small as possible, and the impedance must be sensibly the same as the direct-current resistance. Also, when the inductance is reduced to a minimum by making the net ampere-turns due to the current-carrying conductor practically zero, the stray field, and therefore the disturbing effect on neighboring instruments, is correspondingly decreased; this may be of importance when large currents are dealt with.

If low resistances are employed, the amount of inductance that can be tolerated is exceedingly small. For instance, at 60 cycles per second, a phase displacement of 0°.04 will be produced in a resistance of 0.001 ohm

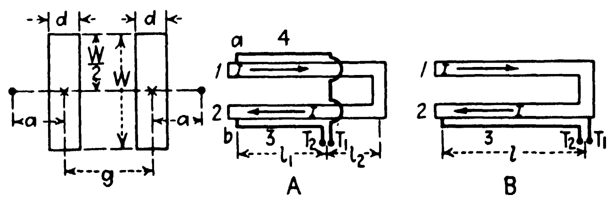


FIG. 62.—Diagrams for alternating-current shunts.

by an inductance of 0.000000002 henry or 0.002 microhenry or 2 abhenrys. At low power factors, even this inductance is of importance when power measurements are made with the electrostatic wattmeter. These small inductances, having a magnitude of only a few abhenrys, can be attained only by special care in design.

To avoid uncertain inductive effects, all the conductors of the current and potential circuits of alternating-current shunts must occupy fixed relative positions; the potential binding posts should be close together, and connection with other apparatus should be made by a twisted pair.

It is important that the leads from the potential terminals to the current-carrying member be properly located, for, as pointed out by A. Campbell,[11] they may be so placed that the net e.m.f. induced in the complete potential circuit by the flux due to the current member is zero, in which case the shunt acts like a noninductive resistor.

Figure 62 shows diagrammatically two shunts formed by bending uniform strips of resistance material into narrow U's. The current is assumed to enter at 1 and leave at 2. The potential circuit in shunt *A* is from *a* to *b*, coinciding in this part of its course with the current member; then from *b* to T_2 and from T_1 to *a*. It is evident that flux due to the current in the strip links the potential circuit; its variation will induce a net e.m.f. dependent on the position of that circuit and on the lengths l_1 and l_2. To determine the net e.m.f. injected into the potential circuit by the flux variation, the mutual inductance between two long, narrow circuits composed of straight and parallel conductors will be deduced.[12]

The shunts under discussion are intended for use at power frequencies 25 or 60 cycles per second and are made of a material of high resistivity; if of the U form, the sides of the U are several centimeters apart. Consequently skin and proximity effects will be assumed to be absent. The validity of this assumption must be tested by experiments on the finished shunt. A shunt such as shown in Fig. 62 consists of two parallel current-carrying conductors and two parallel potential leads. Figure 63 may be taken as the generalized cross section of such an arrangement of four conductors.

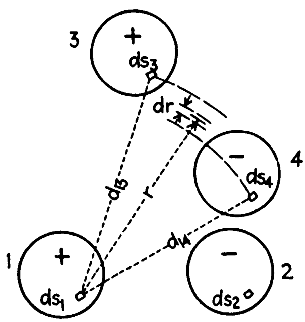


FIG. 63.—Pertaining to inductance effects in shunts.

Conductors 1 and 2 form a loop; likewise conductors 3 and 4. A filament of current in the loop 1, 2 flows in at dS_1 and out at dS_2, and another in the loop 3, 4 flows in at dS_3 and out at dS_4. The currents are assumed to be uniformly distributed in the conductors. Their densities i_1, i_2, i_3, i_4 are taken as positive when the current flowing from the observer enters the plane of the paper. Then

$$\begin{aligned} i_1S_1 &= -i_2S_2 = I_1, \\ i_3S_3 &= -i_4S_4 = I_2. \end{aligned} \tag{26}$$

It is assumed that the conductors are so long and so near together that end effects may be neglected. The current filament i_1dS_1 produces a field at the distance r, which is given by $2i_1dS_1/r$. The flux threading the circuit formed by dS_3 and dS_4 and due to i_1dS_1 is

$$2i_1ldS_1\int_{13}^{14}\frac{dr}{r} = 2i_1ldS_1(\log_\epsilon d_{14} - \log_\epsilon d_{13}),$$

where l is the length of the current filament. That due to i_2dS_2 is

$$2i_2dS_2(\log_\epsilon d_{24} - \log_\epsilon d_{23}).$$

The total flux threading the elementary circuit dS_3, dS_4 is

$$2l[i_1ds_1(\log_\epsilon d_{14} - \log_\epsilon d_{13}) + i_2ds_2(\log_\epsilon d_{24} - \log_\epsilon d_{23})]. \tag{27}$$

This flux links with the current $i_3dS_3 = -i_4dS_4$. The mutual magnetic energy is found by multiplying the flux by the current with which it links. Therefore

$$dW = 2l[i_1dS_1(-i_4dS_4 \log_\epsilon d_{14} - i_3dS_3 \log_\epsilon d_{13}) + i_2dS_2(-i_4dS_4 \log_\epsilon d_{24} - i_3dS_3 \log_\epsilon d_{23})]. \tag{28}$$

Substituting the values of the current densities from (26) gives

$$dW = 2lI_1I_2\left[\frac{1}{S_1S_4}\log_\epsilon d_{14}dS_1dS_4 - \frac{1}{S_1S_3}\log_\epsilon d_{13}dS_1dS_3 - \frac{1}{S_2S_4}\log_\epsilon d_{24}dS_2dS_4 + \frac{1}{S_2S_3}\log_\epsilon d_{23}dS_2dS_3\right]. \quad (29)$$

The total energy is found by summing the elementary values of W over the indicated areas. Therefore

$$W = 2lI_1I_2\left[\frac{1}{S_1S_4}\int_{S_1}\int_{S_4}\log_\epsilon d_{14}dS_1dS_4 - \frac{1}{S_1S_3}\int_{S_1}\int_{S_3}\log_\epsilon d_{13}dS_1dS_3 - \frac{1}{S_2S_4}\int_{S_2}\int_{S_4}\log_\epsilon d_{24}dS_2dS_4 + \frac{1}{S_2S_3}\int_{S_2}\int_{S_3}\log_\epsilon d_{23}dS_2dS_3\right]. \quad (30)$$

Also,

$$W = mI_1I_2.$$

The geometrical mean distance[13] D_{ab} between areas S_a and S_b is defined by the equation

$$\log_\epsilon D_{ab} = \frac{1}{S_aS_b}\int_{S_a}\int_{S_b}\log_\epsilon d_{ab}dS_adS_b.$$

Consequently, the mutual inductance m may be written

$$m = 2l[\log_\epsilon D_{14} - \log_\epsilon D_{13} - \log_\epsilon D_{24} + \log_\epsilon D_{23}].$$

If I is the current flowing in the loop 1, 2, $m\frac{dI}{dt}$ is the induced voltage fall in the loop 3, 4 reckoned in the direction of I. It is seen to consist of four parts which might be considered as contributions due to the action of the current in 1 on conductor 4 and on conductor 3 and due to the action of the current in 2 on conductor 4 and on conductor 3:

$$\Sigma_{\text{fall}} = V_{14} - V_{13} - V_{24} + V_{23}.$$

If conductor 3 coincides with 1, and 4 with 2, the expression becomes that for the self-inductance of the loop so formed:

$$L = 2l[\log_\epsilon D_{12} - \log_\epsilon D_{11} - \log_\epsilon D_{22} + \log_\epsilon D_{21}]. \quad (31)$$

$$D_{12} = D_{21}$$

If conductors 1 and 2 have the same dimensions,

$$D_{11} = D_{22},$$

and

$$L = 4l[\log_\epsilon D_{12} - \log_\epsilon D_{11}]. \quad (32)$$

If the loop is traversed by a current I, $L\frac{di}{dt}$ is the voltage fall in the loop reckoned in the direction of I. By (31), it is seen to consist of four contributions which might be considered as due to the action of the current in 1 on conductor 1 and on conductor 2 and due to the current in 2 on conductor 2 and on conductor 1:

$$\Sigma_{\text{fall}} = -V_{11} + V_{12} - V_{22} + V_{21}.$$

The algebraic signs of the various contributions to the voltage fall, which are the same as those of the corresponding term in the inductances, are those obtained by assuming that each inducing current sets up falls of potential in its own and all the parallel conductors which are oppositely directed to itself in space.

In applying these results to shunts, it is noted that some parts of the two circuits (current and potential) coincide. For example, in Fig. 62A, the part from a to b through the resistance strip is common to both the current and the complete potential circuits.

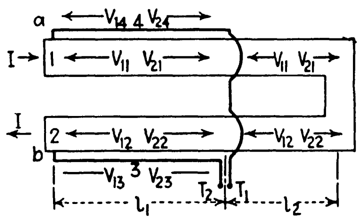


FIG. 64.—Showing directions of voltage contributions in a U shunt.

Referring to Fig. 64, the directions of the voltage contributions are as indicated. Starting from a and proceeding in the direction of the current, the sum of the contributions around the complete potential circuit is

$$[-V_{11} + V_{21}]_{l^1} + [-V_{11} + V_{21}]_{l_2} + [-V_{22} + V_{12}]_{l_2} + [-V_{22} + V_{12}]_{l_1} + [-V_{13} + V_{23}]_{l_1} + [-V_{24} + V_{14}]_{l_1}. \quad (33)$$

Consequently the inductance of the shunt may be written:

$$L = 2l_1[-\log_\epsilon D_{11} + \log_\epsilon D_{21} - \log_\epsilon D_{22} + \log_\epsilon D_{12} - \log_\epsilon D_{13} + \log_\epsilon D_{23} - \log_\epsilon D_{24} + \log_\epsilon D_{14}] + 2l_2[-\log_\epsilon D_{11} + \log_\epsilon D_{12} - \log_\epsilon D_{22} + \log_\epsilon D_{12}]. \quad (34)$$

In this case,

$$D_{11} = D_{22}, \qquad D_{21} = D_{12}, \qquad D_{14} = D_{23}, \qquad D_{13} = D_{24}.$$

So the inductance of the shunt becomes

$$L = 4l_1[-\log_\epsilon D_{11} + \log_\epsilon D_{12} - \log_\epsilon D_{13} + \log_\epsilon D_{14}] + 4l_2[\log_\epsilon D_{12} - \log_\epsilon D_{11}]. \quad (35)$$

Geometrical mean distances have been determined for the configurations usually encountered in inductance calculations.[12] Assuming that the shunt of Fig. 64 is made of wide, thin strips placed close together and that the potential leads are near the surface of the strip and at the middle of its width, the necessary formulae, for this restricted case, are as follows:

For two wide, thin rectangles close together,

$$\log_\epsilon D = \log_\epsilon w - \frac{3}{2} + \frac{\pi g}{w} \cdots. \quad (36)$$

This neglects terms containing g^2/W^2 and d^2/W^2.

For two coincident thin rectangles,

$$\log_\epsilon D = \log_\epsilon W - \frac{3}{2} + \frac{\pi d}{3W} \cdots. \quad (37)$$

This neglects terms containing d^2/W^2.

For a point near a narrow rectangle on a prolongation of the shorter axis,

$$\log_\epsilon D = \log_\epsilon W - 1 + \log_\epsilon 2 + \frac{\pi a}{W} \cdots \quad (38)$$

a may be replaced by $a + g$, g being small. This neglects terms containing a^2/W^2 and d^2/W^2.

Substituting these approximate values in (35) gives, for the length l_1,

$$L_1 = -\frac{4\pi l_1}{W}\left(\frac{d}{3}\right);$$

and for the length l_2,

$$L_2 = \frac{4\pi l_2}{W}\left(g - \frac{d}{3}\right).$$

The effective inductance of the shunt is

$$L = L_1 + L_2 = \frac{4\pi}{W}\left[-\frac{l_1 d}{3} + l_2\left(g - \frac{d}{3}\right)\right]. \quad (39)$$

Evidently, if $\frac{l_1}{l_2} = \frac{g - \frac{d}{3}}{d/3}$, the shunt accords with Campbell's suggestion of balancing the self-induction by mutual inductance to the potential leads and becomes in effect noninductive. If the potential leads are brought off in an equipotential plane near the current terminals, the effective inductance is positive and given by $L = \frac{4\pi l}{W}\left(g - \frac{d}{3}\right)$. If they are carried along the middle of the strip to the far end and then brought off, the effective inductance is negative and given by $L = -\frac{4\pi l}{W}\left(\frac{d}{3}\right)$. If both potential leads are attached to the same side of the U, as in Fig. 62*B*, the effective inductance is

$$L = 2l[\log_\epsilon D_{12} - \log_\epsilon D_{13} - \log_\epsilon D_{22} + \log_\epsilon D_{23}] \quad (40)$$

$$= -\frac{2\pi l}{W}\left(\frac{d}{3}\right). \quad (41)$$

If the potential lead in Fig. 62*B* is displaced parallel to itself, it is evident that the flux leakages will change. When its distance from the edge is very closely two-tenths the width of the strip,[12] the shunt becomes in effect noninductive. This gives a convenient means of adjustment when the thickness and width of the strip are considerable, as in shunts for large currents. It must be shown experimentally that the skin effect is negligible. Work at the Bureau of Standards on a shunt composed of two parallel manganin strips 10 cm. wide and 0.2 cm. thick indicates that at 60 cycles per second the skin-effect ratio differs from unity by about 350 parts in 1,000,000 when the spacing is 10 cm. and by about 50 parts in 1,000,000 when the spacing is 1 cm., showing the skin effect to be negligible in shunts approximating this design.

Figure 65 gives the plan and cross section, and Table VI the data pertaining to a group of air-cooled shunts constructed at the Bureau of Standards. Forced circulation is employed, the air velocity being about 1,600 cm. per second. The heat thus dissipated is about 0.017 watt per square centimeter per degree rise of temperature. The maximum full-load rise of 5° occurs in the 500-ampere shunt.

The construction indicated in Fig. 65 is not used for primary standards, as it is not possible to determine the dimensions with sufficient accuracy. Its use is restricted to secondary shunts, the final adjustment of which is made experimentally by comparison with primary standards.

TABLE VII.—DATA ON NATIONAL BUREAU OF STANDARDS AIR-COOLED SHUNTS

Resistance, ohms	0.05	0.02	0.01	0.005	0.0025	0.001
Current rating, amperes	10	25	50	100	200	500
Length, centimeters	75	75	75	65	76	72
Width, centimeters	1.6	4.0	8.0	7.0	13.3	20.0
Thickness, centimeters	0.056	0.056	0.056	0.081	0.10	0.14
Watts per square centimeter at rated current	0.021	0.021	0.021	0.055	0.050	0.083
Phase angle at 60 cycles, observed, minutes	+0.4	+0.3	+0.1	−0.1	−0.4	−2.0

In all but the 200- and 500-amp. shunts, the potential lead is carried along the middle of the resistance strip close to its surface. This gives a sufficiently close compensation for inductance and skin effect. In the 200-amp. shunt, the lead is placed two-tenths the width of the strip from the edge, as already mentioned. In the 500-amp. shunt, two sets of potential leads in parallel are used. Both start from *a* (or *b*), and each is placed two-tenths the width of the strip from the edge (*lm* and *nk*, Fig. 65). This is superior to a single unsymmetrically placed lead on account of immunity to stray fields.

Above 500 amp., the bureau employs tubular shunts.[12] The resistance element is a thin manganin tube; the return lead, an internal concentrically placed copper tube. Such a structure produces no external field. The cooling is by the forced circulation of oil. The results obtained by Russell in his study of the concentric main give the starting point for this design, which is an advantageous one because the inductance can be calculated accurately.

It would be natural to use a single set of potential leads lashed in place along an element of the cylinder, but this arrangement would render the shunt susceptible to stray fields. If a second set is added in a diametrically opposite position to the first, and the corresponding wires of the two sets connected in parallel at the potential posts, the desired immunity to stray fields is approached. It is made practically complete by adding another complete set of leads displaced 90 deg. around the circumference of the cylinder. All four leads at each end are supposed to be attached to the tube on the same equipotential plane.

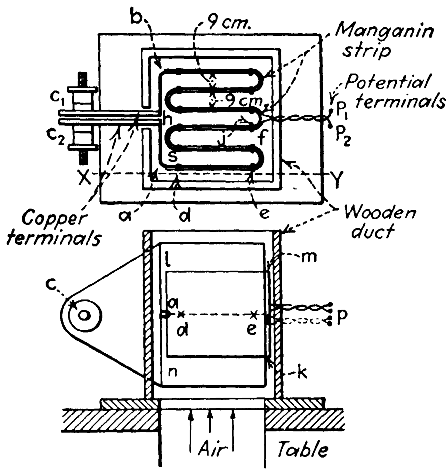


FIG. 65.—Plan and cross section of air-cooled shunt.

This cannot be exactly accomplished; so a resistance of a few tenths of an ohm, alike for all, is included in each lead to cut down the circulating current. The potential difference at the binding post is thus an average value.

To facilitate the adjustment of the resistance, a shunt of comparatively large resistance is applied to a short length of the tube. The lead to the potential post is displaced along the shunt until the desired value is obtained and then soldered in position.

Study of this arrangement at the National Bureau of Standards has revealed discrepancies in the phase of the order of 0.4 minute at 60 cycles per second, between two large concentric tube shunts. It is believed that the discrepancy is due to the nonhomogeneity of the manganin tubes, which prevents a uniform distribution of current around their peripheries. Possibly this effect sets a limit to the extension of methods involving the use of resistance standards with very high currents.

Figure 66 shows the noninductive shunt designed at the National Physical Laboratory.[11] The resistance element is a manganin tube enameled on the inside.

The tube is hard soldered to hollow copper terminals, and these, in turn, are soft soldered to the current leads, which are carried back parallel to the tube, to about the middle of its length, where they are terminated by binding posts. This cuts down the stray field of the shunt itself and brings the current leads so close together that their field has little effect on the neighboring apparatus. A thin copper ring is hard soldered to the tube at the middle of its length. When the shunt is used in connection with the electrostatic wattmeter, this ring is used as a terminal (see Fig. 208).

The tube is covered with a layer of varnished cambric about 0.2 mm. thick, and outside this are the potential leads, in the form of thin sheaths

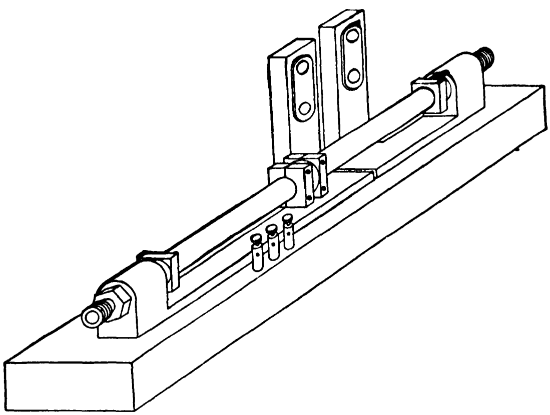


FIG. 66.—Noninductive shunt. Designed at National Physical Laboratory.

of copper foil about 0.04 mm. thick extending from the ends of the tube to near its middle, where they are terminated in potential posts.

The fluxes which produce inductive effects are those in the insulating medium between the tube and the potential leads and in the main tube itself. The insulation therefore should be as thin as practicable, and the potential leads should very closely surround it. The resistance tubes should be very thin and of large diameter. As the resistivity is high, the skin effect is negligible—less than 1 part in 10,000 at power frequencies. This construction reduces the effective inductance to 3 or 4 abh. or to 0.003 or 0.004 μh.

To obtain a high carrying capacity, water from the city mains is briskly circulated through the resistance tube at a rate of about 15 liters per minute. The formation of a layer of hot water in contact with the resistance material is prevented by a centrally located glass rod which nearly fills the tube. For the same change in resistance due to heating, approximately three times as much energy may be dissipated as when simple air cooling is relied upon. As much as 10 kw. may be dissipated by a current of 3,000 amp. in a tube 1½ in. in diameter and 18 in. long.

TABLE VIII.—DATA CONCERNING WATER-COOLED MANGANIN RESISTANCES DESIGNED AT THE NATIONAL PHYSICAL LABORATORY

Resistance, ohms	Outside diam., mm.	Thickness of wall, mm.	Length, cm.	Normal current, amp.	Max. current, amp.	Volts drop at normal current	Kw. at max. current	L, cm.	Time constant L/R	Phase displacement at 60 cycles
0.04	6	0.25	35½	50	115	2	0.53	6.5	1.6×10^{-7}	0°.003
0.02	10	0.30	40	100	260	2	1.35	5.4	2.7×10^{-7}	0°.006
0.01	15	0.40	39	200	450	2	2.00	3.4	3.4×10^{-7}	0°.007
0.002	30	1.00	48	1,000	1,300	2	3.40	3.7	18.5×10^{-7}	0°.040
0.001	40	1.5	42½	2,000	2,500	2	6.25	3.0	30.0×10^{-7}	0°.060

The current density may be as great as 16,000 amp. per square inch of cross section. For manganin tubes, up to 1.5 mm. thick, 10 watts per square centimeter of dissipative area may be allowed as the working load.

To minimize the possibility of accidents, this form of resistance is used in a vertical position, the water inlet being at the bottom so that the tube is always filled.

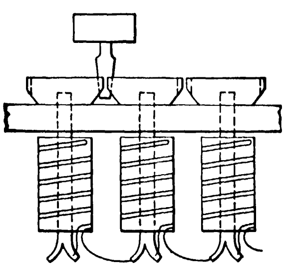


FIG. 67.—Diagram showing series arrangement of resistance coils.

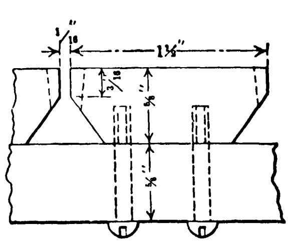


FIG. 68.—Connection block for resistance box.

Resistance Boxes.—Generally speaking, resistance boxes are constructed so that the resistance between their terminals may be varied from zero up to the full capacity of the box by 0.1 ohm or by 1-ohm steps. This must be accomplished by the use of a moderate number of coils. Formerly the series arrangement, shown in Fig. 67, was almost universally employed. It was common to use coils of 1, 2, 2, 5 ohms or 1,2, 3, 4 ohms with similar sets for the tens, hundreds, and thousands. With this arrangement, the withdrawing of a plug removes the short circuit on the corresponding coil and puts it in series with the other active coils. It is essential that the plug contacts be made as perfect as possible; consequently the top of the box must be very rigid, the blocks firmly screwed in place, and the plugs and sockets perfectly made. The blocks should be undercut so that each shall be well insulated from its neighbors.

On account of the inconvenience of the plug arrangement, much attention has been given to the construction of switches with sliding

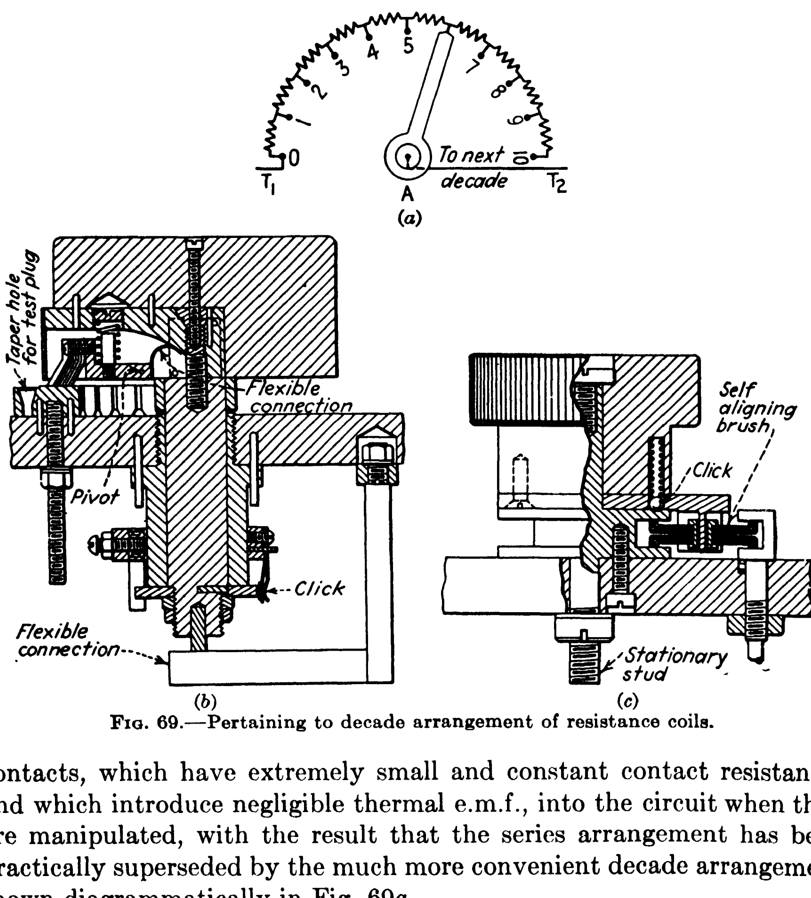


FIG. 69.—Pertaining to decade arrangement of resistance coils.

contacts, which have extremely small and constant contact resistances and which introduce negligible thermal e.m.f., into the circuit when they are manipulated, with the result that the series arrangement has been practically superseded by the much more convenient decade arrangement shown diagrammatically in Fig. 69*a*.

In each decade, there are 10 coils of equal resistance, any number of which from 0 to 10 may be inserted between T_1 and T_2 by the rotative switch. The use of 10 instead of 9 coils contributes to convenience when the desired resistance is very near the end of the decade.

Practically, all the coils for a decade cannot be adjusted to exactly the nominal value; some will be a trifle too high, and some a trifle too low in resistance. This implies that when the coils are assembled in the box, their order should be such that, at any setting, the accumulative positive and negative errors cancel each other as far as possible.

The fact that each coil has been adjusted to have a very small residual effective inductance is no guarantee that the over-all inductance of a decade assembled from these coils will be correspondingly small. The coils have capacitances to ground and to each other and are at potentials above ground dependent on the settings of all the switches in the box. This is well illustrated by Fig. 70,[7] which shows the results of tests on a decade for which the individual coils had been adjusted so that each had an effective inductance of approximately -10 μh. It is seen that the over-all inductance is $-1{,}250$ μh, one hundred and twenty five times, not ten times, that of a single coil. This points to shielding the decades and the adjustment of the coils after they have been assembled in the box, a procedure adopted by the Mervin Tinsley Company in their noninductive resistance boxes. Such a box should be used as directed by the makers.

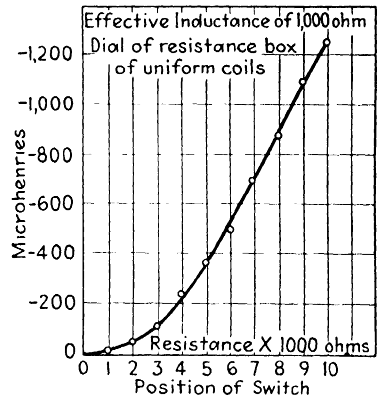


FIG. 70.—Showing aggregate inductance of a decade at various settings. (*Gall.*)

Figure 69*b* and *c* shows two forms of rotative switch developed for use in resistance boxes and bridges of the highest grade. In the Eppley switch, there is only one sliding contact, connection with the next decade being made by a flexible strip, as indicated. In the Tinsley "Dual" switch, connection between the coils and the central terminal, which is connected to the next decade, is made by a self-aligning, floating brush which puts no strain on the rotating member. The contact resistance of a first-class, rotative switch is of the order of magnitude of 0.0001 ohm per contact, with variations of about 0.00004 ohm. At the ends of their travels, it is well to have the brushes slightly overrun the contact blocks in order to maintain uniform wear of the contact surfaces.

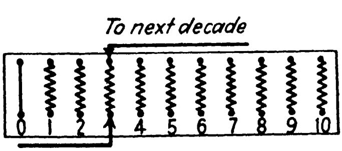


FIG. 71.—Leeds and Northrup single active coil decade.

For alternating-current work, the Leeds and Northrup Company have introduced a shielded resistance box,[8] the values of the coils in any decade being 1, 2, 3, . . . 10 units. The 10 coils are mounted on a rotor operated by a handle projecting through the top of the box. At any setting, only one coil is in use, as indicated in Fig. 71.

This arrangement avoids the effects due to capacitance to ground and to neighboring coils and effects due to "dead ends" which occur in boxes of the usual design. Each setting thus has the inductance and capaci-

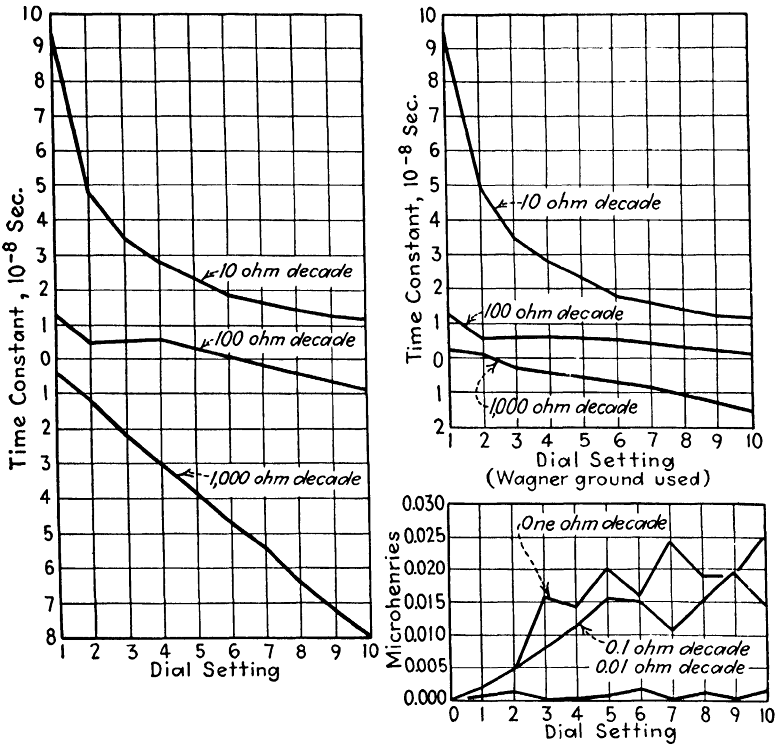


Fig. 72.—Characteristics of Leeds and Northrup shielded six-dial resistance box with single active coil decades.

tance effects of only a single coil. The 100- and the 1,000-ohm decades are supplied with woven resistors. The performance of such a box is shown in Fig. 72.

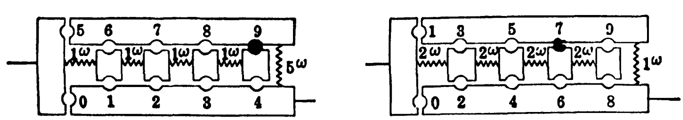


Fig. 73.—Feussner's and Smith's decade arrangements of resistance coils.

Arrangements for Reducing the Number of Coils in a Decade.—A disadvantage of the original decade arrangement lies in the large number of coils that must be made and adjusted; several alternative arrangements are shown in Figs. 73 and 74.

In Feussner's decade arrangement, which gives resistances from 0 to 9 units, the first four coils are arranged as in the ordinary decade system; the fifth value is obtained by using a single coil of 5 units; and the succeeding values by employing this coil in series with the four-step decade. Only one plug is required.

The difference between Smith's and Feussner's arrangements is obvious.

In Northrup's arrangement, four coils having denominations 1, 3, 3, 2 units are used. In Fig. 74, I, II, III, IV, V are terminal posts and taps which may be connected as desired. If all the coils are used in series, the resistance is 9 units. The other values are obtained as shown in Table IX.

The construction necessary for carrying out this scheme by the use of a single plug is shown in Fig. 74.

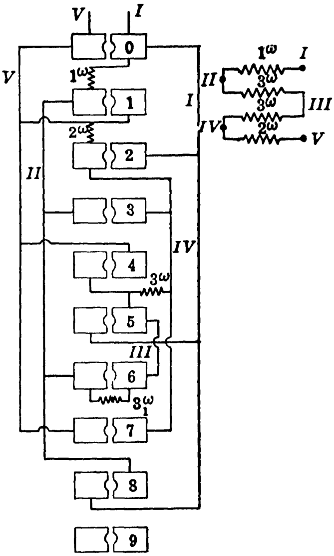


FIG. 74.—Northrup's decade arrangement for resistance coils.

Figure 75 shows one form of decade arrangement employed by the Leeds and Northrup Company[8] for alternating-current resistance boxes. There are six coils in a decade, each having a resistance of two decade units. Five of the coils are mounted in series on a rotor which is provided with appropriate contact blocks and operated by a handle projecting through the top of the box. A brush permanently connects the first coil of the series to the next lower decade. The sixth coil is stationary and is

TABLE IX

Points to be connected	Resistance between terminals I and V Units
I—V	0
II—V	1
IV—I	2
II—IV	3
III—V	4
I—III	5
II—III	6
IV—V	7
I—II	8

inserted between two fixed brushes which connect to the next highe decade. Referring to Fig. 75, at the zero setting the current flows straigh through; for a setting of one unit, the stationary coil and the first movabl coil are in parallel; for a setting of two units, the first coil on the rotor is in circuit alone; for a setting of three units, the second coil on the rotor is in series with the parallel arrangement of the first rotor coil and the station-

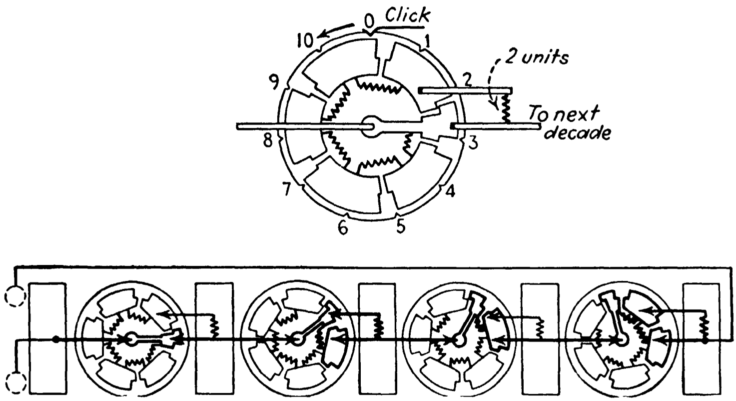
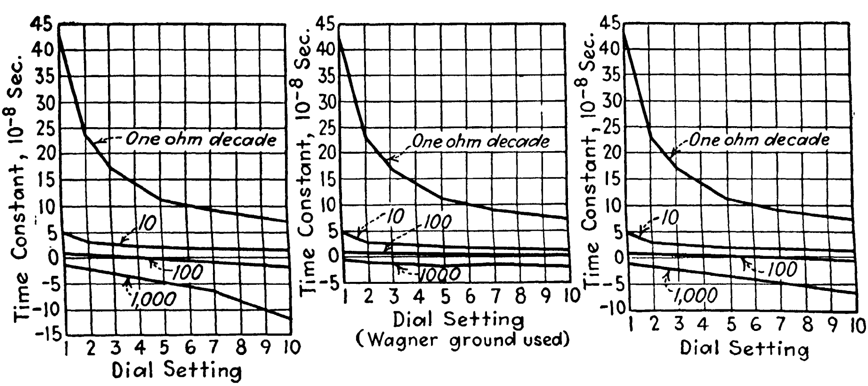


FIG. 75.—Four-dial resistance box. 1,000-, 100-, 10-, and 1-ohm decades. 1,000-ohm decade at left. Box set for 123 ohms. (*Leeds and Northrup Company*).

ary coil; etc. The leads inside the box are arranged to reduce the inductance at the zero setting to a very small value—about 0.4 μh. The box is made in both the unshielded and shielded forms. Figure 76 shows the characteristics of the various decades.



A *B* *C*

FIG. 76.—Characteristics of Leeds and Northrup four-dial resistance box using decade arrangement of Fig. 75.

RHEOSTATS

Water Rheostats.—To control a small current and to be able to give it any value between zero and a maximum, the arrangement shown in Fig. 77 may be used.

The compensating cell renders it possible to bring the current in the derived circuit smoothly down to zero. If the cell is not used, there will be a sudden change in the current when the electrodes *a* and *b* are brought into contact.

Water rheostats are commonly used for absorbing energy during tests of electrical machinery. The "water barrel," shown in Fig. 78, is convenient for temporary arrangements when small amounts of power are to be dealt with.

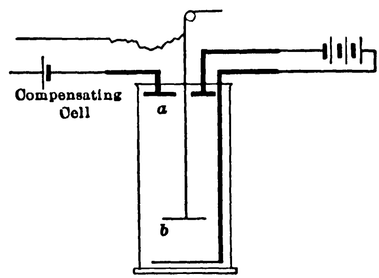


FIG. 77.—Water rheostat for small currents.

A stout wooden barrel is used. An ordinary cast-iron stove grate about 15 in. in diameter is placed at the bottom of the barrel and provided with a terminal of insulated wire. A second grate *S'* is screwed to an iron rod and suspended by a rope which passes over pulleys to a counter-weight. Short wooden pegs prevent the two grates from being brought into contact. Fresh water is used, and the required conductivity obtained by adding a salt, such as sodium carbonate.

Such a "water barrel" will take about 70 amp. at 1,400 volts, absorbing about 100 kw.; the water will boil violently when the rheostat is forced to this extent. An adequate water supply must be provided, and arrangements made by which the barrel may be kept full without danger to the operator. On account of the steam and gases, such rheostats should be used out-of-doors.

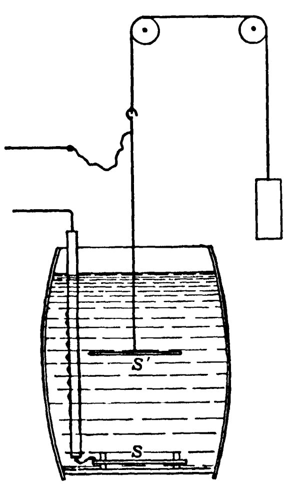


FIG. 78.—Water-barrel rheostat.

When rheostats of this general form are used, there is always more or less slopping over of the water. The ground and surrounding objects often become thoroughly saturated, and the greatest care must be exercised by the attendants that severe or perhaps fatal shocks are not experienced through inadvertently touching some of the wiring. One must not relax his vigilance because the voltage is low, for, with sufficiently good contacts, shocks from 110-volt circuits have proved fatal. The station for the operator should be properly raised from the ground, so that the platform will be dry, and the ropes by which the electrodes are manipulated should be rendered safe by the introduction of strain insulators.

Water Rheostats with Plate and Cylindrical Electrodes.[13]—In case there is a running stream or open canal of fresh water near the station, and the voltage is high, the forms of rheostat shown in Figs. 79 and 80 are convenient.

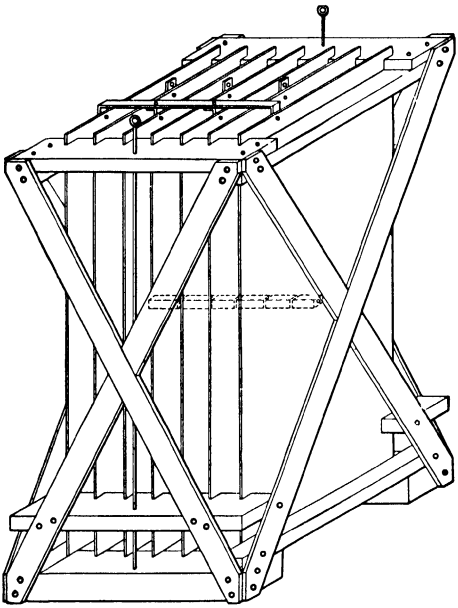


Fig. 79.—Three-phase power-absorbing rheostat with plate electrodes.

That shown in Fig. 79 was used for absorbing power, up to 700 kw., in a 2,300-volt three-phase circuit. There are three terminal and four neutral plates spaced 4 in. apart on centers, all of iron, the dimensions

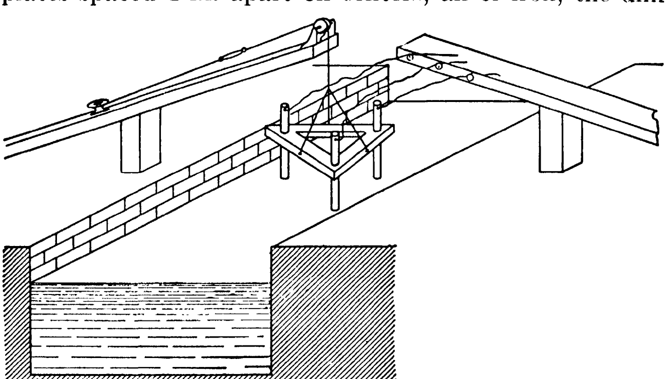


Fig. 80.—Three-phase power-absorbing rheostat with cylindrical electrodes.

being 60 by 24 by $\frac{1}{8}$ in. The frame is hung by a tackle, so that the amount of power may be regulated by varying the immersed area. For these immersion rheostats, the allowable current density at the electrodes is about 3.5 amp. per square inch.

In the rheostat shown in Fig. 80, which is also designed for three-phase loading, a wooden frame made in the form of an equilateral triangle is provided. The three vertical electrodes are of thin metal pipe and are connected by flexible cables to the leads so that the frame may be raised or lowered by means of a tackle, and the immersion of the electrodes varied.

If the conductivity of the water is known, the rheostat may be designed to absorb a given amount of power. The arrangement of electrodes is shown in Fig. 81.

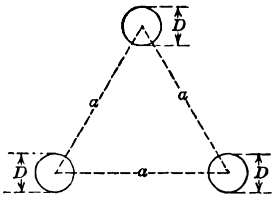


FIG. 81.—Arrangement of electrodes in three-phase water rheostat.

The capacitance of two parallel cylinders in air, diameter D cm., spaced a cm. on centers, length l cm., is*

$$C = \frac{l}{4 \log_\epsilon \left(\frac{a + \sqrt{a^2 - D^2}}{D} \right)}.$$

Hence, the conductance between these cylinders when immersed in an infinite medium of conductivity γ will be

$$g = \frac{\pi \gamma l}{\log_\epsilon \left(\frac{a + \sqrt{a^2 - D^2}}{D} \right)}.$$

Let

$$\frac{a}{D} = K;$$

then

$$g = \frac{1.36 \gamma l}{\log_{10} (K + \sqrt{K^2 - 1})}.$$

The current that will flow between two parallel cylindrical electrodes when they are immersed in a great body of liquid is

$$I' = \frac{1.36 \gamma l E}{\log_{10} (K + \sqrt{K^2 - 1})}, \tag{42}$$

and the power absorbed is

$$P = \frac{1.36 \gamma l E^2}{\log_{10} (K + \sqrt{K^2 - 1})}. \tag{43}$$

In the case of a three-phase rheostat, the line current will be

$$I_L = \frac{2}{\sqrt{3}} I' = 1.15 I', \tag{44}$$

and the power absorbed will be given by

$$P = E I_L \sqrt{3} = \frac{2.72 \gamma l E^2}{\log_{10} (K + \sqrt{K^2 - 1})}. \tag{45}$$

* RUSSELL, "Alternating Currents," vol. 1, p. 102, first edition.

The conductivity γ, which is greatly influenced by local conditions and by temperature, must be found for the water that is to be used. To

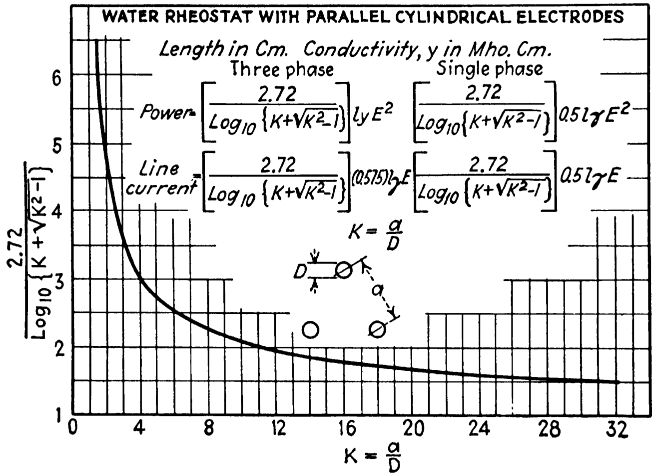


FIG. 82.—Showing constants of water rheostat with different spacings of cylindrical electrodes.

determine it, two conducting cylinders of a known diameter may be fixed at a known distance apart, and the arrangement immersed in the running water. A measured alternating-current voltage is then applied between the two cylinders, and the resulting current determined. γ is calculated by aid of (42).

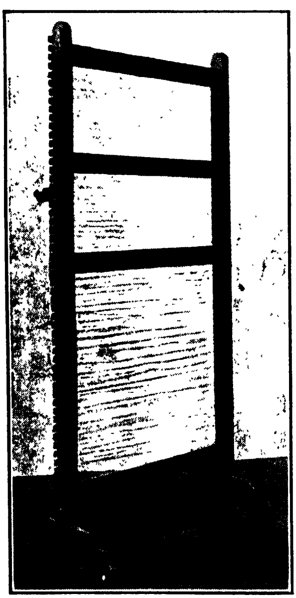


FIG. 83.—Resistance frame.

Wire-wound Rheostats.—For general laboratory purposes, a very convenient rheostat adapted to low voltages is shown in Fig. 83.

The upright frame, 6 ft. high and 3 ft. wide, is strung with about 550 ft. of bare *IaIa* wire, contained in 100 sections. When 110 volts is applied at the terminals of the frame, one can, by means of spring clips, tap off voltages or small currents. By means of four clips and flexible connection wires, the arrangement may be divided into sections and these connected in parallel.

A cheap and convenient form of rheostat, which has proved very useful for loading the small generators used for purposes of instruction in electrical engineering laboratories, is shown in Fig. 84.

The wire is wound in screw threads on molded porcelain cylinders. These cylinders are loosely held in place on the angle-iron frame in such a manner that they may be readily removed. The base and the top,

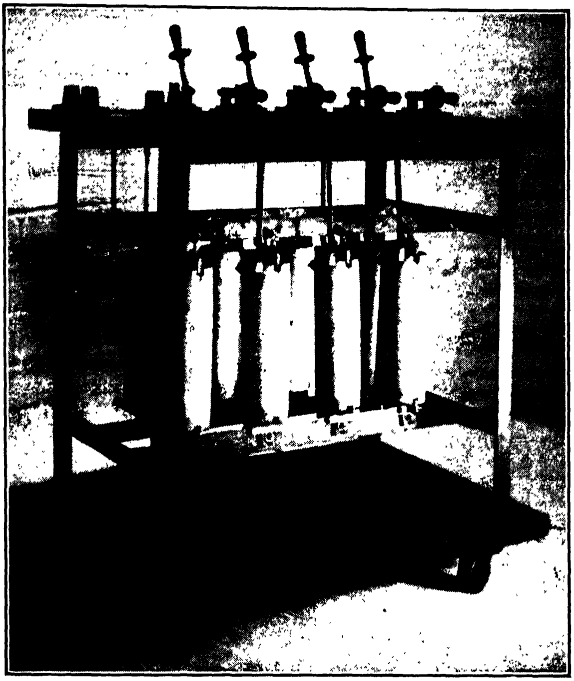


Fig. 84.—Laboratory rheostat for loading small generators.

which carries the switches, are of "ebony asbestos wood." The following sizes of *Ia1a* wire have been employed:

Size of wire	Full-load capacity
No. 17	104 amp. at 110 volts
No. 20	52 amp. at 110 volts
No. 26	20 amp. at 220 volts
No. 23	40 amp. at 220 volts

Immersed-wire Rheostats.—The carrying capacities of wires may be greatly increased by immersing them in water, as will be seen from the following table giving approximate data concerning galvanized-iron wire. Immersed rheostats are useful in temporary arrangements of apparatus. Fine wires may be corroded off after a short time. Provision must be made for safely replacing the water lost by boiling.

The heating of these wires when immersed is so great that there must be no obstruction to a free circulation of the cooling water. Strong strings or fairly sharp edges of wooden sticks will make reliable supports.

TABLE X.—TABLE OF APPROXIMATE DATA CONCERNING CARRYING CAPACITY OF GALVANIZED-IRON WIRE WHEN IMMERSED IN WATER

No. Brown and Sharpe	In air			In water			
	Cir. mils	Amp.	Ft. per 110 volts	Amp.	Ft. per 110 volts	Ft. per 550 volts	Ft. per lb.
20	1,018	2.5	594	36	25	125	369.0
19	1,253	2.9	626	42	27	135	293.0
18	1,624	3.5	673	50	29	145	232.0
17	2,048	4.2	710	60	30	150	184.0
16	2,583	5.0	750	71	32	160	146.0
15	3,257	6.0	790	88	34	170	107.0
14	4,107	7.1	840	103	36	180	91.9
13	5,178	8.5	886	122	38	190	72.1
12	6,530	10.1	941	145	40	200	57.8
11	8,234	12.0	990	173	42	210	45.8
10	10,380	14.3	1,054	205	45	225	36.4
9	13,090	17.1	1,103	245	47	235	33.3
8	16,510	20.3	1,354	293	49	290	25.0

The water used must be clean, to prevent rapid destruction of the wires by electrolysis.

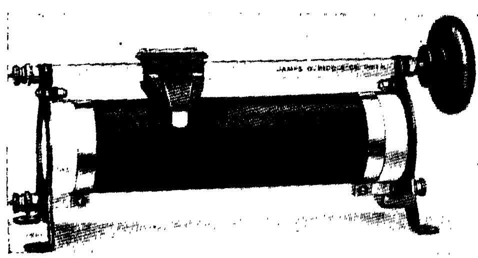


FIG. 85.—Slide-wire rheostat for small currents.

Drop Wires.—A very useful form of drop wire for controlling the voltages applied to the potential coils of instruments may be made by winding a single layer of double cotton-covered resistance wire on a piece of brass tube about a meter long and 5 cm. in diameter, which has been slit lengthwise and covered with stout paper. The insulation is sand-papered off where the slider makes contact. By use of this device, the voltage in a derived circuit may be adjusted from zero to a maximum. It should not be forgotten that the arrangement is a long solenoid and will have a considerable stray field.

Rheostats similar to those shown in Fig. 85 are regularly on the market and are very convenient for general laboratory purposes.

There are numerous stock forms of rheostats which may be obtained from the electrical manufacturing companies and which are useful in particular cases.

Carbon Compression Rheostats.—Carbon compression rheostats are exceedingly useful as laboratory appliances where low-voltage currents are to be controlled, for example in calibration work.

The essential feature is a series of specially molded carbon plates, which can be forced into more or less intimate contact by a screw.

A convenient form of carbon compression rheostat is shown in Fig. 86. It contains 90 plates each $1\frac{1}{2}$ by $1\frac{1}{2}$ by $\frac{1}{8}$ in.

A 4-volt current can be controlled between the limits 1 and 28 amp., the resistance of the circuit outside the rheostat being 0.1 ohm.

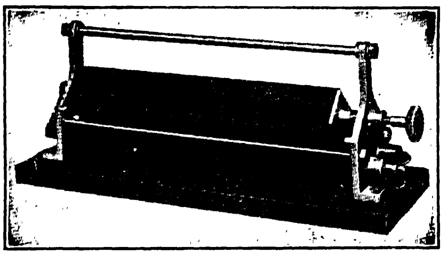


FIG. 86.—Carbon compression rheostat.

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CHAPTER IV

THE MEASUREMENT OF RESISTANCE

Volt and Ammeter Method.—The most obvious method of determining an electrical resistance is by the direct application of Ohm's law. The potential difference between the terminals of the resistor and the current flowing through it are measured by appropriate instruments, which have previously been calibrated.

It is important not to lose sight of the possible influence of the measuring instruments on the results. With the terminal at 1 (Fig. 87), the voltmeter gives the proper potential difference, but the ammeter measures the current through the unknown resistance plus that through the voltmeter. If the resistance X is at all comparable with that of the voltmeter, the error may be great unless allowance is made for the voltmeter current. In this case, the voltmeter resistance must be known. If the terminal is at 2, the ammeter gives the proper current, but the measured potential difference includes the drop in the ammeter and its connections; if this is an appreciable fraction of that in X, the error will be large unless this drop is subtracted from the voltmeter reading. These considerations should be given weight when making connections for any particular test.

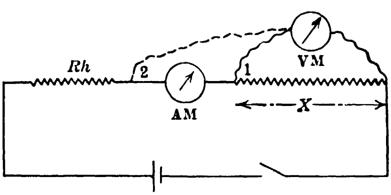


FIG. 87.—Volt and ammeter method for measuring resistance.

If the voltmeter be of low resistance, care must be taken that the resistances of the leads and contacts are negligible. The voltammeter method finds frequent employment in emergency work where comparatively rough measurements will suffice, for instance in determining armature resistance.

Substitution Method.—This method is based on the assumption that the e.m.f. and resistance of the battery employed are constant.

With the connections as in Fig. 88, the galvanometer current is

$$I_G = \frac{ER_S}{\left(R_B + Rh + \frac{R_G R_S}{R_G + R_S} + X\right)(R_G + R_S)}.$$

If, by means of a switch, S be substituted for X and adjusted until the deflection is the same as before, then obviously

$$S = X.$$

Any error that might be due to the law of deflection of the galvanometer is eliminated. The shunt R_S serves to vary the sensitivity of the galvanometer to suit different conditions. The resistances of the other parts of the circuit should be small compared with S and X; for this reason arrangements should be made so that the number of battery cells may be varied. The substitution method in a modified form is frequently used in dealing with very high resistances (see "Insulation Resistance").

Direct-deflection Method.—Two resistors that are to be compared may be connected in series, and the potential differences between their terminals measured by voltmeters of the proper range. If the current is

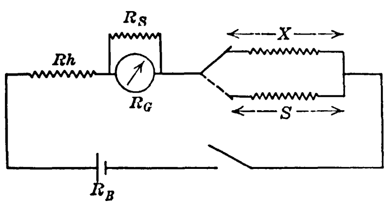


FIG. 88.—Substitution method for measuring resistance.

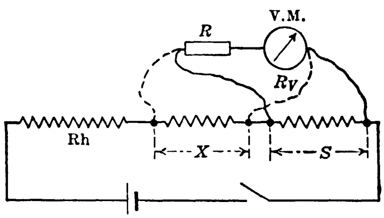


FIG. 89.—Direct-deflection method for measuring resistance.

constant, a single instrument may be used; its deflection should be proportional to the current, and it should be so arranged that the terminals can be quickly transferred from S to X (see Fig. 89).

R is a variable resistance for changing the range of the voltmeter; if the current taken by the voltmeter is negligible,

$$X = S\frac{D_X}{D_S}\left(\frac{R_V + R_X}{R_V + R_S}\right).$$

D_X and D_S are the readings, and R_X and R_S the resistances inserted in R when the terminals are on X and on S, respectively. R_V is the voltmeter resistance. If the current fluctuates, two voltmeters should be used, simultaneous readings being taken by two observers. This procedure, millivoltmeters being employed, is frequently used for testing rail bonds *in situ*, the resistance of a given length of rail including a bond being compared with that of the same length without a bond. The voltages measured are those due to the return current through the rail.

Potentiometer Method.—Instead of determining the potential difference between the terminals of S and of X by a voltmeter, the potentiometer (see page 275) may be used. Obviously, the current in S, in X, and in the potentiometer must remain constant during the test. As the processes of balancing and checking the constancy of the potentiometer current require some time, this condition is very difficult of practical realization; while the method may be made to give accurate results, the measurement becomes a time-consuming operation. In very accurate measurements, the possibility of a heating error is considerable, for the current must be kept on continuously during the process of balancing.

In order that resistances may be determined with accuracy and dispatch, it is necessary to have methods that are independent of fluctuations of the testing current. Such methods will now be discussed.

Differential-galvanometer Method.—A differential galvanometer has two distinct windings which are thoroughly insulated from each other, of equal magnetic strength, of equal resistance, and as nearly coincident as possible. To attain these conditions, the wires are wound throughout their length side by side and in layers. Any residual magnetic effect, if the instrument is of the Kelvin type, may be annulled by a small coil placed outside the case of the instrument and connected in series with the weaker coil in such a manner that its effect is additive; this adjusting coil is mounted so that its position may be altered by sliding it along a rod which is coaxial with the main coil.

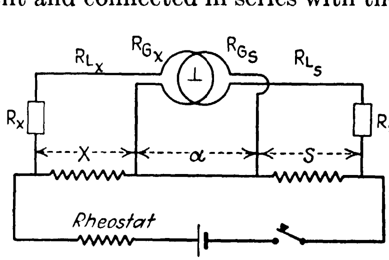


FIG. 90.—Simple method of using differential galvanometer.

The simplest method of using the instrument is shown in Fig. 90.

On the diagram, R_{G_x} and R_{G_s} are the resistances of the two galvanometer coils; R_{L_x} and R_{L_s}, the total lead resistances; R_x and R_s, the resistances inserted in the boxes. To avoid leakage and capacitance effects, the positions of the boxes R_x and R_s should be such that the potential difference between the two galvanometer coils is a minimum. The resistance of α should be low. High insulation of the leads, etc., is necessary.

First, the adjustment of the instrument must be tested. To do this, the coils are connected in series and opposed magnetically; the maximum working current is then sent through them. No deflection should be observable. Perfect adjustment is obtained by adjusting the auxiliary coil.

With the connection shown in Fig. 90, after having inserted a suitable resistance in R_x, the value of R_s is adjusted until the galvanometer stands at zero; then the currents in the two coils are equal, and

$$X = S\frac{R_{G_x} + R_x + R_{L_x}}{R_{G_s} + R_s + R_{L_s}}.$$

The galvanometer and lead resistances must be known.

An alternative method is to insert a small resistance R_x and obtain a balance by adjusting R_s, then to make R_x large and repeat the balance. If all other resistances remain constant, and the values inserted are R_x, R'_x and R_s, R'_s,

$$X = S\frac{R'_x - R_x}{R'_s - R_s}.$$

A practical difficulty is that the exact adjustment of a sensitive instrument is somewhat troublesome and when made is not permanent, being subject to changes in the leveling of the galvanometer. Therefore, methods have been suggested where the deflection is not brought exactly to zero.

Kohlrausch Method of Using a Differential Galvanometer.[1]—For work of the highest class, such as the precision comparison of resistance standards, the method employed must be free from errors due to variations in the resistances of the galvanometer circuits. These might be caused by changes of connections, involving the alteration of contact resistances, or they might be due to the inclusion of potential terminals of the resistances during the test but not when making the preliminary adjustment for differentiality.

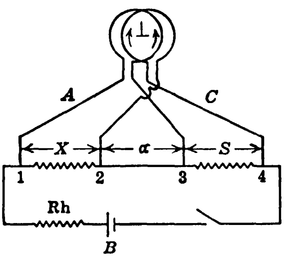


FIG. 91.—Diagram for Kohlrausch method of using a differential galvanometer.

Kohlrausch's method of overlapping shunts fulfills the desired conditions. It is designed for the comparison of nominally equal resistances. The scheme of connections is shown in Fig. 91. In carrying out the test, some means of adjusting either S or X, as well as one of the galvanometer circuits, is required.

It is also necessary to be able to interchange B and α (equivalent to interchanging the galvanometer circuits A and C). This may conveniently be done by a commutator with mercury contacts, such as is shown in Fig. 92. The parts L are insulating strips of ebonite; the other parts of the rocker are of copper; the two middle arcs are in electrical connection. The parts of the commutator are so large that the resistance of the circuit, and therefore the battery current, is not appreciably altered by the interchange.

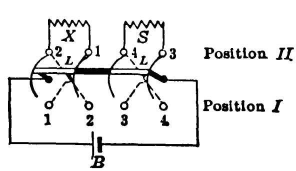


FIG. 92.—Commutator for interchanging the galvanometer coils.

Referring to Fig. 93, N is a shunt. A good resistance box may be used; in practice, it is applied to the larger of the two resistances S, X. By it, the resistance between 3 and 4 is reduced to equality with that between 1 and 2. The resistance of one of the galvanometer circuits is adjusted by means of g and n; trial determines which one should be varied.

As S and X are supposed to be nearly equal, the galvanometer is made as nearly differential as convenient but need not be exactly adjusted.

The connections for the two positions of the commutator are shown in Fig. 93.

In order to find the conditions necessary for balance, the galvanometer deflections for the two positions of the rocker must be determined. Let the connections be as shown in Fig. 94. The resistances of the various circuits are indicated on the diagrams; i_5 and i_6 are the galvanometer currents, i_B the battery current. Let S_1 be the resistance

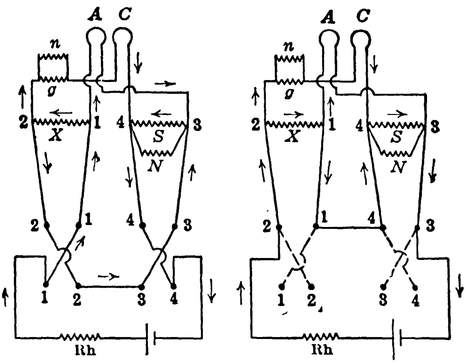


FIG. 93.—Showing connections for both positions of the commutator in Kohlrausch method.

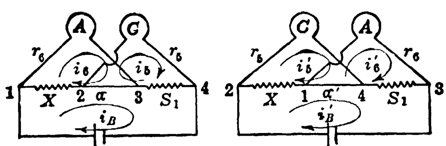


FIG. 94.—Mesh diagram for Kohlrausch method.

between 3 and 4; with the connections shown in the diagram, it is the parallel resistance of S and N (Fig. 93).

$$i_6(r_6 + \alpha + X) - i_B(X + \alpha) + i_5\alpha = 0.$$
$$i_5(r_5 + \alpha + S_1) - i_B(\alpha + S_1) + i_6\alpha = 0.$$
$$i'_6(r_6 + S_1 + \alpha') - i'_B(S_1 + \alpha') + i'_5\alpha' = 0.$$
$$i'_5(r_5 + X + \alpha') - i'_B(X + \alpha') + i'_6\alpha' = 0.$$

Solving for the desired currents and letting

$$M = \frac{i_B}{\alpha(r_5 + r_6 + S_1 + X) + (r_5 + S_1)(r_6 + X)},$$
$$M' = \frac{i'_B}{\alpha'(r_5 + r_6 + S_1 + X) + (r_5 + X)(r_6 + S_1)},$$
$$i_6 = M\{\alpha(r_5 + X) + X(r_5 + S_1)\},$$
$$i_5 = M\{\alpha(r_6 + S_1) + S_1(r_6 + X)\},$$
$$i'_6 = M'\{\alpha'(r_5 + S_1) + S_1(r_5 + X)\},$$
$$i'_5 = M'\{\alpha'(r_6 + X) + X(r_6 + S_1)\}.$$

Now, let the deflection per ampere due to the coil carrying i_5 be A, and that for the coil carrying i_6 be B. Then, as the two coils oppose each other, and their effects are very nearly equal, the resultant deflection will be:

For position *I*,

$$D_I = M[A\{\alpha(r_6 + S_1) + S_1(r_6 + X)\} - B\{\alpha(r_5 + X) + X(r_5 + S_1)\}].$$

For position *II*,

$$D_{II} = M'[A\{\alpha'(r_6 + X) + X(r_6 + S_1)\} - B\{\alpha'(r_5 + S_1) + S_1(r_5 + X)\}].$$

Conditions for Balance.—Suppose that by adjusting the resistances the deflection is made *nil* for *both* positions of the rocker; that is,

$$D_I = D_{II} = 0.$$

Then with the first position of the rocker,

$$\frac{A}{B} = \frac{\alpha(r_5 + X) + X(r_5 + S_1)}{\alpha(r_6 + S_1) + S_1(r_6 + X)};$$

and with the second position of the rocker,

$$\frac{A}{B} = \frac{\alpha'(r_5 + S_1) + S_1(r_5 + X)}{\alpha'(r_6 + X) + X(r_6 + S_1)}.$$

Equating these two expressions for A/B, an equation results of the form

$$C(X - S_1) = 0.$$

C is a function of the various resistances; all the algebraic signs entering into it are positive, so the condition $D_I = D_{II} = 0$ shows that

$$X = S_1.$$

It will be noted that this result is obtained regardless of the values of A and B, that is, without making the galvanometer exactly differential.

Suppose that D_I and D_{II} are equal but not zero, that is, that there are deflections of the same amount and toward the same end of the scale for both positions of the rocker. In the case where $i_B = i'_B$ and $\alpha = \alpha'$, that is, where there is no alteration of the circuit resistance, the relation between X and S is still

$$X = S_1 \quad \text{or} \quad X = \frac{NS}{N + S}$$

If the battery current and α alter slightly, owing to the different positions of the rocker, and the adjustments are made so that $D_I = D_{II}$, the departure from the relation $S_1 = X$ is so slight that it may be neglected even in precision work.

The Kohlrausch method of employing the differential galvanometer is the only one adapted to work of the highest precision.

The differential galvanometer method of comparing resistances has now been superseded by methods based on the Wheatstone bridge.

The Wheatstone Bridge.—This instrument which is so universally used in the determination of electrical resistance was invented by S. Hunter Christie, of the Royal Military Academy at Woolwich, England. He published an account of it in the *Philosophical Transactions*,* calling his invention "a differential arrangement." The variable ratio arms were added by Werner Siemens. In 1843, Sir Charles Wheatstone recalled attention to Christie's device, giving him full credit. At that time, Wheatstone was one of the leading scientists of Great Britain, and his name became associated with the instrument and has so remained.

Having the conductors arranged as in the diagram (Fig. 95), the current through the galvanometer will be zero only when $M/N = X/P$, for to have zero current in the galvanometer, the potential difference between the galvanometer terminals must be zero, or, in other words, the fall of potential along M must be equal to that along X, and the fall along N equal to that along P.

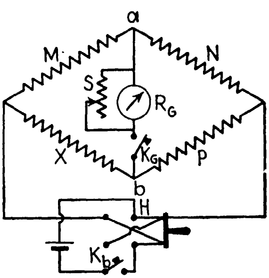


FIG. 95.—Diagram for the Wheatstone bridge.

Let I_M, I_N, I_X, I_P be the unvarying currents in the respective branches; then when

$$I_G = 0,\ MI_M = XI_X, \text{ also } NI_N = PI_P;$$

therefore

$$\frac{MI_M}{NI_N} = \frac{XI_X}{PI_P}.$$

But if no galvanometer current flows,

$$I_M = I_N \quad \text{and} \quad I_X = I_P;$$

so

$$\frac{M}{N} = \frac{X}{P} \quad \text{or} \quad X = \frac{M}{N}P.$$

Consequently, if three of the resistances are known, the fourth may be determined. M and N are called the balance or ratio arms, and P the rheostat arm of the bridge.

Auxiliary Apparatus.—Besides the bridge box, the other necessary pieces of apparatus are the battery (usually two or three cells), the keys K_B and K_G, the galvanometer, shunt, and reversing switch (see Fig. 95).

Keys.—Keys K_B and K_G are usually combined to form what is called a bridge key, which when depressed throws in first the battery and then the galvanometer (to eliminate the effects of inductance and capacitance). To avoid the chance of leakage to the galvanometer and also thermoelectric effects, care must be taken when manipulating this key not to touch the metalwork. The switch H is used to reverse the battery current and so to eliminate the effects of thermo-e.m.fs.

* Feb. 28, 1833.

The Galvanometer.—The galvanometer should be one that is not affected by variations of the local field and, if possible, should be critically damped; either a shielded Thomson or a D'Arsonval instrument may be used.

In selecting a D'Arsonval galvanometer for bridge work, the peculiarities of the instrument should be considered; for, suppose that the resistance in the bridge arms between the galvanometer terminals is low and that the instrument is one that, for critical damping, requires that it be in series with a high external resistance. On the passage of the current, as soon as the coil begins to move, an e.m.f. will be set up in the circuit, and the motion will be damped. Consequently instead of a sharp, decided movement of the index the motion will be so deliberate as greatly to increase the difficulty of deciding when the bridge is in balance. Obviously, it is impossible to select a galvanometer that will be critically damped for all combinations of the bridge arms, but with care a good working compromise may be secured. In general, with a given sensitivity, the shorter the period of the instrument the more satisfactory will its action be.

The Shunt.—The shunt S is a by-pass for the current and is placed between the galvanometer terminals. It is used during preliminary adjustments to protect the galvanometer against currents of abnormal strength. By means of the movable arm the value of the shunt resistance may be altered so that as the adjustment of the bridge nears perfection, a greater proportion of the current can be sent through the galvanometer. During the final adjustment, when full sensitiveness is desired, the movable arm should be turned so far to one side that it breaks the shunt circuit, and the entire current flows through the galvanometer. The various positions of the movable arm are usually so arranged that the fractional parts of the full current that can be sent through the galvanometer are 0.001, 0.01, 0.1, and 1. When using a Wheatstone bridge, one should *always begin measurements with the galvanometer heavily shunted. Violent deflections of the instrument are thus avoided.*

The Null Method of Making a Measurement.—The coil of unknown resistance is inserted as indicated in Fig. 96. All connections must be electrically perfect; all binding posts should be screwed up tightly but without using undue force; and the galvanometer should be heavily shunted. A rough idea of the magnitude of X is obtained as follows: Make $M = N$; insert 1 ohm in P; depress the key with care, being ready to release it immediately should the deflection of the galvanometer be violent. The deflection will be assumed to be toward the left. Note this deflection; replace the 1 ohm in P by a large resistance, for example, 5,000 ohms; and proceed as before. The deflection may be toward the right; it is then known that P, and consequently X, is between 1 ohm and 5,000 ohms. If one deflection is greater than the other, it shows that the proper value of P is nearer the resistance that gives the smaller deflection. Next, try 10 ohms in P. Suppose the deflection to be still toward the left; the proper value of P is between 10 and 5,000 ohms. Proceed in this manner, always narrowing the limits between which the right value of P must be located. Having obtained an apparent balance, the shunt resistance is increased, and a better approximation obtained. Suppose that the bridge finally balances with $P = 25 +$ ohms, 25 ohms being too small and 26 too large; then X is between 25 and 26 ohms. It is obvious that the determination of X is good to only about 2 or 3 per cent. Suppose that X is desired to 0.1 per cent; then, as the smallest coil in P is 1 ohm, P must be between 2,500 and 2,600 ohms in order that the

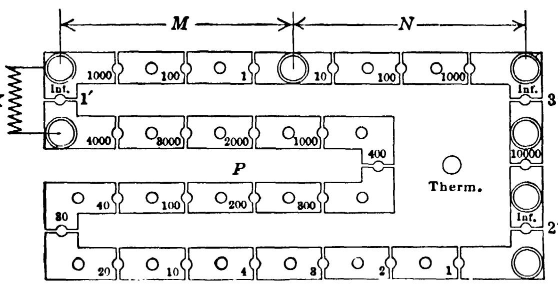


FIG. 96.—Wheatstone bridge top with plug connections.

smallest step may represent most nearly the desired precision. Accordingly, make P 2,500 ohms, and *alter the balance arms M and N to correspond.*

Make $M = 10$ ohms and $N = 1{,}000$ ohms; gradually increase P from 2,500 ohms until exact balance is obtained, with shunt removed; then

$$X = \frac{10}{1{,}000}P.$$

In the foregoing it has been supposed that with $M = N$ the deflections with $P = 1$ ohm and $P = 5{,}000$ ohms were one to the right, the other to the left. If they had both been to the right, and the one with $P = 1$ ohm of the lesser magnitude, then the proper value of P would have been below 1 ohm, and X less than 1 ohm. In such a case, proceed at once to change the ratio of M and N so that 1 ohm in P balances 0.01 ohm in X. In other words, make $M/N = 1/100$, and proceed as before with the adjustment of P. If P should be greater than 5,000 ohms, the proper procedure may be decided upon from the preceding discussion.

As a final precaution, all connections should be gone over to see if they are tight; then the final balance may be taken. The battery current should be reversed, and the test repeated; this is necessary in order to eliminate the effect of thermo-e.m.fs. The average result for P is used in calculating X.

The Deflection Method.—Referring again to the example just discussed, with $M = N$, P was between 25 and 26 ohms, and X could be determined only to 2 or 3 per cent. Now, suppose that with $P = 25$ ohms, the galvanometer deflects from its zero position 13 divisions to the left, and with $P = 26$ ohms 9 divisions to the right; then we may interpolate, for a change of 1 ohm in P causes the spot of light to vary 22 divisions, and the proper value of P for exact balance will be 25+ ohms, or 25.59 ohms. As the readings of the deflections cannot generally be taken with great accuracy, 25.6 ohms would be the value of P to be accepted. It is

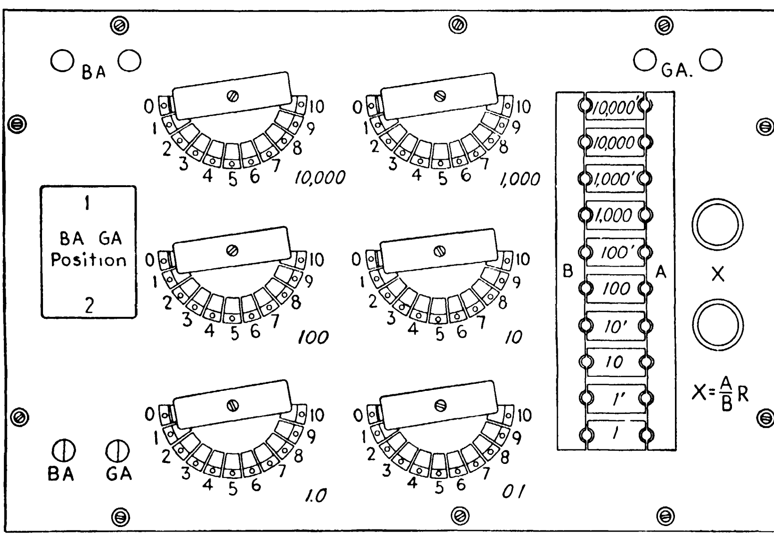


FIG. 97.—Wheatstone bridge with sliding contact switches and Schöne arrangement of ratio coils. (*Eppley Laboratory, Inc.*)

obvious that if this procedure is followed, X may be determined to ½ per cent without changing the ratio from $M = N$. If the ratio is changed, the precision may be still further increased. This method is used to gain precision when X is so small that P must be of small value. This method is slower in its application than the null method, and the arms of the bridge are more likely to be overheated. The battery e.m.f. must remain constant.

Arrangement of Bridge Top.—Figure 96 is the diagram of a bridge top with plug contacts. All the connections made in setting up the instrument are outside the box, the wires being attached to the appropriate binding posts.

It will be noted that there are three gaps at 1′, 2′, 3′, each marked "Inf."; removing the plug from any one of these gaps breaks the circuit.

The 10,000-ohm coil may be used in either the balance or the rheostat arm. Reversal of the bridge arms is accomplished by placing X at $3'$ instead of $1'$, the galvanometer terminal being changed also. By removing the plugs at $1'$ and $2'$ and $3'$, the coils are divided into two independent sections; occasionally, this is very convenient where the box is used for general laboratory purposes.

The rheostat arms now employed in the best bridges are arranged on the decade plan with rotative sliding contact switches (see Fig. 97). Rotative switches are now universally employed in portable bridges for field work.

In the bridge of Fig. 97, the ratio coils are arranged as suggested by A. Schöne[2] (see Fig. 98). There are two coils of each of the following denominations: 1, 10, 100, 1,000 ohms. All the coils have one terminal attached to the central copper bar which is inside the box, the other

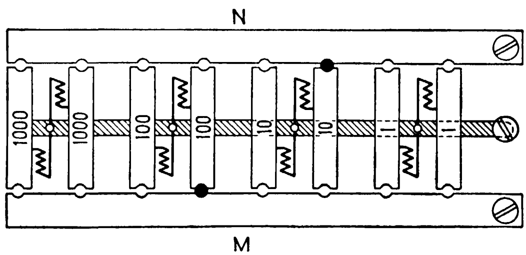


FIG. 98.—Schöne arrangement of ratio coils for Wheatstone bridge.

terminals being connected to the plug blocks. Two plugs are ordinarily used. When inserted as shown, $M/N = 100/10$. The advantages of this arrangement are the ease with which the arms may be reversed, the reduction in the number of plug contacts, and the possibility of obtaining the same ratio by using different coils of the same denomination, thus giving a check on the value of M/N.

Mueller Bridge.[3]—In resistance thermometry, (page 212) it is necessary to determine *changes* in the resistances of coils of comparatively low resistance (10 to 20 ohms, for example); and in work of high grade, specially designed Wheatstone bridges are employed. As changes are to be measured to 0.0001 ohm, it is necessary that uncertainties introduced by contacts within the bridge itself be eliminated. Bridges of ordinary design, which have a number of plug or sliding contacts in low-resistance arms, cannot be relied upon, for the variation in the contact resistances may be of the same order of magnitude as the accuracy desired in the measurement of the resistance changes of the thermometer coil.

To obtain the necessary fine adjustment of the rheostat arm of the bridge and at the same time remove uncertainty as to contact resistances, Mueller employs the shunt decade arrangement of Waidner and Dickinson, as shown in Fig. 99 in the form made by Eppley.

Use is made of the fact that when a shunt s is applied to a resistance r, the change in resistance is

$$r - \frac{rs}{r + s} = \frac{r^2}{r + s}.$$

Referring to Fig. 99, it is seen that in the x0.1 decade, for instance, a fixed resistance is shunted by one that can be varied in 10 steps, from a minimum to infinity. Each step *changes* the resistance of the decade by 0.1 ohm.

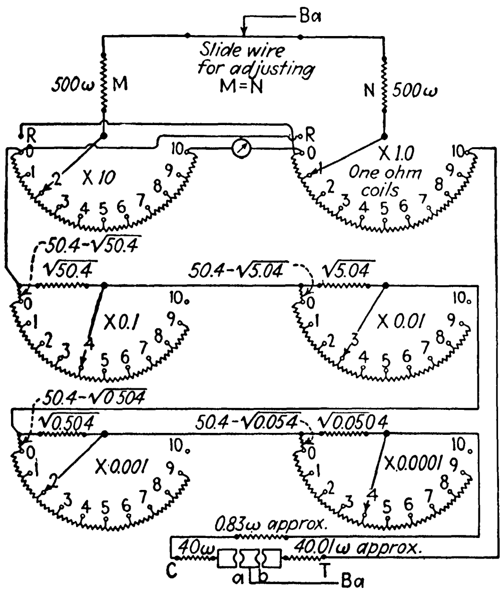


FIG. 99.—Mueller bridge; ratio and rheostat coils, Eppley arrangement.

If Δr is the change in decade resistance per step, for example 0.1, 0.01, etc., and n is the number of the step, then the diminution of resistance from that of the unshunted coil is $(10 - n)\Delta r$; consequently,

$$\frac{r^2}{r + s} = (10 - n)\Delta r,$$

and the required value of the shunt resistance is

$$s = \frac{r^2}{(10 - n)\Delta r} - r.$$

It is seen that r^2 should be divisible by all the integers below 10. One possible value of r^2 is 1.26; another, on which the bridge shown in Fig. 99 is based, is 50.4, the corresponding value of r being 7.099.

The minimum shunt resistance is obtained when the switch is at 0, in which case, for the x0.1 decade, $s = 50.4 - \sqrt{50.4}$. The resistance of a decade is

$$R_{\text{decade}} = r - (10 - n)\Delta r.$$

The sum of all four special decades when set at 0 is 9.167 ohms. The *change* in decade resistance is evidently Δr per step. The resistance of the coil between contact n and contact $n + 1$ is, by the preceding,

$$R_{n,n+1} = \frac{r^2}{\Delta r}\left(\frac{1}{90 - 19n + n^2}\right).$$

Evidently, if the ratio $r^2/\Delta r$ is made the same for all the decades, identical sets of coils may be used for the variable parts of the shunt resistances. The values are:

0 to 1	5.6 ohms
1 to 2	7.0 ohms
2 to 3	9.0 ohms
3 to 4	12.0 ohms
4 to 5	16.8 ohms
5 to 6	25.2 ohms
6 to 7	42.0 ohms
7 to 8	84.0 ohms
8 to 9	252.0 ohms
9 to 10	∞

The resistances that must be put in series with the variable portions of the shunts, that is, the minimum shunt resistances, are given by

$$\frac{r^2}{10\Delta r} - r.$$

As $r^2/\Delta r$ is the same for all the decades, the minimum shunt resistances are:

$50.4 - \sqrt{50.4}$ for the x0.1 decade

$50.4 - \sqrt{5.04}$ for the x0.01 decade

$50.4 - \sqrt{.504}$ for the x0.001 decade

$50.4 - \sqrt{.0504}$ for the x0.0001 decade.

Inspection of Fig. 99 will show that there must be at least 43 ohms in series with any sliding contact. The effects of variations of contact resistances are thus rendered negligible. The x0.1 dial is subtractive, when the switch is set at 0. A resistance of 10 ohms is inserted in series with the unknown. To bring the balance to zero when the other rheostat coils are set at 0, the resistance of 0.83+ ohm is included.

The Mueller bridge has equal ratio arms and is intended for use with four-terminal resistance thermometers. The lead resistance is eliminated by taking two balances (see page 214). The slide wire, the interchanging switches R, and the 40- and 40.01-ohm resistances are for the purpose of testing and of adjusting the bridge ratio to unity.

To make the test, the thermometer is disconnected from C and T, the battery connected to the block between a and b, the x10 and x1.0 dials set at zero, and plugs inserted at a and b. The bridge is then adjusted to balance. The arms M and N are then interchanged by setting the contacts on R, and a second balance obtained; if a difference is found, the setting of the slide wire is altered, and so on until the balance is not upset by the interchange of M and N.

Calibration of a Resistance Box.—In order that any changes in the coils may be detected, all resistance boxes and Wheatstone bridges should be calibrated occasionally. A convenient method of doing this with sufficient accuracy for general laboratory work, and one for which the apparatus is readily assembled, is shown in Fig. 100. It is a substitution method involving the use of the bridge principle.

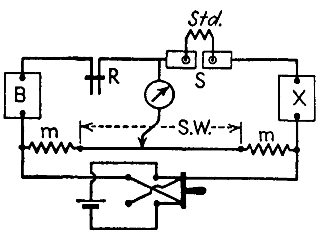


FIG. 100.—Connections for comparing resistance boxes.

Definiteness is the only requirement in the balancing resistances B and R. They must not change through heating or be erratic through ill-fitting plugs or defective sliding contacts.

The box to be calibrated is placed in series with the standard as shown. If the resistances of the equal extension coils m are properly adjusted to that of the slide wire, a given displacement of the slider may be made to correspond to an assigned percentage difference of X and S.

Suppose that the slide wire has a length of 1 meter and a resistance of 10 ohms. It is desired that a difference of $\frac{1}{10}$ per cent between X and S shall correspond to a displacement of the balance point of 10 cm. When $X = S$, the balance point is to be at the middle of the slide wire.

A displacement of 10 cm. to the left takes 1 ohm from the left-hand side of the bridge and adds it to the right-hand side. Then

$$\frac{1}{1.001} = \frac{m+4}{m+6} \qquad \text{and} \qquad m = 1{,}996 \text{ ohms.}$$

Assume a balance to be obtained with the 1-ohm coil in X by unplugging the corresponding coil in the balancing resistance, the standard S having been cut out. If needful, R may be used to bring the balance point to the middle of the slide wire. The reading of the slider is taken, and then the standard S is substituted for X. If $X = S$, there will be no change in the balance point; if X differs from S, balance is restored by moving the slider, and another reading taken. The difference of the two readings and the known displacement of the slider corresponding to 1/10 per cent allows the percentage difference of the two coils to be calculated nearly enough for practical purposes. The 1-ohm coil is thus compared with the standard ohm, then the 2-ohm coil with the sum of the 1-ohm and the standard, then the second 2-ohm coil with the first, and so on.

The 10-ohm coil may be compared with a 10-ohm standard, and the values of the 20-, 30-, · · · , 100-ohm, and other coils determined by comparison with those previously calibrated; or the whole series may be built up from the standard ohm.

After one box has been calibrated, others may readily be compared with it by this method.

Compensation for Large Thermo-e.m.f.—Occasionally, in special bridge arrangements, the unavoidable inequalities of temperature in the apparatus may cause thermo-e.m.fs. of sufficient magnitude to drive the galvanometer spot off the scale. In many cases, such e.m.fs. may be compensated, if they are reasonably constant, and the galvanometer can be used on closed circuit. Naturally, a key in this circuit is to be avoided.

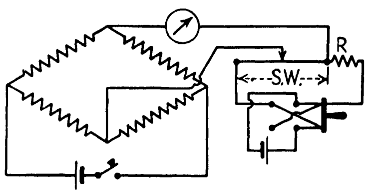


FIG. 101.—Arrangement for compensating large thermo-electromotive forces.

Referring to Fig. 101, R is a high resistance of several thousand ohms, and $S.W.$ a slide wire; by adjusting the slider, the galvanometer may be brought to zero; then the bridge is balanced as usual.

Slide-wire or Divided-meter Bridge.—In this simple form of Wheatstone bridge, shown in Fig. 102, two of the arms are replaced by the two sections of a uniform wire.

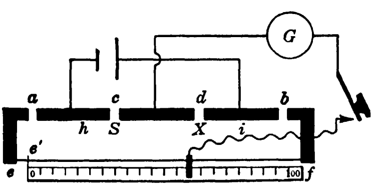


FIG. 102.—Diagram for slide-wire bridge.

In the diagram, the heavy lines represent copper strips of low resistance with gaps at a, b, c, d. On each side of each gap is a binding post so that the gap may be closed by a strap of low resistance or by a resistance coil, as is desired. Between e and f is stretched a wire of high resistance. It is about a millimeter in diameter and, in this form of bridge, intended to be just 1 meter long. A slider s makes contact at any desired point along the wire, and its position may be read off on a scale divided into millimeters. It is intended that e and the zero of the scale shall be coincident. The connections of battery and galvanometer are generally as shown, and this is usually the more sensitive arrangement. If the battery and galvanometer are interchanged, the resulting arrangement will be less disturbed by thermal e.m.fs. at the contact s.

To make a measurement, the simplest process would be to close the gaps at a and b by straps and place at d the unknown resistance X and at c the known resistance S, the best value of which would be about the same as X. The slider s would then be pressed carefully down upon the wire at one point after another until one was found where the galvanometer remained undeflected, that is, where opening and closing at s did not cause motion of the galvanometer index. Let l represent the scale reading in millimeters, that is, l = distance $e's$. As $e'f = 1{,}000$ mm., $sf = 1{,}000 - l$. Then, by the bridge principle, $X = S\frac{\text{res. of } sf}{\text{res. of } l}$. But the wire is assumed to be of uniform resistance per unit length, so that the resistances of the two parts of the wire are proportional to their lengths. Hence, if the resistances of the leads from e' to c and from f to d are assumed to be zero, $X = S\frac{(1{,}000 - l)}{l}$.

Resistances at Ends of Bridge.—The leads at the ends of the bridge may not be of negligible resistance, and there may be an inaccuracy in placing the zero of the scale opposite the end of the slide wire. The resistances between e' and the battery terminal h and between f and the battery terminal i act as prolongations of the corresponding ends of the slide wire. Let n_1 denote the number of millimeters of the wire that would have the same resistance as $e'h$, and n_2 the corresponding number for fi. Then if n_1 and n_2 were known, their effect would be allowed for by writing

$$X = S\frac{1{,}000 - l + n_2}{l + n_1}.$$

The values of n_1 and n_2 may be determined as follows:

After thoroughly cleaning the surfaces of contact, insert the straps at a and b. Place at c a coil of A ohms and at d a coil of B ohms. A and B must be different. Call the reading of the slider when the balance has been obtained l_1. Then

$$\frac{A}{B} = \frac{l_1 + n_1}{1{,}000 - l_1 + n_2}.$$

Interchange A and B, and obtain a new balance at l_2. Then

$$\frac{B}{A} = \frac{l_2 + n_1}{1{,}000 - l_2 + n_2}.$$

$$\frac{A}{A + B} = \frac{l_1 + n_1}{1{,}000 + n_1 + n_2}.$$

$$\frac{B}{A + B} = \frac{l_2 + n_1}{1{,}000 + n_1 + n_2}.$$

Therefore

$$n_1 = \frac{Bl_1 - Al_2}{A - B}.$$

Similarly,

$$n_2 = \frac{B(1{,}000 - l_2) - A(1{,}000 - l_1)}{A - B}.$$

Extension Coils.—If, in measuring X, the balance point falls near one end of the bridge, the shorter section of the wire cannot be determined with accuracy. Again, if equal coils are being compared, greater accuracy in setting s may be desired. In either case, it would be advantageous to use a longer wire. As this would be inconvenient, the same result is attained by inserting resistance coils at the gaps a and b. The effect of these, so far as the balance is concerned, is the same as if the wire had been extended on each side by the addition of such a length as would have the same resistance as the coils. Therefore, the equivalent lengths of these coils in millimeters of the slide wire must be determined. Let m_1 denote this quantity for the coil at a and m_2 that for the coil at b. Then, when measuring X,

$$X = S\frac{1{,}000 - l + n_2 + m_2}{l + n_1 + m_1}.$$

The values of m_1 and m_2 may be determined as follows: Place at c a resistance of E ohms and at d another known resistance of B ohms. Close both a and b by the straps, and balance the bridge. Call the reading l_1. Then

$$\frac{E}{B} = \frac{l_1 + n_1}{1{,}000 - l_1 + n_2}.$$

Now insert at a the coil, the equivalent length of which is desired, and obtain a new balance. Call the reading l_2. Then

$$\frac{E}{B} = \frac{l_2 + n_1 + m_1}{1{,}000 - l_2 + n_2}.$$

Therefore

$$m_1 = (l_1 - l_2)\left(\frac{E}{B} + 1\right);$$

and similarly for m_2.

In using equal extension coils, it is necessary to have the known resistance S (at c) so nearly equal to X that the balance point will come upon the slide wire. If the extension coils are unequal, then the ratio of S to X must be such as to accomplish this; or if S is of a fixed value, then the ratio of m_1 to m_2 must be properly adjusted. If S, m_1, and m_2 are all fixed, the range of the apparatus is limited.

With a more elaborate construction, and when used with due precautions to eliminate thermo-currents, contact resistances, etc., the slide-wire bridge becomes useful in work of the highest precision.

Carey Foster Method for Comparing Two Nearly Equal Coils.—This method is primarily designed for the comparison of nearly equal resistances; it therefore lends itself readily to the determination of temperature coefficients and to the verification of standard coils.

The coils to be compared are inserted at a and b (Fig. 102). Let their resistances be A and B; two approximately equal resistances S and S' are inserted at c and d; this insures that the balance point will fall near the middle of the slide wire. Let r be the resistance per unit length of the slide wire; l_0, the total length of slide wire in divisions; l_1 and l_2, the readings of the slider at balance. Then

$$\frac{S}{S'} = \frac{A + n_1r + l_1r}{B + n_2r + (l_0 - l_1)r}.$$

If A and B are interchanged,

$$\frac{S}{S'} = \frac{B + n_1r + l_2r}{A + n_2r + (l_0 - l_2)r}.$$

So

$$\frac{S}{S + S'} = \frac{A + n_1r + l_1r}{A + B + n_1r + n_2r + l_0r},$$

and

$$\frac{S}{S + S'} = \frac{B + n_1r + l_2r}{A + B + n_1r + n_2r + l_0r}.$$

Therefore

$$A - B = r(l_2 - l_1) = rD.$$

D is the difference of the two readings of the slider.

The difference of the resistances of the coils is seen to be equal to the resistance of the portion of the slide wire between the two balance points. It is to be noticed that this result is independent of S and S' and of n_1 and n_2, as well as of contact resistances, *if these quantities remain constant during the test.* Some convenient device must be adopted for interchanging the coils without removing them from their cooling baths and without handling; and as it is the difference of two nearly equal quantities that is involved, extraneous resistances due to change of connections and contacts must be carefully avoided. Figure 103 shows a form of bridge especially designed for carrying out the Carey Foster test.

The interchange of the coils is effected by raising the contact switch K, turning it through one-half a revolution, and then lowering it.

To adapt the device to the comparison of high as well as low resistances, several pairs of coils must be provided in order that a sensitive bridge arrangement may be maintained. A number of slide wires of different resistance per unit length, together with means for readily inserting them in the circuit, are also required. Three such wires are provided, any one of which may be used at will; and to obtain the effect of a wire of very low resistance, the slide wire proper may be shunted. This is effected by the link seen at the front of the switchboard.

Determination of the Resistance per Unit Length of the Slide Wire.—To determine r, the resistance per unit length of the slide wire, the process of measurement may be inverted. Let A and B be two coils of nearly equal resistance, so that the balance points will fall near the middle

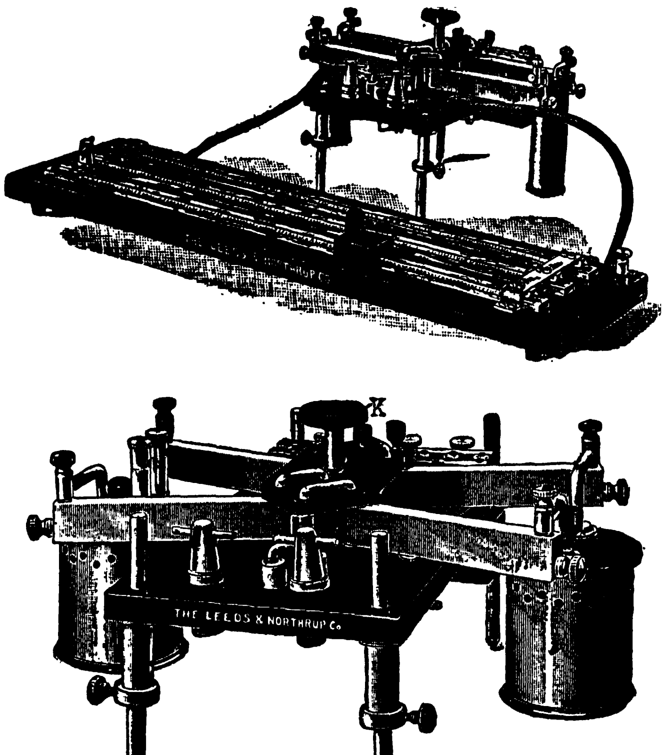


Fig. 103.—Carey Foster bridge.

of the bridge; only one of the coils need be known with exactness. A balance is effected in the usual way. Then, by the law of the bridge,

$$A - B = rD_1.$$

The known coil is then shunted by a resistance C which is of such a value that the balance points fall near the ends of the slide wire. This shunted coil is then compared with the coil A, which gives

$$A - \frac{BC}{B + C} = rD_2;$$

therefore

$$r = \frac{B^2}{(B + C)(D_2 - D_1)}.$$

When comparing coils of moderate resistance, it may happen that their difference is so great that the slide wire is not of sufficient length; in such a case, the larger resistance may be shunted, a comparison effected in the usual way, and the shunt allowed for. The accuracy with which the shunt must be known depends on its ratio to the resistance of the coil around which it is placed.

Calibration of a Slide Wire.—In dealing with the slide-wire bridge, it has been assumed that the wire is of uniform resistance per unit length. In order that troublesome corrections may be avoided, every effort should be made to have this assumption strictly true. If a case arises where the wire must be tested for uniformity, the bridge connections shown in Fig. 104 may be used. R is a set of coils of equal resistance, for instance the coils of a potentiometer of the Crompton type or those of a simple decade; α and β are adjustable resistors with an intermediate slide wire; γ and δ are resistors, the major portion of γ being a resistance box, while the major portion of δ is a slide wire. The contacts a and b are provided with fine adjustments. The resistances α, β, γ, δ are first adjusted until the galvanometer is in balance in both of the dotted positions. The total length of the slide wire is 1,000 mm., and, as indicated in the figure, R consists of 10 equal coils. After the preliminary adjustments are made, the galvanometer is connected to contact n, and balance obtained by moving the contact on the slide wire. If the wire were perfectly uniform, the reading would be $100n$. However, the actual reading is l_n; consequently, the correction at this point is $C = 100n - l_n$. A better distribution at calibrated points will be obtained if R is a Kelvin-Varley slide (page 273).

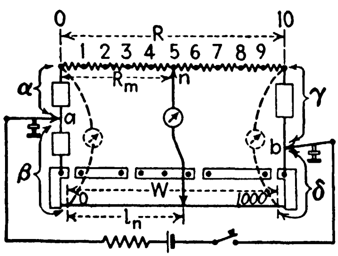


FIG. 104.—Connections for testing slide wire.

General Discussion of the Wheatstone Bridge.—In order to set up and use a Wheatstone bridge to the best advantage, certain points brought out by a study of the theory of the instrument must receive attention.

Galvanometer Current.—The general expression for the current through the galvanometer in terms of the resistances of the various bridge arms and the e.m.f. and resistance of the battery is readily deduced. However, a simpler and more useful formula is that connecting the total bridge current with the current through the galvanometer, for in many cases the bridge current is regulated by a rheostat rather than controlled solely by the e.m.f. of the battery and by the various resistances.

Suppose the connections to be those given in Fig. 105*A*, and assume, following Maxwell, that the meshes are traversed by currents $(x + y)$, x, and I_B, as shown in the figure; also that the resistances of the various branches are M, N, X, P, R_G, and B and that the e.m.f. of the battery is E. By Kirchhoff's corollaries,

$$(x + y)(M + R_G + X) - xR_G - I_BX = 0.$$
$$(x)(N + P + R_G) - (x + y)R_G - I_BP = 0.$$

Solving for y, which is the true galvanometer current, gives

$$y_1 = I_{G_1} = \frac{I_{B_1}(NX - MP)}{R_G(M + N + X + P) + (N + P)(M + X)}. \qquad (1)$$

The important case is when the bridge is nearly balanced. Then if B is

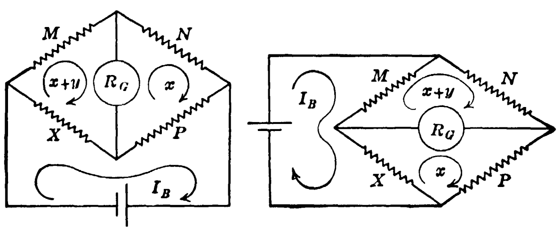


FIG. 105.—Mesh diagrams for Wheatstone bridge.

the total resistance in the battery circuit outside the bridge,

$$I_{B_1} = \frac{E}{B + \frac{(M + N)(X + P)}{M + N + X + P}} \quad \text{nearly enough.} \qquad (1a)$$

If the connections are as shown in Fig. 105*B*,

$$y_2 = I_{G_2} = \frac{I_{B_2}(MP - NX)}{R_G(M + N + X + P) + (M + N)(X + P)}, \qquad (2)$$

and

$$I_{B_2} = \frac{E}{B + \frac{(M + X)(N + P)}{M + N + X + P}} \quad \text{nearly enough.} \qquad (2a)$$

By use of these equations, it is easy to determine whether or not any galvanometer will give results of the desired precision, for the maker will furnish a statement of the sensitivity of the instrument, that is, the deflection per unit current at a scale distance of 1 meter, the galvanometer being in proper adjustment.

The Best Resistance for a Thomson Galvanometer When Used with a Wheatstone Bridge.—In general terms, the galvanometer should have a high or a low resistance depending on whether high or low resistances are to be measured. The magnitudes of the bridge arms being fixed, the galvanometer having the best resistance is that one that will give the greatest deflection when the arm P is changed from the condition of perfect balance by a given amount. It has previously been shown that if the coils of a Thomson galvanometer are always wound on the same bobbin, the galvanometer constant is given by $G = K\sqrt{R_G}$, the effect

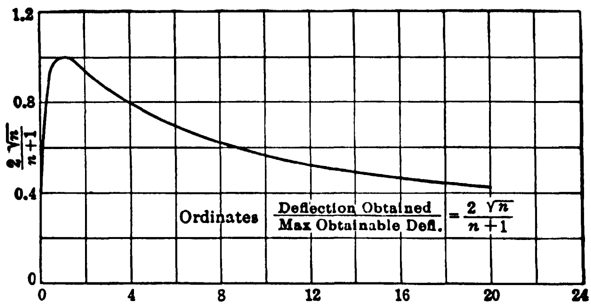


FIG. 106.—Showing effect of change of galvanometer resistance on the sensitivity of a Wheatstone bridge when a Thomson galvanometer is used.

of the insulation being neglected; consequently, if the time of vibration is kept constant, the deflection may be represented by

$$D = KI_G\sqrt{R_G}. \tag{3}$$

K is seen to be the deflection per ampere for an instrument having a resistance of 1 ohm. Using (1),

$$D = KI_G\sqrt{R_G} = \frac{KI_{B_1}(NX - MP)\sqrt{R_G}}{R_G(M + N + X + P) + (N + P)(M + X)}. \tag{3a}$$

This is to be made a maximum, R_G being the only variable.

It will be found that

$$\frac{dD}{dR_G} = 0 \quad \text{if} \quad R_G = \frac{(M + X)(N + P)}{M + N + X + P}. \tag{4}$$

Inspection shows that this result corresponds to a maximum value. Therefore, the galvanometer should have a resistance equal to the parallel resistance of the bridge arms between its terminals.

The effect of a departure from this best value of the galvanometer resistance may be seen from the following: Let the actual resistance of the galvanometer be n times that of the ideal instrument; that is, let

$$R_G = n\frac{(M + X)(N + P)}{M + N + X + P}. \tag{5}$$

Substitution of the value of R_G in (3a) gives as the corresponding value of the deflection

$$D = \frac{KI_{B_1}(NX - MP)\sqrt{n}}{(n+1)\sqrt{(M+X)(N+P)(M+N+X+P)}}.$$

The maximum deflection will be attained when $n = 1$. Consequently, the ratio of the actual to the maximum possible deflection of the galvanometer is

$$\frac{D}{D_{\text{max.}}} = \frac{2\sqrt{n}}{n+1}. \tag{6}$$

The values of this ratio are plotted in Fig. 106.

Best Position for the Galvanometer.—If the galvanometer is of fixed resistance, and if a definite e.m.f. is impressed at the bridge terminals, the magnitude of the galvanometer current may be considerably influenced by the relative positions of the galvanometer and battery.

For suppose that the actual value of P differs from that necessary for a perfect balance by an amount δP; then, referring to Eqs. (1) and (2) and substituting the approximate values of I_B,

$$I_{G_1} = \frac{-EM\delta P}{R_G(M+N)(X+P) + \dfrac{(M+N)(X+P)(N+P)(M+X)}{M+N+X+P}} = \frac{-EM\delta P}{\text{denom.}_1},$$

and

$$I_{G_2} = \frac{EM\delta P}{R_G(M+X)(N+P) + \dfrac{(M+N)(X+P)(N+P)(M+X)}{M+N+X+P}} = \frac{EM\delta P}{\text{denom.}_2}.$$

The better arrangement will be the one corresponding to the equation having the smaller denominator.

$$\begin{aligned}\text{Denom.}_1 - \text{denom.}_2 &= R_G[(M+N)(X+P) - (M+X)(N+P)] \\ &= R_G[(M-P)(X-N)]. \end{aligned} \tag{7}$$

From this it is seen that if the opposite resistances are equal, the sensitiveness is the same for both arrangements. If the algebraic sign of the bracket is +, denom.$_2$ is less than denom.$_1$ and the second connection should be used; if it is −, the first is to be employed.

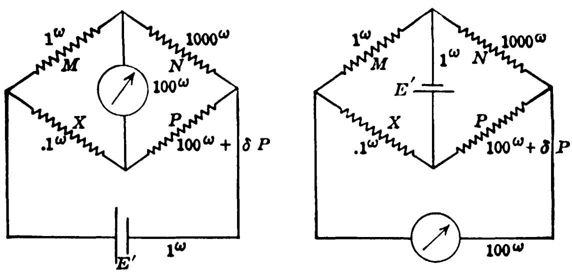
The better arrangement is the one where the galvanometer joins the junction of the two highest resistance bridge arms to the junction of the two lowest. An exact discussion of the more complete formula which includes the e.m.f. and the resistance of the battery gives

$$\text{Denom.}_1 - \text{denom.}_2 = (R_G - B)(M-P)(X-N). \tag{8}$$

Considering the battery and the galvanometer, the one having the higher resistance should join the junction of the two highest to the junction of the two lowest bridge arms. The importance of the proper relative position of the battery and galvanometer increases with the disparity of the bridge arms.

As an illustration, consider the bridge shown in Fig. 107 where there is a great difference in the bridge arms. Denom.$_1$ − denom.$_2$ is +, so the second connection should be used.

The second arrangement is about twenty-three times as sensitive as the first. However, as the total bridge current is about forty-four times as great, the possibility of overheating a low-resistance arm of the bridge



$I_B = \frac{E'}{92}$ Approx. $\qquad I_B = \frac{E'}{2.1}$ Approx.

$I_G = \frac{E'\delta P}{10{,}200{,}000} \qquad I_G = \frac{E'\delta P}{440{,}000}$

FIG. 107.—Showing effect of relative positions of battery and galvanometer on the sensitiveness of a Wheatstone bridge.

should be kept in mind. This is seen from the following table, where E' has been assumed as 2 volts.

WATTS DISSIPATED IN THE BRIDGE ARMS

	M	*N*	*X*	*P*
With the first connection	0.000004	0.004	0.00004	0.04
With the second connection	0.9	0.0009	0.09	0.00009

In the second case, there is a possibility that the arm M may be overheated if the current be kept on continuously.

Sensitiveness Attainable with the Wheatstone Bridge.—The sensitiveness of a bridge arrangement may always be increased by increasing the bridge current. The limit to this increase is fixed by the carrying capacity of the bridge arm that will safely stand the least current.[4]

Equations (1*a*) and (1) show that the battery resistance always diminishes the sensitiveness of the bridge by reducing the current I_B. However, the statement is sometimes made that a high-resistance source should be used when measuring high resistances. The reason for the statement is that with a given number of cells all having the same e.m.f. and all having the same resistance, the maximum current in the external circuit will be obtained if the series-parallel arrangement of the cells is such that the net battery resistance is equal to that of the external circuit. The battery resistance is simply a necessary evil; if it were zero, the maximum current would be obtained by arranging the cells in series.

When a bridge assembly is used, the unknown resistance x has a predetermined value, and a measurement to a designated percentage is usually required. If, for example, this is 0.1 per cent, and the rheostat arm of the bridge P is adjustable to single ohms, the minimum allowable resistance of P is 1,000 ohms, if the zero method of balancing is employed. The ratio $X/P = M/N$ is thus, in a measure, determined. Referring to Fig. 105A, if the battery is of negligible resistance, the Thévenin voltage which would appear across a break in the galvanometer circuit has a definite value. The resistance in series with this voltage decreases as N is decreased; consequently, if there were no limitations imposed by heating of the bridge coils, the resistances of the arms N and M would be made small. For a discussion of bridge sensitiveness when the heating of the coils is not considered, see the paper by Oliver Heaviside.* In several of the reprints of Heaviside's collected papers, there are typographical errors in the final equations.

With coils of like construction but of different resistances, the allowable energy losses are the same; that is,

$$(I')^2R = k \qquad \text{or} \qquad I'\sqrt{R} = \sqrt{k}, \qquad \text{a constant.} \tag{9}$$

I' is the allowable current and k the allowable heating loss in watts; k depends upon the construction and manner of using the coils.

In planning new work, it is frequently of importance to calculate the deflection that would be obtained if a particular galvanometer were used.

Suppose that the maximum allowable bridge current I_B' is fixed by the heating in the arm X and that I_X' is the greatest current that can be employed in that arm.

If a Kelvin galvanometer of the best resistance is employed and the bridge is out of balance by a small amount δP, then, using Eqs. (1), (3), (4) and the approximate relation

$$I_B = \left(\frac{M + X}{M}\right)I_X,$$

the deflection of the galvanometer is given by

$$D = KI_G\sqrt{R_G} = K\left(\frac{\delta P}{P}\right)(I_X'\sqrt{X})\frac{P}{2\sqrt{(P + X)(N + P)}}. \tag{10}$$

The sensitiveness that can be obtained is seen to be proportional to the square root of the allowable power loss in the arm that limits the bridge current.

* *Phil. Mag.*, vol. 45, 1873, p. 114.

Now, suppose that a critically damped moving-coil galvanometer is to be used.

In this case, let R_G' be the total resistance of the galvanometer branch of the circuit. R_G' will be made up of the resistance of the galvanometer itself plus any resistance that it is necessary to add in order to bring about critical damping. Let R_C be the *total* resistance of the galvanometer circuit which is required for critical damping; then, as the bridge is nearly balanced,

$$R_G' + \frac{(M+X)(N+P)}{M+N+X+P} = R_C, \qquad \text{a definite resistance;}$$

also

$$I_B = \left(\frac{M+X}{M}\right)I_X',$$

and

$$\frac{M}{N} = \frac{X}{P}.$$

Suppose that the resistance of the arm P is increased from the value necessary for a perfect balance by a small amount δP; then the galvanometer current becomes

$$I_G = \frac{(M+X)I_X'\delta P}{R_C(M+N+X+P)} = \frac{XI_X'\delta P}{R_C(X+P)}$$
$$= \left(\frac{1}{R_C}\right)\left(\frac{\delta P}{P}\right)(I_X'\sqrt{X})\left(\frac{P\sqrt{X}}{X+P}\right),$$

and the deflection is

$$D = I_G S_I = S_V\left(\frac{\delta P}{P}\right)(I_X'\sqrt{X})\left(\frac{P\sqrt{X}}{X+P}\right). \tag{11}$$

S_V is the volt sensitivity and S_I is the current sensitivity of the galvanometer when it is critically damped.

As in the previous case, the sensitiveness of the bridge is proportional to the square root of the allowable power loss in the arm that limits the bridge current.

If the resistance of the bridge R_B is so great that the galvanometer is underdamped, a resistance must be put in parallel with the bridge between the galvanometer terminals. Let the value of this parallel resistance be R_S, and let R_K be the resistance that it is necessary, in any case, *to put in series* with the galvanometer in order to attain critical damping. That is, let

$$R_C = R_G + R_K.$$

In this particular case, R_K is the parallel resistance of the bridge and the resistance with which it is shunted, or

$$R_K = \frac{R_S R_B}{R_S + R_B};$$

also,

$$R_B = \frac{(M + X)(N + P)}{M + N + X + P}.$$

The galvanometer current when the bridge is slightly out of balance is

$$I_G = \frac{(M + X)I_X' R_S \delta P}{[R_G(R_S + R_B) + R_B R_S](M + N + X + P)}$$
$$= \left(\frac{\delta P}{P}\right)(I_X'\sqrt{X})\left(\frac{P\sqrt{X}}{X + P}\right)\left(\frac{R_K}{R_B}\right)\left(\frac{1}{R_G + R_K}\right).$$

Therefore

$$D = S_I I_G = S_V\left(\frac{\delta P}{P}\right)(I_X'\sqrt{X})\left(\frac{P\sqrt{X}}{X + P}\right)\left(\frac{R_K}{R_B}\right). \tag{12}$$

The sensitivity is seen to be diminished in the ratio R_K/R_B.

MEASUREMENT OF LOW RESISTANCE

Wheatstone-bridge Method.—When the value of X becomes small—0.1 ohm or less—it is difficult to determine it accurately, owing to the uncertainty due to contact resistances introduced at the binding posts where X is clamped to the bridge. For example, such resistance for a No. 12 wire clamped by a binding post may be about 0.00015 ohm. Again, all standard low resistances and the shunts used in current measurements are provided with potential terminals, and what is desired is their four terminal resistances. The following method of comparing such resistances with a standard is very useful, since it requires no special apparatus.

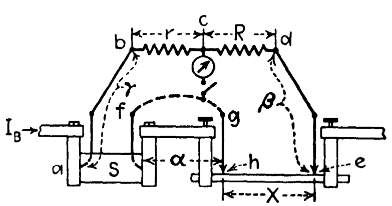


FIG. 108.—Wheatstone-bridge method for comparing low resistances.

Let it be required to measure the resistance of a given length of the bar X. Connect it in series with the standard resistance S, as indicated, and attach two potential terminals h and e at the proper points (Fig. 108). These terminals may be clamps making contact with the bar by pointed screws or knife-edges.

In order to make the measurement, the intermediate resistance α between the potential terminals f and g must be eliminated. To do this, the galvanometer lead is first connected at f, and a balance obtained. The necessary values of r and R may be denoted by r_f and R_f. Then

$$\frac{r_f}{R_f} = \frac{S}{X + \alpha}.$$

A second balance is taken with the galvanometer lead connected to g. The necessary values of r and R may be denoted by r_g and R_g; then

$$\frac{r_g}{R_g} = \frac{S + \alpha}{X}.$$

α can be eliminated from these two equations, giving as a result

$$X = S\left(\frac{R_g}{r_f}\right)\left(\frac{R_f + r_f}{R_g + r_g}\right),$$

or, if $R + r$ is constant,

$$X = S\left(\frac{R_g}{r_f}\right).$$

It has been assumed that the resistance α remains constant. Consequently, all clamps and connections must be firmly set up, and α itself should be of so large a cross section that it will not heat with the largest

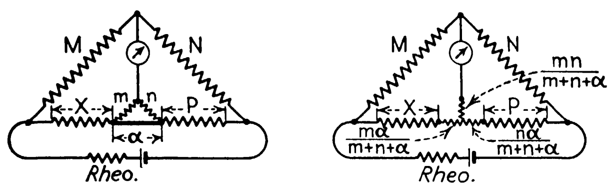


FIG. 109.—Kelvin bridge network and transformation to Wheatstone network.

current which is used. The following points should be attended to: The resistance of α must be made as small as possible. The total bridge current I_B should be as large as is consistent with absence of heating in the various bridge arms. r and R should be small; their magnitude is limited by the fact that they are usually adjustable to single ohms, and a certain definite percentage precision is usually required in the results; they should be large enough so that there is no danger of heating and so large that the resistances of the connection wires γ and β are negligible. The standard resistance S should have ample current-carrying capacity. It may be necessary to keep the temperature of X down by immersion in an oil bath.

Kelvin or Thomson Double Bridge.[5]—The elimination of the intermediate resistance α may also be accomplished by means of the Kelvin bridge. The scheme embodied in this instrument is that most frequently employed for the precision comparison of low resistances; it is also commonly used for special bridges designed for the rapid measurement of the conductivity of samples of wire.

Inspection of the theoretical diagram will show that this arrangement differs from the Wheatstone bridge in the addition of two auxiliary resistances m and n, which are placed in series and shunted around the resistance α, which is to be eliminated; one galvanometer terminal is connected to the junction of m and n (see Fig. 109).

The conditions necessary for a balance may be shown thus:

Referring to Fig. 109, the delta formed by m, n, and α is transformed to the equivalent wye by the method given on page 391, and the resulting network is seen to be that of a Wheatstone bridge, the value of the arms being as indicated. Therefore

$$\frac{M}{N} = \frac{X + \dfrac{m\alpha}{m + n + \alpha}}{P + \dfrac{n\alpha}{m + n + \alpha}}.$$

$$X = \frac{MP}{N} + \frac{\alpha(Mn - Nm)}{m + n + \alpha}.$$

Obviously, if the resistances are adjusted so that $m/n = M/N$, the last member of the second equation becomes zero, and $X = MP/N$. The measurement is then independent of α.

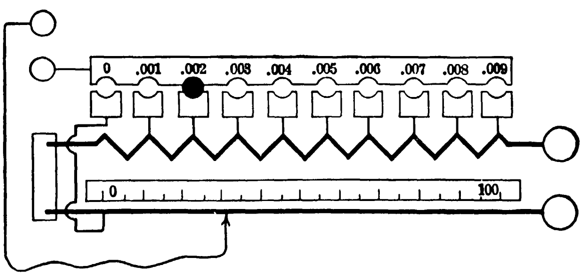


FIG. 110.—Variable standard resistance for use with Kelvin bridge.

For general laboratory purposes, P may be a variable standard and is frequently a slide wire or, better, a resistance divided into tenths, the last tenth being a slide wire of perhaps 0.001 ohm. This standard, shown diagrammatically in Fig. 110, should have ample carrying capacity—50 amp. at least.

A convenient form of this bridge, employing such a low-resistance standard and adapted to commercial work, is shown diagrammatically in Fig. 111. In this arrangement, it is assumed that the resistances of the connecting leads cc', dd', ee', and ff' are so small, compared with the resistance with which they are in series, that their effects are negligible.

With the Kelvin bridge as it is actually used, X, P, and α are always small, so that it is necessary to control the bridge current by a rheostat; the current should be as large as is compatible with accuracy. Its limiting value, and therefore the sensitivity of the bridge, is fixed by the carrying capacity of the resistances employed; consequently, ample provision must be made for dissipating the heat generated in the various arms.

The resistance to be measured must be provided with potential terminals or their equivalent. In dealing with rods, the contacts at c and d may be made by soldering small wires across the rod, the superfluous solder being carefully removed, or, more conveniently, by point or knife-edge clamps.

With massive conductors it is absolutely necessary that the relative positions of the potential and current terminals be such that the stream-lines between c and d are in their normal position; for instance, in measuring a low-resistance shunt, such as is used for large currents on

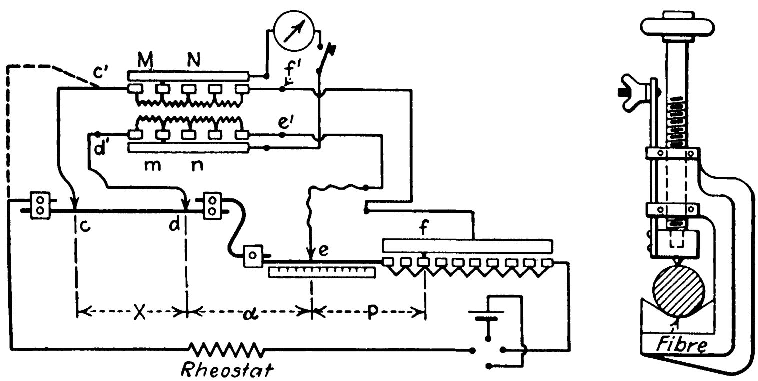


FIG. 111.—Diagram of Kelvin bridge.

switchboards, a serious error may be introduced, even if c and d are at the proper points, if the current is not led into the short, heavy terminals exactly as it is to be in the subsequent use of the instrument.

Expression for Galvanometer Current.—Let the arrangement of the conductors and the mesh currents be as shown in Fig. 112. The mesh equations are:

$$(x + y)(M + R_G + m + X) - yR_G - zm - I_BX = 0.$$
$$y(N + P + n + R_G) - (x + y)R_G - zn - I_BP = 0.$$
$$z(m + n + \alpha) - (x + y)m - yn - I_B\alpha = 0.$$

Solving for x, which is the galvanometer current, gives

$$x = I_G = I_B \frac{NX - MP + \dfrac{\alpha(mN - nM)}{m + n + \alpha}}{R_G(M + N + X + P) + (M + X)(N + P) + [\ ]} \quad (13)$$

where [ ] =

$$\frac{n\alpha(M + X + R_G) + m\alpha(N + P + R_G) + mn(M + N + X + P + \alpha)}{m + n + \alpha}. \quad (14)$$

If the galvanometer stands at zero, I_G is zero, and

$$X = \frac{MP}{N} - \frac{\alpha(mN - nM)}{N(m + n + \alpha)} = \frac{MP}{N} - \alpha n \frac{\left(\frac{m}{n} - \frac{M}{N}\right)}{m + n + \alpha}. \tag{15}$$

Convenience dictates that the second term on the right-hand side of the equation be made zero; this is accomplished if m and n are so adjusted that $m/n = M/N$. In other words, M, N, m, n should fulfill the conditions for the resistances in an ordinary Wheatstone bridge, in which case X is independent of the intermediate resistance α and of the auxiliary conductors m and n, as has just been shown in a somewhat less general fashion.

In the commercial instrument, the ratio arms are usually mounted in the same box and are capable of variation, being made up of coils so chosen that the relation $m/n = M/N$ is conveniently attained. If this condition is not exactly fulfilled, the error due to neglecting the last term in (15) will diminish as α is decreased; therefore the resistance of the intermediate connection should be made as small as possible, especially when measuring small resistances. One obvious test for the accuracy of the relation $m/n = M/N$ may be made by temporarily increasing α or, better, by altogether removing the connection, thus breaking the circuit. If the galvanometer remains in balance with α both open and closed, the adjustment is correct.

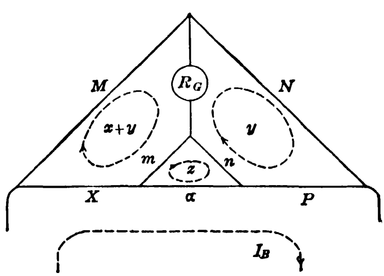


FIG. 112.—Mesh diagram for Kelvin bridge.

Best Resistance for a Thomson Galvanometer When Used with a Kelvin Bridge.—The best resistance for the galvanometer may be found as follows: The resistance α is always made as small as possible; assume that it is negligible in comparison with both m and n; then [ ] in Eq. (14) reduces to

$$[\ ] = \left(\frac{mn}{m + n}\right)(M + N + X + P),$$

and

$$I_G = I_B \frac{(MP - NX)}{\left(R_G + \frac{mn}{m + n}\right)(M + N + X + P) + (M + X)(N + P)}.$$

With Thomson galvanometers having coils of equal dimensions, the relation between the current, the resistance, and the deflection is

$$D = KI_G\sqrt{R_G}.$$

If the resistance of the arm P differs from that necessary for a perfect balance by a small amount δP, the galvanometer deflection is

$$D = KI_B \frac{M\,\delta P\sqrt{R_G}}{\left(R_G + \frac{mn}{m+n}\right)(M+N+X+P) + (M+X)(N+P)}.$$

D is to be made a maximum by adjusting R_G. On differentiating and equating the result to zero, it will be found that the maximum value of D will be obtained when

$$R_G = \frac{mn}{m+n} + \frac{(M+X)(N+P)}{M+N+X+P}. \qquad (16)$$

That is, the galvanometer resistance should be equal to that of the remainder of the circuit as viewed from the galvanometer.

Sensitiveness Attainable in Measurements with the Kelvin Bridge.—The sensitiveness of the bridge increases proportionally to I_B, the limit being reached when one of the arms begins to heat unduly. As the bridge is usually employed, the limit will be set by either X or P.

When the bridge is nearly balanced, and the resistance α is small compared with m and n,

$$I_B = I_X\left[\frac{M+X}{M} + \frac{\alpha}{M+N}\right] = \left[\frac{M+X}{M}\right]I_X \qquad \text{approx.}$$

If a Thomson galvanometer of the best resistance is employed, the value for R_G reduces to

$$R_G = \frac{P(M+X+m)}{X+P},$$

and

$$I_G = \frac{I_B M\,\delta P}{2(N+P)(M+X+m)}.$$

Substituting these values in the expression for D and reducing gives

$$D = K\left(\frac{\delta P}{P}\right)(I'_X\sqrt{X})\frac{P\sqrt{X}}{2\sqrt{(P+X)[X(N+P)+mP]}} \qquad (17)$$

If $m = 0$, this reduces to Eq. (10), which applies to the Wheatstone bridge.

If the same conductors M, N, X, P are arranged as a Wheatstone bridge and then as a Kelvin bridge, it will be found from (10) and (17) that with a Thomson galvanometer the Wheatstone-bridge arrangement will be the more sensitive in the ratio

$$\frac{D_W}{D_K} = \sqrt{1 + \frac{n}{N+P}}.$$

If a critically damped moving-coil instrument is to be used, and the resistance of the bridge is so low that the instrument is overdamped, it will be necessary to increase the resistance of the galvanometer circuit. It will be assumed that the resistance α is low compared with m and n. The arrangement then becomes the equivalent of a Wheatstone bridge where the resistance of the galvanometer branch is

$$R'_G = R_G + \frac{mn}{m+n} + R,$$

R being the resistance added in order to obtain critical damping.

Let the resistance of the circuit necessary for critical damping be R_C; then

$$R_C = R_B + R'_G,$$

where

$$R_B = \frac{(M+X)(N+P)}{M+N+X+P}.$$

When the bridge is slightly out of balance,

$$I_G = \frac{I'_X(M+X)\delta P}{(R_C - R_B)(M+N+X+P) + (M+X)(N+P)}$$

$$I_G = \frac{1}{R_C}\left(\frac{\delta P}{P}\right)\left(I'_X\sqrt{X}\right)\left(\frac{P\sqrt{X}}{X+P}\right)$$

$$D = S_I I_G = S_V\left(\frac{\delta P}{P}\right)\left(I'_X\sqrt{X}\right)\left(\frac{P\sqrt{X}}{X+P}\right). \tag{18}$$

If the instrument is underdamped, it will be necessary to place a shunt around the bridge between the galvanometer terminals. In this case, if R_G is the resistance of the galvanometer; and R_S, that of the shunt,

$$I_G = \frac{I'_X(M+X)R_S\delta P}{\left[\left(\frac{mn}{m+n}\right)(R_G+R_S) + R_G R_S\right](M+N+X+P) + (M+X)(N+P)(R_G+R_S)}.$$

The bridge resistance is

$$R_B = \frac{mn}{m+n} + \frac{(M+X)(N+P)}{M+N+X+P}.$$

$$I_G = \frac{I'_X(M+X)R_S\delta P}{[R_G(R_S+R_B) + R_S R_B](M+N+X+P)}.$$

The resistance which, in any case, it is necessary to add to that of the galvanometer to produce critical damping is R_K; that is,

$$R_C = R_G + R_K.$$

The value of R_K is

$$R_K = \frac{R_S R_B}{R_S + R_B}.$$

Therefore

$$I_G = \frac{I'_X X R_K \delta P}{(R_G + R_K)(X + P)R_B},$$

and

$$D = S_I I_G = S_V\left(\frac{\delta P}{P}\right)\left(I'_X\sqrt{X}\right)\left(\frac{P\sqrt{X}}{X + P}\right)\left(\frac{R_K}{R_B}\right). \tag{19}$$

Compare formula (12) on page 185.

Precision Measurements with the Kelvin Bridge.[5]—In the proofs already given, M, N, m, n are the total resistances of the various bridge arms; that is, they are the sums of the resistances of the coils in the arms and of the necessary leads. The lead resistance can never be zero; consequently, though the coils themselves are adjusted so that the relation, $M/N = m/n$ is fulfilled; yet when they are connected into circuit by the necessary leads, this relation will be slightly disturbed, and the elimination of α from the results will not be complete. In the careful comparison of resistance standards, this matter is of importance, for of necessity the resistances of the potential terminals are included in the bridge arms.

If the resistances in the arms are separated into two parts, that in the coils being denoted by the subscript $_C$, and that in the leads by the subscript $_L$, Eq. (15) becomes

$$X = P\left(\frac{M_C + M_L}{N_C + N_L}\right) - \frac{\alpha\left(\frac{m_C + m_L}{n_C + n_L} - \frac{M_C + M_L}{N_C + N_L}\right)(n_C + n_L)}{m_C + m_L + n_C + n_L + \alpha}. \tag{20}$$

If the elimination of α from the result is complete, the balance of the bridge will not be upset when α is greatly increased or even made infinity by breaking the connection between X and P. Therefore the test for the proper adjustment of the auxiliary ratio is that the bridge remain in balance with α closed and with α open. In precision measurements, it is essential that α be made as low as possible and less than X.

Reeves Method for Adjusting the Ratio Arms to Eliminate α.—Based on the foregoing, the process of adjustment to eliminate α is, with α in place, to adjust the main ratio M/N until the bridge is balanced, then to remove α and rebalance by changing the ratio m/n. This second adjustment will throw out the first, so α must be replaced and M/N readjusted, and so on until by successive approximations such an adjustment is attained that the balance is maintained with α either closed or open. This process of successive balances eliminates all questions as to the exact values of m and n and their leads.

When the elimination of α is complete,

$$X = P\left(\frac{M_C + M_L}{N_C + N_L}\right).$$

The lead resistances to M and N must be determined and allowed for.

Wenner Method for Eliminating the Effects of Lead Resistances and α.—For this method of working the Kelvin bridge, it is necessary that the slides on the main and auxiliary ratio arms be mechanically connected so that the relation $M_C/N_C = m_C/n_C$ is always maintained. The coils are adjusted with this in view.

Inspection of formula (20) shows that if, in addition, the resistances of the leads to the ratio coils are adjusted so that

$$M_C N_L = N_C M_L$$

and

$$m_C n_L = n_C m_L,$$

then

$$X = P\left(\frac{M_C}{N_C}\right).$$

Therefore M_L and m_L are made adjustable by including in each a mercury slide resistance (a and b, Fig. 113). This consists of an ebonite tube about 12 cm. long with a 3-mm. bore. The terminals are at the upper and lower ends of the tube, and an amalgamated copper plunger serves to displace and short-circuit more or less of the mercury. This form of adjustable resistance is remarkably definite in its action.

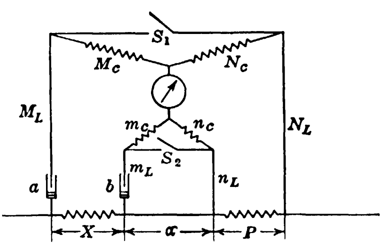


FIG. 113.—Wenner arrangement for eliminating the effect of lead resistances in the Thomson bridge.

To carry out the adjustment, it is necessary to add two switches S_1 and S_2, as shown in Fig. 113, by which the arms $M_C + N_C$ and $m_C + n_C$ may be short-circuited.

The final balance is attained by four steps:

1. With both S_1 and S_2 open, the bridge is balanced as usual by adjusting the ratio arms.

2. The switch S_1 is closed; the balance will be upset; it is restored by adjusting M_L by means of the rheostat a. This makes

$$M_L N_C = N_L M_C, \qquad \text{very closely.}$$

3. The switch S_1 is opened and S_2 is closed, and the balance restored by adjusting the value of m_L by means of the rheostat b. This makes

$$m_C n_L = n_C m_L, \qquad \text{very closely.}$$

4. With both S_1 and S_2 open, the bridge is finally balanced by adjusting the double ratio slides. If this last adjustment requires a considerable change in the setting of the ratio slides, the adjustments are repeated.

Measurement of Resistances in Permanently Closed Circuits.—For a method of measuring a resistance included in a circuit that cannot be opened, see page 83.

MEASUREMENT OF HIGH AND OF INSULATION RESISTANCE[6]

The measurement of insulation resistance, using direct-current potentials of a few hundred volts, is of great practical importance because of its utility as a means of separating the good from the defective insulated wires during the process of manufacture. Also, specifications as to insulation resistance as measured by direct currents are inserted in contracts.

It is to be understood that the results of this test are not "resistances" in the same sense as those obtained for metallic conductors by use of the Wheatstone bridge. As the test is ordinarily carried out, the results give no means of calculating the current which will finally flow through the insulation of the wire under the prolonged application of a direct-current e.m.f. (see page 198, "Absorption Effects"). It is to be understood that the current that will flow through the dielectric when applied e.m.f. is periodic, especially at audio- and radio-frequencies, involves a very different "resistance" from that determined by this method; at a given voltage it is distinctly lower, due to the energy dissipated in the dielectric (see page 368).

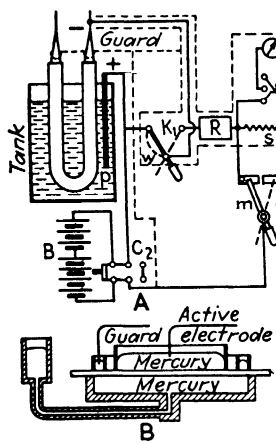


FIG. 114.—*A*, connections for measuring insulation resistance; *B*, electrodes for measuring plates.

Direct-deflection Method.—Insulation resistances have very high values and may be several hundred or several thousand megohms, a megohm being 1,000,000 ohms. The method usually employed in these measurements is really one of substitution; the necessary apparatus is shown in Fig. 114. The tank is filled with water in which the sample is immersed.

The galvanometer G should be of the moving-coil type and very sensitive; an instrument having a current sensitivity of about 1×10^9 and of approximately 1,000 ohms resistance will be satisfactory.. This galvanometer must have a good law of deflection; that is, the deflection must be directly proportional to the current, and it must have a definite zero reading. It should be so supported that it is free from mechanical vibration and thoroughly insulated electrically. C_1 is a double-pole, double-throw switch for reversing the galvanometer current, as it is necessary to keep the deflections in the same direction on account of possible irregularities of the zero reading due to the coil being slightly magnetic. The shunt S should be of the Ayrton universal type, for by selecting one of the proper resistance the galvanometer may be critically damped, thus enabling the readings to be taken in the shortest possible time. With this type of shunt, when used in the manner indicated, the damping of the galvanometer is independent of the multiplying power; consequently, the instrument will not be overdamped even though it is heavily shunted. Also, though there will usually be some thermal e.m.fs. due to inequalities of temperature in the galvanometer circuit G, C_1, and S, no difficulty will arise, as this circuit is of constant resistance;

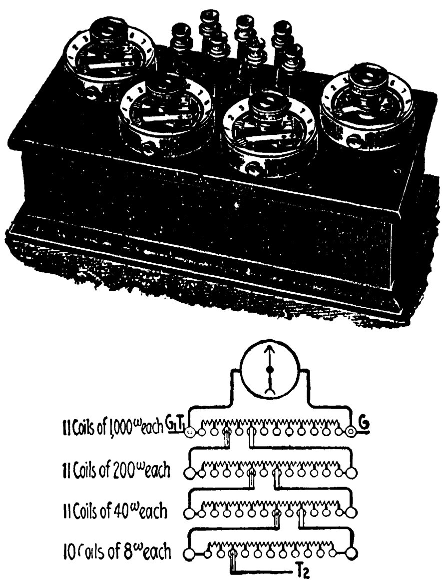


FIG. 115. —Universal shunt box.

the only effect will be that the deflections will be read from a zero that may differ slightly from the mechanical zero of the instrument.

R is a fixed known resistance of 100,000 ohms, $\frac{1}{10}$ megohm.

An exceedingly convenient form of Ayrton shunt is shown in Fig. 115. The constant resistance in the box between the galvanometer terminals is 10,000 ohms. By means of the four handles, the movable terminal may be carried from 0 to 10,000 by steps each one of which changes the multiplying power by one ten-thousandth part of the value that it has when all the slides are at the extreme right hand, a value taken as unity; for with the Ayrton-Mather arrangement we are concerned only with relative multiplying powers. The multiplying power to be used is obtained by dividing 10,000 by the sum of the readings of the four slides. The three posts at the rear are the shunt terminals; the four in the second line connect with the resistance R, of 100,000 ohms, which is divided into three sections of 10,000, 30,000 and 60,000 ohms, respectively.

The key K_2 (Fig. 114) is kept in the position shown, by a spring; when in this position, no current can flow through the galvanometer, and the instrument is thoroughly protected. If the key is held in the dotted position, the galvanometer is in service. The construction should be such that the circuit is not broken when the key is thrown from one position to the other.

The key K_1 serves to throw the cable in circuit and, when in the position shown, short-circuits it, thus ensuring thorough discharge. The battery B is connected to one side of a double-pole, double-throw switch C_2, the other side being short-circuited to enable discharge deflections to be taken. One of the middle connections of C_2 is carried to the tank plate P, the other to the middle post of K_2. The battery should be fairly well insulated to prevent its running down. It must be capable of giving a constant e.m.f. of, possibly, 500 volts.

It is usual to connect the negative pole of the battery to the core of the cable, the idea being that with this connection the effect of electrolysis is to open up any fault that may exist in the insulation. Many specifications require tests with the $-$ pole and with the $+$ pole successively connected to the core and require that the two results check.

To measure X, the "constant" of the apparatus must first be determined; this is the number of megohms at X which will correspond to a deflection of 1 mm. on the galvanometer scale. To do this, short-circuit X at K_1 by a piece of wire W; the resistance of the circuit will then be $R + P_R$, where P_R is the resistance of the shunted galvanometer, the leads, and the battery. S should be set at its smallest value, and the deflection of the galvanometer noted. Now, if necessary, alter S to obtain a good deflection; call this D_R; and let m_R be the corresponding multiplying power of the shunt. Then $E = I_R(R + P_R)$. When X is in place, $E = I_X(X + R + P_X)$. As the deflection of the galvanometer is proportional to the current,

$$I_R = Km_RD_R;$$
$$I_X = Km_XD_X;$$

therefore

$$X = \frac{m_RD_R}{m_XD_X}(R + P_R) - (R + P_X).$$

In general, $R + P_X$ is negligible compared with X, and P_R negligible compared with R; so

$$X = \frac{Rm_RD_R}{m_XD_X}.$$

The quantity Rm_RD_R is the constant of the apparatus. As R is expressed in megohms, this is the number of megohms for unit deflection of the galvanometer when the relative multiplying power of the shunt is unity.

The resistance R is left in circuit continuously, in order that there may be no possibility of a current being sent through the galvanometer of sufficient strength to burn it out. The magnitude of R, $\frac{1}{10}$ megohm, is so small that no material error is introduced by this procedure, as X is some hundreds or thousands of megohms.

Precautions.—In order that a measurement of insulation resistance may possess any value, one must be sure that the only current that passes through the galvanometer is that which flows through the insulation of the cable. Therefore, the galvanometer *must* be connected to the core of the cable and *not* to the tank plate. If it is connected to the tank plate, as the tank is grounded, any leakage current due to imperfect insulation of battery and leads will be measured by the galvanometer.

Any current that leaks over the surface of the insulation from the projecting wire of the cable to the tank will cause error in the results. This leakage current *must* be reduced to zero. Therefore any protective covering, such as braid or armor, should be removed from the ends of the wire for a distance of at least 18 in., thus laying bare the insulating coating. This latter should not be handled and must be kept scrupulously clean. As an additional safeguard, the insulation should be cut back from the end of the wire, as indicated in Fig. 114. This must be done with a sharp, *clean* knife, so as to leave a perfectly clean surface of considerable length (2 or 3 in.). In order to be absolutely certain that all surface leakage has been eliminated, recourse should be had to Price's guard wire. This device is shown dotted in Fig. 114.

A few turns of bare and flexible wire are closely wrapped around the insulation a few inches from its end so that they make perfect contact with it; the wire is then carried to the battery side of the galvanometer. The potential difference between the core and the guard wire is practically *nil*, so that any leakage will be from the guard wire to the tank; consequently, the leakage current will not be measured by the galvanometer. The galvanometer, reversing switch C_1, shunt, resistance R, and the key K_1 should be well insulated and supported by a metal plate which is insulated from ground and connected to the guard wire. The galvanometer lead should be protected in a similar manner.

Even after all precautions have been taken, it will not do to assume that leakage is not present. A test must be made to determine this point. To do this, wire up exactly as for a test; close K_1; then disconnect the lead from the cable, and leave it hanging free. Be sure not to introduce a new source of error by the arrangement for supporting the free end. Now throw on the battery. If the e.m.f. is high, the galvanometer will probably give a slight deflection and then settle back to its original reading. If it does this, the deflection is due to the electrostatic action incident to charging the apparatus to a high potential. If there is a permanent deflection, the guard circuit is defective and must be rearranged.

When measuring X, the key K_2 must not be thrown to the dotted position until the cable is charged electrostatically. Therefore, after closing K_1, allow at least 20 sec. to elapse before throwing K_2 to the right; this will prevent injury to the galvanometer.

Immersion.—After the specimen has been immersed, sufficient time must be allowed before the test is made for the cable to become thoroughly saturated, for moisture to work into any defects that may exist in the insulation, and for the insulation to attain the temperature of the tank. In practice, no tests should be made until after 12 hr.

Absorption Effects.—When the battery is first applied to the cable, there will be a sudden rush of current due to the charging of the cable

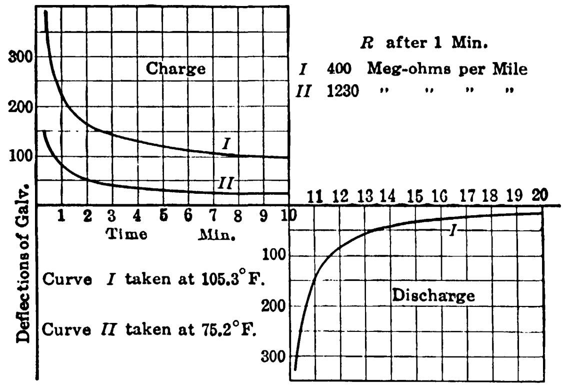


FIG. 116.—Showing the effect of time of electrification on the galvanometer deflection when measuring insulation resistance.

electrostatically. Therefore it is absolutely necessary to have the key K_2 in the position shown when the circuit is made, in order to prevent possible injury to the galvanometer. After the static charging, a current flows into the cable, rapidly diminishing to a nearly constant value. This current furnishes the "absorbed" charge and includes the current that actually flows through the insulation. The first portion diminishes toward zero, while the latter tends to become constant. If the switch C_2 were now thrown to the discharge position (dotted), C_1 reversed (to keep the deflections in the same direction), and the deflection observed, it would be found that at first there is a sudden rush due to the condenser discharge of the cable. This is followed by a current which gradually diminishes toward zero, this latter being due to the gradual working out of the absorbed charge. The various phenomena are illustrated by the curves shown in Fig. 116.

From the curves it is seen that the apparent resistance of the insulating covering is a function of the time of electrification and that it is necessary to state this time when quoting values of the insulation resistance; otherwise, they possess no meaning. It is customary to calculate the resistance at the end of 1 min. electrification; from this result and the known length of the sample the insulation resistance per mile or per 1,000 ft. is determined.

Effect of Temperature.—The substances classed as insulators have very large *negative* temperature coefficients; that is, an increase of temperature lowers their resistance. This is shown in Fig. 117 which gives the result of tests on a sample of rubber-covered wire. In this work, it is customary to express the temperature in degrees F.

For purposes of comparison, it is necessary to reduce the results of insulation resistance measurements to a standard temperature, 15.5°C. This is usually done by dividing the resistances at the temperatures of observation by experimentally determined factors the values of which will be different for different insulating compounds. Consequently, the various reduction factors quoted in electrical handbooks should not be applied indiscriminately.

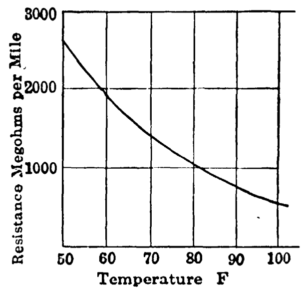


FIG. 117.—Illustrating effect of temperature on insulation resistance.

The great difficulty with tests for insulation resistance as a guide to the condition of underground feeders after installation is the uncertainty as to the temperature, due to the feeders having been in use or to the heating by currents in neighboring ducts.

Insulation Testing by Voltmeter.—Insulation resistances of the magnitude of 1 or 2 megohms may be measured by aid of a direct-current voltmeter of known resistance. Two readings are taken, the first when the instrument is directly across the line; the second, when the line voltage is applied to the instrument and the unknown resistance in series. The testing voltage must remain constant.

Call the reading D_1 when the voltmeter is across the line; and D_2 when it is in series with X, the unknown resistance. Then if the resistance of the voltmeter is R_V, and the constant of the instrument considered as a current galvanometer is K,

$$\frac{E}{R_V} = KD_1.$$

$$\frac{E}{R_V + X} = KD_2 = \frac{ED_2}{R_V D_1}.$$

Therefore

$$X = R_V\left(\frac{D_1 - D_2}{D_2}\right).$$

If the power may be shut off, this method lends itself to the determination of the insulation resistance between the conductors of a two-wire distribution circuit and the ground, that is, the water and gas pipes. Such tests are necessary when investigating the wiring of buildings. The circuit may be opened by removing the main fuses, and the necessary e.m.f. obtained by using either the supply voltage or a portable battery of dry cells.

Loss of Charge Method.—The loss of charge method of measuring insulation resistance is based on the following considerations.

If a perfect condenser is charged and then discharged through a resistance, it can be shown that $V_t = V_0 \epsilon^{-\frac{t}{CR}}$, where V_0 is the initial potential difference of the plates; V_t, the potential difference at any subsequent time t; C and R, the capacitance of the condenser and resistance of the circuit. Solving this for R,

$$R = \frac{t}{C \log_e \frac{V_0}{V_t}} = \frac{0.4343t}{C \log_{10} \frac{V_0}{V_t}}.$$

The relation of its units is such that if C is in microfarads, and t in seconds, R will be given in megohms. If R is large—several hundred or thousand megohms—the time of discharge will be sufficiently prolonged, so that it is possible to follow the variation of V_t with an electrometer or electrostatic voltmeter. From the foregoing it is seen that it is possible to measure a large resistance by discharging through it a condenser of known capacitance and noting the potential difference at the beginning and end of a time t.

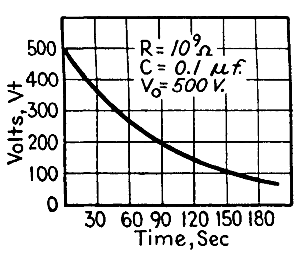


FIG. 118.—Illustrating fall of potential in loss of charge method for measuring insulation resistance.

The curve illustrates the phenomena for the case where R = 1,000 megohms,

$$C = 0.1 \ \mu\text{f}, \ V_0 = 500 \text{ volts}.$$

If the resistance is exceedingly high, the potential difference of the condenser may fall so slowly that, in any reasonable time, V_t may not differ greatly from V_0, and consequently the ratio V_0/V_t will be seriously affected by errors of observation. By observing V_0 and the *fall* of potential, much more accurate results may be obtained. Let the fall from V_0 in time t be denoted by v; then

$$V_0 = V_t + v,$$

and

$$R = \frac{0.4343t}{C \log_{10} \frac{V_0}{V_0 - v}}.$$

Only the ratio of voltages enters the formula, and it is possible to use a ballistic galvanometer instead of an electrometer or electrostatic voltmeter in the work. Connections are made so that the condenser is *charged* from the battery through the ballistic galvanometer. Let the elongation be $D_0 = KQ_0 = KCV_0$. After the cable has leaked for t sec., it is again connected to the battery through the galvanometer, and the elongation due to the quantity that is necessary to replace that which has leaked out is observed. Call this elongation D_t; then

$$D_t = KQ_t = KCv;$$

consequently,

$$R = \frac{0.4343t}{C \log_{10} \frac{D_0}{D_0 - D_t}}.$$

In this formula, an error in D_0 does not seriously affect the results, as it occurs in both the numerator and denominator, and D_t is a comparatively

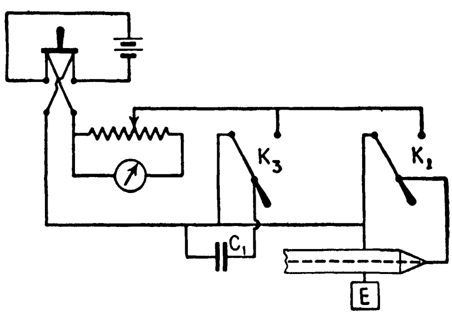


FIG. 119.—Connections for measuring insulation resistance by loss of charge method.

small quantity directly observed, so inaccuracies in it will not greatly affect the results. The ballistic galvanometer should be so sensitive that D_t will be large and consequently can be read with accuracy. This will probably mean that the instrument is used with the Aryton shunt adjusted for $m = 1$. When D_0 is observed, it will be necessary to shunt the galvanometer (the Ayrton shunt is used, as it keeps the damping constant) in order to keep the deflection on the scale. In this case, D_0 and D_t are the deflections corrected for the multiplying power of the shunts; that is, *both* are reduced to the value that they would have if $m = 1$.

To apply this method to the determination of insulation resistance, the connections shown in Fig. 119 are required.

Care should be taken that the connections are such that the current necessary to charge the wiring does not pass through the galvanometer; the connection from K_1 to the core should be short. Air lines should be used when possible.

It is first necessary to determine the capacitance C of the cable, for as a matter of convenience the cable is charged and allowed to leak through its own insulation resistance. The measurement is made by the direct deflection method; K_1 is placed in the mid-position; by throwing K_3 to the right, the condenser C_1 is charged through the galvanometer, and the elongation noted. When the key is against the left-hand stop, the condenser is discharged.

The key K_1 is now thrown to the position shown, thus making sure that the cable is discharged. The elongation D_0, on charging the cable, is now taken by throwing K_1 to the right; several observations should be made; between them the cable should be short-circuited long enough to ensure its thorough discharge. The elongation D_0 is used in computing both the capacitance C and the resistance R. To determine D_t, charge the cable; and at a noted minute and second, place K_1 in the mid-position, thus insulating the cable and allowing it to discharge by leakage through its own insulation resistance. At a noted minute and second, again charge the cable, noting the elongation D_t, which is due to the passage of the quantity Q_t through the galvanometer to replace that which has leaked out.

In thus applying the loss of charge method to cables, the assumption has been made that the current flow through the insulating covering obeys Ohm's law. On account of the nonfulfillment of the assumed conditions, the results are subject to errors; but in many cases of industrial testing, the results attained by a definite method of procedure are sufficient.

Loss of Charge Method, Using Quadrant Electrometer.—The fall of potential may also be obtained by the use of a quadrant electrometer, the needle of which is charged to a definite and constant potential. The connections are shown in Fig. 120.

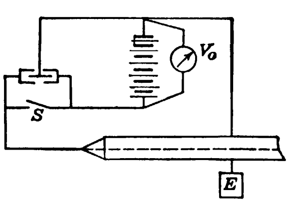


FIG. 120.—Connections for loss-of-charge method, using quadrant electrometer.

By closing the switch S the cable is charged, and both sets of quadrants brought to the *same* potential. The electrometer will therefore remain undeflected. When S is opened, the right-hand quadrants are kept at a potential V_0 by the battery, while the potential of the left-hand quadrants gradually falls, as the cable discharges through its own resistance. A deflection appears which is sensibly proportional to v.

Evershed "Megger."—It is highly desirable for measuring insulation resistance, to have a convenient portable instrument, which shall be rugged in construction, direct reading, capable of giving results with rapidity, and so simple in manipulation that it may be employed by persons not accustomed to the use of delicate instruments. In addition, the device must be capable of furnishing a testing voltage so high that it will search out defects of high resistance.

The Evershed "Megger" (Fig. 121) was designed with these requirements in view.

Referring to the scheme of connections, M and M are permanent magnets which furnish the fields both for the instrument proper and for the small hand-driven magneto D, which gives the testing voltage. The movable element consists of the coils BB' and A which are rigidly connected; flexible leads are taken to the coils, but there are no controlling springs.

The coils BB' are in series and, together with a suitable resistance, are connected across the terminals of the magneto D. The coil A is so connected that, when the selector switch is on $\div 1$, it is traversed by any current that flows through the specimen when the latter is joined between the external terminals. If the external circuit is open, the movable element, under the action of the current through R, will take up such a position that the plane of the coils BB' is along the dotted line. This

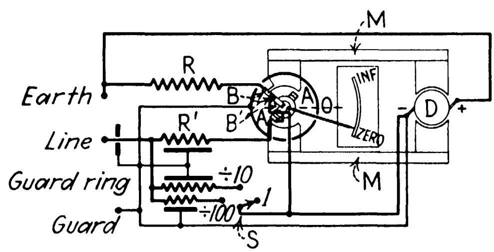


FIG. 121.—Circuits of Evershed Megger.

position corresponds to an infinite external resistance and is marked infinity on the scale. If there is a current in the external circuit, due to the imperfect insulation resistance of the specimen, a current will flow through the coil A and in such a direction that it turns the movable system toward the right, carrying the coils BB' with it until the latter, which move in a nonuniform field, are in a field so strong that the turning moments due to A and BB' are balanced. By use of the selector switch S, current may be diverted from the control coil A to the resistance R', and the deflections for small insulation resistances increased. To prevent surface leakage from the high-potential lead, a guard system is employed both within the instrument and at the specimen.

The indications are independent of the voltage of the magneto; for if that changes, both the deflective and restoring moments are altered in the same ratio.

In one design of the instrument, the magneto is driven through a clutch arrangement controlled by a centrifugal governor so that the voltage cannot rise above a definite maximum; then, when the crank is turned fast enough, a constant testing voltage is obtained. This is of importance when apparatus having a considerable electrostatic capacitance is tested.

MEASUREMENT OF INSULATION RESISTANCES OF COMMERCIAL CIRCUITS WHEN POWER IS ON

It is sometimes necessary to measure the insulation resistance to "ground" (the water and gas pipes) of a distribution system, for instance, that of an office building, where the conditions are such that the power is on and the supply must not be interrupted. This case is illustrated by Fig. 122.

Voltmeter Method.—Direct voltages are employed. The resistances to ground will be represented by x_1 and x_2.

If the insulation resistances are not above 1 or 2 megohms, recourse may be had to the voltmeter method (page 199). Three voltage measure-

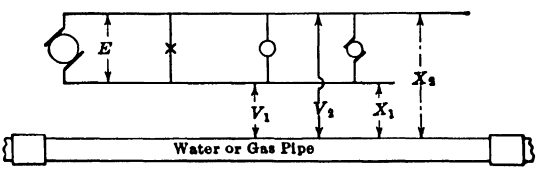


FIG. 122.

ments suffice to determine both x_1 and x_2. The supply voltage should be constant, and the readings made as expeditiously as possible. First measure E, the supply voltage; then V_1 and V_2, the voltages to ground from leads 1 and 2. When the voltmeter is connected from lead 1 to ground, the voltage E sends a current I_1 through the resistance x_2 plus the parallel resistance of x_1 and the voltmeter. Then, if R_V is the voltmeter resistance,

$$I_1 = \frac{E}{x_2 + \dfrac{x_1 R_V}{x_1 + R_V}}.$$

Similarly, when the voltmeter is between lead 2 and the ground,

$$I_2 = \frac{E}{x_1 + \dfrac{x_2 R_V}{x_2 + R_V}};$$

also

$$E - I_1 x_2 = V_1,$$
$$E - I_2 x_1 = V_2.$$

Hence

$$x_1 = R_V\left(\frac{E - V_1 - V_2}{V_2}\right),$$

and

$$x_2 = R_V\left(\frac{E - V_1 - V_2}{V_1}\right).$$

As the voltages enter as a ratio, any galvanometer that gives a deflection directly proportional to the current through it may be used in place of the voltmeter, a suitable series resistance being employed. In this case, the scale readings may be used in place of the voltages. Inspection of the formulæ shows that the method is not applicable when one side of the circuit is grounded.

Northrup Method.—If the insulation resistances are too high for the voltmeter method, a galvanometer may be used according to a scheme due to Northrup.[7]

The connections are shown in Fig. 123.

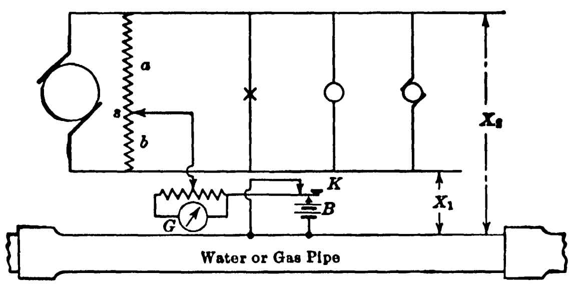


FIG. 123.—Northrup connections for measuring resistance to ground when power is on.

A fairly high resistance ab, which is provided with a sliding tap s, is placed across the circuit. The galvanometer G is shunted, preferably by an Ayrton shunt as shown, so that its sensitivity may be varied to suit the conditions.

When the key K is against the backstop, connection is established between the ground and s. By moving s, a position may be found where the galvanometer will stand at zero, the potential of s being that of the ground. It will be seen that this is a Wheatstone bridge arrangement. Consequently,

$$\frac{x_1}{x_2} = \frac{b}{a},$$

and

$$\frac{x_1}{x_1 + x_2} = \frac{b}{a + b},$$

$$\frac{x_1 + x_2}{x_2} = \frac{a + b}{a}.$$

If the key be depressed, any deflection will be due to the current from the battery B, which flows to s and there divides, part going through the resistances $a + x_2$, and part through $b + x_1$ and back to the battery via the ground connection.

The total resistance encountered will be

$$R = R_G + \frac{1}{\frac{1}{a + x_2} + \frac{1}{b + x_1}}.$$

R_G, a, and b are negligible with respect to x_1 and x_2, so

$$R = \frac{x_1 x_2}{x_1 + x_2} \quad \text{very approximately.}$$

Therefore

$$x_1 = R\left(\frac{a + b}{a}\right);$$

$$x_2 = R\left(\frac{a + b}{b}\right).$$

If the voltage of the battery B is E, then

$$I_G = \frac{E}{R} = KD,$$

where K is the current necessary for unit deflection of the galvanometer, and D is the deflection. Then

$$R = \frac{E}{KD}.$$

Therefore

$$x_1 = \frac{E}{KD}\left(\frac{a + b}{a}\right);$$

$$x_2 = \frac{E}{KD}\left(\frac{a + b}{b}\right).$$

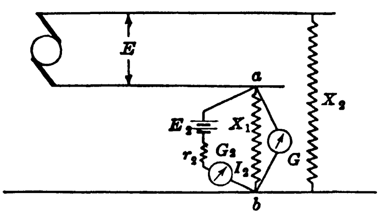


FIG. 124.—Connections for measuring resistance to ground.

Changes in circuit conditions, such as throwing on or off appliances not perfectly insulated, will change x_1 and x_2 and shift the neutral point. This may introduce difficulties in the execution of the test.

Other Methods of Measuring Resistance to Ground.—The current that flows to ground from one of the line wires may be measured by the method given on page 82 for measuring the current in a permanently closed circuit. The connections are shown in Fig. 124. By varying r_2, the potential difference between the points a and b may be made zero; when this adjustment is complete, the deflection of the galvanometer G is zero, and the current which then flows through x_2 is given by the galvanometer G_2. It follows that

$$x_2 = \frac{E}{I_2} = \frac{Er_2}{E_2}.$$

Mance's method, originally designed for measuring the internal resistance of batteries, may be so modified that it can also be used for measuring insulation resistances while under the working voltage.

THE TEMPERATURE COEFFICIENT OF ELECTRICAL RESISTANCE

The effect of change of temperature on the electrical resistance of various materials is shown in Fig. 125. It is seen that, with few exceptions, the resistance increases with increase of temperature. For pure metals, an approximate figure is 0.4 per cent per degree centigrade. It may be noted that metals that at certain temperatures undergo changes of structure, for instance iron at about 780°C., show alterations of curvature in the temperature-resistance curve at those points.

From this it is obvious that if a statement of an electrical resistance is to possess definiteness, the temperature at which the measurement was made must be given. Also, when a resistance is measured, it is necessary to know the temperature and temperature coefficient of the coils with which it is compared, as well as their value at some particular temperature, in order that their true resistance at the time of use may be known.

Experiments show that, in general, when the temperature is altered, the change of the electrical resistance of a conductor to which terminals are rigidly attached, and which therefore possesses a constant mass, is not quite linear; that is, the plot connecting resistance and temperature is not exactly a straight line. For the ordinary ranges of atmospheric temperatures, the curvature is very slight but may be considerable if the temperature is carried to high values (see page 208).

In this and similar cases, an empirical relation of the form here given is frequently employed to represent the results of physical measurements:

$$R_t = R_0(1 + at + bt^2 + ct^3 + \cdots). \tag{21}$$

R_t is the resistance at $t°$; and R_0, that at 0°. The departure from a straight line is determined by the constants $b, c, \ldots$ For the special case of copper, the variation of which is practically linear, between 10 and 100°C., the constants $b, c, \ldots$ are zero within that range. The constants $a, b, c, \ldots$ are best determined by applying the method of least squares to a series of measurements of the resistance made at different temperatures.

The temperature-resistance curve for a coil of very pure platinum is shown in Fig. 126; it is represented by the following equation:

$$R_t = R_0(1 + 0.00392t - 0.000000588t^2);$$

therefore

$$a = +0.00392.$$
$$b = -0.000000588.$$

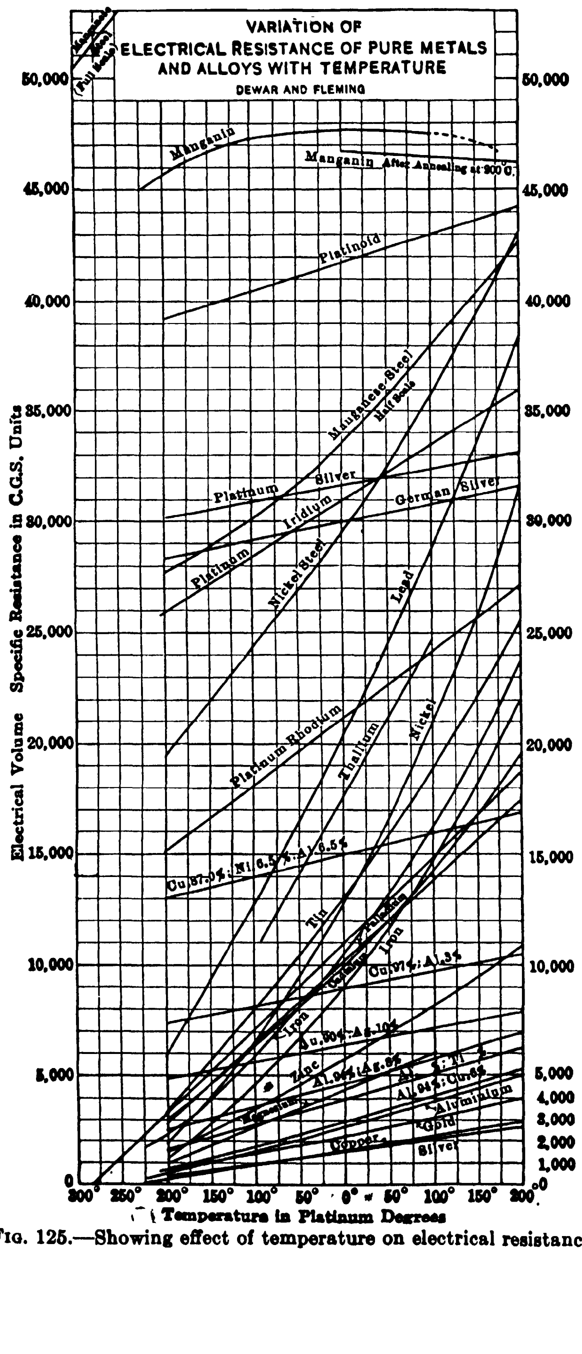


FIG. 125.—Showing effect of temperature on electrical resistance.

On account of its use in resistance thermometers, the variation of the resistance of platinum with temperature is important.

