ENGINEERING SOCIETIES MONOGRAPHS
HARRISON W. CRAVER, *Consulting Editor*

HYDRAULICS OF OPEN CHANNELS

ENGINEERING SOCIETIES MONOGRAPHS

Bakhmeteff: *Hydraulics of Open Channels*
Bleich: *Buckling Strength of Metal Structures*
Nadai: *Theory of Flow and Fracture of Solids*
Rich: *Hydraulic Transients*
Timoshenko: *Theory of Elastic Stability*
Timoshenko: *Theory of Plates and Shells*
Timoshenko and Goodier: *Theory of Elasticity*

HYDRAULICS OF OPEN CHANNELS

HYDRAULICS OF OPEN CHANNELS

BY

BORIS A. BAKHMETEFF

First Edition

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK AND LONDON
1932

PRINTED IN THE UNITED STATES OF AMERICA

X

PREFACE

Engineering practice nowadays exacts deeper and more subtle methods of approach than those which were customary in the past and are still embodied in the usual treatises on hydraulics, constituting the subject of the traditional courses in schools of engineering. In fact, structural mechanics has evolved into "applied elasticity." Machine design is permeated with the study of vibrations and other features of higher dynamics. The notions of turbulence, cavitation, and circulation are in the forefront of hydrodynamical research, which is being successfully applied to problems of aeronautics and hydraulic machinery.

In the field of hydraulic engineering and, in particular, in that most important realm where the civil engineer deals with open flow, the trend has been away from the rudimentary notions of uniform movement. Broader viewpoints, embracing varied flow in open channels, the hydraulic jump, surges in canals, etc., have become topics of discussion in engineering literature and are gradually wedging their way into advanced instruction. On the other hand, most fruitful avenues of approach have been opened by laboratory work on models of hydraulic structures. Here again, the elementary notions, as presented in the old-time treatises with relation to flow of water through orifices, over weirs, etc., have been replaced by a deeper and more detailed study of the physical circumstances of the movement. Illustrative of the present trend in this field is the capital summary, "Hydraulic Laboratory Practice," the American version of which has recently appeared under the enlightened leadership of John Freeman. A crowning result finally is the creation of the National Hydraulic Laboratory in Washington, D. C.

In regard to hydraulic design as envisaged by the civil engineer, the fact is that uniform movement scarcely ever occurs in actual practice. So a comprehensive understanding of the functioning of a hydraulic system would seem possible only when the phenomena were to be considered and the design carried out in terms of varied flow. Unfortunately, as Professor Daugherty expresses it in his "Treatise on Hydraulics," "a satisfactory and reliable treatment of the problem of non-uniform flow is lacking." This means, of course, a treatment opening the way to practical applications, as the theoretical basis of varied flow is well advanced and stands out as a momentous contribution to the art on the part of the great French hydraulicians of the nineteenth century.

The present book is an attempt to supply the need, at least in part, and to offer a manual presenting the subject of varied flow in a manner which makes it useful for engineering practice and design. The origin of this work goes back to pre-war days. The author, then connected with vast hydraulic enterprises in Russia, was faced with the task of finding means of throwing light on the many puzzling and elusive phenomena connected with varied flow, and of solving in a comprehensive manner the different practical problems. The result was a book on "Varied Flow in Open Channels,"* published in Russian in 1912.

In dealing with the physical aspects of flow, considerable use was made in that book of the notion of the "specific energy of flow," which is the energy head referred to the bottom of the canal cross-section. This simple notion was found to give a lucid explanation of many complex features heretofore interpreted solely from an abstract, analytical point of view. Thus a physical basis was given to what is known as the critical depth; a simple explanation was made available for the hydraulic jump together with a comprehensive interpretation of the different types of surface curves. Since 1912, this "energetic criterion"

* "O Neravnomernom Dwijenii Jidkosti v Otkritom Rusle," St. Petersburg, 1912.

has been unmasked and advantageously used by different authors, seemingly independently of each other. In fact, the energetic criterion is mentioned by Rehbock* and very lucidly presented by Hinds.† The author, however, is not aware of any disclosure previous to the Russian publication.

The Russian book also carried a suggestion of a novel method of computing the different surface curves in varied flow. Heretofore the differential equation had been applied and integrated only for certain "idealized" profiles, for which numerical tables had been computed by Bresse, Tolkmitt, Rühlmann, and others. Although these tables presented a substantial advance, their usefulness was nevertheless greatly limited by the simple fact that the "idealized" cross-section underlying the tables had little in common with the practical forms of canals that the engineer deals with in everyday practice.

The method suggested by the author is applicable to canals of any practical form. It is based on an exponential relation which was found to govern (with sufficient approximation) flow in an open conduit under varying stages. The practicability and usefulness of the method depended, however, on computing tables of what is designated in this book as the "varied-flow function." The preparation of such tables for different exponents involved a long and tedious procedure. This work was undertaken and performed for the first time under the author's direction, during 1914–1915, by the Research Board of the then Russian Reclamation Service. In the turmoil of the revolution the data so computed became unavailable to the author, in fact, for a while he considered them totally lost; so the task of computing had to be done over again and was carried out in the form offered here by Professor Kholodovsky and partly by Dr. Pestrečov.‡

* "Betrachtungen über Abfluss, Stau und Walzenbildung," Berlin, 1917.

† *Eng. News-Record*, Vol. 85, p. 1034, 1920.

‡ In the meanwhile the Russian book was reprinted in Leningrad in 1928. This reprint, of which the author was unaware, is complemented by the 1914–1915 tables. The tables as computed and offered in this volume are more precise and complete.

The treatise as presented to the reader in this volume has been completely rewritten. The major part of the material is new and heretofore unpublished. Generally speaking it aims at practical results, and therefore the author has refrained from expositions of purely theoretical character. The presentation is most elementary, having in view, first of all, a clear interpretation of the physical aspects of varied flow.

A large part of this book is taken up with practical examples. Experience shows that in the study of varied flow, as in many other branches of applied mechanics, complete mastery of the subject cannot be attained without familiarizing oneself thoroughly with numerical procedures. In fact, there are many important features which escape deduction from general formulas and which may be stipulated only as rules and conclusions, reached through summarized experience. For such reasons it is better not to consider the numerical examples as mere illustrations, but to regard at least some of them as an organic part of the general presentation of the matter. Experience has also taught the author that, no matter how simple in essence, the notions related to varied flow and the methods of handling the problems require, on the part of the novice, time and a certain amount of persevering apprenticeship, before they become natural and familiar tools. That will explain and partly justify the detail in presenting the examples. Computations by necessity have to be at times laborious, and repetition cannot be entirely avoided.

The author does not claim to have given an exhaustive treatment of the subject. Many problems are too complicated to be dealt with on a practicable basis; in other instances, the actual state of the art is inadequate. However, as learned from experience, the ways and means presented permit the solution of many problems in a comparatively simple and comprehensive manner. It has been said that progress in theoretical engineering is an advance in the way of "thinking." The author is deeply convinced of the importance for the hydraulic engineer of learning to "think" in terms of *varied flow* and of practicing such "thinking" in the everyday approach toward practical problems.

BORIS A. BAKHMETEFF.

GREENHILLS,
BROOKFIELD, CONNECTICUT,
January, 1932.

CONTENTS

SYMBOLS

Throughout this book the foot, pound, second system of measurement is used.

Cross-sectional elements and friction factor C values have been largely taken from "Hydraulic and Excavation Tables" of the United States Reclamation Service.

All computations were performed by means of an ordinary slide rule.

y, d	Depth or stage of flow.
a	Cross-sectional area of a canal.
b	The top width.
p	The wetted perimeter.
R	The hydraulic radius.
C	The Chézy friction factor.
v	Average velocity.
Q	Discharge.
s_0	Bottom slope.
s	Surface slope.
y_o, d_o	Normal depth of flow, being the depth of flow in uniform movement.
W	Work.
W_r	Work of resistances.
N	Power.
y_{cr}, d_{cr}	The critical depth.
$\delta = a/b$	The average depth.
v_{cr}	Critical velocity.
Q_{cr}	Critical discharge.
$\sigma = \dfrac{g \cdot p}{C^2 \cdot b}$	Critical slope.
σ_o	Critical slope at the normal depth.
σ_{cr}	Critical slope at the critical depth.
Δ	The weight of a cubic unit of liquid.
e	Energy head, being the energy per unit weight of the flowing liquid referred to a datum line.
ϵ	Specific energy, being the energy head referred to the bottom line of a cross-section.
$\mathfrak{K} = aC\sqrt{R}$	Conveyance of a cross-section.
n	The hydraulic exponent.
$\mathfrak{M} = a\sqrt{a/b}$	The $\mathfrak{M}$ function.

λ	The kinetic flow factor.
l_{m-n}	Length of a reach, between the cross-sections m and n.
L_m	Distance of the cross-section m from an initial zero point.
$\eta = y/y_o$	The relation of the varying depth of flow y to the normal depth y_o.
$B(\eta) = -\int_o^\eta \frac{d\eta}{\eta^n - 1}$	The varied flow function.
$\beta = s_o/\sigma$	The relation of the bottom slope to the critical slope.
$\tau = y/y_{cr}$	The relation of the depth of flow y to the critical depth y_{cr}.
$\delta = \sigma_o/\sigma$	The relation of the critical slope at the normal depth y_o to the varying critical slope at other stages of flow.
$T(\tau)$	The varied flow function in a canal with horizontal bottom.
d_1 and d_2	The conjugated depths before and after the jump.
j	The height of the jump.
ϵ_1 and ϵ_2	The specific energy before and after the jump.
ϵ_j	The energy head lost in a jump.
c	Celerity of propagation of a translation wave.
$w = v - c$	Relative velocity of propagation with regard to the embankment.
h	The height of a surge or of a translation wave.
z_o	The depth of the center of gravity of a section.
$M(y) = az_o + \frac{Q^2}{ag}$	The M function.
P	The height of the body of a weir.
H	The head over the crest of a weir.
Z	The difference between the levels above and below a weir.
t	Tail-water depth.
α	Contraction coefficient.
φ	Velocity coefficient.
ζ	Resistance coefficient.
μ	Discharge coefficient.

Reference in the examples to "tables" means the Tables of the *Varied Flow Function*, pp. 308 to 312.

HYDRAULICS OF OPEN CHANNELS

CHAPTER I

DEFINITIONS

1. Uniform Flow.—The flow of a liquid in an open channel (Fig. 1) is said to be uniform when the depth and the other elements of flow, such as the cross-sectional area a, the velocity v, and the hydraulic slope s, remain constant from section to section. The surface line in this case is parallel

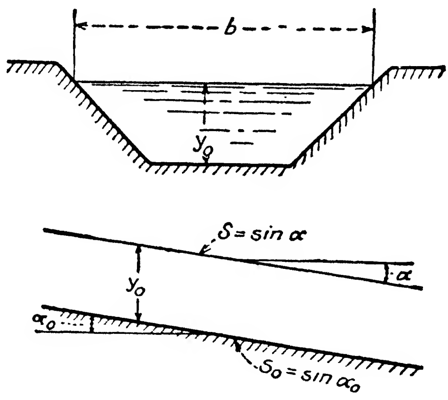

FIG. 1.—Uniform flow in a canal.

to the bottom line, the surface slope $s = \sin \alpha$ being equal to the bottom slope $s_0 = \sin \alpha_0$.

Obviously, flow may be strictly uniform only in case the channel itself is prismatical, that is, when and if the channel is built so that the cross-sectional forms do not vary from section to section and the bottom is laid with a permanent slope. Rivers and watercourses in natural state scarcely ever answer such qualifications and therefore practically never feature strictly uniform flow.

In a canal between two reservoirs (Fig. 2*a*), flow will be uniform if and when levels *A* and *B* are so positioned that the depths y_1 and y_2 at the beginning and at the end of the canal are equal. In this event, assuming the canal to be prismatical, the depth of flow in any section between 1 and 2 will be the same, so that for any section $y = y_1 = y_2$.

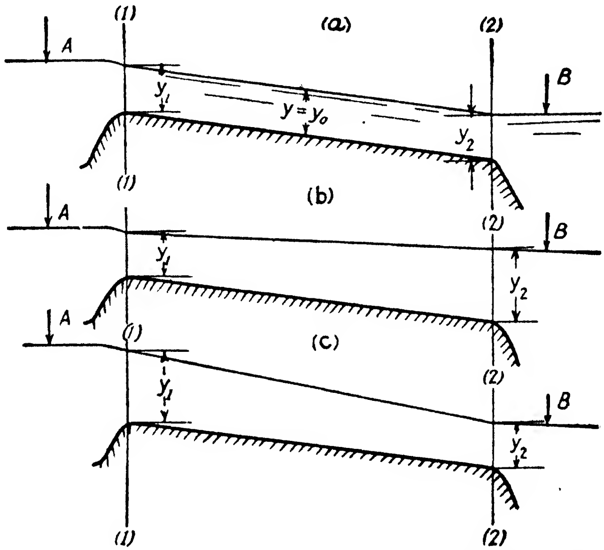

FIG. 2.—Uniform and varied flow in a canal connecting two reservoirs.

2. Non-uniform or Varied Flow.—Whenever the depth and the other features of flow, such as the cross-sectional area, the velocity, and the hydraulic slope, vary from section to section, flow becomes *non-uniform* or *varied.*

The classical example presented is that of a backwater curve (Fig. 3) produced by a dam. The natural surface line *ABC* . . . is lifted to a position *A′*, *B′*, . . . *E′*. The lift *Z* diminishes stream upward, the backwater curve approaching asymptotically the free surface line. Backwater in a natural watercourse is practically the only case of varied flow dealt with in textbooks on hydraulics. Yet, it is but one of the manifold problems of varied flow which the engineer has to face in designing or operating hydraulic structures. As an example, take flow in the canal (Fig. 2), in the event that the depths y_2 and y_1 are no longer equal. Suppose $y_2 > y_1$, as in Fig. 2*b*; the depth will be increasing stream downward, and flow will be varied and said to be taking place with a *rising surface curve*. On the other hand, in Fig. 2*c*, with $y_2 < y_1$, the depth will be

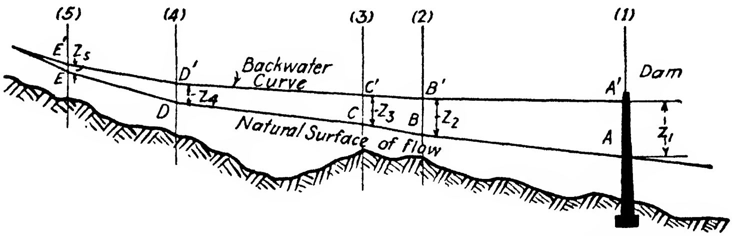

FIG. 3.—Backwater curve in a natural watercourse.

decreasing stream downward with a *falling surface curve*. Another important case is that of Fig. 4, where flow in the canal is regulated by means of a sluice. Depending on the opening of the sluice and the volume of the discharge, the depth y_2 will vary. Varied flow with a rising curve is

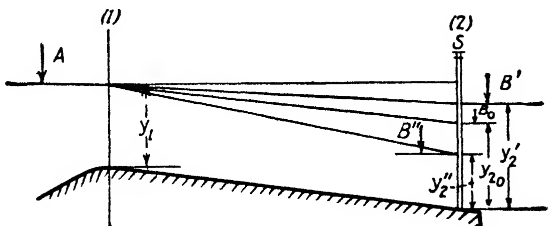

FIG. 4.—Varied flow in a canal regulated by a sluice.

featured by AB', while AB'' corresponds to a falling surface curve. In between them is uniform flow with $y_2 = y_1$. Naturally, the lower the level B, that is the smaller the depth y_2, the greater the discharge Q drawn from reservoir A.

The case outlined in Fig. 4, as well as that of Fig. 2, typifies one of the most important problems related to varied flow, namely, that of determining the *varying* *discharge* of a canal under varying conditions of levels at the extremities of the structure.

From comparing Fig. 2 and Fig. 3, another distinction is gained. In a natural watercourse (Fig. 3), the cross-sections and practically all the other elements of flow vary from section to section. Flow thus is non-uniform in the broadest sense of the word. The same applies to Fig. 5 which features a canal with diverging (Fig. 5*a*) and con-

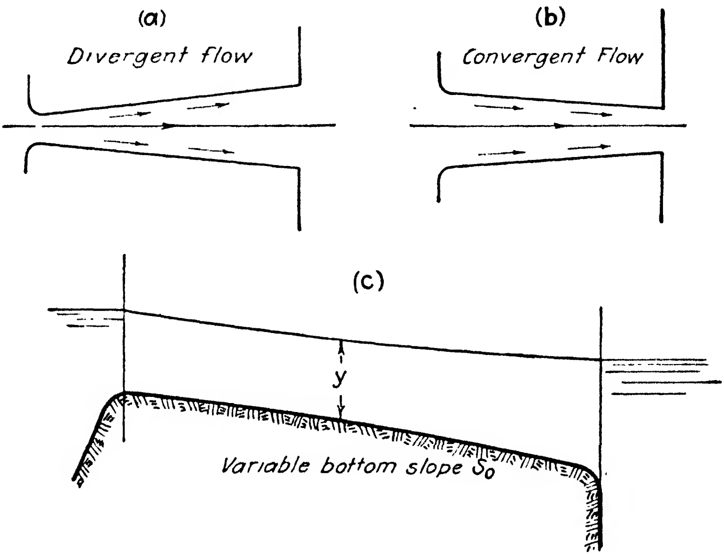

FIG. 5.

verging cross-sectional forms (Fig. 5*b*), or, as in Fig. 5*c*, a canal with a varying bottom slope.

In contradistinction to Fig. 5, the canal in Figs 2 and 4 is assumed to be built with regular and unvarying cross-sectional forms and also with a constant bottom slope s_0, thus featuring *varied flow in a prismatic channel.* In the present state of the art, varied flow in prismatic channels constitutes the most important case, a case which primarily is dealt with in this book.

3. Variable Flow.—Varied and implicitly uniform flows are further taken as being permanent; that is, as movements which do not change in time. As a matter of fact, the depth, the velocity, and all other features, while varying from section to section, remain constant and unchangeable in time. As opposed to varied flow, movement in which the elements of flow are subject to change in the course of time is said to be *non-permanent* or *variable.* An outstanding example of variable movement is waves; also surges produced in canals by a sudden stoppage or a change of the discharge. Again in Fig. 2, in the case when levels A and B should not remain permanent and the depths y_1 and y_2 would be changing in time, the permanent character of the movement would no longer prevail and the flow would become variable.

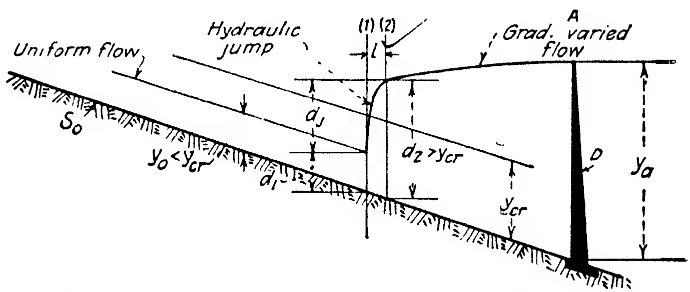

FIG. 6.—Backwater in a watercourse with steep bottom slope featuring a hydraulic jump.

In the present state of the art, practical problems of variable flow are soluble only in a limited number of cases, and then only in a simplified and approximate form.

4. The Hydraulic Jump.—As early as 1820 Bidone showed that backwater does not always assume the forms illustrated by Fig. 3, namely those of a continuous curve tangent to the natural surface profile. In fact, whenever the bottom slope of the watercourse is sufficiently steep, the phenomena unfold as pictured in Fig. 6. After flowing uninterruptedly until a certain section, marked in Fig. 6 as section 1, the surface suddenly jumps from the natural depth d_1 to the depth d_2 in section 2. This curious and, for a long time, paradoxical phenomenon is known as the *hydraulic jump.* The jump in most cases features a disruption of the continuity of flow. Beyond section 2 in

Fig. 6, the surface is once more continuous and the change of depth is gradual and smooth.

Another example of the jump is given in Fig. 7, in which case water is flowing with great velocity from under a

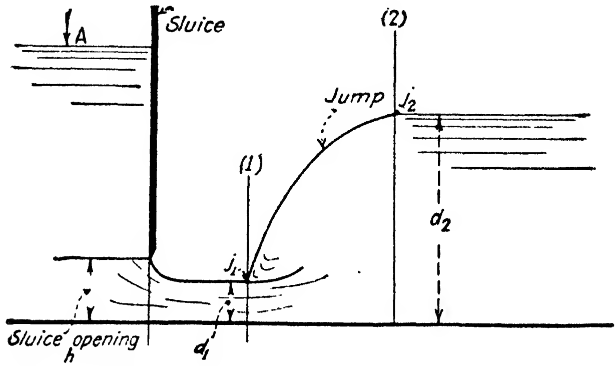

FIG. 7.—Hydraulic jump in a canal below a regulating sluice.

sluice. The jump takes place between the depth d_1 near the *vena contracta* and the depth d_2 which under these circumstances may be many times the multiple of d_1.

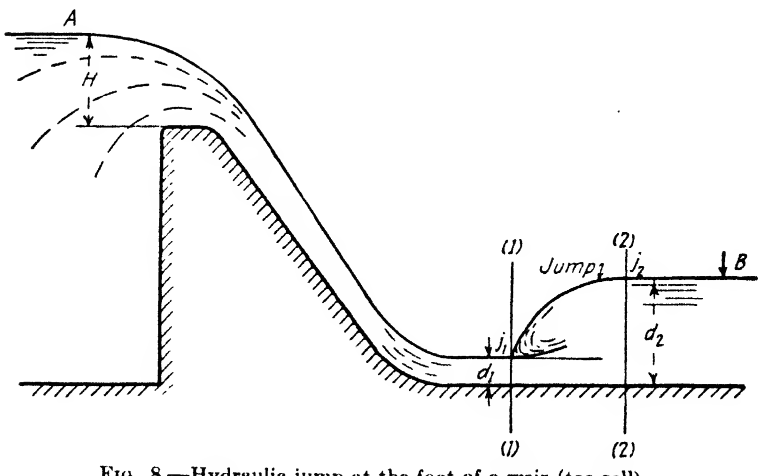

FIG. 8.—Hydraulic jump at the foot of a weir (toe-roll).

Figure 8 pictures the case of a toe roll at the foot of a weir. Tail-water is repelled by the stream falling over the dam; the free vein connecting with the level of the tail-water by means of a jump.

5. The Hydraulic Drop.—In Fig. 9, the canal is laid with a sudden enlargement in section 0, the depth of flow in the respective portions above and below the enlargement being y_1 and y_2. The transition from the narrow to the broad canal takes place by means of a rather steep and abrupt lowering of the surface, a phenomenon which we shall call the *hydraulic drop*. Characteristic of the drop is the fact among others, that the surface of flow in the narrow portion of the canal will not sink below a certain depth y_c; and that, no matter what the stage of flow is in the

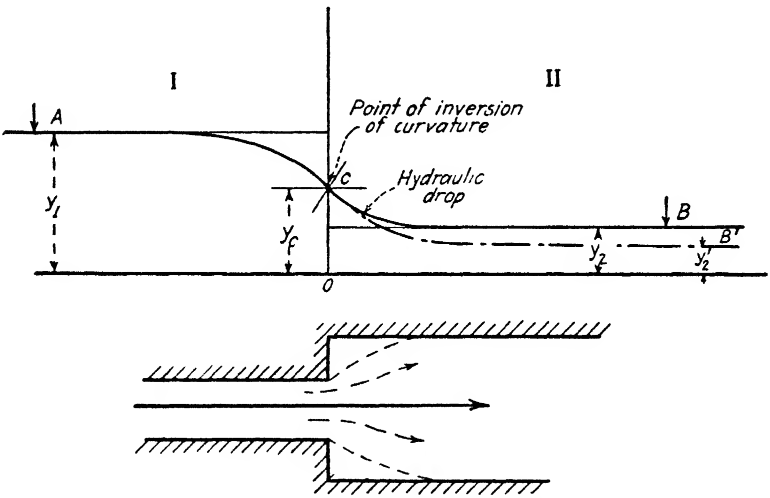

Fig. 9.—The hydraulic drop, caused by an enlargement of the canal cross-section.

broad portion of the structure. Thus, in case the level of flow at the lower stage was B' instead of B, with the depth y'_2, such lowering of stage B would not affect the surface curve above point C. In this light, point C with the depth y_c appears as a natural limit to which the surface in the canal may sink in unobstructed outflow by means of a hydraulic drop. As indicated in Fig. 9, C is, moreover, the point of inversion of curvature of the dropping surface.

Another example of a hydraulic drop is that of a canal of a mild bottom slope, emptying into a chute with a *steep* slope (Fig. 10). Here again the connecting curve between flow in the canal I at y_{01} and flow in the chute II at y_{02} features a hydraulic drop. Again, there is a depth y_{cr} over the breaking point of the bottom which is the lowest possible depth to which the surface in the upper reach can sink, and again point C is a point of inversion of curvature of the surface line.

6. Local Phenomena and Gradually Varied Flow.—The hydraulic drop and the hydraulic jump are both characterized by a rapid change of circumstances of flow which takes place over a comparatively short length. In this aspect they are distinguishable from flow as shown in Figs. 2, 3 and 4, where an appreciable change of depth is evi-

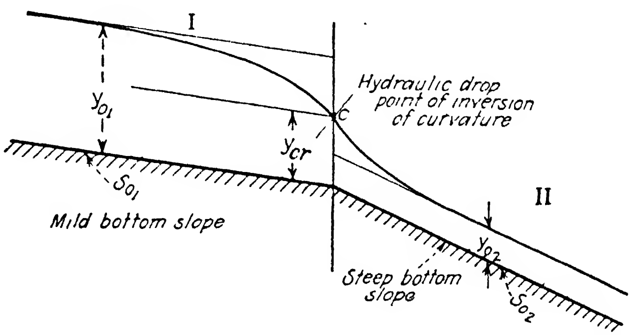

FIG. 10.—The hydraulic drop, caused by a change of the bottom slope from *mild* to *steep*.

denced only over a considerable length. Accordingly flow, as illustrated by Figs. 2, 3 and 4, may be qualified (Boussinesq) as *slowly* or *gradually varied flow,* while cases where the change is effected in abrupt manner may be covered by the general term of *local phenomena.*

A watercourse with varying circumstances of flow may usually be divided into successive reaches in which gradually varied flow alternates with the local phenomena. The case is illustrated by the somewhat artificial but illuminating example of Fig. 11, where the reaches with gradually varied flow are separated by comparatively short stretches featuring phenomena of local character.

May it be remarked at this point that throughout the following it will be customary to designate depths relating to local phenomena by the letter *d*, while depths in gradually varied flow will be designated by the letter *y*. Also, regarding the nomenclature, the word *depth* may be substituted by the word *stage*. In fact, a surface curve may be determined by the consecutive depths of the water over the bottom line, as well as the consecutive stages, meaning the consecutive elevations of the surface level in each and

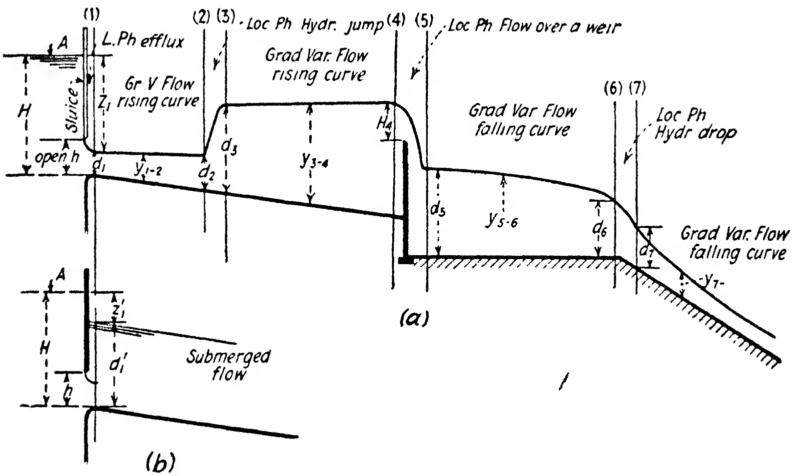

FIG. 11.—Gradually varied flow alternating with local phenomena.

every section. In the following the two terms will be alternately used.

Figure 11 serves also to illustrate the character of the problems which a "theory" of varied flow is expected to solve. Assume, in fact, that the respective locations, the dimensions, and other characteristic features of the different structures are known. Assume further that the position of the initial level *A* is given as well as the aperture of the sluice *h*. The problem first would be to determine the general picture of flow; that is, the type and the general outline of the different surface curves in the reaches, and the specific form in which the different local phenomena unfold. Just as an example one should determine whether, under the circumstances as given, the toe roll below the weir is *covered* as in Fig. 11*a*, section 4–5, or *free* as in Fig. 8; or whether the flow from under the sluice is *free* as in Fig. 11*a* (section 1–2) or is *submerged* as in Fig. 11*b*. Obviously the fact whether the outflow from under the sluice is submerged or free will materially affect the volume of the discharge, and thus the flow in the whole system.

After the general type of movement is established, the next step will be to determine with sufficient precision the numerical features, meaning the depths in the sections which separate the local phenomena from the adjoining stretches and then the precise outline of the surface curves in the reaches where flow is of the gradually varied type. Methods devised in this book permit, speaking generally, solution of problems of this character with a degree of certainty and precision sufficient for practical purposes.