Mean Temperature Coefficient.—Denoting particular temperatures by subscripts, it is usual to write formula (21) thus:

$$R_{t_1} = R_0(1 + \beta_{t_1}t_1), \tag{21a}$$

where $\beta_{t_1} = a + bt_1 + ct_1^2$. It will be seen that β_{t_1} is the mean or average

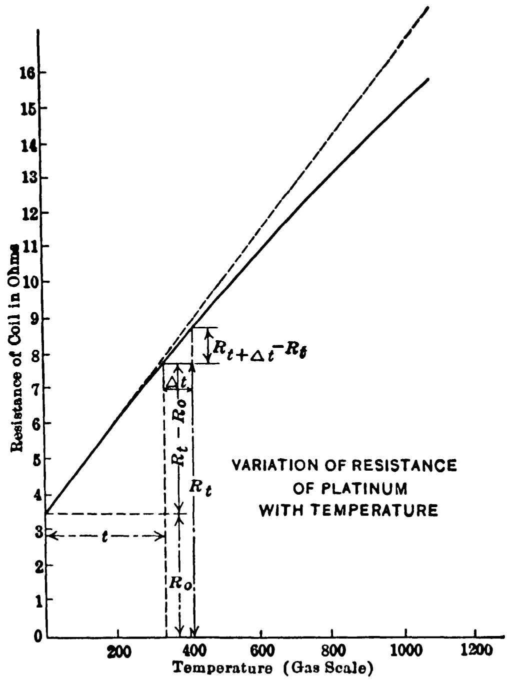


FIG. 126.—Temperature-resistance curve of platinum.

fractional rate of increase of resistance per degree between 0° and t_1, referred to the resistance at 0°, for

$$\beta_{t_1} = \frac{R_{t_1} - R_0}{R_0 t_1}.$$

β_{t_1} is called the *mean temperature coefficient of resistance increase.*

To compute the value of a resistance at any temperature $t°$, from that at some given temperature $t_1°$,

$$R_{t_1} = R_0(1 + \beta_{t_1}t_1);$$
$$R_t = R_0(1 + \beta_t t);$$

therefore,

$$R_t = R_{t_1}\frac{1 + \beta_t t}{1 + \beta_{t_1}t_1}. \tag{22}$$

Temperature Coefficient of Resistance.—As, in general, the graph connecting R_t and t is a curve, the true rate of increase of resistance will have a particular value at each temperature; consequently, a very small temperature interval must be used in computing it.

The *temperature coefficient of resistance increase* at any temperature is the fractional rate of increase of resistance for a very small temperature increment referred to the resistance at that temperature. It will be denoted by α; then

$$\alpha_{t_1} = \frac{1}{R_{t_1}}\left(\frac{dR_t}{dt}\right)_{t_1} = \frac{a + 2bt_1 + 3ct_1^2 + \cdots}{1 + at_1 + bt_1^2 + ct_1^3 + \cdots}. \tag{23}$$

$$\alpha_{t_1} = \frac{R_{t_1+\Delta t} - R_{t_1}}{R_{t_1}\Delta t} \qquad \text{approx.}$$

For a small finite temperature interval, $\Delta t = t - t_1$,

$$R_t = R_{t_1}(1 + \alpha_{t_1}[t - t_1]). \tag{24}$$

On account of the approximations involved, Eq. (24) applies, in general, only to short ranges of temperature.

Strictly speaking, in order to compute α or β, one must know the values of the constants a, b, and c for the particular sample of material under discussion; such data are rarely at hand.

Special Case: Temperature Correction for Copper.[8]—Careful experiments made at the Bureau of Standards upon commercial copper of high conductivity, such as is used for electrical purposes (varying from 96 to 101 per cent conductivity, when referred to the annealed-copper standard, page 217), show that between 10 and 100°C. the *variation of resistance with temperature is linear* and also that the temperature coefficient at 20° is directly proportional to the fractional conductivity. For example, if the conductivity is 95 per cent when referred to the annealed-copper standard, the temperature coefficient at 20°C. is

$$\alpha_{20} = \frac{R_t - R_{20}}{R_{20}(t - 20)} = 0.00393 \times \left(\frac{95}{100}\right) = 0.00373. \tag{25}$$

The *temperature coefficient of resistance* at any other temperature may be calculated from that at 20° as follows, the variation of resistance being linear:

$$\alpha_t = \frac{1}{R_t}\left(\frac{dR_t}{dt}\right)_t = \frac{a}{1 + at} = \frac{1}{\frac{1}{a} + t};$$

similarly,

$$\alpha_{t_1} = \frac{1}{\frac{1}{a} + t_1},$$

and

$$\frac{1}{a} = \frac{1}{\alpha_{t_1}} - t_1;$$

therefore

$$\alpha_t = \frac{1}{\frac{1}{\alpha_{t_1}} - t_1 + t}.$$

Let the original temperature of reference t_1 be taken as 20°; then, at some other temperature, t,

$$\alpha_t = \frac{1}{\frac{1}{0.00393 \times (\text{fractional conductivity})} + [t - 20]}. \tag{26}$$

As the resistance variation of copper is linear, measurements to determine α_{t_1} may be made at any two temperatures which may both differ from t_1, the temperature of reference; for let the measurements be made at temperatures t_2 and t_3; then

$$R_{t_1} = R_{t_3} - \frac{(R_{t_3} - R_{t_2})}{t_3 - t_2}[t_3 - t_1],$$

and

$$\alpha_{t_1} = \frac{R_{t_3} - R_{t_2}}{R_{t_1}[t - t_2]} = \frac{R_{t_3} - R_{t_2}}{R_{t_2}[t_3 - t_1] - R_{t_3}[t_2 - t_1]}.$$

By use of (25) and (26), the following table has been calculated. The standard 100 per cent conductivity copper is taken as having a resistivity of 0.15328 ohm (meter, gram) at 20°C. (see page 217).

TABLE XI.—TEMPERATURE COEFFICIENTS OF STANDARD ANNEALED COPPER AT VARIOUS TEMPERATURES OF REFERENCE

Ohms (meter, g.) at 20°C.	Per cent conductivity	α_0	α_{15}	α_{20}	α_{25}	α_{30}	α_{50}	Inferred absolute zero T
0.16134	95.0	0.00403	0.00380	0.00373	0.00367	0.00360	0.00336	−247.8
0.15966	96.0	0.00408	0.00385	0.00377	0.00370	0.00364	0.00339	−245.1
0.15802	97.0	0.00413	0.00389	0.00381	0.00374	0.00367	0.00342	−242.3
0.15753	97.3	0.00414	0.00390	0.00382	0.00375	0.00368	0.00343	−241.5
0.15640	98.0	0.00417	0.00393	0.00385	0.00378	0.00371	0.00345	−239.6
0.15482	99.0	0.00422	0.00397	0.00389	0.00382	0.00374	0.00348	−237.0
0.15328	100.0	0.00427	0.00401	**0.00393**	0.00385	0.00378	0.00352	−234.5
0.15176	101.0	0.00431	0.00405	0.00397	0.00389	0.00382	0.00355	−231.9

Experiments have shown that distortions of the wire, such as occur in winding and ordinary handling, do not alter the temperature coefficient.

For windings in general, *annealed* copper, conductivity 100 per cent, may be assumed. Hard-drawn copper may be assumed to have a conductivity of 97.3 per cent. These values are approximations to be used only when data concerning the copper in question cannot be obtained.

Measurement of Rise of Temperature.—In the testing of electrical machinery, it is customary to find the average rise of temperature of copper windings by measurements of their resistance. This is facilitated by the linear relation between temperature and resistance; for, *assume* that this relation holds for all temperature intervals, and prolong the line connecting temperature and resistance downward. It will cut the axis of temperatures $T°$ below 0°C. Now, let R_{t_1} and R_{t_2} be the resistances of the windings at temperatures t_1 and t_2; then

$$\frac{R_{t_2} - R_{t_1}}{t_2 - t_1} = \frac{R_{t_1}}{T + t_1},$$

or

$$t_2 - t_1 = \frac{(R_{t_2} - R_{t_1})}{R_{t_1}}[T + t_1]. \qquad (27)$$

Again, from the foregoing,

$$\frac{R_{t_2}}{R_{t_1}} = 1 + \frac{t_2 - t_1}{T + t_1}.$$

The quantity T is usually called the "inferred absolute zero." It may be calculated from the data given in the table. Since

$$R_t = R_{20}(1 + \alpha_{20}[t - 20]),$$

if the resistance becomes zero, and 100 per cent conductivity is taken,

$$t - 20 = -\frac{1}{\alpha_{20}} = -\frac{1}{0.00393} = -254.5,$$

therefore

$$T = 234.5 \quad \text{below } 0°\text{C}.$$

These values are entered in the last column of the table on page 211.

The Resistance Pyrometer.[3,9]—Following a suggestion made in 1871 by W. Siemens, the variation of the electrical resistance of platinum with temperature is utilized in pyrometry. The first experiments in this direction were not successful, and in 1874 the British Association report on the instruments submitted was not favorable, it being found that after exposure to a high temperature the platinum coils did not return to their original resistances and that the changes were progressive. In 1886, Callendar proved that these changes were due to the absorption by the platinum of silica from the coil support and of furnace gases which at high temperatures penetrated the iron tubes then employed to protect the resistance coils. In view of these facts, the coils are now wound on very light frames of mica, which touch each turn of the wire at only four points; the areas of the surfaces of contact are thus reduced to a minimum. The coils are now protected by porcelain tubes glazed externally.

In order to make the relation of resistance to temperature comparatively simple and one that may be determined by measurements at three known temperatures, pure platinum must be employed.

If the resistance of the coil be measured at 0 and at 100°C., the average change per degree will be

$$\frac{R_{100} - R_0}{100};$$

and if R is the resistance at some other temperature,

$$100\frac{R - R_0}{R_{100} - R_0}$$

will be the corresponding temperature, on the assumption that the change of resistance is linear. A temperature so defined is called the "platinum temperature," so

$$t_p = 100\frac{R - R_0}{R_{100} - R_0}.$$

Callendar showed that for high-temperature measurements it was not correct to assume a linear variation of the resistance but that a correction must be applied in order to give the proper temperature on the scale of the gas thermometer. He found that if t is the temperature on the gas scale, the difference of t and t_p could be expressed by the following empirical relation:

$$t - t_p = \delta\left\{\left(\frac{t}{100}\right)^2 - \left(\frac{t}{100}\right)\right\},$$

where if *pure* platinum is used, δ is a constant, its value being about 1.505. This form of expression holds with great exactness for pure platinum but is not general; for instance, it fails in the case of palladium. For impure platinum, δ must be expressed thus: $\delta = a + bt$.

To determine δ, pure platinum being used, the resistance must be measured at three known temperatures, which for high-temperature measurements are usually taken as 0, 100, and 444°.70, the last being the boiling point of sulphur under carefully specified conditions, this having been determined with great care by many experimenters. With the δ correction applied, the platinum thermometer reproduces temperatures on the gas (nitrogen)-thermometer scale to within the limits of accuracy of the latter between −80 and +1100°C.

The preceding empirical formula does not admit of extrapolation downward below −100°; so for measurements of extremely low temperatures, either the formula must be modified, or the coil resistance taken at such points as the temperature of melting ice, melting solid CO_2, and the boiling point of liquid oxygen, in order that the extrapolation may not be so excessive. The platinum thermometer, when used for precise work at high temperatures, must be calibrated frequently; the purer the platinum the less the likelihood of a change in its constants. Also, the changes in the coils are minimized if the wire is large and is supported free from strains. Annealing at a temperature above that at which the instrument is to be used contributes to constancy.

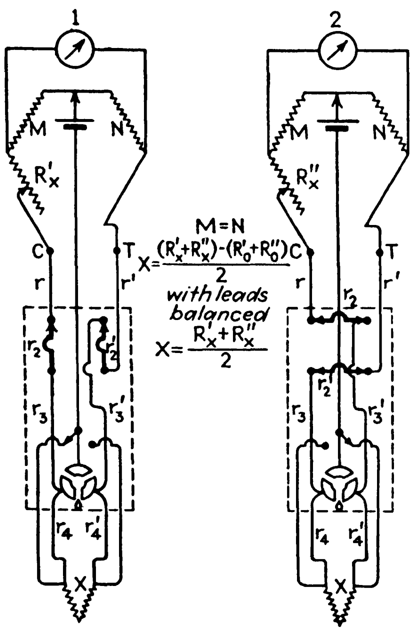


FIG. 127.—Connections for four-terminal resistance thermometer; used with Mueller bridge.

The bridge connections for measuring the resistance of a four-terminal resistance thermometer are shown in Fig. 127. The change from connection 1 to connection 2 is made by a mercury contact switch.

RESISTIVITY, CONDUCTIVITY[10]

It is well known that the electrical resistances of conductors having the same dimensions but made of different materials will differ. Each metal has its characteristic resistance, which, when determined for a unit specimen of the material and at a stated temperature, gives the resistivity or specific resistance. Various methods of expressing this property are used; the three most commonly employed are shown below.

1. The resistivity is frequently expressed as the resistance, in ohms or microhms, of a wire 1 cm. long and having a cross section of 1 sq. cm. This has commonly been called the "centimeter-cube resistivity." This name frequently gives rise to false notions as to the relations of the quantities involved. This quantity is better designated as the ohm-centimeter or microhm-centimeter resistivity, terms descriptive of the units involved. If it is represented by ρ_A, then a wire l cm. long and A sq. cm. in section will have a resistance

$$R = \frac{\rho_A l}{A}. \qquad (28)$$

2. Another unit in common use is the resistance in ohms of a wire 1 ft. long and 1 mil or 0.001 in. in diameter. This is commonly called

the "mil-foot resistivity." It should, however, be designated as the ohms (mil, foot) resistivity. If it is represented by ρ_D, then for circular wires, if the length l is expressed in feet and the diameter D in mils,

$$R = \frac{\rho_D l}{D^2}. \tag{29}$$

3. The third method of expressing resistivity is in terms of the resistance in ohms of a wire of uniform cross section, 1 meter long and 1 g. in mass. This has commonly been called the "meter-gram resistivity." The designation "ohm (meter, gram) resistivity" should, however, be used. If this quantity is represented by ρ_M, then when the length l is expressed in meters, and the mass M in grams,

$$R = \frac{\rho_M l^2}{M}. \tag{30}$$

ρ_A and ρ_D are frequently referred to as the "length-section" and "volume" resistivities, and ρ_M as the "length-mass resistivity" or the "mass resistivity."

The relation of ρ_A to ρ_M involves the density d; for, expressing ρ_M in ohms,

$$\rho_M = \frac{RM}{l^2_{m.}} = \frac{10{,}000 R A l_{cm.} d}{l^2_{cm.}} = 10{,}000 \rho_A d.$$

A careful study by the Bureau of Standards[10] of all available data gives for the density of commercial copper having a conductivity of over 94 per cent the value 8.89 at 20°C. The limits of variation, neglecting extreme values, were found to be 8.87 and 8.91. The density of annealed and hard-drawn copper is sensibly the same.

To find ρ_A, ρ_D, or ρ_M, the resistance of the sample R must be determined at some standard temperature, and the necessary mechanical measurements made in the units stated above. To determine with accuracy the section of a wire of supposed uniform cross section from measurements of its diameter is a very difficult matter and laborious of execution, for a great many observations must be taken by means of the micrometer caliper, and even then the result is likely to be seriously in error, since the square of the diameter enters the formula. An error of 1 per cent in the diameter means 2 per cent in the area. For this reason, diameter measurements should be used only when the wire is large. In case the cross section of the sample is uniform but irregular in outline, such measurements are, of course, impossible. If the wire is small, its average area or average diameter is best determined by weighing a known length of it in air and then in water, thus finding the mass of the displaced water and consequently the volume of the wire. This, however, requires that all the precautions taken in an exact specific gravity determination be observed.

The determination of ρ_M, or the length-mass resistivity, presents the least experimental difficulty, as the mechanical measurements are simply the determination of a length and a weight, both of which can be very accurately made; consequently, the ohm (meter, gram) resistivity is very commonly employed and is recommended in preference to the length-section or volume resistivities denoted above by ρ_A and ρ_D.

The standard temperature at which results are expressed should not differ greatly from the ordinary average atmospheric temperature; therefore 20°C. is to be taken.

The electrical qualities of copper are frequently stated in terms of the conductivity, which is the reciprocal of the resistivity; and in business transactions, guarantees are given as to percentage conductivity. In order that such guarantees may possess definiteness, some standard must be adopted. Obviously, the conductivity of pure copper would be the most proper standard, but this is unknown. So for the greatest convenience, some reasonable figure must be agreed upon. What this figure may be is not of consequence, but that it should be universally recognized is of the greatest moment.

Many different standard values of the resistivity of annealed copper have been in use and sanctioned by various electrotechnical societies. Generally, these values have been based on the work of Matthiessen on supposedly pure copper (1862), but the results on *annealed* copper involve an assumption as to the ratio of the resistivity of the hard-drawn to the annealed wire. There are also uncertainties as to the temperature coefficients, the many digits usually given in the constants being without significance; consequently, the values derived by various persons from Matthiessen's work do not agree, and the so-called Matthiessen standard has had no universal significance.

To obviate this difficulty, the Bureau of Standards, at the request of the American Institute of Electrical Engineers, has investigated the subject, and, as the result of measurements upon 89 samples of commercial copper, procured from 14 different refiners, an average result of 0.15292 ohm (meter, gram) at 20°C. was obtained. This value is seen to be in close agreement with the figure 0.15302 ohm (meter, gram) at 20°, which had previously been adopted by the Bureau. This last figure was suggested for international adoption, but the German engineers had already in use a standard of conductivity, $58\frac{1}{\text{ohm}}$ (meter, square millimeter) at 20°, which is slightly different. The corresponding figure for the resistivity was the one finally recommended for adoption in America and Germany. It is

INTERNATIONAL ANNEALED COPPER STANDARD

Mass resistivity.	0.15328 ohm (meter, gram) at 20°C.
	875.20 ohm (mile, pound) at 20°C.
Volume resistivity.	1.7241 microhm (cm.) at 20°C.
	0.017241 ohm (meter, mm.^{2}) at 20°C.
	0.67879 microhm (inch) at 20°C.
	10.371 ohms (mil, foot) at 20°C.
Density, grams per cubic centimeter.	8.89 at 20°C.

The international standard was originally approved by the International Electrotechnical Commission in March, 1914, and restated without any real modification in March, 1925. It is officially included in American Standard C8a—1932, "Definition and General Standards for Wires and Cables" (Rules 30-100 and 30-101) and also in the various standards of the British Standards Institution. The German Standards Rules do not include it in specific terms; instead, maximum and minimum tolerances, within which the standard falls, have been adopted.

Resistivity-temperature Constant.—The changes in the resistivity of copper due to alterations of temperature are affected by the expansion of the material. This effect is very small and is readily allowed for.

Assuming that the resistance is measured between terminals rigidly attached to the specimen, in general for the ohm (meter, gram) resistivity,

$$\rho_M = \frac{MR}{l^2}.$$

Introducing the temperatures and denoting the coefficient of linear expansion by κ,

$$[\rho_M]_t = \frac{MR_{20}(1 + a_{20}[t - 20])}{l_{20}{}^2(1 + \kappa[t - 20])^2}.$$

For copper, κ is a very small quantity, 0.000017, and so

$$[\rho_M]_t = [\rho_M]_{20}\{1 + (a_{20} - 2\kappa)[t - 20]\} \qquad \text{approx.}$$

For standard copper, 100 per cent conductivity,

$$\begin{aligned}[\rho_M]_t &= 0.15328\{1 + (0.00393 - 0.000034)[t - 20]\};\\ &= 0.15328 + 0.000597[t - 20].\end{aligned}$$

The change per degree in the resistivity is seen to be 0.000597 *ohm;* this figure is independent of the temperature of reference, and in consequence of (25) applies to coppers of all conductivities. It is called the "resistivity-temperature constant."

If the effects of expansion had been neglected, the result would have been 0.000602.

Using the volume resistivity, in general,

$$\rho_A = \frac{RA}{l}.$$

At $t°$,

$$[\rho_A]_t = \frac{R_{20}A_{20}(1 + a_{20}[t - 20])(1 + 2\kappa[t - 20])}{l_{20}(1 + \kappa[t - 20])}$$
$$= [\rho_A]_{20}\{1 + (a_{20} + \kappa)[t - 20]\} \qquad \text{approx.}$$

Using microhms,

$$[\rho_A]_t = 1.7241\{1 + (0.00393 + 0.000017)[t - 20]\}$$
$$= 1.7241 + 0.00681[t - 20].$$

In this case, the "resistivity-temperature constant" is 0.00681. Again, using the ohm (mil, foot) resistivity,

$$[\rho_D]_t = 10.371 + 0.0409[t - 20].$$

Here the resistivity-temperature constant is 0.0409.

Relation between Resistivity and the Temperature Coefficient of Resistance.—As shown above, the change in the ohm (meter, gram) resistivity per degree centigrade is 0.000597; consequently, the temperature coefficient of the ohm (meter, gram) resistivity $= 0.000597/[\rho_M]_{t_1}$.

The resistance of a wire at $t°$, if t_1 is the temperature of reference, is given by

$$R_t = \frac{[\rho_M]_t l^2_{t_1}}{M}\left\{1 + \frac{0.000597}{[\rho_M]_t}(t - t_1)\right\}\{1 + 2\kappa(t - t_1)\}$$
$$= R_{t_1}\left\{1 + \left(\frac{0.000597}{[\rho_M]_{t_1}} + 0.000034\right)(t - t_1)\right\} \qquad \text{approx.}$$

For the copper met with in practice, this is approximately

$$R_t = R_{t_1}\left\{1 + \frac{0.000602}{[\rho_M]_{t_1}}(t - t_1)\right\}.$$

Therefore the temperature coefficient of resistance at the reference temperature $t_1°$ is $0.000602/[\rho_M]_{t_1}$.

Similarly for ρ_A,

$$a_{t_1} = \frac{0.00678}{[\rho_A]_{t_1}}.$$

For ρ_D it is

$$a_{t_1} = \frac{0.0407}{[\rho_D]_{t_1}}.$$

Percentage Conductivity.—The percentage conductivity is obtained by dividing the resistivity of the annealed-copper standard at 20° by that of the sample at 20°.

It is to be noticed that on account of the relation of the temperature coefficient to the conductivity, the percentage conductivity of a sample when referred to the standard copper will depend somewhat on the temperature at which the conductivity is computed. For instance, if copper of resistivity 0.15328 ohm (meter, gram) at 20° is taken as a standard, the resistivity at 0° will be $0.15328 - 0.000597 \times 20 = 0.14134$; a copper that has 95 per cent conductivity at 20° will have a resistivity of 0.16134 and at zero a resistivity of $0.16134 - 0.000597 \times 20 = 0.14940$. Therefore the percentage conductivity at 0° is

$$100\frac{0.14134}{0.14940} = 94.6 \text{ per cent.}$$

To avoid possible confusion, percentage conductivities are to be computed at 20°C.

Resistivity of Aluminum.—There is no international standard of resistivity for aluminum. The question has been studied by various national committees, but no international agreement has been reached. As a result of the study of American practice, the American Standards Association has adopted the following as the tentative American Standard. No standard has as yet been set for annealed aluminum.

TENTATIVE AMERICAN STANDARD FOR HARD DRAWN ALUMINUM CONDUCTORS

Resistivity at 20°C.	2.828 microhm-cm.; 17.01 ohm circular mil per foot 0.07644 ohm gram per meter²
Conductivity, at 20°C. as compared with the annealed copper standard at 20°C.	60.97 per cent
Resistance-temperature coefficient at 20°C.	0.00403 per degree centigrade
Density at 20°C.	2.703 g. per centimeter cubed
Length-temperature coefficient at 20°C.	0.000023 per degree centigrade
Change in resistivity with change in temperature	0.115 microhm-cm. per degree centigrade 0.069 ohm circular mil per foot per degree centigrade 0.000305 ohm gram per meter² per degree centigrade. Standard temperature 20°C.

Conductivity Bridges.—In wireworks and in the testing laboratories of large consumers of wire, it is necessary to have special apparatus for the determination of conductivity, the requirements being:

1. Convenience of manipulation, allowing speed to be attained.
2. No calculation required; that is, the instrument must be direct reading in terms of the accepted standard material.
3. Freedom from all temperature corrections.
4. Accuracy to $\frac{1}{10}$ or $\frac{1}{20}$ per cent.

One form of such an apparatus is the Hoopes conductivity bridge. This device is an adaptation of the Kelvin double bridge, the scheme being as follows:

Hoopes Conductivity Bridge.—The standard P and the unknown X are of the same metal; consequently, *if care is taken that they are at the same temperature,* all corrections due to temperature are avoided. The arms M, N, m, n are in the same case and made of material of low temperature coefficient, so that their relative values will not change. The

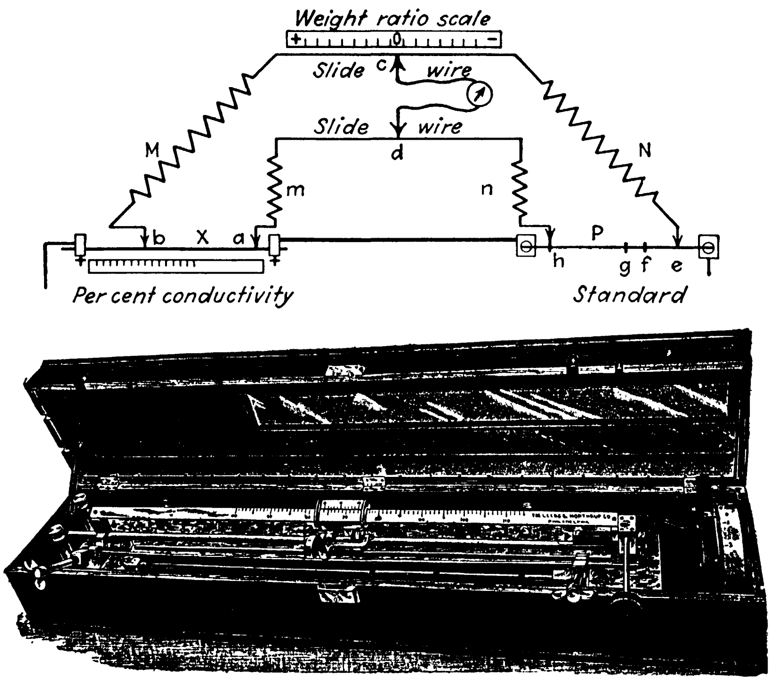


Fig. 128.—Hoopes conductivity bridge.

sliders c, d are rigidly connected so that when they are moved the ratio M/N is altered, while the relation $M/N = m/n$ is maintained.

The sample shown at X is placed alongside of a scale divided into 100 equal parts; the graduations therefore represent percentages of the total length of the scale.

Consider that X is of uniform cross section and 100 per cent conductivity; that a and b are set at 0 and 100, respectively; and that the resistance of P equals that of X; for balance, c and d must be set so that $M = N$. Now, suppose that the sample at X is changed for one of the same diameter but of 50 per cent conductivity; the length required to balance P, contacts c and d remaining fixed, will be only 50 per cent as great as in the first case, and b must be moved along the percentage scale to the 50 per cent mark.

However, the samples X vary in diameter, while the resistance of P is fixed; consequently, the ratio M/N must be variable, so that it may be made to correct for the cross section of the sample. To obtain the relative diameters of wires, it is more accurate and convenient to weigh samples of the same length than to caliper them; accordingly, all samples are cut to a length of 38 in. in a special machine and weighed. The contact c is then set at the graduation corresponding to the percentage excess or defect in the weight, referred to a sample of correct size, thus making $(M/N)P$ equal to the resistance of a sample of 100 per cent conductivity—length 0 to 100 on the percentage scale—and of the same diameter as X. The bridge is then balanced by moving b, and the conductivity is read from the scale.

Several standards are provided; they are removable, and by the use of the taps e, f, g each has a range of three consecutive numbers on the Brown and Sharpe gage. For rapid work, the stock of samples must be kept at the temperature of the testing apparatus.

The coarse adjustment of the slider b is effected by the handle projecting toward the front of the bridge; the final balancing is made by turning the milled head at the front.

The percentage scale is seen at the back of the instrument, the weight-ratio scale at the right hand.

The instrument is covered by a metal-lined wooden case, which serves to keep the temperature constant; the reading is made through a glazed window in the top. Provision is made in the case for storing the sample to be tested.

If desired, the case may be filled with a light noncorrosive oil. The samples are weighed on a special balance which gives directly the percentage excess or defect in weight of the sample compared with a wire of standard size.

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CHAPTER V

THE MEASUREMENT OF POTENTIAL DIFFERENCE AND ELECTROMOTIVE FORCE

The most obvious method of determining the potential difference between two points in a circuit is to connect them through a suitable galvanometer which is in series with a high resistance. The potential difference is the product of the galvanometer current and the total resistance of the galvanometer circuit. The galvanometer and the resistance may be combined in a single instrument, and such "potential galvanometers" were in common use before the introduction of voltmeters or instruments where the products are taken once for all and marked on the scales. The various types of instruments that have been described as ammeters may be used as voltmeters, provided the total resistance of the instrument be made sufficiently high by the use of proper windings and series resistances. There are differences of detail; for example, the resistance of the controlling springs need not be kept so low as in ammeters. For direct-current work, the moving-coil type of instrument has become the standard; for alternating currents, the electrodynamic type is usual, though there are some soft-iron instruments.

It is customary to adjust all the galvanometer elements in a series of direct-current voltmeters to give a full-scale deflection with 10 milliamp. or, in more recent instruments, 1 milliamp. This is accomplished by the choice of the proper controlling springs, the adjustment of the strengths of the magnets by demagnetization with alternating current, and finally the use of adjustable magnetic shunts. When multipliers are used, it is most convenient to have the same resistance per volt for all the instruments. The resistances per volt for alternating-current voltmeters of the electrodynamic type are comparatively low—20 to 30 ohms per volt.

In direct-current portable instruments having ranges up to about 750 volts, it is usual to place the series resistance within the base. Self-contained alternating-current portable voltmeters having a range of 300 volts are common.

When using a voltmeter, it should be kept in mind that it shows only the potential difference between its own terminals and that this is not necessarily the same as the potential difference that previously existed between the points on the circuit to which the terminals are applied. For example, suppose that there is a large resistance, 32,000 ohms, across which the drop is 200 volts and that it is desired to measure the potential difference between one terminal and a tap at the middle of the resistance. Obviously, the potential difference in question is 100 volts; however, if a voltmeter of 16,000 ohms resistance is applied between one terminal and the tap, it will read 66.6 volts. The application of the voltmeter has changed by 33 per cent the quantity that it is desired to measure. The disturbance of the circuit conditions diminishes as the resistance of the voltmeter is increased and would be *nil* with an instrument that operated on open circuit, that is, an electrostatic voltmeter, direct voltages being assumed. In heavy engineering work, this difficulty is not often met, but one should not lose sight of the possibility.

In any given case, the test for the adequacy of the resistance of a voltmeter (other than an electrostatic instrument) is to make two readings: the first with the instrument "as is"; the second, when the resistance is temporarily increased in a definite ratio. The second reading, when reduced to volts, should agree with the first. A similar remark applies to the vacuum-tube voltmeter and to the use of a volt box with a potentiometer.

Allowance for the Effect of a Voltmeter.[1]—If the potential difference to be measured is constant, its value before the application of an electromagnetic voltmeter may be calculated from two readings: the first taken with the instrument in the usual manner; the second, when the resistance of the voltmeter circuit has been increased to n times that of the voltmeter.

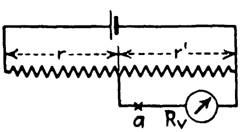


FIG. 129.—Pertaining to effect of voltmeter.

Referring to Fig. 129, if the circuit is opened at a, the voltage that appears across the break is V, the voltage to be measured. Using Thévenin's theorem (page 387), it is seen that the current that will flow through the voltmeter when the circuit is closed at a is

$$I_1 = \frac{V}{R_V + \dfrac{rr'}{r + r'}}.$$

The voltmeter reading is

$$V_1 = \frac{VR_V}{R_V + \dfrac{rr'}{r + r'}}.$$

When the resistance of the voltmeter circuit has been increased to nR_V,

$$I_2 = \frac{V}{nR_V + \dfrac{rr'}{r + r'}};$$

and the reading becomes

$$V_2 = \frac{VR_V}{nR_V + \frac{rr'}{r + r'}}.$$

Therefore

$$V = \left(\frac{V_1V_2}{V_1 - V_2}\right)(n - 1).$$

A resistance in parallel with the voltmeter is an alternative arrangement.

Effect of Temperature.—It is evident that a high resistance, which should not be subject to changes due to the heating action of the current or to alterations of room temperature, is an essential part of all voltmeters other than those of the electrostatic type. As the instrumental errors should be practically independent of temperature, the major portion of the resistance must be of a material having a negligible temperature coefficient. In addition, the effect of temperature on the controlling springs, and, in direct-current instruments, the effect on the magnets, must be small. The springs grow weaker by about 0.04 per cent per degree centigrade as the temperature is raised. The magnets grow weaker with an elevation of temperature, an average value for the temperature coefficient being −0.02 per cent per degree centigrade. The ambient temperature effect on a direct-current voltmeter is the resultant of the changes in the magnet, the springs, and the total resistance. It will depend on the range of the instrument and be comparable to the following:

Reading $\times$ $[1 + 0.00005(t - 25°)]$ for 3-volt range R_v = 300 ohms
Reading $\times$ $[1 - 0.00011(t - 25°)]$ for 50-volt range R_v = 5,000 ohms
Reading $\times$ $[1 - 0.00012(t - 25°)]$ for 150-volt range R_v = 15,000 ohms

It is to be remembered that in low-range voltmeters the movable coil, which is always wound with copper or other wire of high temperature coefficient, forms a relatively large proportion of the total resistance.

Multipliers.—Frequently it is necessary to measure potential differences higher than those for which the voltmeter was originally intended In this case, a properly constructed resistor is joined in series with the instrument so that the voltage necessary to force a given current through the voltmeter circuit is increased; then, if R_M and R_V are the resistances of the multiplier and of the voltmeter, respectively,

$$\text{P.D.} = \text{reading} \times \left(\frac{R_V + R_M}{R_V}\right).$$

When the range is very greatly extended, to several thousand volts, the multiplier is subdivided and mounted in a number of boxes; this makes possible a reduction of the voltage drop between neighboring wires and renders it easier to insulate them. Also, capacitance effects which might be serious in alternating-current work are much reduced, for high-range multipliers may be subject to errors due to distributed capacitance and to capacitance to ground.

The condensation of moisture on the resistors of very high-range multipliers, when they are used in air, frequently gives rise to burnouts. These may be obviated by immersing the multiplier in transformer oil. Multipliers often contain soft rubber insulation, and this must be removed before the immersion.

Electrodynamic Voltmeters.—These instruments are primarily designed for the measurement of alternating potential differences. The indicating portion is a comparatively delicate electrodynamometer with a pivoted movable coil.

The current through an alternating-current voltmeter is given by

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}},$$

where R and L are the total resistance and total effective inductance of the voltmeter, and ω is $2\pi f$. At the ordinary power frequencies, the indications must be practically independent of the frequency; but as L can never be zero, the resistance must be made so high that the reactance can be neglected in comparison with it. The instrument may then be used for both direct and alternating potential differences. This is a convenience, for it may then be calibrated with direct currents, using reversals.

As the electrodynamometer is a comparatively insensitive instrument, considerable current is required. It is not possible, while retaining the characteristics of portability and solidity of construction necessary in order that the instrument may have a long life and maintain its accuracy under the trying conditions of everyday work, to give this form of voltmeter so high a resistance as is common in direct-current instruments. This is a disadvantage, for it increases the liability of altering the circuit conditions by the application of the instrument. The resistance of a 150-volt instrument of this type is usually from 3,000 to 4,500 ohms. Low ranges are obtained by reducing the series resistance; and as the inductance remains the same, the likelihood of a frequency error is much increased.

It is sometimes necessary to measure voltages on power circuits of abnormally high frequency, 500 to 1,000 cycles per second. In this case, if an electrodynamic voltmeter is used, the inductance term will *not* be negligible. Its value will depend on the position of the movable coil. Eddy currents in the metal frames as well as capacitance effects between the coils and in the series resistance also modify the action of the instrument.

Measurements on a certain unshielded electrodynamic voltmeter of a design in common use gave the following results, the resistances, in ohms; the inductances, in millihenrys.

F	At 60 c.p.s.		At 3,000 c.p.s.	
S	At 0 Reading	At 110 Reading	At 0 Reading	At 110 Reading
R	3,337	3,337	3,407	3,417
L	97	107	93	103

Frequency error at 3,000, 17 per cent approximate.

In electrodynamic voltmeters, both the fixed and the movable coils are wound with copper or other material having a high temperature coefficient. Their resistance forms an appreciable portion of the whole. A rise in the ambient temperature increases the resistance of the coil windings, if of copper, 0.4 per cent per degree centigrade, while the series resistance remains practically unaltered. The springs at the same time decrease in stiffness about 0.04 of one per cent per degree centigrade.

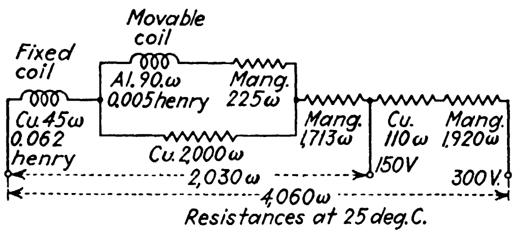


FIG. 130.—Temperature compensation circuit for P_3 voltmeter. (*General Electric Co.*)

The two effects tend to balance, and at first sight it would appear that. with a simple series circuit, the compensation would be exact if the copper contributed one-tenth of the total voltmeter resistance. However, as instruments are of necessity frequently left in circuit for hours, the self-heating error becomes most important, for then the temperatures of the different parts of the instrument are not the same.

Various circuits have been proposed for effecting temperature compensation, that employed by the General Electric Company in their compensated P_3 voltmeter being shown is Fig. 130. The compensation is mainly for the self-heating error and is worked out experimentally for each rating of voltmeter, so that the error due to standing under full voltage for 4 hr. is less than 0.1 per cent of full-scale value. The ambient temperature errors are accepted as they happen to come, provided they are reasonable. The copper resistor spools are positioned near the windings or springs, depending on which predominates in causing the error. The $^{150}\!/_{300}$-volt voltmeter has the 2,000-ohm copper spool placed as closely as convenient to the windings. The results attained are as follows:[1]

Four Hour Self-heating Test at Full Voltage

150-volt connections	±0.1 per cent of full scale
300-volt connections	±0.1 per cent of full scale
Ambient temperature errors	
150-volt connection	+0.006 per cent per degree centigrade
300-volt connection	+0.010 per cent per degree centigrade

Figure 132 shows the general construction of an electrodynamic voltmeter. The working parts of the instrument are surrounded by a

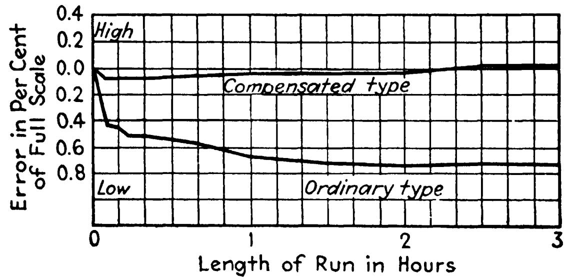


Fig. 131.—Variation of instrument indication with time under different maintained load conditions for electrodynamic voltmeters.

laminated magnetic shield. Electromagnetic damping is obtained by having the movable system carry a fan-shaped sector of aluminum, which swings between the poles of two small permanent magnets. Other makers employ air damping. The arrangement consists of two very light, symmetrically disposed aluminum vanes attached to the movable-coil spindle and moving in carefully finished chambers which are rigidly attached to the frame carrying the jewels (see Fig. 29). The vanes are of exceedingly thin metal stiffened by ribs stamped into them and by the edges which are turned over. The useless leakage to the outside air is reduced to a minimum, and the desired degree of damping is attained by a suitably designed clearance space between the vanes and the walls of the chamber. The moment of inertia of the moving parts of this arrangement is very small.

Fig. 132.—Electrodynamic instrument.

Thermovoltmeters.—Most field measurements of voltages at power frequencies are made by either electrodynamic or soft-iron vane voltmeters. These instruments must of necessity contain coils and consequently have an appreciable inductance, the effect of which becomes apparent if the frequency is raised to the audio range. To obviate this difficulty, thermovoltmeters are used. In the early instruments, the current was passed through a long, thin wire of platinum-silver; and by a suitable mechanism the expansion of the wire, due to its rise of temperature, caused the index to move over the scale. Later designers introduced more effective means for translating the expansion of the wire into the movement of the index. But these instruments never attained great practical importance, for, in general, they are sluggish in action, have unstable zero readings, and require frequent calibration.

The expansion principle has now been discarded, and vacuum thermocouples in series with high resistances are employed. Instruments so constructed are useful for frequencies up to about 2,000 or 3,000 cycles per second. If an attempt is made to employ them at still higher frequencies, they will be found unreliable. This is shown by the fact that if they are connected to measure IR drops at high frequencies, they will give deflections when only one terminal is connected to the circuit or even when not conductively attached to the circuit at all. The errors are introduced by the capacitances to ground of the various parts of the instrument and also capacitances between adjacent parts of the circuit and part of the instrument. At high frequencies, the impedances of these capacitance paths become so low that the heater is traversed by spurious currents of sufficient strength to vitiate the readings of the instrument. The errors may be made commercially negligible, at least up to a frequency of 1,500,000 cycles per second, by properly arranging the series resistor and by the application of electrostatic shields. This has been shown by L. T. Wilson,[2] whose suggestions are embodied in the instrument made by the Weston Electrical Instrument Company.

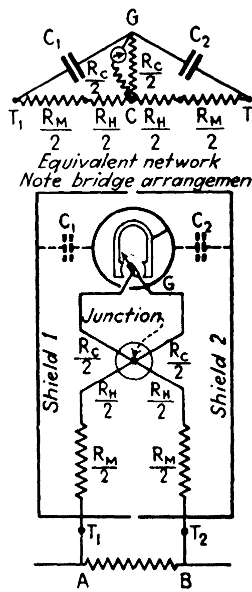


FIG. 133.—Shielded thermal voltmeter for high frequencies. (*Weston Instrument Co.*)

Figure 133 shows the instrument diagrammatically. The fundamental idea is to arrange the resistances and capacitances symmetrically with respect to the two terminals T_1 and T_2. The frame, magnets, moving parts of the indicator, and one terminal of the thermojunction are connected to an inner shield at G. Two symmetrically disposed outer shields are connected to terminals T_1 and T_2. The series resistance, or multiplier, is also divided into two equal parts which are connected to T_1 and T_2. In the diagram, the capacitances between the inner shield and shields 1 and 2 are indicated by C_1 and C_2. Inspection of the equivalent circuit shows that if the capacitances and resistances are symmetrically arranged with respect to GC, no capacitance current will flow through the thermocouple, for the arrangement is then that of a balanced impedance bridge.

The frequency error at 600,000 cycles per second is about one-fourth of 1 per cent; at 1,000,000 cycles per second, it is about 1 per cent; and at 1,500,000 cycles per second, about 1.5 per cent.

For instruments below the 5-volt range, the resistance is 130 ohms per volt; for ranges 5 to 20 volts, it is 500 ohms per volt. If the range is above 20 volts, the frequency error is accentuated. The current taken by the instrument is from 2 to 8 milliamp., depending on the voltage range. In addition, there is the charging current of the shields, which is from 2 to 8 milliamp. at 1,000,000 cycles per second.

The advantages of thermoinstruments are that they are practically noninductive, have no heating error, are not influenced by local magnetic fields, and, when properly designed, are commercially independent of changes of frequency and of wave form. Their disadvantages are that they are easily burned out by overvoltages and that the resistance of the voltmeter is low.

ELECTROSTATIC INSTRUMENTS

In instruments of this class, advantage is taken of the electrostatic attraction existing between bodies charged to different potentials. The magnitude of the force depends on the geometry of the system of conductors, the relative potentials of its parts, and the dielectric coefficient of the medium separating the attracting bodies.

The Attracted-disk Electrometer.—The first suggestion of the attracted-disk electrometer was due to Sir William Snow-Harris, who used an instrument of this type. Its essential members were a fixed circular plate electrode supported by an insulating standard and a movable plate electrode hung directly over the fixed plate from the arm of a gravity balance. By putting weights in the scale pan, a balance could be secured, and a measure of the electrostatic attraction and consequently of the potential difference between the electrodes obtained. A defect of any such simple arrangement, which renders it useless as an absolute instrument, is that the distribution of the charges over the surfaces will not be uniform, owing to the influence of the edges of the plates. This renders inexact the application of a simple formula for the attraction between the two plates, based on the assumption of a uniform density of distribution of the charge.

The distribution of the charges over the central portions of two parallel plates, whose dimensions are large compared with their distance apart, will be practically uniform. Therefore, if the force exerted on the central portion of one of the plates is measured, the use of a formula that assumes a uniform distribution will be legitimate.

Lord Kelvin secured a practically uniform distribution by the use of the guard ring. This is a broad ring closely surrounding, but not touching, the movable member and in electrical connection with it. The stationary, or attracting, plate has the same diameter as the guard ring.

In the Kelvin absolute electrometer, the attraction on the movable member is weighed by a calibrated spring balance.

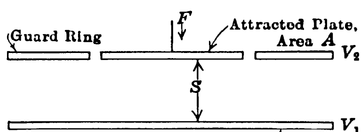


FIG. 134.—Pertaining to attracted-disk electrometer.

The relation between the difference of potential of the plates and the force of attraction may be established thus: Referring to Fig. 134, the attracted plate has an area of A sq. cm. and is distant S cm. from the attracting plate. V_2 and V_1 are the potentials of the two plates. The arrangement forms an electrical condenser; and if S is small compared with the size of the plates, the capacitance will be

$$C = \frac{KA}{4\pi S}.$$

K for air is very nearly unity.

The energy necessary to raise one plate to the potential V_1 and the other to the potential V_2 will be

$$E = \frac{1}{2}Q(V_1 - V_2) = \frac{1}{2}C(V_1 - V_2)^2 = \frac{A(V_1 - V_2)^2}{8\pi S}.$$

Suppose that the upper plate is given a small displacement, V_1 and V_2 being kept constant by connection to a source of potential difference. There will be a change in the energy of the condenser which will be numerically equal to the mechanical work necessary to displace the plate in the direction S.[3]

$$dE = FdS.$$

$$F = \frac{dE}{dS} = \frac{A(V_1 - V_2)^2}{8\pi S^2}. \tag{1}$$

Therefore

$$V_1 - V_2 = S\sqrt{\frac{8\pi F}{A}} \quad \text{in electrostatic units.} \tag{2}$$

If $(V_1 - V_2)$ is in volts,

$$\text{P.D.} = 300S\sqrt{\frac{8\pi F}{A}} = 1{,}504S\sqrt{\frac{F}{A}}. \tag{3}$$

It has been assumed that all of the electrostatic lines of force are straight and normal to the plane of the disk. A few lines will stray into the very narrow gap between the guard ring and the attracted plate.

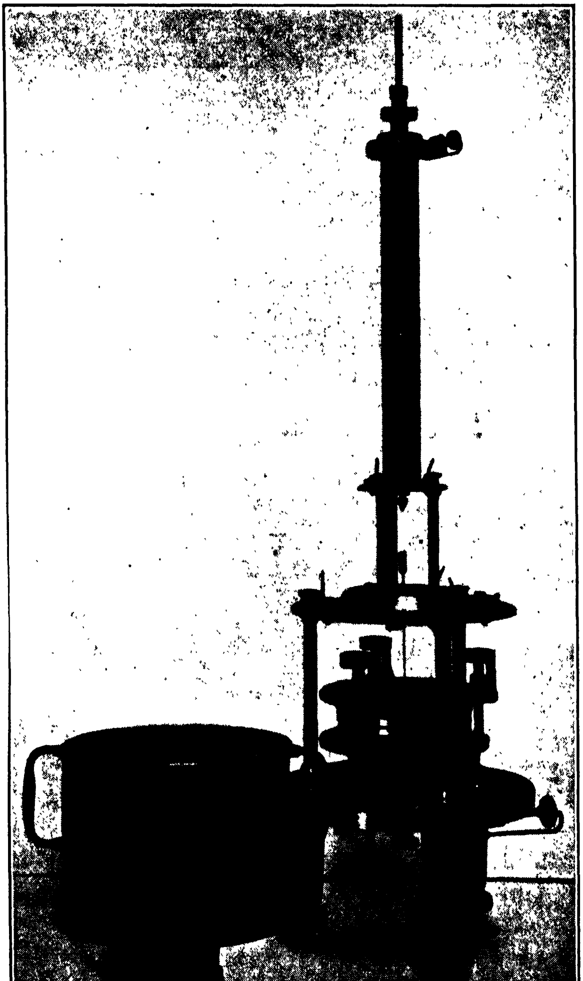


FIG. 135*A*.—Quadrant electrometer used at National Physical Laboratory as an electrostatic wattmeter.

They may be assumed to divide equally between the ring and the plate, so, to make an approximate allowance, the effective area of the plate may be taken as the actual area of the plate plus one-half the area of the air gap.

It is well to emphasize the fact that unless the voltages are high, the forces to be dealt with in electrostatic instruments are small. If

$$A = 100 \text{ sq. cm.},$$
$$S = 1 \text{ cm.},$$
$$\text{P.D.} = 150 \text{ volts},$$

then

$$F = 1 \text{ dyne, approx.}$$

That is, the force is about the same as the attraction of gravity on a mass of 1 mg. To increase this force, the plates must be brought very near together, or the use of the instrument restricted to measuring high potentials.

A very highly developed form of the attracted-disk electrometer, intended for the absolute measurement of high voltages, is now being carefully studied at the National Bureau of Standards.

The Quadrant Electrometer.[3]—The quadrant electrometer, the invention of Lord Kelvin, is much more sensitive than the attracted-disk instrument. Of late years, it has been employed in investigations concerning the energy losses in dielectrics intended for high-voltage insulations. At the National

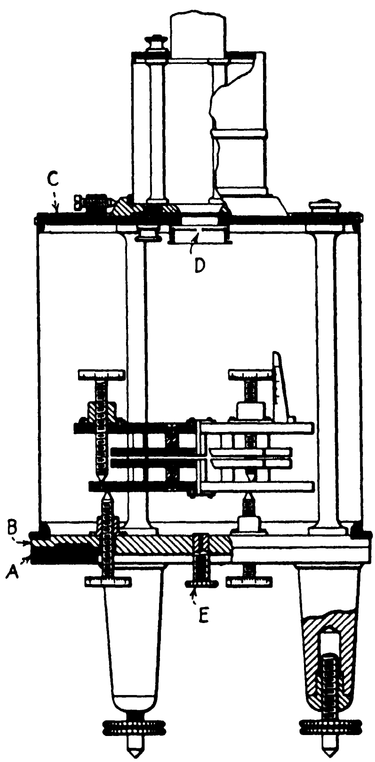


FIG. 135*B*.—Section through the lower portion of Fig. 135*A*.

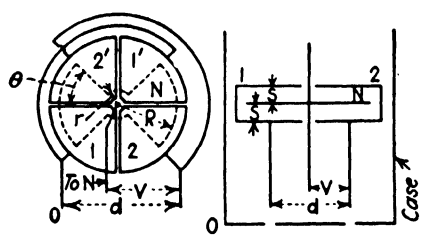


FIG. 136.—Arrangement of conductors in quadrant electrometer.

Physical Laboratory, it is employed in the testing of power and energy meters—a unique installation. The perfected form of the instrument designed at that laboratory for work of high precision is shown in Figs. 135*A* and 135*B*.

The arrangement of conductors in the quadrant electrometer is indicated in Fig. 136.

The quadrants, 1, 1′, and 2, 2′ are made by cutting into quarters two flat, parallel, and coaxial circular plates. For low potentials, their diameters may be from 3 to 5 in. In the instrument shown in Figs. 135*A* and 135*B*, the distance between the plates is adjustable. The quadrants are supported on insulating standards and are cross-connected electrically, as shown in Fig. 136. The thin needle N is made of a stiff metal, of the form indicated and suspended midway between the upper and lower quadrants. If the instrument is intended for use in deflection methods (power measurement), every effort is made to obtain exact symmetry. Referring to Figs. 135A and 135B, each set of four quadrants, of copper-aluminum alloy and 12 cm. in diameter, is attached to a rigid brass back plate by three pillars of amberite, a material that does not deteriorate under the action of light. The cuts between the quadrants are 0.5 mm. wide, being accurately adjusted by use of a special gage. After attachment to the back plate, the quadrants are lightly machined and then ground flat on a surface plate. This is a most important detail, for it renders the active surfaces of each set of quadrants parts of the same plane and eliminates a major cause of departure from the behavior of the ideal instrument. Reassembling of the instrument after it has been dismounted is facilitated by the hole, slot, and plane arrangement of the three fine-pitched screws that support the lower set of quadrants. The upper set of quadrants is supported on the lower back plate by a hole, slot, and plane arrangement of micrometer screws. It is thus possible to raise and lower the quadrants, tilt them, or adjust their distance apart to 0.001 mm. The needle is of copper-aluminum alloy foil, 0.015 mm. thick and ribbed to obtain the necessary stiffness. It is essential that it be bounded by straight radial lines and circular arcs centered on the axis of rotation and that it lie in a horizontal plane, not being tilted with respect to the short rigid shank by which it is attached to the suspension. The needle is too delicate to be handled with the fingers. With the quadrants 2 mm. apart, the air damping brings the needle to rest in one and one-half swings. The time of a damped swing is 16 sec. The needle may be precisely raised or lowered, turned, or displaced laterally in any direction with reference to the quadrants by appropriate adjusting screws. The whole instrument above the subbase, which is supported by the leveling screws, may be turned in azimuth, and the zero reading brought to the zero of the scale. To avoid contact potentials, no "finish" is used on the inside of the instrument.

For high-voltage work, it is necessary to employ much larger spacings, and the quadrants may be as much as 14 in. in diameter and 4 in. apart. In this case, the needle is made more rigid and the electrical stresses reduced by forming the edge of very thin aluminum tubing about ¼ in. in diameter. Such electrometers are oil immersed to increase the dielectric strength.

Theory of the Quadrant Electrometer.—Referring to Fig. 137, let

C_1 = partial capacitance between quadrant pair 1 and the case,—a constant.

C_2 = partial capacitance between quadrant pair 2 and the case,—a constant.

C_{12} = partial capacitance between quadrant pair 1 and quadrant pair 2.

C_{CN} = partial capacitance between movable system and the case, or internal metal frames at the potential of the case.

C_{1N} = partial capacitance between quadrant pair 1 and the movable system.

C_{2N} = partial capacitance between quadrant pair 2 and the movable system.

V_N = rise of potential from case to movable system.

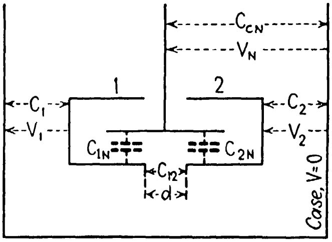


FIG. 137.—Indicating the potential differences and partial capacitances concerned in the action of a quadrant electrometer.

V_1 = rise of potential from case to quadrant pair 1.

V_2 = rise of potential from case to quadrant pair 2.

V = rise of potential from quadrant pair 2 to the movable system.

d = rise of potential from quadrant pair 1 to quadrant pair 2.

θ = angular deflection of the movable system.

Then

$$\begin{aligned} d &= V_2 - V_1 \\ V &= V_N - V_2 \\ V + d &= V_N - V_1 \\ V_1 + V_2 &= 2V_N - (2V + d) \\ \frac{V_1 + V_2}{2} &= V_N - \frac{2V + d}{2}. \end{aligned}$$

C_{CN}, C_{1N} and C_{2N} are dependent on the position of the movable system; they are functions of the deflection. For this form of electrometer, C_{12} is a constant.

When potentials are applied, the movable system swings toward either quadrant pair 1 or quadrant pair 2 according to the distribution of voltages. The variable capacitances, and therefore the energy of the system, are thus altered.

The quantity on the movable system is

$$q_N = C_{CN}V_N + C_{1N}(V_N - V_1) + C_{2N}(V_N - V_2),$$

and similarly for the quantities on quadrants 1 and 2.

The expression for the energy in terms of the voltages and the partial capacitances is

$$W = \frac{1}{2}\begin{bmatrix} +C_{CN}V_N^2 + C_{1N}V_N^2 - C_{1N}V_1V_N + C_{2N}V_N^2 - C_{2N}V_2V_N \\ +C_1V_1^2 + C_{1N}V_1^2 - C_{1N}V_1V_N + C_{12}V_1^2 - C_{12}V_1V_2 \\ +C_2V_2^2 + C_{2N}V_2^2 - C_{2N}V_2V_N + C_{12}V_2^2 - C_{12}V_1V_2 \end{bmatrix}.$$

The derivative of the energy W, with respect to the change in configuration, that is θ, is equal to the electrical turning moment acting on the movable system which at equilibrium is equal to the restoring moment of the suspension. Therefore, as C_1 and C_2 are constant,

$$\tau\theta = \frac{1}{2}\left[\left(\frac{\partial C_{CN}}{\partial\theta}\right)V_N^2 + \left(\frac{\partial C_{1N}}{\partial\theta}\right)(V_N - V_1)^2 + \left(\frac{\partial C_{2N}}{\partial\theta}\right)(V_N - V_2)^2 + \left(\frac{\partial C_{12}}{\partial\theta}\right)(V_1 - V_2)^2\right] \quad (4)$$

or

$$\tau\theta = \frac{1}{2}\left[\left(\frac{\partial C_{CN}}{\partial\theta}\right)V_N^2 + \left(\frac{\partial C_{1N}}{\partial\theta}\right)(V + d)^2 + \left(\frac{\partial C_{2N}}{\partial\theta}\right)V^2 + \left(\frac{\partial C_{12}}{\partial\theta}\right)d^2\right]. \quad (5)$$

If C_{12} is constant, as it is in the instrument shown in Fig. 136, the influence of the "case" negligible, and the variations of C_{1N} and C_{2N} linear, then as the needle swings from a position within one pair of quadrants to a position within the other pair of quadrants,

$$\frac{\partial C_{NC}}{\partial\theta} = 0, \qquad \frac{\partial C_{1N}}{\partial\theta} = -\frac{\partial C_{2N}}{\partial\theta},$$

and

$$\tau\theta = \frac{1}{2}\left(\frac{\partial C_{1N}}{\partial\theta}\right)(2Vd + d^2).$$

Referring to Fig. 136 and assuming that all the lines of force are perpendicular to the needle,

$$C_{1N} = \frac{4\left(\frac{\pi r^2}{4} + \frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta\right)}{4\pi S} = \frac{r^2}{4S} + \left(\frac{R^2 - r^2}{2\pi S}\right)\theta.$$

Hence

$$\tau\theta = \frac{1}{2}\left(\frac{R^2 - r^2}{2\pi S}\right)(2Vd + d^2), \qquad \text{absolute electrostatic units.}$$

If volts are used,

$$\tau\theta = \frac{1}{1{,}130{,}000}\left(\frac{R^2 - V^2}{S}\right)(2Vd + d^2). \quad (6)$$

The quadrant electrometer is always used as a secondary instrument, but (6) is useful in preliminary design. The readings are made by a mirror and scale method, so, for a symmetrical electrometer, if the deflection D is in scale divisions,

$$D = K(2Vd + d^2) = 2Kd\left(V + \frac{d}{2}\right). \tag{7}$$

$V + \frac{d}{2}$ is the potential difference measured from the mid-potential of the quadrants to the needle.

If the potential differences are alternating, the instantaneous turning moment is proportional to $2Vd + d^2$, and

$$D = K\frac{1}{T}\int_0^T (2Vd + d^2)dt = 2K\frac{1}{T}\int_0^T d\left(V + \frac{d}{2}\right)dt. \tag{8}$$

The expression given by Maxwell,

$$D = K\left[(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)\right], \tag{9}$$

which is equivalent to (7), is frequently used when discussing the quadrant electrometer. In it V_1, V_2 and V are the potentials of the two sets of quadrants and the needle.

If one attempts to use a quadrant electrometer as ordinarily constructed and employs the simple theory just given, it will be found, in general, that the factor K which should be constant is, in reality, variable and dependent on the voltage applied to the needle. This shows that the assumptions $\partial C_{NC}/\partial\theta = 0$ and $\frac{\partial C_{1N}}{\partial\theta} = -\frac{\partial C_{2N}}{\partial\theta}$, independently of θ, are not tenable.

In an instrument intended to be symmetrical and in which the departures from symmetry are small, there is no reason for believing that the partial capacitances depart abruptly from the typical relations $C = K_1$ or $C = K_2 + K_3\theta$. In addition, the departures from these relations are not likely to be very great. In such cases, the partial capacitances may be written as follows:

$$\begin{aligned} C_{CN} &= C'_{CN} + C''_{CN}\theta + C'''_{CN}\theta^2 + \cdots \\ C_{1N} &= C'_{1N} + C''_{1N}\theta + C'''_{1N}\theta^2 + \cdots \\ C_{2N} &= C'_{2N} + C''_{2N}\theta + C'''_{2N}\theta^2 + \cdots \end{aligned} \tag{10}$$

The quantities C, with various primes and subscripts, are constants depending on the construction and adjustment of the particular instrument under consideration. Some of them may be zero, some are positive, and some negative. For instance, C''_{1N} and C''_{2N} have opposite signs, for if the needle swings toward quadrant pair 1, C_{1N} is increased while C_{2N} is diminished. On account of uncertainties as to the geometry of the active parts and mathematical difficulties, the instrumental constants are not amenable to accurate calculation. In general, the value of the constants depend not only on the dimensions of the instrument but on the differences in the heights of the various quadrants above a horizontal plane, on the tilting of the quadrants and the tilting of the needle with respect to a horizontal plane, on the height of the needle within the quadrants, on the centering of the needle within the quadrants, on the width of the slits between the quadrants, and in the relative widths of the slits. Usually, the simultaneous occurrence of two of these possible sources of error is required to produce an effect on the behavior of the instrument. For instance, if the quadrants are perfectly symmetrical, a tilting of the needle about its long axis produces no effect; but if some of the quadrants are displaced vertically, the law of the instrument is affected (see page 241). "Contact" differences of potential may occur in the instrument due to soldered joints, the finish, etc. In a properly designed electrometer, they are reduced by making all the working parts of the same corrosion-resisting alloy and avoiding "finish" of any sort.

Let P_N = contact difference of potential between the case and needle;
P_1 = contact difference of potential between the case and quadrant pair 1;
P_2 = contact difference of potential between the case and quadrant pair 2.

The quantities P_N, P_1, P_2 are to be added to the corresponding potential differences due to outside sources, $V_N + P_N$ replacing V_N etc., in (4).

Substituting in (4) the new values of the potential differences and the derivatives obtained by use of (10) and introducing the following abbreviations:

$$A'' = \frac{C''_{CN} + C''_{1N} + C''_{2N}}{2\tau}$$

$$B'' = \frac{C''_{1N}}{2\tau}$$

$$C'' = C''_{2N}$$

$$E'' = \frac{C''_{1N}(P_1 - P_N)}{2\tau}$$

$$F'' = \frac{C''_{2N}(P_2 - P_N)}{2\tau}$$

$$G'' = \frac{P_N(C''_{CN} + C''_{1N} + C''_{2N}) - P_1C''_{1N} - P_2C''_{2N}}{2\tau}$$

$$A''' = \frac{C'''_{CN} + C'''_{1N} + C'''_{2N}}{2\tau}$$

$$B''' = C'''_{1N}$$

$$C''' = C'''_{2N}$$

gives, when terms on the left-hand side of the equation which involve small differences are neglected,

$$\theta\{1 - V_N^2A''' - V_1^2B''' - V_2^2C''' + 2V_N(V_1B''' + V_2C''')\} = V_N^2A'' + V_1^2B'' + V_2^2C'' - 2V_N(V_1B'' + V_2C'') + 2V_1E'' + 2V_2F'' + 2V_NG''; \quad (11)$$

or, if $V = V_N - V_2$ and $= V_2 - V_1$,

$$\theta\{1 - V_N^2A''' + (V_1B''' + V_2C''')(2V + d) + V_1V_2(B''' + C''')\} = V_N^2A'' - (V_1B'' + V_2C'')(2V + d) - V_1V_2(B'' + C'') + 2V_1E'' + 2V_2F'' + 2V_NG''. \quad (12)$$

The general formulae (11) and (12) apply to a quadrant electrometer which is intended to be symmetrical, the quadrants being conductively connected. In many cases, either V_1 or V_2 is zero.

It is seen that the restoring moment consists of two parts: one due to the suspension, the other to the electrical reactions within the electrometer and depending for its value not only on the construction and adjustment of the instrument but on the applied voltages. The terms following the quantity 1 on the left side of Eqs. (11) and (12) represent the electrostatic control. Their net effect may be either positive or negative. If it is positive, their effect is the same as if the stiffness of the suspension were increased. The sensitivity is thus diminished, and there is said to be positive electrostatic control. If their net effect is negative, there is negative electrostatic control, and the sensitivity is increased.

When only alternating potentials are employed, the terms involving the first power of the voltages average out during a cycle. In quadrant electrometers intended for power measurement by the deflection method, every effort is made to have the instruments symmetrical. In certain other instruments, for instance those of the Compton type (page 241), the asymmetry is adjustable, with the result that the electrostatic control may be varied. If d is small, one set of quadrants grounded, and A'' adjusted to zero by the method given below, then, with alternating voltages,

$$\theta\{1 - V_N^2A'''\} = B'''\frac{1}{T}\int_0^T(2Vd + d^2)dt = 2B''\frac{1}{T}\int_0^T d\left(V + \frac{d}{2}\right)dt \quad \text{approx.} \quad (13)$$

V_N^2 is the mean square of the needle voltage. Equation (13) differs from Eq. (8) by the term V_N^2A''', which is due to the fact that the actual instrument does not, in general, fulfill the assumption of a symmetrical arrangement of parts made in deducing (8).

Adjustment.—1. *Leveling.*—The construction of the instrument should be such that it can be turned in azimuth above the subbase which is supported by the leveling screws. The screws should be adjusted until the instrument is level in all azimuths.

A system of accurate levels attached directly to both the upper and the lower quadrant structures greatly facilitates the adjustment of the instrument.

2. *Adjustment for Coincidence of Mechanical and Electrical Zeros.*—If the instrument is to be used for power measurement, it is leveled, the needle accurately centered and carefully set midway between the upper and lower quadrant faces with its long axis directly below a quadrant slit. Both quadrant pairs are connected together and to ground, and a high alternating voltage applied to the needle. There should be no deflection; if one appears, that is, if the mechanical and electrical zeros are not coincident, the quadrants are given a very slight tilt by the micrometer screws until the deflection disappears.

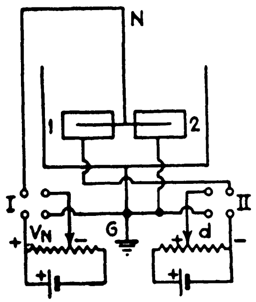


FIG. 138.—Connections for determining constants of quadrant electrometer.

This is a most important adjustment which should be checked whenever the instrument is used, for, when attained, $A'' = 0$, and the term V_N^2A'' may be neglected in Eqs. (11) and (12). The term $2V_NG''$ is not reduced to zero by this adjustment.

Determination of Constants.—The connections to be used, if continuous voltages are employed, are shown in Fig. 138.

V_N is made large, and d small; quadrant pair 2 is connected to ground and to the case. I and II are two commutators with bridging pieces which may be placed in either the vertical or the horizontal positions, thus changing the algebraic signs of V_N and d. Readings are taken with

I vertical, II vertical, deflection θ_1, $V_N \equiv +$, $d \equiv +$
I horizontal, II vertical, deflection θ_2, $V_N \equiv -$, $d \equiv +$
I horizontal, II horizontal, deflection θ_3, $V_N \equiv -$, $d \equiv -$
I vertical, II horizontal, deflection θ_4, $V_N \equiv +$, $d \equiv -$,

then

$$\theta_1[1 - V_N^2A'''] = 2VdB'' + d^2B'' - 2dE'' + 2V_NG''$$
$$\theta_2[1 - V_N^2A'''] = -2VdB'' + d^2B'' - 2dE'' - 2V_NG''$$
$$\theta_3[1 - V_N^2A'''] = +2VdB'' + d^2B'' + 2dE'' - 2V_NG''$$
$$\theta_4[1 - V_N^2A'''] = -2VdB'' + d^2B'' + 2dE'' + 2V_NG''.$$

Therefore

$$(1)\ [1 - V_N^2A'''][(\theta_1 + \theta_3) - (\theta_2 + \theta_4)] = 8VdB''.$$
$$(2)\ [1 - V_N^2A'''][(\theta_1 + \theta_4) - (\theta_2 + \theta_3)] = 8V_NG''.$$
$$(3)\ [1 - V_N^2A'''][(\theta_1 + \theta_2) - (\theta_3 + \theta_4)] = -8dE''.$$
$$(4)\ [1 - V_N^2A'''][(\theta_1 + \theta_2) + (\theta_3 + \theta_4)] = 4d^2B''. \qquad (14)$$

These equations enable the constants to be determined by taking observations with two different sets of values of V and d. In alternating-current work, A''' and B'' are the only constants required; in this case, it is more convenient to calibrate with alternating voltages, the arrangement being such that V and d are in time phase. The effect of contact potentials is then eliminated.

Compton Electrometer.[4]—This form of quadrant electrometer was designed primarily for the measurement of ionization currents by the accumulation of charge method. When thus employed, one pair of quadrants is permanently grounded. The second pair is connected to the collecting plate of the grounded metal ionization chamber, the other plate of which is kept at a constant potential by a battery; for instance 200 volts positive. The second pair of quadrants and the collecting plate form an insulated system of conductors, the capacitance of which must be known. The needle is kept at a constant and relatively high potential by batteries. At the start, the two sets of quadrants and the collecting plate are connected together. The difference of potential between the quadrants d is thus made 0, and when the instrument is properly adjusted there will be no deflection. The quadrant pairs are then separated; one is kept at zero potential by the ground, the other gradually changes in potential as the charge of positive ions accumulates on the collecting plate. Thus d, the voltage between the quadrants, increases from 0, and a correspondingly increasing deflection appears. The ionization current is calculated from the rate of growth of d:

$$i = C\left(\frac{\Delta d}{\Delta t}\right)$$

It is desirable that the capacitance C be small and the instrument very sensitive. The order of magnitude of C is 10 $\mu\mu$f, and a practical sensitivity of 10,000 scale divisions per volt may be attained.

In this form of electrometer, to obtain a high and adjustable sensitiveness, the needle is given a slight—just noticeable—tilt about its long axis, and one quadrant pair (or, as the instrument is actually constructed, one quadrant) arranged so that it can be displaced vertically by a micrometer screw. The rate of variation of capacitance as the needle deflects is thus controlled.

The needle is ordinarily cut from a disk of aluminum foil or very thin mica, sputtered to make it conducting, about 10 mm. in diameter. Its long axis is made perpendicular to the axis of rotation. The vertical distance h between the active faces of a quadrant is about 5 mm.

The rates of variation of the capacitances with the deflection which are to be used in deducing the equation for the deflection may be determined with sufficient accuracy to explain the action of the instrument.

Referring to Fig. 139, h is the distance between the active faces of a quadrant; δ, the vertical displacement of the movable quadrant pair with respect to the fixed quadrant pair; p, the vertical displacement of the center of the needle with respect to the medial line; S, the vertical distance from a quadrant edge to the needle; and s, the slope of the needle $s = dS/rd\theta$.

Suppose the needle to be deflected through an angle θ and then given a small additional deflection $d\theta$ toward quadrant 1. The part already

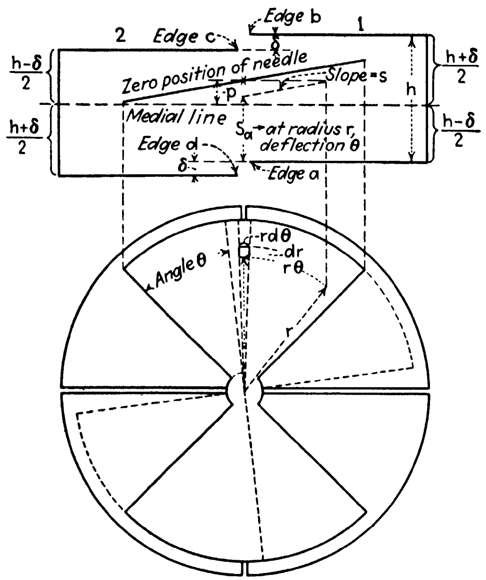


FIG. 139.—Pertaining to Compton electrometer.

within 1 moves to a slightly different position, and no change of capacitance is produced. The element $rd\theta dr$ moves into the influence of quadrant pair 1 and moves out of the influence of quadrant pair 2. This element forms an elementary condenser, the vertical distance between the plates being, for the lower quadrant edge a,

$$S_a = \frac{h - \delta}{2} + p - s\theta r.$$

The elementary capacitance is therefore

$$\frac{rdrd\theta}{4\pi\left(\dfrac{h - \delta}{2} + p - s\theta r\right)} = \frac{rdrd\theta}{2\pi(h - \delta + 2p - 2s\theta r)} = \frac{rdrd\theta}{2\pi(k_1 + k_2 r)},$$

when, for quadrant edge a, $k_1 = h - \delta + 2p$, $k_2 = -2s\theta$.

For simplicity, it is assumed that the active edge of a quadrant extends to the axis of rotation. For one quadrant pair,[4]

$$\frac{\partial c}{\partial \theta} = \frac{1}{\pi}\int^{R}\frac{r\,dr}{k_1 + k_2 r} = \frac{1}{\pi}\left[\frac{1}{k_2{}^2}(k_1 + k_2 r - k_1 \log_\epsilon\ (k_1 + k_2 r)\right]_0^R$$

$$= \frac{1}{\pi}\left[\frac{R}{k_2} + \frac{k_1}{k_2{}^2}\log_\epsilon\left(\frac{1}{1 + (k_2/k_1)R}\right)\right].$$

The logarithm may be expanded into a convergent series,[4] and

$$\frac{\partial c}{\partial \theta} = \frac{R^2}{\pi}\left(\frac{1}{2k_1} - \frac{1}{3}\frac{k_2}{k_1{}^2}R + \frac{1}{4}\frac{k_2{}^2R^2}{k_1{}^3} - \frac{1}{5}\frac{k_2{}^3R^3}{k_1{}^4}\cdots\right).$$

For edge a, $k_1 = h - \delta + 2p$ $k_2 = -2s\theta$.
For edge b, $k_1 = h + \delta - 2p$ $k_2 = +2s\theta$.
For edge c, $k_1 = h - \delta - 2p$ $k_2 = +2s\theta$.
For edge d, $k_1 = h + \delta + 2p$ $k_2 = -2s\theta$.

For quadrant pair 1,

$$\frac{\partial C_{1N}}{\partial} = \frac{R^2}{\pi}\left[\frac{1}{2}\left(\frac{1}{h - \delta + 2p} + \frac{1}{h + \delta + 2p}\right) - \frac{2}{3}Rs\theta\left(-\frac{1}{(h - \delta + 2p)^2} + \frac{1}{(h + \delta - 2p)^2}\right) + R^2s^2\theta^2\left(\frac{1}{(h - \delta + 2p)^3} + \frac{1}{(h + \delta - 2p)^3}\right) - \frac{8}{5}R^3s^3\theta^3\left(-\frac{1}{(h - \delta + 2p)^4} + \frac{1}{(h + \delta - 2p)^4}\right)\right].$$

Similarly for quadrant pair 2.

The fractions may be expanded into series, and the results inserted in the formula for the deflection. When terms are collected according to power of θ, d being assumed small compared with V_N,

$$\tau\theta = \frac{R^2}{\pi h}\left\{V_N d\left[1 + \frac{\delta^2}{h^2} + \frac{4p^2}{h^2}\cdots\right] + V_N{}^2\left[-\frac{4p\delta}{h^2}\left(1 - \frac{2\delta^2}{h^2} + \frac{8p^2}{h^2}\cdots\right) + \frac{8Rs\delta}{3h^2}\left(1 + \frac{2\delta^2}{h^2} + \frac{8p^2}{h^2}\cdots\right)\theta + \frac{64}{5}\frac{R^2s^2\delta}{h^4}\left(1 + \frac{5\delta^2}{h^2} + \frac{20p^2}{h^2}\cdots\right)\theta^3\right]\right\}, \quad (15)$$

which may be written

$$\tau\theta = KV_Nd - Cp\delta V_N{}^2 + As\delta V_N{}^2\theta + Bs^3\delta V_N{}^2\theta^3. \quad (16)$$

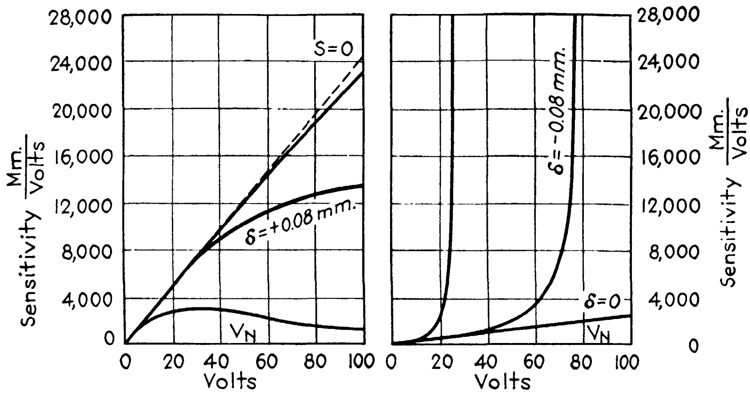
It is seen from (16) that when the two quadrant pairs are connected together and grounded, thus making $d = 0$, in general there will be a deflection when the needle is electrified, due to the term—$Cp\delta V_N{}^2$. This deflection may be eliminated by adjusting p, the height of the needle within the quadrants. Reference to Eq. (15) shows that this adjustment affects only small correction terms in the constants A and B. With the height of the needle properly adjusted,

$$\tau\theta = KV_Nd + As\delta V_N{}^2\theta + Bs^3\delta V_N{}^2\theta^3. \quad (17)$$

The sensitivity is

$$\frac{\theta}{d} = \frac{KV_N}{\tau - As\delta V_N^2 - Bs^3\delta V_N^2\theta^2}. \tag{18}$$

If s and δ are small, and the needle at a constant potential, the deflection is sensibly proportional to the voltage. The algebraic signs of the A and B terms depend on the relative signs of s and δ. In Fig. 139, they both have the same sign, the A and B terms annul in part the stiffness of the suspension, and the sensitivity is increased; that is, there is negative electrostatic control. If s and δ have opposite signs, there is positive electrostatic control, and the sensitivity is decreased. It is seen that the sensitivity can be adjusted between wide limits by simply



A, positive electrostatic control. *B*, negative electrostatic control.
FIG. 140.—Showing effect of positive and negative electrostatic control in Compton electrometer.

moving one quadrant pair vertically. As the instrument is constructed, only one quadrant is made movable, and the constants have one-half the values given in Eq. (15).

Figures 140*A* and *B* show the results attained in a particular case. The suspension used in obtaining the data for Fig. 140*B* was about ten times as stiff as that used in obtaining the data for Fig. 140*A*. The B term in Eq. (18) may become important at high sensitivities. When a high sensitivity is desired, it is advisable to use a weak suspension rather than to rely on the negative electrostatic control given by the A and B terms; for if s is small, and h large, the B is small, and the dependence of the sensitivity on the deflection is reduced. To obtain high sensitivity coupled with quick action, the needle should be small. Numerous practical suggestion are given in the original paper. It is necessary that the instrument be mounted on an antivibration support.

Inspection shows that if $\delta = 0$ and $p = 0$, a great simplification results; then $\theta = KV_Nd/\tau$. The bearing of this on a quadrant electrometer intended for power measurements is obvious.

Lindemann Electrometer.[5]—This electrometer was originally designed to be mounted, in conjunction with a photoelectric cell, on an equatorial telescope; consequently, it must be small, and its performance unaffected by changes of level.

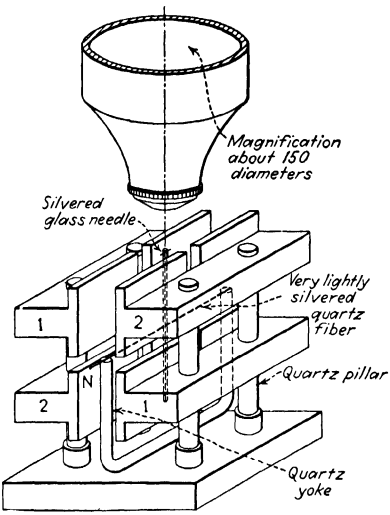


FIG. 141.—Lindemann electrometer about actual size. Case omitted.

Figure 141 is a full-size drawing of the instrument. The following data apply to the instrument as described by Lindemann. The quadrants 1 and 2 are two sets of plates about 6.0 mm. apart and mounted on quartz pillars, which give an exceedingly high insulation. A taut silvered-quartz suspension about 14 mm. long and 0.006 mm. in diameter is used. The taut suspension fixes the center of rotation of the needle, which consists of two silvered-glass fibers about 9 mm. long and 0.007 mm. thick. To secure balance, they are attached one on either side of the suspension fiber. This construction gives a needle sufficiently stiff to withstand the gravitational and electrical forces without appreciable

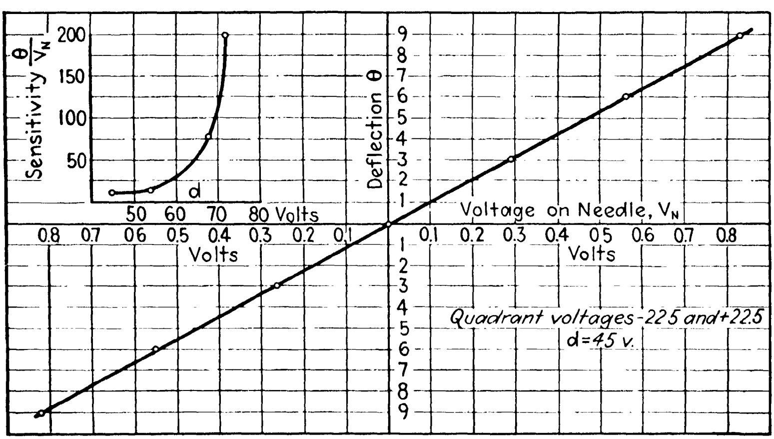


FIG. 142.—Calibration and sensitivity curves for Lindemann electrometer.

bending. The instrument, which has a capacitance of only 1.3 cm., is so small that it can be placed on the stage of an ordinary compound microscope and read by observing the motion of the end of the needle, the microscope being provided with a graduated eyepiece.

With a field of 160 volts per centimeter between the plates, the deflection reaches 0.99 of its final value in about 1 sec. The end of the needle moves 0.76 mm. per volt with this field; with a ¼-in. objective and a × 20 eyepiece, a sensitivity of 450 scale divisions per volt is attained. The reading can be made to one-tenth of a scale division. The needle becomes unstable when the field is raised to 166 volts per centimeter. The sensitivity is uninfluenced by tilting the instrument. The free period *in vacuo* is about 0.15 sec.; the air damping is considerable. The instrument is protected by a metallic case which can be earthed to form an electrostatic screen. The leads to the needle and to the quadrants are carried through the sides of the case in quartz bushings, thus insuring high insulation. Any of the connections common in electrometer practice may be employed. If the quadrants are held at fixed potentials, and the voltage on the needle varied, a linear calibration curve is obtained, as shown in Fig. 142, which applies to an instrument similar to that described. Owing to the arrangements of the parts, the theory of the instrument differs from that of the quadrant electrometer in that C_{12} is an even function of θ rather than constant; that is,

$$C_{12} = C'_{12} + C'''_{12}\theta^2.$$

When the instrument is used, the needle voltage is low, and the quadrant voltage high, which is the reverse of the voltage distribution usually employed with the quadrant electrometer. The voltage to be measured, which in many cases is only a fraction of a volt, is applied to the needle, and d is made relatively large; for instance, V_1 might be $+22.5$ volts, and V_2 might be -22.5 volts. Under these circumstances, the negative electrostatic control is practically proportional to d^2,[5] and the sensitivity is

$$S = \frac{\theta}{V_N} = \frac{K_1 d}{\tau - K_2 d^2}. \qquad (19)$$

K_1 and K_2 are factors depending on the construction of the instrument.

For the curve shown in Fig. 142,

$$S = \frac{0.168d}{\tau - 0.000185d^2}, \qquad \text{roughly.}$$

It is seen that the sensitivity increases when d is increased and that the rate of increase is very rapid as d approaches the value $\sqrt{\tau/K_2}$ at which the needle becomes unstable. The equivalent capacitance of the instrument increases with corresponding rapidity. Figure 142 shows an experimentally determined sensitivity curve. The very small capacitance and high insulation of this electrometer render it especially useful in ionization experiments for measuring the current by the rate of accumulation of charge method. Currents of the magnitude of 10^{-15} amp. may be measured; this is far below the range of any galvanometer.

To obtain permanent records, the shadow of a very fine cross fiber at the upper end of the needle may be projected on a uniformly moving strip of bromide paper, as in the recorder for the string galvanometer.

Evidence of the stability and general reliability of the instrument is afforded by its incorporation as the recording element in automatic portable cosmic-ray meters which operate for weeks without attention.[5]

Electrostatic Voltmeters.—On the supposition that the law of the quadrant electrometer is that previously deduced, if the needle, the case, and one set of quadrants are connected together, $V_N = 0$, and

$$D = Kd^2.$$

Or, if the potential difference is rapidly alternating,

$$D = K\frac{1}{T}\int_0^T (d^2)dt.$$

As one pair of quadrants produces no effect, it may be omitted in an instrument designed primarily for voltage measurements.

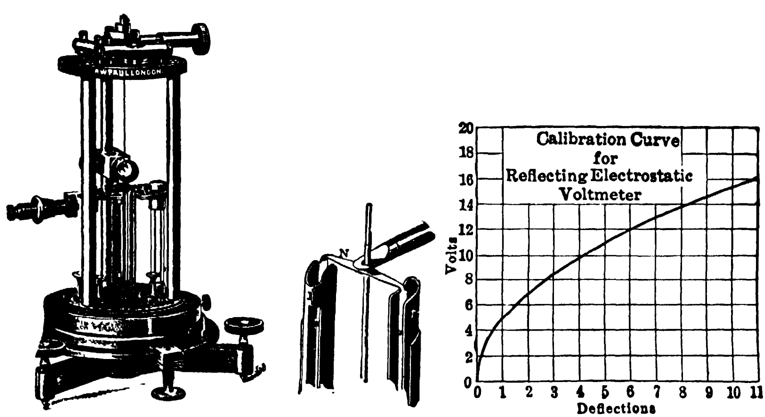


Fig. 143.—Low-range electrostatic voltmeter and the calibration curve. Damping magnet not used in this instrument.

As the deflections are proportional to the mean square value of the potential difference, electrostatic voltmeters are particularly applicable to the measurement of alternating potential differences.

These instruments, if properly designed, absorb no appreciable power and at low frequencies produce no appreciable disturbance of the potential difference to which they are applied. Their action is not complicated by inductance effects, so there are no frequency or wave-form errors. They have no self-heating errors and are uninfluenced by local magnetic fields. On the other hand, at low voltages the forces to be dealt with are very small, and consequently the instruments are much more delicate than those based on the electrodynamic principle. If of low range and used on direct-current circuits, the effects of contact differences of potential are to be eliminated by reversals.

Ayrton, Mather, and others have developed the electrostatic voltmeter so that it has become an instrument of value in laboratory work. Figure 143 shows one of Ayrton and Mather's instruments for low voltages, up to 16 volts.

This instrument is intended to be read by a mirror and scale method. The suspended system consists of a light aluminum needle, made in the form of a portion of a cylinder. The "quadrants" are portions of two cylinders, concentric with the needle. The needle is drawn into the space between them by the electrostatic attraction. The controlling force is given by a flat strip suspension, and the zero may be set by means of a tangent screw.

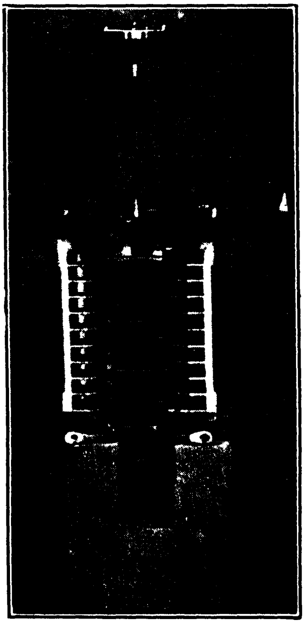


Fig. 144.—Working parts of Kelvin multicellular electrostatic voltmeter.

In order to damp the instrument, the needle, which from its construction forms a closed loop, is frequently arranged to turn between the poles of a permanent magnet.

To prevent extraneous electrostatic action, the needle, the magnet, and the case are connected together electrically. With this construction, the outside case is at the potential of one side of the line. In later instruments, the shielding is accomplished by an inner case which is insulated from the outside or protective case. The construction of the instrument is such that accidental contact between the quadrants and the movable system is impossible.

Instruments of this general design are available which give a full-scale deflection with 7 volts.

In cases where the voltage is high, above 800 volts, the forces become great enough so that the cylindrical needle may be pivoted on jeweled bearings with its axis horizontal. In this case, the controlling moment is obtained by using a small weight attached to a short arm which projects from the axis. Electrical connection with the needle is made by a very fine wire, wound in a flat spiral.

Voltmeters of this type have been manufactured for switchboard use.

To increase the forces acting on the movable systems of electrostatic voltmeters, up to about 1,000 volts, Lord Kelvin devised the multicellular instrument, an example of which is shown in Fig. 144.

A torsion-wire suspension is used, and the increase of the deflecting force, which is in proportion to the number of cells, is sufficient so that a pointer and scale may be used for reading the deflections. The instrument, however, is not portable in the ordinary sense. In the voltmeter shown in Fig. 144, the damping is effected by a disk which turns in a viscous oil contained in a little glass vessel at the bottom of the instrument.

Two vertical plates are connected to the movable system and screen it from the action of the set of quadrants near which they are placed.

Use of a False Zero Reading.—It is sometimes desirable to measure a small direct-current voltage without drawing any current. The use of the electrostatic voltmeter suggests itself, but the normal curve of the reflecting form of this instrument is very nearly a parabola, as is shown by Fig. 143, so that a low potential difference gives a very small deflection which cannot be read with accuracy. The difficulty may sometimes be overcome by superposing the potential difference to be measured on a fixed and higher voltage. For instance, if the instrument gives a scale reading of 50 cm. with 50 volts, a potential difference of 2 volts applied directly to the instrument will give a deflection of 0.08 cm. However, if it is superposed on a potential difference of 50 volts, the *increase* in deflection will be 4.08 cm. As the upper part of the calibration curve is nearly straight, the deflections from the false zero are practically proportional to the voltage.

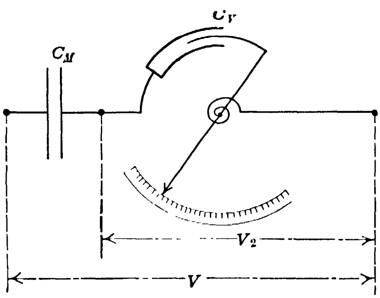


Fig. 145.

Fig. 146.

Figs. 145 and 146.—Condenser multipliers for electrostatic voltmeter.

Condenser Multipliers for Extending the Range of Electrostatic Voltmeters.—The range of an electrostatic voltmeter may be extended by means of condenser multipliers as indicated in Figs. 145 and 146.

A condenser of the proper capacitance may be joined in series with the voltmeter. Then $V = V_2\frac{(C_M + C_V)}{C_M}$, where V_2 is the reading of the instrument; and C_M and C_V, the capacitances of the condenser and the voltmeter, respectively. As the capacitance of the instrument C_V depends on the deflection, the factor $(C_V + C_M)/C_M$ will not be a constant, and the voltmeter must be calibrated with the condenser in place.

An alternative method is to join a number of condensers in series and to place the electrostatic voltmeter around one of them, as shown in Fig. 146.

Here, again, the whole arrangement should be calibrated as a unit.

The Westinghouse Company manufactures the high-range electrostatic voltmeter[6] shown diagrammatically in Fig. 147.

The voltage is applied at *I* and *I'*; between *ab* and *bc* are two condensers which can be short-circuited at will, thus altering the range of the instrument. They are placed within the highly insulated condenser terminal *T* and are brought into action by means of a silk cord. The fixed attracting elements are at *B* and *B'*. The movable element consists of two hollow metal cylinders *M* and *M'* which are united by a

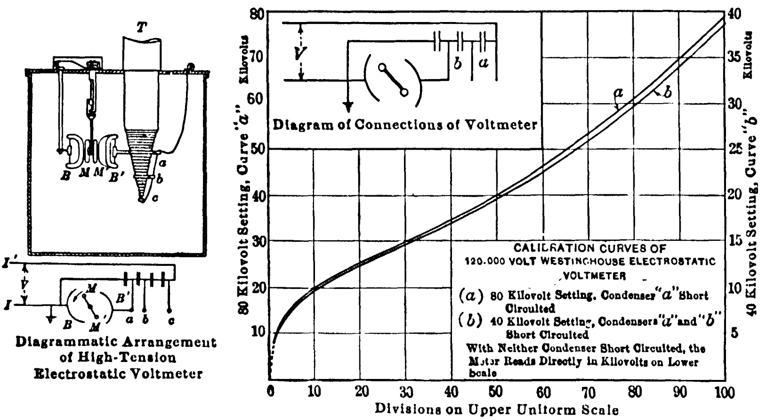


FIG. 147.—Westinghouse high-range electrostatic voltmeter.

suitable web. This system hangs freely from a single pivot resting in a jewel. The usual controlling spring and zero adjustment are provided.

There is no electrical connection to the movable element; on the application of the voltage, charges are induced on it. As the curved plates *B* and *B'* and the movable parts are not concentric, the latter will move so as to increase the capacitance of the arrangement, that is, in the direction of the arrow. The plates *B* and *B'* are so bent that the scale is approximately uniform over a considerable portion of its length. All the working parts are immersed in oil; and as the movable element is hollow, the weight is practically removed from the jewel and pivot.

These voltmeters are made for potentials as high as 200,000 volts. Instruments having a range of 25,000 volts, both condensers being short-circuited, will read up to 100,000 volts with both condensers in service.

Vacuum-tube Voltmeters.—On account of the alteration of circuit conditions, electrodynamic, electrostatic, and thermal voltmeters are unsuited to the measurement of alternating potential differences in high-impedance circuits, especially at high frequencies. The resistance of an electrodynamic or a thermal instrument, intended primarily for use at power frequencies is low; on the other hand, the electrodynamic voltmeter, of necessity, has considerable inductance. The electrostatic voltmeter, if well designed, absorbs negligible energy. However, its capacitance may disturb the circuit conditions at high frequencies. An additional possible source of error in the thermal voltmeter or any shielded voltage-measuring device at high frequencies is the charging current taken by the electrostatic shields. For these reasons, vacuum-tube voltmeters are employed especially on communication work, where the amount of available power is small.

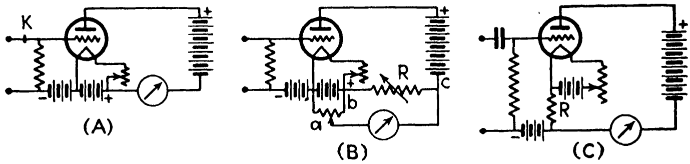


FIG. 148.—Fundamental circuits for rectifier-type vacuum-tube voltmeters.

Rectifier Type.—Figure 148*A* shows a typical vacuum-tube voltmeter circuit employing plate rectification. When the grid is negative, by reason of the curvature of the e_g-i_p characteristic, a symmetrical potential difference produces an increase in the average plate current and therefore an increase in the deflection of the milliammeter (Fig. 149). It is sometimes advantageous to balance out the original deflection and use a more sensitive indicator. The arrangement is shown in Fig. 148*B*. The circuit is so adjusted that when the input voltage is zero, the rise of voltage from *a* to *b* is equal to the fall in voltage from *b* to *c*. The initial deflection of the indicator is therefore zero. After this preliminary adjustment, the entire deflection is due to the voltage impressed at the terminals. To preserve the sensitivity, *R* should be high compared with the resistance of the indicating instrument. Critical damping, but at a reduced sensitivity, may be obtained by shunting the galvanometer. It is evident that any variation within the amplifying circuit will upset the initial balance and cause a drift in the deflection of the indicator.

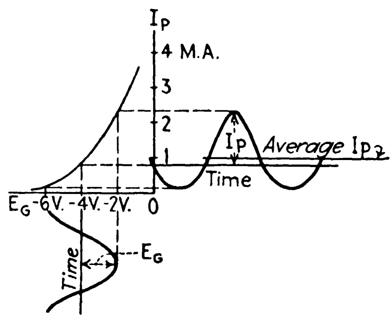


FIG. 149.—Illustrating plate rectification. Utilized in Fig. 148.

If it is desired to measure effective values, as is usually the case, it will be seen that with a rectifying vacuum-tube voltmeter there is a possibility of wave-form errors. Let the relation between Δi_p and Δe_g be represented by the empirical equation

$$\Delta i_p = A(\Delta e_g) + B(\Delta e_g)^2 + C(\Delta e_g)^3 + D(\Delta e_g)^4 + \cdots \quad (20)$$

If a wave having sinusoidal components is impressed, it is evident that the terms involving the odd powers will average out when taken over a cycle, while all the even-power terms will contribute to the average. It is thus seen that if effective values are desired, any term of the series above that involving $(\Delta e_g)^3$ must be negligible. If the impressed voltage has different positive and negative maxima, which may be due to even harmonics, the indication will change when the input terminals are reversed.[7]

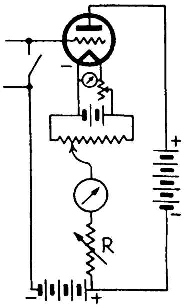


FIG. 150.—Self-biasing arrangement for a vacuum-tube voltmeter.

The sensitivity of the voltmeter may be increased by placing a blocking condenser at K (Fig. 148A) and operating the tube with a positive grid bias, using the grid voltage-grid current characteristic to obtain the rectifying action and the linear portion of the grid voltage-plate current characteristic to obtain amplifying action. This procedure, involving the use of a shunt grid-leak resistor, reduces the impedance of the input circuit and therefore may sometimes restrict the usefulness of the instrument.

The range of a rectifying voltmeter may be extended upward by the self-biasing arrangement in Fig. 150.[8] The drop in the resistor R, which is traversed by the plate current, is added to the grid bias due to the battery, thus cutting down the plate current and allowing a higher voltage to be applied at the terminals. The initial negative bias may be such that the deflection of the indicator is brought to zero; then current will flow only during the positive half wave. It is evident that the assembly must be calibrated with sinusoidal voltages and that it is subject to wave-form errors. The range may also be extended, and the calibration curve straightened, by inserting a resistor in the plate circuit. The range may be made to cover very high values, by interchanging the function of the grid and plate—an arrangement studied by Terman.[9]

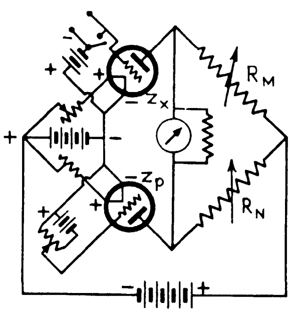


FIG. 151.—Bridge arrangement to reduce initial deflection to zero and compensate for battery changes.

Figure 151 shows an application of the bridge principle to bring the initial deflection of the indicator to zero and to reduce the effects of fluctuation in filament and plate voltage. The two tubes X and P are supposed to be identical. They are operated from the same battery. Beginning with $R_M = R_N$, the grid potential of the tube P is adjusted until the bridge is balanced. The B battery is then altered by 1 or 2 volts, and the galvanometer deflection is read. R_N is then changed by several hundred ohms, and the bridge rebalanced by again adjusting the grid potential of P; and so on until a small alteration in the B battery does not upset the balance.

The effects of small changes of filament potential are minimized by operating the filaments in parallel from the same battery. The filament potentials are adjusted until a change of 1 or 2 per cent does not upset the balance. This bridge arrangment has one-half the sensitivity of a single-tube device, critical damping being employed in both arrangements The advantages of the bridge arrangement are practically perfect B-battery compensation over a comparatively wide range, filament-battery compensation over a fairly wide range, and approximate compensation for aging of the tubes. For direct voltages, the straight portion of the characteristic may be utilized. For alternating voltages, an alternating-current galvanometer may be used for the indicator, or the tubes operated as rectifiers themselves, as in single-tube circuits. Very satisfactory compensation for changes of plate voltage, etc., may be obtained by bridge arrangements having a tube in only one arm, the three other arms containing resistors.[10]

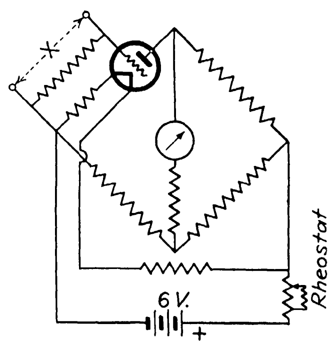


FIG. 152.—Portable bridge-type vacuum-tube voltmeter. (*General Radio Co.*)

Figure 152 shows the circuit for a commercial form of bridge-type voltmeter,[10] operated from a single battery which supplies the plate voltage, the filament voltage, and the grid-bias voltage. The resistances are so proportioned that with no applied voltage at X, the indicator may be brought to zero by manipulating the rheostat. The tube is then at the desired operating point. The range is from zero to 5 volts. The meter is calibrated by using sinusoidal currents, and there is likelihood of error if a complex wave is applied.

Peak voltages may be measured by so arranging the biasing circuit of an amplifier tube that it is readily adjustable and capable of reducing the deflection of the plate milliammeter to substantially zero. After this preliminary adjustment has been made, the unknown voltage is applied, and the deflection returned to the same point by again adjusting the grid bias. The amount of the alteration measures the peak value of one-half of the voltage wave; the other half is dealt with by reversing the connections.

When using a vacuum-tube voltmeter, one must be sure of its calibration at the time that it is used; also that the input impedance is so high that the circuit conditions are not disturbed by the use of the instrument. Figure 153 indicates that the voltmeter should be used as a transfer instrument which is first applied to the voltage to be measured, the indication noted, and then transferred by the double-pole, double-throw switch to the calibration circuit, and the reading reproduced by moving the sliding tap in $R_1 + R_2$.

To determine if the input impedance is of adequate magnitude, two resistors R' and R'' (Fig. 153) of perhaps 5 and 1 megohm, respectively, depending on the circuit under measurement, are joined in series with the input terminals. If the reading of the voltmeter does not alter when the 1-megohm resistor is short-circuited, the input impedance of the voltmeter is adequate.

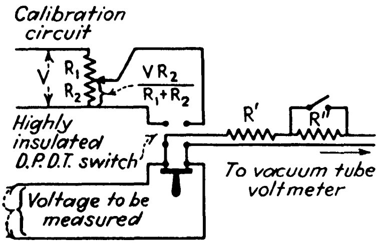


FIG. 153.—Pertaining to use of vacuum-tube voltmeter.

It is frequently convenient to operate either the voltmeter as a whole or the indicator through transformers of the proper ratio.

Figure 154 shows diagrammatically an assembly primarily designed for high-frequency measurements in which the excitation is supplied by rectified alternating currents of power frequency. The full-scale reading is 3 volts. The filament and plate-current supplies are shown at the left of the line *ab*, the voltmeter proper at the right. The filaments are operated at subnormal voltages. Rectification for the plate supply

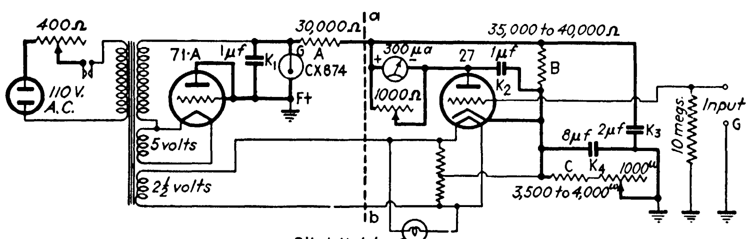


FIG. 154.—Diagram for vacuum-tube voltmeter. (*General Radio Company.*)

is obtained by the use of a 71 *A* tube, while a gas-filled tube (*CX*-874) is used as a voltage regulator. At first, the voltage across the gas-filled tube rises to about 125; the tube then breaks down, and the voltage drops to about 90. The condenser K_1 then charges during the part of the cycle when the instantaneous voltage in the transformer secondary is greater than 90 and discharges during the remainder of the cycle. The gas-filled tube maintains substantially 90 volts across the condenser at all times, regardless of small changes of alternating voltage or load current, drawing more or less current as the conditions demand.

The indicating device is a shunted microammeter with its mechanical zero so depressed that with normal plate current the index stands at the zero mark on the scale. The proper grid bias is obtained by the drop through the resistor C, due to the plate current, supplemented by the current through the resistor B. At high frequencies, the impedances of the condensers K_1, K_2 are so low that the high-frequency component of the current is shunted from the grid resistor, while the low impedance of K_3 shunts the high-frequency current from the microammeter. A rheostat is provided so that all values of line voltage between 100 and 120 volts may be used. A line variation of 1 volt produces a change in scale reading of less than 0.015 volt. Rapid fluctuations of line voltage do not affect the instrument. Since the voltmeter is intended primarily for use at high frequencies, the by-pass condensers for the grid and plate circuits are small; consequently, at low frequencies, error is introduced by the combined effects of increase in impedance of the circuit connected to the plate and the effect of the alternating component of the plate current flowing through the grid-biasing resistor. The effect is negligible at 60 cycles per second and approximately 2 per cent at 30 cycles per second. The frequency error between 60 cycles and 1,500 kc. per second is negligible. It gradually increases with the frequency, being about 2 per cent at 3,000 kc. per second. The instrument is calibrated with sinusoidal voltages. Unsymmetrical wave forms produce an appreciable error. Reversing the leads to the meter and averaging the readings tend to decrease this error.

Amplifier Type.—The insertion of an amplifier between the measuring instrument and the points in the circuit between which the potential difference is desired tends to remove errors due to alteration of circuit conditions, for the input impedance may conveniently be made megohms, if a grid resistor is used; and higher still, if the return from grid to filament is provided by the circuit being measured, so that the grid resistor may be omitted. The effective input capacitance may be made only a few micromicrofarads.

A wide range of voltages may be measured if the gain of the amplifier is adjustable; in that case, a calibrating circuit may be provided, and the voltage at the input terminals determined by comparison with a known potential difference. This is particularly advantageous if the gain is high, for in that case the amplification does not generally remain constant from day to day, owing to changes in battery e.m.f., aging of the tubes, etc. To provide stability, convenience of calibration, and portability, or to adapt the voltmeter to a particular purpose, special circuits have been devised. The indicator, or output meter, may be of any suitable type, such as a thermocouple, copper oxide rectifier instrument, or a direct-current galvanometer. The copper oxide rectifier is an exceedingly sensitive arrangement and has a reasonable overload capacity.

Figure 155 shows the circuit of a variable-gain assembly originally designed to measure the alternating e.m.f. of a delicate thermocouple immersed in a sound field.[11] It is necessary that the noise level in such an amplifier be reduced to a minimum, for this determines the lowest measurable voltage. All connections, including those in the batteries and at the terminals of the tubes, should be soldered, and only wire-wound resistors and condensers with small leakages employed. The tubes themselves should have a low noise level and be operated at reduced voltages to prevent ionization of the gases. Those selected for use should have a very small grid or positive-ion current (10^{-12} amp.). It is advisable to shield from mechanical vibrations and to avoid coupling

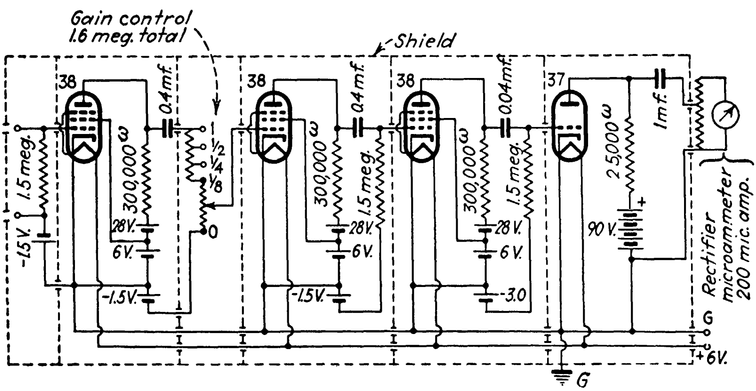


FIG. 155.—High sensitivity amplifier for vacuum-tube voltmeter.

between stages by using separate batteries. As shown in Fig. 155 three No. 38 heater-type power-amplifier pentode tubes are employed. They are operated with a plate-battery voltage of 34 volts, a screen grid bias of 6 volts, and control-grid biases as indicated in the figure. The final stage is a 37-type triode with a normal amplification factor of 9.2. The output-indicating device is a 200-microamp. copper oxide rectifying meter having a resistance of 10,000 ohms. The noise level is such that it produces a just perceptible disturbance of this instrument. As normally used, the input resistance is about 20 megohms, and the input capacitance about 5$\mu\mu$f. The approximate over-all voltage amplification at 1,000 cycles per second is 80,000 with full gain. An input voltage of 1×10^{-6} volt produces a deflection of about 8.0 divisions on the meter. At 10,000 cycles per second, the amplification is reduced about one-half.

The use of an amplifier in the measurement of small direct voltages, such as the drop through a high resistance due to a very small current or the amplification of small currents, is complicated by the presence of the grid current taken by the tube itself. This current depends on a number of factors,[12] in particular, on the leakage between filament and grid and between plate and grid, on the positive-ion current due to ionization of the residual gas in the tube, and on the electron current to the grid when it is only slightly negative and the filament is at a high temperature and therefore emitting electrons with high velocities. The resulting e_g-i_g curve is shown in character in Fig. 156*a*.

Figure 156*b* shows a single-tube amplifier applied to a photocell rendered conducting by a steady source of light. The relation between

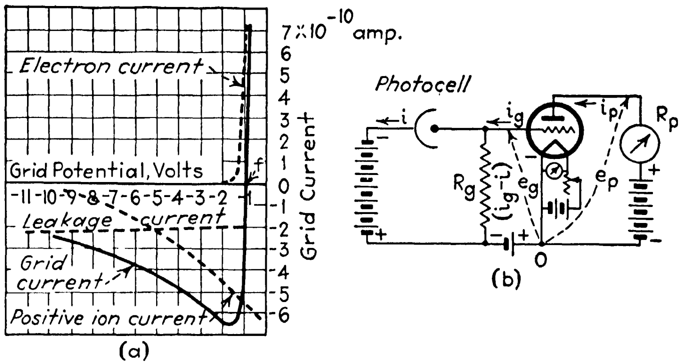


FIG. 156.—*a*, Grid characteristic UX-201-A tube. *b*, Simple single-tube amplifier for photocell.

the change in plate current and the photoelectric current when light falls on the cell is given by[12]

$$\Delta i_p = \left(\frac{g_m}{1 + \dfrac{R_p}{r_p}}\right)\left(\frac{R_g}{1 + \dfrac{R_g}{r_g}}\right)\Delta i \tag{21}$$

where $g_m = \partial i_p/\partial e_g$ = mutual conductance.
$r_p = \partial e_p/\partial i_p$ = internal plate resistance of tube.
$r_g = \partial e_g/\partial i_g$ = grid impedance.

The change in plate current due to a small change in grid voltage is given by

$$\Delta i_p = \left(\frac{g_m}{1 + \dfrac{R_p}{r_p}}\right)\Delta e_g. \tag{22}$$

The first denominator in (21) is always positive; the second may be either positive or negative according to the slope of the grid-current curve at the negative bias employed. It is seen that it is possible to obtain a very sensitive arrangement by using the negative slope of the curve and properly adjusting R_g. This arrangement, like all measurement devices that depend primarily on the difference of important quantities, is greatly affected by slight circuit variations and therefore lacks stability. Although the negative slope of the grid voltage-grid current curve may be utilized if great resistivity is required, it is usually better, on account of the lack of stability, to use values of e_g and R_g such that the term R_g/r_g may be neglected.

For the measurement of minute direct currents, both the Western Electric Company[13] and the General Electric Company[12] have produced vacuum tubes specially designed to reduce the grid current to the lowest practicable magnitude.

In a highly evacuated tube, the grid current may be due to any or all of the following causes[12] which must be considered in the design:

1. Leakage over glass or insulation, both inside and outside the tube.
2. Ions formed by gas present in the tube.
3. Thermionic grid emission due to heating of the grid by the filament power.
4. Ions emitted by the filament.
5. Photoelectrons emitted by the control grid under the action of light from the filament.
6. Photoelectrons emitted by the control grid under the action of soft X rays produced by the normal anode current.

External surface leakage may be minimized by operating the tube in a chamber from which the air has been exhausted. Internal leakage may be minimized by using fused quartz as an insulator, the design and method of assembly being such that surface contamination is avoided. Ionization is prevented by using low voltages. Grid emission is minimized by using low-filament power and large open structures.

Characteristic	D 96,475, for high sensitivity	FP 54
Filament voltage	1.0	2.5
Filament current, milliamp	270	100
Plate voltage, volts	4.0	6
Plate current, microamp	85	40
Control grid voltage, volts	−3.0	−4.0
Space-charge grid voltage, volts	4.0	+4.0
Space-charge grid current, microamp	520	
Mutual conductance, microamp. per volt	40	25
Amplification factor	....	1.0
Plate resistance, ohms	....	40,000
Control grid current, amp., approx	10^{-15}	10^{-15}
Input resistance, approx	10^{16}	10^{16}
Input capacitance, farads	4.5×10^{-12}	2.5×10^{-12}

The effect of the positive ions emitted by the filament is overcome by placing a space charge grid between the filament and the control grid and keeping it at a positive potential. The positive ions are thus repelled.

The emission of photoelectrons from the grid is reduced by using a filament operated at a low temperature. The constants of the Western Electric Company No. D 96,475 and the General Electric Company FP 54 tubes are as shown in the table on page 258.

Tubes of this class are generally called "electrometer tubes," for the grid current has been so reduced that they may be used in place of electrometers for the measurement of minute currents by the rate of accumulation of charge method (page 202). When so employed, the grid of the tube, which has a known capacitance, replaces the insulated quadrant. The rate of change of grid voltage is determined from the rate of change of the plate current.

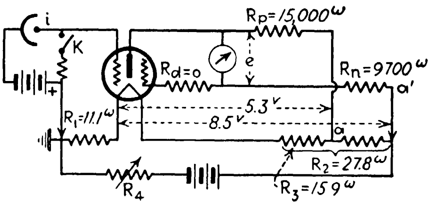


FIG. 157.—Barth circuit for electrometer tube, D 96,475.

In order to employ an electrometer tube to the best advantage, it should be excited from a constant source. Several networks which are self-compensating for small changes of power-supply voltage, due to temperature changes, normal discharge effects, etc., have been devised.[13] The underlying principle may be illustrated by the DuBridge and Brown network[13] which is applicable to the FP 54 tube. This network is obtained by transferring, in Fig. 157, the lead from R_P from a to a' and inserting resistances of appropriate values. It will be noted that only one battery is employed, the separate biasing and filament batteries being dispensed with.

It is desired that the detector galvanometer be unaffected by small changes of filament emission. When the network is nearly balanced, the voltage e at the galvanometer terminals is

$$e = R_P i_P - R_n i_n.$$

If the galvanometer current is to be zero, e must be zero, and

$$\frac{i_P}{i_n} = \frac{R_n}{R_P}. \tag{23}$$

It is seen that any change of filament emission which changes i_P and i_n in the same ratio will not affect the galvanometer. The voltage e should not be affected by changes of battery voltage; that is, de/di_f should be zero. The condition is fulfilled if

$$\frac{di_P/di_f}{di_n/di_f} = \frac{R_n}{R_P}. \tag{24}$$

If (23) and (24) are to be satisfied, it is evident that the i_n-i_f and the i_P-i_f characteristics should cut the i_f axis at the same point. However, the characteristics are not straight, and it is evident for approximate compensation over a short range that the tangents of the two curves should cut the i_f axis at the same point. Figure 158, from the paper by DuBridge and Brown, is illustrative. It is seen that compensation is obtained in the neighborhood of i_f = 0.087 amp. The rated filament current is 0.090 amp. The position of the compensation point may be shifted by varying the control-grid bias or the plate potential by varying the position of the sliding taps indicated on R_1 and R_2, Fig. 157.

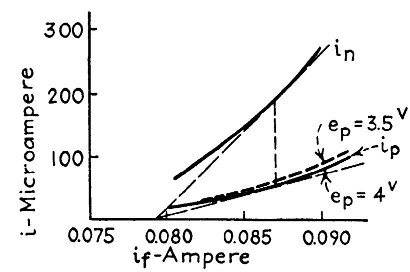


FIG. 158.—Characteristics for FP 54 tube in DuBridge and Brown circuit.

Various compensating networks are discussed in a paper by Penick[13] who recommends for the D 96,475 tube that due to Barth[13] and shown in Fig. 157; a fairly high battery voltage, necessitating high values of R_P and R_n, is desirable, as the adjustment of the network is then not critical. Penick gives the theory of the network and the procedure for adjustment. The network is not applicable to the FP 54 tube.

For measuring minute currents, the electrometer tube with its network may be used as a voltmeter across a very high resistance (that in series with K in Fig. 157). The use of the electrometer tube has been developed to such a degree by A. L. Bennett in his stellar photometer that photoelectric currents of the order of 10^{-15} amp. may be measured to about 1 per cent.[14]

Measurement of Average Values of Alternating Voltages. The Flux Voltmeter.—In the accurate determination of the core losses of transformers, the exploration of alternating-current magnetic networks, and the determination of the form factors of alternating-current voltage waves, it is necessary to employ an instrument that indicates *average*, rather than r.m.s., values of the voltage. For example, the efficiency rating of a transformer is based on sine-wave operation. However, it is difficult to obtain an undistorted wave for core-loss tests, and an instrument that will enable a reduction to the sine-wave basis is necessary. The maximum flux density, on which the hysteresis loss depends, should correspond to the rated sine-wave excitation, even though the impressed voltage is badly distorted.

The flux voltmeter utilizes the fact that the maximum flux density is proportional to the arithmetical average value of the voltage due to the flux variation.

In general, the induced voltage due to a flux variation is given by

$$E = K\frac{d\varphi}{dt},$$

where K is a constant depending on the circuit; consequently

$$\int_{t_1}^{t_2} Edt = K\int_{\varphi \text{ lower limit}}^{\varphi \text{ upper limit}} d\varphi = K \times (\text{max. change in flux})$$

$$\int_{t_1}^{t_2} Edt = \text{area of half cycle of voltage wave}$$
$$= E_{\text{av.}}(t_2 - t_1),$$

where $t_2 - t_1$ is the duration of a half cycle.
Therefore

$$\text{Max. change in flux} = K''E_{\text{av.}}$$

The flux density is proportional to the total flux; consequently the maximum alternating-current flux density $B_{\text{max.}}$ is proportional to the maximum total alternating-current flux, and

$$B_{\text{max.}} = K'''E_{\text{av.}} \tag{25}$$

The simplest arrangement for utilizing (25) is an electronic rectifier tube in series with a direct-current voltmeter of so high a resistance that variations of tube impedance are negligible. To obtain $E_{\text{av.}}$, Camilli[15] uses in his flux voltmeter a full-wave electronic rectifier and a direct-current voltmeter as indicated in Fig. (159).

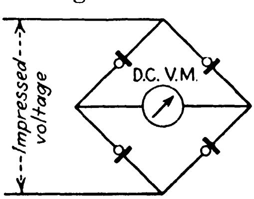


Fig. 159.—Diagram for flux voltmeter. Filament-heating arrangements omitted. (*General Electric Co.*)

As originally described, four UX-120 tubes are employed, the grid and plate of each tube being connected together to act as a single electrode for rectification. The filaments are heated by current from a special transformer, actuated by the system voltage and provided with four low-voltage secondaries, one for each tube. With the connections as used, the resistance of a tube is of the order of 5,000 ohms, while that of the voltmeter is 178,000 ohms. The scale of the meter is graduated in terms of effective sine-wave voltages. Consequently, for any given voltage wave shape, the meter indicates the equivalent sine-wave value that would cause the same maximum flux or flux density.

The meter is subject to the limitation that the wave must not cross the zero line more than twice in a cycle, for the entire wave is rectified. A small negative portion occurring in a positive half wave will be rectified and will add to the reading, rather than subtract from it, as it should, in determining the average value. However, wave forms so badly distorted seldom occur in practice.

In core-loss tests on transformers at the rated frequency, the excitation voltage is set at its rated value by means of the flux voltmeter; thus the maximum flux density in the core corresponds to the rated sine-wave excitation, even though the impressed voltage is badly distorted.

As the form factor of a sine wave is 1.11, and the flux voltmeter is graduated in equivalent sine-wave voltages, the form factor of any voltage wave is given by

$$\text{F.F.} = 1.11 \frac{\text{a.-c. voltmeter reading}}{\text{flux voltmeter reading}}.$$

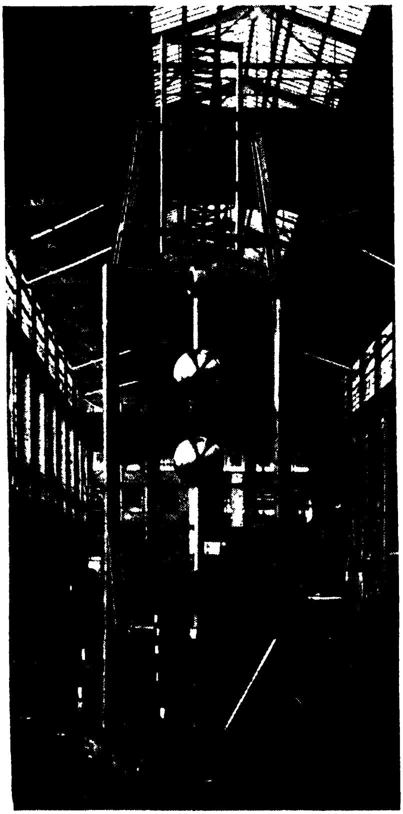


FIG. 160.—Fifty-centimeter sphere gap. (*Westinghouse Electric and Manufacturing Company.*)

The Spark-gap Method of Measuring High Peak Voltages.—It is difficult to design indicating instruments for directly determining extra-high voltages because of corona effects, disruptive discharges, and extraneous electrostatic attractions. Also, in testing the dielectric strengths of insulations, it is desirable to know the maximum rather than the effective voltage to which any sample is subjected. Consequently, a method of measurement depending on the dielectric strength of air has been developed and has been employed for many years. The necessary apparatus is termed a spark gap, and its use as a means of determining high voltages is sanctioned by the American Institute of Electrical Engineers.[16] This method is frequently used in acceptance tests of new apparatus where the dielectric strength of the insulation is guaranteed.

Both needle-point and sphere gaps[17] have been employed. The Standardization Rules (Standard 4, 1928) specify that the sphere spark gap (see Fig. 160) shall be used for voltages above 50 kv. and is preferred for those down to 10 kv. The needle gap may be used for voltages from 10 to 50 kv. It is not recommended, however, on account of the effects of humidity, etc., on its breakdown voltage. Irrespective of the wave form at power frequency, a spark gap breaks down at the crest of the wave; therefore if the relation between sparking distance and crest voltage at spark-over has been determined for a specified gap, the maximum voltage to which an insulation has been subjected may be readily determined.

A voltmeter actuated through a potential transformer or from a special voltage coil on the high-voltage testing transformer is always used in conjunction with a spark gap. In testing apparatus of low capacitance, the relation between the voltmeter reading and the spark-over voltage of the gap is first determined. The distance apart of the electrodes is set to correspond with the required voltage. The first 50 per cent of the test voltage is rapidly applied; the second, in not less than 30 sec. The reading of the voltmeter on the low-tension side of the testing transformer at the instant of breakdown is noted. The apparatus to be tested is then connected in parallel with the gap which is set for a voltage about 20 per cent too high. The voltmeter reading is then reproduced and held for the required time. If the apparatus under test has sufficient capacitance to distort the wave form or alter the effective ratio of the testing transformer, the spark gap should be set, connected in parallel with the apparatus, the voltage then raised to spark-over, and the voltmeter reading taken. The transformer and voltmeter are thus calibrated as a unit with the distorted wave. The gap is left in parallel with the apparatus to be tested but set for a voltage about 20 per cent higher than the required test voltage, which is then applied for the specified time.

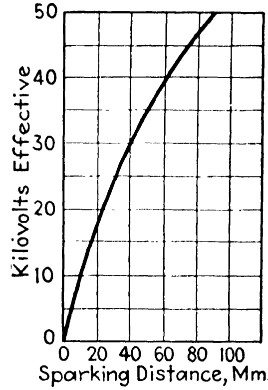


FIG. 161.—Plot of A.I.E.E. values for needle-point gap.

Care must be taken that overvoltage oscillations from sparking in the apparatus under test do not occur. A current-limiting, noninductive resistor of about 1 ohm per volt should be used in series with a spark gap to prevent oscillations at breakdown and damage to the electrodes. If one electrode is grounded, the resistor is inserted on the high-voltage side

TABLE XII.—NEEDLE-POINT SPARK-OVER VOLTAGES WITH NO. 00 DOUBLE LONG SEWING NEEDLES

At 25°C. and 760 mm. barometer—relative humidity 80 per cent

R.m.s., kv.	Millimeters	R.m.s., kv.	Millimeters
10	11.9	40	62
15	18.4	45	75
20	25.4	50	90
25	33.0		
30	41.0		
35	51.0		

of the gap, as shown in Fig. 160 at the top, where the resistor is composed of "zircon" rods, a ceramic material which maintains its resistance under high voltages. A water-tube resistor is also satisfactory. Carbon rods are avoided, for their resistance may become low if the voltage is high.

Standard 4 specifies that "the needle gap shall be between new sewing needles, supported axially at the ends of linear conductors which are at least twice the length of the gap. There must be a clear space around the gap for a radius at least twice the gap length." The gap is set according to the values given in Table XII.

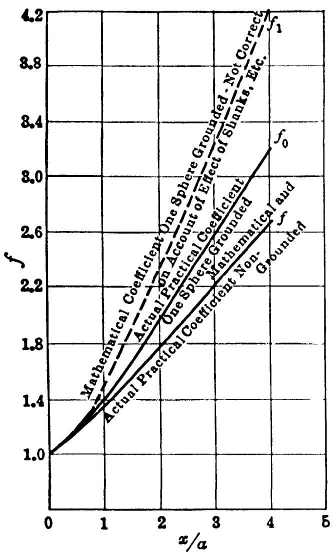


FIG. 162.—Plots of f for spark gap with spherical electrodes.

A new set of needles must be used after each breakdown of the gap, for irregularities arise from varying sharpness of the needles. The needle gap is affected by changes of humidity. This is a major reason why its use is not recommended. Under modern conditions, the range of usefulness of the needle gap is so small that it has become a relatively unimportant device.

For voltages above 10 kv., the use of two spherical electrodes of equal diameters is recommended by the American Institute of Electrical Engineers. The advantage is that if the distance between the electrodes is less than three times the radius of the spheres, corona does not form before the gap breaks down. The erratic effects due to broken-down air around the electrodes are thus avoided. Humidity has no influence on the results.

An additional advantage at high voltage is that the length of the gap is much reduced. The sphere gap is not well adapted for measuring low voltages, for the spheres must be brought very near together.

If the current is limited to less than 1 amp. by high resistors, pitting of the spheres is avoided, and they need be repolished only occasionally.

When neither sphere is grounded, it is possible to calculate mathematically the relation between the breakdown voltage and the length of gap. The dielectric stress will be a maximum at the points where the line joining the centers of the spheres cuts their surfaces. If the value of the potential gradient at this point is denoted by g, it may be shown that

$$g = \left(\frac{V}{x}\right)f \qquad \text{kv./cm.} \tag{26}$$

V is the potential difference, and x the distance apart of the surfaces.

V/x is therefore the average potential gradient; f is a factor that depends on x and the radius of the spheres a; it is the quantity that must be multiplied into the average gradient to give the maximum gradient and has been expressed in the form of an infinite series by A. Russell.[18]

TABLE XIII.—VALUES OF f, NEITHER SPHERE GROUNDED
Computed by A. Russell

x/a	f	x/a	f
0.0	1.000	0.8	1.283
0.1	1.034	0.9	1.321
0.2	1.068	1.0	1.359
0.3	1.102	1.5	1.559
0.4	1.137	2.0	1.770
0.5	1.173	3.0	2.214
0.6	1.208	4.0	2.677
0.7	1.245		

These values are plotted in Fig. 162.

When the potential between the spheres is gradually raised, it should naturally be expected that the gap would break down when the maximum gradient reached a definite value g_s and that if this value is known, the voltage corresponding would be

$$V = g_s\frac{x}{f}. \tag{27}$$

Experiment shows that g_s, the maximum gradient at spark-over, depends upon the radius of the spheres, being larger as the radius is diminished. F. W. Peek has shown that between the limits $x = 0.54\sqrt{a}$ and $x = 2a$, and at standard air density, g_s may be represented by the empirical equation[17]

$$g_s = 27.2\left(1 + \frac{0.54}{\sqrt{a}}\right), \tag{28}$$

where a is in centimeters. This relation is shown graphically in Fig. 163. If x/a is greater than 4, corona forms before the breakdown, and the preceding equations are no longer applicable.

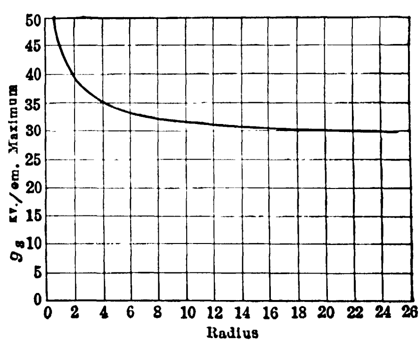


FIG. 163.—Showing relation of maximum potential gradient at spark-over to radius of spheres.

If one sphere is grounded, the electrostatic field is so distorted by the supporting stems and surrounding objects that the value of f deduced from purely mathematical considerations is no longer applicable. In this case, experimentally determined values f_0 (see Fig. 162) must be used. They are deduced on the assumption that at spark-over, g_s is as given in Fig. 163. Variations in frequency, at least up to 1,000 cycles per second, have no influence on the results.

As an illustration of the foregoing, assume the spheres to be 25 cm. in diameter and 25 cm. apart, one of them being grounded. At standard air density,

$$g_s = 27.2\left(1 + \frac{0.54}{\sqrt{12.5}}\right) = 31.4 \qquad \text{kv./cm.}$$

$$\frac{x}{a} = \frac{25.0}{12.5} = 2$$

$$f_0 = 2, \qquad \text{from Fig. 162.}$$

Therefore

$$V = \frac{31.4 \times 25}{2} = 392 \qquad \text{kv., max.}$$

For a sinusoidal wave

$$V = 277 \qquad \text{kv. effective.}$$

Reference to Fig. 163 shows that it is best to employ large spheres and thus avoid the use of the steep part of the curve and the consequent uncertainty in g_s.

In practical work, spark-over voltages should be taken from experimentally determined curves or an equivalent table, rather than determined by the empirically established formulae.

The use of a spark gap is not without danger to the apparatus under test; high-voltage surges may be set up when the gap breaks down, hence the use of the current-limiting resistor. In using any form of spark gap, it is essential that all chance of accidental circuit variations be eliminated. If this is not done, the observer may be misled by the breaking down of the gap, due to high-voltage oscillations. The sphere gap is especially susceptible to circuit variations; consequently, all accidental spark discharges from the testing circuit must be avoided.

To eliminate discrepancies due to grounds, adjacent conducting bodies, walls, etc., the maximum sphere-gap spacing should be limited to one-half the diameter of the spheres.[16]

The use of the sphere spark gap as a reference standard for measuring 60-cycle frequency and impulse voltage (page 268) requires two separate calibration curves above one-fourth to one-third diameter spacings, one calibration for the positive impulse voltage, and another for 60-cycle and negative impulse voltages.

The sphere spark gap flashes over on the average at practically the same crest voltage for waves of 1 or 2 microsec. duration and longer. For small spheres and spacings, the sphere spark gap may be expected to measure correctly voltages reaching flash-over in times of the order of ½ microsec. It therefore can be used to measure the amplitude of such waves requiring only two calibration curves. For shorter impulses, the actual crest voltage is greater than indicated by the two normal calibration curves by an amount depending on the form and duration of the wave.

Corrections for Barometric Pressure and Temperature.—Air density affects the results obtained with any form of spark gap. The spark-over voltage decreases with decreasing barometric pressure and with increasing temperature. At high altitudes, this effect may be considerable. The relative air density is given by

$$\delta = \frac{0.392b}{273 + t} \tag{29}$$

b is the barometric pressure in millimeters; and t, the temperature in degrees centigrade.

TABLE XIV.—AIR-DENSITY CORRECTION FACTORS FOR SPHERE GAPS
(From A.I.E.E. Standard 4)

Relative air density	Diameter of standard spheres, mm.					
	20	62.5	125	250	500	750
0.50	0.573	0.547	0.535	0.527	0.519	0.517
0.55	0.617	0.594	0.583	0.575	0.567	0.565
0.60	0.661	0.640	0.630	0.623	0.615	0.613
0.65	0.705	0.686	0.677	0.670	0.663	0.661
0.70	0.748	0.732	0.724	0.718	0.711	0.709
0.75	0.791	0.777	0.771	0.766	0.759	0.757
0.80	0.833	0.821	0.816	0.812	0.807	0.805
0.85	0.875	0.866	0.862	0.859	0.855	0.854
0.90	0.917	0.910	0.908	0.906	0.904	0.903
0.95	0.959	0.956	0.955	0.954	0.952	0.951
1.00	1.000	1.000	1.000	1.000	1.000	1.000
1.05	1.041	1.044	1.045	1.046	1.048	1.049
1.10	1.082	1.090	1.092	1.094	1.096	1.097

To obtain the gap spacing for a required spark-over voltage, divide the required voltage by the correction factor from Table XIV, and use the new voltage thus obtained to find the corresponding spacing from Table XIII.

To determine the spark-over voltage for a given gap spacing, multiply the voltage corresponding to the gap spacing obtained from Table XIII by the corresponding correction factor from Table XIV.

Measurement of Surge Voltages.[19]—The experimental investigations being made concerning lightning discharges have necessitated the development of apparatus for measuring the crest voltages of surges. In the beginning, the only means available was the sphere gap whose behavior under impulses was unknown. In 1931, Fiedler showed that the calibration of the gap for positive and negative surges differed.* The difference appears when the gap spacing is from one-fourth to one-third of the diameter of the spheres, and investigation has shown that the calibrations for negative surges and alternating voltages at 60 cycles per second are the same. Questions also arise as to a possible time effect as noted on page 270.

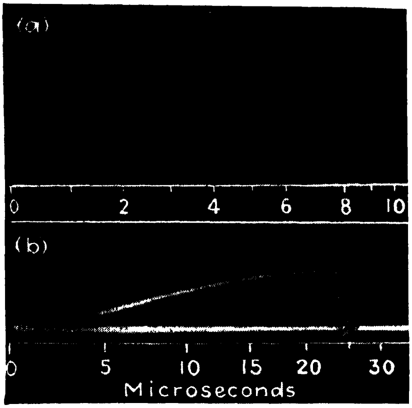


FIG. 164.—Surge wave forms.

Figure 164 indicates the nature of the problem. From the figure it is seen that the time involved is a matter of a few microseconds, a microsecond being 0.000001 sec. To give an idea of the abruptness or shape of surges, they are designated thus: 1.5–40. microsec., etc., signifying that the voltage rises to its crest value in 1.5 microsec. and falls to one-half the crest value in 40 microsec.

In order to study impulse phenomena, it is necessary to delineate the form of the surges. On account of the extreme quickness of the phenomena, the cathode-ray oscillograph is the only form of instrument that can be employed for the purpose. One assembly for making surge calibrations of the sphere gap is indicated in Fig. 165.

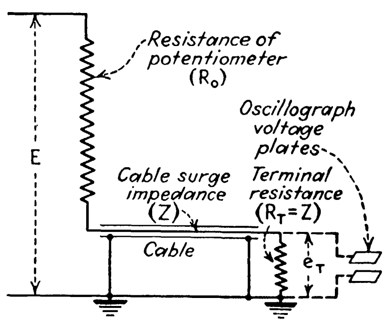


FIG. 165.—Resistance potentiometer for delineating voltages.

It is seen that the voltage for the deflecting plates of the cathode-ray oscillograph is obtained by taking the drop around a comparatively small nonreactive resistor, 44 ohms, which is equal to the surge resistance of the cable. It is necessary that the drop e_T be, as nearly as possible, a replica of the applied surge voltage E. Referring to the figure, R_0 is a noninductively wound oil-immersed resistor. Connecting R_0 and the measuring

* Probably due to the asymmetrical electric field at the gap due to the fact that the gap and the other apparatus are all grounded at one terminal.

apparatus, is a length, as short as practicable, of high-grade low-loss cable. This resistance voltage divider has been used in this country to measure directly impulse voltages at the cathode-ray oscillograph for seven years or more. Hence it has been developed into a reliable measuring equipment. The important design and application features for good performance of this voltage divider require the following conditions:

1. Constant resistance.
2. Low inductance.
3. Low distributed capacitance.
4. Low cable losses.
5. Accurate determination of cable surge impedance.
6. Fixed and beneficial location with respect to other high-voltage parts.

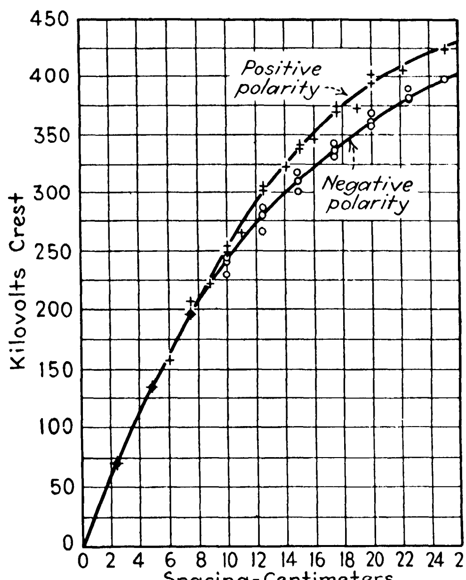


FIG. 166.—Curves for a 25-cm. gap.

To analyze the response of this circuit to surges, as has been done by Bellaschi,[19] involves the application of the methods used in investigating transients and the use of analysis which lies beyond the scope of the present work. To avoid reflections, the resistance R_T is made equal to the surge impedance of the cable. The cable introduces a time delay. Mathematical analysis is a help and a guide in the design of the apparatus, but it is not relied upon in obtaining the final results. The results are verified by the use of other apparatus of radically different design, namely, a potential divider involving the use of condensers in place of resistors. Concordant results have been obtained by engineers of the Westinghouse Company, using both types of divider. The resistance divider has the great advantage of being a means of direct, quantitative measurement of phenomena, *not too fast for its response*, and therefore its use was essential along with the capacitance divider. A subcommittee composed of engineers particularly conversant with this form of measurement is working (1936) on a revision of the A.I.E.E. Standard 4 and the impulse calibration of the sphere gap. Their report "Recommended Sphere Gap Calibration Standards," revised to Oct. 1, 1935, contains the recommended sphere-gap calibrations, which are as shown on page 271. A deviation tolerance of ±3 per cent is recommended. Proposals for irradiating short gaps with ultraviolet light are being considered.

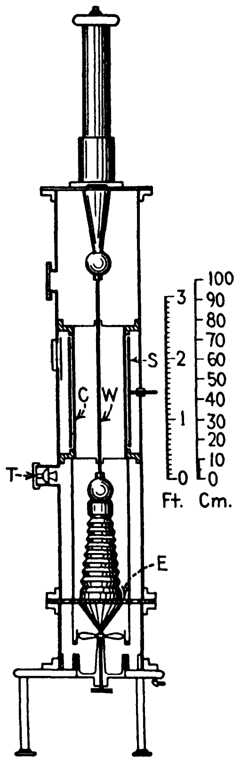


FIG. 167.—Corona voltmeter with subbase.

Whitehead Corona Voltmeter.[20]—The voltage at which corona forms is dependent on the density of the air surrounding the conductor. The density, in turn, is dependent both on the pressure and on the absolute temperature of the air, the relation given by Peek being $\delta = 3.921(P/T)$, where P is the pressure in centimeters of mercury reduced to 0°C., and T is the absolute temperature $T = 273.1 + t$°C. The standard conditions are evidently $P = 76.0$ cm. and $t = 25$°C., for in that case $\delta = 1$.

Whitehead utilizes these facts in his corona voltmeter for measuring high potentials. This instrument, as arranged at the National Bureau of Standards, is shown in section in Fig. 167. W is a straight rod of circular cross section to which the high voltage is applied. It is placed coaxial with the circular cylinder C, which is perforated with many small holes and grounded. At T is a condenser microphone for aurally detecting the establishment of corona. It is connected to the observer's telephone through an amplifier. The establishment of corona may be observed visually or detected by ionization tests, to facilitate which the insulated conducting cylinder S with an external terminal is provided. The subbase, which contains provisions for heating, cooling, drying, and circulating the air within the case of the voltmeter, was added by Brooks and Defandorf in their researches on the instrument at the National Bureau of Standards.[20] The whole device is surrounded by a tight, heat-insulated metal case so that the pressure and temperature of the air may be maintained constant or varied as desired.

Table XV.—Proposed Sphere-gap Spark-over Voltages
(At 25°C. and 760 mm. Barometric Pressure)
One Sphere Grounded

Sphere-gap spacing, cm.	Sphere diameter, cm. 6.25: 60-cycle and negative impulse, kv. crest	6.25: Positive impulse, kv. crest	12.5: 60-cycle and negative impulse, kv. crest	12.5: Positive impulse, kv. crest
0.5	16.2	16.2	.....	.....
1.0	31.0	31.0	31.7	31.7
1.5	44.5	44.8	44.9	44.9
2.0	57.0	57.4	58.0	58.0
2.5	68.8	69.3	70.8	70.8
3.0	78.8	79.4	83.5	83.5
3.5	86.6	88.0	95.0	95.3
4.0	93.6	....	106.0	108.0
4.5	99.8	....	117.0	120.0
5.0	105.5	....	127.0	132.3
5.5	.....	....	135.3	142.5
6.0	.....	....	143.5	153.8
6.25	.....	....	147.5	158.0
7.0	.....	....	157.7	171.0
8.0	.....	....	170.5	.....
9.0	.....	....	182.0	.....
10.0	.....	....	.....	.....
11.0	.....	....	.....	.....
12.0	.....	....	.....	.....
12.5	.....	....	.....	.....

Sphere-gap spacing, cm.	Sphere diameter, cm. 25: 60-cycle and negative impulse, kv. crest	25: Positive impulse, kv. crest	50: 60-cycle and negative impulse, kv. crest	50: Positive impulse, kv. crest	75: 60-cycle and negative impulse, kv. crest	75: Positive impulse, kv. crest
....	...	...	...	...	.....	.....
2.5	72	72	...	...	.....	.....
5.0	136	136	136	136	136	136
7.5	192	196	197	197	200	200
10.0	241	252	260	260	261	261
12.5	278	260	317	319	324	324
15.0	309	334	367	374	380	380
17.5	338	364	411	426	433	443
20.0	362	390	451	474	484	499
22.5	379	409	486	511	528	548
25.0	393	426	519	547	573	597
30.0	...	...	573	605	653	687
35.0	...	...	615	655	721	755
40.0	...	...	651	698	777	816
45.0	...	...	681	732	827	870
50.0	...	...	707	758	870	917
55.0	...	...	...	...	910	960
60.0	...	...	...	...	945	999
65.0	...	...	...	...	977	1,031
70.0	...	...	...	...	1,003	1,058
75.0	...	...	...	...	1,025	1,081

Sphere-gap spacing, cm.	Sphere diameter, cm. 100: 60-cycle and negative impulse, kv. crest	100: Positive impulse, kv. crest	150: 60-cycle and negative impulse, kv. crest	150: Positive impulse, kv. crest	200: 60-cycle and negative impulse, kv. crest	200: Positive impulse, kv. crest
10	261	261	261	261	261	261
20	504	504	505	505	506	506
30	700	715	736	736	746	746
40	862	888	947	955	973	973
50	985	1,024	1,120	1,140	1,172	1,178
60	1,084	1,124	1,254	1,293	1,346	1,364
70	1,163	1,209	1,360	1,400	1,505	1,533
80	1,234	1,284	1,458	1,502	1,635	1,671
90	1,295	1,344	1,552	1,597	1,752	1,788
100	1,338	1,390	1,628	1,678	1,857	1,896
110	.....	.....	1,695	1,755	*1,944	*1,990
120	.....	.....	1,760	1,824	2,027	2,069
130	.....	.....	1,815	1,880	2,103	2,146
140	.....	.....	1,865	1,920	2,169	2,220
150	.....	.....	1,900	1,944	2,231	2,291
160	.....	.....	.....	.....	2,293	2,360
170	.....	.....	.....	.....	2,343	2,415
180	.....	.....	.....	.....	2,395	2,464
190	.....	.....	.....	.....	2,436	2,507
200	.....	.....	.....	.....	2,475	2,543

* The values for the 200-cm. sphere gap for spacings over 100 cm. are extrapolated.

A practically sinusoidal, 60 cycles per second voltage wave derived from a generator capable of furnishing a leading current without wave distortion was employed by Brooks and Defandorf. Extreme care was used in the measurement of the effective voltage, the determination of the crest factor, and the determination of wave form. The pressure was determined by a mercury manometer; the air temperature, by thermocouples.

For concentric circular cylinders, such as are here employed, the voltage gradient at the surface of the inner cylinder is

$$g = \frac{V_c}{r \log_\epsilon \frac{R}{r}},$$

where V_c is the crest voltage between the two cylinders at which corona begins to form; and R and r, the radii of the outer and inner cylinders, respectively. From their observations, Brooks and Defandorf deduced that the gradient g, at which corona forms, is given by the empirical equation

$$g = A\delta + \frac{B\sqrt{\delta}}{\sqrt{r}} - \frac{C}{r},$$

where A, B, and C are constants. This expression differs from that given by other investigators in the addition of the term $-\frac{C}{r}\cdot$ Combining the two expressions for g gives

$$V_c = \left(A\delta + \frac{B\sqrt{\delta}}{\sqrt{r}} - \frac{C}{r}\right)\left[2.303r \log_{10}\left(\frac{R}{r}\right)\right],$$

or, inserting the expression for δ,

$$V_c = A \times 9.03\left[r \log_{10}\left(\frac{R}{r}\right)\right]\left(\frac{P}{T}\right) + B \times 4.56\left[\sqrt{r} \log_{10}\left(\frac{R}{r}\right)\right]\sqrt{\frac{P}{T}} - C \times 2.303\left[\log_{10}\left(\frac{R}{r}\right)\right].$$

From the experiments, representative values of the constants are

$$A = 27.95.$$
$$B = 11.18.$$
$$C = 0.365.$$

Consequently,

$$V_c = 252.4\left[r \log_{10}\left(\frac{R}{r}\right)\right]\left(\frac{P}{T}\right) + 5.11\left[\sqrt{r} \log_{10}\left(\frac{R}{r}\right)\right]\sqrt{\frac{P}{T}} - 0.81\left[\log_{10}\left(\frac{R}{r}\right)\right]; \quad (30)$$

or, for any one rod,

$$V_c = K_1\left(\frac{P}{T}\right) + K_2\sqrt{\frac{P}{T}} - K_3. \tag{31}$$

A nomogram for facilitating calculations is readily constructed. It is of prime importance that the corona rods (of Stubb's steel drill rod) be kept free of all dust and dirt which would localize the formation of corona. This is emphasized by the elaborate method of cleaning the rods before insertion in the voltmeter. (1) Grease and oil are removed by wiping with a clean rag. (2) The rod is washed with high-grade benzene. (3) It is polished with crocus paper. (4) It is washed with redistilled ethyl alcohol. (5) It is wiped with specially cleaned chamois skin from which tanning residues have been removed by washing with soap and distilled water, then washing and drying with redistilled alcohol; the chamois skin is kept free of dirt, and the surfaces used for cleaning the corona rods are not touched with the hand. The air within the case must be kept dry and free from dust. This necessitates the use of calcium chloride in the subbase and an air filter in the pipes connecting with the pressure apparatus.

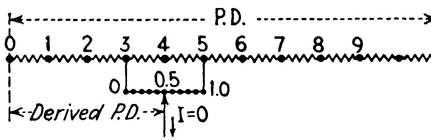


FIG. 168.—Kelvin-Varley slide.

Potential Dividers. Kelvin-Varley Slide.—If a potential difference is spanned by 10 equal coils in series, taps being carried to each junction point, the potential difference will be divided into 10 equal parts provided no current is drawn from the taps.

If 11 equal coils are used, and 2 of them are spanned by a movable rider having a resistance equal to that of the two coils, the total resistance will again be that of 10 equal coils. If this rider is divided into 10 equal sections along which a sliding tap can be displaced, the rider can be set in 9 different positions, and the sliding tap in 11 different positions, so that the potential difference may be divided into 100 equal parts by the use of only 21 coils.

POTENTIOMETER ARRANGEMENTS

Poggendorf Method of Comparing a Potential Difference and an E.M.F.—All of the various methods now used for the rapid standardization of direct-current instruments depend on the ability to compare a potential difference with an e.m.f. This may be accomplished by Poggendorf's method, which is shown diagrammatically in Fig. 169. E_1 and E_2 are the e.m.fs. of the batteries at E_1 and E_2. The cell at E_1 will of necessity have the higher e.m.f. R_1 and R_2 are two variable resistances; K, a key which is normally open; and G, a suitable galvanometer. The circuit of E_1 is closed through the resistance $R_1 + R_2$, across which there will be established a potential difference V. Suppose the key K to be open. Then the current in R_1 is the same as that in R_2, and its value is

$$I = \frac{V}{R_1 + R_2}.$$

The potential difference between the ends of R_1 will consequently be

$$\frac{R_1 V}{R_1 + R_2}.$$

This may be varied by altering either R_1 or R_2. The battery E_2 is so inserted that, when the key is depressed, its e.m.f. opposes the potentia difference due to the passage of the current through R_1. If by varying R_1 or R_2 this potential difference is made equal to E_2, no current will flow through the galvanometer and the battery E_2 when the key is closed Consequently, the absence of a deflection of the galvanometer when the circuit is closed shows that

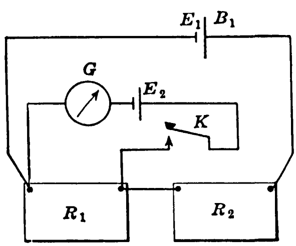


FIG. 169.—Connections for Poggendorf method of comparing a potential difference with an electromotive force.

$$E_2 = \frac{R_1 V}{R_1 + R_2},$$

or

$$\frac{E_2}{V} = \frac{R_1}{R_1 + R_2}.$$

If $R_1 + R_2$ are so high that very little current flows through the battery E_1, the fall of potential in E_1, which is given by IB, where B is the battery resistance, will be so small that the potential difference, which is equal to $E_1 - IB$, may be taken as equal to E_1, and

$$\frac{E_2}{E_1} = \frac{R_1}{R_1 + R_2} \qquad \text{very nearly,}$$

or

$$E_1 = E_2 \frac{R_1 + R_2}{R_1}.$$

The larger $R_1 + R_2$ the better the approximation.

It will be noticed that current can flow through E_2 only when the key is depressed and that when the adjustment is perfect, there can be no current through the battery E_2. This is of importance, for, if care is exercised, it allows cells to be used at E_2 without danger of altering their e.m.fs. by polarization.

Much labor has been expended in the development and study of galvanic cells, suitable for use at E_2, which shall have perfectly definite e.m.fs. and consequently can be used as standard cells with which potential difference or E_1 may be compared. On account of its high degree of reproducibility and reliability, the Weston cell is now universally used. Standard cells are of the open-circuit type, and no appreciable current can be drawn from them without temporary alteration of their e.m.f.

As it is important to avoid short-circuiting E_1 or connecting it through a small coil which might be overheated, it is well to insert large resistances (1,000 ohms) in R_1 and R_2 before making the final connections.

The key should not be left depressed but released as soon as the galvanometer needle begins to move. The direction of the motion of the needle must be noted, and another trial made with a different resistance in R_1. The direction of the deflection depends on whether R_1 is too large or too small.

After repeated trials, a resistance will be found such that the galvanometer will not deflect. Then

$$E_1 = E_2\frac{R_1 + R_2}{R_1}.$$

To make the final balancing, either R_1 or R_2 may be adjusted. One must be sure that the plug contacts, or switches, in R_1 and R_2 do not introduce spurious resistances and that all connections are perfect.

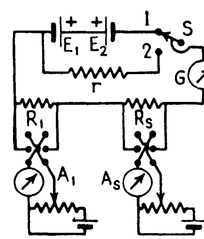


FIG. 170.—Comparison of standard cells, difference method.

If in any particular case the standard cell is higher in e.m.f. than the cell to be compared with it, or if it is desired to compare the e.m.fs. of two standard cells, the procedure must be changed, for standard cells cannot be used in closed circuits. In this case, an auxiliary battery having a higher e.m.f. than either E_1 or E_2 is used in position E_1. The cells may then be compared with it in succession, or else the circuit so arranged that both cells can be compared with the auxiliary battery *at the same time.*

Figure 170 shows the connections for determining the difference of the e.m.fs. of two standard cells; r is a resistance which replaces that of the cells when the switch is in position 2. With the switch in this position, any parasitic e.m.fs. are balanced by adjusting the current through R_1 until the galvanometer is brought to zero. Then, with the switch in position 1, the circuit of A_S is closed, and the current through R_S adjusted until the galvanometer again reads zero.

$$E_1 - E_2 = I_S R_S. \tag{32}$$

Potentiometers.—In the section on "Calibration of Instruments" (see page 624) are discussed methods of employing standard cells the apparatus for which may be assembled in any well-appointed laboratory for electrical measurements. In general, it is much more convenient to use for the purpose pieces of commercial apparatus called "potentiometers," which give the result directly without calculation. The principle involved is illustrated in Fig. 171*A*.

It is desired to measure potential difference. With the circuit shown, there are two possible procedures: (1) Keep the resistance $a'b'$ fixed, and vary the current I_{ab} until the drop in $a'b'$ is equal and opposite to potential difference; then P.D. = $I_{ab} \times r_{a'b'}$. This method necessitates the use of an accurate current-measuring instrument at *A*. It was given by Poggendorf and has been used by many experimenters. It has lately been developed in a practical form by Brooks and Spinks.[21] (2) Keep the current I_{ab} constant, and vary the resistance $a'b'$ until balance is obtained. This is the method employed in one form or another in the majority of commercial potentiometers. (3) Balance the unknown voltage against the *difference* of two *IR* drops, as indicated in Fig. 171*C*. Then P.D. = $I_{ab} \times r_{ab} - I'_{ab}r'_{ab}$. I_{ab} and I'_{ab} have fixed values and a predetermined ratio.

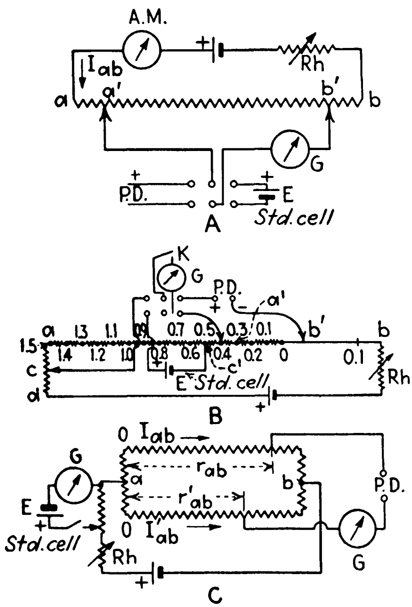


FIG. 171.—Illustrating principle of potentiometer.

Referring to Fig. 171*A*, *ab* is a definite resistance along which the sliders $a'b'$ can be displaced. *Rh* is a rheostat by which the current in *ab* can be adjusted. In all forms of potentiometer the rheostat should be capable of varying the current *continuously* between the required limits; plug resistors are to be avoided. Let r_1 be the resistance from a' to b' when the galvanometer deflection is zero and the standard cell is in circuit; let r_2 be the corresponding resistance when the galvanometer deflection is zero and the switch is on potential difference, *the current in ab being as before.* If *E* is the e.m.f. of the standard cell, the potentiometer current will be $I_{ab} = E/r_1$. Consequently,

$$\text{P.D.} = \frac{Er_2}{r_1}.$$

To ascertain if I_{ab} has remained constant necessitates throwing the switch to *E* and resetting the sliders to a predetermined reading. It is evident that the resistance in the I_{ab} circuit must remain constant irrespective of the setting of the sliders. Early potentiometers were constructed practically as just indicated.

It is desirable to employ for *ab* a practical equivalent of a slide wire which will permit the setting to be made with a high precision; also it is obviously a great inconvenience to be obliged to reset the slides when I_{ab} is to be checked, which must be done very frequently.

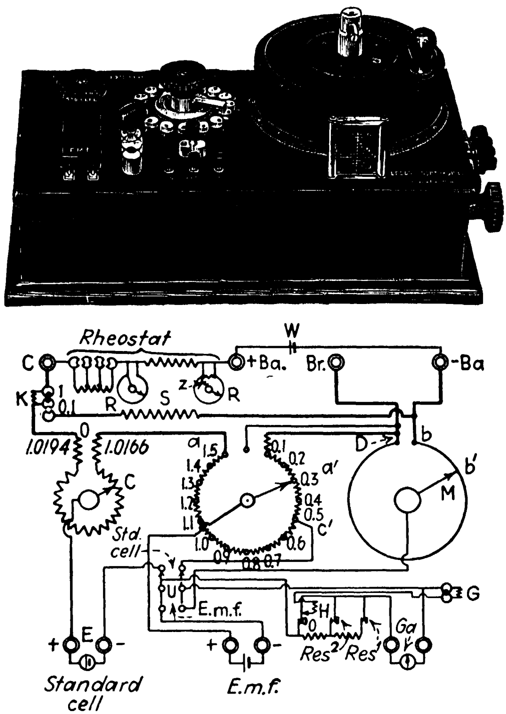


FIG. 172.—Leeds and Northrup type K potentiometer.

The circuit shown in Fig. 171*B* is employed in the Leeds and Northrup type K potentiometer (Fig. 172).

The slide wire is replaced by a set of 15 five-ohm coils in series with a slide wire having a resistance of 5.5 ohms. The standard potentiometer current I_{ab} is 0.02 amp.; therefore the drop in each coil is 0.1 volt, and in the slide wire 0.11 volt, making the total drop 1.61 volts. The instrument is graduated to read volts directly. The slide wire *Db* has 11 turns which are wound in a screw thread on a marble cylinder 6 in. in diameter. Protection from dust and mechanical injury is secured by a movable hood mounted on a screw coaxial with and having the same pitch as the winding on the cylinder. The slider, which is always in contact with the wire, is carried by the hood, which, at its lower edge, has 100 equal graduations.

Fractions of a turn may thus be read. The whole turns of the slide wire correspond to the divisions on the vertical scale. As one division on this scale corresponds to one turn, it is equivalent to 0.01 volt, while one division on the hood corresponds to 0.0001 volt or 100 microvolts.

To avoid resetting the slides when I_{ab} is checked, the resistor *ad* is added. The tap *c* from the standard cell is adjustable so that the resistance *c'c* may be varied in short steps through a limited range. This is necessary, for at balance I_{ab} must have a predetermined value, and some standard cells have temperature coefficients, while the Weston cell, which is the one now universally employed, is, in its commercial form, a secondary standard, the e.m.fs. being slightly different for different cells. (The actual e.m.f. is stated on the calibration certificate.) The necessary value of the resistance *c'c* is E/I_{ab}. It is convenient to mark the settings on *ad* with the standard cell voltages to which they correspond, rather than the resistances between the tap and the point *c'*. The standard cell is connected between the movable arm and the point 0.5 volt. Cells having e.m.fs. from 1.0166 to 1.0194 volts may be employed. The arm *c* is set to correspond to the voltage of the particular cell used. By means of a double-throw switch, the galvanometer may be quickly transferred from the standard cell circuit to that marked "E.m.f."; as the two circuits are entirely distinct, no resetting of the instrument is necessary when checking the potentiometer current. This is a very great convenience.

The process of making a measurement is to set the standard-cell dial at the voltage of the cell to be employed and to put the double-throw switch on the point marked "Standard Cell," then to vary the rheostat at *Rh* to obtain a zero deflection of the galvanometer. This adjusts the potentiometer current to its standard value. The switch is then thrown to E.m.f., and balance obtained by using the voltage slides without altering the rheostats. The reading gives the potential difference directly in volts. To check the potentiometer current, it is necessary simply to throw the switch to Standard Cell and press the key. No resetting is necessary.

For satisfactory action, it is necessary to apply occasionally a little vaseline to the wire.

Low-scale Arrangement.—If the current through the coils and the slide wire is reduced to 0.002 amp., the drop through each coil becomes 0.01 volt, and the entire range of the potentiometer is 0.161 volt. After the potentiometer current has been checked with the plug *K* in socket 1, the plug is transferred to socket 0.1. The resistance of the standard-cell adjustment, the 5-ohm coils, and the slide wire is then shunted by *S*, which is of such a value that one-tenth of the total current flows through the 5-ohm coils and the slide wire. The resistance *K* is so adjusted that the total current is kept at 0.02 amp.

In using any form of potentiometer, it is absolutely necessary to check the potentiometer current before taking a reading.

Leeds and Northrup Type K-2 Potentiometer.—The circuit of this potentiometer differs from that of the instrument just described chiefly in the use of a range-changing device in which there are no plug contacts. The sliding contacts of the circular range-changing switch are in battery circuits where their contact resistances have no effect on the accuracy of the measurements.