PART I

THEORY OF GRADUALLY VARIED FLOW

CHAPTER II

UNIFORM FLOW

The notions relating to uniform flow, familiar from elementary hydraulics, are presented in this chapter from a somewhat different aspect which makes them more convenient for the subsequent study of varied flow.

7. The Conveyance of a Canal Cross-section.—Assuming uniform flow taking place in a canal (Fig. 1) with the depth y_0, the average velocity of flow in accordance with the Chézy formula is:

$$v = C\sqrt{R}\sqrt{s_0} \tag{1}$$

And the discharge Q:

$$Q = aC\sqrt{R} \cdot \sqrt{s_0} \tag{2}$$

In these formulas:

a = the cross-sectional area.
$R = a/p$ = the hydraulic radius; p = the wetted perimeter.
s_0 = the bottom slope.
C = the Chézy resistance factor, to be determined by means of the Ganguillet-Kutter, Bazin, Manning, or other empirical formulas.

Designating

$$aC\sqrt{R} = \mathfrak{K} \tag{3}$$

we obtain, instead of Eq. (2)

$$Q = \mathfrak{K}\sqrt{s_0}; \quad s_0 = Q^2/\mathfrak{K}^2 \tag{4}$$

For a given canal, the value of $\mathfrak{K}$ is a function of the depth y. It can be traced as a curve $\mathfrak{K} = f(y)$ (Fig. 12), which curve features the capacity of the canal to convey water depending on the stage of flow. The actual discharge Q, at a certain stage y is obtainable by multiplying the value of $\mathfrak{K}$, corresponding to the respective depth, by $\sqrt{s_0}$.

As $s_0 = \sin \alpha_0$ is a dimensionless quantity, the physical dimension of $\mathfrak{K}$ is that of a discharge (L^3/T). In fact, $\mathfrak{K}$

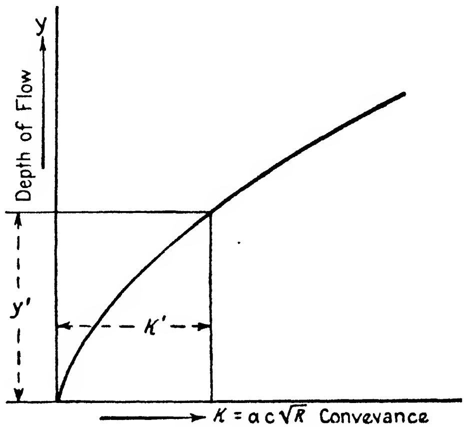

FIG. 12.—The conveyance curve $\mathfrak{K} = aC\sqrt{R} = f(y)$.

measures the quantity of liquid delivered by the canal per unit of time in the hypothetical case of $\sqrt{s_0} = 1$. We

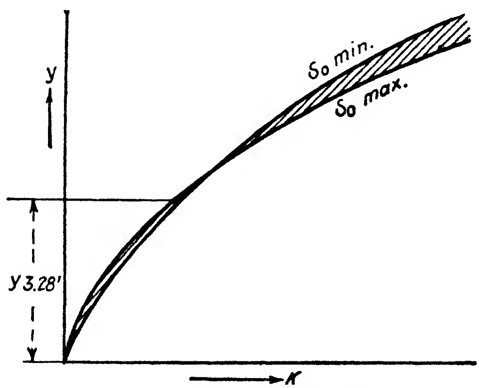

FIG. 13.—Conveyance curves in case the Ganguillet-Kutter formula is used.

shall designate $\mathfrak{K} = f(y) = aC\sqrt{R}$ by the term *conveying capacity*, or briefly *conveyance* of a cross-section.

Speaking generally $\mathfrak{K}$ is a general characteristic, inherent in a canal cross-section as such. If the friction factor C

is determined by means of a formula which does not contain s_0, such as the Bazin or the Manning formula, then in such case the *conveyance* curve features the conveying capacity of a certain cross-section for the whole range of practically usable slopes. If the Ganguillet-Kutter formula is used, which makes C vary (even if only slightly) with the bottom slope s_0, then the conveyance diagram is represented (Fig. 13) by the shaded area contained between the conveyance curves, corresponding to certain limiting values of s_{0min} and s_{0max}.* The following examples will

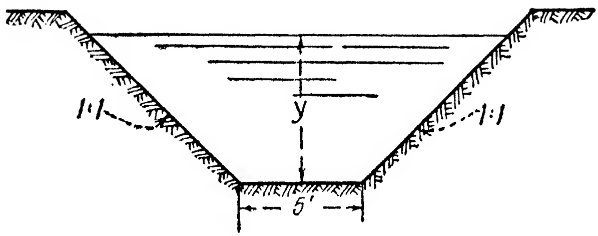

FIG. 14.—Canal cross-section type B.

help the reader to become familiar with the practical use of the notion.

Example 1

Question 1. Trace the conveyance curves for a canal cross-section (Fig. 14), assuming concrete lining, corresponding to a G.K. coefficient $n = 0.013$, and a Bazin coefficient $\gamma = 0.16$. When applying the G.K. formula, consider the limits of bottom slopes to be $s_{0max} = 0.001$ and $s_{0min} = 0.0001$.

The computations referring to the Bazin formula with

$$C = \frac{87}{0.55 + \dfrac{0.16}{\sqrt{R}}}$$

* We have no intention at this time to enter into the much discussed field of the relative value of the different empirical formulas. The great advantage of the G.K. formula, in many other ways untenable, is the abundance of experimental data, which have been reduced to the form of G.K. coëfficients. The G.K. experimental coefficients, however, may be expediently used in an exponential formula of the Manning type (see Art. 32).

are assembled in Table I, which is self-explanatory; while the respective $\mathfrak{K}$ curve is traced in an unbroken, drawn line in Fig. 15.

TABLE I

y	a	p	R	C	$\mathfrak{K}$
0.5	2.75	6.41	0.429	109.0	196
1.0	6.00	7.82	0.768	118.0	623
1.5	9.75	9.23	1.053	123.0	1,228
2.0	14.00	10.64	1.312	126.0	2,019
2.5	18.75	12.05	1.558	128.5	3,000
3.0	24.00	13.46	1.783	130.0	4,165
3.5	29.75	14.87	2.010	132.0	5,560
4.0	36.00	16.28	2.215	132.7	7,100
4.5	42.75	17.69	2.410	133.5	8,870
5.0	50.00	19.10	2.620	134.3	10,880
6.0	66.00	21.95	3.005	135.6	15,500

The values of C's in accordance with the G.K. formula and the respective $\mathfrak{K}$'s are given in Table II and traced in broken lines in Fig. 15.

TABLE II

y	$s_0 = 0.001$		$s_0 = 0.0001$	
	C	$\mathfrak{K}$	C	$\mathfrak{K}$
0.5	97.0	174	86.0	154.5
1.0	110.0	578	103.0	541
1.5	117.0	1,170	110.0	1,100
2.0	121.0	1,940	116.0	1,875
2.5	124.5	2,910	120.0	2,840
3.0	127.0	4,070	124.0	3,980
3.5	130.0	5,490	127.0	5,360
4.0	132.0	7,060	129.5	6,940
4.5	133.5	8,870	131.0	8,700
5.0	135.0	10,910	133.5	10,800
6.0	138.0	15,780	137.0	15,640

Question 2. In the canal as above, determine the discharge under $y_0 = 3.7$ ft. and $s_0 = 9\,{}^0\%_0$.

Remark: The sign, ${}^0\%_0$, used to indicate the slope, means that the incline is given in units of 1/10,000.

The convenience of using such notation becomes apparent when figuring out $\sqrt{s_0}$. In fact $\sqrt{1^0\%_0}$ is 1% or 0.01. Thus, if the slope is given as $s_0{}^0\%_0 = s'_0 \cdot 10^{-4}$, the square root is $\sqrt{s_0} = \sqrt{s'_0} \cdot 10^{-2}$.

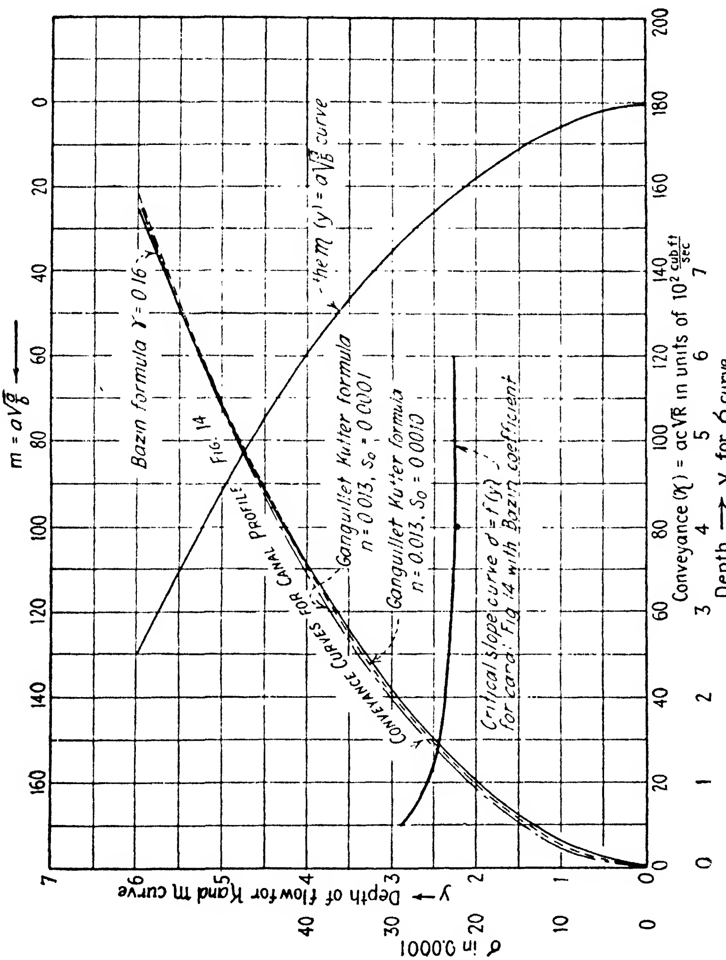

FIG. 15.—Characteristics of canal cross-section type B. Fig. 14.

If, on the other hand, the $\mathfrak{K}$ values, which are usually quite large, are given in units of hundreds, so that $\mathfrak{K} = \mathfrak{K}' \cdot 10^2$, then, for figuring discharges, we have

$$Q = \mathfrak{K}\sqrt{s_0} = \mathfrak{K}' \cdot 10^2 \times \sqrt{s'_0} \cdot 10^{-2} = \mathfrak{K}'\sqrt{s'_0}$$

In other words, the $\mathfrak{K}'$ values should be simply multiplied by the square root of s'_0. This practice is maintained throughout the book.

From curve (Fig. 15), for $y_0 = 3.7$ ft.:

$$\mathfrak{K}\text{ (Bazin)} = 61 \cdot 10^2 \text{ cu. ft. per second.}$$
$$\mathfrak{K}\text{ (G.K.)} = 60 \cdot 10^2 \text{ cu. ft. per second.}$$

Hence with $s_0 = 9 \cdot 10^{-4}$:

$$Q\text{ (Bazin)} = 61 \cdot \sqrt{9} = 183 \text{ cu. ft. per second.}$$
$$Q\text{ (G.K.)} = 60 \cdot 3 = 180 \text{ cu. ft. per second.}$$

Exercise:

Determine Q for $y_0 = 2.5$ ft. and $y_0 = 4.8$ ft. under $s_0 = 2^0\!/_{00}$, and respectively $s_0 = 6^0\!/_{00}$.

Question 3. In the canal, as given, determine the slope required to convey a discharge of 200 cu. ft. per second at a depth of 4 ft.

a. Using Bazin coefficients, the conveyance at $y_0 = 4$ ft. is $\mathfrak{K} = 71 \cdot 10^2$; hence the required slope

$$s_0 = Q^2/\mathfrak{K}^2 = 200^2/71^2 \times 10^4 = (200/71)^2 \cdot 10^{-4} = 7.9^0\!/_{00}$$

b. If G.K. coefficients are used, start in first approximation with an average value of $\mathfrak{K} = 70 \times 10^2$; for which

$$s_0 = (200/70)^2 \cdot 10^{-4} = 8.2^0\!/_{00}$$

In further approximation with $\mathfrak{K} = 70.5 \times 10^2$ the slope

$$s_0 = (200/70.5)^2 \times 10^{-4} = 8.05^0\!/_{00}$$

Exercise:

Determine the slope required to make the canal carry Q = 500 cu. ft. per second and 250 cu. ft. per second, respectively, at the depths of $y_0 = 5.4$ ft. and 2.7 ft.

Question 4. *a.* Determine the depth, required to convey 300 cu. ft. per second at a slope of $10^0\!/_{00}$. Use G.K. coefficients.

The required conveyance is: $\mathfrak{K} = \dfrac{Q}{\sqrt{s_0}} = \dfrac{300}{\sqrt{10}} 10^2 = 95.1 \times 10^2$. From the curve, Fig. 15, we find the corresponding depth $y_0 = 4.7$ ft.

b. Assuming, in accordance with Question 2, that a discharge $Q = 183$ cu. ft. per second is being conveyed by a canal with

$y_0 = 3.7$ ft. and $s_0 = 9\,^0\!/_{000}$; determine the depth required to maintain the same discharge with $s_0 = 5\,^0\!/_{000}$. Use Bazin coefficients.

The required conveyance is $\mathfrak{K} = Q/\sqrt{s_0} = 183/\sqrt{5} \cdot 10^{-2} = 81.9 \times 10^2$. The corresponding depth from curve, Fig. 15, is $y_0 = 4.3$ ft.

Exercise:

Determine the depth, required to convey 450 cu. ft. per second under $s_0 = 12\,^0\!/_{000}$, and respectively $4\,^0\!/_{000}$.

General Exercise:

It is suggested that the student select a certain canal cross-section which he shall subsequently use for different exercises. At this point the student should compute and trace the con-

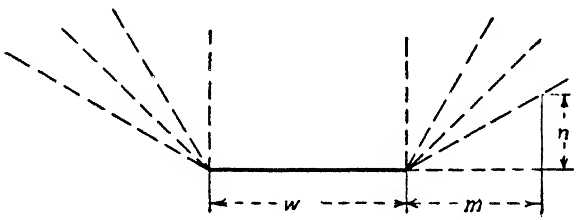

FIG. 16.

veyance curve for the canal as selected and practice in solving problems, similar to those covered by the above examples. In selecting the cross-section, take a trapezoidal canal with a bottom width w (Fig. 16) between 3 and 25 ft. and side slopes m/n between 0.5/1 to 2.5/1. In addition to the trapezoidal canal, work on a rectangular cross-section of the same bottom width w. In this case, assume the canal concrete lined.

8. Resistance Losses.—Referring the movement to a horizontal datum line 0–0 (Fig. 17), with the X-axis parallel to the line of the bottom and pointing in the direction of the flow, the energy heads (energy per unit of weight of liquid) in sections 1 and 2 are, respectively,

$$e_1 = h_1 + \frac{v_1^2}{2g} \text{ and } e_2 = h_2 + \frac{v_2^2}{2g} \qquad (5)$$

The loss of energy head over the stretch dx is equal to the work spent per unit of weight of liquid on hydraulic resistances over the same stretch. Designating the energy head lost by e_r, we obtain:

$$e_1 - e_2 = -de = de_r$$

or

$$-de/dx = de_r/dx \qquad (6)$$

In the uniform movement, as the velocities remain without change, we have

$$de = e_2 - e_1 = h_2 - h_1 = -dh = -s_0 dx$$

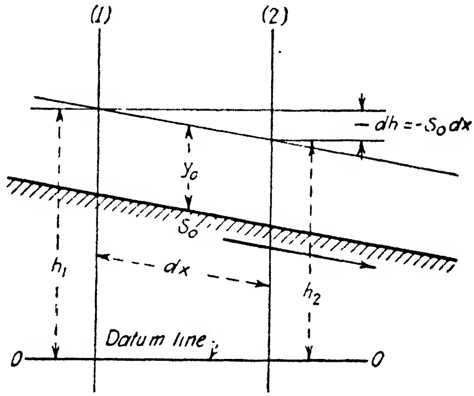

FIG. 17—Uniform flow referred to a horizontal datum line.

which means, that the work of gravity is wholly spent on overcoming resistances.

Substituting (Eq. [4]) $s_0 = Q^2/\mathfrak{K}^2$, we obtain instead of Eq. (6)

$$\left.\begin{aligned} de/dx &= -s_0 = -Q^2/\mathfrak{K}^2 \\ de_r/dx &= Q^2/\mathfrak{K}^2 \end{aligned}\right\} \qquad (7)$$

The quotient $Q^2/\mathfrak{K}^2$ measures thus the rate of energy lost in hydraulic resistances. In the foot, pound, second system, $Q^2/\mathfrak{K}^2$ indicates *the work in foot pounds spent on hydraulic resistances by every pound of liquid in its movement over a stretch* 1 *ft. long.*

If a volume V of liquid of specific gravity Δ moves over a stretch x, the total work spent in overcoming resistances over such stretch will be

$$W_r = \Delta \cdot V \cdot Q^2/\mathfrak{K}^2 \cdot x \tag{8}$$

The *power* N (work per unit of time) spent in the flow of a discharge Q over the stretch x will be:

$$N = \Delta \cdot Q \cdot \frac{Q^2}{\mathfrak{K}^2} \cdot x = \Delta \cdot \frac{Q^3}{\mathfrak{K}^2} \cdot x \text{ ft.} \cdot \text{lb. per second} \tag{9}$$

or in differential form

$$dN/dx = \Delta \cdot Q^3/\mathfrak{K}^2 \tag{10}$$

which is the rate of power lost on resistances by the discharge Q per unit of length. For water, with $\Delta = 62.4$ lb. per cubic foot and expressing N in horsepower, we have

$$dN/dx = 62.4/550 \cdot Q^3/\mathfrak{K}^2 = 0.1135\ Q^3/\mathfrak{K}^2 \text{ hp.} \tag{10a}$$

or, in view of Eq. (4),

$$\frac{dN}{dx} = \frac{Q}{8.81} \cdot \frac{Q^2}{\mathfrak{K}^2} = \frac{Q}{8.81} \cdot s_0 \text{ hp.} \tag{10b}$$

Example 2

With reference to the canal (Fig. 14) and the curves (Fig. 15).

Question 1. What is the power, spent in overcoming resistances per mile length, with 120 cu. ft. per second, flowing at the depth of $y_0 = 3$ ft., and $y_0 = 5$ ft. and $y_0 = 4$ ft. respectively? Use, as an average, G.K. coefficients for $s_0 = 10‰$.

For $y_0 = 4$ ft., the value of $\mathfrak{K} = 70.6 \times 10^2$.

Hence the power lost

$$N \text{ hp.} = 0.1135 \frac{120^3}{70.6^2 \times 10^4} \cdot 5{,}280 = 20.6 \text{ hp.}$$

For the other given depths:

$$y_0 = 3 \text{ ft.};\ \mathfrak{K} = 40.7 \times 10^2;\ N = 20.6\ (70.6/40.7)^2 = 62 \text{ hp.}$$
$$y_0 = 5 \text{ ft.};\ \mathfrak{K} = 109.1 \times 10^2;\ N = 20.6\ (70.6/109.1)^2 = 8.7 \text{ hp.}$$

Question 2. Figure the energy spent in resistances per 1-ft. length of canal per 24 hr., in case 200 cu. ft. per second are flowing at $y_0 = 5$ ft. Use G.K. coefficients for $s_0 = 10‰$. With $\mathfrak{K} = 109.1 \times 10^2$, we have, in accordance with Eq. (8),

$$W_r = 62.4 \times 200 \times 3{,}600 \times 24 \times (200/109.1)^2 1/10^4 \times 1 = 36.1 \times 10^4 \text{ lb.} \cdot \text{ft.}$$

Question 3. Using Bazin coefficients, determine the rate of loss of energy per pound of water over a foot length with 100 cu. ft. per second flowing under each of the following depths: $y_0 = 1$ ft.; $y_0 = 2$ ft.; $y_0 = 3$ ft.

The respective conveyances are:

$$6.23 \times 10^2;\ 20.19 \times 10^2, \text{ and } 41.65 \times 10^2.$$

The rate of loss of energy is determined by Eq. (7):

$$y_0 = 1;\ de_r/dx = Q^2/\mathfrak{K}^2 = 100^2/(6.23)^2 \times 10^4 = 0.02560$$
$$y_0 = 2;\ de_r/dx = Q^2/\mathfrak{K}^2 = 100^2/(20.19)^2 \times 10^4 = 0.00246$$
$$y_0 = 3;\ de_r/dx = Q^2/\mathfrak{K}^2 = 100^2/(41.65)^2 \times 10^4 = 0.00057$$

9. The Normal Depth.—In treating problems of varied flow, uniform movement will be frequently used as *one of reference*. Assume a discharge Q flowing in a canal of given dimensions with a bottom slope s_0. The different possible surface curves in Fig. 18, each determined by the respective depths y_1 and y_2 in section 1 and section 2, illustrate the innumerable possible ways in which the discharge Q may flow between the above sections.

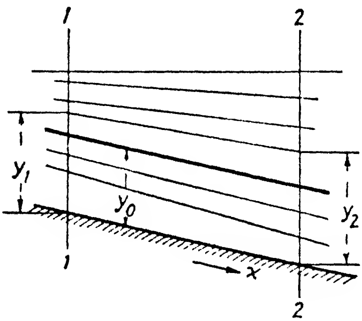

FIG. 18.

Among these movements the one indicated by a heavy line, parallel to the bottom, represents *uniform* movement. The characteristics of such movement in contradistinction to all other possible forms of flow are

$$y = \text{const.},\ dy/dx = 0 \qquad (11)$$

The depth of uniform movement constitutes a *parameter*, fully determined once the discharge Q and the features of the canal are given. We shall call the depth of uniform movement *the normal depth*, and shall designate it by y_0. The index *o* will be used, in general, to designate elements pertaining to uniform flow. So we shall have a_0, R_0, C_0, and finally $\mathfrak{K}_0 = a_0C_0\sqrt{R_0}$, which are respectively the cross-sectional area, the hydraulic radius, the friction factor, and, finally, the conveyance at the normal depth y_0. For a given canal and a given discharge Q, the normal depth is determined by the procedure indicated in Example 1, Question 4. In fact, dividing the discharge by the square root of the bottom slope, one finds the conveyance, corresponding to the normal depth

$$\mathfrak{K}_0 = Q/\sqrt{s_0} \tag{12}$$

after which the corresponding y_0 is taken from the respective $\mathfrak{K}$ curve.

The Normal Discharge Curve $Q_0 = f(y_0)$.—In handling problems with varying discharges in a given canal, it will be found handy at times to use a chart, giving the relation between Q_0 and y_0. This is accomplished by tracing what we shall call *the normal discharge curve* $Q_0 = f(y_0)$.

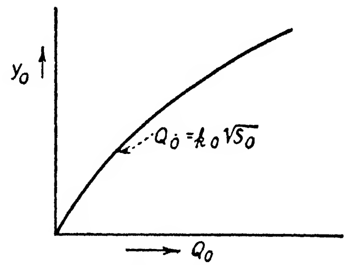

FIG. 19.—The *normal discharge* curve.

This curve, Fig. 19, represents for each stage y the value of the discharge in uniform movement corresponding to the respective depth $y_0 = y$. As for each depth y_0 the discharge is $Q_0 = \mathfrak{K}_0\sqrt{s_0}$, the normal discharge curve is the conveyance curve $\mathfrak{K} = aC\sqrt{R}$ multiplied by $\sqrt{s_0}$.

Exercise:

1. Draw the $Q_0 = f(y_0)$ curve for canal (Fig. 14).
   a. For $s_0 = 4^0/_{00}$ with Bazin coefficients.
   b. For $s_0 = 10^0/_{00}$ with G.K. coefficients.
2. Draw a normal discharge curve for the canal cross-section selected for general exercise at the end of Example 1; in doing so, select a slope at your discretion between $s_0 = 1^0/_{00}$ and $s_0 = 15^0/_{00}$.

CHAPTER III

EQUATION OF VARIED FLOW

10. Geometric Relations between Surface and Bottom Slopes.—In varied flow, the surface line is not parallel to the bottom. The relation between the surface slope $s = \sin \alpha$ and the bottom slope $s_0 = \sin \alpha_0$ follows from Fig. 20:

$$s = \sin \alpha = \frac{bd}{ba} = \frac{cd - cb}{ba} = \sin \alpha_0 - \frac{dy}{dx}$$

Hence

$$s = s_0 - \frac{dy}{dx} \tag{13}$$

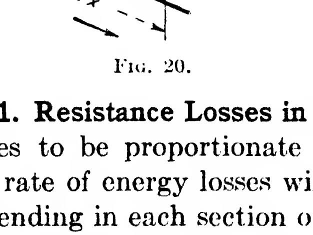

FIG. 20.

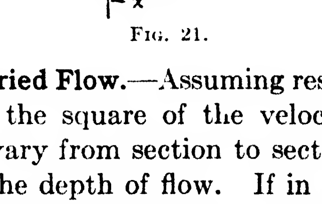

FIG. 21.

11. Resistance Losses in Varied Flow.—Assuming resistances to be proportionate to the square of the velocity, the rate of energy losses will vary from section to section, depending in each section on the depth of flow. If in any section (Fig. 21) the stage of flow y' is greater than the normal depth y_0, the velocity v will be less than v_0 and the loss of energy will be proportionately reduced. On the contrary, with $y'' < y_0$, the velocity of flow will be greater than that in uniform movement, and the loss of energy will exceed that of uniform flow.

The rate of loss in varied flow at a certain stage y may be compared with the losses which would take place in uniform movement, provided the same discharge were flowing at the same depth and, therefore, at the same average velocity. Evidently, one should expect that the losses in varied flow would be somewhat different from the losses in uniform movement. The difference would probably be caused, in the first place, by a somewhat different distribution of the velocities over the cross-section as compared with the case of uniform flow. Then, and this is probably the most important factor, there would be the general effect which divergence or convergence of flow exercises on the degree of turbulence in the flowing liquid.* So far there is little, if any, reliable experimental material available on this subject. Neither do we know anything about the comparative value of the friction factor, as caused by the roughness of the canal walls in ordinary *tranquil* and in so-called *rapid* movement (Art. 24).† On the other hand in most practical cases, the change of depth takes place rather gradually; so the picture of movement at a certain depth cannot be very different from that which would be taking place under similar conditions in uniform flow. Therefore, the basic assumption is made that the rate of resistance losses in varied flow in a certain section characterized by the depth y (Fig. 22) is identical with the losses which would take place were the same discharge to flow with the same depth $y = y_0$ in uniform movement. The rate of energy losses in uniform movement being $Q^2/\mathfrak{K}^2$ (y_0 line in Fig. 22) an identical expression is used to qualify

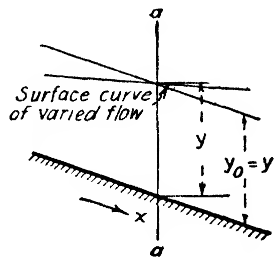

FIG. 22.—Resistance losses in varied flow at the depth y, assumed to be equal to the losses in uniform movement at the same depth $y_0 = y$.

* REYNOLDS, O., *Phil. Trans. Roy. Soc.*, 1883.

† See, in part, Darcy Bazin, "Recherches hydrauliques." An attempt to approach the subject theoretically was further made by Boussinesq, "Théorie des eaux courantes," 1877, and other works.

losses in varied flow. Accordingly, the energy losses per pound of liquid over the stretch dx will figure as

$$de_r = \frac{Q^2}{\mathfrak{K}^2}\,dx;\ \frac{de_r}{dx} = \frac{Q^2}{\mathfrak{K}^2} \qquad (14)$$

The rate de_r/dx, depending on $\mathfrak{K}^2 = f(y)$, will be itself a function of the depth. With reference to $s_0 = Q^2/\mathfrak{K}_0^2$, which measures the rate of loss of energy in uniform flow, the rate of loss in varied flow will be $Q^2/\mathfrak{K}^2 > s_0$ or $Q^2/\mathfrak{K}^2 < s_0$, depending on whether $y < y_0$ or $y > y_0$.

✓12. Equation of Varied Flow.—The differential equation of varied flow follows from the energy Eqs. (5) and (6). Applying the latter to the two sections 1 and 2 (Fig. 23) at a small distance dx, we have

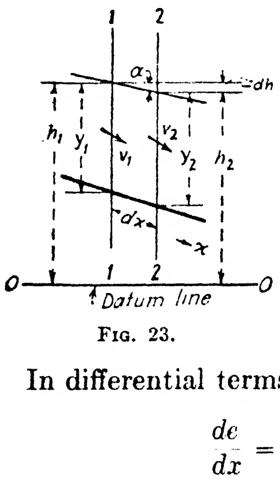

Fig. 23.

$$-de = e_1 - e_2 = \left(h_1 + \frac{v_1^2}{2g}\right) - \left(h_2 + \frac{v_2^2}{2g}\right) = de_r$$

In differential terms the equation reads

$$\frac{de}{dx} = \frac{dh}{dx} + \frac{d}{dx}\frac{v^2}{2g} = -\frac{de_r}{dx} \qquad (15)$$

From Fig. 23, we have $-dh/dx = \sin\alpha = s$. On the other hand (Eq. [14]).

$$de_r/dx = Q^2/\mathfrak{K}^2 = v^2/C^2R \qquad (16)$$

Substituting into Eq. (15), we obtain

$$s = \frac{Q^2}{\mathfrak{K}^2} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) = \frac{v^2}{c^2R} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) \qquad (17)$$

which is the classical form in which the varied flow equation is usually given in textbooks on hydraulics.

13. Limitations of the Applicability of the Equation of Varied Flow.—It is most important to make clear the specific conditions under which Eq. (17) is applicable. In Eq. (5) the expression of energy head $e = h + \frac{v^2}{2g}$ was applied to the flow as a whole; meaning that $h + \frac{v^2}{2g}$ was assumed to represent the energy which is contained, on the average, in every pound of liquid flowing through the section.

Such will be the case, if and when the potential energy head for each and every point of the cross-section will be identical. Now, the potential energy head at a certain point *a* (Fig. 24) is measured (in the so-called Bernoulli

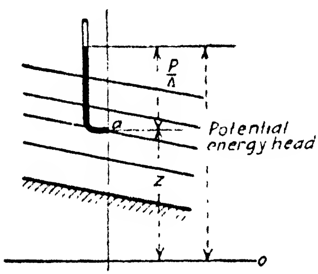

FIG. 24.—Case of nonhydrostatic distribution of pressure.

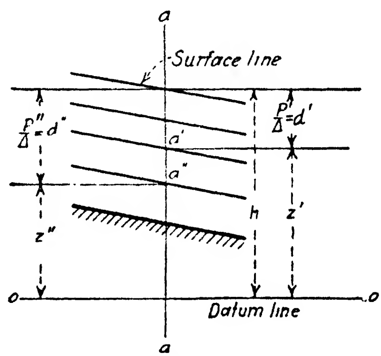

FIG. 25.—Case of hydrostatic distribution of pressure in a moving liquid.

equation) by the height $z + \frac{p}{\Delta}$; where z is the elevation of the point over the datum line, while p/Δ is the piezometric head, meaning the height of the liquid column equivalent to the pressure p at the respective point.

If the flow were to take place in such a manner that the pressure p in any point of a certain cross-section (a' or a'' in Fig. 25) were to be equal to the hydrostatic pressure, corresponding to the depth d of such point below the free surface, then, as follows from Fig. 25, the sum $z + \frac{p}{\Delta}$ would be the same for all points of the cross-section and always be equal to $z + \frac{p}{\Delta} = z + d = h$. In this case the condition above stated would prevail, and the energy head would be expressed by Eq. (5). The condition that in a moving liquid the pressure in each point of a cross-section is to be equal to the pressure corresponding to the depth of the respective point below the surface, is equivalent to saying that the distribution of pressure over a section in a moving liquid is affected solely by gravity, and that the pressures are distributed in the same way and manner as if the liquid were at rest; in other words that the distribution of pressure over a cross-section of moving liquid follows the hydrostatic law.

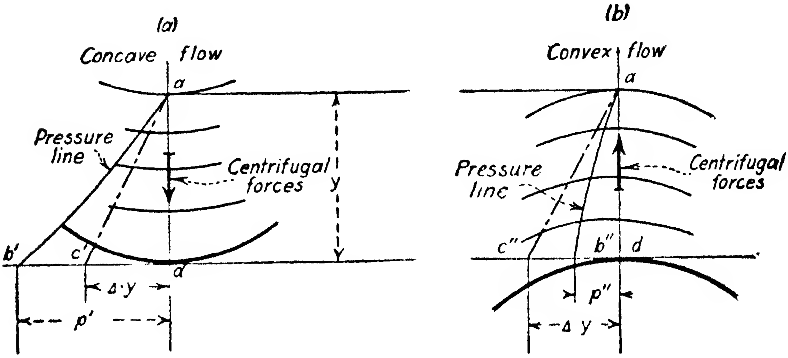

FIG. 26.—Effect of curvature on the distribution of pressure in a moving liquid

Elementary hydromechanics teaches that the distribution of pressure in a cross-section of moving liquid will obey the hydrostatic law and will be affected solely by gravity, when, and only when, flow takes place in such a manner that the fluid filaments have *no acceleration components in the plane of the cross-section.* Movement of such kind, *i.e.*, flow, where there are no accelerations to disturb the distribution of pressures in the plane perpendicular to the axis of the flow, is known as *parallel movement.* The specific requirements of parallel movement were clearly defined by Bélanger in 1828 in his celebrated paper which is justly considered as the foundation of the theory of varied flow.*

*See bibliographical notice Appendix I.

These conditions are:

1. That the stream lines have no substantial curvature.
2. That the stream lines be not substantially divergent.

In curvilinear movement (Fig. 26), depending on whether the stream lines are concave or convex, the centrifugal forces will act either in the direction of gravity or opposite to it. As a result, instead of the hydrostatic pressure triangle *acd*, the pressure will be represented by a curve *ab*.

In the case of divergent flow (Fig. 27), when the stream lines are substantially inclined towards the cross-sectional plane, the acceleration *oa* may have a noticeable component *oa′* in the cross-sectional plane, the effect of which again will be to modify the distribution of pressures as caused by gravity alone.

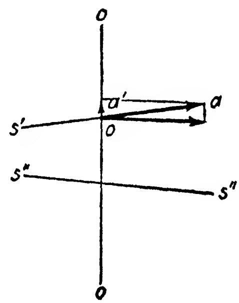

Fig. 27.

It may be said that the effect of divergence is usually comparatively negligible. On the other hand, the deviations from hydrostatic distribution caused by curvature are usually quite substantial, so that whenever curvilinear flow is on hand, Eq. (5), and thus Eq. (17), cannot be strictly applied.

In Art. 6, the distinction is made between *gradually varied flow* and *local phenomena*. We are in a position now to specify the mechanical qualifications underlying such distinction.

Gradually varied flow is a term, introduced by Boussinesq, to describe more appropriately movements which conform with the Bélanger qualifications of parallel flow. While, strictly speaking, Bélanger's conditions are met only by rectilinear uniform movement, in practice, nevertheless, as stated before, the change of circumstances of flow may take place so gradually and so slowly that the stream lines may be said to possess no *appreciable* curvature or divergence. In other words, the curvature and divergencies are small enough to make the effect of the acceleration components in the cross-sectional plane negligible. In gradual varied flow of such kind the distribution of pressures may, therefore, be assumed to take place as if caused by gravity alone. The energy equation in the form of Eq. (5) will prevail and as a result the varied flow equation (Eq. [17]) may be applied.

In contradistinction, in most local phenomena, one meets with substantial curvature or divergence of the stream lines. The hydrostatic pressure law does not hold and Eq. (17) cannot be applied.

It is most important to keep this distinction, together with the underlying premises, clearly in mind.

✓14. Prismatic Channels.—Outside the limitations imposed in the preceding paragraph, Eq. (17)

$$s = \frac{Q^2}{\mathfrak{K}^2} + \frac{d}{dx}\left(\frac{v^2}{2g}\right)$$

is quite general and, therefore, can be applied to describe varied flow in the broadest sense, including the case where the form of the canal itself would gradually change from section to section (see Art. 2). However, analytical treatment of these broader cases scarcely leads to generalized solutions of practical character; so it will not be further pursued.*

In the case of a prismatic channel (see Art. 2) with the discharge Q given, the velocity and all the other elements of flow in a certain cross-section depend on the stage of flow y. The problem, therefore, becomes one of two variables: the depth y and the axial distance x. Thus, when the equation of the surface line $y = f(x)$ is established, a complete description of the movement is available, as the depth y determines the value of a, which in turn defines $v = Q/a$ and thus the other elements of movement.

* For an approach to the treatment of the more general case, see Casler. *Trans., A. S. C. E.*, 1930.

For prismatic canals the member $\frac{d}{dx}\left(\frac{v^2}{2g}\right)$, featuring the variation of kinetic energy in Eq. (17), can be expressed as:

$$\frac{d}{dx}\left(\frac{v^2}{2g}\right) = \frac{d}{dx}\left(\frac{Q^2}{2ga^2}\right) = -\frac{Q^2}{ga^3}\frac{da}{dy}\frac{dy}{dx} \tag{18}$$

In da/dy, the numerator da measures the increase of the cross-sectional area due to the increase of the depth by dy.

Neglecting members of higher degrees of smallness, this increase of area (Fig. 28) is $da = bdy$, where b is the *width of the free surface* of the liquid in the profile. We have, therefore,

$$da/dy = b \tag{19}$$

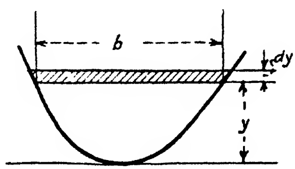

FIG. 28.

and thus the rate of change of kinetic energy

$$\frac{d}{dx}\left(\frac{v^2}{2g}\right) = -\frac{Q^2}{g}\frac{b}{a^3}\frac{dy}{dx} \tag{20}$$

In Eq. (17) we substitute $\frac{d}{dx}\left(\frac{v^2}{2g}\right)$ by Eq. (20); we express s by Eq. (13). Finally in view of Eq. (12) we make $Q^2/\mathfrak{K}^2 = s_0\mathfrak{K}_0^2/\mathfrak{K}^2$, where $\mathfrak{K}_0$ is the conveyance corresponding to the normal depth. We have now

$$s = s_0 - \frac{dy}{dx} = \frac{Q^2}{\mathfrak{K}^2} + \frac{d}{dx}\frac{v^2}{2g} = s_0\frac{\mathfrak{K}_0^2}{\mathfrak{K}^2} - \frac{Q^2}{g}\frac{b}{a^3}\frac{dy}{dx}$$

Wherefrom

$$\frac{dy}{dx} = s_0\frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - \frac{Q^2}{g}\frac{b}{a^3}} \tag{21}$$

which is the differential equation of gradually varied flow in *prismatic channels.*

CHAPTER IV

GENERAL FEATURES OF FLOW

15. The Specific Energy of Flow.—When a discharge Q is flowing in uniform movement under the normal depth y_0, the work of gravity s_0dx is wholly spent on hydraulic resistances $\frac{Q_0^2}{\mathfrak{K}_0^2}dx$. Therefore, the circumstances of flow (Fig. 29) in section 2 remain the same as they were in section 1.

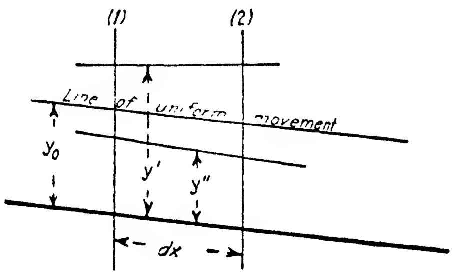

FIG. 29.

If, on the other hand, liquid were to flow with a depth $y' > y_0$, the energy loss $\frac{Q^2}{\mathfrak{K}^2(y')}dx$ spent on resistances over the stretch dx would be less than the work of gravity over the same stretch $s_0dx = \frac{Q^2}{\mathfrak{K}_0^2}dx$. Therefore, in its movement over the stretch from section 1 to section 2, the liquid would gain a certain increment of energy, which per unit weight would be

$$s_0dx - \frac{Q^2}{\mathfrak{K}^2(y')}dx = \left(\frac{Q^2}{\mathfrak{K}_0^2} - \frac{Q^2}{\mathfrak{K}^2(y')}\right)dx =$$

$$s_0\left(1 - \left[\frac{\mathfrak{K}_0}{\mathfrak{K}(y')}\right]^2\right)dx \quad (22)$$

In the event of $y'' < y_0$, the energy loss $\frac{Q^2}{\mathfrak{K}^2(y'')}dx$ would exceed the work of gravity. Therefore, in the movement of liquid over the stretch dx, a certain amount of energy would have to be withdrawn from the stock of energy, contained in the moving liquid and available in section 1. The decrease in the content of energy carried by the flow would be measured by the same expression (Eq. [22]) only that with $\mathfrak{K}(y'') < \mathfrak{K}_0$ the sign of $1 - (\mathfrak{K}_0/\mathfrak{K}(y''))^2$ would be negative. We see now that depending on whether

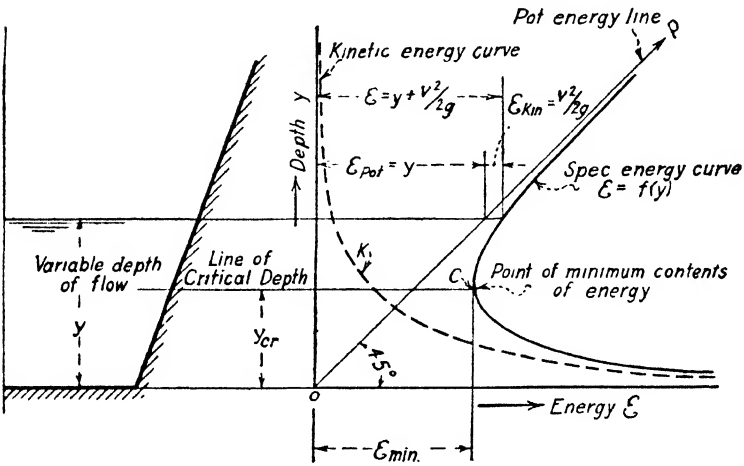

FIG. 30.—The specific energy diagram $\epsilon = y + \frac{Q^2}{2ga^2} = f(y)$.

$y > y_0$ or $y < y_0$ energy is added to or withdrawn from the flowing liquid, and thus the energy content from section to section changes.

A very clear insight into the whole mechanism of the phenomenon is gained by means of the notion of the *specific energy of flow.*

Assume a discharge Q flowing in the canal (Fig. 30) under varying stages y. Referring the flow to a datum line passing through the bottom of the section, the average energy head of the flowing liquid is:

$$\epsilon = y + \frac{v^2}{2g} = y + \frac{Q^2}{2ga^2} \tag{23}$$

Obviously ϵ is a function of the depth of flow and can be drawn in the form of a curve as $\epsilon = f(y)$. We shall call ϵ the *specific energy of flow* and the curve $\epsilon = f(y)$ the *specific energy diagram.* It is most important to gain a clear understanding of the situation, and to bear in mind the distinction of the specific energy $\epsilon = y + \frac{v^2}{2g}$, as defined in Eq. (23) from the energy $e = h + \frac{v^2}{2g}$ as defined in Eq. (5). The energy in Eq. (5) is energy referred to a constant datum line. It pictures the changes in energy over a certain stretch in the flow as a whole. The specific energy (Eq. [23]) is referred to a bottom line which changes from section to section. The change of ϵ pictures the variation of energy in a cross-section as such depending on the depth of flow. Uniform flow is characterized by ϵ constant. In other words in uniform flow $\delta\epsilon/\delta x = 0$. In varied flow the excess of the work of gravity as against the resistance losses over the stretch dx adds to, or subtracts from the specific energy. Accordingly, in view of Eq. (22), we have

$$\delta\epsilon/\delta x = s_0(1 - [\mathfrak{K}_0/\mathfrak{K}]^2) \tag{24}$$

Obviously, when

$$\left.\begin{array}{l} y > y_0;\ \mathfrak{K} > \mathfrak{K}_0;\ \delta\epsilon/\delta x > 0;\ \epsilon \text{ grows in the direction of the flow} \\ \text{while when} \\ y < y_0;\ \mathfrak{K} < \mathfrak{K}_0;\ \delta\epsilon/\delta x < 0;\ \epsilon \text{ diminishes in the direction of the flow} \end{array}\right\} \tag{25}$$

For a given discharge and a given canal, the specific energy curve $\epsilon = y + \frac{Q^2}{2ga^2}$ can be traced as $\epsilon = f(y)$. The first member, the potential energy is represented by a straight line *op* (Fig. 30) at 45° to the x-axis. The second member $v^2/2g$, the kinetic energy head, is a curve K, asymptotically tangent to the y- and the x-axis. The $\epsilon = f(y)$ curve is the sum of *op* and of the K curve, being assymptotic to *op* and to *ox*. The curve has a minimum point *c*, corresponding to a certain depth to be designated as y_{cr}.

16. Critical Depth.—The particular depth which makes the specific energy a minimum; in other words, the depth under which a certain discharge Q flows in a given canal with a minimum content of specific energy is called *the critical depth.* It is designated as y_{cr}.

It is most important to gain a clear conception of the whole matter. A certain discharge Q may flow in a given canal in an innumerable number of ways, each characterized by a certain depth y. To each depth there corresponds a definite value of the specific energy ϵ. In general, ϵ varies with the depth; but, under no circumstances, can the content of energy per unit weight of liquid fall below a certain value ϵ_{min}, which minimum is attained at the critical depth. In other words, ϵ_{min} is the least possible content of specific energy with which the discharge Q is able to flow in the canal as given. Obviously, the critical depth y_{cr} and the minimum possible energy ϵ_{min} constitute definite parameters inherent in the flow.

To determine the value of y_{cr}, we make

$$\frac{\delta \epsilon}{\delta y} = 1 - \frac{Q^2}{ga^3} \cdot \frac{\delta a}{\delta y} = 0$$

With

$$\delta a / \delta y = b \text{ (see Eq. [19])}$$

$$\frac{\delta \epsilon}{\delta y} = 1 - \frac{Q^2}{g} \cdot \frac{b}{a^3} = 0 \tag{26}$$

which corresponds to a depth determined by the equation

$$Q^2/g \cdot b/a^3 = 1 \tag{27}$$

In other words, the critical depth for a given discharge Q is the depth y_{cr}, for which the value of a^3/b is equal to

$$\left.\begin{aligned} (a^3/b)_{cr} &= Q^2/g \\ (a\sqrt{a/b})_{cr} &= Q/\sqrt{g} \end{aligned}\right\} \tag{28}$$

The $\mathfrak{M}$ Function.—For a given profile the value of a^3/b is a function of the depth only. We shall designate

$$a\sqrt{a/b} = \mathfrak{M};\ a^3/b = \mathfrak{M}^2 \tag{29}$$

and call $\mathfrak{M}(y) = a\sqrt{a/b}$ the $\mathfrak{M}$ function. Traced as a curve for a given cross-sectional form (Fig. 31), the $\mathfrak{M}$ function permits determination of critical depth for any

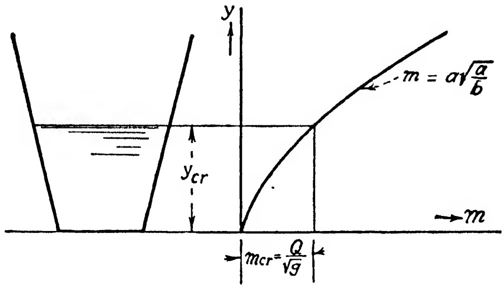

FIG. 31.—Determining the critical depth by means of the $\mathfrak{M} = a\sqrt{\frac{a}{b}}$ curve.

discharge flowing in the canal. In fact, for a given discharge Q, we determine (Eq. [28]) the critical value of

$$\mathfrak{M}_{cr} = Q/\sqrt{g} \tag{30}$$

and, then, from the $\mathfrak{M}$ curve the corresponding value of the critical depth y_{cr}.

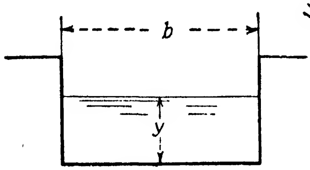

FIG. 32.

Rectangular Profile.—In the case of a rectangular profile (Fig. 32), the discharge per unit width of the canal being

$$q = Q/b \tag{31}$$

the $\mathfrak{M}(y)$ function is

$$\mathfrak{M} = \sqrt{a^3/b} = \sqrt{b^3y^3/b} = b\sqrt{y^3} \tag{32}$$

while the critical depth follows from the relation

$$Q/\sqrt{g} = bq/\sqrt{g} = \mathfrak{M}_{cr} = b\sqrt{y^3_{cr}}$$

Hence

$$\left.\begin{aligned} y_{cr} &= \sqrt[3]{q^2/g} \\ q^2 &= gy^3_{cr} \end{aligned}\right\} \tag{33}$$

Example 3

Question 1. Draw the $\mathfrak{M} = a\sqrt{a/b}$ curve for the canal cross-section (Fig. 14).

The elements of the figuring, partly taken from Table I, are assembled in Table III.

TABLE III

y	a	b	a/b	$\mathfrak{M} = a\sqrt{a/b}$
0.5	2.75	6	0 458	1.86
1.0	6.00	7	0.858	5.56
1.5	9.75	8	1.220	10.76
2.0	14.00	9	1.560	17.45
2.5	18.75	10	1 875	25 65
3.0	24.00	11	2.180	35.50
3.5	29.75	12	2.480	46.85
4.0	36.00	13	2.770	59.90
4.5	42.75	14	3.050	74.60
5.0	50.00	15	3.330	91.40
6.0	66.00	17	3.880	130.00

In Fig. 15 the curve is traced in the direction opposite to $\mathfrak{K}$.

Question 2. Determine the critical depth for $Q = 100$ cu. ft. per second and, respectively, for $Q = 200$ and $Q = 300$ cu. ft. per second.

In accordance with Eq. (30), taking $\sqrt{g} = 5.67$, we have for:

$Q = 100$ cu. ft. per second; $\mathfrak{M}_{cr} = 100/5.67 = 17.6$;
hence from curve $y_{cr} = 2.01$ ft.

$Q = 200$ cu. ft. per second; $\mathfrak{M}_{cr} = 200/5.67 = 35.2$;
hence from curve $y_{cr} = 2.99$ ft.

$Q = 300$ cu. ft. per second; $\mathfrak{M}_{cr} = 300/5.67 = 52.8$;
hence from curve $y_{cr} = 3.74$ ft.

Question 3. In a rectangular canal, 10 ft. wide, determine the critical depth corresponding to a discharge of 120, 180, and 300 cu. ft. per second.

The respective discharges per unit of width are $q = 12$, 18, and 30 cu. ft. per second. From Eq. (33), we have for:

$$q = 12 \text{ cu. ft. per second; } y_{cr} = \sqrt[3]{12^2/32.2} = 1.65 \text{ ft.}$$
$$q = 18 \text{ cu. ft. per second; } y_{cr} = \sqrt[3]{18^2/32.2} = 2.16 \text{ ft.}$$
$$q = 30 \text{ cu. ft. per second; } y_{cr} = \sqrt[3]{30^2/32.2} = 3.04 \text{ ft.}$$

Exercises:

1. Draw the $\mathfrak{M} = a\sqrt{a/b}$ curve for the canal profile, selected for the purpose of general exercise in Art. 7.

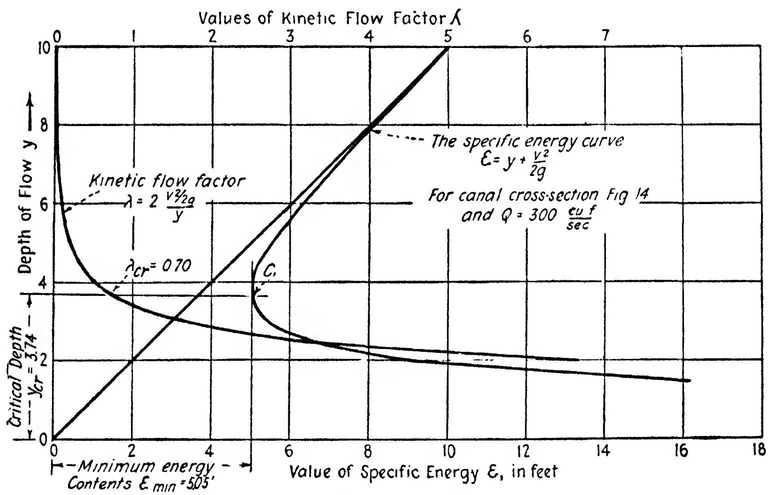

FIG. 33.—The specific energy diagram for canal type B with $Q = 300$ cu. ft. per second.

2. Determine the critical depth for a series of discharges, and trace the curve of critical depths $y_{cr} = f(Q)$.
3. Trace the $y_{cr} = f(q)$ curve for a rectangular canal.

Example 4

Draw the specific energy curve for the canal (Fig. 14) for the discharge of $Q = 300$ cu. ft. per second.

The figuring of $\epsilon = y + \frac{v^2}{2g}$ is given in Table IV; the energy curve is traced in Fig. 33.

TABLE IV

y	a	$v = \frac{300}{a}$	$\frac{v^2}{2g}$	$\epsilon = y + \frac{v^2}{2g}$	λ
0.50	2.75	109.00	185.000	185.50	740.000
0.75	4.31	69.60	75.200	75.95	200.000
1.00	6.00	50.00	38.900	39.90	77.800
1.25	7.81	38.40	22.900	24.15	36.600
1.50	9.75	30 80	14.700	16.20	19.600
2.00	14.00	21.40	7.140	9.14	7.140
2.50	18.75	16.00	3.980	6.43	3.090
3.00	24.00	12 50	2.430	5.43	1.620
3.25	26.81	11.12	1.940	5.19	1.195
3.50	29.75	10.11	1.580	5.08	0.903
3.60	30.96	9.69	1.460	5.06	0.810
3.70	32.19	9.32	1.350	5.05	0.730
3.74	32.69	9.08	1.310	5.05	0.700
3.80	33.44	8.96	1.250	5.05	0.658
3.90	34.70	8.64	1.160	5.06	0.594
4.00	36.00	8.33	1.080	5.08	0.540
4.25	39.31	7.63	0.905	5.15	0.425
4.50	42.75	7.02	0.765	5.26	0.340
5.00	50.00	6.00	0.560	5.56	0.224
5.50	57.75	5.20	0.420	5.92	0.153
6.00	66.00	4.55	0.320	6.32	0.107
6.50	74.75	4.01	0.250	6.75	0.077
7.00	84.00	3.57	0.198	7.20	0.056
8.00	104.00	2.88	0.129	8.13	0.013
9.00	126.00	2.38	0.086	9.09	0.019
10.00	150.00	2.00	0.062	10.06	0.012

17. Physical Interpretation of Phenomena.—The notion of specific energy gives a lucid and simple explanation of many phenomena of varied flow.

The Hydraulic Jump.—As illustrated by Fig. 34, the jump is an abrupt transition of flow from the lower to the upper branch of the energy curve. ϵ_1 and ϵ_2 are the specific energies of flow, corresponding to the depths d_1 and d_2 before and after the jump. $\epsilon_j = \epsilon_1 - \epsilon_2$ is the loss of energy inherent to the jump, a loss which under certain circumstances may be quite large.

Flow over a Fall. The Hydraulic Drop.—Figure 35 illustrates flow over a fall. To make the case particularly clear we shall assume the bottom of the canal horizontal, so that the effect of the bottom slope on accelerating the flow and on overcoming resistances is nil. The movement

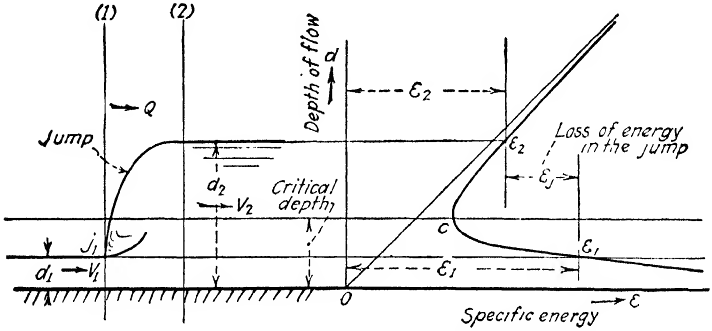

FIG. 34.—Physical interpretation of the hydraulic jump.

under such conditions must take place entirely at the expense of the specific energy stored in the liquid. Accordingly, the passage from section 1 to section 2 on the falling

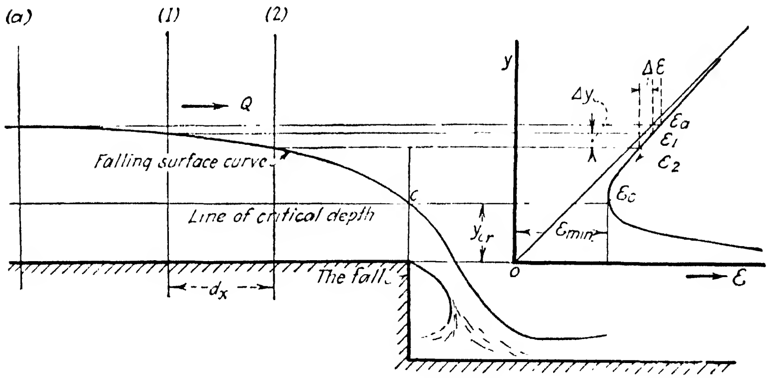

FIG. 35.—Flow over a *fall*.

curve corresponds to a shift down the upper branch of the energy curve in which the loss in specific energy $-\Delta\epsilon$ is accompanied by an appropriate lowering of the depth by $-\Delta y$. As a matter of course, the surface of the moving liquid in its natural tendency to lower cannot drop below the critical depth, which depth corresponds to the least possible content of energy in the falling liquid. Any further lowering of the surface beyond y_{cr} would mean the passing of the movement into the lower branch of the energy curve, which could be possible only if energy were added from the outside. Thus, the critical depth is the lowest limit to which the surface may sink in the natural process of dissipating energy. Therefore the critical depth is the lowest depth which by reason of natural circumstances establishes itself at the end of the canal over a fall.