The ranges provided, 0 to 1.6 volts, 0 to 0.16 volt, and 0 to 0.016 volt, correspond to the markings of 1, 0.1, and 0.01 on the range-changing switch. The regulating rheostats are continuously adjustable, coils and

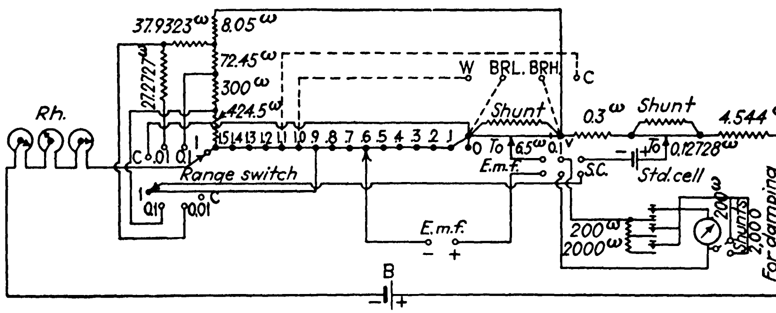


FIG. 173.—Connections of Leeds and Northrup type K-2 potentiometer.

plugs being avoided. There are two damping shunts for the galvanometer, one of 200. and one of 2,000. ohms which may be used as desired. The change from Standard Cell, to E.m.f. is made by a dial switch. All the switches are protected by housings, so that there are no exposed metal contacts.

Feussner Potentiometer.[22]—With the Feussner potentiometer, settings to 0.00001 volt are obtainable without recourse to a slide wire. The distinguishing feature of the circuit is the arrangement of coils by which the potentiometer current is maintained at its predetermined value when the setting is changed. Figure 174 is diagrammatic of the instrument as made by Wolff. Referring to the figure, the battery current from B flows through all the coils marked $\times 100$, then to the lower group marked $\times 0.1$, where it traverses the coils to the left of the contact D and on to the group $\times 1$, traversing the coils to the left of the contact E, thence to the coils to the left of the contact F in the group $\times 10$, and on through all the coils in the group $\times 1{,}000$.

The function of the upper sets of coils in the groups marked $\times 10$, $\times 1$, and $\times 0.1$, which are connected in series with those in the lower or measuring sets, is to maintain the potentiometer current at a fixed value irrespective of the position of the contacts D, E, F. The contacts on the upper and lower sets of these resistances are rigidly connected, so that if a coil is removed from the lower set an equal coil is added in the upper set.

When the switch K is on X, the derived circuit containing the unknown potential difference is connected between A and C via the galvanometer. By manipulating the switches, the resistance between A and C may be varied from 0 to 18,999.9 ohms; so if the potentiometer current is kept at 0.0001 amp., any potential difference between 0 and 1.89999 volts may be balanced, in steps of 0.00001 volt.

The standard-cell circuit is connected between the eighth and ninth coils in group ×1,000 and the contact at H; by moving this contact, the resistance between the terminals of the standard-cell circuit may be varied

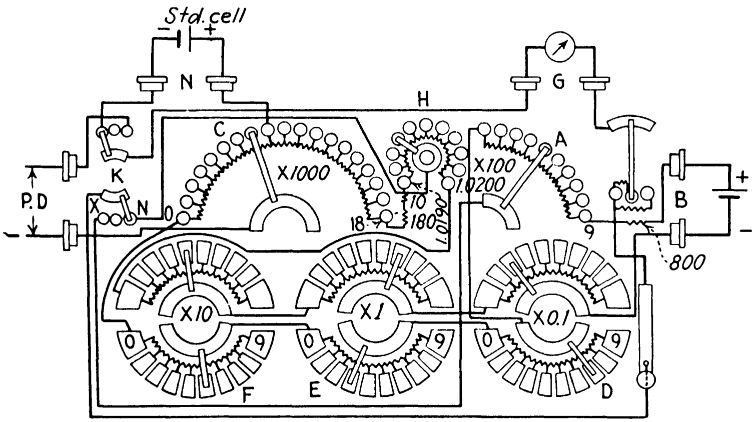


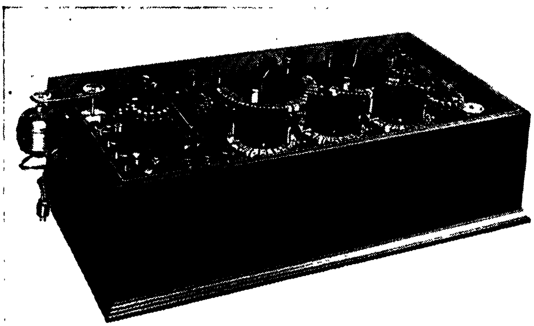
FIG. 174.—Wolff-Feussner potentiometer.

from 10,190 to 10,200 ohms in 1-ohm steps. The instrument may thus be adjusted so that standard cells having e.m.fs. from 1.0190 to 1.0200 volts may be employed. To check the potentiometer current, it is necessary merely to throw the switch K to the position marked N and to depress the key.

When all the dials are set at zero, there are included in the measuring circuit 12 sliding contacts and the necessary internal connection wires; consequently a theoretically perfect zero reading is not obtainable, even though the resistance of a sliding contact of high grade is only about 0.0001 ohm. Inspection of Fig. 174 shows that six sliding contacts on the nearly semicircular sectors are employed in the compensating arrangement. In the instrument as made by the Eppley Laboratory[22] (Fig. 175), insulated leads from the contact brushes pass down through the spindle of each switch and are carried to the next set of coils by flexible metallic connections. Six sliding contacts are thus eliminated. Also, in the Eppley potentiometer, the standard-cell adjustment may be set to five decimal places to correspond with the main dials setting.

Figure 175 shows the general appearance of the Eppley instrument which is oil immersed. At the extreme right is the regulating rheostat for the potentiometer current; and at the extreme left, the temperature control for the oil bath.

The Eppley Laboratory has developed a four-dial potentiometer for general laboratory purposes, the connections of which are shown diagrammatically in Fig. 176. Feussner compensating coils are used with the two



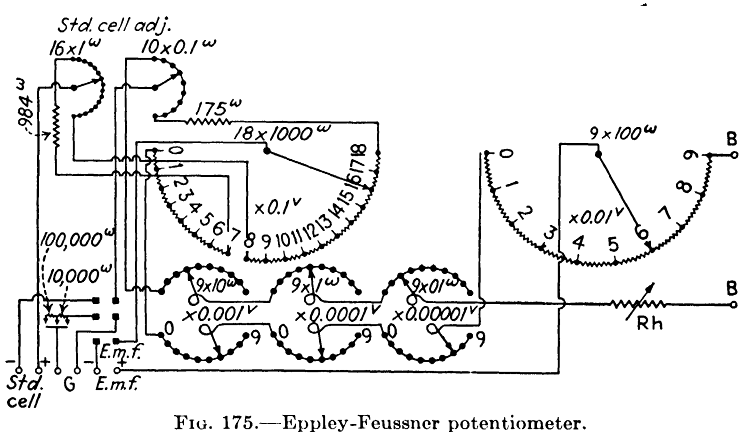


FIG. 175.—Eppley-Feussner potentiometer.

middle dials. It will be noted, however, that the instrument has a real zero reading; for if the connections are traced when all the dials are set at zero, it will appear that the e.m.f. circuit has only one point in common with the main potentiometer circuit and therefore spans no part of that circuit in which there is an *IR* drop. The range of the instrument is from 0. to 2.111 volts in steps of 0.0001 volt, or 100 microvolts.

Volt Box.—The usual range of a potentiometer is less than about 2 volts; so for higher voltages, it is necessary to use it in conjunction with a volt box. This consists of a high resistance which is connected across the potential difference to be measured. It is divided by taps so that the potentiometer measures a definite fraction, one-tenth, one-hundredth, one-thousandth of the potential difference.

It is essential that the division of the resistances be very exact and that there be no leakage around high-resistance coils. In work of exceptional refinement, it may be found that the voltage ratio is, to a certain extent, dependent on the magnitude of the voltage being measured. This arises from the fact that the coils of the volt box are wound with wire of various sizes and on bobbins of different diameters which may be so placed that their temperature environment is not the same, the result being that if the coils have an appreciable temperature coefficient there will be a gradual drift of the voltage ratio dependent on the time that the current has been flowing through the volt box.

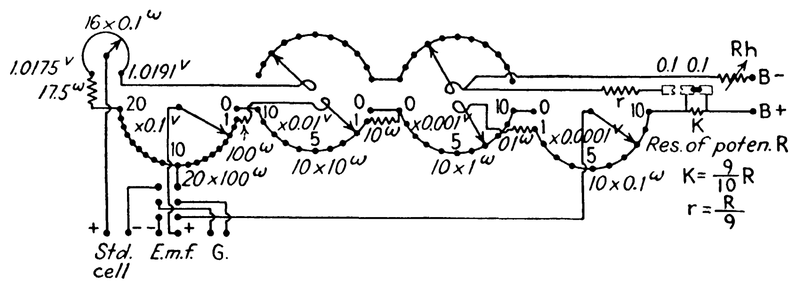


FIG. 176.—Eppley four-dial potentiometer with real zero.

When a volt box of unknown characteristics is first put in service, the voltage ratio should be checked, first, by cold-resistance measurements; second, by resistance measurements after the normal working voltage has been applied for a considerable time. In the subsequent use of the instrument, the voltage ratios of the various taps should be verified frequently.

Attention has been called to the fact that in certain cases the application of a voltmeter alters the potential difference that is to be measured. A volt box introduces the same sort of error, which is minimized by making the resistance of the volt box very high.

The Brooks Deflection Potentiometer.[23]—The potentiometers thus far described are read by the null method, an exact balance being obtained between the potential difference in the instrument, due to the potentiometer current, and the potential difference to be measured. The objection to this method is that, while it gives results of the highest precision, the potential difference to be measured must be steady, and repeated trials have to be made before the balance point is obtained. When the unknown potential difference is not steady, many trials must be made before the null point is hit upon by mere chance, and the expenditure of time and patience becomes so great as to be almost prohibitive in much commercial work

In the case of a large electrical engineering laboratory, where many instruments must be checked and kept in adjustment, it is imperative that the work be done with great speed, combined with the accuracy necessary in engineering work.

The Brooks potentiometer was designed with this in mind. By it, results may be obtained even though the potential difference under measurement is not perfectly steady. In this instrument, no attempt is made to obtain an exact balance; the slides are set so near to the null point that the galvanometer deflection is small. The galvanometer is so graduated that it gives the amount that must be added to the reading of the slides in order to obtain the unknown potential difference.

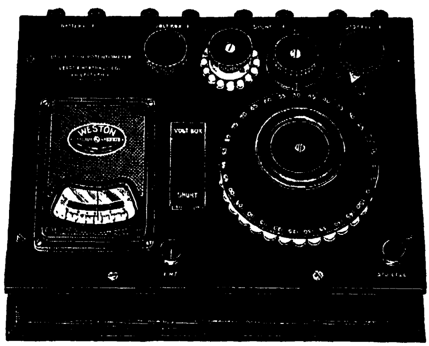


FIG. 177.—Brooks deflection potentiometer, Model 7, Leeds & Northrup Co.

That certain conditions must be fulfilled may be seen from the following discussion.

In general, the potentiometer is used:

I. To determine potential differences which are within the normal range of the instrument.

II. To determine, by the use of a volt box, potential differences which are above the normal range of the instrument.

III. To measure currents by the use of shunts.

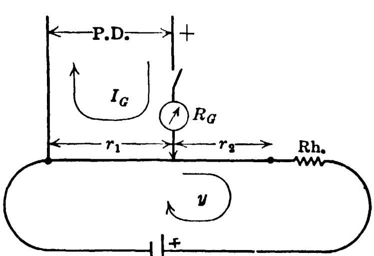


FIG. 178.—Diagram for Brooks deflectional potentiometer, Case I.

CASE I. DIRECT MEASUREMENT OF POTENTIAL DIFFERENCE.

A storage cell is used at e (Fig. 178), so its resistance may be neglected.

R_G is the resistance of the galvanometer plus any resistance placed directly in series with the instrument.

The mesh equations are

$$I_G(R_G + r_1) - yr_1 - \text{P.D.} = 0;$$
$$y(r_1 + r_2 + Rh) - I_G r_1 + e = 0.$$

Therefore

$$I_G = \frac{\text{P.D.} - \dfrac{er_1}{r_1 + r_2 + Rh}}{R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}.$$

The standard potentiometer current when it has been adjusted, as in the ordinary potentiometer, by bringing the galvanometer to zero with the standard cell in circuit, is $\frac{e}{r_1 + r_2 + Rh}$; therefore $\frac{er_1}{r_1 + r_2 + Rh}$ is the graduation marked on the slides of the instrument.

If the null method is employed,

$$\frac{er_1}{r_1 + r_2 + Rh} = \text{P.D.}, \qquad \text{reading on slides.}$$

The instrument being in exact balance, suppose that the applied potential difference P.D. is increased by a small amount δ[P.D.], so that it becomes P.D. + δ[P.D.], the slides being kept as they were. A current will now flow through the galvanometer, and its value will be

$$I_G = \frac{\delta[\text{P.D.}]}{R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}. \tag{33}$$

This shows that in measuring any potential difference, the major portion of it may be read from the slides as usual, and to this may be added the voltage $I_G\left(R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$ in order to obtain the total value.

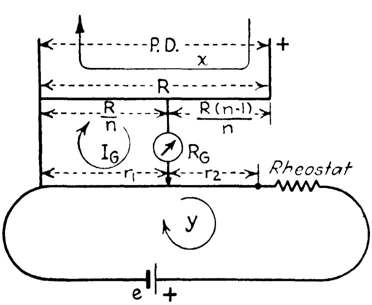


FIG. 179.—Diagram for Brooks deflection potentiometer, Case II.

The quantity $\left(R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$ is the total resistance of the galvanometer circuit with P.D. and e short-circuited. If *this resistance is kept constant for all positions of the slides and the adjusting rheostat*, and a D'Arsonval galvanometer is used, the galvanometer scale may be graduated to read δ[P.D.] directly in volts. In order that this graduation, as a voltmeter, may be correct for all three of the cases mentioned above, the resistance of the galvanometer circuit must always be kept the same, irrespective of the use to which the potentiometer is put. This is the cardinal point in the design of the deflection potentiometer and is attained by an ingenious arrangement of coils and switches.

CASE II. POTENTIAL DIFFERENCE BY USE OF VOLT BOX.

The mesh equations are

$$xR - I_G\frac{R}{n} - \text{P.D.} = 0;$$

$$I_G\left(\frac{R}{n} + R_G + r_1\right) - x\frac{R}{n} - yr_1 = 0;$$

$$y(r_1 + r_2 + Rh) - I_G r_1 + e = 0.$$

Therefore

$$I_G = \frac{\dfrac{\text{P.D.}}{n} - \dfrac{er_1}{r_1 + r_2 + Rh}}{\dfrac{R(n-1)}{n^2} + R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}.$$

If the slides are set so that I_G is zero, then

$$\frac{er_1}{r_1 + r_2 + Rh} = \frac{\text{P.D.}}{n}, \qquad \text{reading on slides.}$$

If, now, the potential difference is increased by a small amount δ[P.D.], the slides remaining fixed, a current will flow through the galvanometer

$$I_G = \frac{\delta[\text{P.D.}]/n}{\dfrac{R(n-1)}{n^2} + R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}. \tag{34}$$

The total potential difference is, therefore, n times the sum of the reading of the slides and the reading of the galvanometer in volts, *provided* the coils are so arranged that

$$\left(\frac{R(n-1)}{n^2} + R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$$

is a constant for all settings of the slides, and the galvanometer has been calibrated as a voltmeter with this resistance. The expression in parentheses is the total resistance of the galvanometer circuit when e and the volt box are short-circuited.

CASE III. CURRENT MEASUREMENT.

R_G includes the resistance of the galvanometer and the potential leads from the shunt.

The mesh equations are

$$I_G(S + R_G + r_1) - yr_1 - IS = 0;$$

$$y(r_1 + r_2 + Rh) - I_G r_1 + e = 0.$$

Therefore

$$I_G = \frac{IS - \dfrac{er_1}{r_1 + r_2 + Rh}}{S + R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}.$$

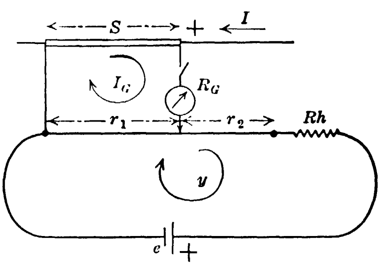


FIG. 180.—Diagram for Brooks deflectional potentiometer, Case III.

If the current is increased by a small amount $\delta[I]$ from that necessary for an exact balance, the slides remaining fixed, the galvanometer current will be

$$I_G = \frac{\delta[I]S}{S + R_G + \dfrac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}}, \tag{35}$$

or

$$\delta[I]S = I_G\left(S + R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right).$$

The expression in parentheses is the resistance of the galvanometer circuit with e short-circuited and the main-current circuit open beyond the shunt. The quantity to be added to the reading of the slides before dividing by S is seen to be the galvanometer reading reduced to volts, *provided* the switches and coils are so arranged that

$$\left(S + R_G + \frac{r_1(r_2 + Rh)}{r_1 + r_2 + Rh}\right)$$

has a fixed value.

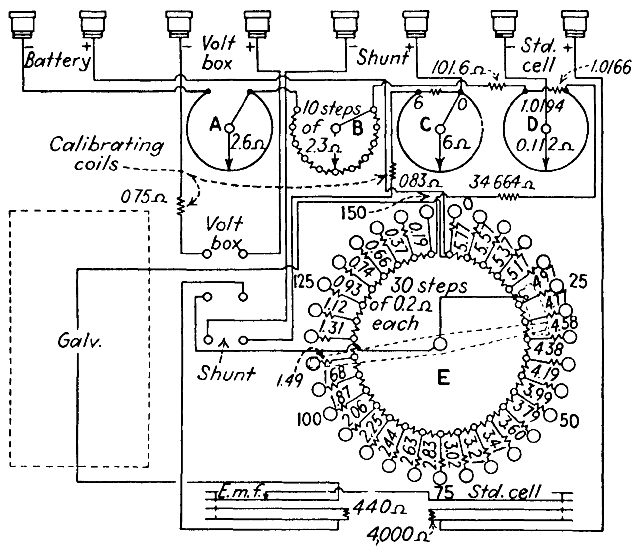


FIG. 181.—Circuit of Brooks deflection potentiometer, Model 7 L. & N. Co.

The galvanometer is the determining feature in the design of the deflection potentiometer. An instrument for the purpose having a resistance of 7.4 ohms and a period of about 1 sec. has been designed by the Weston Instrument Company.

The range of the Brooks Model 7 potentiometer is from 0 to 153 millivolts, 150 being given by the dials, and 3 by the deflection of the galvanometer. The circuit diagram is given in Fig. 181. A Weston unsaturated standard cell is to be used.

The potentiometer current of 0.025 amp. is derived from two lead-storage cells or three Edison cells. It flows, in turn, through the 30 potentiometer coils, each of 0.2 ohm resistance; an added resistance of 34.664 ohms; the standard-cell rheostat D; an added resistance of 101.6

ohms; and the adjusting rheostats *B* and *A*. Each tap on the potentiometer is seen to correspond to 0.05 volt. The standard cell is tapped around the 30 potentiometer coils, the resistance of 34.664 ohms, and whatever is necessary in the standard-cell rheostat.

When currents are to be measured, the potential terminals of the shunt are connected to the binding posts marked "Shunt." Referring to the expression for I_G, if the main dial setting is altered, that is, if r_1 and r_2 are changed, the denominator, which should be constant for all settings of the main dial, is altered. It is therefore necessary to arrange the connections so that compensating resistances of the proper values are automatically inserted in series with the galvanometer. As shown in the figure, when the setting is 110 millivolts the compensating resistance is 1.49 ohms; and when the setting is 120 millivolts it is 1.12 ohms.

The sum of the resistances of the galvanometer circuit, the shunt, and the potential leads from the shunt must be kept constant. Therefore the rheostat *C* is introduced. Any increase of the shunt resistance or of the lead resistance is compensated for by cutting out resistance in *C*.

When the settings of the adjusting rheostats *A* and *B* are altered, *Rh* may be changed between the limits 136.376 and 161.976 ohms; however, because of the manner in which *Rh* appears in the formula, the resulting change in the denominator is negligible.

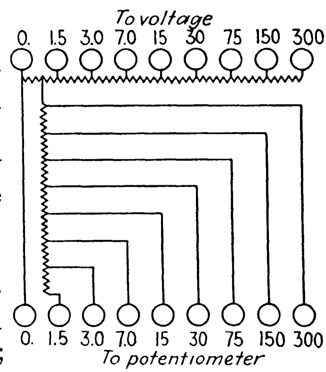


FIG. 182.—Diagram of volt box for Brooks deflection potentiometer.

Any shunt suitable for current measurement may be used, but a special compensating volt box having an upper limit of 300 volts must be employed. Referring to Fig. 182, it will be seen that each tap has in series with it a compensating resistor, so that the resistance measured from a break in the galvanometer circuit when the main battery and the volt box are short-circuited is constant, irrespective of the value of *n* that appears in the formula. The range from 300 to 750 volts is obtained by connecting a multiplier in series with the volt box.

The potentiometer as here described requires a volt box of rather low resistance. This difficulty has been overcome by Brooks in a later design.

The galvanometer is so arranged that the undeflected position of the index is at the middle of the scale, which has two sets of numbering, in millivolts. If the dial setting ends in 5, the lower scale is to be used; for instance, if the dial setting is 75, the mid-point of the scale corresponds to a voltage of 75 millivolts. If the dial setting ends in 0, the upper scale is used; for instance, for a dial setting of 80, the mid-point of the scale corresponds to 80 millivolts.

The "Thermokraftfrei" Potentiometer.[24]—In the measurement of small potential differences such as those of thermocouples, a low-resist-

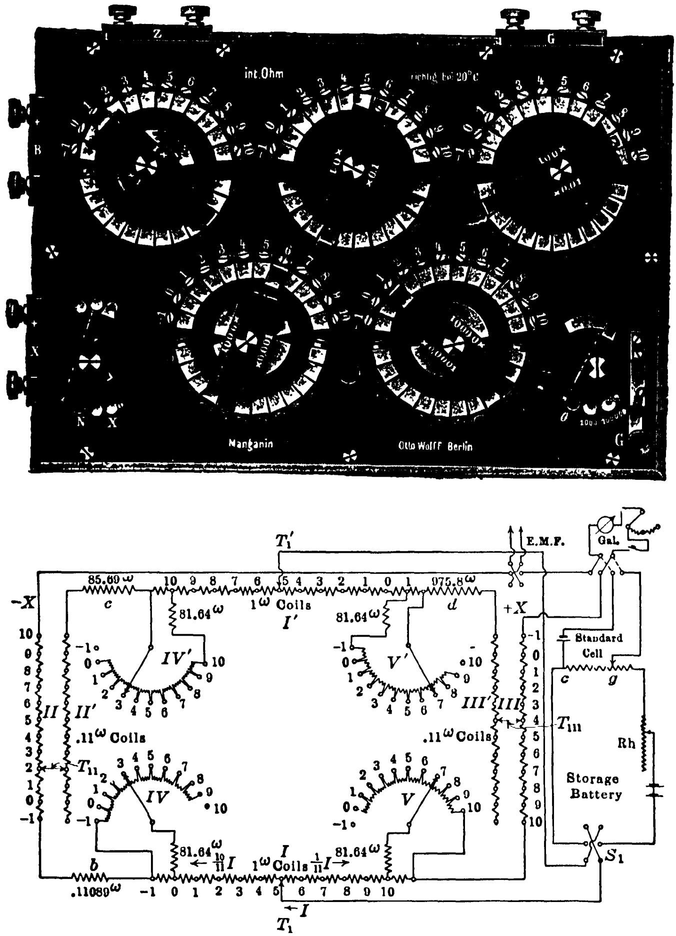


FIG. 183.—Thermokraftfrei potentiometer.

ance potentiometer is used, and it is necessary to eliminate all thermoelectric disturbances due to contact of dissimilar materials and inequalities of temperature in the potentiometer itself; therefore the metal employed in the coils and in their terminals must be carefully selected with this result in view. For the same reason, the design should be such that the effect of thermo-e.m.fs. introduced by the manipulation of the necessary sliding contacts and switches will be reduced to a minimum. This is accomplished in the instrument under discussion, which includes elements of design due to H. Hausrath, W. P. White, and H. Diesselhorst. The potentiometers previously discussed have been series arrangements; that is, the compensating potential difference is the *sum* of the potential differences existing between the terminals of various groups of coils of which the circuit is composed. Hausrath suggested the use of a divided circuit in place of coils in series. In that case, the compensating potential difference is due to the *difference* of the potential drops along the two branches measured from the point where the current enters the apparatus. Figure 183 shows in diagram the arrangement of the coils.

The potentiometer current is kept at its standard value, 0.001 amp., in the usual manner by equating the drop in *cg* to the e.m.f. of the standard cell; *Rh* is the regulating rheostat; S_1 is a reversing switch in the main circuit.

The current entering at T_{I} divides, the resistances being so arranged that ten-elevenths of the total current flows to the left, and one-eleventh to the right. The design is such that these relative values are maintained for all positions of the switches.

Each coil in the decade I and in the group of compensating coils I′ has a resistance of 1 ohm, and the sliding terminals T_{I} and T'_{I} are mechanically connected so that if T_{I} is shifted to the right, T'_{I} is shifted an equal number of coils to the left. The resistances of both paths between the terminals T_{I} and T'_{I} are thus kept constant.

As all the coils in decades II and III and all the compensating coils in II′ and III′ are alike, the resistances of both the left-hand and the right-hand branches of the circuit are independent of the positions of T_{II} and T_{III}.

The two sliding contacts on decades IV and the compensating coils IV′ are mechanically connected so that they must move together, as are the two contacts on decade *V* and its compensating coils *V′*. If the contact in decade IV is set on a terminal having a certain number, then automatically the contact in group IV′ is set on the terminal of the same number, and similarly with V and V′. The group of coils IV consists of a 1-ohm coil (marked −1 in group I), shunted by a variable resistance which consists of a fixed portion 81.64ω and a variable part included between −1 and the position of the sliding contact. The coils in the variable portion of IV are

Between	Ohms	Between	Ohms
−1 and 0	8.264	4 and 5	30.30
0 and 1	10.101	5 and 6	45.41
1 and 2	12.626	6 and 7	75.80
2 and 3	16.234	7 and 8	151.51
3 and 4	21.645	8 and 9	454.54
		9 and 10	∞

In group IV′, the order is reversed; that is, the coil having a resistance of 8.264 ohms is between contacts 9 and 10. The resistances are given these particular values in order that, when the contact in IV is moved one number, the resistance between T_{I} and T_{II}, may always be altered by a definite amount, 0.0011 ohm, which is one one-hundredth of the alteration that would be obtained by moving T_{II} one number. For instance, if the decade contact is on number 4, the resistance of group IV is

$$R_{IV} = \frac{1 \times 150.51}{1 + 150.51} = 0.99342 \text{ ohm.}$$

When it is set on number 5, this becomes

$$R_{IV} = \frac{1 \times 180.81}{1 + 180.81} = 0.99452 \text{ ohm.}$$

The difference of these two values is 0.0011 ohm, while the value of a single step in II is 0.11 ohm.

The minimum value of R_{IV} is

$$R_{IV} = \frac{1 \times 89.904}{1 + 89.904} = 0.9890 \text{ ohm,}$$

so the general value is

$$R_{IV} = 0.9890 + 0.0011 \times n_{IV} \text{ ohm,}$$

where n_{IV} is the number of the contact on dial IV.

Similarly, the general value of $R_{IV'}$, is

$$R_{IV'} = 0.9989 - 0.0011 \times n_{IV'} \text{ ohm.}$$

In decade V, the coil of 8.264 ohms is between contacts 9 and 10; in the group of compensating coils V', it is between -1 and 0; so, in general,

$$R_{V} = 0.9989 - 0.0011 n_{V} \text{ ohm.}$$
$$R_{V'} = 0.9890 + 0.0011 n_{V'} \text{ ohm.}$$

The coils *b*, *c*, *d* have such values that taken in conjunction with the other resistances they divide the current flowing in at T_{I} in the ratio 10:1.

The voltage between T_{II} and T_{III} is the difference of the ohmic drops measured from T_{I}, so for any setting (potentiometer current, 0.001 amp.)

$$\text{P.D.} = {}^{10}\!/\!_{11} \times 0.001[n_{I} \times 1 + 0.9890 + 0.0011 n_{IV} + 0.11089 + 0.11(n_{II} + 1)]$$
$$- {}^{1}\!/\!_{11} \times 0.001[(10 - n_{I})1 + 0.9989 - 0.0011 n_{V} + 0.11(10 - n_{III})]$$
$$= 0.001\left[n_{I} + \frac{n_{II}}{10} + \frac{n_{III}}{100} + \frac{n_{IV}}{1{,}000} + \frac{n_{V}}{10{,}000}\right] \text{ volt.}$$

Therefore when the dials are properly graduated the unknown potential difference is measured by the sum of the dial readings, as in the usual instruments.

A study of the network shows that if the battery circuit is open, the resistance between the galvanometer terminals $+X$ and $-X$ is, to a good degree of approximation, 14.35 ohms and that when the battery circuit is closed through a series resistance B, which is external to the potentiometer, the resistance becomes, using a second approximation,

$$14.35 - \frac{n_1^2}{B + R}, \tag{36}$$

where R is the resistance of the potentiometer between T_I and T_I' with the galvanometer circuit open.

The resistance of the right-hand path between T_I and T_I' is 990 ohms; that of the left-hand path, 99 ohms; the resistance of the whole apparatus is therefore $R = 90$ ohms.

If a storage cell is used, and the potentiometer current is 0.001 amp., $R + B$ must be approximately 2,000 ohms; therefore, the maximum variation in the resistance of the galvanometer circuit will be only 0.05 ohm or about 0.3 per cent. This constancy of the resistance allows one to obtain the last figure in the potential difference under measurement, by the deflection method, the reading of the last decade n_V being kept at zero. By properly setting up the apparatus, the full reading of the last decade $n_V = 10$ may be made to correspond to 1, 10, or 100 divisions on the galvanometer scale, and thus the necessity for exact balancing may be obviated. Where the potential difference to be measured is fluctuating slightly, this is a decided advantage (see page 282, "Brooks Deflection Potentiometer").

Another advantage, if a moving-coil galvanometer is used, is that the damping remains constant, irrespective of the setting of the potentiometer.

Reference to the table on page 389 shows that although the use of a slide wire to obtain the fine adjustment has been avoided, it has become necessary to adjust the coils to odd values. Consequently, if doubt arises as to the accuracy of the potentiometer, the difficulty of checking it has been much increased.

Contact Resistances and Thermo-e.m.f.—As this potentiometer is especially designed for the precise measurement of small e.m.fs., the effect of variable resistances at the sliding contacts and of parasitic e.m.fs. set up by the manipulation of the switches must be negligible. The order of magnitude of the variation of the resistance of a well-made switch such as is used in this instrument, is about 0.0002 ohm, and of the parasitic e.m.f. 0.000001 volt (1 μv).

The contacts T_I and T_I' are in the battery circuit, the resistance of which is about 2,000 ohms. The double contacts at T_{II} and T_{III} are in series with resistances in excess of 85 and 975 ohms, respectively. Their resistances have a negligible effect on the galvanometer deflection. The sliding contacts at dials IV and V and at IV′ and V′ are in circuits of comparatively high resistance, which are shunted around 1-ohm coils. If R_s is the value of the resistance in the dial, and R the resistance around which it is shunted, the resistance of the combination is

$$R' = \frac{RR_s}{R + R_s}.$$

if a small change ΔR_s is made in R_s, the resulting change in R' is

$$\Delta R' = \Delta R_s \left(\frac{R}{R + R_s}\right)^2.$$

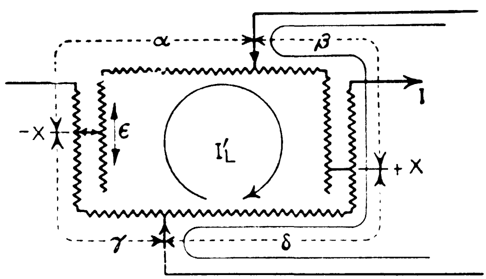


FIG. 184.—Pertaining to effect of thermal e.m.f. in thermokraftfrei potentiometer.

If the switch is set at 0 in dial IV and $\Delta R_s = 0.0002$, then

$$\Delta R' = 0.0002\left(\frac{1}{89.9}\right)^2,$$

a quantity that is insignificant compared with the shunted resistance of the 1-ohm coil.

Any e.m.fs. arising from manipulating I and I' are added to the battery e.m.f. (2 volts) and will be negligible. The effect of a thermo-e.m.f. of magnitude ϵ, due to moving contact II, will be very small.

Referring to Fig. 184, by Kirchhoff's rules the current in the left-hand branch, if I is the total battery current coming to the potentiometer, is

$$I'_L = \frac{I(\beta + \delta) \mp \epsilon}{\alpha + \beta + \gamma + \delta};$$

and in the right-hand branch

$$I'_R = \frac{I(\alpha + \gamma) \pm \epsilon}{\alpha + \beta + \gamma + \delta}.$$

Therefore, the change in the potential difference between the terminals

$+X$ and $-X$ due to $\pm\epsilon$ will be

$$\frac{\pm\epsilon(\gamma + \delta)}{\alpha + \beta + \gamma + \delta}.$$

The maximum value of $\gamma + \delta$ is 14.42 ohms, and the value of

$$\alpha + \beta + \gamma + \delta$$

is 1,089 ohms, so

$$\frac{\epsilon(\gamma + \delta)}{\alpha + \beta + \gamma + \delta} = \epsilon \times 0.013.$$

Therefore the error introduced is only 1.3 per cent of ϵ.

Parasitic e.m.fs. in IV, IV′ and V, V′ also have very small effects. Suppose that dial V is set at 10 and that the parasitic e.m.f. is ϵ; this e.m.f. sends a current through 81.64 ohms and the 1-ohm coil shunted by nearly 1,100 ohms. It therefore produces a potential drop of approximately $\epsilon/82.6 = \epsilon \times 0.012$. Therefore the error introduced is of the order of only 1.2 per cent of ϵ. The error due to parasitic e.m.fs. having been reduced to negligible amounts, the apparatus is said to be free from thermo-e.m.fs.

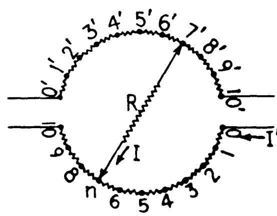


FIG. 185A.—Wenner decade arrangement.

Wenner Potentiometer.[25]—Wenner has designed a difference potentiometer in which the number of coils adjusted to odd values is greatly reduced. The characteristic feature is the arrangement of the three lowest dials.

Referring to Fig. 185*A*, there are 10 active coils between 0 and 10 and 10 compensating coils between 0′ and 10′, each of the 20 coils having the same resistance r. R is the resistance permanently inserted between the two contacts of the circular sliding switch. Inspection shows that when the switch is displaced, the total resistances to the right and left of the sliding contacts are not altered. The current distribution is therefore independent of the setting of the switch. When the contact is on the nth stud, the fall in potential from 0 to 10 is

$$(I + I')(10 - n)r + I'_n r = 10(I + I')r - nIr.$$

When the contact is at 0, the fall of potential from 0 to 10 is

$$10(I + I')r.$$

Consequently, the *change* in the fall of potential when the switch is displaced from 0 to n is nIr, that is the product of the cross current in the switch and the resistance between 0 and n. Figure 185*B* is a simplified diagram of the complete network on which the values of the various coils

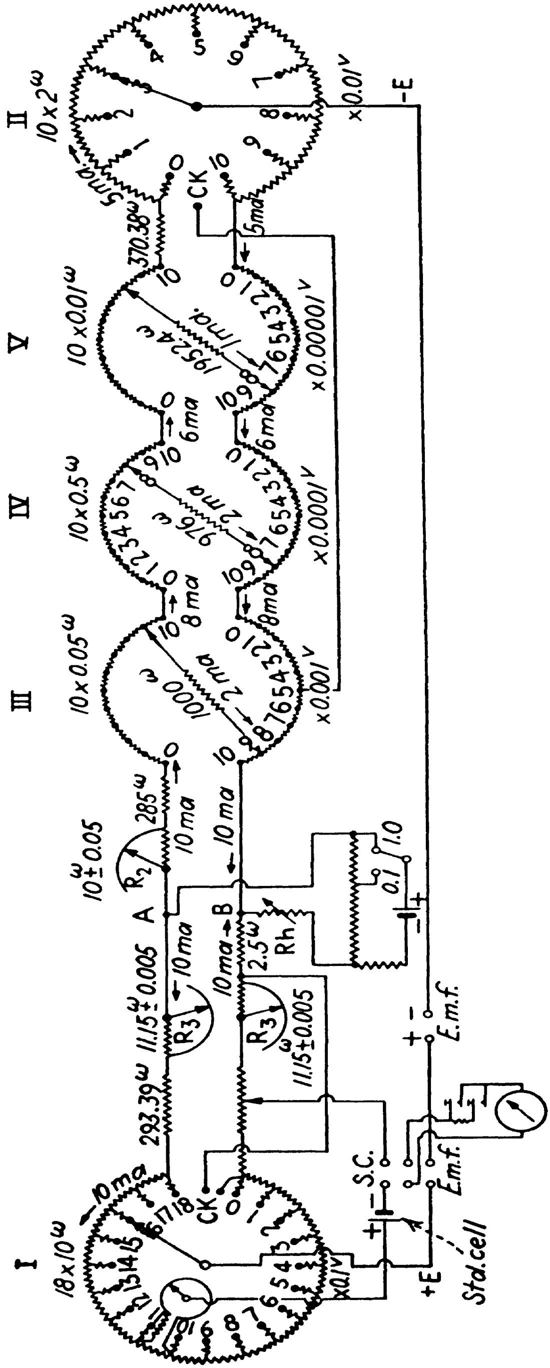


FIG. 185B.—Connections for Wenner potentiometer.

are indicated. The arrangement of the two upper dials is as usual, compensating resistances being employed to keep the resistance of the galvanometer circuit at approximately 174 ohms.

A current of 0.02 amp. from the battery flows in at A and out at B. At A it divides, 10 ma. flowing to the right, and 10 ma. to the left. The resistance on either side of AB is 500 ohms. If all the contacts are set at 0, the rise of potential from B to $E-$ is 0.1546 volt; that from B to $E+$ is also 0.1546 volt. Consequently, the difference of potential between $E-$ and $E+$ is zero. If all the contacts at the right of AB are set at 10 while the one contact at the left of AB is set at 18, the rise of potential from B to $E-$ is 0.0435 volt, while the rise from B to 18 is 1.9546 volt. The difference, 1.9111 volt, gives the upper limit for the potentiometer. This is attained in steps of 10μv.

Parasitic e.m.fs. set up by manipulating dials I and II are not a source of error, although they are directly in series with the unknown e.m.f., since they die out before the final balance is obtained by use of the three lower dials. The effect of a parasitic e.m.f. at the contacts of dial III may be shown as follows: The resistance of the battery branch between A and B will be assumed to be 100 ohms. Suppose that all the dials are set at 0; the resistance to the right of dial III is 250 ohms, while that to the left is 383 ohms. If ϵ is the value of the parasitic e.m.f., the current flowing to the right is $\epsilon \times 0.00052$ amp. approximately, to the left, $\epsilon \times 0.00034$ amp. approximately. On account of the shunting action of the 100-ohm battery resistance, only one-sixth of the current flows through the coils of dial I. The current to the right divides in the same proportions as the regular potentiometer current, producing a fall of potential from $E-$ to B of less than $\epsilon/100$. When dial I is set at 18, the fall of potential from 18 to B is very slightly greater than $\epsilon/100$. As the error is the difference of these two quantities, each of which is of the order of magnitude of $\epsilon/100$ the net error will certainly be less than $\epsilon/100$. Parasitic e.m.fs. in dials IV and V produce somewhat less error in the measured voltage.

The effect of contact resistances in dial III may be estimated by noting that if the resistance of the two contacts is about 0.0004 ohm, the fall of voltage introduced is 0.8 μv. Consequently, the error introduced into the measurement of a voltage would be of the order of magnitude of 0.8/100 microvolt.

The potentiometer current is standardized by balancing the e.m.f. of the standard cell against the drop in a portion of the coils of dial I plus an added resistance, the range-changing switch being set at $\times 1$. Cells having e.m.fs. from 1.01800 to 1.01960 may be used. The range is changed from 1.9111 to 0.19111 by reducing the potentiometer current from 20 to 2 milliamp. by appropriate changes in the battery connections.

As the readings of the potentiometer depend on the difference of two *IR* drops, the currents to the right and left of *AB* must always be equal. When the switches on dials I and II are set at *CK*, and the contacts on dial III set below 5, the galvanometer is tapped into the circuit so that it spans equal resistances to the right and left of *AB*. Absence of a deflection shows that the two currents are equal. The current in the dials to the right of *AB* is subject to a slight adjustment by means of the rheostat R_2. The zero may be adjusted by the two rheostats R_3, which

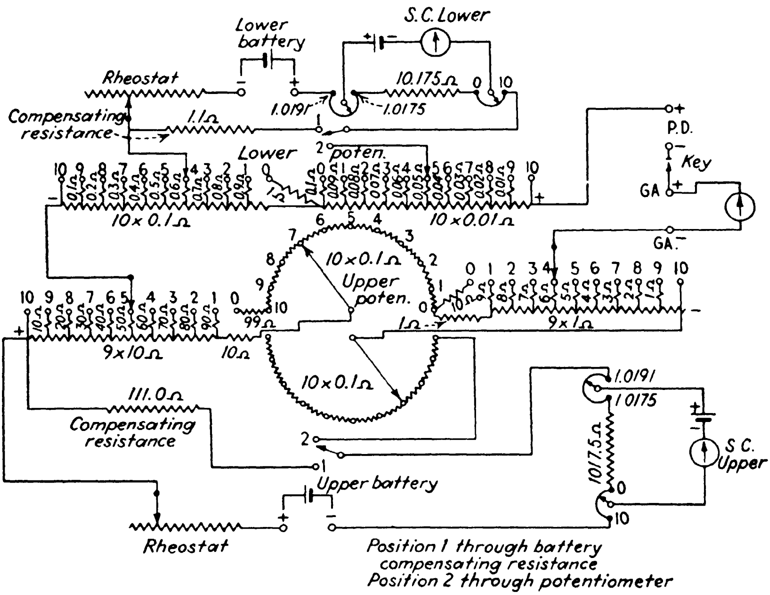


FIG. 186.—Diagram for double potentiometer. Eppley Laboratory. Range 0 to 111, 110 microvolts.

are so arranged in the same spindle that resistance may be transferred from one to the other without altering the total.

White, or Combination, Potentiometers.[26]—The construction of the "Thermokraftfrei" potentiometer (page 288) involves the accurate adjustment of the resistance for a large number of coils to particular and highly special values. A disadvantage of this instrument is that few laboratories possess the equipment necessary for the subsequent verification of the coils in case doubt arises as to their accuracy. This difficulty is avoided in the "combination potentiometer" of W. P. White, particularly designed for use with thermocouples in temperature measurements. The required range of such an instrument might be from 0 to about 0.1 volt, or 100,000 microvolts. In the White scheme, two complete potentiometers are employed: one adapted to the measurement of voltages up to a little over 100 microvolts in 1-microvolt steps; the other, to measure up to a little over 100,000 microvolts in 100-microvolt steps. The arrangement is such that the value of the unknown e.m.f. is given by the sum of the readings of the two potentiometers. Figure 186 is a simplified diagram of the connections of a highly developed combination potentiometer; the details of the design are due to F. W. Keyes of the Massachusetts Institute of Technology and the staff of the Eppley Laboratory. The lower potentiometer measures up to 110 microvolts in 1-microvolt steps; the upper potentiometer measures up to 111,000 microvolts in 100-microvolt steps. Each potentiometer has its own supply battery and standard cell adjustment. The standard current in the lower range instrument is 0.0001 amp., while that in the upper range instrument is 0.001 amp. The instrument is immersed in sulphur-free oil which is kept at a constant temperature by a thermostat. It is found best not to have the studs of the dials covered by the oil. Two unsaturated Weston standard cells are also immersed in the same oil bath and are thus kept under constant-temperature conditions; one is the working cell, the other being used as a check. In the actual instrument, the cells are transferred from one potentiometer to the other by a circular switch.

Parasitic e.m.fs. due to the manipulation of the upper potentiometer are not of consequence, for though they are directly in the galvanometer circuit, they die out before the final balance is effected by the lower instrument. Parasitic e.m.fs. in the lower potentiometer are not of consequence, for they are directly in series with the supply battery.

The resistance of a single contact in a first-class sliding switch may be of the order of 0.0001 ohm. As the current in the upper potentiometer is 0.001 amp., the voltage drop at the contact is of the order of 0.1 microvolt, subject to a variation of perhaps something like one-third this value. This drop is included between the contacts of dials 9×10 ohms and 9×1 ohm. Resistance variations at the contacts of the lower potentiometer are of negligible effect, as the resistance of the circuit is over 10,000 ohms.

Only copper and manganin are used in the electrical circuits, brass being excluded, and the galvanometer key is supplied with gold contacts. The circuit of the galvanometer is entirely of copper. It is impossible to detect any spurious e.m.fs. in the instrument. A dummy coil having the same resistance as the thermojunction may be substituted for it when testing for the presence of spurious thermal e.m.fs. The galvanometer, supplied by Leeds and Northrup Company, has a resistance of 45 ohms, a critical damping resistance of 150 ohms, and a period of 10.9 sec. The deflection, with a scale distance of 2 meters, is 12 mm. per microvolt. In order that the storage batteries, used as a source of supply, may be kept discharging at the normal rate when adjustments are being made, the compensating resistances are supplied.

Application of the Potentiometer to Alternating-current Measurements.[27]—The principle involved in the potentiometer was applied to alternating-current measurements many years ago, but at that time the necessary adjunct of a convenient phase-shifting device had not been developed by C. V. Drysdale, and recourse was had to an arrangement of two small dynamos on the same shaft, one of which could be displaced in phase with reference to the other. The arrangement, while satisfactory in many ways, is much less convenient and more costly than the phase-shafting transformer now used. The development of the alternating-current potentiometer as a distinct instrument is due to C. V. Drysdale, whose instrument is shown diagrammatically in Fig. 188.

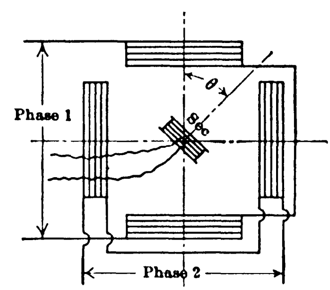


FIG. 187.—Theoretical diagram for Drysdale phase shifter.

In applying the potentiometer principle to alternating-current measurements, it is obvious that to balance two potential differences at every instant, they must be of the same frequency, the same wave form, and in the same time phase. The first two conditions demand that the potentiometer current be derived, through a suitable transformer, from the same source as the current to be measured. The third implies the use of some form of phase-shifting device. In addition, there must be some means of insuring that the potentiometer current, when it is alternating, is of such a magnitude that the r.m.s. value of the potential difference between the terminals of each of the coils of the instrument is given by the potentiometer scale. As the coils are wound noninductively, this may be accomplished if the potentiometer current is measured by an electrodynamometer of the astatic form. Such an instrument gives r.m.s. values and is equally accurate on direct and alternating-current circuits; therefore it is very readily calibrated.

The Drysdale Phase Shifter.—The principle involved in the Drysdale phase-shifting transformer may be illustrated by the following ideal arrangement of the apparatus (Fig. 187). The two sets of coils are of equal magnetic strength and may have their axes at right angles, in which case they are energized by currents in quadrature, as from the two phases of a two-phase circuit. The secondary is so mounted that it may be turned by hand and clamped in position; thus θ may be given any desired value. Let the coils in phase 2 be traversed by a current $I \sin \omega t$, and the coils in phase 1 by a current 90 deg. out of phase with the first, or $I \cos \omega t$. The rectangular components of the resultant field at the center are

$$x = H \sin \omega t;$$
$$y = H \cos \omega t.$$

The resultant is

$$R = \sqrt{x^2 + y^2} = H\sqrt{\sin^2 \omega t + \cos^2 \omega t} = H, \qquad \text{a constant.}$$

At any instant, the tangent of the inclination of this resultant to the vertical axis is x/y;

$$\tan \gamma = \frac{x}{y} = \tan \omega t;$$

Therefore

$$\gamma = \omega t.$$

Therefore the resultant field at the center is of constant magnitude and revolves with a constant angular velocity.

The flux threading the secondary coil is proportional to $\cos (\omega t - \theta)$, and the induced e.m.f. to $\sin (\omega t - \theta)$, so the time-phase displacement of the induced e.m.f. is equal to the angular displacement of the secondary from its zero position.

In the actual construction of the phase shifter, the four stationary coils are replaced by stator windings of the induction-motor type; the secondary is wound on an iron core. As built by some makers, the apparatus has the fault—serious in some methods of measurement—that change of phase is accompanied by an alteration of secondary e.m.f. This is obviated in Drysdale's own design by a proper arrangement of the windings. If the currents supplied to the phase shifter are not sinusoidal, the wave form in the secondary will depend on the value of θ. The device may be wound so as to operate on either two- or three-phase circuits, or it may be arranged to be operated on a single-phase circuit by means of a phase-splitting device. The polar phase shifter or its equivalent is a necessary adjunct of the alternating-current potentiometer.

The Drysdale-Tinsley Alternating-current Potentiometer.—The Drysdale-Tinsley alternating-current potentiometer is shown diagrammatically in Fig. 188. It consists of a regular Tinsley potentiometer, such as is used for direct-current work (included in the dotted rectangle), supplemented by the electrodynamometer necessary for the measurement of the potentiometer current; a selector switch, for quickly transferring the instrument from one set of terminals to another; a connection board, by which the potentiometer is attached to the outside circuits; a change-over switch, for quickly substituting alternating for direct current; and the phase shifter; this last device is best operated from a single-phase circuit by means of a phase-splitting condenser and resistance. This arrangement is best because any single-phase supply, of good wave form, can be used, and all doubt as to the exact quadrature of the two phases eliminated.

The phases can be adjusted to within 0°.1, the procedure being as follows: Join the 100-volt supply to the two terminals marked "1" on the phase-shifting transformer. The condenser and resistance, in series with the terminals marked "2" are also connected across the 100-volt supply. The secondary is then turned by the tangent screw until the pointer marked "Axis" is at 0 deg. on the dial. By means of the rheostat of the potentiometer, the current is adjusted until the dynamometer reads exactly 50. Then the tangent screw is turned until the axis pointer is at 45 deg. *leading*. If the dynamometer reads

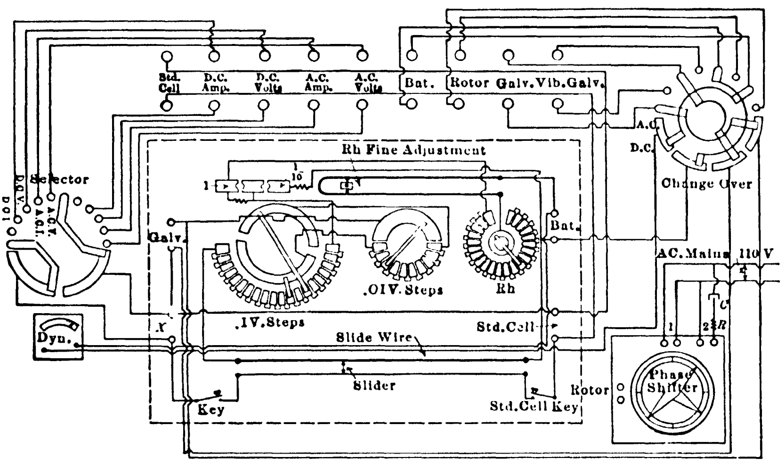


FIG. 188.—Diagram for Drysdale-Tinsley alternating- and direct-current potentiometer.

higher or lower than 50, the *capacitance* is altered until the reading is correct. The axis pointer is then turned to 90 deg., and the *resistance* altered until 50 is again registered. After this, the dynamometer should remain exactly at 50, while the secondary is turned through 180 deg. If it does not so remain, the process is repeated until it does remain at 50 for all positions of the secondary.

In measuring currents or e.m.fs., it is not necessary to split the phase with great accuracy, although it is more convenient to do so; but the utmost care must be taken in doing this when vector diagrams are being constructed or when an accurate knowledge of phase angles is required.

Above all things, it is necessary that the supply voltage and frequency remain perfectly steady.

After the adjustment of the phase splitter, the first step in using the instrument is to calibrate the electrodynamometer at the reading corresponding to the standard potentiometer current. To do this, the battery is used as a source, and the adjustment made as with the ordinary potentiometer. The Tinsley instrument has no separate standard-cell tap, so it is necessary to set the slides at the voltage of the cell and then adjust the rheostat; this process must be repeated whenever the standard potentiometer current is checked. When the galvanometer is in balance, the reading of the dynamometer is taken; if the instrument is not astatic, reversed readings must be taken, and the two results averaged. By means of the change-over switch, alternating is substituted for direct current, and the reading brought to the same value and held there. Then the graduation in volts on the potentiometer scale gives r.m.s. values of the potential difference. First the phase of the potentiometer current is roughly adjusted; then the unknown potential difference is balanced as nearly as possible by the potentiometer slides. The balance is then improved by shifting the phase of the potentiometer current and still further improved by resetting the slides; thus, by a process of double adjustment, the vibration galvanometer which is used as the detector is brought to rest. As the vibration galvanometer is a tuned instrument which responds freely to currents of only one frequency, *the periodicity of the supply current must be kept constant* if the sensitivity of the potentiometer is to be maintained. With any potentiometer, the deflection of the detector is dependent on the difference of the two potential differences that are being balanced. As the vibration galvanometer which is used as a detector is tuned to the fundamental frequency of the circuit, a balance indicates that the values of the fundamentals and not the mean square values of the potential differences are equal. If the wave forms are very bad, the vibration galvanometer may be forced to vibrate in other than its natural period, in which case an exact balance cannot be obtained. It is seen that sinusoidal currents are necessary for the successful operation of the alternating-current potentiometer.

The Drysdale potentiometer was originally devised for use at power frequencies 50 to 60 cycles per second. The present design is such that frequencies up to 2,000 cycles per second may be employed. By separating the phase-shifting transformer from the potentiometer and taking care to eliminate leakage and capacitance currents between the stator of the transformer and potentiometer by means of an isolating transformer with screened windings, the range may be still further increased. Designs have been prepared for an instrument to operate at 15,000 cycles per second. However, it is to be noted that the phase-shifting transformer consumes about 60 watts, so that in many cases the instrument cannot be employed. The Drysdale instrument is a polar potentiometer, as it gives the magnitude of a potential difference and its angular time-phase displacement from some other potential difference which is taken as a datum.

Campbell-Larsen Rectangular Potentiometer.[27]—In 1910, Larsen suggested a form of potentiometer in which the measured voltage is resolved into two perpendicular components. The fundamental idea is to balance the unknown potential difference by another which is made up of an adjustable IR drop plus an adjustable e.m.f. in quadrature with the IR drop. This e.m.f. is derived by induction from the potentiometer current I. As the quadrature component at power frequencies is $jm\omega I$, it is apparent that in its simple form such a potentiometer would be correct at only one frequency. This difficulty has been overcome by A. Campbell, whose arrangement is shown diagrammatically in Fig. 189A. Referring to the figure, AOC is a potentiometer which gives the in-phase

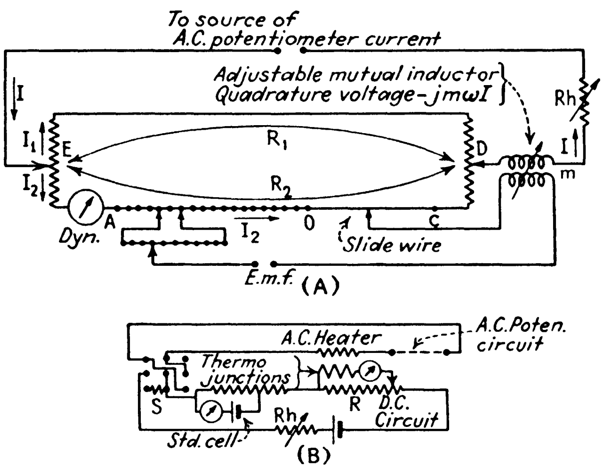


FIG. 189.—A, diagram for Campbell-Larsen alternating-current potentiometer. B Campbell alternating-current and direct-current comparator.

component. At A is a current-indicating device by aid of which the potentiometer current is kept at the proper value.

The quadrature component is furnished by an adjustable mutual inductor. If the primary of the mutual inductor were connected directly in series with the potentiometer AOC, its scale could be graduated to give the quadrature component at a particular frequency, for instance 60 cycles per second. If the potentiometer current is kept constant, and the frequency reduced to 50 cycles per second, the quadrature component would be only five-sixths of that marked on the scale. However, if it is so arranged that the current in the primary of the mutual inductor can be increased in the same proportion as the frequency is decreased, the potentiometer current in AOC being maintained constant, the graduation of the scale will still be correct. With this in view, Campbell employs what he calls a "loop shunt." The circuit $AOCDE$ is closed on itself and has a fixed resistance. At D and E are two sliding contacts through which the current enters and leaves the loop. At E, the current divides, part flowing through the in-phase potentiometer and part through the upper branch of the loop. These two parts of the current unite and flow through the primary of the quadrature mutual inductor. If R_1 is the resistance of the upper branch of the loop, and R_2 that of the lower branch,

$$R_1 + R_2 = \text{a constant,}$$

and

$$I = I_2\left(\frac{R_1 + R_2}{R_1}\right).$$

The quadrature component is

$$jm\omega I = jmI_2(R_1 + R_2)\left(\frac{\omega}{R_1}\right).$$

If the arrangement is such that R_1 can be set at a value proportional to the frequency, the integrity of the graduated scale can be maintained. The voltage between the e.m.f. terminals is $I_2(r + jmk)$, and both scales can be graduated to read in volts at all frequencies. The magnitude of the measured voltage is $\sqrt{V^2_{\text{inphase}} + V^2_{\text{quadrature}}}$.

Instead of using an electrodynamometer at A, an alternating- and direct-current comparator may be employed. Such an arrangement is shown diagrammatically in Fig. 189B. The circuit of the potentiometer contains a heater coil which acts on the hot junctions of a thermopile placed in series with a galvanometer and shunted around an adjustable resistance R in a direct-current circuit. Switches are arranged so that a direct current equal to the normal potentiometer current can be sent through R and the heater coil in series. The e.m.f. of the junction is proportional to the square of the current, while the voltage drop in R, which opposes it, is proportional to the current. By varying R, a balance can be obtained when the direct current has a value equal to the standard potentiometer current as shown by reference to the standard cell. After a balance has been obtained, the two circuits are separated, and the direct current maintained at the standard value. Any variation in the potentiometer current is shown by a deflection of the galvanometer and can be corrected.

Tinsley-Gall Alternating-current Potentiometer.[27]—This instrument is a combination of two similar potentiometers, the exciting currents being in quadrature. Figure 190A is a simplified diagram of the connections. The currents in the two potentiometers are maintained accurately in quadrature and of equal magnitude. The adjustment is obtained as follows: A direct current is sent through the "Direct Poten." and adjusted to the proper value by aid of the standard-cell circuit; the deflection of the electrodynamometer is then noted. Alternating current is then sent through the Direct Poten., and the deflection of the electrodynamometer reproduced. This potentiometer is then ready for use. To insure that the current in the quadrature potentiometer is properly adjusted in phase and magnitude, the primary of a mutual inductor is included in its circuit. The value of the mutual inductance is such that the voltage induced in its secondary is just 0.5 volt at normal frequency and current. The transfer switch is set in position 1, so that the voltage from the quadrature potentiometer is cut out. The "Direct Poten.," is set at exactly 0.5 volt. Adjustments of the magnitude and phase of the current in the "Quad. Poten." are then made until the vibration galvanometer is balanced. The transfer switch is then brought to position 2, which cuts out the secondary of the mutual inductor and puts the two potentiometers in series for the measurements of the rectangular components of the unknown voltage. The double-throw

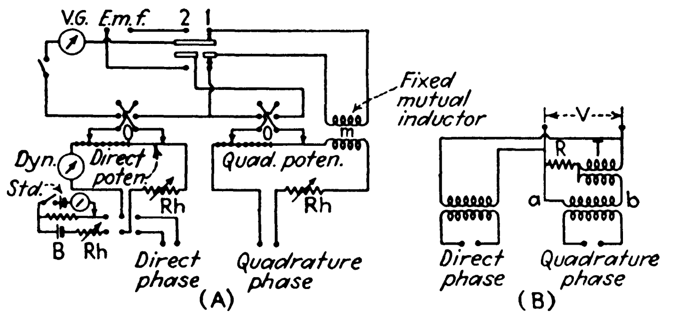


FIG. 190.—*A*, diagram for Tinsley-Gall alternating-current potentiometer. *B*, Gall phase splitter.

switches are for the purpose of making possible measurements in all four quadrants. It is sometimes convenient, for instance in iron testing, to excite the two potentiometers from a phase-shifting transformer with a two-phase secondary. In the actual instrument, it is possible to reduce the current in the Quad. Poten. to 0.1 of its normal value. This is convenient when small phase displacements are involved.

By completely screening each section of the potentiometer and all the potential circuits so that pickup between the current-carrying and the potential circuits is eliminated, using fully screened supply transformers and earthing one point of the circuit under test, it is possible to employ frequencies as high as 50,000 cycles per second.

The phase-shifting device commonly used with this potentiometer is shown diagrammatically in Fig. 190*B*. The potentiometer is connected to the supply circuit through the two isolating transformers. The primary of that in the quadrature phase is supplied from the transformer *T*, which is wound on an iron core having an air gap and which has in its primary circuit an adjustable resistance *R*. It is evident that the voltage supplied at *ab* is the sum of the *IR* drop in *R* and the induced voltage. By adjusting R, the voltage supplied to the isolating transformer may be brought into quadrature with the supply voltage V.

STANDARD CELLS[28]

In order to realize and maintain the international volt in such a manner that it will be a practical unit, easily applied for purposes of measurement, recourse must be had to some form of galvanic cell. Reference to the section on the legal definitions of the electrical units will show that in the Act of Congress approved July 12, 1894, the Clark normal cell was mentioned; and this act is still in force. At that time, this cell was the only one that had been carefully investigated and shown to have, in sufficient degree, the necessary characteristics of *reproducibility* and *permanence*. It has since been shown that the Weston normal cell is as reproducible and much more permanent; it possesses the practical advantage of having a small temperature coefficient—only about one-twentieth of that of the Clark cell.

The Weston normal cell was recommended by the London Conference on Electrical Units and Standards, 1908, for use in voltage and current measurements and was adopted by the Bureau of Standards as the working standard for the United States on Jan. 1, 1911.[28]

A standard cell should be free of local action, should exhibit no hysteresis effect between specified temperatures, should not appreciably alter its e.m.f. when it delivers a minute and momentary current, and after delivering such a current should quickly recover its original e.m.f.

The Clark Cell.—This cell, the invention of Latimer Clark, was described by him and recommended as a standard of e.m.f. in a paper read before the Royal Society in 1872.[28] The cell consisted of a zinc-rod electrode in a neutral, saturated zinc sulphate solution opposed to a mercury electrode covered with a paste consisting of mercurous sulphate and zinc sulphate in saturated zinc sulphate solution and containing finely divided mercury. The function of the mercurous sulphate is that of a depolarizer. In later forms of the Clark cell, the zinc was used in the form of an amalgam. This form of cell has been studied by Kahle and later, in this country, by Wolff and Waters. With skilled manipulation, it is reproducible to within a few parts in 100,000. The value of the e.m.f. at 15°C. as stated in the act of 1894 is 1.434 volts, but subsequent investigation showed that the value is really 1.4328 international volts.

Figure 191 shows a hermetically sealed Clark cell in which the zinc is used in the form of an amalgam, and Lord Rayleigh's H form of container is employed. The sealing prevents changes of concentration by evaporation. The H form of Clark cell has the disadvantage of short life, as the glass is likely to crack where the platinum electrode is fused in at the amalgam terminal,[28] owing to the alloying of the zinc amalgam with the platinum and the consequent increase in the diameter of the sealed-in wire. Also, there is likely to be a gradual accumulation of gas above the amalgam which increases the internal resistance and often eventually interrupts the electrical continuity of the cell. That these defects are serious in standards which must be maintained for long periods is shown by the fact that approximately 94 per cent of a lot of 128 cells set up at the National Bureau of Standards in 1906 had become useless by 1920 because of one or both of the foregoing defects. Such failures have caused the gradual disappearance of Clark cells from among the e.m.f. standards preserved at such institutions as the National Bureau of Standards, the National Physical Laboratory, and the Reichsanstalt. The saturated cadmium cell is now universally employed.

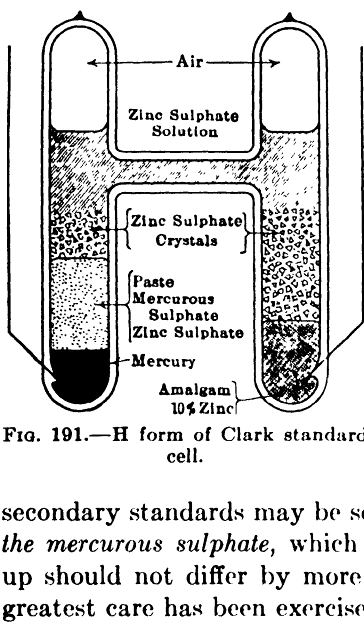


FIG. 191.—H form of Clark standard cell.

Materials Used in Standard Cells. In order that the e.m.f. of a primary standard cell may accord with the stated value, great care must be exercised in the preparation of the materials. The processes have been carefully worked out, particularly by Kahle and later by Wolff and Waters and still more recently by Eppley.[28]

Many of the impurities ordinarily found in the chemicals employed have but a small effect on the e.m.f., so that secondary standards may be set up with the best of c.p. materials, *except the mercurous sulphate*, which must be specially prepared. Cells so set up should not differ by more than 0.01 per cent from those where the greatest care has been exercised in the preparation of the materials.

The mercury should be purified by letting it run through a column of nitric acid in a very fine spray. After this, it should be distilled twice under reduced pressure in a stream of air. The oxides of zinc, cadmium, bismuth, tin, copper, lead, etc., so formed float on the surface and are removed by passing the mercury through a pinhole in a filter paper. The gold, silver, and platinum remain in the mercury.

The *purity of the mercurous sulphate is of prime importance.* Lack of purity of this salt has been considered the chief cause of variations in the e.m.f. The usual impurities are basic mercurous sulphate, basic mercuric sulphate, traces of mercuric nitrate, etc., according to the method of preparation. This salt is subject to the action of light and must be prepared in subdued light and preserved in the dark under dilute (1:6) sulphuric acid. There are a number of methods of preparation, all of which give satisfactory results, the best one apparently being an electrolytic method devised by Wolff and Waters.[28] The mercury anode is at the bottom of a tall jar; the cathode is a piece of platinum foil suspended above the anode; and the electrolyte is dilute sulphuric acid. The mercury is violently stirred during the passage of the current and for some time after the circuit is broken. To insure uniform size of grain, the mercurous sulphate should be digested upon the steam bath under 1 to 6 sulphuric acid for about four days. It appears that new cells attain their equilibriums more quickly if the mercurous sulphate paste is slightly acid. The cadmium sulphate must be freed from the impurities zinc, lead, ferrous and ferric iron, and occasionally nickel and manganese. The manganese impurity has been studied especially by Eppley. Its effect is to give the cell an abnormally high and inconstant e.m.f.

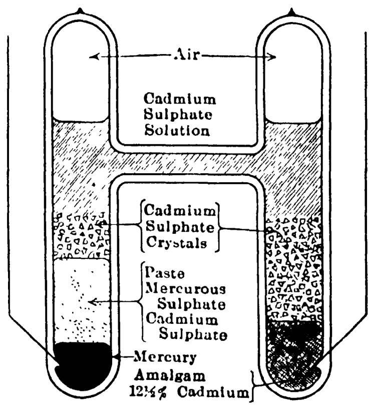


FIG. 192.—Weston normal standard cell.

The Cadmium, or Weston, Cell.—As early as 1884, Czapski called attention to the low temperature coefficients of cells with cadmium electrodes and cadmium chloride electrolytes. However, the e.m.fs. that he obtained were not constant, and no advantage was taken of his observations. In 1892, Edward Weston produced a cadmium cell which had a very small temperature coefficient and an e.m.f. so constant and so reproducible that the cell could be used as a standard of e.m.f.

The cadmium cell is composed of cadmium in cadmium sulphate solution opposed to mercury in mercurous sulphate paste. The great advantages of this form of cell are its long life and very small temperature coefficient—the last by reason of the fact that the solubility of the cadmium salt is only slightly influenced by the temperature—consequently, the changes in concentration of the solution are small. Also, the temperature effects on the two limbs of the cell tend toward compensation. For the + limb, the effect is +0.00031 volt per degree centigrade, while the effect on the − limb is −0.00035 volt per degree, the net effect being 0.00004 volt per degree. The cadmium is used in the form of an amalgam made by dissolving with the aid of heat 1 part of Kahlbaum's best cadmium in seven parts of mercury. If necessary, the cadmium may be purified by distillation at reduced pressure. The normal form of this cell has been carefully studied, especially at the Reichsanstalt, by Jaegar, Kahle, Wachsmuth, and Lindeck; at the National

Physical Laboratory by F. E. Smith; at the National Bureau of Standards by Wolff and Waters; and at the Eppley Laboratory by Eppley and Vosburgh.

For the temperature range of from 15 to 40°C., amalgams containing from 8 to 13 per cent of cadmium are satisfactory. Irregularities in the behavior of the cell are noted at about 15°C.

The e.m.f. at 20°C. of the normal cadmium cell containing saturated solution is, when derived from the international ampere and the inter-

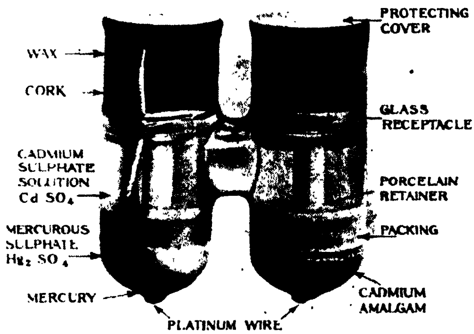


FIG. 193.—Weston secondary standard cell; a working standard.

national ohm, 1.01830 international volts. Its e.m.f. at any temperature t is

$$E_t = E_{20} - 0.0000406[t - 20] - 0.00000095[t - 20]^2 + 0.000000001[t - 20]^3.$$

The cells adjust themselves to changes of temperature very slowly. At the National Bureau of Standards, they are kept in "constant-temperature baths" maintained within 0.01°C. of the specified temperature. This avoids appreciable hysteresis effects due to changes of temperature. The advantages of the cadmium cell are low-temperature coefficient, small lag error, long life due to freedom from cracking at the negative electrode, and continuity of action due to the fact that gas is not formed at the negative electrode.

The standard of e.m.f. is maintained the world over by use of normal cadmium cells. The difference in the voltage of this type of cell as determined at the National Physical Laboratory in England and at the National Bureau of Standards in the United States, using radically different procedures, is only ten-millionths volt.

Weston Secondary Standard Cell.—The convenient portable form of cadmium cell shown in Fig. 193 was originated by Edward Weston. The solution is saturated at or about 4°C. and therefore does not contain an excess of cadmium sulphate crystals. This type of cell is to be used between the temperatures of 15 and 35°C. The temperature coefficient is much smaller than that of the normal cell and is negligible for any ordinary measurements. For cells of the Weston Instrument Corporation,

$$E_t = E_{20} - 0.000005(t - 20°) \quad \text{approx.}$$

The resistance depends on the size of the cell and may be 100 or 200 ohms. The extreme values of the e.m.f. from a large number of carefully prepared cells were found to be 1.01865 and 1.01910, making the extreme variation 0.00045 volt.

Each secondary Weston cell is accompanied by a certificate stating its e.m.f. The Weston unsaturated cell is the best form of working standard, being remarkably permanent.

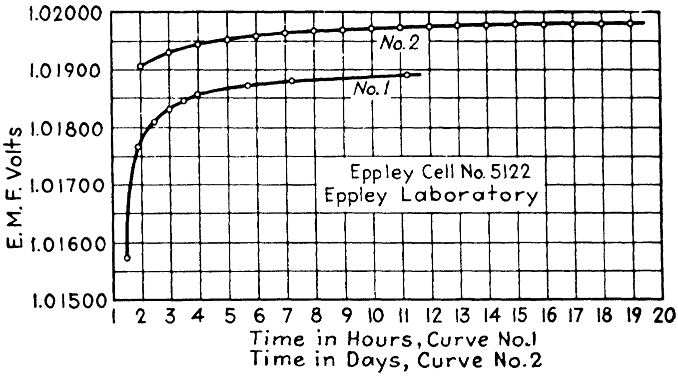


FIG. 193*A*.—Curves showing time required and values during this time for an unsaturated cadmium cell to recuperate after it was short-circuited for 30 min.

Curve No. 1 voltage *vs.* hours 1st day
Curve No. 2 voltage *vs.* days up to 19 days
Total time to recuperate 36 days
Value before short circuit 1.019875 volts
Value after recuperation 1.019800 volts

Difference 0.000075 volt

Precautions in Using Standard Cells.—No appreciable current can be taken from a standard cell without alteration of its e.m.f. It is found that, after delivering current, the cell gradually recovers its e.m.f. The recovery curve for an unsaturated cadmium cell is shown in Fig. 193*A*. Cells that have been short-circuited should be viewed with suspicion.

Unsaturated cells that are in general laboratory use should be checked against each other frequently and calibrated at the National Bureau of Standards once a year. Temperature changes and irregularities of temperature between various parts of the cell are to be avoided.

Standard cells are used only in compensation methods and must always be protected by a key, and the manipulation must be such that the key is closed for but an instant.

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CHAPTER VI

THE MEASUREMENT OF POWER

In direct-current circuits, the measurement of power, or rate of expenditure of electrical energy, is best accomplished by the use of calibrated ammeters and voltmeters. In alternating-current circuits where both the e.m.f. (or potential difference) and the current are varying from instant to instant, passing through a definite cycle of values, the "instantaneous power," or the power being given to the circuit at a

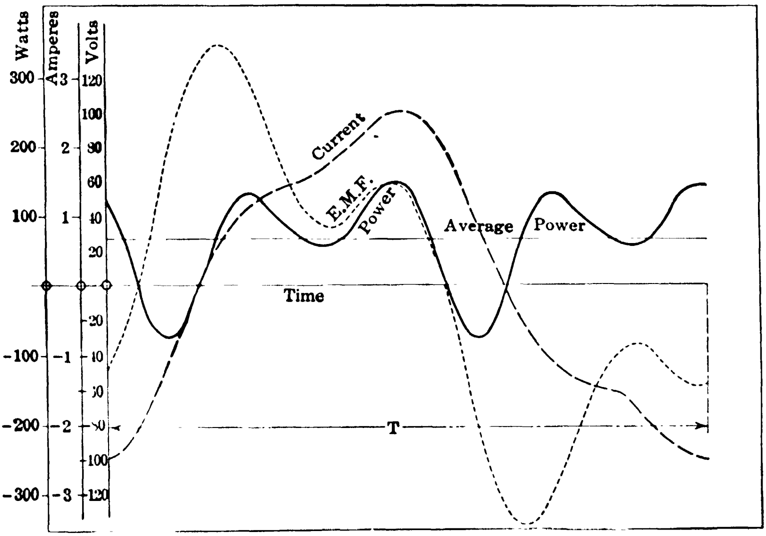


FIG. 194.—Illustrating instantaneous and average power.

particular instant, is vi. The power to be measured is the average value of vi during the cycle

$$P = \frac{1}{T}\int_0^T vidt.$$

To illustrate, let the curves of voltage, current, and vi be as shown in Fig. 194.

The net area under the power curve is proportional to $\int_0^T vidt$, which is the energy. The time of a complete cycle T is proportional to the length of the base line; $\frac{1}{T}\int_0^T vidt$ is therefore, to the proper scale, the average ordinate of the power curve.

In general, when dealing with alternating-current circuits, it is necessary to have methods of evaluating $\frac{1}{T}\int_0^T vidt$. The expression for the power must be left in this general form, for in practical measurements it is not permissible to make any assumptions as to the forms of the voltage and current waves.

For general purposes, the simplest and most satisfactory method of measuring the power in an alternating-current circuit is by the use of the electrodynamic wattmeter. Other methods will be discussed, but their usefulness is restricted to particular cases.

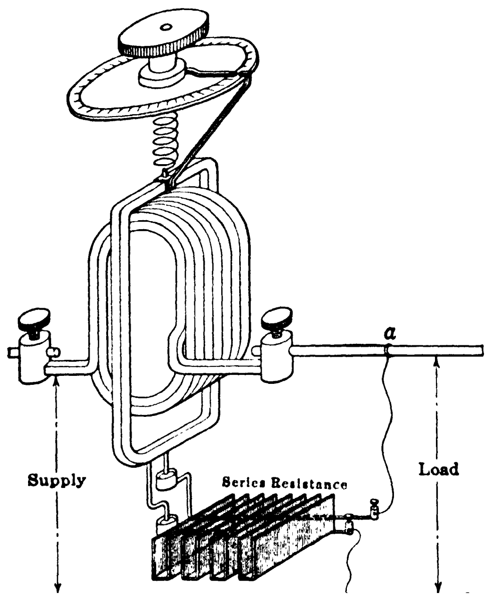


FIG. 195.—Showing connections of wattmeter.

The Electrodynamic Wattmeter.—It has been shown previously that any electrodynamometer measures the mean product of the currents flowing in the fixed and movable coils; it evaluates

$$\frac{1}{T}\int^T i_F i_M dt.$$

If a Siemens dynamometer is connected to a circuit in the manner shown in Fig. 195, the fixed coils being put in series with the load, while the movable coil, in series with a suitable nonreactive resistance, is placed across the line, the current in the fixed coil i_F is the instantaneous load current, and the current in the movable coil i_M is proportional to the instantaneous voltage.

If R is the total resistance of the movable coil circuit, $i_M = v/R$, the inductance of the coil being considered negligible.

In the case of the Siemens dynamometer with a uniformly graduated scale,

$$KD = \frac{1}{T}\int_0^T i_F i_M dt.$$

K is a constant depending on the windings and on the strength of the controlling spring, and D is the angle through which it is necessary to twist the spring in order to bring the coil back to its initial position.

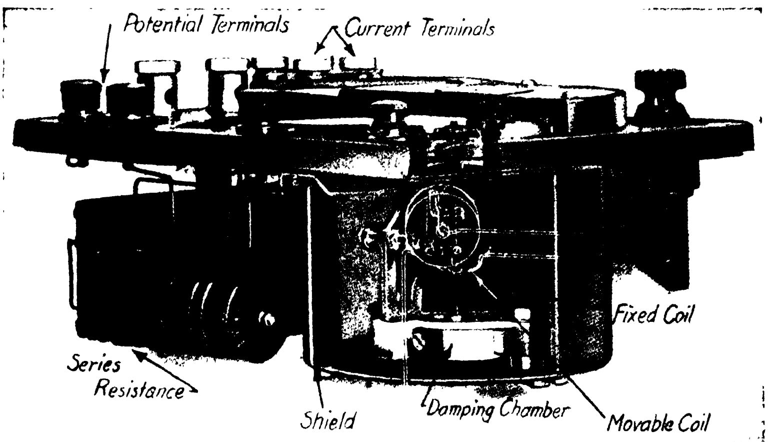


FIG. 196.—Phantom view showing construction of Weston portable wattmeter.

Substituting the foregoing values for the currents in the fixed and movable coils,

$$KD = \frac{1}{R}\cdot\frac{1}{T}\int_0^T vidt,$$

or

$$RKD = P. \qquad (1)$$

If the spring is perfect, and the scale is uniform, the power is proportional to the deflection.

The influence of reactance in the potential circuit will be discussed later.

The relation $P = RKD$ will hold for any wattmeter where the movable coil is always brought back to a position where there is no mutual induction between the fixed and movable coils. When the movable coil is allowed to deflect against the action of the spring, K is no longer a constant, for it depends on the geometry of the system of coils, that is, on the angle between the coils, and this angle varies with every change of load. In consequence, the scales of portable wattmeters are generally nonuniform. Proper proportioning of the relative diameters of the fixed and movable coils will do much toward correcting this (see page 69).

Figure 196 shows, in section, one form of portable wattmeter which is in common use.

Heating Losses in Wattmeters.—It is important to note just what a wattmeter measures. *P* in formula (1) is the power corresponding to the current that flows through the fixed coils, that power being expended between the two points at which the potential terminals are attached to the circuit. *P* therefore must include the heating loss either in the current coils or in the potential circuit, depending on whether the point *a* (Fig. 195) is on the supply or the load side of the current coils.

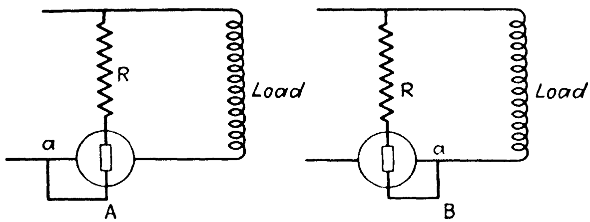


Fig. 197.—Showing wattmeter connections.

Where the total amount of power is small, it may be necessary to correct the readings for the loss in the instrument itself. The two possible methods of connection are shown in Fig. 197.

With connection *A*, the indication of the wattmeter includes the I^2R loss in the current coils; with connection *B*, the loss in the entire potential circuit is included. When a small output is measured, and the connections of Fig. 197*A* are used, the loss in the potential coil circuit must be added to the wattmeter reading. When *a* is on the load side of the current coil, the loss in the current coil must be added.

Compensation for Energy Loss in the Potential Circuit.[1]—In some cases, it is necessary to measure small amounts of power—a few watts—and in this case the loss in the potential circuit of the wattmeter may be a large percentage of the power to be determined. To avoid the necessity of making a correction, instruments are sometimes so designed that this error is compensated. In this case, connection *B* is used. The current that flows through the fixed coil is made up of two components: one due to the load; the other, to the current in the movable-coil circuit. The effect of this last must be compensated, for the magnetic field in which the potential coil moves is due to both components. If a second winding, coincident at all points with the regular current coil, could be put on the bobbin carrying the fixed coil and be connected into the potential circuit so that its effect opposed that of the main-current coil, the compensation would be exact; for instance, if the load circuit were broken, the net ampere-turns acting on the movable coil would be zero, and there would be no deflection. As the two coils cannot be made coincident, care must be exercised in placing the compensating turns so that when the load circuit is broken, the net magnetic field at the movable coil will be zero for all positions of the movable coil. Otherwise, the degree of compensation will vary with the scale reading. There is a small transformer action due to the mutual inductance of the two windings on the fixed coil; but in commercial measurements, at the usual voltages, this does not cause an appreciable error.

This compensation may be extended so that the power lost in a voltmeter connected directly across the load may be allowed for, a second compensating winding being added and connected in series with the voltmeter; allowance must be made for the changed resistance of the voltmeter circuit.

Grouping of Instruments.—The heating losses have a special bearing on the grouping of the instruments when the voltage, current, power, and power factor of a *small* reactive load are to be determined.

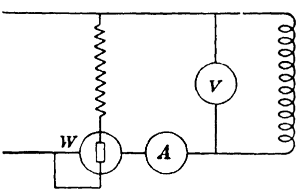


FIG. 198.—Showing grouping of instruments for measuring, power, current, and voltage.

Electrical apparatus is generally sold to be operated at a definite voltage, so the voltmeter is placed across the load as in Fig. 198.

As shown, the wattmeter measures, in addition to the load, the power in its own current coils, in the ammeter, and in the voltmeter. The ammeter gives the vector sum of the currents in the load and in the voltmeter.

Errors Due to Local Fields.—In industrial testing, it is not safe to assume that a wattmeter will be uninfluenced by local magnetic fields. In the ordinary portable instruments, this error, if present, will depend on the deflection. Its presence or absence may be made obvious by connecting the potential coil alone to the circuit and observing the deflection when the instrument is turned to several different azimuths. If the error is present, and it is not feasible to change the location of the observing station, the instrument may be turned to the position where the pointer is undeflected, that is, where the movable coil is threaded by the maximum number of lines of force, due to the stray field. When the proper position has been found, the direction of the pointer may be noted. If in the subsequent use of the wattmeter the pointer is always kept in approximately this direction, the body of the instrument being turned as the load is varied, the error will be rendered negligible, provided the direction of the local field does not change.

Direct-current stray fields have no influence on the readings when alternating currents are being dealt with, and alternating stray fields, to have any effect, must be of the frequency of the current under measurement.

As high-capacity instruments have comparatively few turns on the fixed coil, special care must be taken that there are no loops in the leads near the instrument and that the current leads occupy the same position with respect to the instrument during its calibration and subsequent use.

To obviate stray field errors, the working parts of the instrument are now surrounded by a soft-iron shield built up of stampings (see Fig. 196).

Voltage between Current and Potential Coils.—The movable coil of a wattmeter, being in series with a large resistance, may inadvertently be connected to the circuit so that practically the full-line potential exists between the current and the potential coils. This may give rise to errors due to the electrostatic attraction between these two coils; also, there is danger that the insulation between the coils may be punctured. The connections should be so made that the current and potential coils are, as nearly as possible, at the same potential. If, when so connected, the deflection is in the wrong direction, the current coil should be reversed. The proper position of a multiplier, when one is used, is governed by the same consideration.

It is well to mark the terminals of a wattmeter once for all so that there can be no mistake in making the connections. Such a marking also obviates any question as to the algebraic sign of the readings when measuring polyphase power.

When instrument transformers are used, electrostatic troubles may be avoided if the two coils of the wattmeter are connected by a piece of very fine fuse wire.

The Effect of Reactance in the Potential Circuit.[2]—It has been assumed that the potential circuit is nonreactive; this can never be strictly true, since it must contain the movable coil. In commercial instruments, when used at ordinary frequencies and power factors, the resistance of the potential circuit is so high in comparison with its reactance that the effect of the inductance is entirely negligible. In special investigations, however, cases arise where the utmost care must be exercised if reliable results are to be obtained. The presence of reactance has two effects: It cuts down to a certain extent the current in the potential coil and shifts its phase by an amount dependent on the frequency. Thus the mean product of the currents in the fixed and movable coils is altered from its proper value. If *sinusoidal currents* are assumed, the magnitude of the error thus introduced may readily be computed.

Symbols Used in the Discussion of the Theory of the Wattmeter

Maximum values of currents and voltages are denoted by large letters; instantaneous values, by small letters. Referring to Fig. 199,

$ab = V$ = voltage at terminals of potential-coil circuit.

I_P = current in potential coil.

R_P = resistance of potential-coil circuit.

$ac = I_P R_P$ = ohmic drop in potential-coil circuit.
I_L = current in load.
R_L = resistance of load.
R_C = resistance of current coils.
I_C = current in current coils.
$ad = I_L(R_L + R_C)$ = ohmic drop in load circuit between potential terminals.
θ_P = phase displacement in potential-coil circuit.
θ_L = phase displacement in load circuit between potential terminals.
L_P = inductance of potential-coil circuit.
L_L = inductance of load.
L_C = inductance of current coils.
Z_L = impedance of load.
Z_P = impedance of potential-coil circuit.
Z_C = impedance of current coils.
ω = 2π times frequency.
m = mutual inductance of fixed- and movable-coil circuits.
P_L = power in load.
P_W = power as read from the wattmeter.
H_P = heating loss in potential circuit.
H_C = heating loss in current coils.
K = constant of dynamometer.
D = deflection of instrument.

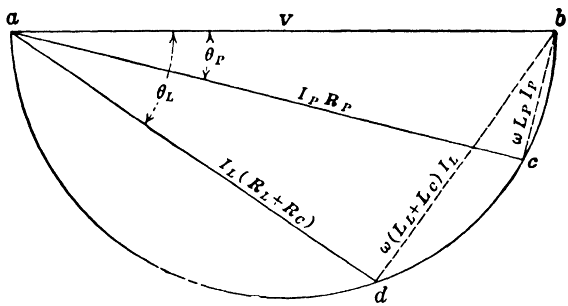


FIG. 199.—Pertaining to effect of inductance in the potential circuit of a wattmeter.

Consider Case A (Fig. 197), where the current coils are traversed by the load current only, and the power measured includes the heating loss in the current coils.

On account of the power factor of the reactive load circuit, the current I_L will lag behind V by an angle θ_L; and in consequence of the reactance of the potential-coil circuit, the current in it will be out of phase with V by an angle θ_P and will be altered from the value V/R_P to $(V/R_P) \cos \theta_P$.

The deflection of the instrument is proportional to the mean product of the instantaneous values of I_P and I_L; therefore,

$$KD = \frac{1}{T}\int_0^T i_L i_P dt = \frac{I_L I_P}{2} \cos (\theta_L - \theta_P).$$

So the apparent power or the scale reading is

$$R_P K D = I_L \frac{V \cos \theta_P}{2} \cos (\theta_L - \theta_P) = P_W. \tag{2}$$

The true power, that is, the power that would be read from the scale if there were no reactance in the potential circuit, is given by

$$P_L + H_C = \frac{I_L V}{2} \cos \theta_L. \tag{3}$$

A correction to be subtracted from the apparent power in order to obtain the true power may be determined, for, by (2),

$$P_W = \frac{I_L V \cos \theta_P}{2} \cos (\theta_L - \theta_P) = \frac{I_L V}{2}$$
$$(\cos^2 \theta_P \cos \theta_L + \cos \theta_P \sin \theta_P \sin \theta_L).$$

Therefore

$$P_L + H_C = \frac{P_W}{\cos^2 \theta_P} - \frac{I_L V}{2} \tan \theta_P \sin \theta_L.$$

In portable instruments, the angle θ_P is a very small fraction of a degree. Using the values of the current and voltage given by indicating instruments, instead of maximum values, and remembering that ordinarily θ_P is very small,

$$P_L + H_C = P_W - I_L V \tan \theta_P \sin \theta_L = P_W - I_L V \left(\frac{L_P \omega}{R_P}\right) \sin \theta_L. \tag{4}$$

The effect of the phase displacement in the potential circuit evidently depends on the frequency and on the characteristics of the load as well as on the wattmeter itself. It increases as the power factor of the load is decreased, and at extremely low power factors great precautions must be taken in order to secure accurate results.

If the resistance of the potential circuit is exceedingly high, there may be capacitance effects in the series resistance. Some instruments are so designed that a part of the potential circuit is coiled on a metal bobbin; and although this is split lengthwise, it is not entirely free from eddy currents. Both the capacitance and the eddy currents modify the phase displacement.

If the load current is leading, and the power factor very low, the wattmeter readings tend toward zero, and at some particular value of the power factor the reading will reverse.*

The ratio of the true to the apparent power is

$$\frac{P_L + H_C}{P_W} = F = \frac{\cos \theta_L}{\cos \theta_P \cos (\theta_L - \theta_P)}, \tag{5}$$

* For an interesting case in point, see *Jour. Inst. Elec. Eng.*, vol. 30, 1901, p. 467.

and the total power is obtained by multiplying the apparent power by the correction factor F.

The trigonometrical form of this expression may be changed so that the tangents rather than the cosines of the angles may be used; then

$$F = \frac{1 + \tan^2 \theta_P}{1 + \tan \theta_P \tan \theta_L}. \tag{6}$$

This is the usual form of the correction factor.

If there are no modifying causes such as capacitance or eddy-current effects, $\tan \theta_P$ may be expressed in terms of the inductance and resistance of the potential circuit. Then

$$F = \frac{1 + (L_P\omega/R_P)^2}{1 + (L_P\omega/R_P)(\tan \theta_L)}. \tag{7}$$

It is preferable to use the correction term of Eq. (4) rather than the factor given in Eq. (7), for the latter becomes practically indeterminate at low power factors.

Compensation for the Inductance of the Potential Circuit.[3]—It naturally suggests itself that the effect of inductance in the potential circuit may be annulled by the use of capacitance. Simple tuning of the

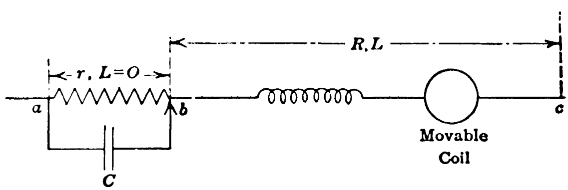


FIG. 200.—Showing method of compensating the inductance of the potential circuit of a wattmeter.

circuit is manifestly inadequate, for the arrangement must be such as to be practically independent of the frequency, in order that there may be no errors when dealing with nonsinusoidal waves.

The arrangement used for this purpose by Abraham and Rosa[3] is a condenser inserted in series with the movable coil and shunted by a noninductive resistance. Such an arrangement may be adjusted so that, for all practical purposes, the net reactance of the circuit is reduced to zero.

The connections of the potential circuit then become those shown in Fig. 200.

The impedance between a and b is

$$Z_{ab} = \frac{1}{\frac{1}{r} + j\omega C} = \frac{r}{1 + j\omega Cr} = \frac{r - j\omega Cr^2}{1 + \omega^2 C^2 r^2}.$$

The impedance between b and c is

$$Z_{bc} = R + j\omega L.$$

Therefore the total impedance is

$$Z_{ac} = R + j\omega L + \frac{r - j\omega Cr^2}{1 + \omega^2 C^2 r^2} = \left(R + \frac{r}{1 + \omega^2 C^2 r^2}\right) + j\omega\left(L - \frac{Cr^2}{1 + \omega^2 C^2 r^2}\right).$$

If $\omega^2 C^2 r^2$ can be neglected in comparison with unity,

$$Z_{ac} = R + r + j\omega(L - Cr^2) \qquad \text{approx.} \tag{8}$$

If r is adjusted so that

$$L = Cr^2,$$

then

$$Z_{ac} = R + r,$$

which is the resistance of the circuit as measured by a bridge. As long as the term $\omega^2 C^2 r^2$ is negligible, the compensation will not be affected by the frequency. As an example, consider the following data:

$$L = 0.000328 \text{ henry.}$$
$$R = 20 \text{ ohms.}$$
$$C = 1\ \mu\text{f.}$$
$$r = \sqrt{\frac{L}{C}} = 18.11 \text{ ohms.}$$

Using these values, the impedances and phase displacements at 60 and 1,000 cycles per second are:

$$Z_{60} = 38.109 + j0.000016.$$
$$\theta_{60} = 0°.000024.$$
$$Z_{1,000} = 37.878 + j0.0265.$$
$$\theta_{1,000} = 0°.040.$$

If no compensation is applied, the values are

$$Z_{60} = 38.11 + j0.124.$$
$$\theta_{60} = 0°.186.$$
$$Z_{1,000} = 38.11 + j2.06.$$
$$\theta_{1,000} = 3°.1.$$

To determine whether compensation is needed and to make the necessary adjustments, the potential circuit is short-circuited on itself, and a large alternating current sent through the fixed coil. The instrument will remain undeflected:

1. If there is no mutual inductance between the two coils.

2. If the effective inductance of the movable-coil circuit is zero, for then the currents in the movable and fixed coils will be in quadrature.

Having made sure that the mutual inductance is not zero, the resistance r may be adjusted until the deflection disappears. The adjustment is then complete, provided there are no eddy-current effects in the fixed coil or in the frame of the instrument (see page 324). These effects are discussed in the original paper by Rosa.

Effect of Mutual Inductance between Fixed- and Movable-coil Circuits.[4]—In the previous discussion, no reference has been made to the possible effects of mutual inductance between the current and potential circuits. In commercial instruments, these effects are so small as to be negligible. They might, however, be worthy of consideration in special low-voltage wattmeters where an attempt is made to attain a maximum of sensitivity, especially if the frequency is high.

When the instrument is read by a torsion head, the mutual inductance can be made zero; but if the movable coil is allowed to deflect, the mutual

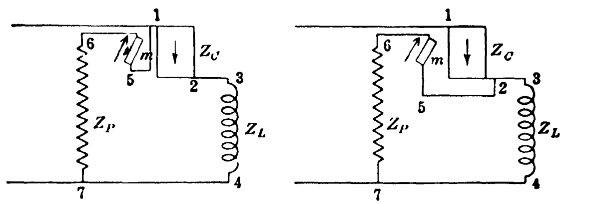


FIG. 201.—Pertaining to effect of mutual induction in wattmeter.

inductance will have an effect dependent on the reading. In instruments where the axis of the movable coil lies in the medial plane of the fixed coil, the induced e.m.f. due to mutual induction becomes zero and changes sign when the coil deflects through the position of zero mutual inductance, which is about mid-scale.

Take the connections shown in Fig. 201*A*, which are the same as those assumed in Fig. 199. Equating the values of the potential difference between 1 and 7 reckoned via the load and via the potential circuit gives, when the deflection is above mid-scale and the mutual and self-induction fluxes are in the same general direction,

$$I_{23}(Z_L + Z_C) + jm\omega I_{56} = I_{56}Z_P + jm\omega I_{23}$$

$$I_{56} = I_{23}\left[\frac{(R_L + R_C) + j\omega(L_L + L_C - m)}{R_P + j\omega(L_P - m)}\right]$$

$$I_{56} = I_{23}\left[\frac{R_P(R_L + R_C) + \omega^2(L_L + L_C - m)(L_P - m)}{R_P^2 + \omega^2(L_P - m)^2}\right] + j\omega I_{23}\left[\ \right].$$

The current in the potential coil is seen to consist of two components: one in phase and one in quadrature with the current in the fixed coils I_{23}, which is also the load current I_L.

The turning moment and therefore the deflection are proportional to the mean product of the currents I_{23} and I_{56}, and this mean product multiplied by the resistance of the potential circuit is the reading of the wattmeter. Using effective values, power by wattmeter is

$$R_P KD = I_L^2\left[\frac{R_P^2(R_L + R_C) + \omega^2 R_P(L_L + L_C - m)(L_P - m)}{R_P^2 + \omega^2(L_P - m)^2}\right].$$

The instrument should give the power in the load circuit between the points 1 and 4, that is, the power in the load, plus the heating in the current coils.

$$P_L + H_C = I_L^2(R_L + R_C).$$

Therefore

$$P_L = P_W\left[\frac{R_P^2 + \omega^2(L_P - m)^2}{R_P^2}\right] - \frac{I_L^2\omega^2(L_L + L_C - m)(L_P - m)}{R_P} - H_C. \quad (9)$$

Eddy-current Errors.[4]—Another source of error, one not amenable to calculation, may be found in instruments of faulty design. It is that due to currents induced in masses of metal, such as the frame of the instrument or, in instruments of large current capacity, in the current coils themselves, for these coils must be made very massive in order to give the requisite carrying capacity.

This error should be reduced to a minimum by the design of the instrument. In high-capacity instruments, it will be necessary to wind the current coil with a stranded conductor or its equivalent, the strands being insulated from one another. Great care must be taken in arranging them. The average position of each strand in the cross section should be the same as that of every other strand; otherwise the heating of the coil may change the current distribution and therefore the calibration of the instrument.

In laboratory instruments intended for heavy currents, the current coil may be made of a small and thin copper tube through which there is a rapid circulation of water.

To detect the presence of eddy-current errors in research instruments where the inductance of the potential-coil circuit is compensated, the zero reading is brought to a particular point on the scale, and the compensation for the inductance of the moving-coil circuit made as shown on page 321. The zero is then changed by use of the torsion head, and the compensation tested at various points along the scale. If the eddy-current error is absent, the adjustment of the potential circuit for zero effective inductance will be the same at all points.

Self-heating Errors.[5]—When electrodynamic wattmeters are left in circuit under load, inequalities of temperature develop within the instrument which introduce errors owing to the weakening of the spring and the increase of the resistance of the potential circuit. The instrument may be compensated by properly locating copper-wound heating coils (see page 227). In order to avoid errors due to mutual inductance and consequent false phase relations, it is best to employ in the wattmeter a simple series circuit. Consequently, the circuit shown on page 227 is not recommended for this instrument.

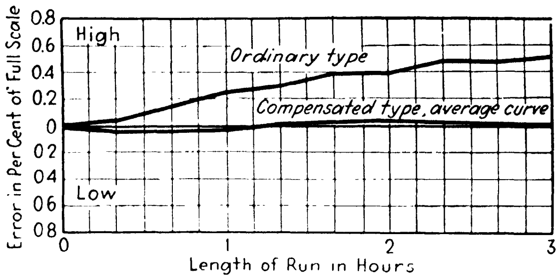


FIG. 202.—Electrodynamic wattmeters. Variation of instrument indication with time under maintained load conditions.

Figure 202 shows what may be accomplished by placing a small copper-wound resistor in series with the movable coil and near the spring.[5]

OTHER METHODS OF MEASURING POWER

Thermal Wattmeter.[6]—On account of the reactances of its coils, the electrodynamic wattmeter is manifestly unsuited to the measurement of power at high frequencies. For this reason, the thermal wattmeter has been developed. One scheme of connections is shown in Fig. 203.

Referring to the figure, two vacuum thermocouples connected in opposition are employed. The current through the indicator G, and therefore the deflection, is proportional to the difference of the direct-current e.m.fs. set up by the junctions, that is, to

$$K_2\frac{1}{T}\int_0^T i_2^2dt - K_1\frac{1}{T}\int_0^T i_1^2dt,$$

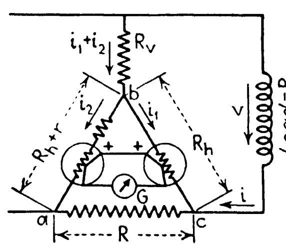


FIG. 203.—Diagram for thermal wattmeter.

where K_1 and K_2 are the constants of the junctions 1 and 2. R_V and R are shielded, nonreactive resistances. R_h is the resistance of the heater circuit of junction 1 between b and c; $R_h + r$ is the resistance of the heater circuit of junction 2 between b and a, r being a small, variable, nonreactive resistance.

By Kirchhoff's rules,

$$i_1(R_h + R_V) + i_2R_V - v = 0.$$
$$i_1(R_h + R) - i_2(R_h + r) + iR = 0.$$

Therefore

$$i_1 = \frac{v(R_h + r) - iRR_V}{(R_h + r)(R_h + R_V) + (R_h + R)R_V}. \tag{10}$$

$$i_2 = i_1\left(\frac{R_h + R}{R_h + r}\right) + i\left(\frac{R}{R_h + r}\right). \tag{11}$$

If the deflection of the indicator is θ, then

$$K_2\frac{1}{T}\int_0^T\left[i_1^2\left(\frac{R_h + R}{R_h + r}\right)^2 + 2ii_1R\frac{(R_h + R)}{(R_h + r)^2} + i^2\left(\frac{R}{R_h + r}\right)^2\right]dt - K_1\frac{1}{T}\int_0^T i_1^2dt = K\theta,$$

or

$$\left[K_2\left(\frac{R_h + R}{R_h + r}\right)^2 - K_1\right]\frac{1}{T}\int_0^T i_1^2dt + K_2\left[\frac{2R(R_h + R)}{(R_h + r)^2}\right]\frac{1}{T}\int_0^T ii_1dt + K_2\left(\frac{R}{R_h + r}\right)^2\frac{1}{T}\int_0^T i^2dt = K\theta.$$

The first left-hand term, which depends on v^2, will be eliminated if r is so adjusted that

$$K_2\left(\frac{R_h + R}{R_h + r}\right)^2 = K_1.$$

With this adjustment made,

$$\frac{1}{T}\int_0^T ii_1dt + \left[\frac{R}{2(R_h + R)}\right]\frac{1}{T}\int_0^T i^2dt = \left[\frac{K(R_h + R)}{2K_1R}\right]\theta = K_3\theta.$$

Inserting the value of i_1 from (10) and carrying the constant factor to the right-hand side of the equation gives

$$\frac{1}{T}\int_0^T vidt + \frac{R}{2}\left[\frac{R_h + R_V}{R_h + R} - \frac{R_V}{R_h + r}\right]\frac{1}{T}\int_0^T i^2dt = K_3\left[\frac{(R_h + r)(R_h + R_V)R_V}{R_h + r}\right]\theta,$$

or

$$P_L + \frac{I^2R}{2}\left[\frac{R_h + R_V}{R_h + R} - \frac{R_V}{R_h + r}\right] = K_3\left[\frac{(R_h + r)(R_h + R_V)R_V}{R_h + r}\right]\theta. \tag{12}$$

It is seen that the importance of the power loss in R increases as the power factor of the load diminishes.

In consequence of the development of the electrodynamic wattmeter, the three methods now to be described are merely of historical interest as methods for power measurement. However, two of them have other applications which are still of practical value.

The Three-dynamometer Method.—The connections are as shown in Fig. 204. At D_1, D_2, and D_3 are three electrodynamometers each with its fixed and movable coils in series. R is the total resistance of the circuit of D_2. The instantaneous values of the currents are as indicated. In general,

$$P = \frac{1}{T}\int_0^T v i_3 dt.$$

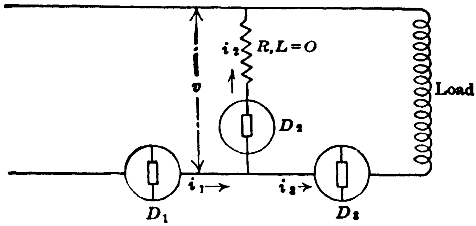


FIG. 204.—Connections for three-dynamometer method for power measurement.

R is assumed to be noninductive, so that

$$P = R\frac{1}{T}\int_0^T i_2 i_3 dt.$$

At any instant,

$$i_1 = i_2 + i_3,$$

and

$$i_1^2 = i_2^2 + 2i_2i_3 + i_3^2,$$

so

$$i_2i_3 = \frac{i_1^2 - i_2^2 - i_3^2}{2},$$

and

$$P = \frac{R}{2}\left(\frac{1}{T}\int_0^T i_1^2 dt - \frac{1}{T}\int_0^T i_2^2 dt - \frac{1}{T}\int_0^T i_3^2 dt\right). \quad (13)$$

The three integrals are the mean square values of the three currents and will be given by the readings of the dynamometers. If Siemens instruments are used,

$$K_1D_1 = \frac{1}{T}\int_0^T i_1^2 dt,$$

where K_1 is the constant of the instrument, and D_1 its deflection; and similarly for the other instruments. Hence

$$P = \frac{R}{2}(K_1D_1 - K_2D_2 - K_3D_3). \quad (14)$$

The result involves no assumption as to wave form. It does assume that the resistance R is noninductive. This cannot be exactly true, for the potential circuit contains the electrodynamometer. The use of hot-wire ammeters would render this assumption practically true. P includes the power in D_3.

The results are greatly affected by the errors of reading. To obtain the best precision in the measurement at unity power factor, the power wasted in R must be equal to the load, an obviously impracticable condition.

The Three-voltmeter Method.—In this analogous method, the three instruments capable of measuring the mean square values of the currents

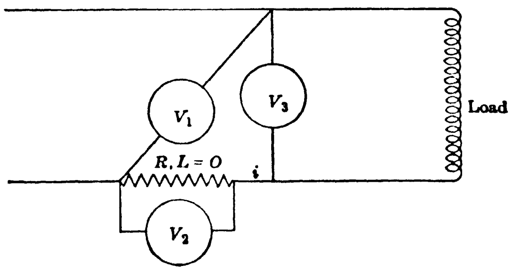


FIG. 205.—Connections for three-voltmeter method for power measurement.

are replaced by three instruments that measure mean square voltages. The connections are as shown in Fig. 205.

The noninductive resistance R is joined in series with the load, and the three voltages read, as indicated.

$$P = \frac{1}{T}\int_0^T iv_3dt = \frac{1}{R}\cdot\frac{1}{T}\int^T v_2v_3dt.$$

At any instant,

$$v_1 = v_2 + v_3 \quad \text{and} \quad v_1^2 = v_2^2 + 2v_2v_3 + v_3^2.$$

Therefore

$$P = \frac{1}{2R}\left(\frac{1}{T}\int_0^T v_1^2dt - \frac{1}{T}\int_0^T v_2^2dt - \frac{1}{T}\int_0^T v_3^2dt\right). \tag{15}$$

If the voltmeters are either of the electrodynamic or of the electrostatic type,

$$P = \frac{1}{2R}(K_1D_1 - K_2D_2 - K_3D_3). \tag{16}$$

For the greatest precision, the power wasted in R must be equal to the load. P includes the power in V_3.

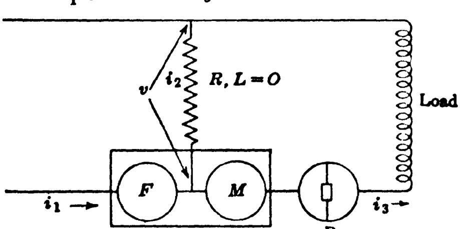


FIG. 206.—Connections for split-dynamometer method for power measurement.

The Split-dynamometer Method.—The connections are shown in Fig. 206.

D_3 is an ordinary current electrodynamometer with its coils in series. *FM* is another dynamometer with a noninductive resistance R, tapped in between the fixed and movable coils.

$$P = \frac{1}{T}\int_0^T v i_3 dt = R \cdot \frac{1}{T}\int_0^T i_2 i_3 dt.$$

At any instant,

$$i_1 = i_2 + i_3.$$

Therefore

$$P = R\left(\frac{1}{T}\int_0^T i_1 i_3 dt_t - \frac{1}{T}\int_0^R i_3^2 dt\right). \tag{17}$$

The split dynamometer gives the mean product of the currents in its coils, so

$$P = R(K_1D_1 - K_3D_3). \tag{18}$$

The Electrostatic Wattmeter.[7]—The quadrant electrometer, used as an electrostatic wattmeter, is useful in research work, particularly in investigations concerning dielectric losses where small amounts of power

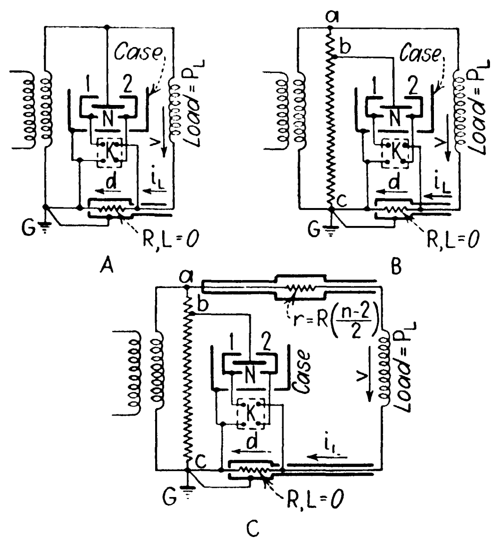


FIG. 207.—Connections for electrostatic wattmeter. *A*, full voltage on needle; *B*, partial voltage on needle; *C*, Walker's connection.

at high voltages and low power factors must be measured. It is employed at the National Physical Laboratory in testing electricity meters. Both deflection and zero methods of using the instrument have been developed.

Referring to Fig. 207*A*, the voltage d applied between the quadrants 1 and 2 is taken from the terminals of a nonreactive resistor in series with the load; therefore, in the ideal case, d is proportional at every instant to the load current i. By page 237, the deflection is proportional to the mean product of d, which is proportional to i, and $v + d/2$; that is, it is proportional to the power P_L expended in the load plus one-half the power expended in the nonreactive resistor. Therefore

$$D_1(1 - V_{Nc}^2A''') = 2B''R\left(P_L + \frac{I_2^2R}{2}\right).$$

If two readings are taken, one with the vertical and one with the horizontal position of the switch K,

$$P_L = \frac{(D_1 - D_2)(1 - V_{Nc}^2A''')}{4B''R} - \frac{I_L^2R}{2}. \qquad (19)$$

In high-voltage work, it is customary to apply only a part of the total voltage between the quadrants and the needle. In Fig. 207B, abc represents some form of potential divider, such that at every instant

$$v_{bc} = \frac{v_{ac}}{n} = v_{Nc}.$$

It may be a resistor shielded against the effects of capacitance to ground or the secondary of the high-voltage transformer tapped at the proper point. In either case, one must be sure that both the magnitudes and the phases of the two sections of the voltage are correct. In general, the characteristics of a potential divider may change with the amount of current drawn from the tap, with the load on the transformer, and with the voltage if a tapped secondary is used. The divider must be thoroughly investigated under actual working conditions before reliance can be placed upon it. Referring to Fig. 207B, with the commutator vertical the rise of voltage from the second set of quadrants to the needle is $-Ri + \dfrac{v + Ri}{n}$ and $d = Ri$. If the switch K is horizontal, the rise of voltage from the second set of quadrants to the needle is $(v + Ri)/n$ and $d = -Ri$. Therefore, when deflections are taken with both positions of the switch,

$$\frac{(D_1 - D_2)(1 - V_{Nc}^2A''')n}{4B''R} = P_L - I^2R\left(\frac{n - 2}{2}\right). \qquad (20)$$

If small amounts of power are being measured, the correction for the loss in the series resistor may be so large that the accuracy of the determination of P_L is impaired. It is desirable to avoid the necessity for this correction. It is seen that if P_L were zero, there would still be a deflection but in the negative direction; also, that if $n = 2$ in (20), no correction is required. This is the great advantage of connecting the needle to the mid-point of the supply voltage.

Walker Connection.[7]—Miles Walker has shown that this great advantage may be retained even though the needle is not connected to the mid-point of the supply voltage if a nonreactive resistor of a particular value is connected in the high-voltage lead. The connections are shown in Fig. 207*C*. From the figure, it will be seen that the load that is really measured is P_L plus the I_L^2r loss in the added resistance r; so in (20), P_L is replaced by $P_L + I_L^2r$. Therefore

$$\frac{(D_1 - D_2)(1 - V_{Nc}^2A''')n}{4B''R} = P_L + I^2\left[r + \frac{R(2 - n)}{2}\right]. \quad (21)$$

If n is greater than 2, and r is given the value $r = R\left(\frac{n-2}{2}\right)$, the I^2 term disappears, and

$$\frac{(D_1 - D_2)(1 - V_{Nc}^2A''')n}{4B''R} = P_l, \quad (22)$$

irrespective of where the supply voltage is tapped.

When the electrostatic wattmeter is used in connection with a fictitious load (see page 520) in the calibration of power and energy

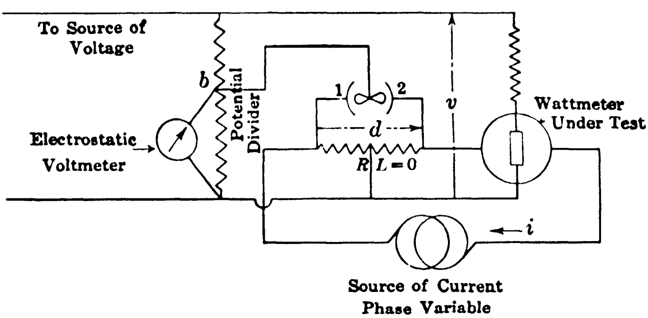


Fig. 208.—Connections for electrostatic wattmeter with fictitious load.

meters, the correction term due to the power loss in the series resistance will be eliminated if the current and voltage circuits are electrically connected at the middle of the resistance R, as shown in Fig. 208.

In this case, the rise of voltage between quadrant 2 and the needle is

$$v_{2b} = -\frac{d}{2} + \frac{v}{n}.$$

Therefore

$$D(1 - V_{Nc}^2A''') = 2B''\frac{1}{T}\int_0^T\left[2\left(-\frac{d}{2} + \frac{v}{n}\right)d + d^2\right]dt = \frac{2B''R}{n}\frac{1}{T}\int_0^T vidt.$$

$$P = \frac{D(1 - v_{Nc}^2A''')n}{2B''R}. \quad (23)$$

This method of connection is used in a highly developed form at the National Physical Laboratory for calibrating commercial instruments.[7]

Its advantage is the high degree of accuracy that can be attained over a wide range of current and voltage. The wide current range is obtained by using different shunts, which are designed for a full-load drop of from 1 to 2 volts.

Effect of Capacitance to Ground.—It will be seen from Fig. 207*C* that the resistor r and the lead to the high-voltage terminal of the load are at full voltage above ground. Therefore they should be protected by shields which at every instant are kept, as nearly as may be, at the potentials of the parts of the circuit that they protect. The other parts of the circuit must be shielded and guarded so that capacitance and leakage currents to ground do not find their way into the measuring circuit.

The low-voltage terminal of the load (Fig. 207*B*) is at a potential I_LR above ground; therefore R is shunted by the capacitance between the quadrants, the capacitance between one set of quadrants and the case, the capacitance to ground of the low-voltage terminal, and the distributed capacitance to ground in R. These effects may be considerable if the resistance of R is high, as it must be if small specimens are used.

The effect of shunting R by a capacitance C is to change its impedance operator from

$$z = R$$

to

$$z = \frac{R}{1 + R^2\omega^2C^2} - j\frac{R^2\omega C}{1 + R^2\omega^2C^2}. \tag{24}$$

The fall of voltage from quadrant pair 1 to quadrant pair 2 is

$$d = -Iz = -\frac{IR}{1 + R^2\omega^2C^2} + j\frac{IR^2\omega C}{1 + R^2\omega^2C^2}. \tag{25}$$

The voltage applied to the load is $V = V_1 - jV_2$, when V_1 is in phase with the current I. If the needle is so connected that it receives $1/n$th of the total voltage, the fall of potential from the second set of quadrants to the needle is

$$V_{2N} = Iz\left(\frac{n-1}{n}\right) - \frac{1}{n}(V_1 - jV_2).$$

Therefore

$$\begin{aligned} 2Vd + d^2 &= I^2z^2\left(\frac{2-n}{n}\right) + \frac{2Iz}{n}(V_1 - jV_2) \\ &= I^2\left(\frac{2-n}{n}\right)\left(\frac{R^2}{1 + R^2\omega^2C^2}\right) + \frac{2I}{n(1 + R^2\omega^2C^2)} \\ &\qquad (V_1R - jV_1R^2\omega C - jV_2R - V_2R^2\omega C). \end{aligned}$$

Averaging the indicated products over a whole cycle and expressing the result in terms of effective values gives

$$\frac{D(1 - V_{Nc}^2A''')(1 + R^2\omega^2C^2)n}{2B''R} = I^2R\left(\frac{2-n}{2}\right) + (V_1I - R\omega CV_2I)$$
$$= I^2R\left(\frac{2-n}{2}\right) + P_L + R\omega CVI \sin\theta, \qquad (26)$$

where θ is the power-factor angle.

Generally, in dielectric tests, the power factor is desired. From (26):

$$\cos\theta = \frac{D(1 - V_{Nc}^2A''')(1 + R^2\omega^2C^2)n}{2B''RVI} + \left(\frac{n-2}{2}\right)\frac{IR}{V} - R\omega C \sin\theta. \qquad (27)$$

θ is very close to 90 deg. when dielectrics are tested. It is seen that the effect of the capacitance to ground of the low-voltage terminal and the resistance R has added the term $R\omega C \sin\theta$ and the factor $(1 + R^2\omega^2C^2)$.

Another source of error when very small specimens of dielectric are tested is the current taken by the needle of the electrometer. The needle and the quadrants form a condenser across which there is a considerable voltage. A part of the current that flows through the condenser is added vectorially to the current from the specimen. Also, if the suspension has a high resistance, the potential of the needle may be shifted in phase with respect to that of the quadrants. Simons and Brown have discussed these effects[8] and have made use of a controlled displacement of the needle voltage to obtain an effective zero deflection method for determining the power factor of insulating materials.

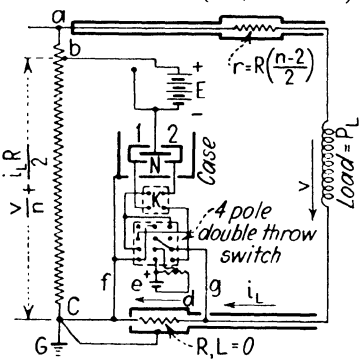


FIG. 209.—Kouwenhoven and Betz method with Walker's added resistance.

The great advantage of zero methods is the elimination of all questions as to the law of deflection of the electrometer.

Zero Method, Using Quadrant Electrometer.[9]—The connections are shown in Fig. 209.* The fundamental idea is to superpose on the alternating potential differences, which would ordinarily produce the deflection, direct potential differences of such magnitudes and relative directions that the torque due to the alternating voltages is balanced, and the deflection brought to zero.

The direct voltages are derived from the two batteries E and e. The four-pole double-throw switch serves to insert the voltage e in

* As given by KOUWENHOVEN and BETZ (*Z. für Instrumentenkunde*, vol. 23, 1903, p. 112).

either lead f or lead g. When both direct- and alternating-voltages are applied simultaneously, the contributions to the deflecting moment of all first-power terms in the general formula for the electrometer are due entirely to the direct voltages. The signs may be positive or negative according to circumstances, for during a complete cycle the mean value of each of the alternating terms is zero. The contributions from terms involving products of alternating and direct voltages are also zero when averaged over a cycle. The magnitudes and directions of the direct voltages are adjusted until the deflection is brought to zero. Referring to Fig. 209, with the commutator vertical and the four-pole switch to the left,

$$d = V_2 - V_1 = Ri + e_1$$
$$V_1 = -e_1$$
$$V_2 = Ri.$$

The voltage from the second set of quadrants to the needle is, with Walker's added resistance,

$$V_{2N} = \frac{V}{n} - \frac{Ri}{2} - E.$$

With the commutator horizontal and the four-pole switch to the right,

$$d = V_2 - V_1 = -Ri - e$$
$$V_1 = Ri + e$$
$$V_2 = 0.$$

The voltage from the second set of quadrants to the needle is

$$V_{2N} = \frac{V}{n} + \frac{Ri}{2} - E.$$

Inserting the first set of values in the general formula for the quadrant electrometer and taking the mean value over the whole cycle gives

$$0 = \frac{2B''R}{n}\frac{1}{T}\int_0^T Vidt - 2E''e_1 - 2G''E + B''e_1^2 - 2B''Ee_1. \quad (28)$$

The second set of values gives

$$0 = -\frac{2B''R}{n}\frac{1}{T}\int_0^T Vidt + 2E''e_2 - 2G''E + B''e_2^2 + 2B''Ee_2. \quad (29)$$

Subtracting (29) from (28),

$$0 = \frac{4B''R}{n}\frac{1}{T}\int_0^T Vidt - 2E''(e_1 + e_2) + B''(e_1^2 - e_2^2) - 2B''E(e_1 + e_2).$$

Therefore

$$P_2 = \frac{1}{T}\int_0^T Vidt = n\left[\frac{E''(e_1 + e_2)}{2B''R} - \frac{(e_1^2 - e_2^2)}{4R} + \frac{E(e_1 + e_2)}{2R}\right]. \quad (30)$$

e_1 and e_2 are very nearly equal; let $(e_1 + e_2)/2 = e$;

$$P_L = \frac{n}{R}\left[Ee + \frac{eE''}{B''}\right] \quad \text{very nearly.} \quad (31)$$

The only electrometer constants involved are in the correction term $e\frac{E''}{B''}$, the importance of which is decreased by using a large value of E, for this decreases the required value of e. E''/B'' may be experimentally determined by using the method to check the known power supplied to a nonreactive resistor.

Power-diagram Indicator.[10]—The power-diagram indicator is a device in which the cathode-ray tube (see page 667) is employed to trace diagrams, the areas of which are proportional to the power supplied to the load. Its particular field of usefulness is the measurement of small amounts of power at high voltage. It has been applied to the measurement of corona losses and to the total losses in samples of insulating materials. Losses as small as 0.03 watt at 9,000 volts have been measured. The accuracy of the original arrangement was about 5 per cent. The application of the device (sometimes called the cyclograph) to the investigation of the behavior of insulating materials has been developed by Minton whose connections are shown in Fig. 210.

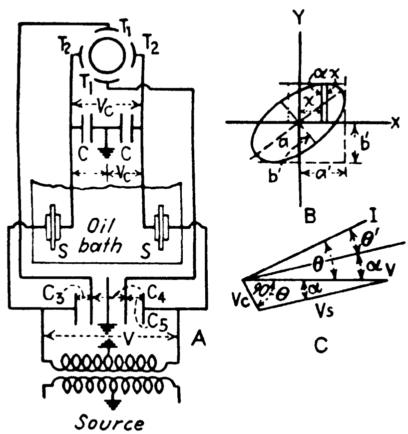


FIG. 210.—Power-diagram indicator.

In order to obtain the diagram, it is necessary to employ two sets of electrodes. One set is arranged to deflect the fluorescent spot along the X-axis on the screen; another, along the Y-axis.

Looking along the axis of the tube, the electrodes are placed as shown in Fig. 210, which also shows the other connections. The connections are arranged as symmetrically as practicable with respect to the ground connection. Two identical, oil-immersed specimens S are employed. They are joined in series with the two equal condensers C, which are grounded at their junction point. The current-deflecting plates T_1 are connected across C and C; consequently only a moderate voltage is applied to them. C_3, C_4, C_5 is a condenser multiplier by means of which a low voltage, at every instant proportional to that on the specimen, is applied to the plates T_1 which deflect the spot vertically. The electrodes T_2 cause the spot to be deflected horizontally proportionally to the instantaneous values of the potential difference between the outer terminals of the condensers C, C, that is, proportionally to the current through the specimen.

Let the displacement along y be

$$y = Kv,$$

and that along x be

$$x = K'v_c.$$

K and K' are constants. Then, for an elementary area,

$$dA = ydx = KK'vdv_c.$$

The current i through the condensers C is that taken by the specimen, and

$$i = \frac{C}{2}\frac{dv_c}{dt}.$$

Then

$$dA = \frac{2KK'}{C}vidt,$$

and the area of the diagram is

$$A = K''\int_0^T vidt = k'P. \tag{32}$$

$$P = VI\cos\theta = k''A.$$

For sinusoidal currents and voltages, the figure is an ellipse. If the power factor is unity, the semiaxes are along x and y, and $P_1 = VI = k''A_1$.

When dealing with insulating materials, the power factor rather than the power loss in the specimen is usually desired.

$$\text{Power factor} = \frac{P}{VI} = \frac{A}{A_1} = \frac{ab}{a'b'}, \tag{33}$$

and calibration is avoided.

The power factor so obtained is that of the whole circuit. To determine that of the sample, use is made of Fig. 210C, where θ is the power-factor angle of the specimen, and V_c and V_s the voltage applied across the condenser C and across the specimen.

$$\cos\theta' = \cos(\theta - \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha;$$

$$\frac{\sin\alpha}{\sin(90 - \theta)} = \frac{V_c}{V_s}; \qquad \sin\alpha = \frac{V_c}{V_s}\cos\theta;$$

$$\cos\theta' = \cos\theta\sqrt{1 - \left(\frac{V_c}{V_s}\right)^2\cos^2\theta} + \left(\frac{V_c}{V_s}\right)\sin\theta\cos\theta;$$

$$\cos\theta' = \cos\theta + \frac{1}{2}\left(\frac{V_c}{V_s}\right)\sin 2\theta, \qquad \text{approx.} \tag{34}$$

V_c is determined by an electrostatic voltmeter of low capacitance. The voltage applied to the specimen is

$$V_s = \sqrt{V^2 + V_c^2 - 2VV_c \sin \theta}. \tag{35}$$

POWER MEASUREMENT IN POLYPHASE CIRCUITS

Blondel's Theorem.[11]—If energy is supplied to a network through n wires, the total power in the system is given by the algebraic sum of the readings of n wattmeters, so arranged that each of the n wires contains one current coil, the corresponding potential coil being connected between that wire and a point on the system that is common to all the potential circuits. If this common point is on one of the n wires and coincides with the point of attachment of the potential lead to that wire, only $n - 1$ wattmeters are required.

The receiving and generating circuits may be arranged in any desired manner, and no assumption is made as to the way in which the e.m.fs. and currents vary.

To prove the theorem, denote by the subscripts 1, 2, 3, . . . , n the different supply wires; by $v_1, v_2, \ldots, v_n$ the instantaneous *potentials* of the points on the various wires that form the terminals of the absorbing device; and by $i, i_2, \ldots, i_n$ the instantaneous currents at these same points. Then the rate of displacement of electricity through wire 1 will be i_1; and the rate of doing work, or the instantaneous power, will be i_1v_1; and similarly for all the others. Therefore,

$$p = i_1v_1 + i_2v_2 + \cdots i_nv_n. \tag{a}$$

In practice, it is necessary to deal with potential differences rather than with individual potentials. Let v_0 be the *potential* of a designated point on the system. In general,

$$i_1 + i_2 + i_3 \cdots + i_n = 0.$$

Consequently,

$$i_1v_0 + i_2v_0 \cdots + i_nv_0 = 0. \tag{b}$$

Subtracting (b) from (a),

$$p = i_1(v_1 - v_0) + i_2(v_2 - v_0) \cdots i_n(v_n - v_0).$$

The average power will be

$$P = \frac{1}{T}\int_0^T i_1(v_1 - v_0)dt + \frac{1}{T}\int_0^T i_2(v_2 - v_0)dt \cdots + \frac{1}{T}\int_0^T i_n(v_n - v_0)dt. \tag{36}$$

But $\frac{1}{T}\int_0^T i_1(v_1 - v_0)dt$, etc., are the readings of the n wattmeters connected as above. If the common point is on one of the n wires at one of the terminal points of the absorbing device, then one of the quantities in parenthesis will be zero, the corresponding wattmeter will read zero, and only $n - 1$ wattmeters will be required.

The preceding demonstration is perfectly general and therefore applies in all cases that can arise in polyphase power measurements. However, the consideration of the cases that are of frequent occurrence in practice is instructive.

Designation of Wattmeter Terminals.—As some of the mean products in (36) may be negative, it is necessary that the connections be so arranged that a negative deflection of the wattmeter signifies that the reading should be subtracted when computing the power. That there may be no confusion, the potential and current terminals of the wattmeters through which the currents should enter when flowing from the generator to the load should be determined and marked on the instruments once for all. The proper marking may be determined by putting the instruments in a single-phase circuit. Then, whenever the instruments are used, the currents, as they flow from the generator to the load, must enter both the current and the potential coils at the marked terminals.

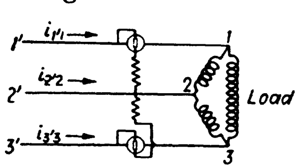


FIG. 211.—Power measurement; two-phase three-wire system.

When the instruments are so connected, if the pointer deflects up the scale, the mean product *vi* is positive; if the deflection is in the contrary direction, the mean product is negative; and to obtain its numerical value, the *current* coils must be reversed. The reading so obtained is regarded as negative. This simple procedure avoids all uncertainty as to the algebraic signs of the readings and renders unnecessary any special tests for their determination. The terminals of current and potential transformers should be similarly marked.

Two-phase Three-wire System.—By the theorem, two wattmeters are required, the connections being as in Fig. 211.

If the two phases are separately loaded, it is obvious that the power is the sum of the wattmeter readings. A load might, however, be connected between leads 1 and 3 as indicated, and then the instantaneous power would be

$$p = v_{12}i_{12} + v_{23}i_{23} + v_{31}i_{31}.$$
$$v_{31} = v_{32} + v_{21}.$$
$$p = v_{12}i_{12} + v_{23}i_{23} + v_{32}i_{31} + v_{21}i_{31}$$
$$= v_{12}(i_{12} - i_{31}) + v_{32}(i_{31} - i_{23}).$$
$$i'_{11} = i_{12} + i_{13} = i_{12} - i_{31}.$$
$$i'_{33} = i_{32} + i_{31} = i_{31} - i_{23}.$$

Therefore

$$P = \frac{1}{T}\int_0^T v_{12}i'_{11}dt + \frac{1}{T}\int_0^T v_{32}i'_{33}dt. \tag{37}$$

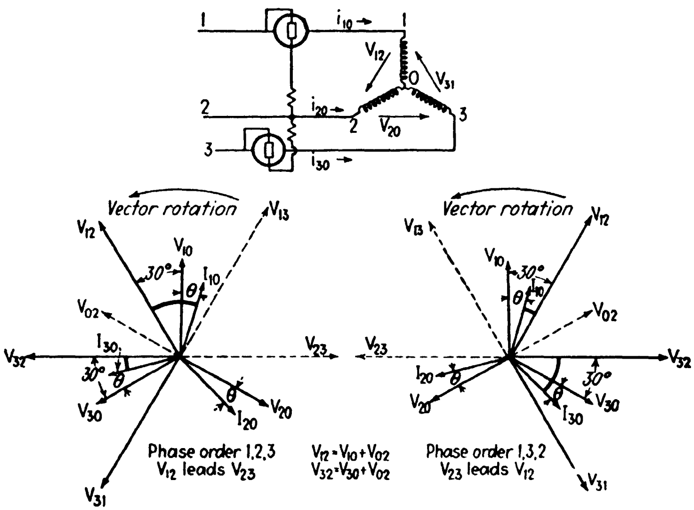
The two wattmeters evaluate the integrals.

Three-phase Three-wire System.—By Blondel's theorem, two wattmeters are required. Referring to Fig. 212, the instantaneous power is

$$p = v_{10}i_{10} + v_{20}i_{20} + v_{30}i_{30}.$$
$$v_{12} = v_{10} + v_{02}.$$
$$v_{32} = v_{30} + v_{02}.$$
$$i_{10} + i_{20} + i_{30} = 0.$$
$$p = i_{10}(v_{12} + v_{20}) + i_{20}v_{20} + i_{30}(v_{32} + v_{20})$$
$$= i_{10}v_{12} + i_{30}v_{32} + (i_{10} + i_{20} + i_{30})v_{20}.$$

Therefore

$$P = \frac{1}{T}\int_0^T i_{10}v_{12}dt + \frac{1}{T}\int_0^T i_{30}v_{32}dt. \tag{38}$$



$R_1 + R_3 = V_{12}I_{10}\cos[-(30° + \theta)] + V_{32}I_{30}\cos[+(30° - \theta)]$
$= VI\sqrt{3}\cos\theta$
$R_1 - R_3 = -VI\sin\theta$
$\tan\theta = -\sqrt{3}\dfrac{R_1 - R_3}{R_1 + R_3} \equiv +$ for lagging current
R_1 smaller than R_3
Current lags the voltage

$R_1 + R_3 = V_{12}I_{10}\cos[+(30° - \theta)] + V_{32}I_{30}\cos[(-30° + \theta)]$
$= VI\sqrt{3}\cos\theta$
$R_1 - R_3 = +VI\sin\theta$
$\tan\theta = +\sqrt{3}\dfrac{R_1 - R_3}{R_1 + R_3} \equiv +$ for lagging current
R_1 larger than R_3
Current lags the voltage

Balanced inductive load—the wattmeter with its voltage coil across the leading line voltage has the smaller reading.

FIG. 212.—Two-wattmeter method.

The two wattmeters evaluate the integrals. The vector diagram when the phase order is 1, 2, 3 and also when it is 1, 3, 2 is shown in Fig. 212. Let R_1 be the reading of the wattmeter with its current coil in line 1, and R_3 the reading of the wattmeter with its current coil in line 3. Irrespective of the phase order the sum of R_1 and R_3 gives the power. Referring to the vector diagram, inspection will show that when the phase order is 1, 2, 3, the wattmeter with its current coil in line 1 has the smaller reading, it being proportional to cos $(30° + \theta)$. As the lag angle is increased, the reading decreases and becomes zero when $\theta = 60$ deg. (corresponding to power factor 0.5), for then V_{12} and I_1 are in quadrature. If $\theta > 60$,

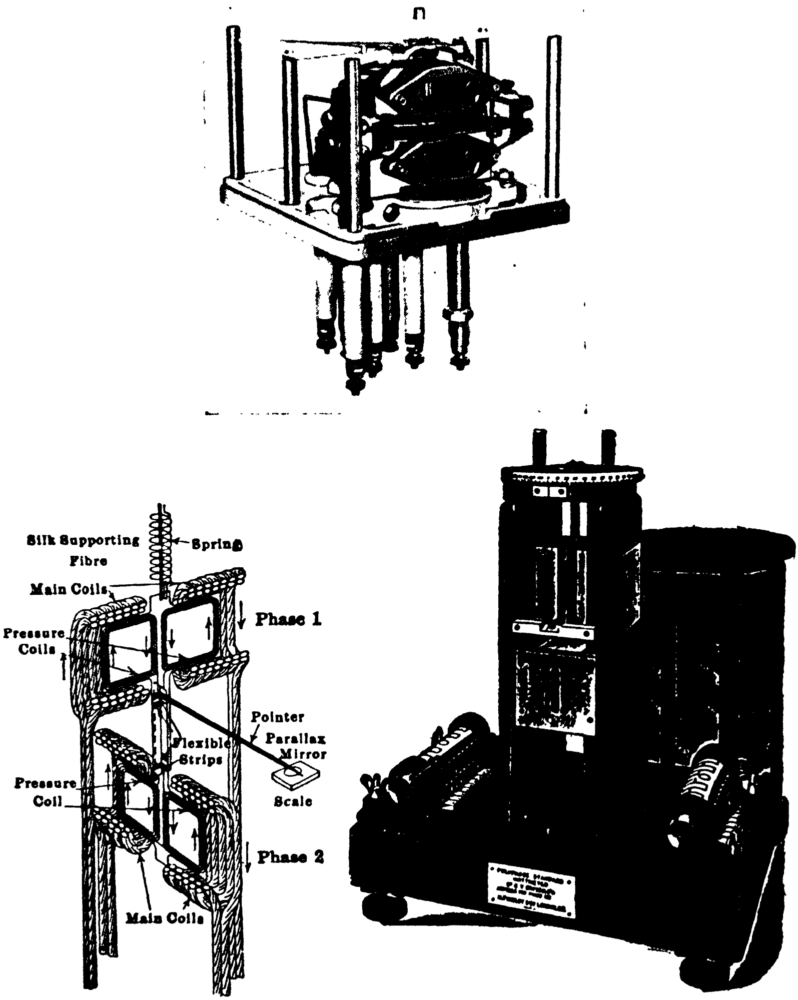


FIG. 213.—Drysdale polyphase wattmeter for laboratory work.

the reading becomes negative; in this case, the current coil is reversed, and the reading subtracted when determining P. When the phase order is 1, 3, 2, wattmeter 1 has the larger reading, it being proportional to cos $(30° - \theta)$.

It will be noted that the phase difference of V_{32} and I_3 is the same as that of V_{13} and I_1; and as the maximum values of the current and voltage are the same in both cases, one wattmeter will suffice for measurements on a *balanced* load. For example, two readings may be taken with wattmeter 1, the first with the potential terminals connected between mains 1 and 2, the second with the potential terminals connected between mains 1 and 3.

The Polyphase Wattmeter.[12]—To avoid the necessity of using two separate instruments, the polyphase wattmeter has been devised.

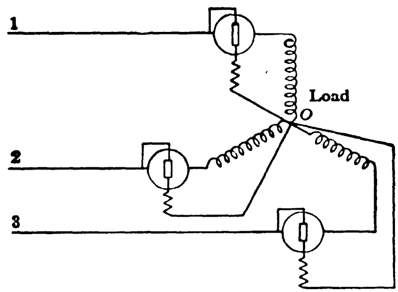


FIG. 214.—Connections for measurement of three-phase power by three wattmeters.

The instrument consists of two complete wattmeters mounted in the same case. The two movable coils are attached to the same rigid stem and consequently act against the same spring. The deflection is thus made to depend on the sum of the torques of the two elements, so that the total power is read directly from the scale. The electrical connections are the same as for two single-phase wattmeters (see Fig. 212).

Ample insulation must be provided between the two elements, and it is imperative that the stray field from one element have no influence on the torque generated by the other. Protection from both external and internal stray fields may be obtained by the use of laminated shields.

In careful tests at low power factors, the use of the polyphase wattmeter with instrument transformers is to be avoided, since the necessary corrections for the ratio and phase angle of the current transformers cannot be made under these conditions.

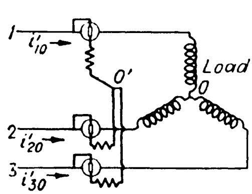


FIG. 215.—Connections for three-phase power measurements with artificial neutral.

Figure 213 shows two forms of polpyhase wattmeter. The laboratory instrument is read by means of a torsion head, and the effect of the stray field from the upper element on the torque of the lower element, and vice versa, is minimized by placing the elements with their axes perpendicular. In the switchboard instrument, a magnetic shield is inserted between the two elements.

Three-phase Power Measurement by Three Wattmeters.—If the neutral is accessible, the sum of the readings of three wattmeters connected as in Fig. 214 will give the power.

It is not necessary that the neutral be accessible, for by the theorem one has only to connect the potential coils at a common point, as O' in Fig. 215.

As previously shown,

$$P = \frac{1}{T}\int_0^T v_{12}i_{10}dt + \frac{1}{T}\int_0^T v_{32}i_{30}dt,$$

but

$$v_{12} = v_{10'} + v_{0'2},$$
$$v_{32} = v_{30'} + v_{0'2},$$

so

$$P = \frac{1}{T}\int_0^T (v_{10'} + v_{0'2})i_{10}dt + \frac{1}{T}\int^T (v_{30'} + v_{0'2})i_{30}dt$$
$$= \frac{1}{T}\int^T v_{10'}i_{10}dt + \frac{1}{T}\int^T v_{30'}i_{30}dt + \frac{1}{T}\int_0^T v_{0'2}(i_{10} + i_{30})dt.$$
$$i_{10} + i_{30} = -i_{20}.$$

Therefore

$$P = \frac{1}{T}\int_0^T v_{10'}i_{10}dt + \frac{1}{T}\int_0^T v_{30'}i_{30}dt + \frac{1}{T}\int_0^T v_{20'}i_{20}dt = \text{sum of wattmeter readings.} \quad (39)$$

No assumptions are made as to the resistances of the potential circuits of the wattmeters.

If the load is balanced, the readings on all three instruments will be the same; this leads to the use of the Y-box for the measurement of *balanced* three-phase loads.

The Y-box.—The Y-box consists of a small case, like that of an ordinary multiplier, containing two resistors in series, *each of which has a resistance equal to that of the potential circuit of the wattmeter with*

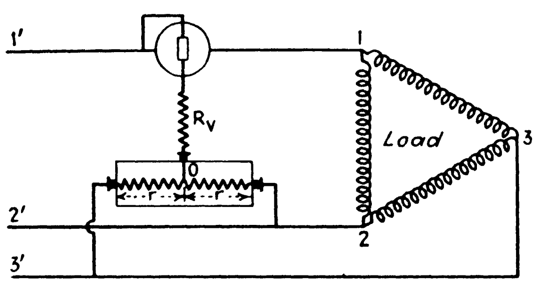


FIG. 216.—Connections of Y-box for measuring three-phase power under balanced loads.

which the box is to be used. As a tap is carried to the junction of the two resistors, the Y-box has three terminals. It is connected in circuit as indicated in Fig. 216.

If the load is balanced, the power is given by 3 times reading of wattmeter.

It may be convenient to use a Y-box with an instrument other than that for which it was designed; in this case, the factor is no longer 3.

Referring to Fig. 216, by Kirchhoff's rules the current i_V through the potential coil of the wattmeter is

$$i_V = \frac{v_{12} + v_{13}}{2R_V + r},$$

where R_V is the resistance of the potential circuit of the wattmeter; and r, the resistance of each section of the Y-box. If the line current is i_L, the reading of the wattmeter is given by

$$R_V \frac{1}{T}\int_0^T i_V i_L dt.$$

Therefore,

$$\text{Reading} = \frac{R_V}{2R_V + r}\left[\frac{1}{T}\int_0^T i_{V1} v_{12} dt + \frac{1}{T}\int_0^T i_{V1} v_{13} dt\right].$$

For a *balanced load*

$$\text{Power} = \frac{1}{T}\int_0^T i_{V1} v_{12} dt + \frac{1}{T}\int_0^T i_{V1} v_{13} dt.$$

Therefore

$$P = \frac{2R_V + r}{R_V} \times \text{reading of wattmeter.} \tag{40}$$

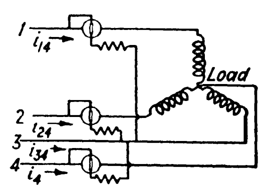


FIG. 217.—Connections for power measurement in a four-wire three-phase circuit.

Four-wire Three-phase System.—The power will be given by the sum of the readings of three wattmeters connected between the leads 1, 2, 3 and the neutral point. Figure 217 shows another arrangement which also conforms to Blondel's theorem.

$$p = v_{14}i_1 + v_{24}i_2 + v_{34}i_3;$$

but

$$v_{13} = v_{14} + v_{43};$$
$$v_{23} = v_{24} + v_{43};$$
$$i_1 + i_2 + i_3 + i_4 = 0;$$

so

$$\begin{aligned} p &= (v_{13} + v_{34})i_1 + (v_{23} + v_{34})i_2 + v_{34}(-\,i_1 - i_2 - i_4) \\ &= v_{13}i_1 + v_{23}i_2 + v_{43}i_4 + v_{34}(i_1 + i_2i_1 - i_2). \end{aligned}$$

Therefore

$$P = \frac{1}{T}\int_0^T v_{13}i_1 dt + \frac{1}{T}\int_0^T v_{23}i_2 dt + \frac{1}{T}\int_0^T v_{43}i_4 dt, \tag{41}$$

which is the sum of the readings of the three wattmeters. The theorem shows that the power in any four-wire system may be measured by three wattmeters.

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CHAPTER VII

MEASUREMENT OF INDUCTANCE AND CAPACITANCE

STANDARDS OF INDUCTANCE

For carrying out the methods of measurement described in this chapter, standards of inductance and capacitance are required, and convenience dictates that, in many cases, they be made adjustable. These standards may be divided into two classes: primary standards, whose values are calculated from their dimensions; and secondary standards, whose values are determined experimentally.

The subject of the calculation of primary standards of self- and mutual inductance is beyond the scope of this work. Readers are referred to the papers of Rosa and Grover and to later papers by Grover in the publications of the U. S. Bureau of Standards.[1] In these papers, all the available formulae are collected and tested, and illustrative examples given.

Standards of Mutual Inductance.—Primary standards of mutual inductance, which have a single value, are useful in the calibration of variable working standards of mutual and self-inductance for use in the laboratory and in the calibration of ballistic galvanometers for use in iron testing.

The considerations governing the design are:

1. The value must be accurately calculable from the geometrical dimensions.
2. The construction must be such that permanence is assured.
3. The value must be sufficiently large to give high sensitivity when comparisons are made.
4. The resistances of the coils must be kept as low as possible.
5. Eddy-current effects must be eliminated as far as possible.
6. The capacitance effect between the primary and the secondary must be a minimum.
7. The bobbins upon which the coils are wound must be free from magnetic materials.

To eliminate eddy-current effects, all conductors that carry large currents must be made of insulated strands, and all metal frames, etc., near the coils must be avoided. The binding posts should be small and near together.

In the past, inductance coils have frequently been wound on bobbins of serpentine. It has been found, however, that a coil so wound has an inductance that depends, to a slight extent, upon the strength of current flowing in the conductor, thus showing that the permeability of serpentine is not unity and that it depends on the magnetic field in which the serpentine is placed. Coils wound on mahogany bobbins are satisfactory for working standards.

For primary standards of *self-inductance* it is customary to use single-layer coils which are wound in a screw-cutting lathe on accurately ground cylinders of marble, or fused quartz. Such a construction facilitates the exact determination of the geometry of the coil. In a standard of *mutual inductance*, this construction is not admissible for *both* the primary and the secondary coils, since the product of the primary and secondary turns must be large, of the order of 100,000 for a mutual inductance of 0.01 henry.

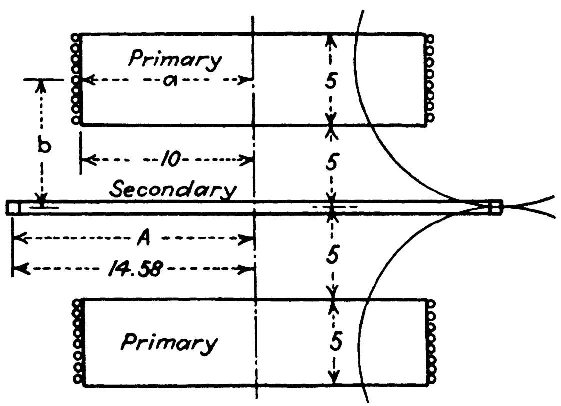


FIG. 218.—Campbell single-valued primary standard of mutual inductance.

Campbell Fixed Standard of Mutual Inductance.[2]—It is necessary that the arrangement of the primary and secondary coils in a standard of mutual inductance be such that errors arising from slight displacements of the coils from their supposed relative position will be reduced to a minimum. By dividing the primary into two equal sections, properly separating them, and placing the secondary midway between them, all three coils being coaxial, a satisfactory arrangement may be obtained. If the diameter of the secondary coil is such that the mutual inductance is a maximum, the secondary will be so placed that its circumference is in a zero field. For this reason a small variation in the diameter or a small axial displacement of the coil will produce only a slight variation in the mutual inductance, the value of which then depends, in the main, on the carefully determined dimensions of the primary. The construction is indicated in Fig. 218, where the proper relative dimensions are shown.

With the proportions indicated, a multiple-layer secondary having a considerable cross section (0.5 by 0.5 cm.) may be used. The effect of variations of the diameter of the secondary coil is shown below. The mutual inductance is a maximum when $A = 14.58$ cm. If the secondary circuit is displaced from the mid-position between the primary coils by 0.35 cm., m is reduced by less than 1 part in 10,000.

CAMPBELL STANDARD OF MUTUAL INDUCTANCE
$a = 10$ cm., $b = 7.5$ cm., $n_1 n_2 = 100{,}000$

A, centimeters	14.1	14.3	14.5	14.7	14.9	15.0
m, millihenrys	9.1630	9.1728	9.1759	9.1759	9.1696	9.1567

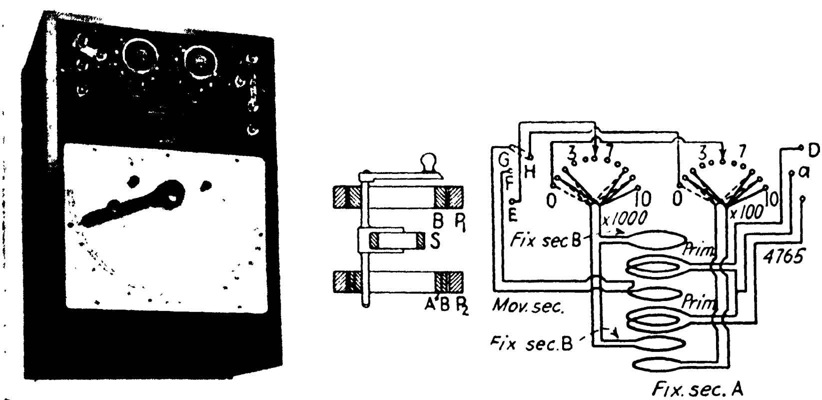
The National Physical Laboratory has adopted the Campbell form of primary mutual inductance as the standard for both self- and mutual inductance measurements. Such a standard is superior to a standard of self-inductance constructed with equal care, for its value can be calculated from the dimensions with a higher degree of accuracy; also, capacitance and eddy-current effects which cause the effective inductance to depend on the frequency are less. Both sections of the N.P.L. primary standard have 75 turns of bare copper wire wound on an accurately machined marble cylinder 30 cm. in diameter. The secondary of 488 turns has a mean diameter of 43.8 cm. and is wound in a groove 1 × 1 cm. cut in a marble ring. The value of the mutual inductance at 15°C. is 10.0177_8 millihenrys.

Variable Mutual Inductances.—Variable mutual inductances, in other words air-core transformers of variable ratio, are extremely useful in alternating-current measurements, as, for example, in determining self-inductances and in measuring the ratios of instrument transformers.

For the highest utility, the coils should be wound astatically, especially if the apparatus is to be used in an electrical engineering laboratory, and even then one should satisfy himself by tests that stray field effects, due to nonuniform fields, are absent. If an astatic arrangement is not used, great care must be taken that the standard is not set up where there are alternating stray fields or where its field will influence other instruments.

Variable mutual inductometers frequently have a single primary coil and a secondary wound in a number of sections which can be put in series by appropriate switches. The value of m is thus built up to the desired point. The sections are so proportioned that there are no gaps in the scale. To obtain a fine adjustment, the smallest section of the secondary is mounted so that it may be made to link different amounts of the primary flux. By this means m may be varied continuously through zero to a small negative value. The ability to reach a zero value is one point of superiority of the variable mutual over the variable self-inductometer. It is to be noted that if a secondary turn or section is reversed, its effect becomes subtractive. This is sometimes very convenient. For instance, if a decade consists of three coils having the values of 1, 3, and 6 dial units, any value from 1 to 10 dial units may be obtained by different combinations; for instance, if the secondary giving 1 dial unit is connected in opposition to that giving 6 units, the effective value is 5 dial units. $6 - 1 = 5$.

A. Campbell was the pioneer in developing variable mutual inductometers and methods for their use. To Butterworth and Hartshorn of



A B

Fig. 219.—Campbell variable mutual inductometer. (*Cambridge Instrument Co.*)

the National Physical Laboratory are due later developments and a critical study of the device.

In the earlier form of Campbell inductometer (Fig. 219), the primary consists of two coils *PRIM.* and *PRIM.*, which are fixed in position. From *a* a tap is carried to the mid-point of the primary. This is convenient in certain bridge networks where it is desired to insert the two parts of the primary in contiguous bridge arms. As indicated in the figure, the fixed secondary coils are in two groups, each having 10 strands thoroughly interlaced and brought out to terminals. In the instrument illustrated, the m of each strand in the $\times 1{,}000$ group is 1,000 μh exactly. Each strand in the $\times 100$ group has exactly one-tenth the effect of a strand in the $\times 1{,}000$ group, or 100 μh. The total m of the fixed sections is therefore 0.011000 henry. The movable section varies m from $+105$ to -3 μh. The total range of this particular instrument is therefore from $+0.011105$ to -0.000003 henry. The movable secondary is connected in series with the fixed secondary by the link *GH*. The instrument is made in two ranges, the maximum readings being 11 and 1.1 millihenrys approximately.

The winding of the secondary with 10 interlaced strands gives a ready means of making the first adjustment, for, on the average, each strand occupies the same position with respect to the primary as every other strand. However, it has the disadvantage that there is considerable capacitance between the strands, which may become troublesome if the frequency is much above the power range, that is, 500 cycles per second. It would be better to use a number of separate secondaries, as is done in a later design by Campbell, in which there is also an ingenious arrangement of the variable section to obtain a long scale (260 deg.) and an almost constant percentage accuracy of reading. Referring to Fig. 220*A*, the fixed coils are at *C*. The movable coil *ms* turns about the

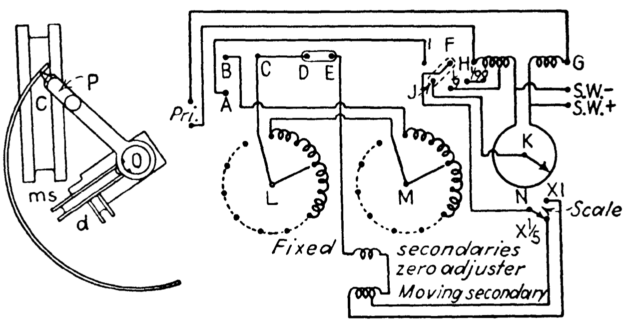


FIG. 220.—Campbell variable mutual inductometer, second design.

eccentrically located axis *O*. *d* is a small coil which is attached to and in series with *ms*; its function is to improve the higher readings by opening out the scale. Figure 220*B* indicates the internal connections. In Fig. 220*B*, the "moving secondary" includes *ms* and *d* of Fig. 220*A*.

Defects of Mutual Inductometers. Impurity.[4]—In elementary discussions, it is assumed that a mutual inductor injects into its secondary an e.m.f. in exact quadrature with the primary current and represented symbolically by e.m.f. $= +jm\omega I_1$ (self- and mutual fluxes opposing) or e.m.f. $= -jm\omega I_1$ (self- and mutual fluxes aiding), as the case may be. If the frequency is low, this assumption is so nearly true that no difficulties arise. However, Campbell noted that if the frequency was high, there were divergencies from the elementary law of action which depended on the design and construction of the device. A divergence from the elementary law, which is readily guarded against, may also be due to improperly connecting the mutual inductor into the measuring circuit. To insure a definite distribution of potentials, the primary and secondary windings are so connected that they have a common point. If by some error or by intent, as in the Hartshorn method for comparing mutual inductance (page 447), a resistance that terminates at this point is included in both the primary and secondary circuits, the effect of, for instance, the secondary current on the primary circuit is no longer in quadrature with the secondary current, as will be seen from the following.

Referring to Fig. 221*A*, and assuming that the only source of error is the resistance σ, the expression for the fall of voltage around the primary circuit in the direction of I_1 is, as the self- and mutual fluxes are opposed,

$$I_1(R_1 + j\omega L_1 + \sigma) - I_2\sigma - jm\omega I_2 - E_1 = 0,$$

or

$$I_1(R_1 + j\omega L_1 + \sigma) - I_2(\sigma + jm\omega) - E_1 = 0.$$

It is seen that the fall of voltage in the direction of I_1 due to the action of the secondary current, is $-I_2(\sigma + jm\omega)$. The rise of voltage, or e.m.f. introduced into the primary by the action of the secondary, is

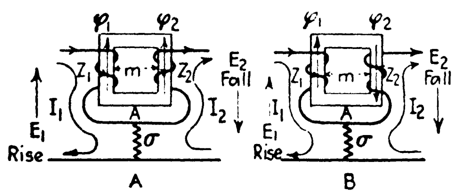


FIG. 221.—Pertaining to an impurity due to a resistance common to both primary and secondary.

$I_2(\sigma + jm\omega)$. The phase of this voltage evidently differs from the ideal 90 deg. by the phase-defect angle $\tan^{-1}\dfrac{\sigma}{m\omega}$; σ is called the impurity of the mutual inductance. In general, σ is the in-phase component of the voltage, and $m\omega$ the quadrature component. In this particular case, σ is an ohmic resistance. For the secondary circuit, the voltage drop in the direction of I_2 is $I_2(R_2 + j\omega L_2 + \sigma) - I_1(\sigma + jm\omega) - E_2 = 0$, and the rise of voltage in the direction of I_2 is $I_1(\sigma + jm\omega)$. If the self- and mutual fluxes are aiding (Fig. 221*B*),

$$I_1(R_1 + j\omega L_1 + \sigma) - I_2(\sigma - jm\omega) - E_1 = 0,$$
$$I_2(R_2 + j\omega L_2 + \sigma) - I_1(\sigma - jm\omega) - E_2 = 0;$$

it is seen that the algebraic sign prefixed to the m term depends on whether the self- and mutual fluxes are opposed or in the same direction. This difference is to be kept in mind. Butterworth, in his discussions, assumes the fluxes to be opposed and uses the plus sign, for the rise of voltage.

The most important source of impurity is distributed capacitance in both the primary and the secondary windings and distributed capacitance between them. It is customary to assume that the effects of distributed capacitance in a coil may be simulated by a perfect condenser connected between its terminals. This arrangement is indicated in Fig. 222*A*. It will be noted that six condensers are required, this being a four-terminal network. If the primary and secondary have one common terminal, as indicated in Fig. 222*B*, the potential differences between the coils are rendered definite, and the equivalent network much simplified, only three condensers being required.

Butterworth and Hartshorn of the National Physical Laboratory have studied this structure theoretically and experimentally. It is obvious that it must possess a frequency coefficient, for the impedances of the condensers decrease while those of the self-inductances increase with an increase of frequency. Butterworth's procedure in reducing the network of Fig. 222*B* to a simpler, equivalent form was as follows: It was first assumed that the self- and mutual fluxes were opposed. All of Butterworth's formulae are based on this assumption, which determines the algebraic sign of the mutual inductance terms in the equations. By the use of the transformation given on page 392, the mutual inductance is replaced by its equivalent three-pointed star of impedances, which is then transformed to its equivalent Δ of impedances and then combined with the Δ formed by the three condensers C_1, C_2, C_{12}. This combined Δ is then transformed into the equivalent three-pointed star of impedances, which is finally transformed by the inverse of the equations on page 392 into an equivalent mutual inductance which is without the extraneous condensers, their effects being included in the modified resistance and inductance of the primary and secondary and the modified mutual inductance. In making the transformations, the squares and higher powers of the small capacitances involved are neglected.

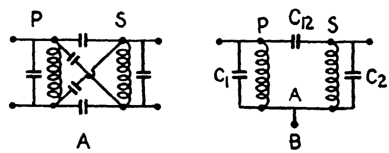


FIG. 222.—Pertaining to capacitance impurity.

It is thus found that the fractional increase in m due to capacitance is, when the self- and mutual fluxes are opposed,

$$\frac{\Delta m}{m_0} = \omega^2\left[C_1L_1 + C_2L_2 - \frac{C_{12}(L_1 - m_0)(L_2 - m_0)}{+m_0}\right],$$

where m_0 is the mutual inductance at low frequencies. The angle of phase defect due to capacitance is

$$\delta_c = \tan^{-1}\frac{\sigma}{\omega m} = R_1C_1\omega + R_2C_2\omega - C_{12}\left[\frac{R_1(L_2 - m_0)\omega}{+m_0} + \frac{R_2(L_1 - m_0)\omega}{+m_0}\right].$$

Here σ is the in-phase component of the mutual inductance operator and not the ohmic resistance of Fig. 221. If the self- and mutual fluxes are aiding, the sign preceding m_0 is reversed. Thus both $\Delta m/m_0$ and δ_c are larger when both the self- and mutual fluxes are aiding than when they are opposed. For this reason it is better, if possible, to use a mutual inductance with opposed fluxes, provided the capacitances are the same in both cases.

Capacitance effects, though the most important, are not the only sources of impurity. Losses in the insulation and eddy currents in the conductors and neighboring metal parts, which set up fluxes out of phase with the main fluxes, are sources of error. Eddy-current effects which may be either positive or negative are reduced to negligible amounts by winding the coils with finely stranded conductors and avoiding all massive binding posts and metal fastenings. Butterworth has suggested that by intentionally introducing eddy-current and capacitance effects of the proper value an inductometer without frequency error, over the range where the impurity is proportional to ω^2, may be produced.

It is seen that no mutual inductometer should be used for accurate work in the audio range and above unless it is accompanied by data giving, with a stated relative direction of the self- and mutual fluxes, the effective mutual inductance and the impurity for the entire scale and at the frequency to be employed.