In Fig. 9, illustrating the hydraulic drop, the depth designated by y_c is obviously the critical depth. The same reasoning will apply to the case illustrated by Fig. 2_c where the canal empties into a reservoir *B*. Assume that the level in reservoir *A* remains permanent, while

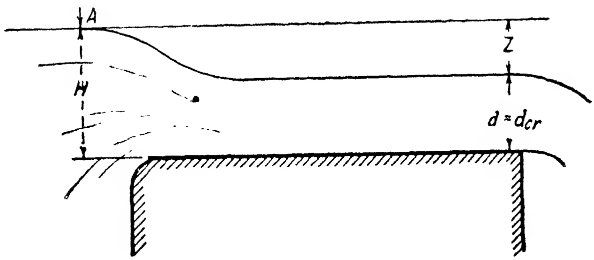

Fig. 36.

the level in reservoir *B* is lowered. Within a certain range, the depth y_2 will follow the sinking of level in the reservoir *B*. But as soon as the natural limit of $y_2 = y_{cr}$ is reached, the depth y_2 at the extremity of the canal will continue to remain permanent and equal to the critical depth, no matter how much further the level *B* is lowered.

The connection between the surface of the liquid outflowing from the canal and the level in reservoir *B* will take place in such case by means of a hydraulic drop.

Broad-crested Weir.—Figure 36 schematically represents a broad-crested weir. In case the outflow is free, the depth *d* which will establish itself at the end of the weir will be the critical depth, so that $d^3 = q^2/g$, where q is the discharge per unit width. If H is the head over the weir, corrected for the velocity of approach, the discharge is $q = m\sqrt{2g}H^{3/2}$, where m is a weir discharge coefficient. Eliminating q, we obtain $d^3 = 2m^2H^3$, so that

$$d/H = d' = \sqrt[3]{2m^2};\ m = \sqrt{\tfrac{1}{2}(d/H)^3} \tag{34}$$

We see that for a given H, the depth d which establishes itself over the weir depends on the value of m. The smaller the discharge coefficient, meaning the higher the resistances, the smaller the relative value of $d' = d/H$. This is in conformity with experiment, but in contradiction to the maximum discharge theory, traditionally presented in textbooks. According to the maximum discharge principle, $d' = d/H$ should under all circumstances be two-thirds. Such, however, will be the case only when the flow is ideal with no resistance losses. In all practical cases, $d' = d/H$ is less than two-thirds. To make this clear, determine the velocity v at the extremity of the weir in the section where the depth is d. By introducing a velocity coefficient φ, to take account of the resistance losses, we have

$$v = \varphi\sqrt{2g(H - d)} = \varphi\sqrt{2gH}\sqrt{1 - \frac{d}{H}} = \varphi\sqrt{2gH}\cdot\sqrt{1 - d'}$$

Accordingly, the discharge

$$q = v \cdot d = \varphi\sqrt{2g}H^{3/2}\cdot d'\sqrt{1 - d'}$$

and the discharge coefficient

$$m = \varphi d'\sqrt{1 - d'} \tag{34a}$$

By comparing with Eq. (34), we obtain

$$m = \varphi d'\sqrt{1 - d'} = \sqrt{\tfrac{1}{2}(d')^3} \tag{34b}$$

from which there follows:

$$\left.\begin{aligned} \varphi^2 &= \frac{d'}{2(1 - d')} \\ d' &= \frac{2\varphi^2}{1 + 2\varphi^2} \end{aligned}\right\} \tag{34c}$$

Obviously, if $\varphi^2 = 1$, d' in Eq. (34c) becomes $d' = \frac{2}{3}$, and the discharge coefficient in Eq. (34) will be $m = 0.385$. For any values of the weir discharge coefficient, the corresponding values of the reduced depth and of the velocity coefficients follow from Eq. (34a–34c). The parameters of flow are all organically interconnected. Following are some numerical data:

m	φ	d'
0.385	1.0	$\frac{2}{3}$
0.350	0.915	0.625
0.320	0.85	0.59

Curvilinear Flow.—Experiments show that in actual movement over a broad-crested weir, the critical depth $d_{cr} = \sqrt[3]{q^2/g}$ is attained by the surface at a certain distance before the edge of the weir (section C, Fig. 37), and that the depth d'_{cr} over the edge is somewhat smaller. The

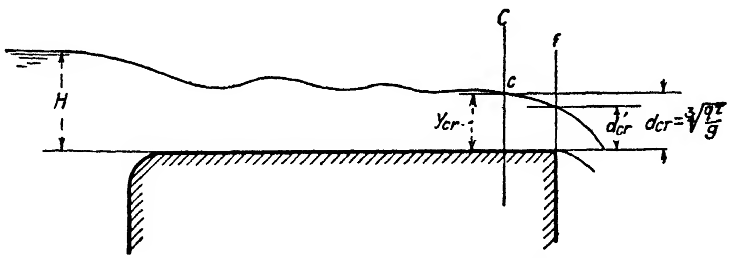

FIG. 37.—Flow over a broadcrested weir.

explanation lies in the fact that the critical depth $d_{cr} = \sqrt[3]{q^2/g}$ was determined in Art. 16 under the assumption of parallel flow. In other words, $d_{cr} = \sqrt[3]{q^2/g}$ corresponds to flow with a possible minimum content of energy in parallel flow, and in parallel flow only.

For curvilinear flow the minimum possible content of energy, corresponding to a discharge q differs from $\epsilon_{min} = 1.5\sqrt[3]{q^2/g}$ corresponding to parallel flow. It is greater for concave flow (Fig. 26a) and smaller for convex movement (Fig. 26b).

Also the critical depth, determined as the depth at which flow with a minimum content of energy takes place, is smaller than $\sqrt[3]{q^2/g}$ for convex streaming, and greater than $\sqrt[3]{q^2/g}$ for concave flow. Between sections C and F in Fig. 37, the filaments gradually increase their convex curvature; the critical depth accordingly diminishes from $d_{cr} = \sqrt[3]{q^2/g}$ in section C to a smaller value d'_{cr} over the edge.*

Critical-depth Water Meter.—Inasmuch as the critical depth is a definite parameter of flow, independent of the roughness of the walls and other uncontrollable circumstances, the idea naturally arose to use flow in critical state as a meter to determine the discharge in an open channel.

In fact, assuming that a device was so laid out that flow would be critical in a definite section, all that one would have to do in order to determine the discharge would be to measure the depth d in the section where the depth is supposed to be the critical depth.

If the cross-section of the canal in question were to be rectangular, then the volume of flow corresponding to the measured depth would be

$$Q = b\sqrt{gd^3} = bd\sqrt{gd}$$

or in a more general case, the discharge would be

$$Q = \sqrt{g} \cdot \mathfrak{M}_{cr} = a\sqrt{g\frac{a}{b}}$$

where $\mathfrak{M}_{cr}$ is the value of $a\sqrt{a/b}$ corresponding to the measured d.

Different attempts have been made to incorporate the idea into a practical device.†

Critical flow is usually produced either by a retrenchment of the cross-section with a subsequent enlargement or by

* For more detailed analysis of the case, see Bakhmeteff, *Proc. A.S.C.E.*, No. 2, 1931.

† See discussion by Hinds and others of a paper by Parshall, *Trans. A.S.C.E.*, Vol. 89, p. 840, 1926.

a break in the bottom. In all cases the phenomenon is related to an elongated hydraulic drop.

In designing critical-depth water meters it is particularly important to bear in mind that the relations $q = bd\sqrt{gd}$ or $Q = a\sqrt{ga/b}$ are strictly limited to parallel flow only. In other words, the purpose of the meter would be achieved only if flow in the section where the critical depth is to be measured will be practically parallel.

Equation of Gradually Varied Flow.—The notion of specific energy and the energy diagram (Fig. 30) may also be directly used for establishing the general differential equation of varied flow. In fact, with reference to Fig. 35, the change of energy between sections 1 and 2, assuming the distance dx between the sections is small, is $-\frac{\delta\epsilon}{\delta x}dx = -d\epsilon$, with $-\frac{\delta\epsilon}{\delta x}$ determined by Eq. (24). This gain or loss of specific energy obviously is equal to a change of the specific energy on the diagram corresponding to a change of depth dy, namely:

$$-d\epsilon = -\frac{\delta\epsilon}{\delta y}dy.$$

wherein $\delta\epsilon/\delta y$ is determined by Eq. (26). Combining, we obtain

$$\frac{\delta\epsilon}{\delta x}dx = \frac{\delta\epsilon}{\delta y}dy$$

which, through proper substitution, gives:

$$\frac{dy}{dx} = \frac{\delta\epsilon/\delta x}{\delta\epsilon/\delta y} = s_0\frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - \frac{Q^2}{g}\frac{b}{a^3}}$$

This is Eq. (21), with the physical meaning of the numerator and denominator properly unveiled.

May it also be mentioned, that prior to the introduction of the notion of specific energy of flow, which notion in spite of its simplicity was not unmasked until recently (see Preface), no direct and immediate insight into the physical essence, underlying the phenomena of varied flow was available. The presence of the jump, for example, was explained by purely analytical considerations. In fact, the denominator of Eq. (21), for the particular value of $\frac{Q^2 b}{g a^3} = 1$, that is, for the critical depth, becomes zero, so that $dy/dx = \sim$. This indicates a disruption in the continuity of the surface curve, the latter becoming perpendicular to the X-axis. This disruption was supposed to find experimental confirmation in the formation of a jump.

18. Critical Flow.—Liquid, flowing in a given canal under the critical depth ($y = y_{cr}$), is described as being in *critical flow* or in *critical state*. For a given discharge, the critical depth, determined as in Art. 16, indicates the particular depth which makes the discharge Q flow in critical state.

Critical Discharge Q_{cr}.—Reversing the reasoning, one may say that for every depth y in a given canal there is a certain discharge, to be designated as Q_{cr}, which makes flow under the chosen depth critical. We shall call Q_{cr} the critical discharge. Determined by Eq. (30) it is equal to

$$Q_{cr} = \sqrt{g}\mathfrak{M}(y) = \sqrt{g}a\sqrt{a/b} \tag{35}$$

For a given profile, $Q_{cr} = f(y)$ may be traced as a curve. The Q_{cr} curve is obviously the $\mathfrak{M}(y)$ curve (Fig. 31) multiplied by $\sqrt{g}$.

Critical Velocity v_{cr}.—The velocity corresponding to critical flow is called the critical velocity and will be designated as v_{cr}. From Eq. (35) we have:

$$v_{cr} = Q_{cr}/a = \sqrt{g}\sqrt{a/b} \tag{36}$$

The quotient a/b has a simple physical meaning; it is (Fig. 38) the depth of a rectangular canal $AB'D'C$, having the same cross-section area a as the given canal $ABCD$ and the same top width, b. We shall designate:

$$a/b = \delta \tag{37}$$

and shall call δ the *average depth* of a cross-section.

The critical velocity is thus:

$$v_c = \sqrt{g\delta} = \sqrt{2g\frac{\delta}{2}} \tag{38}$$

or the velocity corresponding to a head equal to one-half of the average depth δ. A comparison between the average depth δ and the hydraulic radius $R = a/p$ is gained from

$$\delta/R = a/b \cdot p/a = p/b \tag{39}$$

Rectangular Profiles.—For a rectangular profile (Fig. 32), we have from Eq. (33)

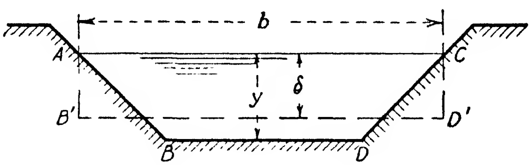

FIG. 38.—The average depth of flow $\delta = a/b$.

$$\begin{aligned} q_{cr} &= \sqrt{g}\sqrt{y^3} \\ v_{cr} &= \sqrt{g}\sqrt{y} \end{aligned} \tag{40}$$

In this case obviously

$$\delta = a/b = y \tag{41}$$

19. Critical Slope. (Figure 39.)—The value of the bottom slope which makes uniform flow at a chosen depth y critical, will be called the *critical slope* and in contradistinction to s and s_0 will be designated by the Greek letter σ. The critical slope σ is a function of the depth. To determine σ we have by definition: $Q^2 = \sigma\mathfrak{K}^2$ and simultaneously $Q^2 = g\mathfrak{M}^2$. From which follows

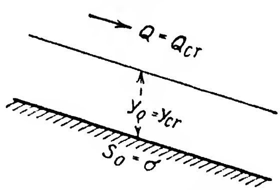

FIG. 39.—The critical slope σ.

$$\sigma\mathfrak{K}^2 = g\mathfrak{M}^2$$

or

$$\sigma = g\frac{\mathfrak{M}^2}{\mathfrak{K}^2} \tag{42}$$

In this equation $\mathfrak{M}$ and $\mathfrak{K}$ are the particular values of the $\mathfrak{M} = a\sqrt{a/b}$ function, and of the conveyance $\mathfrak{K} = aC\sqrt{R}$

for the chosen depth y. For a given cross-section the critical slope $\sigma = f(y)$ may be traced as a curve, an example of which for canal cross-section (Fig. 14) is given in Fig. 15.

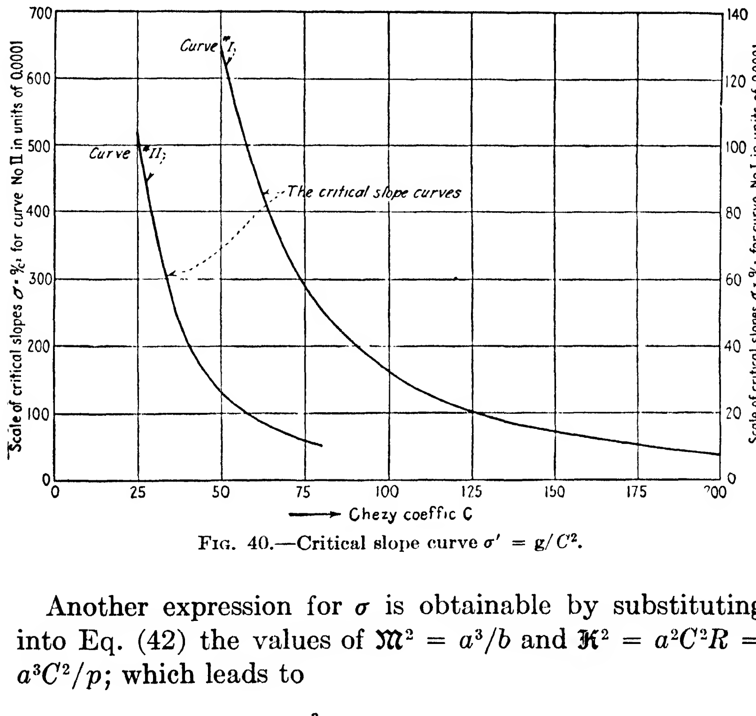

FIG. 40.—Critical slope curve $\sigma' = g/C^2$.

Another expression for σ is obtainable by substituting into Eq. (42) the values of $\mathfrak{M}^2 = a^3/b$ and $\mathfrak{K}^2 = a^2C^2R = a^3C^2/p$; which leads to

$$\sigma = g\frac{a^3}{b} \cdot \frac{p}{a^3C^2} = \frac{g}{C^2} \cdot \frac{p}{b} \tag{43}$$

which may be presented also as

$$\sigma = \sigma' \cdot p/b$$

where

$$\sigma' = g/C^2 \tag{43a}$$

To get an appreciation of concrete values, the curve of $\sigma' = g/C^2$ as function of C is traced in Fig. 40. In case of a profile the width of which is large compared to its depth, so that p/b in Eq. (43) does not differ much from unity (which, by the way, is the case of most natural water-courses) the curve $\sigma' = g/C^2$ gives the values of the critical slope directly. Otherwise the curve figure σ' must be multiplied by p/b; the latter is obviously always > 1.

To appraise the effect of p/b in raising the value of σ over σ', there are given in Fig. 41 some examples of canal cross-sections with the respective values of p/b.

Normal Critical Slope.—The critical slope curve as traced in Fig. 15 is a characteristic inherent in a canal cross-section as such. It depends on the form of the cross-section and

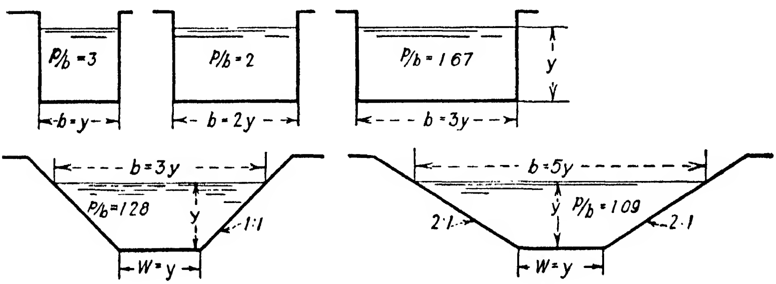

FIG. 41.

on the roughness of the walls, being a function of the depth y.

For a given discharge, it is expedient to fix as characteristic parameters certain particular values of the critical slope, namely: (1) the critical slope σ_0 at the normal depth y_0 and (2) the critical slope σ_{cr} at the critical depth y_{cr}. We shall designate σ_0 as the *normal critical slope*.

Obviously σ_{cr} is the slope, which would make the given discharge flow in uniform movement in critical state.

Example 5

Question 1. Draw the critical slope curve $\sigma = f(y)$ for the canal profile, Fig. 14. Use Bazin coefficients.

Either Eq. (42) $\sigma = g\frac{\mathfrak{M}^2}{\mathfrak{K}^2}$ or Eq. (43) $\sigma = \frac{g}{C^2}\cdot\frac{p}{b}$ may be used.

Using Eq. (43) and taking the C, p and b values from Table I and Table II, and the g/C^2 values from Fig. 40, we obtain

TABLE V

y	p	b	C	g/C^2 in $1 \cdot 10^{-4}$	p/b	σ in $1 \cdot 10^{-4}$
0.5	6.41	6	109.0	27.2	1.070	29.1
1.0	7.82	7	118.0	23.2	1.126	25.9
1.5	9.23	8	123.0	21.2	1.152	24.6
2.0	10.64	9	126.0	20.2	1.182	23.9
2.5	12 05	10	128.5	19.6	1.205	23.6
3.0	13.46	11	130 0	19.0	1.223	23.2
3.5	14.87	12	132.0	18.4	1.239	22.8
4.0	16.28	13	132.7	18.1	1.252	22.7
4 5	17.69	14	133.5	18 0	1.261	22.7
5.0	19.10	15	134.3	17.9	1.272	22.8
6.0	21.95	17	135.7	17.6	1.292	22.8

The curve is traced in Fig. 15.

Question 2. Determine σ_0 and σ_{cr} for a flow of $Q = 240$ cu. ft. per second with $s_0 = 5 \cdot 10^{-4}$.

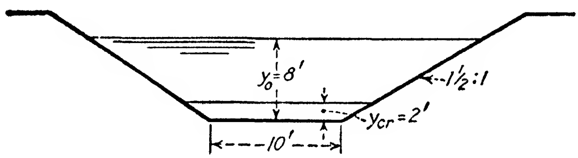

FIG. 42.

For the normal depth y_0 we have

$$\mathfrak{K}_0 = Q/\sqrt{s_0} = 240/\sqrt{5} \cdot 10^{-2} = 107.2 \times 10^2$$

from the $\mathfrak{K}$ curve (Fig. 15) the corresponding $y_0 = 4.96$ ft.

For the critical depth

$$\mathfrak{M}_c = Q/\sqrt{g} = 240/5.67 = 42.4$$

from the $\mathfrak{M}$ curve (Fig. 15)

$$y_{cr} = 3.31 \text{ ft.}$$

The values of σ_0 and σ_{cr}, which on the σ curve (Fig. 15) correspond to $y_0 = 4.96$ and $y_{cr} = 3.31$ are practically equal and close to $23\,{}^0\!/_{00}$.

Question 3. Assume that in canal, Fig. 42, $y_0 = 8$ ft. and $y_{cr} = 2$ ft. Determine σ_0 and σ_{cr}. Use G.K. coefficients for $s_0 = 10\,{}^0\!/_{00}$, with $n = 0.013$ and $n = 0.025$, respectively.

1. The geometric elements in the case are:

$y = 2$ ft.; $a = 26$; $b = 16$; $p = 17.2$; $p/b = 1.073$; $R = 1.51$
$y = 8$ ft.; $a = 176$; $b = 34$; $p = 38.8$; $p/b = 1.14$; $R = 4.54$

2. The C and σ' values.

	$n = 0.013$	$n = 0.025$
For $y = 2$ $R = 1.51$	$C = 124$ $\sigma' = 21 \times 10^{-4}$	$C = 62$ $\sigma' = 84 \times 10^{-4}$
For $y = 8.00$ $R = 4.54$	$C = 145$ $\sigma' = 15.5 \times 10^{-4}$	$C = 77$ $\sigma' = 54.5 \times 10^{-4}$

3. The σ values ($\sigma = \sigma' \cdot p/b$):

σ_{cr} for $y_{cr} = 2$ ft.
With $n = 0.013$; $\sigma_{cr} = 21 \cdot 10^{-4} \cdot 1.073 = 22.4^0/_{00}$
With $n = 0.025$; $\sigma_{cr} = 84 \cdot 10^{-4} \cdot 1.073 = 90.0^0/_{00}$

σ_0 for $y_0 = 8$ ft.
With $n = 0.013$; $\sigma_0 = 15.5 \cdot 10^{-4} \cdot 1.14 = 17.7^0/_{00}$
With $n = 0.025$; $\sigma_0 = 54.5 \cdot 10^{-4} \cdot 1.14 = 62.2^0/_{00}$

20. Other Forms of Varied Flow Equation.—Equation (21)

$$\frac{dy}{dx} = s_0 \frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - \dfrac{Q^2}{g}\dfrac{b}{a^3}}$$

may be now presented in a somewhat different form:

1. In the denominator, in accordance with Eqs. (28) and (29), substitute:

$$Q^2/g = \mathfrak{M}_c^2; \; b/a^3 = 1/\mathfrak{M}^2$$

obtaining

$$\frac{dy}{dx} = s_0 \frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - (\mathfrak{M}_c/\mathfrak{M})^2} \tag{44}$$

To make things clear, bear in mind that $\mathfrak{K}$ and $\mathfrak{M}$ are the respective variable values of the conveyance $\mathfrak{K} = aC\sqrt{R}$ and of the $\mathfrak{M} = a\sqrt{a/b}$ function, varying with the depth y; while $\mathfrak{K}_0$ and $\mathfrak{M}_c$ are certain parametric values of the two functions for the normal depth y_0 and for the critical depth y_{cr}, respectively.

2. Another form is obtained by replacing in the denominator $\mathfrak{M}$ by $\mathfrak{K}$. From Eq. (42) we have $\mathfrak{M}^2 = \frac{a^3}{b} = \mathfrak{K}^2\frac{\sigma}{g}$; while, on the other hand, $Q^2/g = \mathfrak{K}_0^2 s_0/g$. Hence

$$1 - \frac{Q^2 b}{g\,a^3} = 1 - \frac{s_0\mathfrak{K}_0^2}{g}\cdot\frac{g}{\sigma\mathfrak{K}^2} = 1 - \frac{s_0}{\sigma}\left(\frac{\mathfrak{K}_0}{\mathfrak{K}}\right)^2 \tag{45}$$

With $s_0/\sigma = s_0/\sigma_0 \cdot \sigma_0/\sigma$, where σ_0 is the *normal* critical slope, we have instead of Eq. (21)

$$\frac{dy}{dx} = s_0\frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - \dfrac{s_0}{\sigma}\left(\dfrac{\mathfrak{K}_0}{\mathfrak{K}}\right)^2} = s_0\frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - \dfrac{s_0}{\sigma_0}\cdot\dfrac{\sigma_0}{\sigma}\cdot\left(\dfrac{\mathfrak{K}_0}{\mathfrak{K}}\right)^2} \tag{46}$$

It is in this latter form (Eq. [46]) that the varied flow equation will be used for integration and, therefore, for determining the surface curves $y = f(x)$. When the shape of the canal, the roughness of the walls, the bottom slope s_0, and the discharge Q are given, Eq. (46) gives the rate of change of the depth as depending on only two varying factors, namely, $\mathfrak{K}_0/\mathfrak{K}$ and σ_0/σ. Both these factors, in turn, are functions of the depth y, or functions of y/y_0, the ratio of the varying depth y to the constant parametric value of the normal depth y_0.

For $y = y_0$, both $\mathfrak{K}/\mathfrak{K}_0$ and σ_0/σ are $= 1$. Designating, in particular,

$$\frac{s_0}{\sigma} = \beta \text{ and } \frac{s_0}{\sigma_0} = \beta_0, \text{ so that } \beta = \beta_0\frac{\sigma_0}{\sigma} \tag{47}$$

we obtain Eq. (46) as

$$\frac{dy}{dx} = s_0\frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - \beta(\mathfrak{K}_0/\mathfrak{K})^2} \tag{48}$$

In Eqs. (47) and (48), β_0 again is a constant parametric value, the relation of the given bottom slope s_0 to the critical slope σ_0 at the normal depth y_0, while σ_0/σ and, therefore,

β reflect the change of the critical slope within the range of the varying depths. Usually, this variation is not very substantial, and within certain limits (see Question 2, Example 5) the value of σ_0/σ and, therefore, the β value may be accepted as practically constant. In the integration procedure, the practice will be to divide the range of depths into certain reaches, assuming for each of such reaches a certain constant average value of β.

CHAPTER V

RECAPITULATION: THE CHARACTERISTICS OF FLOW

To avoid possible confusion in regard to the many notions, introduced in the course of the preceding analysis, it may be useful to assemble, by way of a summary, the different characteristics of flow, as developed above:

21. Characteristics of a Canal Cross-section.—A *canal* is said to be *defined* when there are given:

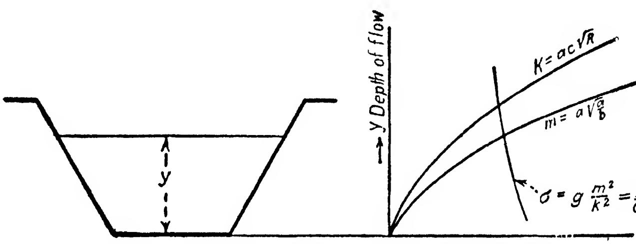

FIG. 43.—Characteristics of a canal cross-section.

The shape and the dimensions of the cross-section.

The nature of the walls (roughness coefficient).

A canal cross-section, once given as above, possesses the following characteristics, inherent in the cross-section as such and being functions of the depth (Fig. 43; see also Fig. 15):

1. The conveyance curve . . . . $\mathfrak{K} = aC\sqrt{R}$
2. The $\mathfrak{M}$ curve . . . . $\mathfrak{M} = a\sqrt{a/b}$
3. The critical slope curve . . . . $\sigma = \dfrac{g}{C^2} \cdot \dfrac{p}{b} = g\dfrac{\mathfrak{M}^2}{\mathfrak{K}^2}$

Also

4. The critical velocity curve . . . . $v_c = \sqrt{g}\sqrt{\delta} = \sqrt{g}\sqrt{a/b}$

5. The critical discharge curve. $Q_{cr} = \sqrt{g}\mathfrak{M} = a\sqrt{g}\sqrt{a/b}$

The $\mathfrak{M}$, v_c and Q_c curves depend solely on the geometric forms of the cross-section.

The $\mathfrak{K}$ and σ curves depend on the roughness of the walls. If a C formula is used, like the G.K., where C is supposed to vary with the bottom slope, the possible effect of such variation will have to be taken into consideration.

Sets of characteristic curves for certain canal cross-sections which are used for practical examples in this book have been computed and presented in the special plates assembled at the end of the book.

22. Parameters of Flow.—A *case* of *flow* is said to be *defined,* when there are given:

The canal cross-section (defined as in the preceding article).

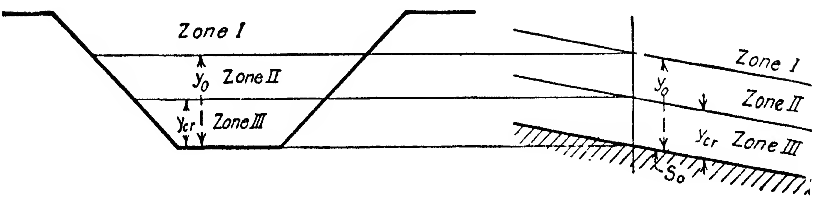

FIG. 44.—The zones of flow.

The bottom slope s_0 with which the canal is laid.

The discharge Q, flowing in the canal.

The parameters of flow then will be as follows:

1. The normal depth y_0, being the depth of uniform flow for the discharge Q under s_0. y_0 is determined from the $\mathfrak{K}$ curve as a depth corresponding to $\mathfrak{K}_0 = Q/\sqrt{s_0}$

2. The critical depth y_{cr}, being the depth at which the discharge Q would flow in the given canal with the minimum possible content of specific energy. y_{cr} is determined by means of the $\mathfrak{M}(y)$ curve, as the depth corresponding to $\mathfrak{M}_c = Q/\sqrt{g}$

3. The normal critical slope σ_0 and σ_c which are the critical slope values corresponding to y_0 and to y_{cr}, respectively.

Zones of Flow.—The normal and critical depths, when drawn into the cross-sectional and into the longitudinal profiles, divide the plane of the respective profiles into zones (Fig. 44).

Auxiliary Curves.—In practical computation it will be occasionally useful to have as auxiliary curves:

4. The normal discharge curve $Q_0 = f(y_0)$; the curve of discharges in uniform movement $Q = \mathfrak{K}_0\sqrt{s_0}$, in the given canal under the bottom slope s_0.

5. The σ_0/σ and eventually the σ_{cr}/σ curve, representing the ratio of σ_0 and σ_{cr} to the varying critical slope.

6. The specific energy curve $\epsilon = y + \frac{v^2}{2g} = y + \frac{Q^2}{2ga^2}$.

Exercises:

It is earnestly recommended, that before proceeding further, the student should become thoroughly familiar with the different characteristics and parameters of flow. He should assemble into a complete set the different curves as enumerated above. This should be carried out for the cross-section selected for the purpose of general exercise in Art. 7. The selection of Q and of s_0 is left to the student's discretion.

CHAPTER VI

CLASSIFICATION OF FLOW

23. "Mild" and "Steep" Slopes.—It was Bélanger (1828) who, in commenting on Bidone's experiments on the hydraulic jump, distinguished natural watercourses in which jumps would occur (Fig. 6), from streams where the backwater curve produced by a dam would connect with the undisturbed surface line by means of a continuous curve

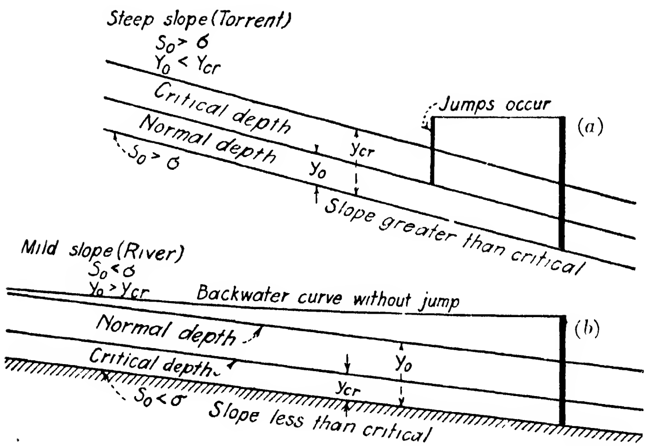

FIG. 45.—Distinguishing between *mild* and *steep* bottom slopes.

(Fig. 3). Naturally (Fig. 34), a jump may take place only if water would be flowing in natural condition with a depth less than the critical ($y_0 < y_{cr}$, as in Fig. 45*a*). On the other hand, in case the depth of flow in natural condition is above the critical ($y_0 > y_{cr}$, as in Fig. 45*b*), then the backwater curve lies wholly within the upper branch of the energy curve. There can be no jump and the curve will be of the continuous type as shown in Fig. 3.

As the relative values of y_0 and y_{cr} depend on the size of the bottom slope, the suggestion naturally arose to distinguish between *steep* and *mild* bottom slopes: the slope s_0 to be designated as *mild* when it is smaller than the critical ($s_0 < \sigma_0$) and when it makes $y_0 > y_{cr}$; while a slope is *steep* when it is greater than the critical slope ($s_0 > \sigma_0$) and when it makes $y_0 < y_{cr}$.

In tabular form:

$$\left.\begin{array}{l}\text{Mild slopes} = s_0 < \sigma_0;\ y_0 > y_{cr}\\ \text{Steep slopes} = s_0 > \sigma_0;\ y_0 < y_{cr}\end{array}\right\} \quad (49)$$

Saint Vénant* called natural streams of *mild* slope, exhibiting calm, steady flow, *rivers*. Streams of *steep* slope, featured by jumps, cataracts, and other irregularities, he named *torrents*.

24. States of Flow.—While the above distinction is useful in many instances, it does not wholly bear on the

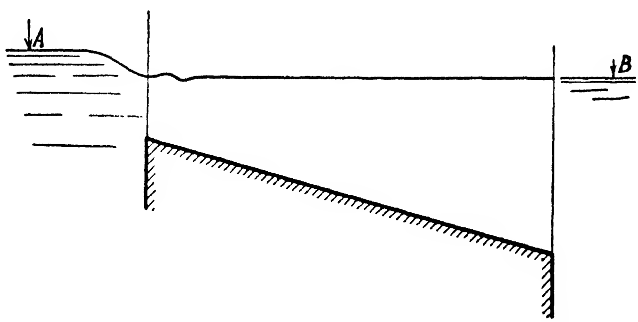

FIG. 46.—Continuous surface curve without a jump in a canal with steep bottom slope.

essence of the case. In fact, Boussinesq rightfully pointed out that it is essential in the situation to distinguish between different possible *states of flow*.† A jump, as shown in Figs. 7 and 8, may take place in a channel of mild slope with $s_0 < \sigma_0$ while there may be a continuous backwater curve without any jumps (Fig. 46) in a flume the bottom slope of which is steep.

* Ann. des mines, 1851.

† "Théorie des eaux courantes," 1877.

In fact all depends on whether the velocity of flow is above or below the critical; or, in other words, on whether the depth of flow is *above* or *below* the critical depth.

A very clear picture of the states of flow is gained by means of the specific energy curve. In fact, in Fig. 47, the critical depth divides all possible conditions of flow into two *zones*, corresponding to the two principal states of flow:

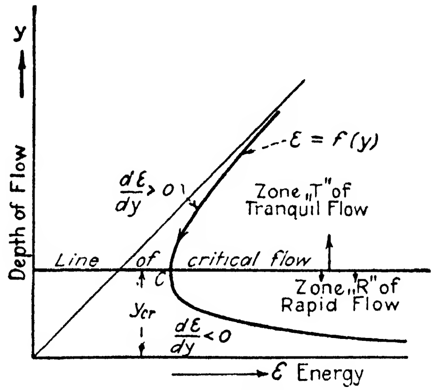

FIG. 47.—The *states* of flow in their relation to the specific energy diagram.

1. *Zone T*, embracing the upper branch of the ϵ curve, with $y > y_{cr}$ and $v < v_{cr}$, a zone within which flow is said to be in *tranquil state*.

2. *Zone R*, embracing the lower branch of the ϵ curve, with $y < y_{cr}$ and $v > v_{cr}$, a zone within which flow is said to be in *torrential* or *rapid state*.

3. In between the *T* and *R* zones lies the separating point *C*, corresponding to critical flow, or flow in *critical state*.

The names applied to characterize the *states* are suggestive of the nature and what one might call the different behavior of flow. Rehbock most fittingly applies the word *flowing* (*fliessend*) to describe "tranquil" flow; while he speaks of a stream as *shooting* (*schiessend*) when water rushes down a chute or races from under a sluice in "rapid" state.

The basic physical distinction between the different states of flow is determined by the outline of the specific energy curve. Namely:

$$\left.\begin{array}{l}\text{In tranquil state: } \epsilon \text{ increases with } y;\ \delta\epsilon/\delta y > 0 \\ \text{In } rapid \text{ flow: } \epsilon \text{ decreases with } y;\ \delta\epsilon/\delta y < 0 \\ \text{In } critical \text{ state: } \epsilon - \text{minimum};\ \delta\epsilon/\delta y = 0\end{array}\right\} \quad (50)$$

These simple relations help to explain some interesting features, inherent in flow in general.*

25. Submerged Obstacles.—One of them is the effect, produced on the surface of a flowing stream by a submerged obstacle, such as a large boulder or a local elevation at the bottom. In a *river*, such an obstacle either produces no visible effect or results in additional surface eddies and, at times, in a slight local depression of the surface; a kind of a hollow which is rapidly smoothed out by the pouring in of

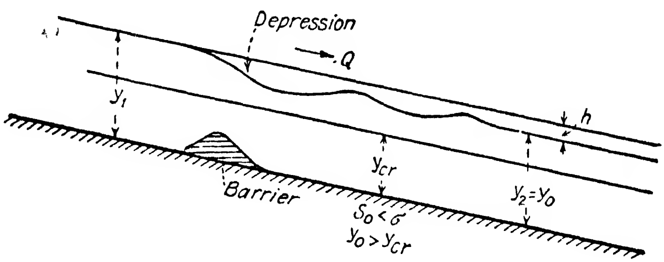

Fig. 48.—Barrier in a stream of *mild* slope.

water from the adjacent streaming. In a *torrent*, a bottom obstacle usually produces a foaming center, often marked by a local jump in the nature of a surface swell. Speaking generally, the surface of a torrent when streaming over a shallow rocky bed is all covered by such foaming centers. The presence of a swell in *rapid* flow in contradistinction to a depression in *tranquil* movement is explained by the fact that the crossing of an obstacle is accompanied by loss of energy. Now, loss of energy in tranquil state calls for a *lowering* of the surface, while in rapid state dissipation of energy is accompanied by an *increase* in depth.

The difference is particularly conspicuous in the case of a submerged barrier. In a *river* (Fig. 48), if the barrier is not too high, there usually develops what Bazin called an

* Boussinesq, *loc. cit.*

undulated nappe, namely a sequence of gradually diminishing undulations which follow the initial depression. The depth y_1 above the barrier is somewhat larger than the depth y_2; the difference $h = y_1 - y_2$, is the head, lost in

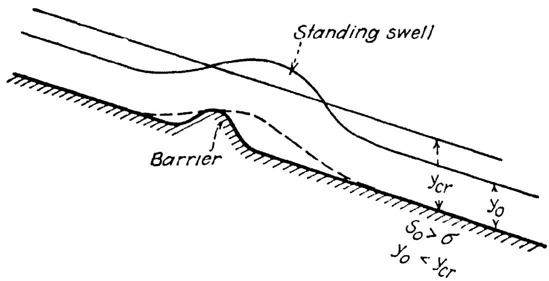

FIG. 49.—Barrier in a *torrent;* the *standing swell.*

passing over the submerged weir. In a *torrent*, the barrier under certain conditions is crossed by what might be termed a *standing swell* (Fig. 49), in the form of a single

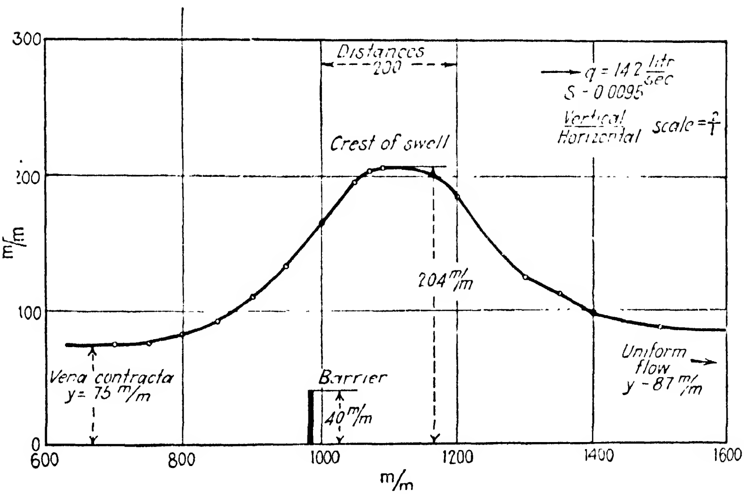

FIG. 50.—Example of a *standing* swell.

undular surface rise, unaccompanied by further undulations. In this case, at least in case of uniform flow, the depths above and below the barrier are the same. Figure 50 gives the profile of a standing swell observed by the author.* The height of the swell was nearly twice the original depth. The highly rapid flow was produced by streaming from under a sluice (see Fig. 7).

As may be expected, a standing swell will take place only if the barrier does not exceed a certain height. Beyond that, the type of the phenomenon changes; the swell becomes replaced by the usual backwater curve (Fig. 51), preceded by a jump. The surface of a standing swell is continuous, allowing unimpeded passage to a small flowing object, such as a block of wood, a small ship model, etc. On the contrary, the whirling roll at the foot of the jump

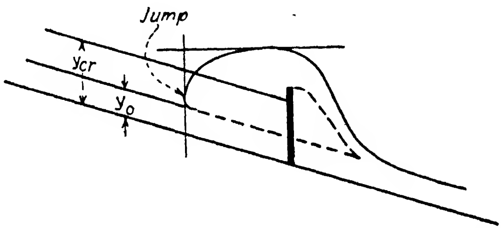

FIG. 51.—Flow over a barrier in a canal with steep slope featuring a jump.

(Fig. 51) disrupts the continuity of the surface. A flowing body is usually caught by the whirls and tossed around in continuous rotations.

26. The Establishing of Flow.—The absence of undulations in Fig. 49, as compared to Fig. 48, manifests another feature of general character, also revealed theoretically by Boussinesq, and being in good consonance with observation. It refers to the forms of the surface within the *transitory sections,* where flow "establishes" itself. An example is the entrance portion of a flume (Fig. 52). Above section *a* there lies the undisturbed level of the reservoir. Below section 1, flow is uniform. The stretch between sections *a* and 1 is the transitory part in which uniform flow establishes itself and to which the Boussinesq reasoning applies. In Fig. 52 this transitory stretch presents an undulatory surface. Such is always the case when the

* Hydr. Laboratory, Polytechnic Institute, St. Petersburg, 1911.

slope of the flume is mild or, to be more specific, when flow, which is in process of establishing itself, is tranquil. Another example is that of a broad-crested weir (Fig. 37); here the depth is near the critical. With reference to the energy diagram, we see that in the neighborhood of the

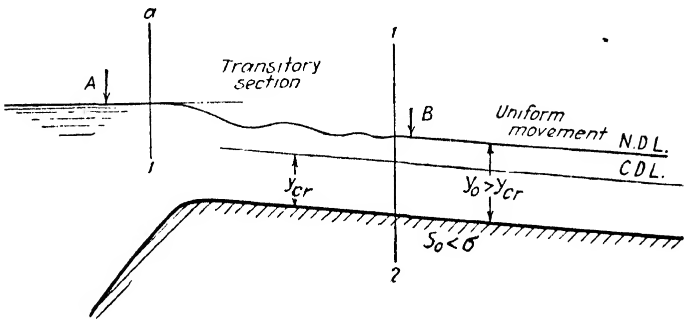

FIG. 52.—The *establishing* of *tranquil* flow.

critical depth the energy curve is very steep, so that practically an unnoticeable difference in energy corresponds to a substantial variation of depth. This circumstance, together with the effect of curvature on the size of the critical

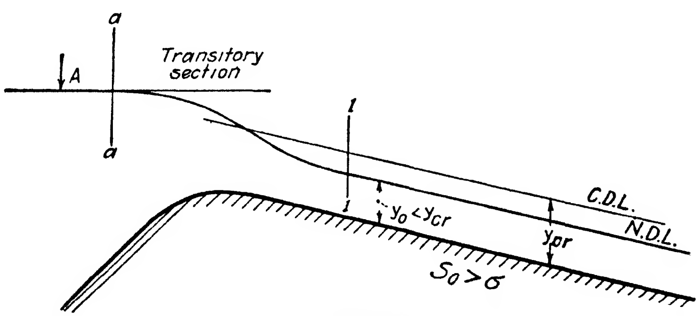

FIG. 53.—The *establishing* of *rapid* flow.

depth (Art. 17), explains the rather pronounced undulations characteristic of the case.

Figure 53 pictures entrance conditions at the head of a steep-sloped flume. Rapid uniform movement below section 1 is preceded by a transitory section where there are no undulations; this is *always* the case in a transitory stretch when the movement to be installed is of the "rapid" kind. Another instance is given in Fig. 54, where a stream in rapid state from under a sluice emerges into a horizontal flume, the flume being sufficiently short to allow the water to leave before it reaches the critical depth. Here again the surface features no undulations.

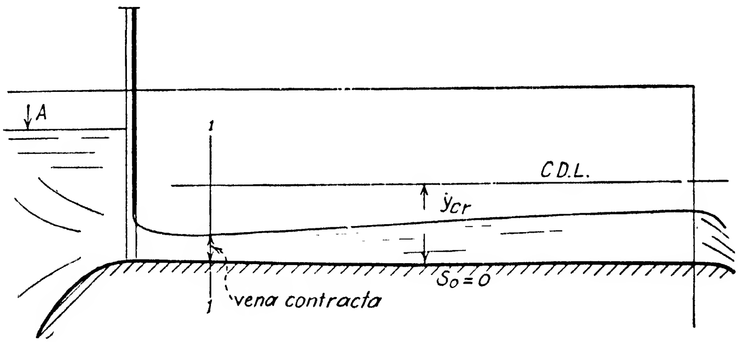

FIG. 54.—Rapid flow in a flume below a sluice

27. The Kinetic Flow Factor.—In tranquil flow, it is potential energy that prevails. In rapid state, as the velocity grows, the kinetic energy head becomes predominant.

To measure the *degree of rapidity* or that of *tranquility* of flow; to obtain, in general, a standard by means of which the state of flow may be qualified numerically, the author uses the notion of the *kinetic flow factor*, which will be designated by λ and which is defined by means of the specific energy equation, as follows:

$$\epsilon = y + \frac{v^2}{2g} = y\left(1 + \frac{1}{2}\frac{v^2}{gy}\right) = y\left(1 + \frac{1}{2}\lambda\right) \qquad (51)$$

The kinetic flow factor is thus equal to

$$\lambda = 2 \cdot \frac{v^2/2g}{y} = 2 \cdot \frac{Q^2}{2ga^2y} = \frac{Q^2}{ga^2y} \qquad (52)$$

or twice the ratio of the kinetic energy head to the potential energy head. The kinetic flow factor thus is a measure of the *kineticity of flow*. One may speak of flow being in a state of "high" or "low" kineticity. In each case the degree of kineticity will be qualified by the respective value of λ, just as the thermic state is qualified by temperature.

Rectangular Profile.—In the case of a rectangular profile, applying Eq. (52) to a unit width of the canal and remembering that $q^2/g = y^3_{cr}$, the kinetic flow factor is

$$\lambda = q^2/gy^3 = (y_{cr}/y)^3 \tag{53}$$

and the specific energy equation

$$\epsilon = y\left(1 + \frac{\lambda}{2}\right) = y\left(1 + \frac{1}{2}\left[\frac{y_{cr}}{y}\right]^3\right) \tag{54}$$

$$\left.\begin{array}{l}
\textit{In critical state}\text{, in particular,} \\
\qquad \lambda_{cr} = 1;\ \epsilon_{cr} = y_{cr}\left(1 + \frac{\lambda}{2}\right) = 1.5y_{cr} \\
\textit{Tranquil state}\text{ is characterized by} \\
\qquad \lambda < 1;\ \epsilon < 1.5y \\
\textit{Rapid state}\text{ is featured by} \\
\qquad \lambda > 1;\ \epsilon > 1.5y
\end{array}\right\} \tag{55}$$

The particular circumstance that, in the practically so important case of a rectangular canal, flow in critical state is characterized by the simple symbol $\lambda = 1$, explains the reason for choosing the definition as given in Eq. (52).

Cross-sections of Any Form.—A general expression for the kinetic flow factor applicable to a cross-section of any form is obtained by substituting in Eq. (52) Q^2/g by the equivalent value of $\mathfrak{M}^2_{cr}$ (see Eq. [30]); and, on the other hand (Eqs. [29] and [37]), by putting $\mathfrak{M}^2 = a^2\frac{a}{b} = a^2\delta$, so that $a^2 = \frac{\mathfrak{M}^2}{\delta}$. This leads to

$$\lambda = Q^2/ga^2y = \mathfrak{M}^2_{cr}/\mathfrak{M}^2 \times \delta/y \tag{56}$$

and the specific energy

$$\epsilon = y(1 + \tfrac{1}{2}[\mathfrak{M}_{cr}/\mathfrak{M}]^2 \cdot \delta/y) \tag{57}$$

Naturally, Eq. (54) is but a particular case of Eq. (57); in fact, for a rectangular profile

$$\delta/y = 1 \text{ and } (\mathfrak{M}_{cr}/\mathfrak{M})^2 = (y_{cr}/y)^3$$

For critical flow $\mathfrak{M}$ in the denominator of Eq. (56) becomes $\mathfrak{M}_{cr}$. Hence, for critical state

$$\left.\begin{aligned} \lambda &= 1 \cdot \delta/y; \ \epsilon_{kin} = \tfrac{1}{2}y \cdot \delta/y \\ \epsilon_{cr} &= y_{cr}(1 + \tfrac{1}{2}[\delta/y]) \end{aligned}\right\} \quad (58)$$

As compared to conditions in a rectangular channel, the kinetic energy content in critical flow differs by the factor δ/y, this being the relation of the "average depth" of a cross-section ($\delta = a/b$) to the actual depth of flow, y.

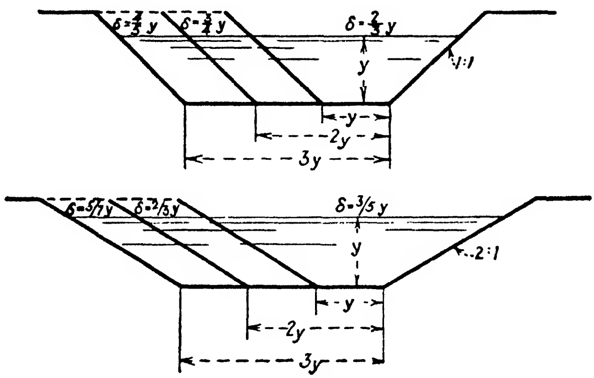

FIG. 55.

For an open canal of the usual type, which broadens with the depth, δ is always smaller than y, so $\delta/y < 1$ and $\lambda_{cr} < 1$.

Some actual values of δ/y for a series of canal cross-sections are given in Fig. 55. It is seldom, that δ/y falls below 0.5; usually δ/y is between 0.5 and 1.

For such limits, the specific energy at the critical depth fluctuates between

$$1.25y < \epsilon_{cr} < 1.5y$$

Flow with $\lambda > 1$ and $\epsilon > 1.5y$ under all circumstances will be in *rapid* state. On the other hand, flow with $\lambda < 0.5$ and $\epsilon < 1.25y$ will be *practically* always in *tranquil* state.

For closed cross-sections, as in the case of a sewer (Fig. 56) the top width of which decreases with the depth, the average depth δ may be larger than y. In such case, the kinetic flow factor for the critical depth λ_{cr} may be greater than 1, and accordingly $\epsilon > 1.5y_{cr}$.

From the general expression

$$\epsilon = y\left(1 + \frac{\lambda}{2}\right); \quad \epsilon_{pot} = y; \quad \epsilon_{kin} = \frac{1}{2}\lambda y$$

there follow the relations:

$$\left.\begin{aligned} \frac{\epsilon_{pot}}{\epsilon} &= \frac{2}{\lambda + 2} \\ \frac{\epsilon_{kin}}{\epsilon} &= \frac{\lambda}{\lambda + 2} \\ \epsilon_{kin}/\epsilon_{pot} &= \lambda/2 \end{aligned}\right\} \quad (59)$$

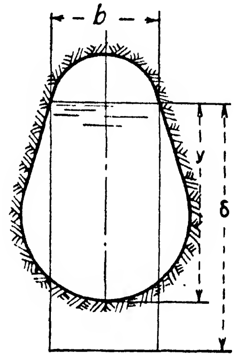

FIG. 56.

Example 6

Question 1. Assuming a discharge of $Q = 200$ cu. ft. per second, flowing in a rectangular canal 10 ft. wide, determine the kinetic flow factor and the energy content for flow at $y = 1$ ft.; as well as at $y = 0.5$, 2, 5 and 10 ft.

$q = {}^{200}/_{10} = 20$ cu. ft. per second; $y_{cr} = \sqrt[3]{q^2/g} = \sqrt[3]{400/32.2} = 2.32$ ft.

The kinetic factor value for the depth $y = 1$ ft. is $\lambda = (y_{cr}/y)^3 = (2.32/1)^3 = 12.4$.

For the other depths, respectively,

$\lambda = (2.32/0.5)^3 = 99.2$; $(2.32/2)^3 = 1.56$; $(2.32/5)^3 = 0.1$; $(2.32/10)^3 = 0.0125$

The energy content for

$$y = y_{cr} \text{ is } \epsilon_{cr} = 2.32 \times 1.5 = 3.48 \text{ ft.}$$

For

$$y = 0.5; \epsilon = y\left(1 + \frac{\lambda}{2}\right) = 0.5\left(1 + \frac{99.2}{2}\right) = 25.3 \text{ ft. } \epsilon_{kin} = 24.8 \text{ ft.}$$

$$y = 1.0; \epsilon = y\left(1 + \frac{\lambda}{2}\right) = 1.0\left(1 + \frac{12.4}{2}\right) = 7.2 \text{ ft. } \epsilon_{kin} = 6.2 \text{ ft.}$$

$$y = 2.0;\ \epsilon = y\left(1 + \frac{\lambda}{2}\right) = 2.0\left(1 + \frac{1.56}{2}\right) = 3.56\text{ ft. } \epsilon_{kin} = 1.56\text{ ft.}$$

$$y = 5.0;\ \epsilon = y\left(1 + \frac{\lambda}{2}\right) = 5.0\left(1 + \frac{0.1}{2}\right) = 5.25\text{ ft. } \epsilon_{kin} = 0.25\text{ ft.}$$

$$y = 10.0;\ \epsilon = y\left(1 + \frac{\lambda}{2}\right) = 10.0\left(1 + \frac{0.012}{2}\right) = 10.06\text{ ft. } \epsilon_{kin} = 0.06\text{ ft.}$$

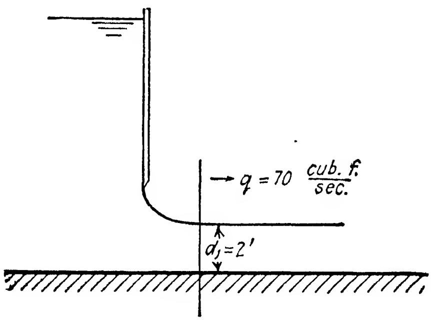

FIG. 57.

Question 2. What is the state of rapidity in a stream flowing from under a sluice (Fig. 57) into a rectangular channel at the rate of $q = 70$ cu. ft. per second with $d_1 = 2$ ft.?

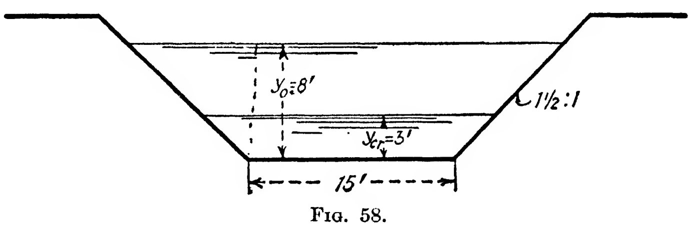

FIG. 58.

The kinetic flow factor may be determined directly from Eq. (53), as

$$\lambda = \frac{q^2}{gd_1^3} = \frac{70^2}{32.2 \cdot 8} = 19.05;\ \epsilon = 2\left(1 + \frac{19.05}{2}\right) = 21.05$$

Question 3. A discharge of 520 cu. ft. is flowing in a canal (Fig. 58) in critical flow at $y_{cr} = 3$ ft. Determine the energy content. Also the kinetic flow factor and the energy content for the same discharge at $y = 1$ and $y = 8$ ft.

For $y = 3$ ft., the average depth $\delta = a/b = 58.5/24 = 2.44$; hence

$$\lambda_{cr} = 1 \cdot \frac{\delta}{y} = \frac{2.44}{3} = 0.813; \quad \epsilon_{cr} = 3\left(1 + \frac{0.813}{2}\right) = 4.22 \text{ ft.}$$

For $y = 1$; $a = 16.5$; hence

$$\lambda = Q^2/ga^2y = (520/16.5)^2 \cdot 1/32.2 \times 1 = 30.8$$

$$\epsilon = 1\left(1 + \frac{30.8}{2}\right) = 16.4$$

For $y = 8$; $a = 216$; hence

$$\lambda = (520/216)^2 \cdot 1/32.2 \times 8 = 0.0225$$
$$\epsilon = 8(1 + 0.01125) = 8.09 \text{ ft.}$$

Question 4. Compute and trace the kinetic factor curve $\lambda = f(y)$ for canal (Fig. 14) with $Q = 300$ cu. ft. per second. With $\lambda = 2 \cdot v^2/2g \cdot 1/y$, we may use directly the figures of Table IV. Thus, for

$$y = 0.5; \lambda = 2 \cdot 185/0.5 = 740.$$

The respective values of λ are given in the last column of Table IV. The λ curve is traced in addition to the ϵ curve in Fig. 33.

CHAPTER VII

PROPERTIES AND TYPES OF SURFACE CURVES*

28. Nomenclature.—In Art. 2, the distinction was made between "rising" and "falling" surface curves, depending on whether the depth of flow *increased* or *decreased* in the

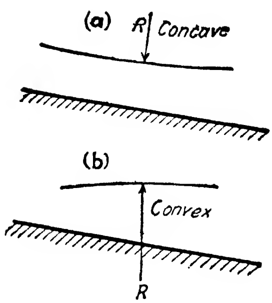

FIG. 59.