Secondary Standards of Self-inductance.—Secondary standards of self-inductance are usually wound on mahogany or marble bobbins. Shawcross and Wells[5] have reviewed Maxwell's work on the best form to give such a coil and find that to obtain the maximum value of L the cross section of the winding should be square and that the mean diameter of the winding should be practically three times the side of the square. More exactly, the value is 2.95, as Grover has shown by direct calculation, but a variation from 2.80 to 2.95 changes the value of L only four-hundredths of 1 per cent, while the variation from 2.95 to 3.00 changes the value only five one-thousandths of 1 per cent. Procedure for the design of such coils has been given by Brooks,[6] the starting point being a selected value of the time constant.

Ayrton and Perry Inductometer.—The Ayrton and Perry variable standard of self-inductance has long been used for general laboratory purposes. This standard consists of two coils of slightly different diameters wound on spherical surfaces; the smaller coil is pivoted within the larger in such a manner that it can be rotated about a vertical diameter. The coils are connected in series, and a pointer shows their relative position; as each position corresponds to a particular value of the self-inductance of the combination, the dial may be graduated in henrys. When the index stands at the lower end of the scale, the inductance of the combination is nearly zero, owing to the fact that the currents in the two coils are circulating in opposite directions. When the movable coil is turned through 90 deg., the inductance becomes the sum of the self-inductances of the two coils, as there is no mutual induction in this position. When turned through 180 deg., the currents in the coils are in the same direction, and the total inductance becomes the sum of the self-inductances of the coils and twice their mutual inductance. Thus the inductance of the standard may be varied continuously from a minimum, which is nearly zero, to a maximum. The scale is irregular, and the instrument is not astatic.

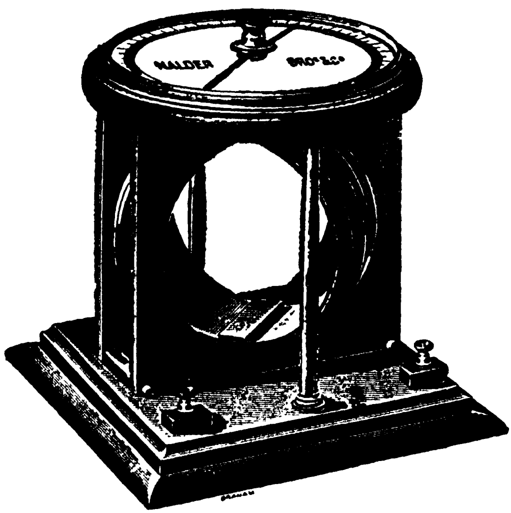


FIG. 223.—Ayrton and Perry variable inductor.

Brooks Variable Inductor.[3]—The Brooks variable inductor very closely fulfills the requirements for a variable standard of mutual and self-inductance. It was developed for use in testing current transformers (see page 609) but is applicable in any measurement where such a variable standard, having a constant resistance, is necessary.

The particular advantages of the instrument are its large current carrying capacity, low resistance and therefore high time constant, and practically uniform scale. The range of the instrument, as described, is from 125 to 1,225μh.

Figure 224 shows the instrument complete and in section, as well as the form of the coils. Referring to the diagram, the four coils F are fixed; by means of the handle H the two movable coils M can be displaced in their own plane about the axis A. The number of flux linkages between F and M can thus be varied. The coils of stranded wire are arranged astatically. Current is carried to the movable coils through heavy copper spirals, thus eliminating all contact resistances. The cross-hatching in the diagram indicates the numbers of turns in the coils. The four fixed coils are permanently connected in series and provided with binding-post terminals; likewise the two movable coils.

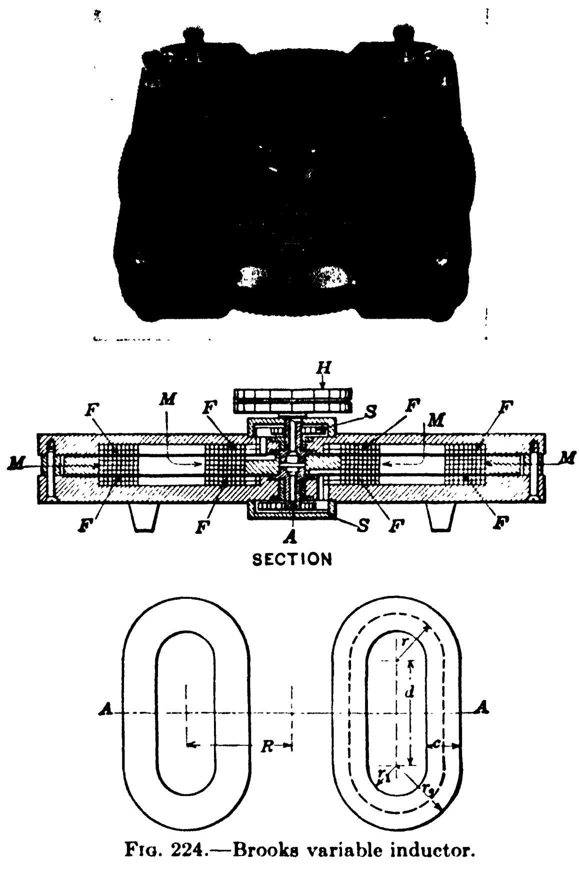


FIG. 224.—Brooks variable inductor.

When the fixed and movable elements are connected in series, the instrument may be used as a standard of self-inductance; and when they are separated, as a standard of mutual inductance.

The self-inductance, or scale reading, is

$$L = L_1 + L_2 \pm 2m,$$

where L_1 and L_2 are the inductances of the fixed and movable elements, and m is the mutual inductance of the elements. L_1 and L_2 are constants, and, for the instrument as described, $L_1 + L_2 = 669$ μh. The expression for the mutual inductance is therefore

$$\pm m = \frac{L - (L_1 + L_2)}{2} = \frac{L - 669}{2}\ \mu\text{h}.$$

The whole device is about 14 in. in diameter. The interleaving of the fixed and movable coils is important, for if M recedes axially from one fixed coil it approaches the other, so that the net effect on the inductance of a slight axial displacement of M is zero.

The distinctive feature of this inductor is that the scale divisions are of equal length throughout the greater part of the useful range. For instance, in an instrument having a useful range of from 125 to 1,225 μh, the scale is uniformly divided between 325 and 1,025 μh; outside these limits the length of the divisions gradually decreases, but there are no sudden changes.

The uniform scale is attained by using link-shaped coils; the proper proportions were determined experimentally and are given below.

Referring to Fig. 224:

r = mean radius of semicircular end of coil.
$c = 0.78r$.
$d = 2.2r$.
$R = 2.26r$.
$r_1 = 0.61r$.
$r_2 = 1.39r$.

The combined cross section of the fixed and movable coils is a square having a side c units long.

Sources of Error.—In using inductometers, both self- and mutual, it is to be remembered that their actions are complicated by a number of secondary effects. The neighboring turns are separated by the insulation, which is an imperfect dielectric, and there is a considerable voltage between turns; consequently, there are capacitance effects and energy losses in the dielectric, both of which at high frequencies affect the equivalent inductance and the equivalent resistance. The higher the inductance the greater these effects become.

A general idea of the effects of the capacitance may be gained by considering that the actual inductive coil with its distributed capacitance may be represented by a coil of inductance L and resistance R, in parallel with a condenser C. Assuming that the coil resistance is small, and neglecting the losses in the dielectric, it will be seen from the expression for the impedance of the combination that

The equivalent inductance is $L' = L(1 + CL\omega^2)$ approx.

The equivalent resistance is $R' = R(1 + 2CL\omega^2)$ approx.

The equivalent resistance is affected twice as much as the equivalent inductance, and both effects depend on the square of the frequency. The net inaccuracies may be negligibly small but become greater and greater as the frequency is raised to the audio range and beyond, especially if the inductance is large.

The possibility of inductive effects from neighboring coils in the same network and from alternating stray fields of the same frequency as that being employed in the measurements should be kept in mind. In general, the second but not the first effect may be obviated by making the inductor astatic. Inductors of fixed value should be as widely separated as practicable from other coils in the network and carefully oriented so that mutual inductance effects are minimized. There is a large stray field from inductors with straight cores, so in many cases the wire is uniformly wound on a nonmagnetic toroidal core.

If it is necessary to adjust an inductor to a particular value, the winding is terminated in a small loop the size and position of which and therefore its mutual inductance with respect to the main winding may be altered until the desired over-all value of L is attained. The loop is firmly stowed away in a cavity in the bobbin on which the main coil is wound.

STANDARDS OF CAPACITANCE

As examples of primary standards of capacitance, that is, condensers whose capacitances in electrostatic units are calculated from their dimensions, those used by Rosa and Dorsey[6] in their determination of v, the ratio of the electromagnetic to the electrostatic unit of charge, may be taken. It is in connection with the determination of v that the possible sources of error in such primary standards have been most carefully studied.

In order to be able to calculate the capacitance of a condenser with a high degree of accuracy, the dielectric coefficient of the medium between the plates must be definitely known, and the medium must be free from absorption and from dielectric losses. For these reasons, air is always used as the dielectric in primary condensers. Also, the distance between the plates must be so great that the thickness of the dielectric may be determined with accuracy. In consequence of these facts, the capacitances of primary condensers are very small.

Three forms of primary condensers have been developed, namely, those with spherical, cylindrical, and plate electrodes.[6] A section of the spherical condenser used by Rowland in the determination of v in 1879, by Rosa in 1889, and by Rosa and Dorsey in 1905 is shown in Fig. 225.

The capacitance in electrostatic units of such a spherical air condenser is

$$C = \frac{Rr}{R - r}$$

where R and r are the radii bounding the dielectric.

The internal radius of the spherical shell in the Rowland condenser is 12.67158 cm., and the radius of the ball is 10.11806 cm. (at 20 deg.). The capacitance is, therefore, 50.2095 electrostatic units.

When the condenser is used, the ball must be carefully centered, and corrections made for the holes in the shell, the bushings, and the cord by which the ball is suspended. The error in the final calculated value of the capacitance was estimated at about 2 parts in 100,000. When measured by Maxwell's method (see page 375), the capacitance was found to be 5.59328×10^{-20} absolute electromagnetic units, or 0.0000559328 μf.

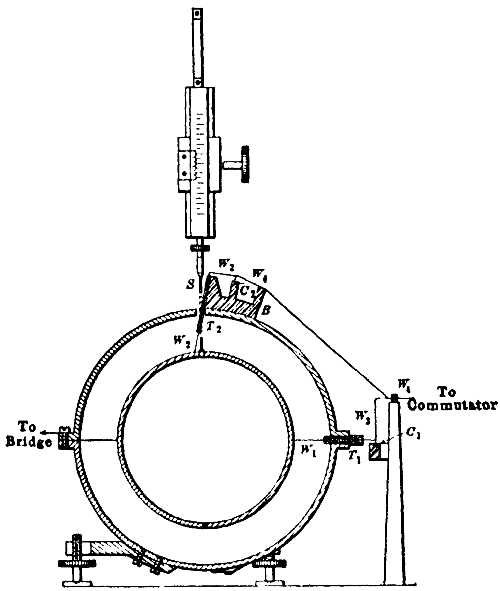


FIG. 225.—Section of spherical air condenser.

In any experimental work with such a very small capacitance, it is necessary to make allowances for the capacitance of the wire by which the charge is imparted to the ball and for that of all leads and of the commutator by which the charging and discharging are effected.

The capacitance of an air condenser with coaxial cylindrical electrodes, if the charge is uniformly distributed, is given in electrostatic units by

$$C = \frac{l}{2 \log_e \frac{R}{r}},$$

where l is the length of the cylinder, and R and r are the radii bounding the dielectric. For precision work, on account of the effect of the ends of the condenser, the assumption of a uniform density of charge is not tenable, so recourse is had to guard cylinders, shown in Fig. 226 at G. The effect of the ends is thus removed from the central section, which is the one connected to the measuring apparatus, to the guard cylinders, where it does no harm. In order to make practically all the lines of force radial, the air gaps between the main section and the guard cylinders must be made as small as possible, and the measuring apparatus so arranged that the guard cylinders are always at the same potential as the main or central section.

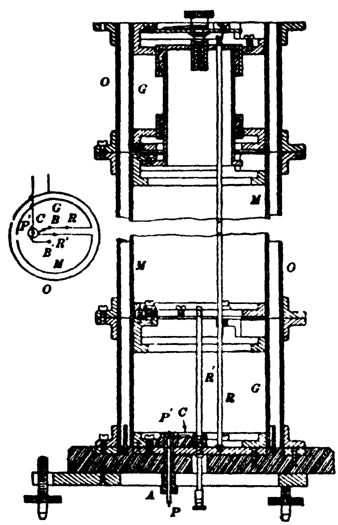


FIG. 226.—Section of cylindrical air condenser with guard cylinders.

For one of the condensers used by Rosa and Dorsey,

$$l = 20.00768 \text{ cm.}$$
$$R = 7.23831 \text{ cm.}$$
$$r = 6.25740 \text{ cm.}$$

Then, as a first approximation,

$$C = \frac{20.00769}{2 \log_e \frac{7.23831}{6.25740}} = 68.696 \text{ e.s.u.}$$

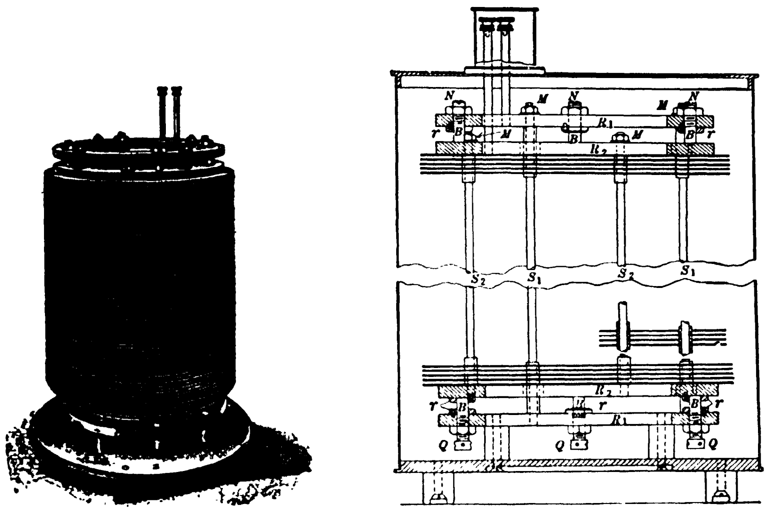
This result must be corrected for the effects of the gaps between the main cylinder and the guard cylinders and for the different thicknesses of the dielectric on the two sides of the gaps due to differences in the diameters of the cylinders. The deduction of these corrections involves the use of the higher forms of analysis.[6] The numerical value of the net correction for the condenser just mentioned, when the lower gap is 0.6 mm. and the upper one 0.5 mm., is +0.182 electrostatic unit. Thus, in the case of this short condenser, the corrections with even these small air gaps amount to over one-fourth of 1 per cent. The longer the central section the smaller is the percentage correction.

For the most refined work, the parallel-plate condenser is inferior to these just referred to, for comparatively large errors may be introduced if the adjustments are not perfect. Flat-plate condensers are provided with guard rings (see page 194).

Secondary Air Condensers.—Secondary air condensers are useful in experimental work in those cases where the dielectric losses must be reduced as near zero as is possible, for instance in determining the phase angles of mica condensers or in the investigation of the dielectric losses occurring in insulating materials. As such condensers must be calibrated, it is possible to use for each electrode a number of plates in parallel and to make the distance between the electrodes only a few millimeters. By this means, capacitances of a few hundredths of a microfarad may be obtained without undue bulk.

In any air condenser, the ohmic resistance between the terminals and the condenser proper must be kept low in order that there may be no appreciable internal I^2R losses which would cause an alternating current to lead the applied voltage by less than 90 deg.

Investigations made at the Massachusetts Institute of Technology[10] show that, contrary to the generally accepted idea, there is a small



A *B*

FIG. 227.—Secondary air condenser.

energy loss in air condensers. This loss depends on the character of the surfaces of the plates and on the voltage gradient, and it gives rise, with carefully cleaned brass plates and a gradient of 5,000 volts per inch, to a power factor of the order of magnitude of 10^{-6}.

A secondary air condenser,[7] having a capacitance of 0.01 μf, is shown (with the case removed) in Fig. 227*A*. The plates (of magnalium) are 20 cm. in diameter, 1 mm. thick, and so spaced that the thickness of the dielectric is 2 mm.; 35 plates are used in one electrode, and 36 in the other. In order to insure permanence, a very solid construction must be employed.

The plates are supported as shown in Fig. 227*B*. The bronze ring R_1 is firmly screwed to the base of the instrument; through it pass four adjusting screws of fine pitch Q, which support a second ring R_2 by means of the little amber cylinders B, which move in the guides r. Four equally spaced brass rods, 5 mm. in diameter and having screw threads cut on their upper ends, are firmly attached to ring R_1 and pass upward, with ample clearances, through holes in the ring R_2. Four equally spaced vertical rods are attached to R_2. Each plate is pierced with eight holes, four of them 5 mm. and four 12 mm. in diameter. The holes are so placed that they accommodate the eight vertical rods.

The condenser is built up by first putting on a member of electrode 2. The smaller holes in this plate just fit the rods S_2, and the larger ones allow ample clearances for the rods S_1. Distance pieces of the proper diameter and of a length sufficient to give a 2-mm. air space between the plates are then slipped over the rods S_1. They rest on the ring R_1 and pass with ample clearances through the holes in R_2; on these four distance pieces rests the first plate of electrode 1. Distance pieces 8 mm. in diameter and 5 mm. long are then slipped over the rods S_2 and rest on top of the first plate of electrode 2; the second plate of electrode 2 is then slipped on and rests on the top of the distance pieces; and so on.

When the pile has been completed, electrode 1 is firmly clamped between the rings R_1R_1 by means of the nuts M. Electrode 2 is clamped between the rings R_2R_2. The spaces between the two electrodes are finally adjusted by raising or lowering electrode 2 by means of the adjusting screws Q, which are then locked. The top of 2 is firmly held by tightening and then locking the screws N which bear on the ring R_2 by means of the amber cylinders B.

The assembled condenser is about 30 cm. high and weighs approximately 37½ lb.

By making the air space 1 instead of 2 mm. and employing 107 plates, condensers having a capacitance of 0.03 μf have been constructed. With this extremely small thickness of the dielectric, trouble was experienced in insulating the two sets of plates; for when voltage was applied, fine particles of dust from the air bridged the space between the plates, thus reducing the insulation resistance. It is not possible to remove the dust after the condenser is assembled, but the insulation may be improved by placing a drying material in the case of the instrument.

The breakdown voltage of the condenser with 2 mm. air space is 3,000 volts; and with 1 mm. air space, 900 volts. All sharp edges on the plates and internal fittings must be avoided, in order to prevent brush discharges.

To render the condenser independent of the surroundings, one set of plates is connected to the case, the other set being connected directly to the measuring apparatus.

Variable capacitances are necessary for general laboratory purposes, but a difficulty presents itself when one attempts to put a number of very small condensers in parallel by the ordinary means, since the connections possess an unknown capacitance which may be enough to introduce serious errors. For this reason, it is necessary that the design of the small sections from which the larger capacitances are built up be such that this uncertainty is eliminated.[8]

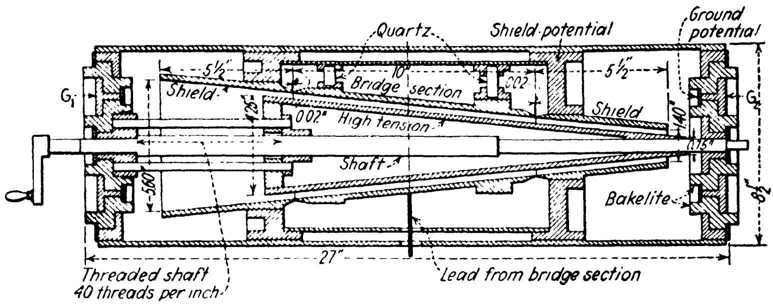


FIG. 228A.—Variable air condenser.

In the most refined work, the temperature coefficient of an air condenser, due to the change of dimensions and change in the dielectric coefficient of the air, must be considered. It may amount per degree to 2 or 3 parts in 100,000. Air condensers capable of withstanding high voltages are used in bridges intended for dielectric tests.[10] Two designs are shown in Fig. 228. Condenser A[10] is intended for precision work up to 5,000 volts. The capacitance may be varied over a limited range, from 30 up to 100 $\mu\mu$f, by displacing the inner or high-voltage electrode axially. The over-all length of the condenser is 27 in. Attention is called to the conducting rings G_1 and G_2, interposed between the high-voltage and shield electrodes. These rings are connected to earth and prevent surface leakage from the high-voltage electrode to the shielding system. This leakage would render it difficult, if not impossible, to maintain the shield at the proper potential.

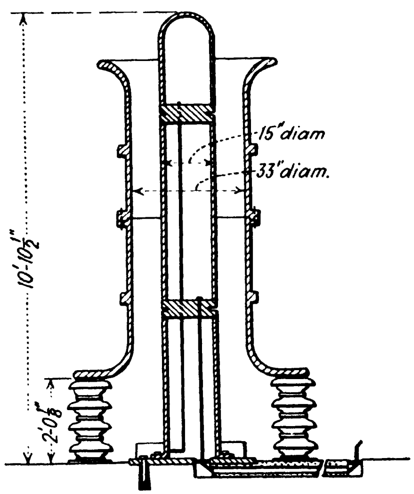


FIG. 228B.—High-voltage standard air condenser.

The insulation of the structure that supports the various parts of the condenser must be nonhygroscopic and have the minimum dielectric loss. Amber, amberoid, and fused quartz are employed.

Condenser B[11] is intended for work up to 150 kv.; although it has a capacitance of only 100 $\mu\mu$f, it is nearly 11 ft. tall.

The capacitances of these condensers must be determined by measurements. However, for rough preliminary calculations, the formula $C = 0.0886A/t$ μμf, where A is the area in square centimeters and t the distance apart of the plates in centimeters, may be used for flat-plate condensers, while the capacity of cylindrical air condensers is given, nearly enough for the purpose, by $C = \dfrac{l}{4.15 \log_{10} \dfrac{R}{r}}$ μμf. R and r are the radii of the outer and inner cylinders; l is the length of the active section in centimeters. It is essential that there be no appreciable losses in such condensers; therefore all potential gradients must be kept low by using large spacings, flaring end bells, and well-rounded corners wherever the potential gradient would naturally be high. As the clearances, across which there are potential differences, must be large, these condensers, though of low capacitance, are very bulky. The maximum voltage gradient in a cylindrical condenser occurs at the surface of the inner cylinder and is given by

$$g_V = \frac{V}{r \log_\epsilon \dfrac{R}{r}}.$$

V is the voltage across the condenser. The denominator is a maximum when $R/r = \epsilon = 2.72$. With a given diameter of the outer cylinder and an assigned value of g_V, the voltage V which can be applied is a maximum when $R/r = 2.72$. This relation is roughly adhered to. Calculation as to breakdown voltage can be only approximate on account of the corner at the ends of the active section. At *standard density*, the voltage gradient at which air between concentric cylinders breaks down is given by

$$g_V = 31.\left(1 + \frac{0.308}{\sqrt{r}}\right) \quad \text{kv./cm. approx.}$$

The dielectric strengths of "air" condensers may be greatly increased if the dielectric is a dry compressed gas under a pressure of 60 lb. per square inch or greater.[9] This eliminates brush discharges and energy losses at high voltages. For example, with an air pressure of 175 lb. per square inch, a condenser with plates 2.1 mm. apart showed no appreciable energy loss at 27,500 volts. It broke down at 28,500 volts.

In another case, with the plates 3.2 mm. apart, the breakdown voltage at atmospheric pressure was 6,000 volts; when the air pressure was raised to 260 lb. per square inch, the breakdown voltage became 30,000.

It is necessary to use a drying material in the condenser cases. The use of compressed gas, of course, necessitates a strong and, therefore, a very heavy metal case. A practical difficulty arises in the introduction of the lead to the insulated set of plates; it must be perfectly insulated, and all joints must be airtight as well. Practically, the casing cannot be made absolutely tight, so a pressure gage must be supplied, and the gas pressure renewed from time to time.

Experience has shown that with careful construction the pressure may not fall more than 20 lb. during a year from an initial value of 220 lb. per square inch.

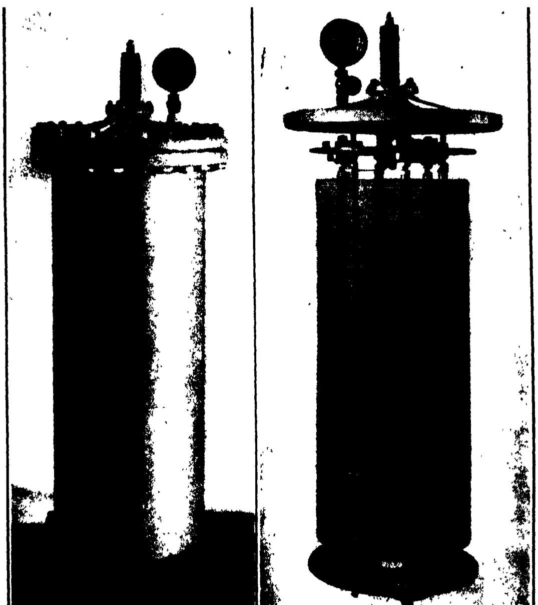


FIG. 229.—Compressed-gas condenser for use in radio-telegraphy.

Compressed-gas condensers are used in connection with radio-telegraphic apparatus. Figure 229 shows one of those installed at Arlington, Va., by the United States government.

Continuously Variable Air Condensers.—Continuously variable air condensers having a maximum capacitance of about 1,500 $\mu\mu$f are useful for obtaining the final balance in low-voltage capacitance bridges, etc. Figure 230 shows the design developed by the Leeds and Northrup Company.

Capacitance Boxes.—Though the determination of the capacitance and phase angle of working standards of capacitance depends ultimately upon the use of secondary air condensers, such instruments are not suitable for general use in the laboratory on account of their size, and more convenient arrangements must be employed. Ordinarily, the capacitance of laboratory standards is of the order of magnitude 1 μf. Such standards should be subdivided and made adjustable by arrange-

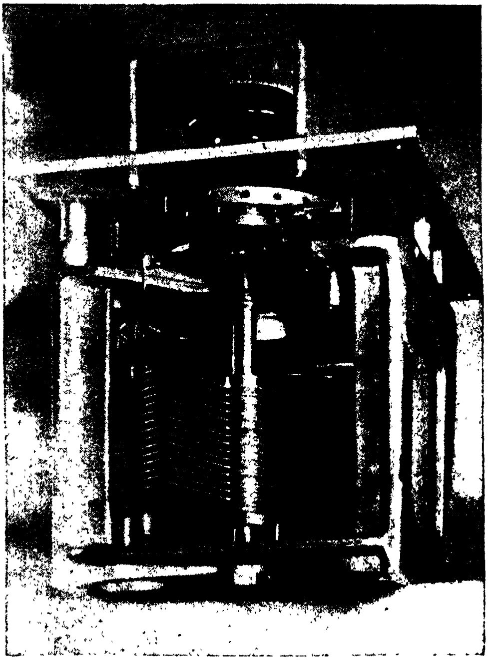


FIG. 230.—Variable air condenser. (*Leeds and Northrup Co.*)

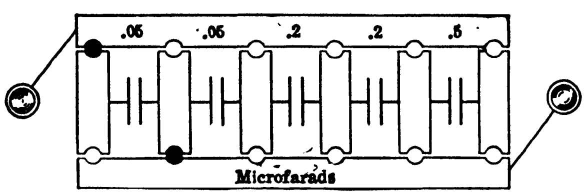


FIG. 231.—Subdivided condenser.

ments for placing the various sections in parallel. When this is done, the net capacitance in terms of the capacitance of the various sections is

$$C = C_1 + C_2 + C_1 + \cdot \cdot \cdot .$$

Figure 231 shows a common arrangement of the terminal blocks. By the proper insertion of the plugs, the sections may be put in parallel or in series, and any section may be discharged, or the whole condenser may be short-circuited.

An alternative scheme is to connect the sections to the terminal buses by small switches. The insulation is thereby improved, and the capacitance of the top reduced. Such condensers, variable in 0.001-μf steps up to 1 μf, are most convenient for general laboratory purposes,

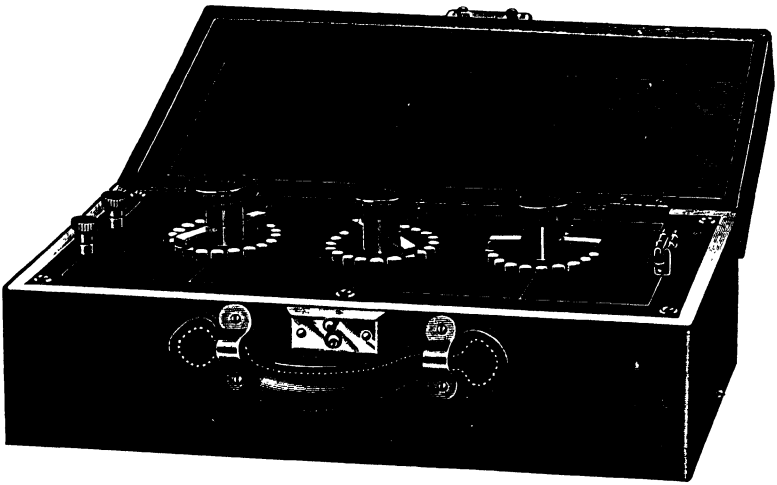


FIG. 232.—Three dial mica condenser. (*Leeds and Northrup Co.*)

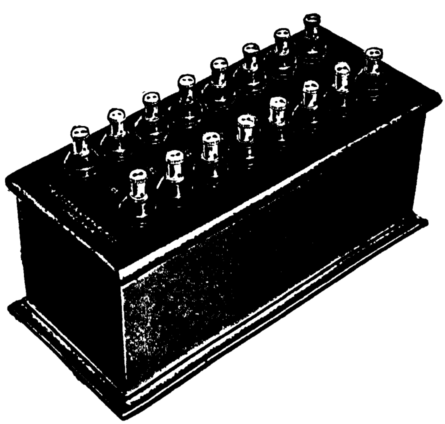


FIG. 233.—Subdivided precision condenser.

especially if paralleled by a suitable continuously variable air condenser, for it is then possible to vary the capacitance continuously from about 20 $\mu\mu$f to a little more than 1 μf.

Of necessity, when all the switches are set at zero, there is a small residual capacitance, the value of which is given by the makers. In work of high accuracy, the switch capacitance is not negligible. The various sections are adjusted with this in mind.

For precision standards, these schemes of connection are open to the objection that the capacitance of the top itself, which depends upon the position of the plugs or switches, is included between the terminals. For precision work, it is better to have each section provided with small and well-insulated terminal posts, spaced as far apart as practicable. This also allows the sections to be connected in series as well as in parallel.

With the series connection, the capacitance will be

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}.$$

The capacitance in electrostatic units of a condenser with parallel plates separated by a medium having a dielectric coefficient K is

$$C = \frac{KA}{4\pi t}.$$

A is the total active area of one set of plates in square centimeters and t is the thickness of the dielectric in centimeters. The capacitance in microfarads of a parallel-plate condenser is

$$C = \frac{0.886KA}{10^7 t}$$

Obviously, to increase the capacitance without an undue increase of bulk, a dielectric having a high dielectric coefficient and capable of being used in very thin sheets must be employed. The material chosen should have an exceedingly high resistivity and a high dielectric strength.

The preceding formula for capacitance is convenient for rough preliminary calculations; as t cannot be known with any great certainty, all condensers with dielectrics of small thickness must be calibrated.

All materials used in the construction of condensers must be clean and perfectly dry, and the finished instrument must be sealed so that the access of moisture is prevented. Mica and paraffined paper are the materials commonly used for the dielectric. Air pockets in the dielectric must be entirely eliminated to reduce losses.

Temperature has an appreciable effect on the behavior of condensers having solid dielectrics. It is not possible to give a definite statement as to the temperature coefficient of the capacitance of a particular condenser, for the temperature effects are dependent on the particular cycle of operations to which the condenser is subjected. This is illustrated in Figs. 236A and 236B, which are typical of good and poor mica condensers. The best mica condensers when subject to the ordinary fluctuations of room temperature may show variations in the capacitance of 2 or 3 parts in 10,000.

The active portion of any condenser intended for use as a standard must be firmly confined between clamps, so that its geometry and, consequently, the capacitance of the condenser may be definite. Condensers *without clamps* are greatly affected by temperature and, when taken through a cyclic variation of temperature (for instance, 17°, 30°, 17°), do not return to their initial capacitances. This permanent alteration may be as much as 3 or 4 parts in 10,000.

Condensers on Direct-current Circuits.—In the use of condensers with direct currents, difficulties arise from "absorption" and its related effects. It is found that the discharge of any condenser having a solid dielectric consists of two portions—a sudden rush of current at the instant of closing the circuit, due to the free charge; and a small, gradually decreasing current, due to the liberation of the "absorbed" charge. This latter current complicates the various methods of measurement when direct currents are employed (see "Direct-deflection Method," page 379). If high voltages be used, the absorbed charge continues to be given up for a long time.

When the condenser is charged, the first rush of current consists of two portions: one furnishing the free charge; the second, a diminishing current, furnishing the absorbed charge. This latter current, for a short time—about 0.01 sec.—may be relatively large. If the charging circuit is broken too soon, before the dielectric is "saturated," the absorption goes on; and if there is a delay in discharging the condenser, the free charge will be diminished below its proper value. Thus the apparent capacitance depends on the previous history of the condenser, on the time of charging, on the length of time between disconnecting from the battery and discharging, and on the time of discharge.

An arbitrary measure of the absorption may be obtained by subjecting the condenser to a definite series of operations, for example by charging for 1 sec., insulating for 30 sec., discharging instantaneously through a ballistic galvanometer, insulating for 30 sec., discharging again, and so on, until five residual deflections have been obtained. The measure of the absorption is the total quantity in the five residuals expressed as a fraction of the free charge. The absorption curves in Fig. 236 were obtained in this manner.

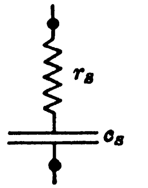
As the absorption increases very markedly with the increase in temperature, while the insulation resistance decreases, condensers are preferably used at low temperatures, about 20°.

The measured capacitances of condensers that have large absorption are greatly affected by the time of discharge.

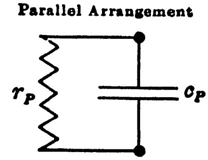
Condensers on Alternating-current Circuits.—If an air condenser, which is perfectly insulated and the resistance of whose leads is zero, is subjected to a sinusoidal potential difference, the current flowing into the condenser will lead the potential difference across its terminals by practically 90 deg. (see page 372) there being an exceedingly small expenditure of energy; if the dielectric is solid, energy is expended in the condenser, as is shown by its rise of temperature under continuous operation. If energy is expended, the current flowing into the condenser must have an energy component, or, in other words, the current and potential difference will no longer have a phase difference of 90 deg. The amount of departure from the 90-deg. relation will be denoted by ϕ; the power factor of the condenser is then $\sin \phi$. The angle ϕ, called the *phase-defect angle*, is the best single criterion as to the quality of a condenser. For a first-class instrument with mica as the dielectric, it may not be more than a few minutes of arc, possibly 5, and may be much below this. If the condenser is of poor quality with a paraffined-paper dielectric, this angle may in extreme cases be as much as 20 deg. The power factor of such inferior condensers is very sensitive to changes of frequency. It must not be assumed that mica condensers are of necessity characterized by very small phase-defect angles, for such condensers from well-known makers may occasionally show phase-defect angles of several degrees. Such abnormal values are found most frequently in the small sections (1/1,000 μf) and show the condenser to be of poor quality. In a divided condenser, the different sections may have very different phase-defect angles. The measured capacitances of condensers that have large phase-defect angles will be found to be very dependent on the frequency.

Various methods for the measurement of capacitance by means of alternating currents are to be given, and it is of practical importance to be able to apply them to condensers with imperfect dielectrics such as are met with in practice, that is, to find the equivalent capacitances of such condensers. As there is a dissipation of energy in the condenser, its equivalent arrangement should be a perfect condenser in connection with such a resistance that the energy dissipated in the combination is the same as that in the actual condenser. The energy loss may be duplicated by assuming the resistance to be either in parallel or in series with the perfect condenser, as is indicated below.

Series Arrangement



Parallel Arrangement



Given V, I, P, ω, and assuming sinusoidal currents,

$$P = I^2 r_S.$$

Therefore

$$r_S = \frac{P}{I^2}.$$

$$z_S = r_S - \frac{j}{\omega C_S}.$$

$$\tan\theta = \frac{1}{r_S \omega C_S} \text{ leading.}$$

Phase-defect angle,

$$\varphi = \tan^{-1} r_s \omega C_S.$$

$$P = \frac{V^2}{r_P}.$$

Therefore

$$r_P = \frac{V^2}{P}.$$

$$z_P = \frac{1}{\frac{1}{r_P} + j\omega C_P}.$$

$$\tan\theta = r_P \omega C_P \text{ leading.}$$

Phase-defect angle,

$$\varphi = \tan^{-1} \frac{1}{r_P \omega C_P}.$$

In cases where the dielectric losses are large, the equivalent capacitances for the series and parallel arrangements are slightly different.

$$\text{Power factor} = \text{P.F.} = \cos\theta = \frac{P}{VI}$$

$$C_S = \frac{I}{\omega V \sqrt{1 - \text{P.F.}^2}}. \qquad C_P = \frac{I}{\omega V}\sqrt{1 - \text{P.F.}^2}.$$

$$\frac{C_S}{C_P} = \frac{1}{\sin^2\theta}.$$

$$r_s = \frac{r_P}{1 + r_P{}^2\omega^2 C_P{}^2}. \qquad C_S = C_P + \frac{1}{r_P{}^2\omega^2 C_P}.$$

If $r_P{}^2\omega^2 C_P{}^2 >> 1$,

$$r_S = \frac{1}{r_P \omega^2 C_P{}^2} \quad \text{approx.} \qquad C_S = C_P \quad \text{approx.}$$

The series loss resistance r_S is commonly denoted by ρ. r_P has no relation to the insulation resistance as measured with direct current. A very small but measurable loss occurs in air condensers; it appears to be a surface phenomenon. In addition, there may be a loss in the insulation which supports the plates; consequently, the design should be such that this insulator is in a very weak electrostatic field. In variable air condensers, it is essential that the necessary insulation that supports the structure be so disposed that it is in a very weak electrostatic field which is not subject to variation when the capacitance is changed. Therefore a variable air condenser may be approximately represented as in Fig. 234.

Referring to the figure, C is a variable "no-loss" condenser formed by fixed and movable plates; C_1 is the unavoidable capacitance due to the solid insulation used in the supporting structure, ρ_1 is the equivalent series resistance necessary to represent the energy loss in this insulation. The impedance of the whole model between the terminals is

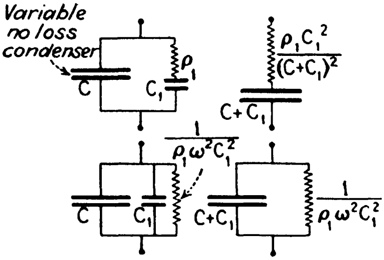


FIG. 234.—Networks approximately representing a variable air condenser.

$$z = \frac{\rho_1 C_1^2\omega - j[(C_1 + C) + \rho_1^2 C_1^2\omega^2 C]}{\omega(C_1 + C)^2 + \rho_1^2 C_1^2\omega^2 C^2\omega}.$$

ρ_1 and C_1 are small, so $\rho_1 C_1\omega$ is small, and its square still smaller; consequently,

$$z = \frac{\rho_1 C_1^2}{(C_1 + C)^2} - j\frac{1}{\omega(C_1 + C)} \qquad \text{approx.}$$

Therefore the equivalent series capacitance is $C' = C_1 + C$. The equivalent series resistance is $R' = \rho_1 C_1^2/(C_1 + C)^2 = \rho_1 C_1^2/[C']^2$. It is seen that independently of the setting, the product of the equivalent series resistance and the square of the equivalent series capacitance is

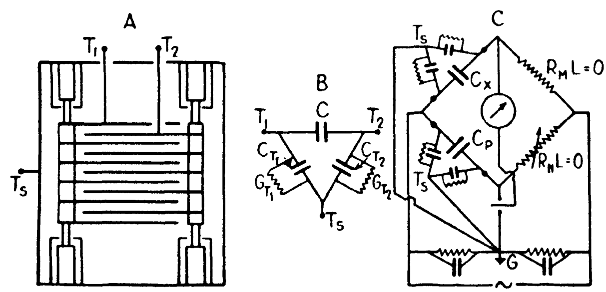


FIG. 235.—Diagrams for Ogawa no-loss air condenser.

equal to $\rho_1 C_1^2$, a constant at a given frequency. Denoting the tangent of the phase-defect angle of the whole condenser by η, the product of η and the capacitance of the whole condenser is $\eta C' = \omega\rho_1 C_1^2$, a constant at any given frequency.

In the Ogawa form of secondary air condenser,[8] no solid dielectric is interposed between the active plates which, as shown in Fig. 235A, are supported from a shield kept at ground potential by connection to the Wagner earthing device. The insulators supporting the active plates are surrounded by grounded shields. Figure 235B shows the equivalent network. C is the active capacitance; C_{T_1} and C_{T_2}, the capacitances between the active plates and the shield; G_{T_1} and G_{T_2}, their associated conductances (see page 368). Ogawa has used this arrangement for both fixed and variable condensers. Figure 235*C* shows two Ogawa condensers, represented by their equivalent networks, inserted in an impedance bridge with truly nonreactive ratio arms. It is seen that C_{T_1} and C_{T_2} either form parts of the grounding device or are in parallel with the detector. Hence for this ideal case

$$C_x = C_P\left(\frac{R_N}{R_M}\right).$$

If it is necessary to introduce a loss, C may be shunted by a suitable high resistance connected between T_1 and T_2.

Mica Condensers.[12]—Mica is used as the dielectric in the best working standards of capacitance. Its specific resistance is about 1×10^{10} (megohms, cm.). Its dielectric coefficient varies between 6 and 8. The puncturing voltage of selected specimens 0.1 mm. thick, when tested between plates, may be as high as 12,000 volts (r.m.s.). The average strength is much lower.

Mica condensers are not ideally perfect and vary greatly in their properties, so that in careful work the characteristics of the particular condenser employed as a standard must be known. However, a mica condenser always behaves in the same manner if the same conditions are maintained. For this reason, in work of high precision, the cycle of operations to which the condenser is subjected, both when its capacitance is determined and in subsequent use, should be the same. The capacitance of a good mica condenser is independent of the voltage.

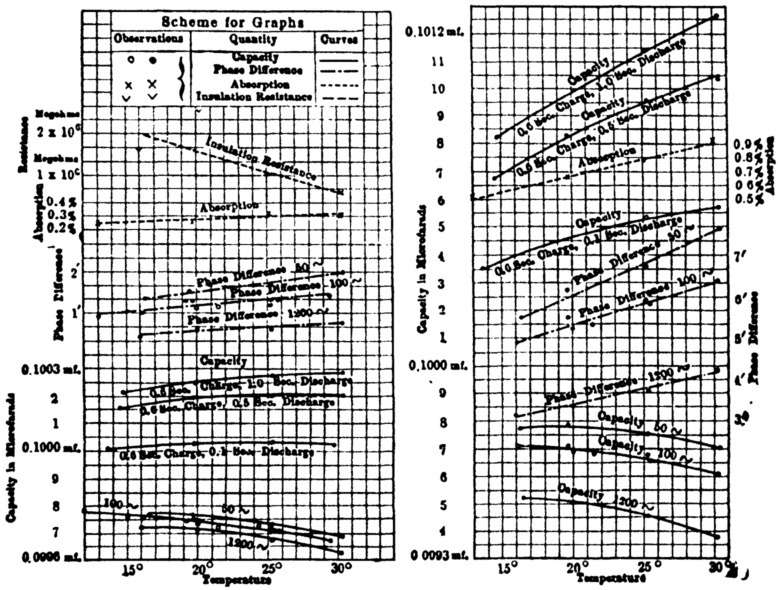
To be useful as a standard, a mica condenser must be firmly clamped.

Experiments show that the capacitance of a good mica condenser, when determined at higher and higher frequencies by a method of rapid charge and discharge using direct currents, approaches the same value as that obtained by the use of alternating currents, the period with alternating currents and the time of discharge with direct currents being the same. The two curves connecting the reciprocal of the frequency and the capacitance and the time of discharge and the capacitance, when extrapolated for infinite frequency and zero time of discharge, apparently cut the capacitance axis at the same point. The capacitance determined by this process of extrapolation is called the "instantaneous" or, by some writers, the "geometric" capacitance, being independent of absorption and depending only on the dielectric coefficient and on the dimensions of the condenser.

Changes of atmospheric pressure cause in mica condensers minute changes of capacitance which may be detected by the most refined methods of measurement. The changes are subject to a considerable time lag and may be of the order of magnitude 1 or 2 parts in 100,000 for 1 cm. change of pressure. Usually, if the pressure is reduced, the condenser expands; and as the increase in the distance between the plates produces more effect than their increase of size, the capacitance is decreased. Firmly clamped condensers are but very slightly affected.

The characteristics of good and poor mica condensers are illustrated by Fig. 236.

Condensers of silvered mica are sometimes used but are inferior to those of the ordinary construction, being more unstable and having a capacitance dependent upon the voltage. This unstable character is



A *B*

FIG. 236.—Characteristics of good (*A*) and poor (*B*) mica condensers.

probably due to deposits of silver, under flakes of mica, which are imperfectly attached to the main deposit.

Paraffined-paper Condensers.[13]—On account of the possibility of large absorption effects and frequency errors, paraffined-paper condensers should not be employed as standards. No general rule concerning their behavior can be formulated. When used with alternating current, the capacitance of a paraffined-paper condenser decreases with an increase of frequency and very markedly if the phase-defect angle is large. An increase of temperature usually causes an increase in the capacitance; for an exception, see Fig. 237. The phase-defect angle is much larger than that of a good mica condenser. It generally increases with a rise of temperature, and more and more rapidly as the temperature becomes higher. The phase-defect angle is very susceptible to changes in frequency. Usually, an increase of frequency causes a decrease in the angle. Figure 237 shows the characteristics of a good paraffined-

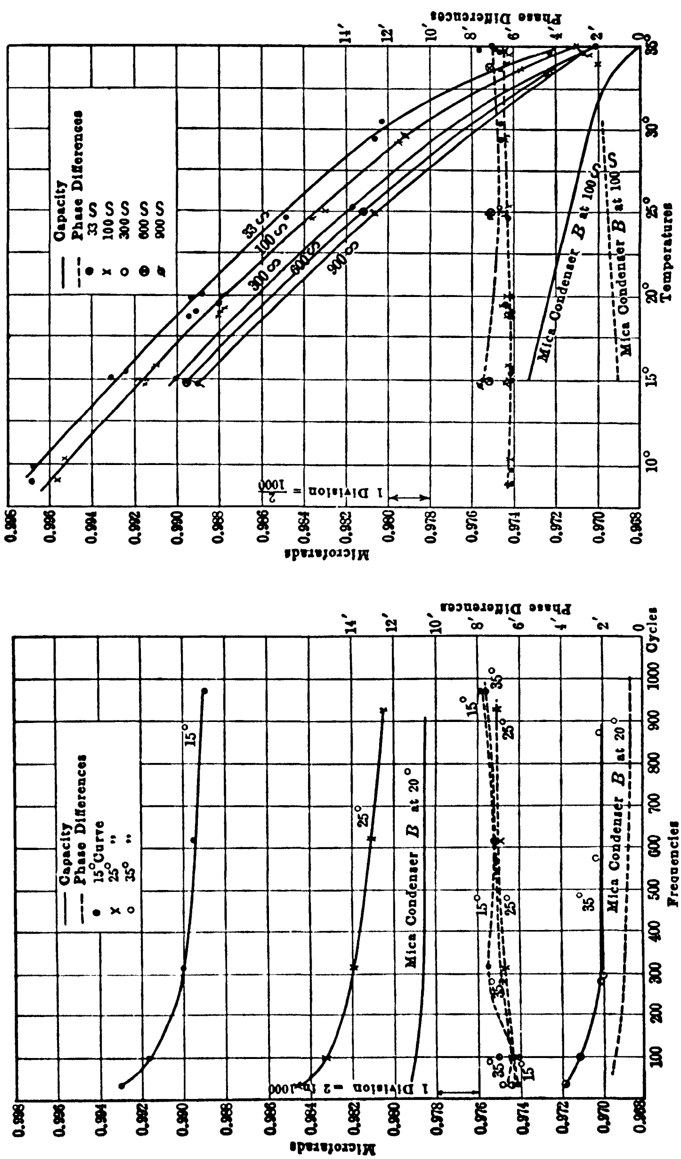


FIG. 237.—Characteristics of a good paraffined-paper condenser.

paper condenser. Figure 238 applies to a rolled condenser. An inspection of the curves will show that this is a poor condenser and that in some respects its behavior is the reverse of that of the better condenser.

The internal resistance of a condenser, that is, the resistance of the connections from the binding posts to the plates and of the plates themselves, may cause an abnormal phase-defect angle. This is the case in that form of condenser which is made by rolling up long strips of tin foil together with the paper dielectric, the electrical connections being made

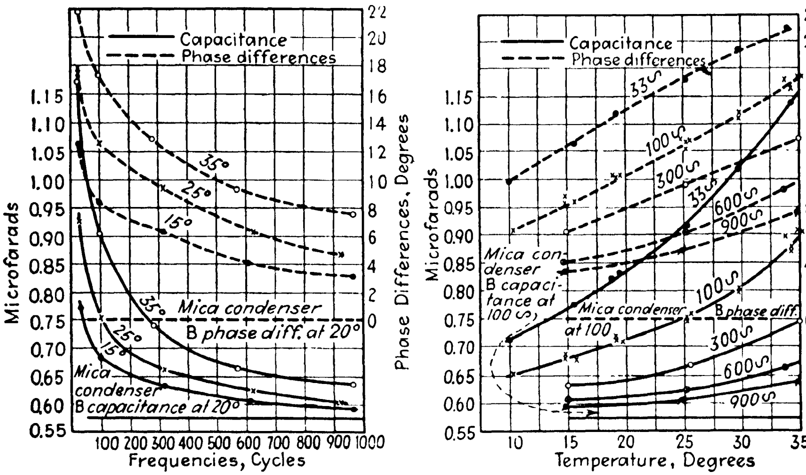


Fig. 238.—Characteristics of a poor paraffined-paper condenser.

at the ends of the strips. High internal resistance causes excessive heating and an increase of phase-defect angle with an increase of frequency. Condensers of the "short-path" type in which the terminals are soldered to the sides of the tin-foil strip show smaller errors.

Modification of Effective Capacitance and Inductance by Non-inductive Shunts.—It is sometimes desirable to be able to modify the

Effective capacitance

$$C' = \left(1 + \frac{1}{r^2w^2C^2}\right)C$$

Effective resistance

$$R' = \left(\frac{1}{1 + r^2w^2C^2}\right)r$$

Effective inductance

$$L' = \left(\frac{r^2}{(R + r)^2 + w^2L^2}\right)L$$

Effective resistance

$$R' = \left(\frac{R(R + r) + w^2L^2}{(R + r)^2 + w^2L^2}\right)r$$

Fig. 239.—Pertaining to application of non-inductive shunts to condensers and inductances.

effective value of a capacitance or an inductance that is to be used as a standard or is to be measured, in order that it may be more readily dealt with by the apparatus at hand. In some cases, this may be accomplished by the use of noninductive shunts, as will be seen by reference to Fig. 239.

Reference to the expressions of this figure shows that the successful application depends on the maintenance of a constant frequency. It is seen that the effective capacitance may be greatly increased by a low-resistance shunt, very slight changes in which produce large changes in the capacitance. Any increase in the shunt resistance lowers the effective value toward C; at the same time, the combination becomes less sensitive to resistance changes. For low values of the shunt, the effective resistance is low. It increases to a maximum when $r = 1/\omega C$ and then diminishes as r is still further increased.

It is seen that the effective inductance is always less than L, a fact that is conveniently utilized when very large inductances are to be measured. The effective resistance is small for small values of the shunt. It increases to a maximum as the shunt resistance is increased, finally diminishing toward R when r is greatly increased.

METHODS OF MEASUREMENT

Determination of Capacitance in Absolute Measure.[14]—Maxwell in his "Treatise on Electricity and Magnetism"* gives a bridge method

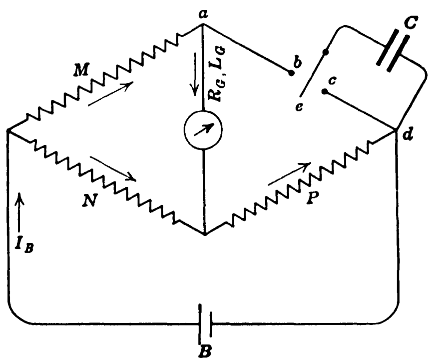


FIG. 240.—Connections for Maxwell method for determining a capacity in absolute measure.

for determining the electrostatic capacitance of a condenser in electromagnetic measure. This method has been employed in many determinations of the velecity of light and is probably the best yet devised for determining pure capacitances in absolute measure. The connections are shown in Fig. 240.

The condenser to be measured is at C; M, N, and P are the resistances of the bridge arms, B is the battery resistance, and R_G and L_G the resistance and self-inductance of the galvanometer. The resistances ab and cd are negligibly small. The condenser is rapidly charged and discharged by the switch e, which is a rotating commutator, driven at a constant and known speed, or a vibrating commutator.

When contact is made at c, the condenser is discharged and so remains until the tongue e touches b. Until e touches b, the currents in the

* Art. 776, 3d ed.

network are determined by the e.m.f. of the battery and the resistances, and a steady current flows through the galvanometer as indicated by the arrow. When contact is made with b, a varying current i_C will flow into the condenser until it is fully charged. A part of this varying current flows through M, and a part flows upward through the long-period galvanometer, tending to deflect it in a direction opposite to that produced by the steady current. When C is fully charged, the currents return again to their steady values. By properly adjusting the arms of the bridge and the number of times per second that the condenser is charged and discharged, the galvanometer can be brought to zero, though a slight and definite unsymmetrical oscillation about the zero position will persist, for the coil is subjected to an impulse followed by a push in the opposite direction during each cycle. A zero deflection means that the net quantity displaced through the galvanometer in a second is zero.

When e and b are not in contact, the currents have the steady values I_M, I_N, I_P, I_G.

Call the quantity of electricity on the condenser when it is fully charged Q_C. Then

$$Q_C = C(\text{P.D.})_{ad}$$

As the currents have arrived at the steady state,

$$\frac{I_G}{I_N} = \frac{N}{M + R_G}.$$

Consequently,

$$\frac{I_G}{I_G + I_N} = \frac{I_G}{I_P} = \frac{N}{M + R_G + N}.$$

Therefore

$$(\text{P.D.})_{ad} = I_G R_G + I_P P = I_G\left(R_G + \frac{P(M + R_G + N)}{N}\right),$$

and

$$Q_C = C\left(R_G + \frac{P(M + R_G + N)}{N}\right)I_G, \qquad (1)$$

where I_G is the galvanometer current when the circuit is in the steady state.

When e touches b, a varying current i_C, which finally becomes zero, flows into the condenser. and all the other currents are temporarily altered.

Let the alteration in I_M be δI_M
in I_N be δI_N
in I_P be δI_P
in I_B be δI_B
in I_G be δI_G.

If Q_G is the quantity of electricity displaced through the galvanometer by the current δI_G at each contact of e and b, and there are n such contacts per second, the quantity per second so displaced will be nQ_G.

Therefore, if the galvanometer stands at zero under the combined action of I_G and nQ_G, the average current is zero.

$$I_G + nQ_G = 0,$$

and

$$Q_C = -Cn\left(R_G + \frac{P(M + R_G + N)}{N}\right)Q_G. \qquad (2)$$

To find Q_G, the quantity displaced through the galvanometer by the variable current δI_G when the condenser is charged, suppose C to be discharged and the steady currents to be flowing as indicated in Fig. 240. The potential differences between the terminals of all the resistances will have definite values. As soon as e makes contact with b, the varying current i_C flows into the condenser, causing temporary alterations in the currents through M, N, R_G, B, and P. The change in potential difference between the terminals of any one of the resistances is the change in the current multiplied by the resistance. The currents δI_B, δI_G, etc., are variable and become zero when the condenser is fully charged.

Referring to Fig. 240, it will be seen that

$$\begin{aligned} \delta I_M &= i_C + \delta I_G. \\ \delta I_B &= \delta I_M + \delta I_N. \\ \delta I_P &= \delta I_B - i_C. \\ \delta I_P &= \delta I_G + \delta I_N. \\ \delta I_N &= \delta I_B - i_C - \delta I_G. \end{aligned}$$

Using the changes in the currents, Kirchhoff's laws may be applied to the meshes $M\ R_G\ N$ and $B\ N\ P$; and by using the preceding relations, the resulting equations may be expressed in terms of the resistances and the variable currents δI_B, δI_G, and i_C.

For the mesh $M\ R_G\ N$ at any instant during charging,

$$M(\delta I_M) + R_G(\delta I_G) + \frac{L_G d(\delta I_G)}{dt} - N(\delta I_N) = 0,$$

or

$$M(i_C + \delta I_G) + R_G(\delta I_G) + \frac{L d(\delta I_G)}{dt} - N(\delta I_B - i_C - \delta I_G) = 0.$$

Uniting terms gives

$$i_C(M + N) + \delta I_G(M + R_G + N) - N(\delta I_B) + \frac{Ld(\delta I_G)}{dt} = 0. \quad (3)$$

For the mesh $B\ N\ P$ at any instant during charging,

$$B(\delta I_B) + N(\delta I_B - i_C - \delta I_G) + P(\delta I_B - i_C) = 0,$$

or, uniting terms,

$$-i_C(N + P) - \delta I_G(N) + \delta I_B(N + P + B) = 0. \quad (4)$$

Equations (3) and (4) may be integrated to obtain the total quantities displaced by the variable currents during the charging of the condenser; δI_G is zero at both limits. Therefore

$$Q_C(M + N) + Q_G(M + R_G + N) - Q_B(N) = 0, \quad (3a)$$

and

$$-Q_C(N + P) - Q_G(N) + Q_B(N + P + B) = 0. \quad (4a)$$

Eliminating Q_B gives

$$Q_C = \left[\frac{M + R_G + N - \dfrac{N^2}{N + P + B}}{\dfrac{N(N + P)}{N + P + B} - (M + N)}\right] Q_G. \quad (5)$$

The values of Q_C from (2) and (5) may be equated, thus eliminating Q_G, and the resulting equation solved for C.

$$C = \frac{N}{nPM}\left[\frac{1 - \dfrac{N^2}{(B + N + P)(M + R_G + N)}}{\left(1 + \dfrac{NB}{M(B + N + P)}\right)\left(1 + \dfrac{NR_G}{P(M + R_G + N)}\right)}\right]. \quad (6)$$

With very small capacitances it is necessary to use a high value of n (500 cycles per second) and high voltages (100 to 200 volts) in order to obtain sufficient sensitiveness.

The capacitance as determined includes that of the commutator and the leads to the condenser under measurement. The correction due to these capacitances is determined by a separate measurement, the leads being disconnected from the condenser without altering their position more than is absolutely necessary.

Curtis and Moon have lately examined the method both theoretically and experimentally[14] and conclude from their experience with two independently assembled bridges, one employing a rotating, the other a vibrating, commutator, that under favorable conditions the error in the measurement of a 0.1 μf capacitance is 2 or 3 parts in 100,000. The source of the error is obscure.

Direct-deflection Method for Comparing Capacitances.—The most obvious method for comparing the capacitances of two condensers is to charge the condensers from the same battery and determine the relative quantities that they have accumulated. This may be done either by charging or by discharging them in turn through the same ballistic galvanometer. To carry out this test in its simplest form, the connections shown in Fig. 241 may be used.

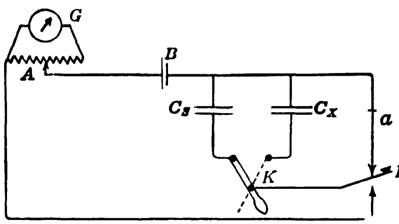


FIG. 241.—Connections for direct-deflection method for comparing capacitances.

When K is thrown to the right-hand stop, and K_1 is depressed, the condenser C_X is charged through the ballistic galvanometer, giving rise to a deflection θ_{1X}; this must be corrected for the multiplying power of the shunt m_X. When K_1 is released against the back stop, C_X is discharged. A similar procedure with K thrown to the left gives θ_{1S}, the multiplying power of the shunt now being m_S. If the damping of the galvanometer is the same in the two cases,

$$\frac{C_X}{C_S} = \frac{Q_X}{Q_S} = \frac{\theta_{1X}m_X}{\theta_{1S}m_S},$$

or

$$C_X = \frac{\theta_{1X}m_X}{\theta_{1S}m_S}C_S.$$

If the damping is not the same in the two cases, the deflections must be corrected (see page 103). Constant damping may be attained by use of the Ayrton universal shunt A.

If the standard and the unknown are of very different capacitance, instead of shunting the galvanometer, different known voltages may be used in the two tests, and an allowance made. This procedure is convenient if an Ayrton shunt is not at hand.

The proper value of the standard is one that will give a deflection about equal to that due to the unknown capacitance. Enough battery should be used so that the deflections may be read with good precision.

If the galvanometer is placed at a, the capacitances may be compared by discharging the condensers.

In arranging the apparatus, care must be taken that the capacitances to earth of long leads and of the instruments, as well as the capacitances between the leads to the condensers, do not introduce errors. For instance, the leads from K_1 to the condensers should be short, of small wire, and well separated from the leads to the other side of the condenser; otherwise, a separate test must be made to determine their capacitance. To prevent errors from leakage, the battery and all the wiring should be well insulated, especially the keys K and K_1 and the leads from K_1 to the condensers via K.

Fleming and Clinton have devised a rotating commutator[15] by means of which the discharges may be sent through the galvanometer in such rapid succession that the instrument takes up a steady deflection.

Sources of Error.—In reality, this test is not so simple as might appear, for the assumption has been tacitly made that the condenser is entirely charged or discharged before the needle of the ballistic galvanometer has moved appreciably. Reference to the section on con-

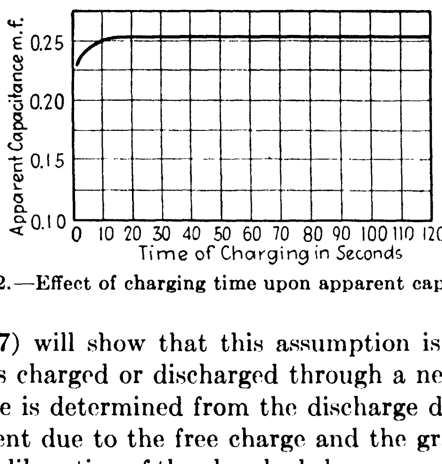


FIG. 242.—Effect of charging time upon apparent capacitance.

densers (page 367) will show that this assumption is strictly true only for air condensers charged or discharged through a negligible resistance. If the capacitance is determined from the discharge deflection, both the first rush of current due to the free charge and the gradually decreasing current due to the liberation of the absorbed charge are active in producing the deflection, and the galvanometer needle is acted on by two forces—a sudden blow due to the passage of the free charge and a long-continued and gradually diminishing push due to the absorbed charge. Therefore, when there is considerable absorption, the apparent capacitance as determined by this simple method is dependent on the period of the galvanometer employed. In working with the discharge deflection, any delay after the condenser is disconnected from the battery and before it is connected to the galvanometer will cause an error if the time of charging has not been sufficiently long for the dielectric to become saturated, for absorption goes on during this delay, thus reducing the free charge.

In industrial testing, the direct-deflection method is very commonly applied to cables, but it is obvious that to obtain results that are of value as a basis of comparison between samples that are nominally the same, some definite procedure must be adopted.

This point is emphasized by Fig. 242, which shows the apparent capacitance, on discharge, of a piece of rubber-covered wire when subjected to different times of charging. Specifications for rubber-covered wires commonly call for a charging period of 10 sec.

The Zeleny Discharge Key.—In order to obtain the free charge capacitance of a condenser with an imperfect dielectric, the condenser must be disconnected from the galvanometer after it has parted with its free charge and before any appreciable portion of the absorbed charge has been given up. This may be accomplished by the Zeleny discharge key,[12] shown diagrammatically in Fig. 243. In this key, there are three flexible leaves L_1, L_2, and L_3. As shown, the condenser C is being charged from the battery B. When the key is depressed, the battery circuit is broken at a and the discharge circuit completed at b. On continuing the depression, mechanical contact is made at c, and the discharge circuit broken at d. The period of delay before the galvanometer is taken out of circuit is con-

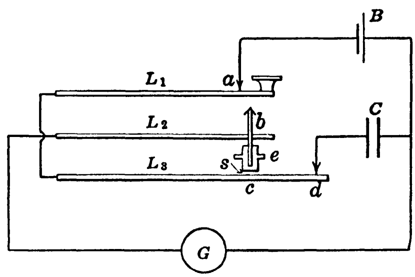


FIG. 243.—Diagram for Zeleny discharge key.

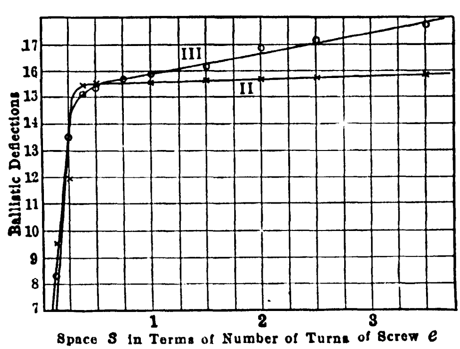


FIG. 244.—Illustrating results attained by use of Zeleny discharge key.

trolled by varying the distance s, by turning the milled head e. The key must be kept depressed until the first elongation has been completed.

In using this key, one starts with the distance s large (5 mm.) and, maintaining as nearly as may be a constant velocity of tapping, successive throws of the galvanometer needle observed as s is diminished. The result will be as shown in Fig. 244. The deflections fall off regularly until the point is reached where sufficient time has not been allowed for the condenser to part with its free charge. After this, the deflections rapidly decrease, and the ordinary variations in the velocity of tapping begin to cause irregularities. The results are independent of the period of the galvanometer.

Thomson Method for Comparing Capacitances.[16]—If two condensers are charged with equal quantities of electricity, the voltages required will be inversely as the capacitances. To take advantage of this relation, a ready means must be provided for indicating when the charges are equal and for showing the relative voltages applied to the condensers. The arrangement shown in Fig. 245 is that necessary for carrying out the measurements according to Thomson's method. The battery current flows through R_1 and R_2 in series. If K_1 and K_2 are both depressed at the same time, C_X is charged to a voltage IR_1 while C_P is charged to a voltage IR_2. When K_1 and K_2 are released and make contact with their backstops, the condensers are connected in series + to −, so that their charges tend to neutralize each other, or "mix." To see if the neutralization is perfect, that is, to see if there were equal quantities on the two condensers, K_3 is depressed, and the unneutralized portion of the charge sent through the galvanometer, the deflection of which will be to the right or to the left according to which charge preponderates. By successive trials, altering R_1/R_2, the galvanometer deflection may be reduced to zero. Then

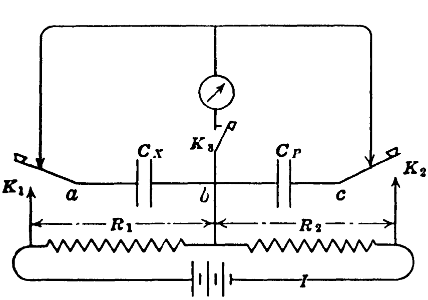


FIG. 245.—Connection for Thomson method of comparing capacitances.

$$IR_1C_X = IR_2C_P;$$
$$C_X = C_P\frac{R_2}{R_1}.$$

To carry out the test, a special key which combines K_1, K_2, and K_3 on a single base is usually employed. In manipulating this key, care must be taken that it perform its functions properly and that cross-contacts by the fingers of the observer be avoided.

To obtain a good precision, the variable resistances R_1 and R_2 should be high, so that their adjustment may be sufficiently flexible. If resistance boxes are used, the smallest step is usually 1 ohm; so when comparing ordinary condensers, the resistance which is adjusted should be at least 1,000 ohms. In submarine cable work, much higher resistances are employed; $R_1 + R_2$ may be as much as 100,000 ohms. The galvanometer must be very sensitive, and the battery voltage as high as is consistent with safety of the apparatus. With high voltages, to avoid throwing an unduly large potential on either condenser, the known and unknown capacitances should be about the same.

When cables are tested, the core is connected to *a*, and *b* then becomes the common "ground"; in this case, perfect insulation of the battery is essential.

The resistances R_1 and R_2 and their leads must be thoroughly insulated.

The effect of any considerable leakage in C_X during the period of charging is practically to shunt R_1 by the leakage resistance. To get equal quantities on the condensers, R_1 as inserted in the resistance box must be made larger than if the condenser had been of the same capacitance but devoid of leakage. Thus the apparent capacitance will be too small.

In this case, the procedure recommended is to use, instead of the resistance inserted in the box, the parallel resistance due to R_1 and the condenser under test. This necessitates a measurement of the insulation resistance of the condenser. The period between charging and mixing must be made as short as possible, since leakage during this time will cause the charge on the unknown to be too small, which will necessitate an increase in R_1 above the proper value. Leakage during mixing will reduce, by shunting, the quantity to be discharged through the galvanometer and will thus diminish the sensitiveness of the method.

If absorption is present, it is necessary to adopt a definite cycle of operations in order to obtain comparable results, for the behavior of an imperfect condenser depends to a certain extent on its previous history, that is, on the voltage to which it has been subjected, the time of electrification, and the duration of the charging and mixing periods, together with the completeness of the discharge.

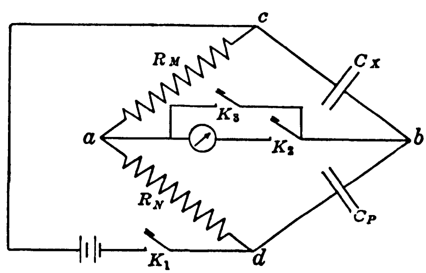


FIG. 246.—Connections for Gott method for comparing capacitances.

As the absorbed charges reappear gradually, and as it is the free charges that are to be neutralized, the key K_3 must be closed for only an instant, when observing the galvanometer. The condensers should be completely discharged between the observations.

Gott Method for Comparing Capacitances.—The connections for Gott's method are shown in Fig. 246. If condensers are being compared, *acbd* including the galvanometer circuit is a perfectly insulated system. The condensers being discharged, the key K_1 is depressed, thus sending a current through the circuit *cad* and charging the condensers C_X and

C_P in series. Then

$$\frac{V_{da}}{V_{ac}} = \frac{R_N}{R_M},$$

and

$$C_X = C_P\frac{V_{db}}{V_{bc}}.$$

If K_2 is depressed, still keeping K_1 closed, there will, in general, be a deflection of the galvanometer, due to the difference in the potentials of a and b. If there is a deflection, K_1 and K_2 are raised, and the condensers completely discharged by the use of K_3. After this, another test is made with a different value of R_N/R_M. By successive trials, adjusting either R_M or R_N, the deflection due to the difference of potential of a and b at the instant of closing K_2 may be reduced to zero. When the adjustment is complete,

$$C_X = C_P\frac{R_N}{R_M}.$$

As anything that gives rise to a false distribution of potentials in the network *acbd* will cause errors, all parts must be carefully insulated, and K_2 especially, as it is handled. Leakage within the battery or between the battery leads is not a source of error, as it merely alters the potential difference applied to the network. Direct leakage from the battery to the condenser circuits must be avoided. Any leakage between the terminals of either condenser is a source of error; this leakage may be through the dielectric of the condenser or between the leads. If C_X is imperfect in this respect, and K_1 is kept depressed, the potential of b gradually approaches that of c. The value found for C_X will then be too great and will increase with the time of charging.

When cables are measured, the core should be connected to b, as the sheath is necessarily grounded. Because of the small effect of battery leakage, this method is very commonly employed in submarine cable work.

If the rates of absorption of the two condensers are not the same, the results obtained will be dependent on the time of charging. For condensers, and for cables up to a length of about 1,000 nautical miles, a correction devised by Muirhead and intended to correct for both absorption and leakage is applicable; it fails, however, in the case of cables of greater length, supposedly on account of the retardation due to the resistance of the cable itself.

The methods of Thomson and of Gott are of importance in submarine cable testing.[17] In this work, exceedingly large capacitances must be measured—frequently several hundred microfarads. It is in connection with these measurements that the complications due to leakage and absorption become most troublesome.

Elementary Methods of Determining Inductance and Capacitance by Alternating Currents.[16]—If a sinusoidal current of known frequency is used, the most obvious method of measuring an inductance, which is uninfluenced by the current strength, is to determine the current and the potential difference between the terminals of the coil. Then, if ω is 2π times the frequency; R, the resistance; V, the applied voltage; and I, the current, the inductance is given by

$$L = \frac{\sqrt{V^2 - I^2R^2}}{\omega I}$$

or, if the resistance is negligible, by

$$L = \frac{V}{\omega I}.$$

When the current wave is nonsinusoidal, it is possible to allow for the effect of the harmonics. Suppose that the maximum values of the various components are I_1, I_3, I_5, etc. Then

$$i = I_1 \sin \omega t + I_3 \sin (3\omega t - \theta_3) + I_5 \sin (5\omega t - \theta_5) + \cdots .$$

The effective value of the current will be

$$I = \sqrt{\frac{I_1^2}{2} + \frac{I_3^2}{2} + \frac{I_5^2}{2} + \cdots} .$$

The fundamental equation for the flow of current through an inductive resistance is

$$v = Ri + L\frac{di}{dt}.$$

From the foregoing,

$$v = R[I_1 \sin \omega t + I_3 \sin (3\omega t - \theta_3) + I_5 \sin (5\omega t - \theta_5) + \cdots]$$
$$+ \omega L[I_1 \cos \omega t + 3I_3 \cos (3\omega t - \theta_3) + 5I_5 \sin (5\omega t - \theta_5) + \cdots].$$

The mean square value of the potential difference will be

$$V^2 = I^2R^2 + \omega^2L^2\left[\frac{I_1^2}{2} + 9\frac{I_3^2}{2} + 25\frac{I_5^2}{2} + \cdots \right].$$

Therefore,

$$L = \frac{\sqrt{V^2 - I^2R^2}}{\omega\sqrt{\frac{I_1^2}{2} + 9\frac{I_3^2}{2} + 25\frac{I_5^2}{2} + \cdots}}$$
$$= \frac{\sqrt{V^2 - I^2R^2}}{\omega I}\sqrt{\frac{I_1^2 + I_3^2 + I_5^2 + \cdots}{I_1^2 + 9I_3^2 + 25I_5^2 + \cdots}}. \tag{7}$$

When the resistance is negligible,

$$v = L\frac{di}{dt}.$$

In this case, suppose that the potential difference wave has been analyzed. Then, at any instant,

$$v = V_1 \sin \omega t + V_3 \sin (3\omega t - \theta_3) + V_5 \sin (5\omega t - \theta_5) + \cdots.$$

Therefore,

$$-Li = \frac{V_1}{\omega} \cos \omega t + \frac{V_3}{3\omega} \cos (3\omega t - \theta_3) + \frac{V_5}{5\omega} \cos (5\omega t - \theta_5) + \cdots.$$

The mean square value of Li will be

$$L^2I^2 = \frac{1}{\omega^2}\cdot\frac{V_1^2}{2} + \frac{1}{9\omega^2}\cdot\frac{V_3^2}{2} + \frac{1}{25\omega^2}\cdot\frac{V_5^2}{2} + \cdots,$$

and

$$L = \frac{V}{I\omega V}\sqrt{\frac{V_1^2}{2} + \frac{1}{9}\cdot\frac{V_3^2}{2} + \frac{1}{25}\cdot\frac{V_5^2}{2} + \cdots},$$

or

$$L = \frac{V}{I\omega}\sqrt{\frac{V_1^2 + \frac{1}{9}V_3^2 + \frac{1}{25}V_5^2 + \cdots}{V_1^2 + V_3^2 + V_5^2 +}}. \tag{8}$$

Capacitance Measurements.—The current through a condenser with a perfect dielectric is given by

$$i = C\frac{dv}{dt}.$$

Assuming sinusoidal currents, the capacitance is

$$C = \frac{I}{\omega V}.$$

If the applied e.m.f. is not sinusoidal, the harmonics will be exaggerated in the current wave, for the current is
$i = \omega C[V_1 \cos \omega t + 3V_3 \cos (3\omega t - \theta_3) + 5V_5 \cos (5\omega t - \theta_5) + \cdots].$
Using root-mean-square values,

$$I = \omega C\sqrt{\frac{V_1^2}{2} + \frac{9V_3^2}{2} + \frac{25V_5^2}{2} + \cdots}.$$

Therefore,

$$C = \frac{I}{\omega V}\sqrt{\frac{V_1^2 + V_3^2 + V_5^2 + \cdots}{V_1^2 + 9V_3^2 + 25V_5^2 + \cdots}}. \tag{9}$$

Corollaries from Ohm's Law. Transformations.—From the experimentally established law of Ohm and the principle of continuity come a number of corollaries, some of which are essential and some convenient when dealing with networks. The most important of these are Kirchhoff's rules which have been used repeatedly in previous pages. When a linear alternating-current network with lumped constants is in the steady state, Kirchhoff's rules may be applied if the e.m.fs. and currents are put in symbolic form and complex impedance operators are used in place of resistances.

Any solution for a direct-current network is applicable to a steady-state alternating-current network of the same configuration if the e.m.fs. and currents are symbolically expressed and the resistances of the various branches are replaced by the corresponding impedance operators. For instance, the galvanometer current in a Wheatstone bridge is

$$Y = \frac{E(R_N R_X - R_M R_P)}{D_{\text{a function of its resistances}}}.$$

For the impedance bridge this becomes

$$Y = \frac{E(z_N z_X - z_M z_P)}{D_{\text{a function of the complex impedances}}}.$$

Superposition.—An e.m.f. acting in a network having constant lumped parameters which are independent of current produces the same effect whether it acts alone or in conjunction with other e.m.fs. Accordingly, when several e.m.fs. act simultaneously in a *linear* network, their combined effect is found by calculating the effect of each one separately and taking the algebraic sum. This is a general principle which has innumerable applications in engineering and physics. It can be proved for linear electrical networks by applying the theory of simultaneous linear differential equations to the mesh equations.

Reciprocal Theorem.—The current in branch *a* of a network due to an e.m.f. in branch *b* is equal to the current in branch *b* due to an equal e.m.f. in branch *a*. This may be shown by writing the solution for the network in the determinant form for the two cases. The solutions will differ only in the interchange of the rows and columns in the numerator of the resulting determinant. It is not to be inferred that the *sensitivity* of a measuring network, for instance a Wheatstone bridge, is not altered by interchanging the battery and the galvanometer, for they both possess resistance, and the parameters in the mesh equations would be altered by the interchange.

Thévenin's Theorem.[18]**—Sometimes Referred to as Pollard's Theorem.**—Although this theorem was published by Thévenin in 1883 and was frequently used by the engineers of the American Telephone and

Telegraph Company from about 1904, its existence was almost forgotten, and its usefulness was appreciated by few outside that company until Wenner employed it in 1927 in his paper "A Principle Governing the Distribution of Current in a System of Linear Conductors."[18]

Theorem.—The current in any branch of a network is that which would result if an e.m.f., equal to the potential difference that would appear across the break if the branch were opened, were introduced into the branch, all other e.m.fs. being suppressed. As an example, take the Poggendorf network, shown in Fig. 247. To find the galvanometer current, the circuit is supposed to be opened at a. The voltage that appears across the break is the Thévenin voltage

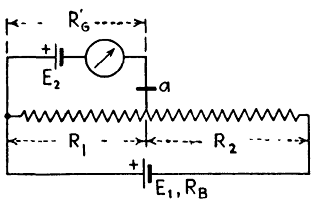


FIG. 247.—Illustrating application of Thévenin's theorem.

$$E_T = \frac{E_1R_1}{R_1 + R_2 + R_B} - E_2.$$

By the theorem, the galvanometer current is equal to this voltage divided by the resistance of the network measured from a, E_1 and E_2 being suppressed; therefore

$$I_G = \frac{\dfrac{E_1R_1}{R_1 + R_2 + R_B} - E_2}{R_G + \dfrac{R_1(R_2 + R_B)}{R_1 + R_2 + R_B}} = \frac{E_1R_1 - E_2(R_1 + R_2 + R_B)}{R_G(R_1 + R_2 + R_B) + R_1(R_2 + R_B)}.$$

Evidently the balance condition is $E_T = 0$.

Wenner's Proof.—(1) Each e.m.f. in a network produces its effect independently of all other e.m.fs., and the current in any branch is the algebraic sum of the currents in that branch that would be caused by all the e.m.fs. in the network taken separately. (2) From 1 it follows that if in any branch there is introduced an e.m.f. of the proper sign and magnitude to reduce the branch current to zero, the current in the branch caused by the e.m.f. is equal in magnitude but opposite in sign to the branch current caused by all the other e.m.fs. acting in conjunction. (3) With the current in any branch equal to zero, the branch may be opened without changing the current in any branch or the potential difference between any two points in the network. Consequently, for the condition given in (2), opening the branch causes no potential difference to appear across the break. (4) With the branch open, any or all of the e.m.fs. in it may be removed without changing the current in any branch of the network but with the appearance of a potential difference across the break equal in magnitude but opposite in sign to the e.m.f. removed. (5) Consequently, when a branch is opened, the potential difference that appears across the break is equal in magnitude but opposite in sign to the e.m.f. that, if placed in the branch, would reduce the current to zero. (6) Therefore if an e.m.f. equal to the potential difference that would appear across the break, should the branch be opened, were introduced into the branch under consideration, all other e.m.fs. regardless of their location, being suppressed, leaving all branches closed, the current would be the same as that caused by the combined effect of all the actual e.m.fs.

G. A. Campbell Transformation of a Star Arrangement of Admittances to the Equivalent Arrangement of Direct Admittances.[19]—The solution of a number of network problems is facilitated by the use of this transformation which enables one to substitute for a star of admittances an equivalent arrangement of direct admittances (page 425), thus perhaps avoiding the direct application of Kirchhoff's rules. The most familiar transformation of this sort is that for replacing a three-pointed star of admittances, or, in this case, of impedances, by three direct admittances. This is a special case of the more general transformation for an n-pointed star. This transformation was used without comment by G. A. Campbell in his 1904 paper on the shielded balance. It apparently was not generally known until 1924 when Rosen recalled attention to it.

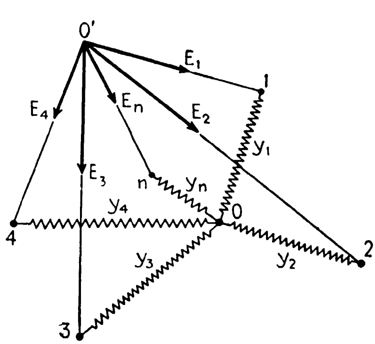


FIG. 248.—Pertaining to G. A. Campbell star to direct-admittances transformation.

Referring to Fig. 248, the component admittances of the star are $y_1, y_2, y_3, \ldots, y_n$. The rises of potential from the common point 0 to 1, 2, 3, . . . , n are $E_1, E_2, E_3, \ldots, E_n$. Suppose all the rises of potential except E_1 to be suppressed, and denote by $(I_1)_{E_1}$ the current that flows in at 1 due to E_1. The admittance from 1 to 0 is

$$\frac{y_1(y_2 + y_3 + \cdots y_n)}{y_1 + y_2 + y_3 + \cdots y_n} = \frac{y_1(y_2 + y_3 + \cdots y_n)}{\Sigma y}.$$

Therefore

$$(I_1)_{E_1} = \frac{E_1 y_1(y_2 + y_3 + \cdots y_n)}{\Sigma y}.$$

Next, suppose that all the rises of potential except E_2 are suppressed; then

$$\frac{V_{20}}{V_{01}} = \frac{y_1 + y_3 + y_4 \cdots y_n}{y_2}.$$

$$\frac{V_{20} + V_{01}}{V_{01}} = \frac{\Sigma y}{y_2} = \frac{E_2}{V_{01}}.$$

$$V_{01} = \frac{E_2 y_2}{\Sigma y}.$$

Therefore the current that flows out at 1 due to E_2 is

$$(I_1)_{E2} = \frac{E_2 y_2 y_1}{\Sigma y}.$$

Similarly, the currents that will flow out at 1 due to E_3, E_4, . . . , E_n acting separately are

$$(I_1)_{E_3} = \frac{E_3 y_3 y_1}{\Sigma y}.$$

$$(I_1)_{E_4} = \frac{E_4 y_4 y_1}{\Sigma y}.$$

$$(I_1)_{E_n} = \frac{E_n y_n y_1}{\Sigma y}.$$

The total current that will flow in at 1 due to the simultaneous action of E_1, E_2, E_3, . . . , E_n is

$$I_1 = (I_1)_{E_1} - (I_1)_{E_2} - (I_1)_{E_3} \cdot \cdot \cdot - (I_1)_{E_n}.$$

$$I_1 = \frac{(E_1 - E_2)y_2 y_1}{\Sigma y} + \frac{(E_1 - E_3)y_3 y_1}{\Sigma y} + \frac{(E_1 - E_4)y_4 y_1}{\Sigma y} \cdot \cdot \cdot \frac{(E_1 - E_n)y_n y_1}{\Sigma y}.$$

In a similar manner, the currents that will flow in at points 2, 3, n are

$$I_2 = \frac{(E_2 - E_1)y_1 y_2}{\Sigma y} + \frac{(E_2 - E_3)y_3 y_2}{\Sigma y} + \cdot \cdot \frac{(E_2 - E_n)y_n y_2}{\Sigma y}.$$

. . . .

$$I_n = \frac{(E_n - E_1)y_1 y_n}{\Sigma y} + \frac{(E_n - E_2)y_2 y_n}{\Sigma y} + \cdot \cdot \cdot \frac{(E_n - E_3)y_3 y_n}{\Sigma n}.$$

That is, the effect of a star of admittances external to its points is the same as that of a mesh-with-chords arrangement of direct admittances formed by joining every point of the star with every other point by an admittance of the form $y_{ab} = y_a y_b / \Sigma y$, where y_a and y_b are the admittances terminating at the points a and b, and Σy is the sum of all the admittances terminating at the middle of the star. This transformation is always possible.

The number of different elements of direct admittance necessary to replace an n-pointed star is $n(n - 1)/2$, for any point can be connected to all the other points by $(n - 1)$ admittances, and the process repeated for each of the n points. However, when this is done, each admittance will be counted twice. As an example, a four-pointed star of admittances is replaceable for all external points by six direct admittances. This particular transformation is useful when discussing earth capacitance effects in impedance bridges.

The transformation when $n = 3$ is very useful. It was treated as an isolated case by Kennelly[19] in 1899. By the general theorem,

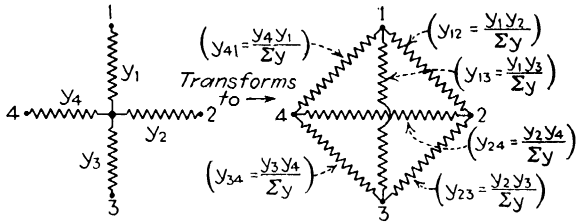


FIG. 249.—Transformation of four-pointed star of admittances.

If n is greater than 3, a simple mesh-connected system, that is, one without chords, cannot be transformed into an equivalent star-connected system. As there are n points, the number of elements in the mesh is only n instead of $n(n - 1)/2$.

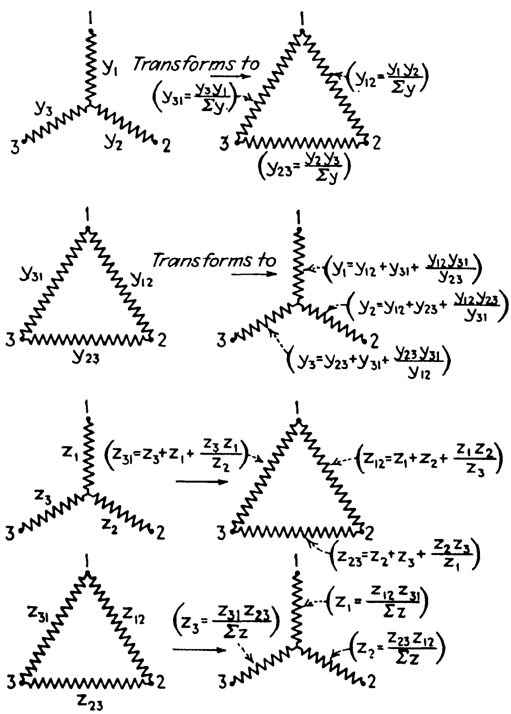


FIG. 250.—Transformations for three-element network.

If chords are present, the transformation is practically impossible on account of the limitations imposed.

For a further discussion of equivalent networks, see the paper by Starr.[19]

G. A. Campbell Star Transformation for a Three-terminal Mutual Inductance.—This transformation was in use by engineers of the Bell Telephone Company in the latter part of 1899.[20] As given below, it is subject to two conditions.

1. The primary and secondary coils have one common terminal.
2. The mutual and self-inductance fluxes are opposed.

Referring to Fig. 251, the terminal, common to both the primary and secondary circuits, is at A. With the windings as indicated, the mutual flux from one coil opposes the self-inductance flux in the other coil. The impedance operators z_a, z_A, z_b are to be so defined that their reaction

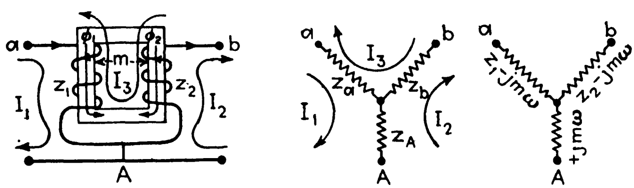


FIG. 251.—Pertaining to star equivalent of a three-terminal mutual inductance.

on a network external to a, A, b is to be the same as that of the mutual inductance. Referring to the figure,

$$I_1z_1 + I_2(-jm\omega) + I_3(-z_1 + jm\omega) = \text{fall of voltage } a \text{ to } A.$$
$$I_1(-jm\omega) + I_2z_2 + I_3(-z_2 + jm\omega) = \text{fall of voltage } A \text{ to } b.$$

Referring to the equivalent star of impedances,

$$I_1(z_a + z_A) - I_2z_A - I_3z_a = \text{fall of voltage } a \text{ to } A.$$
$$-I_1z_A + I_2(z_A + z_b) - I_3z_b = \text{fall of voltage } A \text{ to } b.$$

Equating the coefficients of like terms in I gives, for the impedance operators of the star,

$$z_a = z_1 - jm\omega,$$
$$z_A = jm\omega,$$
$$z_b = z_2 - jm\omega,$$

as indicated in the figure at the right.

It is seen that the impedance operator between a and b is

$$z_a + z_b = z_1 + z_2 - 2jm\omega;$$

therefore the equivalent inductance between a and b is $L = L_1 + L_2 - 2m$. The impedance operator between a and A is $z_a + z_A = z_1$ and that between A and b is $z_A + z_b = z_2$. These are the relations that should hold as the self- and mutual fluxes are opposed.

BRIDGE MEASUREMENTS OF CAPACITANCE AND INDUCTANCE

As originally devised, many of the methods for comparing capacitances and for comparing inductances, as well as methods for determining an inductance in terms of a capacitance, depended on the employment of variable currents. As the industrial uses of alternating currents have developed, especially in connection with telephony, it has become important that tests be made under conditions as nearly as possible those pertaining to the ordinary use of the apparatus. Hence, alternating currents have replaced the variable currents formerly employed, and the methods for capacitance measurement have been so modified that they give data of value, in addition to determining the capacitance of the condenser under measurement.

Condition for Zero Indication of Detector.—When variable currents are used in balance methods for measuring inductance and capacitance, a long-period galvanometer is employed as the detector. The arrangement of the circuits is such that at balance no permanent current flows through the instrument; this being so:

1. The deflection will certainly be zero if no current flows through the galvanometer at any time during the establishment of the permanent state of the circuit.

2. Presumably, the deflection will also be zero when the net quantity of electricity displaced through the instrument during the establishment of the permanent state of the circuit is zero, or when

$$\int_0^t i_G dt = Q_G = 0.$$

If $i_G = 0$ continuously, then necessarily $Q_G = 0$. The converse is not true, for the net quantity may be made zero by a current that flows through the detector first in one and then in the reverse direction. When deducing the conditions that must be fulfilled in order that the galvanometer may remain undeflected, it is best to impose the condition that no current shall flow through the galvanometer at any time, for in some cases the galvanometer needle will be disturbed even though the integral current is zero.[21] The disturbance depends on the alteration of the strength of the galvanometer needle by the transient current. In a general way, the reason for the deflection may be seen by supposing the instrument to be traversed by an alternating current. If the magnetism of the needle is affected by the current, the galvanometer becomes, in effect, a soft-iron instrument with a magnetic control, and there will be a deflecting moment proportional to the square of the current. In the case of the steadily applied alternating current, the needle will come to rest in a deflected position depending upon the strength of the current.

As an example of a method where the phenomenon is of importance, take the comparison of two mutual inductances by the method given on page 445. The integral flow of current through the galvanometer will be zero if

$$\frac{m_X}{m_P} = \frac{r_X}{r_P}.$$

That the current through the galvanometer may be zero continuously it is necessary that

$$\frac{m_X}{m_P} = \frac{r_X}{r_P} \tag{A}$$

and

$$\frac{m_X}{m_P} = \frac{L_X}{L_P}. \tag{B}$$

On trying the experiment, it will be found that unless the relation (B) is approximately fulfilled, the needle will be slightly disturbed.

In the following proofs, except in those for De Sauty's method for comparing capacitances, where the application of both conditions for balance will be illustrated, the condition $i_G = 0$ continuously will be imposed.

Moving-coil instruments are subject to this same error, when used as detectors for integral currents.[14] For instance, in Maxwell's method for the absolute measurement of capacitance (page 375), the apparatus is so arranged that the galvanometer is subjected to a uniform train of identical impulses due to transient currents, the integrated value of which is supposed to be zero. Curtis and Moon show that after a short time the small resulting deflection attains a *cyclic state* in which the angular velocity and angular position of the movable coil are the same at the end as at the beginning of the cycle.[14]

If the galvanometer field H is unaltered by the transient currents, the equation $P\frac{d}{dt}\left(\frac{d\theta}{dt}\right) + k\left(\frac{d\theta}{dt}\right) + \tau\theta = AHi_G$ applies. Integrating with respect to t from $t = 0$ to $t = T$, the time of a complete cycle, gives $P\left(\frac{d\theta}{dt}\Big]_T - \frac{d\theta}{dt}\Big]_t\right) + k(\theta_T - \theta_t) + \tau\int_0^T \theta dt = AH\int_0^T i_G dt$. Therefore, in this case, $\tau\int_0^T \theta dt = AH\int_0^T i_G dt$; that is, the integrated current is zero if the integrated deflection is zero.

If the galvanometer field is altered by the transient currents in the coil, the factor H is replaced by $H + H_1 i_G + H_2 i_G^2$, where H_1, H_2, . . . are due to the m.m.f. set up by the current and depend on the angular position of the coil. Inserting this value of H in the fundamental equation, assuming that the motion of the coil is very small so that H_1,

$H_2, \ldots$ may be regarded as constant, and integrating subject to the same conditions as before gives

$$\tau\int_0^T \theta dt = AH\int_0^T i_G dt + AH_1\int_0^T i_G^2 dt + AH_2\int_0^T i_G^3 dt \cdots.$$

The integral of i_G^2 can be zero only if i_G is always zero, while that of i_G^3 can be zero only if i_G is symmetrical about the time axis. $H_1, H_2, \ldots$ can be zero only if $\theta = 0$. It is difficult, if not impossible, to make the necessary adjustments. It is seen that only when the galvanometer field is unaltered by the coil current is $\int_0^T \theta dt = 0$ the criterion for $\int_0^T i_G dt = 0$.

It is evident that the magnet of the galvanometer must be permanent in spite of the m.m.f. due to the current in the movable coil. A properly designed and aged cobalt-steel magnet is indicated.

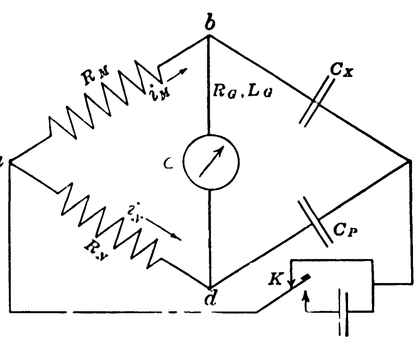


FIG. 252.—Connections for De Sauty method for comparing capacitances.

A galvanometer may be tested for correct integration by rapidly charging and discharging a perfect condenser through the instrument by means of a perfectly insulated commutator. Absence of a deflection indicates that the galvanometer integrates correctly.

De Sauty Method for Comparing Capacitance.—This method is adapted to the comparison of condensers without leakage, which are either free from absorption or have equal rates of absorption. Figure 252 shows the connections.

The two bridge arms R_M and R_N are noninductive resistances. C_X and C_P are the two condensers that are to be compared. The detector, which will be considered as having both inductance and resistance, is at G.

When the key K is against the backstop, the condensers are discharged. The arms R_M and R_N are adjusted until on depressing K the detector gives no indication. Then

$$C_X = C_P\frac{R_N}{R_M}.$$

To prove this relation, condition 1 (page 393) will first be applied. In that case, the variable current i_M is that flowing into condenser C_X, and the variable current i_N is that flowing into C_P, and at every instant

$$\frac{i_M}{i_N} = \frac{R_N}{R_M}.$$

The potential difference between b and c is

$$V_{bc} = \frac{1}{C_X}\int_0^t i_M dt,$$

and between d and c it is

$$V_{dc} = \frac{1}{C_P}\int_0^t i_N dt.$$

V_{bc} must equal V_{dc}, since at no time during the establishment of the steady state does any current flow through G. Then

$$\frac{1}{C_X}\int_0^t i_M dt = \frac{1}{C_P}\int_0^t i_N dt = \frac{R_M}{R_N}\frac{1}{C_P}\int_0^t i_M dt.$$

Therefore

$$C_X = C_P\frac{R_N}{R_M}.$$

If the condition $\int_0^t i_G dt = Q_G = 0$ is applied, the demonstration is more complicated, since the expression for Q_G must first be deduced, and then the condition found that renders it zero. Consider that the bridge is arranged as in Fig. 253.

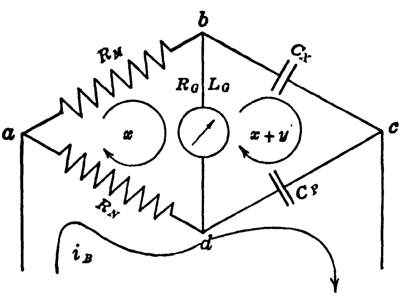


FIG. 253.—Mesh diagram for De Sauty method for comparing capacitances.

Assume that during the time of charging the condensers, that is, until the steady state has been established, the meshes are traversed by the variable currents x, $x + y$, and i_B, which finally become zero. Taking the mesh abd,

$$x(R_M + R_G + R_N) + L_G\frac{dx}{dt} - (x + y)R_G - L_G\left(\frac{dx}{dt} + \frac{dy}{dt}\right) - i_B R_N = 0;$$

or, uniting terms,

$$x(R_M + R_N) - yR_G - L_G\frac{dy}{dt} - i_B R_N = 0. \qquad (10)$$

Considering the mesh bcd, the potential difference between b and c is

$$V_{bc} = \frac{1}{C_X}\int_0^t (x + y)dt,$$

and similarly for C_{cd}. Hence

$$\frac{1}{C_X}\int_0^t (x+y)dt + \frac{1}{C_P}\int_0^t (x+y)dt - \frac{1}{C_P}\int_0^t i_B dt + (x+y)R_G + L_G\left(\frac{dx}{dt}+\frac{dy}{dt}\right) - xR_G - L_G\frac{dx}{dt} = 0;$$

or, uniting terms,

$$\frac{1}{C_X}\int_0^t (x+y)dt + \frac{1}{C_P}\int_0^t (x+y-i_B)dt + yR_G + L_G\frac{dy}{dt} = 0. \quad (11)$$

(10) and (11) may be integrated from $t = 0$, when the key K is closed, to the time t when the permanent state has been established. Call Q_X the quantity displaced by the current x in that time; Q_G, the quantity displaced through the galvanometer by y, the true galvanometer current; and Q_B, the quantity displaced by i_B. Integrating (10), and remembering that y is zero at the start and zero at the finish,

$$Q_X(R_M + R_N) - Q_G R_G - Q_B R_N = 0. \quad (10a)$$

Integrating (11) gives

$$(Q_X + Q_G)\frac{1}{C_X} + (Q_X + Q_G - Q_B)\frac{1}{C_P} = 0. \quad (11a)$$

From (10*a*) and (11*a*),

$$Q_X = \frac{Q_G R_G + Q_B R_N}{R_M + R_N} = -Q_G + Q_B\left(\frac{C_X}{C_X + C_P}\right).$$

Therefore

$$Q_G = \frac{Q_B\left[(R_M + R_N)\dfrac{C_X}{C_X + C_P} - R_N\right]}{R_M + R_G + R_N}. \quad (12)$$

If $Q_G = 0$,

$$\frac{C_X}{C_X + C_P} = \frac{R_N}{R_M + R_N},$$

or

$$C_X = C_P\frac{R_N}{R_M}, \qquad \text{as before.} \quad (13)$$

Maxwell Method for Comparing Inductances.—An inductance may be compared with a variable standard by an analogous method given by Maxwell.*

The arrangement of the circuits is shown in Fig. 254. As before, R_M and R_N are adjustable noninductive resistances. L_X and L_P are the

* "Treatise on Electricity and Magnetism," 3d ed., Art. 757.

inductances to be compared; they have resistances R_X and R_P, respectively. In order to make the adjustment expeditiously, it is necessary to include a variable noninductive resistance R which can be thrown into the arm L_X if necessary, by changing the battery lead from c to e, and to use for L_P a variable standard of inductance having a constant resistance.

The adjustment is made in two steps: A probable value of the ratio R_M/R_N is chosen; and, keeping K_1 closed, the resistance R is adjusted until balance is obtained. In this case, the arrangement is an ordinary Wheatstone bridge, only the resistances coming into play. When the adjustment is complete,

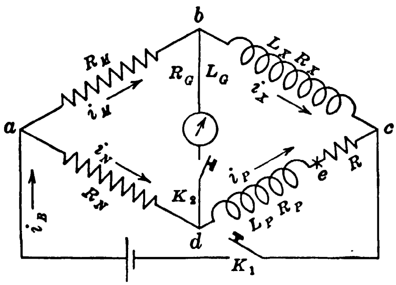


FIG. 254.—Mesh diagram for Maxwell method for comparing inductances.

$$\frac{R_M}{R_N} = \frac{R_X}{R_P + R}. \tag{14}$$

After the balance has been effected, K_1 is released, and K_2 kept closed. L_P is now varied until the detector gives no indication when contact is made and broken at K_1. Then

$$L_X = L_P\frac{R_M}{R_N}. \tag{15}$$

Several trials with various values of R_M/R_N may be necessary before the proper ratio is found.

In this and other methods where the balance is independent of the frequency, a "buzzer" operating through a telephone induction coil is often a convenient source of supply for the interrupted current, the detector being a telephone.

To prove the relation (15), the first condition stated on page 393 may be imposed. After the adjustment is complete, no current passes through the detector at any time, so for all values of t,

$$V_{bc} = V_{dc}.$$
$$V_{ab} = V_{ad}.$$
$$i_M = i_X.$$
$$i_N = i_P.$$
$$\frac{i_M}{i_N} = \frac{R_N}{R_M} = \frac{i_X}{i_P}.$$

Therefore

$$i_X R_X + L_X\frac{di_X}{dt} = i_P(R_P + R) + L_P\frac{di_P}{dt};$$

so

$$i_X\left[R_X - \frac{R_M}{R_N}(R_P + R)\right] + \frac{di_X}{dt}\left[L_X - \frac{R_M}{R_N}L_P\right] = 0,$$

which must be true for all values of t. Therefore

$$L_X = L_P\frac{R_M}{R_N},$$

and

$$R_X = \frac{R_M}{R_N}(R_P + R).$$

In order to balance the bridge when all four of the arms are inductive, three instead of two conditions must be satisfied, as will be seen from the following.

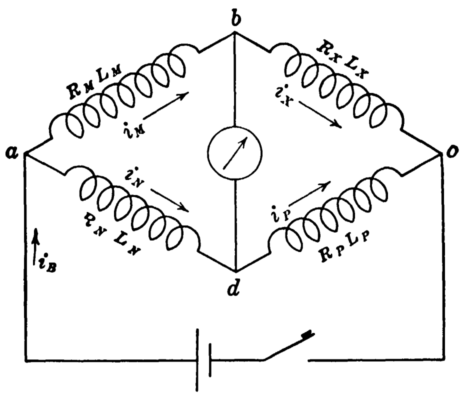


FIG. 255.—Mesh diagram for Maxwell bridge with four inductive arms.

Suppose that no current flows through the detector at any time. Then, referring to Fig. 255,

$$V_{ab} = V_{ad}.$$
$$V_{bo} = V_{dc}.$$
$$i_X = i_M.$$
$$i_P = i_B - i_X = i_B - i_M.$$
$$i_N = i_B - i_M.$$

Therefore, making no assumption as to the relation of i_B to t,

$$(R_M + R_N)i_M + (L_M + L_N)\frac{di_M}{dt} = R_Ni_B + L_N\frac{di_B}{dt}, \quad (16)$$

and

$$(R_X + R_P)i_M + (L_X + L_P)\frac{di_M}{dt} = R_Pi_B + L_P\frac{di_B}{dt}. \quad (17)$$

Eliminating i_M between (16) and (17) gives

$$\frac{di_M}{dt} = \frac{\frac{di_B}{dt}[-R_PL_N + R_NL_P - R_XL_N + R_ML_P] + i_B[R_MR_P - R_NR_X]}{(R_M + R_N)(L_X + L_P) - (R_X + R_P)(L_M + L_N)}. \quad (18)$$

Eliminating di_M/dt between (16) and (17) gives

$$i_M = \frac{\frac{di_B}{dt}[L_NL_X - L_ML_P] + i_B[-R_PL_N + R_NL_P + R_NL_X - R_PL_M]}{(R_M + R_N)(L_X + L_P) - (R_X + R_P)(L_M + L_N)}. \quad (19)$$

The value of di_M/dt derived from (19) when equated to that in (18) gives

$$\frac{d^2i_B}{dt^2}[L_NL_X - L_ML_P] + \frac{di_B}{dt}[-R_PL_M + R_NL_X + R_XL_N - R_ML_P] + i_B[R_NR_X - R_MR_P] = 0. \quad (20)$$

By supposition, (20) must hold for all values of t and, therefore, for the steady state, so

$$R_NR_X - R_MR_P = 0, \quad (21)$$

which is the ordinary condition for the balance of the Wheatstone bridge. In order that (20) may be true for all values of t, the coefficients of d_2i_B/dt_2 and di_B/dt must also be zero; so

$$L_NL_X - L_ML_P = 0. \quad (22)$$

$$-R_PL_M + R_NL_X + R_XL_N - R_ML_P = 0. \quad (23)$$

As no assumption has been made concerning the relation of i_B *to* t, Eq. (20) holds when the bridge is supplied with sinusoidal as well as with variable currents.

The Secohmmeter and Synchronous Commutator.[22]—To increase the sensitiveness of the older bridge methods for the measurement of self-inductance and capacitance which depended upon the use of variable currents, Ayrton and Perry devised the secohmmeter, by which the impulses on the galvanometer needle can be made to follow one another so rapidly that the instrument takes up a steady deflection. The arrangement is essentially a double commutator. One of the commutators reverses the battery connections, while the other reverses the galvanometer terminals so that the impulses on the needle of the instrument are always in the same direction. Referring to Fig. 256, the shaded portions of the two commutators are made of an insulating material. The unshaded portions are conducting segments. The brushes *aa′*, *bb′* and *cc′*, *dd′* are so placed that the circuits are manipulated in the proper sequence. The secohmmeter is driven at a constant speed by a small motor.

In using this device, the speed must not be so high that sufficient time is not allowed for the establishment of the steady state at each reversal.

An analogous device is the synchronous commutator sometimes used with the alternating-current bridges about to be described. In this device, the commutator *cdc'd'* is omitted, for the reversal of the supply current is automatically effected. The commutator *abb'a'*, which makes contact for one-half wave is driven by a synchronous motor connected to the bridge supply. The brushes are mounted on a rocker

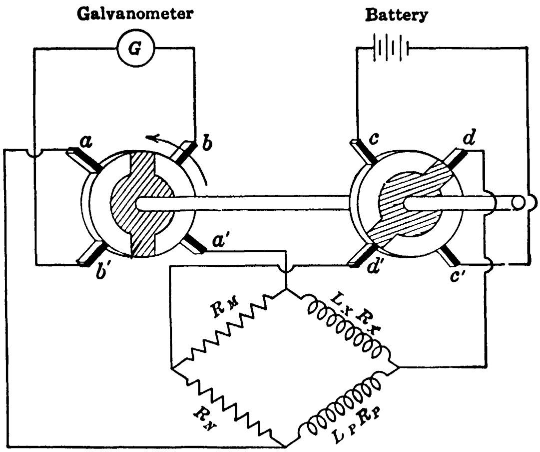


FIG. 256.—Showing connections for secohmmeter.

arm so that the time phase of the contacts may be altered. A less troublesome arrangement and the one actually employed is a cam-operated switch with a phase adjustment. The synchronous commutator renders the direct-current moving-coil galvanometer available as a detector in alternating-current measurements. As the phase of the contact can be adjusted, it may be so set that a current in phase produces a deflection while one in quadrature produces no deflection. Advantage of this is taken in some methods of measurement, for instance, in the Sharp and Crawford method for determining the constant of instrument transformers.[22]

The Impedance Bridge.[23]—Because of the nearer approach to actual working conditions, capacitance and inductance measurements are now made by aid of alternating currents, preferably sinusoidal.

Practically all the recent researches on dielectrics and condensers as well as the precision measurements of inductances have been made by bridge methods, using either the impedance bridge or the Anderson bridge.

In the impedance bridge, which may be applied to the measurement of either inductance or capacitance there are four main conductors arranged as in the Wheatstone bridge. Alternating currents are employed, and usually either two or four of the conductors are reactive.

To deduce the condition for balance, the arrangement shown in Fig. 257 may be taken. It will be assumed that the bridge arms have impedance operators z_M, z_N, z_X, z_P, and z_G and are traversed by sinusoidal currents. The mesh currents will be taken as indicated. As cognizance must be taken of their phase relations, these currents must be in the symbolic form, and all referred to the same axis, for instance, that of I_B. Applying Kirchhoff's rules, for the X mesh,

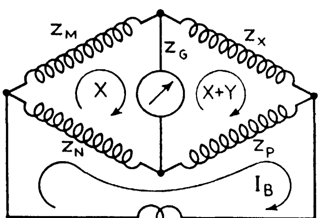


FIG. 257.—Mesh diagram for impedance bridge.

$$X(z_M + z_G + z_N) - (X + Y)z_G - I_B z_N = 0;$$

for the $(X + Y)$ mesh,

$$(X + Y)(z_X + z_P + z_G) - Xz_G - I_B z_P = 0.$$

Solving for Y, the current through the detector,

$$Y = \frac{I_B(z_P z_M - z_N z_X)}{z_G(z_X + z_P + z_M + z_N) + (z_M + z_N)(z_X + z_P)} = I'(z_P z_M - z_N z_X).$$

If the detector and generator are interchanged, the detector current becomes

$$Y = \frac{I_B(z_N z_X - z_P z_M)}{z_G(z_X + z_P + z_M + z_N) + (z_M + z_X)(z_N + z_P)} = I_B''(z_N z_X - z_P z_M).$$

For zero detector current, the result of operating on I_B' and I_B'' with $z_N z_X$ must produce a result identical with that given by operating with $z_M z_P$. The condition for no current in the detector is

$$z_N z_X = z_P z_M. \tag{24}$$

If admittances are used in place of impedances,

$$Y_N Y_X = Y_P Y_M.$$

Compare the foregoing with corresponding deduction for the Wheatstone bridge (page 179). The generator used as a power source should give a sinusoidal e.m.f. wave.

The ratio arms M and N may be noninductive resistances, highly inductive resistances, or perfect condensers. Bridges with highly inductive ratio arms have been used by Giebe in inductance measurements[24] and by Grover[25] in measurements of the capacitance and power factor of condensers. Air condensers were used as ratio arms by Fleming and Dyke[26] in their bridge for investigating the behavior of dielectrics at telephonic frequencies.

The detector may be a telephone,[27] a vibration galvanometer,[28] or an alternating-current galvanometer.[29] At low frequencies, the vibration galvanometer (page 465) is very satisfactory, for it is a tuned instrument responding freely to currents of only one frequency. With it an accurate balance may be obtained, even though the currents are not exactly sinusoidal.

As the maximum frequency obtainable with the vibration galvanometer is about 1,800 cycles per second, a limit is set above which the telephone must be used. The sensitivity of a telephone detector may be greatly increased by having it tuned to the frequency of the supply, especially if that is near the frequency at which the ear is most sensitive (800 to 1,000 cycles per second). Of course, with any tuned detector, the periodicity of the current must be kept constant.

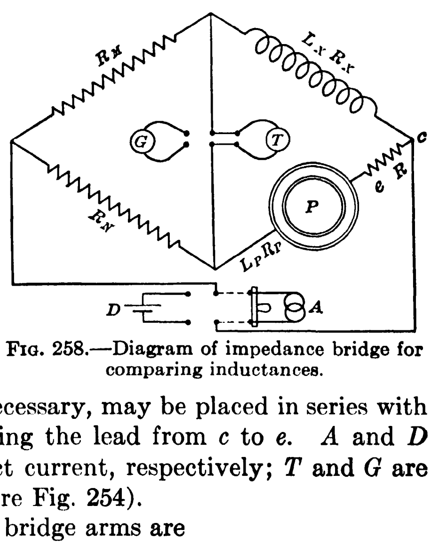


FIG. 258.—Diagram of impedance bridge for comparing inductances.

Inductance Measurements.—In the comparison of an inductance with a variable-standard inductance, the connections shown in Fig. 258 may be used.

The ratio arms R_M and R_N are noninductive resistances. The variable standard of self-inductance, of constant resistance, is at P (see page 353). L_X is the unknown inductance, and R is an adjustable noninductive resistance which, if necessary, may be placed in series with the unknown inductance by changing the lead from c to e. A and D are sources of alternating and direct current, respectively; T and G are the corresponding detectors (compare Fig. 254).

The impedance operators of the bridge arms are

$$z_M = R_M.$$
$$z_N = R_N.$$
$$z_P = R_P + j\omega L_P.$$
$$z_X = R_X + j\omega L_X.$$

Substituting these values in the general equation (24) gives

$$R_M R_P + j\omega L_P R_M = R_N R_X + j\omega L_X R_N.$$

This one equation being complex is really equivalent to two, for when all the terms are transposed to the left-hand side the sum of all the horizontal components, "the real terms," must be zero; and the sum of all the vertical components, "the imaginary terms," must also be zero. From the quadrature components, the two conditions that must be fulfilled in order that the bridge may be balanced follow:

From the horizontal components,

$$R_X = \frac{R_M}{R_N} R_P. \tag{25}$$

From the vertical components,

$$L_X = \frac{R_M}{R_N} L_P. \tag{26}$$

Therefore, when measuring an inductive coil, a perfect balance implies two things—that the ohmic resistances are balanced, as in the ordinary Wheatstone bridge, and that the inductances are in the ratio of the corresponding bridge arms. If there are other than i^2r losses in the arm X, they appear in R_X which in this case is an equivalent resistance.

With a bridge properly constructed, its coils being free from inductance and capacitance, it is thus possible to make a simultaneous measurement of the inductance and *resistance to alternating currents* of a coil or piece of apparatus.

To assist in carrying out the necessary adjustments in an expeditious manner, it may be noted that nonmagnetic conductors of *small cross section*, used with currents of ordinary frequencies, have practically the same resistance with alternating as with direct current. Therefore, to save time, a preliminary balance may be made with direct current, using the apparatus as an ordinary Wheatstone bridge, thus satisfying the condition

$$R_X = \frac{R_M}{R_N} R_P.$$

If the order of magnitude of the inductance under measurement is entirely unknown, one may begin with $R_M = R_N$ and balance by varying R. It may be necessary to transfer this resistance to the other side of the bridge by changing the lead from c to e. Alternating is now substituted for direct current. The detector will, in general, give an indication that must be reduced to a minimum by adjusting the variable standard of inductance. The chances are that on account of the limited range of the standard, L_P cannot be made of such a value as to obtain even a minimum of sound in the telephone. In this case, one notices at which end of the scale the indication is the smaller and then alters the ratio so that the balance point will be thrown toward the middle portion of the scale. The bridge is then rebalanced for direct and for alternating currents. Two or three trials may be necessary in order to obtain a good reading on L_P. If with $R_M = R_N$ the indication is apparently the same at all points of the scale, a large change should be made in the ratio, for instance, to $R_M/R_N = 10$. If this does not give results, $R_M/R_N = 1/10$ may be tried, and so on. Last, a final attempt to obtain a perfect zero indication of T may be made by altering R and L_P *slightly.*

The Impedance Bridge with Four Inductive Arms.—It is desirable to inquire as to the conditions necessary for a balance if all four arms of the impedance bridge contain inductances. This arises from the fact that if very small inductances are to be compared, using currents of high frequency, *the residual inductances existing in the ordinary double-wound resistance coils become of great moment.* The reactances of the bridge coils may be positive or negative according as the inductance or capacitance component preponderates *and at times may be of the same order of magnitude as the reactance under measurement.* In a bridge with four inductive arms,

$$z_M = R_M + j\omega L_M;$$
$$z_N = R_N + j\omega L_N;$$
$$z_P = R_P + j\omega L_P;$$
$$z_X = R_X + j\omega L_X;$$

and, at balance,

$$z_M z_P = z_N z_X.$$

Substituting,

$$R_P R_M + j\omega R_P L_M + j\omega R_M L_P - L_M L_P \omega^2 = R_X R_N + j\omega R_X L_N + j\omega R_N L_X - L_N L_X \omega^2.$$

The horizontal component gives as one condition for balance

$$(L_N L_X - L_M L_P)\omega^2 + R_P R_M - R_X R_N = 0. \quad (27)$$

The vertical component gives as the other necessary condition

$$R_P L_M - R_N L_X - R_X L_N + R_M L_P = 0. \quad (28)$$

L_M, L_N may be the residual inductances of the balance arms; L_P, the inductance of the standard plus any residual inductance in the P arm. If $R_M = R_N$ and $L_M = L_N$, that is, if the balance arms are identical, then $L_X = L_P$, and $R_X = R_P$. The effect of the residual inductances in the balance arms is thus eliminated, hence the advantage of using a perfectly constructed equal-arm bridge. It may be noted that, in such a bridge, mutual induction between the balance arms has no effect on the results. The symmetrical bridge and the substitution method should be used in precision measurements.

In some cases, it is convenient to express z as a polar operator; consequently,

$$z_M = z_M\underline{|\theta_M};$$
$$z_N = z_N\underline{|\theta_N};$$
$$z_X = z_X\underline{|\theta_X};$$
$$z_P = z_P\underline{|\theta_P};$$

where, if inductances are measured,

$$z = \sqrt{R^2 + \omega^2 L^2};$$
$$\theta = \tan^{-1} \frac{\omega L}{R}.$$

In this case,

$$z_N\underline{|\theta_N} \times z_X\underline{|\theta_X} = z_P\underline{|\theta_P} \times z_M\underline{|\theta_M};$$
$$z_X\underline{|\theta_X} = \left(\frac{z_M\underline{|\theta_M}}{z_N\underline{|\theta_N}}\right)(z_P\underline{|\theta_P});$$
$$z_X\underline{|\theta_X} = \left(\frac{z_M}{z_N}\right)(z_P\underline{|\theta_M + \theta_P - \theta_N}). \tag{29}$$

If the balance arms are nonreactive,

$$z_X = \left(\frac{R_M}{R_N}\right)(z_P);$$
$$\theta_X = \theta_P.$$

Capacitance Measurements.—If two perfect condensers are to be compared, they may be placed in the arms P and X (compare with Fig. 252). The arms M and N may be noninductive resistances. In this case,

$$z_M = R_M;$$
$$z_N = R_N;$$
$$z_X = \frac{1}{j\omega C_X};$$
$$z_P = \frac{1}{j\omega C_P}.$$

Substituting in (24) gives

$$\frac{R_N}{j\omega C_X} = \frac{R_M}{j\omega C_P}.$$

Therefore

$$C_X = C_P \frac{R_N}{R_M}.$$

In practice, the comparison of ordinary condensers is not so simple, for an energy loss may occur in one or both of them. As the behavior of a condenser with an imperfect dielectric depends on the frequency, it is important that the correct periodicity be employed.

If energy losses are present, the phase of the current in R_M will probably not be the same as that in R_N, and no adjustment of these resistances can be found that will cause a zero indication of the detector. If a telephone is used, there will be a considerable range of adjustment over which the sound is faint but never entirely disappears.

If an energy loss is introduced into the arm of the bridge having the smaller power factor, the currents in R_M and R_N may be brought into phase, and an exact balance obtained. Wien accomplishes this by the use of a series resistance in the arm containing the better condenser.[23]

The bridge is arranged as shown in Fig. 259. All the resistances are supposed to be noninductive. The condensers to be compared are at C_X and C_P. One or both may have an imperfect dielectric; and to duplicate their behaviors, it will be assumed that perfect condensers having effective capacitances C_X and C_P are in series with resistances ρ_X and ρ_P, respectively. This is in accordance with the convention mentioned on page 369. ρ_X and ρ_P are hypothetical resistances assumed simply as an aid in the demonstration in order to introduce energy losses, their values being such that under the condition of the test the behavior of the combination of the perfect condenser C_X and the resistance ρ_X will be the same as that of the actual condenser at C_X, and the behavior of C_P and ρ_P the same as that of the actual condenser at C_P. The balance arms R_M and R_N are variable, and R_X and R_P are the adjustable resistances used to bring the potential differences V_{cb} and V_{cd} into phase. Two resistances are included because it may not be known at the start which of the condensers has the lower power factor. Only one of the resistances will be used.

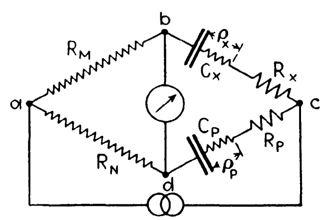


FIG. 259.—Diagram for Wien impedance bridge.

The balancing is effected as follows: with R_X and R_P both zero, the ratio R_M/R_N is adjusted until the indication of the detector is a minimum. The balance is then improved by adjusting R_X or R_P as the case may require and still further improved by readjusting R_M/R_N, and so on, thus obtaining a perfect balance by successive adjustments. When the balance is effected,

$$C_X = C_P \frac{R_N}{R_M}.$$

This may be proved as follows; in general, by (24),

$$z_P z_M = z_N z_X.$$

In this case,

$$z_M = R_M;$$
$$z_N = R_N;$$
$$z_P = R_P + \rho_P - \frac{j}{\omega C_P};$$
$$z_X = R_X + \rho_X - \frac{j}{\omega C_X}.$$

Substituting in (26) gives

$$R_M(R_P + \rho_P) - j\frac{R_M}{\omega C_P} = R_N(R_X + \rho_X) - j\frac{R_N}{\omega C_X}.$$

Equating the vertical components,

$$\frac{R_M}{\omega C_P} = \frac{R_N}{\omega C_X},$$

or

$$C_X = C_P\frac{R_N}{R_M}. \qquad (30)$$

Equating the horizontal components,

$$\frac{R_M}{R_N} = \frac{R_X + \rho_X}{R_P + \rho_P}. \qquad (30a)$$

Determination of Phase-defect Angle of Condenser.—As previously defined, the *phase-defect angle of a condenser* φ is the deviation of the phase of the current from the ideal lead angle of 90 deg. which would exist in a perfect condenser. To determine the difference of the phase-defect angles of C_X and C_P, $\varphi_X - \varphi_P$, from (30) and (30a),

$$\frac{R_X + \rho_X}{R_P + \rho_P} = \frac{R_M}{R_N} = \frac{C_P}{C_X}.$$

When multiplied out and then multiplied through by ω this becomes

$$\omega C_X\rho_X - \omega C_P\rho_P = \omega C_P R_P - \omega C_X R_X,$$

but

$$\omega C_X\rho_X = \tan\varphi_X \quad \text{and} \quad \omega C_P\rho_P = \tan\varphi_P,$$
$$\tan\varphi_X - \tan\varphi_P = \omega C_P R_P - \omega C_X R_X. \qquad (31)$$

In general, $\tan a - \tan b = (1 + \tan a \tan b)(\tan(a - b))$, so if the *phase-defect angle* φ_P of the standard condenser is small, as it usually will be,

$$\tan(\varphi_X - \varphi_P) = \omega C_P R_P - \omega C_X R_X. \qquad (32)$$

Either R_X or R_P may be zero, as previously noted. If φ_P is known, the power factor of the unknown condenser is readily computed. The values of φ_P and C_P would be determined by a process of stepping up from an air condenser. The curves on page 372 were determined by this method. As it is customary to use a high frequency—800 cycles per second or greater—residual inductances and capacitances in the apparatus must be reduced to a minimum. The sources of error to be considered when refined measurements are to be made are:

1. Residual inductance of M and N.
2. Error in ratio of M and N.
3. Residual inductance of R_X and R_P.
4. Stray capacitances.

Measurements of Inductance in Terms of Capacitance.—An obvious method of making this measurement is to use an impedance bridge having three nonreactive arms, while the fourth arm contains a tunable series combination of inductance and capacitance of adjustable resistance. Balance is effected by bringing the fourth arm to resonance and adjusting its resistance; then

$$\frac{R_M}{R_N} = \frac{R_X}{R_P}. \qquad L_X = \frac{1}{\omega^2 C_X}. \tag{33}$$

$$\omega L_X = \frac{1}{\omega C_X}. \qquad \omega = 2\pi f = \frac{1}{\sqrt{L_X C_X}}. \tag{34}$$

If an untuned detector is employed, a pure sinusoidal e.m.f. of constant and known frequency is obviously necessary. The resonant arm bridge, using known values of inductance and capacitance, may be used to determine the frequency of pure sinusoidal e.m.fs.

Belfils[32] has used this form of bridge in determining the purity of the e.m.f. wave of a generator under test. If the wave is a pure sinusoid, the detector, which is an electrodynamic voltmeter, can be brought exactly to zero. If harmonics are present, there will be a minimum deflection due to the aggregate of the harmonics, that is, to

$$\sqrt{V_3^2 + V_5^2 + V_7^2 \cdots},$$

which is an index as to the purity of the wave.

Variants of the resonant arm bridge are obtained by shunting the condenser with a noninductive resistance, thus altering its effective capacitance and resistance, or by shunting the condenser around the inductance and noninductive resistance in series. In all cases when balance is attained, the fourth arm of the bridge has, in effect, been made nonreactive.

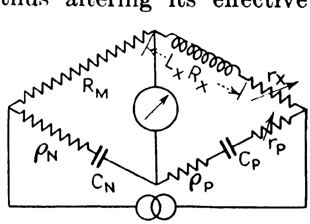


FIG. 260.—Owen bridge network.

Owen Bridge.[33]—The circuit of the Owen impedance bridge, which has lately been studied by Ferguson,[33] is shown in Fig. 260, where L_X and R_X are the *over-all* inductance and resistance of the unknown. R_M is a nonreactive resistance variable in a few steps. r_X and r_P are nonreactive resistances variable in very small steps. C_N and C_P are high-quality condensers, C_N being variable in a few steps. ρ_N and ρ_P are the equivalent series loss resistances of C_N and C_P. The range of the bridge is varied by changing either R_M or C_N or both.

Assuming that the condensers are without losses and that all the resistances are truly nonreactive, it will be found, by using the general formula (24) and separating the horizontal and vertical components, that at balance

$$L_X = R_M C_N r_P, \tag{35}$$

$$R_X = \left(\frac{C_N}{C_P}\right) R_M - r_X. \tag{36}$$

It is seen that in this ideal case the balance is independent of the frequency. A wide range of inductances may be covered with a single pair of condensers. Fine adjustment of r_X and r_P may be had by following Owen's procedure of using about a meter of constantan wire doubled back on itself, the two branches being about 0.5 cm. apart. The effective length is altered by a movable short-circuiting bridge piece. When dealing with very small inductances, it would be necessary to know the inductance of such a rheostat.

A study of the effect of residual inductances in the bridge arms and losses in the condensers is very instructive, for it reveals a weakness which is not at all apparent from equations (35) and (36) and emphasizes the necessity of a thorough investigation of the effects of residuals in *any* method of measurement before the results can be accepted with confidence.

Let the inductance to be determined be denoted by L_X, and let each of the residual inductances, which include the inductances of the leads, be represented by l with the proper subscript to show the bridge arm in which it is located. Let ρ_N and ρ_P be the series loss resistances of the condensers; then, at balance,

$$\frac{R_M + j\omega l_M}{\rho_N - \dfrac{j}{\omega C_N} + j\omega l_N} = \frac{R_X + j\omega L_X + r_X + j\omega l_X}{r_P + j\omega l_P - \dfrac{j}{\omega C_P} + \rho_P}. \tag{37}$$

Separating the vertical and horizontal components gives, from the vertical component,

$$L_X = R_M C_N r_P - l_X + C_N[\rho_P R_M - \rho_N(R_X + r_X)] + \left(\frac{C_N}{C_P}\right) l_M - \omega^2 C_N l_M l_P + \omega^2 C_N l_N (L_X + l_X); \tag{38}$$

from the horizontal component,

$$R_X = \left(\frac{C_N}{C_P}\right) R_M - r_X - \omega^2 l_M (r_P + \rho_P) C_N - \omega^2 l_P R_M C_N + \omega^2 \rho_N (L_X + l_X) C_N + \omega^2 l_N (R_X + r_X) C_N. \tag{39}$$

The formulae may be somewhat simplified if a second balance is taken with the unknown short-circuited. Denoting the second values by primes and taking the difference of the first and second balances gives

$$L_X = R_M C_N (r_P - r'_P) - (l_X - l'_X) + \rho_N C_N [r' - R_X - r_X] - \\ \omega^2 l_M C_N [l' - l_P] + \omega^2 l_N C_N [L_X + l_X - l'_X]. \quad (40)$$

The last three terms all contain factors that are very small. The condenser C_N is supposed to be of high quality, so ρ_N is small, and l_N is simply the inductance of the connecting wires; so for practical purposes,

$$L_X = R_M C_N (r_P - r'_P) - (l_X - l'_X). \quad (41)$$

Usually, the term $l_X - l'_X$ is negligible. The condenser losses and residual inductances have a much more serious effect on R_X than on L_X. From the two balances,

$$R_X = r'_X - r_X + \omega^2 l_M C_N (r_P - r'_P) + \omega^2 R_M C_N (l'_P - l_P) + \\ \omega^2 \rho_N C_N (L_X + l_X - l'_X) + \omega^2 l_N C_N (R_X + r_X - r'_X). \quad (42)$$

The last term is negligible, for l_N is simply the inductance of the leads by which the condenser is connected into the circuit. The result is not affected by the loss in C_P. Using relations derived from (41),

$$R_X = r'_X - r_X + \omega[L_X + l_X - l'_X]\left\{\omega\rho_N C_N - \frac{\omega l_M}{R_M} + \omega\left(\frac{l'_P - l_P}{r_P - r'_P}\right)\right\}. \quad (43)$$

The correction to be applied to $r'_X - r_X$ to obtain R_X may be a large percentage of R_X. Its importance increases with the magnitude of the inductance being measured, with the frequency, and with the loss resistance in C_N; therefore C_N should be of the highest quality. The important term is usually $\omega L_X \cdot \omega\rho_N C_N$; for example, in a certain measurement,[31]

$f = 3{,}000.$ $L_X = 1$ henry $\omega L_X = 18{,}850.$ $r'_X - r_X = 297.0.$

$$\omega C_N \rho_N = 0.0023.$$

$$-\frac{\omega l_M}{R_M} = 0.0013.$$

$$\omega\left(\frac{l'_P - l_P}{r_P - r'_P}\right) = 0.0000$$

$$\overline{}$$

$$0.0036$$

$$\text{Correction} = 18{,}850 \times 0.0036 = 67.8 \text{ ohms.}$$

$$R_X = 297.0 + 67.8 = 365 \text{ ohms.}$$

The correction is nearly 23 per cent of $r'_X = r_X$, which might at first be taken as equal to R_X. It is evident that the Owen bridge is particularly adapted to inductance measurements. If equivalent resistances are desired, the residuals and condenser loss resistances must be evaluated, and the foregoing corrections applied. When this is done with care, Ferguson estimates that an accuracy of 2 or 3 per cent may be attained in R_X.

The Anderson Bridge.[34]—The determination of an inductance in terms of a capacitance may conveniently be made by means of the Anderson bridge, which is the most serviceable of several developed from Maxwell's original arrangement.*

The connections are shown in Fig. 261.

All the resistances except R_X are supposed to be noninductive. The condenser is placed at C and r is an adjustable resistance.

When variable currents are used, as was originally intended, the bridge is first balanced for steady currents, the battery circuit being kept closed. After balance has been attained, the capacitance C and the resistance r are adjusted until there is no deflection of the galvanometer when the battery circuit is made and broken. It will be noted that the adjustment of C and r does not disturb the steady current balance but does affect the rate at which the potential of the junction e rises. As the initial values of the potentials of b and e are the same, and the final values are the same, there will be no current in the detector at any time if the potentials of these two points rise at the same rate.

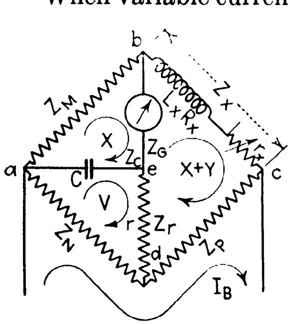


FIG. 261.—Anderson bridge network.

The Anderson bridge is now used with alternating currents, and a vibration galvanometer is employed as the detector. This arrangement has been used at the Bureau of Standards in much of the very accurate work on the measurement of inductance.

The necessary conditions for balance are shown below.

The impedance operators of the various branches are denoted by z with the proper subscript. The mesh equations are

$$X(z_M + z_C) - Yz_G - Vz_C = 0.$$
$$V(z_C + z_r + z_N) - X(z_C + z_r) - Yz_r - I_Bz_N = 0.$$
$$Y(z_X + z_P + z_r + z_G) + X(z_X + z_P + z_r) - Vz_r - I_Bz_P = 0.$$

Solving for Y, the galvanometer current,

$$Y = \frac{I_B(z_Mz_Cz_P + z_Mz_rz_P + z_Mz_Nz_P + z_Mz_Nz_r - z_Cz_Nz_X)}{\text{denominator, a function of the impedance operators}}.$$

Therefore, for balance,

$$z_Mz_Cz_P + z_Mz_rz_P + z_Mz_Nz_P + z_Mz_Nz_r - z_Cz_Nz_X = 0. \qquad (43a)$$

* "Treatise on Electricity and Magnetism," 3d ed., Art. 778.

In the ideal case where the coils of the bridge proper are entirely free from inductance and distributed capacitance,

$$z_M = R_M.$$
$$z_X = R_X + r_X + j\omega L_X.$$
$$z_P = R_P.$$
$$z_N = R_N.$$
$$z_C = \frac{1}{j\omega C}.$$
$$z_r = r.$$

Substituting in (43*a*),

$$\frac{R_M R_P}{j\omega C} + R_M r R_P + R_M R_N R_P + R_M R_N r = \frac{R_N}{j\omega C}(R_X + r_X + j\omega L_X).$$

Separating the quadrature components, the horizontal component gives

$$L_X = R_M C\left[r\left(1 + \frac{R_P}{R_N}\right) + R_P\right]. \qquad (44)$$

The vertical component gives

$$R_M R_P = R_N(R_X + r_X). \qquad (45)$$

Thus a perfect balance of the vibration galvanometer implies that both (44) and (45) are satisfied.

The value of L_X is an equivalent value and contains the effect of the distributed capacitance within the coil itself.

Butterworth has investigated the sensitivity of the Anderson bridge[34] and finds that a small variation in r produces the greatest effect on the balance when the arrangement is such that

$$R_M = R_X + r_X = \sqrt{\frac{L}{2C}}, \qquad R_N = R_P = \frac{R_X + r_X}{2} = \frac{R_M}{2}.$$

It is seen that the most sensitive bridge arrangement is one having two equal contiguous arms N and P. The effect of residuals in these arms, which are of identical mechanical construction, is eliminated by interchanging them, taking a second balance, and using the average in computing L. It is found that a small residual in r has a negligible effect unless the frequency is very high. The effect of the residuals in M and the r_X portion of the arm X may be minimized by making them as nearly alike in characteristics as is practicable.

In arranging the bridge according to the foregoing, a reasonable value of R_M is selected. From it and an appropriate value of L, the capacitance C, for maximum sensitivity, is computed. The arms R_N and R_P are each made equal to one-half of R_M, and balance attained by adjusting r and r_X, the resistance in series with the unknown inductance. If the resistances involved have the same values for alternating as for direct currents, work may be expedited by taking a preliminary balance using direct current, thus satisfying (45).

Effect of Dissipation of Energy in the Condenser.—In the demonstration, a perfect condenser has been assumed. As an energy loss occurs in most condensers, it is important to see how this loss will influence the results. As previously shown, an energy loss is equivalent to an increase of the conductance of the condenser. Suppose that the condenser has been measured with alternating currents and that its equivalent *parallel* capacitance is C, and its equivalent conductance $1/R_C$; then

$$z_C = \frac{R_C}{1 + jR_C C_P \omega}.$$

Substituting this value in (43*a*),

$$\frac{R_M R_C R_P}{1 + jR_C C_P \omega} + R_M r R_P + R_N R_M R_P + R_M R_N r = \frac{R_C R_N(R_X + r_X + j\omega L_X)}{1 + jR_C C\omega}.$$

The horizontal component gives

$$(R_X + r_X)R_N - R_M R_P = \frac{R_M}{R_C}\left[r(R_P + R_N) + R_N R_P\right].$$

The vertical component gives

$$L_X = R_M C\left[r\left(1 + \frac{R_P}{R_N}\right) + R_P\right].$$

That is, the energy loss does not complicate the measurement of the inductance.

If the equivalent circuit of the condenser is assumed to consist of a capacitance C_S and a resistance ρ in *series*,

$$z_C = \rho - \frac{j'}{\omega C_S},$$

and

$$L_X = \frac{R_M C_S}{1 + \rho^2\omega^2 C_S^2}\left[r\left(1 + \frac{R_P}{R_N}\right) + R_P\right].$$

The equivalent parallel and series capacitances are slightly different: $C = C_S \sin^2 \theta$, where θ is the power-factor angle of the condenser; consequently,

$$C = \frac{C_S}{1 + \rho^2\omega^2 C_S^2}.$$

The value for L_X is the same in both cases, the appropriate constants being used. The difference of C and C_S is exceedingly small in good condensers (see page 369).

Butterworth's paper[34] should be consulted by those interested in the precision application of the Anderson bridge.

Stroude and Oates Bridge.[34]—Stroude and Oates modified the Anderson bridge by interchanging the source of current and the detector.

The advantage of the rearrangement is that when the conditions are such that r is high, it is possible to increase the applied voltage and thus maintain the sensitiveness of the bridge by keeping the bridge current at a high value.

As the only alteration has been to interchange the source of current and the detector, the formula connecting the self-inductance and the capacitance is the same as for the Anderson bridge.

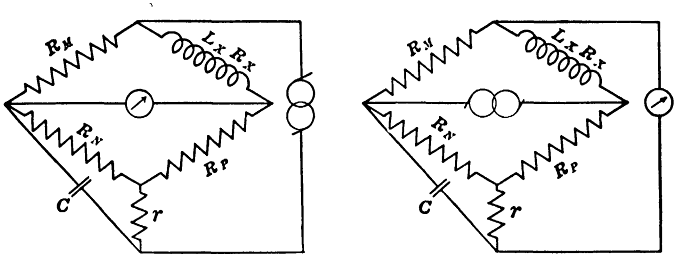


FIG. 262.

Stroude and Oates arrangement of the Anderson bridge. Anderson bridge, Fig. 261 redrawn.

High-voltage Impedance Bridges.—There has been much interest in the behavior of imperfect dielectrics, especially when subjected to high voltages. One problem of practical importance is the power losses in the insulations of cables under normal operating conditions. These losses should be reduced to a minimum by the proper choice of materials and methods of construction. The development of accurate methods for determining small amounts of power at high voltages and very low power factors has been of great assistance in perfecting the modern high-voltage cable. Power determinations of this sort may be made in a number of ways, for instance, by an electrostatic wattmeter or a special electrodynamic wattmeter; but it has become common to employ an indirect method in which the power factor of the insulation is determined by some form of alternating-current bridge. As the sample must be subjected to high voltages without danger to the operator, it is seen that only bridges in which the adjustable arms are of low impedance and connected to ground at a common point are available.

The very high and very pure impedances necessary for the remaining known arm of such a bridge are most readily and inexpensively obtained by use of shielded air condensers (Fig. 228), for, except in rare cases, they may be considered to be without losses (see page 359). High resistances are impracticable above about 10,000 volts on account of the effects of capacitance to ground. Shielding high resistances introduces too many complications and too much expense. Several bridges particularly adapted to high-voltage work have been devised, of which the Schering impedance bridge[31] (page 416), the Dawes-Hoover mutual inductance bridge[39] (page 424), and the Atkinson split-voltage impedance bridge[35] (page 418) are examples.

Schering Impedance Bridge.[31]—The bridge network with which the name of Schering has become associated was originally devised by

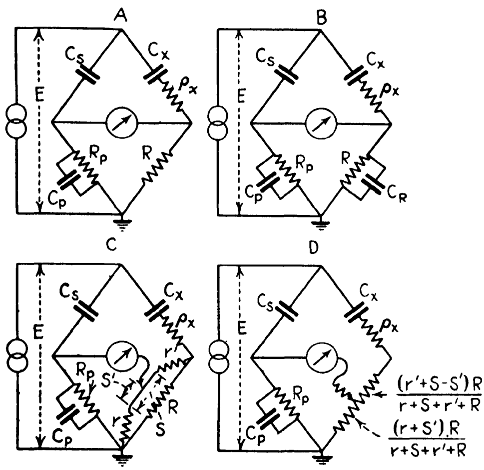


FIG. 263.—Schering bridge networks.

Phillips Thomas of the research laboratory of the Westinghouse Company whose patent application was filed Dec. 4, 1913. Previous to the patent disclosure, the bridge had been in use for a considerable time in investigations concerning high-voltage bushings. The diagram for this bridge, which has a wide field of usefulness, and some variations are shown in Fig. 263.

Referring to Fig. 263*A* and *B*, C_X and ρ_X are the capacitance and equivalent series loss resistance of the sample under test. C_S is an air condenser of fixed value, *assumed to be perfect*. R and R_P are variable, nonreactive resistors. C_P and C_R are adjustable condensers assumed to be without appreciable losses. Then, for Fig. 263*A*,

$$z_X = \rho_X - \frac{j}{\omega C_X}.$$

$$z_S = -\frac{j}{\omega C_S}.$$

$$z_R = R.$$

$$y_P = \frac{1}{R_P} + j\omega C_P.$$

At balance,

$$z_X = z_S z_R y_P = -\frac{jR}{R_P C_S \omega} + \frac{C_P R}{C_S}.$$

Equating the corresponding terms in the impedance operators,

$$C_X = \left(\frac{R_P}{R}\right) C_S \qquad \rho_X = \left(\frac{C_P}{C_S}\right) R.$$

$$\text{Power factor} = \cos\left(\tan^{-1}\frac{1}{\omega C_X \rho_X}\right) = \omega R_P C_P \tag{46}$$

as the angle of defect is small.

At high voltages, the potential difference across the specimen is practically E, the impedance of R being insignificant compared with that of X; consequently,

$$I_X = EC_X\omega = E\left(\frac{R_P}{R}\right) C_S \omega \qquad \text{approx.,}$$

and the power loss is, nearly enough,

$$W = EI_X \cos\theta_X = E^2\left(\frac{R_P}{R}\right) C_S \omega R_P C_P \omega = \frac{E^2 \omega^2 R_P{}^2 C_S C_P}{R}.$$

In some cases the resistance R is shunted by a condenser C_R (Fig. 263B). This renders the bridge more symmetrical, which is advantageous when electrostatic shielding is applied. With this arrangement,

$$C_X = C_S\left(\frac{R_P}{R}\right)\left(\frac{1 + \omega^2 C_R{}^2 R^2}{1 + \omega^2 C_R C_P R R_P}\right).$$

$$\rho_X = \left(\frac{R}{R_P}\right)\left(\frac{C_P R_P - C_R R}{C_S}\right)\left(\frac{1}{1 + \omega^2 C_R{}^2 R^2}\right).$$

$$\text{And P.F.} = \cos\theta_X = \omega C_X \rho_X = \frac{\omega(C_P R_P - C_R R)}{1 + \omega^2 C_R C_P R R_P}.$$

Compare these expressions with (46). If the capacitance of the specimen is large, the charging current may be much greater than can be taken through the branch R if it consists of ordinary resistance boxes. In this case (Fig. 263C), R is a resistance of ample carrying capacity shunted by the resistance rSr', where r and r' are variable, and S is a slide wire. $r + S + r'$ is kept at a convenient value. One galvanometer terminal is brought to the slide wire, as indicated. Using the transformation given on page 391, Fig. 263C becomes Fig. 263D.

From Fig. 263*D*, which has the form of a simple impedance bridge,

$$C_X = C_S R_P \frac{[r + S + r' + R]}{(r + S')R}.$$

$$\rho_X = \frac{R}{r + S + r' + R}\left[\frac{C_P}{C_S}(r + S') - (r' + S - S')\right].$$

$$\text{P.F.} = C_X \rho_X \omega = C_P R_P \omega - \left(\frac{C_S R_P \omega}{r + S'}\right)(r' + S - S') \qquad \text{very nearly.}$$

The second term is usually negligible.

For high-voltage work, the capacitance C_S might well be of the order of 100 $\mu\mu$f. The impedance of such a condenser at 60 cycles per second is about 2.6×10^7 ohms, while that of the arm P may be about 1,000 ohms. It is apparent that the voltage from R_P to ground is very small; consequently, the low-voltage branches may be manipulated with safety *provided* that neither the specimen nor the standard condenser breaks down. To provide against this contingency, protective film gaps across the low-impedance arms are provided. In case of a breakdown, they spill over, and the current is carried directly to ground. Additional protection is obtained by surrounding the low-voltage arms with grounded screens through which project thoroughly insulated handles for manipulating the resistances and condensers. As in all alternating-current bridges, the network must be thoroughly shielded against extraneous electrostatic effects.

Atkinson High-voltage Bridge.[35]—The Wien impedance bridge, as shown in Fig. 263, has been used by Monasch up to about 10,000 volts.[30] However, it is not well adapted to convenient and safe manipulation by the observer. For this reason, Atkinson has devised the modification indicated by Fig. 264*A* in which a low voltage is supplied to the arms P and N.

If V_1 and V_2 are in phase, the condition for balance is evidently

$$\frac{V_1 z_M}{z_X + z_M} = \frac{V_2 z_N}{z_P + z_N}.$$

Inserting the various impedance operators gives, if $V_1/V_2 = n$,

$$z_X = n\left(\frac{R_M}{R_N}\right)\left(R_P + R_N - \frac{j}{\omega C_P}\right) - R_M,$$

from which the equivalent series capacitance and loss resistance are

$$C_X = \left(\frac{1}{n}\right)\left(\frac{R_N}{R_M}\right)C_P, \qquad \rho_X = n\left(\frac{R_M}{R_N}\right)(R_P + R_N) - R_M.$$

The power-factor angle is given by $\tan\theta_X = 1/\omega C_X \rho_X$.

$$\text{P.F.} = \cos\theta_X = \frac{1}{\sqrt{1 + \tan^2\theta_X}} = \frac{\omega C_X \rho_X}{\sqrt{1 + \omega^2 C_X{}^2 \rho_X{}^2}} = \omega C_X \rho_X \qquad \text{approx.}$$

Therefore

$$\text{P.F.} = \omega C_P\left[R_P + R_N\left(1 - \frac{1}{n}\right)\right].$$

When high-voltage cables are dealt with, the term $1/n$ is negligible compared with unity. R_N in the formula may be replaced by a constant if the connections are as shown in Fig. 264*B*. The detector is an alter-

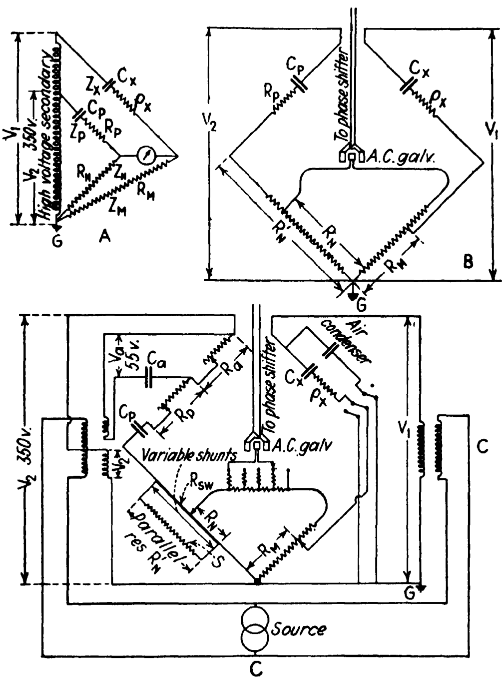


FIG. 264.—Pertaining to Atkinson high-voltage bridge.

nating-current galvanometer, the fixed coils being supplied from a phase shifter (page 521). With the connection of Fig. 264*B*,

$$\frac{V_2R_N}{R'_N + R_P - \dfrac{j}{\omega C_P}} = \frac{V_1R_M}{R_M + \rho_X - \dfrac{j}{\omega C_X}}.$$

From this

$$C_X = \left(\frac{1}{n}\right)\left(\frac{R_N}{R_M}\right)C_P. \qquad \rho_X = n\left(\frac{R_M}{R_N}\right)(R_P + R'_N) - R_M.$$

$$\text{P.F.} = \omega C_P\left[R_P + R'_N\left(1 - \frac{R_N}{nR'_N}\right)\right].$$

If $R_N/R'_N n$ is negligible compared with unity,

$$\text{P.F.} = \omega C_P[R_P + R'_N].$$

R'_N is constant.

In Fig. 264*C*, two additional features are introduced. The resistance R_a with the condenser C_a and the winding V_a on the low-voltage transformer are for the purpose of injecting into the low-voltage circuit a voltage *rise* to balance the voltage *fall* in R'_N. The variable shunt around the slide wire serves to change the range of the bridge.

Let

$$\frac{S + R_{SW}}{S} = m.$$

Then

$$I_{SW} = \frac{I_P}{m}.$$

If the voltage rise in R_a balances the voltage fall in R'_N

$$\frac{V_2 R_N}{\left(R_P - \frac{j}{\omega C_P}\right)m} = \frac{V_1 R_M}{R_M + \rho_X - \frac{j}{\omega C_X}}.$$

$$V_2 R_M R_N + V_2 R_N \rho_X - j\frac{V_2 R_N}{\omega C_X} = V_1 R_M m R_P - j\frac{V_1 R_M m}{\omega C_P}.$$

$$\rho_X = nm\left(\frac{R_M}{R_N}\right)R_P - R_M.$$

$$C_X = \frac{1}{nm}\left(\frac{R_N}{R_M}\right)C_P.$$

$$\text{P.F.} = \omega C_P\left[R_P - \frac{R_N}{nm}\right].$$

If R_N/nm is negligible compared with R_P

$$\text{P.F.} = \omega C_P R_P.$$

In this case, the resistance R_P may be graduated so that the power factor may be read directly from the scale.

The foregoing is on the assumption that V_1 and V_2 are in phase; if they are not, compensation may be made by adjusting the R_a, C_a circuit. To assist in this the "no-loss" air condenser is used in place of C_X the low-voltage side of which is grounded. The power-factor scale is set at zero, the voltage raised to the test value, and R_a, R_N, and R_M adjusted until balance is obtained. The air condenser and the unknown are then interchanged, and the final balance taken by adjusting R_N and R_M. The shunts on the alternating-current galvanometer enable substantially constant damping to be obtained.

For the Dawes-Hoover high-voltage bridge, see page 424.

The Heaviside or Mutual Inductance Bridge.[36]—If variable or alternating currents are used, a Wheatstone bridge which has three noninductive arms and one inductive arm cannot be made to balance, for the potentials at the two ends of the detector circuit can never be in the same time phase. A balance can be obtained, however, by the addition of an adjustable mutual inductance, or air-core transformer of variable ratio, the secondary of which is connected in series with the detector while the primary is placed in one of the leads from the source of supply to the bridge. The primary, therefore, carries the entire bridge current, and the mutual inductance introduces into the detector circuit a small e.m.f. which is in quadrature with that current.

An apparatus so arranged was used in 1886 by Hughes, and the results obtained were given by him in his inaugural address on assuming the presidency of the British Institution of Electrical Engineers. The discussion[37] that followed the presentation of this paper should be read by every student who has any doubts on the question of practice versus theory plus practice. It is sufficient to say here that, on account of an inadequate theory of his bridge, Hughes misinterpreted the readings that he obtained. Weber, Rayleigh, and Heaviside showed that the observations obtained by means of the Hughes apparatus are, when correctly reduced, in entire accord with the accepted theory of induction. Heaviside, in particular, studied and classified bridges of this general type. He showed that the Hughes arrangement is only a particular case, for the mutual inductance may be between any two specified branches.

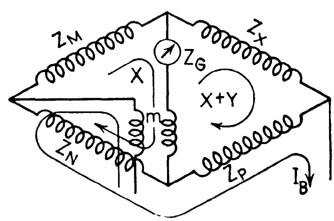


FIG. 265.—Mesh diagram for Hughes bridge.

The connections for the Hughes bridge are shown in Fig. 265. They are much like those for the Wheatstone bridge, but in the galvanometer circuit is included the secondary of the air-core transformer of variable ratio, the primary of this transformer being connected in the lead running from the source of current to the bridge. The mutual inductance of the air-core transformer will be represented by m. Assuming sinusoidal currents, the mesh equations are

$$(X + Y)(z_X + z_P + z_G) - Xz_G - I_Bz_P - jm\omega I_B = 0.$$
$$X(z_M + z_G + z_N) - (X + Y)z_G - I_Bz_N + jm\omega I_B = 0.$$

In respect to the sign given to the term involving the mutual inductance, in this and other methods of measurement, it may be either positive or negative depending on the manner in which the device is connected into the circuit. However, the particular connection and the corresponding sign in the equations must be used which will enable a balance to be obtained.

Solving the preceding equations for Y, the galvanometer current, and substituting the values of the impedances, the arms M, N, and P being noninductive,

$$Y = \frac{I_B[(R_P + jm\omega)(R_M + R_N) - (R_N - jm\omega)(R_X + R_P + jL_X\omega)]}{\text{denominator}}.$$

If only the condition of balance is required, it is not necessary to know the expression for the denominator. For balance, the numerator must be zero, or

$$R_P(R_M + R_N) - R_N(R_X + R_P) - m\omega^2 L_X + j[(R_M + R_N)m\omega + (R_X + R_P)m\omega - L_X R_N \omega] = 0.$$

Separating the quadrature components, the horizontal component gives

$$R_P R_M - R_N R_X = m\omega^2 L_X,$$

and the vertical component gives

$$m(R_M + R_N + R_X + R_P) = R_N L_X.$$

In order to obtain a balance, both these equations must be satisfied.

Solving for L_X and R_X,

$$L_X = \frac{R_P R_M}{R_N} m \left[\frac{\frac{1}{R_N} + \frac{R_M + R_N + R_P}{R_P R_M}}{1 + \frac{m^2\omega^2}{R_N^2}} \right].$$

$$R_X = \frac{R_P R_M}{R_N} \left[\frac{1 - \frac{m^2\omega^2(R_M + R_N + R_P)}{R_P R_M R_N}}{1 + \frac{m^2\omega^2}{R_N^2}} \right].$$

In his paper, Hughes treated his observations as if he were dealing with an ordinary bridge, that is, as if R_X were equal to $R_P R_M / R_N$.

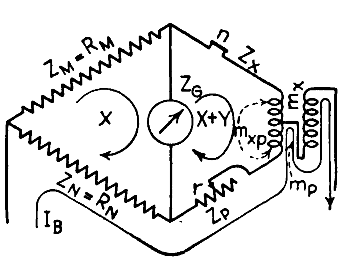


FIG. 266.—Mesh diagram for one form of Heaviside bridge.

Heaviside,[37] in his examination of the work of Hughes, pointed out that balance may be obtained if the secondary of the mutual inductance is introduced into a main bridge arm as well as if it is used in the galvanometer circuit. Figure 266 shows a bridge arranged in this manner; as before, z with the proper subscript denotes the total impedance operator of the corresponding bridge arm.

The mesh equations are

$$X(z_M + z_G + z_N) - (X + Y)z_G - I_B z_N = 0,$$

or

$$X = \frac{Yz_G + I_B z_N}{z_M + z_N}, \tag{47}$$

and

$$X(z_X + z_P) + Y(z_X + z_P + z_G) - I_B z_P - j(m_X + m_P)\omega I_B + jm_{XP}\omega(X + Y) + jm_{XP}\omega(X + Y - I_B) = 0. \tag{48}$$

Assuming the bridge to be balanced, $Y = 0$; and substituting the corresponding value of X from (47) in (48) gives, as M and N are supposed to be nonreactive,

$$I_B R_N z_X - I_B R_M z_P = j(m_X + m_P)(R_M + R_N)\omega I_B + jm_{XP}(R_N - R_M)\omega I_B.$$

Separating the quadrature components, the horizontal component gives

$$R_N R_X = R_M R_P. \tag{49}$$

The vertical component gives

$$R_N L_X - R_M L_P = (m_X + m_P)(R_M + R_N) + m_{XP}(R_N - R_M). \tag{50}$$

It is seen that if the bridge ratio is unity, the result is independent of the mutual inductance between the X and P arms, and

$$R_X = R_P. \tag{49a}$$

$$L_X - L_P = 2(m_X + m_P). \tag{50a}$$

$(m_X + m_P)$ is the reading m of the inductometer; consequently,

$$L_X - L_P = 2m.$$

This arrangement is useful in measuring small inductances, for, as suggested by Campbell,[38] a method of differences can be used, thus eliminating the effects of residual inductances in the bridge arms.

Referring to Fig. 266, the inductance to be measured is inserted in the gap n.

Let L_{X_0}, L_{P_0}, R_{X_0}, R_{P_0}, and m_0 be the values of the inductances, resistances, and mutual inductance when the bridge is balanced with n short-circuited.

$$R_{X_0} = R_{P_0}.$$

$$L_{X_0} - L_{P_0} = 2m_0.$$

When the unknown inductance L_{X_1} of resistance R_{X_1} is inserted at n, and balance obtained by adjusting the mutual inductance and R_P, then

$$R_{X_0} + R_{X_1} = R_{P_1}. \tag{49b}$$

$$R_{X_1} = R_{P_1} - R_{X_0} = R_{P_1} - R_{P_0}. \tag{49c}$$

$$L_{X_0} + L_{X_1} - L_{P_0} = 2m_1. \tag{50b}$$

$$L_{X_1} = 2m_1 - (L_{X_0} - L_{P_0}).$$

$$L_{X_1} = 2(m_1 - m_0). \tag{50c}$$

The inductance is twice the change in the setting of the mutual inductance. The resistance is equal to the change in the setting of r.

Obviously, inductances from zero up to a maximum of about twice the full value of the mutual inductance can be measured.

In obtaining the inductance, it is not necessary to know the values of any of the resistances. The variable resistance r may be made of two small and straight wires placed a few millimeters apart and short-circuited by a bridge piece. The inductance of such an arrangement may be calculated, if it is necessary to allow for it.

The apparatus should be so arranged that the equality of the ratio arms may be tested, and their adjustment to exact equality facilitated by interchanging them. Any lack of equality affects the value of R_X much more than that of L_X.

Dawes and Hoover High-voltage Bridge.[39]—Dawes and Hoover devised a bridge of the Heaviside type particularly adapted for research

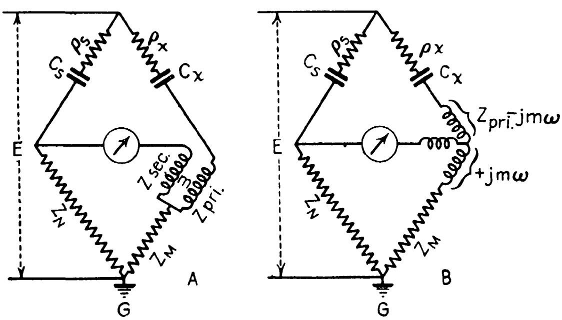


FIG. 267.—Diagram for Dawes-Hoover high-voltage Heaviside bridge.

work on high-voltage cables at power frequencies. The network is shown in Fig. 267.