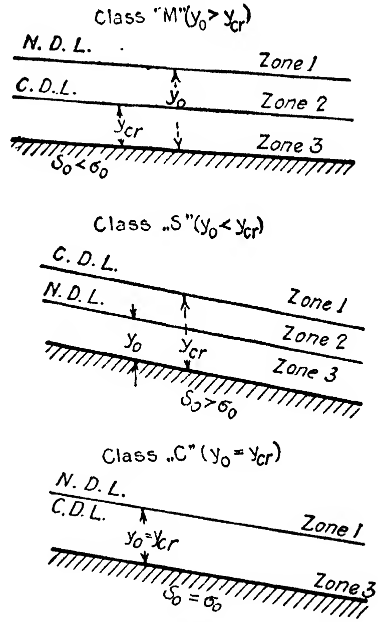

FIG. 60.—Classification of surface curves.

direction of the current. In the future we shall denote by the superior "+" or "−" sign whether a certain factor is increasing or decreasing downstream. We shall designate, accordingly:

* A general and complete classification of surface curves in varied flow was first given by Boudin. See bibliographical note, Appendix 1.

$$\left.\begin{array}{l}\text{A rising curve with } dy/dx > 0\text{, by the symbol } y^{+} \\ \text{A falling curve with } dy/dx < 0\text{, by the symbol } y^{-}\end{array}\right\} \qquad (60)$$

Concave and Convex Curves.—To designate the curvature of a surface curve, it will be assumed that the observer is located above the stream. Hence, curves with the radius of curvature directed upwards (Fig. 59*a*) will appear as *concave*, while curves, as shown in Fig. 59*b*, with the radius of curvature pointing downwards, will be *convex*.

Classes of Curves.—With reference to the zones of flow, as pictured in Fig. 44, all the possible surface curves can be divided into certain classes, depending on the relative position of the y_0 and y_{cr} lines. The three possible combinations are given in Fig. 60, the respective classes being designated by the capital letters M, S, and C. For each class, there will further be three zones which are marked by numbers placed at the foot of the respective letter which indicates the class.

The relative position of the y_0 and y_{cr} lines depends on the value of the bottom slope. Flow belonging to Class M, with y_0 greater than y_{cr}, will take place when the bottom slope will be *mild* with $s_0 < \sigma_0$. The letters designating the classes have been chosen accordingly.

Presenting the matter in tabular form, we have:

Class M: Bottom slope—*mild;* $y_0 > y_{cr}$; $s_0 < \sigma_0$
uniform flow—*tranquil*
Class S: Bottom slope—*steep;* $y_0 < y_{cr}$; $s_0 > \sigma_0$
uniform flow—*rapid*
Class C: Bottom slope—*critical;* $y_0 = y_{cr}$; $s_0 = \sigma_0$
uniform flow—*critical*
(61)

The location of the zones is clear from Fig. 60. Zone 1 in each and every case *lies above* both the *uniform* and the *critical* depth. Zone 3 in each and every case lies *below* both y_0 and y_{cr}.

Zone 2 lies between y_0 and y_{cr}, with the relative position of y_0 and y_{cr} depending on the value of $s_0 \gtrless \sigma_0$.

In the C class, with $s_0 = \sigma_0$, Zone 2 vanishes. There remain only Zones 1 and 3.

Types of Curves.—To each zone there belongs one, and only one, "type" of surface curve. The type will be designated by the sign of the respective zone. So, for example an M_2 curve will mean, that the particular surface curve is located in Zone 2, Class M. It will be a surface curve in a channel characterized by $y_0 > y_{cr}$; $s_0 < \sigma_0$. The curve itself, due to the sub. 2 mark position, will embrace depths in the limits $y_{cr} < y < y_0$. Obviously, there can be only *eight possible types* of surface curves, one to each zone; namely, three of the M class, three of the S class, and two of the C class.

We shall proceed, now, to establish the properties and features of such curves. We shall make use of the specific energy diagram (Fig. 47) in its relation to the states of flow; also of the rules governing the change of energy in a flowing stream, summarized in Eq. (25).

29. Energy Balance in Y^+ and Y^- Curves.—With reference to the change of specific energy from section to section in the direction of flow, one should distinguish

1. Flow with a gain of specific energy.
2. Flow with a loss of specific energy.
3. Flow with constant specific energy.

That or other contingency depends on the *position* of the *varying depth* y with regard to the *normal depth* y_0. With reference to the rules (Eq. [25]), we have in tabular form:

$$\left.\begin{aligned} y > y_0,\ \delta\epsilon/\delta x > 0;\ \epsilon^+ \text{ curve} \\ y < y_0,\ \delta\epsilon/\delta x < 0;\ \epsilon^- \text{ curve} \\ y = y_0,\ \delta\epsilon/\delta x = 0;\ \epsilon^0 \text{ curve} \end{aligned}\right\} \qquad (62)$$

The question whether a particular curve will be a y^+ or a y^- curve, or, in other words, whether ϵ^+ and ϵ^- will lead to $dy/dx > 0$ or to $dy/dx < 0$, and *vice versa*, will depend on whether the movement under consideration is on the *upper* or on the *lower* portion of the ϵ *curve; i.e.*, whether the movement in question is in tranquil or in rapid state.

In *tranquil* movement, with $y > y_{cr}$, the energy increases with the depth, $\delta\epsilon/\delta y > 0$, hence, when

$$\left.\begin{array}{l|c|l} y > y_0;\ \delta\epsilon/\delta x \text{ is } > 0 & \text{which} & dy/dx > 0 \\ y < y_0;\ \delta\epsilon/\delta x \text{ is } < 0 & \text{leads to} & dy/dx < 0 \end{array}\right\} \tag{63}$$

we have in the case of $y > y_0$ a y^+, and in the case of $y < y_0$ a y^- curve; the signs of $\delta\epsilon/\delta x$ and dy/dx are thus *identical*.

In *rapid* state, the contrary is the case. With $y < y_{cr}$, a gain in energy translates itself into a decrease of depth, $\delta\epsilon/\delta y < 0$. Hence, when

$$\left.\begin{array}{l|c|l} y > y_0;\ \delta\epsilon/\delta x \text{ is } > 0 & \text{which} & dy/dx < 0 \\ y < y_0;\ \delta\epsilon/\delta x \text{ is } < 0 & \text{leads to} & dy/dx > 0 \end{array}\right\} \tag{64}$$

Thus, in rapid movement, in case of $y > y_0$, we have a y^- curve, while in case of $y < y_0$ the curve is y^+; the signs of $\delta\epsilon/\delta x$ and dy/dx are *opposite*.

With this in mind, it will be easy to determine the sign of dy/dx in each and every zone.

Zone 1. As this zone lies always above the normal depth ($y > y_0$), there will always be $\delta\epsilon/\delta x > 0$. Hence, curves in Zone 1 in all classes will be ϵ^+ curves.

As, on the other hand, with $y > y_{cr}$, the movement in this zone is always tranquil, the gain of energy will lead to an increase of depth. Therefore, in all cases $dy/dx > 0$. So, the surface curves in Zone 1 in all the classes will be of the y^+ type.

Zone 3. For Zone 3, which in all cases lies below the normal depth with $y < y_0$, we always have $\delta\epsilon/\delta x < 0$. All curves are of the ϵ^- type. The flow in Zone 3, on the other hand, with $y < y_{cr}$, will be always rapid. Loss of energy leads to an increase of depth. Hence, in all classes, $dy/dx > 0$, the curves being always of the y^+ type.

Zone 2. In Zone 2, dy/dx is always negative. In fact for curve M_2, with $y < y_0$, we have $\delta\epsilon/\delta x < 0$, so that M_2 is an ϵ^- curve. On the other hand, with $y > y_{cr}$, the movement is tranquil; hence loss of energy results in a lowering surface curve, so that $dy/dx < 0$; a y^- curve. In the case of curve S_2, $y > y_0$, so that $\delta\epsilon/\delta x > 0$; the curve is ϵ^+. On the other hand, with $y < y_{cr}$, the movement is rapid; gain of energy requires a decrease in depth, leading to $dy/dx < 0$; a y^- curve. Tabulating, we obtain as Eq. (65):

Zone	Class M $y_0 > y_{cr}$ $s_0 < \sigma_0$	Class C $y_0 = y_{cr}$ $s_0 = \sigma_0$	Class S $y_0 < y_{cr}$ $s_0 > \sigma_0$
1	$y > y_0$, $\delta\epsilon/\delta x > 0$: ϵ^+ curve $y > y_{cr}$—Tranquil state—$\delta\epsilon/\delta y > 0$ $dy/dx > 0$—Rising curve: y^+		
2	$y < y_0$, $\delta\epsilon/\delta x < 0$: ϵ^- curve $y > y_{cr}$, Tranquil state—$\delta\epsilon/\delta y > 0$ Falling curve: $dy/dx < 0$: y^-	No curve	$y > y_0$, $\delta\epsilon/\delta x > 0$: ϵ^+ curve $y < y_{cr}$, Rapid state: $\delta\epsilon/\delta y < 0$ Falling curve: $dy/dx < 0$: y^-
3	$y < y_0$, $\delta\epsilon/\delta x < 0$: ϵ^- curve $y < y_{cr}$—Rapid state: $\delta\epsilon/\delta y < 0$ $dy/dx > 0$—Rising curve: y^+		

Conditions at Boundaries.—The following general properties are common to all surface curves:

1. The curves are asymptotically tangent to the uniform depth line y_0.
2. The curves are perpendicular to the critical depth line y_{cr}.
3. With the increase of depth, the curves tend to become tangent to a horizontal pool line.

(66)

To demonstrate, we shall make use of the varied flow equation (Eq. [44])

$$\frac{dy}{dx} = s_0 \frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - (\mathfrak{M}_c/\mathfrak{M})^2}$$

In the following it will be assumed, that the values of both the $\mathfrak{K} = aC\sqrt{R}$ and the $\mathfrak{M} = a\sqrt{a/b}$ function continuously increase with the depth, so that for all values of y, both $\frac{d\mathfrak{K}}{dy} > 0$ and $\frac{d\mathfrak{N}}{dy} > 0$. This practically is no restriction, as all *open* profiles comply with the above requirements. The exception would be only in case of closed profiles, and then only for the very small range near the top of a conduit, beyond a certain depth y_m (Fig. 61), which is known to correspond to a maximum discharge and therefore to a $\mathfrak{K}_{max}$. Exceptions of this kind, however, are of no practical importance so far as varied flow is concerned.

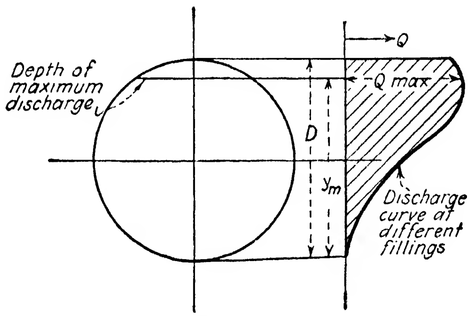

FIG. 61.

1. The asymptoticity of the curves to the normal depth line follows from the fact that with the approach of y towards y_0, the value of $\mathfrak{K}$ approaches $\mathfrak{K}_0$, which makes the numerator in Eq. (44) have a limit

$$\text{lim. } (1 - [\mathfrak{K}_0/\mathfrak{K}]^2)_{y=y_0} = 0$$

Hence

$$\text{lim. } (dy/dx)_{y=y_0} = 0$$

2. The perpendicularity of the curves to the critical depth line follows from the fact, that with $y = y_{cr}$, $\mathfrak{N}$ approaches $\mathfrak{N}_c$. The denominator becomes

$$\text{lim. } (1 - [\mathfrak{N}_c/\mathfrak{N}]^2)_{y=y_{cr}} = 0$$

which makes

$$\text{lim. } (dy/dx)_{y=y_{cr}} = \sim$$

3. Notice that with the increase of the depth, both $\mathfrak{K}$ and $\mathfrak{M}$ grow, making $(\mathfrak{K}_0/\mathfrak{K})^2$ and $(\mathfrak{M}_c/\mathfrak{M})^2$ small.
Hence

$$\lim. \left(\frac{1 - [\mathfrak{K}_0/\mathfrak{K}]^2}{1 - [\mathfrak{M}_c/\mathfrak{M}]^2}\right)_{y=\sim} = 1$$

and

$$\lim. (dy/dx)_{y=\sim} = s_0$$

which corresponds to a horizontal line at an angle α_0 ($\sin\ \alpha_0 = s_0$) with the bottom line.

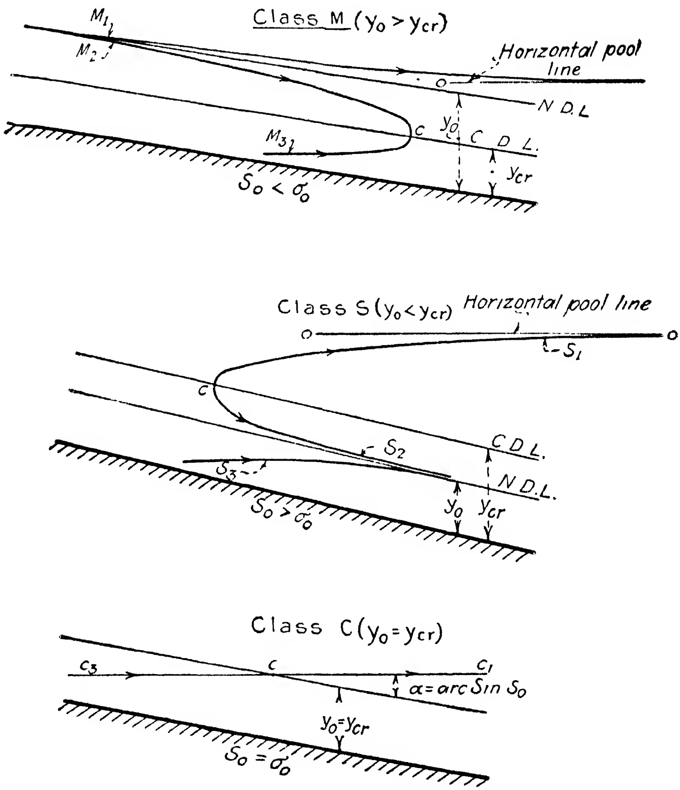

FIG. 62.—General outline of the different types of surface curves.

30. The Outline of Curves.—The properties of the curves as established in Eq. (66), together with the sign of dy/dx (tabulated in Eq. [65]), determine the outline of the curve of each particular type. A summary is given in Fig. 62. May it be said for clarity that while the different parameters affect numerical features, the general outline of the curves remains identical and is completely covered by the schematic presentation given in Fig. 62. We now shall review in brief the different curves, making particular reference to the practical instances where each type may take place.

1. *Class M. Watercourses of Mild Slope;* $y_0 > y_{cr}$. *Type* M_1. A concave rising curve, tangent *from above* to the normal depth line y_0 and to the horizontal pool level line 0. This curve is the most important type from the practical point of view. It is featured in the case of a backwater curve in a natural river with mild slope in Fig. 3; in a canal in Fig. 2*b*; AB' in Fig. 4; and stretch 3-4 in Fig. 11.

Type M_2. A convex falling curve, tangent *from beneath* to y_0 and ending in a hydraulic drop near *c*. The curve is featured in Fig. 9, stretch *Ac*; in stretch 5-6 in Fig. 11; also in Fig. 2*c*; and AB'' in Fig. 4.

Type M_3. A concave rising curve, leading to a jump near the critical depth. This type occurs when flow in a highly rapid state enters a flume of mild slope. It takes place in a stream below a sluice in stretch 1-2, Fig. 11, at the foot of a weir (Fig. 8); another instance is a flume (Fig. 63), with a break in the bottom slope from steep ($s_{01} > \sigma$) to mild ($s_{02} < \sigma$). The M_3 curve lies between the breaking point *a* and the foot of the jump j_1.

2. *Class S. Watercourses of Steep Slope;* $y_0 < y_{cr}$. *Type* S_1. A convex rising curve, beginning with a jump and tangent *from beneath* to the horizontal pool level 0-0. The curve is featured in Fig. 6, stretch 2-A; also in Fig. 46 and, finally, in Fig. 64, stretch *a*-*b*, which represents a *submerged* flume with a steep slope.

Type S_2. A concave falling curve, usually comparatively short and more in the nature of a *transition section* between a hydraulic drop and uniform flow. Such is the particular case in Fig. 9, stretch cB, and beyond section 7 (Fig. 11). Another case is Fig. 65, a flume with a break

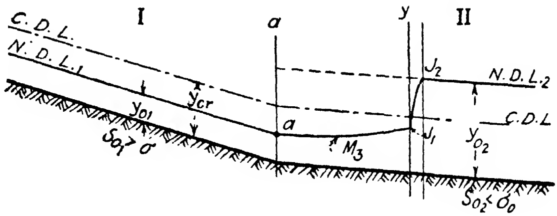

FIG. 63.

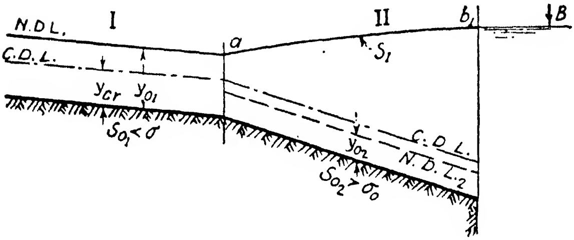

FIG. 64.

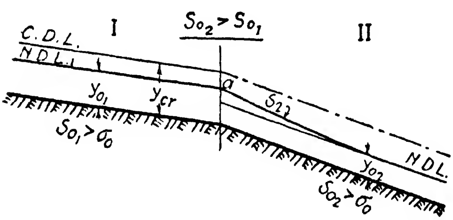

FIG. 65.

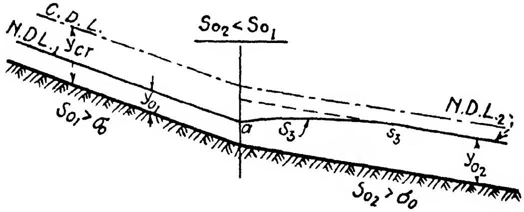

FIG. 66.

FIGS. 63–66.—Flow in a canal with a break in the bottom slope.

in the bottom slope, from steep to steep, with s_{02} in particular being greater than s_{01}.

Type S_3. A convex rising curve, also of the transition type, between a source of highly rapid movement and the uniform depth line, to which the curve is *tangent from beneath*. An example is given in Fig. 66, where the order of succession of two steep-sloped flumes is reversed by comparison with Fig. 65. Another instance is Fig. 67, where flow from under a sluice with the depth in the *vena contracta* d_1, continues in a steep-sloped channel, the normal depth y_0 being $> d_1$.

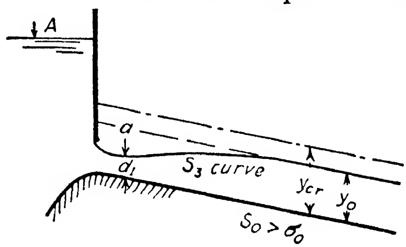

FIG. 67.—Example of an S_3 type surface curve.

3. *Class C. Watercourses of Critical Slope;* $y_0 = y_{cr}$.—With the bottom slope $s_0 = \sigma_0$, this case is intermediary between Class M and Class S. Naturally, the C_1 curve would be an intermediary between the concave M_1 and the convex S_1 curves; and the C_3 curve an intermediary between the concave M_3 and the convex S_3 type. Such an intermediary must be a straight line.

The varied flow equation (46), because of $s_0 = \sigma_0$, becomes in this case

$$\frac{dy}{dx} = s_0 \frac{1 - (\mathfrak{K}_0/\mathfrak{K})^2}{1 - \dfrac{\sigma_0}{\sigma}\left(\dfrac{\mathfrak{K}_0}{\mathfrak{K}}\right)^2}$$

For $y = y_0$, $\sigma_0/\sigma = 1$. For $y > y_0$, σ_0/σ is usually slightly greater than 1; the opposite is the case for $y < y_0$. The deviation of σ_0/σ from unity, however, is usually not substantial. Now, $\sigma_0/\sigma = 1$ makes

$$dy/dx = s_0 \tag{67}$$

which represents a *horizontal* line intersecting with the $y_0 = y_{cr}$ line at the angle $\alpha = \text{arc sin } s_0$. An example of the C curves is given in Fig. 68. A horizontal C_3 surface line connects the *vena contracta* with the foot of the jump j_1; while a horizontal surface C_1 line connects the end of the jump j_2 with level B above the weir. Another instance is that of a flume with a critical bottom slope emptying into a pool (Fig. 69*b*), or into a mild-slope channel (Fig. 69*a*).

In this last case the point of intersection of the horizontal C line with the normal depth lines y_0 would seem to

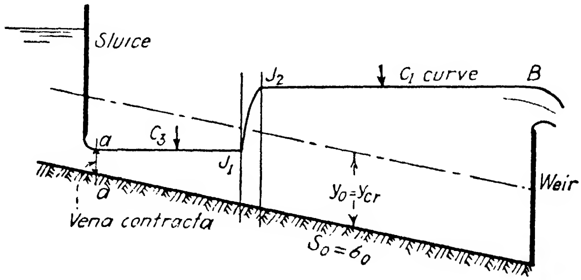

FIG. 68.—Surface curves of the C type.

feature a break of continuity in the junction points a and c. Also, when viewed in the light of the general properties of surface curves as specified in (66), there is an apparent inconsistency between the fact that at the junction point,

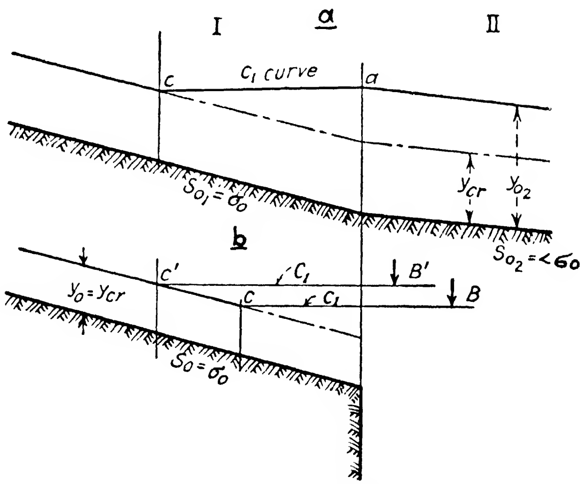

FIG. 69.—Examples of C_1 surface curves.

where y is simultaneously equal to y_0 and to y_{cr}, the surface curve should be simultaneously perpendicular and tangent to the $y_0 = y_{cr}$ line. This contradiction finds its analytical expression in the undetermined value of dy/dx which in accordance with Eq. (44) for $y = y_0 = y_c$ becomes

$$dy/dx = 0/0$$

The physical essence of the phenomenon, however, becomes clear, if one were to follow the formation of a junction point c in Fig. 69 in its evolution out of a case of flow, as presented in Fig. 68. In fact, suppose that level B were to be gradually lowered by reducing the height of

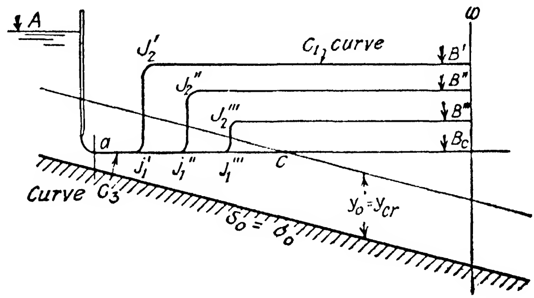

FIG. 70.

the weir. The lowering of the level from B' to B'', B''', etc., in Fig. 70 would result in the jump receding stream downwards, from j'_1 to j''_1, j'''_1 etc., the height of the jump being each time reduced. The vertical distance between the C_1 and the C_3 line would become smaller and smaller, until, with B reaching B_c, the lines would merge, the jump at that becoming *infinitely small.* The intersection point c in Fig. 69 corresponds, therefore, to the limiting case of a jump *infinitely small in height.*

CHAPTER VIII

INTEGRATION OF THE VARIED FLOW EQUATION

31. Introductory. Historical.—By separating the variables in Eq. (48), one obtains

$$s_0 dx = \frac{1 - \beta(\mathfrak{K}_0/\mathfrak{K})^2}{1 - (\mathfrak{K}_0/\mathfrak{K})^2} dy = dy + (1 - \beta)\frac{dy}{(\mathfrak{K}/\mathfrak{K}_0)^2 - 1} \quad (68)$$

The length of the reach $l_{2,1} = x_2 - x_1$ (Fig. 71) between the two sections, the depths in which are respectively y_2 and y_1, is

$$l_{2,1} = x_2 - x_1 = \frac{1}{s_0}\left[(y_2 - y_1) + \int_{y_1}^{y_2} \frac{(1 - \beta)dy}{(\mathfrak{K}/\mathfrak{K}_0)^2 - 1}\right] \quad (69)$$

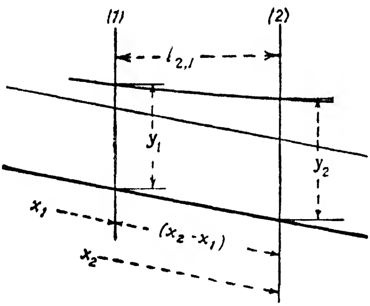

FIG. 71.—The length of a reach $l_{2,1}$ between integration limits y_2 and y_1.

With the elements of flow given, in the sense of Arts. 21 and 22, the expression under the integral $\frac{(1 - \beta)}{(\mathfrak{K}/\mathfrak{K}_0)^2 - 1}$, is a function of y only and may be designated as $\Theta(y)$. It may be computed point by point and drawn as a curve, Fig. 72. The value of the integral $\int_{y_1}^{y_2}\Theta(y)dy$ is the shaded area in Fig. 72; its numerical value may be computed by any of the usual processes of approximate integration, analytical or graphical. With the value of the integral known, the distance $l_{2,1} = x_2 - x_1$ is determined from Eq. (69). The method as outlined is general and is applicable without any limitations whatsoever.

Approximate integration, on the other hand, either graphical or analytical, is always cumbersome and justly unpopular. It is natural, therefore, that from early days methods were sought by which the task of figuring out a surface curve could be reduced to some simple analytical procedure. Since Dupuit (1848) the preferred method has been to substitute for the actual canal cross-section some "idealized" profile of simple form which, with other simplifying assumptions, would reduce the integration of the equation of varied flow to a quadrature. Dupuit and later Rühlmann and Bresse chose as such idealized profile that of a rectangular canal of great width (Fig. 73).

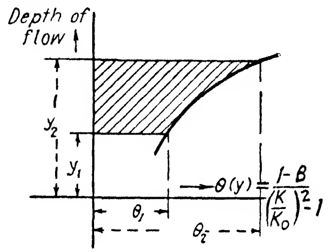

FIG. 72.—Graphical representation of the integral $\int_{y_1}^{y_2} \theta(y)dy$

On the other hand, Tolkmitt, who was particularly interested in natural watercourses, selected a parabolic profile (Fig. 74). The above authors assumed, moreover, in their cases a constant value of the Chézy friction factor C throughout the whole range of depths. To facilitate practical calculations special Tables were prepared giving the numerical values of the quadrature in question.

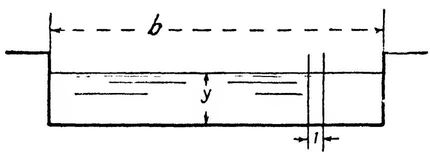

FIG. 73.

The main defect of these methods lies obviously in the fact that the idealized cross-sections have little in common with the cross-sectional forms which the engineer deals with in actual practice. Moreover, there were no means to judge of the degree of approximation and error inherent in the procedure.

About 1912, the author, inspired in general by the work of Bresse and Tolkmitt, devised a method which allows us to approach the solution with much closer precision, and which moreover permits us to estimate the magnitude and character of the possible errors.

32. The Hydraulic Exponent.—The method suggested by the author is based on the empirically established fact, that the conveyance function $\mathfrak{K} = aC\sqrt{R}$, within a reasonable range of depths, follows sufficiently close the exponential relations:

$$\left.\begin{aligned} \mathfrak{K}^2(y) &= a^2C^2R = \text{const. } y^n \\ (\mathfrak{K}/\mathfrak{K}_0)^2 &= a^2C^2R/a_0{}^2C_0{}^2R_0 = (y/y_0)^n \end{aligned}\right\} \quad (70)$$

We call the exponent n in Eq. (70) the *hydraulic exponent.* Obviously it is another characteristic of a cross-section, to be added to those summarized in Art. 21. Equation (70), as mentioned, is approximate. Some cross-sections comply more, others less. However, the probing of cross-sections of the most different forms and nature confirms one in the view that Eq. (70) holds remarkably well. In fact, one becomes impressed by the absence and the unimportance of the deviations from the law, rather than by the departures and exceptions to the rule.

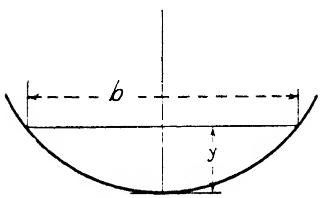

FIG. 74.

To determine the value of n, plot $\mathfrak{K} = aC\sqrt{R}$ in logarithmic scale. By drawing a straight line (Fig. 75) to follow the points, one obtains the value of the exponent as twice the value of $tg\alpha$. Examples for a series of canal cross-sections are given in Plate II.

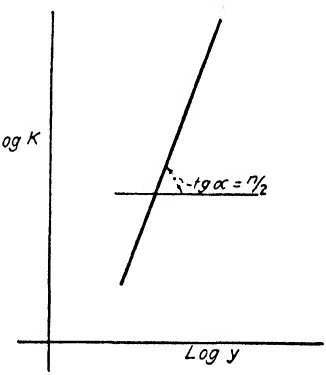

FIG. 75.—Logarithmic plotting of the conveyance curve.

The lines, as drawn in Plate II, give the average value of n for the cross-section as a whole. In most practical cases one has to determine the surface curve only within a certain limited range of depths.

From Eq. (70), we have for any depth

$$n = 2\frac{\text{Log }(\mathfrak{K}[y]/\mathfrak{K}[y_0])}{\text{Log }(y/y_0)} \tag{71}$$

Applying Eq. (71) to the limiting depths y_a and y_b of the range, one finds the limiting values of n_a and n_b. Usually they will be sufficiently close to assume an average. If, by chance, greater precision is advisable, the range of depths in question can be subdivided; however, as practice shows this is scarcely ever required.

It can be easily shown, further, that the cases treated by Bresse and Tolkmitt constitute particular cases of Eq. (70), corresponding respectively to a hydraulic exponent of $n = 3$ and $n = 4$.

Rectangular Section of Great Width (Bresse).—The term is applied to a cross-section (Fig. 73), the width of which is sufficiently large compared to the depth to make the hydraulic radius

$$R = \frac{a}{p} = \frac{by}{b + 2y} = y\left(1 - \frac{2y}{b + 2y}\right) = \sim y$$

Assuming further a constant value for the friction factor C, one obtains

$$\mathfrak{K} = aC\sqrt{R} = \text{const.}\cdot y^{3/2}$$

Hence

$$(\mathfrak{K}/\mathfrak{K}_0)^2 = (y/y_0)^3 \text{ and } n = 3 \tag{72}$$

Parabolic Section (Tolkmitt).—Assuming in a parabolic section (Fig. 74) the width sufficiently large compared to the depth, so that $p = \sim b$, one has

$b = \text{const. } \sqrt{y}$; $a = \frac{2}{3}b \cdot y = \text{const.}\cdot y^{3/2}$;

$R = a/p = \sim\frac{2}{3}y$; assuming a constant value for C,

$$\mathfrak{K} = \text{const.}\cdot y^2$$

Hence

$$(\mathfrak{K}/\mathfrak{K}_0)^2 = (y/y_0)^4;\ n = 4 \tag{73}$$

Correction for a Varying Friction Factor C.—For this purpose an exponential expression of the friction factor is best used

$$C = C_0 R^p \tag{74}$$

as recommended, for example, by Manning:

$$C = C_0 R^{1/6} = \frac{1.49}{n} R^{1/6} \tag{75}$$

where $1/n$ is the reciprocal of the Ganguillet-Kutter friction factor.

It appears, however, that better compliance with experimental results may be obtained by using instead of a constant exponent, as used by Manning, a varying exponent value, which increases with the roughness of the walls. Thus, for canals in earth or of gravelly soil, the value of p has been found to lie between 0.20 and 0.25. If, instead of a constant C, one were to make in the Bresse and Tolkmitt equations

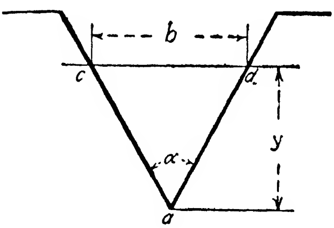

Fig. 76.

$$C = C_0 R^{0.25}$$

the hydraulic exponent instead of $n = 3$ and $n = 4$ would become $n = 3.5$ and $n = 4.5$, respectively. This would correspond to the solution suggested by Schaffernack.*

Trapezoidal Canals. The Limiting Values of n.—With regard to trapezoidal canals, in most practical cases the hydraulic exponent will be found to be between $3 < n < 4$. The surface curve, therefore, will be intermediary between the solutions obtained from the Bresse and the Tolkmitt Tables, respectively.

The *largest possible value of n* is that for a triangular canal (Fig. 76). The geometrical elements being similar, the cross-section is proportionate to y^2, and p and R are proportionate to y, which makes $\mathfrak{K} = \text{const.}\ C \cdot y^{5/2}$ or, with Eq. (74),

$$\mathfrak{K}^2 = \text{const.}\ y^{5+2p} \tag{76}$$

* See Appendix I.

With p varying between 0.15 and 0.25, the maximum value of the hydraulic exponent for a triangular section becomes

$$n = 5.3\text{–}5.5 \tag{77}$$

The *smallest value of n* corresponds to the case of a very narrow rectangular section (Fig. 77), the width of which is very small compared to the depth. In such case we may put approximately:

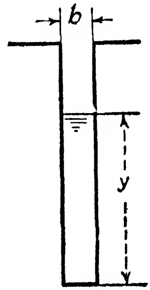

Fig. 77.

$$p = 2y + b = 2y\left(1 + \frac{b}{2y}\right) = \sim 2y;$$

$$R = \frac{by}{2y} = \sim\frac{b}{2} = \text{const.}$$

R constant makes C constant, hence

$$\mathfrak{K} = \text{const. } y;\ n = 2 \tag{78}$$

Obviously, $n = 2$ and $n = 5.5$ are extreme values. They seldom occur in practice. However, a deep canal with vertical walls oftentimes features a hydraulic exponent

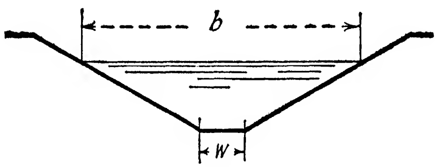

Fig. 78.

below $n = 3$; while, on the other hand, a trapezoidal canal, the bottom width of which is small in comparison with the depth (Fig. 78), may give $n = 4.5$ and over.

33. The Varied-flow-function Tables.—With

$$(\mathfrak{K}/\mathfrak{K}_0)^2 = (y/y_0)^n$$

Eq. (68) becomes

$$s_0 dx = dy + (1 - \beta)\frac{dy}{(y/y_0)^n - 1} \tag{79}$$

Designating

$$y/y_0 = \eta;\ \text{so that } dy = y_0 d\eta \tag{80}$$

we obtain

$$dx = \frac{y_0}{s_0}\left[d\eta + (1 - \beta)\frac{d\eta}{\eta^n - 1}\right] \tag{81}$$

As mentioned previously, the value of $\beta = s_0/\sigma$ does not change substantially and, moreover, the change is usually gradual and slow. So one may subdivide the integration range into intervals going by the depths, and assume for each depth interval a certain constant average value of $1 - \beta$. Within each such interval, $1 - \beta$ becomes then an integration constant.

Assuming now, with reference to Fig. 71, that the depths y_1 and y_2 lie within an interval for which an average constant value for $1 - \beta$ has been assumed, and integrating Eq. (81) between the limits y_2 and y_1, the length of the reach $l_{2,1}$ (being the distance between the two sections 2 and 1, the depths in which are respectively y_2 and y_1) will be

$$l_{2,1} = x_2 - x_1 = \frac{y_0}{s_0}\left[(\eta_2 - \eta_1) + (1 - \beta)\int_{\eta_1}^{\eta_2} \frac{d\eta}{\eta^n - 1}\right] \qquad (82)$$

In this equation, in accordance with Eq. (80), the integration limits y_1 and y_2 are substituted by $\eta_1 = y_1/y_0$; $\eta_2 = y_2/y_0$.

The problem has thus been reduced to a *quadrature.* Designating the value of the integral

$$\int_0^{\eta} \frac{d\eta}{\eta^n - 1} = \text{const.} - B(\eta) \qquad (83)$$

and assuming that the values of $B(\eta)$ for the different η are known, we obtain:

$$x_2 - x_1 = l_{2,1} = \frac{y_0}{s_0}[(\eta_2 - \eta_1) - (1 - \beta)(B(\eta_2) - B(\eta_1))] \qquad (84)$$

By designating

$$\eta - (1 - \beta)B(\eta) = \Pi(\eta) \qquad (85)$$

Eq. (84) may be presented in the form

$$x_2 - x_1 = l_{2,1} = \frac{y_0}{s_0}[\Pi(\eta_2) - \Pi(\eta_1)] \qquad (86)$$

To make Eq. (84) or (86) usable in actual practice, the values of $B(\eta)$ have been computed for a series of exponents, covering the whole practical range. The respective

$B(\eta)$ values are given in table form at the end of the book. In figuring the $B(\eta)$ values, the integration constant in Eq. (83) was assumed const. = 0, so that the table figures represent the numerical value of

$$B(\eta) = -\int_0^\eta \frac{d\eta}{\eta^n - 1} \tag{87}$$

We call the $B(\eta)$ function (Eq. [87]) the *varied flow function,* and the *tables* will be referred to accordingly. For the range of n from 2.8 to 4.2, which is the most important, the $B(\eta)$ values have been computed for lines of exponents at an interval of 0.2. Beyond $n = 4.2$, the exponent intervals are larger. Wherever the exponent value, inherent in a particular case, falls in between the table lines, a simple interpolation allows us to obtain the intermediary solutions with sufficient precision.*

Example 7

The particular purpose of this example is to familiarize the reader with the use of the *varied-flow-function tables* as well

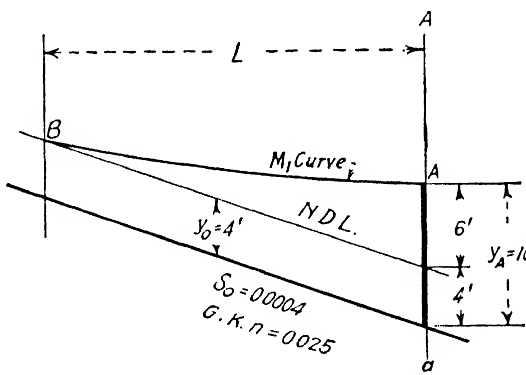

FIG. 79.—Layout of a canal for Example 7. The *end* of the curve at point B corresponding to η_B = 1,001 or 1,01 η_0.

as with the general technique of varied flow computations. Recourse, therefore, is taken to the simplest case of a rectangular canal of great width (Fig. 73).

* The methods used for computing the $B(\eta)$ tables and other particulars are given in Appendix II.

The canal (Fig. 79) is laid with $s_0 = 4^0/_{00}$; the roughness of the walls is taken to correspond to G.K., $n = 0.025$. Uniform flow in natural condition occurs with $y_0 = 4$ ft., corresponding to a discharge per unit width of

$$q = C_0 y_0^{3/2} \sqrt{s_0} = 76 \times 4^{3/2} \times \sqrt{4} \cdot 10^{-2} = 12.2 \text{ cu. ft. per second.}$$

It is assumed, further, that the level in A is raised by $z = 6$ ft., making $y_a = 10$ ft.

Question 1. Determine and trace the varied flow surface curve.

First, one should establish the type of flow. One of the depth parameters, $y_0 = 4$ ft., is given; the other, the critical depth, is

$$y_{cr} = \sqrt[3]{q^2/g} = \sqrt[3]{12.2^2/32.2} = 1.66 \text{ ft.}$$

With $y_0 = 4$ ft. $> y_{cr}$, flow is of the M class. With $y_a > y_0$, the curve is of the M_1 type—a backwater curve in a mild-sloped channel.

To apply Eq. (84), one should determine:

a. The value of the hydraulic exponent n.

b. The 1-β curve.

Exponent n.—For a unit width, with $R = \sim y$, the conveyance is $\mathfrak{K} = C \cdot y^{3/2}$. Accordingly

TABLE VI

y	1 ft.	1.5 ft.	2 ft.	3 ft.	4 ft.	6 ft.	10 ft.
C	55.00	61.00	65.00	71.00	76.00	81.0	88.0
$\mathfrak{K}$*	0.55	1.12	1.84	3.70	6.08	11.9	27.9

* In units of 10^2.

Logarithmic plotting (canal type E in Plates I and II) gives tan $\alpha = 1.70$ and $n = 3.40$.

The 1-β Curve.—For a canal of large width, $p/b = 1$; hence, the critical slope (Eq. [43]) is simply $\sigma' = g/C^2$, the values of which are given in Fig. 40. The normal critical slope (σ for $y_0 = 4$ ft.) is

$$\sigma_0 = 55.6^0/_{00};\ \beta_0 = s_0/\sigma_0 = 4/55.6 = 0.072;\ 1 - \beta_0 = 0.928$$

For other depths, $\beta = 0.072\frac{\sigma_0}{\sigma}$, which gives

TABLE VII

y	2 ft.	3 ft.	4 ft.	6 ft.	10 ft.
σ in $^0\!/_{00}$	76.2	63.9	55.6	49.0	41.5
σ_0/σ	0.730	0.872	1.000	1.130	1.340
$\beta = 0.072\frac{\sigma_0}{\sigma}$	0.052	0.063	0.072	0.082	0.096
1-β	0.948	0.937	0.928	0.918	0.904

The 1-β curve is traced in Fig. 80. For integration purposes, the total depth range, from 4 to 10 ft., is divided into three

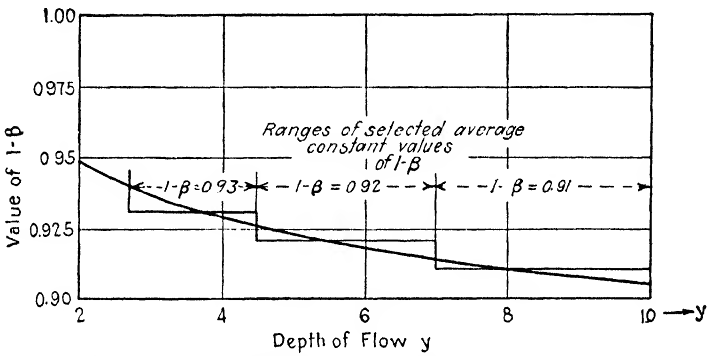

FIG. 80.—Selecting average constant values of $1 - \beta$ for depth intervals in Example 7.

portions, each corresponding to an average constant value of 1-β = 0.91, 0.92, and 0.93.

Integration Limits. The Length of a Curve.—The lower limit (in the direction of the flow) is given through $y_a = 10$ ft. For this limit $\eta_a = y_a/y_0 = {}^{10}\!/_4 = 2.5$. The upper limit is the "end" of the backwater curve. Strictly speaking, as the curve approaches the normal depth asymptotically, its length is infinite. In practice, however, one assumes the curve as "ending" in a certain section (B in Fig. 79), where the difference between the varied flow depth y and the normal depth line y_0 dwindles down to a certain assumed small value. Usually such difference is fixed in relative terms, to be for example 1 per cent or

0.1 per cent of y_0. Accordingly the end of the curve is determined by $\eta = y_b/y_0 = 1.01$ or $\eta = 1.001$, respectively.

Assuming for our example, $\eta = 1.001$, we obtain the integration limits:

$$y_a = 10 \text{ ft}; \ \eta_a = {}^{10}\!/_4 = 2.5$$
$$\eta_b = 1.001; \ y_b = 4.004 \text{ ft}.$$

Integration Procedure.—The expedient way is to divide the integration range into a series of intervals, going by the depths, and then to determine, by means of Eq. (84) or Eq. (86), the length of the reach, corresponding to each depth interval.

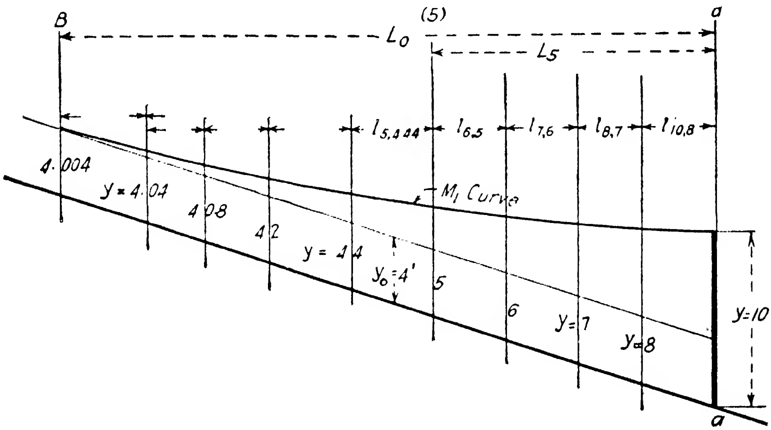

FIG. 81.—Dividing the depth range of Fig. 79 into intervals.

For example, in our particular case, the depth range may be divided as in Fig. 81, making the successive depths $y = 10$ ft., 8 ft., 7 ft., . . . ; 4.08, 4.04, and finally 4.004 ft.

Remark: It will be found expedient to designate a section by the value of the depth in that particular section. So section 5 means a section in which $y = 5$ ft. The distance between two successive sections will be designated by $l_{n,m}$; the first of these marks will correspond to the section which is *lowest* in the direction of the flow. So $l_{6,5}$ will mean the distance $x_6 - x_5$ between section 6 and section 5, with section 6 lying *stream downward* of section 5.

In applying Eqs. (84) and (86) to an interval between two sections with the depths y_n and y_m (Fig. 82), keep in mind that Eq. (82) is established under the assumption of Fig. 71; namely, that the depth, noted as y_2, was situated *downward* of y_1 in the direction of the flow. Accordingly, in Fig. 82, y_n with $\eta_n = y_n/y_0$ will be the upper limit and y_m with $\eta_m = y_m/y_0$ the lower limit of the integral in Eq. 82. Thus, in Eqs. (84) and (86) y_n corresponds to y_2; and y_m to y_1.

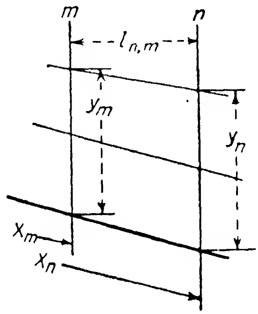

FIG. 82.

Remembering this simple rule, use the *varied-flow-function tables* and carry out the computations strictly algebraically. If the location of the depths as in Fig. 82 was assumed correctly, the distance $l_{n,m}$ will result from Eq. (84) as *positive*. If, by any chance, the distance were to come out *negative*, it would mean that the relative position of the depths y_n and y_m was not assumed correctly, and that the reverse position was the case, meaning that y_m is to lie stream downwards of y_n.

By adding the successive partial distances l, beginning from the initial section, one obtains for any section m the total distance L_m and, thus, the location of the particular section with regard to the initial section. For example, in Fig. 81, the initial section being naturally section a, L_5 will designate the distance of the section with the depth $y = 5$ ft. from section a.

The distance from the initial section to the end of the curve shall be the total length of the curve and will be designated by L_0.

Returning to our example, compute the distance $l_{10,8}$ between $y_a = 10$ ft. and $y = 8$ ft.

The exponent, $n = 3.4$ in our case, corresponds directly to a table line; therefore, $B(\eta)$ may be taken straight from the tables.

We have, with reference to Fig. 71,

$$y_2 = 10 \text{ ft.}; \eta_2 = {}^{10}\!/\!_4 = 2.5; B(2.5) = 0.047$$

$$y_1 = 8 \text{ ft.}; \eta_1 = 8/4 = 2.0; B(2.0) = 0.082$$

For the interval 10–8, the value of $1-\beta$, from Fig. 80, is 0.91. Further, $y_0/s_0 = 4/4.10^{-4} = 10{,}000$ ft. Applying Eq. (84)

$$l_{10,8} = 10{,}000[(2.5 - 2) - 0.91(0.047 - 0.082)] = 10{,}000[0.5 + 0.032] = 5{,}320 \text{ ft.}$$

Repeating the procedure for the other intervals, we obtain:

TABLE VIII

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
y	$\eta = y/y_0$	$B(\eta)$ with $n = 3, 4$	$1-\beta$	$\Delta\eta$	$\Delta B(\eta)$	$-(1-\beta)\Delta B$	$\Delta\Pi$	l	L
					−	+			
10.000	2.500	0.047							0
				0.500	0.035	0.032	0.532	5,320	
8.000	2.000	0.082							5,320
			0.91	0.250	0.034	0.031	0.281	2,810	
7.000	1.750	0.116							8,130
				0.250	0.061	0.055	0.305	3,050	
6.000	1.500	0.177							11,180
				0.250	0.136	0.125	0.375	3,750	
5.000	1.250	0.313							14,930
			0.92	0.150	0.217	0.200	0.350	3,500	
4.400	1.100	0.530							18,430
				0.050	0.193	0.177	0.229	2,290	
4.200	1.050	0.723							20,720
				0.030	0.259	0.251	0.281	2,810	
4.080	1.020	0.982							23,530
			0.93	0.010	0.200	0.186	0.196	1,960	
4.040	1.010	1.182							25,490
				0.009	0.674	0.626	0.635	6,350	
4.004	1.001	1.856							31,840

In the above table, $\Delta\eta$ (Col. 5) is the difference between the neighboring values of η (Col. 2), so that $\Delta\eta = \eta_2 - \eta_1$ in Eq. (84). The same applies to $\Delta B(\eta)$ (Col. 6). Column 7 is the multiple of Col. 6 by Col. 4, taken in accordance with Eq. (84) with a minus sign. The result being positive, the figures of Col. 7 are added to those of Col. 5, giving in Col. 8, $\Delta\Pi = \Pi(\eta_2) - \Pi(\eta_1)$, see Eq. (86), which is identical with the value of the big parenthesis in Eq. (84).

The lengths of the reach l between the respective sections $l = \frac{y_0}{s_0}\Delta\Pi$ are obtained by multiplying Col. (8) values by $y_0/s_0 = 10^4$. Column 10 gives the total distance L of the particular section from the initial section corresponding to $y = 10$ ft. For any depth, L (Col. 10) is the sum total of all preceding reach lengths l (Col. 9). The surface curve $y = f(x)$ is traced in Fig. 83.

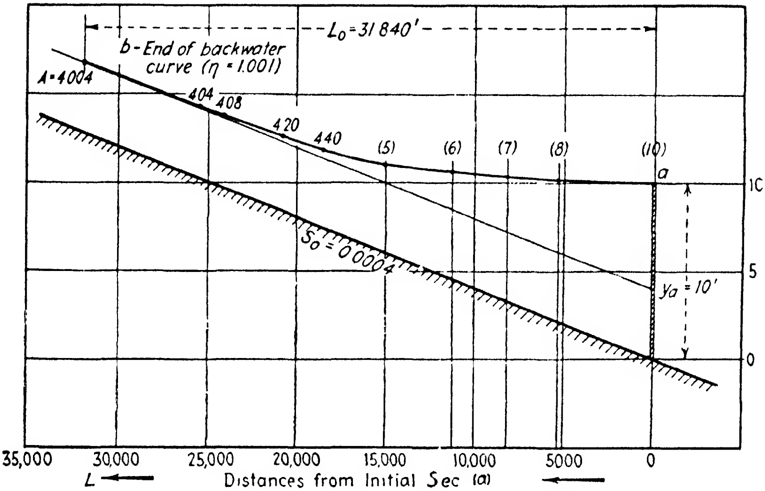

FIG. 83.—Tracing of the M_1 type backwater curve in Example 7, Question 1.

Question 2. Determine the depth y at a distance $L = 12{,}000$ ft. from section a.

This problem is the reverse to that treated in Question 1. There being given L and y_2 (Fig. 84), which means knowing $\Pi_2 = \eta_2 - (1 - \beta)B(\eta_2)$ (Eq. [86]), the solution lies in determining the particular value of η_1 which in

$$\Pi_1 = \eta_1 - (1 - \beta)B(\eta_1)$$

will satisfy the equation

$$\Pi_1 = \Pi_2 - L\frac{S_0}{y_0} \qquad (88)$$

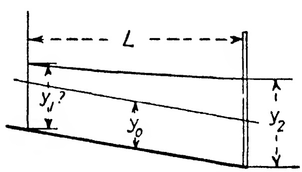

FIG. 84.—Determining the depth in a given section.

In applying Eq. (88), one should be certain to operate within an interval, where 1-β is an integration constant.

So in our case, knowing that $L_6 = 11{,}180$ ft., it will be expedient to make $y_2 = 6$ ft., and to determine the depth in a section (Fig. 85) situated at a distance $l = 12{,}000 - 11{,}180 = 820$ ft. above section 6.

With $y_2 = 6$ ft.:

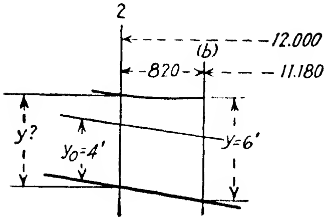

FIG. 85.

$$\Pi_2 = \Pi(1.5) = 1.5 - 0.92 \times 0.177 = 1.337,$$

and

$$\Pi(\eta_2) - L\frac{s_0}{y_0} = 1.337 - \frac{820}{10{,}000} \cdot 4 = 1.255$$

The solution now lies in finding the value of η, which will make

$$\Pi(\eta_1) = \eta_1 - 0.92B(\eta_1) = 1.255$$

By tentative probes, always using the tables with $n = 3.4$, we find:

η_1	$B(\eta_1)$	$\Pi_1 = \eta_1 - 0.92B(\eta_1)$
1.46	0.191	1.284
1.44	0.199	1.257
1.42	0.208	1.229
1.40	0.217	1.200

The values of $\Pi(\eta_1)$ are plotted in Fig. 86. From the graph we obtain the sought η, which makes $\Pi(\eta) = 1.255$, as $\eta = 1.438$ and, accordingly, $y_1 = \eta_1 y_0 = 1.438 \times 4 = 5.74$ ft. The outline of the $\Pi(\eta)$ curve (Fig. 86) indicates that simple arithmetical interpolation could be expediently used.

34. Simplified Solution. The $\beta = 0$ Curve.—Already Dupuit pointed out that in slow-flowing streams the value of $v^2/2g$ (Eq. [17]) is small and that therefore, in figuring out backwater curves, the possible effect derived from restored kinetic energy may be reasonably omitted.

In fact, in the preceding example the original velocity of uniform flow $v_0 = 12.2/4 = 3.05$ ft. per second is retarded to $v_a = 12.2/10 = 1.22$ ft. per second in section a, resulting in a "release" of a velocity head of $\dfrac{(3.05)^2 - (1.22)^2}{2g} =$ 0.122 ft., or a head of 1½ in. over a distance of more than 6 miles and a total surface fall, from b to a, of nearly 7 ft.

Moreover, one should bear in mind certain physical features characterizing retarded flow in general. As mentioned before, retarded, *i.e.*, divergent, flow is accompanied by increased turbulence, with the result that only a portion, and often only a small portion, of the kinetic energy head, which is theoretically released through retardation, is actually regained. Under all circumstances, the actual gain in restored kinetic energy head over a long stretch of a spacious canal is most uncertain. On the other hand, as will be seen later, the Dupuit suggestion substantially simplifies the figuring.

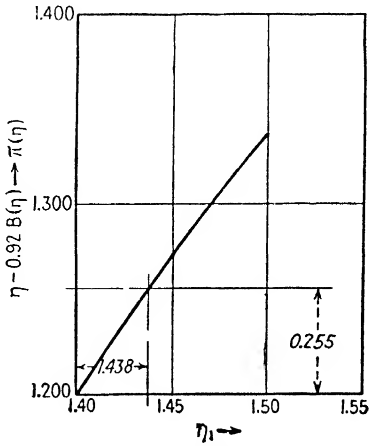

FIG. 86.—Graphical solution of equation $\eta - 0.92B(\eta) = 1{,}255$.

With regard to Eqs. (84) and (86), to omit the effect of restored kinetic energy will mean making $\beta = 0$; in other words, neglecting the term $\frac{d}{dx}\left(\frac{v^2}{2g}\right)$ in Eq. (17) translates itself into dropping β in the parenthesis $(1 - \beta)$ of the final equation, so that the simplified equation reads:

$$l_{2,1} = x_2 - x_1 = \frac{y_0}{s_0}[(\eta_2 - \eta_1) - \{B(\eta_2) - B(\eta_1)\}] \tag{89}$$

For proof, one should simply follow the evolution of Eqs. (82) and (86) in their development from Eq. (21) through Eqs. (46) and (48) and take notice how the original member $\frac{Q^2}{g}\frac{b}{a^3}$, which reflects the change of the kinetic energy head $\frac{d}{dx}\left(\frac{v^2}{2g}\right)$, finally evolves into $\beta(\mathfrak{K}/\mathfrak{K}_0)^2$.

Designating

$$\eta - B(\eta) = \Phi(\eta) \tag{90}$$

one obtains Eq. (89), as

$$l_{2,1} = x_2 - x_1 = \frac{y_0}{s_0}[\Phi(\eta_2) - \Phi(\eta_1)] \tag{91}$$

To facilitate computation, tables of $\Phi(\eta) = \eta - B(\eta)$ have been prepared for the range of exponents from $n = 2.8$ to $n = 4.2$. These tables are simply derived from the $B(\eta)$ tables and constitute a complement to the latter. We shall designate curves obtained by this simplified method as "$\beta = 0$ curves."

Among all the possible types of surface curves, it is curve M_1 which occurs most frequently and figures most prominently in practical engineering design. In the great majority of cases, when dealing with an M_1 curve, the simplified method, omitting the effect of restored kinetic energy and leading to a $\beta = 0$ curve, is justified and recommended; particularly, as in most cases it adds to the margin of safety by making the surface curve somewhat longer.

It should be clearly understood, on the other hand, that this simplified method is applicable to the M_1 curve *only*. On the falling M_2 or S_2 curve, the velocity head increases in the direction of flow and thus potential energy is absorbed for increasing the kineticity of flow. With regard to the other y^+ curves,* one should bear in mind, that the restoration of potential energy plays a decisive rôle in causing the particular forms of such curves. Thus the S_1 curve owes its *convex* form and its general outline to the very fact of kinetic energy being restored. As to the M_3 and the S_3 curves, the very essence of their nature lies in the fact of a rapid and energetic release of kinetic energy from a state of originally highly kineticized flow.†

In fact, even in the case of M_1 curves, the simplified method should be used only when the kineticity, expressed

* The importance of thorough experimental research in the domain of varied flow, with the particular purpose of unmasking the process of restoration of kinetic energy in open divergent streaming is strongly emphasized. Another most important object is the comparative value of friction losses in mild and rapid flow.