Referring to Fig. 267A, C_X and ρ_X are the capacitance and series loss resistance of the sample. C_S and ρ_S are the capacitance and series loss resistance of the standard condenser. z_{PRI} and z_{SEC} are the impedance operators pertaining to the primary and secondary of the mutual inductance, whose coefficient of mutual inductance is m. z_M and z_N are adjustable nonreactive resistances. Application of the transformation for mutual inductances, given on page 392, converts the network into the equivalent impedance bridge shown in Fig. 267B. For balance,

$$\frac{z_S}{z_N} = \frac{z_X + z_{PRI} - jm\omega}{z_M + jm\omega}. \tag{51}$$

In this case,

$$z_M = R_M. \qquad z_S = \rho_S - \frac{j}{\omega C_S}.$$

$$z_N = R_N.$$

$$z_X = \rho_X - \frac{j}{\omega C_X}. \qquad z_{PRI} = R_{PRI} + j\omega L_{PRI}.$$

Inserting these values in (51) and separating the vertical and horizontal components gives

$$\frac{1}{C_X} = \frac{R_M}{R_N C_S} + \omega^2\left[L_{PRI} - m\left(1 + \frac{\rho_S}{R_N}\right)\right].$$

$$\rho_X = \frac{m}{R_N C_S} + \frac{\rho_S R_M}{R_N} - R_{PRI}.$$

As C_S is an air condenser, $\rho_S = 0$, nearly enough for work on cables; also, $\omega^2[L_{PRI} - m]$ is negligible compared with $R_M/R_N C_S$; consequently

$$\rho_X = \frac{m}{R_N C_S} - R_P = \frac{m}{R_N C_S} \qquad \text{approx.} \tag{52}$$

$$C_X = \left(\frac{R_N}{R_M}\right) C_S. \tag{53}$$

The power factor of the sample is

$$\text{P.F.} = \omega C_X \rho_X = \frac{\omega m}{R_M}. \tag{54}$$

$$I_X = E C_X \omega \qquad \text{approx.}$$

Consequently, the power loss in the specimen is

$$W = \frac{E^2 \omega^2 m C_X}{R_M}. \tag{55}$$

An advantage of this form of bridge is the ease with which an exceedingly wide range of power factors may be covered. In some cases, three variable mutual inductors having ranges 0 to 30, 0 to 200, 0 to 2,000 millihenrys are supplied. By using the appropriate inductor it is possible to measure samples having power factors as low as 0.1 per cent and as high as 80 per cent. This wide range is necessary when investigating the behavior of insulations at high temperature or when subjected to voltages beyond the ionization point.

As with all high-voltage bridges, the proper shields and guards must be applied (see page 437).

Direct Admittances and Capacitances.[40]—Many cases arise in which it is necessary to determine the equivalents of complicated networks which have only a few accessible terminals and internal structures which are unknown and unknowable.

The existence of the star-mesh transformations implies that it is possible to replace a complicated network of admittances by another network consisting of admittances of the proper values connected only between the accessible terminals taken in pairs, all possible pairs being utilized; that is, by direct admittances which G. A. Campbell defines as follows:

> The direct admittances of an electrical system with n accessible terminals are defined as the $n(n - 1)/2$ admittances which, connected between each pair of terminals, will be the exact equivalent of the system in its external reactions upon any other system with which it is associated only by conductive connection through the accessible terminals. All direct coupling, either electromagnetic or electrostatic, between the system and conductors external to it is excluded.

To illustrate, consider the familiar Y-Δ arrangement of admittances shown in Fig. 268*A*. Referring to Fig. 268*A*, the number of direct admittances involved is $4 \times 3/2$, or 6; these are obviously the admittances shown in the figure. If only the terminals 1, 2, 3 are accessible,

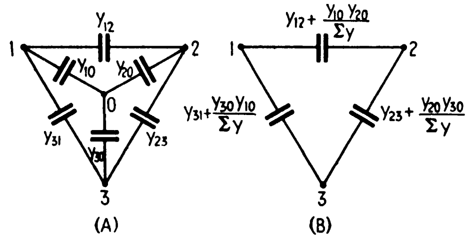


FIG. 268.—*A*, star-delta arrangement of admittances; *B*, equivalent three-terminal network of direct admittances.

the number of direct admittances is $3 \times 2/2$, or 3. By transforming the Y to the equivalent Δ, the arrangement is seen to be as shown in Fig. 268*B*. Either of these three terminal networks will produce the same electrical reactions at all points external to 1, 2, 3. What goes on internally to these three points is very different in the two cases.

In dealing with direct admittances, it is to be noted[40] that connecting an admittance between two terminals adds that admittance to the direct admittance existing between the two terminals and leaves all the other direct admittances unchanged. Connecting the accessible terminals of two distinct electrical systems in pairs gives a system in which each direct admittance is the sum of the corresponding direct admittances. Joining two terminals of a single electrical system to form a single terminal adds together the two direct admittances from the two merged terminals to any third terminal and short-circuits the direct admittance between the two merged terminals, leaving all other direct admittances unchanged. Combining the terminals into any number of merged groups leaves the total direct admittance between any pair of groups unchanged and short-circuits all direct admittances within the group.

Measurement of Direct Admittances.—As an illustration, consider the direct capacitances in a four-conductor lead-covered cable (Fig. 269). There are five accessible terminals, so the equivalent network will consist of 10 elements or direct capacitances, as shown in Fig. 269*B*.

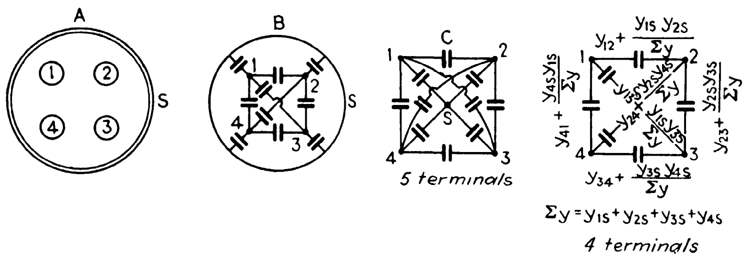


FIG. 269.—Pertaining to direct admittances.

Suppose that it is necessary to measure y_{13} or C_{13}. It cannot be isolated but must be measured while associated with nine other direct capacitances. If the currents are sinusoidal, the volt-ammeter method may be employed (page 385). In this and other cases, it is possible by the proper connections to short-circuit some of the direct admittances, throw others into positions where they do not affect the measurements, and by the proper procedure eliminate the contributions to the measured current of the currents through all the remaining superfluous direct

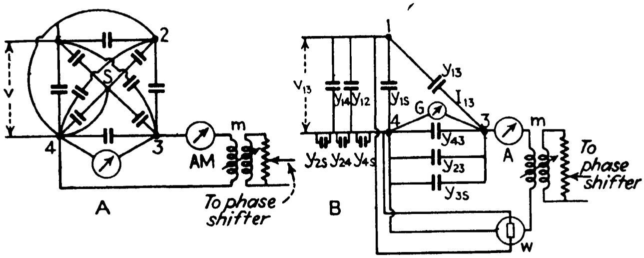


FIG. 270.—Pertaining to measurement of direct capacitance.

admittances. To obtain y_{13}, it is necessary to measure the current I_{13} and the voltage V_{13}; then

$$I_{13} = y_{13}V_{13} = j\omega C_{13}V_{13}.$$

$$C_{13} = \frac{I_{13}}{\omega V_{13}} \qquad \text{r.m.s. values used.}$$

Figure 270*A* shows possible connections; 270*B* indicates more clearly the location of the various capacitances after the connections have been made. Terminals 2 and *S* are connected to 4. The ammeter to measure I_{13} is connected between 3 and 4 via the secondary of a variable mutual inductance *m*, which is actuated from a phase shifter (page 521). The net work thus becomes that of Fig 270*B*. By varying *m* and the phase shifter, the detector G is brought to zero. Then no current flows between 3 and 4, there being no difference of potential. The current through A is I_{13}; the reading of the voltmeter is V_{13}; consequently,

$$C_{13} = \frac{I_{13}}{\omega V_{13}}.$$

If there is a power loss P_{13}, in y_{13}, it may be measured by connecting the potential coil of a wattmeter across V and the current coil in series with A. Then

$$C_{13} = \frac{1}{\omega V_{13}}\sqrt{I_{13}^2 - \frac{P_{13}^2}{V_{13}^2}} \qquad G_{13} = \frac{P_{13}}{V_{13}^2}.$$

The foregoing is an application to sinusoidal alternating currents of the scheme given on page 159 for direct currents.

Colpitts Bridge Method.[40]—Direct admittances are most accurately and conveniently measured by means of the Colpitts bridge. The

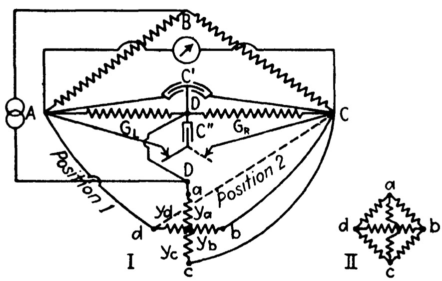


FIG. 271.—Illustrating Colpitts bridge method for direct admittances.

fundamental idea is to arrange an alternating-current bridge so that all the direct admittances except that to be determined are short-circuited; placed in shunt with the detector, where they do not affect the balance; or permanently connected in an arm of the bridge, so that when the unknown is transferred from one arm of the bridge to the other and the difference of the readings taken, their effect is eliminated.

To illustrate, take the case of a four-pointed star of admittances with an inaccessible junction point. It is desired to determine the component admittances of a four-terminal network which will exactly replace it, the new admittances being connected only between the accessible terminals; that is, they are to be direct admittances. The principle involved may be illustrated by Fig. 271.

The star in I and the mesh with chords in II are exactly equivalent at all points external to *abcd*, and it would be impossible to tell from measurements made between the terminals which of the two networks was being dealt with.

Referring to Fig. 271I, AB and BC are equal nonreactive ratio arms; C' is a continuously variable double condenser; C'' is a variable condenser which can be shifted from the left to the right side of the bridge by the switch; G_L and G_R are adjustable conductances. As the bridge has equal ratio arms, it will balance when the total admittance to the left of D, y_L, is equal to the total admittance to the right of D, y_R. One terminal of the direct admittance which is to be determined is connected permanently to D, while the other is first connected to A, while all the other accessible terminals are permanently connected to either C or A. As shown, y_{ab} is in parallel with DC; y_{bc} is short-circuited; y_{cd} is in parallel with the detector; y_{ac} is in parallel with DC; y_{db} is in parallel with the detector.

At balance, with the lead in position 1,

$$y_L + y_{ad} = y_R + y_{ab} + y_{ac}. \tag{56}$$

At balance, with the lead in position 2,

$$y'_L = y'_R + y_{ab} + y_{ac} + y_{ad}. \tag{57}$$

Subtracting (56) from (57),

$$y_{ad} = \frac{(y'_L - y_L) - (y'_R - y_R)}{2}. \tag{58}$$

If C_L and C_R and G_L and G_R are the total capacitances and conductances at balance, to the left and right of D,

$$G_{ad} + j\omega C_{ad} = \frac{G'_L + j\omega C'_L - G_L - j\omega C_L - G'_R - j\omega C'_R + G_R + j\omega C_R}{2}$$
$$= \frac{(G'_L - G_L) + (G_R - G'_R) + j\omega[(C'_L - C_L) + (C_R - C'_R)]}{2}.$$

In the Colpitts bridge,

$$C_L + C_R = \text{a constant.}$$
$$G_L + G_R = \text{a constant.}$$

Therefore

$$G'_L - G_L = G_R - G'_R.$$
$$C'_L - C_L = C_R - C'_R.$$

Consequently,

$$G_{ad} + j\omega C_{ad} = (G'_L - G_L) + j\omega[C'_L - C_L]$$

and

$$G_{ad} = G'_L - G_L, \qquad C_{ad} = C'_L - C_L. \tag{59}$$

The coarse variations of capacitance are obtained by means of a subdivided condenser C'', which can be thrown into either the left or the right side of the bridge. The fine variations are obtained by the continuously variable double condenser C', which adds as much capacitance to one side of the bridge as it removes from the other.

The sum of the variable conductances G_L and G_R must be kept constant. To obtain this result, the arrangement shown in Fig. 272 is used for the larger steps.[40] Between A and C are 10 equal, nonreactive coils in series each of resistance R. If the radial arm simply made contact with taps at $0'$, $1'$, $2'$, etc., the conductances between A and D and D and C would not vary by equal steps, so the arm is arranged to make its contacts along a fringe of supplementary resistors so proportioned that $G_{AD} + G_{DC}$ is constant and the steps are of equal value.

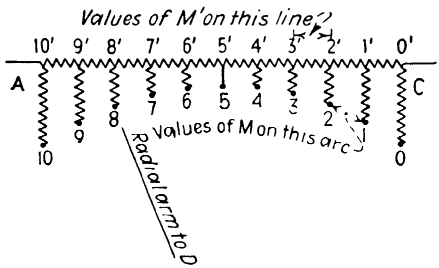


FIG. 272.—Diagram for variable conductance.

If the arm is at No. 5, then

$$G_{AD} = \frac{1}{5R} \qquad G_{DC} = \frac{1}{5R}.$$

$$G_{AD} + G_{DC} = \frac{2}{5R}.$$

This value is to be maintained irrespective of the position of the arm, while G_{AD} and G_{DC} are varied by equal steps. Suppose that the arm is set at any terminal n; then

$$G_{An'} = \frac{1}{R(10 - n)} \qquad G_{n'C} = \frac{1}{nR}.$$

At any setting of the movable arm, the conductance between n and n' must be so proportioned that $G_{AD} + G_{DC} = 2/5R$. Let G_n be the conductance which must be inserted in series with the radial arm to maintain this relation. By the transformation given on page 391,

$$G_{AD} = \frac{G_n(1/R(10 - n))}{\dfrac{1}{R(10 - n)} + \dfrac{1}{nR} + G_n}, \qquad G_{DC} = \frac{G_n(1/nR)}{\dfrac{1}{R(10 - n)} + \dfrac{1}{nR} + G_n},$$

and

$$G_{AD} + G_{DC} = \frac{2}{5R}.$$

Therefore

$$G_n = \frac{20}{R[50 - 2n(10 - n)]}.$$

If the contact is on No. 3,

$$G_3 = \frac{20}{8R} = \frac{5}{2R}.$$

$$G_{AD} = \frac{\frac{1}{7R} \times \frac{5}{2R}}{\frac{1}{7R} + \frac{5}{2R} + \frac{1}{3R}} = \frac{3}{25R}.$$

$$G_{DC} = \frac{\frac{1}{3R} \times \frac{5}{2R}}{\frac{1}{7R} + \frac{5}{2R} + \frac{1}{3R}} = \frac{7}{25R}.$$

$$G_{AD} + G_{DC} = \frac{10}{25R} = \frac{2}{5R}.$$

For other settings, the sequence of the conductances is

n	G_n	G_{AD}	G_{DC}
0	$\frac{2}{5R}$	0	$\frac{10}{25R}$
1	$\frac{5}{8R}$	$\frac{1}{25R}$	$\frac{9}{25R}$.
2	$\frac{10}{9R}$	$\frac{2}{25R}$	$\frac{8}{25R}$.
. . .	. . .	. . .	. . .
8	$\frac{10}{9R}$	$\frac{8}{25R}$	$\frac{2}{25R}$.
. . .	. . .	. . .	. . .
10	$\frac{2}{5R}$	$\frac{10}{25R}$	0

Finer subdivisions of conductance are obtained by a slide-wire arrangement which also has the characteristic that $G_{AD} + G_{DC}$ = constant.

Extraneous Effects in Bridges.[41]—In the previous demonstrations and those that follow, it is assumed that all the current paths in the networks are shown in the diagrams. Consideration will show that in reality the complete diagrams are much less simple.

1. There is the possibility of electromagnetic induction between any arm of the bridge and all the other arms. The e.m.fs. in the bridge arms so produced may be sources of error unless they are controlled. In general, they must be reduced to negligible amounts by the proper design and placing of any inductors used in the bridge. For instance, fixed inductors should be of the toroidal form. It may be noted that inductors that are astatic in uniform stray fields will not be astatic in the nonuniform field near a conductor carrying a current. Variable inductometers of the Ayrton and Perry form are potential sources of error and

should be used accordingly. Frequently, the coils being measured produce a large stray field which may act inductively on the bridge arms. With certain forms of detector, for instance a telephone or an unshielded moving-needle vibration galvanometer, the direct action of this stray field on the detector will give rise to errors unless the unknown is placed at a distance, properly oriented, and the effects of the long test leads allowed for.

In an important group of bridges, first carefully studied by Heaviside, the electromagnetic induction between two designated arms is controlled and utilized in obtaining the balance (see page 424). Generally, the effects of electromagnetic induction do not give trouble if the network is properly designed. However, there is always a chance of "pickup" from stray fields originating outside the network. To avoid this, if a vibration galvanometer or other tuned detector is used, the bridge should be operated at a frequency slightly different from that of the neighboring commercial circuits, for instance at 55 or 65 rather than at the normal 60 cycles per second.

2. As the bridge arms and their elements are at different potentials and separated by dielectrics, there is the possibility of electrostatic induction between them. However, in a well-designed network, the spacings are so great that this source of error is not of consequence. The most important source of error is in the self-capacitances and capacitances to ground and neighboring objects of the bridge parts, which may, in effect, shunt the bridge arms. This source of error becomes of extreme importance when the frequency is raised, for the admittances of the electrostatic paths are then increased and may become comparable with the admittances of the bridge arms which they shunt. Unless the proper precautions are taken, every change in environment, such, for example, as a change in the position of the observer, alters these extraneous admittances and causes the bridge to give variable and incorrect results. The environment of three of the bridge arms can be controlled. The fourth or X arm in many cases must be accepted "as is." If practicable, it should be far removed from neighboring objects. The immediate environment of resistors, inductors, and capacitors used in bridge arms is rendered definite by surrounding them with conducting shields, as indicated in Fig. 274.

The Wagner Ground Connection.[42]—The power sources used in bridge work have two obvious terminals; however, they all contain more or less complicated networks, the various parts of which have capacitances to ground. Consequently, the assembly is not necessarily the equivalent of an isolated source of e.m.f. in series with an admittance. The ideal power source is one in which the admittances to ground may be considered as concentrated at the two terminals.

In many cases, a telephone is a satisfactory detector for use with impedance bridges. It will be found that although the sound in the telephone may be brought to a minimum, it does not disappear as it should according to the theory based on the assumption that the only impedances concerned are those shown in Fig. 257.

These "head noises" are due to the fact that the power source is capacitively connected to the ground upon which the observer stands so that there is a potential difference between the observer and the windings of the telephone. There is an admittance to ground via the observer's body which prevents an exact balance from being obtained. Evidently when the bridge is balanced, if both b and d are brought to ground potential, the head noises will disappear. The Wagner earth connection was devised with this end in view; but when used with a properly arranged bridge, it accomplishes much more than this.

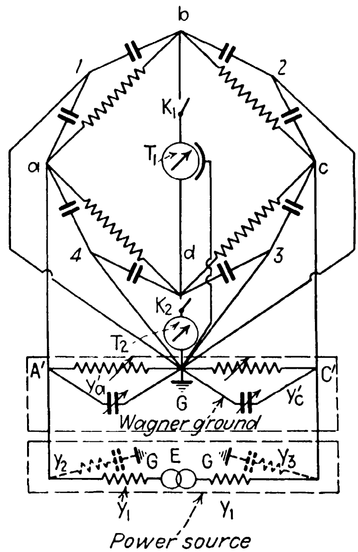


FIG. 273.—Diagram for impedance bridge with four single-element shielded arms and Wagner ground.

Figure 273 shows a Wagner ground applied to an impedance bridge arrangement in which the four main arms consist of three terminal networks, in this case shielded resistors. The Wagner ground is shown at $A'GC'$. It consists of two admittances y_a' and y_c' placed in series across the terminals of the power source, their junction being grounded at G. The two sections are made up of combinations of resistance, inductance, and capacitance, as is necessary. Any combination that will produce the desired result may be used. Generally, y_a' would be similar in its make-up to y_{ad}, and y_c' similar to y_{dc}. T_1 and T_2 are two detectors (or one, if supplied with the proper switching arrangement.)

The ratio arms ad and cd may first be balanced to ground, using the detector T_2. Examination of the connections will show that the direct admittances $d4$ and $d3$ shunt the detector and so have no effect on the balance, while the admittances $a4$ and $c3$ are added to the admittances of the Wagner ground and the power source to the right and left of G. The net admittance from A' to G and C' to G is adjusted by varying y_a' and y_c' until T_2 is balanced. The arms ab and bc may then be added. The direct admittances $b1$ and $b2$ are connected to ground in parallel with the detector, while $a1$ and $c2$ increase the net admittance to the left and right of G. By successive adjustments of the bridge and the Wagner ground, both T_1 and T_2 are brought to zero, the switch at T_2 being finally left open. The points b, d, G are all at the same (ground) potential. There will be no current through the direct admittances $b1$ and $b2$, so they do not influence the balance. The admittances $a1$ and $c2$ are virtually parts of the Wagner ground.

The admittances $b1$ and $b2$, $c2$ and $c3$, etc., may be combined to give an admittance to ground at each of the four bridge corners. If both the bridge and the Wagner ground are balanced, the three points b, d, G are at ground potential. The admittance dG shunts the detector T_2; the admittance bG shunts T_1 and T_2; while the admittances aG and cG are virtually parts of the Wagner ground, and the result is as before. When the Wagner ground is employed, both ends of the detector winding and the observer's body are at ground potential, and no current will flow through any of the telephone windings. Head noises are thus eliminated. At final balance,

$$\frac{y_{ab}}{y_{bc}} = \frac{y_{ad}}{y_{cd}},$$

superfluous direct admittances having been eliminated.

The successful application of the Wagner ground is dependent on the use of a power source in which the admittances to ground may be regarded as concentrated at the two visible terminals. If this is not so, the admittances of the Wagner ground cannot be placed in parallel with the admittances to ground of the power source, and it will not be possible to obtain a balance with the ground connection.

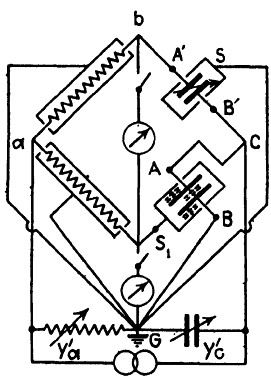


FIG. 274.—Showing use of Wagner ground in measuring partial capacitances.

As it is desirable not to be restricted in the choice of power sources, the bridge is usually actuated through a transformer. The secondary should be screened electrostatically from the primary.

As shown in Fig. 273, all four bridge arms are shielded. The unknown arm is generally not shielded. In this case, it is assumed to be so distant from neighboring movable objects that the direct admittances to these objects may be neglected. With the Wagner ground, one terminal of the unknown is necessarily at ground potential. The working conditions for some apparatus require that tests be made where the mid-point rather than the terminal is at ground potential.

Measurement of Partial Capacitances.—The balance of a bridge provided with a Wagner ground depends only on the direct impedances of the four bridge arms. This device makes possible the determination of partial and direct admittances and capacitances. Figure 274 shows the application to the measurement of the capacitance of a three-terminal shielded air condenser between one of the main plates and the shield. The two terminals, between which the desired admittance is located, are connected between C and S_1; the remaining term'nal is connected to the Wagner ground.

Referring to the figure, the main capacitance, between A and B, virtually becomes a part of the grounding device, being in parallel with y'_c. No current flows between B and S_1 for at balance they are at the same potential. Thus the only capacitance that affects the balance is that between A and S.

SHIELDED BRIDGES

Shielded Capacitance Bridge.[43] **Substitution Method.**—A form of capacitance bridge adapted to the investigation of the dielectric properties of small samples of insulation at low voltages by the substitution method is shown diagrammatically in Fig. 275A, and with shields and a Wagner ground applied in Fig. 275B.

The bridge has ratio arms of equal resistance shunted by continuously variable condensers to obtain phase adjustment. The shields are permanently grounded, with the exception of those on C_1, R_1, C_2, R_2, C_3, R_3, and C_4, R_4, which are permanently connected to the junction points b and c. The Wagner ground serves to adjust the potentials of the bridge parts relative to the grounded shield. In Fig. 275A, C_3R_3 is a continuously variable air condenser of high quality —capacitance about 1,500 $\mu\mu f$. The specimen is represented by C_XR_X.

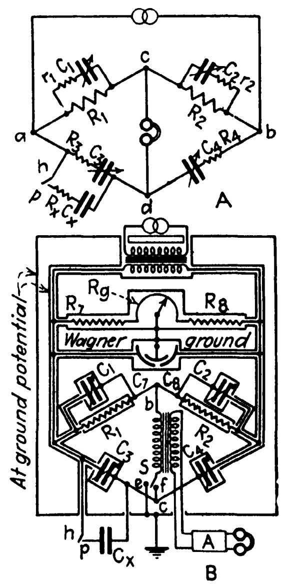


Fig. 275.—Capacitance bridge arranged for substitution method.

With p and h in contact, the switch on f, and C_4 set at a convenient value, the bridge is balanced by varying C_3, C_1, and C_2. The switch is then transferred to e, and the Wagner ground adjusted, after which the switch is transferred back to f, and a final balance taken without disturbing C_4. The contact between p and h is then broken, and a second bridge balance obtained.

Denoting the second values by primes, the capacitance, C_X, and power factor, η_X, are given by

$$C_X = C'_3 - C_3.$$

$$R_X\omega C_X = \eta_X = \omega\left(\frac{C'_3}{C_X}\right)[R_2(C_2 - C'_2) - R_1(C_1 - C'_1)]. \quad (60)$$

It is to be noted that on account of the approximations involved (60) is not a general formula; it applies when the insulating material is of good quality.

From the first balance with the unknown X in arm 3,

$$\frac{z_1}{z_2} = \frac{z_{3X}}{z_4}.$$

From the second balance,

$$\frac{z_1'}{z_2'} = \frac{z_3'}{z_4}.$$

Therefore

$$\left(\frac{z_1}{z_2}\right)\left(\frac{z_2'}{z_1'}\right) = \frac{z_{3X}}{z_3'}.$$

The condensers C_1, C_2, and C_3 are of high quality but are not assumed to be perfect; r_1, r_2, and R_3 are their series loss resistances.

The impedance operator for arm 1 is

$$z_1 = \frac{R_1\left(r_1 - j\dfrac{1}{\omega C_1}\right)}{R_1 + r_1 - j\dfrac{1}{\omega C_1}} = \frac{R_1 - jR_1{}^2\omega C_1 + R_1 r_1 \omega C_1{}^2[R_1 + r_1]}{1 + [R_1 + r_1]^2(\omega C_1)^2}$$

$$= R_1 - jR_1{}^2\omega C_1 \qquad \text{approx.}$$

Let $\eta = R_1\omega C_1$; then $z_1 = R_1(1 - j\eta_1)$. Similarly,

$$z_2 = R_2(1 - j\eta_2), \qquad \text{and} \qquad \frac{z_1}{z_2} = \left(\frac{R_1}{R_2}\right)[1 + j(\eta_2 - \eta_1)];$$

also

$$\frac{z_1'}{z_2'} = \left(\frac{R_1}{R_2}\right)[1 + j(\eta_2' - \eta_1')].$$

When X is in place, the impedance operator of arm 3 is

$$z_3 = R_{3X} - j\frac{1}{\omega C_{3X}} = \frac{\eta_{3X} - j}{\omega C_{3X}};$$

from the second balance,

$$z_3' = \frac{\eta_3' - j}{\omega C_3'}.$$

Therefore

$$\frac{1 + j(\eta_2 - \eta_1)}{1 + j(\eta_2' - \eta_1')} = \left(\frac{C_3'}{C_{3X}}\right)\left(\frac{\eta_{3X} - j}{\eta_3' - j}\right).$$

When products of very small quantities are neglected,

$$1 - j(\eta_2' - \eta_1') + j(\eta_2 - \eta_1) = \left(\frac{C_3'}{C_{3X}}\right)[1 + j(\eta_{3X} - \eta_3')].$$

Separating the horizontal and vertical components gives

$$C_{3X} = C_3' \qquad \text{and} \qquad (\eta_2 - \eta_1) - (\eta_2' - \eta_1') = \left(\frac{C_3'}{C_{3X}}\right)(\eta_{3X} - \eta_3').$$

For the arrangement shown in Fig. 275,

$$C_{3X} = C_3 + C_X \qquad \text{approx.}$$

Therefore

$$C_X = C_3' - C_3 = \Delta C_3.$$

For the arm 3,

$$\frac{1}{R_3 - j\frac{1}{\omega C_3}} + \frac{1}{R_X - j\frac{1}{\omega C_X}} = \frac{1}{R_{3X} - j\frac{1}{\omega C_{3X}}},$$

from which

$$C_{3X}\eta_{3X} = C_3\eta_3 + C_X\eta_X \qquad \text{approx.}$$

Therefore

$$(\eta_2 - \eta_1) - (\eta_2' - \eta_1') = \left(\frac{C_3'}{C_{3X}}\right)\left[\frac{\eta_X C_X}{C_{3X}} + \frac{\eta_3 C_3}{C_{3X}} - \frac{\eta_3' C_3'}{C_{3X}}\right].$$

For a continuously variable air condenser of high quality, the product of the equivalent series resistance and the square of the equivalent series capacitance is constant (page 370). Consequently,

$$\eta_X = \omega\left(\frac{C_3'}{C_X}\right)[R_2(C_2 - C_2') - R_1(C_1 - C_1')]. \tag{61}$$

$\eta_X = R_X\omega C_X$ is the tangent of the angle of defect of the sample. It is to be kept in mind that the formulae are not general, for they rest on the assumption that $R^2\omega^2C^2$ is negligible compared with unity. If the angle of defect of the sample is 1 deg.,

$$\eta_X = R_X\omega C_X = 0.0175, \qquad \text{and} \qquad \eta_X{}^2 = 0.0003.$$

The paper by Siskind[43] also discusses the effects of bridge grounding in capacitance bridges when the adjustment is made by a resistor in series with an air condenser.

Shielded Schering Bridge.[31]—Figures 276 and 277 show diagrammatically the connections of a shielded Schering bridge, developed by Balsbaugh for the precision investigation of small samples of insulating oils at voltages up to 5,000 volts.

Shielded air condensers are used for the two high-potential arms; that at the left is cylindrical and of fixed value, while that at the right consists of two continuously variable units (one, a vernier), connected in parallel. The construction of the units is shown in Fig. 228*A*. The resistances R_7 and R_8 are intended to be equal. C_7 and C_8 are high-quality, continuously variable air condensers. The capacitances of arms 7 and 8 include the capacitances of the condensers plus the unknown shunt capacitances to ground of the high-potential plates of C_7 and C_8 plus the

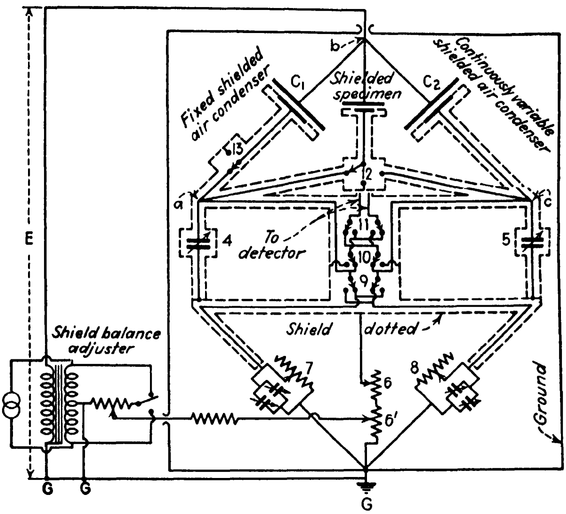


FIG. 276.—Indicating shielding for Schering bridge.

unknown equivalent shunt capacitances to ground of R_7 and R_8. The loss resistances R_{C1} and R_{C2} are the total loss resistances of the net capacitances. The bridge corners a and c are connected to switch 9, by which the bridge arms may be interchanged. The detector may be connected between a and c or between bridge and shield by switch 10 and reversed by switch 11. By means of switch 12 the shielded specimen may be placed in parallel with either high-potential arm or connected to the shield. The fixed air condenser C_1 may be connected to the bridge or the shield by switch 13. The shield is indicated by the dotted line. The adjustable condensers 4 and 5 form parts of the capacitances between a and c and the shield. Inspection of the diagram will show that the high-potential plates of C_1 and C_2 and the shield form air condensers which are connected to ground through the resistances 6 and 6′. The current through these two resistors would naturally be nearly in quadrature with the applied voltage E. It is necessary to be able to

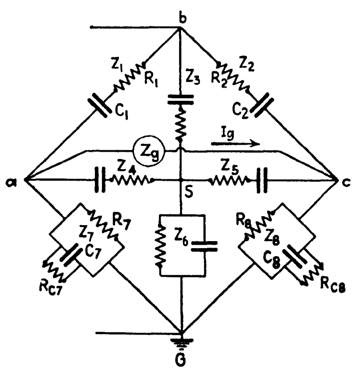


FIG. 277.—Diagram for shielded Schering bridge.

adjust the potential of the shield in both magnitude and phase. This might be accomplished by replacing the resistances 6 and 6′ by a suitable network consisting of inductances or condensers and resistances. However, in the arrangement shown, the adjustment is made by injecting into the connection 6, 6′ a component of voltage from the shield balance adjuster, which is in phase with E. In this bridge, balance is obtained by altering the capacitance of the high-potential condenser C_2 and the constants of arms 7 and 8.

The general expression for the detector current I_g is found by solving the network diagram (Fig. 277). This gives

$$I_g = E\frac{N}{D},$$

$$\left.\begin{aligned} \text{where } N = {} & z_2z_7z_3z_4z_5z_6\left(\frac{z_1}{z_3z_4} + \frac{z_8}{z_5z_6} + \frac{1}{z_3} + \frac{1}{z_4} + \frac{1}{z_5} + \frac{1}{z_6}\right) \\ & - z_1z_8z_3z_4z_5z_6\left(\frac{z_2}{z_3z_5} + \frac{z_7}{z_4z_6} + \frac{1}{z_3} + \frac{1}{z_4} + \frac{1}{z_5} + \frac{1}{z_6}\right). \end{aligned}\right\} \quad (62)$$

$$\left.\begin{aligned} D = {} & z_3z_4z_5z_6\left(\frac{1}{z_3} + \frac{1}{z_4} + \frac{1}{z_5} + \frac{1}{z_6}\right)[z_1z_2(z_7 + z_8 + z_g) \\ & + z_7z_8(z_1 + z_2 + z_g) + z_g(z_1z_8 + z_2z_7)] \\ & + z_2z_3z_4z_6z_8z_g(z_1 + z_7)\left(\frac{1}{z_3} + \frac{1}{z_4} + \frac{1}{z_6}\right) \\ & + z_1z_3z_5z_6z_7z_g(z_2 + z_8)\left(\frac{1}{z_3} + \frac{1}{z_5} + \frac{1}{z_6}\right) \\ & + z_1z_2z_7z_8(z_3 + z_6)(z_4 + z_5 + z_g). \end{aligned}\right\} \quad (63)$$

These complicated algebraic expressions are necessary if the effects of unbalanced conditions are to be studied. Consideration of (62) will show that the detector current will be zero when any two of the following conditions are satisfied:

$$z_1z_8 = z_2z_7. \quad (a)$$

$$z_1z_6 = z_3z_7. \quad (b)$$

$$z_2z_6 = z_3z_8. \quad (c)$$

If (b) and (c) are satisfied, the shield has been balanced. If these two conditions are not satisfied, that is, the shield is not balanced, but condition (a) and the condition $z_4z_2 = z_1z_5$ are satisfied, the detector current is also zero. $I_g = 0$ if

$$\frac{z_2z_7}{z_1z_8} = \frac{\dfrac{z_2}{z_3z_5} + \dfrac{z_7}{z_4z_6} + \dfrac{1}{z_3} + \dfrac{1}{z_4} + \dfrac{1}{z_5} + \dfrac{1}{z_6}}{\dfrac{z_1}{z_3z_4} + \dfrac{z_8}{z_5z_6} + \dfrac{1}{z_3} + \dfrac{1}{z_4} + \dfrac{1}{z_5} + \dfrac{1}{z_6}}.$$

If the shield is balanced, that is, if (*b*) and (*c*) hold,

$$\frac{z_2 z_7}{z_1 z_8} = 1 \qquad \text{or} \qquad \frac{z_1}{z_2} = \frac{z_7}{z_8}.$$

The ordinary bridge balance equation and the terms z_3, z_4, z_5, and z_6, which it would be very difficult if not impossible to determine, are eliminated.

The impedance operators for the various bridge arms are

$$z_1 = \rho_1 - j\frac{1}{\omega C_1},$$

$$z_2 = \rho_2 - j\frac{1}{\omega C_2},$$

$$z_7 = \frac{R_7 k_7 - j\omega R_7^2 k_7^2 C_7}{1 + \omega^2 R_7^2 k_7^2 C_7^2},$$

$$z_8 = \frac{R_8 k_8 - j\omega R_8^2 k_8^2 C_8}{1 + \omega^2 R_8^2 k_8^2 C_8^2},$$

where

$$k_7 = \frac{R_{C7}}{R_7 + R_{C7}}, \qquad \text{and} \qquad k_8 = \frac{R_{C8}}{R_8 + R_{C8}},$$

R_{C7} and R_{C8} being the parallel loss resistances of arms 7 and 8.

From these relations

$$\rho_X = \rho_2 = \frac{R_8 k_8}{C_1 R_7 k_7}\left[\frac{R_7 k_7 C_7 - R_8 k_8 C_8 + \rho_1 C_1 + \omega^2 \rho_1 R_7 R_8 k_7 k_8 C_1 C_7 C_8}{1 + \omega^2 R_8^2 k_8^2 C_8^2}\right], \tag{64}$$

$$C_X = C_2 = \frac{C_1 R_7 k_7}{R_8 k_8}\left[\frac{1 + \omega^2 R_8^2 k_8^2 C_8^2}{1 + \omega^2 \rho_1 C_1 (R_8 k_8 C_8 - R_7 k_7 C_7) + \omega^2 R_7 R_8 k_7 k_8 C_7 C_8}\right]. \tag{65}$$

These expressions should be compared with those on page 417 in which the standard condenser is assumed to be perfect, and the parallel loss resistance to ground infinite.

The tangent of the phase-defect angle of arm 2 is

$$\eta_2 = \omega \rho_2 C_2$$

or

$$\eta_2 = \frac{\omega R_7 k_7 C_7 - \omega R_8 k_8 C_8 + \omega \rho_1 C_1 + \omega^3 \rho_1 R_7 R_8 k_7 k_8 C_1 C_7 C_8}{1 + \omega^2 \rho_1 C_1 (R_8 k_8 C_8 - R_7 k_7 C_7) + \omega^2 R_7 R_8 k_7 k_8 C_7 C_8}.$$

If C_7 and C_8 are small and of high quality, $k_7 = 1$, and $k_8 = 1$, then

$$\eta_2 = \omega R_7 C_7 - \omega R_8 C_8 + \omega \rho_1 C_1 \qquad \text{approx.}$$

or

$$\eta_2 - \eta_1 = \omega R_7 C_7 - \omega R_8 C_8.$$

One method of determining η_S, the tangent of the phase-defect angle of a specimen of oil, is to connect to the shield by the switch 12 the filled oil cell which, to avoid the necessity of subsequently changing C_2 appreciably, should have a capacitance nearly equal to C_1; and then obtain the bridge and shield balances. As previously shown, this eliminates the effect of z_3, which contains the cell, and of z_4, z_5, and z_6. $\eta_2 - \eta_1$ is thus determined.

$$\eta_2 - \eta_1 = \omega R_7 C_{7d1} - \omega R_8 C_{8d1}, \tag{66}$$

in which the capacitances C_{7d1} and C_{8d1} are unknown. The sample is placed in arm 1 by switch 12, and C_1 connected to the shield. The bridge and shield balances are then taken, C_2 or C_8 being adjusted. Then, if C_8 is varied,

$$\eta_2 - \eta_S = \omega R_7 C_{7d1} - \omega R_8 C_{8d2}. \tag{67}$$
$$R_7 = R_8 = R.$$

Subtracting (67) from (66) gives

$$\eta_S - \eta_1 = \omega R(C_{8d2} - C_{8d1}). \tag{68}$$

As the only change made in arm 8 is to alter the condenser setting, $\eta_S - \eta_1$ is determined by the change in this setting. It is seen that to determine η_S, the quantity η_1 must either be known or be negligible. It may be made negligible by having the surfaces of C_1 clean, free from oxides, and under a low-potential gradient. If, on account of a slight difference between R_7 and R_8, a small change is made in C_2 in order to obtain a balance, η_2 is slightly altered but usually not sufficiently to affect appreciably the result. In the original paper by Balsbaugh and Herzenberg,[43] the bridge is thoroughly examined theoretically. See also the paper by Kouwenhoven and Baños.[43]

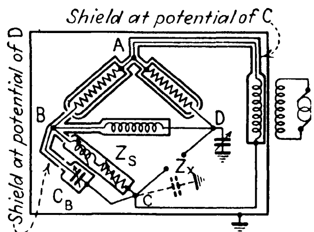


FIG. 278.—Shielded-bridge circuit with balancing condenser.

Extraneous admittances are effectively dealt with in the shielded bridges developed by G. A. Campbell[41] and his associates[33,44] for use in communication work. The fundamental idea is to concentrate as far as possible the capacitance effects between the diagonal corners of the bridge where they do not affect the balance and to provide an adjustment for any unbalance to ground of the terminals of the detector circuit.

Referring to Fig. 278, power is supplied through a shielded transformer, the shield being at the potential of the bridge corner C. The ratio arms are provided with double shields, the inner being connected to A; the outer, to C. The capacitance between the shields is thus put between A and C, where it does not affect the balance. The double shield thus renders the ratio arms independent of any change of environment. The outer shields on the ratio arms and the shield on Z_S are both at the potential of C. The junction point B and the primary of the detector transformer are protected by a shield connected to D. Any capacitance between B and D has no effect on the balance. There will be capacitance from C and D to ground which will affect the balance. Its value is controlled by a condenser connected from either C or D to ground, whichever has the lower capacitance, usually D. Capacitance from D to C, which would shunt the unknown Z_X, is balanced by the condenser C_B. If the mid-point of the unknown must be at ground potential, the capacitances from C to ground and D to ground are made equal.

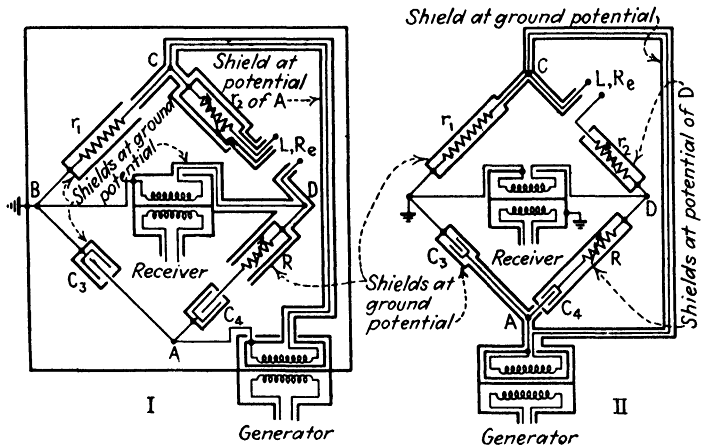


FIG. 279.—Shielding for Owen bridge.

A preliminary balance is made with C_B, both Z_X and Z_S being removed; then, without disturbing this adjustment, Z_S and Z_X are inserted, and balance obtained by adjusting Z_S. As the bridge ratio is unity, the resistance and inductance in Z_S are equal to the resistance and inductance in Z_X. If the test is to be made with one terminal of Z_X grounded, the point C is earthed, and the capacitance to ground of D balanced, Z_S and Z_X both being removed. Then Z_S and Z_X are inserted, and balance obtained as before.

A more complicated shielding problem is presented by the Owen bridge.[33] Figure 279I shows the complete solution as given by J. G. Ferguson, and Fig. 279II his practical compromise made to obtain simplicity of construction. Corresponding points in the two diagrams bear the same letter. In Fig. 279I it will be seen that all capacitance effects are concentrated across AC and across BD, where they do not affect the balance; or are added to C_3, where, as the shields are fixed in position, they can be included in the calibration. In the compromise of

Fig. 279II, there is capacitive shunting of r_1 due to the lead Cr_1, of R due to the lead RC_4, and of AB due to the lead from A to C_3.

The Owen bridge[33] is particularly suited for inductance measurements. Equivalent resistance measurements are greatly affected by the residuals, so for such measurements a careful determination of the bridge constants is necessary.

It is to be remembered that while the application of proper shielding renders coil residuals definite, it increases their values, and this may put a limit to the frequency at which the bridge may be used.

Figure 280 indicates the shielding arrangement for the Dawes-Hoover high-voltage bridge[39] (page 424) used in investigating the dielectric properties of cables and cable insulations. The high-voltage transformer

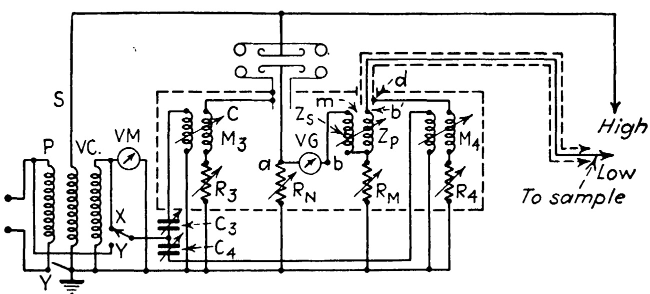


FIG. 280.—Shielding for Dawes-Hoover high-voltage bridge. Balance a to b, a to c, b' to d. Connection y may be used if one side of primary may be grounded.

is provided with a voltage coil VC for determining the potential difference applied to the sample and, if desired, for supplying the voltage to the shields. The magnitude and the phase of the voltage for the bridge shield and for the guard on the low-potential side of the standard condenser are regulated by the variable resistor R_3 and the variable mutual inductor and capacitor M_3 and C_3, while R_4, M_4, and C_4 serve to control the voltage applied to the guard on the sample and its lead. A preliminary balance is obtained with the vibration galvanometer connected between a and b. The galvanometer is then connected between a and c, and the shield balance obtained. It is then connected between b' and d, and the guard surrounding the low-potential lead to the sample brought to the same potential as the lead. All three balances are then perfected to obtain the final bridge balance.

Wilson Method for Measuring Inductance.[45]—In this method, the reactive component of the potential difference between the terminals of the unknown inductance is measured by a quadrant electrometer.[45] The connections are shown in Fig. 281, where, in this case, L_X and R_X are the inductance and resistance of a shunt such as is used in alternating-current testing.

The two sets of quadrants are connected to the terminals of the inductance X to be measured. One end of the needle circuit is attached to R_X, preferably at the middle. TR is an air-core transformer of mutual inductance m; and V, an electrostatic voltmeter for determining the potential of the needle. A is an ammeter for measuring the main current; and d, the voltage across X.

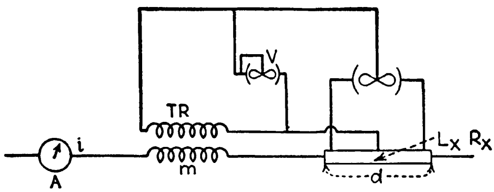


FIG. 281.—Connections for Wilson method for measuring inductance.

When applied to this case, the elementary formula for the deflection of the quadrant electrometer becomes

$$D = \frac{K}{T}\int_0^T\left(2d\left[\pm m\frac{di}{dt} - \frac{d}{2}\right] + d^2\right)dt.$$

$$D = \frac{2Km}{T}\int_0^T (d)\left(\frac{di}{dt}\right)dt.$$

$$d = R_X i + L_X\frac{di}{dt}.$$

Let the readings of the electrostatic voltmeter and of the ammeter be V and I. Assuming sinusoidal currents,

$$D = \frac{2KV}{I\omega}\left[\frac{1}{T}\int_0^T R_X i\left(\frac{di}{dt}\right)dt + \frac{1}{T}\int_0^T L_X\left(\frac{di}{dt}\right)^2 dt\right].$$

$$\frac{1}{T}\int_0^T i\left(\frac{di}{dt}\right)dt = 0,$$

and only the reactive component produces a turning moment. Then

$$L_X\frac{1}{T}\int_0^T\left(\frac{di}{dt}\right)^2 dt = L_X I^2\omega^2.$$

Therefore

$$D = 2\omega KVIL_X \qquad \text{or} \qquad L_X = \frac{D}{2\omega KVI}. \tag{69}$$

This method may be applied to the measurement of small inductances having a large current-carrying capacity and a small resistance; for example, it has been applied to shunts such as are used for alternating-current measurements.

The secondary of the air-core transformer may be the secondary of an ordinary induction coil. The primary may be wound to have a number of turns depending on the current to be used.

The Measurement of Inductances Containing Iron.—The preceding methods are adapted to the measurement of coils with air cores, for in that case the self-inductance is constant, as has been assumed in all the demonstrations. If the coils have iron cores, then, owing to the dependence of the permeability on the degree of saturation of the iron, the self-inductance is no longer constant but varies during the cycle.

In such cases, the effective inductance may be determined from measurements of the applied voltage, current, frequency, and power.

$$L = \frac{1}{\omega I}\sqrt{V^2 - \frac{P^2}{I^2}}. \tag{70}$$

The current should be adjusted to the value that it has in the ordinary use of the apparatus. With telephonic apparatus it is possible to obtain satisfactory results by bridge methods, for the saturation is so low that the permeability is practically constant.

Measurement of Mutual Inductance.—An obvious method of determining the mutual inductance of two coils is to connect them in series and to measure, by any convenient method, the net self-inductance of the combination; then to reverse one of the coils and again measure the net inductance. One measurement gives the sum, the other the difference, of the mutual and self-inductance effects. If the two net inductances are L' and L'', and the self-inductances of the coils are L_1 and L_2, respectively,

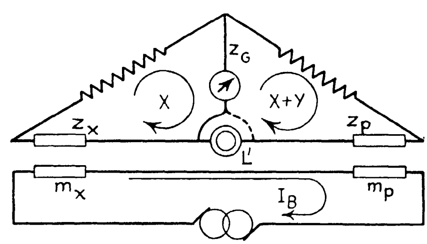


FIG. 282.—Mesh diagram for Maxwell method of comparing mutual inductances.

$$L' = L_1 + L_2 + 2m.$$
$$L'' = L_1 + L_2 - 2m.$$

Consequently the mutual inductance is given by

$$m = \frac{L' - L''}{4}. \tag{71}$$

Maxwell Method of Comparing Mutual Inductances.—The connections necessary for comparing two mutual inductances by Maxwell's method are shown in Fig. 282, where it is indicated that alternating currents replace the variable currents assumed in the original demonstration.* The two mutual inductances are m_X and m_P; z_X and z_P are

*"Treatise on Electricity and Magnetism," 3d ed., Art. 775.

the impedances of the respective meshes, *omitting* the detector; L' is a variable self-inductance of constant resistance which can be used in series with either m_X or m_P. Its inductance need not be known.

The mesh equations are

$$Xz_X - Yz_G - j\omega m_X I_B = 0,$$
$$Xz_P + Y(z_P + z_G) - j\omega m_P I_B = 0,$$

from which the detector current is

$$Y = j\omega I_B\left[\frac{z_X m_P - z_P m_X}{z_P z_G + z_X(z_P + z_G)}\right].$$

In order that the detector current may be zero

$$z_X m_P = z_P m_X.$$

When the values of z_X and z_P have been substituted, and the quadrature components separated, the horizontal component gives

$$\frac{m_X}{m_P} = \frac{r_X}{r_P},$$

or

$$m_X = \frac{r_X}{r_P} m_P, \qquad (72)$$

and the vertical component gives

$$\frac{m_X}{m_P} = \frac{L_X}{L_P}. \qquad (73)$$

As (73) must be satisfied, and the self-inductances of the secondaries of the mutual inductances cannot be varied at will, it is necessary to include the variable self-inductance L'.

If the source of current and the detector are interchanged, and the secondaries of the mutual inductances are connected in opposition, the arrangement will be balanced if the two secondary e.m.fs. are equal at every instant, that is, when

$$-j\omega\frac{V}{Z_X}m_X = -j\omega\frac{V}{Z_P}m_P;$$

therefore

$$\frac{m_X}{m_P} = \frac{z_X}{z_P}, \qquad \text{as before.}$$

A disadvantage of the method is that r_X and r_P include the resistances of the copper secondaries of the mutual inductances. Also the resistance of the copper wound inductance L' is included in either r_X or r_P; therefore immediately a balance has been obtained r_X and r_P must be determined by a bridge measurement.

Campbell[38] has developed the method so that these resistances are replaced in the formula for m_X by those of carefully calibrated bridge coils. The connections are shown in Fig. 283. Here m_P is a variable standard of mutual inductance; the inductance and the resistance of its primary circuit are fixed.

In carrying out the measurement, the switches are first thrown to position 1, the desired values of R_M and R_N inserted, and a balance

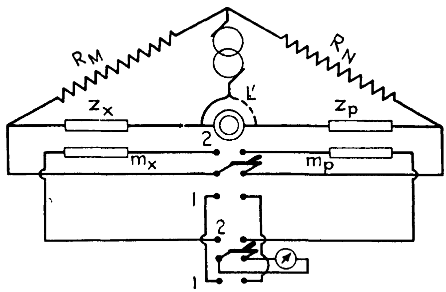


FIG. 283.—Connections for Campbell method of comparing mutual inductances.

obtained by adjusting L'. The arrangement is then a simple impedance bridge, and at balance

$$\frac{z_M}{z_X} = \frac{z_N}{z_P} \quad \text{or} \quad \frac{z_M}{z_N} = \frac{z_X + z_M}{z_N + z_P}.$$

The opposed secondaries of the mutual inductances are next introduced into the detector circuit by throwing the switches to position 2, and a balance obtained by adjusting m_P.

At balance, the two secondary e.m.fs. must be equal, or

$$-j\frac{\omega m_X V}{z_X + z_M} = -j\frac{\omega m_P V}{z_N + z_P}.$$

Therefore

$$\frac{m_X}{m_P} = \frac{z_X + z_M}{z_N + z_P} = \frac{z_M}{z_N} = \frac{R_M}{R_N},$$

and

$$m_X = \frac{R_M}{R_N} m_P. \tag{74}$$

Hartshorn Method for Comparing Unequal, Impure Mutual Inductances.[46]—The methods previously given are not applicable to impure mutual inductances, that is, to those where by reason of capacitance and eddy-current effects the voltages induced in the secondary are not in exact quadrature with the primary current. That those methods must fail is obvious, for they rest on the assumption that the induced voltage drop is given by $jm\omega I_P$. If this is not true, it will, in general, be impossible to obtain a perfect balance. The Hartshorn method obviates this difficulty and furnishes a measure of the impurity. The connections are as shown in Fig. 284.

The primaries are connected to the source in parallel; the current through one of them can be adjusted by R and L. R_1 and R_2 are two resistors. The ratio of R_1 to R_2 is at first made equal to the nominal ratio of m_1 to m_2. With K open, one of the currents is adjusted until G_1 stands at zero. The key K is then closed, and impurity added to the better coil by adjusting the slider in an attempt to balance G_2. By repeated trials, adjustment is made until both G_1 and G_2 are balanced. Then, if l_1 and l_2 are the residual inductances in R_1 and R_2, and $jm_1\omega + \sigma_1$ and $jm_2\omega + \sigma_2$ are the impure mutual impedances,

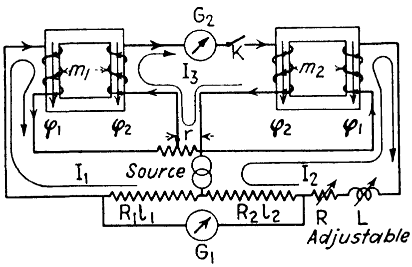


FIG. 284.—Hartshorn method for comparing impure mutual inductances.

$$-I_1[jm_1\omega + (\sigma_1 + r)] - I_2[jm_2\omega + \sigma_2] = 0,$$
$$-I_2(R_2 + j\omega l_2) - I_1(R_1 + j\omega l_1) = 0,$$

from which

$$m_2 = \left(\frac{R_2}{R_1}\right)m_1 + \frac{l_2(\sigma_1 + r) - l_2\sigma_2}{R_1}; \tag{75}$$

$$\sigma_2 = \left(\frac{R_2}{R_1}\right)(\sigma_1 + r) + \frac{\omega_2(m_2l_1 - m_1l_2)}{R_1}. \tag{76}$$

Dividing the value of σ_2 by $m_2\omega$ and substituting the approximate value $m_2 = (R_2/R_1)m_1$ gives

$$\frac{\sigma_2}{m_2\omega} = \frac{\sigma_1}{m_1\omega} + \frac{r}{m_1\omega} + \frac{\omega l_1}{R_1} - \frac{\omega l_2}{R_2}; \tag{77}$$

or, using phase angles, as they are small,

$$\delta_2 = \delta_1 + \frac{r}{m_1\omega} + \theta_1 - \theta_2, \tag{78}$$

where θ_1 and θ_2 are the phase displacements in the ratio arms.

Reliance should not be placed on a mutual inductometer until its impurity has been shown to be negligible for the measurements in hand. The presence of short-circuited turns may be detected by this test.

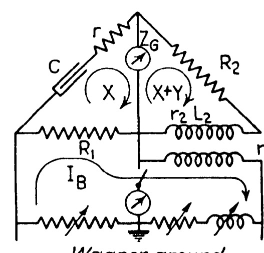


FIG. 285.—Connections for measurement of mutual inductance in terms of capacitance, with Wagner ground.

Measurement of Mutual Inductance in Terms of Capacitance.—The connections for the Carey Foster method of comparing a mutual inductance with a capacitance, as modified by Heydweiller,[47] are shown in

Fig. 285. C is a condenser; r, R_1, and R_2 are noninductive resistances. r_2 is the resistance, and L_2 the self-inductance of the secondary of the mutual inductance. The Wagner ground is essential to eliminate earth capacitance effects.[47] The impedance operators are

$$z_1 = R_1.$$
$$z_2 = (R_2 + r_2) + j\omega L_2.$$
$$z_C = r + \frac{1}{j\omega C}.$$

The mesh equations are

$$X(z_C + z_G + z_1) - (X + Y)z_G - I_B z_1, = 0.$$
$$(X + Y)(z_2 + z_G) - Xz_G \pm j\omega m_X I_B = 0.$$

Solving for the galvanometer current,

$$Y = \frac{I_B[z_1 z_2 - j\omega m_X(z_C + z_1)]}{z_2 z_G + (z_2 + z_G)(z_C + z_1)}.$$

Therefore the equation for balance is

$$R_1(R_2 + r_2) - \frac{m_X}{C} = j\omega[m_X(R_1 + r) - R_1 L_2].$$

On separating the quadrature components, the horizontal component gives

$$m_X = CR_1(R_2 + r_2). \tag{79}$$

The vertical component gives

$$L_2 = \frac{m_X(R_1 + r)}{R_1}. \tag{80}$$

The adjustment of R_2 and C does not disturb the second condition for balance, and an adjustment of r does not disturb the first condition. The two adjustments are thus independent, and the arrangement a convenient one. The resistance R_1 should have a large current-carrying capacity. From (80) it is seen that if the resistance r is not present, $m_X = L_2$. This is the lowest usable value of L_2. If the inductance in the branch 2 is below this value, it must be increased by adding an inductive coil. In Carey Foster's original method, the resistance r was absent, and variable currents were used.

Butterworth Corrections.[47]—Because of its importance, this method has been carefully studied at the National Physical Laboratory. Dye added the Wagner grounding device, and Butterworth has studied the effect of residual inductances and of impurity in the mutual inductometer. As the frequency is raised, the effective value of m and the impurity increase, the effective capacitance changes, and both the effective resistance and inductance of the primary and secondary are changed, and residual inductances in the so-called nonreactive resistances begin to affect the results. Whether these effects are important or not depends on what is to be accomplished by the measurement. If it is desired simply to measure a capacitance in terms of a mutual inductance at, for instance, 1,000 cycles per second, then for results of an accuracy of 1/10 per cent the corrections are of little consequence, as will be illustrated. On the other hand, if the equivalent series resistance or the power factor of the condenser is desired, the corrections must be applied; otherwise, the results have no significance whatsoever.

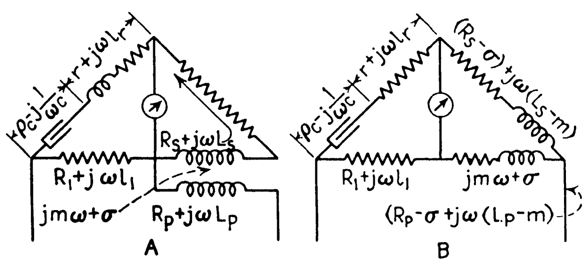


FIG. 286.—*A*, original network. *B*, transformed network. Transformation for Carey Foster method.

Referring to Fig. 286, ρ_C is the series loss resistance of the condenser; l_r, the residual inductance in r; l_1, the residual inductance in R_1; and σ, the impurity of the mutual inductance. By use of the transformation given on page 392, the original network is transformed into the impedance bridge shown in Fig. 286*B*. Then

$$\frac{(r+\rho_C)+j\left(\omega l_r-\frac{1}{\omega C}\right)}{R_1+j\omega l_1}=\frac{(R_S-\sigma)+j\omega(L_S-m)}{jm\omega+\sigma}.$$

Separating the horizontal and vertical components and solving for C gives, when terms depending on the products of very small quantities are neglected,

$$C=\frac{m}{R_1R_S\left[1-\frac{\sigma L_S}{mR_S}-\frac{\omega^2 l_1(L_S-m)}{R_1R_S}+\frac{ml_r\omega^2}{R_1R_S}\right]}.$$

The equivalent resistance of the secondary R_S depends on the frequency. Let R_{S0} be the value at low frequencies; then $R_S=R_{S0}+r_S$, where r_S is the increase due to the higher frequency. Inserting this value gives

$$C = \frac{m}{R_1R_{S0}\left[1 + \frac{r_S - \frac{\sigma L_S}{m}}{R_{S0}} - \frac{\omega^2 l_1(L_S - m)}{R_1R_{S0}} + \frac{\omega^2 l_r m}{R_1R_{S0}}\right]}.$$

$$C = \left(\frac{m}{R_1R_{S0}}\right)\left[1 - \frac{r_S - \frac{\sigma L_S}{m}}{R_{S0}} + \frac{\omega^2 l_1(L_S - m)}{R_1R_{S0}} - \frac{\omega^2 l_r m}{R_1R_{S0}}\right] \quad \text{approx.} \quad (81)$$

Solving for ρ_C gives

$$\rho_C = R_1\left(\frac{L_S}{m} - 1\right) + \frac{l_1R_S}{m} - r + \frac{\sigma}{mC\omega^2}. \quad (82)$$

The impurity σ introduces a constant error into ρ_C, since σ is proportional to ω^2.

The term L_S/m depends on the frequency. Let the value at low frequencies be L_0/m_0; then

$$\frac{L_S}{m} = \frac{L_0}{m_0} + \mu\omega^2.$$

μ is the frequency coefficient of L_S/m; $\mu\omega^2$, the change in L_S/m in passing from low frequency to a high frequency of angular velocity ω. Substituting in (82),

$$\rho_C = \left[R_1\left(\frac{L_0}{m_0} - 1\right) + \frac{l_1R_S}{m} - r\right] + \frac{\sigma}{mc\omega^2} + \mu R_1\omega^2. \quad (82a)$$

$$\text{P.F.} = C\omega\left[R_1\left(\frac{L_0}{m_0} - 1\right) + \frac{l_1R_S}{m} - r\right] + \frac{\sigma}{m\omega} + \mu R_1C\omega^3. \quad (83)$$

The effect of the corrections may be illustrated by data taken from Butterworth's paper.

$m_0 = 0.01$ henry.
$m = m_0(1 + \alpha\omega^2) = m_0(1.00098)$.
$\sigma = 0.0175$ ohm
$r_S - \frac{\sigma L_S}{m} = -0.0035$.
$\omega = 6{,}280$.
$l_1 = 0.0000028$ henry.
$L_r = 0.0000065$ henry.
μ = frequency coefficient of $L/m = -1.40 \times 10^{-11}$.

$L_{S0} = 0.02611$ henry.
$R_{S0} = 400.0$ ohms.
$r_S = 0.042$ ohm.
$R_1 = 50.0$ ohms.

$$R_1\left(\frac{L_{S0}}{m_0} - 1\right) + \frac{l_1R_S}{m} - r = 0.098.$$

Using these data

$$C = \frac{m_0(1.00098)}{R_1R_{S0}}[1.0 + 0.0000088 + 0.000088 - 0.000128]$$

$$C = \frac{m_0(1.00098)}{R_1R_{S0}}[0.99997].$$

$C = 0.5$ μf normal value.

It is seen that when measuring capacitances at a frequency of 1,000 cycles per second, it is necessary to apply the corrections only when results of high accuracy are desired. The corrections become greater as the frequency is raised.

The result is very different if the equivalent series loss resistance or the power factor is desired. Insertion of the data in (82a) gives for the equivalent series loss resistance

$$\rho_C = +0.098 + 0.089 - 0.028 = 0.159 \text{ ohm.}$$
$$\text{P.F.} = \rho_C C\omega = 0.00050.$$

In this case, even at the comparatively low frequency of 1,000 cycles per second, the results for the series loss resistance and the power factor are worthless unless the corrections are applied. This example gives a good illustration of the care with which any method of measurement must be scrutinized before it can be relied upon. Omission of the careful study of any proposed method of measurement may lead to serious results. For example, in the absence of such a study, one might use the relation $\rho_C = R_1\left(\frac{L_{S0}}{m_0} - 1\right) - r$ for the series loss resistance. However, it is known that ρ_C is small, and it is seen that this formula involves the difference of two comparatively large quantities. This fact alone would arouse suspicion as to the availability of this simple formula and lead one to investigate the possible sources of error. Use of the simple formula gives the absurd value $\rho_C = -0.014$ ohm, a value that might at first be attributed to faulty manipulation rather than to a defective knowledge of the theory of the method.

It is seen that it is a formidable task to obtain an accurate measurement of the power factor of a single sample of insulation by the Carey Foster method.

DEVICES FOR MAINTAINING CONSTANT SPEED

Occasionally, in inductance or capacitance measurements, the frequency and the amount of power required are such that it is best to employ a small sine wave alternator directly coupled to a motor. If a tuned detector is used, or if the bridge balance depends on the frequency, it is necessary that the speed be kept constant.

Gradual changes in the speed of a shunt or separately excited direct-current motor may be produced by

1. Changes in temperature of the windings.
2. Gradual changes of applied voltage.
3. Gradual changes of load.
4. Gradual changes of friction.

Sudden changes of speed may be produced by

1. Sudden changes of applied voltage.
2. Sudden changes of load.
3. Sudden changes of friction, such as might result from axial oscillation.
4. Poor brush contacts or erratic sparking.

Sudden changes of applied voltage are obviated by operating the motor from a storage battery.

Synchronizing Tuning Forks.—A properly driven tuning fork kept at a constant temperature maintains a fixed rate of vibration. This fact is utilized in a number of arrangements for maintaining a constant speed of rotation by means of small direct-current motors.

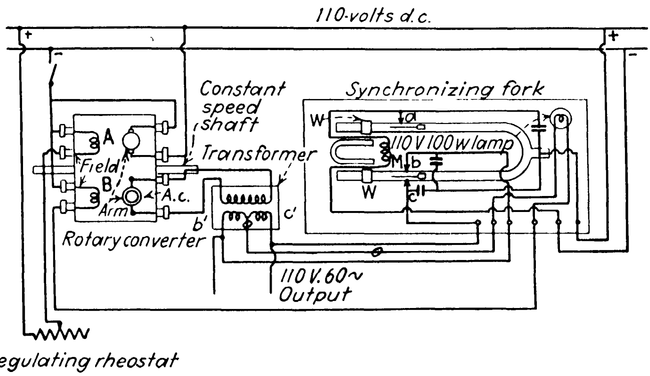


FIG. 287.—Diagram of connections of constant-speed and -frequency sets.

Connections for the device of the Leeds and Northrup Company, which is operated from a 110-volt direct-current circuit and capable of delivering a few amperes at 110 volts at the constant frequency of 60 cycles per second, are shown in Fig. 287. The alternating current is derived from a rotary converter which has two independent field windings. *A* is the regular winding; the current in it is controlled by the regulating rheostat. *B* is an auxiliary winding; the magnitude of the current in it is controlled by the action of the synchronizing fork which is maintained in vibration by the electromagnet *M* and the "buzzer action" of the contact at *a*. The converter delivers alternating current to the primary of the transformer. The lead *O* proceeds from the middle of the secondary through the lamp and through the auxiliary winding *B* to the − side of the direct-current line. Suppose the fork to be vibrating and that the primary of the transformer is open. When contact is made at either *a* or *b*, an intermittent unidirectional current will flow from + through the handle of the fork, one-half the winding of the transformer, the lead *O*, the lamp, and the auxiliary winding *B* to −. If the primary is closed, and the contacts at b and a are made at the zero points of the alternating-current waves, it is evident that the action of the current from + is not modified. If the contact at b occurs when the secondary e.m.f. of the transformer is directed from b' to c', a current will flow outward from the center tap whenever contact is made at b or c and the action of the direct current from + is reinforced. If contact at b is made when the secondary e.m.f. is directed from c' to b', the effect of the direct current from + is annulled in part. Thus the average field of the converter is increased or decreased, and the speed regulated accordingly. The amount of the regulating action is controlled by the positions of the points on the alternating-current waves at which the contacts are made.

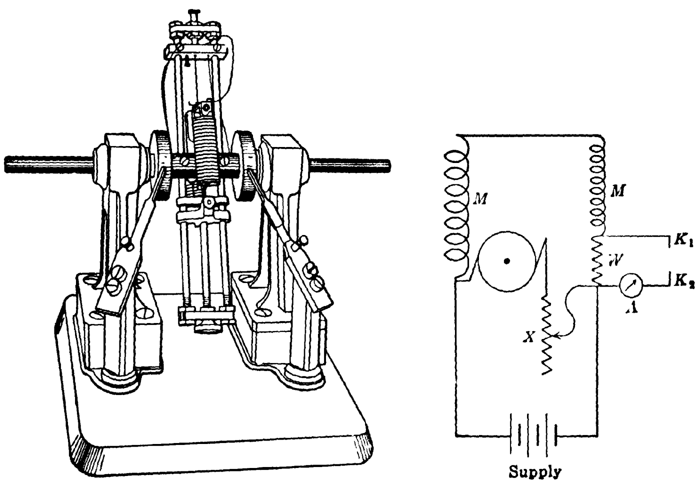


FIG. 288.—Giebe speed regulator.

It is essential that the contacts be in good condition. The precision of the control is to within 0.1 to 0.05 per cent. A 10 per cent change of the direct-current voltage may cause speed variations of 0.1 per cent. It is to be remembered that tuning forks have a temperature coefficient depending upon the material from which they are made.

A certain amount of adjustability is obtained by means of two movable weights, one on each prong of the fork. Each weight carries a thin metal plate provided with a slit, so that the fork may be used stroboscopically.

R. H. Frazier has described a convenient method of applying a synchronizing fork to control a small shunt motor.[48]

For very precise work it is best to avoid the use of spring contacts on the fork and employ an arrangement of photoelectric cells and vacuum tubes.

The Giebe-Helmholtz Speed Regulator.[48]—A shunt-wound direct-current motor may be controlled by a form of centrifugal governor suggested by Helmholtz and developed by Giebe.

Figure 288 shows the diagram of the circuits and a general view of the governor. The resistance X is for the rough adjustment of the speed.

When the speed rises too high, the governor short-circuits a resistance in the field circuit and thus decreases the speed.

Referring to Fig. 289, the frame D carries the mechanism and is rigidly attached to the shaft R which is directly connected to the motor. The electrical connections to the contacts K_1 and K_2 are made through the slip rings V. The weight P slides on a guide wire W and must move without appreciable friction; it is drawn inward by the helical springs F, the tension of which may be regulated. When the motor is started, the centrifugal force caused the weight to move out in opposition to the control exercised by the springs. If the speed is high enough, contact between K_1 and K_2 will be made and the resistance short-circuited. The motor then slows down a little and the contact is broken; the speed then rises and the cycle is repeated. Thus the speed is kept constant, subject to very slight oscillations about its mean value.

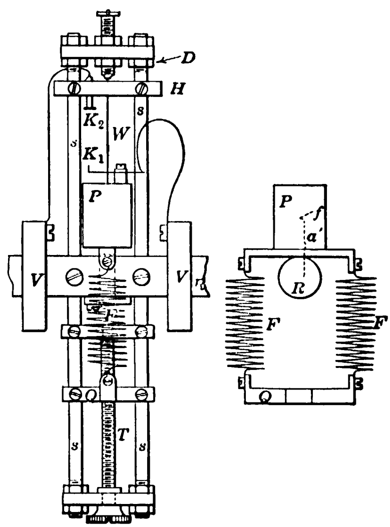


FIG. 289.—Details of Giebe speed regulator.

The position of the piece Q to which the rear ends of the springs are attached may be adjusted by the micrometer screw T (pitch, 1 mm.) and, when properly adjusted, may be clamped in position on the rods ss. The platinum contact K_1, which is carried by a flat spring of moderate strength, is insulated and mounted on the weight P and is connected to one of the slip rings by a flexible wire. The contact K_2 is a flat plate of platinum, and the bar H which carries it may be set at any desired distance from the axis. As the shaft runs through the device, two springs are used as indicated in the smaller figure.

It is obvious that on account of the shaft the center of gravity of the weight can never be brought to the axis of rotation. Let a' be its minimum possible distance from the center of the shaft, M be the mass of the weight, and C the constant of the springs, that is, the force in dynes necessary to extend the springs 1 cm. Let r be the distance of the center of gravity of the weight from the center of the shaft when the latter is rotating with an angular velocity ω. It will be assumed that the springs have been given an initial tension T_1; that is, when the device is at rest in a horizontal position and the weight is in contact with the shaft, a tension T_1 has been applied to the springs by means of the micrometer screw.

When the angular velocity is ω, and the center of gravity is r cm. from the axis, the centrifugal force will be

$$K_C = M\omega^2 r,$$

and the tension on the springs will be

$$K_S = C(r - a') + T_1.$$

At equilibrium, $K_C = K_S$, and

$$M\omega^2 r = C(r - a') + T_1.$$

The equilibrium may be stable, neutral, or unstable, depending on the initial tension T_1. For, suppose that at an instant when the center of gravity of the weight is distant r from the axis, K_C happens to be equal to K_S. If the weight is given a slight displacement outward δr,

$$\frac{\delta K_C}{K_C} = \frac{\delta r}{r}, \qquad \text{and} \qquad \frac{\delta K_S}{K_S} = \frac{C\,\delta r}{C(r - a') + T_1} = \frac{\delta r}{r - \left(a' - \dfrac{T_1}{C}\right)}.$$

T_1/C is the initial extension of the spring. It will be denoted by e'; then

$$\frac{\delta K_S}{K_S} = \frac{\delta r}{r - (a' - e')}.$$

If a' is greater than e', the denominator of the expression for $\delta K_S/K_S$ is less than r, $\delta K_S/K_S > \delta K_C/K_C$, and the weight will return toward its original position; that is, the equilibrium is stable. If $a' = e'$, then $\delta K_S/K_S = \delta K_C/K_C$, and the equilibrium is neutral. If a' is less than e', the denominator is greater than r, and $\delta K_S/K_S < \delta K_C/K_C$. In this case, the weight will suddenly fly outward to the extent of its travel, the spring being insufficient to make it return to its original position. This is the case of unstable equilibrium.

If the springs are given increasing tension, the equilibrium remains stable until $T_1 = Ca'$ or $e' = a'$, that is, until the tension is that which would bring the center of gravity of the weight to the center of the shaft if the motion of the weight in that direction were not limited. This may be called the critical value of the tension, since, if it is exceeded, the weight will suddenly fly outward.

While the device may be operated with any one of the three adjustments implied above, it is most sensitive and regulates best when the tension is near its critical value. Referring to the equation of equilibrium,

$$M\omega^2 r = C(r - a') + T_1,$$

or

$$\frac{M}{C}\omega^2 r = r - (a' - e').$$

If $Ca' = T_1$, or, what is equivalent, if $a' = e'$,

$$\omega = \sqrt{\frac{C}{M}}.$$

This may be called the *critical speed,* since for any higher speed the equilibrium is unstable.

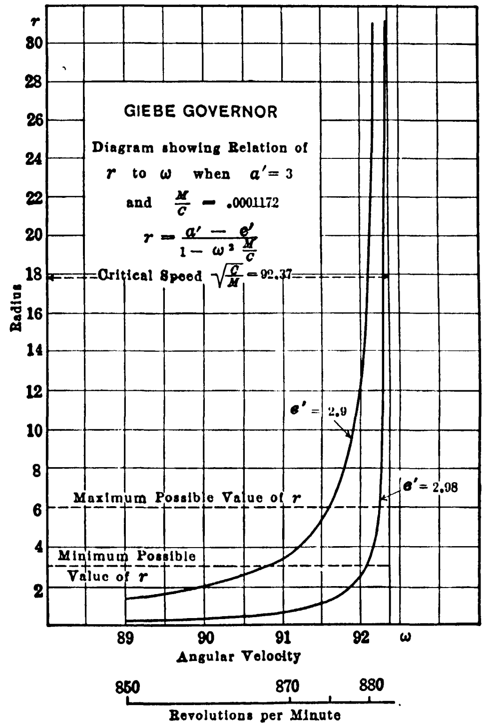


FIG. 290.—Pertaining to Giebe speed regulator.

Why the apparatus regulates most satisfactorily in the neighborhood of the critical speed becomes evident if values of ω and r are plotted; this has been done in Fig. 290, two values of e' being used.

It has been assumed in Fig. 290 that the construction of the regulator is such that the center of gravity of the weight can never be nearer the axis than 3 cm. or more distant than 6 cm. It is seen that as the critical speed is approached, the change in the position of the weight for a given increase in the angular velocity becomes vastly increased. This means a corresponding increase in the sensitiveness of the apparatus.

If $a' > e'$, the weight arrives at its ultimate position slowly, and the contact of K_1 and K_2 may be uncertain. If $a' < e'$, the motion is sudden, and the pressure between the contacts considerable. For the best results, e' should be slightly larger than a'; that is, the initial tension on the springs should be such that the weight is in unstable equilibrium at the speed that it is desired to maintain.

Any given regulator has only one speed at which it works with entire satisfaction, that is, at $\omega = \sqrt{C/M}$.

If it is desired to regulate the motor at another speed, either C or M must be altered.

To adjust the tension of the springs so that the center of gravity is in the proper initial position, that is, to give $a' - e'$ its correct value, it is necessary to experiment with the completed apparatus. Tests show that the regulator will keep the speed constant to about 1/10,000 part of its mean value.

POWER SOURCES

Microphone Hummer. Audio Oscillator.—If the amount of power required is small, the microphone hummer is a convenient power source for bridge measurements at a fixed frequency and low voltage. One form of this device is shown diagrammatically in Fig. 291, where F is a tuning fork rated at, for instance, 1,000 cycles per second—an advantageous frequency if a telephone is to be employed as the detector. The gap BC can be closed by a strap, so that the internal battery AB furnishes current for two otherwise independent circuits: one for driving the fork, the other for furnishing energy to the bridge. They are simultaneously controlled by the switch S. The driving current flows through the magnet M and through the solid-back microphone M_1, the movable member of which rests against the fork, as indicated.

If, by tapping, the fork is set in vibration, the changes in the resistance of M_1 cause variations of current in the driving magnet, and the vibration is sustained. The second circuit from AB is through the solid-back microphone M_2 and through the primary of the transformer T. The motion of the fork is communicated to the movable member of M_2 and causes variations in the current through the filter arrangement to the right of the dotted line, which frees the output of direct current and reduces its harmonic content. To obtain as pure a wave as practicable, the amplitude of vibration should be kept small by employing a battery of not over 4.5 volts at AB. The output is then 10 or 15 milliwatts. The driving circuit takes about 25 milliamp. at 4.5 volts; the output circuit, about 60 milliamp. External batteries of higher voltage may be employed, but this reduces the purity of the output wave. The numbering at the output terminals gives the proper load impedances. It is to be kept in mind that the wave form is far from sinusoidal. There are various other forms of microphone hummer. That designed by Campbell employs a straight bar instead of a tuning fork for the vibrating

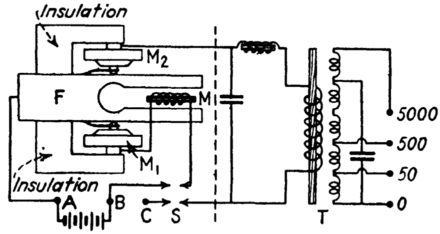


FIG. 291.—Simplified diagram for microphone hummer or audio oscillator. (*General Radio Co.*)

member. By changing the bar for another of different dimensions, the frequency is readily altered.

Variable-frequency Oscillator.—Figure 292 shows the circuit for a variable-frequency oscillator as assembled in the Communications Laboratory of the Massachusetts Institute of Technology. The oscillator proper is at the left of *AB*; the amplifier for transferring the oscil-

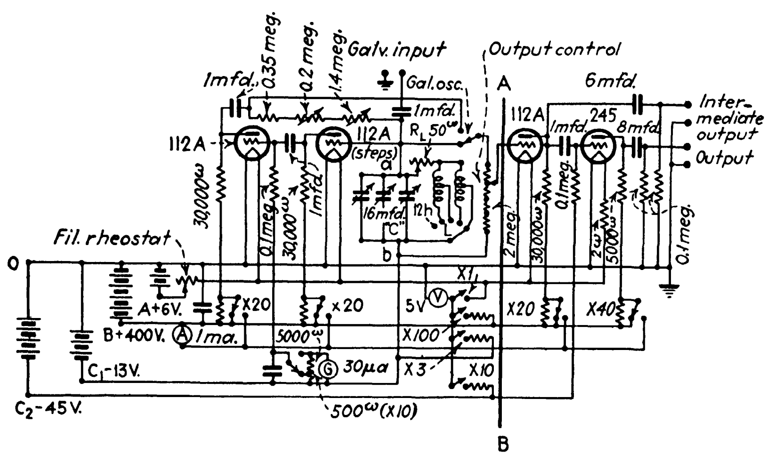


FIG. 292.—Oscillator and tuned-circuit galvanometer. (*Massachusetts Institute of Technology.*)

lations to the external circuit, at the right. The frequency is continuously variable from 12 to 20,000 cycles per second, the power output being 0.2 watt at all frequencies.

The oscillating circuit between *a* and *b* consists of an adjustable bank of paper and mica condensers supplemented by a continuously variable air condenser, the whole being in parallel with an adjustable assembly of six inductances. For frequencies below 800 cycles per second, the inductance coils have nicaloi and A-metal cores. Air-core coils are employed for higher frequencies. All the coils are tapped so that each value of the inductance is used for one octave, thus keeping the variation of reactance with frequency within the limits of 2:1. The design is such that the ratio of reactance to resistance is always greater than 20, and additional resistance may be added to keep it exactly at that value, so that the operation of the oscillator is independent of the frequency. Control of the amplitude of oscillation is obtained by letting the grid of the second oscillating tube swing slightly positive, thus automatically introducing a grid current which reduces the bias. The production of harmonics is thus confined to a tube once removed from the oscillating circuit. The combination of high feed-back resistance and tuned circuit is highly selective, so that only a small part of the harmonics produced in the second oscillating tube appears across the tuned circuit. For, as the impedance of the oscillating circuit for the tuned frequency is very high, the voltage impressed between a and b is practically the fundamental due to the second oscillating tube. The voltage impressed between a and b, due to the harmonics, is small, owing to the fact that for them the impedance of the tuned circuit is low. The amplitude of oscillation is adjusted by means of the feed-back resistance to a predetermined value of grid current in the second tube.

The output amplifier is controlled by an adjustable tap from the high resistance shunted around the oscillating circuit. The tubes are so chosen that they operate over only small fractions of their possible ranges. The harmonic content of the output voltage is less than 0.1 per cent over the entire frequency band, full output. This circuit, without the variable feature, is employed in commercially obtainable oscillators.

By throwing the switch to "Galv.," placing a high resistance in series with the "Galv. Input" terminals, and operating just below the point of oscillation, a highly selective circuit is obtained, and the apparatus converted into the equivalent of a tuned galvanometer.

Beat-frequency Oscillator.—A beat-frequency oscillator is a convenient laboratory power source, for it allows any desired frequency within its range to be obtained by manipulating a single, continuously variable condenser. One form of the apparatus, which is arranged throughout as a balanced push-pull assembly and is excited by rectified and filtered 115-volt, 60-cycle current, is shown diagrammatically in Fig. 293.

In this particular case, the frequency of the fixed oscillator is 180,000 cycles per second, while that of the variable oscillator may be set at anything between 180,000 and 200,000 cycles per second. Thus beat frequencies of from 0 to 20,000 are obtainable. The frequency of the variable oscillator is regulated by altering a continuously variable condenser in the resonating plate circuit, thus giving large variations of the output frequency which is read from the main dial, the graduations of which are in intervals of 100 cycles per second. Smaller variations of output frequency may be obtained by altering the "Cycles Increment" condenser, which changes the rate of the fixed oscillator by an amount which is read from the cycles-increment dial. The frequency is obtained by taking the algebraic sum of the two dial readings.

The two component oscillators are made as nearly alike as practicable and placed in similar environments, so that any drift in their frequencies,

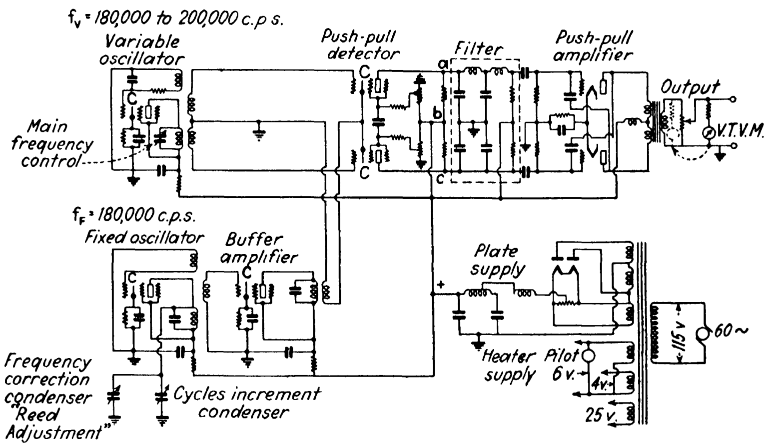


FIG. 293.—Diagram for alternating-current operated beat-frequency oscillator. (*General Radio Co.*)

due to heating, etc., will tend to be the same in both and thus keep the beat frequency constant. However, it is necessary to provide a means of checking and adjusting the frequency. To aid in this, a reed (not shown in the diagram) tuned to 100 cycles per second and excited from the output is included in the assembly. With both the main and the frequency-increment dials set at 0, the reed should not vibrate, or, if the main dial is set at 100 and the frequency-increment dial set at 0, the reed should vibrate at maximum amplitude. These conditions are realized by manipulating the "Reed Adjustment" condenser. It is necessary that the two oscillators act independently; so care is exercised that there is no direct coupling, and a buffer amplifier is employed. The fixed and variable frequencies are impressed simultaneously on the grids of the balanced push-pull detector. The detector output contains frequencies of [illegible] $f_v + f_F$ and other high-frequency terms. The reduction of the terms other than $f_v - f_F$ to within the limits of tolerance is an important element in the designing of beat-frequency oscillators. The filter is intended to remove frequencies above 20,000 cycles per second. The filtered current furnishes the excitation for the grids of the balanced and stabilized push-pull amplifier, the plate currents of which, through a transformer, furnish the voltage for the drop wire in the output circuit by which variable voltages are obtained.

The internal output impedance is about 2,000 ohms. The maximum open-circuit r.m.s. voltage is 130 volts. The maximum output is 2 watts into 2,000 ohms. In this case, however, the harmonic distortion is about 5 per cent. To preserve a good wave form, the load should be at least 6,000 ohms; the maximum output is then 1.5 watts, and the harmonic distortion less than 1 per cent for frequencies above 100 cycles per second. At 10 cycles per second and full voltage, the harmonic content is less than 5 per cent. When the output voltage is reduced by a factor of 10, the harmonic content is less than 1 per cent over the entire frequency range. The voltage supplied to the load may be measured by the vacuum-tube voltmeter (VTVM). By means of a switch, this meter may be transferred to the dotted position across the transformer terminals. To ensure a good wave form, the reading should not then alter by more than 25 per cent when the load is thrown on and off.

DETECTORS

The Telephone.—If an impedance bridge can be set up in a soundproof room, and the frequency of the power source is well within the audio range, a telephone is a satisfactory detector. If the bridge is of high impedance, the detector current will, of necessity, be small. Consequently, the winding of the telephone must consist of many turns; that is, the receiver must also have a high impedance. The instrument should be one tuned to the frequency of the power source, which, for best results, should be that at which the ear is highly sensitive—in the neighborhood of 900 cycles per second. The addition of a tuned acoustical resonator will greatly increase the over-all sensitivity. To do away with head noises, a Wagner ground should be employed. Beginners should not forget that stray fields, possibly from the apparatus being measured, may affect the telephone. The effective impedance of the telephone may be adjusted to that of the bridge by the use of a transformer of proper ratio.

The Alternating-current Galvanometer.[29]—This instrument is, in reality, a separately excited electrodynamometer. The fixed coils are provided with a laminated iron core to increase their effect and are excited by alternating current from a source having a definite voltage. Current is taken to the movable coil through the suspension. The return circuit is through a flexible ribbon hanging loosely alongside of the suspension and therefore outside the alternating field due to the fixed coils. The movable-coil structure must be nonmagnetic, and all metal parts, other than the coil itself must be eliminated. The movable system must be surrounded by a fixed electrostatic shield which is kept at the potential of one of the coil terminals. Shielding from air currents set up by the heating of the electromagnet is essential. The sensitivity of the instrument may be made comparable with that of the moving-coil galvanometer for direct currents. Like that instrument, its behavior depends on the constants of the circuit in which it is used. The field in the air gap should be uniform, and a short suspension should be employed.

As the instrument is an electrodynamometer, its indications depend both on the magnitude of the current in the movable coil and on the phase difference of this current and the alternating flux due to the excitation which is derived from a phase shifter (page 521). If sinusoidal currents are employed, and the flux in the air gap is brought, by use of the phase shifter, nearly into phase coincidence with the current in the movable coil, the magnitude of the deflection depends chiefly on the size of this current and but little on its phase relative to the air-gap flux. If the flux is brought nearly into phase quadrature with the movable-coil current, the deflection depends chiefly on the phase difference of the flux and this current. In bridge work, these are important characteristics, for, when balance is nearly attained, they allow practically independent adjustments of the bridge for power factor and reactance. Transformer action between the fixed- and movable-coil circuits is troublesome and necessitates working from a false zero. This effect may be balanced by using a variable mutual inductance, its secondary being in series with the movable coil, while the primary is traversed by the fixed-coil current. If nonsinusoidal currents are employed, the deflection depends on the mean products of all the components common to the air-gap flux and the movable-coil current. This galvanometer has been carefully studied by Weibel,[29] who gives specifications for successful instruments.

Amplifiers.—It is customary to increase the sensitiveness of an impedance bridge by actuating the detector through an electronic amplifier. One such device which is operated by alternating current of power frequency and particularly adapted to measurements at audio frequencies, using a telephone as a detector, is shown in Fig. 294.

As this is a high-gain amplifier, special precautions against pickup must be taken. Also, it is essential to employ tubes having low microphonic actions. In alternating-current operated amplifiers, a serious problem is presented by the hum due to the power supply. In this particular arrangement, the equivalent input voltage from this cause is less than 10 microvolts r.m.s. with the input terminals short-circuited and less than 20 microvolts r.m.s. with them open-circuited.

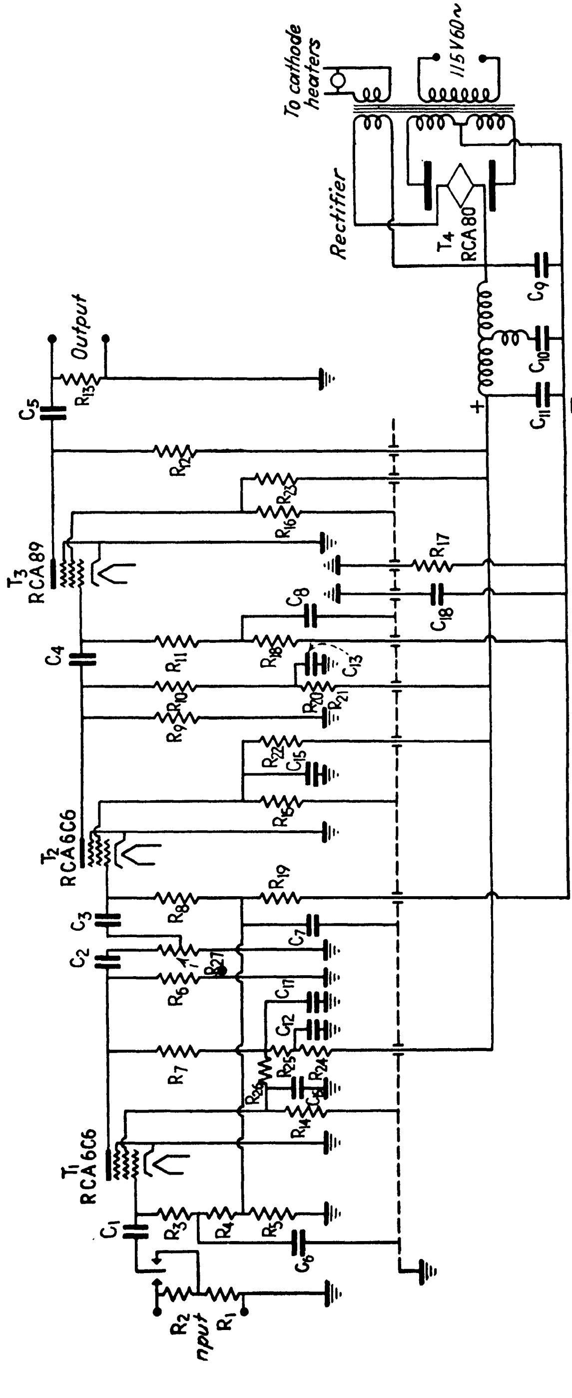


FIG. 294.—Diagram for alternating-current operated amplifier for bridge work, for use at audio frequencies. (General Radio Co.)

It is essential that the voltage supply should not fluctuate. The amplification is continuously adjustable between 20 and 80 db. The maximum input voltage is 10 volts r.m.s.; the maximum output voltage is 100 volts r.m.s.; the load resistance should not be less than 10,000 ohms for an undistorted output of 100 volts. The maximum gain is obtained when the load is about 20,000 ohms. The total shunt capacitance is about 20$\mu\mu$f.

Tests made under the most favorable conditions in a soundproof room, using telephones, indicated that voltages of 10 microvolts could be detected over the frequency range of 60 to 8,000 cycles per second,

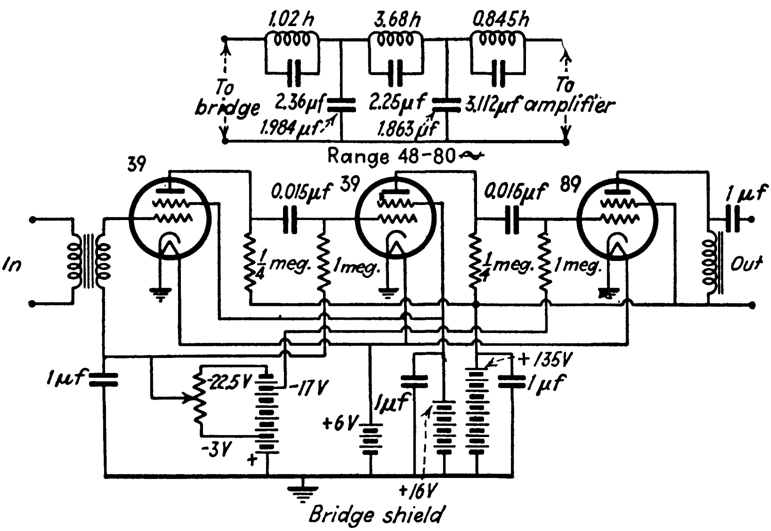


FIG. 295.—Filter and amplifier for Dawes-Hoover high-voltage bridge.

1 microvolt over the range of 200 to 4,000 cycles, and 0.1 microvolt over the range 500 to 2,500 cycles. The ordinary working sensitiveness is about one-tenth as great.

Another amplifier operated by direct current and intended for bridge work on cables at power frequencies is shown in Fig. 295. In this case, the detector is a vibration galvanometer of the form shown in Fig. 300. As the samples may be nonlinear, a filter is used between the bridge terminals and the amplifier.

The Vibration Galvanometer.—In 1891, Max Wien suggested that it was possible greatly to increase the sensitivity of the detectors used in alternating-current measurements, where zero methods are employed, by taking advantage of the principle of resonance. To do this, the moving member of the detector is mechanically tuned so that its natural period is the same as that of the alternating electromagnetic forces which cause its deflection. The idea was realized by Wien in his "optical telephone." The later development of this instrument into the vibration galvanometer has been due more especially to Wien, Rubens, Campbell, Duddell, Drysdale, Schering and Schmidt who have utilized both the moving-needle and the moving-coil principles.[27]

The instrument is read by the mirror-and-scale method and the optical arrangement should be such that when no current is passing, a *sharply* defined image may be seen on the screen. When the bridge, or other apparatus to which the galvanometer is connected, is out of balance, the galvanometer coil will be set in vibration, and this image will become extended into a band of light.

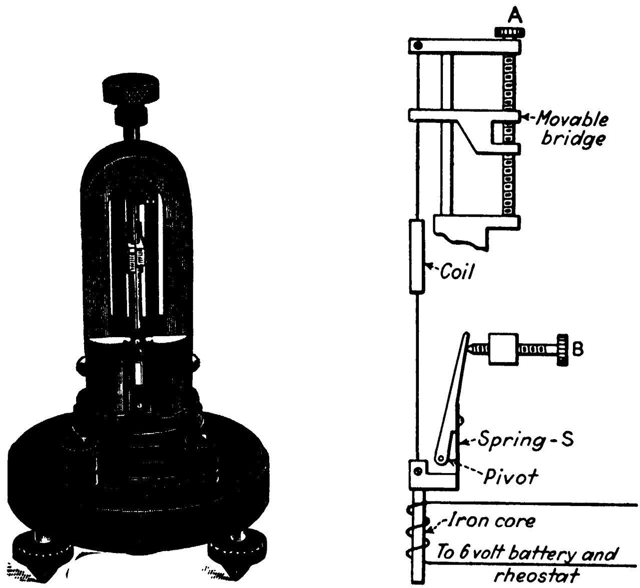


FIG. 296.—Vibration galvanometer. (*Leeds and Northrup Co.*)

The instrument shown in Fig. 296 is a D'Arsonval galvanometer so designed that the coil may be given a high rate of vibration. The small sketch at the side shows how the rate may be varied, that is, how the instrument may be tuned to the frequency of the circuit. A flat strip suspension is used, the effective length of which may be adjusted by turning the milled head *A* at the upper end of the vertical screw, thus securing a coarse adjustment. A finer adjustment is made by turning the milled head *B* which, by means of the spring *S*, controls the tension on the suspension. To perfect the adjustment and allow for minor variation in the frequency of the supply, a remote-control device is used. By means of a rheostat the observer can keep the instrument in tune without mechanical disturbance.

On account of the large restoring moment which must be employed to obtain a high rate of vibration, the sensitivity of a vibration galvanometer for direct currents is very small. It is only when the period of the galvanometer and that of the current coincide that the current

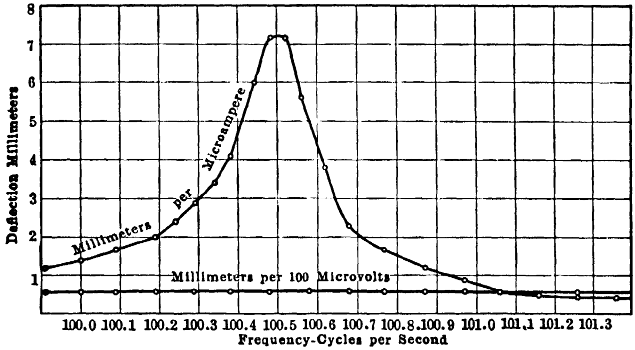


FIG. 297.—Showing effect of tuning, on current and voltage sensitivities of a vibration galvanometer.

sensitivity rises to a high value. This is well illustrated by Fig. 297. From the figure it is clear that if the sensitivity is to be maintained, the frequency of the current must be constant; in the case shown, a change of 0.2 per cent in the frequency reduces the current sensitivity by about 70 per cent.

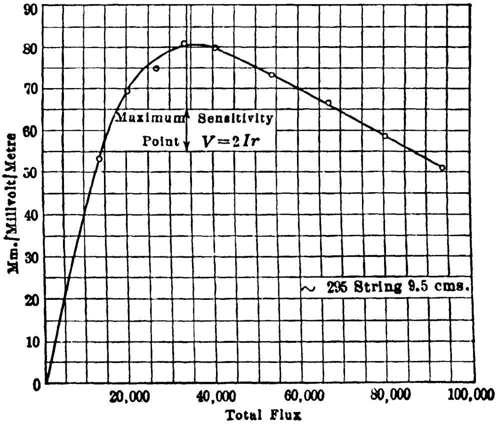


FIG. 298.—Showing effect on the voltage sensitivity of a vibration galvanometer when the flux cut by the coil is varied.

The characteristic of responding freely to only one frequency permits many measurements to be made with nonsinusoidal currents, provided the harmonics are not so pronounced that they "force" the vibration of the movable system. In one very good commercial form of vibration galvanometer, the sensitivity for the third harmonic is only 1/4,000, and for the fifth harmonic, only 1/12,000 of that for the fundamental. This selective sensitivity is one reason why, within the range where they are both effective, the vibration galvanometer is superior to the telephone as a detector, unless the telephone is tuned to the frequency of the current.

However, it may be noted that under certain rather unusual conditions, this selectivity in effect disappears; for instance when a high-voltage bridge is applied to measurements on a nonlinear sample, and the galvanometer is actuated through an amplifier, which is nonlinear. If, in the case where the bridge is balanced for the fundamental frequency so that the voltage applied to the amplifier contains both the third and the fifth harmonics but no fundamental, the voltage that reaches the galvanometer may contain a fundamental component together with higher harmonics. A false balance indication is thus obtained. Errors due to this cause, as great as 7 per cent, have been observed when testing high-voltage cables stressed beyond the ionizing potential. To obviate this difficulty, it is necessary to use a filter, in parallel with the amplifier and galvanometer, which will pass only a narrow band near the nominal frequency of the circuit.

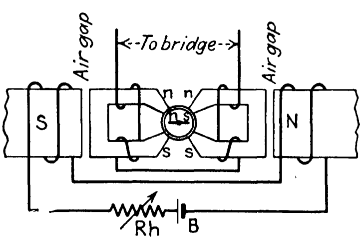


FIG. 299.—Diagram for Sehering and Schmidt moving-needle vibration galvanometer with remote control.

As current of constant frequency is essential, it is not always possible to use commercial electric circuits as sources of power in those alternating-current measurements where the vibration galvanometer is employed unless the variations are small, take place slowly, and the instrument is provided with a remote tuning device. No vibration galvanometer can be considered thoroughly satisfactory for general use unless it can be tuned without handling the instrument itself and thus setting it in vibration mechanically. Figures 299 and 300 show two examples of moving-needle vibration galvanometers.

Referring to Fig. 299, the soft-iron needle *ns* and the mirror are carried by a taut phosphor-bronze suspension, about 4 cm. long and 0.02 cm. in diameter. The needle is polarized by the electromagnet *NS*. Tuning is effected by adjusting the strength of the current from the battery *B*, while large changes in the rate of vibration are made by altering the size of the needle.

The deflecting field is due to the laminated alternating-current magnet *nn*, *ss*. Owing to the air gaps, the needle system is thoroughly insulated from the controlled circuit which must be manipulated. Figure 300 is diagrammatic of a simple and convenient instrument intended for work at power frequencies and is so constructed that the coils can readily be removed and replaced by others of the impedance best adapted to the work in hand. Moving-needle vibration galvanometers are very susceptible to stray fields of the frequency to which they are tuned and should therefore be protected by magnetic shields.

Current Sensitivity.—For an instrument such as that shown in Fig. 296, which has 1 deg. of freedom, the relation between current and deflection for the vibration galvanometer is obtained by solving Eq. 11 (page 15), in which $i = I_N \sin (N\omega)t$. $N\omega$ is $N2\pi$ times the *fundamental* frequency of the current; for the fundamental $N = 1$, for the third harmonic $N = 3$, etc.

The equation according to which the vibration of an instrument of the type mentioned takes place is

$$P\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + \tau\theta = CI_N \sin (N\omega)t. \quad (84)$$

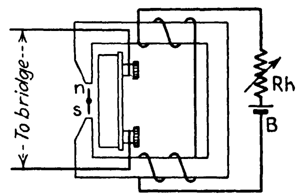


FIG. 300.—Diagram for moving-needle vibration galvanometer with remote control and removable coil.

The following analysis does not apply to instruments that have multiple resonance points[28] such as may be due to a lateral displacement of the movable system and an asymmetrical distribution of the vibratory mass. The action of instruments of the string galvanometer class cannot be represented by a simple differential equation of the type shown in Eq. (84).

An idea of the magnitudes of the constants in (84) will be given by data applying to an instrument investigated by Wenner.

$$C = 1.4 \times 10^5 \text{ c.g.s.}$$
$$\tau = 5{,}700 \text{ c.g.s.}$$
$$k = 0.018 \text{ c.g.s.}$$
$$P = 0.015 \text{ c.g.s.}$$
$$\text{Resistance} = 30 \text{ ohms.}$$
$$\text{Resonating frequency} = 100 \text{ cycles per second.}$$

The symbolic solution of (84) is

$$\theta_N = \frac{CI_N}{[\tau - (N\omega)^2P] + jkN\omega}. \quad (85)$$

If ω_0 is 2π times the frequency of the vibrating coil of the galvanometer,

$$\theta_N = \frac{CI_N}{P[\omega_0^2 - (N\omega)^2] + jkN\omega}. \quad (86)$$

The maximum sensitivity will be obtained when the instrument is exactly tuned to the frequency of the circuit by changing τ or when $\omega_0 = \omega$, $N = 1$. When tuned, the galvanometer is but little affected by the harmonics in the current wave. For example, the deflection due to the fundamental will be

$$\theta_1 = \frac{CI_1}{k\omega}; \tag{87}$$

and that due to a current of N times the fundamental frequency will be, as $kN\omega$ is then negligibly small,

$$\theta_N = \frac{CI_N}{P\omega^2(1 - N^2)}.$$

That is, for the same value of the current,

$$\frac{\theta_1}{\theta_N} = \frac{P\omega(1 - N^2)}{k}. \tag{88}$$

Using the preceding data for a particular instrument,

$$\frac{\theta_1}{\theta_3} = 4{,}200 \qquad \text{approx.};$$

or the sensitivity for the third harmonic is less than 1/4,000 that for the fundamental.

From (87) it is seen that for high sensitivity the damping must be small, the free period of the coil low, and the coil constant large.

The resonance range of a vibration galvanometer is an arbitrary measure of the exactness of tuning required if a high sensitivity is to be maintained and is defined as the fractional change in the frequency of the current that will reduce the sensitivity of the instrument to one-half its maximum value. It is highly desirable if the galvanometer is to be used for general laboratory purposes, that the resonance point be not too sharply defined. That is, the resonance range of the instrument should be large, as great as two-tenths of 1 per cent.

To express the resonance range in terms of the constants of the galvanometer: When the instrument is perfectly tuned,

$$\theta_1 = \frac{CI_1}{k\omega}.$$

If the frequency of the supply is slightly raised, that is, if N is made a little greater than 1, the deflection becomes

$$\theta_N = \frac{CI_N}{\omega\sqrt{P^2\omega^2(1 - N^2)^2 + k^2N^2}}.$$

In determining the resonance range, the change in N is supposed to be such that $\theta_N/I_N = \theta_1/2I_1$, so

$$k^2(4 - N^2) = P^2\omega^2(1 - N^2)^2;$$

and as N is very nearly 1,

$$1 - N^2 = \frac{k\sqrt{3}}{P\omega} \qquad \text{approx.}$$

$$N = \frac{\text{frequency of current which halves the maximum amplitude}}{\text{resonating frequency}} = 1 + R_i,$$

where R_i is the resonance range for current. Therefore

$$R_i = \frac{k\sqrt{3}}{2P\omega} \qquad \text{approx.} \tag{89}$$

Voltage Sensitivity.—When a vibration galvanometer is so used that the voltage sensitivity is important, it should be noted that as the coil vibrates it cuts the flux in the air gap and thus sets up a back e.m.f. which is in time quadrature with the deflection and has a component in opposition to and a component in quadrature with the current.

The e.m.f. that is effective in forcing the current through the circuit is the vector difference of the applied and the back e.m.fs.

The back e.m.f. is given by

$$\begin{aligned} E_B &= -C\frac{d\theta}{dt} = -j\omega C\theta = -j\frac{\omega C^2 I}{P(\omega_0^2 - \omega^2) + jk\omega} \\ &= \frac{-I\omega^2 C^2 k - jI\omega C^2 P(\omega_0^2 - \omega^2)}{P^2(\omega_0^2 - \omega^2)^2 + k^2\omega^2}. \end{aligned} \tag{90}$$

If r and L are the resistance and inductance of the reactive circuit in which the galvanometer is inserted, and V is the applied voltage,

$$I(r + j\omega L) = V + E_B,$$

and

$$\frac{\theta[P(\omega_0^2 - \omega^2) + jk\omega](r + j\omega L)}{C} = V - j\omega C\theta.$$

Therefore

$$\theta = \frac{CV}{rP[\omega_0^2 - \omega^2] - k\omega^2 L + j\omega[kr + LP(\omega_0^2 - \omega^2) + C^2]}. \tag{91}$$

In this case, where the circuit is reactive, the sensitivity is not a maximum when $\omega_0 = \omega$ but when r is so adjusted that

$$P(\omega_0^2 - \omega^2) = -\frac{C^2\omega^2 L}{r^2 + \omega^2 L^2}. \tag{92}$$

The corresponding value for the magnitude of θ is

$$\theta = \frac{CV\sqrt{r^2 + \omega^2 L^2}}{\omega[rC^2 + k(r^2 + \omega^2 L^2)]}. \tag{93}$$

From (92) and (93) it is seen that if C is large, it may be possible to increase the deflection by placing an inductance in series with the galvanometer and slightly raising the frequency of the supply, for the fractional increase in the numerator of (93) may be greater than that in the denominator.

To obtain the greatest possible deflection when the instrument is used in a circuit of fixed inductance, both the torsional constant of the suspension τ and the coil constant C must be adjusted. C may be varied by changing the strength of the flux threading the coil. When both τ and C are adjusted, the magnitude of θ is a maximum when

$$\left.\begin{aligned} P(\omega_0^2 - \omega^2) &= -\frac{C^2\omega^2 L}{r^2 + \omega^2 L^2}, \\ \text{and} \qquad C^4 &= \frac{r^2 + \omega^2 L^2}{\omega^2}[P^2(\omega_0^2 - \omega^2)^2 + k^2\omega^2]. \end{aligned}\right\} \tag{94}$$

The corresponding maximum value of θ, obtained by (94) and (91), is

$$\theta = \frac{V}{2\omega\sqrt{kr}}.$$

In general, the equivalent impedance of the circuit including the galvanometer is

$$Z = \left[r + \frac{C^2\omega^2 k}{P^2(\omega_0^2 - \omega^2)^2 + k^2\omega^2}\right] + j\omega\left[L + \frac{C^2 P(\omega_0^2 - \omega^2)}{P^2(\omega_0^2 - \omega^2)^2 + k^2\omega^2}\right]. \tag{95}$$

When conditions (94) are imposed, this reduces to

$$Z = 2r + jo.$$

Therefore, when the deflection has been made a maximum by adjusting both τ and C, the current is in phase with the applied voltage, and

$$I = \frac{V}{2r}. \tag{96}$$

In this case, half the energy supplied to the instrument is expended mechanically, and half in electrical heating.

The sensitivity to a voltage whose frequency is N times the resonating frequency may be seen from (91). If the voltage V_N is applied at the galvanometer terminals,

$$\theta_N = \frac{CV_N}{Pr\omega^2[1 - N^2]} \qquad \text{approx.} \tag{97}$$

By a process similar to that used on page 470, it may be shown that the resonance range for voltage R_v is

$$R_v = \frac{\sqrt{3}(kr + C^2)}{2Pr\omega} \qquad \text{approx.} \tag{98}$$

Use of Transformers with Power Sources and Detectors.—Frequently the statement is made that with a bridge, or other load of high impedance, a high-impedance power source should be used. This sometimes misleads beginners. Impedance in the power source is a necessary evil; it reduces the sensitivity by cutting down the bridge current, to which the sensitivity is proportional. It is merely associated with the high e.m.f. necessary to force the required current through the impedance of the bridge. If the impedances of the power source and the bridge differ greatly in magnitude, the bridge current, and therefore the sensitivity, may be increased by interposing a transformer of the proper ratio between the generator and the bridge.

To obtain high efficiency, a nickel-iron core should be used, and the coils wound with wire of such a size that the resistances are as small as practicable. Magnetic leakage should be reduced to a minimum, the windings arranged to reduce the capacitance effects to a minimum, and a grounded shield should be cemented between the primary and secondary. For general use, a flat frequency characteristic is desirable. When the secondary is open, the voltage at the primary terminals should approximate closely the e.m.f. of the generator and have a very small resistance component. This implies that the impedance of the primary of the transformer should be much greater than that of the power source.

Let E_S = e.m.f. of power source.
Z_S = impedance of power source.
L_S = inductance of power source.
R_S = resistance of power source.
L_1 = inductance of primary.
Z_1 = impedance of primary.
N_1 = number of primary turns.
Z_L = impedance of load.
L_L = inductance of load.
R_L = resistance of load.
L_2 = inductance of secondary.
Z_2 = impedance of secondary.
N_2 = number of secondary turns.

If no transformer is used, $I'_L = \dfrac{E_S}{Z_S + Z_L}$.

Using the transformation for mutual inductance (page 392), when a transformer is employed the load current is

$$I_L = \frac{jE_S\omega m}{(Z_2 + Z_L)(Z_1 + Z_S) + \omega^2 m^2}. \tag{99}$$

If the resistances of the transformer coils are small compared with their reactances, and there is no appreciable leakage,

$$\left.\begin{aligned} Z_1 &= j\omega L_1 \\ Z_2 &= j\omega L_2 \end{aligned}\right\}\text{approx.} \qquad \frac{Z_2}{Z_1} = \frac{L_2}{L_1} = \left(\frac{N_2}{N_1}\right)^2. \qquad m = \sqrt{L_1L_2}.$$

Then

$$I_L = \frac{jE_S\omega m}{Z_2Z_S + Z_1Z_L + Z_SZ_L} \quad \text{approx.} \tag{100}$$

If $Z_S Z_L$ can be neglected,

$$I_L = \frac{(N_2/N_1)E_S}{(N_2/N_1)^2 Z_S + Z_L} \quad \text{approx.} \tag{101}$$

The optimum ratio of turns is given by

$$\left(\frac{N_2}{N_1}\right)^2 = \sqrt{\frac{R_L^2 + \omega^2 L_L^2}{R_S^2 + \omega^2 L_S^2}}.$$

$$\frac{N_2}{N_1} = \sqrt{\frac{|Z_L|}{|Z_S|}}. \tag{102}$$

If the load is of high and the power source of low impedance, the use of a transformer with $N_2/N_1 > 1$ between the power source and the load increases the effective e.m.f. in series with the load in the ratio N_2/N_1 and increases the effect of the low impedance of the source in the ratio $(N_2/N_1)^2$. If the impedance of the source is small, there will be a net gain in the load current in spite of the increase in the effective source impedance.

The primary and secondary coils of the transformer should be subdivided so that different ratios may be obtained. A transformer that gives satisfactory results under the conditions for which it was designed may prove unsatisfactory when subjected to very different conditions. For instance, if the frequency is greatly reduced, the approximations given on page 473 may not be sufficiently close.

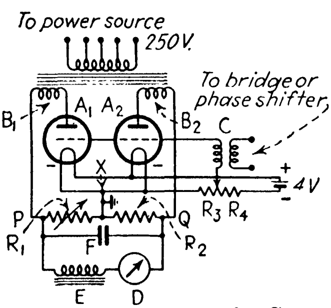


Fig. 301.—Diagram for Cosens bridge detector. (*Cambridge Instrument Co.*)

Wenner calls attention to the possibility of increasing the effective voltage sensitivity of vibration galvanometers by use of transformers. Instrument makers can supply transformers of required ratios which have practically flat frequency characteristics between specified limits.

Cosens Detector.[49]—This electronic frequency-selective bridge detector has a sensitivity comparable with that of the vibration galvanometer or the telephone, which it is intended to replace. It has the advantage that when applied to an impedance bridge in conjunction with a phase shifter, the resistance and reactance balances are practically independent.

The connections are shown in Fig. 301, where A_1 and A_2 are two triodes which are supposed to be identical. Their plate currents, which differ in phase by 180 deg., are supplied via the transformers B_1 and B_2 from the power source that actuates the bridge. The negative grid bias for both tubes is supplied from the 4-volt filament battery through the resistor R_3 and is of such a magnitude as to bias the tubes to the middle of the straight parts of their grid-voltage, plate-current characteristics with the normal plate voltage of 150. D is a center-zero, unipivot, direct-current galvanometer. E is a choke coil, and F a condenser; their function is to prevent the flow of alternating current in the galvanometer. Both grid voltages are supplied via a phase shifter (Fig. 301A) and the transformer C from the detector terminals of the bridge.

The bridge being out of balance, suppose that the plate of A_1 is positive and that the grid voltage is also positive, or only slightly negative; a current will flow through A_1 and R_1. At the same time, the plate of A_2 is negative, and there will be no current from A_2 in R_2. During the next half wave, the plate of A_1 will be negative, while that of A_2 will be positive. The grid of A_2 will be strongly negative, so that while A_2 delivers current to R_2, this current will be smaller than that delivered by A_1 to R_1 during the previous half wave; consequently, the average potential difference of P and Q is not zero, and the galvanometer will be deflected. A preliminary adjustment is necessary, for the two tubes are not likely to be identical. To make the adjustment, the galvanometer is short-circuited, and the proper voltage applied at the power-source terminals. Sufficient time is then allowed for the tubes to come to the steady state. The short circuit is removed; the primary of the transformer C connected to the unenergized bridge; and R_1, which is provided with a fine adjustment, is altered until the galvanometer stands at zero. The bridge may then be energized by connecting it to the power source.

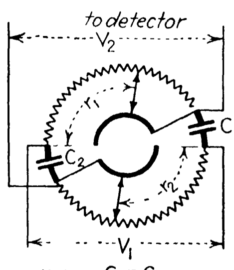


FIG. 301A.—Diagram for phase shifter.

If the bridge arms are nonreactive, the phases of the grid and plate voltages will be as previously assumed. However, if the bridge arms are highly reactive, there may be a phase difference of nearly 90 deg. between the plate and grid voltages, and there will be practically no deflection of the galvanometer. In this case, the grid voltage is shifted 90 deg. by the phase shifter, and balance obtained by altering the reactance of the adjustable bridge arm. In an impedance bridge, where there are both reactive and nonreactive components of impedance present, the two adjustments are practically independent.

By the simple phase shifter shown in Fig. 301A, the phase difference between the voltage V_1 derived from the bridge and the voltage V_2 applied to the detector may be changed from zero to about 120 deg., without greatly changing the ratio of V_2 to V_1, provided a very small current is drawn from V_2.

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CHAPTER VIII

ELECTRICITY METERS

Watt-hour Meter.—In connection with the supply of electrical energy for lighting and power, it is necessary to have some form of integrating meter, that is, a meter that will give not the rate at which energy is supplied to the circuit but the total amount of energy supplied during a given time, as, for instance, a month. Such a meter must evaluate $\int_{t_1}^{t_2} vidt$; v and i are the instantaneous values of the voltage and current; the time during which the energy is supplied is $t_2 - t_1$. It is customary to express this time in hours. The energy is then stated in watt-hours or, more frequently, in kilowatt-hours.

The necessity for accurately measuring electrical energy is apparent from the fact that in the United States alone, the charges for the electrical energy furnished for light and power for the year 1934 were approximately $1,837,000,000.

The essentials of the watt-hour meter will be better appreciated if one approved form of the instrument is described. The Thomson watt-hour meter for direct currents will be selected, for this was the first successful commutating meter. It was placed on the market in the latter part of 1889, has passed through the usual processes of development, and is still regarded as one of the best of its class.

The instrument consists of a small motor which is provided with a magnetic brake. The motor drives a counter whose indications on a system of dials are proportional to the total number of revolutions that have been executed by the armature.

No iron is used in either the field or the armature of the motor; hence all magnetic effects are directly proportional to the currents.

The fields of the motor are placed in the main circuit in series with the load. The armature, in series with a suitable resistance, is connected across the supply mains. The driving torque of the motor is proportional to VI, and the retarding torque due to the brake is proportional to the angular velocity of the disk. Broadly speaking, therefore, the energy supplied to the circuit during a given time will be proportional to the number of revolutions of the armature during that time —in other words, to the reading on the dials of the counter.

Referring to Fig. 302 the main current passes through two similar field coils of low resistance F. In a two-wire meter, these coils are connected in series. The potential circuit, which contains the armature A, is connected on the line side of the main coils. The armature (now made in a spherical form to reduce the weight) is carried by an upright spindle which, at its upper end, gears by means of a worm into the very light train of wheels which moves the pointers of the counter over the dials. The current is carried to the armature through silver-tipped brushes, which rest with a very slight pressure on a silver commutator of small diameter. The brushes are adjusted before the meter leaves the factory and should be very carefully handled.

In the early designs of this meter, the series resistance R was wound noninductively on cards and carried in an envelope at the back of the

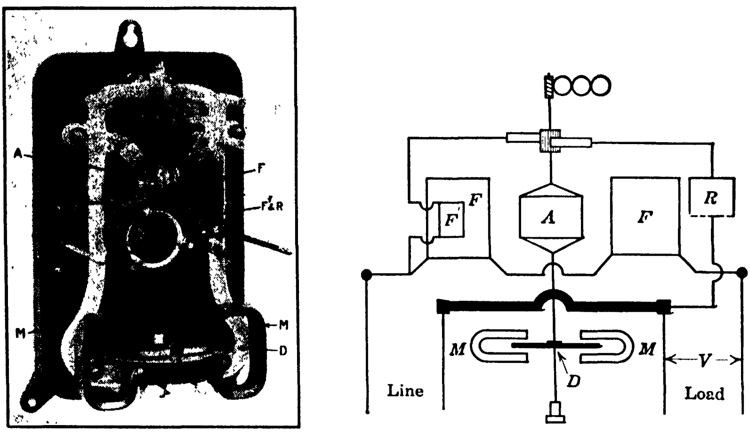

Fig. 302.—Thomson watt-hour meter for direct currents.

instrument. It is now combined with the light-load coil F', mentioned below. In a 110-volt meter, the total resistance of the potential circuit is about 2,500 ohms.

The lower end of the shaft is provided with a removable steel pivot which rests in a sapphire or diamond jewel carried by a spring support in the end of the jewel screw (see Fig. 302). This screw can be turned back so that the disk D is clamped against the magnets M; the pivot is thus relieved of all strain during transportation.

The brake disk D, now made of aluminum, moves through the fields of the permanent magnets M. To change the retarding torque and therefore the speed of the meter, the distance of the poles of the magnets from the axis of the disk may be altered.

In order to compensate at light loads for the effects of mechanical friction, a field coil of fine wire F' is connected in series with the armature. A small permanent driving torque is thus obtained. By adjusting the position of the coil with respect to the armature this torque may be made such that the registration at light loads is commercially correct; at the same time, the load current necessary to start the meter is much reduced. The effect of the light-load coil at the higher loads is insignificant.

To show that the number of revolutions during a given time is practically proportional to the energy supplied to the load via the meter:

Let V = line voltage.
I = line current.
I_a = current in potential circuit or armature.
R = resistance of potential circuit.
ω = angular velocity of armature.
h = field due to drag magnets.
r = resistance to eddy currents in brake disk.
K_S = driving torque due to light-load coil.
K_M = initial friction torque.

K and k, with various subscripts, are constants or proportionality factors.

The flux through the armature due to the main coils F will be proportional to I, and that due to the starting or light-load coil F' will be proportional to I_a, so

$$\text{Total flux through armature} = k_F I + k_{F'} I_a.$$

The back e.m.f. in the armature circuit will be proportional to the product of the flux and the angular velocity:

$$\text{Back e.m.f.} = k(k_F I + k_{F'} I_a)\omega.$$

The armature current will be

$$I_a = \frac{V - kk_F I\omega}{R + kk_{F'}\omega} = \frac{V - kk_F I\omega}{R} \qquad \text{approx.}$$

The driving torque due to the main coils may be represented by $K_F I I_a$, and that due to the light-load coils by $K_{F'} I_a^2$. As the meter is operated at a constant voltage, and the torque due to the light-load coil is small, it is allowable to consider this quantity as constant; it will be denoted by K_S. The total accelerating torque is

$$\left(\frac{K_F}{R}\right)VI - \left(\frac{K_F k k_F}{R}\right)I^2\omega + K_S. \tag{1}$$

The total retarding torque is that due to the mechanical friction of the meter (including windage) plus that due to the magnetic brake. The friction torque has a certain initial value, denoted by K_M, and increases more rapidly than the speed; its value may be represented by $K_M + K_{M'}\omega^2$. The brake torque is proportional to the angular velocity

of the disk and, if the temperature of the disk and the magnets is constant, may be expressed by $k_D h^2 \omega / r = K_D \omega$. (2)
Consequently, the total retarding torque is

$$K_M + K_D\omega + K_{M'}\omega^2. \tag{3}$$

For steady motion, the accelerating and retarding torques must be equal. Equating (1) and (3) gives for the angular velocity of the disk

$$\omega = K_1 VI - K_2 I^2 \omega + K'_S - K'_M - K'_{M'}\omega^2. \tag{4}$$

The terms on the right-hand side of the equation which involve ω are small corrections due to the back e.m.f. and the change of friction torque with the speed.

It is seen that if the light-load coil is adjusted so that at no load the meter is just on the point of starting (K'_M very slightly greater than K'_S), the angular velocity of the armature will be practically proportional to the power supplied to the load; and it at once follows that the total number of revolutions N executed by the armature in a given time is proportional to the energy supplied to the load during that time.

For meters as actually constructed,

$$\text{Watt-hours registered on dials} = K_h N. \tag{5}$$

This is fundamental and applies to all watt-hour meters. The factor K_h is called the watt-hour constant and is the number of watt-hours of energy necessary to cause one revolution of the movable system.

General Discussion of Essential Characteristics.[1]—A consideration of the uses to which the watt-hour meter is put will show that the instrument should possess certain characteristics.

In order that the first cost and the expense of the maintenance may not be too great, electricity meters must be simple in design and must contain no parts that are subject to rapid deterioration.

It is desirable that the reading in kilowatt-hours be given directly by the dials, especially in small meters; this avoids the necessity for multiplying the dial readings by a constant. A possible source of misunderstanding between the consumer and the supply company is thus avoided.

The meter should be protected by a case which can be sealed and which is dust-, water- and insect-proof; the arrangement should be such that there is little likelihood that a short circuit can occur during the removal and the replacement of the meter cover. It is desirable that the electrical connections to the meter be so made that it is not possible to tamper with the instrument; in certain cases, special devices are used for covering all the connections from the service wires to the meter so that it is impossible for the customer to draw current that does not pass through the meter.

The National Electrical Code now permits the connection of meters, up to 50 amp. capacity, directly to the supply mains without fuses. In this case, the necessary protection from the effects of continuous overloads is afforded by a miniature circuit breaker in each circuit, the breaker being so constructed that it cannot be held in a closed position as long as an overload continues. Owing to the time lag of the breaker, momentary overloads do not interrupt the service.

To carry this protective idea into effect the Westinghouse Company has developed for alternating-current services what is the equivalent of a large socket into which the meter, when provided with the proper back plate carrying switch blades, can be slipped and automatically connected between the service and the customer. The line is brought to the socket through metallic conduits, and the whole protected by the meter seals. Opportunity for theft of current is thus removed; also, there is no delay in reestablishing service as soon as a short circuit or overload is removed. Under this system, the meter may be installed out of doors and is thus accessible at all times for reading, testing, and repairs. If current transformers are used, a device is added that automatically closes the secondary circuit when the meter is removed. To protect the operator during testing, a special test jack with a "dead front" has been developed, which carries the necessary connections and can be safely interposed at the socket between the supply and the meter.

Permanency of calibration is a prime requisite. On a large system, it is not practicable to test and adjust the majority of the meters oftener than once a year and under certain conditions the period is much longer. In the case of large consumers where the amount of money involved is considerable, the inspections are more frequent.

To attain permanence of calibration, the friction at the pivots and commutator and the retarding torque of the magnetic brake must remain constant. It is essential that the commutator and brushes be capable of operating continuously for long periods without attention and without undue increase of friction. To secure this result, Elihu Thomson after experimenting with many materials was led by a knowledge of the mechanical and electrical properties of silver and experience with contacts made of it to adopt the pure metal for both the commutator and the brushes. This solved the greatest problem in the design of commutating meters.

The magnets used in connection with the retarding disk or brake must not age and the effect of temperature should be small. This involves the choice of a proper magnet steel, a correct design (a nearly closed magnetic circuit) and the artificial aging of the magnets by partial demagnetization with alternating current and by the proper heat treatment at moderate temperatures. From Eq. (2) in the demonstration already given (page 484), it will be seen that a 1 per cent change in the strength of the retarding magnets affects the accuracy of the meter by 2 per cent.

The magnets should be so placed that their strength is not likely to be altered by the field due to the current coil.

Besides the natural deterioration of the magnets there is the chance of an accidental change due to short circuits, and the magnets should be so placed that this effect will be minimized. The enormous and sudden rush of current through the current coils during a short circuit sets up a magnetic field which may be strong enough to change entirely the distribution of magnetism in the drag magnets and cause the meter to overregister. Such a change in the distribution of magnetism is illustrated in Fig. 303.

Normal distribution. Distribution after a short circuit.

FIG. 303.—Showing effect of a short circuit on the distribution of magnetism in the drag magnets of a direct-current watt-hour meter.

The field coils must be firmly held and kept apart by spacing blocks so that they cannot alter their positions or draw together and crush the armature if a short circuit occurs.

As friction losses in the instrument are unavoidable, they must be reduced to a minimum and must remain practically constant over long periods of time. Therefore the moving parts should be light; the jewels and pivots should be of the best and kept in good order. The jewels should be carried by spring supports, as this construction increases the life of the jewels by the elimination of the effect of "hammering" and consequently assists in maintaining accuracy at light loads.

Natural sapphire jewels are commonly used for alternating-current meters and for low-capacity, direct-current meters; however, some manufacturers use synthetic sapphires.

Cupped diamond jewels are now used in direct-current meters of large capacity. This contributes materially to the maintenance of accuracy at light loads.

In order to reduce the wear on the moving parts of a meter, the full-load speed is limited to about 50 r.p.m.

The commutator should be of small diameter and smooth and must be kept free from oil and dust of all kinds; the brushes must be smooth, and the brush pressure properly adjusted.

The counter should have the minimum possible friction, and the worm-and-wheel connection to the armature spindle should be properly adjusted. Steady-pins should be used to insure permanence of the adjustment.

The torque of the meter should be high, so that the unavoidable irregularities in friction may not cause inaccuracies, for in time the pivot and jewel as well as the commutator become rough, especially if the meter is subject to vibration and sudden jars. The commutator is very

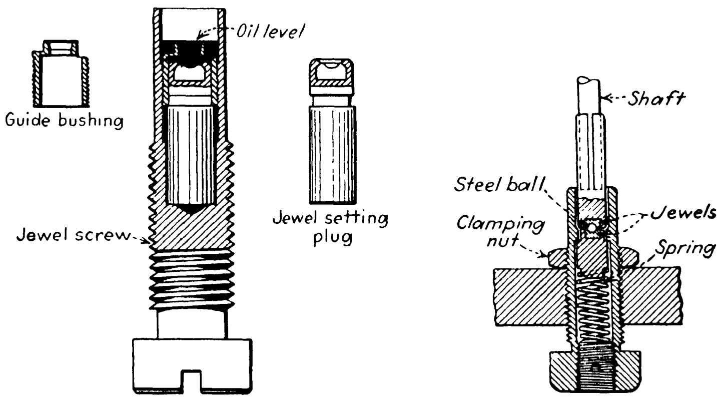

Fig. 304.—Jewel supports for watt-hour meters.

likely to be troublesome, especially if any sparking occurs due to the presence of dust or oil.

The fact that a meter has a high torque is advantageous only when it is associated with a light moving element. The ratio

$$\frac{\text{Full-load torque}}{\text{Full-load speed} \times \text{friction}}$$

should be large if the accuracy of the meter is to be affected only slightly by the wear of the jewel, pivot, and commutator.

The weight of the movable element has been much decreased by the use of a spherical self-supporting armature and an aluminum-alloy brake disk.

The effect of changes of room temperature on the accuracy of the meter should be reduced to a minimum. It is evident that the net effect of temperature on the resistance of the disk and on the drag magnets should be balanced by the change in the resistance of the armature circuit. The average temperature coefficient of a direct-current meter between 20 and 40°C. should not be more than 0.2 per cent per degree centigrade at either 10 or 100 per cent of full-load current. Tests show that the temperature coefficients of representative direct-current watt-hour meters of American manufacture vary between +0.26 and +0.07 per cent per degree centigrade, most of them being about +0.1 per cent.

As the armature circuit carries considerable current and is in part made of copper, its resistance [R in formula (1)] will rise and decrease the driving torque when the meter is first connected in circuit; consequently, tests should not be made until the permanent state of temperature has been reached, which may require about 20 min.

When a load is thrown on the meter, the heat liberated in the current coils also raises the temperature of the copper-wound armature and

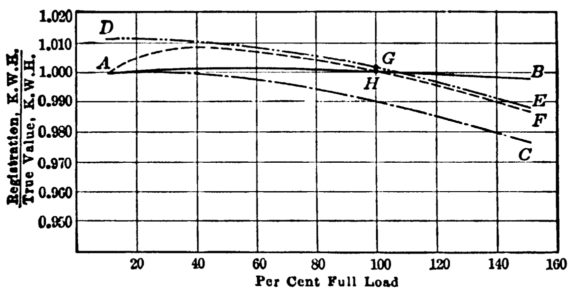

FIG. 305.—Pertaining to self-heating error due to current coils of direct-current watt-hour meter.

increases the resistance of the potential circuit, thus decreasing the registration. This effect increases with the load current and up to the time of attainment of temperature equilibrium with the length of time that the current is left on.

It will be seen that the effect of the self-heating of the meter and the consequent increase in the resistance of the disk decreases the retarding torque and causes the meter to speed up. However, as the brake is at a considerable distance from both the current and the potential coils, the error is practically that due to the change in temperature of the potential circuit.

The net result of the heating due to the current coils is that if the meter is adjusted at full load by changing the position of the drag magnets and then at light load by means of the light-load coil, the registration will be correct at light and at full load, a little too great at intermediate loads, and a little too small at overloads. The general form of the percentage-registration curve is shown at *AHF* in Fig. 305.

In detail, if there were no errors due to the heat from the current coils the percentage registration curve, if the meter were adjusted to register correctly at full load and at light load, would be that derived from

Eq. (4), all the coefficients being constant. If the adjustments were made at 10 per cent load and at full load, the curve would be *AHB*.

The effect of the heating is to increase *R*, the resistance of the potential circuit. This causes the upper parts of the curve to droop, the result being the curve *AC*. By moving the drag magnets, the curve *AC* may be shifted bodily so that it takes the position *DGE*, and by means of the light-load coil the registration at a low load (10 per cent load) may be made correct. The percentage registration curve is then *AHF*. The registration is correct at 10 per cent load and at full load. These self-heating errors are important in portable rotating standard watt-hour meters such as are referred to on page 515.

The meter should be unaffected by local magnetic fields. Trouble may be experienced from improper wiring, leads carrying large currents

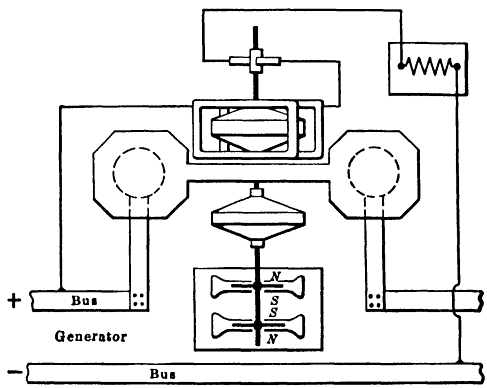

Fig. 306.—Diagram for astatic watt-hour meter.

being placed too near the meter. This would be likely to occur in heavy direct-current switchboard work, for in this case the meter coils consist of only a few turns, and the bus bars at the back of the switchboard may be very near the meter. To obviate this trouble a special astatic watt-hour meter has been designed. It is shown diagrammatically in Fig. 306.

The spindle carries two equal armatures: one operating in the field above, and the other in the field below a straight conductor. In consequence of this arrangement, variations of the local field which affects its strength equally at the upper and lower armatures have no effect on the registration. The drag magnets are so placed that if the strength of one is increased by the extraneous field, that of the other is diminished. The whole retarding device is enclosed in an iron shield.

Any watt-hour meter should maintain its accuracy under varying conditions of voltage and load. In general, in the neighborhood of the power station, the voltage on a system of electrical supply will remain practically constant, especially if the system is large; but at a distance, owing to insufficient copper in the conductors, the voltage variations may be considerable, and the accuracy of the meter should not be affected by them. Because of heating, the resistance of the potential circuit of the meter is dependent on the line voltage. Consequently, a change in line voltage does not produce a proportionate change in the speed of the armature. At moderate and full loads, an increase of voltage will tend to make the meter register too little. At light loads, where the light-load coil F' (Fig. 302)furnishes a considerable part of the driving torque, an increase of voltage tends to make the meter register too high, for the torque of the light-load coil, which increases as the square of the current in the armature circuit, may more than counterbalance the tendency of the meter to

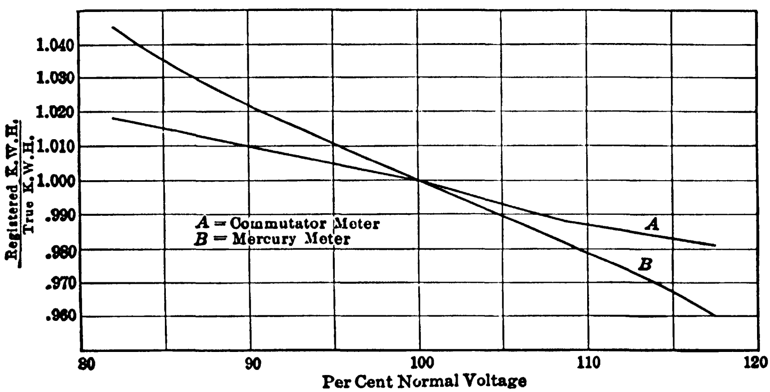

FIG. 307.—Showing effect of voltage variation on the registration of direct-current watt-hour meters at full-load current.

register too low owing to the increase in the resistance of the potential circuit. Practically, the effect of voltage variations will depend on the load on the meter, for the position of the drag magnets and of the light-load coil is adjusted at a standard voltage. For direct-current meters, at full-load current, a variation of from 10 per cent above to 10 per cent below the normal voltage should not affect the accuracy by more than 3 per cent; and at 10 per cent of the rated full-load current, the effect should not be more than 5 per cent.

Accuracy at light loads is of great importance, for it is seldom that the meter in an installation is worked at full capacity. Indeed for a great portion of the time the load on the meter may be but a small fraction of its rated capacity, and under these conditions it is essential that the energy be measured as accurately as possible.

A direct-current meter should rotate continuously with 2 per cent of rated full-load current, and at 10 per cent of rated full-load current should register correctly to within 3 per cent. The light-load adjustment must be such that the meter does not "creep," that is, rotate continuously when the consumer is not using energy. This should be true even when the supply voltage is 10 per cent higher than that at which the meter was adjusted. Electrical supply companies which give careful attention to their meters instruct testers to leave them so adjusted that they register correctly to within 1 per cent at from 5 to 10 per cent of full load and to within 1 per cent at full load.

The periodic service tests on a large distribution system where careful attention is given to the upkeep of the meters may be expected to show results comparable with the following:

Commutating meters, percentage of total numbers registering between	Light load, 5 to 10 per cent of full load	Full load
98 and 102 per cent of correct value	60.0	90.5
95 and 105 per cent of correct value	91.9	98.3
90 and 110 per cent of correct value	98.3	98.9
Induction meters, percentage of total numbers registering between		
98 and 102 per cent of correct value	94.1	98.3
95 and 105 per cent of correct value	98.5	99.4
90 and 110 per cent of correct value	99.3	99.8

The tests were made at intervals of from six months to a year. The aim of careful supply companies is to maintain their meters so that a full-load accuracy of 98 per cent or better is obtained.

The energy loss in the meter must be small, for the potential coil of the instrument is in circuit continuously even though the consumer is using no energy. The expense of energizing the potential coils falls on the supply company.

When the consumer uses energy, the voltage at the load is diminished by the IR drop in the current coils of the meter. This, of course, varies with the load. For a small direct-current meter (5 amp.) the drop in the field coils may be about 1 per cent of the line voltage when the rated full-load current is used. The expense of energizing the field coils falls on the consumer.

Use of Watt-hour Meters on Three-wire Circuits.—For metering on low-tension, three-wire, direct-current circuits such as are used in congested districts in cities, two ordinary two-wire meters may be used, one with the current coils in the positive lead, the potential circuit being connected between this lead and the neutral wire, while the other is similarly connected on the negative side of the circuit. Such an arrangement is free from errors due to the unbalancing of the voltage of the circuit and to unequal currents in the positive and negative leads; it is, therefore, a desirable arrangement when the circumstances are such that these effects may become very large.

Ordinarily, a single three-wire meter is employed. In this instrument, one of the current coils is in the positive while the other is in the negative lead.

When traversed by equal currents, the two current coils should have equal effects on the armature. The potential circuit may be connected between the positive or the negative lead and the neutral or between the

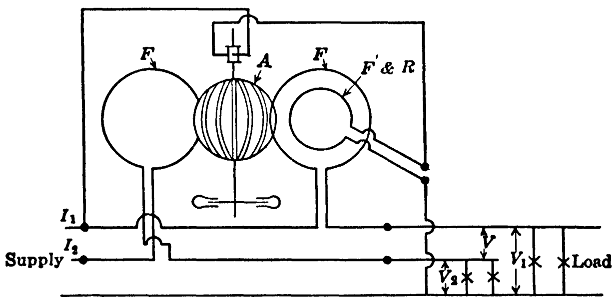

FIG. 308.—Diagram for three-wire direct-current watt-hour meter.

positive and negative leads. Figure 308 shows diagrammatically a three-wire meter with the first connection.

Theoretically, the three-wire meter is subject to certain errors; for instance, with the connection shown in Fig. 308, the potential lead being on main 1, an error will occur if the voltages are unbalanced, for the power given to the circuit is

$$P = V_1 I_1 + V_2 I_2.$$

The angular velocity of the armature is supposed to be proportional to this quantity, while in reality it is proportional to

$$V_1(I_1 + I_2).$$

Therefore, with a steady load, the correction that must be added to the reading, reduced to watts, to obtain the true power, is

$$C = I_2(V_2 - V_1).$$

This correction will be positive or negative depending on whether V_2 or V_1 is the larger. With the potential lead connected to main 2,

$$C = I_1(V_1 - V_2).$$

If the potential circuit is connected between the positive and negative mains, the angular velocity of the disk will be proportional to

$$\tfrac{1}{2}V(I_1 + I_2),$$

where V is the potential difference between the positive and negative mains. The correction in watts that must be added to the reading to obtain the true power will be the difference between P and this quantity, or

$$C = \frac{(I_2 - I_1)(V_2 - V_1)}{2}.$$

It will be noted that the three-wire meter violates the rules established by Blondel's theorem (page 337) for the correct measurement of power in a three-wire circuit and consequently would be expected to show error due to unbalance.

The error in the registration will be zero if the currents in the current coils are equal, even though the voltages are unbalanced. It will also be zero if the voltages are equal, even though the currents flowing in the two current coils are unbalanced. If the leads to the meter and load are of considerable resistance, and the voltages are balanced before any current is drawn, the meter will always read too high when unequal currents are taken by the two sides of the load; for if I_1 is greater than I_2, the quantity $(V_2 - V_1)$ is positive, and the correction negative. If I_1 is less than I_2, the factor $(V_2 - V_1)$ is negative, and the correction is negative.

If alternating currents are used and the loads are reactive, these relations are affected by the phase displacements of the currents.

Practically it is impossible to make allowance for these errors, for the loads on the two sides of the installation are continually shifting; the best that can be done is to see that the load distribution is such that the two sides are well balanced.

INDUCTION METERS

The Induction Watt-hour Meter.[2]—Commutating watt-hour meters may be made to register commercially correctly on circuits of all power factors. Formerly, lagged meters of this class were in common use on alternating-current circuits; they have now been superseded by induction watt-hour meters for the following reasons: The moving element of the induction meter may be made very light, and at the same time the torque may be kept high. This reduces the wear on the lower pivot and jewel and lessens the chance of errors due to pivot friction. There is no commutator to become rough through wear and sparking thus increasing the friction, and there are no brushes to keep in order. The net result is a great decrease in the first cost of the meters and in the cost of maintaining them, a decrease in the current necessary to start the meters, and an increase in the accuracy of the registration at light loads (see page 491). This last point is of the utmost importance. The losses in the potential coils are less in the induction than in the commutating meter; and as the loss goes on continuously 24 hr. a day, this fact is of importance.

The induction principle has been utilized in alternating-current ammeters, voltmeters, and wattmeters; however, as these instruments in the portable form are likely to be used under all sorts of circuit conditions, they are less reliable than those based on the electrodynamometer, for they are subject to errors inherent in the induction principle. The present-day application of the induction principle is in the induction

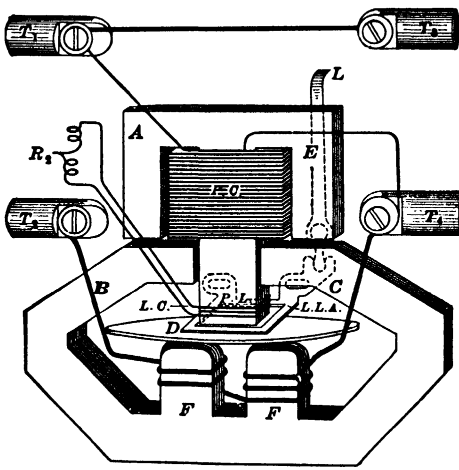

FIG. 309.—Showing electric and magnetic circuits of induction watt-hour meter.

watt-hour meter, the standard instrument for measuring energy in supply circuits. Figure 309 shows schematically the essential parts of the motor element of an induction watt-hour meter. As now made, meters differ in detail but not in principle from that shown in the figure, which is used because it clearly indicates the essential parts. The most obvious differences are in the means for obtaining the light-load and lag adjustments.

The terminals T_3 and T_4 are connected to the supply, while the load is connected between T_1 and T_2.

PC is the coil of a highly inductive potential circuit; it is connected across the line. Most of the flux through this coil passes down the central core and returns via *BA* and *CE*. Some of it, however, goes to the potential-coil lug *PL* and thus magnetizes it. The flux which proceeds outward from *PL* cuts the pivoted disk *D* which forms the movable element of the meter.

The line current flows through the oppositely wound series coils *FF*. On the passage of currents in all the coils, the flux from *PL* induces currents in the disk which are acted upon by the flux due to *F*, and the flux from *FF* induces currents in the disk which are acted on by the flux due to *PL*.

The principle involved may be explained in a general way by the aid of Fig. 310 which is schematic for the driving portion of the meter.

Referring to Fig. 310, the current I circulates around the lower laminated electromagnet; the current I_P, around the upper magnet. The positive directions for the currents and their attendant fluxes will be assumed as indicated. The fluxes φ_I by their variation set up a system of eddy currents in the disk D. Let I' be their equivalent value,

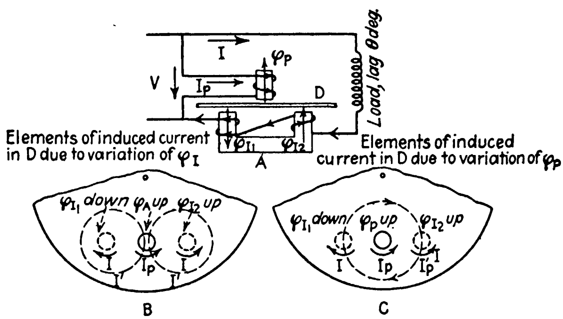

FIG. 310.—Pertaining to induction watt-hour meter.

and y their equivalent phase displacement which will depend on the impedance of the eddy-current paths and therefore on the frequency and the resistance and inductance of these paths. These currents are acted upon by the flux φ_P. Likewise, φ_P induces currents in D, their value and phase displacement being I'_P and y. These currents react with the flux φ_I. The current-coil fluxes will be assumed to be in phase with the m.m.fs. In reality, there is a slight phase displacement between the current and the current-coil flux.

Let v equal $V \sin \omega t$ and i equal $I \sin (\omega t - \theta)$. θ is the power-factor angle of the load.

$$\varphi_P = \Phi_P \sin (\omega t - \Delta_P), \qquad \text{upward.}$$

Δ depends on the reactance and resistance of the potential-coil circuit, on the iron losses in the potential-coil core, and on the reactance and resistance of the eddy-current paths in the disk, since the eddy currents contribute to the total m.m.f. active in producing the potential-core flux.

$$\varphi_{I_1} = \Phi_I \sin (\omega t - \theta) \qquad \text{downward}$$
$$\varphi_{I_2} = \Phi_I \sin (\omega t - \theta) \qquad \text{upward.}$$

If z is the impedance of the eddy-current path, the induced current due to φ_{I_1} will be

$$i'_{\varphi_{I_1}} = \frac{\Phi_I \omega}{z} \sin (\omega t - \theta - y + 90°),$$

and the total current along the medial line will be

$$i'_{\varphi_I} = \frac{2\Phi_I \omega}{z} \sin (\omega t - \theta - y + 90°).$$

The reaction of this current with φ_P is found by averaging $\varphi_P i_{\varphi_I}$ over a cycle.

$$T_1 = \frac{2\Phi_P \Phi_I \omega}{z} \frac{1}{T} \int_0^T \sin (\omega t - \Delta) \sin (\omega t - \theta - y + 90°) dt;$$

$$T_1 = \frac{\Phi_P \Phi_I \omega}{z} \cos [(y - 90°) - (\Delta - \theta)] \qquad \text{to the right.}$$

Similarly, the induced current due to φ_P will be

$$i'_P = \Phi_P \omega \sin (\omega t - \Delta - y + 90°).$$

The reaction of the current with φ_{I_1} and φ_{I_2} is

$$T_2 = \frac{2\Phi_P \Phi_I \omega}{z} \frac{1}{T} \int_0^T \sin (\omega t - \theta) \sin (\omega t - \Delta - y + 90°) dt;$$

$$T_2 = \frac{\Phi_P \Phi_I}{z} \omega \cos [(y - 90°) + (\Delta - \theta)] \qquad \text{to the left.}$$

The total turning moment is proportional to

$$T = \frac{\Phi_P \Phi_I}{z} \omega \{\cos [(y - 90°) - (\Delta - \theta)] - \cos [(y - 90) + (\Delta - \theta)]\}. \quad (6)$$

If f is the frequency,

$$T = \frac{K \Phi_P \Phi_I f}{z} \sin (\Delta - \theta) \cos y. \quad (7)$$

The accelerating torque is proportional to the product of the two fluxes, to the sine of the time-phase angle between them, and to the frequency.

The retarding torque is due to the movement of the disk through the air gaps of the drag magnets, which in this meter are placed diametrically opposite *PL*.

For steady motion the driving torque must equal the retarding torque.

If the disk is in motion, there will be a retarding effect due to its movement through the alternating fields. This retarding effect will be proportional to the mean square values of the fields and to the angular velocity of the disk. Under a fixed voltage, the field from the potential coil will be of constant mean square value. That due to the current coils will vary with the load. The retarding effect will vary with the load on the meter and is a source of error. By correct design, its effect may be made negligibly small under the usual operating conditions. Evidently the disk speed should be low. As Φ_P and Φ_I are proportional to V and I, the line voltage and current, and the frequency is fixed, the angular velocity of the disk is

$$\omega' = K'VI \sin(\Delta - \theta). \quad (8)$$

In order that ω' may be proportional to the power, Δ must, in effect, be made 90 deg. Referring to Fig. 311, if $\alpha = 0$, that is, if the useful potential-coil flux is adjusted so that it is exactly 90 deg. out of time phase with the applied voltage, the torque will be proportional to the power in the circuit. This, of course, is the proper adjustment and must be attained by the addition of special phase-shifting or lagging devices by which the *useful potential-coil flux* in the air gap is brought into quadrature with the line voltage.

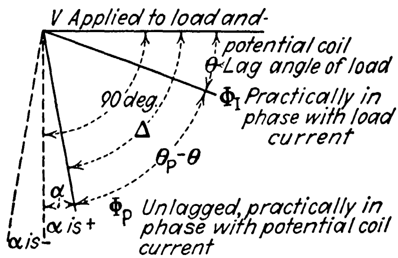

FIG. 311.—Time-phase diagram for induction watt-hour meter.

If the useful flux lags behind the applied voltage by less than 90 deg., that is, if α is $+$, the wattmeter is said to be underlagged; if $\alpha = 0$, it is exactly lagged; and if α is $-$, that is, if the flux has been caused to lag more than 90 deg., the instrument is overlagged.

The construction of the magnetic circuit of the potential coil should be such that the useful potential-coil flux is naturally nearly 90 deg. out of phase with the applied voltage; the required amount of additional lagging is thus reduced. The less the reliance placed on the lagging device the better will the meter behave when it is subjected to wide variations of frequency and to distorted wave form.

Referring to Fig. 311, $\theta_P = 90° - \alpha$; then

$$\omega' = K'VI \cos(\theta + \alpha). \quad (9)$$

If the meter is properly lagged, that is if $\alpha = 0$, the angular velocity of the disk is proportional to the power, and the total number of revolutions executed in any time interval is proportional to the kilowatt-hours of energy supplied during that time.

It is evident that Δ should be made as near 90° deg. as is practicable, and hence the resistance of the potential winding should be low, and the iron losses in the core small.

The Lag Adjustment.—Exact time quadrature between the useful potential-coil flux and the voltage applied to the load is obtained by the use of a lag coil *LC* which is wound about the potential pole tip. The circuit of this coil is completed by the resistance R_2 which is adjusted until the desired phase relation ($\Delta = 90°$) is established. A simplified diagram of the potential circuit is shown in Fig. 312. It will be seen that the arrangement is equivalent to a transformer with large leakage reactances. For simplicity a 1:1 ratio is assumed. The disk, which serves as the rotor, projects into the air gap a_2 and is cut by the total flux in the gap Φ_2. The mutual flux which cuts the primary and secondary is Φ_M, and the primary and secondary leakage fluxes are Φ_{L1} and Φ_{L2}, respectively, Φ_{L2} being the leakage flux which cuts through the disk. The total flux through the primary is Φ_1. When the secondary is open, the angle Δ is less than 90 deg. owing to the *IR* drop and losses in the

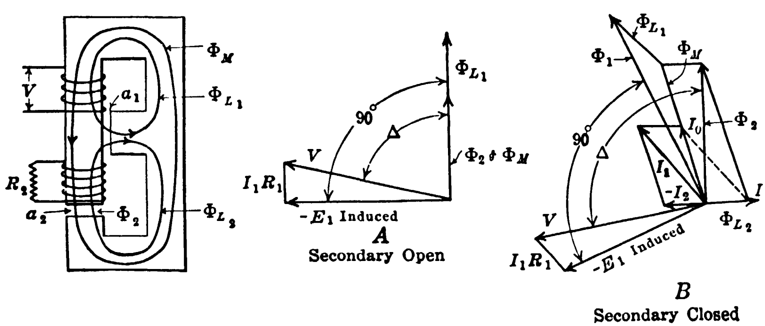

FIG. 312.—Diagram for lag adjustment of induction watt-hour meter.

potential coil. When the secondary is closed, an m.m.f. proportional to and in phase with I_2 is introduced. The leakage flux Φ_{L2} will be in time phase with and substantially proportional to I_2; likewise, the leakage flux Φ_{L1} will be in time phase with and substantially proportional to I_1. The mutual flux Φ_M is proportional to and in phase with I_o, I_o being the vector sum of I_1 and I_2, as in any transformer. Φ_1 is the vector sum of Φ_M and Φ_{L1}, and Φ_2 is the vector sum of Φ_M and Φ_{L2}. $-E_1$, the induced voltage in the primary, will obviously be 90 deg. ahead of Φ_1. By the proper adjustment of I_2, by varying R_2, the flux Φ_{L2} may be made of such magnitude that Φ_2, the total flux cutting the disk is swung clockwise so that angle Δ becomes 90 deg.

In some meters of later design, the winding *LC* and the resistor R_2 (Fig. 309) are replaced by a single stamping made of a material of the proper resistivity and placed directly below the potential lug and immediately above the light-load plate *LL*. The effect of the currents induced in the stamping may be varied by moving it radially inward or outward. The pole is thus shaded to a greater or lesser extent, and the net phase displacement of the useful potential-coil flux thus adjusted. A secondary effect is a slight radial thrust on the spindle. The lag plate and the light-load plate are assembled in a single structure, the adjustable lag plate riding on the adjustable light-load plate. The light-load plate has a fine-pitch screw adjustment accessible from the front of the meter when the cover is removed. The lag plate is reached by removing the back of the meter. As the two adjustments are not entirely independent, the full- and light-load adjustments should be checked after an adjustment of the lag plate. In order to make the lag adjustment, one must have at command a source of sinusoidal current which has the voltage and frequency for which the meter was designed and from which loads can be taken at unity power factor and a lower power factor, 0.5 or thereabouts. One must also know whether the power factor is due to a lagging or a leading current.

The meter is set up and adjusted by moving the drag magnets so that it registers correctly at unity power factor. If the registration is correct, the constant K, calculated by the formula

$$K = \frac{Pt}{N3{,}600},$$

will agree with that stated by the makers. (P is the power; t is the time in seconds required for N revolutions.) The light-load adjustment is now made.

Keeping these adjustments the same, the meter is then tested at the lower power factor. If it is correctly lagged, the values of K from the two tests will be the same. Suppose, however, that K given by the second test is greater than that obtained at unity power factor. This shows that the meter runs too slow at low power factors. Therefore, if the current lags [see Eq. (9)],

$$\cos(\theta + \alpha) < \cos\theta;$$
$$\theta + \alpha > \theta.$$

Here α is a positive angle (see Fig. 311), and the meter is *underlagged.* This means that the resistance R_2 in Fig. 309 must be decreased or the position of the lag plate changed. After the change has been made the test is repeated until the two agree.

If the current had been leading, θ negative, the result would have been

$$\cos(-\theta + \alpha) < \cos(-\theta);$$
$$-\theta + \alpha > -\theta.$$

Here α must be a negative angle, and consequently the meter is *overlagged.*

In this connection, attention may be called to the fact that the statement that a power factor is 0.5, for example, may give little indication of the conditions under which the meter is operating, for both the potential difference and the current waves may be irregular. With a distorted potential-difference wave, one may obtain various current waves, depending upon the method of regulation used, the power factor always being 0.5. For instance, if inductances are used, the upper harmonics in the current wave will be suppressed to a certain extent. If the change from unity to a low power factor (0.5) is made by using a three-phase circuit, as shown on page 522, the *fundamental* will be lagged 60 deg., but the harmonics will not appear in their proper phase relations.

The most exacting test for the lagging is when $\theta = 90$ deg., for in that case the meter will register unless $\alpha = 0$. It is difficult to adjust θ to exactly 90 deg. A natural method is to take the voltage and current from the two phases of a two-phase circuit, but the two e.m.fs. may not be exactly 90 deg. apart, and the regulating devices together with the current coils of the instruments may shift the phase of the current slightly.

Correct lagging is especially important when induction meters are used for measuring energy supplied for industrial purposes. Induction motors which are commonly used may be only partially loaded and therefore operating at low power factors. This is the condition for which it is most necessary to keep the potential and current fluxes of the meter in the proper phase relation.

Light-load Adjustment.—The principle underlying the devices used for the light-load adjustment is that of the shaded-pole motor. In this type of motor, the flux from the stator is split into two portions which are displaced in time phase with respect to each other. Consequently, the forces acting upon the movable element are unbalanced, and a tendency towards rotation results. Referring to Fig. 309, which shows the electric and magnetic circuits of a type of watt-hour meter formerly made by the General Electric Company, *PL* is the potential lug. Immediately below it is a stamping *LLA* which forms a short-circuited coil of a single turn; it is made of sheet metal of the appropriate resistivity and is mounted so that it can be displaced in its own plane either to the right or to the left by moving the lever *L*.

Suppose that the potential coil is energized, that there is no load on the meter, and that *LLA* is placed symmetrically with respect to the potential pole. Currents will be induced in *LLA* which will cause a back m.m.f.; but as *LLA* is symmetrically placed with respect to the pole tip, the flux cutting the disk will be symmetrical with respect to the pole and all in the same time phase. Consequently, there will be no tendency for the disk to turn. Now, suppose the loop to be displaced toward the left; the part of the pole covered by it will be "shaded," that is, owing to the induced currents in the loop, the flux from that portion of the pole will be decreased and displaced in phase when compared with that from the unshaded portion at the right of the loop. Thus the disk is acted on by two sets of fluxes which differ in time phase, and there is a traveling field and a tendency to rotation. By giving the loop the proper displacement, the friction may be compensated so that the disk will begin to move as soon as a very small load is applied.

Sources of Error in Induction Watt-hour Meters.—The readings of induction instruments are subject to a number of errors inherent in their construction. These errors are not found in instruments based on the electrodynamic principle.

Errors in the driving torque may be due to

1. *Incorrect magnitudes of the fluxes.* These may arise from abnormal voltages and load currents. The potential-coil flux may be in error owing to changes in the resistance of the coil or to abnormal frequencies.

2. *Incorrect phase relation of the fluxes.* These may arise from defective lagging, abnormal frequencies, changes in the resistance of the potential-coil circuit due to temperature changes, changes in the iron losses of the potential element due to temperature changes or saturation.

3. *Changes in the resistance of the eddy-current paths in the disk, light load, and lag plates.*

4. *Lack of symmetry in the magnetic structure.* This produces a driving effect similar to that resulting from the light-load adjustment and tends to make the meter creep.

Errors in the retarding torque may be due to

1. *Changes in the strength of the drag magnets.* These may be due to temperature or aging. A 1 per cent change of magnet strength produces a 2 per cent change in the retarding torque.

2. *Changes in the resistance of the disk.* These are due to temperature changes.

3. *Changes in the retarding torque due to the disk moving through the field of the current coils.* This field changes with the load. There may also be retardation due to the disk moving through the field of the potential coil. This field changes with the line voltage.

4. *Abnormal friction of the moving parts.*

In addition, there may be errors due to badly distorted wave forms. Phase-angle errors become increasingly important as the power factor is lowered. The results attained in modern induction meters are shown in Figs. 313 and 314, which are average results from four single-phase, 5-amp., 115/120-volt, 60-cycle, two-wire meters.

Induction-meter Characteristics. *Normal Curve.*—By adjusting the position of the drag magnets, the meter is made to register correctly at full-load current and unity power factor. The light-load compensator is then adjusted until the meter registers correctly at 10 per cent of full line current. As the compensator is actuated from the voltage element, it produces a constant driving torque, which has an insignificant effect at full load. However, for currents of less than 10 per cent of full load the meter will be slightly overcompensated for friction. If the power factor is lowered, the driving torque is decreased, and this effect is more marked. The reluctances of the magnetic paths are mostly in the air gaps, but with very low line currents the decreased permeability of the iron may introduce an appreciable effect.

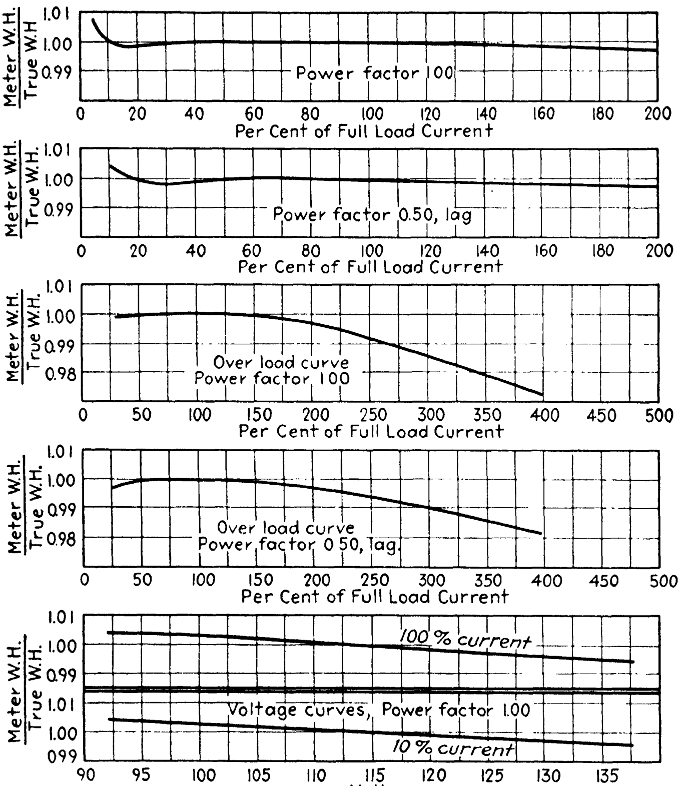

FIG. 313.—Characteristics of induction watt-hour meter.

Overload Curve.—An increase in the line current increases the retarding effect due to the movement of the disk through the increased current-coil flux and causes the meter to underregister at overloads. This effect is minimized by employing a small current-coil flux and a low disk speed.

Voltage Curve.—An increase of the line voltage above normal increases the drag due to the disk moving through the increased potential-coil flux. This causes the meter to underregister. The self-heating will be increased, and the consequent rise in temperature will tend to reduce the potential-coil current and the total potential-coil flux below normal. The increased resistance also changes the phase of the potential-coil flux, making Δ smaller and thus reducing the registration at low power factors. For the ordinary range of line voltages, on a circuit normally operated at constant potential, the net effect of all these changes is commercially inappreciable.

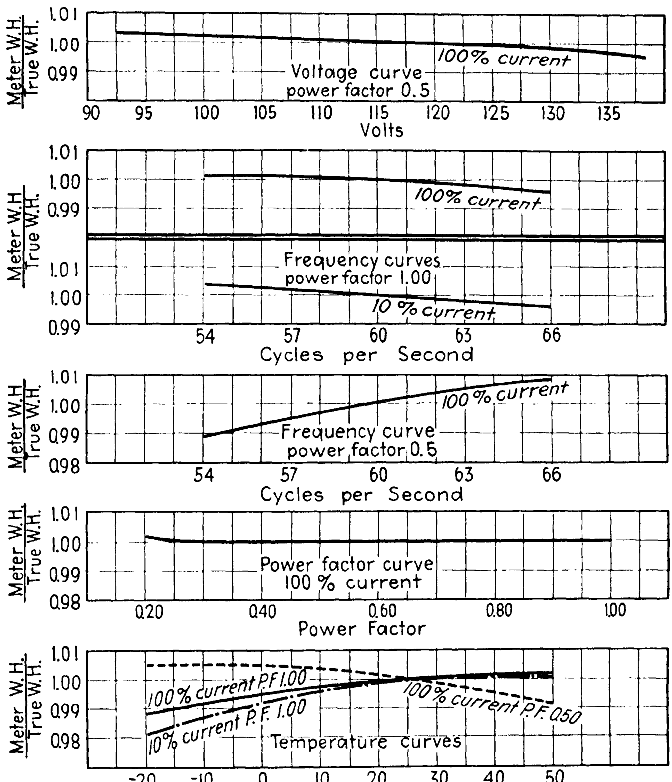

FIG. 314.—Characteristics of induction watt-hour meter.

Frequency Curve.—Suppose the frequency to be doubled, the voltage, current, and power factor of the load remaining fixed. Assuming that the resistance of the potential coil is small, the potential-coil flux will be halved; however, the currents induced in the disk by the flux will remain as before, for, while the flux is only one-half as great, it is varying at twice the normal frequency. The induced currents react with the current-coil flux, which is fixed. The net result is that the alteration in this portion of the accelerating torque is that due to the changed time-phase relation of the fluxes. The currents induced in the disk by the current-coil flux are doubled, for though the value of the flux is not changed, it is varying at twice the normal frequency. These doubled currents react with the halved potential-coil flux, so again the effect is that due to the changed time-phase relation of the fluxes.

At unity power factor changed phase relations of the fluxes produce little effect, but at low power factors their effect is marked. An increase of frequency lowers the potential-coil current and total flux. The decreased current lowers the temperature and diminishes the ohmic resistance of the potential windings. At the same time, the iron losses are increased. If the net effect is an increase in the equivalent resistance of the potential circuit, Δ becomes smaller, and the registration is decreased. If saturation effects are present, the changed total flux may alter the division of flux between the disk gap and the leakage path. The lowered disk-gap flux reduces the braking action and tends to make the meter run faster. At the higher frequencies the increased reactances of the eddy-current paths in the disk increase Z and diminish the registration. The angle y is also increased. The magnitudes and phases of the currents in the light-load and lag coils are altered, the lag of the fluxes being increased. If the frequency is above normal, the net effect of these possibilities is to make the meter overregister slightly with low, lagging power factors.

Power-factor Curve.—The meter is originally adjusted to register correctly at unity and at a lower power factor—usually 0.5. The registration is then commercially correct for all power factors above 0.2.

Temperature Curve.—Temperature effects are important where, for convenience of reading, meters are installed out of doors, for in some places the seasonal variation of temperature is from about 110 to $-40°$F. They may be of special importance in the use of portable test meters. With those of old design it is necessary to insert a thermometer in the instrument in such a manner as to give the mean temperature and also to provide a calibration card which will give the necessary corrections for the ordinary range of atmospheric temperature. Temperature variations affect the resistance of the disk, the resistances of the light load and the lag plates, and that of the potential circuit; the permeability and the iron losses in the magnetic circuits are also affected, so both the magnitude and the *phase relation* of the fluxes may be altered. These effects are especially important at low power factors. However, the more important cause of error, which is active at *all power factors*, is the variation of the strength of the drag magnets. To correct this error, Kinnard and Faus[2] have developed the use of a thermalloy magnetic shunt. The material is a copper-nickel-iron alloy which decreases very uniformly in permeability with increase in temperature until near the transformation point, when the rate of decrease of permeability becomes less. The alloy becomes nonmagnetic at the low temperatures of 20 to 100°C., according to the composition and heat treatment to which the material has been subjected. The hysteresis effects in the material are extremely small. Two grades of thermalloy are used in a properly proportioned magnetic shunt, which is indicated in Fig. 315 at *s*.

As the temperature rises, the net permeability of the shunt decreases, and more of the flux is thrown into the air gaps, thus compensating for the normal decrease in magnetic effect. In one case, for a temperature change of 25 to 55°C., the average change in the meter speed, using magnets thus compensated, was +0.11 per cent. With uncompensated magnets the change was +2.3 per cent. In modern meters, the temperature errors at both unity and 50 per cent power factor are practically negligible, three- or four-hundredths of 1 per cent per degree centigrade.

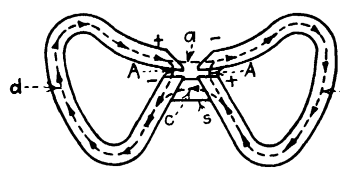

FIG. 315.—Schematic diagram of magnetic compensation with thermalloy magnetic shunt. (*General Electric Company*.

Effect of Wave Form.—As the registration is affected by variations in frequency, one would naturally expect that changes of wave form would have an effect on the accuracy of the meter, especially at low power factors. The theory of the induction meter, given on page 493, rests on the assumption of sinusoidal waves of current, voltage, and flux. As the flux wave is the time integral of the voltage wave, the form of the flux wave due to the potential coil will not be the same as that of the voltage applied at the potential terminals unless the voltage is sinusoidal. The currents induced in the disk by the current coils depend on the time rate of change of the flux due to the current coils, and will differ from the current wave in form, unless the current is sinusoidal. However, the e.m.f. waves of modern generators are very closely sinusoidal, and usually the wave forms encountered in distribution systems are also sinusoidal, so that wave-form errors are not generally important. If it is necessary to measure the energy given to a nonlinear device, the existence of wave-form errors should be kept in mind.

Polyphase Watt-hour Meters.—For metering in polyphase circuits where there are n conductors between the generator and the load, a special form of watt-hour meter has been developed. An example of this, adapted for use on a four-wire circuit, is shown in Fig. 316.

The instrument consists of $n - 1$ complete induction watt-hour meters, (see page 337) with the disks rigidly fastened to the same shaft and acting on the same register. The total driving torque is therefore the sum of the torques due to the $n - 1$ members; that is, it is proportional at any instant to the power in the circuit. The retarding torque is furnished by drag magnets, one being applied to each disk. Of course there must be no interference between the elements.

Each element is complete in itself and must be adjusted so that it registers correctly at both high and low loads, as well as at both unity and low power factors. It is essential, when carrying out these adjustments, that all potential circuits be kept energized; otherwise the moving element will experience an abnormal retarding torque.

Suppose that there are two elements and that the upper one is under adjustment; both sets of drag magnets are placed in what seems to be a

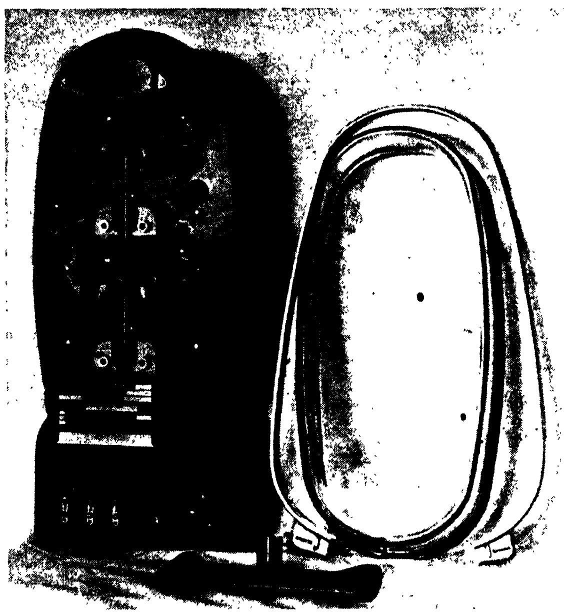

FIG. 316.—Polyphase watt-hour meter. (*Westinghouse Electric & Manufacturing Company.*)

reasonable position, and the adjustment is made as in a single-phase meter. When it is completed, attention is given to the lower element, the constant of which must be varied and made equal to that of the upper element without changing the constant of the latter.

It is not permissible to change the position of the drag magnets; the change in the constant must be effected by altering the driving torque of the lower element. This may be done by varying the fluxes, and for this purpose taps are sometimes brought out from the potential coil by which the number of active turns may be altered. In the polyphase induction meters now made by the General Electric Company, this adjustment is effected by changing the magnitude of the working potential-core flux by moving an iron armature in the air gap between the outer and center poles of the potential elements.

A very good check on the equality of the two elements may be obtained by operating the potential circuits in parallel and the current circuits in series and opposed; under these conditions the disk should not rotate.

When induction meters are used on loads that have a rectifying effect, such as three-phase arc furnaces, they must be inserted in the *primary* of the transformer that supplies the load.

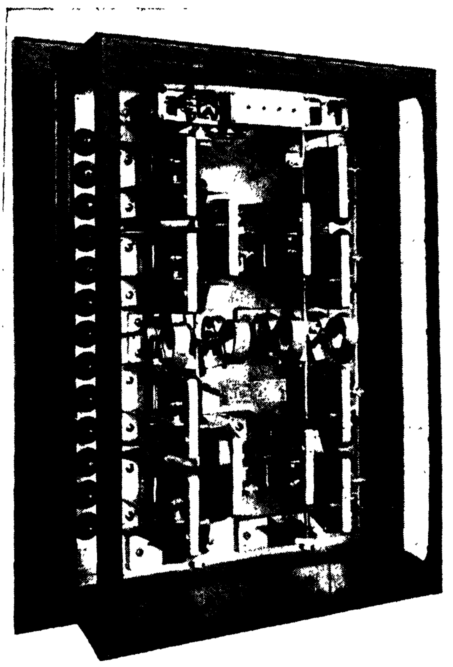

FIG. 317.—Twelve-element totalizing watt-hour meter. (*General Electric Company.*)

Totalizing Watt-hour Meters.—In case it is necessary to read from a single register the total kilowatt-hours of a group of independent circuits, recourse may be had to a multiple-element totalizing watt-hour meter. Such a meter provided with 12 complete driving elements is shown in Fig. 317. A group of six elements actuates each vertical shaft. The summation of the two groups is accomplished by a differential gear arrangement working into the register proper. As there are 12 elements, this particular meter is competent to deal with six independent three-wire, three-phase circuits or four independent four-wire circuits. The elements are "staggered" to prevent interference. Each vertical shaft is provided with its own magnetic brake, and the full-load adjustment is effected by moving the drag magnets as usual. Each element is provided with a torque-adjusting device, together with the usual light-load and lag adjustments. All the adjustments are made by means of finely pitched screws. Light-load compensation to offset friction is provided by a constantly excited torque-producing element on each disk. The instrument transformers and the windings of the series coils must be such that the torque per kilowatt on the line is the same for all the elements.

MERCURY MOTOR METERS

The principle utilized in the mercury motor meters is that illustrated by the familiar Barlow's wheel, in which a current flows radially in a pivoted copper disk so placed between the poles of a magnet that it is cut by the flux. On the passage of the current the disk is set in rotation.

The advantages claimed for the mercury motor meter for direct current are the elimination of the commutator, as well as the wire-wound armature and the brushes, and the decrease of the wear on the lower pivot and jewel. These things tend to decrease the cost of maintenance.

A practical difficulty has been that in time the mercury is very likely to become contaminated and to cause an increase in the friction of the meter.

The Mercury Ampere-hour Meter.—Aside from special uses, some of which will be referred to later, ampere-hour meters are intended for use on constant-potential circuits; for if the potential is *kept constant*, their readings form as just a basis for that part of the charges which are dependent on the amount of energy furnished as do those of the watt-hour meter. When the ampere-hour meter is so used, the register is graduated to read in kilowatt-hours, the voltage of the circuit having a definite value.

In America, the watt-hour meter is now used almost exclusively in lighting installations; but in Great Britain, the ampere-hour meter is extensively employed.

Ferranti Ampere-hour Meter.—The Ferranti ampere-hour meter was one of the earliest forms of electricity meters, its development having been begun as early as 1883.

Figure 318 shows the essential parts of this meter. The motor is of the Faraday disk type.

The current enters at the + terminal C_1, flows through the mercury to the amalgamated edge of the copper disk *CD*, then through the disk to its central portion, which is amalgamated, and out by the terminal C_2. Thus the current in the disk is in the field of the permanent magnets *SD*, and a driving torque is imparted to the disk armature. To protect the copper from the action of the mercury, the top and bottom surfaces of the disk are platinum plated and enameled, except directly above C_2. As the armature moves through the fields of the two magnets SB and SD, there will be the usual braking action due to eddy currents. The fluid friction of the mercury also contributes a retarding action; and as this increases with the speed, that is, with the customer's load, the meter is compounded by a coil of a few turns CC on the lower iron crossbar. When the current is increased, the strength of the field SD is also increased, and hence the driving torque becomes larger. However, the action of the magnetic brake remains the same, for the poles at SD are so arranged that when the field at SD is increased, that at SB is diminished.

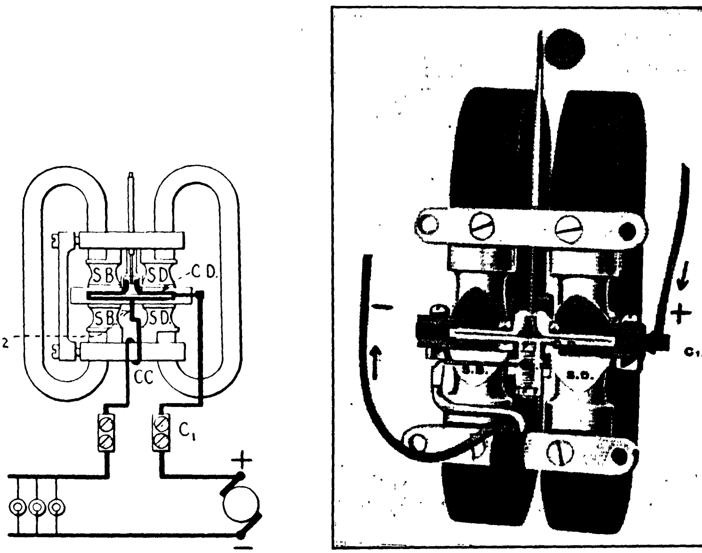

Fig. 318.—Working parts of Ferranti ampere-hour meter.

The buoyancy of the armature is adjusted by a weight on the spindle until the disk just sinks. Friction between the pivot and jewel is thus reduced to a minimum. A sealing device is used so that the mercury will not be spilled during transportation.

Sangamo Meter.—In America, the mercury motor meter has been developed by the Sangamo Electric Company which began the work in 1904.

The main body of the mercury chamber is made of a molded insulating compound (see Fig. 319). The two current terminals E_1, E_2 are diametrically opposite each other, and above the lower part of the chamber that contains the copper disk armature is a spirally laminated ring of soft iron (return plate). On the spindle above the disk is a hardwood float; this takes the pressure from the lower bearing, which becomes merely a guide; in fact, a slight thrust is exerted against the bearing plate of the upper ring jewel.

In all Sangamo meters, the copper armature disks are now slit radially. The current is thus caused to flow directly from terminal E_1 to E_2 without spreading over the disk. By this means the torque is increased about 40 per cent.

The cover of the mercury chamber is made with a central tube projecting downward. The clearance of the spindle in the tube is about

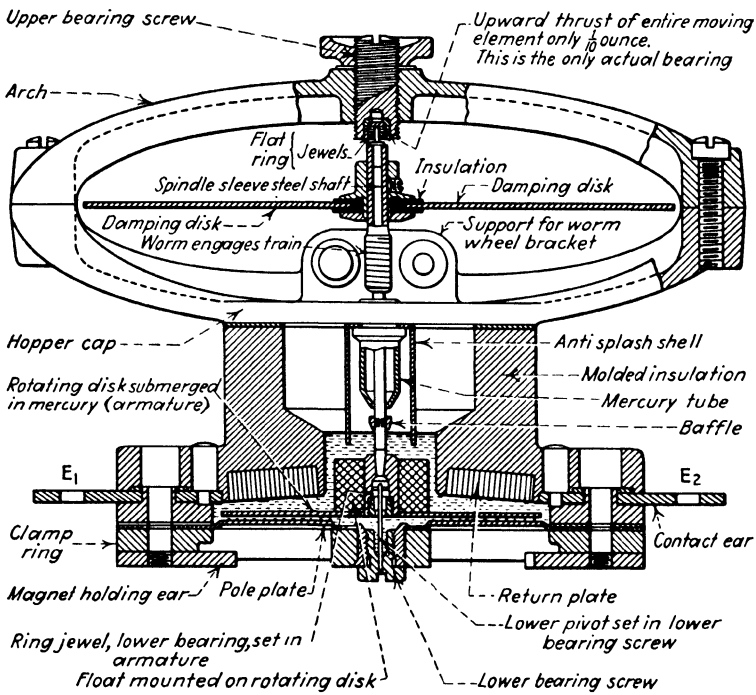

FIG. 319.—Cross section of mercury motor of Sangamo Type D-5 direct-current watt-hour meter with register, field magnets and damping magnets removed. Mercury indicated by short dashes.

0.006 in., and the form of the chamber is such that the mercury cannot be spilled even though the instrument is inverted.

As the current flows diametrically across the disk, the flux must be directed upward on one side of the spindle and downward on the other. The driving field is furnished by either a permanent magnet or an electromagnet, according to circumstances; the poles are immediately beneath the chamber and contiguous to the current lugs. The magnetic circuit is completed by the spirally laminated soft-iron return plate. The necessary "braking" action is due to induced currents in the armature disk and in the usual aluminum damping disk provided for that purpose.

The Sangamo Ampere-hour Meter.—Aside from the ordinary lighting and power installations, there are certain operations, such as electroplating and the charging and discharging of storage batteries, where it is desirable to register the total quantity of electricity rather than the energy. For this purpose, the Sangamo ampere-hour meter has been developed. In this instrument, the driving field is produced by a large permanent magnet.

In electric automobile and truck work, an ampere-hour meter will give an indication of the state of charge of the battery.

The Mercury Watt-hour Meter.—The electrical connections for the Sangamo direct-current watt-hour meter are shown in Fig. 320. The main-line current passes across the copper armature disk in the direction E_1E_2. The U-shaped electromagnet Y, which furnishes the necessary field, is connected across the line.

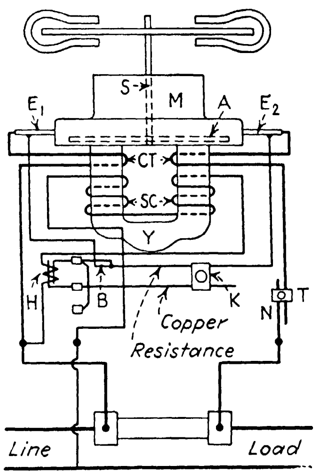

FIG. 320.—Mercury watt-hour meter with shunt. (*Sangamo Company.*)

The light-load adjustment is obtained by the use of a thermocouple H which is inserted in a shunt circuit between E_1 and E_2 and heated by a resistance coil which forms a part of the potential circuit of the meter. The couple sends a small current through the disk in the same direction as the load current. The effect of the thermocouple is controlled by altering the position of the connecting link K. The couples are now made reversible, so that the same meter may be used in either the positive or the negative side of the line.

As the fluid friction naturally becomes unduly large with increase of armature speed, the instrument is compounded by taking the main circuit around the U-magnet at CT. This improves the action of the instrument at heavy load and at overload.

For high capacities, meters of 10 amp. are used with shunts provided with heavy connecting cables. For obtaining the final adjustment of the multiplying power of the shunt, a high-resistance wire N with a sliding terminal T is included in the lead from E_2 to the shunt. This adjustment is made at the factory.

The full-load drop through the armature of a 10-amp. meter without a shunt is about 30 millivolts; in the 20- to 80-amp. meters with internal shunts it is about 60 millivolts; for the external shunts the drop is about 75 millivolts. This higher drop is necessitated by the resistance of the connecting cables. The loss in the potential circuit of a 110-volt meter is about 4.5 watts; with 220- and 550-volt meters it is about 9 and 22 watts, respectively. In the last two, the larger part of the loss is in the added wire resistance. The full-load torque is about 6 cm.-g.

In this meter, the drag magnets are fixed in position, and a 25 per cent variation in the braking action may be obtained by the use on them of a magnetic shunt. The shunt is a disk of soft iron mounted on a fine-pitched screw. The drag magnets are shielded by the cast-iron frame of the instrument.

The ability of the Sangamo meter to withstand severe mechanical shocks and jars and its freedom from the influence of stray fields, which if they do cut the armature are directed either upward or downward on both sides of the spindle, render it applicable to car tests in street-railway work. For this purpose a special form of register has been developed, with a resetting device for registering the consumption of energy during a single trip. The register has also the ordinary totalizing dials.

METER TESTING[3]

To maintain the accuracy of the meters in any distribution system, it is necessary that they be tested periodically. On account of the risk of altering the constant of any form of motor meter during transportation, tests should be made on the meters as installed.

In many states, laws have been enacted which permit a customer, in case he is dissatisfied with his bill, to request the services of the appropriate public service commission in order that a test of his meter may be made by a disinterested party.

Referring to the fundamental formula for the watt-hour meter (page 484), for meters as actually constructed, the watt-hours registered $= K_h \times$ number of revolutions of disk. K_h is the watt-hour constant of the meter; its value is usually marked on the meter disk. In some types of meters, the constant is expressed in watt-seconds for each revolution of the disk. In any case, the meter constant is a fixed ratio depending on the arrangement and ratio of worm, wormwheel, gear train, and dial units of the watt-hour meter. In any meter tests which are made by timing the disk as it rotates, one must be certain that the register used on the meter has the proper constant.

In case of a dispute between the customer and a supply company, the meter must be tested as found, that is, before any adjustments are attempted. The records of these tests are necessary in order that the customer and the company may arrive at a satisfactory understanding.

To test a watt-hour meter, it is necessary merely to determine the rate of revolution of the disk, then to multiply this value by the test constant K_t of the meter, and to compare the result with the number of watts indicated by standard instruments which are so connected in the circuit as to measure the same amount of power as the watt-hour meter under test. The energy is supposed to be supplied at a constant rate.

$$\text{Watts by watt-hour meter} = P' = \frac{K_t N 3{,}600}{t}.$$

For direct-current meters:

$$\text{Correct watts} = P = VI.$$

N equals number of revolutions of the disk; t, time in seconds for N revolutions; V, corrected average voltage measured at the potential terminals of the meter; I, corrected average current flowing through the series coils of the watt-hour meter.

If the voltage and current fluctuate badly, VI should be replaced by the *average watts* during the test.

If a three-wire meter, with the potential circuit connected between one side of the main circuit and the neutral conductor is calibrated with both current coils connected in series, the value of K_t to be used in the preceding formula for P' is one-half that marked on the disk.

The different manufacturers of meters have used various modifications of the fundamental formula, and one should be sure that the test constant given by the maker is used in the proper manner.

In modern meters, the test constant is identical with the watt-hour constant K_h, the number of watt-hours per revolution.

The watt-second constant K_s is the number of watt-seconds of energy necessary to cause one revolution of the movable element. It is equal to the watt-hour constant multiplied by 3,600, the number of seconds in an hour.

The register constant K_r is the factor by which the reading of the register must be multiplied in order to ascertain the total amount of electrical energy that has been supplied to the load via the meter. For meters of small size, such as are used in the majority of cases, the modern practice is to make this factor unity, for it is likely that the small consumer will fail to understand why the supply company, in making out his bill, should multiply his meter reading by a factor of 2 or 4, for example. In meters of large size, it is necessary to use register constants of 10, 100, and so on, for otherwise the value in kilowatt-hours of one dial division becomes too large.

The register ratio R_r is the number of revolutions of the wheel, meshing with the worm or pinion on the shaft of the movable element, that is necessary to cause the first or most rapidly moving dial hand to make one revolution.

The gear ratio R_g is the number of revolutions of the movable element required to cause the first dial hand to make one revolution.

Common Sources of Inaccuracy.—If the meter is very slow or cannot be brought up to speed, the trouble may be due to:

1. Commutator and brushes pitted, oily, and dirty.
2. Commutator segments short-circuited.
3. Lint or magnetic particles between drag magnet and disk.
4. Disk may not run true or may be out of position.
5. Pivot worn.
6. Jewel rough or cracked.
7. Dirt in jewel.
8. Undue friction in the worm and the registering train.
9. Upper guide bearing pressed down on shoulder of spindle.

The meter may overregister owing to weakening of the magnets, through aging, or a short circuit on the customer's premises.

Methods of Testing.—There are several methods of making tests to determine whether the meter is registering correctly. They differ in the arrangement employed for ascertaining the true amount of power or energy delivered to the load via the meter. The arrangements commonly used for this purpose are:

1. Indicating instruments and stop watch.
2. A portable standard.
3. The stroboscopic method.

When indicating instruments are employed, the time in seconds required for a whole number of revolutions of the moving element of the watt-hour meter is determined by means of a stop watch. In direct-current work, calibrated ammeters and voltmeters of the moving-coil type are used to determine the true watts. In alternating-current work, a calibrated indicating wattmeter is used. If small meters are tested, one must be sure that the results are not complicated by the loss occurring in the voltmeter or in the potential coil of the indicating wattmeter. The watts given by the meter are calculated by the appropriate test formula and compared with the results given by the indicating instruments. "The percentage of accuracy" is given by $\frac{\text{meter watts}}{\text{true watts}} \times 100$.

The "rate" of the meter is given by $\frac{\text{meter watts}}{\text{true watts}}$.

In carrying out the test, the meter should be timed for as much as 60 sec. if accurate results are desired. This tends to reduce the errors due to the personal equation in timing and counting as well as the errors due to the stop watch.

Great care must be exercised in the purchase and in the maintenance of the stop watches, for they are the weakest element in this method of testing. A watch may keep good time but be inaccurate as a stop watch. It is important that the indicating hand start and stop promptly without jumping and reset to exactly zero. The starting and stopping errors are of great importance. In order that one may be sure that the watch is in good condition, it should be tested at several points before beginning work.

The index hand of a stop watch moves forward by a succession of jumps separated by intervals during which the hand is at rest, so that though the watch beats $\frac{1}{5}$ sec., the hand is in motion only about $\frac{1}{100}$ sec. at each beat, that is, while the escapement is in action. There is thus a possibility of an error of nearly $\frac{1}{5}$ sec. due to the peculiar mechanism of the watch. In timing for 30 sec., this might give rise to an error of about two-thirds of 1 per cent, in addition to all the other errors due to the imperfect mechanical action of the mechanism and the personal equation of the observer.

Portable Watt-hour Meter Standards.—Routine tests are now made by means of portable test meters instead of by indicating instruments and a stop watch. The standard is a portable watt-hour meter with a special register, readable to 0.01 revolution, which allows the number of revolutions of the movable element to be read with precision. This register must be so arranged that it may be started and stopped promptly by the use of a push button.

In case such an instrument is used, after having connected its current coils in series and its potential coils in parallel with those of the meter under test, one has merely to compare the number of revolutions made by the standard during a certain time with the number made by the meter under test during an equal time. Naturally, the meter constants must be taken into consideration. For example, denote by x the meter under test, and by s the standard meter. The average powers given by the two meters are P_x and P_s; then

$$P_x = \frac{(K_t)_x N_x 3{,}600}{t}. \qquad P_s = \frac{(K_t)_s N_s 3{,}600}{t}.$$

The "percentage of accuracy" is $\frac{P_x}{P_s} 100 = \frac{(K_t)_x N_x 100}{(K_t)_s N_s}$.

It would be an obvious convenience if the meters had equal watt-hour constants.

The advantages of this method are the elimination of the use of the stop watch, independence of load and voltage fluctuations, and the reduction of the working force, for only one man is necessary. Independence of load and voltage variations is a most decided advantage, for at times, especially if high-capacity meters are being tested, it is necessary to use the consumer's load, and this may be fluctuating.

Portable watt-hour meter standards are now made for both alternating and direct currents.

The alternating-current standard is started and stopped by making and breaking the potential circuit. In the direct-current instrument, the potential circuit is kept closed so that the armature and disk rotate continuously; the register is thrown into and out of gear by an electrically operated clutch.

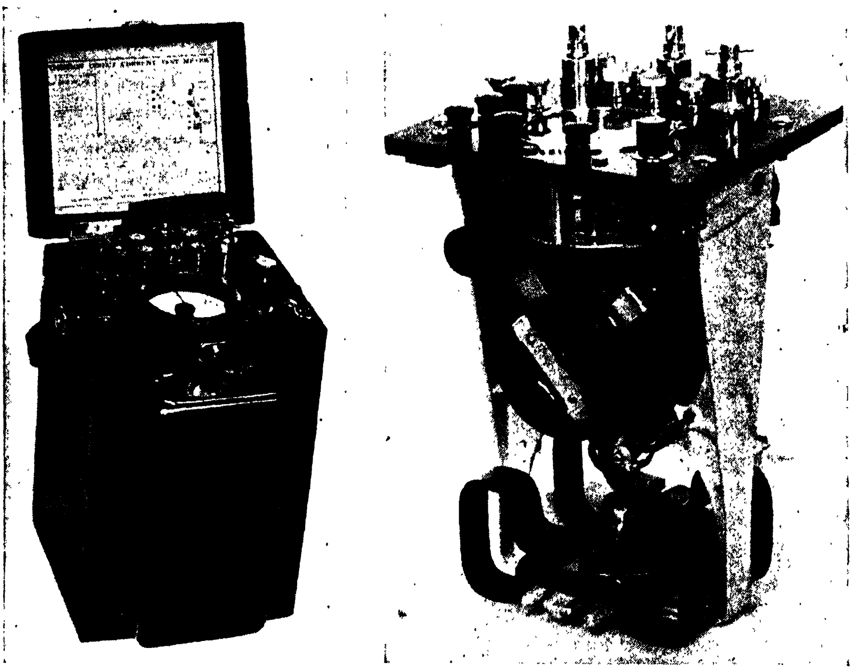

FIG. 321.—Rotating standard watt-hour meter for direct currents. (*General Electric Company.*)

It is essential that the construction of portable rotary standards be such that their accuracy will not be affected by the necessary handling during transportation. This means that the geometry of the coil system and also of the brake must not alter and that the friction must remain constant. To insure this last, it is necessary to provide means for raising and clamping the movable system so that the pivots and jewels may not be injured.

To reduce the irregularities due to unavoidable friction, the commutators used in direct-current standards should be of small diameter, and the brush pressure constant. A high ratio of torque to weight of the moving element is most desirable.

When the instrument is connected into the circuit, care should be taken that the current and the voltage coils are at practically the same potential, especially if the voltage is so high that a multiplier is used.

For the greatest utility, the current range of the rotary standard should be large, so that it may be used for testing meters of a number of different capacities. At the same time, the ampere-turns due to the fixed coils must be large even when small meters are tested. This insures that the standard will not be operated on what is the equivalent of a light load. Therefore, the current coils must be wound in sections so arranged that they can be connected conveniently in various series-

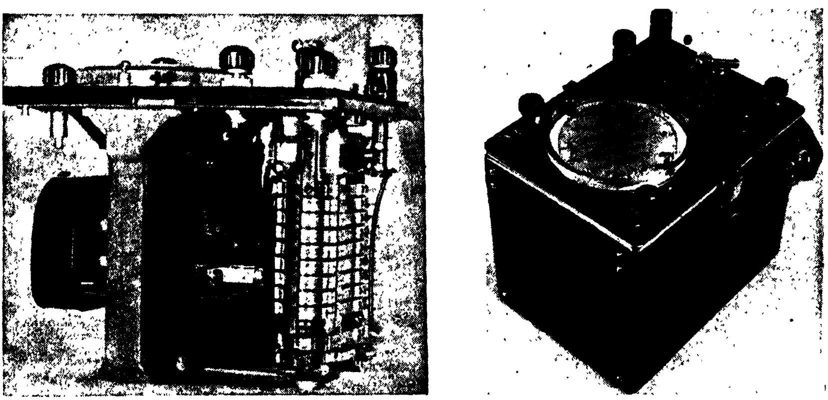

FIG. 322.—Rotating standard watt-hour meter for alternating currents. (*Westinghouse Electric and Manufacturing Company.*)

parallel combinations by a reliable means. The full-load ampere-turns of all the sections should be the same.

Large electrical companies now use rotary standards, which are accurately maintained by their laboratory departments, as secondary standards when checking and adjusting service meters before they are sent out for installation on the consumer's premises.

While the rotary standard is very convenient and in some cases necessary, one must not forget that care must be taken if accurate results are to be obtained.

The direct-current meter is heavier and less convenient than the alternating-current instrument. When it is calibrated, and when it is used, it is necessary to keep the potential-coil circuit of a direct-current rotary standard energized for a considerable time (about 30 min.) before any readings are taken—long enough for the entire armature circuit, the disk, and the drag magnets to attain their steady state of temperature, since the resistance of the armature circuit, the resistance of the disk to eddy currents, and the strength of the drag magnets are all dependent upon temperature. The heat liberated in the current coils also influences the accuracy of the meter, for it, too, affects the temperature of the potential coil, the disk, and the drag magnets. This self-heating error may be of importance in careful tests if the meter is so used that it must carry a large current for a long time. The alternating-current standard is subject to the errors found in meters of the induction type. Progress in design has reduced them to very small values (see pages 501, 524). However in case of a serious dispute between the consumer and the supply company, these errors should be considered.

The use of rotary standards takes the determination of the time element from the tester, who must of necessity use a stop watch, and hands it over to the laboratory department, where much more accurate timing devices may be maintained.

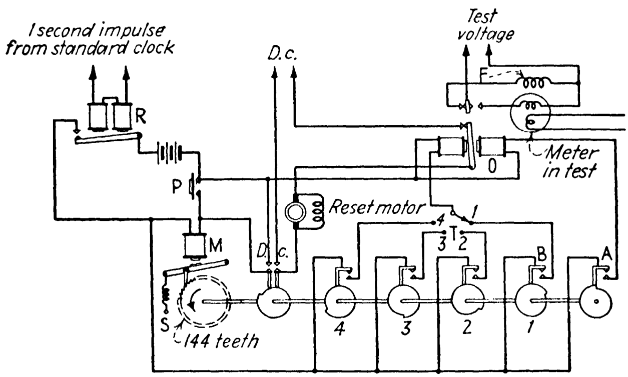

FIG. 323.—Diagram for meter timing device. (*Brooklyn Edison Company.*)

A timing device designed for use in calibrating rotary standards is shown diagrammatically in Fig. 323. With the switches in the positions shown, immediately *P* is closed the impulses from the standard clock begin to advance the timing shaft. The first effect is to close the contact *A* and start the meter under test. After a predetermined time, the contact *B* is closed; this stops the meter and starts the reset mechanism, which through gearing not shown advances the timing shaft to its original position.

Stroboscopic Method.[3]—The principle underlying all stroboscopic devices for studying recurrent motion, such, for instance, as that of the valve gear of an explosion engine or, in this case, that of a meter disk provided with the proper equally spaced markings, is that if the parts in question are illuminated by a series of short, sharp flashes of light which always occur at a definite point in the motional cycle, the markings appear to stand still. If the flashes occur at equally spaced and progressively different points in successive cycles, the object is caught at correspondingly different points and appears to have a slow motion forward or backward, as the case may be. The Westinghouse Company has developed stroboscopic devices for testing both switchboard and house meters. It was necessary to develop a portable test meter, the speed of which could be varied between 5 per cent fast and 5 per cent slow in accurately determinable amounts. The adjustment is made by varying the position of the drag magnets by a finely pitched screw. The percentage above or below normal speed is read directly from a scale. Figure 324 shows the electrical connections for the stroboscopic portion of the portable test meter. The periphery of the disk of this meter is notched, as shown in the figure, and the disk of the tested meter carries a series of markings. Light from an incandescent lamp within

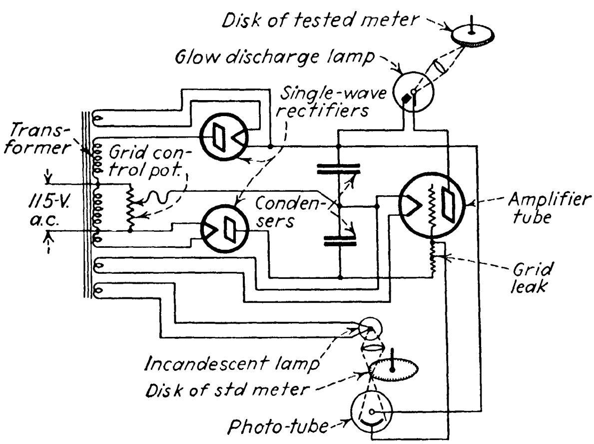

FIG. 324.—Schematic diagram of wiring for portable stroboscopic watt-hour meter tester.

the test meter passes through the notches in the disk and falls on a photoelectric tube which controls the flashes via the amplifier circuit. If the standard is set for zero error, and the disk of the test meter appears to stand still, the adjustment is correct. If it appears to be in slow motion, the registration adjustment screw is turned until the motion ceases. The error is then read directly from the scale. The use of the stroboscopic principle greatly expedites routine testing. The standard may be used in the ordinary manner, if so desired.

Fictitious Loads and Arrangements for Phase Shifting.—In the laboratory, it is often convenient, and sometimes necessary, especially when meters of high capacity are tested, to avoid the consumption of energy that would result from loading the meter in the ordinary way. Also, in service tests after the meter has been installed, it is often necessary to test at definite loads and under constant conditions. This is frequently impossible if the customer's load is relied upon. It is not feasible to use large rheostat loading boxes on account of their expense and inconvenience. In such cases, the potential and current circuits may be separately excited from two sources: the potential circuit from the line as usual, and the current circuit from a low-voltage source.

For direct-current work, up to 500 amp., two Edison storage cells and a compact carbon rheostat, as indicated in Fig. 325, are very convenient.

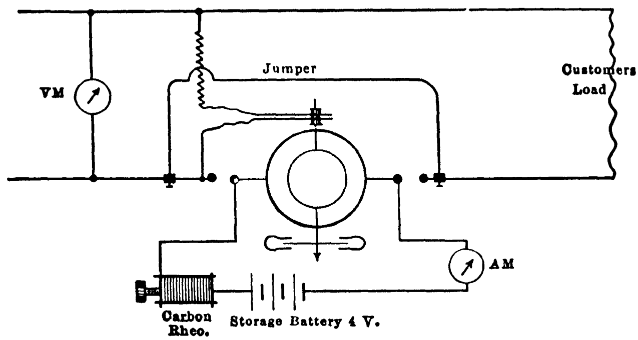

FIG. 325.—Connections for testing a watt-hour meter by use of a fictitious load.

By turning back the handle of the rheostat, the circuit is broken when the readings are not being taken. The weight of the cells for testing 500-amp. meters is about 180 lb.

It will be noticed that the customer's load is carried by the jumper, which is put on before the meter is taken out of service, thus avoiding any interruption of the circuit. *In this and other cases where jumpers are used, it is essential that they be so applied that the normal field at the armature of the meter is not disturbed.*

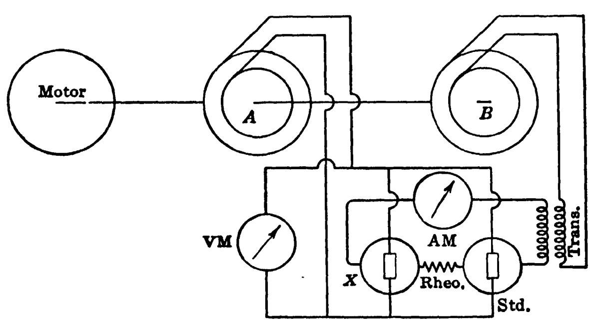

FIG. 326.—Diagram for phase-shifting motor-generator set.

For tests of alternating-current meters after installation, it is possible to obtain large fictitious loads by the use of a special stepdown transformer connected across the mains. This implies that the controlling devices may be made simple and compact, and the whole apparatus portable. Such devices are on the market and are sold under the name of *phantom-*

load boxes. It is to be remembered that the percentage accuracy of an alternating-current meter depends on the power factor of its load, so it is necessary to be sure that the transformer arrangement does not introduce complications due to phase displacements.

Phase-shifting Devices.—In testing and adjusting alternating-current meters in the laboratory, one must be able to vary the effective power

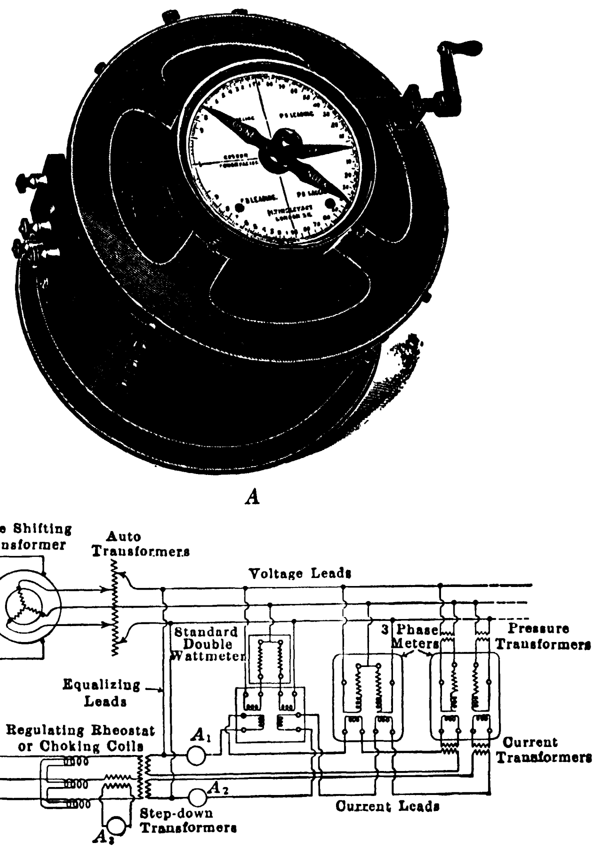

FIG. 327.—Drysdale phase-shifting transformer.

factor of the load, preferably without any attendant alteration in current, voltage, or wave form. An arrangement for this purpose is shown diagrammatically in Fig. 326.

It consists of two motor-driven machines, the armatures of which are rigidly coupled; one field is stationary, while the other is so mounted that it can be displaced about the axis of the shaft by a worm wheel and sector, and its angular position read on a graduated arc. The displacement may be effected by a remote-control arrangement. Machine *A* energizes the potential coils of the meters, while machine *B* supplies the current coils; *B* is either of low voltage and large current capacity or else works through a stepdown transformer. Both machines should give sinusoidal waves, for the operation of induction meters, especially at low power factors, is influenced by wave form. To obtain good wave forms, specially designed three-phase machines with Y-connected armatures are necessary.

A much simpler and less expensive device for accomplishing the same purpose is the Drysdale phase-shifting transformer, the principle of which is explained on page 298.

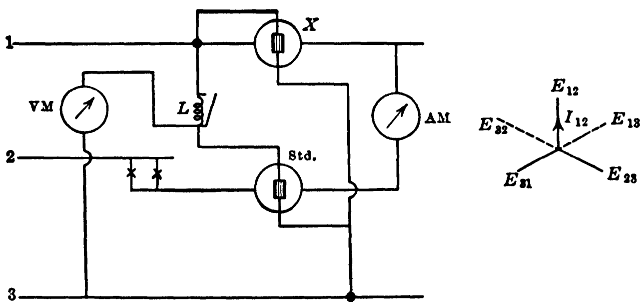

FIG. 328.—Arrangement for obtaining power factor 0.5 from a three-phase circuit.

This transformer as designed for meter tests, together with the connections necessary in testing three-phase meters, is shown in Fig. 327.

The phase-shifting transformer should be used on circuits that have sinusoidal voltage waves; otherwise the wave form in the secondary will change with the adjustment of the phase displacement.

In order to lag an induction meter, it is necessary to operate it at two power factors, and usually 1 and 0.5 are chosen. The double motor-generator set or the phase-shifting transformer previously described may be used, but these two particular power factors may be obtained from a three-phase circuit. Figure 328 shows the connections.

As shown, the current is in phase with E_{12}, the voltage at the meters in phase with E_{13}, and the power factor is 0.5 leading. To obtain 0.5 power factor with lagging current the voltage coils would be connected between leads 3 and 2.

To determine whether one is dealing with a lagging or a leading current, a small inductance *L*, of low resistance, may be included in the potential circuit of the standard dynamometer wattmeter; *normally this inductance is short-circuited.* If the current is lagging, the insertion of the inductance will slightly increase the apparent power factor and will decrease it when the current is leading.

A power factor of zero may be obtained from a balanced three-phase circuit, as shown in Fig. 329.

The currents I_{12} and I_{13} must be equal, and the resistances nonreactive.

A power factor of zero may be obtained also from a two-phase circuit, the voltage being taken from one phase, and the current, through a

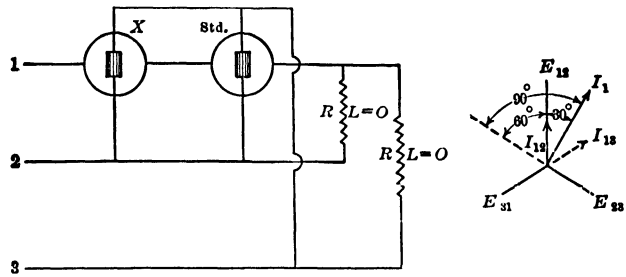

FIG. 329.—Arrangement for obtaining zero power factor from a three-phase circuit.

nonreactive resistance, from the other. It must be assured at the beginning that the two phases are really in time quadrature and that the inductances of the current coils do not cause an appreciable phase displacement. Where a two-phase current is obtained from a three-phase circuit by Scott transformers, unless the wave forms of the primary supply are sinusoidal, the wave forms in the secondaries may be badly distorted, one being flat-topped; the other, peaked.

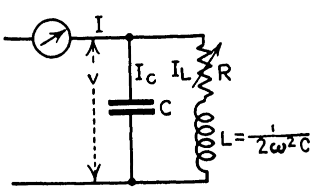

FIG. 330.—Turner constant-impedance circuit.

With any of these phase-shifting devices it is important that the voltage and current waves be sinusoidal, for a 60-deg. displacement of the fundamental in the current wave with respect to the fundamental in the voltage wave implies a 180-deg. displacement of the third harmonics, a 300-deg. displacement of the fifth harmonics, and so on. A statement that the load has a power factor of 0.5 gives little idea of the conditions under which the watt-hour meter is operating. The changed phase relation greatly complicates the behavior of induction meters.

Figure 330 shows a parallel circuit so proportioned[3] that, in the ideal case, any phase displacement with respect to the applied voltage of from 90-deg. leading to 90-deg. lagging may be obtained by simply varying R, the line current remaining constant. The line current is

$$I = I_C + I_L = V\left[\frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right)\right].$$

The ampere value is

$$|I| = V\sqrt{\omega^2C^2 + \frac{1 - 2L\omega^2C}{R^2 + \omega^2L^2}}.$$

If the circuit is adjusted so that $2L\omega^2C = 1$,

$$|I| = V\omega C, \qquad \text{a constant, irrespective of } R.$$

The phase displacement is $\tan^{-1} \dfrac{R^2\omega^2L^2}{2R\omega L}$; the current lags by the angle $2 \tan^{-1} \dfrac{\omega L}{R} - 90°$.

For the ideal case of $R = 0$, the current lags 90 deg. If the inductive circuit is opened, $R = \infty$; the current leads by 90 deg.

In the ideal case, for a current of 5 amp. at 115 volts and 60 cycles per second, $C = 115$ μf, and $L = 0.0306$ henry. The inductor should have an air core, be of low resistance, and must obviously have a large carrying capacity.

Testing Polyphase Induction Meters.—When a single-phase induction watt-hour meter is used on a noninductive load, the error due to incorrect lagging is negligible. If a polyphase induction watt-hour meter is used on a three-phase load of power-factor unity, the error due to incorrect lagging may be appreciable, for in this case, although the power factor of the load is unity, one of the elements of the meter operates at a power factor 0.866 leading, while the other operates at a power factor 0.866 lagging (see page 339).

For other three-phase power factors, the conditions under which the elements operate are shown by Fig. 331.

Suppose that the upper element is underlagged, while the lower is overlagged. Then, when the upper element operates at a lagging power factor, and the lower element at a leading power factor, both elements tend to make the meter register too low. If the elements are interchanged, both tend to make the meter register too high.

Polyphase induction watt-hour meters, operated through instrument transformers, are often used in determinations of the water rates of three-phase turbogenerators, water rheostat loads being employed. The three-phase power factor is then unity. In calibrating the meter, with the transformers, a three-phase noninductive load should be used, and the connections so made that the element that operated with the lagging power factor during the test is traversed by a lagging current during calibration.

Checking Polyphase Watt-hour Meter Connections.[4] **Woodson Method.**—Three-phase watt-hour meters are actuated through two potential and two current transformers which must be connected to the line in the proper relative position and with the proper polarities. The secondary of each current transformer and of each potential transformer must be connected to the correct watt-hour meter element through leads and *with the correct polarity.* The chances of error are manifold, especially as the leads come to the switchboard meter through conduit and cannot be traced.

That the meter rotates in the proper direction is no indication that the connections are correctly made. A number of methods of checking have been proposed. As each has its own peculiar limitations, an

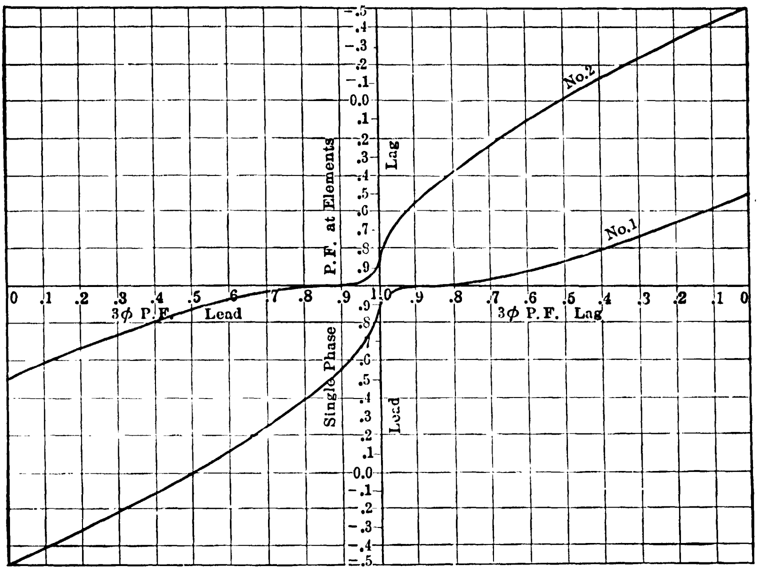

FIG. 331.—Showing the power factors at which the two elements of a polyphase watt-hour meter operate when the balanced three-phase load has different power factors.

alternative method should be available in doubtful cases. Only the Woodson test will be discussed here. The general requirements are:

1. No connections from the transformers to the meter are to be disturbed.

2. The method should permit testing without interrupting the service. One must know:

1. The direction of vector rotation.
2. Whether the *load power factor* is *lagging* or *leading.*
3. Whether the load power factor is near unity or is very small.
4. That the load is not badly out of balance.

The direction of vector rotation is obtained by a phase-sequence indicator; the other facts are established from a knowledge of the load conditions. Figure 332 shows a three-phase watt-hour meter connected to a load through current and potential transformers. Three indicating wattmeters are connected to measure the power by the three-wattmeter method (page 341).

With the three-wattmeter method, questions as to the algebraic sign of the deflections can arise only in cases of extreme unbalance. If

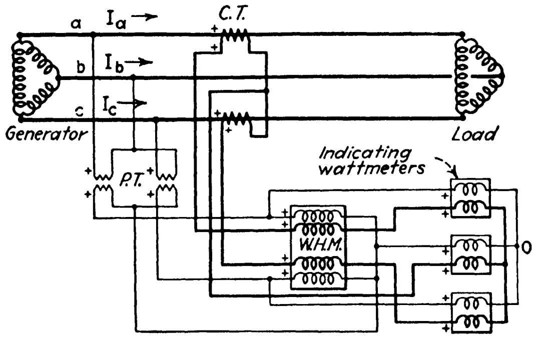

FIG. 332.—Connections for checking three-phase watt-hour meter.

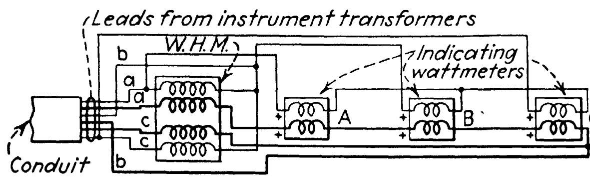

FIG. 333A.—Woodson method of determining exact location of a current vector through three simultaneous wattmeter readings.

on making the test the readings are not all positive with the exception noted, the watt-hour meter is not properly connected. Generally, a negative reading shows that an instrument transformer is reversed on either the primary or the secondary side or that the leads are interchanged at the meter. However, the test is not conclusive, for there are several combinations of reversed connections which will give positive readings and cause the meter to register forward. The necessary additional facts are furnished by locating the three line-current vectors with respect to the three corresponding star or Y voltages. Consider line a; it is desired to locate I_a with respect to V_{ao}. Figure 333A shows the necessary connections. The current coils of three wattmeters A, B, C are connected in *series* in line a; the three potential circuits, of equal impedance, are connected in Y as indicated.

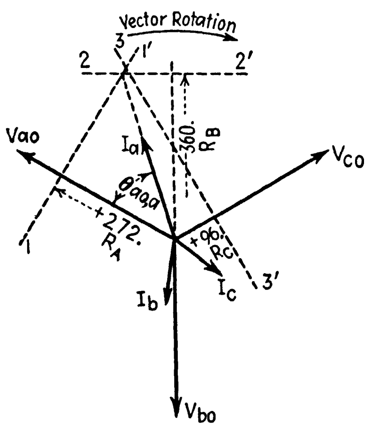

FIG. 333B.—Vector diagram for the method showing that in this case the connections are incorrect, for the load consisted of induction motors.

The voltages applied to the meters are V_{ao}, V_{bo}, V_{co} as indicated. If $\theta_{ao,a}$, $\theta_{bo,a}$, and $\theta_{co,a}$ are the phase displacements of I_a with respect to V_{ao}, V_{bo}, and V_{co}, respectively, the three readings are

$$R_A = V_{ao}I_a \cos \theta_{ao,a}.$$
$$R_B = V_{bo}I_a \cos \theta_{bo,a}.$$
$$R_C = V_{co}I_a \cos \theta_{co,a}.$$

These equations are most conveniently solved graphically. The readings are plotted from O (Fig. 333*B*), regard being had to algebraic signs. R_A is along V_{ao}, R_B is along V_{bo}, R_C is along V_{co}. The perpendiculars 11′, 22′, 33′ are drawn as indicated. As an example, let the readings be as follows:

Load induction motors		Phase sequence V_a, V_b, V_c	
	Current coils in *a*	Current coils in *b*	Current coils in *c*
Amperes	5.7	3.4	3.2
Reading of *A*	+272	− 84	−192
Reading of *B*	−360	+208	+116
Reading of *C*	+ 96	−128	+ 68

The volt-ampere line of length $V_{ao}I_a$ must terminate somewhere on 11′. With the given data there are two possible values of θ: one lagging, the other leading. The vector of length $V_{bo}I_a$ must terminate somewhere on the line 22′. Therefore I_a lies on a line joining O and the intersection of 11′ and 22′ or on a line joining the intersection of 11′ and 33′. This fixes the phase relation of I_a and V_{ao}. The current coils are then transferred successively to positions where they are traversed by the currents I_b and I_c, and the phase relations of I_b and I_c to V_{bo} and V_{co} are established, with the results shown in Fig. 333*B*.

The graphical solution shows that I_a leads V_{ao}. In this particular case, it was known that the load consisted of induction motors which take lagging and balanced currents. The meter is then shown to be incorrectly connected, although it registers forward.

In general, if it is definitely known that the power factor is either lagging or leading and that the power-factor angle is less than 60 deg., the test shows positively whether the connections are correct or not. If the load power factor is not known, but if something is known of the load balance, the method will show whether the connections are correct or not except in two cases: (1) when the potential or current (not both) is shifted at the meter clockwise or counterclockwise, 120 electrical deg.; (2) when the potential or current (not both) is shifted clockwise or counterclockwise 120 electrical deg. with the additional error of reversing the polarity of both potential transformers or both current transformers.

More involved cases arise when the phases are unsymmetrically loaded. However, similar comparisons of the results from the diagram with known facts concerning the load enable the check to be made. In the original report, the results of incorrect connections are analyzed, and various proposed checks are critically examined. The paper of Kouwenhoven[(4)] and the Weston Instrument Company "Data on Model 551 Watthour Meter Test Set" should be consulted. For the execution of the Woodson test the Weston Instrument Company has devised a convenient assembly (shown in Fig. 334) containing the

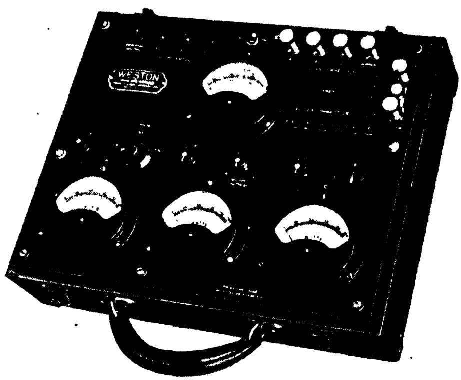

FIG. 334.—Weston assembly for checking polyphase watt-hour-meter connections.

necessary switches and instruments, a phase-sequence indicator, and an extra resistance. This resistance may be inserted in the current transformer secondary, thus raising the voltage sufficiently to reveal the existence of intermittent short-circuited turns by a sudden diminution of the wattmeter reading.

Testing of Large Direct-current Watt-hour Meters on Fluctuating Loads.[3]—On account of the great revenue per meter that may be involved, it is very important for both the supply company and the consumer that the meters that measure large amounts of power be kept in an accurate condition. The necessary tests must be made with the meters in place; and if they are used on a rapidly fluctuating load, such as a street-railway system, difficulties are experienced in making the test and the necessary adjustments.

Owing to the large number of readings of current and voltage which it is necessary to take in order to obtain a good average, the ordinary method of using a stop watch and of measuring the line voltage and current is a time-consuming operation, and in some cases the fluctuations are so rapid that the use of the ammeter is quite out of the question.

An alternative procedure is to take the meter out of service and to send through its coils the current from a storage battery (see page 520). This current may be controlled by resistors, so that tests at light load and up to about 500 amp. may be made without the apparatus being too unwieldy to be managed by two persons. For the test, two Edison cells are convenient, being readily portable. It is, however, desirable to avoid taking the meter out of service, for the test may occupy an hour or more, and the loss of revenue may be as much as $5 to $10 for each hour that the meter is out of service. Also, it is desirable to make the test using the customer's regular load.

The very convenient portable standard watt-hour meters developed for alternating-current work naturally suggested similar devices for use

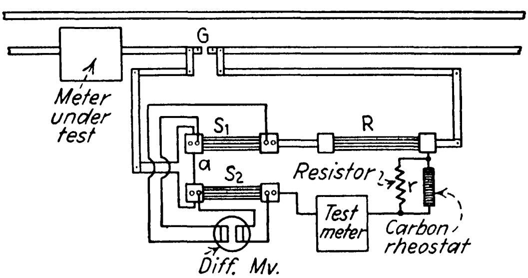

FIG. 335.—Arrangement for testing large direct-current watt-hour meters by use of two shunts and differential millivoltmeter.

on direct-current circuits. Their development, however, has been attended with difficulty. Nevertheless, the problem has been solved quite successfully, and the best of these instruments, when carefully used, are of great service where load conditions are extremely variable. Such test meters are now made in capacities up to 150 amp.

In railway work, it is frequently necessary to test meters of several thousand amperes capacity. The direct application of shunts to a portable standard watt-hour meter, of the commutating type, is not permissible on account of the change of the multiplying power of the shunt through heating and the uncertainty due to bad contacts.

As it is desirable to retain this type of meter as a standard, methods have been devised whereby shunts are applied to the portable standard watt-hour meter in such a way that errors due to heating and to contact resistances are eliminated. One method is shown in Fig. 335, where for clearness the potential circuits are omitted.

The station bus bar is arranged so that it has a narrow gap at *G*. This gap is ordinarily closed by plates firmly bolted in position. It should be narrow, and the leads so arranged that the field at the meter is not altered when the gap is opened. The test circuit is clamped to the bus bars, and the gap opened without interrupting the service. The entire current then flows to a, where it divides, a comparatively small portion flowing through S_2, and the large portion through S_1 and R, which must be designed to take the maximum current involved. The potential drops in S_1 and S_2 are made equal by the adjustable resistor r, and this equality is indicated by a differential millivoltmeter of the D'Arsonval pattern. Any shunts that are suited to the purpose may be temporarily bolted together and used for S_1 and S_2. They should be free from thermal errors, as these are troublesome in some cases.

The standard meter is set up where it will be as free as possible from stray fields. The leads to it are flexible, and readings are taken with the meter in four different azimuths 90 deg. apart. This is usually sufficient; but conditions may arise where, owing to the change in the distribution of current between feeders that are at different distances from the test meter, this procedure would not eliminate stray-field errors. In such cases, an astatic meter is desirable.

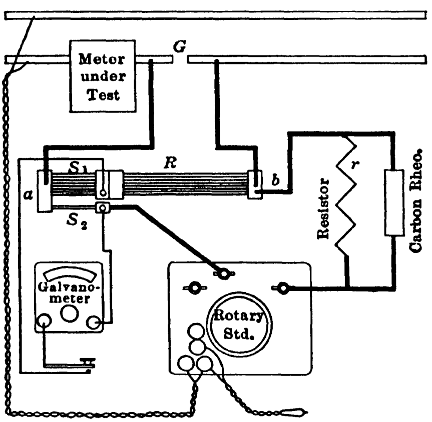

FIG. 336.—Connections for testing large direct-current watt-hour meters by the bridge method.

The differential millivoltmeter may be replaced by a pivoted D'Arsonval galvanometer if a special double shunt is constructed for the purpose. The connections are shown in Fig. 336. Inspection of the figure will show that the arrangement has become a Wheatstone bridge, the low-resistance sides being composed of fixed resistors. One of the high-resistance sides is a resistor of fixed value; the other is made up of the rotary standard and the necessary adjustable resistor for maintaining the bridge in balance when contact and coil resistances change. The sections of the shunt S_1 and S_2 have a common terminal at a. If the galvanometer stands at zero, $I_1/I_2 = S_2/S_1$, and the corrected reading of the test meter is its actual indication multiplied by $(I_1 + I_2)/I_2$. By the use of two potentiometers to measure $I_1 + I_2$ and I_2 when the galvanometer is balanced, the multiplying factor can be very accurately determined in the laboratory.

In a particular case, the capacity of the test meter used is 40 amp., and there are two sets of shunts and auxiliary resistances R mounted on the same base, the ratings being 1,000 and 2,000 amp. The voltage drop in the shunts at full load is 100 millivolts, and in the resistor R it is 400 millivolts. The adjustable resistor r is a strip of Boker metal, the effective length of which can be altered by the use of screw clamps.

To obtain a fine adjustment, a carbon compression rheostat is placed in parallel with the strip.

DEMAND METERS[5]

The business of supplying electrical energy is peculiar because, broadly speaking, the product to be sold cannot be stored. It must be used as generated, and the supply company must stand ready to furnish its product to customers at any hour.

The demand of the individual consumer for the company's product passes through a fairly well-defined daily and seasonal variation, being naturally the greatest when the days are the shortest. It is necessary to install generating machinery of sufficient capacity to carry safely the greatest aggregate demand, or the peak of the load, as it is called, and provide a sufficient reserve. This means that machinery, representing a considerable investment, must stand idle for a large portion of the time. This peculiarity of the business has led electrical companies to divide into two parts the cost of supplying their customers: "fixed costs," which are independent of the amount of the product delivered to the consumer; and "running costs," which depend directly upon the amount of energy delivered.

On account of the large amount of time during which a portion of the machinery and the distribution system is idle, the fixed costs are large, and efforts have been made to establish systems of rates that are in accordance with Hopkinson's maxim that "the charge for a service rendered should bear some relation to the cost of rendering it."

The investment necessary in order that a company may stand ready to supply any group of consumers is dependent on the maximum demand that the consumers make for the company's product; and in certain systems of charging, maximum-demand indicators are used in conjunction with the watt-hour meters as an aid in apportioning the fixed costs among the consumers.

Demand indicators record the greatest *sustained* amount of current, power, or kilovolt-amperes that the consumer uses. They are not supposed to indicate demands that are of such short duration that no serious burden is placed thereby on the generating machinery or distribution system. The length of time during which the demand must be sustained depends upon the character of the service, 15-, 30-, and 60-min. intervals being commonly employed. The demand, which is expressed as an average, might be determined by the inspection of the chart from a recording meter and the integration of selected portions of the record for the required time interval. However, such a procedure is altogether too laborious, and recourse is had to specially designed demand meters, of which there are two classes.

1. Integrated demand meters, which integrate the desired quantity over a definite time, thus furnishing the data for an arithmetical average. Such meters are said to operate on the block demand system.

2. Lagged demand meters, which are so arranged that the indications respond only gradually to the load, much as a greatly damped galvanometer responds to a steadily applied current. In such meters, the indications are asymptotic to the value that would be obtained if the quantity under measurement were steadily applied for infinite time. The Lincoln thermal storage meter[6] and the Westinghouse "RL" demand meter[7] are of this class. Such meters are said to furnish a *logarithmic average.*

Strictly speaking, to furnish adequate data for use in determining rates, a demand meter should give not only the demand but the hour at which it occurs; for a consumer who takes a large demand at a time when the generating machinery and distribution system would otherwise be quite idle necessitates no additional investment and can be given a better rate than a consumer who makes the same demand at the time of peak load. It is only in the case of large consumers that a supply company is justified in installing an expensive form of demand meter which will show the time at which the maximum demand occurs as well as its magnitude. With small consumers, allowance for the fact that all demands do not occur simultaneously is made by use of the *diversity factor* when the rates are originally determined. This is defined as the ratio of the sum of the maximum power demands of the subdivisions of any system or part of a system to the maximum demand of the whole system or of the part of the system under consideration, measured at the point of supply.

Diversity factors can be determined only by actual observation of the consumers' maximum demands and the corresponding maximum demand on the station. They will be different for different classes of service.

Integrated Demand Meters.—For billing purposes, it is necessary that the supply company know both the kilowatt-hours of energy supplied to the customer and his maximum demand in kilowatts, kilovolt-amperes, or reactive kilovolt-amperes, as the case demands. For small installations, it is usual to employ, on alternating-current circuits, an induction watt-hour meter provided with a special register which gives *both* the kilowatt-hours and the maximum kilowatts demand. To obtain the demand, a train of gears driven by the watt-hour meter turns a shaft provided with a dog which pushes a friction-controlled pointer up the demand scale at a velocity proportional to that of the meter disk, that is, to the power being delivered. At the end of the predetermined demand period, the pusher dog is disengaged and returned to its zero position, leaving the demand pointer held at its maximum reading by the friction. The dog is then reengaged, and the cycle repeated continuously during the entire billing period, at the end of which the demand indicator is read, unsealed, and reset to zero by the meter reader. There is no record of the time when the maximum demand occurred, and, after resetting, no trace of the reading remains, and no opportunity exists for its subsequent verification in case of a dispute. These are characteristics of this form of demand meter.

Many mechanisms have been devised for carrying out the operations as outlined. They differ much in mechanical details, but all give the same final result. All of them contain as an essential element a device for timing the demand period. In modern distribution systems, the

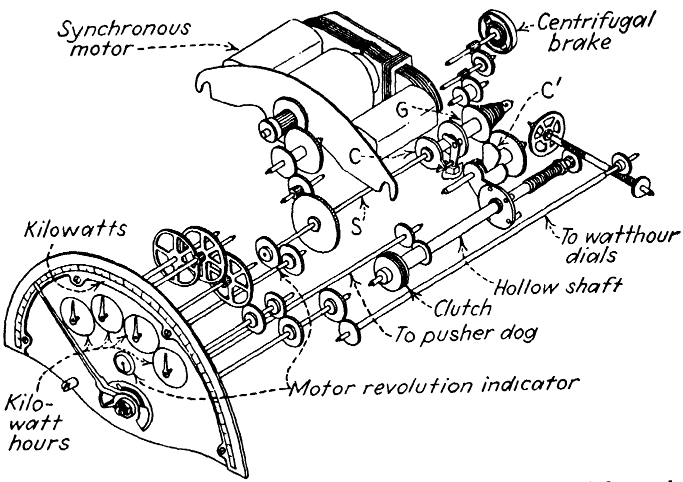

FIG. 337.—Schematic diagram for General Electric Company "M20" demand register.

average frequency is maintained with a high degree of accuracy. Consequently, the most convenient timing device is a form of self-starting, synchronous motor. The device used on direct-current systems is a motor-wound clock.

The General Electric Company's "M20" demand register is a typical device for use on alternating-current systems. Referring to Fig. 337, the revolution indicator shows at a glance whether or not the synchronous timing motor is operating. The register mechanism for kilowatt-hours is operated by the usual worm on the spindle of the induction meter. The worm also turns a shaft *within the hollow shaft,* thus continuously rotating the front element of the clutch. The rear element of the clutch is attached to the hollow shaft and geared to the pusher dog. The timing motor turns the shaft S, which makes one revolution in each integrating period. The rear end of S is attached to one end of the conical-coil spring, the other end being attached to the element G, which is loose on S and consists of a gear and a plate carrying a latch, which normally rests against a stop on the frame of the register and prevents G from turning. The motor thus winds up the spring. The cam C is fast on S and once every revolution raises the pawl on the latch and allows G to make one revolution under the influence of the coiled conical spring, too violent an uncoiling of which is prevented by the centrifugal brake. As G revolves, it carries with it the cams C' which engage with pins on the disk attached to the hollow shaft; the clutch slips, and the pusher dog is returned to its zero position ready again to begin its upward travel.

If it is desired to have a permanent record of the demand on a single circuit and the time when it occurred, a recording type of meter is used, the record being made by a pen on a strip of paper or circular chart. The Westinghouse "RB" recording watt-hour demand meter (Fig. 338) is a typical device of this sort, which is applicable to any three-wire, three-phase circuit. Although the mechanical details are different, the result obtained is much the same as if the pusher dog of a typical demand register were replaced by an arm, about 5 in. long, which carries at its extremity a marking pen. Under the influence of the watt-hour meter, the pen moves across the paper with a velocity proportional to the power being delivered. At the end of each integrating period, the timer disengages the pen, which returns quickly to zero, in this case under the influence of a weight. Simultaneously the synchronous motor moves the paper forward a definite amount and reengages the pen mechanism. The cycle is then repeated again and again. As the hours of the day are printed along the edge of the paper, the time when the maximum demand occurred is readily determined by inspection. An illustrative record is shown in Fig. 338.

Impulse or Contact-operated Demand Meters.—For the larger installations, and where the total of the simultaneous demands on a number of circuits is to be obtained, the demand meter is made a separate unit and actuated by electrical *impulses* (hence the designation *impulse demand meters*) sent out from the watt-hour meter by a contact-making device geared to a worm on the spindle, which drives the watt-hour register. An impulse occurs each time that a definite number of revolutions of the meter disk has been completed, that is, each time that a specified amount or "block" of energy is delivered to the customer. If the kilowatt-hour value of a "block" is multiplied by the number of impulses in a demand interval and divided by the length of the interval in hours, the demand will be obtained; that is, the desired result is given by multiplying the number of impulses by the appropriate constant. It is seen that fundamentally this form of meter is a device for counting the number of impulses in the demand period. Positive action of the sending contacts and the receiving relay is essential. The equivalent of a three-wire channel between the sender and receivers is advisable, for then it can be so arranged that an impulse positively advances

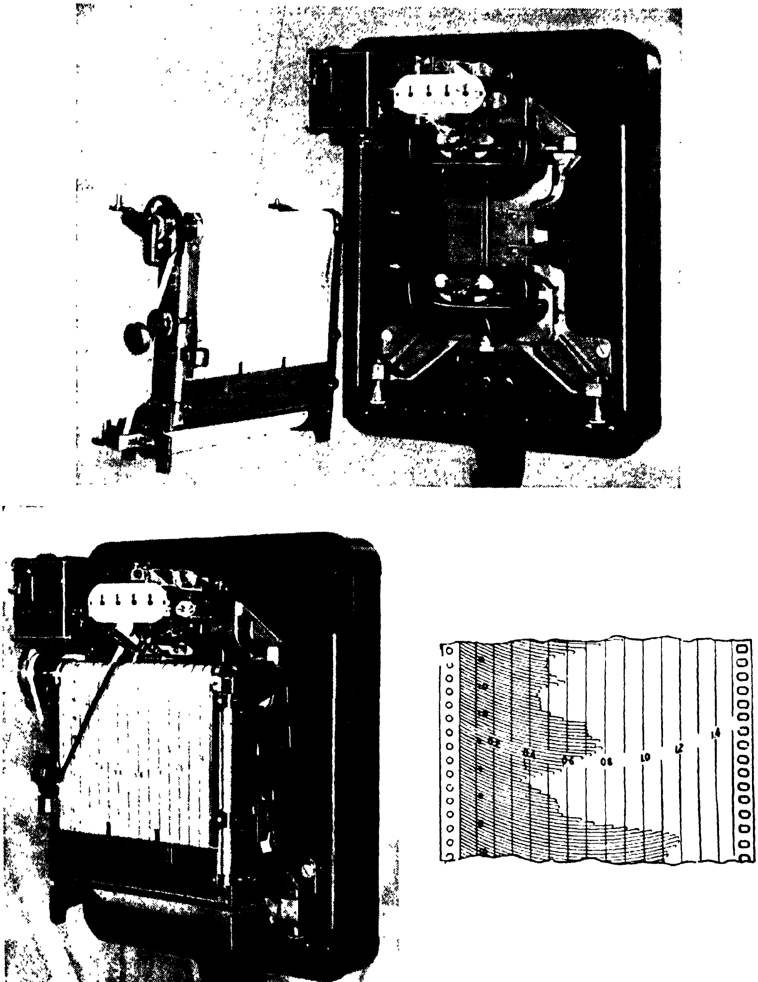

FIG. 338.—Westinghouse "RB" watt-hour recording demand meter, and record.

the demand indicator while the next impulse positively draws back the pawl and makes the device ready for the next forward impulse. The contacts should be required to carry only a small current (measured in milliamperes) and at a low voltage. Figure 339 illustrates one application of this principle.

The impulses from the contact maker in the watt-hour meter are received by the operating coil, the armature of which is connected through the ratchet and gearing to the pusher dog. At the end of the demand period, the timing cam, which is actuated by the synchronous motor, unmeshes the sliding gear and allows the spring to return the dog to its zero position, leaving the demand pointer at the highest point of its travel. The sliding gear is then remeshed. The reading on the scale is proportional to the greatest number of impulses that have been received in any demand interval. The complete return of the pusher dog to its proper initial position is assured by the armature return coil.

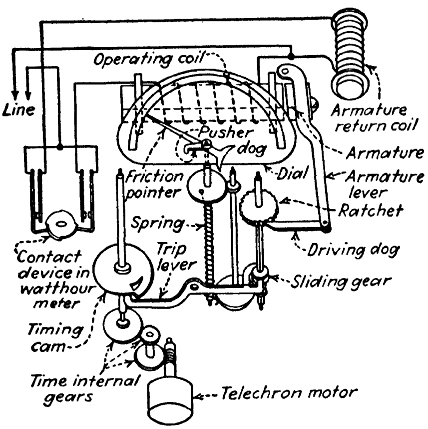

FIG. 339.—Diagram for General Electric M16 demand meter.

Impulse demand meters of this class are converted into recorders by replacing the pusher dog by an arm carrying a marking pen and adding a chart mechanism which properly advances the record paper at the end of each demand interval. The record may be made either on a strip or on a circular chart.

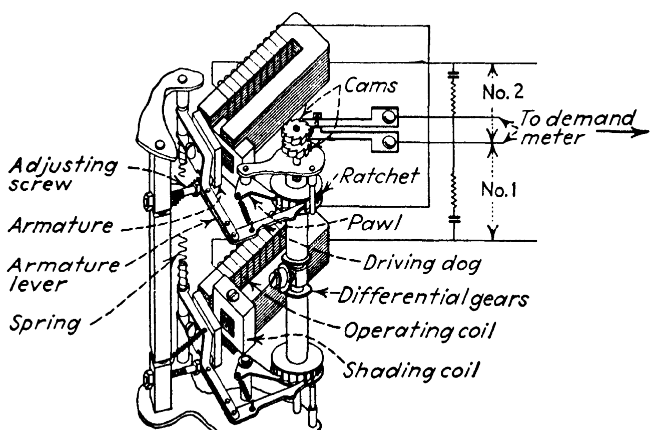

FIG. 340A.—Showing principle of totalizing relay. (*General Electric Company.*)

Totalizing Devices.—The demands on two circuits may be added by bringing the leads from the contact-making devices in the two watt-hour meters to a totalizing relay, one form of which is shown diagrammatically in Fig. 340*A*. The leads from the contact makers on circuits 1 and 2 are carried to two electromagnets which actuate the pawls on the two ratchet wheels. The summation is effected by the differential gears which give motion to the cam which actuates the contact in the demand meter circuit. If two signals arrive simultaneously, each produces its effect independently of the other. Obviously, the kilowatt constants for circuits 1 and 2 must be the same.

If more than two demands are to be totalized, for instance three, the leads from two of the contact devices are brought to a totalizing relay, the output from which is carried to a second totalizing relay along with the leads from the third contact device, as indicated in the single line diagram (Fig. 340*B*).

The mechanism of the Westinghouse RA impulse-operated demand meter includes two totalizing relays as indicated in Fig. 341. The impulses arriving along circuit 2, for instance, actuate an escapement device which is not unlike that employed in clocks. The escapement controls one side of the differential; the other side is similarly controlled from circuit 1. The power for driving the differential gearing is derived from a spring kept coiled by the torque motor; thus no heavy duty is imposed on the electrical contacts. The driving shaft of the differential is geared to operate the recording mechanism. The record is made on a strip chart similar to that shown in Fig. 338. The chart mechanism is driven by a synchronous electric motor or hand-wound spring clock. The assembly gives a graphic record of the integrated block interval demand, a dial record of the totalized integrated energy, and a cyclometer dial reading of the total number of impulses received.

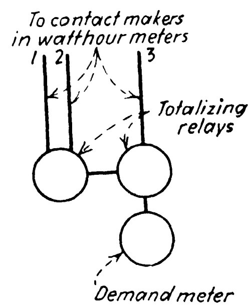

FIG. 340*B*.—Diagram of arrangement for totalizing three demands.

Printing Demand Meters.—Figure 342 shows diagrammatically one form of meter used for large installations. In this case, the number of impulses occurring in each demand interval is printed on a paper tape marked on its edge with the hours of the day (see Fig. 342).

The impulses from the contact maker in the watt-hour meter are received alternately by the two electromagnets. The walking beam is thus set in motion, and the two pawls advance the arbitrarily graduated type wheel. At the conclusion of a block interval, a separate contact-making device operated by a synchronous clock energizes the printer magnet, and a platten presses the inking and record tapes against the type, thus recording the positions to which the wheel has been advanced. As soon as the printing mechanism has functioned, the reset coil raises the pawls and holds them clear of the type wheel, which is returned to the zero position by a spring. At the same time, the record and inking ribbons are advanced.

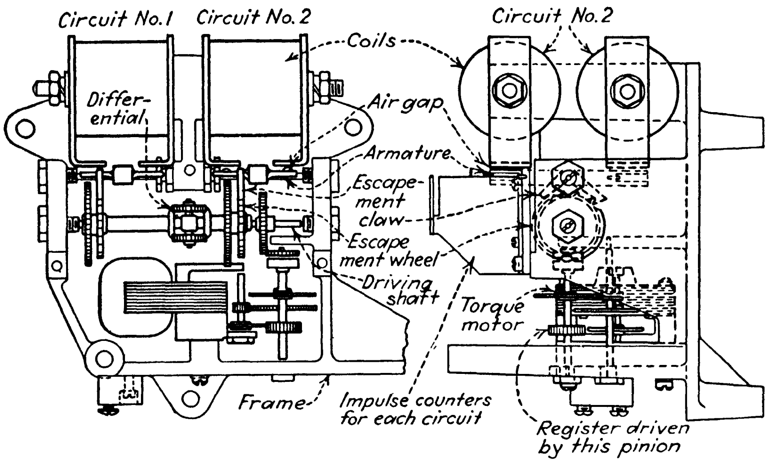

FIG. 341.—Diagram for Westinghouse RA impulse demand meter.

When the type wheel reaches the zero position, the pawls reengage, and the wheel again begins its upward travel. An indicator shows the most advanced position at which the wheel has arrived during any part of the time interval since it was manually reset, and a second indicator

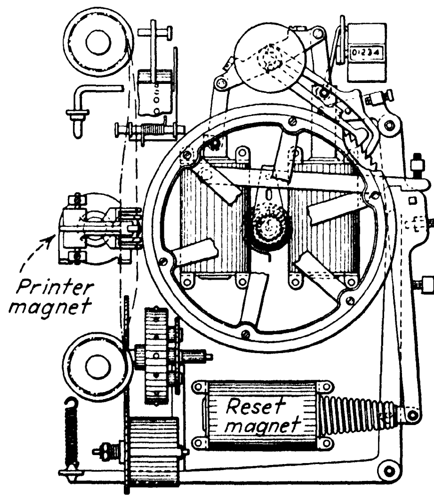

FIG. 342.—Diagram for "PD5" demand meter. (*General Electric Company.*)

shows the degree of advancement of the wheel at any time. The cyclometer reading shows the total number of times that the walking beam has oscillated. As the motion of the record tape is positive and controlled by the timing arrangement, an inspection of the tape shows where the maximum demand occurred.

Ingalls Demand Meters.—In this form of meter, the impulses from the watt-hour meter are received by an electromagnetic device which prints a mark on a uniformly moving paper tape which carries at its edge a numbering that gives the hours of the day. A record is made each time that the customer has received a definite amount or block of energy. A similar printing arrangement receives the impulses from a reactive watt-hour meter and prints them alongside of the watt-hour record. Each week the tape is removed and replaced by a new one. The portions where the watt-hour markings are most closely grouped are selected for measurement. If the equivalent kilowatt-hours of each space and the number of spaces in a demand interval of specified length are known, the maximum demand is readily calculated. The time when it occurred is read from the tape. The average power factor may be calculated by combining the watt-hour and reactive meter readings.

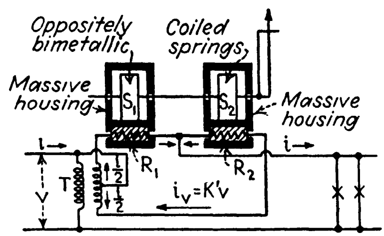

FIG. 343.—Diagram for thermal-storage meter.

Lagged Demand Meters.[6] **Lincoln Thermal-storage Demand Meter.** This device is a heavily lagged thermal wattmeter, as will be seen from Fig. 343, which shows diagrammatically the electrical circuit. Referring to the figure, T is a small transformer with the primary placed across the line. It circulates a current proportional to the voltage through two identical heater coils R_1 and R_2, which are of equal resistance at all temperatures. The value of this current may be taken as $K'v$. The line current i is brought to the mid-point of the secondary and divides—one half flowing through R_1, and one half through R_2. The total current in R_1 is $K'v + i/2$, while that in R_2 is $K'v - i/2$. The rates of heat production in R_1 and R_2 are

$$H_1 = K''\left[(K')^2v^2 + K'vi + \frac{i^2}{4}\right].$$

$$H_2 = K''\left[(K')^2v^2 - K'vi + \frac{i^2}{4}\right].$$

The difference in the rates of heat production is

$$H_1 - H_2 = K'''vi = K''' \times \text{(instantaneous power)}.$$

The heat liberated at R_1 and R_2 raises the temperatures of the oppositely coiled bimetallic springs S_1 and S_2. Any difference in their expansions rotates the pusher arm and moves the index up the scale. Changes in ambient temperature affect both springs alike and produce no net effect. The springs and their housings are intentionally given a large heat capacity, so that under the influence of the current they rise in temperature very slowly. The effects of momentary changes of current are thus "ironed out." A part of the heat liberated at R_1 and R_2 is stored in the spring structures and raises their temperatures, a part is lost to their surroundings and a part is conducted from the structure at the higher to that at the lower temperature. The result is that under

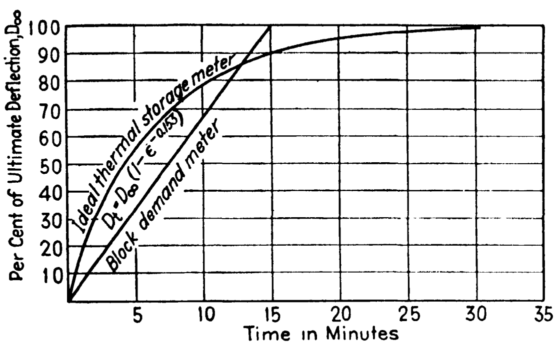

FIG. 344.—Comparison of responses of block demand and thermal, storage meters, 15-minute intervals.

constant load the difference of temperature of the two springs at a time t, denoted by $\theta_1 - \theta_2$, is given by

$$\theta_1 - \theta_2 = K(H_1 - H_2)(1 - \epsilon^{-kt}),$$

where k is a function of the thermal constants of the storage arrangement. As the deflection at any line t is proportional to $\theta_1 - \theta_2$,

$$D_t = K'(H_1 - H_2)(1 - \epsilon^{-kt}) = K'' \text{ (power)}(1 - \epsilon^{-kt}).$$

That is, under constant load conditions and starting from zero, the deflection rises according to the logarithmic law, much as the current in an inductive circuit rises when a constant voltage is impressed, and theoretically attains the final value only after infinite time, when the deflection would be $D_\infty = K''$(power). Therefore under constant load

$$D_t = D_\infty(1 - \epsilon^{-kt}).$$

The ideal form of the deflection-time curve is shown in Fig. 344, where it is contrasted with that obtained by the block system.

In its lower portions, the curve actually obtained by the Lincoln meter lies slightly below the ideal curve; however, the differences are perfectly regular and introduce no difficulties.

Evidently it is necessary to define the demand interval arbitrarily. If the deflection attains 90 per cent of its ultimate value in 15 min.. the meter might be said to be operating on a 15-min. demand interval. It is evident that on a fluctuating load the deflection depends on the integrated effect of previous loads and not on the instantaneous power.

Westinghouse "RL" Watt Demand Meter.[7]—This demand meter is a special register which may be substituted for that usually employed on any alternating-current watt-hour meter. It is a purely mechanical device which duplicates the results that would be obtained by a perfect thermal-storage watt-demand meter; that is, it gives the demand based on the so-called *logarithmic average*. Figure 345 shows the arrangement in diagrammatic form.

The bevel gear of A is driven by the watt-hour meter and hence the linear velocity of its periphery is proportional to the power being consumed. The disk E is driven at a constant angular velocity by the synchronous motor. The bevel gear B is driven by a worm on the spindle of the cylinder F. The solid aluminum ball is supported at three points by contact with the disk E, the slightly inclined cylinder F, and the involute cam C which carries the planetary gear of the differential and is attached to the shaft of the pusher dog D. As an involute cam is used, the vertical displacement of the ball is proportional to the angle of rotation of C. When D is at its zero position, the ball is in contact with the disk at the center; and if no load is on the meter no motion is imparted to C, and the demand pointer remains undisturbed. When the customer takes a load, the meter disk and A begin to turn, and a difference in the velocities of A and B is established. This turns the cam; and the ball, which rotates on an axis parallel to F, is displaced upward, thus imparting a greater and greater velocity to B. As A and B turn in opposite directions, the ball continues to rise at a slower and slower rate, until A and B have equal velocities, when the ball comes to rest. The demand pointer approaches its ultimate position exponentially, as in the ideal thermal-storage meter. If a load is suddenly removed, the pusher dog returns to zero—at first rapidly, then more and more slowly.

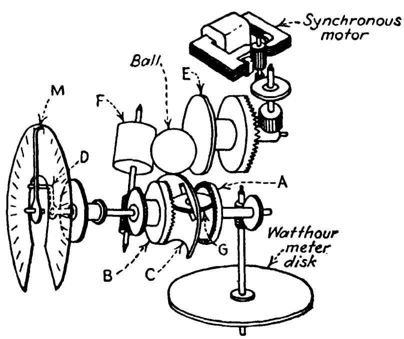

FIG. 345.—Diagram for Westinghouse "RL" demand meter.

Under a constant load,

$$\text{Linear velocity of } A = K_1 \times (\text{power})$$

If h is the distance of the point of contact of the ball from the center of the disk E, and θ is the angular deflection of the involute cam,

$$\text{Linear velocity of } B = K_2h = K_2K_3\theta.$$

$$\text{Angular velocity of } C = [K_1(\text{power}) - K_2K_3\theta]K_4 = \frac{d\theta}{dt}. \tag{10}$$

Therefore

$$\theta = (\text{power})\left(\frac{K_1}{K_2K_3}\right)[1 - \epsilon^{-[K_2K_3K_4]t}].$$

The final value of θ is $\theta_F = (\text{power})(K_1/K_2K_3)$,
and

$$\theta = \theta_F\left[1 - \epsilon^{-\left[\frac{(\text{power})K_1K_4}{\theta_F}\right]t}\right].$$

If the synchronous motor is blocked, making the velocity of B zero, the time T required for the pusher dog to come to full-scale deflection, if full load is being applied, is, using (10),

$$T = \frac{\theta_{\text{full scale}}}{K_1K_4(\text{power})_{\text{full scale}}}.$$

Therefore

$$\theta = \theta_F\left[1 - \epsilon^{-\frac{t}{T}}\right].$$

If t' is the time required for the deflection to come to 90 per cent of its final value,

$$10 = \epsilon^{\frac{t'}{T}}.$$

or

$$2.303 = \frac{t'}{T}.$$

If it is desired that the deflection come to 90 per cent of its final value in a definite time, for instance 15 min.,

$$T = \frac{15}{2.303} = 6.51 \text{ min.}$$

That is, if the gearing is so designed that with a blocked motor the full-scale deflection is attained in 6.51 min. the meter will attain under constant load and normal operating conditions 90 per cent of its ultimate deflection in 15 min. and might be said to have a 15-min. demand period. Like the thermal-storage meter, in fluctuating loads the deflection depends on the previous load history.

The advent of the RL meter puts the lagged demand meter among those devices the performance of which can be accurately predetermined from mechanical principles. It is still a matter of discussion whether the block or the lagged average system is the more satisfactory for billing purposes.

Peak Splitting in Demand Meters.—Up to the present time, the majority of demand meters have been designed to operate on the block interval or Merz system. Their acknowledged fault is that with an overload of short duration they may "split the peak," that is, divide it between two consecutive block intervals, thus giving too small a reading. As it is a matter of chance where the division between successive block intervals falls in relation to the overload, an infinite number of different readings is possible. However, the principle of the meters is readily explained and understood, and they can be built with mechanical precision. The proponents of the lagged meter maintain that their instrument avoids peak splitting and that its law of action approximates that limiting the allowable flow of current in a system.

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CHAPTER IX

PHASE METERS, POWER-FACTOR INDICATORS, SYNCHROSCOPES, FREQUENCY METERS, REACTIVE METERS, VOLT-AMPERE METERS, SEQUENCE INDICATION, MEASUREMENT OF SEQUENCE COMPONENTS, EFFECTS OF UNBALANCE

In the mathematical discussion of alternating currents, it is usual to assume sinusoidal waves, in which case

$$\text{Power factor} = \cos\theta = \frac{\text{watts}}{\text{volt-amp.}}$$

where θ is the time-phase displacement of the current wave with respect to the e.m.f. wave, that is, the angular distance between the zero points of the waves. With nonsinusoidal waves the power factor is taken as the ratio of the watts to the volt-amperes. In this case, θ is without significance.

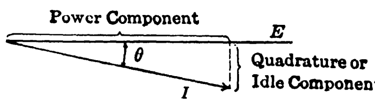

FIG. 346.—Showing power and quadrature components of current.

The current in a reactive circuit may be resolved into two components: one the power component, in time phase with the voltage; the other the quadrature component, which is wattless.

Evidently, for sinusoidal currents,

$$\text{Power component} = I\cos\theta$$
$$\text{Quadrature component} = I\sin\theta.$$

The quadrature component $I\sin\theta$ is sometimes called the idle or wattless current. It will be seen that operating a circuit at unity power factor is equivalent to operating it so that the idle current is zero.

The output of a generator is limited by the heating due to the currents in its coils, and the financial return on this output is primarily based on the true watts. For this reason alone it is highly desirable to operate the system supplied by the generator at as high a power factor as possible. Also, the power factor of the load influences the voltage regulation of the system on which the quality of the salable product depends.

To maintain the power factor near unity, the system may be so operated that the idle current is practically zero, the reactive volt-amperes near zero, or the volt-amperes in close agreement with the true power.

The loads on the various phases of a polyphase system should be balanced, for this reduces the trouble from heating of the rotor of the generator and renders possible the maximum return on the investment for the machinery and distribution system. Also, it tends to preserve the voltage regulation of the system. Unbalance as well as power factor thus becomes an economic factor.

It is not unusual to employ a form of synchronous apparatus as a transforming device between the generator and the load. As the power factor of the synchronous apparatus may be controlled by varying the excitation, it is necessary to have on the switchboard a power-factor meter, or its equivalent, as an aid to the proper handling of the synchronous machinery.

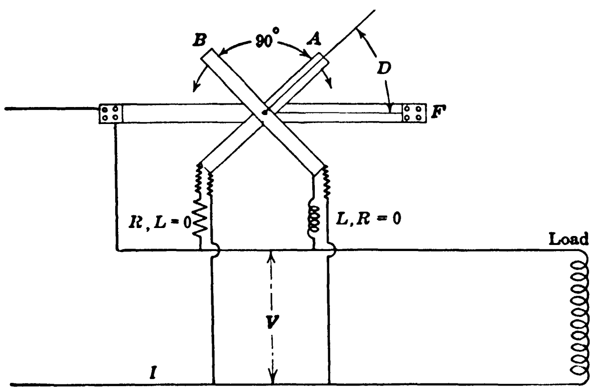

FIG. 347.—Diagram of Tuma phase meter.

Tuma Phase Meter.[1]—Power-factor meters, as well as various forms of synchroscopes, are developments from the Tuma phase meter, the essential portions of which are shown in Fig. 347.

In its original form, the ideal Tuma phase meter is applicable only to single-phase circuits and gives a deflection equal to the power-factor angle of the load. By a trifling alteration it may be adapted to polyphase circuits, as will be seen later.

The fixed coil F is traversed by the load current. The coils A and B are of equal magnetic strength and together form a single movable system which is pivoted in the field due to F; in the ideal instrument, A and B are inclined at an angle of 90 deg. to each other.

The current in coil A is supposed to be controlled by a pure resistance and consequently is in phase with the applied voltage V. The current in coil B is supposed to be controlled by a pure inductance and hence is in quadrature with the voltage V.

The currents are taken into the movable system at the pivot, through flexible connections of annealed silver foil which resemble ordinary controlling springs in appearance but exercise no appreciable torque on the system.

When no currents are flowing, the crossed coils are perfectly neutral and will remain in any position to which they are turned. The position of the crossed coils from which the deflections are reckoned is that assumed by them when the device is applied to a load of power factor unity. In that case, the planes of coils A and F will coincide, for as the currents in B and F are in quadrature, on account of the inductance L, the average turning moment on B is zero.

In general, on the passage of currents through the coils F, A, and B, a field will be set up by F, and the coils A and B which form the movable element will both experience turning moments. As A and B are rigidly connected, the movable element will turn to such a position that the resultant moment acting on it is zero. The deflection D, from the initial position occupied by the coils when the power factor is unity, will be equal to the power-factor angle of the load.

It will be assumed that the coil F is so large compared with A and B that the crossed *coils move in a sensibly uniform field;* also that the circuits of A and B are inductionless and resistanceless, respectively. If the potential difference wave is taken as the datum for measuring phase displacements, the turning moment acting at any instant on coil A will be

$$K_A[I \sin (\omega t - \theta)]\left[\frac{V}{R} \sin \omega t\right] \sin D,$$

where K_A is a constant depending on the windings.

The turning moment acting at any instant on coil B will be

$$K_B[I \sin (\omega t - \theta)]\left[\frac{V}{L\omega} \sin (\omega t - 90°)\right] \sin [D + 90°].$$

The currents in the coils are such that A and B tend to turn in opposite directions. When the movable system has come to rest, the average turning moment on coil A must equal that on coil B, so that

$$\left[\frac{K_AIV}{R}\right] \sin D \frac{1}{T}\int_0^T \sin (\omega t - \theta) \sin (\omega t) dt =$$

$$\left[\frac{K_BIV}{L\omega}\right] \cos D \frac{1}{T}\int_0 \sin (\omega t - \theta) \sin (\omega t - 90°) dt;$$

$$\left[\frac{K_AIV}{R}\right] \sin D \cos \theta = \left[\frac{K_BIV}{L\omega}\right] \cos D \sin \theta.$$

If, by the construction of the apparatus

$$\frac{K_A}{R} = \frac{K_B}{L\omega}, \tag{a}$$

then

$$\tan D = \tan \theta,$$

and

$$D = \theta. \tag{1}$$

That is, the movable system turns through an angle equal to the power-factor angle of the load.

The assumptions made in obtaining this result are that the frequency is constant, that the coils A and B are small compared with F, that the planes of coils A and B are 90 deg. apart in space, that the coils are traversed by currents that differ 90 deg. in time phase, and that the current in coil A is in phase with the line voltage V. In practice, these current relations cannot be attained; the lag in the circuit B can never be exactly 90 deg. Nevertheless, by a proper adjustment of the angle between the coils, the instrument can be made to read correctly.

To investigate this matter, suppose that the current in coil B lags Δ deg. behind V and that the mechanical angle between the coils is β instead of 90 deg.

Assuming that the crossed coils are alike and have the same number of ampere turns,

$$\sin D \cos \theta = \sin (D + \beta) \cos (\theta - \Delta).$$

The fiducial point on the scale corresponds to the reading when the power factor of the load is unity; in that case, the deflection D_0 will be given by

$$\cot D_0 = \frac{1}{\sin \beta \cos \Delta} - \cot \beta.$$

When the power-factor angle of the load is θ, the reading will be given by

$$\cot D = \frac{1}{\sin \beta \cos \Delta + \tan \theta \sin \beta \sin \Delta} - \cot \beta.$$

The change of deflection is $D - D_0$,

$$\cot (D - D_0) = \cot \theta \left\{ \frac{1 - \cos \beta \cos \Delta + \left[(\cos^2 \Delta - \cos \beta \cos \Delta) + \left(\cos \beta \cos \Delta + \sin^2 \beta \frac{\cos \Delta}{\cos \beta} - 1 \right) (\tan \theta \cos \beta \sin \Delta) \right]}{\sin \beta \sin \Delta} \right\}$$

Inspection shows that if $\beta = \Delta$, this equation reduces to

$$\cot (D - D_0) = \cot \theta \frac{1 - \cos^2 \beta}{\sin^2 \beta} = \cot \theta,$$

or

$$D - D_0 = \theta. \tag{2}$$

Consequently, if the crossed coils are adjusted once for all so that the angle between their planes is equal to the electrical angle between their currents, the deflection from the initial position will be equal to the power-factor angle of the load.

An explanation of the action of the Tuma phase meter may also be based on the fact that the crossed coils set up a rotating field, for in the original design of the instrument these coils are 90 deg. apart in space and are traversed by currents differing 90 deg. in time phase.

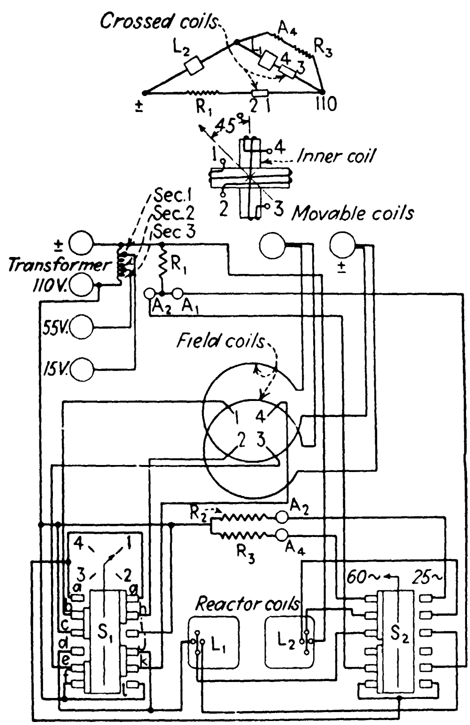

FIG. 348.—Diagram for Weston phase-angle meter.

Weston Phase-angle Meter.—This instrument is a special form of phase meter calibrated to read directly in electrical degrees and so designed that phase displacements up to 360 deg. may be measured. As it is desirable for structural reasons not to have the crossed coils deflect more than 90 mechanical deg., the directions of the currents in them is controlled by a switch S_1 (Fig. 348), the position of which indicates the quadrant in which the angle lies and allows one to select the proper scale; for while there is but one set of graduations, there are four sets of numbers—up the scale from 0 to 90 electrical deg., printed in black; down the scale from 90 to 180 deg., printed in red; up the scale from

180 to 270 deg., printed in black; down the scale from 270 to 360 deg., printed in red. The 0-, 180-, and 360-deg. marks coincide at the lower end of the scale; the 90- and 270-deg. marks coincide at the upper end of the scale. The angle as read from the scale is from the vector $+V$ in the counterclockwise or positive direction.

From Fig. 348 it will be noted that there are two reactors with iron cores L_1 and L_2 in series with the coil 34, which together with L_1 is shunted by the resistances R_3 and A_4 in series. This construction allows the currents in the crossed coils to be brought into quadrature in spite of the fact that there are resistance and iron losses in the circuits. Magnetic leakage in minimized, and astaticism is secured by placing the windings of the reactor on two parallel legs of the core and keeping the same number of turns on each leg. The switch S_2 controls the number of active turns and allows measurements to be made at either 60 or 25 cycles per second. Taps on the autotransformer allow the meter to be used at 15, 55, or 110 volts.

To measure the phase difference of two currents I_1 and I_2, the potential terminals are connected to a voltage of the proper value derived from the same source as I_1 and I_2. The positive directions of both I_1 and I_2 are designated. The current I_1 is sent through the current coils in the positive direction, and S_1 manipulated until the deflection is on the scale. The reading is then taken, after which, without disturbing the potential connections, I_2 is sent through the current coils in the positive direction, and a second reading taken. Suppose that the first reading is taken when the quadrant switch indicates the second quadrant and that the reading is 110 deg., also that the second reading is taken when the quadrant switch indicates the first quadrant and is 28 deg. The phase difference is then 82 deg., and I_1 leads I_2.

Single-phase Power-factor Meters.[2]—In the application of the principle of the Tuma phase meter to the construction of power-factor meters for use on single-phase circuits, a difficulty is encountered. For though the windings and the angle between the crossed coils may be adjusted so that the instrument reads correctly at the normal frequency, any change of frequency will render the readings inaccurate, because both the phase and the magnitude of the current in coil B are controlled by an inductance and, therefore, depend on the frequency. At low frequencies, coil B carries too much current; at high frequencies, too little; and condition (*a*) (page 547) is not fulfilled. The result of frequency changes on a single-phase power-factor meter is illustrated by Fig. 349. It is, however, to be noted that in modern distribution systems the frequency is very accurately maintained.

Polyphase Power-factor Meters.—The indications of polyphase power-factor meters which are adaptations of the phase meter are correct only on balanced circuits. If the circuit is unbalanced, the reading is without significance. The application of the Tuma phase meter to a balanced two-phase circuit is obvious. The two crossed coils are placed 90 deg. apart in space; their currents, 90 deg. apart in time phase, are obtained by using a resistance in series with each coil

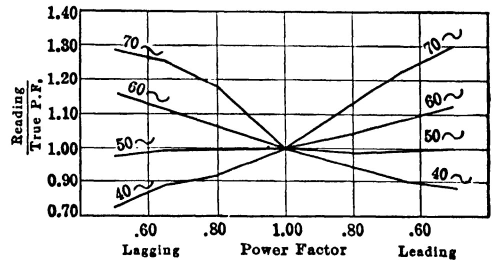

FIG. 349.—Showing effect of frequency on a single-phase power-factor meter.

and energizing a coil from each of the two phases. To adapt the instrument to balanced three-phase circuits, it is to be remembered that the angle between the planes of the movable coils should be made equal to the electrical angle between the currents in these coils. The fixed coil is placed in one of the line wires, while the movable coils are connected from this wire through resistances to the other two wires of the circuit (see Fig. 351).

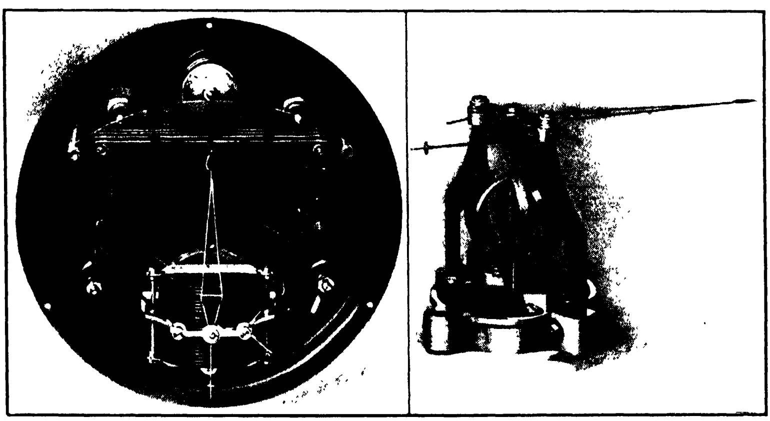

FIG. 350.—Indicating portion of Weston power-factor meter.

Reckoning from lead 1, the currents in A and B differ by 60 deg. in phase. The fiducial position of the coils is given when the power factor of the load is unity; if the power factor is other than unity, the current in lead 1 is shifted θ deg., where θ is the power-factor angle, and the crossed coils turn through an equal angle.

The polyphase instruments are independent of frequency, for no reliance is placed on the use of reactances to shift the phases of the currents properly in the crossed coils. On high-voltage circuits, the meters are operated through instrument transformers.

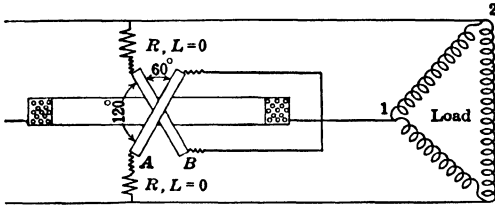

FIG. 351.—Connections for three-phase power-factor meter.

In power-factor meters as actually constructed, the fixed coils are made to surround closely the movable system. Economy of space and of materials is thus attained. The scale is graduated by experiment.

Power-factor Charts.—In tests of industrial plants, it is frequently important to gain an idea of the power factor under ordinary operating conditions. In three-phase work the two-wattmeter method of measur-

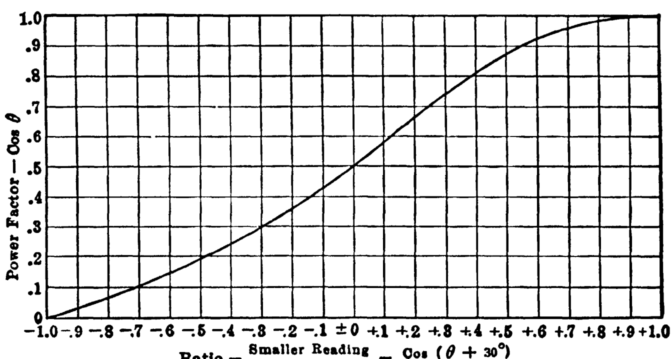

FIG. 352.—Power-factor chart, two-wattmeter method, balanced three-phase load.

ing power will usually be employed. For a balanced load the power indicated by the two wattmeters is

$$P_1 = EI \cos (\theta + 30°);$$
$$P_2 = EI \cos (\theta - 30°).$$

It is convenient to calculate and plot once for all a curve such as is shown in Fig. 352.

The ordinates of the curve are power factors or values of cos θ; the abscissæ are values of the ratio

$$\frac{\text{Smaller reading}}{\text{Larger reading}} = \frac{\cos (\theta + 30°)}{\cos (\theta - 30°)} = R. \qquad (3)$$

The relation between the power factor and the ratio R is

$$\text{P.F.} = \frac{1}{\sqrt{1 + 3\left(\frac{R-1}{R+1}\right)^2}}. \tag{4}$$

For use in unbalanced circuits, power-factor meters should give indications that are in accordance with the A.I.E.E. definition.

Power Factor in Polyphase Circuits.—The power factor of a polyphase circuit, either balanced or unbalanced, is the ratio of the total active power in watts to the total vector volt-amperes.

The total vector volt-amperes is the square root of the sum of the squares of the total active power and the total reactive power.

The total reactive power is the algebraic sum of the reactive power corresponding to the separate harmonic components of the system.

$$\text{P.F.} = \frac{\text{active power}}{\text{vector volt-amp.}} = \frac{\text{active power}}{\sqrt{(\text{active power})^2 + (\text{reactive power})^2}}. \tag{5}$$

Measurement of Reactive Volt-amperes.[6]—With sinusoidal single-phase currents a two-circuit electrodynamometer, with the movable circuit placed across the line and the fixed coils traversed by the line current, will give a deflection

$$D = KI_FI_M \cos \theta,$$

where θ is the phase difference of I_F and I_M. If the circuit of the movable coil is noninductive, the instrument is an ordinary wattmeter; but if this circuit could be made perfectly reactive, the phase of I_M would be shifted 90 deg. with respect to the line voltage, and

$$D = KI_FI_M \cos (90° - \theta) = KI_FI_M \sin \theta. \tag{6}$$

At a constant voltage the deflection would be proportional to the idle current $I \sin \theta$, or at any voltage to the reactive volt-amperes which are defined as follows: "The reactive volt-amperes in a circuit are the square root of the difference between the square of the apparent power (volt-amperes) and the square of the power (watts)." For sinusoidal currents

$$\text{Reactive volt-amperes} = \sqrt{(VI)^2 - (VI)^2 \cos^2 \theta} = VI \sin \theta. \tag{7}$$

On account of the energy dissipated in the reactor, it is impossible to shift the phase of the potential-coil current by exactly 90 deg., unless the coil is shunted by a condenser. Such a combination is very sensitive to changes of frequency.

Reference to the theory of the induction watt-hour meter will show that this instrument would be converted into a reactive volt-ampere-hour meter if its potential circuit were made perfectly *noninductive.*

The reactive volt-amperes in a *balanced* three-phase circuit may be measured by the use of wattmeters if the coils are connected in circuit as shown in Fig. 353.

From the vector diagram it is seen that each wattmeter gives a deflection proportional to $EI \cos (90° - \theta) = EI \sin \theta$. As the readings

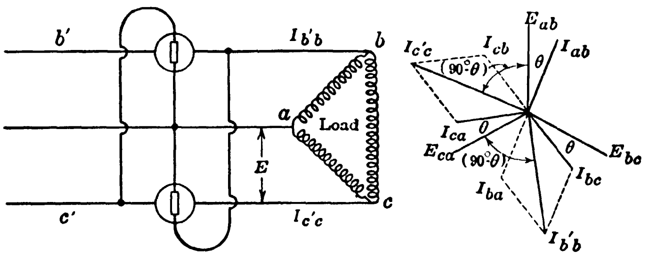

Fig. 353.—Connections for measuring idle volt-amperes in balanced three-phase circuit.

are alike, one wattmeter is sufficient, the proper multiplying factor being used. If the two wattmeters in Fig. 353 are the two elements of a polyphase wattmeter, the reading of that instrument will give $2EI \sin \theta$. As the total reactive volt-amperes in the load is $\sqrt{3}EI \sin \theta$,

$$\text{Reactive volt-amp.} = (\text{reading}) \frac{\sqrt{3}}{2} = (\text{reading})\ 0.866. \qquad (8)$$

The scale of the instrument may be graduated so that the reactive volt-amperes may be read directly. (Compare with two-wattmeter method for measuring three-phase power, page 339.)

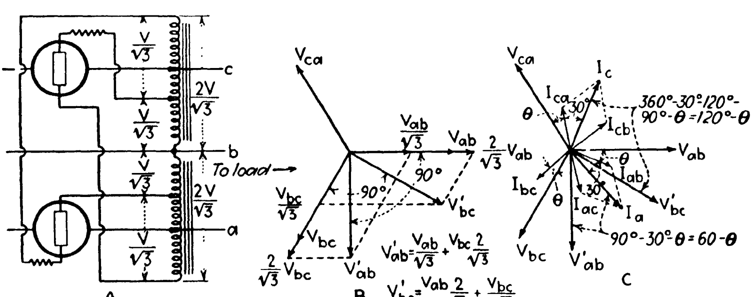

Fig. 354.—Determination of reactive volt-amperes in a balanced three-phase circuit using General Electric phase-shifting transformer.

Usually the potential circuits of the meters are actuated through phase-adjusting transformers. Those supplied by the General Electric Company for three-phase, three-wire circuits are autotransformers whose arrangement is shown diagrammatically in Fig. 354.

The two autotransformers, which give terminal voltages equal to $2/\sqrt{3}$ times the line voltage, are tapped at their centers, and the voltages of the sections added vectorially, as shown in Fig. 354*B*. The voltages applied to the potential coils of the meters are unchanged in magnitude but are shifted 90 deg. from the positions that they would occupy if power were being measured. The two wattmeters or the two-element polyphase watt-hour meter requires no special modification. The current coils are usually supplied from current transformers. The relations of the currents and voltages are shown in Fig. 354*C*, from which it appears that the readings of the two instruments are

$$P'_a = VI \cos (60° - \theta),$$
$$P'_c = VI \cos (120° - \theta),$$
$$P'_a + P'_c = \sqrt{3} VI \sin \theta$$

The phase-adjusting autotransformer supplied by the Westinghouse Company for use in three-phase, three-wire circuits is a double-Δ arrangement having six accessible terminals, as indicated in Fig. 355, which shows the internal connections. Referring to the figure, the balanced line voltages are

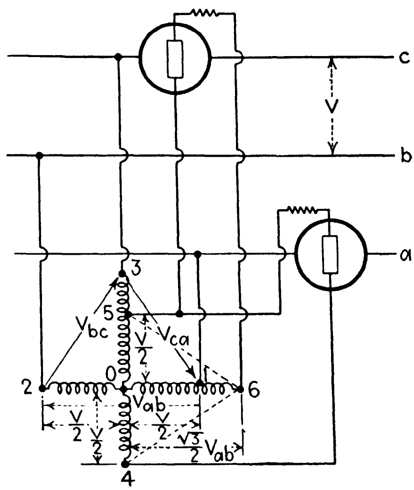

FIG. 355.—Determination of reactive volt-amperes in a balanced three-phase circuit, using Westinghouse phase-shifting transformer.

$$V_{ab} = V_{ab};$$
$$V_{bc} = V_{ab}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right);$$
$$V_{ca} = V_{ab}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right).$$

Then

$$V_{10} = \frac{V_{ab}}{2};$$
$$V_{03} = -V_{ca} - V_{10} = -V_{ab}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) - \frac{V_{ab}}{2} = -j\frac{\sqrt{3}}{2}V_{ab};$$
$$V_{05} = -j\frac{V_{ab}}{2};$$
$$V_{04} = +j\frac{V_{ab}}{2};$$
$$V_{60} = +\frac{\sqrt{3}}{2}V_{ab}.$$

The voltage applied to wattmeter in line a is

$$V_{40} + V_{05} = -j\frac{V_{ab}}{2} - j\frac{V_{ab}}{2} = -jV_{ab}. \tag{9}$$

The voltage applied to wattmeter in line c is

$$\begin{aligned} V_{60} + V_{05} &= V_{ab}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) \\ &= jV_{bc}. \end{aligned} \tag{9a}$$

These voltages are seen to be equal in magnitude and in quadrature with the corresponding line voltages; that is, they occupy the positions shown in Fig. 354*C*.

These devices are correct only when used in circuits where both the voltages and currents are balanced. Various other arrangements for three-phase circuits are given in a report on the subject printed by the National Electric Light Association.[6] The effects of unbalanced conditions have been studied by Knowlton[6] and are summarized in his book "Electrical Power Metering."[6]

Measurement of Symmetrical Components or Symmetrical Coordinates.—The method of symmetrical-component analysis was devised by C. L. Fortescue[7] of the Westinghouse Electric and Manufacturing Company to aid in the solution of problems involving unbalanced polyphase machinery. For a comprehensive treatment of symmetrical coordinates the student should refer to the book by C. F. Wagner and R. D. Evans[7] and to that by Waldo V. Lyon.[7]

By way of illustration, refer to Fig. 356, where three sets of vectors, all having the same direction of vector rotation, are shown. Inspection shows that the sequence in set A is a, b, c, while that in B is a, c, b. In set C, there is no sequence, all three vectors being coincident in direction. The vectors in A are those of a balanced three-phase system of currents; their vector sum is therefore zero. For the same reason, the sum of the vectors in B is zero. In set C, all three vectors are of the same magnitude and coincident in direction. Their vector sum is therefore three times any one of the components. Figure 356*D* shows the result of vectorially adding the three sets. The result is obviously a badly unbalanced system of vectors I_a, I_b, I_c having the same direction of vector rotation as its components.

Suppose the vectors I_a, I_b, I_c to be given, their magnitudes and directions being as shown. To find I°, the zero-sequence component, note that, as A and B are balanced systems of vectors, they contribute nothing to the vector sum of I_a, I_b, I_c, the value of which must therefore be $3I^\circ$, as illustrated in Fig. 356*E*.

If I° is subtracted from I_a and I_b and I_c in turn, the system becomes that given by the dot-dash lines I_a'', I_b'', I_c'', which consist of two balanced three-phase systems. To determine the magnitude of the positive-sequence component, I_a^+, I_b'' may be rotated 120 deg. in the counterclockwise direction. I_b^+ then coincides in direction with I_a^+, while I_b^- moves to the position occupied by I_c^-. Next, if I_c'' is rotated 120 deg. in the clockwise direction, I_c^+ becomes coincident in direction with

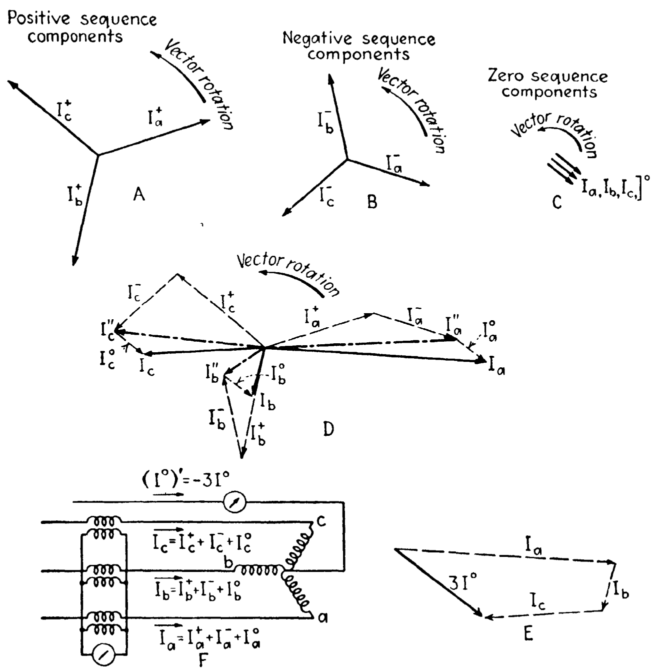

FIG. 356.—Showing positive-, negative-, and zero-sequence component vectors and synthesis to form an unbalanced system, and one arrangement of ammeters to measure zero-sequence current in a four-wire system and construction for obtaining I°.

I_a^+, and I_c^- moves to the position formerly occupied by I_b^-. The sum of the vectors in their new positions is obviously $3I_a^+$; for as the reversed-sequence components I_b^- and I_c^- have merely interchanged places, the vector sum still remains zero, and all three of the direct components coincide in direction, and one-third their vector sum gives I_a^+. Similarly, I_a^- is determined by rotating I_b'' 120 deg. clockwise and I_c'' 120 deg. counterclockwise.

Measurement of Sequence Currents.[7]—One arrangement for determining the zero-sequence component is shown in Fig. 356F, where current transformers in the three lines feed into a single ammeter, which thus measures the sum of the line currents, $3I^\circ$.

If only the positive- and negative-sequence components are present, they may be separated by the low-impedance network (due to Fortescue) shown in Fig. 357, where A_1 and A_2 are two identical ammeters: A_1 for the positive-sequence current, and A_2 for the negative-sequence current. From the diagram,

$$\text{Current in } A_1 = I$$
$$\text{Current in } A_2 = I'_a + I'_b - I$$
$$I'_a = I'_a]^+ + I'_a]^-$$
$$I'_b = I'_a]^+\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + I'_a]^-\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$
$$I'_a + I'_b = I'_a]^+\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + I'_a]^-\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right).$$

The network is so proportioned that the numerical values of z_{ba} and z_{ac} are the same, and the phase angle of I'_a and I'_b differs by 60 deg.

Then
$$z_{bd} = (R + R_A) + jX_A, \tag{10}$$
and
$$z_{ac} = [(R + R_A) + jX_A]\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right). \tag{11}$$

If the circuit is opened at ϵ, the Thévenin voltage (page 387) which appears across the break is

$$V_T = \left\{I'_b\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + I'_a\right\}[(R + R_A) + jX_A]. \tag{12}$$

The impedance in series with this voltage is

$$z_T = \left\{\frac{3}{2} + \frac{j\sqrt{3}}{2}\right\}[(R + R_A) + jX_A].$$

Therefore

$$I = \frac{I'_b\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + I'_a}{\frac{3}{2} + j\frac{\sqrt{3}}{2}}. \tag{13}$$

Substituting for I'_b and I'_a,

$$\text{Current in } A_1 = I = \frac{I'_a]^+\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) + I_a]^-(-1 + 1)}{\frac{3}{2} + j\frac{\sqrt{3}}{2}}$$
$$= I'_a]^+\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = I'_a]^+\overline{\backslash 60°}. \tag{14}$$

No reversed-sequence current flows in ammeter A_1. I is in phase with the direct-sequence current in line C. The current in ammeter A_2 is

$$I_{A2} = I'_a]^+\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + I'_a]^-\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) - I'_a]^+\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) \quad (15)$$

$$= I'_a]^-\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = I'_a]^-\underline{/60°}. \quad (16)$$

No direct sequence current flows in ammeter A_2.

I_{A2} is in phase with the reversed-sequence current in line C. If present, the zero-sequence current may be diverted from the network in several ways. One way, involving the use of two pairs of identical current transformers with cross-connected secondaries, is shown in

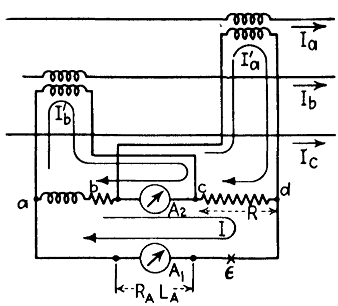

FIG. 357.—Connections for determining positive- and negative-sequence currents, no zero-sequence current present.

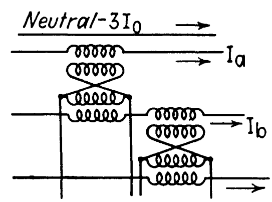

FIG. 358.—Connections to exclude zero-sequence current from network.

Fig. 358. The zero-sequence current circulates in the secondaries but does not appear in the measuring circuit. The current supplied to the network is $\sqrt{3}$ times the line or phase current.

Measurement of Sequence Voltages.[7]—The line-to-neutral voltages in a three-phase system with neutral are

$$\begin{aligned} V_a &= V_a^+ + V_a^- + V_a^0. \\ V_b &= V_b^+ + V_b^- + V_a^0. \\ V_c &= V_c^+ + V_c^- + V_a^0. \end{aligned} \quad (17)$$

The positive- and negative-sequence voltages both form balanced systems and their vector sums are therefore zero. Consequently, the vector sum $V_a + V_b + V_c = 3V^0$. The zero-sequence voltage may be measured by using three identical potential transformers connected as shown in Fig. 359. The voltmeter responds to the sum of the line-to-neutral voltages and consequently reads three times the zero-sequence voltage.

It will serve as a ground detector on a normally ungrounded three-wire, three-phase circuit.

The connections for measuring the positive-sequence component of voltage are shown in Fig. 360, where V is a voltmeter having a total resistance R_V. The impedance of the coils alone is $Z_C = R_C + jX_C$. The instrument is provided with a tap between the coils and the series resistance $(R_V - R_C)$. The tap circuit consists of an inductance and a resistance so proportioned that the magnitude of its impedance is equal to $(R_V - R_C)$, while its phase angle differs from that of $R_V - R_C$ by 60 deg. Referring to the figure, the regular circuit of the voltmeter is

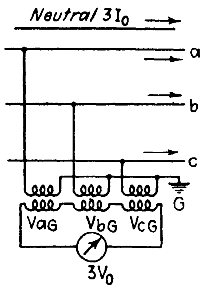

FIG. 359.—Connections for determining zero-sequence voltage.

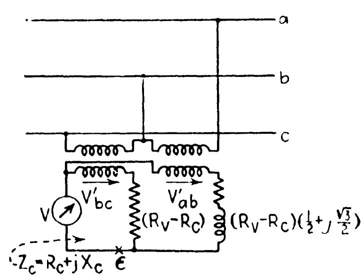

FIG. 360.—Connections for determining positive-sequence component of line voltage.

actuated by V'_{bc}; the inductive tap circuit, by V'_{ab}, which is the voltage of the leading phase.

If the circuit is opened at ϵ, the Thévenin (page 387) voltage which appears across the break is

$$V_T = \frac{-V_{bc} + V_{ab}}{\frac{3}{2} + j\frac{\sqrt{3}}{2}} + V_{bc} = +\frac{V_{ab} + V_{bc}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)}{\frac{3}{2} + j\frac{\sqrt{3}}{2}} \tag{18}$$

The Thévenin impedance in series with this voltage is

$$Z_T = Z_C + \frac{(R_V - R_C)\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)}{\frac{3}{2} + j\frac{\sqrt{3}}{2}}. \tag{19}$$

Therefore the current in the coils is

$$I_C = +\frac{V_{ab} + V_{bc}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)}{(R_V - R_C)\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + Z_C\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)} = + \frac{V_{ab}\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + V_{bc}}{(R_V - R_C) + Z_C\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)}.$$

$$V_{ab} = V_{ab}^{+} + V_{ab}^{-}.$$

$$V_{bc} = V_{ab}^{+}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) + V_{ab}^{-}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right).$$

Therefore

$$I_C = \frac{-j\sqrt{3}V_{ab}^{+} + 0.}{(R_V - R_C) + Z_C\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)}. \qquad (20)$$

No reversed-sequence current flows in the coils of the meter.

If it is desired to express V_{ab}^{+} in terms of volts to neutral,

$$V_{ab}^{+} = \sqrt{3}V_{a0}^{+}\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right).$$

$$I_C = -\frac{3V_{a0}^{+}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)}{(R_V - R_C) + Z_C\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)}.$$

$$\text{Reading of meter} = \frac{3V_{a0}^{+}}{(R_V - R_C) + Z_C\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)}. \qquad (21)$$

If the reactive and nonreactive elements are interchanged, it can be shown in a similar manner that

$$I_C = +\frac{V_{ab}^{+}0 + V_{ab}^{-}\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)}{(R_V - R_C) + Z_C\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)};$$

that is, no positive-sequence current flows in the instrument coils.

$$\text{Reading of meter} = \frac{3V_{a0}^{-}}{(R_V - R_C) + Z_C\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)}. \qquad (22)$$

Symmetrical components may be much more elegantly dealt with by use of the three-phase operator α, which when applied to a vector rotates it through 120 deg.[6,7] Figure 361 shows an unbalanced system of voltages and currents and the symmetrical components of both voltage

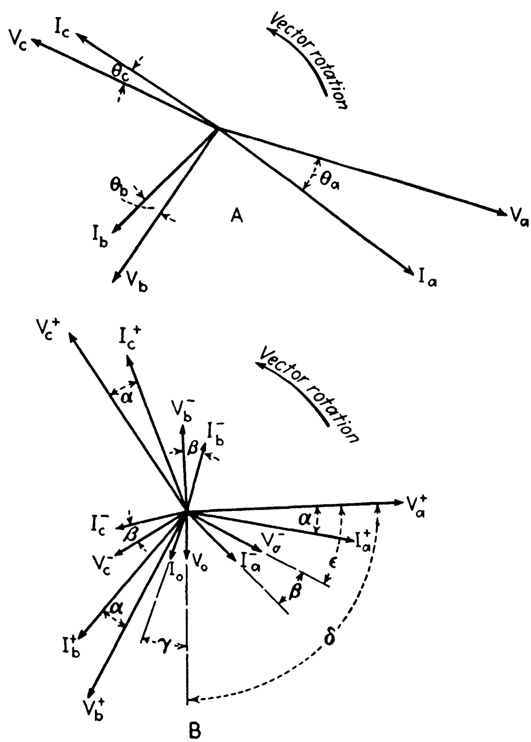

FIG. 361.—Showing an unbalanced system of voltages and currents and their symmetrical components.

and current. Referring to the figure, it will be seen that the power in a three-phase, four-wire current is

$$P = 3V^+I^+ \cos \alpha + 3V^-I^- \cos \beta + 3V^0I^0 \cos \gamma; \tag{23}$$

or, if there is no zero-sequence component,

$$P = 3V^+I^+ \cos \alpha + 3V^-I^- \cos \beta. \tag{24}$$

Similarly, the reactive volt-amperes are given by

$$Q = 3V^+I^+ \sin \alpha + 3V^-I^- \sin \beta + 3V^0I^0 \sin \gamma. \tag{25}$$

If there is no zero-sequence component,

$$Q = 3V^+I^+ \sin \alpha + 3V^-I^- \sin \beta. \tag{26}$$

Unbalance introduces no difficulties into the measurement of power. This is not the case when reactive volt-amperes are measured. At first sight, it might appear that to arrive at the desired result it would be necessary merely to change by 90 deg. the phase of the voltage applied to the meter, using one of the devices previously described. However, investigation shows that on unbalanced loads such arrangements give erroneous results, each being subject to its own peculiar errors, the principal one being that the negative-sequence terms in the reactive volt-amperes appear with the − instead of the + sign, as will be seen from the following.

Effect of Unbalance on Measurements.[7]—In the deduction of Blondel's theorem (page 337), no assumptions were made concerning balanced conditions; and as the theorem is perfectly general, any method of power measurement that adheres strictly to the rule established by the theorem gives correct results. Methods of measurement that depart from the rule will give erroneous results when the circuit is unbalanced. For instance, the three-wire watthour meter is sensitive to unbalanced conditions, as shown on page 493, whereas the energy would be correctly measured by two two-wire meters connected according to the theorem. In general, if an attempt is made to reduce the number of instruments or meters by using short-cut methods, the result will be an arrangement that, while it gives correct results under *balanced* conditions, will be subject to errors if the circuit is *unbalanced*.

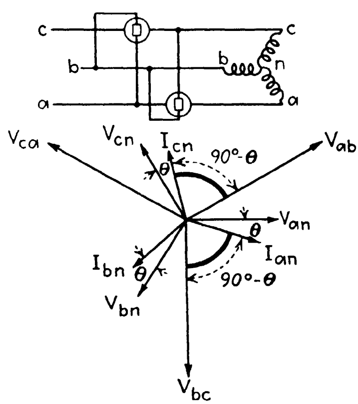

FIG. 362.—Pertaining to measurement of reactive voltamperes by cross phasing. The vector diagram is for balanced conditions.

Figure 362 shows the connections for determining reactive volt-amperes in a balanced Y-connected circuit by cross phasing the voltage coils of the wattmeters. Inspection of the diagram shows that

$$R_c = V_{ab}I_{cn}\cos(90° - \theta) = \sqrt{3}V_{cn}I_{cn}\sin\theta.$$

$$R_a = V_{bc}I_{an}\cos(90° - \theta) = \sqrt{3}V_{an}I_{an}\sin\theta.$$

$$\text{Reading} = R_c + R_a = 2\sqrt{3}V_{an}I_{an}\sin\theta. \tag{27}$$

The true value of the reactive volt-amperes is

$$Q = 3V_{an}I_{an}\sin\theta = (\text{reading}) \times \frac{\sqrt{3}}{2}. \tag{28}$$

If the circuit is *unbalanced*, and the same cross-phasing arrangement is employed, the diagram of Fig. 362 is distorted, and in order to obtain the desired results it becomes necessary to use the symmetrical components of the various voltages and currents. Referring to the figure, the readings are

$$R_c = V_{ab}I_{cn} = (V_{an} - V_{bn})I_{cn} \quad \text{mean product.}$$
$$R_a = V_{bc}I_{an} = (V_{bn} - I_{cn})I_{an} \quad \text{mean product.}$$

Referring to Fig. 361,

$$R_c = \{V_{an}^+ + V_{an}^- - V_{bn}^+ - V_{bn}^-\}\{I_{cn}^+ + I_{cn}^-\} \quad \text{mean product.}$$
$$R_a = \{V_{bn}^+ + V_{bn}^- - V_{cn}^+ - V_{cn}^-\}\{I_{an}^+ + I_{an}^-\} \quad \text{mean product.}$$

Therefore, taking the indicated mean products,

$$\begin{aligned} R_c = {} & V^+I^+ \cos (120° - \alpha) + V^+I^- \cos (120° + \epsilon + \beta) + \\ & V^-I^+ \cos (120° + \epsilon - \alpha) + V^-I^- \cos (120° + \beta) - \\ & V^+I^+ \cos (120° + \alpha) - V^+I^- \cos (\epsilon + \beta) - V^-I^+ \cos (\epsilon - \alpha) - \\ & V^-I^- \cos (120° - \beta); \end{aligned} \quad (29)$$

and

$$\begin{aligned} R_a = {} & V^+I^+ \cos (120° - \alpha) + V^+I^- \cos (120° - \epsilon - \beta) + \\ & V^-I^+ \cos (120° - \epsilon + \alpha) + V^-I^- \cos (120° + \beta) - \\ & V^+I^+ \cos (120° + \alpha) - V^+I^- \cos (120° + \epsilon + \beta) - \\ & V^-I^+ \cos (120° + \epsilon - \alpha) - V^-I^- \cos (120° - \beta). \end{aligned} \quad (29a)$$

Introducing the sine and cosine of 120 deg. and adding R_c and R_a,

$$\text{Reading} = 2\sqrt{3}V^+I^+ \sin \alpha - 2\sqrt{3}V^-I^- \sin \beta + V^+I^-\left[-\frac{3}{2}\cos (\epsilon + \beta) + \frac{\sqrt{3}}{2}\sin (\epsilon + \beta)\right] + V^-I^+\left[-\frac{3}{2}\cos (\epsilon - \alpha) + \frac{\sqrt{3}}{2}\sin (\epsilon - \alpha)\right]. \quad (30)$$

Using the multiplier $\sqrt{3}/2$, as was done in the balanced case, by meter,

$$Q' = 3V^+I^+ \sin \alpha - 3V^-I^- \sin \beta + \frac{\sqrt{3}}{2}V^+I^-\left[-\frac{3}{2}\cos (\epsilon + \beta) + \frac{\sqrt{3}}{2}\sin (\epsilon + \beta)\right] + \frac{\sqrt{3}}{2}V^-I^+\left[-\frac{3}{2}\cos (\epsilon - \alpha) + \frac{\sqrt{3}}{2}\sin (\epsilon - \alpha)\right]. \quad (31)$$

The result should be

$$Q = 3V^+I^+ \sin \alpha + 3V^-I^- \sin \beta. \quad (32)$$

It is seen that, in the expression for the reading, the negative-sequence term has its algebraic sign reversed. The error from the reversed-sequence term alone is $6V^-I^- \sin \beta$. This is the characteristic error of cross-phasing arrangements. In addition, there are the last two terms in (31) which arise from interaction in the meters of the direct- and reversed sequence components; they represent no real reactive volt-amperes.

The other reactive metering arrangements may be similarly analyzed. The results, in the cases of the two phase-shifting transformers described, show that both the errors are $6V_rI_r \sin \beta$, for again the reversed-sequence terms have the incorrect algebraic sign. Knowlton[6] discusses several other reactive metering devices which all possess the characteristic error $6V_rI_r \sin \beta$, and in most cases there are terms that are analogous to the last two in (31).

Determination of Kilovolt-ampere-hours.[8]—The kilovolt-ampere-hours in a circuit may be determined by the Westinghouse type RI kva recording demand watt-hour meter. This self-contained device is intended for use where large amounts of energy are concerned. It gives simultaneously the kilovolt-ampere-hours, the kilowatt-hours, the kilovolt-ampere demand, the kilowatt demand, and the power factor. The power factor and reactive kilovolt-ampere-hours for any time interval may be calculated from the chart of the instrument. The assembly includes two meter elements which are similar, one being activated by the true power, and the other by the reactive volt-ampere. The watthour element is provided with a register so that the kilowatt-hours may be read from the dials as usual. A second register, actuated by a ball mechanism, to be described, shows the integrated kilovolt-amperes or kilovolt-ampere-hours. As the pen, which records on the strip chart, is also controlled by the ball mechanism, the record shows for each block the integrated kilovolt-amperes and the time to which the record corresponds. The mechanism is such that at the end of each block interval the driving mechanism of the recording pen is disengaged, and the pen begins its return to zero under the influence of a small weight. The motion is arrested for a moment by a stop controlled by the watthour element; the integrated kilowatt-hours are thus recorded, after which the pen is allowed to return to zero. The record obtained is illustrated by Fig. 363.

The vectorial addition of the power and the reactive power is effected by the ingenious ball mechanism shown diagrammatically in Fig. 363.

Referring to the figure, B is a carefully ground ball which, to increase the friction, is made of aluminum. It is supported at P and R on two aluminum wheels which are turned in the same direction by both the watt-hour meter and the reactive-volt-ampere meter.

The angular distance between P and R is 90 deg., measured from the center of the ball. The third point of support is furnished by the aluminum wheel W, mounted in a carriage which can turn on an axis passing through the center of the ball. The linear velocities at P and R are along tangents to the wheels. In order that there may be no slipping of the ball on the wheels, the paths on the ball traced by the contacts at P and R must be, in general, small circles whose planes are perpendicular to the axis of rotation XY. Also, the carriage for the summation wheel W must have so adjusted itself that the point of contact with the ball is on a great circle having XY as its axis. Under these circumstances, the linear velocities of the sphere at the point of contact with W and of the periphery of W coincide in direction.

Let the angular velocity of the sphere about the axis XY be denoted by A. The two right triangles marked abc are equal, since their sides are mutually perpendicular with a side in each of the same length.

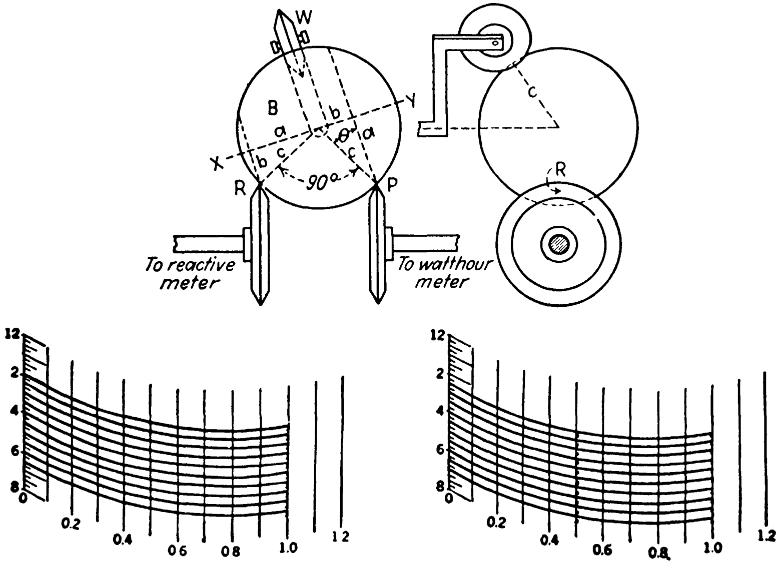

FIG. 363.—Diagrams for Westinghouse "RI" kva recording demand watt-hour meter. Arrangement for vectorially adding power and reactive power.

The linear velocity at P is K times the true power $= Aa$.
The linear velocity at R is K times the reactive power $= Ab$.
The linear velocity of the periphery of W =

$$Ac = A\sqrt{a^2 + b^2} = A\sqrt{\frac{K^2\ (\text{power})^2}{A^2} + \frac{K^2\ (\text{reactive power})^2}{A^2}}.$$

The linear velocity at $W = K\sqrt{(\text{power})^2 + (\text{reactive power})^2}$. (33)

Therefore the angular velocity of the wheel W is proportional to the volt-amperes; and the total angle turned through, or the number of revolutions of W in a given time, is proportional to the integrated volt-amperes. The revolutions of W are transmitted through suitable gearing to the kilovolt-ampere register and the recording device. There are other devices for determining volt-amperes—notably the Landis and Gyr "Trivector" volt-ampere-hour meter[8] and the Westinghouse

"Pantograph" volt-ampere-hour meter.[8] The student should refer to the description of Karapetoff's substitute method of metering;[10] also to the suggestion of Knowlton for registering only the negative loop of the power wave.[9]

Phase-order Indicators.—There are many possible devices for indicating phase order. Frequently, a miniature three-phase induction motor

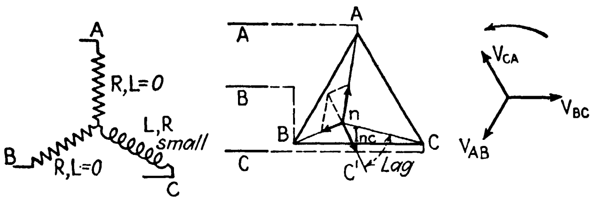

FIG. 364.—Diagrams for sequence indicator, leading voltage the smaller.

is employed, the phase order of the terminals and the direction of rotation having been correlated once for all. When measuring the power in a *balanced*, inductive three-phase load by the two-wattmeter method or when measuring the energy by a two-element watt-hour meter, the wattmeter with the voltage coil across the leading line voltage makes the greater contribution to the result.

The behavior of unsymmetrical Y arrangements of resistances and reactances depends on the phase order of the three-phase voltages and is utilized in phase-sequence indicators. For instance, the Y may have one highly reactive and two equal nonreactive arms, as indicated in Fig. 364. Referring to the diagram, V_{nA}, V_{nB} and V_{nC} must terminate at the vertices A, B, C of the voltage triangle. The arms nA and nB may be two electrodynamic voltmeters of equal resistance, while nC may be the potential coil of an induction watt-hour meter, or nA and nB may be two incandescent lamps of equal resistance, and nC any appropriate inductive resistance. If voltmeters are used, their readings appear on the diagram as V_{nA} and V_{nB}. Joining n and C gives the voltage V_{nC} across the inductance. The currents in nA and nB are proportional to and in phase with the voltages, and the *angular* position of $-I_{nC}$ is given by adding *vectorially* V_{nA} and V_{nB}. Hence I_{nC} is in the position shown. As it is known that nC is inductive, the angle CnC' must be lag angle. It is seen that the reading of the leading voltmeter is the smaller. If incandescent lamps are used, the leading lamp is the less brilliant. The phase order of the Y is then nB, nA, nC.

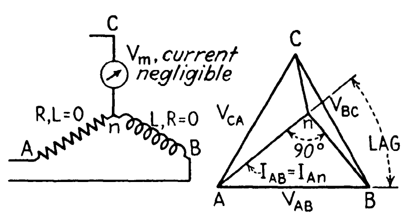

FIG. 365.—Diagrams for phase-sequence indicator.

Another arrangement is shown in Fig. 365. A nonreactive resistance is connected between A and n, and a highly inductive resistance between n and B. A voltmeter having so high a resistance that it draws a negligible current is connected between C and n. If An is an electrodynamic voltmeter, and nB is the potential coil of an induction watt-hour meter, an electrostatic voltmeter may be employed for nC, since the instrument draws a negligible current. From the character of the circuit, the current I_{AB} lags the voltage AB. Referring to the diagram, if the voltage V_{nC} is less than the line voltage, the line I_{AB} lies above V_{AB}, and the designated angle must correspond to a lag. The phase order is then V_{AB}, V_{BC}, and V_{CA}. If V_{nC} is larger than the line voltage, the phase order is reversed.

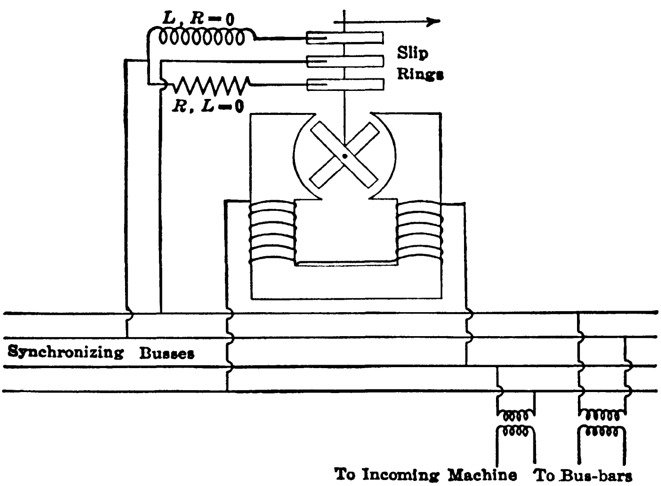

FIG. 366.—Diagram for Lincoln-type synchroscope.

Synchroscopes, Synchronizers.—When alternators are operated in parallel, it is necessary in putting a machine on the system that the voltage of the incoming machine be equal in magnitude and opposite in phase to the voltage of the buses. To avoid accidents and injurious stresses in the machines, it is necessary to have an instrument that will show when the proper phase relation has been attained and, in the case of large machines, whether the speed of the incoming machine must be increased or diminished before the main switches are closed.

The Lincoln-type Synchroscope.[5]—The Lincoln and kindred forms of synchroscope furnish the switchboard attendant with the desired information. This instrument is in principle a Tuma phase meter with the slight modification that the currents are carried to the crossed coils by brushes and slip rings or an equivalent arrangement so that the movable element can rotate continuously. The essential electrical connections are indicated in Fig. 366.

In the actual instrument, the shaft carrying the slip rings and the movable element may be mounted in ball bearings and, as shown, is perpendicular to the plane of the paper. To increase the turning moment acting on the movable system and thus reduce the effect of the brush friction, both the field and movable coils are wound on laminated iron cores. The similarity of the arrangement to the Tuma phase meter is at once apparent.

The index tries to point to the angle of phase difference between the machines, and its rate of movement is dependent on the difference of the machine speeds. It will move forward or backward or come to rest depending on whether the incoming machine is running faster or slower than, or at the same speed as the other machine. The pointer may come to rest in any position on the dial. This merely means that both machines are running at the same speed, though not necessarily in phase.

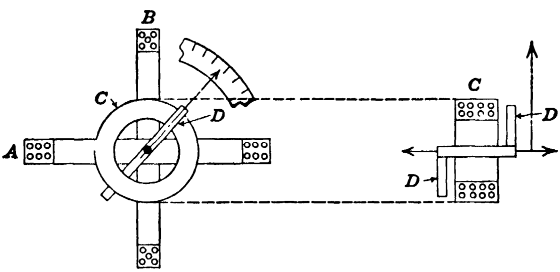

FIG. 367.—Westinghouse arrangement of coils for power-factor meter.

The speed of the incoming machine must be altered very gradually, and the main switch closed as the pointer slowly drifts across the index mark.

To avoid the use of movable coils and slip rings, the Westinghouse company employs the construction indicated in Fig. 367. The stationary crossed coils A and B are fed from a phase-splitting device activated from the bus bars, and within the coils is a third fixed coil C with its axis perpendicular to the plane of the paper and activated by the incoming machine. This coil magnetizes the balanced soft-iron element D which is mounted in jeweled bearings and is perfectly free to move. The iron D thus forms the core of an alternating-current electromagnet and acts as if it were a movable coil carrying a current derived from the incoming machine.

Phasing Lamps.—Before the invention of synchronizing devices based on the phase meter, it was customary to depend upon synchronizing lamps. With small low-voltage machines it is sufficient to place an incandescent lamp across the gap of the single-pole switch by which the incoming machine is to be connected to the bus bars. If, when the voltage has been adjusted, the incoming machine is not running at the proper frequency, the lamp will be alternately light and dark. As the frequency of the incoming machine is brought toward its proper value, the flicker of the lamp becomes slower and slower. The proper time for closing the switch is when the lamp remains dark, for then the voltages on the machine circuits are in opposition.

When high-voltage machines are used, it becomes necessary to employ transformers. They may be connected so that the proper time for closing the main switch is shown either when the lamp is dark or when it is at full brilliancy. The latter is the better practice, as it avoids mistakes due to the failure of the lamp. The two transformers may be combined into one with two primaries wound on two different branches of the magnetic circuit and a single secondary wound on a third branch.

A fault of these arrangements of phasing lamps is that they give no indication as to whether the speed of the incoming machine should be

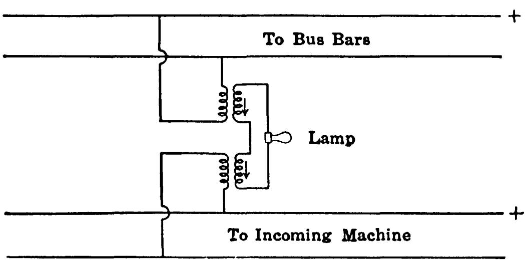

FIG. 368.—Arrangement of phasing lamp actuated by two transformers.

increased or diminished. This is obviated by the arrangement sketched in Fig. 369, where the transformers necessary on high-voltage systems are omitted.

The dotted connection simply denotes that the two neutral points are at the same potential. The noticeable feature of the arrangement is that phase *oc* is connected to *o'b'*, and phase *ob* to *o'c'*.

Suppose that the two machines are in synchronism. The vector diagrams for the e.m.fs. are as shown.

Synchronism is indicated when lamp 1 is dark and lamps 2 and 3 are glowing. If the machines are not in phase, the vectors being in the dotted position, lamp 1 begins to glow, No. 2 grows dimmer, and No. 3 grows brighter, so the dark lamp is passed from position 1 to position 2. If the phase displacement is in the other direction, lamp 1 begins to glow, No. 2 grows brighter, and No. 3 is dimmed, so that the dark lamp is passed from position 1 to position 3.

The lamps are arranged on the switchboard at the corners of an equilateral triangle; by noticing whether the order of brilliancy of the lamps proceeds around the triangle in the right-hand or the left-hand direction, one can tell whether the speed of the incoming machine must be increased or diminished.

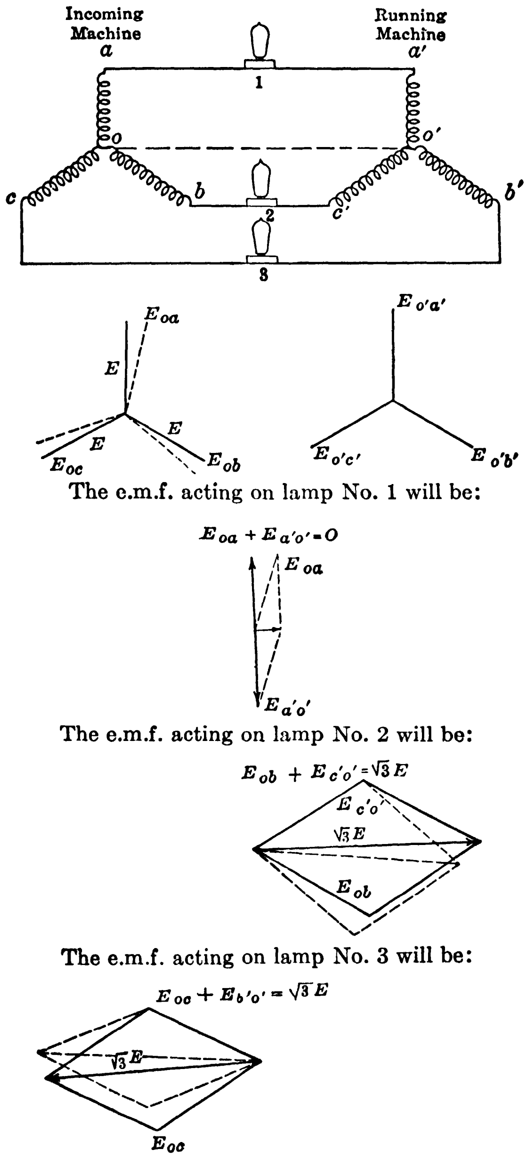

FIG. 369.—Vector diagrams, Siemens and Halske phasing lamps.

Frequency Meters.—Instruments of this class should be independent of wave form and also of variations of the line voltage. Because of the latter requisite, certain forms of frequency meter are constructed so that *both* the controlling and the deflecting moments acting on the movable system depend on the current through the instrument, that is, on the line voltage.

As indicated in Fig. 370, the General Electric Company frequency meter employs two resonating circuits. They are tuned to different frequencies, one above and one below the normal. For instance, circuit 1 might resonate at 65 cycles per second, while circuit 2 resonates at 55 cycles per second.

The fixed coils are so arranged that their resultant field at the movable coil is proportional to $i_1 - i_2$. The current $i_1 + i_2$ is led to the movable coil by nondirective spirals. Within the movable coil is a permalloy vane (shown dotted). The vane can be set at an angle with the movable-coil plane; this affects the scale distribution. The vane can be rotated about a horizontal axis, altering the effective length of the vane and changing the sensitivity of the instrument.

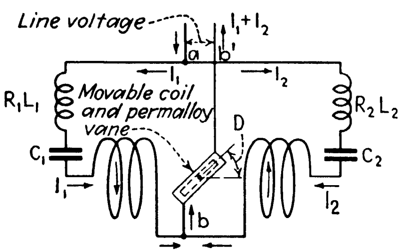

FIG. 370.—Diagram for frequency meter. (*General Electric Company.*)

At any instant the turning moment due to the action of the coils is

$$T_C = K(i_1 - i_2)(i_1 + i_2) \cos D.$$

It is to be noted that the permalloy vane is in a *weak field.* The flux through the vane is sensibly proportional to the field. At any instant the turning moment due to the action of the fixed coils on the vane is

$$T_V = K'(i_1 - i_2)^2 \cos D \sin D.$$

As T_C and T_V are opposed, for equilibrium

$$\cos D\frac{K}{T}\int_0^T (i_1 - i_2)(i_1 + i_2)dt = \cos D \sin D\frac{K'}{T}\int_0^T (i_1 - i_2)^2 dt.$$

Let θ be the phase difference of the currents i_1 and i_2. One of them lags, while the other leads, voltage *ab*.

$$i_1 = I_1 \sin \omega t.$$
$$i_2 = I_2 \sin (\omega t - \theta).$$

Consequently,

$$\cos DK\left(\frac{I_1^2}{2} - \frac{I_2^2}{2}\right) = \cos D \sin DK'\left(\frac{I_1^2}{2} + \frac{I_2^2}{2} - \frac{2I_1I_2}{2}\cos \theta\right);$$

and

$$\sin D = \frac{I_1^2 - I_2^2}{I_1^2 + I_2^2 - 2I_1I_2 \cos \theta}. \tag{34}$$

If the ratio I_2/I_1 is denoted by r,

$$\sin D = K\frac{1 - r^2}{1 + r^2 - 2r \cos \theta}. \tag{35}$$

Both r and θ are functions of the frequency.

The constants of the instrument are so proportioned that at the normal operating frequency, $r = 1$. As the deflection D is independent of the magnitudes of the currents, depending only on their ratio and their phase difference, the meter is unaffected by variations of line voltage so long as the permalloy vane is in a weak magnetic field. The inductances are wound on laminated iron cores provided with air gaps. The gaps are necessary, since in order that the tuning may be accomplished, the power factors must be low, and wave distortion must be avoided.

The circuit employed by the Weston Instrument Company is shown diagrammatically in Fig. 371. 1, 1 and 2, 2 are stationary crossed coils within which is pivoted the soft-iron needle N. As the two sets of coils are traversed by currents which differ in phase, they set up an elliptical, rotating field, and the needle takes up a position along the major axis, the direction of which is dependent on the frequency. The coils 2, 2, the inductance X, and the condenser together form a resonating circuit which is in tune at the normal operating frequency. As the resistance of X is small, the current in coils 2, 2 is relatively large at resonance, while that in coils 1, 1 is relatively small. Consequently, at standard frequency the position of the needle is sensibly along the axis of coils 2, 2. Variations of frequency, above and below normal, cause the needle to swing to right or left of the normal position.

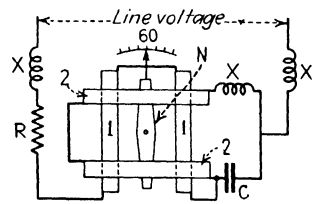

FIG. 371.—Diagram for frequency meter. (*Weston Electrical Instrument Company.*)

In 1888, Ayrton suggested that it was possible to determine the frequency of an alternating current by employing the principle of mechanical resonance, a suggestion that has been developed into commercial forms of frequency meters. In these instruments, there is a bank of steel reeds so tuned that the natural periods of successive reeds differ by, for example, one cycle per second. This bank of reeds is acted upon by an electromagnet traversed by current taken from the line. Only the reeds very nearly in tune with the frequency of the circuit respond visibly, the reed most nearly in tune showing the maximum amplitude. If it is exactly in tune, the amplitude is very large. This is well illustrated by Fig. 372, which shows the amplitude of vibration of a reed tuned to a frequency of 90 alternations when currents of various frequencies are sent through the magnet. A variation from 90 to 89.5

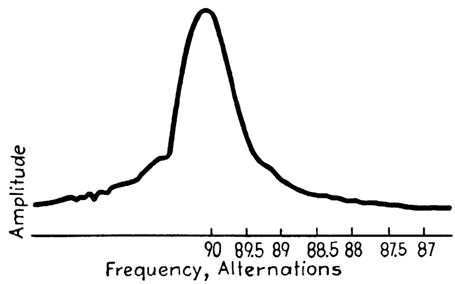

FIG. 372.—Showing effect of frequency on the amplitude of vibration of a reed in a frequency meter.

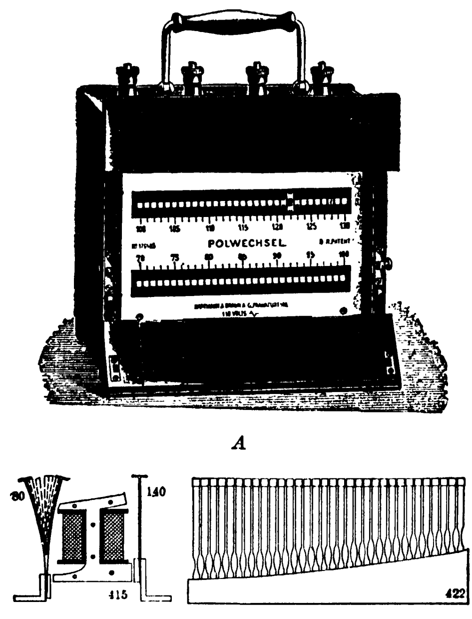

FIG. 373.—Hartmann and Braun frequency meter.

alternations per second reduces the amplitude over 50 per cent (compare with the vibration galvanometer, page 465).

In order to insure reliability and long life, the butts of the reeds must be firmly held. The arrangement adopted by the firm of Harmann and Braun and one form of the complete instrument are shown in Fig. 373.

If the reeds are *unpolarized*, they will be drawn toward the magnet at each alternation, so that the reed having twice the frequency of the current is the one to respond. If they are *polarized*, by either permanent or electromagnets, the reed having the same frequency as the current will have the maximum amplitude of vibration.

The reeds are approximately tuned by making them of different lengths, the final tuning being effected by altering their weights by filing away drops of solder placed at their outer ends just behind the white indexes. In the instrument made by Siemens and Halske, all the reeds are fixed to a single metallic bar which is so mounted on a spring support that it may be gently vibrated by the action of an electromagnet excited from the circuit. The particular reed which is in tune with the circuit responds with the maximum amplitude.

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CHAPTER X

GRAPHIC RECORDING OR CURVE-DRAWING INSTRUMENTS

Graphic recording instruments are those that automatically record their indications on a uniformly moving strip, or circular sheet of paper. Continuous and permanent records of the quantity that the instruments are adapted to measure are thus obtained.

Such instruments are extremely useful in investigating the power conditions in factories and in studying the cycles of operations of single machines,[1] where in many cases, the load fluctuates so rapidly that the use of indicating instruments is impracticable.

Continuous records are frequently important in central-station work. For instance, a registering ammeter in a feeder gives a record

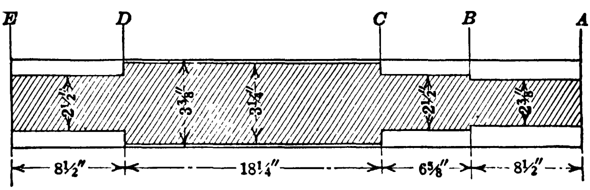


FIG. 374.—Piece to which cycle shown in Fig. 375 applies.

of the current and shows, if the clock is of good quality and properly regulated, when the feeder is put in and taken out of service, as well as the time when any abnormal conditions arise. Such data, if systematically kept, may be of great importance as evidence in adjusting disputes arising from accidents. The records obtained by a registering wattmeter show the customer's power consumption throughout the day and are useful in the determination of rates.

Figure 375 illustrates the application of a graphical recording instrument to the study of a particular machine. It shows the current taken by a direct-current motor which drives a roughing lathe. The cycle of operations is to be referred to Fig. 374 which shows the piece that is being turned. Corresponding points in the two figures bear the same letter.

The variation of power with depth of cut and also the time required for each operation are clearly shown.

A simple form of recording ammeter, in use for many years, is shown in Fig. 376. The current flows through the coil A giving rise to an attraction of the soft-iron disk B carried by the rod CD which passes freely along tne axis of the coil. The rod is supported on knife-edges by two flat springs *CC* and *DD* which are fixed at their lower ends; *DD* carries the pointer *E* to which the pen is attached. The record is made on a circular sheet of paper which is rotated at a uniform rate by the clockwork.

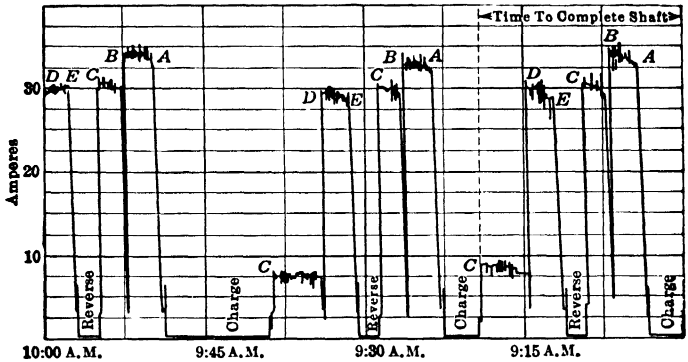


FIG. 375.—Curve showing typical cycle on a roughing lathe.

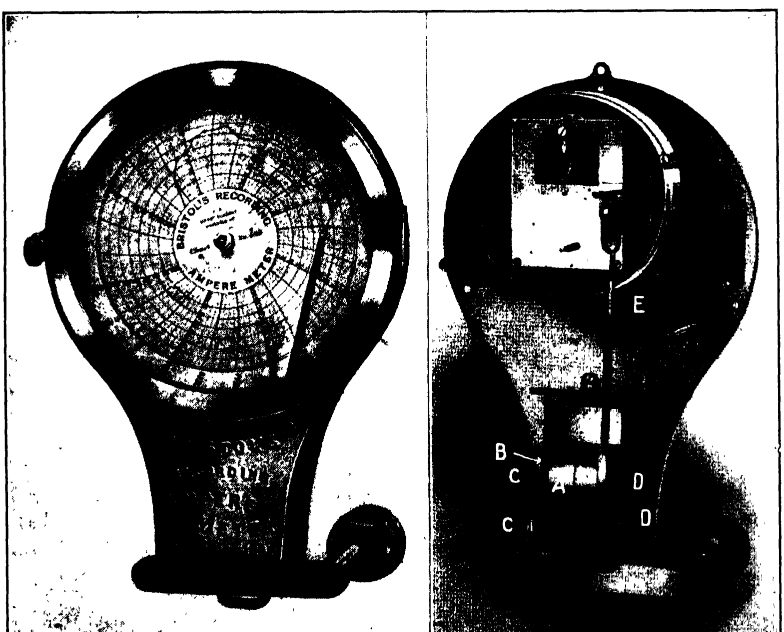


FIG. 376.—Bristol curve-drawing ammeter.

Ordinarily for central-station work the records are for a 24-hr. period. For special work this may be varied by using the appropriate clockwork. The pen, which rests on the paper continuously, is a little V-shaped trough cut away at one end so that only a fine point at the apex of the

V drags on the paper. The trough is supplied with an aniline-glycerin ink which is carried to the paper by capillary action.

Another instrument of the same class is shown in Fig. 377. A counter-balanced soft-iron core, consisting of a tube which projects perpendicularly from a disk of the same material, is attracted into the coil against the action of a spiral spring. By a simple lever the motion of the core causes the pen to travel over the chart. The tube and disk are slit to reduce eddy currents. An adjustable iron plug in the lower part of the solenoid allows the deflection, when the pointer is at the upper end of the scale, to be adjusted. The pointer is damped by a magnet and an aluminum damping disk of the usual form actuated by gearing from the pivot

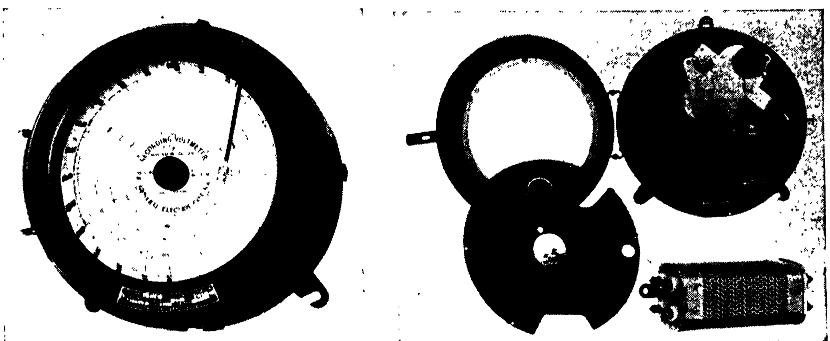


FIG. 377.—Curve-drawing voltmeter. *(General Electric Company.)*

carrying the pointer. The clock is ordinarily arranged so that the chart is for either a 12-hr. or a 24-hr. period.

Instruments like the two just described are useful when a moderate accuracy will suffice, for instance on a set of feeder panels where many such instruments must be installed, and considerable expense is not justified. If a high degree of accuracy is desired, more complicated arrangements must be used.

In many cases, a registering instrument should be capable of operating for a considerable period without attention, 1 or 2 weeks for example.

The clock should be of good quality, preferably of the synchronous-motor type or else motor wound, the motion of the paper should be positive, and the time scale uniform.

The special difficulties in the design of accurate instruments of this sort come from the pen friction which impedes the motion of the pointer. It is best that the scale be uniform and that the records be given on rectangular coordinates so that they may be integrated readily by a planimeter. A uniform time coordinate may be attained by driving the paper by a metal drum having projecting pins which engage with perforations at the edges of the record paper.

In the better class of instruments, the effect of pen friction is minimized or eliminated:

1. By giving the movable system a high torque and employing a very strong controlling spring. By using soft-iron instruments of proper design, ammeters and voltmeters may be constructed in which the pen-friction error is reduced to 1 or 2 per cent of the full-scale deflection, with a reasonably small consumption of power (20 watts in a voltmeter; 25 watts in an ammeter).

2. By providing the pointer with a stile which ordinarily swings clear of the paper but which is periodically pressed against it by an electromagnet and imprints a dot. This arrangement is useful where the phenomena under investigation vary slowly. It has proved of service in those forms of registering thermoelectric pyrometers which are registering millivoltmeters in reality. A modification is to have an arrangement by which a high-tension spark is caused to pass periodically from the stile through the paper, thus giving the record in the form of dots.

3. By using the relay principle; in this case, the movable system has to control only the position of the pen, the power necessary to move it being supplied from an external source. Relay instruments are, in general, somewhat complicated, but wattmeters, direct-current ammeters, and voltmeters, as well as self-balancing potentiometers, may be designed to have uniform scales and thus give records on rectangular coordinates. The relay principle has been applied very successfully in recording pyrometers. These instruments are, in reality, self-balancing potentiometers adapted to the measurement of the small e.m.fs. (15 to 50 mv.) set up by thermojunctions. It is a cardinal principle in the design of such self-balancing arrangements that the tendency to return the device to balance shall be proportional to the departure from the balanced conditions. This prevents troublesome "overshooting" when the departures are small.

4. By having recourse to electronic devices, thus avoiding the use of ordinary electrical contacts in the relays. This class of recorders may be made very sensitive.

A direct-action recording wattmeter for polyphase circuits is shown in Fig. 378. Here the friction is overcome by the high torque of the movable system. The two rings surrounding the coils are laminated soft-iron shields. The working parts consist of two substantial electrodynamic wattmeters, both movable coils being rigidly attached to the same stem which is supported by a regular watt-hour meter pivot resting on a ring-stone, end-stone bearing; the upper end of the stem is provided with a watt-hour meter pivot guided by a ring stone. The weight of the movable system is about 270 g. The power consumption per element at 115 volts, 5 amp. is, for the potential circuit, 12 watts; for the current circuit, 27 watts.

Magnetic damping is employed, the magnets and damping sector being just below the lower element and to the front. Strong nonmagnetic controlling springs are used giving a full-load torque of 340 g.-mm. The pen consists of a siphon of capillary tubing, one end of which dips into the ink reservoir while the other end rests lightly on the record strip. The iridium pen tip is very hard, takes a high polish, and does not corrode. With the proper quality of paper and the pressure correctly adjusted, the pen friction is reduced to about 0.65 g.-mm.

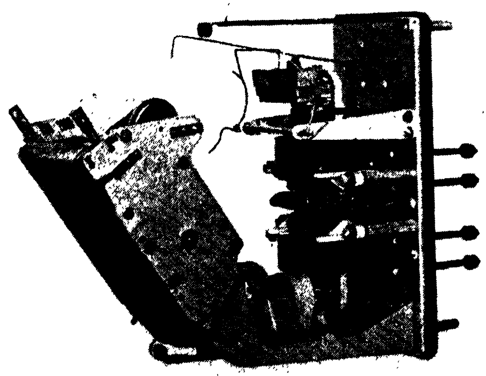


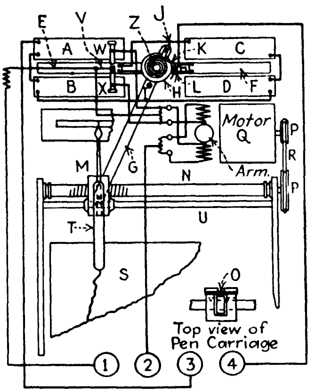
FIG. 378.—Curve-drawing polyphase wattmeter. (*General Electric Company.*)

Relay Instruments.—A relay-type voltmeter is shown in Fig. 379. The six coils are arranged as in the Kelvin balance.

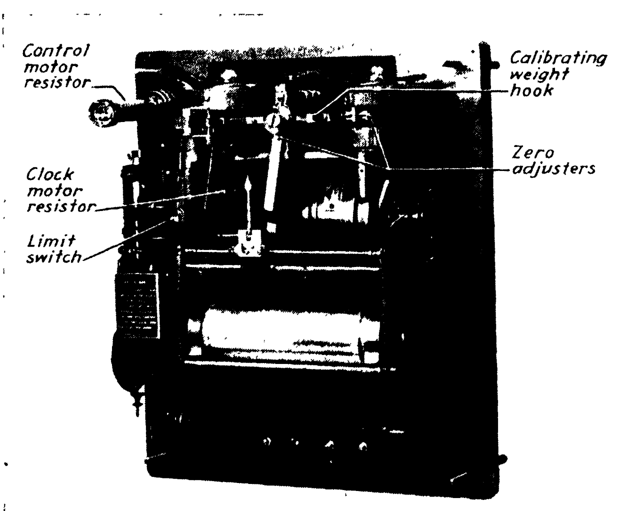
The two movable coils are pivoted at Z. The inner end of the controlling spring K is attached to the pivot of the coils, the outer end being attached to the spring arm G. The lower end of this arm slides in a properly shaped slot in the nut M. The reversible motor Q moves the pen across the record by means of the screw N, carrying with it the lower end of the spring arm G. W and X are the fixed contacts of the relay. The movable contact V is carried by the beam of the balance. If, through variation of voltage, contact is made between V and W, the armature turns in such a direction that the tension on the spring tends to return the beam to the mid-position. This continues until the contact is broken. If the contact is between V and X, the armature turns in the opposite direction, changing the tension on the spring accordingly. The net result is that the beam is kept in balance.

Registering Self-balancing[4,5] Potentiometers and Bridges.—The essential portions of the self-balancing device made by the Leeds and Northrup

Company and used by them in various potentiometer and bridge recorders, as well as in industrial controllers, are shown in Fig. 380. A shaft, which is free to turn, carries the large wheel 1 and through a clutch device carries a cylinder on which the potentiometer wire is wound. A second



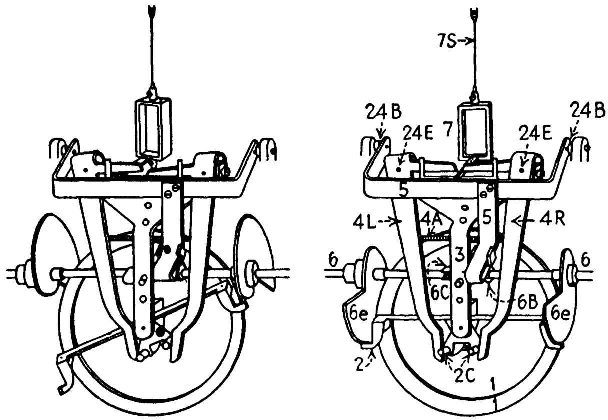
(a)



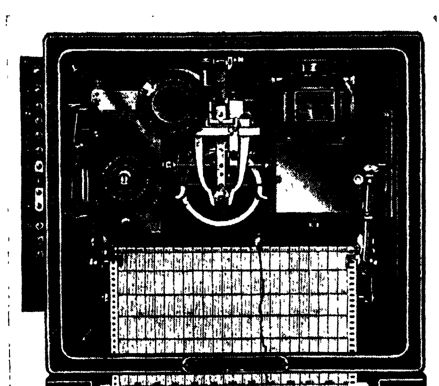
(b)

Fig. 379.—Recording voltmeter, Westinghouse Electric and Manufacturing Company. (a) Diagram of Type R recording instrument elements. (b) Type R recording instrument with cover removed.

cylinder, also controlled by the clutch, carries the rheostat wire for adjusting the potentiometer current. Contacts with the slide wires are made by stationary contact springs. Therefore, with the first clutch in gear, any rotation of the wheel 1 changes the position of the contact on the slide wire, altering the balance of the potentiometer or Wheatstone bridge if resistance devices are involved. Pivoted coaxially with 1, but entirely separate from it, is the crossbar 2, provided at both ends with little cork shoes and carried by the spring 3 which is attached to the frame of the recorder. Two pins 2*C* are attached to 2 at some distance



A



B

FIG. 380.—Self-balancing arrangement for recording potentiometer. (*Leeds and Northrup Company.*)

from the axis of rotation. Pressing against either one of the pins causes a tilting of 2. The whole structure of which 2 forms a part is given a slight motion in the direction of the axis of 1 by means of the cam 6*C*. This cam is located in the horizontal shaft, which is rotated continuously by a motor at the rate of about 30 r.p.m. The shaft also carries two cams 6*e* which bear against the crossbar 2 and at each revolution of the shaft return 2 to the horizontal position. The galvanometer is a robust moving-coil instrument with a taut suspension and is provided with a substantial pointer. Pivoted on the frame of the recorder at 24*E* are two bell-crank levers which extend downward and may press against either of the pins 2*C*. The structure 5 is pivoted in the frame of the recorder at 24*B*. Once at every revolution of the shaft, a cam 6*B* raises 5 a definite amount, bringing it in contact with the galvanometer pointer and giving the pointer a slight vertical displacement. If the pointer is at its zero position, it falls between the two inner extremities of the bell-crank levers, and the raising and lowering of 5 have no effect. But if the potentiometer is out of balance, the galvanometer is deflected, and one of the bell-crank levers is tilted by an angular amount proportional to the deflection. Consequently, the arm 2 is rotated about its pivot in the proper direction by an amount proportional to the lack of balance. Immediately this rotation has taken place, the cam 6*C* causes the cork shoes at the extremity of the arm 2 to engage with the wheel 1, so that 1 must turn, carrying the slide wire with it. One of the cams 6*e* then returns 2 to the horizontal position, thus adjusting the potentiometer contact. The shoes are then disengaged and leave 1 free again. Thus, thirty times a minute, the contact point on the slide wire is displaced toward the balance position by an amount proportional to the lack of balance, and thus equilibrium is quickly established. The galvanometer is in its free position about two-thirds of the time. Every 12 min. the first clutch releases the setting mechanism from the slide wire and attaches it to the standard-cell adjustment. The potentiometer current is thus kept at its standard value. The adjusting mechanism is then transferred to the potentiometer wire. The cams are so adjusted that the necessary sequence of operations is maintained. The record strip is given a uniform motion in the usual manner. The carriage with the recording pen is carried across the record strip by an endless band wrapped around the slide-wire cylinder, and thus a continuous record is obtained. It is obvious that the voltage scale is uniform. A switch is provided which permits records to be obtained from several different thermojunctions in the form of dots, each dot having an identifying number.

Electronic Recorders.[2,3]—Both the General Electric Company and the Westinghouse Company have developed recorders in which the marking pen is controlled by the deflection of a sensitive moving-coil instrument of regular design.

In the General Electric Company instrument, this is accomplished by the ingenious combination of mirrors and photocells indicated in Figs. 381 *A* and *B*. By means of the condensing lens *L*, the light from a strong source is directed upon the mirror *A* which is attached to the shaft of the pilot, or basic instrument. If desired, the regular pointer may be removed to reduce the moment of inertia. The light is reflected by A upon the stationary spherical mirror B, the curvature and location of which are such that whatever the angular displacement of A, the light is always reflected to the mirror C. The mirror C is attached to the shaft of a powerful recording galvanometer which moves the pen across the record strip.

Although the light always strikes C, its angle of incidence varies with the deflection of A, and the beam leaving C strikes the curved mirror D at a point determined by the angular deflection of C. The curvature of mirror D is such that if the beam falls to the right of point D it is reflected to the photocell P_1; if it falls to the left of D, it is reflected to the photocell P_2. Whenever A and C are parallel, that is, have the same angular displacement, the reflected beam falls on the dividing

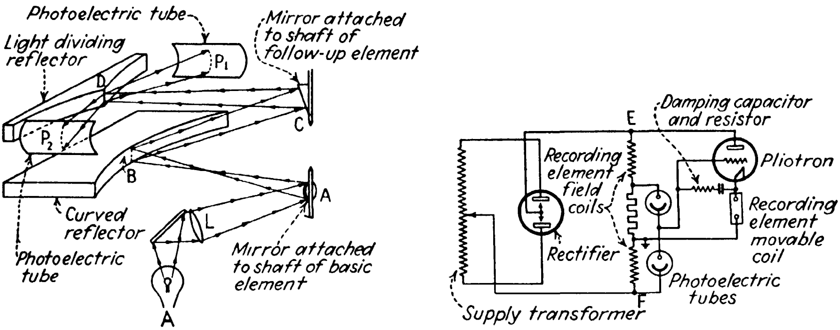


FIG. 381.—Diagram for electric recorder. (*General Electric Company.*)

line of D_1 and both photocells receive light. The recording element is a powerful, moving-coil galvanometer in which the usual permanent magnet is replaced by a strong electromagnet. The movable coil is traversed by the plate current of the pliotron. The circuit EF which is supplied from the rectifier, consists of the field coils of the recorder galvanometer together with an added resistance. The drop in EF furnishes the plate voltage for the three-element vacuum tube or pliotron, and also furnishes the bias for the two photoelectric cells. These cells are connected in series and operated beyond the saturation point, so that the tube current is independent of the applied voltage. If the angular deflections of C and A are the same, the reflected beam falls on the dividing line of D, and the photoelectric currents in the two tubes are the same. There is then no resultant current to flow to the grid capacitance. If the angular displacement of the pilot element A changes, the beam is deflected to the right or left of D, and either P_1 or P_2 receives the greater light. In consequence, the grid voltage of the pliotron would normally go immediately to the extreme positive or negative value. To obviate the hunting between the electrical circuit and the recording element which in consequence would ensue, the speed of response must be slowed down. This is accomplished by inserting between the filament and the grid a capacitor in series with a resistor. The charge of the capacitor must alter before the grid voltage can change. It is thus possible to introduce any desired amount of time lag. Stability is thus secured, and the recording pen follows the pilot element without hunting.

For direct-current work the pilot element may be a miniature instrument with a mirror substituted for the pointer. A full-scale deflection may then correspond to 75 microamp., the resistance of the instrument being 350 ohms. If a low-watt-consumption element is employed, the

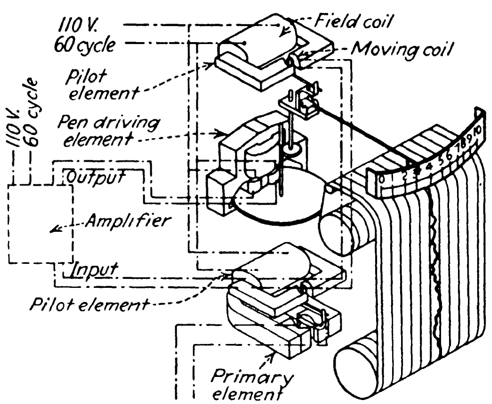


FIG. 382.—Schematic diagram of electric recorder. (*Westinghouse Electric and Manufacturing Company.*)

full-scale deflection may be obtained with 300 microamp., the resistance being 10 ohms.

The repeater mechanism designed by the Westinghouse Company for transferring the indications of a sensitive millivoltmeter or milliammeter to the recording pen is shown diagrammatically in Fig. 382, where the sensitive instrument is designated as the "primary" element. Coaxial with the moving coil and controlled by it is a second coil on the "pilot element" which turns in the field of an alternating-current electromagnet, the magnet being actuated by the regular alternating-current supply service. The field coil of a second similar pilot element is connected in parallel with the field coil of the first pilot element. The position of the moving coil of the second element is controlled by the pointer which carries the marking pen. The power for moving the pen is supplied through gearing from the pen-driving motor, which is similar in construction to the motor element of an induction watt-hour meter.

The potential coil is energized by the alternating-current supply circuit. The current coils are energized by the output of the amplifier. The input to the amplifier is derived from the two moving coils of the pilot element which are connected in series opposition.

Thus when the movable coils of the two pilot elements occupy the same relative positions in the alternating fields, the induced e.m.fs. in these coils are balanced. If the coil of the primary element is deflected, the balance is disturbed, and there is a resultant e.m.f., about 50 millivolts per degree, the magnitude and direction of which are dependent on the relative displacements of the two movable coils. This resultant e.m.f. is applied to the grid of the power amplifier and thus controls the magnitude and phase of the current in the current coils of the reversible driving element. The fields set up by the potential and current coils differ in phase as well as in magnitude. Therefore, as in the induction watt-hour meter, a driving torque is exerted on the disk. The torque is to the right or left, depending on the relative directions of the potential-coil and current-coil fields. The magnitude of the torque is dependent on the relative displacements of the two moving coils in the pilot elements. As the disk rotates, moving the marking pen on the record strip, the coil of the upper pilot element is turned in a direction to establish the balance of the induced e.m.fs. The speed of turning is proportional to the difference in the displacements of the coils of the two pilot elements; that is, the speed is proportional to the need for adjustment. Hunting is thus avoided. The motor stops when equality of induced e.m.fs. has been established. Thus the pen follows the motion of the pointer of the primary element. The only extra load on the primary element is that due to the restoring torque of the leading-in wires of the lower pilot element. The pen requires about 1 sec. to traverse the entire scale, with an overshoot of 2 per cent. If the time is increased to 1.25 sec., the overshoot is inappreciable; with normal excitation the torque gradient is about 2 g.-cm. per degree. This corresponds to the torque of a 720 g.-cm. spring and is from five to ten times that of the ordinary direct-acting recorder. A 20 per cent decrease in the alternating-current voltage still leaves the torque gradient higher than that of the usual direct-acting instrument. Changes in the vacuum tubes affect the speed but not the accuracy of the device. By combining a moving-coil element with a Rectox rectifier, alternating currents as low as 0.5 milliamp. may produce a full-scale deflection. By the use of such a rectifier, alternating-current voltages as low as 1 volt may give a full-scale deflection with a potential circuit resistance of 500 ohms.

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CHAPTER XI

INSTRUMENT TRANSFORMERS

In the development of high-voltage alternating-current systems of transmission and distribution, it has been found necessary to remove the various instruments, as well as the devices used to actuate the switch gear, from direct contact with the line circuits and to operate them by means of properly constructed transformers, since direct connection between the high-tension lines and the devices on the front of the switchboard must be avoided. This method of operation through transformers reduces to a minimum the possibility of personal injury to the station attendants and enables them, especially in emergencies, to operate the apparatus with confidence, thus contributing to maintaining continuity of service.

It is frequently necessary to meter very large alternating currents, and as it is highly desirable to avoid the expense of carrying heavy leads to the switchboard, current transformers are used.

By properly choosing the current and potential transformers, it is possible to use instruments and switchboard devices wound for 5 amp. and 110 volts, for installations of all capacities. This reduces the instrument cost and is now the accepted American practice.

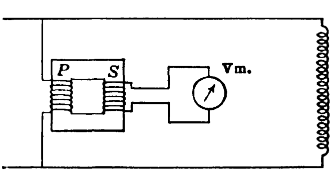


FIG. 383.—Showing manner of using potential transformer.

Where it is necessary to measure a high voltage, a potential transformer is used to reduce this voltage to a more convenient and safe value for measurement.

The potential transformer in Fig. 383 is diagrammatic only; as actually constructed, the primary and secondary windings are superposed.

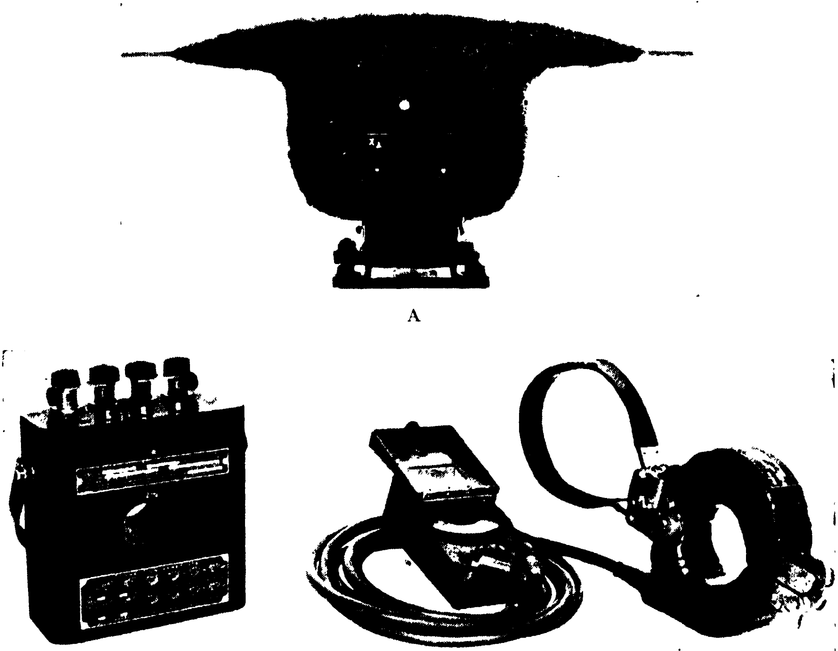
As potential transformers are usually operated under practically fixed conditions of applied voltage, frequency, and number and character of the instruments in the secondary circuit, one would expect them to be instruments of precision, and experience shows this to be the case. They are much more permanent than the instruments that they actuate. When used for voltage measurements, only the ratio of transformation is important, and this should be constant under the varying operating conditions. The line voltage is given by

$$V = (\text{ratio}) \times (\text{volts at instrument}).$$

Switchboard voltmeters are graduated so that the line voltage is read directly from the dial. If the transformer ratio is not constant, the combination of transformer and voltmeter may be calibrated as a unit.

The current transformer is used in cases where very large alternating currents must be measured and also where the current coils of instruments must be isolated from high-voltage lines.

Different designs of current transformers are shown in Fig. 384.



A

B C

Fig. 384.—Current transformers.

Transformer A is for use on 7,500-volt installations. The distance between the primary and the secondary terminals and the frame is to be noted. Transformer B is a portable instrument designed for general testing purposes. The ratio is variable, for the primary is formed by thrusting a flexible cable through the central opening, the number of primary turns being thus readily altered. Transformer C has its iron core made in two parts which are hinged together so that the magnetic circuit can be opened when the screw clamp is loosened. This allows the transformer to be placed around a cable and permits the current in a single conductor cable to be measured without interrupting the service.

Figure 385 indicates the connections for a simple current measurement. For obvious reasons, the current transformer is often called a series transformer in distinction from a potential or shunt transformer.

For a current measurement,

$$I = (\text{ratio}) \times (\text{amperes through instrument}).$$

Convenience dictates that the ratio be constant. This requirement involves a difficulty, for, as the load changes, the transformer must operate under widely varying conditions. Experiment shows that the ratio is not constant, being to a certain extent dependent on the strength of the current that is being measured and also on the number and the character of the instruments in the secondary circuit.

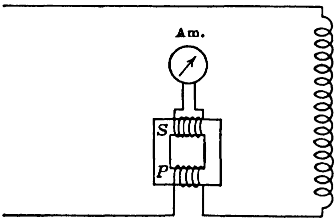


FIG. 385.—Showing manner of using current transformer.

In power measurements on high-voltage circuits, it is necessary to use both current and potential transformers. As shown in Fig. 386, the connections are such that the current and voltage as well as the power are measured.

With the connections shown, at high power factors, the power is given to a fair degree of approximation by

$$P = (\text{ratio of current transformer}) \times (\text{ratio of potential transformer}) \times (\text{reading of wattmeter}).$$

Another difficulty is encountered here. In the discussion of power measurement, it was repeatedly emphasized that for accurate work, the

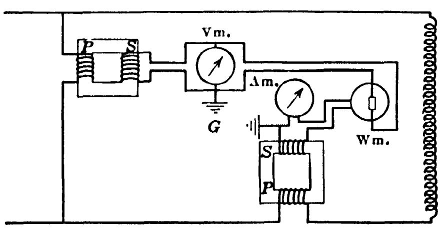


FIG. 386.—Showing connections for measuring power, voltage, and current, using instrument transformers.

currents in the fixed and movable coils of an electrodynamic wattmeter must have the same phase relation as the current and voltage of the load.

It is one of the imperfections of instrument transformers that they introduce false phase relations. With the potential transformer, the voltage at the secondary terminals is not in *exact* opposition to the voltage applied to the primary. The departure from exact opposition is small, to be sure, and of the order of magnitude of 10 min. of arc under normal operating conditions. This phase angle may be either a lag or a lead and depends on the frequency as well as on the number and character of the instruments in the secondary circuit.

The current transformer is normally subject to a much greater phase-angle error than the potential transformer, but by careful design the phase angle may be reduced to a few minutes of arc (see Fig. 388). The displacement depends on the magnitude of the primary current, on the frequency, and on the number and character of the instruments in the secondary circuit. The burden, that is, the load in the secondary of a current transformer, is specified in terms of impedance in ohms, or volt-amperes, and the power factor.

It will be seen that the errors introduced into power measurement by the use of transformers are those due to the variation of ratio of both the current and potential transformers, as well as those due to the phase displacement in both transformers.

It is possible to determine the ratio and phase angle and to make the corresponding correction so that accurate results may be obtained even at low power factors, where the phase-angle errors are most pronounced. These matters are of great practical importance, for instrument transformers are used with wattmeters in acceptance tests of alternating-current apparatus, as well as with watt-hour meters on all high-capacity alternating-current circuits.

In all instrument transformers, the primary must be thoroughly insulated from the secondary and from the core and case, so that there is little chance of puncturing the insulation. In addition, the secondary circuit should be grounded so that the operator is protected even though the insulation between the primary and secondary breaks down. Grounding the secondary circuit prevents errors due to accumulation of electrostatic charges on the instruments. The coils must be held in place so firmly that there is no chance of mechanical injury when short circuits occur. The primary and secondary terminals must be so far apart that there is no liability of an arc forming between the two circuits due to line surges.

The ordinary vector diagram for a transformer is shown in Fig. 387. It is not drawn to scale and gives no idea of relative numerical magnitudes.

By means of the diagram, a general explanation may be obtained of the phenomena occurring in instrument transformers.

For the potential transformer, the ratio used is

$$\frac{V_1}{V_2}.$$

The magnitude and phase of its components differ from E_1/E_2, which is the ratio of the number of turns, or the true "ratio of transformation." The phase angle of the potential transformer is designated on the diagram by γ. This angle is the departure from exact opposition of V_1 and V_2. It is usually very small and in reality may be an angle of either lead or lag. The phase angle of the current transformer will be denoted by β.

Figure 388 shows the results of experimental determinations of the constants of certain commercial current and potential transformers. It gives an idea of the order of magnitude of the changes to which the ratios and phase angles are subject. Similar curves, giving representative values of ratios and phase angles, which have been determined by testing a few transformers all made in accordance with the same specifications,

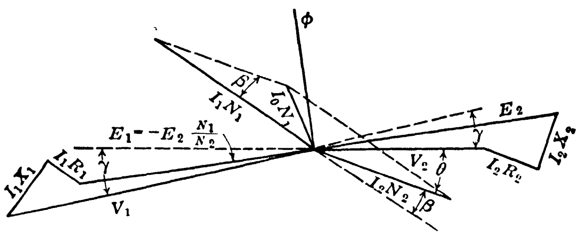


FIG. 387.—Vector diagram for transformer.

may be obtained from the makers of such transformers and are sufficiently accurate for much commercial work.

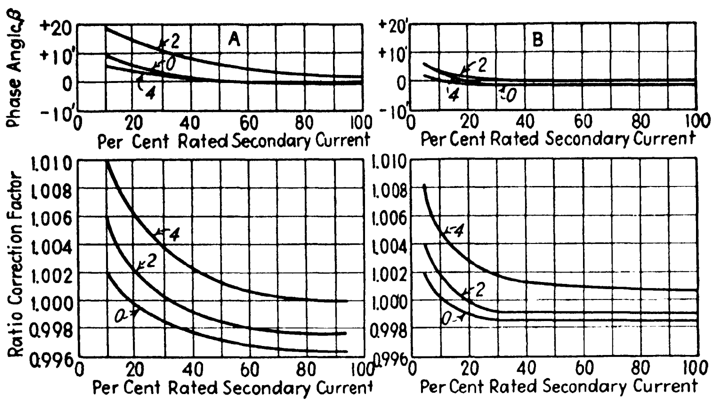
In tests where the greatest accuracy is desired, the ratios and phase angles for the transformers should be determined at the frequency and voltage and with the same connected burden of instruments and leads as are to be used in the subsequent work.

When using instrument transformers in connection with wattmeters and power-factor meters, it is necessary to know the relative polarities of the secondaries of the transformers with respect to their primaries, as otherwise the meters may be connected so that they will not read properly.

Some manufacturers arrange the internal connections so that the corresponding terminals of the primary and secondary are always of like polarity. Thus in Fig. 389*A* the + and − signs indicate the polarities at a particular instant. The primary and secondary currents are shown diagrammatically as flowing in opposite directions.

The + on the primary is the terminal at which the current enters the transformer, and the + on the secondary is the terminal at which the current leaves the transformer to enter the external circuit.

Some manufacturers cross the secondary connections giving the relative polarity shown in Fig. 389*B*.



Secondary Burden at 5.0 Amp., 60 cycles

No.	Z, ohms	VA	PF
0	0.240	6 0	1.00
2	0.768	19 2	0.89
4	1.804	45 1	0 54

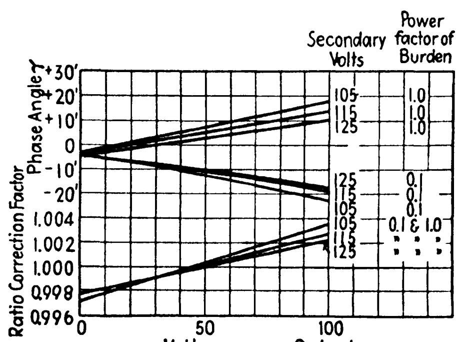


FIG. 388.—Examples of instrument-transformer characteristics. Silicon steel cores.

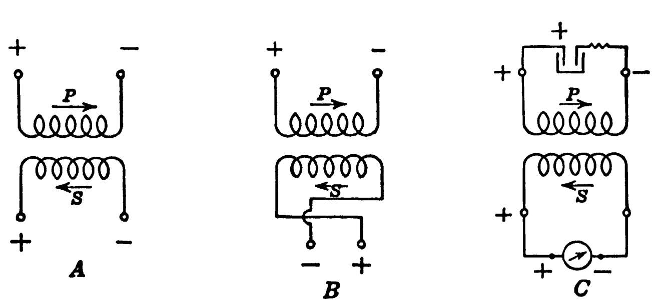


FIG. 389.—Pertaining to polarity tests of instrument transformers.

A simple method of testing the polarity is as follows: Connect a direct-current voltmeter to one of the windings (Fig. 389*C*), noting which terminal is connected to the + post of the voltmeter. Touch *for an instant* the terminals of a dry cell to the terminals of the other winding, making the polarity such that the meter reads up scale on closing the circuit. The two corresponding + terminals will be the carbon of the cell and the + post of the voltmeter. A very small current should be used, as otherwise the iron may be left in a highly magnetized condition, and the ratio and phase angle of the transformer may be altered from their normal values. Demagnetization after the test is an advisable precaution.

When stating the conditions under which instrument transformers are used (especially current transformers), it is necessary that the inductance and resistance, that is the burden, of the external secondary circuits be specified, for then there can be no misunderstanding of the conditions under which the transformer is operating.

If the conditions are such that the readings of the 5-amp. wattmeters and ammeters, which are commonly used in the secondary circuits of current transformers, are in the lower parts of the scales, there is a temptation to substitute lower range instruments, for instance 3 amp., in order to obtain a good scale reading. It must not be forgotten that such instruments will heavily tax the current transformer and alter both its ratio and phase angle, for the burden placed on the transformer by the 3-amp. equipment is such that the voltage at the transformer terminals must be increased to approximately 2.8 times its original value.

Resistance and Inductance of the Current Coils of Typical Alternating-current Instruments

Range, amp.	Ammeters		Wattmeters	
	Resistance, ohms	Inductance, henrys	Resistance, ohms	Inductance, henrys
3	0.17	0.00033	0.16	0.00036
5	0.059	0.00012	0.056	0.00013

Current Transformers.[1]—It is important that when the transformer is being operated the secondary circuit always be kept closed. If it is opened, there will be no demagnetizing effect due to the secondary, and, as the primary current is fixed by the load on the line, the flux will rise to a high value. This will increase the iron losses to such an extent that the insulation may be injured by the heat so that at some subsequent time it may be punctured by a moderate voltage or perhaps burned out. The voltage at the secondary terminals will be large, and the secondary insulation may be injured. There is also the possibility of disagreeable, if not fatal, shocks.

Opening the secondary circuit when the transformer is being operated may alter both ratio and phase angle, for the circuit opening may occur when the iron is fully magnetized. In the subsequent use of the instrument, the iron will not be put through its normal hysteresis cycle, and the exciting current therefore will be altered. At low loads, the alteration may amount to several per cent. For the same reason, direct current, used for the purpose of calibrating the instruments, must never be sent through either the primary or secondary of the transformer. Under ordinary operating conditions, these changes in the magnetic state of the core will persist, since, in instrument transformers, the magnetic circuit

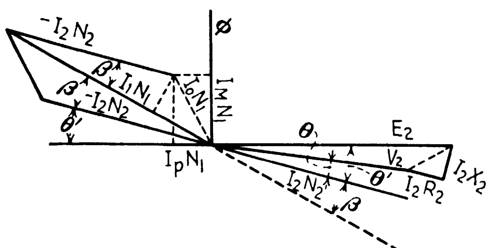


FIG. 390.—Diagram for current transformer.

is unusually good. The core may be demagnetized in the usual manner, an alternating current being sent through the primary, and gradually decreased from its full-load value to zero, the secondary being open.

Owing to the necessity of having considerable insulation between primary and secondary coils and of having the terminals of the coils widely separated, there may be a pronounced stray field in the neighborhood of current transformers. Hence shielded instruments should be used.

Theory of the Current Transformer.[1]—To examine the theory of the current transformer, transfer from Fig. 387 those parts of the diagram shown in Fig. 390,

Let N_1 and N_2 = number of turns in primary and secondary.
I_1 and I_2 = primary and secondary current.
I_0 = total exciting current.
I_M = magnetizing component of exciting current.
I_P = power component of exciting current.
θ' = angle between E_2 and secondary current.
β = phase-defect angle of transformer.

Throughout the discussion of instrument transformers, the phase-defect angle will be called simply the *phase angle.*

Reference to the figure will show that both the ratio and phase angle are dependent on the exciting current, for obviously, if I_0 were zero, $I_1N_1 = I_2N_2$, and β, the phase angle of the transformer, or the departure of the primary and secondary currents from exact opposition, would also become zero. Such an ideal transformer can never be realized, for there must be enough ampere-turns to give the requisite flux through the core; and with the iron core, the exciting current must have an energy component sufficient to account for the hysteresis and eddy-current losses. When an iron core is used, I_0N_1 may be resolved into two components: the magnetizing component I_MN_1 along the flux, and the power component I_PN_1 along E_1.

Referring to Fig. 390 and projecting I_2N_2 and I_0N_1 on the line I_1N_1,

$$I_1N_1 = N_2I_2 \cos \beta + I_0N_1 \cos \left[90 - \theta' - \beta - \sin^{-1} \frac{I_P}{I_0} \right].$$

Therefore

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \cos \beta + \frac{I_M \sin (\theta' + \beta) + I_P \cos (\theta' + \beta)}{I_2}. \qquad (1)$$

As β is a small angle, its cosine is very nearly unity; the second member on the right-hand side of the equation is therefore a correction term. Hence

$$\text{Ratio} = \frac{I_1}{I_2} = \frac{N_2}{N_1} + \frac{I_M \sin \theta' + I_P \cos \theta'}{I_2} \qquad \text{approx.}$$

The expression for the phase angle is determined as follows: From the diagram,

$$\tan \beta = \frac{I_0N_1 \sin \left[90 - \theta' - \beta - \sin^{-1} \frac{I_P}{I_0} \right]}{I_2N_2 \cos \beta}$$

$$= \frac{I_MN_1 \cos (\theta' + \beta) - I_PN_1 \sin (\theta' + \beta)}{I_2N_2 \cos \beta}. \qquad (2)$$

As β is a small angle,

$$\beta = \frac{N_1}{N_2}\left[\frac{I_M \cos \theta' - I_P \sin \theta'}{I_2} \right] \qquad \text{approx.}$$

As shown in Fig. 387, I_2 leads I_1 reversed. However, β may change sign when the load is increased. The change is dependent on the characteristics of the core, the character of the burden, and the leakage flux.

The properties of the core material are conveniently shown in curves similar to those of Fig. 391. Assuming sine waves, let

B = flux density in coil.
A = area of coil in centimeters squared.
W = Mass of core, in kilograms.
E_2 = induced e.m.f. in secondary necessary to produce I_2 in the burden.
I_2 = secondary current.
V.A. = wattless volt-amperes.
f = frequency.

Then

$$B = \frac{E_2 10^8}{AN_2 f 4.44}$$

$$I_M = \frac{(\text{wattless volt-amp. per kilogram})W}{(E_2/N_2)N_1} \quad \text{at flux density } B, \tag{3}$$

$$I_P = \frac{(\text{Watts per kilogram})W}{(E_2/N_2)N_1} \quad \text{at flux density } B, \tag{4}$$

$$\frac{I_P}{I_M} = \frac{\text{watts per kilogram}}{\text{wattless volt-ampere per kilogram}} \quad \text{at flux density } B, \text{ variable}$$

with change of B, that is with change of E_2.

Therefore

$$\text{Ratio} = \frac{N_2}{N_1}\left[1 + \frac{W\{(\text{wattless volt-ampere per kilogram}) \sin \theta' + (\text{watts per kilogram}) \cos \theta'\}}{E_2 I_2}\right]. \tag{1'}$$

$$\text{Phase angle} = W\left[\frac{(\text{Wattless volt-ampere per kilogram}) \cos \theta' - (\text{watts per kilogram}) \sin \theta'}{E_2 I_2}\right]. \tag{2'}$$

By the use of Eqs. (1′) and (2′) the performance of straight-through or bushing-type transformers, with secondaries uniformly and closely wound on cores of ring stampings, may be calculated if certain provisions are fulfilled. Either the primary conductor is centrally located with the return lead so distant that the current in this lead does not magnetically affect the core or the primary conductor is divided into conductors symmetrically located with respect to the core. In both these cases, the secondary reactances are negligible.

The cores of this type of transformer, the straight through, may be made of ring stampings or of a long ribbon of core material coiled like a clock spring. Also, they may be built up in the rectangular form, using rectangular or L-shape stampings. The secondary winding may be concentrated, or distributed as uniformly as practicable, along the magnetic circuit. The effect of changes in the arrangement or position of the primary turns depends on the distribution of the secondary winding. By such changes Price and Duff[7] showed that it is possible to introduce an equivalent negative reactance into the secondary, resulting in changes in the ratio and phase angle. The straight-through type of current transformer is used for very heavy currents and has recently been studied by Park.[7] In general, a symmetrical location of the primary and secondary windings with respect to the core is best. Park gives numerous curves showing the effect on the ratio and phase angle of changes in the arrangement of the windings together with data concerning the transformers tested.

The dependence of the ratio and phase angle on the properties of the core is shown clearly in (1′) and (2′). Both I_M and I_P should be reduced to a minimum, and both should be proportional to I_2 if the ratio and phase angle are to be made as nearly as possible independent of the secondary current and of the character of the secondary load. In the current transformer, I_0 varies with the saturation of the core, that is, with the load current. To reduce I_M, the core should be of high permeability, of large cross section, and of short magnetic length. I_P is rendered small by choosing for the core a material with small hysteresis and eddy-current losses and operating at a very low flux density. The impedance of the instruments forming the secondary load or burden should be small so that the requisite secondary e.m.f. is furnished by a small flux density.

As there is iron in the magnetic circuit, the wave form of the current in the secondary cannot be the same as that in the primary. But with periodic phenomena, the distortion while measurable by refined methods is so small that it is of no practical importance even though the wave form is very much distorted. Owing to the action of the iron, large currents of a transient nature, such as occur in short-circuit tests, which rise to values much higher than those for which the transformer was designed, are not correctly reproduced.

The characteristics of current transformers are shown in two curves: (1) The *ratio-correction factor curve*, which shows, for specified burdens, the factor, at various percentages of full-load current, by which the *marked* ratio must be multiplied to obtain the *true* ratio; (2) the *phase-angle curve*, which shows, for specified burdens and at various percentages of full-load current, the departure of the primary and secondary currents from exact opposition. Examples of these curves are given in Fig. 388.

Inspection of Eqs. (1) and (2) (page 597) will show that a certain amount of control over the ratio-correction factor may be had by varying the number of secondary turns. For instance, in a transformer having a marked ratio 8:1, the actual number of turns were $N_1 = 25$, $N_2 = 196$. By varying the secondary turns, the ratio-correction curve may be moved up or down as a whole, while the phase-angle curve is not greatly affected by small changes in the number of secondary turns. The magnitudes of the ratio-correction factor and the phase-angle errors depend not only on the number of primary and secondary turns but on the characteristics of the burden, the impedance of the secondary of the transformer itself, and the magnitudes of I_M and I_P. The *variation* of the correction factor and the phase-angle error is due to the fact that I_M and I_P do not increase and decrease together and do not vary proportionally with I_2. If the ratio I_P/I_M were constant, and I_M were proportional to I_2, the ratio-correction curve and the phase-angle curve would become straight lines parallel to the horizontal axis.

Figure 391 shows the characteristics of silicon steel and of hipernik, the two materials used for current-transformer cores. The values were

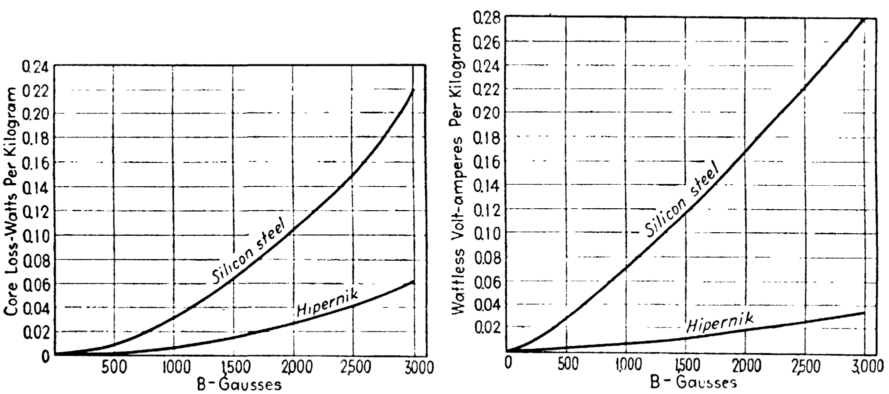


FIG. 391.—Characteristics of core materials for instrument transformers. Wattless volt-amperes per kilogram. Hipernik and 4 per cent silicon steel at 60 cycles per second.

determined at 60 cycles per second by the Tinsley and Gall alternating-current potentiometer.[1] The development by engineers of the Westinghouse Company of hipernik, an approximately 50-50 nickel-iron alloy containing a little manganese, to which *after fabrication* a special heat treatment has been applied, has furnished a material by the use of which I_M and I_P may be much reduced. Figure 391 shows what may be accomplished in reducing core losses and wattless volt-amperes by substituting hipernik for silicon steel.

The characteristics of hypernik are high permeability at low flux densities, high maximum permeability, and low hysteresis loss. The resistivity and therefore the eddy-current loss is comparable with that of 4 per cent silicon steel. To reduce the eddy-current loss, which in 14-mil lamination is about 80 per cent of the total loss (at $B = 10{,}000$ and $f = 60$), thin laminations should be employed. There is no aging effect.

As hipernik saturates at a comparatively low flux density, its particular field of usefulness is in the construction of measurement transformers of high accuracy. It is not of service if subjected to excessive m.m.fs., as, for instance, in tripping devices for oil switches, but for such uses silicon steel, a much less expensive material, is entirely satisfactory.

The behavior of current transformers may be modified by the use of shunts.[6] If the shunts are nonreactive and are applied to either the primary or the secondary, the ratio is increased and the phase angle is reduced. However, the ratio becomes more sensitive to changes of burden. A primary shunt either noninductive or capacitive protects the transformer and its burden against high-frequency surges.

Primary and secondary capacitive shunts tend to decrease the ratio and also to decrease the phase angle. They make the transformer sensitive to variations in frequency and wave form. Inductive shunts are not generally useful in reducing ratio and phase-angle errors.

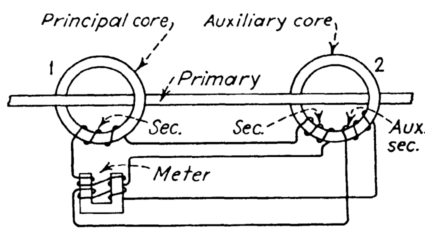


FIG. 392.—Diagram for Brooks two-stage transformer. Shown as two separate straight-through or bushing-type transformers.

The defects of current transformers being due to the presence of the exciting ampere turns, any device that compensated for their effect in the measuring circuit would render the over-all accuracy that of a perfect transformer. This compensation may be effected to a high degree by the use of the multistage principle devised by H. B. Brooks,[3] which is applicable to any current transformer of good design. In Fig. 392 the application to a straight-through or bushing type of transformer is indicated. For high-voltage circuits this form of current transformer is advantageous on account of the high dielectric strength of the bushing which separates the primary and secondary, and on account of the necessarily wide separation of the primary and secondary terminals. The likelihood of trouble from punctured insulation and flashover is thus minimized. There are, however, serious handicaps, for the ampere-turns may be low, and the magnetic length of the coil is large. The performance of this type of current transformer may be greatly improved, if ordinary coil material is used, by adopting the multistage principle; also, nickel-iron cores may be employed.

Referring to Fig. 392, at 1 is a high-grade transformer of the usual construction; at 2 is a second transformer having a ratio of secondary to primary turns equal to the desired ratio of primary to secondary current. The primaries of the two transformers are in series. The two secondaries are also in series and are in series with one set of current coils on the meter. The primary and secondary ampere-turns on transformer 2 act jointly and differentially on the core and serve as the primary ampere-turns for the auxiliary secondary. This auxiliary secondary delivers to a second and duplicate set of current coils on the meter a close approximation to the current which must be added to the secondary current produced by transformer 1 to obtain a combined effect in the meter which will approximate closely to that which would be due to a perfect transformer. There is a slight second-order error, since the compensating device is not itself compensated. However, this error may be neglected. In practice, the two transformers are combined in a single structure. An objection to the original Brooks two-stage arrangement is that it requires a specially wound meter with two independent current coils of equal strengths. Brooks points out and gives experimental results showing that the main and auxiliary secondaries should act in practical independence, and suggests that interaction between them may be compensated by inserting a mutual inductance of the proper value and sign.

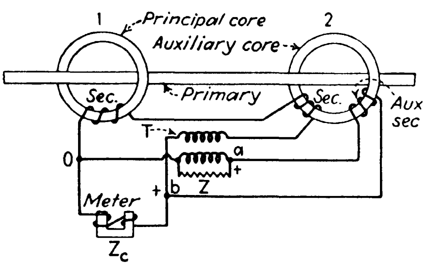


FIG. 393.—Boyajian and Skeats development of Brooks two-stage transformer.

Boyajian and Skeats[3] have shown how to utilize the two-stage principle so that only a single wattmeter current coil is required. This development is shown diagrammatically in Fig. 393.

A mutual-inductance or transformer T has its primary in the secondary circuit of transformers 1 and 2, while its secondary is in the auxiliary secondary circuit of 2. There will be a fall of voltage from b to O through the meter coils Z_c and a rise of voltage from O to a due to the transformer T. With the circuit from the auxiliary secondary disconnected, the inductive shunt Z is adjusted until the potential difference between a and b is 0, that is, until the main and auxiliary secondaries can act independently. The current from the auxiliary secondary on transformer 2 is thus free to circulate in Z_c without interference from the current from transformer 1. The necessity for a specially wound meter is thus avoided.

In some types of current transformers the General Electric Company employs the method of compensation devised by M. R. Wilson.[4] Reference to Fig. 388 will show that the ratio-correction factor decreases as the load is increased. If it were possible properly to increase the number of secondary turns as the load is increased, the effect would be to hold the ratio-correction factor at the value existing at low loads. In effect, the Wilson method accomplishes this. It was first shown experimentally that if the secondary turns were placed on the core in two unequal groups, both wound in the same direction, and the part of the core through the smaller group was provided with a magnetic shunt of a material that has a low reluctance at low flux densities (permalloy), then, as the line current increases, the flux due to the m.m.f. of the smaller group of turns acting in conjunction with the main flux increases the flux in the permalloy shunt, decreasing its permeability relative to that of the main core. As saturation is approached, this effect throws a greater portion of the flux into the secondary winding and increases the number of effective secondary turns. In order to produce this effect, it is necessary to operate the shunt on the descending portion of its permeability curve so that its reluctance will increase more rapidly than that of the portion of the main core which it shunts and which operates lower down on its permeability curve.

Although the ratio errors can thus be compensated, this construction was not considered commercially feasible, and the alternative arrangement shown in Fig. 394 was adopted.

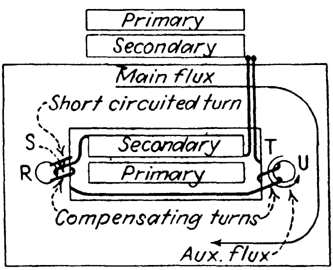


FIG. 394.—Wilson method of compensation.

Two holes are made in the core, one at *RS* and one at *TU*, and a few of the secondary turns (1 or 2 per cent) are wound about either *T* or *S*. The same results are attained with either location of these turns, provided the direction of the windings is such that the flux due to them is opposite to the main flux in the larger section of the core, as indicated in Fig. 394. The sections *T* and *S* function like the permalloy shunt in the original arrangement. Assuming, for purposes of explanation, that the added turns embrace the smaller sections *T* and *S*, when the current is increased the flux through *T* and *S* increases relatively to that through *U* and *R*, the permeability at *T* and *S* is decreased, and flux is transferred from *T* to *U* and from *S* to *R*. The effect is to increase the effective secondary turns and raise the ratio-correction factor for the larger values of the current. The ratio-correction factor as a whole is then adjusted by changing the *total* number of secondary turns.

To improve the phase angle, a lag band is employed at *S*. The current induced in this lag band causes the main flux, and therefore the secondary current, to lag slightly and thus tends to compensate for the natural lead of the secondary current. The action is analogous to that of the lag band used to shift the phase of the useful potential-coil flux in induction watt-hour meters (page 498). An exact analytical explanation of this method of compensation is not possible, because of the complexity of the saturation effects.

Another method of compensation has been suggested by Schwager,[5] who has shown how to design a small corrective burden to be placed in parallel with the instruments in the transformer secondary. This burden automatically changes its characteristics when the secondary terminal voltage changes, that is, when I_2 changes. The lack of proportionality between I_M, I_P, and I_2 is thus compensated. The design is shown in Fig. 395*A*.

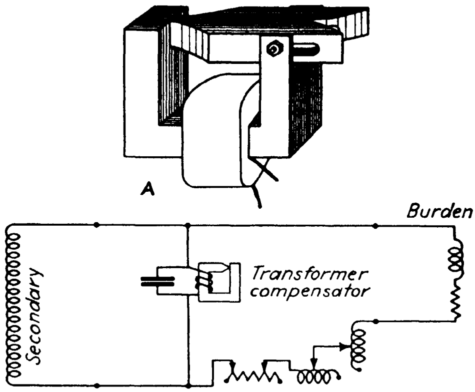


FIG. 395*A* and *B*.—Schwager method of compensation.

The parts of this reactive transformer compensator, or burden, saturate at different values of secondary voltage and can be adjusted to give the result desired. To complete the adjustment, a capacitive shunt is used in parallel with the reactive burden. Figure 395*C* shows the over-all characteristics of a bushing-type transformer with a silicon-steel core when the Schwager device is used.

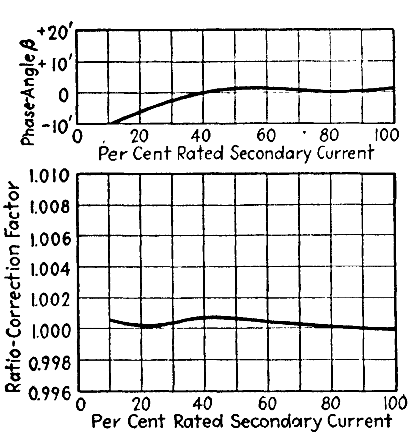


FIG. 395*C*.—Ratio-correction and phase-angle curves. Bushing-type transformer and compensator. Ratio 400:5. Silicon steel core. Schwager system. 60 cycles. 50 per cent power factor, 50 volt-amp.

To maintain a high degree of accuracy with this method of compensation, the current transformer should be operated with the secondary burden for which the reactor was originally adjusted. To make this possible, the equipment includes a variable compensating burden of resistors and reactors (Fig. 395*B*). Any change in the burden of instruments is offset by making an equal and opposite change in the burden compensator.

Theory of Potential Transformer.—In the theory of the potential transformer, the two most important quantities are the equivalent reactance and the equivalent resistance, both being determined by the usual short-circuit test. The exciting current and the reactance and resistance of the primary windings must also be taken into account, but their combined influence is much less than that of the equivalent impedance of the transformer.

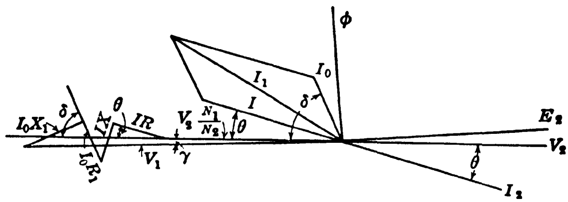


FIG. 396.—Diagram for potential transformer.

Referring to Fig. 396,

Let N_1 and N_2 = number of primary and secondary turns.
R and X = equivalent resistance and equivalent reactance of the transformer.
R_1 and X_1 = resistance and reactance of the primary windings.
I_0 = exciting current.
V_1 and V_2 = primary and secondary terminal voltages.
θ = power-factor angle of the secondary load.
δ = angle between I_0 and V_2 reversed.
γ = phase angle of the transformer, or the angle between V_1 and V_2.

The component of the primary current which is in opposition to the secondary current is $I_2\frac{N_2}{N_1} = I$. To determine the transformation ratio, project V_1 on V_2, and add the components of the projection.

$$V_1 \cos \gamma = V_2\frac{N_1}{N_2} + IR \cos \theta + IX \sin \theta + I_0R_1 \cos \delta + I_0X_1 \sin \delta.$$

Therefore

$$\frac{V_1}{V_2} = \frac{1}{\cos \gamma}\left[\frac{N_1}{N_2} + \frac{IR \cos \theta + IX \sin \theta + I_0R_1 \cos \delta + I_0X_1 \sin \delta}{V_2}\right].$$

In potential transformers, γ is usually considerably less than 1°, so $\cos \gamma = 1$ very nearly, and

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} + \frac{IR \cos \theta + IX \sin \theta + I_0R_1 \cos \delta + I_0X_1 \sin \delta}{V_2}. \quad (5)$$

To determine the phase angle, project V_1 on a line perpendicular to V_2.

$$V_1 \sin \gamma = IR \sin \theta - IX \cos \theta + I_0R_1 \sin \delta - I_0X_1 \cos \delta.$$

For small angles $\sin \gamma = \gamma$ approximately, so

$$\gamma = \frac{1}{V_1}[IR \sin \theta - IX \cos \theta + I_0R_1 \sin \delta - I_0X_1 \cos \delta]. \qquad (6)$$

In the potential transformer, the small change of exciting current I_0 between no load and full load produces a negligible effect on the results.

Application of Corrections for Ratio and Phase Angle.[8]—It is customary to express the results of tests of instrument transformers in the form of curves similar to those in Fig. 388. Reference to curves having the proper load characteristics will give the ratio and phase angle to be used in any particular test.

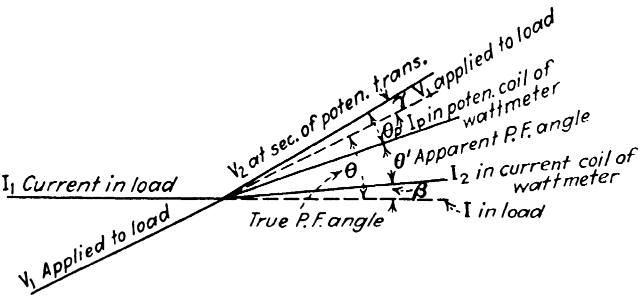


FIG. 397.—Phase diagram for power measurement using instrument transformers.

When power measurements are made, there are to be considered, in addition to the ratios:

1. The phase displacement in the potential transformer.
2. The phase displacement in the current transformer.
3. The phase displacement in the potential coil of the wattmeter.

These phase relations are indicated in Fig. 397.

The phase angle γ of the potential transformer is shown as a lead, though V_2 may either lag or lead V_1. If γ is a lag angle, its algebraic sign is to be reversed.

The wattmeter gives an indication proportional to the mean product of the current in its two coils, or to $\cos \theta'$. The apparent power factor of the load is

$$\cos \theta' = \frac{\text{wattmeter reading}}{\text{volt-amperes}}.$$

The true power-factor angle is

$$\theta = \theta' + \beta + \theta_P - \gamma;$$

and the true power factor is

$$\cos \theta = \cos (\theta' + \beta + \theta_P - \gamma) = \frac{\text{true watts}}{\text{volt-amperes}}.$$

Therefore

$$\text{True watts} = \frac{\cos \theta}{\cos \theta'} \times (\text{wattmeter reading}), \tag{7}$$

or

$$[1 - \tan \theta' \tan (\beta + \theta_P - \gamma)] \times (\text{reading}) \qquad \text{approx.}$$

The true watts must be multiplied by the appropriate current and potential transformer ratios to give the power in the circuit. The effect of the phase angles increases as the power factor of the load decreases.

To illustrate, consider the following data:

The losses in the instruments are neglected.

Inductive load, 25 cycles.

Reading of ammeter, corrected for calibration, 1.5 amp.

Reading of voltmeter, corrected for calibration, 110.5 volts.

Reading of wattmeter, corrected for calibration, 50.0 watts.

θ_P is negligible.

Nominal ratio of current transformer, 8:1.

Ratio of current transformer from its curve, 1.0125:8.

Phase angle β, from current transformer curve, 1°.79.

Nominal ratio of potential transformer, 10:1.

Ratio of potential transformer from its curve, 0.995:10.

Phase angle γ from potential transformer curve, 0°.12.

V_2 lags $-V_1$.

Apparent power factor $= \cos \theta' = 50/110.5 \times 1.5 = 0.3016$.

Apparent power-factor angle, 72°.45.

True power-factor angle $= \theta = 72°.45 + 1°.79 + 0°.12 = 74°.36$.

True power factor $= \cos \theta = 0.2696$.

True value of the load $= (0.2696/0.3016) \times 50 \times 8.1 \times 9.95 = 3{,}602$ watts, which is the corrected reading of the wattmeter multiplied by the proper transformer ratios.

The application of phase-angle corrections is facilitated by tables prepared by H. B. Brooks.[8]

Effect of Phase Angles in Three-phase Power Measurements.—Consider a balanced three-phase load where the power is measured by the two-wattmeter method using transformers of the same characteristics in both phases. When the power factor of the load is low, the wattmeter in which the currents are more nearly in phase indicates the larger part of the load. The reading of the other wattmeter, which works under much more adverse conditions as to the phase displacement of the currents in its coils, is small, so that even if the percentage error in its readings is large, the percentage error introduced by it into the measurement of the power will be small.

It has been shown that the power in a single-phase circuit is given by

$$P = \frac{\cos\theta}{\cos\theta'} \times \text{(reading)}.$$

Assuming the transformer ratios to be unity, the power that *should* be indicated by the two instruments is

$$P_1 = VI\cos(30° + \theta).$$
$$P_2 = VI\cos(30° - \theta).$$

So

$$P = VI[\cos(30° + \theta) + \cos(30° - \theta)] = VI\sqrt{3}\cos\theta.$$

The effect of the phase angles of the transformers and the phase angle of the potential circuit of the wattmeter is to reduce the phase difference of the currents in the fixed and movable coils of the two wattmeters by the angle $\beta + \theta_P - \gamma$. Hence

$$\text{(Reading)}_1 = VI\cos[30° + \theta - \beta - \theta_P + \gamma] = VI\cos[30° + \theta'].$$
$$\text{(Reading)}_2 = VI\cos[30° - \theta + \beta + \theta_P - \gamma] = VI\cos[30° - \theta'].$$
$$\text{(Reading of meters)} = VI\sqrt{3}\cos\theta'.$$
$$\text{So } P = \text{(reading)}\,\frac{\cos\theta}{\cos\theta'}.$$

That is, the fractional error due to the phase angles of the transformer and the potential circuits is the same as the error occurring in a single-phase measurement at the same power factor.

If the load is not balanced, the readings of each instrument should be corrected as in a single-phase measurement.

Use of Transformers with Watt-hour Meters.—It is customary to use instrument transformers with induction watt-hour meters. An additional complication is introduced, especially at low power factors, by the phase angles of the transformers and also by the adjustments of the phase relations of the fluxes in the potential circuits of the meters.

To be ideally perfect when used with a current transformer, an induction meter would have to be lagged so that the sum of the time-phase angle between the potential-coil flux and the current in the secondary of the current transformer and the power-factor angle of the load would be 90 deg. This suggests that the watt-hour meter and the transformers should be treated as a unit when the lag adjustment is made. A perfect adjustment is not possible, for the ratios and the phase angles vary with the load.

In preparation for careful industrial tests, the combination of watt-hour meter and instrument transformers may be calibrated in the laboratory without undue expenditure of power by using fictitious loads (see page 520). The test conditions may be reproduced, *wave form excepted.*

By altering the light-load adjustment of the meter, an attempt is sometimes made to correct for the variation of the ratio of the current transformer, which is most troublesome at light loads. If the ratio increases at light loads, the adjustment is set so that the meter runs a little fast, creeping being avoided. There is no means of making even an approximate adjustment for the variation of the phase angle.

DETERMINATION OF THE RATIO AND PHASE ANGLE OF INSTRUMENT TRANSFORMERS

If accurate measurements are to be made, it is necessary to know the ratios and the phase angles of the current and potential transformers. Among the various methods that have been devised for their determination, those based on the potentiometer principle have become generally accepted. They give results of high accuracy and are convenient because they do not require currents and voltages to be held at fixed values.

Ratio and Phase Angle of Current Transformer.[9]—The determination of the ratio of a current transformer is simply the determination of the ratio of two currents. To make such a measurement using two *direct currents*, the connections shown in Fig. 398 are used.

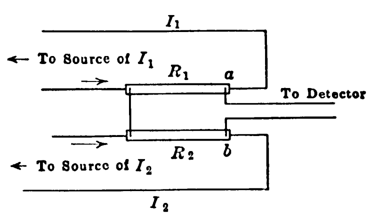


FIG. 398.—Connections for determining the ratio of two direct currents.

If the detector stands at zero,

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}.$$

When two *alternating currents* are to be compared, the resistances R_1 and R_2 should be noninductive; I_1 and I_2 must have the same frequency and wave form and be either in time phase or have a *fixed* phase difference.

In applying this general method to the testing of current transformers, I_1 and I_2 are the currents in the primary and secondary circuits; ordinarily they will be in phase within 2 deg. or less. As a time-phase difference exists, it is impossible by adjustment of the resistances R_1 and R_2 to balance the two IR drops.

In order to bring the detector to zero, unless it is separately excited, it is convenient to introduce into the detector circuit an e.m.f. in quadrature with I_2 by the method used by Hughes and by Heaviside in their inductance bridges, a variable mutual inductance, or air-core transformer, *m* being employed (Fig. 399).

The arrangement of apparatus shown diagrammatically in Fig. 399 has been used by a number of experimenters,[10] the chief differences between them being in the form and manner of using the detector.

Sharp and Crawford use a D'Arsonval galvanometer,[10] the current being rectified by a synchronous reversing key driven by a synchronous

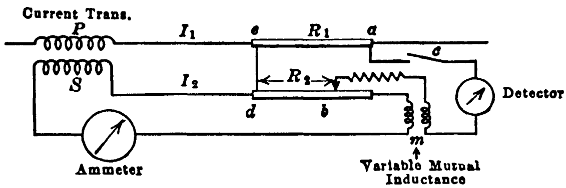


FIG. 399.—Connection for determining characteristics of current transformers.

motor. The brushes or their equivalent are so mounted that the time phase of the commutation may be altered, in order to match with the time phase of the potential difference between a and b.

Agnew and Silsbee[10] use a vibration galvanometer of special design. The circuits are as in Fig. 400.

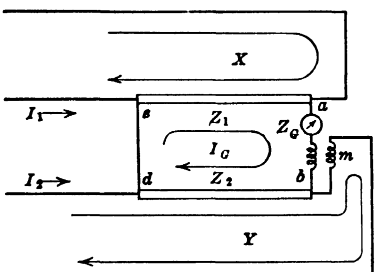


FIG. 400.—Mesh diagram for Fig. 399, pertaining to determination of ratio and phase angle of current transformers.

By a double adjustment of R_2 and m the vibration galvanometer may be brought to zero.

With the mesh currents as indicated,

$$I_G(Z_1 + Z_G + Z_2) - XZ_1 - YZ_2 + j\omega mY = 0$$

$$I_G = \frac{XZ_1 + YZ_2 - jm\omega Y}{Z_1 + Z_G + Z_2}.$$

At balance, $I_G = 0$, or

$$XZ_1 + YZ_2 = +j\omega mY.$$

$$X = -I_1, \qquad Y = I_2,$$

and

$$Z_1 = R_1 + j\omega L_1,$$
$$Z_2 = R_2 + j\omega L_2.$$

Substituting,

$$I_1 = \left[\frac{R_2 - j\omega(m - L_2)}{R_1 + j\omega L_1}\right]I_2 = \left[\frac{R_1R_2 - \omega^2L_1(m - L_2)}{R_1^2 + \omega^2L_1^2}\right]I_2 - j\omega\left[\frac{L_1R_2 + R_1(m - L_2)}{R_1^2 + \omega^2L_1^2}\right]I_2.$$

$$I_1^2(R_1^2 + \omega^2L_1^2) = I_2^2(R_2^2 + \omega^2(m - L_2)^2).$$

$$\text{Ratio} = \frac{I_1}{I_2} = \sqrt{\frac{R_2^2 + \omega^2(m - L_2)^2}{R_1^2 + \omega^2L_1^2}} = \frac{R_2}{R_1}\sqrt{\frac{1 + \dfrac{\omega^2(m - L_2)^2}{R_2^2}}{1 + \dfrac{\omega^2L_1^2}{R_1^2}}}. \qquad (8)$$

Therefore

$$\frac{I_1}{I_2} = \frac{R_2}{R_1} \quad \text{approx.}$$

$$I_2 \text{ leads } I_1 \text{ by } \beta, \qquad \tan\beta = \frac{\omega L_1 R_2 + \omega R_1(m - L_2)}{R_1 R_2 - \omega^2 L_1(m - L_2)}. \qquad (9)$$

The inductances L_1 and L_2 are made as small as possible. If they could be neglected,

$$\tan\beta = \frac{\omega m}{R_2}.$$

The same results may be derived more simply by use of a vector diagram (Fig. 401). Take I_2 along the horizontal axis.

At balance, the P.D._{ea} must equal the P.D._{dc} (see Fig. 399); that is, the points a and c in Fig. 401 must be made to coincide.

$$I_2{}^2R_2{}^2 + I_2{}^2\omega^2(m - L_2)^2 = I_1{}^2R_1{}^2 + I_1{}^2\omega^2L_1{}^2,$$

and

$$\frac{I_1}{I_2} = \sqrt{\frac{R_2{}^2 + \omega^2(m - L_2)^2}{R_1{}^2 + \omega^2L_1{}^2}}.$$

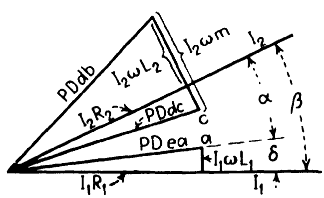


FIG. 401.—Vector diagram for Fig. 399.

When a and c are made to coincide, by varying R_2 and m,

$$\tan\beta = \tan(\alpha + \delta);$$

$$\tan\alpha = \frac{\omega(m - L_2)}{R_2};$$

$$\tan\delta = \frac{L_1\omega}{R_1};$$

or

$$\tan\beta = \frac{+\dfrac{\omega(m - L_2)}{R_2} + \dfrac{L_1\omega}{R_1}}{1 - \dfrac{\omega^2(m - L_2)L_1}{R_1R_2}} = \frac{\omega L_1R_2 + \omega R_1(m - L_2)}{R_1R_2 - \omega^2L_1(m - L_2)}.$$

With good transformers the quantity m is small; both R_1 and R_2 are so-called *noninductive* resistors. Though reduced to a minimum, L_1 and L_2 may be of importance when phase angles are determined.

A variable mutual inductance designed for this measurement is shown in Fig. 224.

The arrangement in Fig. 399 is an alternating-current potentiometer of limited range. The underlying idea, to compensate by mutual inductance for a lack of phase coincidence, is susceptible of being developed to produce an instrument generally applicable to measurements with sinusoidal currents.

Ratio and Phase Angle of Potential Transformers.—With slight modification, Poggendorf's method of comparing an e.m.f. and a potential difference may be applied to the determination of the voltage ratio and phase angle of potential transformers.

For comparing two *direct* potentials, the connections shown in Fig. 402 may be used.

If the detector stands at zero,

$$\frac{V_1}{V_2} = \frac{R_1 + R_2}{R_2}.$$

In testing potential transformers, V_1 and V_2 (Fig. 403) are replaced by the primary and secondary terminal voltages, and either Z_1 or Z_2 must contain an adjustable reactor by which the potential difference across Z_2 may be brought into time phase with V_2. This is necessary on account

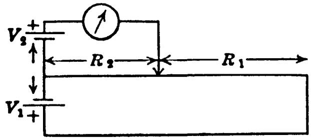


FIG. 402.—Connections for determining the ratio of a direct-current potential difference to a direct-current e.m.f.

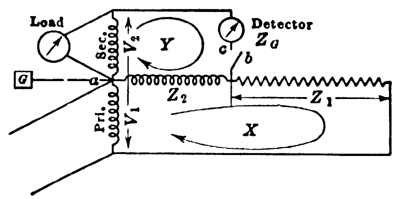


FIG. 403.—Connections for determining ratio and phase angle of potential transformers.

of the phase angle of the transformer. If the adjustable reactor were not used, the detector could be brought to a minimum but not to zero.

The resistor Z_1 should be nonreactive, free from both inductance and capacitance effects. With very high voltages it may be necessary to use shielded resistors (page 127) to avoid capacitance effects. Either Z_1 or Z_2 must be adjustable. The detector may be a vibration galvanometer.

To find the condition of balance, the mesh equations of Fig. 403 are

$$X(Z_1 + Z_2) - YZ_2 + V_1 = 0.$$
$$Y(Z_G + Z_2) - XZ_2 - V_2 = 0.$$

The galvanometer current is

$$I_G = Y = \frac{V_2(Z_1 + Z_2) - V_1Z_2}{Z_G(Z_1 + Z_2) + Z_1Z_2}.$$

In this, as in other similar cases involving networks, the equation for Y may be written at once from the solution for the direct-current case by replacing the resistances by impedance operators.

For balance, $I_G = Y = 0$, and

$$V_1 = \frac{(Z_1 + Z_2)}{Z_2}V_2.$$

If Z_1 is without reactance, Z_2 must be reactive. Then

$$V_1 = \left[\frac{(R_1 + R_2) + j\omega L_2}{R_2 + j\omega L_2}\right]V_2.$$

$$V_1 = \left[\frac{R_2(R_1 + R_2) + \omega^2 L_2^2}{R_2^2 + \omega^2 L_2^2}\right]V_2 - j\left[\frac{\omega L_2 R_1}{R_1^2 + \omega^2 L_1^2}\right]V_2$$

$$V_1^2(R_2^2 + \omega^2 L_2^2) = V_2^2[(R_1 + R_2)^2 + \omega^2 L_2^2].$$

$$\text{Ratio} = \frac{V_1}{V_2} = \sqrt{\frac{(R_1 + R_2)^2 + \omega^2 L_2^2}{R_2^2 + \omega^2 L_2^2}} = \frac{R_1 + R_2}{R_2}\sqrt{\frac{1 + \dfrac{\omega^2 L_2^2}{(R_1 + R_2)^2}}{1 + \dfrac{\omega^2 L_2^2}{R_2^2}}}$$

or

$$\text{Ratio} = \frac{R_1 + R_2}{R_2} \quad \text{approx.} \tag{10}$$

$$V_2 \text{ leads } V_1, \text{ and } \tan\gamma = \frac{\omega L_2 R_1}{R_2(R_1 + R_2) + \omega^2 L_2^2}. \tag{11}$$

As the phase angle may be either positive or negative, it is necessary to be able to give Z_2 either a positive or a negative reactance. This may

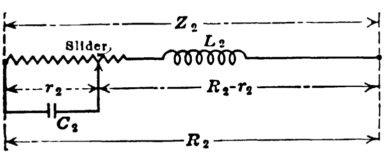


FIG. 404.—Arrangement for obtaining positive and negative reactance in potential-transformer tests.

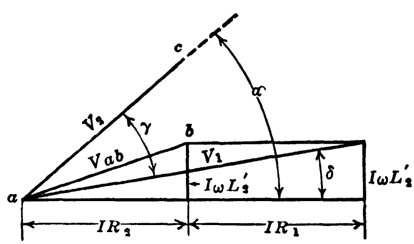


FIG. 405.—Vector diagram for Fig. 403.

be accomplished if Z_2 is made up as in Fig. 404. The condenser C_2 is shunted by a variable noninductive resistance r_2, and L_2 is an inductance.

$$Z_2 = \left[R_2 - r_2 + \frac{r_2}{1 + \omega^2 C_2^2 r_2^2}\right] + j\omega\left[L_2 - \frac{C_2 r_2^2}{1 + \omega^2 C_2^2 r_2^2}\right]. \tag{12}$$

If $\omega^2 C_2^2 r_2^2$ is negligible compared to unity,

$$Z_2 = R_2 + j\omega[L_2 - C_2 r_2^2], \tag{13}$$

or the circuit acts as if it had a resistance R_2 and an inductance

$$(L_2 - C_2 r_2^2).$$

The resistance r_2 may be varied by the slider. The inductance L_2 may have a fixed value and can be placed so that its field will not affect the vibration galvanometer and also so that it is free from the effeot of the field due to other apparatus.

The vector-diagram solution for the ratio and phase angle, when the connections of Fig. 403 are used, is obtained as follows:

Referring to Fig. 405, the direction and magnitude of the line *ab*, representing the drop across Z_2, may be controlled by adjusting R_2 and

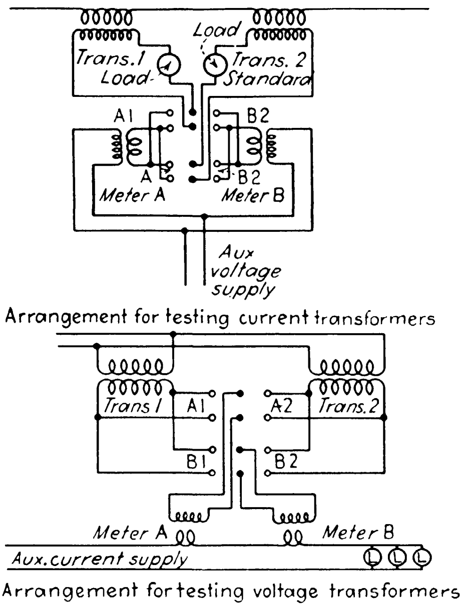


Fig. 406.

L_2, and *b* may be made to coincide with *c*. When this has been done, at balance, if L_2' is the effective inductance of Z_2, or $(L_2 - C_2r_2^2)$,

$$V_2^2 = I^2(R_2^2 + \omega^2 L_2'^2),$$

and

$$V_1^2 = I^2(R_1 + R_2)^2 + I^2\omega^2 L_2'^2.$$

$$\text{Ratio} = \frac{V_1}{V_2} = \sqrt{\frac{(R_1 + R_2)^2 + \omega^2 L_2'^2}{R_2^2 + \omega^2 L_2'^2}} = \frac{R_1 + R_2}{R_2} \quad \text{approx.}$$

To obtain the phase angle,

$$\gamma = \alpha - \delta.$$

For balance, $\tan \alpha = \dfrac{L_2'\omega}{R_2}$, and $\tan \delta = \dfrac{L_2'\omega}{R_1 + R_2}$.

V_2 leads V_1 by γ, and

$$\tan \gamma = \frac{\dfrac{L_2'\omega}{R_2} - \dfrac{L_2'\omega}{R_1 + R_2}}{1 + \dfrac{L_2'^2\omega^2}{R_2(R_1 + R_2)}} = \frac{\omega L_2' R_1}{R_2(R_1 + R_2) + L_2'^2\omega^2}.$$

Comparison Tests of Instrument Transformers.[11]—Current and potential transformers may be compared with standard transformers of the same rating, whose constants are known, by methods developed by Agnew and by Silsbee. No special apparatus is required. Such comparisons may be made also by means of the assemblies devised by Silsbee and by the Leeds and Northrup Company.

When dealing with current transformers, by the Agnew method, the primaries of the standard and the unknown transformers are connected in series, while the secondaries are connected to the current coils of two induction watt-hour meters whose potential coils are excited from a common source. The phase difference between the potential difference applied to the meters and the current in the primaries of the transformers should be adjustable. Figure 406 shows the connections for testing both current and potential transformers by the Agnew method. If the applied voltage and current were in phase, and the meters were perfectly accurate, the relative value of the transformer ratios would be found by taking the ratio of the numbers of revolutions made by the two watt-hour meters in the same time. If the power factor were other than unity, the ratio of the revolutions would be affected by an amount dependent on the difference in the phase angles of the two transformers.

To eliminate differences in the calibration of the meters A and B, two runs are made, the meters being interchanged.

Let θ = phase difference of current (or voltage) in primary of transformer and auxiliary voltage (or current) applied to watt-hour meter. θ is + for lagging current.

m_A and m_B = rates of meters A and B, that is, values of ratio $\dfrac{\text{watt-hours registered}}{\text{true watt-hours}}$.

K_h = watt-hour constant of meters as marked on the disks.

$(N_A)_1$ and $(N_A)_2$ = number of revolutions made by meter A when connected to transformers 1 and 2.

$(N_B)_1$ and $(N_B)_2$ = number of revolutions made by meter B when connected to transformers 1 and 2.

β_1 and β_2 = phase angles of transformers 1 and 2, taken + when current or voltage in secondary lags primary current or voltage reversed.

R_1 and R_2 = ratios of transformers 1 and 2.

Suppose that meter A is connected to transformer 1 and that the phase of the auxiliary voltage is adjusted so that θ is large. The watt-hours registered on the meter dials are given by $(K_h)(N_A)$, or $(K_h)(N_A)(1/m_A)$ when corrected for the rate of the meter. As the meter is operated through a transformer of ratio R_1, this must be multiplied by R_1, giving

$(K_h)(N_A)_1(R_1)\frac{1}{m_A}$. To obtain the true watt-hours by meter A, this last result must be multiplied by $\frac{\cos\theta}{\cos\theta'} = \frac{\cos\theta}{\cos(\theta+\beta)}$ (see page 606). Therefore, from meter A,

$$\text{Corrected watt-hours} = (K_h)(N_A)_1(R_1)\frac{1}{m_A}\frac{\cos\theta}{\cos(\theta+\beta_1)}.$$

Similarly, for meter B connected to transformer 2,

$$\text{Corrected watt-hours} = (K_h)(N_B)_2(R_2)\frac{1}{m_B}\frac{\cos\theta}{\cos(\theta+\beta_2)}$$

Hence,

$$(N_A)_1(R_1)\frac{1}{m_A}\frac{\cos\theta}{\cos(\theta+\beta_1)} = (N_B)_2(R_2)\frac{1}{m_B}\frac{\cos\theta}{\cos(\theta+\beta_2)}. \tag{14}$$

When the meters are interchanged,

$$(N_A)_2(R_2)\frac{1}{m_A}\frac{\cos\theta}{\cos(\theta+\beta_2)} = (N_B)_1(R_1)\frac{1}{m_B}\frac{\cos\theta}{\cos(\theta+\beta_1)}. \tag{15}$$

If the test is made with $\theta = 0$, since β is a small angle,

$$(N_A)_1(R_1)m_B = (N_B)_2(R_2)m_A, \tag{14a}$$

$$(N_A)_2(R_2)m_B = (N_B)_1(R_1)m_A, \tag{15a}$$

and therefore

$$\frac{R_1}{R_2} = \sqrt{\frac{(N_B)_2(N_A)_2}{(N_B)_1(N_A)_1}}.$$

To determine the difference of the phase angles of the two transformers, a test is made with θ large.

Using the relation

$$\frac{\cos\theta}{\cos(\theta+\beta)} = \frac{1}{\cos\beta(1-\tan\theta\tan\beta)},$$

and remembering that β is a small angle, (14) and (15) give

$$\frac{(N_A)_2(N_B)_2}{(N_A)_1(N_B)_1}\left(\frac{R_2}{R_1}\right)^2 = \frac{1-2\tan\theta\tan\beta_2}{1-2\tan\theta\tan\beta_1} = 1 + 2\tan\theta(\tan\beta_1 - \tan\beta_2) \quad \text{approx.},$$

or

$$\tan\beta_2 - \tan\beta_1 = \frac{1}{2\tan\theta}\left[1 - \frac{(N_A)_2(N_B)_2}{(N_A)_1(N_B)_1}\left(\frac{R_2}{R_1}\right)^2\right]. \tag{16}$$

Rotary standard watt-hour meters are convenient for making the tests, as the revolutions may be read accurately from the dials.

Silsbee Comparison Methods for Current Transformers.—For tests in central stations and where large numbers of meters are concerned, it is desirable to have a method capable of giving results of commercial accuracy with great rapidity that does not involve complicated apparatus and connections.

Deflection Method.—Referring to Fig. 407, X is the current transformer under test; S, the standard transformer of known ratio and phase angle. The primaries are placed in series, as are the secondaries, care being taken that the polarities are such that both transformers tend to send current around the secondary circuit in the same direction. The polarities are correct if a 10-amp. ammeter placed at D gives only a very

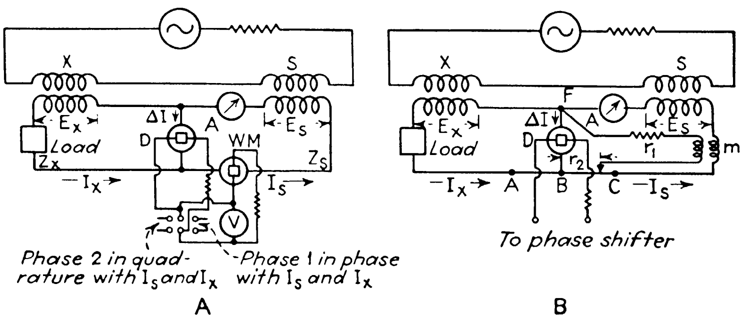


FIG. 407.—Silsbee methods for current transformers. *A*, deflection method; *B*, zero method.

small indication. With the connections shown, if ΔI is the current through the detector D,

$$\Delta I = I_S - I_X;$$

$$\frac{I_X}{I_S} = 1 - \frac{\Delta I}{I_S} \qquad \text{vectorially.}$$

Hence

$$\frac{(\text{Ratio})X}{(\text{Ratio})S} = 1 + \frac{\Delta I}{I_S}, \qquad \text{as } \frac{\Delta I}{I_S} \text{ is a small correction.} \tag{17}$$

As I_X and I_S are very nearly in phase (say, to within a degree), the component of ΔI, which is practically in phase with I_X and I_S, may be used in computing the ratio by Eq. (17).

$$\frac{(\text{Ratio})X}{(\text{Ratio})S} = 1 + \frac{\text{in-phase component of } \Delta I}{I_S}.$$

The tangent of the difference between the phase angles of the transformers is given by the component of ΔI, which is practically in quadrature with I_X and I_S divided by I_S, or

$$\tan (\beta_X - \beta_S) = \frac{\text{quadrature component of } \Delta I}{I_S}.$$

The detector may be a 1-amp. wattmeter connected so that the current circuit is traversed by ΔI, while the potential circuit is connected to either phase of a two-phase system (Fig. 407A).

To determine the in-phase component of ΔI, a voltage V_0, derived from one phase of the two-phase system and practically in phase with I_X and I_S, is applied to the potential circuit. Then

$$V_0 \times (\text{in-phase component of } \Delta I) = \text{reading} = W_1;$$

$$\text{In-phase component of } \Delta I = \frac{W_1}{V_0};$$

$$\frac{(\text{Ratio})X}{(\text{Ratio})S} = 1 + \frac{W_1}{V_0 I_S}.$$

To determine the quadrature component of ΔI, a voltage of value V_0 is taken from the other phase of the two-phase system. Then

$$V_0 \times (\text{quadrature component of } \Delta I) = \text{reading} = W_2;$$

$$\text{Quadrature component of } \Delta I = \frac{W_2}{V_0};$$

$$\tan(\beta_X - \beta_S) = \frac{W_2}{V_0 I_S}.$$

To determine whether β_X or β_S is the larger, β_X may be temporarily increased by adding a resistance to the burden of transformer X and noting the effect on the quadrature component of ΔI.

If the voltage V_0 in one phase of the two-phase system differs from V_0 in the other phase by θ instead of by 90 deg.,

$$(\text{In phase component of } \Delta I)V_0 \cos\theta + (\text{quadrature component of } \Delta I)V_0 \sin\theta = W_2';$$

$$\text{Quadrature component of } \Delta I = \frac{W_2' - (\text{in-phase component of } \Delta I)V_0 \cos\theta}{V_0 \sin\theta};$$

$$\tan(\beta_X - \beta_S) = \frac{W_2' - W_1 \cos\theta}{V_0 I_S \sin\theta}.$$

The angle θ is determined by the readings of the voltmeter, ammeter, and wattmeter.

It is important to know the burden on each transformer, since the performance depends upon it. If Z_D is the impedance of the detector,

$$\text{Burden for transformer } X = \frac{E_X}{I_X} = \frac{I_X Z_X - \Delta I Z_D}{I_X} = Z_X - \frac{\Delta I}{I_X} Z_D \qquad \text{(vectorially).}$$

$$\text{Burden for transformer } S = \frac{E_S}{I_S} = \frac{I_S Z_S + \Delta I Z_D}{I_X} = Z_S + \frac{\Delta I}{I_X} Z_D \qquad \text{(vectorially).}$$

A low-impedance detector is desirable in order that some of the load may not be shifted from the transformer of higher ratio to the transformer of lower ratio, thus changing the conditions of the test from those assumed to hold. This method requires no special apparatus, provided a two-phase or a three-phase supply is accessible.

Zero Method.—Referring to Fig. 407*B*, *ABC* is a slide wire; the possible displacement of the slider is from *A* to *C*; *m* is a mutual inductance; the carrying capacity of the primary is 5 amp.; r_1 is the total resistance of the slider circuit; the detector *D* is an electrodynamic wattmeter, or a separately excited electrodynamometer of about 1 amp. capacity, the current coil of which is connected between *B* and *F*, while the potential circuit may be connected to either phase of a two-phase system or to a phase shifter. If no deflection is obtained when the potential is shifted 90 deg., the voltage between *B* and *F* is zero. An exact knowledge of the phase shift is not necessary.

By properly adjusting the resistance r_1 in the secondary circuit of the mutual inductance, the position of the slider, and the value of *m*, it is possible to bring the detector to zero, that is, to divert ΔI so that it all flows in the r_1 circuit. When the adjustment has been attained, the e.m.f. in the secondary of the mutual inductance is just sufficient to force the current ΔI around the circuit through r_1.

If the sliding contact is to the right of *B*, and no current flows in the detector,

$$\Delta I(r_1 + j\omega L_1) + jm\omega I_S - I_X r_2 = 0.$$

m is considered positive if β_X is larger than β_S as shown by the test to be indicated. r_1 includes the resistance of the secondary of *m*, while L_1 is its inductance. As $\Delta I = I_S - I_X$,

$$\frac{I_S}{I_X} = \frac{r_1 + r_2 + j\omega L_1}{r_1 + j\omega(L_1 + m)}.$$

The value of I_S^2 in terms of I_X^2 and the constants of the network is

$$I_S^2 = I_X^2\left[\frac{(r_1 + r_2)^2 + \omega^2 L_1^2}{r_1^2 + \omega^2(L_1 + m)^2}\right].$$

Hence,

$$\frac{(\text{Ratio})X}{(\text{Ratio})S} = \sqrt{\frac{(r_1 + r_2)^2 + \omega^2 L_1^2}{r_1^2 + \omega^2(L_1 + m)^2}} = 1 + \frac{r_2}{r_1} \qquad \text{approx.}$$

Assuming the reactance terms to be negligible,

$$\tan\,(\beta_X - \beta_S) = \frac{\omega(r_1 + r_2)m + \omega r_2 L_1}{(r_1 + r_2)r_1 + \omega^2 L_1(L_1 + m)}.$$

$$\tan\,(\beta_X - \beta_S) = \frac{\omega m}{r_1} + \frac{\omega r_2 L_1}{r_1^2} \qquad \text{approx.}$$

The resistance AB to the left of B forms part of the burden on transformer X.

To determine if β_X is larger than β_S, the slider being to the right of B, a resistance may be added to the burden on transformer X. Its effect is to increase the lead of I_X with respect to the primary current reversed, or to increase β_X. If it is necessary to increase m to restore the balance, β_X was originally larger than β_S, the angle $\beta_X - \beta_S$ is positive, and m is used as a positive quantity.

If the slider is to the left of B,

$$\frac{(\text{Ratio})X}{(\text{Ratio})S} = 1 - \frac{r_2}{r_1} \quad \text{approx.}$$

$$\tan(\beta_X - \beta_S) = \frac{\omega m}{r_1} - \frac{\omega L_1 r_2}{r_1^2} \quad \text{approx.}$$

The Leeds and Northrup Company has assembled in a convenient and portable form the apparatus necessary for the zero method. In this

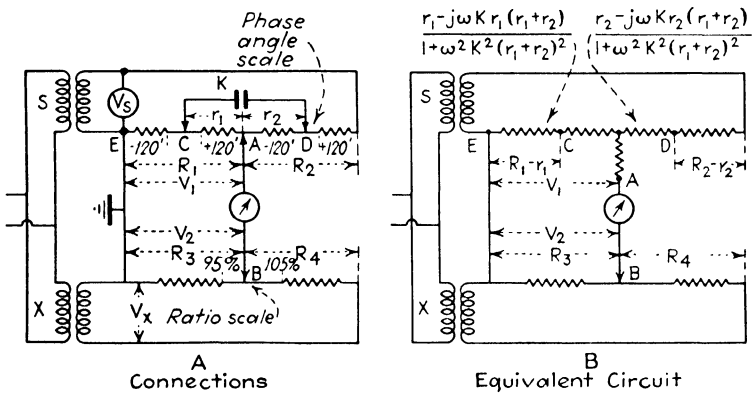


FIG. 408.—Leeds and Northrup method for potential transformers.

assembly, AB and BC are coils of 0.01 ohm each. The detector is connected to B, and a 15-ohm slide wire over which the movable contact may be displaced is connected between A and C.

Leeds and Northrup Comparison Method for Potential Transformer.— Figure 408 shows diagrammatically the Leeds and Northrup apparatus in which

$$R_1 + R_2 = \text{constant},$$
$$R_3 + R_4 = \text{constant},$$
$$r_1 + r_2 = \text{constant}.$$

This apparatus is a direct application of the potentiometer principle to the comparison of the secondary voltages of two potential transformers whose primaries are supplied in parallel from the same source.

The standard transformer is at S; the unknown transformer is at X. The equivalent circuit is obtained by applying the Δ-to-Y transformation to the Δ formed by K, r_1, and r_2. If V_S were in phase with V_X, there would be no need for the condenser K. The condition for balance would then be

$$\frac{V_S R_1}{R_1 + R_2} = \frac{V_X R_3}{R_3 + R_4}.$$

If the resistances R_1 and R_2 are constructed equal,

$$\frac{(\text{Ratio})X}{(\text{Ratio})S} = \frac{V_S}{V_X} = \frac{2R_3}{R_3 + R_4}.$$

If $R_3 + R_4$ is fixed in value, and balance is obtained by adjusting the slider B, the ratio of the two transformer ratios is proportional to R_3, and the scale over which B is displaced may be graduated to give the ratio (ratio)X/(ratio)S directly. In the practical case, V_X will not be exactly in phase with V_S, and it will be impossible to bring the detector to zero by a simple adjustment of the slider B. It is necessary to change the phase of V_1 until V_1 is in phase with V_2. For this purpose, the phase-angle condenser K is added. Its terminals are two rigidly connected sliders C and D which can be moved over portions of the resistances R_1 and R_2, $r_1 + r_2$ being kept constant. When the detector draws no current, the transformer S supplies to its own circuit a current of definite magnitude and bearing a definite phase relation to V_S irrespective of the setting of the phase-angle slides C and D. When C and D are displaced, both the "in-phase" and "quadrature" components of the IZ drop between E and A are altered. The voltage V_1 can be brought into phase with V_2, the change in the phase of V_1 being accompanied also by a change in the magnitude of V_1, dependent on the setting of the phase-angle slide C and D. To perfect the balance, it would be necessary to alter R_3. As this apparatus is intended for rapid commercial work, it is desirable that the ratio scale be made direct-reading, without corrections due to the setting of the phase-angle slides. Consequently, instead of changing R_3 to compensate for the movements of C and D, the condenser slides C and D and the slider A are connected mechanically so that the correction is automatically made. The phase-angle scale is calibrated by experiment, voltages of known phase difference being used. The apparatus gives, without calculation and without corrections, the ratio of the voltage ratios of the two transformers and the difference of their phase angles. As with all potentiometer arrangements, careful attention must be given to the polarity of the voltages to be compared. An incorrect connection will result in burning out the detector. A polarity check is obtained by removing the detector and connecting a voltmeter between A and B. If there is an appreciable deflection, the leads from one of the transformers must be reversed. This preliminary test is imperative and is provided for by a switch which substitutes, for the detector between A and B, the voltmeter normally used to measure the secondary voltage of the standard transformer.

In effect, the detector employed is a separately excited electrodynamometer. The movable coil is connected between A and B, the fixed coils being energized from a phase shifter. If no deflection appears when the phase of the energizing current is shifted through 90 deg., the detector current is zero, and balance has been obtained. With detectors of this type one must guard against errors arising from the mutual inductance between the fixed and movable coils. The test for mutual-inductance effect is (1) to bring the movable coil to its proper mechanical zero, no current flowing in any part of the apparatus; (2) to send full current through the fixed coils of the electrodynamometer, the transformers being disconnected from the source or, if this is not practicable, the ungrounded secondary leads being opened. Reference to the diagram will show that if there is mutual induction, a current will flow around the circuit formed by R_1, R_3, and the movable coil of the detector, giving rise to a deflection. The remedy is to adjust the relative angular position of the fixed and movable coils until the deflection disappears.

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CHAPTER XII

THE CALIBRATION OF INSTRUMENTS

This chapter deals chiefly with electrical indicating instruments intended for use in commercial work.[1]

Accuracy and Precision.—Every experimenter must form his own estimate of the *accuracy*, or approach to the absolute truth, obtained by the use of his instruments and processes of measurement. He must remember that a high *precision*, or agreement of the results among themselves, is no indication that the quantity under measurement has been *accurately* determined. For instance, under favorable conditions, one may be certain of the *reading* of a voltmeter (at about 100 volts) to one-tenth of 1 per cent. At the same time, the instrument may be several per cent in error, and the true value of the measured potential difference will remain unknown until the instrument has been calibrated.

Generally speaking, in any piece of experimental work one should aim at as high a degree of accuracy in the final result as can be attained without undue labor and expense. This conduces to discipline among the observers, as it discourages slipshod methods of observation. It also aids in the collection of reliable engineering data for use on other occasions. But it is to be remembered that the expense and the labor increase very rapidly as the required degree of accuracy is raised. Consequently, the determination of what the required degree of accuracy shall be becomes an economic problem.

Clear ideas concerning the distinction between *accuracy* and *precision* are especially important to those beginning experimental work.

During the progress of many tests, the obvious thing, and therefore the factor on which the beginner's attention is likely to be fixed, is the uncertainty in determining the best representative values of the readings of his instruments. Frequently the circuit conditions are fluctuating, so that close attention in reading the instruments and the averaging of many observations are necessary. After the experimental work has begun, these things may distract the beginner's attention from insidious constant errors and errors of method, so that these errors are scarcely thought of, though they may be of vastly greater importance than the errors of reading. For this reason it is necessary to study carefully any proposed test or method of measurement in order that, as far as possible, all constant errors and errors of method may be eliminated before the experimental work is begun.

Another result of this preliminary study should be a proper division of labor among the various component measurements. Generally, there are some measurements whose effects on the final result are small, and therefore labor expended in making numerous readings is wasted. This, however, does not imply that the readings giving these less important components may be made in a slovenly manner.

In an investigation, the preliminary study is frequently a very difficult part of the work, involving a careful analysis of the workings of the apparatus, a wide range of theoretical knowledge, and an accurate understanding of the behavior and sources of error in many different kinds of instruments.

In a complicated experimental investigation, to complete the elimination of constant errors and errors of method, a second determination of the quantity under measurement should be made, if possible, by *an independent method,* using different apparatus.

The beginner must keep in mind that it is scarcely possible to measure any of the electrical magnitudes, as he finds them in commercial practice, without changing the conditions of the circuit in which the measurement is made and thus altering the very thing that is to be determined. When an ammeter is introduced into a circuit, it alters the current, and an electromagnetic voltmeter alters the potential difference to which it is applied. In the vast majority of cases, these effects are negligible, but the possibilities of error due to the alterations of circuit conditions must not be forgotten.

Calibration before and after Tests.—In making careful acceptance tests of electrical machinery, the indicating instruments should be calibrated before the test, and a check calibration made at the conclusion of the work. This allows the various runs to be worked up while the test is in progress, which is highly desirable, as it insures that all necessary data are being taken and that the procedure of the observers is correct. The check calibration eliminates questions as to the accuracy of the instruments and as to whether or not they have been tampered with or injured in any way during the progress of the test.

Choice of Instruments.—In selecting the instruments for a particular piece of work, those should be chosen which will give good deflections, that is, deflections in a favorable part of the scale and of such a magnitude that the required *precision of reading* is readily attained. The choice, therefore, involves a preliminary study of the conditions of the test in order to determine approximately the magnitudes of the quantities involved. It must then be decided whether or not the desired, and obtainable, degree of accuracy in the final result is such that careful calibrations are necessary.

Often one knows from previous experience that his instruments are correct to within 1 or 2 per cent, and instances are continually arising where the conditions are such that an accuracy of 2 or 3 per cent is sufficient. In such cases, where differences of nearly equal quantities are not involved, there is no point in calibrating the instruments to 0.2 per cent, for example. Again, there are many cases where the magnitudes to be determined cannot be estimated *a priori*, and a certain amount of rough preliminary work is necessary to determine the most advantageous ranges of the instruments. In such cases, the calibrations should be deferred until the proper instruments have been selected.

Attention to these simple matters may save the beginner much valuable time and may prevent his arriving on the ground for a test without the proper equipment.

Sources of Error in Instruments.—The various sources of error which have been referred to in discussing particular instruments will be recapitulated.

Errors of Reading.—In general, the construction of the pointer and the graduation of the scale should be such that under steady conditions the position of the pointer may be read, by estimation, to one-tenth of a scale division. This is readily attained in direct-current instruments of the moving-coil type and in wattmeters over the larger part of the scale. Alternating-current ammeters and voltmeters have scales on which the graduations are crowded together at the lower end and possibly at the upper end also, so this precision of reading may be obtained between 25 and about 90 per cent of the full-scale reading.

Under commercial conditions, one must expect irregular fluctuations in the readings. If the fluctuations are not too great, the readings may be averaged mentally, but it is generally best to record a series of readings taken at regular intervals and calculate the average. In industrial testing, it is surprising how closely the averages from various runs made under the same general conditions will check one another, even though there are great momentary fluctuations.

In selecting instruments for industrial testing, attention must be given to the damping (often very defective in alternating-current instruments); otherwise the swinging of the pointer in its own natural period will be superposed on the deflection due to the load and will render it quite impossible to obtain the true reading. If several instruments have to be read simultaneously, their times of vibration and damping should be such that they will keep step as the load varies.

Mechanical Errors. Friction.—The effects of friction at the jewels and pivots should be reduced to a minimum. This means that the construction must be of the best and the ratio of torque to friction torque large.[5] The friction torque depends on the shape of the pivot point and its deformation by the weight of the movable system. Various formulae for the relation between the friction torque and the weight have been proposed. It is customary to compare similar instruments with vertical shafts by means of the ratio $\frac{\text{torque}}{(\text{weight})^{1.5}}$.

Before taking any instrument from the laboratory for use on a test, one should satisfy himself as to the pivot friction and freedom of motion of its movable element by putting it in circuit and *slowly* carrying the reading over the whole scale, stopping at several points on the scale. If there is undue friction, it will be made evident by a sudden change of reading when the instrument is tapped. Excessive friction may be due to a cracked jewel or to other injury, due to dropping the instrument. Again, the freedom of motion of the movable system may be impeded by the buckling of the paper scale, due to dampness, or to projecting fibers of the paper, which cause the pointer to stick. Air dampers, which have very small clearances, may also give trouble by getting out of adjustment and lightly dragging on the damping box. In direct-current moving-coil instruments, trouble may be due to dust in the air gap. If the dust is magnetic, it is dislodged with difficulty.

Springs.[4]—The assumption that the deflections of all sorts of instrument springs are always proportional to the deflecting moment is not tenable. The exact fulfillment of Hooke's law depends on the shape of the spring and on the method of mounting. In deflectional instruments, the peculiarities of the springs are taken care of in the initial calibration and introduce no trouble unless the spring is subsequently deformed in some manner; but when an equally divided scale and a torsion head are used, as in the Siemens dynamometer, the instrument must be tested at several points on the scale, and a calibration curve drawn.

Zero Shift.—This is due to a gradual yielding of the spring when the instrument is kept at a large deflection for a considerable time, as an hour or several hours. On breaking the circuit, the pointer does not return at once to its original zero position but will gradually assume it.

The magnitude of the zero shift depends on the design and material of the spring and on the nearness with which the elastic limit is approached. Springs that are used in high-resistance circuits, such as those of voltmeters and wattmeters, may be made of a material having good elastic properties, such as a bronze, for no limit is set as to spring resistance. Low-resistance springs, for millivoltmeters, have a high percentage of copper and show a larger zero shift than the bronze springs.

Temperature Coefficients of Springs.—With a rise of temperature, the elasticity of the springs decreases about 0.03 or 0.04 per cent per degree centigrade. This, if uncompensated, would cause an increase of like amount in the deflection. In many cases, electrical and magnetic changes afford a partial compensation.

Balancing.—Accurate balancing of the movable system is essential, for commercial instruments should not require careful leveling. As a test, the instrument should be tilted in various directions from its normal position, and the pointer observed. If lack of balance is found to be present, and the instrument must be used, it should be set up, using a level, the same precaution being taken during the calibration. The rebalancing should be done by an experienced person.

Scale Errors.—For each particular instrument, the cardinal points on the scale are supposed to be laid off by comparison with a standard. However, the subdivisions are frequently very carelessly made, and their irregularities are often apparent on inspection. The calibration curves for such irregular scales are "lumpy," and the calibration points must be taken near together.

Corrosion.—Hard-rubber covers and instrument bases which are imperfectly vulcanized may give trouble. The free sulphur attacks delicate wires, controlling springs, and suspensions, causing gradual deterioration and finally total failure. The effect on the indications of the instrument is progressive. Fine wires insulated with soft-rubber tubes, as is common for internal connections, may suffer in the same way.

Electrical and Magnetic Errors. Shunts.—Care must be taken that shunts suffer no mechanical injury. In making connections for a test, they should be firmly bolted into the circuit, all contacts being clean. Imperfect contact at one end of the shunt may result in unequal heating and a consequent thermo-e.m.f. error; but the overzealous application of the monkey wrench should be avoided, for if the shunt is not properly supported, some of the soldered joints where the resistance strips are sweated into the terminal blocks may be broken, and, though no damage is visible, the shunt may be rendered entirely untrustworthy and the test useless. The current leads should be of ample size, so as to assist rather than to hinder the dissipation of the heat from the shunt. During calibrations, especially where high-capacity shunts are involved, current connections identical with those of regular service must be used so that the current may be properly distributed before the potential terminals are reached.

Millivoltmeter Leads.—External-shunt ammeters *must* be calibrated with the same set of leads connecting the shunt and the millivoltmeter that is to be used in the subsequent test. Frequently special leads 30 or 40 ft. long must be used to remove the millivoltmeter from stray fields or to allow it to be placed where it can be read easily. Such leads should be of large diameter to reduce the resistance and should be provided with proper terminals.

When connecting the millivoltmeter and the shunt, care must be taken that all contacts are clean and firmly set up. The leads should be carefully examined to see that they are not broken inside the insulation or where they are soldered to the terminals.

Thermo-E.M.F.—The material of the resistance strips used in shunts should have a small thermo-e.m.f. when opposed to copper. Manganin is the best. The existence of a thermo-e.m.f. may be demonstrated by sending full current through the shunted instrument for a considerable time and then breaking the main circuit, when the millivoltmeter will not return at once to zero. This effect may be differentiated from the zero set by breaking the millivoltmeter circuit. Switchboard shunts may be defective in this respect and should not be used in very careful work before being investigated.

Effect of External Temperatures.—Variations of room temperature produce only small errors in soft-iron ammeters with a spring control. As the spring weakens, the permeability of the iron decreases by such an amount as practically to compensate for the weakening of the spring. In the voltmeter there is an additional source of error in the alteration of the resistance. The windings are of copper, and the series resistance is of a material with a zero temperature coefficient. The net effect will depend on the relative magnitudes of the resistances. The effect will be small in high-range instruments.

The only effect on current dynamometers, with the coils in series, is to alter the spring by 0.03 or 0.04 per cent per degree centigrade, causing the instrument to read high. The effect on electrodynamic voltmeters is on the resistance as well as on the spring.

With a rise of temperature, the magnets of a moving-coil voltmeter decrease in strength, the springs weaken, and the total resistance increases somewhat. In a 150-volt instrument, the net effect is negligible; but lower-range instruments, made by using the same galvanometer element and a smaller series resistance, will be affected by an amount increasing with the diminution of the range; about 0.4 per cent per degree is the extreme value, for then the copper of the moving coil becomes relatively more important. In accurate work, instruments of very low range should be used with care. Frequently, in laboratory voltmeters, a thermometer is inserted in the case as an aid in making the temperature corrections.

In shunted ammeters from the standpoint of external temperature effects, the shunt and the millivoltmeter should be of the same material, so that they may have practically the same temperature coefficient and render the multiplying power independent of temperature. As the shunt is best made of manganin, a resistance of manganin should be used in series with the copper moving coil, so as to obtain an approximation to the ideal condition. This means that the drop in potential in precision ammeters is considerable, being of the order of 150 to 200 millivolts at full load. The drop in switchboard shunts is 50 millivolts.

Internal Heating Errors.[2]—In shunted ammeters, errors may arise from the unequal percentage increase of resistance for the shunt and the millivoltmeter, due to the passage of the current. In old instruments, with internal copper shunts, this error is pronounced. For instance, in a certain 150-amp. instrument the error was found to be 4 per cent at full-scale deflection. In modern precision ammeters this error is not troublesome.

In voltmeters, if they are kept in circuit, there will be heating of the series resistance and movable coil due to the passage of the current, but on account of the low net temperature coefficient of *direct-current* instruments the resulting error in them will not be great. The energy dissipated at full-scale deflection is about 1.5 watts.

In wattmeters and *alternating-current* voltmeters, together with their accompanying multipliers, much more heat must be dissipated on account of the lower resistances—about 7 watts in a 150-volt instrument at full-scale deflection. In addition, the proportion of copper to zero-temperature-coefficient resistance wire is greater than in direct-current instruments. The liberation of so much heat in a confined space affects not only the resistances but the springs as well. The permanent state of temperature is attained gradually, and as a result there is a progressive change in the corrections when the instrument is left in circuit. An increase in temperature decreases the controlling effect of the springs and increases the resistance of the copper portion of the circuit, affecting the deflection in opposite directions. Some makers form parts of the circuit into small heater coils which are placed in proximity to the copper windings or the springs, as the case requires. It is then possible, by experiment, to proportion and position the parts of the circuit so that the time-lag temperature error is reduced to a negligible amount. When this adjustment has been made, it is generally found that ambient-temperature error is also negligible (see page 227). In all cases, high-resistance multipliers must be properly ventilated.

Stray Fields.—One of the most troublesome sources of error in industrial testing is due to stray fields from bus bars, feeders, motors, masses of iron, etc. These may so modify the strength of the field in which the movable coil swings that the indications of the instrument are untrustworthy. Especial care must be exercised when working near switchboards.

Stray fields due to ordinary working conditions are not likely to produce permanent alterations in the instruments, but violent short circuits on the line may cause permanent changes in the strength of the magnets and occasion very large errors. This is especially true of direct-current watt-hour meters, the stray field being due to the current coils. Figure 301 shows the normal distribution of magnetism in the drag magnets of a watt-hour meter and the distribution after a short circuit. Strong alternating stray fields, due to short circuits, may modify greatly the strength of permanent magnets in their neighborhood. If a short circuit has occurred, with either alternating or direct currents, no reliance should be placed on the instruments until they have been tested.

Direct-current stray fields of ordinary strength cause a percentage change throughout the scale in the indications of moving-coil ammeters and voltmeters. They produce no effect on alternating-current instruments. Alternating stray fields of ordinary strength have no effect on direct-current moving-coil voltmeters and ammeters but will affect ammeters, voltmeters, and wattmeters in which the current is of the same frequency as the stray field.

The effect on electrodynamic instruments will depend on the angular position of the movable coil with respect to the direction of the stray field being a maximum when the plane of the coil is in the direction of the field and zero when it is perpendicular to it. Assuming that the stray field, is fixed in direction, dynamometer instruments with torsion heads should be set up in such a position that the movable coil is perpendicular to the stray field. This position is found by sending full current through the movable coil alone and then turning the entire instrument in azimuth until any deflection disappears.

To detect the presence of a stray field, the instrument should be read, then immediately turned through 180 deg. and read again, the circuit conditions being maintained as nearly constant as possible. If no stray field is present, the readings will check. If a wattmeter is being used, current may be sent through the potential circuit alone, and the instrument slowly turned in azimuth. Any deflection observed will be due to the stray field.

The stray fields due to heavy currents in the leads to the instruments themselves must not be neglected. The leads must be free from loops and coils, should run straight away, and should be twisted. Special care must be exercised, when testing direct-current watt-hour meters *in situ*, that the stray fields from the temporary connections, such as the necessary jumpers, do not vitiate the results, especially at light loads. Do not set moving-coil instruments on sheets of "tin," which are tinned iron, and do not place instruments too near one another. As the field strength in alternating-current instruments is small, they are more susceptible to these errors than direct-current instruments of the moving-coil type. In general, one cannot assume that stray fields are constant in either magnitude or direction.

Careful attention must be given to these points when deciding on the location and arrangement of apparatus for a test; for in any case where results are called in question, the measurements have no standing unless one can prove there were no stray-field errors. Shielded instruments avoid these troubles, but implicit reliance should not be placed on them until the shielding has been proved to be effective under the conditions of actual use. Occasionally the shields themselves become magnetized and cause trouble.

Electrostatic Attraction.—Electrostatic attraction between the fixed and movable members may cause erroneous deflections as, for instance, in wattmeters operated from instrument transformers. The remedy is to connect the current and potential circuits by a bit of the finest fuse wire. Glass and hard-rubber covers sometimes give trouble. They should not be rubbed immediately before a reading is taken. The surface charges may be dissipated by breathing on the instrument. High-range instruments, having metal covers supported by insulating bases, are likely to give trouble. The secondary circuits of instrument transformers should be grounded.

Eddy Currents.—Eddy currents induced in massive coils, in metal frames supporting the coils, or in metal covers may be a source of error in alternating-current work. These effects may be pronounced in wattmeters when working at low power factors. The effects are absent when direct currents are used.

Current Distribution.—Distribution errors may be met with in alternating-current instruments with massive coils, the current not distributing itself uniformly over the cross section of the conductor as it does when direct current is used. This also may cause the alternating-current and direct-current calibrations to differ.

Frequency and Wave Form.—There is a possibility of error due to frequency in electrodynamic voltmeters and wattmeters and in soft-iron voltmeters if the reactance of the instrument becomes unduly high in proportion to the resistance. Especially one should be on his guard in investigation work where abnormal frequencies are sometimes employed. An instrument that is commercially correct at 60 cycles may be in considerable error at 500 cycles. Eddy-current effects are much accentuated at high frequencies.

Soft-iron instruments may be subject to wave-form errors arising from saturation effects, but with good modern instruments no trouble is likely to be experienced.

Induction instruments have errors of their own, due to the fact that when the compensation has been adjusted to suit the fundamental frequency, it will be incorrect, in general, for the various harmonics. These instruments are designed for use under definite conditions as to voltage, frequency, and wave form; and though they are serviceable on distribution systems where these factors are fixed, it is unsafe to apply them indiscriminately in general testing. The wattmeters and watt-hour meters may have frequency and wave-form errors, especially at low power factors. The magnitudes of the frequency errors in portable alternating-current instruments are indicated by the following figures furnished by a prominent manufacturer. The electrodynamic voltmeter adjusted for 60 cycles per second has an error of about 0.055 per cent at 180 cycles and an error of about 0.156 per cent at 300 cycles. The movable iron type of voltmeter for 150 volts and 60 cycles has an error of 0.25 per cent at 100 cycles, an error of 0.8 per cent at 180 cycles, and an error of about 2.25 per cent at 300 cycles. The electrodynamic ammeters with 5 and 10 amp. ranges adjusted for 60-cycle use have an error of about 0.33 per cent at 500 cycles. The electrodynamic wattmeters have negligible error up to 500 cycles, both for unity and 50 per cent power factor.

Use of Transformers.—The use of instrument transformers introduces error due to ratio and phase angle varying with the load. These are discussed on pages 606 and 607.

METHODS OF CALIBRATION[2]

The dates of all calibrations should be recorded and inserted in the legends of the calibration plots, together with the numbers of the instruments.

Direct-current Instruments.—As electrical measuring instruments cannot be relied upon to give absolutely accurate results, it is necessary to have methods for calibrating them. It should be possible to assemble the apparatus necessary for dealing with direct-current ammeters and voltmeters from the instruments found in any laboratory devoted to general electrical measurements.

Voltmeter Calibration by Standard Cell.—The method here given is an application of Poggendorf's method (see page 273).

An electromagnetic voltmeter is, in reality, a galvanometer in series with a high resistance. The scale of the galvanometer is graduated not in current strengths but in the voltages that it is necessary to apply to the terminals of the instrument in order to obtain the various deflections, that is, in values of $I_V R_V$, where I_V and R_V are the current through and resistance of the voltmeter. Therefore, if R_V and I_V have been measured, one may find the true value of the potential difference applied to the instrument, and this may be compared with its value as read from the scale. The difference will be the correction to be added to the observed reading to obtain the true value of the potential difference.

The resistance of the voltmeter is determined by a Wheatstone bridge.

The necessary connections for the measurement of I_V are shown in Fig. 409.

B is a battery capable of giving the desired current; W is a water rheostat by which the current I_V, and consequently the reading of the voltmeter V, may be varied; r is a known and variable resistance. The standard cell has a voltage denoted by E. *One must be sure that the cell is properly inserted so that in the galvanometer circuit its e.m.f. will oppose the potential difference due to the drop in r.* If the cell is properly inserted,

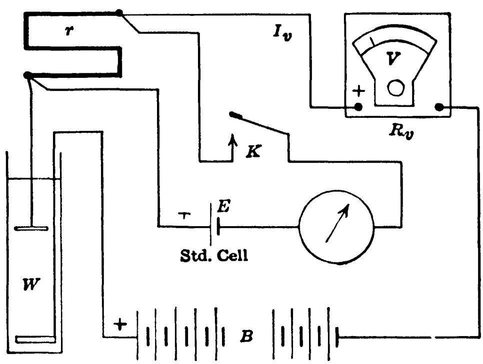

FIG. 409.—Connections for voltmeter calibration.

and $I_V r = E$, the galvanometer will remain undeflected when the key K is depressed.

$$I_V = \frac{E}{r},$$

and the potential difference across the terminals of the voltmeter will be given by

$$\text{P.D.} = \frac{E}{r}R_V.$$

In practice, it is usually desired to calibrate at or near certain predetermined points, at readings of 10, 20, 30, and so on volts. Therefore, it is necessary to know the value of r which must be inserted in order that when the galvanometer is balanced the voltmeter reading may be that desired. This is readily determined. Suppose that the instrument is to be calibrated at or near a reading of 30 volts, that $E = 1.0186$, and that $R_V = 17{,}000$ ohms. The proper value of r is

$$r = \frac{1.0186 \times 17{,}000}{30} = 577 \text{ ohms}.$$

577 ohms are to be inserted at r, and the reading of the voltmeter is to be brought to 30 volts by the water rheostat. It may be necessary to change the number of battery cells. The key K should be depressed cautiously

and released immediately when the deflection appears. In general, there will be a deflection, as the voltmeter will probably have a slight error, so that although it reads 30 volts, the true potential difference between the terminals will differ from that value; also, the resistance r may not be exactly the value corresponding to 30 volts, for r may be adjustable to single ohms only. An exact balance is obtained by varying the water rheostat, and the voltmeter is read immediately thereafter. The reading and the corresponding value of r must be recorded. The difference between the true potential difference at the instrument terminals and the observed reading gives the correction, the quantity that must be added to the reading to obtain the true potential difference.

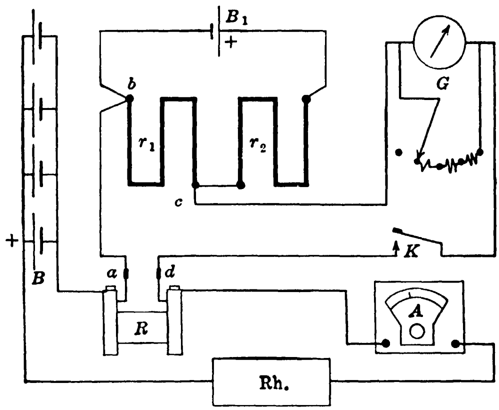

FIG. 410.—Connections for ammeter calibration.

It is important to take the zero reading, for it is subject to variation. If taken and separately allowed for, a calibration may retain its value, if the scale is equally divided, even though the zero reading may alter.

Ammeter Calibration.—To calibrate an ammeter it is necessary to have a method for measuring the current that will be free from such instrumental errors as may affect the indications of even the best direct-reading standard instruments. Such calibrations may be carried out by the aid of standard cells and accurately adjusted resistances in the following manner:

Referring to Fig. 410, R is a known resistance, so constructed that it will not heat appreciably with the passage of the current. This resistance is inserted in series with the ammeter A to be calibrated. Rh is a rheostat for controlling the current, and B is a battery of storage cells to furnish the steady current required.

To measure the ammeter current it is necessary to determine the potential difference between the potential terminals of R. This may be done by the method of projection of potentials, an application of Poggendorff's method (page 273). To apply this, it is necessary to have an auxiliary battery B_1 of *constant* e.m.f. and capable of furnishing a small current, 0.001 amp., continuously; two resistance boxes r_1 and r_2 of a total resistance of about 10,000 ohms each, which should be capable of adjustment in single-ohm steps; and a suitable galvanometer and key. The wire *ab* causes the points *a* and *b* to assume the same potential. If the current through the ammeter is denoted by I, the potential at *d* differs from that at *a* by IR. If the potential difference at the terminals of the battery B_1 is P.D., that between *b* and *c* will be

$$\text{P.D.}\frac{r_1}{r_1 + r_2}.$$

When the potentials at *c* and *d* are the same,

$$IR = \text{P.D.}\frac{r_1}{r_1 + r_2},$$

and the galvanometer will not deflect when the key is depressed. Consequently, if r_1 or r_2 has been adjusted so that the galvanometer gives no deflection,

$$I = \left(\frac{\text{P.D.}}{R}\right)\left(\frac{r_1}{r_1 + r_2}\right).$$

One cell of storage battery is satisfactory for B_1. In order that its potential difference may remain constant, the cell should be partially discharged. A standard cell cannot be used for B_1 because it is incapable of supplying even a small current without alteration of its e.m.f. through polarization.

The first step in carrying out the test is to determine the potential difference of the auxiliary battery B_1. This is to be done by Poggendorff's method.

The connections shown in Fig. 410 are then made; it is necessary that B_1 and R be connected + to +.

In general, on closing the key there will be a deflection which is to be brought to zero by adjusting r_1 or r_2. When a balance is obtained, the ammeter is to be read immediately.

If $r_1 + r_2$ greatly exceeds the battery resistance of B_1, the potential difference will be approximately the e.m.f. of the cell. It is well to remember that imperfect connections to the cell and excessive lead resistance have the same effects on the results as high battery resistance of B_1 and that this battery resistance should be negligible both during the test by Poggendorff's method and the subsequent use of the arrangement in determining the current. By using proper resistances at R, currents of all magnitudes can be measured by this method.

Calibration by Means of Potentiometer.—Direct-current ammeters and voltmeters are calibrated most readily by means of either the *ordinary* or the *deflectional* type of potentiometer, which are discussed in Chap. V. Currents are determined by measuring the potential difference between the terminals of standard resistances. These resistances should be certified by the Bureau of Standards. The certification should be made once a year, since the resistances are subject to slight changes.

For voltage measurements the range of potentiometers may be extended upward from 1.5 volts by the use of volt boxes.

As a source of current for ammeter calibrations, a 4-volt storage battery is most convenient. The cells may be charged in series and discharged in parallel. Variations in the current are obtained by the use of rheostats. These may have metal grids for the large steps and a carbon compression rheostat for the fine adjustments.

For potential differences a storage battery should also be employed. The cells must be large enough so that they may be properly maintained. A drop wire furnishes the most convenient means of regulating the potential difference at the instruments.

Alternating-current[2] Ammeters and Voltmeters.—Most of the alternating-current voltmeters in daily use for engineering work are based on the electrodynamic principle. Such instruments may be calibrated with direct currents, using either a standard direct-current voltmeter, whose errors are accurately known, or a direct-current potentiometer and volt box. On account of the effect of the local field, it is essential that two readings be taken at each point, first with a voltage in one direction, and then in the reverse direction. The two results should be averaged. This procedure ignores the existence of any frequency error. The magnitude of this error may be calculated from the resistance, and the inductance measured at the desired point on the scale.

Except for thermal instruments, which give the same reading with both direct and alternating currents, and for regular electrodynamometers with the two coils in series, it is necessary to use alternating currents when calibrating alternating-current ammeters. Generally, such ammeters are soft-iron instruments, whose indications on direct-current circuits are complicated by the effects of residual magnetism in the iron vane. Also, there may be errors due to wave form. These same remarks apply to soft-iron voltmeters which are intended for alternating currents. Induction ammeters and voltmeters must be calibrated with alternating current.

The calibrations may be made by the alternating-current potentiometer, used in connection with nonreactive volt boxes and shunts, but this potentiometer is not in common use. It would not be serviceable if wave-form errors were being investigated.

The arrangement shown in Fig. 411 does very well for ammeters.

The thermal ammeter, provided with an appropriate set of shunts, is used as a transfer instrument. The ammeter under calibration is compared with it, and then by means of the double-throw switch the thermal instrument may be transferred to the direct-current circuit and calibrated at the *proper point* by means of a potentiometer and standard

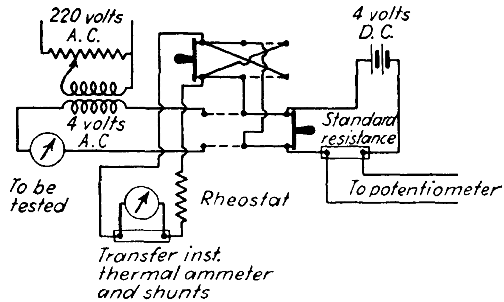

FIG. 411.—Connections used in calibrating alternating-current ammeters.

resistances. This procedure avoids all questions as to the permanence of the calibration of the transfer instrument. A difficulty is that the range of a thermal ammeter is short, the deflection depending upon the square of the current. Consequently the scale is of such a nature that even with care only about the upper 60 per cent of it is readable with sufficient accuracy. Therefore the millivoltmeter part should be sensitive, and the range should be extended by numerous shunts, so that the deflection may be kept in the upper part of the scale.

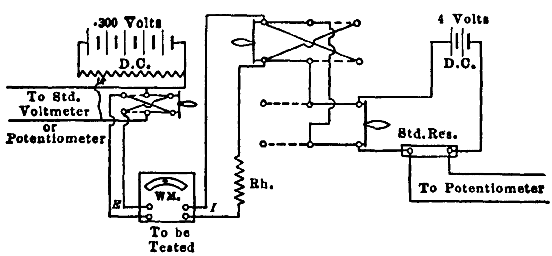

FIG. 412.—Connections for fictitious loading of wattmeters.

Soft-iron voltmeters may be compared with an electrodynamic instrument which has been calibrated with direct current.

Wattmeters.—When two wattmeters are to be compared, the current coils are placed in series, and the potential coils in parallel. If both instruments are of the electrodynamic type, direct current may be used, reversals being taken to eliminate the effects of any local field.

When calibrating high-capacity wattmeters, in order to save power and bring the work within the range of the apparatus found in a well-equipped laboratory, it is necessary to resort to fictitious loading, that is, to supplying the current and potential coils from two distinct sources.

The connections for a calibration are as in Fig. 412.

It is convenient to use storage cells for supplying both current and potential circuits. Two readings are made at each point, both the current and the potential switches being reversed. The results are averaged to eliminate the effect of the local field.

In this procedure, eddy-current and frequency errors are assumed to be negligible. If there is doubt as to this, the wattmeter must be compared with one known to be free from them, using alternating currents of the proper frequency.

If an induction meter is being tested, alternating current of the proper frequency must be used and the instrument compared with an electrodynamic wattmeter which has been calibrated with direct current.

In case it is necessary to test at different power factors, the supply may be derived from two alternating-current machines, having the same number of poles, with their armatures on the same shaft, one field being arranged so that it may be given any desired angular displacement about the axis of rotation. The wave form of both machines should be sinusoidal. A phase-shifting transformer (see page 521) may be used and is more convenient.

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CHAPTER XIII

DETERMINATION OF WAVE FORM

To simplify the mathematical treatment of the flow of alternating currents, it is customary to assume that the wave form is sinusoidal for both applied e.m.f. and current.

Designers aim to produce machines with e.m.f. waves that are sinusoidal, or nearly so, since experience has shown that this form of e.m.f. wave is the most advantageous in practice.

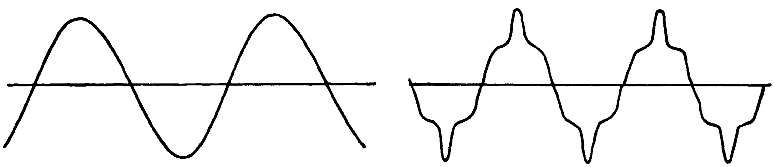


Fig. 413.—Examples of wave forms.

Figure 413*A* shows the e.m.f. wave of a modern turboalternator. The departure from the sinusoidal form is not obvious, and a careful analysis is necessary to determine it. On the other hand, Fig. 413*B* shows the e.m.f. wave of a very old type of machine; this wave form might seriously complicate the behavior of the devices placed in the circuit.

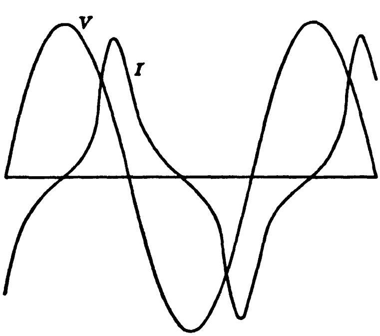


Fig. 414.—Potential difference applied to and current in a coil with iron core.

The form of the current wave is affected by the character of the circuit as well as by the form of the e.m.f. wave. If the e.m.f. wave is not a sine curve, the effect of its various harmonics upon the current wave will be accentuated by capacitance and smoothed by inductance in the circuit.

Saturation effects in iron cores may also materially affect the form of the current wave. This is illustrated in Fig. 414, which shows both the potential difference applied to a coil with an iron core and the resulting current.

In engineering work, cases frequently arise where wave-form determinations are important on account of the assistance that they give in explaining the behavior of electrical apparatus.

Two cases may arise:

1. When the phenomena are periodic, for instance, the ordinary e.m.f. and current waves (Fig. 414).

2. When the phenomena are transient, such as those occurring when the circuit conditions are suddenly altered. This is illustrated by Fig. 425, which shows the potential difference and current curves taken during a short-circuit test of an enclosed fuse.

Contact Method for Determining Wave Forms.[1]—Methods for dealing with case 1 were the first to be developed, the earliest being the contact method, used in 1849 by Lenz in investigating the wave forms of alternators. In 1880, Joubert employed it to determine the wave form of a Siemens machine, and since then it has commonly been called Joubert's contact method.

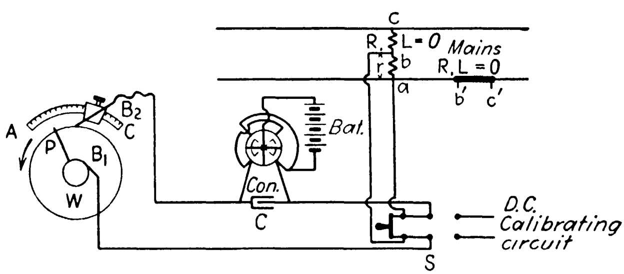


FIG. 415.—Connections for contact method for wave form, using quadrant electrometer.

The fundamental idea is to connect periodically the measuring apparatus to the circuit for a time so short that the current or voltage remains practically unchanged during this time. This is accomplished by an apparatus that is the equivalent of a key operated by a rigid connection from the dynamo shaft. The key is closed for an instant once during each revolution of the dynamo and at a definite point on the wave.

If the voltage is high, a large nonreactive resistance R (Fig. 415) is placed across the circuit, and by means of a tap a definite fraction of the total voltage is impressed on the apparatus.

Referring to Fig. 415, the contact wheel of hard rubber or fiber is at W. In the original arrangement, this wheel was attached directly to the dynamo shaft; B_1 is a brush which rests on a collector ring and gives permanent connection to the contact point P, which projects very slightly from the periphery of the wheel W. B_2 is a thin and very light brush which just clears the contact wheel. It is supported from a movable brush holder which may be set at any desired position along a uniformly graduated arc AC.

The measurements may be made by the aid of an electrostatic voltmeter, a quadrant electrometer, or a ballistic galvanometer. In the arrangements in Fig. 415, the needle of the electrometer is kept charged to a high potential by the battery, and the deflection is sensibly proportional to the applied voltage or to the potential difference between a and b at the instant of contact. The well-insulated condenser C adds to the capacitance of the electrometer, so that the voltage on the instrument will not be appreciably altered by leakage during the time between successive contacts of B_2 and P.

The process is to set the brush at a definite position on the arc and to read the electrometer, then to move the brush forward to another position and take another reading, and so on.

The magnitude of the deflections may be controlled by means of the battery and the resistance r.

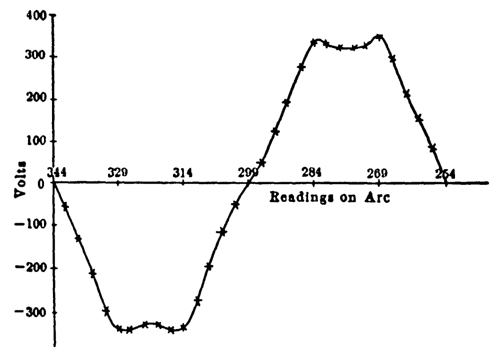


Fig. 416.—Wave form determined by contact method.

The electrometer is calibrated by use of direct-current voltages.

The electrometer readings, reduced to volts, are plotted against the readings on the uniformly graduated arc, as in Fig. 416, and a smooth curve drawn through the points.

If an electrostatic voltmeter is used in place of the electrometer, a difficulty is encountered, since the deflections depend on the square of the voltage; hence, deflections obtained near the zero points of the wave will be very small. This difficulty may be obviated by working from a false zero. A battery of sufficient voltage to give a large deflection is joined in series with the voltmeter, and when readings are taken the voltage to be measured is superposed on the battery voltage. If a reflecting instrument is used, the calibration curve is very closely a parabola, and as the upper part of it is practically a straight line, the deflection from the false zero is sensibly proportional to the voltage between a and b at the instant of contact.

Figure 417 shows the arrangement when a ballistic galvanometer is employed. In this arrangement, S_1 is a double-pole, double-threw switch, by which the apparatus may be connected to the alternating-current circuit or to the direct-current circuit for purposes of calibration. C_1 is a variable condenser which is charged by throwing S_2 to the left and discharged through the ballistic galvanometer *BG* when the switch S_2 is thrown to the right.

The deflection, which is proportional to the instantaneous voltage between *a* and *b*, may be controlled by varying the capacitance.

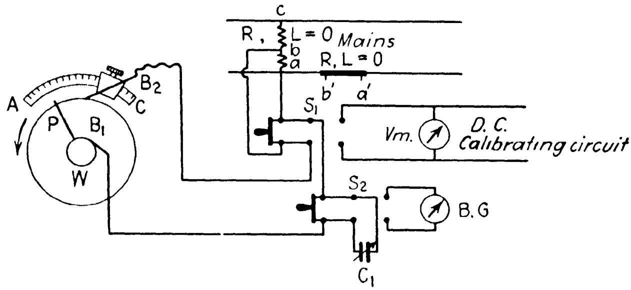


FIG. 417.—Contact method for wave form, using ballistic galvanometer

Use of Potentiometer Principle.—These methods may be improved if a potentiometer arrangement is adopted, as in Fig. 418.

Referring to Fig. 418, *DE* is a slide wire or its equivalent, supplied with direct current from *B*. The voltage from *O* to *D* or to *E* is slightly larger than the maximum between *a* and *b*. A direct-current voltmeter is connected between *O* and the slider *S*; *G* is a detector, which may be a telephone or a moving-coil galvanometer; a Thomson galvanometer is not suitable on account of the possible alteration of the strength of the magnets.

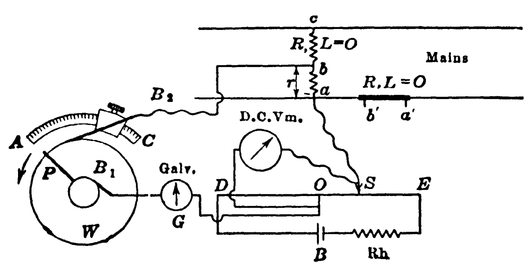


FIG. 418.—Contact method for wave form, using potentiometer principle.

After setting the contact brush, the position of the slider *S* is varied until the detector *G* stands at zero. The instantaneous voltage between *a* and *b* is then equal to the steady voltage between *O* and *S*, as read from the direct-current voltmeter.

The methods given are applied to current waves by placing a non-reactive resistance (shown at *a'b'*) directly in the circuit and determining the instantaneous potential difference between its terminals.

The time consumed in mapping a wave form by the foregoing methods is considerable; this is disadvantageous and necessitates holding the circuit conditions practically constant for a considerable time, since *if the wave is nonsinusoidal*, many points located near together must be taken. Frequently, in industrial testing, the conditions cannot be maintained constant. In addition, much time and labor must be expended in computing and plotting the results. Obviously, simultaneous records of two or more waves cannot be obtained with a single instrument.

In many cases, the necessity for directly connecting the contact disk to the dynamo shaft prohibits the use of the contact method in the forms

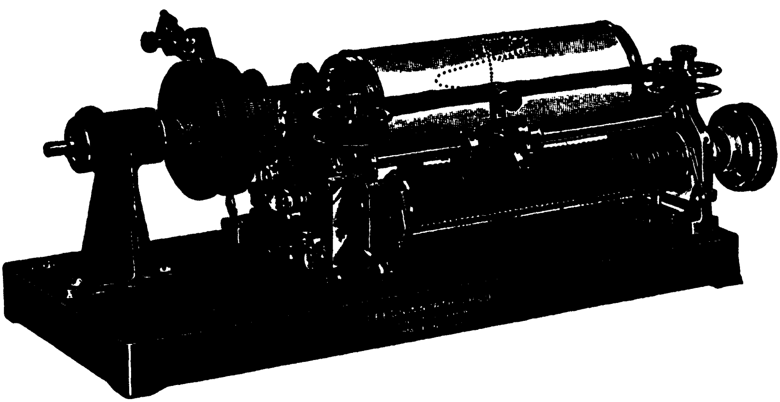


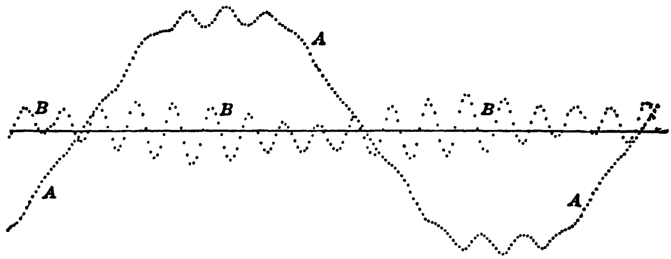
FIG. 419.—Potentiometer and registering apparatus for Rosa curve tracer.

previously given, for it is frequently necessary to determine the wave forms at a place remote from the generating station.

From the potentiometer method, the Rosa curve tracer (Fig. 419) has been developed. The object of this machine is to reduce the time necessary for making the observations and plotting the results. In Fig. 418, if the direct-current voltmeter is omitted, and the potentiometer wire carries a *definite* constant current, the displacement OS of the slider from the zero position will be proportional, at balance, to the instantaneous voltage. The idea is to plot these displacements, as ordinates, on a sheet of paper carried by a drum, the abscissae being proportional to the displacements of the contact brush along the arc AC. This plotting is done in a semiautomatic manner, as will be seen from the following.

Referring to Fig. 418, the potentiometer wire DE (Fig. 418) is wound on a screw thread cut in a bakelite cylinder. When the cylinder is turned, the thread cut in the bakelite moves the carriage to which are attached the contact point S and the stile for registering the results.

A short-period, dead-beat, moving-coil galvanometer is used as a detector. To make an observation, the handle at the right is turned until the galvanometer stands at zero. Then the printing lever at the left is raised, causing the stile, through the medium of the typewriter ribbon, to imprint a dot on the paper carried by the large drum at the rear of the



Curve *A*, Electromotive Force Wave. Curve *B*, Current Wave.
Resonance of Fifteenth Harmonic

FIG. 420.—Wave forms taken with Rosa curve tracer.

apparatus. When the lever is lowered, a ratchet and pawl turn the drum a predetermined amount. At the same time, by means of another ratchet and pawl actuated by an electromagnet, the contact brush B_2, in Fig. 418, is advanced proportionally.

The teeth of the ratchets of the contact maker and drum are numbered correspondingly, so that the brush and recording drum may be set at any desired position. As shown in Fig. 420, the points may be taken close together, so that irregular waves may be plotted.

For the best work, it is necessary to connect the contact wheel directly to the dynamo shaft and to keep the circuit conditions absolutely constant. If this can be done, the Rosa curve tracer furnishes the most accurate apparatus yet devised for mapping periodic electrical phenomena.

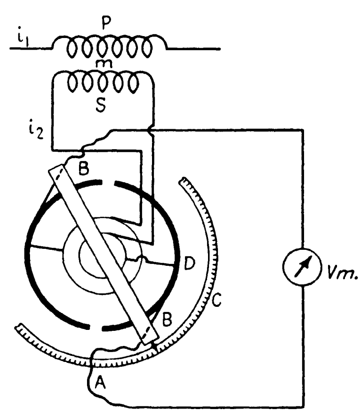


FIG. 421.—Connections for integrating method for determining wave form.

Integrating Methods for Determining Wave Form.[2]—One arrangement for determining wave form by an integrating method rather than by "instantaneous" contacts is shown diagrammatically in Fig. 421.

The current i_1, whose wave form is to be determined, passes through the primary of an air-core transformer, in the secondary of which is a low-range voltmeter, or a galvanometer, and the commutator *D*. The commutator is driven from the dynamo shaft or by means of a synchronous motor and is so constructed that the connections to the galvanometer are reversed at each half wave. As shown, the commutator is suitable for a two-pole machine; if there are more than two poles, it is necessary to increase the number of segments correspondingly.

The distance between the brushes is 180 electrical deg. They are mounted in a holder which can be moved concentrically with the shaft so that they may be set at any point on the wave. Their position can be read from a graduated circle. It is convenient to have the arrangement such that the brushes can be moved readily in a succession of equal steps. Thin metallic brushes, which will not short-circuit the commutator, should be used. The apparatus must be well-insulated to prevent the entrance of stray currents. This form of commutator is applicable only when the two halves of the wave are identical or only odd harmonics are present.

The galvanometer should be of the moving-coil type and should integrate correctly a transient current.

When the commutator is in action, the galvanometer experiences a deflection proportional to the average value of the current during a half cycle. The reading I_2 of the instrument is given by

$$I_2 = \frac{2}{T}\int_t^{t+\frac{T}{2}} i_2 dt.$$

Let R = resistance of secondary circuit of transformer.
m = mutual inductance of air-core transformer.
f = frequency.
e_2 = instantaneous induced e.m.f. in secondary.
V_2 = reading of voltmeter.
i_1 = instantaneous current in primary.
$(i_1)_t$ = instantaneous current in primary at time t.
i_2 = instantaneous current in secondary.
I_2 = average value of current in secondary.

Then

$$e_2 = m\frac{di_1}{dt}.$$

$$m\int_t^{t+\frac{T}{2}} di_1 = \int_t^{t+\frac{T}{2}} e_2 dt = R\int_t^{t+\frac{T}{2}} i_2 dt = \frac{T}{2}RI_2.$$

Hence,

$$2m(i_1)_t = \frac{T}{2}RI_2;$$

$$(i_1)_t = \frac{RI_2}{4fm} = \frac{V_2}{4fm} = K \text{ times the scale reading,}$$

where K is a constant of the instrument. This gives the instantaneous value of the current at a particular point on the wave, and the wave form is traced point by point, as in the contact method.

E.M.F. Waves.—E.m.f. waves are obtained by placing across the line the primary of a suitable air-core transformer in series with a high nonreactive resistance. If the resistance is so high that the circuit is practically nonreactive, a determination of the form of the current wave gives also the wave form for the potential difference applied to the terminals. As the nonreactive resistance must be large, the method is not applicable to low voltages. With this arrangement all the measuring apparatus is entirely separated from the primary circuit. This may be advantageous if the voltage is high.

The transformer should have a variable ratio. If the mutual inductance is not known, the apparatus may be calibrated by use of a sinusoidal current whose maximum ordinate may be calculated from the measured value of the current and compared with the reading I_2 of the galvanometer.

Or a wave may be plotted, using the readings of the galvanometer, its root mean square computed, and compared with the same quantity determined by a standard alternating-current ammeter.

Flux Waves.—Referring to Fig. 421, the flux that threads the secondary is proportional at every instant to the current in the primary whose wave form has been determined. Thus the form of this flux curve has been found also. The total flux through the secondary at any instant will be $(i_1)_t m$.

This suggests that in case the form of a flux wave is desired, it is necessary merely to replace the secondary of Fig. 421 by a coil of N turns wound around the core through which the flux passes.

For the case just considered, the total flux linkages at any instant t will be given by

$$m(i_1)_t = \frac{V_2}{4f}.$$

In the case under consideration, the flux through the core is

$$(\varphi)_t = \frac{V_2}{4fN};$$

or, if V_2 is in volts,

$$(\varphi)_t = \frac{V_2 10^8}{4fN}.$$

If a nonsynchronous commutator is used, the maximum value of the flux will be proportional to the maximum reading of the voltmeter as it goes through its cycle of deflections.

Use of Condenser and Synchronous Commutator.—Another arrangement for determining the forms of potential waves is shown in Fig. 422. A condenser, a commutator, and a galvanometer, or a millivoltmeter, are joined in series across the mains.

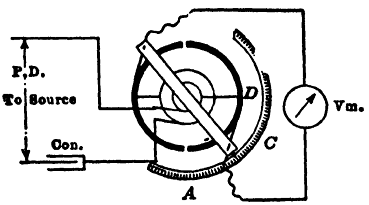


FIG. 422.—Connections for determining wave form by synchronous commutator and condenser.

The current through the circuit is $i = dq/dt$. In the time $\frac{T}{2}$ of half a cycle, the charge on the condenser changes from $+q$ to $-q$, and

$$\int_{+q}^{-q} dq = \int_{t}^{t+\frac{T}{2}} idt.$$

The reading I of the galvanometer is proportional to the average current, or

$$I = \frac{2}{T}\int_{t}^{t+\frac{T}{2}} idt = 2f\int_{t}^{t+\frac{T}{2}} idt;$$

and

$$2q = \frac{I}{2f}.$$

If a good condenser is used, $q = V_t C$;
or

$$V_t = \frac{I}{4fC}.$$

The deflection may be controlled by varying C. Currents may be dealt with in the usual way by determining the potential difference

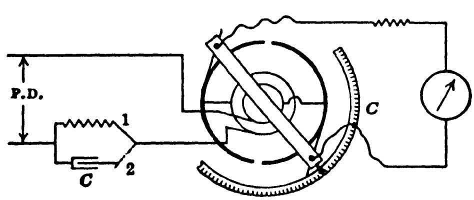


FIG. 423.—Connections for determining average value of alternating-current wave.

between the terminals of a nonreactive resistance through which the current flows.

Determination of the Average Values of Potential Difference and Current Waves.—If the commutator is arranged as in Fig. 423 average values of the potential difference and current may be determined. To do this the brushes must be set so that the reversals are at the zero points of the wave. When the switch is on 1, the galvanometer is subjected to a series of unidirectional impulses and gives a deflection proportional to their average value. The instrument may be calibrated by transferring it to the line side of the commutator and applying a known direct-current voltage.

To set the brushes properly, the switch is placed on 2, so that the condenser C is in circuit, and the position of the brushes is altered until the galvanometer stands at zero. Currents are dealt with by using a nonreactive resistance.

Determination of Form Factor.—To determine the form factor, an electrodynamic voltmeter and a direct-current voltmeter are placed in parallel, as in Fig. 424.

The direct-current instrument gives the mean value, while the electrodynamic instrument gives the root-mean-square value. The deflections are controlled by nonreactive resistances, the values of which need not be known. With the commutator running and the brushes adjusted

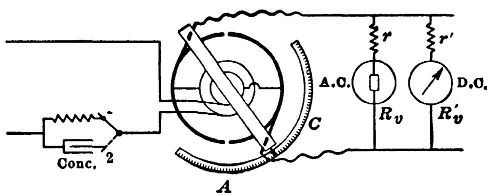


FIG. 424.—Connections for determining form factor.

for reversal at zero point of the wave by the method just given, both instruments are read. Call the alternating-current instrument reading D and the direct-current instrument reading G; then

$$\underset{\text{Effective}}{\text{P.D.}} = D\left(\frac{R_v + r}{R_v}\right); \qquad \underset{\text{Average}}{\text{P.D.}} = G\left(\frac{R'_v + r'}{R'_v}\right).$$

With the commutator stationary, a direct-current potential difference is applied. Then, if D' and G' are the readings,

$$D'\left(\frac{R_v + r}{R_v}\right) = G'\left(\frac{R'_v + r}{R'_v}\right);$$

and the form factor is

$$F = \frac{\text{P.D. effective}}{\text{P.D. average}} = \frac{DG'}{D'G}.$$

The Electromagnetic Oscillograph.[3]—All the methods that have been given for obtaining wave form require that the phenomena be periodic and that the circuit conditions remain fixed for a considerable time. Also, with some of the methods, much time must be spent in calculating and plotting the results. Cases are continually arising where it is desirable to investigate phenomena that are transient in character, as, for example, that illustrated in Fig. 425.

It is obvious in this case that the previous methods are not applicable.

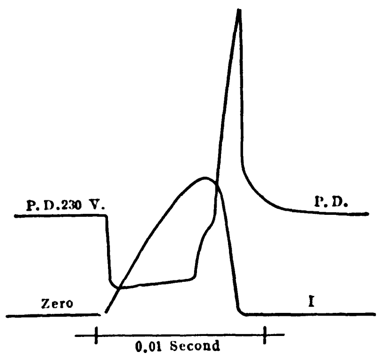
Blondel was the first to state definitely the conditions that must be fulfilled in order that a galvanometer may follow, with sufficient accuracy, the rapid variations of an alternating current and be capable of recording wave forms photographically with but a single traverse over the photographic plate of the spot of light used as an indicator. He applied the term oscillograph to such an instrument.

The conditions are as follows:

1. High free period of oscillation, as great as fifty times the period of the phenomena to be investigated.

2. Damping small and in the neighborhood of the critical aperiodic value.

3. Self-inductance as small as possible.



Nominal bus-bar voltage	230 volts
Minimum bus-bar voltage	61 volts
Maximum bus-bar voltage	586 volts
Maximum current	6,600 amp.
Time required to open circuit	0.0086 sec.
Time required to attain maximum current	0.006 sec.
Average rate of decrease of current	2,500,000 amp. per second.

FIG. 425.—Showing variation of current and voltage during short-circuit test of 100-amp. enclosed fuse.

4. Negligible hysteresis and Foucault current effects.

5. Adequate sensitivity.

In addition, the design and construction must be such that the necessary adjustments and repairs may be made with ease by anyone accustomed to handling electrical instruments.

The moving needle, the iron vane, the moving-coil galvanometer, the string galvanometer, and the hot-wire instruments have all been modified so that they may be used as oscillographs. Blondel's first instrument was of the moving needle type; in its late development, the needle has become a thin strip of soft iron stretched over bridge pieces.

This type of instrument may be given a very high free period of vibration but has the disadvantage that its self-inductance is comparatively large.

Blondel's suggestion for making the moving-coil galvanometer available as an oscillograph is illustrated in Fig. 426.

A single loop of a narrow and very thin conducting strip is stretched over a frame in such a manner that the part between the bridge pieces *a* and *b* is free. A very small and very light mirror is cemented to the two sides of the loop midway between the bridges. The loop is placed between the poles *NS* of a powerful cobalt-steel magnet. Formerly an electromagnet was employed. The poles are large enough so that the free section of the loop is in a practically uniform field.

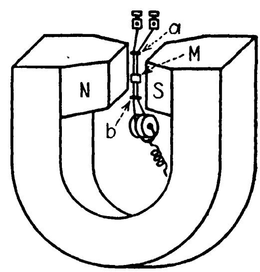


FIG. 426.—Showing principle of bifilar oscillograph.

A current passing around the loop will cause one side to advance while the other recedes; thus the mirror is turned around a vertical axis. For the very small movements that are employed, the deflections of the mirror are proportional to the current.

The movable system is immersed in oil of such a viscosity that at the normal temperature of operation the galvanometer is dead beat or nearly so.

It is important that the damping be constant; oil damping is greatly affected by changes of temperature. Consequently the behavior of an instrument in which electromagnets are used depends on the length of time that the electromagnets have been continuously in the circuit. This is one reason for adopting permanent magnets. At the Massachusetts Institute of Technology, water-cooled magnets were used. Oil damping also reduces the natural frequency from f_0 to $f_0\sqrt{1 - B_0^2}$ (page 20).

The material from which the strip is drawn should have a low resistivity so there will be little heating. This avoids creeping of the spot of light due to the expansion of the strip and reduces the energy consumed by the galvanometer.

This form of instrument has the advantage that the inductance is very small. It is the one to which most attention has been given by designers in the United States and in England.

Figure 427 shows a group of oscillograph vibrators. The corresponding galvanometers, complete, are also shown.

The minimum free period that it is practicable to give this form of vibrator appears to be about 1/10,000 sec. To attain this figure, the parts, especially the mirror, must be exceedingly delicate; this increases the liability to accident and the difficulty of making repairs.

Instruments with such a high rate of vibration are useful and indeed essential in research work, but they must be used by skilled operators. For general engineering work, a more robust vibrator, having a free period of about 1/5,000 sec., is better. Such an instrument will follow closely enough for practical purposes the waves usually met in power work.

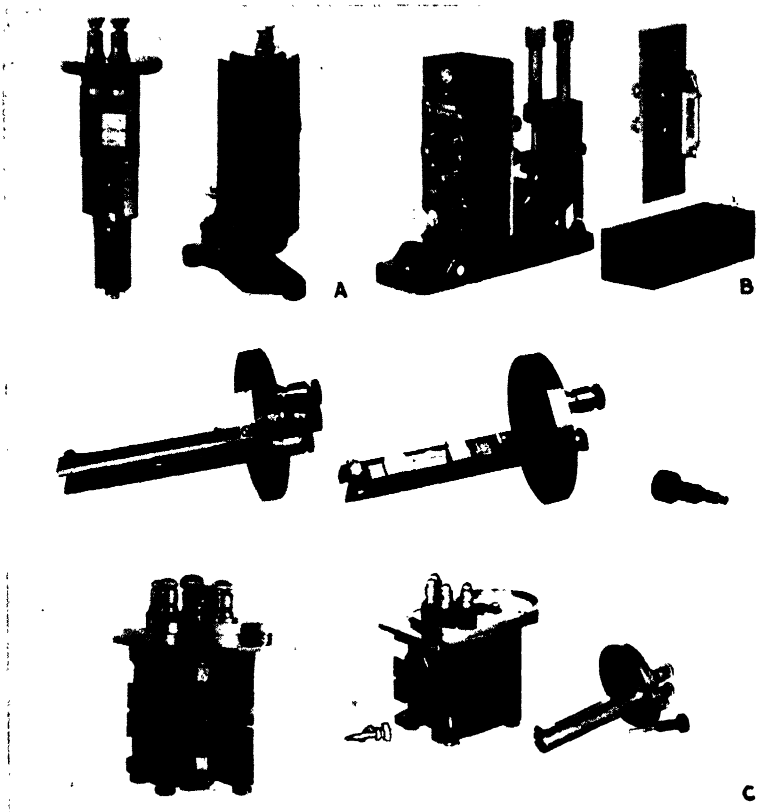


Fig. 427.—Oscillograph vibrators and galvanometers.

For work at power frequencies the Westinghouse Electric and Manufacturing Company has developed a small portable oscillograph in which the galvanometer is of the iron-vane type, shown diagrammatically in Fig. 428.

The iron vane, or needle, is firmly held in the position indicated, by a string torsional control. On the passage of the current, one of the two diagonally located magnet poles is strengthened, while the other is weakened, and the vane is rotated about a vertical axis. A large mirror is employed. The damping is obtained by using a nonreactive

Ayrton shunt, as indicated in the wiring diagram. It is noted (page 470) that this method of damping acts as a sort of corrective network and irons out the sharp response at resonance, thus greatly improving the frequency characteristic which would otherwise show a sharp maximum at about 3,500 cycles per second. The resistance of the galvanometer is about 3.5 ohms, the inductance is about 1.2 millihenrys.

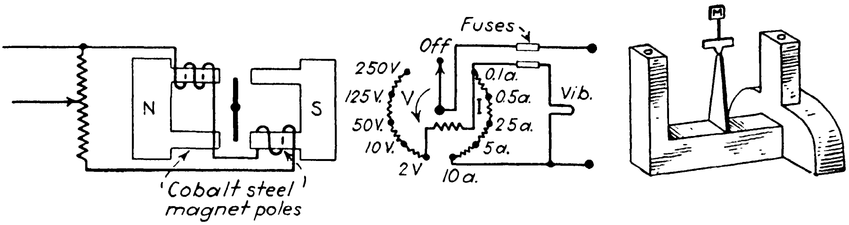


FIG. 428(*A*).—Westinghouse soft-iron-needle oscillograph.

As the period of an oscillograph galvanometer is very short, the sensitivity is low. In the laboratory form of instrument, a deflection on the scale of from 1 to 3 mm. usually corresponds to about 0.01 amp.

As the movable mirror is very small, an intense source of light is required. A direct-current arc was formerly employed, but the late J. W. Legg[6] introduced the use of a straight-filament incandescent lamp with a preheated filament. The beam of light, after being reflected

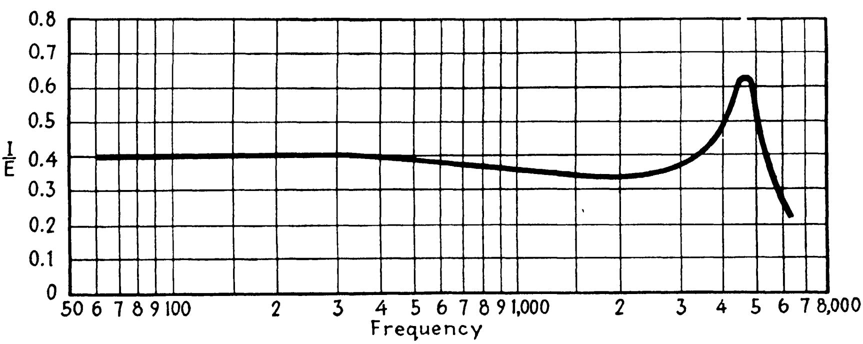


FIG. 428(*B*).—Relative deflection as a function of frequency for the Westinghouse soft-iron-needle oscillograph.

from the oscillograph mirror (Fig. 429), passes through a long cylindrical lens which compresses the beam vertically. This focusing improves the definition of the spot of light on the screen, and a still further improvement is effected by placing a narrow vertical slit between the arc and the mirror.

For visual observations, the beam of light after passing through the cylindrical lens is received either on a multisided mirror which is rotated at the proper speed by a synchronous motor or on a mirror that is tilted with a uniform angular velocity by a cam, also driven by a synchronous motor. With the tilting mirror, a shutter actuated by the motor cuts off the light while the cam is returning the mirror to its initial position.

The revolving mirror furnishes the necessary time coordinates. From it the spot of light is reflected upon a curved translucent screen concentric with the axis of the mirror.

With periodic phenomena, the waves appear in a fixed position on the screen and may be traced on thin paper.

For photographic work, these mirror arrangements are dispensed with, and the spot of light is focused by the cylindrical lens directly on a photographic film carried by a uniformly rotating drum or in a uniformly moving strip of film. With the drum a shutter is employed which remains

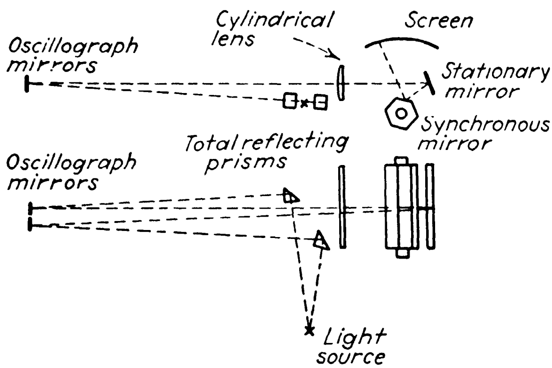


FIG. 429.—Optical system for two-element oscillograph.

open while the drum makes one revolution. The mechanical features of these recording devices are described in the catalogues of instrument makers.

Theory of the Electromagnetic Oscillograph.—As the oscillograph is a damped galvanometer with a high free period, the equation established on page 15 applies, provided it be assumed that the instrument has one degree of freedom. As the current, and consequently the deflecting force, may be any periodic function of the time, it may be expressed analytically by a Fourier series. Consequently,

$$P\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + \tau\theta = C\sum_{n=1}^{n=\infty} I_n \sin(n\omega t - \beta_n), \qquad (1)$$

where ω is 2π times the fundamental frequency of the current, and n is the order of a harmonic. For the fundamental, n is 1; for the third harmonic, n is 3; and so on.

If the second member of (1) is zero, and the relative values of P, k, and τ are such that the motion is oscillatory,

$$\theta' = K\epsilon^{-\left(\frac{2\lambda}{T}\right)} \sin\left[\left(\frac{2\pi}{T}\right)t + \varphi\right]. \tag{2}$$

K and φ are constants which are determined by the initial conditions. λ is the logarithmic decrement, and T is the time of a complete vibration (see page 18).

On account of the damping, the transient portion of the deflection, represented by θ', rapidly diminishes to zero after the circuit is closed.

The particular integral to which θ' must be added to obtain the complete integral is

$$\theta = C\sum_{n=1}^{n=\infty} \frac{I_n}{\sqrt{k^2n^2\omega^2 + (\tau - n^2\omega^2P)^2}} \sin\left(n\omega t - \beta_n - \tan^{-1}\frac{kn\omega}{\tau - n^2\omega^2P}\right). \tag{3}$$

As the instrument must closely follow sudden changes of current, the damping should be near the critical value, or

$$k^2 = 4\tau P.$$

Then

$$\theta = C\sum_{n=1}^{n=\infty} \frac{I_n}{\tau + n^2\omega^2P} \sin\left(n\omega t - \beta_n - \tan^{-1}\frac{2n\omega\sqrt{\tau P}}{\tau - n^2\omega^2P}\right). \tag{4}$$

This expression for θ should be compared with the expression for the flow of current in a circuit containing resistance, inductance, and capacitance after the steady state has been established. In both expressions, there are "impedance" and phase-displacement terms.

Inspection of (3) shows that after the steady state has been reached:

1. The various harmonics have effects on the deflection dependent on their order.

2. The harmonics have different phase displacements.

In consequence, the oscillograph can never give a *mathematically* correct picture of a wave form. This being so, it is necessary to find the conditions that will make the instrument sufficiently correct for practical purposes.

If the moment of inertia and the damping were zero, the wave would be followed exactly. Therefore, the mass of the moving parts must be reduced to a minimum, and an arrangement adopted that will make the moment of inertia as small as possible. At the same time, the directive moment on the movable system must be increased so that the $d^2\theta/dt^2$. and the $d\theta/dt$ terms are small in comparison; hence the usual statement that the rate of free vibration must be high.

The mass of the moving system and the damping restrict the usefulness of the electromagnetic oscillograph to work at low- or power-frequency phenomena. Neither this instrument nor the string oscillograph would be suitable for impulse testing (page 102) where the phenomena to be registered occupy only a few microseconds. For work of this class recourse must be had to the cathode-ray oscillograph (page 667).

If an oscillogram must be analyzed into its sinusoidal components, either the results obtained must be corrected for the distortion produced by the oscillograph, or it must be shown that for the work in hand the distortion effects are negligible. One method of procedure will be outlined.[5]

Let the free angular velocity of the vibrator be $\omega_0 = 2\pi f_0 = \sqrt{\tau/P}$. Let the ratio of the actual damping coefficient to that required for critical damping be B_0, or $k = B_0 k_c = 2B_0\sqrt{\tau P}$. B_0 is termed the bluntness of resonance. For a general-purpose instrument B_0 might be of the order of 0.6.

Let the fundamental angular velocity of the current be ω, and

$$U = n\omega/\omega_0.$$

Then Eq. (3) may be written

$$\theta = \frac{C}{\tau}\sum_{n=1}^{n=\infty} \frac{I_n}{\sqrt{1 + 2U^2(2B_0^2 - 1) + U^4}} \sin\left(n\omega t - B_n - \tan^{-1}\frac{2B_0U}{1 - U^2}\right). \quad (3a)$$

The elongation of the mirror deflection due to the nth harmonic is

$$\theta_n = \frac{C}{\tau}\frac{I_n}{\sqrt{1 + 2U^2(2B_0^2 - 1) + U^4}}$$

If a simple sinusoidal current of very low frequency and maximum value I'' is sent through the instrument, the mirror deflection will be

$$\theta'' = \frac{C}{\tau}I''.$$

Therefore

$$\sqrt{1 + 2U^2(2B_0^2 - 1) + U^4} = \frac{(\theta''/I'')}{(\theta_n/I_n)}. \quad (3b)$$

Hence, the amplitudes of the results obtained from the analysis must be multiplied by $\sqrt{1 + 2U^2(2B_0^2 - 1) + U^4}$ to reduce them to what would be obtained with an instrument that was without distortion.

For example, tests show for a certain instrument that $B_0 = 0.21, f_0 = 3{,}300.$; then for $f = 1{,}000$, $U = 1{,}000/3{,}300 = 0.303$, and

$$\sqrt{1 + 2U^2(2B_0^2 - 1) + U^4} = 0.917,$$

or the instrument indicates 9 per cent too high at 1,000 cycles per second. In addition, the mirror deflection lags its proper position by the angle $\tan^{-1} \dfrac{2B_0U}{1 - U^2} = \tan^{-1} 0.14 = 8°$ at 1,000 cycles per second. It is evident that to correct the oscillogram, two constants are necessary, ω_0 and B_0. To determine them, Kennelly suggested the use of an auxiliary air-damped vibrator of high frequency. For such an instrument, B_0 might be of the order of magnitude of 0.001, while f_0 might be 2,500. Equation (3a) shows that for a continuously applied sinusoidal current, the phase displacement with such a vibrator is negligible for frequencies not in the neighborhood of 2,500 cycles per second. The auxiliary air-damped vibrator, called an *oscillograph meter*, is arranged with the axis of rotation of the mirror perpendicular to the axis of the instrument under test and sufficiently distant so that the magnetic fields of the instruments do not react on each other. A beam of light is reflected successively by the two mirrors to a ground-glass screen, as in Fig. 430.

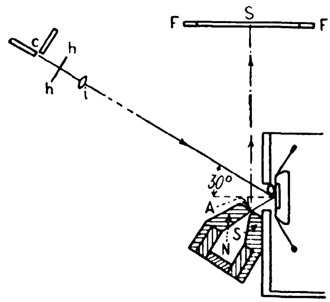


FIG. 430.—Optical system of oscillograph meter A and the tested oscillograph O. Length of A mirror 2.3 mm. Width 0.9 mm. Distance O–A = 1.1 cm. Distance A–S = 50 cm.

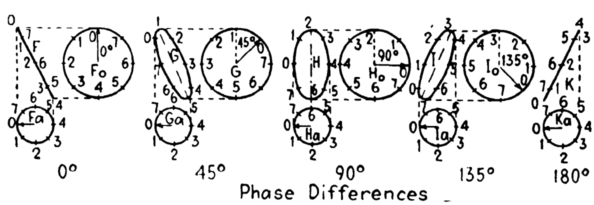


FIG. 431.—Lissajous figures with components in the ratio of 2 to 1, and various phase differences.

A sinusoidal voltage of variable and controlled frequency is supplied to the two vibrators in parallel, the currents being regulated so that one amplitude is two or three times the other. A Lissajous figure, an ellipse, appears on the screen. At very low frequencies, the vibrations of the two instruments will be very nearly cophasal, and the ellipse becomes practically a straight line inclined to the vertical. As the frequency is raised, the two vibrations cease to be cophasal; the ellipse opens up and assumes a more nearly upright position, as in Fig. 431.

If the *relative* directions of the currents are reversed, the result is as in Fig. 432.

When the frequency is made equal to f_0, $U = 1$, the phase displacement is 90 deg., the ellipse is vertical, and ω_0 or f_0 is determined. If the frequency is raised still more, the ellipse begins to contract and to incline in the opposite direction, again becoming a straight line when the phase displacement is 180 deg. Note that f_0 is not the frequency that gives the maximum deflection. If it is more convenient, one of the deflections may be advanced 90 deg. by the insertion of a condenser, and the quadrature

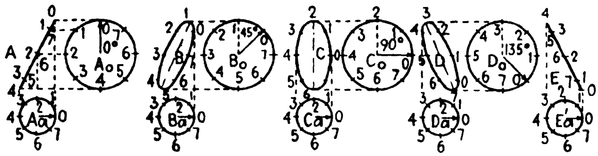


FIG. 432.—Same as Fig. 431, but with one component reversed.

relation is shown by the ellipse becoming an inclined straight line. Having determined f_0, B_0 is found by making two calibrations, one at a low frequency, the other at the resonant frequency f_0. Then, by (3a),

$$B_0 = \frac{1}{2} \frac{(\theta/I) \text{ at low frequency}}{(\theta/I) \text{ at remote frequency } f_0} \tag{5}$$

Corrective Networks.—From the foregoing it is evident that the ordinary electromagnetic oscillograph has characteristic errors. The same is true of the hot-wire instrument and of the string galvanometer when used to trace wave forms. It is possible to correct these errors sufficiently for practical purposes by using in conjunction with the oscillograph a corrective network. This network so distorts the current through the indicating portion of the instrument that the mechanical response is, nearly enough for practical purpose, proportional instant by instant to the current or voltage applied to the combination of instrument and network.

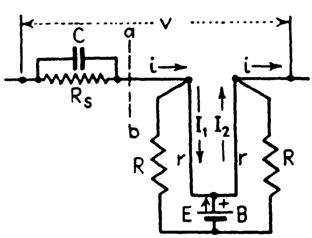


FIG. 433.—Circuit diagram for Irwin hot-wire oscillograph.

An early, if not the earliest, example of such a network is that due to H. Abraham[4] and employed in his rheograph (1897) by which he converted a moving-coil galvanometer into an instrument that in spite of the high moment of its moving parts was capable of delineating alternating-current waves of power frequency.

A striking example of the application of a corrective network is furnished by the Irwin hot-wire oscillograph,[8] the circuit diagram for which is shown in Fig. 433, the indicating portion being to the right of *ab*.

The hot wires are polarized by the battery of internal resistance B. By Kirchhoff's rules,

$$I_1 = \frac{iR\left(1 + \frac{2B}{R + r}\right) - E}{R + r + 2B}, \quad \text{and} \quad I_2 = \frac{iR\left(1 + \frac{2B}{R + r}\right) + E}{R + r + 2B}. \tag{6}$$

As the wires carry different currents, they expand unequally. It is ingeniously arranged that this small differential linear expansion causes a large angular deflection of the mirror proportional to the differences of temperature of the wires, as $\theta = K''(t_1 - t_2)$.

The *difference* of the rates of heating of the two wires is

$$D = \left[\frac{4ERr\left(1 + \frac{2B}{R + r}\right)}{(R + r + 2B)^2}\right]i = Ki, \tag{7}$$

where K is an instrumental constant.

The *difference* in the rates of heating of the two wires is proportional instant by instant to the current supplied to the apparatus at the right of ab. The wires have heat capacity, and heat is dissipated to their surroundings at a rate proportional to the difference of temperature between the wires and their environment. Consequently, though D keeps step with the variations of i, the resultant difference in the expansions of the wires, to which the deflection is proportional, cannot so keep step. The wave form of the current through the wires must be modified so that the difference in expansion is in step at every instant with the line voltage v.

Assume the wires to be alike in all particulars. Let m be the mass of a wire in grams; s, its specific heat in joules per gram per degree; k, the emission constant in joules per second per degree centigrade difference of temperature between the wire and its environment; t_1 and t_2, the temperature of the wires; and t_0, the temperature of the surroundings. The rates at which heat is disposed of are

For wire 1,

$$ms\left(\frac{dt_1}{dt}\right) + k(t_1 - t_0) = \text{rate of supply of energy to wire 1.}$$

For wire 2

$$ms\left(\frac{dt_2}{dt}\right) + k(t_2 - t_0) = \text{rate of supply of energy to wire 2.}$$

The difference of the two rates is

$$ms\frac{d(t_1 - t_2)}{dt} + k(t_1 - t_2) = Ki. \tag{8}$$

As the deflection is proportional to $(t_1 - t_2)$, the quantity should be proportional at every instant to the terminal voltage v, or $(t_1 - t_2) = K'v$. This means that the wave form of i must contain a term proportional to the time derivative of v and a term proportional to v and that the proportionality factors have the proper relative values. This is accomplished by the network at the left of ab. Assuming that the resistance to the right of ab is very small, while that to the left is large,

$$i = C\frac{dv}{dt} + \frac{v}{R_s}.$$

Therefore

$$ms\frac{d(t_1 - t_2)}{dt} + k(t_1 - t_2) = KC\frac{dv}{dt} + \frac{K}{R_s}v. \tag{9}$$

If the corrective network is so adjusted that

$$C = \frac{msK'}{K} \quad \text{and} \quad R_s = \frac{K}{K'k}, \tag{10}$$

where K' is a constant, the difference of temperature of the strips at each instant is proportional to v; that is,

$$(t_1 - t_2) = K'v, \tag{11}$$

which is the relation required to make the deflection follow the voltage wave. Quite a different network is required for current waves.

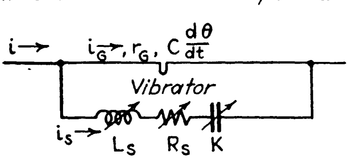


FIG. 434.—Pertaining to Irwin resonant shunt.

Another example of a corrective network, also due to Irwin, is furnished by the resonant shunt as applied particularly to high-period string galvanometers or oscillographs originally having air damping in order to obtain critical damping without the use of damping oil.[8] Referring to Fig. 434, the shunt is seen to be an arrangement of resistance, inductance, and capacitance and is tunable to the natural frequency of the vibrator. In his original demonstration Irwin neglected the back e.m.f. set up by the string as it moves through the magnetic field; consequently, the condition established by him should be modified. In applying the resonant shunt to instruments with metal strings, that is, of low resistance, Butterworth, Wood, and Lakey showed the necessity of taking into consideration the e.m.f. which has an instantaneous value $C\frac{d\theta}{dt}$. They showed the relation between the deflection θ of the midpoint of the string and the main current i to be

$$P\frac{d^2\theta}{dt^2} + k'\frac{d\theta}{dt} + \tau\theta = Ci, \tag{12}$$

where P is the mass per unit length of the string; $k' = k + \frac{Pr_G}{L_S}$, the effective damping coefficient; and τ, the restoring force per unit deflection. k is the original air damping coefficient of the vibrator on open circuit.

Equation (12) is subject to the conditions

$$\frac{1}{L_S K} = \frac{\tau}{P} = \omega_0^2, \tag{13}$$

ω_0 being the free angular velocity of the vibrator; and

$$\frac{R_S}{L_S} = \frac{k + \frac{C^2}{r_G}}{P} = \frac{k_{SC}}{P}. \tag{14}$$

k_{SC} is the effective damping coefficient of the instrument when short-circuited. Equation (14) shows that free rate of decay of an electrical disturbance in the shunt should be identical with free rate of decay of a mechanical disturbance of the vibrator. The term C^2/r_G is due to the back e.m.f. and does not appear in Irwin's original formulae. R_S is an equivalent resistance which accounts for the energy loss in the shunt.

The difficulty in applying the resonant shunt is to obtain sufficiently small time constants.

To obtain the expressions for the constants of the shunt L_S, K, and R_S in terms of quantities readily measured, denote by subscripts the conditions under which B_0 (page 20) is to be determined, and note that

$$\omega_0 = 2\pi x_0 = \sqrt{\frac{\tau}{P}}; \qquad (B_0)_{k'} = \frac{k'}{k_c} = \frac{k'}{2\sqrt{\tau P}};$$

$$\frac{k'}{P} = 2(B_0)_{k'}\omega_0 = \frac{r_G}{L_S} + \frac{k}{P}. \tag{15}$$

For critical damping, $B_0 = 1$;

$$L_S = \frac{r_G}{2\omega_0(B_0)_{k'} - \frac{k}{P}} = \frac{r_G}{2\omega_0[(B_0)_{k'} - (B_0)_{SC}]}. \tag{16}$$

$(B_0)_{k'}$ is the value of B_0 when the instrument is used, and $(B_0)_{SC}$ is the value of B_0 on open circuit. With a taut string the value is very small

By (13),

$$K = \frac{1}{\omega_0^2 L_S}. \tag{17}$$

By (14),

$$R_S = \frac{L_S k_{SC}}{P} = 2\omega_0 L(B_0)_{SC}. \tag{18}$$

To facilitate the determination of B_0, Thomander gives a table[8] (Table XVI) which shows the first fractional overswing (page 18) and the corresponding values of B_0 and the ratio of the resistivity at resonance to that at low frequencies. The first fractional overswing is (page 18) $\epsilon^{-\frac{\pi}{\sqrt{\frac{1}{B_0}-1}}}$. The table is calculated by assuming values for B_0 and then calculating the ratio of resonant and low-frequency sensitivities and the first fractional overswing.

TABLE XVI.—FOR APPLYING IRWIN RESONANT SHUNT[8]
Thomander

$\left(\frac{\theta_r/I}{\theta_E/I}\right)$	B_0	Fractional overswing	$\left(\frac{\theta_r/I}{\theta_E/I}\right)$	B_0	Fractional overswing
10.00	0.05	0.854	1.25	0.40	0.253
5.00	0.10	0.728	1.00	0.50	0.163
3.33	0.15	0.625	0.83	0.60	0.095
2.50	0.20	0.526	0.71	0.70	0.046
2.00	0.25	0.444	0.59	0.85	0.006
1.50	0.33	0.329	0.50	1.00	0.000

A string galvanometer resonates not only at the fundamental frequency of the string but also at approximately odd multiples of that frequency, and Butterworth and his associates added a second resonant shunt tuned for three times the fundamental with resulting improvement of the performance.

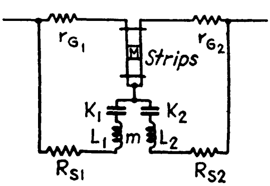


FIG. 435.—Martin and Caris arrangement of Irwin shunt.

An obstacle to the application of the resonant shunt to low-resistance bifilar oscillographs is the fact that each strip has its own free period, and the two strips are coupled by the mass of the mirror. In addition, it is difficult to obtain inductances with sufficiently high effective time constants. Figure 435 shows the arrangement investigated by Martin and Caris[8]. The strips are united just below the lower bridge piece; their resistances are increased by r_{G_1} and r_{G_2}, and each is provided with its own Irwin shunt. The effect of mass coupling is controlled by the mutual inductance of L_1 and L_2.

As shown by Aoki, Tada, and Tomoda, it is possible to obtain satisfactory damping with a low-resistance bifilar instrument if the natural damping is somewhat increased by use of kerosene as the damping fluid and completing the adjustment by means of a single resonant shunt. Using this combination, in a particular case, the adjustment for critical damping reduced the resonant frequency from 2,400 to 1,700 cycles per second; with oil damping alone the reduction was to 1,200 cycles per second. As the kerosene damping is but little affected by temperature, the great advantage of constant damping is retained.

Figure 436 shows the results obtained by Martin and Caris, using kerosene and resonant shunt damping on a bifilar instrument having a resonant frequency of 1,700 cycles per second.

On account of the increase in sensitivity, which may be forty or fifty times that for a steady or a low-frequency current, an electromagnetic oscillograph without some form of corrective network can be employed only when the frequency is well below that for resonance of the vibrator.

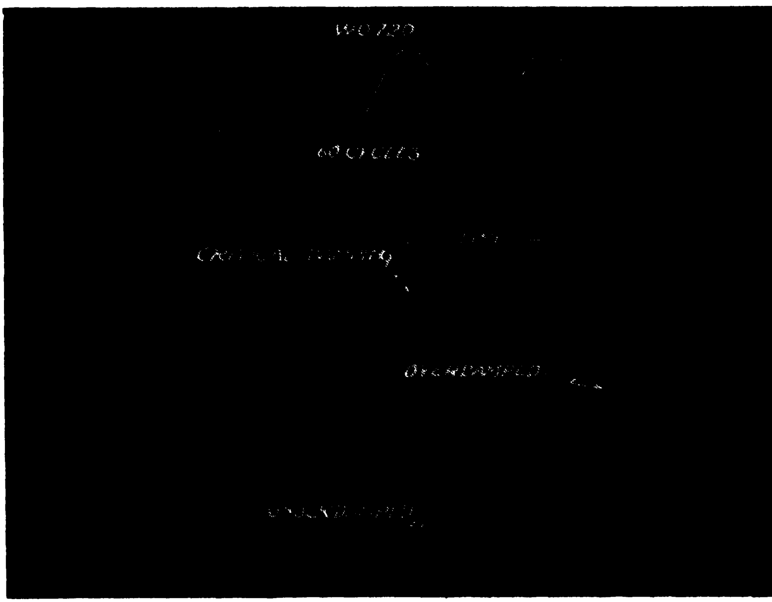
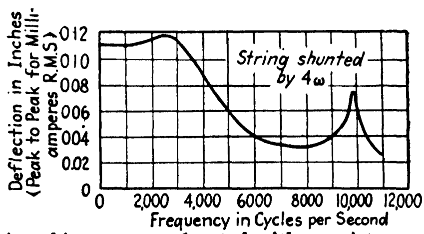


Fig. 436.—Kerosene and tuned shunt damping. Natural frequency = 1,700 cycles per second. Sensitivity = 4.2 mils per inch.

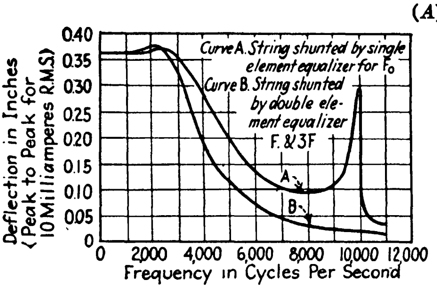
To raise the working range, it has been customary to reduce the mass of the moving element and to increase the tension on the vibrating strips or string, but the limit is soon reached. To provide an instrument for work in the audio range, engineers of the Bell Telephone Laboratories employ a string galvanometer (page 35) having metal strings and a moderate free period, 3,500 cycles per second for example, together with a corrective network,[8] so that a practically flat frequency characteristic is obtained from 0 to 10,000 cycles per second (Fig. 437*C*). The addition of a simple nonreactive shunt to an instrument with only air damping, while it greatly reduces the sensitivity, improves the response curve for frequencies below that of resonance by greatly increasing the damping coefficient (see Fig. 437*A*). The sharp resonance peak for the fundamental has become a rounded hump in the curve, while that for the third harmonic is greatly reduced.

If an Irwin shunt is employed, tuned for the fundamental, the frequency characteristic in Fig. 437*B* is obtained. The sensitivity is about three times that obtained with the nonreactive shunt, while below about 2,300 cycles per second the frequency characteristic is quite as good. But the sensitiveness for the third harmonic has been increased. If a



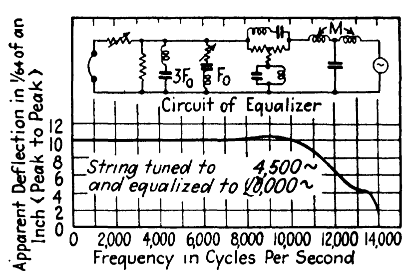
Characteristics of instrument shunted with a resistance of 4 ohms.

(*A*)



Characteristics of galvanometer with resonant shunt, alone for curve *A*, and with an additional shunt to suppress the resonance at $3F_0$, for curve *B*.

(*B*)



Equalizing network with new oscillograph for 10,000 cycles, and the characteristic obtained.

(*C*)

FIG. 437.—Response curve for string oscillograph. *A*, characteristics of instrument shunted with a resistance of 4 ohms; *B*, characteristics of galvanometer with resonant shunt, alone for curve *A*, and with an additional shunt to suppress the resonance at $3F_0$ for curve *B*; and *C*, equalizing network employed with a new oscillograph for 10,000 cycles, and the characteristic obtained.

second shunt tuned to three times the fundamental is added, after the manner of Butterworth and his associates, the third-harmonic peak disappears, as in Fig. 437*B*.

The Bell Telephone Laboratories have developed the corrective network of Fig. 437*C*. The results obtained are shown in the diagram. It is important that the phase displacement of the deflection with respect to the current be proportional to the frequency. When the corrective network is employed, there is some phase distortion, but it is not great enough to be troublesome.

The Electrostatic Oscillograph.[9]—The electrostatic oscillograph is a form of electrometer so designed that the rate of free vibration is high, and the damping of proper value. The instrument is used heterostatically. The high free period necessitates a strong controlling force; and as the electrostatic forces are small when low potentials are concerned, the electrostatic oscillograph is naturally best adapted to high-voltage work. For such work it possesses the advantage that it consumes extremely small energy, and the current required is exceedingly small, being only that necessary to charge it and to charge the condenser multipliers which are used at very high voltages.

In the instrument devised by Ho and Koto (Fig. 438), the potential to be measured is applied between the plates F_1 and F_2, either directly or through a condenser multiplier if the voltages are very high. These plates serve as "quadrants." They were 9 mm. wide and 15 mm. long, exactly alike, and placed 5 mm. apart. The mirror m is observed through one of the openings w_1, the other opening w_2 being added in order that the arrangement may be symmetrical.

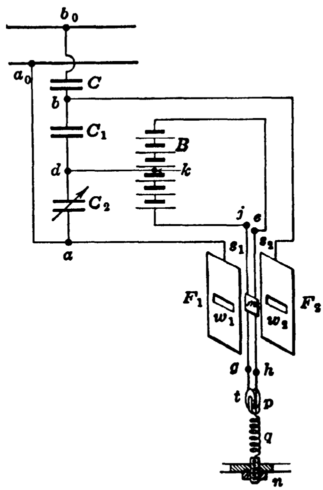


FIG. 438.—Diagram of electrostatic oscillograph of Ho and Koto.

The condensers C_1 and C_2 serve to split the potential which is applied between the quadrants. They are nominally of the same capacitance; one of them must be adjustable. The condenser multiplier is introduced by the insertion of the condenser C.

The moving members, or "needles," are formed by two metal strips s_1 and s_2. As in the ordinary oscillograph, they are stretched by a spring q between insulating bridge pieces. They are insulated from each other at their lower ends by the silk thread gph. The needles are charged from a 300-volt battery of dry cells B. The middle of the battery is connected to the point d, between the condensers C_1 and C_2.

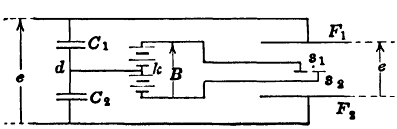


FIG. 439.—Pertaining to demonstration for electrostatic oscillograph.

Assume the electrometers to be perfectly symmetrical; the arrangement is that shown in Fig. 439. Consider a fall of potential to be positive in the direction of the arrow. Let e be the voltage between F_1 and F_2 at any instant, and let B be the constant e.m.f. of the well-insulated battery. F_1, F_2, s_1 may be regarded as forming the elements of one electrometer; and F_1, F_2, s_2, those of another. The two moving elements or needles are mechanically coupled by the mirror m, which is cemented to both.

It has been shown (see page 237) that the force acting on the movable element of an electrometer may be represented by

$$f = K(2Vd + d^2),$$

where d is the fall (or rise) of potential from quadrant 1 to quadrant 2, and V is the fall (or rise) of potential from quadrant 2 to the needle. Applying this to the electrometer F_1, F_2, s_1,

$$f_1 = K\left[2e\left(-\frac{e}{2} + \frac{B}{2}\right) + e^2\right] = +KBe;$$

and for the electrometer F_1, F_2, s_2,

$$f_2 = K\left[2e\left(-\frac{e}{2} - \frac{B}{2}\right) + e^2\right] = -KBe.$$

Therefore, the mirror that is cemented between s_1 and s_2 is acted upon by a couple proportional to Be. If the natural period of the instrument is high (a frequency of 3,500 vibrations per second may be obtained), and if proper damping is employed, the deflection at every instant is $\theta = kBe$, or the instrument follows the wave as an ordinary oscillograph.

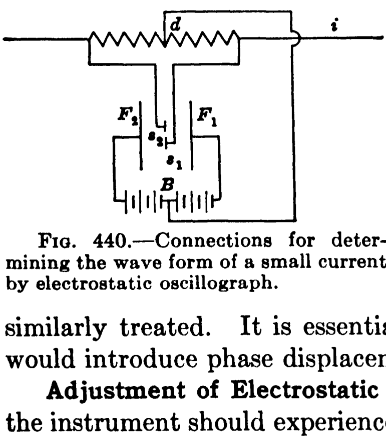


FIG. 440.—Connections for determining the wave form of a small current by electrostatic oscillograph.

It is necessary to prevent sparking between the various parts of the instrument and across the condenser. Consequently, the entire system F_1, s_1, s_2, F_2 is immersed in transparent oil of high dielectric strength, the condenser being similarly treated. It is essential that no dielectric losses occur, as they would introduce phase displacements.

Adjustment of Electrostatic Oscillograph.—If the voltage B is zero, the instrument should experience no deflection; as it is practically impossible, however, to construct the instrument with the mathematical accuracy assumed, the following adjustment is necessary.

Make B equal to zero, or connect both strips to d, and apply the full alternating voltage between F_1 and F_2. In general, the strips will vibrate with a frequency twice that of the voltage. One of the condensers C_1, C_2 is then adjusted until this deflection disappears. If the adjustment is not perfect, the two halves of a symmetrical alternating-current wave will appear dissimilar. If the connection at k is not made at the correct point, the wave will be shifted with respect to the zero line.

The metallic vibrator case should be connected to d. The sensitivity can be varied by altering B.

The sensitivity attained is as follows: With $B = 300$ volts, $e = 2{,}000$ volts, scale distance 70 cm., the amplitude of wave trace is 2 cm.

For the measurement of very small currents, the connections in Fig. 440 may be used. The voltage B is made very large by using either a high-tension battery or two condensers in series continuously charged from an influence machine.

Cathode-ray Oscillograph.—All the oscillographs thus far described have distinct frequency limitations. The bifilar instrument was designed for, and is admirably adapted to, work at power frequency. The string galvanometer may be made effective in the lower range of audio frequencies. However, it is necessary to have an instrument that will respond correctly in the audio range and beyond and to high-period transients or impulses, which last for only a few microseconds.

The cathode-ray oscillograph, developed from the Braun tube[10] (Fig. 441), effectively fulfills these requirements.

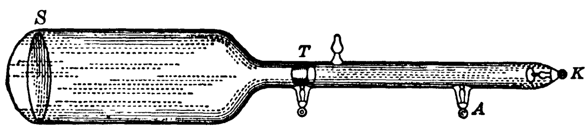


FIG. 441.—Braun tube for determining wave form.

Referring to Fig. 441, a flat aluminum cathode is located at K, the anode being at A. T is a grounded metal target pierced by a small hole about 0.5 mm. in diameter. The transparent screen S is coated with a fluorescent material—zinc sulphide or calcium tungstate. The original Braun tube had a cold cathode and a high vacuum, requiring a constant-potential difference of about 50,000 volts to be impressed between the anode and cathode. Usually, an electrostatic machine was employed.

Cathode rays from K fall on the target T where most of them are stopped. A small pencil of electrons passes through the opening in T and falls on the screen S, where it produces a bluish or greenish fluorescent spot. In passing through a magnetic field, the pencil is deflected so as to cut the lines of force; if the pencil passes through an electrostatic field, furnished by the deflection plates, it is deflected in the plane of the electrostatic lines of force. The deflecting magnetic field may be due to a pair of coils placed with their axis perpendicular to the axis of the tube. The deflecting electrostatic field is best produced by two condenser plates located within the tube. On the passage of a current through the coils, or the application of a voltage between the plates the fluorescent spot is deflected proportionally to the current or voltage. If the pencil passes in succession through two magnetic or two electrostatic fields, or a combination of both, the deflection is dependent on the vector sum of the two component fields. This is a unique and most important feature of the device. As a cathode beam is substantially without inertia, the deflection takes place instantaneously with the establishment of the deflecting field. This is a second unique property of the tube.

Immediately the electrons enter the deflecting field their deflection begins and continues to increase until they leave the field, after which they continue in their final direction straight on to the fluorescent screen. Consider an electron, and assume for purposes of explanation that it suddenly enters a uniform magnetic field of strength β of axial length l between a and b (Fig. 442) and then suddenly emerges. The final deflection d is made up of two parts: d_1, which takes place during the traverse of the field; and d_2, which takes place in the distance l'. Assume that the mass of the electron is m; its charge is e, and its velocity is v. The moving electron is equivalent to a current ev. The force due to the deflecting field is

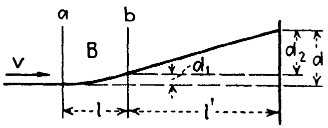


FIG. 442.—Pertaining to cathode-ray deflection.

$$F = ev\beta = ma.$$

The acceleration a is given by

$$a = \left(\frac{e}{m}\right)v\beta.$$

Referring to Fig. 442, the transverse deflection d_1 due to a is

$$d_1 = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{e}{m}\right)v\beta\left(\frac{l}{v}\right)^2.$$

The transverse velocity of the electron as it leaves the field is

$$v_2 = \left(\frac{e}{m}\right)v\beta\left(\frac{l}{v}\right) \quad \text{and} \quad d_2 = \left(\frac{e}{m}\right)v\beta\left(\frac{l}{v}\right)\left(\frac{l'}{v}\right).$$

Therefore

$$d = \left(\frac{e}{m}\right)\left(\frac{\beta}{v}\right)l\left[l' + \frac{l}{2}\right] \quad EM \text{ units, length in centimeters.} \qquad (19)$$

$\left(\frac{e}{m}\right) = 5.30 \times 10^{17}$ electrostatic units $= 1.77 \times 10^7$ electromagnetic units.

The electron velocity in cm./sec. due to a potential difference V between the anode and cathode is

$$v = \sqrt{2\left(\frac{e}{m}\right)V};$$

if V is in volts,

$$v = 5.97 \times 10^7\sqrt{V} \text{ cm./sec.}$$

Consequently, for electromagnetic deflections

$$d = 0.298\frac{\beta}{\sqrt{V}}l\left[l' + \frac{l}{2}\right]. \tag{20}$$

For electrostatic deflection, if E is the potential difference of the deflecting plates, and l and d their length and distance apart,

$$d = \left(\frac{e}{m}\right)\left(\frac{E}{v^2}\right)\left(\frac{l}{d}\right)\left[l' + \frac{l}{2}\right] \tag{21}$$

$$= 0.50\left(\frac{E}{V}\right)\left(\frac{l}{d}\right)\left(l' + \frac{l}{2}\right). \tag{21a}$$

In many cases, it is advantageous to employ the electrostatic deflection, for practically no energy is expended in the deflecting arrangement, and the capacitance between the deflecting plates is only a few micro-microfarads, while the resistance and inductance of coils used to produce electromagnetic deflections are substantial.

In order to view the waves or obtain permanent records, it is necessary to introduce the time coordinate. With recurrent phenomena the spot may be viewed in a rotating mirror or photographed on a film carried by a synchronously rotating drum. For visual observation, it is desirable that the curve appear on the fluorescent screen. This may be accomplished by using an auxiliary set of coils or plates which cause a deflection of the spot perpendicular to that due to the unknown current or voltage. At low frequencies these coils or plates have sometimes, in the past, been excited from a drop wire wound on the circumference of a circular disk which is rotated synchronously with the wave to be delineated. Such a device gives a saw-tooth wave of current or voltage to the deflecting arrangement. Some form of "sweep circuit" (page 673) controlled by the phenomena under investigation is now universally employed. From (20) the electromagnetic sensitivity is inversely proportional to the square root of the anode-to-cathode voltage, while the electrostatic sensitivity is inversely proportional to that voltage. If the electrons strike the screen with a high velocity, the spot is brilliant. If the electron velocity is low, the spot is less brilliant, but the deflection is greater. That is, brilliancy and sensitivity are alternative properties of the tube. There are two kinds of brilliancy, each requiring its appropriate coating on the screen. J. B. Johnson[10] employs a coating of equal parts calcium tungstate (to obtain photographic efficiency) and zinc silicate (to obtain visual efficiency) cemented with water glass. The brilliancy also depends on concentrating or focusing the cathode rays on a small area of the screen, decreasing the width of the trace and rendering the oscillograms more legible.

To convert the Braun tube into a practical laboratory appliance, the lines of development must be (1) the convenient production of a copious supply of electrons, in many cases at comparatively low anode-to-cathode voltages; (2) the development of efficient focusing arrangements; (3) the development of convenient means for introducing the time coordinate; (4) the choice of proper materials for the fluorescent screen for both visual and photographic purposes; (5) the increase of photographic efficiency; (6) the production of a tube having long life. The resulting device must be convenient to manipulate and free from sporadic electrostatic effects due to charges on the glass, etc.

A large supply of electrons is ensured by the use of a hot, coated cathode. The lower the driving voltage the greater the sensitivity, and the less the photographic and visual efficiency. Consequently, a compromise must be made. For general use, the present trend is to use voltages of 1,000 to 5,000 volts. With low voltages and therefore low electron velocities, the mutual repulsions of the electrons tend to make the fluorescent spot large. It is important to concentrate the rays on a small, sharply defined area of the screen.

Early in the development of the cathode-ray oscillograph, it became customary to surround the tube between the cathode and the target with a short coaxial coil, called a *focusing coil,* traversed by a direct current of adjustable magnitude. A magnetic field parallel, in the main, to the axis of the tube is produced. An electron traversing an electric field along a line of force is not deviated; but if its path crosses the lines of force, there is a transverse component of the velocity, and the field acts the same as the deflecting field previously discussed. The result is that the electron moves in a spiral path. After completing one turn, it arrives at the image of its original starting point. This is true irrespective of the angle between the original path of the electron and the lines of force. In practice, the focusing coils are short, and their fields are not uniform, but the focusing property remains. The action of focusing coils has been studied by H. Bush.[10]

An entirely different scheme for focusing is employed by J. B. Johnson in his low-voltage, hot-cathode tube.[10] An oxide-coated cathode, operating at about 300 volts, is employed, and the tube contains a small amount of a heavy gas, argon, at about 0.01 mm. pressure.

As it is necessary to guard against the deteriorating effect of positive-ion bombardment and against arcing, the filament is bent into a circular arc concentric with the axis of the tube; the anode and cathode are enclosed in a small tube, the dimensions of which are less than the mean free path of the electrons in the gas, so that ionization cannot build up.

The passage of the electron beam ionizes some of the gas; the resulting electrons are much more mobile than the heavy positive ions and move out of the beam leaving an excess of positive ions in a beam surrounded by a negative space charge. As a result, the outer electrons in the beam are urged toward the center. The ionization also prevents the accumulation of charges on the glass. It is found that too small an electron current produces a large, unfocused spot, while too large an electron current focuses the beam before it reaches the screen. Hence the focusing is

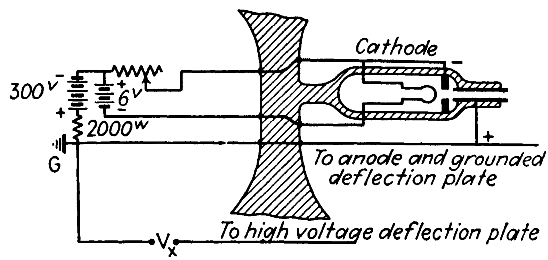


FIG. 443.—Electron gun for Johnson cathode-ray oscillograph

accomplished by adjusting the heating current of the cathode. The cathode arrangement shown in Fig. 443 has been improved, but the details have not been published.

It should be kept in mind that owing to the presence of the gas and the consequent ionization, the tube does not act as a perfect electronic device. When voltage is applied to the deflecting plates, positive ions from the beam are drawn toward the negative plate, and electrons are drawn toward the positive plate. The space charge set up by the ions and electrons produces a field opposing that due to the applied voltage, so there is little deflection of the beam until the potential difference between the plates is greater than that at which all the ions produced are drawn to the plate—2 or 3 volts. This accounts for the slight hesitation observed when the spot moves across the center of the screen.

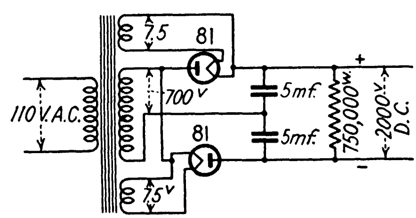


FIG. 444.—Showing a rectifying circuit for supplying direct voltage to cathode-ray oscillograph.

The present trend in the development of the cathode-ray oscillograph is toward the employment of highly evacuated (hard) tubes with hot cathodes and moderately high excitation voltages and of electrostatic focusing obtained by projecting the electron beam axially through the proper arrangement of diaphragms and tubular electrodes which are kept at definite predetermined voltages. Since the current required by a cathode-ray tube is very small, it is obtained conveniently by rectification from the ordinary 110-volt supply circuit, a very simple form of filter being used. Figure 444 shows one such arrangement. The required voltage is considerably in excess of the ratings of the usual small rectifier tubes, and a "voltage-doubling" circuit is generally desirable.

Two separate half-wave rectifiers, each of which keeps a condenser charged practically to the peak value of the alternating-current voltage output of the transformer, are employed. The condensers are connected in series, so the direct-current output voltage will be 2 × 1.4 × the r.m.s. voltage of the transformer. Thus two type-81 rectifier tubes, maximum rating 700 volts r.m.s., will provide a 2,000-volt power supply.

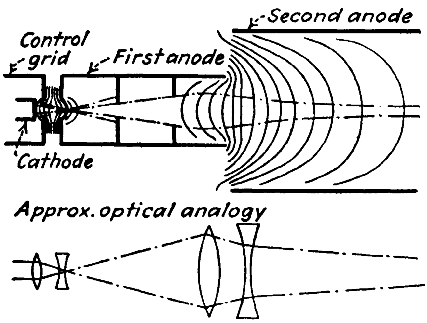


FIG. 445.—Arrangement of electrodes in electron gun for electrostatic focusing.

Figure 445 shows diagrammatically an arrangement for electrostatically focusing the spot of light on the fluorescent screen.[10] The curved lines are the traces of the equipotential surfaces on a plane through the axis. The action is analogous to that of a system of lenses on a beam of light. The optical analogy is shown in the lower part of the figure. The part of the cathode-ray tube for generating, concentrating, controlling, and focusing the electron beam is called an *electron gun*. The principles involved in this device have been discussed by Zworykin[10]

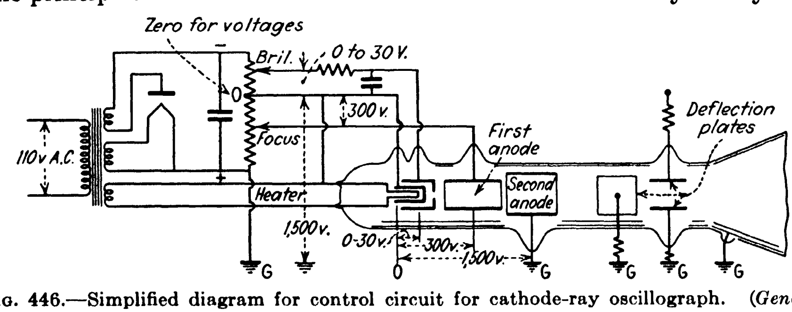


FIG. 446.—Simplified diagram for control circuit for cathode-ray oscillograph. (*General Radio Company.*)

and by Maloff and Epstein.[10] By means of electrostatic focusing it is possible to focus the beam on the screen in a brilliant spot about one mm. in diameter. To prevent injury to the fluorescent screen, such a spot should not be allowed to remain in a fixed position. It should be kept in motion or deflected entirely off the screen except when actual observations are being taken. A simplified diagram of a circuit, arranged to control a modern cathode-ray tube, is shown in Fig. 446.

Sweep Circuits.[11]—The time coordinate necessary for visual observation and for photographing wave forms on a stationary photographic plate is introduced by a form of "sweep circuit." The fundamental idea is to excite a pair of deflecting plates by the voltage between the terminals of a condenser which is charged at a uniform rate and then instantaneously discharged; thus the fluorescent spot moves across the screen

Resistors	Condensers	Tubes	Lamps
$R1$ = 10 k Ω	$C1$ = 8 μf	$T1$ = RCA 80 (or equiv.)	$T1$ = Mazda 41
$R2$ = 100 k Ω	$C2$ = 8 μf	$T2$ = RCA 885 (or equiv.)	
$R3$ = 100 k Ω	$C3$ = 8 μf	$T3$ = RCA 58 (or equiv.)	
$R4$ = ½ M Ω	$C4$ = 0.5 μf		
$R5$ = 500 Ω	$C5$ = 0.005 μf		
$R6$ = 1 k Ω	$C6$ = 0.05 μf		
$R7$, $R8$, $R9$, $R10$ = 30 k Ω			

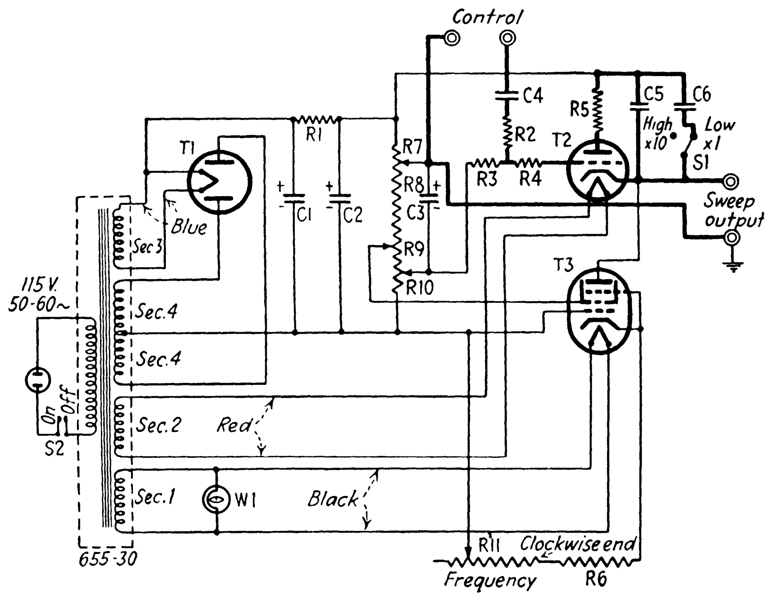


FIG. 447.—Schematic diagram of Bedell sweep circuit. (*General Radio Company.*)

at a uniform rate and then instantaneously returns to its original position. The process is repeated indefinitely. The main deflecting plates to which the phenomenon under investigation is applied deflect the fluorescent spot perpendicularly to the displacement due to the sweep circuit. Thus the trace on the screen is in rectangular coordinates. The frequency of this cycle of operations is governed by the phenomenon under investigation. Figure 447 shows a Bedell sweep circuit as arranged by the General Radio Company.

In Fig. 447, T_1, R_1, C_1, and C_2 constitute a power supply. T_2 is a gaseous discharge tube, and T_3 is a triple-grid supercontrol amplifier tube, which operates as a constant-current device giving a uniform charging rate for condensers C_5 and C_6. The space current of this tube and therefore the rate of charging the condensers is adjusted by varying the potential of the control grid by means of the resistor R_{11}. The rate at which the potential across the condensers builds up may be changed by a factor of 10 by cutting out C_6. At a predetermined plate potential, the tube T_2, breaks down and discharges the condensers practically instantaneously. The exact moment of breakdown may be controlled by applying an alternating potential to the grid of T_2 through the "control terminals." When the plate potential is just about enough to break down, an increase in the grid potential will cause breakdown to occur. The sweep frequency and the control voltage are thus synchronized. The three adjustable contacts on R_7, R_8, R_9, and R_{10} are set at the factory. The lowest contact controls the grid bias on T_2 and therefore the voltage that must build up across the condensers C_5 and C_6 before T_2 will break down. This controls the length of sweep. The next contact controls the shield-to-grid potential of T_3 and hence the space current of that tube and the rate of charging the condensers C_5 and C_6. This gives a preliminary adjustment of the sweep frequency. The top contact controls the direct-current potential between the sweep output terminals and hence the location of the electron beam in the oscillograph. The length of sweep, location of spot, etc., are adjusted once and for all for any particular oscillograph. The control circuit will operate on 5 to 100 volts r.m.s. The sweep frequency may be varied continuously from about 30 to 300 cycles per second by means of a single knob. This range may be multiplied by 10 by cutting out C_6.

To register very rapidly varying nonrecurrent phenomena, for example, a lightning discharge, either natural or artificial, it is necessary to increase enormously the photographic sensitivity and diminish to a few microseconds the time necessary for the image to sweep across the photographic surface. To increase the photographic sensitivity, Dufour[10] placed the photographic surface within the tube, so that the cathode rays would impinge directly upon this surface. He also used a high anode-to-cathode voltage, 50,000 volts, thus giving the electrons a high velocity and great photographic power, though at the expense of sensitivity. As the tube must be opened to change the photographic plates, it is constructed largely of metal, only the electron gun being of glass. It is necessary to have a ready means of exhausting the tube and removing moisture. A diffusion pump is employed in series with a permanently connected mechanical pump, all the connections being of ample diameter to ensure rapid exhaustion. The penetration of the cathode rays through gelatin is very small, and photographic plates with a very thin coating are used. The plates intended for photographing spectra in the ultraviolet region have proved satisfactory. The photographic impression is increased by the fluorescent effect if the plates are dusted with calcium tungstate. Following Dufour, instruments in which the cathode beam falls directly on the photographic surface have been designed by Norinder,[11] A. B. Wood,[10] Rogowski, R. H. George,[11] the Westinghouse Electric and Manufacturing Company, and the General Electric Company.

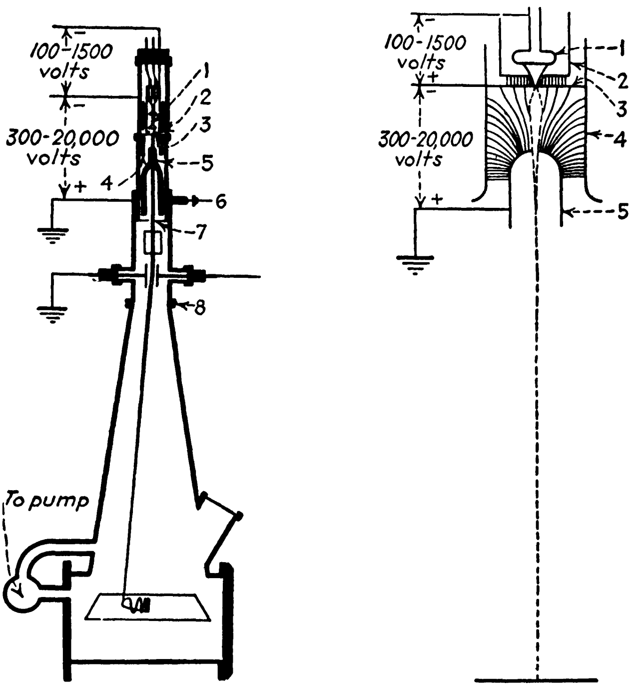


FIG. 448.—Diagram for George portable cathode-ray oscillograph. (1) Ribbon filament, (2) filament shield, (3) positive plate, (4) cup-shaped cathode, (5) cylindrical anode, (6) screw for raising and lowering anode.

Figure 448 shows diagrammatically the instrument and electron gun designed by George for use in lightning investigations. Electrons from the oxide-coated filament 1 pass through a hole in the filament shield 2 and fall upon a positive screen at 3. Some of the electrons pass through a hole in this screen in a divergent beam. The cathode 4 is cup shaped, and the anode 5 projects into it, giving rise to an electrostatic field, as indicated. The divergent beam of electrons projected through the field is accelerated and concentrated toward the axis and, with the proper adjustments, may be brought to a focus on the photographic plate. The anode may be raised or lowered from without by a rack-and-pinion motion, and the configuration of the electrostatic field altered, so that focusing is possible at widely different anode voltages. To focus a high-voltage beam, the anode is raised, and the plate voltage of the gun is increased. If a low-voltage beam is to be focused, the anode is lowered, and the plate voltage is reduced. The adjustments are not critical.

The spacings between the high-voltage electrodes are small compared with the mean free path of the electrons, and the beam is shielded from the field of surface charges on the insulation by the form of the electrodes. For the study of lightning disturbances, equipment is included for automatically applying plate voltage to the gun. The relay circuit devised by George is shown in Fig. 449. The filament is heated, and

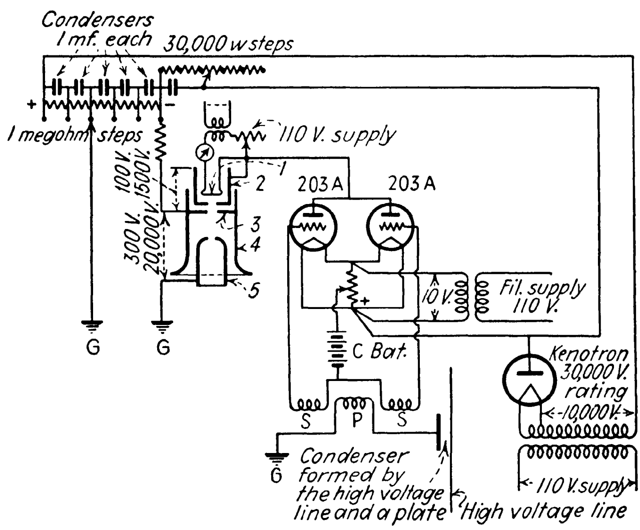


FIG. 449.—George relay for cathode-ray oscillograph. Used in investigating lightning discharges.

the bias on the two 203A tubes is adjusted to near the tripping point. When a lightning disturbance occurs in the neighborhood of the high-voltage line, the condenser formed by the line and a plate connected to ground through the primary P of the transformer is charged, one of the 203A tubes breaks down, and the electron beam is projected on the photographic plate. The oscillograph is actuated in from $\frac{1}{3}$ to $\frac{1}{2}$ a microsec. after the disturbance has occurred.

In instruments of this general type, a high-speed time coordinate may be obtained by employing electromagnetic deflection once across the plate in the direction X (Fig. 450) at a uniform velocity, while a sinusoidal current from another source, having a frequency of perhaps 50,000 cycles per second, deflects the spot in the Y direction. The voltage to be depicted is thrown on the deflecting plates, while the spot is moving along the "straight" part of the sinusoidal trace. The result is shown in Fig. 450. Note that the time scale is not uniform.

The cathode-ray oscillograph records with fidelity, certainly, up to a frequency of 1,000,000 cycles per second. The limit is reached when the time required for the passage of an electron through the deflecting field is comparable with the frequency.

A recent development in cathode-ray oscillography is to employ a tube with a Lenard window,[11] thus avoiding the inconvenience of opening the tube for changing the photographic plate.

FIG. 450A.—High-voltage impulse superimposed on a 50,000-cycle timing-wave 10,000-volt beam.

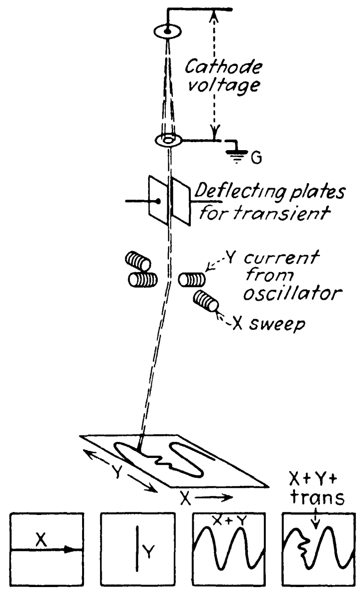


FIG. 450B.—Dufour method of introducing time coordinate for transients.

WAVE ANALYSIS[12]

Curves taken by the foregoing methods show that in practice both the potential difference and the current waves may depart widely from the sinusoidal form. In any particular case, after having obtained the graph of the wave, it is possible to write its equation as the sum of a series of sinusoidal terms of multiple frequencies which have the proper time-phase relations.

To effect such a harmonic analysis, recourse is had to the work of Fourier, who, in 1812, first explicitly showed that a function subject to certain mathematical conditions can be represented by a constant term plus the sum of a sine and a cosine series.[12] This result he published in his "Théorie analytique de chaleur" (1822). Accordingly,

$$\left.\begin{aligned} f(\theta) &= A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + \cdots \\ &+ \frac{B_0}{2} + B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + \cdots . \end{aligned}\right\} \quad (22)$$

The coefficients are given by the following equations:[12]

$$A_k = \frac{1}{\pi}\int^{2\pi} f(\theta) \sin k\theta \, d\theta. \quad (23)$$

$$B_k = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos k\theta \, d\theta. \quad (24)$$

The expression (22) is called a Fourier series.

The sine and cosine terms may be combined, for

$$A \sin\theta + B \cos\theta = \sqrt{A^2 + B^2} \sin\left(\theta + \tan^{-1}\frac{B}{A}\right) = C \sin(\theta + \alpha').$$

This gives

$$f(\theta) = \frac{B_0}{2} + C_1 \sin(\theta + \alpha_1') + C_2 \sin(2\theta + \alpha_2') + C_3 \sin(3\theta + \alpha_3') + \cdots . \quad (25)$$

If the origin is taken at the zero of the fundamental, which is convenient if the waves are to be plotted,

$$f(\theta) = \frac{B_0}{2} + C_1 \sin\theta + C_2 \sin 2(\theta + \alpha_2) + C_3 \sin 3(\theta + \alpha_3) + \cdots , \quad (25a)$$

where

$$\alpha_2 = \frac{\alpha_2'}{2} - \alpha_1' \qquad \alpha_3 = \frac{\alpha_3'}{2} - \alpha_1'; \text{ etc.}$$

Here α_n is measured on the same scale as θ, and the zero point of any component is where it first passes from a negative to a positive value. A positive value of α_n indicates a leading component. The constant term is usually absent in alternating-current waves.

The effect of the odd and even harmonics and of their phase displacements is illustrated in Fig. 451. Odd harmonics produce a wave the second half of which is like the first with the algebraic signs of all the ordinates reversed. Even harmonics produce an asymmetrical wave.

The integrals in (23) and (24) are proportional to the areas under the curves that would be obtained if each value of $f(\theta)$ were multiplied by the corresponding value of sin $k\theta$ or cos $k\theta$, and new curves plotted on the same base 2π. Therefore,

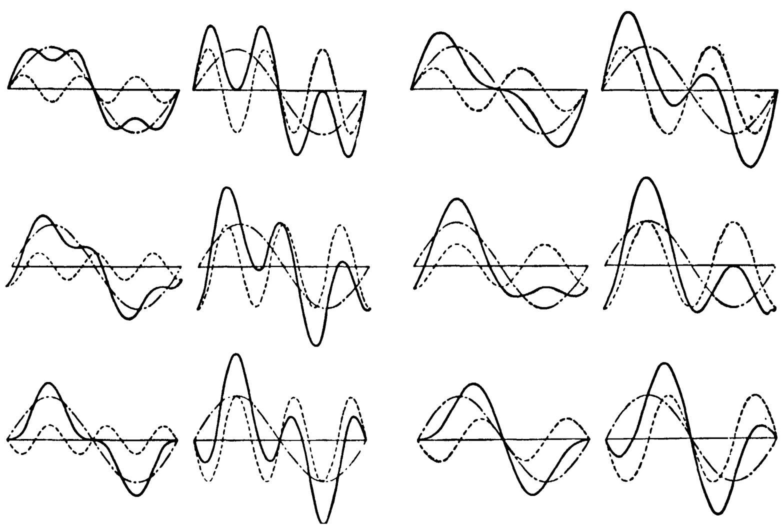
$$A_k = \text{twice the average ordinate of curve } f(\theta) \sin k\theta. \quad (23a)$$

$$B_k = \text{twice the average ordinate of curve } f(\theta) \cos k\theta. \quad (24a)$$

The integration could be performed by a planimeter.

The labor involved in carrying out the process just indicated is prohibitive, but machines called *harmonic analyzers* have been devised, some of which practically effect the determination of A_k and B_k in this manner.

The various coefficients may be determined arithmetically as follows: A complete cycle of the curve to be analyzed is plotted. The base (2π) is then divided into $2n$ equal spaces, and ordinates erected. Let the



Odd Harmonics Even Harmonics

FIG. 451.—Illustrating the effects of odd and of even harmonics.

measured values of the $2n$ ordinates be $y_0, y_1, y_2 \; . \; . \; . \; y_{2n-1}$. The distance between consecutive ordinates is $\Delta\theta = 2\pi/2n = \pi/n$. By (23*a*),

$$A_k = \frac{2}{2n}[y_0 \sin 0k\Delta\theta + y_1 \sin 1k\Delta\theta + y_2 \sin 2k\Delta\theta + \cdots + y_{2n-2} \sin (2n - 2)k\Delta\theta + y_{2n-1} \sin (2n - 1)k\Delta\theta]. \quad (26)$$

If the data are furnished by the contact method, the measurements being made at equal intervals along the wave, the values of y_0, y_1, etc., are given directly, and the curve need not be plotted. Equation (26) may be written

$$A_k = \frac{1}{n}\sum_{m=0}^{m=2n-1} y_m \sin km\Delta\theta, \quad (27)$$

where m is the number of the ordinate concerned in the multiplication; similarly,

$$B_k = \frac{1}{n}[y_0 \cos 0kd\theta + y_1 \cos k\Delta\theta + y_2 \cos 2k\Delta\theta + \cdots + y_{2n-2} \cos (2n - 2)k\Delta\theta + y_{2n-1} \cos (2n - 1)k\Delta\theta] \quad (28)$$

$$= \frac{1}{n}\sum_{m=0}^{m=2n-1} y_m \cos km\Delta\theta. \quad (29)$$

Runge Method of Grouping Terms.[13]—Theoretically, it is easy to calculate the coefficients. The practical difficulty lies in the great expenditure of time necessary for carrying out the process as indicated. For this reason, Runge[13] has introduced an abridged method of calculation which is carried out by aid of a systematically arranged schedule.

In the majority of cases, the two halves of an alternating-current wave are the same except for the algebraic sign; that is, only the odd harmonics are present, and consequently the values of k are odd numbers. It is necessary, therefore, to deal with only one-half of the wave, and $2n$ spaces per *half* wave are used, making $\Delta\theta = \pi/2n$.

The method of grouping the terms so as to economize time may be explained as follows. Referring to (26), y_1 is multiplied by $\sin \frac{k\pi}{2n}$, and y_{2n-1} by $\sin (2n - 1)\frac{k\pi}{2n}$. As k is odd,

$$\sin \frac{k\pi}{2n} = \sin \left(k\pi - \frac{k\pi}{2n}\right) = \sin (2n - 1)\frac{k\pi}{2n};$$

and, in general,

$$\sin (2n - m)\frac{k\pi}{2n} = \sin \frac{mk\pi}{2n}. \quad (30)$$

Consequently,

1. The number of multiplications may be halved by adding, before taking the products, those values of y for which the sum of the subscripts is $2n$.

2. Again, the same products are needed in A_k and A_{2n-k}, that is, in those coefficients for which the sum of the subscripts is $2n$, since

$$\pm \sin (2n - k)\frac{m\pi}{2n} = \sin k\frac{m\pi}{2n}. \quad (31)$$

If the number of the ordinate concerned in the multiplication (m) is even, the sign of the left-hand member is $-$; and if m is odd, the sign is $+$.

For example, suppose that a half period is divided into $2n = 12$ equal parts ($\Delta\theta = \pi/2n \approx 15°$), and the ordinates measured; then, by (26) and (30),

$$A_1 = \tfrac{1}{6}[(y_1 + y_{11}) \sin 15° + (y_2 + y_{10}) \sin 30° + (y_3 + y_9) \sin 45° + (y_4 + y_8) \sin 60° + (y_5 + y_7) \sin 75° + y_6 \sin 90°].$$

$$A_{11} = \tfrac{1}{6}[(y_1 + y_{11}) \sin 165° + (y_2 + y_{10}) \sin 330° + (y_3 + y_9) \sin 495° + (y_4 + y_8) \sin 660° + (y_5 + y_7) \sin 825° + y_6 \sin 990°].$$

For convenience the sines of all the angles may be expressed in terms of the sines of angles of 90 deg. or less.

Accordingly, by the aid of (31) the value for A_{11} may be written

$$A_{11} = \tfrac{1}{6}[(y_1 + y_{11}) \sin 15° - (y_2 + y_{10}) \sin 30° + (y_3 + y_9) \sin 45° - (y_4 + y_8) \sin 60° + (y_5 + y_7) \sin 75° - y_6 \sin 90°].$$

Applying the rules,

$$A_3 = \tfrac{1}{6}[\{(y_1 + y_{11}) + (y_3 + y_9) - (y_5 + y_7)\} \sin 45° + \{(y_2 + y_{10}) - y_6\} \sin 90°].$$

$$A_9 = \tfrac{1}{6}[\{(y_1 + y_{11}) + (y_3 + y_9) - (y_5 + y_7)\} \sin 45 - \{(y_2 + y_{10}) - y_6\} \sin 90°].$$

$$A_5 = \tfrac{1}{6}[(y_1 + y_{11}) \sin 75° + (y_2 + y_{10}) \sin 30° - (y_3 + y_9) \sin 45° - (y_4 + y_8) \sin 60° + (y_5 + y_7) \sin 15° + y_6 \sin 90°].$$

$$A_7 = \tfrac{1}{6}[(y_1 + y_{11}) \sin 75° - (y_2 + y_{10}) \sin 30° - (y_3 + y_9) \sin 45° + (y_4 + y_8) \sin 60° + (y_5 + y_7) \sin 15° - y_6 \sin 90°].$$

Cosine Terms.—The cosine terms may be treated in a similar manner. As

$$- \cos (2n - m)\frac{k\pi}{2n} = \cos \frac{mk\pi}{2n}, \tag{32}$$

the differences of the ordinates are involved. The equation corresponding to (31) is

$$\mp \cos (2n - k)\frac{m\pi}{2n} = \cos k\frac{m\pi}{2n}. \tag{33}$$

The sign of the left-hand member is + if m is even and − if m is odd. Applying the preceding and, for convenience, expressing the results in terms of the sines of the angles, the values of B are

$$B_1 = \tfrac{1}{6}[y_0 \sin 90° + (y_1 - y_{11}) \sin 75° + (y_2 - y_{10}) \sin 60° + (y_3 - y_9) \sin 45° + (y_4 - y_8) \sin 30° + (y_5 - y_7) \sin 15°].$$

$$B_{11} = \tfrac{1}{6}[y_0 \sin 90° - (y_1 - y_{11}) \sin 75° + (y_2 - y_{10}) \sin 60° - (y_3 - y_9) \sin 45° + (y_4 - y_8) \sin 30° - (y_5 - y_7) \sin 15°].$$

$$B_3 = \tfrac{1}{6}[\{(y_1 - y_{11}) - (y_3 - y_9) - (y_5 - y_7)\} \sin 45° + \{y_0 - (y_4 - y_8)\} \sin 90°].$$

$$B_9 = \tfrac{1}{6}[\{-(y_1 - y_{11}) + (y_3 - y_9) + (y_5 - y_7)\} \sin 45° + \{y_0 - (y_4 - y_8)\} \sin 90°].$$

$$B_5 = \tfrac{1}{6}[y_0 \sin 90° + (y_1 - y_{11}) \sin 15° - (y_2 - y_{10}) \sin 60° - (y_3 - y_9) \sin 45° + (y_4 - y_8) \sin 30° + (y_5 - y_7) \sin 75°].$$

$$B_7 = \tfrac{1}{6}[y_0 \sin 90° - (y_1 - y_{11}) \sin 15° - (y_2 - y_{10}) \sin 60 + (y_3 - y_9) \sin 45° + (y_4 - y_8) \sin 30° - (y_5 - y_7) \sin 75°].$$

The calculations may be systematized by the use of properly prepared forms such, for example, as that following. The numerical work may be checked by using particular values of θ.

$$y_0 = (B_1 + B_{11}) + (B_3 + B_9) + (B_5 + B_7).$$

$$y_6 = (A_1 - A_{11}) - (A_3 - A_9) + (A_5 - A_7).$$

$$y_3 + y_9 = 2 \sin 45°[(A_1 + A_{11}) + (A_3 + A_9) - (A_5 + A_7)].$$

$$y_3 - y_9 = 2 \sin 45°[(B_1 - B_{11}) - (B_3 - B_9) - (B_5 - B_7)].$$

$$y_4 + y_8 = 2 \sin 60°[(A_1 - A_{11}) - (A_5 - A_7)].$$

Following the plan outlined above, schedules corresponding to any number of measured ordinates may be prepared. If even harmonics are present, it is necessary to divide the *whole* wave into $2n$ parts.

The values of the coefficients A and B, obtained as above by the use of a definite number of measured ordinates, determine a curve which coincides with the original curve *at the measured points* and diverges from the original curve at intermediate points; consequently, the more complicated the wave form the greater the number of ordinates that must be used. A schedule based on 18 instead of 12 ordinates is frequently necessary. As a check on the sufficiency of the analysis, values of y intermediate between the measured values should be calculated and compared with the actual ordinates at the same points.

ILLUSTRATION OF THE USE OF A TWELVE-POINT SCHEDULE FOR THE ANALYSIS OF WAVES CONTAINING ONLY ODD HARMONICS

To analyze the e.m.f. wave of a small alternator 12 equally spaced ordinates were measured; their values are entered in the form below.

$y_0 = 0.30$	$y_1 = 8.50$	$y_2 = 14.30$	$y_3 = 20.60$
	$y_{11} = 8.70$	$y_{10} = 18.40$	$y_9 = 26.00$
Sums	$y_1 + y_{11} = 17.20$	$y_2 + y_{10} = 32.70$	$y_3 + y_9 = 46.60$
Differences	$y_1 - y_{11} = -0.20$	$y_2 - y_{10} = -4.10$	$y_3 - y_9 = -5.40$
	$y_4 = 26.15$	$y_5 = 29.80$	$y_6 = 32.25$
	$y_8 = 30.70$	$y_7 = 32.90$	$y_0 = 0.30$
Sums	$y_4 + y_8 = 56.85$	$y_5 + y_7 = 62.70$	$y_0 + y_6 = 32.55$
Differences	$y_4 - y_8 = -4.55$	$y_5 - y_7 = -3.10$	

Calculation of A_1 and A_{11}

$(y_1 + y_{11}) \sin 15° = (17.2)\ 0.2588 =$ .... →	4.452		
$(y_3 + y_9) \sin 45° = (46.6)\ 0.7071 =$ .... →	32.951		
$(y_5 + y_7) \sin 75° = (62.7)\ 0.9659 =$ .... →	60.562		
$(y_2 + y_{10}) \sin 30° = (32.7)\ 0.5000 =$	.... →		16.350
$(y_4 + y_8) \sin 60° = (56.85)\ 0.8660 =$	.... →		49.233
$y_6 \sin 90° = (32.25)\ 1.0000 =$	.... →		32.250
Sums..	$(97.965)_1$		$(97.833)_2$

$$\begin{array}{lr} & (97.965)_1 \\ & (97.833)_2 \\ \hline \text{Sum} & 195.798 = 6A_1 \end{array} \qquad \begin{array}{lr} & (97.965)_1 \\ & (97.833)_2 \\ \hline \text{Difference} & 0.132 = 6A_{11} \end{array}$$

$A_1 = 32.633$ $\qquad$ $A_{11} = 0.022$

Calculation of A_3 and A_9

$$\begin{array}{lr} y_1 + y_{11} = & 17.20 \\ y_3 + y_9 = & 46.60 \\ \hline \text{Sum} = & 63.80 \\ y_5 + y_7 = & 62.70 \\ \hline \text{Difference} = & 1.10 \end{array}$$

$$\{(y_1 + y_{11}) + (y_3 + y_9) - (y_5 + y_7)\} \sin 45° = (1.1)\ 0.7071 = (0.778)_1$$

$$\begin{array}{lr} y_2 + y_{10} = & 32.76 \\ y_6 = & 32.25 \\ \hline \text{Difference} = & 0.45 \end{array}$$

$$\{(y_2 + y_{10}) - y_6\} \sin 90° = \ldots\ldots\ldots\ldots (0.450)_2$$

$$\begin{array}{lr} & (0.778)_1 \\ & (0.450)_2 \\ \hline \text{Sum} & 1.228 = 6A_3 \end{array} \qquad \begin{array}{lr} & (0.778)_1 \\ & (0.450)_2 \\ \hline \text{Difference} & 0.328 = 6A_9 \end{array}$$

$A_3 = 0.205$ $\qquad$ $A_9 = 0.055$

Calculation of A_5 and A_7

$(y_1 + y_{11}) \sin 75° = (17.2)\ 0.9659 = 16.613$

$(y_5 + y_7) \sin 15° = (62.7)\ 0.2588 = 16.227$

Sum $= 32.840$

$(y_3 + y_9) \sin 45° = (46.6)\ 0.7071 = 32.951$

Difference $= (-0.111)_1$

$(y_2 - y_{10}) \sin 30° = (32.7)\ 0.5000 = 16.35$

$y_6 \sin 90° = \ldots\ldots = 32.25$

Sum $= 48.600$

$(y_4 + y_8) \sin 60° = (56.85)\ 0.8660 = 49.232$

Difference $= (-0.632)_2$

	$(-0.111)_1$		$(-0.111)_1$
	$(-0.632)_2$		$(-0.632)_2$
Sum	$-0.743 = 6A_5$	Difference	$+0.521 = 6A_7$
	$A_5 = -0.124$		$A_7 = +0.087$

Calculation of B_1 and B_{11}

$y_0 = (+0.30)\ 1.000 =$	$\ldots\ldots \rightarrow$	0.300	
$(y_2 - y_{10}) \sin 60° = (-4.10)\ 0.8660 =$	$\ldots\ldots \rightarrow$	-3.551	
$(y_4 - y_8) \sin 30° = (-4.55)\ 0.5000 =$	$\ldots\ldots \rightarrow$	-2.275	
$(y_1 - y_{11}) \sin 75° = (-0.20)\ 0.9659 =$	$\ldots\ldots$	$\ldots\ldots \rightarrow$	-0.193
$(y_3 - y_9) \sin 45° = (-5.4)\ 0.7071 =$	$\ldots\ldots$	$\ldots\ldots \rightarrow$	-3.818
$(y_5 - y_7) \sin 15° = (-3.1)\ 0.2588 =$	$\ldots\ldots$	$\ldots\ldots \rightarrow$	-0.802
Sums	$\ldots\ldots$	$(-5.526)_1$	$(-4.813)_2$

	$(-5.526)_1$		$(-5.526)_1$
	$(-4.813)_2$		$(-4.813)_2$
Sum	$-10.339 = 6B_1$	Difference	$-0.713 = 6B_{11}$
	$B_1 = -1.723$		$B_{11} = -0.119$

Calculation of B_3 and B_9

$$y_3 - y_9 = -5.40$$
$$y_5 - y_7 = -3.10$$

$$\text{Sum} = -8.50$$
$$y_1 - y_{11} = -0.20$$

$$\text{Difference} = -8.30$$

$$\{-(y_1 - y_{11}) + (y_3 - y_9) + (y_5 - y_7)\} \sin 45° = (-8.30)\,0.7071 = (-5.869)_2$$

$$y_4 - y_8 = -4.55$$
$$y_0 = 0.30$$

$$\text{Difference} = -4.85$$

$$\{(y_4 - y_8) - y_0\} \sin 90° = \ldots\ldots\ldots\ldots (-4.85)_1$$

	$(-4.850)_1$		$(-4.850)_1$
	$(-5.869)_2$		$(-5.869)_2$
Sum	$-10.719 = -6B_3$	Difference	$+1.019 = -6B_9$
	$B_3 = 1.786$		$B_9 = -0.170$

Calculation of B_5 and B_7

$$y_0 \ldots\ldots\ldots\ldots = 0.300$$
$$(y_4 - y_8) \sin 30° = (-4.55)\,0.5000 = -2.275$$

$$\text{Sum} = -1.975$$
$$(y_2 - y_{10}) \sin 60° = (-4.1)\,0.8660 = -3.551$$

$$\text{Difference} = (+1.576)_1$$
$$(y_1 - y_{11}) \sin 15° = (-0.20)\,0.2588 = -0.052$$
$$(y_5 - y_7) \sin 75° = (-3.1)\,0.9659 = -2.994$$

$$\text{Sum} \quad -3.046$$
$$(y_3 - y_9) \sin 45° = (-5.4)\,0.7071 = -3.818$$

$$\text{Difference} \quad (+0.772)_2$$

	$(1.576)_1$		$(1.576)_1$
	$(0.772)_2$		$(0.772)_2$
Sum	$2.348 = 6B_5$	Difference	$0.804 = 6B_7$
	$B_5 = 0.391$		$B_7 = 0.134$

Checks, see page 682

	+	−
B_1	.	1.723
B_3	1.786	.
B_5	0.391	.
B_7	0.134	.
B_9	.	0.170
B_{11}	.	0.119
Sums	2.311	2.012

Net sum 0.299

$y_0 = 0.300$

$A_1 = 32.633$
$A_{11} = 0.022$

Sum = ... $(32.655)_1$
Difference = $(32.611)_2$
$A_3 = 0.204$
$A_9 = 0.055$

Sum = ... $(0.259)_3$
Difference $(0.149)_4$

Sum...................... 32.914
$A_5 = -0.124$
$A_7 = +0.087$

Sum...................... $(-0.037)_5$
Difference = $(-0.211)_6$

Difference................. (32.951)
(32.951)(1.414) = 46.59
From curve $y_3 + y_9 = 46.60$

$(32.611)_2$
$(-0.211)_6$

Diff. [32.822] 1.732
= 56.84

From curve $y_4 + y_8 = 56.85$

$(32.611)_2$
$(-0.211)_6$

Sum 32.400
$(0.149)_4$

Diff. [32.251]
From curve $y_6 = 32.25$

$B_3 = 1.78$
$B_9 = -0.17$

Difference................ $(+1.95)_7$
$B_5 = 0.39$
$B_7 = 0.13$

Difference................ $(0.26)_8$

Sum.......................... 2.21
$B_1 = -1.720$
$B_{11} = -0.119$

Difference....................... $(-1.601)_9$

Difference..................... [+3.81]1.414 = + 5.37
From curve.................. $-(y_3 - y_9) = +5.40$

$C_1 = \sqrt{(32.633)^2 + (1.72)^2} = 32.678$

$C_3 = \sqrt{(0.204)^2 + (1.78)^2} = 1.798$

$C_5 = \sqrt{(0.124)^2 + (0.391)^2} = 0.410$

$C_7 = \sqrt{(0.087)^2 + (0.134)^2} = 0.160$

$C_9 = \sqrt{(0.055)^2 + (0.170)^2} = 0.179$

$C_{11} = \sqrt{(0.022)^2 + (0.119)^2} = 0.121$

$\tan^{-1}\frac{B_1}{A_1} = \tan^{-1}\frac{-1.72}{+32.633} = -3.^\circ 02$

$\tan^{-1}\frac{B_3}{A_3} = \tan^{-1}\frac{+1.78}{+0.204} = +83.^\circ 5$

$\tan^{-1}\frac{B_5}{A_5} = \tan^{-1}\frac{+0.391}{-0.124} = +107.^\circ 6$

$\tan^{-1}\frac{B_7}{A_7} = \tan^{-1}\frac{+0.134}{+0.087} = +57.^\circ 0$

$\tan^{-1}\frac{B_9}{A_9} = \tan^{-1}\frac{-0.170}{+0.055} = -72.^\circ 1$

$\tan^{-1}\frac{B_{11}}{A_{11}} = \tan^{-1}\frac{-0.119}{+0.022} = -79.^\circ 5$

Therefore the equation of the curve is

$$e = 32.678 \sin(\omega t - 3.^\circ 02) + 1.798 \sin 3(\omega t + 27.^\circ 8) + 0.410 \sin 5(\omega t + 21.^\circ 52) + 0.160 \sin 7(\omega t + 8.^\circ 14) + 0.179 \sin 9(\omega t - 8.^\circ 0) + 0.121 \sin 11(\omega t - 7.^\circ 2).$$

Fischer-Hinnen Method of Analysis.[14]—A convenient method of harmonic analysis and one in which the arithmetical work is reduced to a

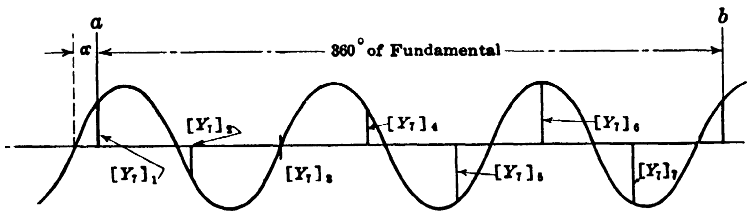


FIG. 452.—Pertaining to Fischer-Hinnen method of harmonic analysis.

minimum is due to Fischer-Hinnen; the procedure is based on two mathematical laws which are demonstrated below.

Suppose that a wave has been plotted, and the length *ab* (Fig. 452) is that of a complete cycle, or 360 deg. of the fundamental. Then between *a* and *b* there will be

1 complete period of the fundamental.
3 complete periods of the third harmonic.
5 complete periods of the fifth harmonic.

Denote by k the number of complete periods of any harmonic comprised between a and b. Then

$$k = 1 \text{ for the fundamental.}$$
$$k = 3 \text{ for the third harmonic.}$$
$$k = 5 \text{ for the fifth harmonic.}$$
$$\cdots\cdots\cdots$$

The equation of the sine curve corresponding to any particular harmonic will be

$$Y = A_k \sin k(\theta + \alpha),$$

where both θ and α are expressed in degrees of the fundamental. Now let ab, which corresponds to a *whole* wave, be divided into P equal parts, and P ordinates erected, the first being coincident with a. In Fig. 452,

$$k = 3, \qquad \text{and} \qquad P = 7.$$

Denote the various ordinates thus: $[Y_P]_1$, $[Y_P]_2$ . . . ; the subscript P within the bracket shows the number of sections into which the base ab is divided, while the subscript outside the bracket shows the number of the particular ordinate under consideration.

$$[Y_P]_1 = A_k \sin k\alpha.$$
$$[Y_P]_2 = A_k \sin\left(k\frac{360°}{P} + k\alpha\right).$$
$$[Y_P]_3 = A_k \sin\left(2k\frac{360°}{P} + k\alpha\right).$$
$$\cdots\cdots\cdots$$
$$[Y_P]_P = A_k \sin\left((P-1)k\frac{360°}{P} + k\alpha\right).$$

Then the sum of the P ordinates is

$$\begin{aligned}[Y_P]_1 + [Y_P]_2 + [Y_P]_3 + \cdots + [Y_P]_P = \\ A_k\Bigg[\sin k\alpha\Bigg\{1 + \cos\left(\frac{k360°}{P}\right) + \cos 2\left(\frac{k360°}{P}\right) + \\ \cos 3\left(\frac{k360°}{P}\right) + \cdots + \cos (P-1)\left(\frac{k360°}{P}\right)\Bigg\} + \\ \cos k\alpha\Bigg\{\sin\left(\frac{k360°}{P}\right) + \sin 2\left(\frac{k360°}{P}\right) + \\ \sin 3\left(\frac{k360°}{P}\right) + \cdots + \sin (P-1)\left(\frac{k360°}{P}\right)\Bigg\}\Bigg]. \qquad (34)\end{aligned}$$

Inspection shows that if k/P is a *whole* number,

$$[Y_P]_1 + [Y_P]_2 + [Y_P]_3 + \cdots + [Y_P]_P = PA_k \sin k\alpha = P[Y_P]_1. \qquad (35)$$

That is, when k/P is a *whole number*, the sum of P equally spaced ordinates is equal to P times the first ordinate. This is the first of the laws referred to above.

If k/P is *not* a whole number, the foregoing series can be summed by aid of the following trigonometrical formulæ:

$$\cos\theta + \cos 2\theta + \cos 3\theta + \cdots + \cos(P-1)\theta = -\tfrac{1}{2} + \frac{\cos(P-1)\theta - \cos P\theta}{2(1-\cos\theta)}. \tag{36}$$

$$\sin\theta + \sin 2\theta + \sin 3\theta + \cdots + \sin(P-1)\theta = \frac{\sin\frac{(P-1)\theta}{2}\sin\frac{P\theta}{2}}{\sin\frac{\theta}{2}}. \tag{37}$$

In this case, when
$\theta = k360°/P$, and k/P is not a whole number, both the series in (34) reduce to zero. Consequently,

$$[Y_P]_1 + [Y_P]_2 + [Y_P]_3 + \cdots + [Y_P]_P = 0. \tag{38}$$

That is, when k/P is not a whole number, the sum of P equally spaced ordinates is zero. This is the second of the two laws upon which this method depends.

These relations are used as follows: The wave is plotted, and any point is taken as the origin.

At the origin, $t = 0$, all the sine terms are zero, and all the cosine terms have their maximum values, that is, B_1, B_3, B_5, . . . , so

$$Y_1 = B_1 + B_3 + B_5 + \cdots.$$

To find B_3: Between a and b are 3 complete periods of the third harmonic, 9 complete periods of the ninth harmonic, 15 complete periods of the fifteenth harmonic, and so on.

Divide the base ab into three equal parts. Then $P = 3$, and

$$\frac{k}{P} = 1 \text{ for the third harmonic.}$$

$$\frac{k}{P} = 3 \text{ for the ninth harmonic.}$$

$$\frac{k}{P} = 5 \text{ for the fifteenth harmonic.}$$

. . . .

These are all whole numbers and by (35)

$$[Y_3]_1 + [Y_3]_2 + [Y_3]_3 = 3[B_3 + B_9 + B_{15} + B_{21} + \cdots].$$

To find B_5: Divide the base *ab* into five equal parts, $P = 5$; between *a* and *b* are 5 complete periods of the fifth harmonic, 15 of the fifteenth harmonic, so

$$\frac{k}{P} = 1 \text{ for the fifth harmonic.}$$

$$\frac{k}{P} = 3 \text{ for the fifteenth harmonic.}$$

. . . .

Consequently,

$$[Y_5]_1 + [Y_5]_2 + [Y_5]_3 + [Y_5]_4 + [Y_5]_5 = 5[B_5 + B_{15} + B_{25} + \cdots].$$

Similarly, by dividing the base into seven and into nine equal parts,

$$[Y_7]_1 + [Y_7]_2 + [Y_7]_3 + \cdots + [Y_7]_7 = 7[B_7 + B_{21} + B_{35} \cdots].$$

$$[Y_9]_1 + [Y_9]_2 + [Y_9]_3 + \cdots + [Y_9]_9 = 9[B_9 + B_{27} + B_{45} \cdots].$$

It is convenient to erect the first ordinate at the point where the curve crosses the axis. In that case, $Y_1 = 0$, and

$$B_1 + B_3 + B_5 + B_7 + \cdots = 0.$$

In practice, the process is somewhat simplified, for except in special cases the harmonics above the seventh are not important. So

$$B_3 = \tfrac{1}{3}\Sigma[Y_3] \quad \text{approx.}$$
$$B_5 = \tfrac{1}{5}\Sigma[Y_5] \quad \text{approx.}$$
$$B_7 = \tfrac{1}{7}\Sigma[Y_7] \quad \text{approx.}$$
$$B_9 = \tfrac{1}{9}\Sigma[Y_9] \quad \text{approx.}$$
$$B_1 = -B_3 - B_5 - B_7.$$

When these approximations are used, by appropriately dividing the base, the tests may be applied to detect the presence of higher harmonics. If they are present, the approximate values of the lower harmonics may be corrected; for instance, if the ninth is present, then

$$B_3 = \tfrac{1}{3}\Sigma[Y_3] - B_9.$$

To find A_1, A_3, *etc.*: As these are the coefficients of the sine terms, which will have their maximum values a quarter period from the initial ordinate $[Y_P]_1$, draw the first of the new set of ordinates $[Y_P]_1'$ a quarter period from $[Y_P]_1$. At this point, all the cosine terms are zero and consequently add nothing to the value of the ordinate.

The initial ordinate of the fundamental, as well as that of the fifth and of the ninth harmonic, is positive, while the initial ordinate of the third and of the seventh harmonic is negative.

$$Y_1' = A_1 - A_3 + A_5 - A_7 + A_9 \cdots.$$

When the base has been divided into three, five, seven, etc., equal parts, by the rules already given,

$$[Y_3]'_1 + [Y_3]'_2 + [Y_3]'_3 = 3[-A_3 + A_9 - A_{15} \cdots].$$
$$[Y_5]'_1 + [Y_5]'_2 + [Y_5]'_3 \cdots + [Y_5]'_5 = 5[A_5 - A_{15} \cdots].$$
$$[Y_7]'_1 + [Y_7]'_2 + [Y_7]'_3 \cdots + [Y_7]'_7 = 7[-A_7 + A_{21} \cdots].$$
$$[Y_9]'_1 + [Y_9]'_2 + [Y_9]'_3 + \cdots + [Y_9]'_9 = 9[A_9 - A_{27} \cdots].$$

Therefore,

$$A_3 = -\tfrac{1}{3}\Sigma[Y_3]' \quad \text{approx.}$$
$$A_5 = \tfrac{1}{5}\Sigma[Y_5]' \quad \text{approx.}$$
$$A_7 = -\tfrac{1}{7}\Sigma[Y_7]' \quad \text{approx.}$$
$$A_9 = \tfrac{1}{9}\Sigma[Y_9]' \quad \text{approx.}$$
$$Y'_1 = A_1 - A_3 + A_5 - A_7 + A_9 \cdots .$$

When dealing with waves containing only the odd harmonics, it is necessary to plot only one-half the wave, since the second half is like the first with the algebraic sign of the ordinates reversed. Suppose that the *half wave* has been divided into $2k$ parts, equivalent to dividing the whole wave into $4k$ parts; then the ordinate $[Y_k]_1$ is identical with $[Y_{4k}]_1$. The relation of the number N_{4k} of any ordinate when the whole base is divided into $4k$ parts to the number of the same ordinate N_k when the whole base has been divided into k parts is given by

$$4(N_k - 1) + 1 = 4N_k - 3 = N_{4k}.$$

Consequently,

$$B_3 = \tfrac{1}{3}[[Y_{12}]_1 + [Y_{12}]_5 + [Y_{12}]_9] = \tfrac{1}{3}[[Y_{12}]_1 + [Y_{12}]_5 - [Y_{12}]_3].$$
$$B_5 = \tfrac{1}{5}[[Y_{20}]_1 + [Y_{20}]_5 + [Y_{20}]_9 - [Y_{20}]_3 - [Y_{20}]_7].$$
$$B_7 = \tfrac{1}{7}[[Y_{28}]_1 + [Y_{28}]_5 + [Y_{28}]_9 + [Y_{28}]_{13} - [Y_{28}]_3 - [Y_{28}]_7 - [Y_{28}]_{11}].$$

When the sine coefficients are determined, the ordinate $[Y_k]'_1$ is identical with $[Y_{4k}]_{k+1}$; for when the sine terms are determined, the initial ordinate is transferred k spaces to the right. In this case, the number of any ordinate N_{4k} when the whole base has been divided into $4k$ parts is related to the number of the same ordinate N'_k when the whole base has been divided into k parts, as follows:

$$4(N'_k - 1) + k + 1 = 4N'_k - 3 + k = N_{4k}.$$

So

$$A_3 = -\tfrac{1}{3}[[Y_{12}]_4 + [Y_{12}]_8 + [Y_{12}]_{12}] = -\tfrac{1}{3}[[Y_{12}]_4 - [Y_{12}]_2 - [Y_{12}]_6].$$
$$A_5 = \tfrac{1}{5}[[Y_{20}]_2 - [Y_{20}]_4 + [Y_{20}]_6 - [Y_{20}]_8 + [Y_{20}]_{10}].$$
$$A_7 = -\tfrac{1}{7}[-[Y_{28}]_2 + [Y_{28}]_4 - [Y_{28}]_6 + [Y_{28}]_8 - [Y_{28}]_{10} + [Y_{28}]_{12} - [Y_{28}]_{14}].$$

. . . .

To combine the sine and cosine terms,

$$
\begin{aligned}
A_k \sin k\omega t + B_k \cos k\omega t &= C_k \sin\left[k\omega t + \tan^{-1}\frac{B_k}{A_k}\right] \\
&= C_k \sin k\left[\omega t + \frac{\varphi_k}{k}\right]. \\
C_k &= \sqrt{A_k^2 + B_k^2}. \\
\tan \varphi_k &= \frac{B_k}{A_k}.
\end{aligned}
$$

φ_k is positive if the ascending portion of the component curve first cuts the axis at the left of the origin.

There are other methods of harmonic analysis, but it is evident that the labor involved is very considerable and becomes formidable in an investigation requiring the treatment of many curves. Cases arise where curves other than those of e.m.fs. and current must be analyzed, and it is necessary to include both the odd and even harmonics; hence the need in practical work of machines by which the analysis may be effected. References to descriptions of a number of harmonic analyzers are given at the end of this chapter.

A simple form of harmonic analyzer, designed particularly for electrical engineering work, has been described by Chubb.[15] Its action may be explained as follows:

On page 678 attention was called to the fact that the exact expressions for A_k and B_k are

$$A_k = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \sin k\theta \, d\theta$$

and

$$B_k = \frac{1}{\pi}\int_0^{2\pi} f(\theta) \cos k\theta \, d\theta.$$

Referring to Fig. 453, if the point P is given a horizontal displacement always equal to $f(\theta)$, and if at the same time P experiences a perpendicular displacement always proportional to sin $(k\theta)$,

$$
\begin{aligned}
x &= f(\theta); \\
y &= R \sin k\theta,
\end{aligned}
$$

where R is the maximum displacement along y;

$$dy = kR \cos k\theta \, d\theta;$$

and the area of the curve traced by P while θ goes through 360 deg. will be

$$\text{Area} = kR\int_0^{2\pi} f(\theta) \cos k\theta \, d\theta.$$

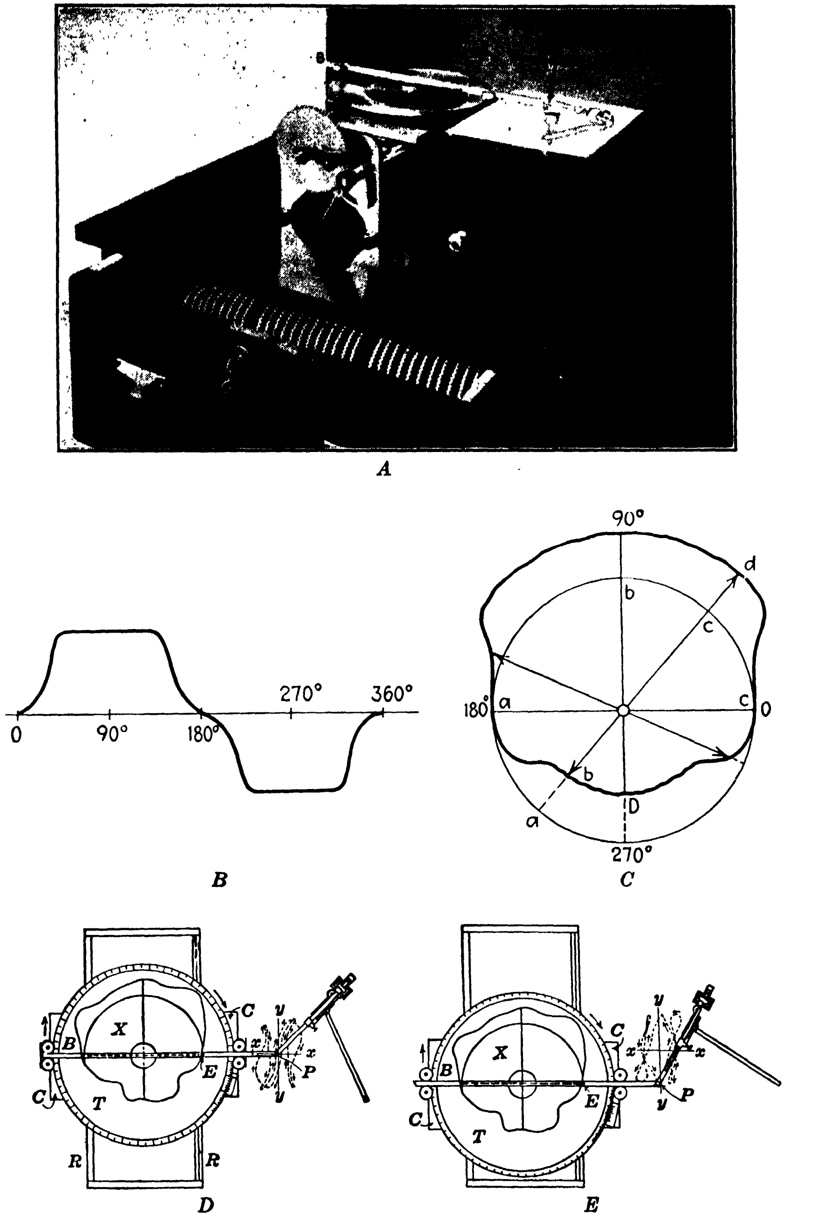


FIG. 453.—Chubb harmonic analyzer. (*Westinghouse Electric and Manufacturing Company.*) *D*, analyzer set for determining cosine components; *E*, analyzer set for determining sine components.

Consequently, by (24),

$$B_k = \frac{\text{area}}{\pi k R}.$$

Similarly, if the vertical displacement of P is given by

$$y = R \sin \left(k\theta - \frac{\pi}{2}\right) = -R \cos k\theta,$$
$$dy = kR \sin \theta \, d\theta,$$

and the area of the curve traced by P will be

$$\text{Area} = Rk \int_0^{2\pi} f(\theta) \sin \theta \, d\theta.$$

Consequently,

$$A_k = \frac{\text{area}}{\pi k R}.$$

The Chubb analyzer is a mechanism for mechanically calculating A_k and B_k. The areas of the curves are determined by a planimeter, as in Fig. 453. The arrangement by which the point P is guided is shown in a general way in Fig. 453 D and E. The first step in using the analyzer is to cut out a bristol-board template which represents $f(\theta)$, such as shown in Fig. 453 C. The circle *abc* is the base line from which the values of $f(\theta)$ are measured, + values of $f(\theta)$ being measured radially outward, and − values radially inward.

The template is mounted on the turntable T, and the pin E on the movable transverse rod BP is forced against the edge of the template by springs. If the turntable rotates, the point P experiences a displacement $x = f(\theta)$.

The frame that carries the turntable and the mounting for the rod BP slides on the ways RR (Fig. 453D) and by means of a crank and slotted crosshead can be given a sinusoidal displacement along these ways.

Rotary motion is communicated to the turntable by means of a worm wheel mounted on the axis of the turntable and a worm which slides on a splined shaft placed parallel to the ways.

By means of a system of change gears, the frame carrying the turntable may be caused to make k complete displacements along the ways, while the table makes one complete revolution. The point P is thus guided in the manner suggested.

When the sine components are to be determined, the crank is placed so that the carriage is at its maximum displacement at the lower end of the ways, $\theta = 0$. If the cosine terms are desired, the carriage is started at its mid-position, $\theta = 0$.

By the use of the proper system of change gears the coefficient of any harmonic, either odd or even, is readily determined with all the accuracy needed in electrical engineering work.

The oscillograph is readily adapted for obtaining curves plotted in the peculiar manner necessary for the construction of the template; the oscillogram is taken on a plate that is rotated about an axis perpendicular to its plane. If undeflected, the spot of light traces the base circle *abc* from which the displacements are measured (Fig. 453*C*).

From the oscillogram one can readily detect the presence of even harmonics, for if they are absent all the "diameters" are of equal length and equal to the diameter of the base circle *abc*.

There are many other forms of harmonic analyzer.

Experimental Analysis: Laws' Method.[16]—When dealing with potential difference and current waves, it is possible to determine the various coefficients experimentally.

The coefficient A_k is twice the mean product, between $+\pi$ and $-\pi$, of curve $f(\theta)$ and the curve $\sin k\theta$; B_k is twice the mean product of $f(\theta)$ and $\cos k\theta$, between the same limits. The deflection of an electrodynamometer is proportional to the mean product of the currents in the fixed and movable coils. Consequently, if the wave to be analyzed, of frequency f, is led through the fixed coil while a sinusoidal current wave of frequency kf and maximum value A'_k is sent through the movable coil, the deflection of the instrument will be proportional to A_k or to B_k, according as the zero point of the unknown wave coincides with the zero or the maximum of the sine wave. The deflection will be proportional to C_k if the phase of the sinusoidal current is adjusted until the deflection is a maximum.

In order to carry out the analysis, the machine from which the sinusoidal current is derived must be driven from the shaft of the generator whose waves are to be analyzed by means of change gears which permit the speed to be varied so that the frequency may be made 1, 3, 5, 7, . . . , times that of the main current. Also, the machine must be so constructed that the phase of the sinusoidal current may be altered at will, a scale being provided so that the phase displacements are readily determined.

The maximum value of the sinusoidal current A'_k is determined from the reading of a current dynamometer. The process of making a measurement is to change the phase of the machine giving the sinusoidal current until the dynamometer stands at zero, then to shift the phase 90 deg. and take the dynamometer reading. A contact arrangement and galvanometer, which by means of a double-throw switch may be connected to the terminals of noninductive resistances in either the main circuit or that of the sine generator, permit the zero points on the waves to be located. The phases of the harmonics may then be determined from the readings on a scale of degrees attached to the movable field frame of the sine dynamo.

The disadvantage of this method is that it requires special apparatus.

Resonance Analysis.—The various harmonic components of a complex wave may be identified and their relative magnitudes determined by properly arranged resonating circuits, one example of which is the harmonic analyzer for use on power circuits, designed by the National Electric Light Association (1928).[17] This assembly gives the relative magnitudes of the fundamental (60 cycles per second) and odd harmonics up to and including the thirty-ninth. Reference to Fig. 454 will show that a doubly tuned resonating circuit is employed. *PS* is a loosely coupled air-core mutual inductance. It is astatically arranged and subdivided to extend conveniently the range of the instrument. The

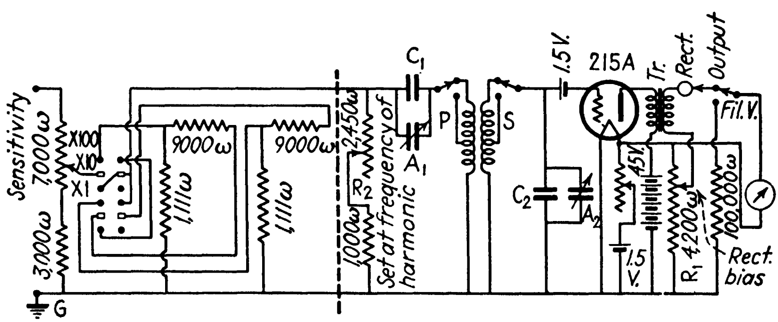


FIG. 454.—N.E.L.A. harmonic analyzer.

primary is tuned by means of the decade condenser C_1 and the continuously variable air condenser A_1 (0.001 μf), the settings of which are made according to an arbitrarily divided scale. The secondary is tuned by the decade condenser C_2 and the continuously variable, arbitrarily graduated air condenser A_2. When tuning for the fundamental, the condensers C_1 and C_2 are supplemented by 0.6-μf condensers inserted by switches. The voltage across C_2 is supplied to a single-tube amplifier, the output of which is delivered via a transformer to a copper oxide rectifier in series with a direct-current microammeter. The proper *A* battery for the amplifier is a "6-in." dry cell. A 45-volt battery block serves for the *B* battery. The *C* battery is a 1.5-volts cell mounted within the case. By means of a switch, the microammeter may be placed in series with 100,000 ohms and thus becomes a voltmeter for measuring the filament voltage, which should be kept at 1.0 volt. A variation of 5 per cent in the filament voltage produces about 2 per cent change in the results. A biasing current of 2.5 microamp., controlled by R_1 and equivalent to a deflection of 10 divisions on the output meter, is used to improve the action of the copper oxide rectifier. This biasing current should be kept within 5 per cent of the specified value. The impedance to the right of the dotted line is maintained always at 1,000 ohms. This is accomplished by setting the dial of the rheostat R_2 at the point marked with the frequency of the harmonic being sought. The network to the left of the dotted line is for the purpose of varying the range of the analyzer. At the extreme left is a sensitive potentiometer whose ratio is continuously variable from 1 to 10. The switch introduces multipliers of 1, 10, and 100, as desired. The input voltage should not be over 150 volts.

In any location, before the analyzer is used, it should be tested for "pickup" by disconnecting the high-potential lead at the supply, the other lead being grounded at the analyzer, and making settings of the various harmonics that may be present in the power circuit. Electrostatic disturbances are eliminated by an internal copper shield. The analyzer is accompanied by a table giving the settings of the condensers for the various harmonic frequencies and the corresponding sensitivities. The table applies to a particular instrument only.

Condenser Settings for N. E. L. A. Harmonic Analyzer

Low Range

F	k	C_1	A_1	C_2	A_2	Sensitivity factor, volts
60	1	0.690	0	0.690	0	0.437
180	3	0.090	0	0.086	0	0.0343
300	5	0.032	20	0.030	40	0.0134
420	7	0.015	90	0.015	50	0.0086
1260	21	0.001	61	0.001	54	0.0042
1380	23	0.001	35	0.001	28	0.0043

High Range

F	k	C_1	A_1	C_2	A_2	Sensitivity factor, volts
1500	25	0.002	46	0.002	39	0.0047
1620	27	0.002	12	0.002	4	0.0047
1740	29	0.001	76	0.001	73	0.0047
2220	37	0.001	6	0.000	91	0.0059
2340	39	0.000	81	0.000	80	0.0066

To make a reading, after checking the various voltages and currents, the sensitivity potentiometer is set at "10," the multiplier switch at "100," and the condensers at the values given in the table for the frequency being sought. The circuit is then accurately tuned, the multiplier being adjusted so that the microammeter does not read off scale.

The sensitivity is then adjusted until the microammeter stands at a point marked "unity." The magnitude of the harmonic is given by

$$E = (\text{sensitivity factor}) \times (\text{multiplier reading}).$$

As the fundamental frequency may not be exactly 60 cycles per second, the observer should try tuning for maximum reading on either side of the prescribed setting. When using the high range, the successive values of the required capacitances differ but little, and the observer must satisfy himself that he is really tuning for the frequency that he desires. To analyze the current in the secondary of a current transformer, a 0.25-millihenry shunt is used, and the voltage is supplied to the analyzer through a "repeat" coil.

More elaborate analyzers capable of dealing with waves of higher than power frequencies and containing both odd and even harmonics are obtainable and are useful in communication work.

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CHAPTER XIV

CABLE TESTING

FAULT LOCATION

Continuity of service is essential to the success of any electrical undertaking, whether it be for supplying light or power or for purposes of communication. So far as the transmission lines are concerned, continuity implies that the risk of interruption has been reduced to a minimum by the use of proper methods of construction and of suitable materials, such as cables and insulators. Even when the greatest care has been exercised in these matters, cables will break down, line wires become crossed or grounded, and insulators are punctured or broken. Therefore it becomes necessary to have some means for locating the position of the defective parts of the lines or cables, in order that repairs may be expeditiously made. Again, newly installed power cables break down not infrequently during the high-voltage acceptance tests, and these breaks must be located.

The special problem of locating faults in long submarine cables is discussed in such works as Kempe's "Handbook of Electrical Testing" and will not be considered here.

In general, the methods treated here will be those employed in dealing with power cables in cities and with telephone and telegraph lines, and it will be assumed that only one fault, or connection to ground, exists.

The theory underlying the methods of fault location is very simple, but, on account of constantly varying circuit conditions, the practical execution of the tests requires a skill and judgment that can be obtained only by actual experience.

Location of Grounds and Crosses.—An earth fault, or ground, is due to any defect in the insulation of the conductor that impairs or destroys its efficiency so that a current may pass from the wire to the earth or to the cable sheath. A cross is due to the impairment of the insulation between two wires so that the current may pass between them.

The location of grounds and crosses is effected by the same methods. In a multiple-conductor cable, the first step is to pick out the conductors that are faulty, for, in general, some conductors remain in perfect condition. For this purpose, both ends of the cable are disconnected from the service apparatus, and the insulation resistances between the various conductors and ground and between the conductors themselves are measured; the voltmeter method may be used. If the faults are of sufficiently low resistance, bridge measurements with reversed currents may be made. Continuity tests should also be made to determine whether any of the conductors have been burned off or broken. The foregoing tests enable one to decide on the subsequent procedure.

If the resistance per unit length of the line is uniform, three things must be known in order that a ground or a cross may be located:

1. The total length of the faulty line.
2. The total conductor resistance of the faulty line *at the time of test.*
3. The resistance of the faulty line from the testing station to the fault.

The length of the line is given by the office records. When there are only two wires connecting the stations at the ends of the line, it is not possible to measure the line resistance after the fault has occurred. The best

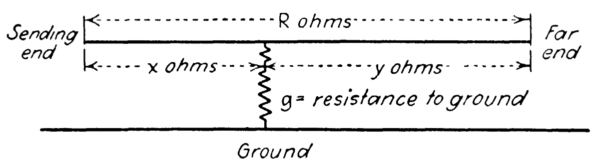

FIG. 455.—Blavier and earth overlap tests for fault location.

approximation possible must then be made by taking the stated resistance per unit length and correcting it for temperature; in this correction, there may be considerable uncertainty, for the temperature coefficient of the copper is large (0.4 per cent per degree centigrade), and it is often difficult to form a just estimate of the temperature of the conductor, especially if it is in a duct near heavily loaded cables.

Blavier Test—In case the faulty wire is the only one connecting the stations, Blavier's method furnishes the only means of locating the fault by measurements made from one end of the line. In order that the test may be carried out, it is necessary that the observer be able to send his instructions over the line to the attendant at the other end.

The total line resistance is supposed to be known from previous measurements made while the line was perfect; denote it by R.

Two measurements of the resistance to ground are made by the observer at the sending end, one, R_1, with the far end insulated; and a second, R_2, with the far end grounded. Then, referring to Fig. 455,

$$R = x + y.$$
$$R_1 = x + g.$$
$$R_2 = x + \frac{gy}{g + y}.$$

Eliminating g and y,

$$x = R_2 - \sqrt{(R - R_2)(R_1 - R_2)}. \tag{1}$$

This gives the resistance from the sending station to the fault. The corresponding distance is calculated by aid of the known resistance per unit length of the cable.

The resistance measurements may be made in any convenient manner, as by the voltmeter and ammeter method. A practical difficulty is that the resistance to ground g is variable, being influenced by the amount of moisture present and the action of the current at the fault. Also, the resistance g may be so high that it exerts very little shunting action when y is placed in parallel with it by grounding the far end of the line.

The Earth Overlap Test.—In applying this test, it is necessary to make resistance measurements from both ends of the line. With the far

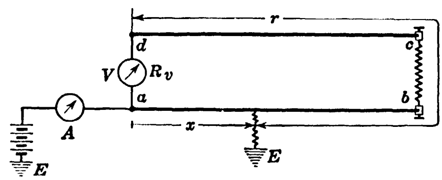

FIG. 456.—Volt-ammeter method for fault location.

end grounded, the resistance R_1 to ground is measured from the near end. The line is then grounded at the near end, and the resistance to ground R_2 is measured from the far end. Then

$$R = x + y.$$

$$R_1 = x + \frac{gy}{g + y}.$$

$$R_2 = y + \frac{gx}{g + x}.$$

Or

$$x = R_1\left[\frac{R - R_2}{R_1 - R_2}\right]\left[1 - \sqrt{\frac{R_2(R - R_1)}{R_1(R - R_2)}}\right]. \tag{2}$$

$$y = R_2\left[\frac{R - R_1}{R_2 - R_1}\right]\left[1 - \sqrt{\frac{R_1(R - R_2)}{R_2(R - R_1)}}\right]. \tag{3}$$

Practical details concerning the application of this test to long submarine cables are given in the *Journal of the Institution of Electrical Engineers*, 1885, vol. 16, page 581.

The Volt-ammeter Test.—In this method, it is necessary to have, between the testing stations, a second and unfaulted conductor which can be used as a potential lead to the far end of x.

In Fig. 456, the faulted wire is shown by ab; the unfaulted one, by dc; V is a voltmeter, and A is an ammeter. With these connections a regular fall of potential measurement of x may be made.

The potential difference V_x, between the terminals of x, will be given by

$$V_x = V_1\left(\frac{R_v + r}{R_v}\right),$$

where V_1 is the reading of the voltmeter, and R_v is the voltmeter resistance. If I_1 is the reading of the ammeter, the current through x will be

$$I_x = I_1 - \frac{V_1}{R_v}.$$

If r is an appreciable fraction of R_v, it may be eliminated; to do this, transfer the ammeter connection to d, and make a second measurement. Call the reading of the voltmeter V_2, and that of the ammeter I_2. Then the voltage across the ends of r is

$$V_r = V_2\left(\frac{R_v + x}{R_v}\right).$$

$$I_r = I_2 - \frac{V_2}{R_v}.$$

The value of x is

$$x = \frac{V_1}{I_1 - \frac{V_1}{R_v} - \frac{I_1}{I_2}\frac{V_2}{R_v}}. \tag{4}$$

If an electrostatic voltmeter is used, no allowance is necessary for the voltmeter current, and

$$x = \frac{V}{I}.$$

Loop Tests.—By a loop test is meant any method of locating grounds or crosses by determining the resistances of the two sections of the loop formed by connecting the faulted conductor at its far end to an unfaulted conductor which returns to the sending station (see Fig. 457).

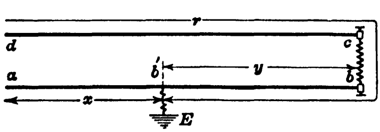

Fig. 457.—Loop test for locating faults.

The grounded conductor is represented by ab, the fault being at b'. The unfaulted conductor is shown by dc; at the far end it is electrically connected with the faulted conductor by a low-resistance jumper which must be insulated from ground. It is essential that the contacts at b and c be perfect.

The superiority of the loop tests is due to the fact that the results are independent of the resistance of the fault itself.

Two-ammeter Loop Test.—In this method, the two sections of the loop are fed in parallel from the same battery, and the ratio x/r is determined from the readings of two ammeters, one placed in series with x, the other in series with r, as in Fig. 458.

The resistances of the two ammeters and the connections are assumed to be negligible. The polarity of the battery should be such that the fault resistance is a minimum. If the readings of the ammeters are I_x and I_r,

$$\frac{I_r}{I_x} = \frac{x}{r} \quad \text{or} \quad \frac{I_r}{I_x + I_r} = \frac{x}{x + r}.$$

$$x = (x + r)\left(\frac{I_r}{I_r + I_x}\right). \tag{5}$$

The total resistance $(x + r)$ of the loop may be determined by the volt-ammeter method.

Equation (5) gives the resistance in ohms from the sending end to the fault. If the conductors are both of the same material and size and at

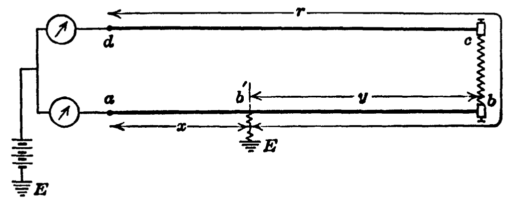

FIG. 458.—Two-ammeter loop test for locating faults.

the same temperature, the resistance per unit length will be the same for both, and

$$\text{Distance to fault} = (\text{total length of loop})\left(\frac{I_r}{I_r + I_x}\right). \tag{6}$$

The resistance y from the far end of the line to the fault may also be obtained, for

$$\frac{I_x}{I_r} = \frac{r}{x},$$

or

$$\frac{I_x - I_r}{I_x + I_r} = \frac{r - x}{r + x} = \frac{\frac{r - x}{2}}{\frac{r + x}{2}}.$$

For a loop of uniform resistance per unit length, the distance from the far end of the line to the fault is

$$(\text{length of } \textit{one wire})\left(\frac{I_x - I_r}{I_x + I_r}\right). \tag{7}$$

Murray Loop Test.—In the Murray loop test, the connections are such that the resistances x and r form two arms of a Wheatstone bridge, the other two arms being made up of resistances under control of the observer. Figure 459 shows the scheme of connections.

The relative positions of the galvanometer and battery are important; with the connections as shown, earth currents have no effect on the readings.

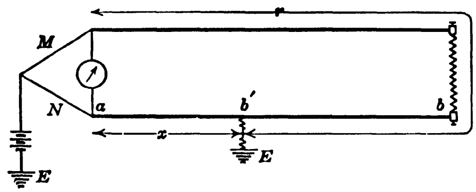

FIG. 459.—Connections for Murray loop test for fault location.

When the galvanometer stands at zero,

$$\frac{M}{N} = \frac{r}{x}, \qquad \text{or} \qquad \frac{M + N}{N} = \frac{x + r}{x}.$$

$$x = (x + r)\left(\frac{N}{N + M}\right).$$

The total resistance of the loop $(r + x)$ is obtained by a bridge measurement.

If uniform wires are being dealt with, the distance to the fault is

$$(\text{total length of the loop})\left(\frac{N}{M + N}\right).$$

A potential divider or a slide-wire arrangement with extension coils may be convenient for $M + N$, in which case $M + N$ is constant.

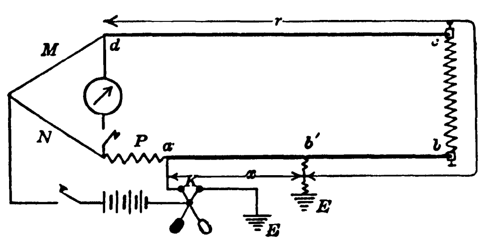

FIG. 460.—Connections for Varley loop test for fault location.

A cross between two conductors in a multiple-conductor cable is located in a similar manner, the only difference being that the battery, instead of being connected to ground, is attached to one of two faulty conductors, the other being looped with an unfaulted wire.

Varley Loop Test.—In this method, a fixed bridge ratio is used, and the balance obtained by adding resistance to the smaller section of the loop, as in Fig. 460.

With the apparatus arranged as in Fig. 460, and the switch in the dark position, the bridge will balance when

$$\frac{M}{N} = \frac{r}{x + P_1},$$

P_1 being the resistance unplugged at P. Hence,

$$x = \frac{(x + r)N - P_1M}{M + N}. \tag{8}$$

To measure $(r + x)$, the total resistance of the loop, the key K is thrown to the light position, and a second balance obtained, using the apparatus as an ordinary Wheatstone bridge.

Convenient portable bridge assemblies especially designed for the execution of the Murray and Varley loop tests and the location of total disconnections are obtainable. For the location of faults in power cables, the bridge must have a large carrying-capacity, 5 amp. for example. The Murray test is employed, and the assembly must be such that errors are not introduced at the contacts where the bridge is connected to the cable.

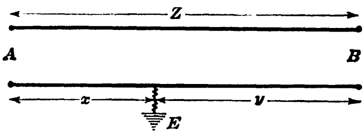

Fig. 461.—Pertaining to determination of resistance of faulty conductor.

Determination of the Total Resistance of the Defective Conductor.—If there is only one perfect wire between the stations, the total resistance of the faulted line cannot be determined by measurements made from one end of the line.

The determination may be made by tests from both ends (see Fig. 461). The line is first looped with Z at B, and x determined by one of the previous methods. Then the loop is made at A, and the observer at B measures y. The total resistance is $(x + y)$.

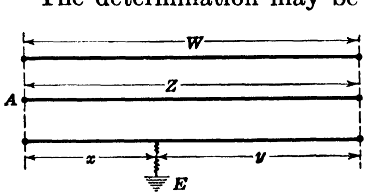

Fig. 462.—Pertaining to determination of resistance of faulty conductor.

When there are two perfect wires between the stations, measurements from one end of the line suffice. When dealing with a multiple-conductor cable, two unfaulted wires in the same cable may be used as the auxiliary wires (see Fig. 462).

The faulty wire is looped with Z, and the resistance R_1 of the loop measured by a bridge.

$$R_1 = (x + y) + Z.$$

Loop the faulty wire with W, and measure the resistance R_2 of this loop.

$$R_2 = (x + y) + W.$$

Finally, loop W and Z, and measure the resistance R_3 of the loop.

$$R_3 = W + Z.$$

Then

$$(x + y) = \frac{R_1 + R_2 - R_3}{2}. \tag{9}$$

Another procedure is to loop the faulty wire with Z and measure the loop resistance R_1.

$$R_1 = (x + y) + Z.$$

W and Z are then looped and grounded at B, and Z measured by one of the previous methods; then

$$(x + y) = R_1 - Z. \tag{10}$$

If there are one perfect and two faulty wires of the same length and resistance connecting the stations, as in Fig. 463, the total resistance of either of the faulty wires may be obtained thus:

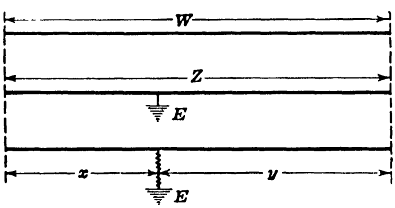

FIG. 463.—Pertaining to determination of resistance of faulty conductor.

Loop Z and W, and measure the resistance R_1 of the loop. Loop $(x + y)$ and W, and measure the resistance R_2 of this loop. Finally, connect Z and $(x + y)$ in parallel, loop the combination with W, and measure the resistance R_3.

$$R_1 = Z + W.$$

$$R_2 = (x + y) + W.$$

$$R_3 = W + \frac{Z(x + y)}{(x + y) + Z}.$$

$$(x + y) = (R_2 - R_3) + \sqrt{(R_3 - R_2)(R_3 - R_1)}. \tag{11}$$

Or,

$$Z = (R_1 - R_3) + \sqrt{(R_3 - R_2)(R_3 - R_1)}.$$

To be strictly accurate, the ratio of the resistance up to the fault to the total resistance of the wire should be the same for both defective conductors, for then there will be no flow of current through the faults.

Fisher Loop Test.—In order to make this test, there must be two unfaulted wires which run from the testing station to the far end of the line. The result obtained is the same as that given by the above methods already described where two perfect conductors are available, and both the resistance up to the fault and the total resistance of the faulty line are measured.

Two balancings are necessary, as indicated in Fig. 464. From the first balancing,

$$\frac{M}{N} = \frac{Z + y}{x}.$$

From the second,

$$\frac{M'}{N'} = \frac{Z}{x + y}.$$

Then

$$x = (x + y)\frac{\frac{M'}{N'} + 1}{\frac{M}{N} + 1}. \tag{12}$$

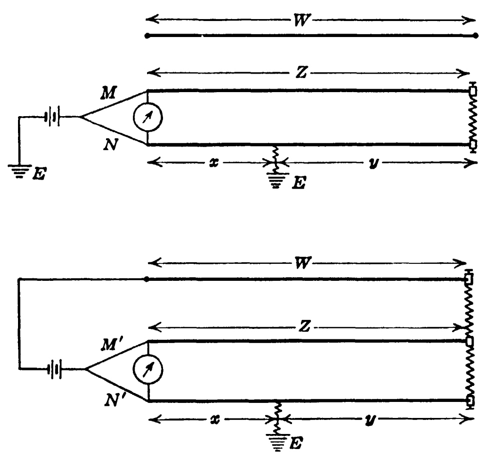

FIG. 464.—Connections for Fisher loop test for fault location.

If a slide wire or its equivalent is used for the balance arms,

$$M + N = M' + N',$$

and

$$x = (x + y)\frac{N}{N'}. \tag{12a}$$

When the resistance per unit length of conductor is uniform,

$$\text{Distance to fault} = \left(\frac{\frac{M'}{N'} + 1}{\frac{M}{N} + 1}\right)(\text{length of cable}). \tag{13}$$

Corrections for Conductors of Different Diameters.—In the foregoing discussion, it has been assumed that the resistance per unit length of conductor is uniform. In some cases, however, the conductor may be made up of a number of wires in series having different diameters. The lengths and sizes of the wires in the different sections will be known from the office records. Let the lengths be l_1, l_2, l_3, . . . , and the corresponding resistances per unit length be K_1, K_2, K_3, ... The resistances of the sections will be as indicated in Fig. 465.

To locate the section in which the fault exists, compare the resistance x, as found by one of the previous methods, with l_1K_1, then with

$$l_1K_1 + l_2K_2,$$

and so on. Suppose that the comparison shows the fault to be in the third section; then

$$x = l_1K_1 + l_2K_2 + l_3'K_3,$$

and the distance of the fault beyond the junction of the second and third sections is

$$l_3' = \frac{x - l_1K_1 - l_2K_2}{K_3}. \tag{14}$$

Uncertainty as to the values of K introduces difficulties. The average values of K for the different sections depend on the diameters of the wires; their conductivities; the temperatures of the sections; and, in underground conductors of large cross section where the lengths of the sections are short, on the number of joints.

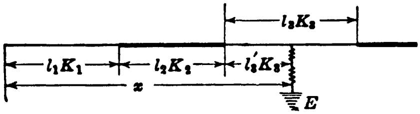

FIG. 465.—Pertaining to the location of a ground in a nonuniform conductor.

In cables for large currents laid in ducts where the temperatures may be high, there may be great uncertainty as to the temperature and a consequent difficulty in correcting K_1, K_2, etc., to obtain their values at the time of test.

Locating Faults in Underground High-tension Cables.[1]—In locating faults in underground high-tension cables, special difficulties are experienced because of the low resistance of the conductor, which may be from 0.10 to 0.01 ohm per 1,000 ft. On the other hand, such cables are readily accessible at the manholes, which may be about 300 feet apart. This makes it possible, before cutting the cable, to verify the location of the fault as given by a loop test, and for this purpose special apparatus has been devised so that the particular length of cable in which the fault is located may be identified with certainty. This verification is necessary, for in a 10-mile length of cable an uncertainty of 0.3 per cent in the resistance measurements corresponds to an uncertainty of 158 ft., or *half a length* of the cable between manholes.

In the long run, time will be saved by adopting an orderly procedure, and the following has been found satisfactory in dealing with this class of faults:[1]

1. Tests to diagnose the trouble and show the tester with what he has to deal.

2. Reduction of the resistance of the fault (if necessary) so that current sufficient to make the location and verification tests will flow through the fault with a moderate voltage.

3. Preliminary location of the fault by a loop test. If the conductor is burned off the loop test is not applicable. In this case, the ground is located by use of an exploring coil (see below).

4. Verification of the location by use of an exploring coil.

a. Taking a three-phase cable, all three conductors are insulated at both ends, and the resistance to ground of each conductor is determined by the voltmeter method, using a direct-current potential of about 110 volts. This shows whether the fault is a ground and, if it is, gives an idea of the fault resistance.

b. If a low-resistance ground is found, verify *a* by using a test lamp, that is, an incandescent lamp with one side of the socket attached to the 110-volt direct-current supply, the other side being provided with a flexible cord so that it may be attached to any of the conductors. If the lamp glows, the resistance of the fault is to be measured by a bridge, two readings, with reversed currents, being taken. This measurement of the fault resistance shows whether it is necessary to reduce it further before making the loop and verification tests.

c. Conductors that show the same insulation to ground should be tested to determine if this is due to the wires being crossed. The test is made by grounding one of the wires and remeasuring the resistance to ground of the others. If the voltmeter method shows that this resistance is low, it should be measured by the bridge, for the reason stated in *b*.

d. The far ends of all three conductors should be carefully connected together, and the resistances of all the uncrossed loops measured by the bridge. A comparison of these results with the resistances computed from the known size and length of the line will show if there are open faults, that is, places where the wires are broken or burned off. If an open fault exists, an idea of its resistance should be obtained. The resistance to ground of the near side of the open fault has been obtained in *a*. The resistance to ground of the far side of the open fault is obtained by measuring it via an unfaulted conductor, which is used as a lead. If this resistance is very low, the resistance across the open fault has been measured practically in *a*. If it is not low, it should be determined by measuring the insulation of the open conductor when the far side is grounded through one of the unfaulted conductors.

2. In order to locate the ground, it is necessary that the fault resistance be low, so that sufficient current for the tests may be obtained with

low voltage. This is convenient in the loop tests and is necessary in the subsequent exact localization by means of exploring coils. For, if considerable voltage is used, the charging current going to the parts of the cable beyond the fault introduces a difficulty.

After having determined the nature of the fault and having gained an idea of its resistance, it will probably be found necessary to reduce the fault resistance. The fundamental idea is to carbonize a sufficient amount of the paper insulation at the fault, so that a current of 1 or 2 amp. may be carried for several hours. Practice is required to accomplish this in the shortest possible time and without any approach to an explosive short circuit at the fault, which would destroy the continuity of the path via the carbonized paper. When the fault is not submerged in water, for in water the paper cannot be carbonized, the procedure is to send a current of from 3 to 5 amp. through the fault for 10 min., in order to dry it out, and to follow this by a current of about 1 amp., for 5 min., to carbonize the paper. High-resistance faults at a considerable distance from the testing station give trouble on account of the charging current. In such cases, the current through the fault may be determined by aid of a wattmeter.

3. After having reduced the fault resistance, a Murray or a Varley loop test is used to determine the approximate location of the fault.

In making this test, great care must be exercised in applying the jumpers at the far end and in joining the bridge to the cable, in order that no extraneous resistances may be introduced. Also, allowance must be made for the leads connecting the bridge to the cable, or else the bridge must be so constructed that these resistances are eliminated.

Irregularities in joint resistances, etc., render it necessary to supplement the loop tests with exploration tests which will show definitely the particular length of cable in which the fault exists. Also, the cable may be burned off, in which case the loop tests are not applicable.

4. The idea of the exploration tests is to send a characteristic signal into the cable and to find by means of a suitable detector the point at which the signal ceases to be heard as the exploring device is moved along the cable.

Taking the case shown in Fig. 466, when the sheaths of the various lengths of cable are bonded to prevent electrolysis, a diminishing current will flow in the sheath to points beyond the ground, as indicated. Currents will also flow in the sheath, due to inequalities of ground potential, produced, for example, by stray currents from street-car lines.

If the detector is a simple coil of wire connected to a telephone and held with its plane parallel to the length of the cable, the sheath currents, from whatever cause, will affect it in the same manner as if they flowed in the conductor, and an exact location of the trouble is not possible.

When dealing with three-phase cables, it is possible to use a longitudinal exploring coil, devised by W. A. Durgin to eliminate the effect of sheath currents.[1]

It depends for its effectiveness on the fact that the conductors in the cable are spiraled, the lay, or length of a complete spiral, being about 20 in. for a No. 00 three-phase paper-insulated cable.

The exploring coil consists of a laminated-iron core, of a length determined by the lay of the cable, over which is wound a coil of insulated

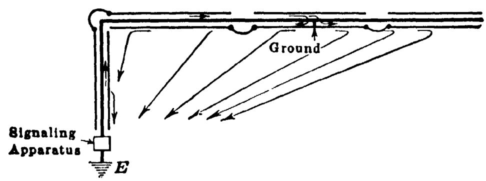

FIG. 466.—Pertaining to locating a ground by exploration tests.

wire with its terminals attached to a telephone. The core is placed *parallel* to the axis of the cable. Any current that flows only in the sheath produces a field that has no longitudinal component, and, therefore, stray currents cause no disturbance of the telephone.

Referring to Fig. 467, when a current flows out along the spiraled conductor *ab* and returns along the sheath, in effect along *cd*, there is a twisted loop which presents alternately its positive and negative side to the observer as he passes along an element of the sheath; that is, positions of maximum and minimum magnetic potential succeed each other in order.

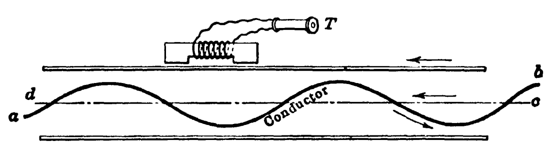

FIG. 467.—Diagram for Durgin exploring coil.

When the longitudinal exploring coil is used in the case shown in Fig. 466, it will be found that no indication is obtained at points near the signaling apparatus, for at these points there is little return current in the sheath, and the effect of a twisted loop is not obtained. On approaching the ground, more and more current flows in the sheath, and the signals increase in intensity until the ground is reached; beyond the ground is silence, for only sheath currents are present, and they produce no longitudinal field.

Another use of the exploring coil is to identify a particular cable in the distributing system. When dealing with three-phase cables, there are three signaling loops which may be utilized.

I. Two conductors, the current flowing out by one and returning by the other.

II. Two conductors in parallel and in series with the third.

III. One conductor and the sheath.

For purposes of explanation, take the first case; the equipotential lines on a plane perpendicular to the axis of the cable due to a steady current are shown in Fig. 468.

It is evident, because of the position of the conductors in the cable, that the magnetic potential varies from point to point around the circumference of the sheath and that the maximum and minimum points are 180 deg. apart. This means on account of the lay of the cable that the distance between the points of maximum and minimum magnetic potential, measured along an element of the sheath, is one-half the lay of the cable. If the iron core of the exploring coil has a length equal to one-half the lay and is applied longitudinally to the cable between these points, it will be traversed by a considerable flux, and a signal sent into the cable will be audible in the telephone. Sheath currents of any sort are not effective in producing sounds in the telephone, since they cause no inequalities of magnetic potential along the length of the cable.

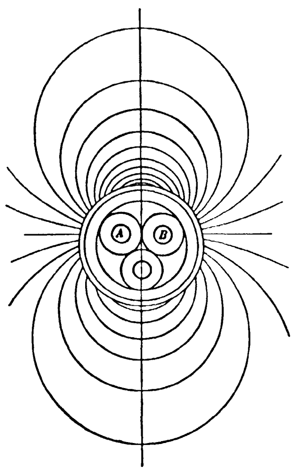

FIG. 468.—Showing magnetic equipotential lines around a three-phase cable when steady current flows in the signalling loop formed by conductors *A* and *B*.

The two-conductor circuit I is best when a cable has to be identified throughout its length, because it gives the maximum difference of magnetic potential and also because the maximum and minimum points are equally spaced.

With connection II, the maximum and minimum points are spaced alternately 40 and 60 per cent of the lay. With III, the corresponding spacing is 35 and 65 per cent of the lay (see Fig. 469).

When locating high-resistance grounds, all the conductors are connected in parallel so that the charging current to the portion of the cable beyond the fault may not produce a sound in the telephone. To send the characteristic signal into the cable, a motor-driven commutator is used which will break the circuit about 3,000 times per minute; in series with it is a make-and-break switch actuated by a cam also driven by the motor. The result is that one hears in the telephone a definite note which is interrupted in a particular manner.

Location of Total Disconnection.—A total disconnection occurs when the wire breaks inside the insulating covering, and the ends are pulled so far apart that the two sections of the conductor are insulated from each other.

Theoretically, the conductors in a cable occupy definite positions with respect to each other and to the sheath. This being so, the electrostatic capacitance measured between two conductors or between a conductor

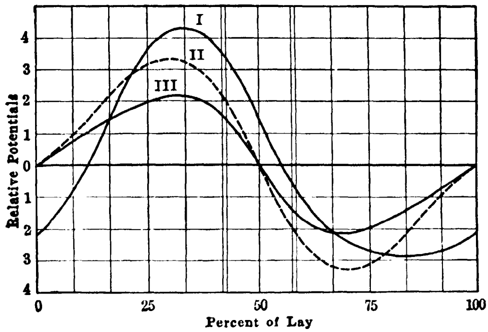

FIG. 469.—Showing variation of magnetic potential when different signaling loops are used.

and sheath should be proportional to the length of the cable. Consequently, when the insulation remains intact, it should be possible to locate a total disconnection by measuring the electrostatic capacitance of the portion of the cable from the testing station up to the break and comparing it with the capacitance of the whole cable. If the total capacitance is not known, measurements must be made from both ends of the line.

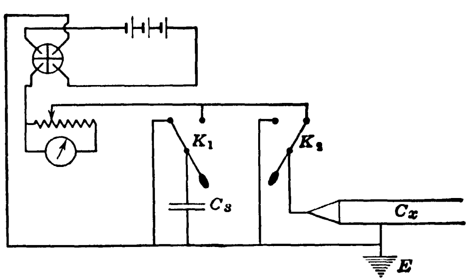

FIG. 470.—Connections for direct-deflection method for measuring electrostatic capacity of cable.

The capacitance may be measured by the direct-deflection method, the connections for which are shown in Fig. 470. A definite procedure should be adopted and used throughout the test, in order that the effects of absorption may be eliminated.

The ballistic deflection of the galvanometer, which occurs when the key K_2 is thrown to the right, is read and compared with the ballistic deflection obtained when the standard condenser is substituted for the cable, the battery being kept constant. It may be necessary to change the effective sensitivity of the galvanometer; consequently an Ayrton shunt may be used, as suggested by Fig. 470. The lead from the key to the cable core should be as short as possible. If necessary, its capacitance may be determined, and a suitable correction made.

The capacitance may be measured also by the simple bridge method (see page 406). The connections for a single-conductor cable or its equivalent are shown in Fig. 471.

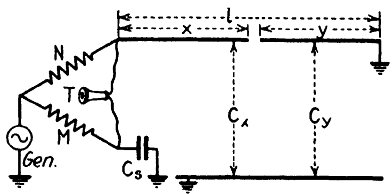

Fig. 471.—Simple bridge arrangement for measuring capacitance of short length of cable.

In this case, it is convenient to use an alternating current or a rapidly interrupted direct current and to employ a telephone as the detector. When the ratio M/N has been adjusted so that there is the minimum sound in the telephone, $M/N = C_x/C_s$, or

$$C_x = C_s\frac{M}{N}.$$

The capacitance of the other section C_y may be similarly determined by tests from the far end of the line. Then

$$\text{Distance to the fault} = (\text{total length of cable})\left(\frac{C_x}{C_x + C_y}\right). \qquad \textbf{(15)}$$

If the fault is in one wire of a pair, $C_x + C_y$ may be measured by interchanging the faulted and unfaulted conductor on the bridge and

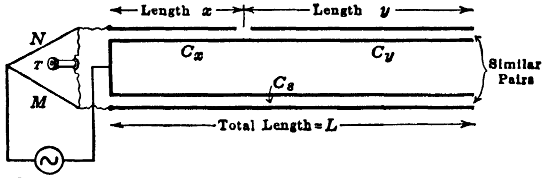

Fig. 472.—Bridge measurement of capacitance to total disconnection, using the capacitance of an unfaulted pair as a standard.

taking a second balance, C_s being unchanged. Then distance to fault = (total length of cable)$(M_1/N_1)(N_2/M_2)$.

The condensers used for C_s should be of good quality, and the procedure adopted should be the same as that employed when their capacitance was measured.

Where there are several pairs of wires in the cable, a pair may sometimes be used in place of the standard condenser, as in Fig. 472.

In this case, from the construction of the cable, the capacitance per unit length for both pairs is nominally the same, and as

$$\frac{M}{N} = \frac{C_x}{C_s}$$

$$\text{Distance to fault} = (\text{total length of cable})\left(\frac{M}{N}\right). \qquad (16)$$

These methods of location by capacitance measurements are convenient when dealing with telephone cables. The difficulty in applying them to power cables is that the disconnection, due to a burnout, may not be total and that the apparent capacitance per unit length may not be uniform.

Locating Irregularities of Line Constants by Impedance Tests.—When plotted, the frequency-impedance characteristic of a long uniform line terminated by a network having the same frequency characteristic as the line itself is a smooth curve without maxima or minima. If, however, there is an irregularity in the line constants, extreme examples being an open circuit or a short circuit, the frequency characteristic shows a succession of maximum and minimum values for the impedance. The frequency interval between the maximum (or minimum) values of the impedance is determined by the distance of the open or short circuit from the testing end. Therefore if the impedance, or its components—the effective resistance and effective inductance—are measured at different frequencies by an impedance bridge, the approximate distance from the open or short circuit may be found.

It can be shown[7] that with an open circuit at a distance X from the testing end, the maximum impedances occur at frequencies that are given by

$$f_1 = \frac{n_1 V}{2X}, \qquad f_2 = \frac{(n_1 + 1)V}{2X}, \qquad f_3 = \frac{(n_1 + 2)V}{2X} \quad \cdots \qquad (17)$$

n_1 is an integer, and V is the velocity of propagation in miles per second. For a uniform line operated at above 800 cycles per second, V is taken as 180,000. For loaded lines, V equals 52,000 miles per second. From (17)

$$X = \frac{V}{2(f_{n+1} - f_n)} \text{ miles.}$$

$f_{n+1} - f_n$ is the average of the frequency intervals between two successive maxima.

If the line is short-circuited at a distance X from the testing end, the maximum impedances occur at frequencies

$$f_1 = \frac{[2n_1 - 1]V}{4X}, \qquad f_2 = \frac{[2(n_1 + 1) - 1]V}{4X}, \qquad f_3 = \frac{[2(n_1 + 2) - 1]V}{4X} \quad \cdots \qquad (18)$$

Hence,

$$X = \frac{V}{2(f_{n+1} - f_n)} \text{ miles.} \tag{19}$$

As the result is the same for these two extreme cases, it is assumed that the formulae apply under less drastic conditions. In reality, the value of V is somewhat dependent on the frequency. Consequently, the irregularity is finally located by impedance measurements at different frequencies when an artificial fault is applied systematically at points near that indicated by the original test.

High-voltage and Breakdown Tests of Cables.[2]—Factory tests of reel lengths of cable are made with alternating voltage. This is the ideal procedure, for it duplicates service conditions. Any frequency between 25 and 100 cycles per second may be used, but 60 cycles per second is preferred.

In the practical execution of high-voltage and breakdown tests, numerous questions arise, for example:

1. Given the size of wire, thickness of insulation, and character of the insulating material, what test voltage should be applied?

2. How should the ends of the cable be prepared so that the insulation is not locally deteriorated or overstressed?

3. At what rate should the voltage be increased? For how long a time should the test voltage be maintained?

4. How can the wave form of the test voltage be assured?

5. How shall the voltage be measured so that the maximum stress to which the insulation is being subjected may be known?

6. What precautions must be taken in order that high-frequency disturbances set up by spark discharges from the testing circuits may be eliminated? The cable breakdown may be due to the high-frequency disturbance rather than to the regular test voltage, and the observer be misled.

7. Is it possible to determine whether a cable, which has not been actually broken down, has been overstressed by the high-voltage test so that it is permanently injured?

A high test voltage is advisable, since it promotes care on the part of the manufacturer; on the other hand, it is possible to overstress a cable, without actually breaking it down, so that it is permanently injured and in consequence may fail at some future time when conditions are normal.

The test voltages and times of application for the different kinds and thicknesses of insulation and conductor sizes are set forth in various specifications, for instance in "Specifications for Impregnated Paper Insulated Lead Covered Cable," prepared by the High Tension Cable Committee of the Association of Edison Illuminating Companies in consultation with the Insulated Power Cable Engineers Association[5] (printed, not published, 1934); "Specifications for Varnished Cambric Insulated Cables," prepared by the Insulated Power Cable Engineers Association, endorsed by the Electrical Manufacturers Association;[3] "Standards for Rubber Insulated Power Cable for the Transmission and Distribution of Electrical Energy,"[4] prepared by the Insulated Power Cable Engineers Association.

It is essential that the wave form of the voltage used in executing the tests given in the specifications be sinusoidal within the limit of tolerance (10 per cent) set by the American Institute of Electrical Engineers. Hence the generators and high-voltage transformers must be of ample capacity and designed to maintain the wave form with a capacitance load. In making the high-voltage tests, a potential difference not greater than the rated voltage of the cable is applied and is then increased at an approximately uniform rate until the desired value is reached, the rate of increase being not over 100 per cent of the rated voltage in 10 sec. and not less than 100 per cent of the rated voltage in 60 sec.

For paper cable rated at 7.5 kv. or less (phase to phase), the time of application of the test voltage is 5 consecutive min. If the rating is above 7.5 kv., the time of application is 15 min. The following table taken from the specifications mentioned gives the proper test voltages.

TABLE XVII.—TEST VOLTAGES FOR STRANDED CONDUCTOR CABLES RATED AT 7.5 KV. OR LESS FROM REF. 5
(Phase to Phase)

Thickness of insulation, 64ths in.	Impregnated paper insulation single-conductor cable				Multiple-conductor cable, belted	
	Test volts, kv. (conductor-sheath)				Test volts, kv. (conductor-conductor)	
	Nos. 1 to 0000	213,000 to 500,000	501,000 to 1,000,000	Over 1,000,000	Nos. 1 to 0000	225,000 and larger
4	5.0	5.0	....	....	6.0	
5	8.0	8.0	6.0	....	14.0	14.0
6	12.5	12.5	11.0	7.5	25.0	25.0
7	18.0	18.0	17.0	13.0	39.0	39.0
8	24.0	24.0	23.0	22.0		
9	31.0	31.0	31.0	31.0		

Note 1: Three-conductor belted cables shall be tested from conductor to sheath at $1/\sqrt{3}$ times the tabulated (c-c) value, except that whenever a separate c-s test is required, it shall be made at an average stress (c-s) equal to 80 per cent of the average stress (c-c) determined from the above table.

Note 2: Cables having stranded conductors smaller than No. 1 A.W.G. shall be tested at 85 per cent of the voltage specified for No. 1 A.W.G. and larger. If such cables have solid conductors, they shall be tested at 70 per cent of the voltage specified for stranded cables with conductors No. 1 A.W.G. and larger.

TABLE XVIII.—TEST VOLTAGES FOR CABLES RATED AT OVER 7.5 KV. From Ref. 5
(Phase to Phase)
Impregnated Paper Insulation

Type of cable	Test volts, volts per mil of specified insulation thickness
Single-conductor:	
Up to 50 kv	220
Over 50 kv	230
Multiple-conductor:	
Shielded	200
Belted, cond.-cond	180
Belted, cond.-sheath	145*

* When approved by the purchaser, the conductor-sheath test voltage for three-conductor cable may be $1/\sqrt{3} \times$ conductor-conductor test voltage.

Note: Concentric conductor cable to be tested as single-conductor cable.

For varnished-cambric cable, the test voltage is to be applied for 5 consecutive min. The voltages to be employed are given in Table XIX.

TABLE XIX.—TEST POTENTIALS—KILOVOLTS From Ref. 3
5-min. Tests
Varnished-cambric Insulation

Size of conductor A.W.G. or cir. mils	Thickness of Insulation—64th in. (and mils)													
	3 (47)	4 (63)	5 (78)	6 (94)	7 (109)	8 (125)	9 (141)	10 (156)	11 (172)	12 (188)	13 (203)	14 (219)	15 (234)	16 (250)
14 to 8	2.5	3.5	4.5											
7	...	3.5	4.5	7.0	9.5	11.5	14.0	16.0	18.0	19.5	21.5	23.0		
6	...	4.5	5.0	7.5	10.0	12.0	14.5	16.5	18.5	20.0	22.0	23.5	25.0	27.0
4 to 2	...	5.0	5 5	8.0	10.5	12.5	15.0	17.0	19.0	20.5	22.5	24.0	26.0	27.5
1 to 0000	...	...	6.0	8.5	11.0	13.5	15.5	18.0	19.5	21.0	23.0	24.5	26.5	28.0
213,000 to 500,000	...	...	6 5	9.0	11.5	14.0	16.0	18.5	20.0	22.0	23.5	25.5	27.0	29.0
501,000 and larger	...	...	...	9.0	11.5	14.0	16.0	18.5	20.0	22 0	24.0	26.0	27.5	29.5

Size of conductor A.W.G. or cir. mils	17 (266)	18 (281)	19 (297)	20 (313)	21 (328)	22 (344)	23 (359)	24 (375)	25 (391)	26 (407)	27 (422)	28 (438)	29 (453)	30 (469)
6	29.0	30.5	32.5	35.0										
4 to 2	29.5	31.0	33.0	35.5	38.0	40.5	43.0	45.0	47.5	49.5	50.5	52.0	53.5	55.0
1 to 0000	30.0	31.5	33.5	36.0	38.5	41.0	43.5	45.5	48.0	50.0	51.0	52.5	54.0	55.5
213,000 to 500,000	30.5	32.5	34.5	37.0	39.0	41.5	44.0	46.0	48.5	51.0	52.0	53.5	55.0	56.5
501,000 and larger	31.0	33.0	35.0	37.5	39.5	42.0	44.5	46.5	49.0	51.5	52.5	54.0	55.5	57.0

Single-conductor or multiple-conductor shielded cables shall be tested from conductor to ground at a voltage specified in the preceding table for the thickness of insulation from conductor to ground.

For multiple-conductor nonshielded cables, the test between conductors shall be twice the foregoing values for a single-conductor cable having the same thickness of insulation as on the conductor. The test pressure to sheath or ground shall be 58 per cent of this value except for multiple-conductor cables for ungrounded circuits over 6,000 volts working pressure, where the test pressure shall be 80 per cent of the test pressure between conductors.

For rubber-insulated cables, the test voltages are based on a ruling of the American Institute of Electrical Engineers—1 kv. per $\frac{1}{64}$ in. thickness of insulation up to $\frac{10}{64}$ in. and 1.5 kv. for each $\frac{1}{64}$ in. if the thickness is over $\frac{10}{64}$ in. The values are definitely stated for various sizes of conductor in a table which forms part of the "Standards for Rubber Insulated Power Cables."[4]

On purchasing cables, samples are taken to determine specific properties such as dielectric strength, power factor, and dielectric loss. The last two are determined by use of a high-voltage impedance bridge (page 415). Every reel length of paper cable intended for voltages over 7.5 kv. is given a power-factor test at a voltage equal to 20 and 100 volts per mil. The difference between the two power factors must not exceed certain values which are given in the following table of so-called *ionization factors*.

TABLE XX.—IMPREGNATED PAPER INSULATION IONIZATION FACTORS From Ref. 5

No. of conductors	Rated volts, kv. (phase to phase)	Ionization factor, per cent
Single	7.5–20.0	0.4
	20.1–35.0	0.3
	35.1 and over	0.2
Multiple shielded	7.5–20.0	0.4
	20.1 and over	0.3
Multiple belted	7.5 and over	0.7

If the ionization factor is measured at a temperature between 10 and 25°C. (both inclusive), the measured value shall be corrected to the corresponding value at 25°C. by deducting from the measured value 4 per cent of that value per degree difference between it and 25°C. If the measurement is made at a temperature above 25°C., no correction shall be made.

The difference between the power factors determined at the two prescribed voltages shall not exceed the values given in Table XX when corrected to a temperature of 25°C.

Kenotron Tests.—In distribution systems, it is necessary to test new cables after installation and to make routine tests of all cables in order to detect if possible those likely to fail. As the cables may be several miles long, the charging current may become so large that heavy, cumbersome, and expensive alternating-current apparatus is required, and in consequence recourse is now had to high direct-voltage tests made with portable apparatus. The high voltage is obtained by rectification from a step-up transformer which need not be of large size, as the current is in milliamperes rather than amperes. Rectification is obtained by use of kenotrons, or high-voltage electron discharge tubes. The vacuum in this device is as high as practicable, and a hot cathode is employed.

Wide variations of the test voltage applied to the cable are rendered possible by employing a transformer with a central tap on the high-tension secondary and four tubes in various combinations, as indicated in Fig. 473.

The kenotrons employed in one test set are rated at 50 kv., 0.25 amp. With connection *A*, the insulation between an electrified conductor and the grounded conductor, or sheath, is subjected to the peak voltage of the transformer, while the insulation between the two electrified conductors is subjected to twice that voltage.

With connection *B*, the insulation between the electrified conductor and the sheath may be subjected to twice the rated voltage of a single tube, while the current may be twice that allowable for a single tube.

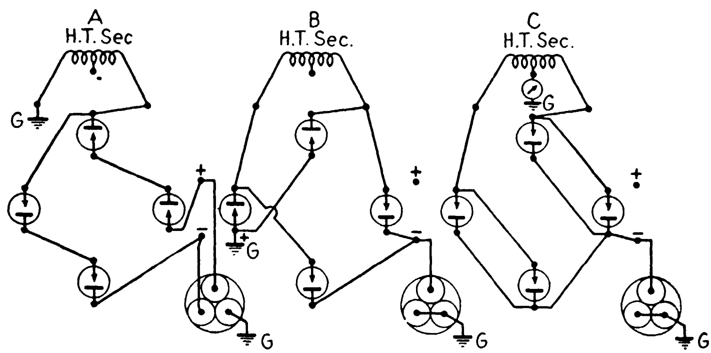

FIG. 473.—Connections for four-tube kenotron testing set.

With connection *C*, the insulation between the electrified conductor and the grounded conductors, or sheath, may be subjected to the rated voltage of a single tube, while the current may be four times that allowable for a single tube. For field work, the entire apparatus is assembled in a truck for transportation and may be energized from either a 230- or a 2,300-volt circuit. Convenient arrangements are made for safely making all connections necessary for operating the kenotron, and regulating and measuring the voltage applied to the cable as well as the leakage current. To avoid current rushes, the voltage is gradually increased from a low value. If a cable breaks down, the fault is reduced, and then located by one of the methods previously discussed. In routine work, a standard procedure must be adopted in order that comparisons with the results of previous tests may indicate the cables that show deterioration and are likely to fail. As the phenomena involved in the direct- and alternating-voltage tests are very different, the determination of the direct test voltage appropriate for various alternating working voltages and the proper times of application can be determined only by comparative tests made at the factory by cable manufacturers. For some the harmonics give rise to currents in such a direction that they, too, increase the deflection.

These difficulties from negative loops are overcome by the connection[6] given in Fig. 476, where grid-controlled, three-electrode tubes

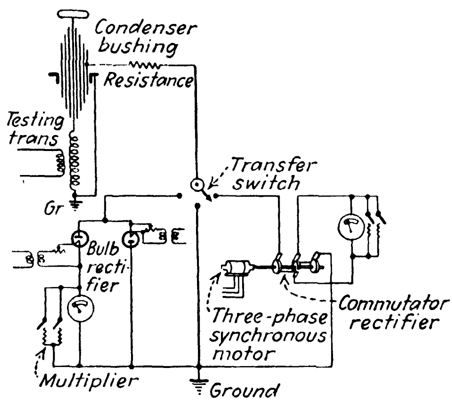

Fig. 475.—Crest-voltmeter connections for use with either simple bulb rectification or mechanical rectification.

are employed. The phase of the grid control is adjusted by means of the phase shifter. During one half-cycle, the grids of the main meter tube passes the positive half wave. If a negative loop occurs, it sends current through the meter in the reversed direction and therefore diminishes the reading as it should. At the same time, the grids of both by-pass tubes are negative and so take no current. During the next half wave, the potentials are interchanged, so that both meter tubes are blocked, while the by-pass tubes carry the current.

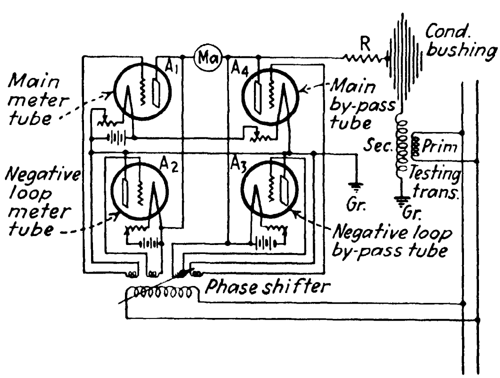

Fig. 476.—Crest voltmeter using three-element vacuum tubes.

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