† The ludicrous results at which one may arrive by neglecting to take into consideration the effect of restored kinetic energy in cases where such restoration is called upon by the very nature of things to play a decisive rôle, are demonstrated for example by the unfortunate treatment of falling curves and subcritical flow, given in the at one time standard "Hydromechanik" of Rühlmann.

by the kinetic flow factor (Art. 27), is actually low. As a matter of fact, kineticity is reflected in the value of β. Thus, when $y_0 = y_{cr}$, so that normal flow is critical, $s_0 = \sigma_0$ and $\beta_0 = 1$. A mild-sloped channel is thus featured by $\beta_0 < 1$ and $1 > 1 - \beta > 0$. On the contrary, a steep-sloped flume makes $\beta_0 > 1$ and $(1 - \beta)$ negative. For steep slopes, moreover, $1 - \beta$ may become many times the multiple of unity.

It is left to the flair of the designing engineer to pass judgment as to when and where the simplified method may be judiciously applied.

Example 8

Assuming the circumstances of Example 7:

Question 1. Determine and trace a $\beta = 0$ surface curve.

The figuring in accordance with Eq. (91) is assembled in Table IX.

TABLE IX

y	η	$\Phi(\eta)$ $n = 3.4$	$\Delta\Phi$	l	L
10.000	2.500	2.453	.....	.....	0
			0.535	5,350	
8.000	2.000	1.918	.....	.....	5,350
			0.284	2,840	
7.000	1.750	1.634	.....	.....	8,190
			0.311	3,110	
6.000	1.500	1.328	.....	.....	11,300
			0.386	3,860	
5.000	1.250	0.937	.....	.....	15,160
			0.373	3,730	
4.400	1.100	0.564	.....	.....	18,890
			0.237	2,370	
4.200	1.050	0.327	.....	.....	21,260
			0.289	2,890	
4.080	1.020	0.038	.....	.....	24,150
		—	0.210	2,100	
4.040	1.010	0.172	.....	.....	26,250
			0.683	6,830	
4.004	1.001	0.855	.....	.....	33,080

Remark: For $\eta = 1.01$, and below, the value of $\Phi(\eta)$ is negative. Remember the rule that computations should be carried out strictly algebraically.

The curve is so close to that of Fig. 83 that it would be difficult to trace them together without confusion. The total length of the $\beta = 0$ curve is about 4 per cent in excess of the curve as computed in Table VIII.

Question 2. On the $\beta = 0$ curve, as above, determine the depth y at a distance of 12,000 ft. from the initial section.

It is in this case, analogous to Question 2 of Example 7, that the particular advantages of the simplified method become apparent. In the first place, with $\beta = 0$, one is no more bound by the limits of an interval, for which 1-β must be an integration constant. Therefore, in applying Eq. (91), we make

$$y_2 = y_a = 10 \text{ ft., with } \eta_2 = 2.5 \text{ and } \Phi(2.5) = 2.453$$

while the solution lies in finding an η_1, which would satisfy the relation

$$\Phi(\eta_1) = \Phi(\eta_2) - L\frac{s_0}{y_0} \qquad (92)$$

In our case, we have

$$\Phi(\eta_1) = 2.453 - \frac{12{,}000}{10{,}000} = 1.253$$

The tables with $n = 3.4$ give as nearest values

$$\eta = 1.44 \qquad \Phi = 1.241$$
$$\eta = 1.46 \qquad \Phi = 1.259$$

Interpolating arithmetically (proportionate parts) we have

$$\eta_1 = 1.44 + \frac{(1.253 - 1.241)}{(1.259 - 1.241)} \cdot (1.46 - 1.44) = 1.453$$

The sought depth

$$y_1 = 1.453 \times 4 = 5.81 \text{ ft.}$$

A comparison with 5.74 in Question 2, Example 7, shows a difference of about ½ in.

35. Intermediary Exponents.—In Examples 7 and 8 the value of the hydraulic exponent taken from Plate II was that of a table line with $n = 3.4$. Naturally such is mostly not the case. Even for the canal in question, if a more precise value of n was sought, for example for the particular range of depths between $y_0 = 4$ ft. and $y_a = 10$ ft., Eq. (71) would give

$$n = 2\frac{\text{Log } (\mathfrak{K}_{10}/\mathfrak{K}_4)}{\text{Log } 10/4} = 2\frac{\text{Log } \dfrac{270.8}{60.8}}{\text{Log } 2.5} = 2\frac{0.6601}{0.3979} = 3.32$$

If $n = 3.32$, which is intermediary between the table lines, were to be used, recourse must be taken to interpolation.

To gain a practical appreciation of the degree of precision gained, we shall first compare the solutions, obtained from using the neighboring table lines $n = 3.4$ and $n = 3.2$, between which the precise solution with $n = 3.32$ lies.

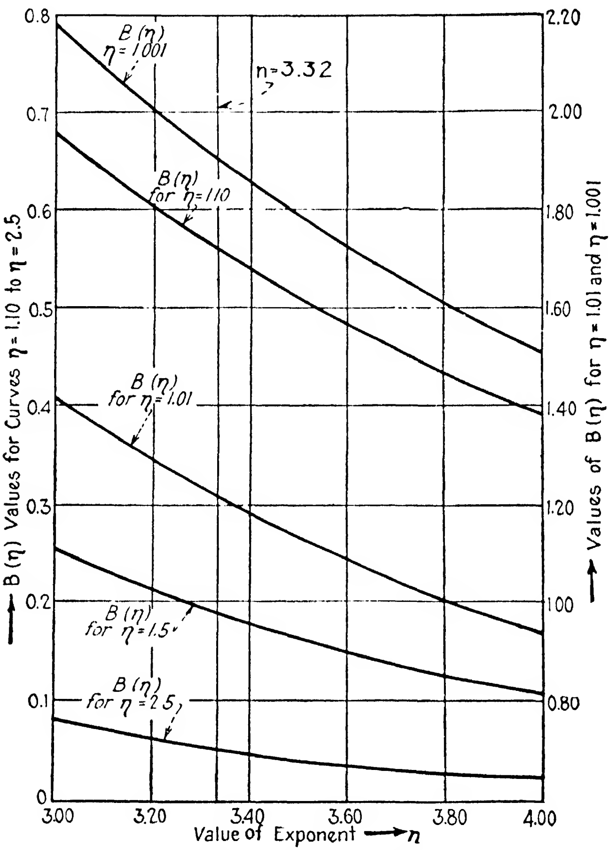

FIG. 87.—Illustrating the effect of the hydraulic exponent on the values of $B(\eta)$.

Using a $\beta = 0$ curve, and limiting the comparison to a few characteristic points, we obtain the result as shown in Table X, page 102.

The last column shows the relative difference of the distances L Evidently, if in first approximation, a solution with any of the two table lines ($n = 3.4$ or $n = 3.2$) were adopted, the mistake would not exceed 2 to 3 per cent.

TABLE X

y	η	$n = 3.4$			$n = 3.20$			Difference, per cent
		$\Phi(\eta)$	$\Delta\Phi$	l	$\Phi(\eta)$	$\Delta\Phi$	l	
10.000	2.500	2 453	.....	......	2 438			
			1 130	11,300	....	1.149	11,490	1.8
6.000	1.500	1.323	.....	......	1 289			
			0.759	7,590	....	0.789	7,890	3.9
4.400	1.100	0 564	.....	......	0 499			
		–	0.736	7,360	– .	0 780	7,800	5.8
4.040	1 010	0.172	.....	......	0.281			
		–	0.683	6,830	–	0.726	7,260	6.1
4.004	1.001	0.855	.....	......	1 007			
			$\Sigma l = L_0 =$	33,080		$L_0 =$	34,450	4 1

Graphical Interpolation.—In Fig. 87, curves of $B(\eta)$ for a series of η's have been drawn as functions of the exponent n. These curves allow one to determine graphically the intermediate value of $B(\eta)$ for $n = 3.32$ resulting in:

TABLE XI

y	η	$B(\eta)$ for $n = 3.32$	$\Phi(\eta) = \eta - B(\eta)$ $n = 3.32$	$\Delta\Phi$	l
10.000	2.500	0.052	2.448		
				1.138	11,380
6.000	1.500	0.190	1.310		
				0.772	7,720
4.400	1.100	0.562	0.538		
			–	0.753	7,530
4.040	1.010	1.225	0.215		
			–	0.697	6,970
4.004	1.001	1.913	0.912		
					33,600

Arithmetical Interpolation.—Proceeding by arithmetical interpolation and making $\Phi(\eta)$ with $n = 3.32$, the proportionate intermediary between the adjoining table lines, one obtains:

TABLE XII

y	η	$\Phi(\eta)$			For $n = 3.32$	
		$n = 3.2$	$n = 3.4$	$n = 3.32$	$\Delta\Phi$	l
10.000	2.500	2.438	2.453	2.447		
					1.138	11,380
6.000	1.500	1.289	1.323	1.309		
					0.771	7,710
4.400	1.100	0.499	0.564	0.538		
					0.754	7,540
4.040	1.010	−0.281	−0.172	−0.216		
					0.701	7,010
4.004	1.001	−1.007	−0.855	−0.917		
						33,640

The difference between the distances in Tables XI and XII is negligible. The example proves the rule, well substantiated by experience, that arithmetical interpolation (using proportionate parts) is a procedure sufficiently precise for general engineering practice

CHAPTER IX

COMPUTATION PROCEDURES

The solving of problems relating to varied flow will be illustrated in this chapter by practical examples. The cases as presented are distinctly elementary and are conceived primarily with the idea of familiarizing the reader with the computation procedures: In their entity, nevertheless, the problems selected indicate the ways of approach to many problems of actual practice and thus serve as an introduction to the more complex cases treated in Part II. Also each and every type of surface curve is given separate treatment. As a preliminary remark it may be useful to emphasize once more, as a feature which repeatedly occurs in computations dealing with varied flow, that in such computations the *discharge may be substituted by the equivalent normal depth.*

In a given canal, meaning a canal of given cross-section and of given bottom slope s_0, the discharge Q and the depth y_0 of uniform flow are connected by the relations

$$Q = \mathfrak{K}(y_0)\sqrt{s_0}; \quad \mathfrak{K}(y_0) = Q/\sqrt{s_0}$$

where $\mathfrak{K}(y_0)$ is the conveyance of the canal cross-section at the stage y_0. For a series of discharges Q_1, Q_2, Q_3, . . . we have thus a series of uniform depths y_{01}, y_{02}, y_{03}, . . . connected with the respective discharges through

$$\mathfrak{K}(y_{01}) = Q_1/\sqrt{s_0}; \mathfrak{K}(y_{02}) = Q_2/\sqrt{s_0}; \text{etc.}$$

Whenever a discharge Q_m is used to characterize flow in a canal, the equivalent normal depth y_{0m} may be used as a substitute. It will be found from the following examples that, generally, the value of the discharge as such does not enter into computations and is represented by the equivalent normal depth.

36. The M_1 Curve.—Throughout this article the simplified method, using the "$\beta = 0$" curve and Eqs. (91) and (92), is applied.

Example 9

A canal, Type A (Plate III), is laid between two reservoirs A and B (Fig. 88/1) at a distance of $L = 2$ miles; $s_0 = 4^0\%_0$; G.K. $n = 0.025$; $Ls_0 = 10{,}560 \text{ ft.} \times 4 \cdot 10^{-4} = 4.224$ ft.

Water is fed into the canal by means of a regulating sluice. The outflow at B is unobstructed. The canal is designed to carry $Q = 2{,}140$ cu. ft. per second in uniform flow, with $y_0 = 8$ ft. The respective

$$\mathfrak{K}_0 = 2{,}140/\sqrt{4} \cdot 10^{-2} = \sim 1{,}068 \cdot 10^2 \text{ cu. ft. per second}$$

(see Table related to Plate III)

Obviously in uniform flow $y_1 = y_2 = 8$ ft.

It will be assumed now that the stages y_1 and y_2 may change. Also, that the discharge may differ from $Q = 2{,}140$ cu. ft. per second. The problem will be to determine in each case how the change of y_1 and y_2, or of Q, respectively, will affect the other elements of flow.

In the computations below, when using the varied-flow-function tables, an average value of the hydraulic exponent $n = 3.6$ will be applied.

Question 1. The discharge remaining $Q = 2{,}140$ cu. ft. per second, stage B is changed, so that $y_2 = 12$ ft. Determine the depth y_1 in section 1.

With reference to Fig. 88/2, flow now will be varied, the surface curve being of the M_1 type. The discharge is represented by the normal depth $y_0 = 8$ ft. The circumstances in section 2 are *given*, and are characterized by $\eta_2 = 12/8 = 1.5$; $\Phi(\eta_2) = 1.351$.

The stage y_1 at section 1 is connected with stage y_2 by Eq. (92)

$$\Phi(\eta_1) = \Phi(\eta_2) - \frac{Ls_0}{y_0} = 1.351 - \frac{4.224}{8} = 0.823$$

From the $\Phi(\eta)$ tables we find the value of η, corresponding to $\Phi(\eta) = 0.823$ to be $\eta = 1.174$, which gives $y_1 = 1.174 \times 8 \text{ ft.} = 9.39$ ft.

Question 2. Assuming the discharge remaining constant and equal to $Q = 2{,}140$ cu. ft. per second while level B changes

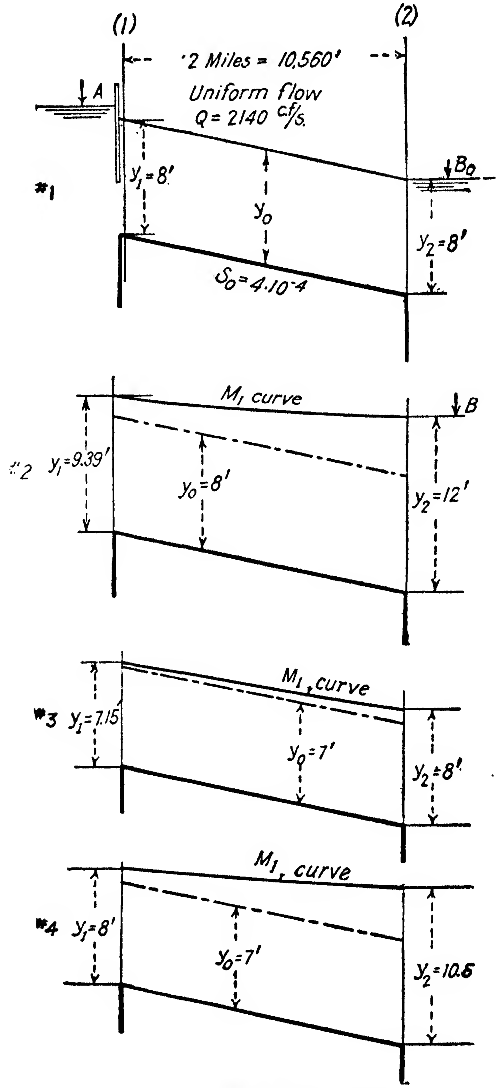

FIG. 88.—Relating to Example 9.

between $y_2 = 8$ ft. and $y_2 = 12$ ft., compute and draw a curve representing the relation between the changing stages y_1 and y_2.

The solution lies in applying the procedure of Question 1 to a series of points. The computations are gathered in the following table.

TABLE XIII

(1)	(2)	(3)	(4)	(5)	(6)
y_2	$\eta_2 = y_2/8$	$\Phi(\eta_2)$	$\Phi(\eta_1) = \Phi(\eta_2) - 0.528$	η_1	$y_1 = \eta_1 \times 8$
12	1.500	1.3510	0.8230	1.1740	9.39
11	1.375	1.1785	0.6505	1.0950	8.76
10	1.25	0.9770	0.4490	1.0495	8.40
9	1.125	0.7275	0.1995	1.0210	8.17
8	Uniform flow				8

The curve representing the relation between the stages is drawn in Fig. 89.

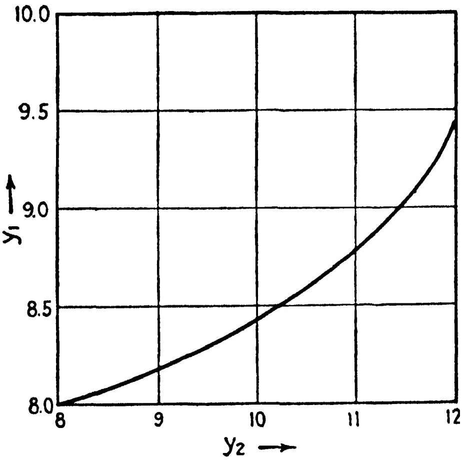

FIG. 89.

Question 3. Assume that the discharge is reduced to $Q =$ 1684 cu. ft. per second, while the stage in section 2 remains to be $y_2 = 8$ ft. Determine the corresponding stage y_1 at section 1.

The discharge $Q = 1{,}684$ cu. ft. per second corresponds to a conveyance $\mathfrak{K} = 1{,}684/\sqrt{4} \times 10^2 = 842 \times 10^2$, which (Plate III) corresponds to the normal depth $y_0 = 7$ ft. Accordingly, in Fig. 88/3, the discharge is represented by the line of uniform flow $y_0 = 7$ ft. For section 2 we have as given $y_2 = 8$ ft.; $\nu_2 = 8/7 = 1.143$. When determining $\Phi(1.143)$ take proportionate parts between table values for $\Phi(1.14)$ and $\Phi(1.15)$. Thus

$$\Phi(1.143) = 0.740 + 3/10(0.766 - 0.740) = 0.7478$$

The stage in section 1 is determined by

$$\Phi(\eta_1) = \Phi(\eta_2) - \frac{Ls_0}{y_0} = 0.7478 - \frac{4.224}{7} = 0.748 - 0.603 = 0.145$$

To find η_1, interpolate between the table values

$$\Phi(1.02) = 0.120 \text{ and } \Phi(1.03) = 0.240$$

Using proportionate parts, we find

$$\eta_1 = 1.02 + \frac{(1.03 - 1.02)(0.145 - 0.120)}{(0.240 - 0.120)} = 1.022$$

Accordingly,

$$y_1 = \eta_1 \cdot y_0 = 1.022 \times 7 = 7.154 \text{ ft.}$$

Question 4. Assume, now, that it is the stage in section 1 below the sluice which is maintained constant at $y_1 = 8$ ft., and find the stage y_2 corresponding to a flow of $Q = 1{,}684$ cu. ft. per second (Fig. 88/4).

We have now for section 1

$$y_1 = 8 \text{ ft.}; \eta_1 = 1.143; \Phi(\eta_1) = 0.748$$

The stage at section 2 is determined through

$$\Phi(\eta_2) = \Phi(\eta_1) + \frac{Ls_0}{y_6} = 0.748 + 0.603 = 1.351$$

which gives

$$\eta_1 = 1.50 \text{ and } y_1 = 1.50 \times 7 = 10.5$$

Question 5. Assuming with a flow at 2,140 cu. ft. per second that initially y_1 and y_2 were both 8 ft., determine the highest stage in section 2, until which level B may be raised without appreciably influencing the stage at section 1.

Recourse in this case has to be taken to the definition of the "end" of a curve as given in Art. 33, Fig. 79. The influence of the depth y_2 on the stage in section 1 will be considered nil, whenever the backwater curve produced by a rise of y_2 over y_0 ends below section 1, or is *shorter* than the length of the canal $L = 10{,}560$ ft.

Assuming the end of the curve to be at $\eta_1 = 1{,}001$ which makes $\Phi(\eta_1) = -0.724$, we find the corresponding stage 2 through

$$\Phi(\eta_2) = \Phi(\eta_1) + \frac{Ls_0}{y_0} = -0.724 + \frac{4.224}{8} = -0.724 + 0.528 = -0.196$$

The $\Phi(\eta)$ table shows that the corresponding value of η_2 lies between $\eta_1 = 1.005$ and $\eta_1 = 1.010$, which means that in our case no appreciable change of level B may take place without affecting the depth y_1

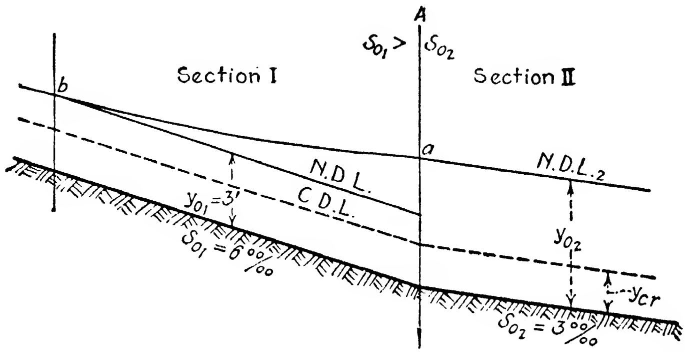

FIG. 90.—Canal with a break in the bottom slope featuring an M_1 curve (Exercise 2, Example 9).

However, if the canal were to be substantially longer, circumstances would be different. For example, with $L = 5$ miles, and $Ls_0/y_0 = 1.320$, the value of $\Phi(\eta_2)$ in the above would become

$$\Phi(\eta_2) = -0.724 + 1.320 = 0.596$$

which corresponds to $\eta_2 = 1.093$ and $y_2 = 1.093 \times 8 = 8.75$ ft.

The level in B could fluctuate 10 per cent without affecting the stage at section 1.

Exercises:

1. Assume in the layout of Fig. 88 the canal to be of Type B (Figs. 14 and 15); $y_0 = 5$ ft.; $L = 10{,}000$ ft.; $s_0 = 2^0\!/_{00}$; and $s_0 = 8^0\!/_{00}$, respectively. Use $n = 3.70$.

a. With reference to Question 5 of the preceding example, determine the stage y_2 until which level B may be raised without affecting the stage at y_1. Take for end of curve $\eta_1 = 1.001$ and $\eta_1 = 1.01$, respectively.

b. For both bottom slopes and $y_0 = 5$ ft., determine and draw the curve as in Fig. 89, showing the relation between y_2 and y_1.

2. Assume a canal, Type *B*, laid with a break in the bottom slope (Fig. 90); $s_{01} = 6^0\%_0$; $s_{02} = 3^0\%_0$. The discharge Q corresponds to uniform flow in section 1, with $y_{01} = 3$ ft. Use $\mathfrak{K}$ curve with Bazin coefficients (Fig. 15). Take $n = 3.70$. Determine the surface profile.

Explanatory Remark: In this problem, as in others to come, where the case is one of a canal with a break in the bottom slope, it is incumbent first to establish the *type* of movement. With reference to Fig. 90, because of s_{02} being less than s_{01}, the surface must rise from the normal depth $y_{01} = 3$ ft. to the normal depth $y_{02} > 3$ ft.

To determine the type and location of the transition curve, a reasoning on the following lines will usually be found expedient. First, by figuring out y_{cr}, one finds that both y_{02} and y_{01} are $> y_{cr}$; the movement is in the *M* class. The only possible y^+ curve will be an M_1 curve, which must lie wholly within section 1 above the y_{01} line. The transition curve *cannot* lie in section 2, as in such case the only possible curve with $y < y_{02}$, would be a y^- curve, Type M_2. The normal depth y_{02} reaches to the dividing section *A*; it acts as a barrier with regard to section 1, causing a backwater curve *a-b* of the M_1 type.

37. The M_2 Curve.

Example 10

A canal, Type *B* (Fig. 14), laid with $s_0 = 10^0\%_0$, terminates in a fall (Fig. 91); $y_0 = 5$ ft.

With G.K. $n = 0.013$, the discharge (Table II and Fig. 15) is

$$Q = \mathfrak{K}_0\sqrt{s_0} = 109.1\sqrt{10} = 346 \text{ cu. ft. per second}; \sigma_0 = 22.8^0\%_0$$

With $s_0 < \sigma_0$, the class of flow is *M*. The curve is a falling M_2 curve, between the normal depth $y_0 = 5$ ft. and y_{cr}, near the crest of the fall.

Question 1. Determine the surface curve.

In figuring out y_{cr}, we have (Eq. [28])

$$\mathfrak{M}_{cr} = Q/\sqrt{g} = 346/5.67 = 61; y_{cr} = 4.01$$

With $\sigma_0 = 22.8^0\!/_{00}$, $\beta_0 = 10/22.8 = 0.438$, $\sigma_{cr} = 22.7^0\!/_{00}$, $\beta_{cr} = 0.441$, a single average value of $\beta = 0.44$ with $1 - \beta = 0.56$ may be used throughout the integration range.

With $(1 - \beta) =$ const., one may conveniently use the equation in form (86)

$$l = \frac{y_0}{s_0}[\Pi(\eta_2) - \Pi(\eta_1)]$$

The integration limits will be:

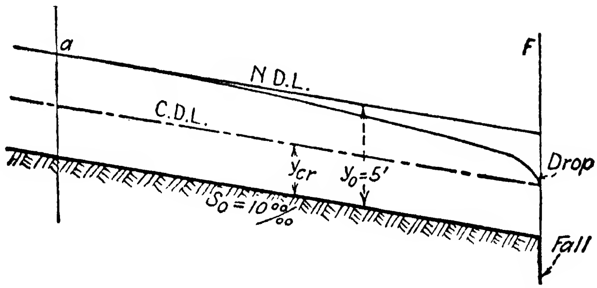

FIG. 91.—The M_2 curve in a canal leading toward a fall (Example 10.)

For the section over the fall

$$\eta_F = \eta_{cr} = y_{cr}/y_0 = 4.01/5 = 0.802$$

For the section corresponding to the end of the curve at a

$$\eta_a = 0.999 \text{ and } y_a = 0.999y_0 = 4.995 \text{ ft.}$$

Following the procedure of Example 7, we shall select a series of depth intervals and determine the lengths of the reaches between them. We shall assume that y_{cr} lies over the crest of the fall.* Section F will be the zero section, from which the distances L will be measured.

For the range of depths 5 to 4 ft., the hydraulic exponent is

$$n = 2\frac{Lg\frac{\mathfrak{K}(5)}{\mathfrak{K}(4)}}{Lg\,5/4} = 2\frac{Lg\frac{109.1}{70.6}}{Lg1.25} = 3.90$$

The middle values between table lines $n = 3.8$ and $n = 4.0$ will be accepted as $B(\eta)$.

$$y_0/s_0 = 5/10 \cdot 10^{-4} = 5{,}000 \text{ ft.}$$

With this, the figuring is given in the following table.

* In other words, the small distance between section C and section F in Fig. 37, is neglected. In the present state of the art, with what little is known of the surface curve forms in curvilinear flow, this approximation is unavoidable.

TABLE XIV

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
y	$\eta = y/5$	$B(\eta)$ $n = 3.9$	$0.56B(\eta)$	$\Pi(\eta) = \eta - 0.56B$	$\Delta\Pi$	l	L
4.010	0.802	0.8950	0.501	0.301			0
					0	0	
4.050	0.810	0.9090	0.509	0.301	.....	.....	0
					0	0	
4.100	0.820	0.9270	0.519	0.301	.....	.....	0
					0.004	20	
4.250	0.850	0.9865	0.553	0.297	.....	.....	20
					0.022	110	
4.500	0.900	1.1155	0.625	0.275	.....	.....	130
					0.058	290	
4.750	0.950	1.3090	0.733	0.217	.....	.....	420
					0.108	540	
4.900	0.980	1.5550	0.871	0.109	.....	.....	960
					0.092	460	
4.950	0.990	1.7375	0.973	0.017	.....	.....	1,420
					0.323	1,615	
4.995	0.999	2.3315	1.305	−0.306	.....	.....	3,035

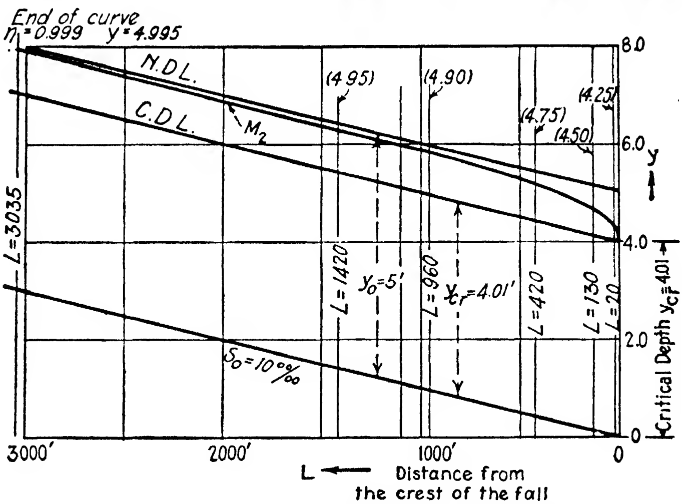

FIG. 92.—Tracing of the M_2 curve in Example 10.

The curve is traced in Fig. 92.

Remark: In Col. 7, the distance between the depths 4.01 and 4.10, respectively, are shown at zero, which means that the horizontal distance in which the respective increment of depth takes place is too small to be noticeable within the degree of precision with which the figuring is conducted.

Example 11

A canal, Type *D* (Plate V), is laid between two reservoirs at a distance of 2 miles (Fig. 93); $s_0 = 1^0\!/_{00}$; G.K. $n = 0.025$. The canal is designed for uniform flow at $y_0 = 8$ ft., corresponding to a discharge of $Q = \mathfrak{K}_0\sqrt{s_0} = 384.5 \times \sqrt{1} = 384.5$ cu. ft. per

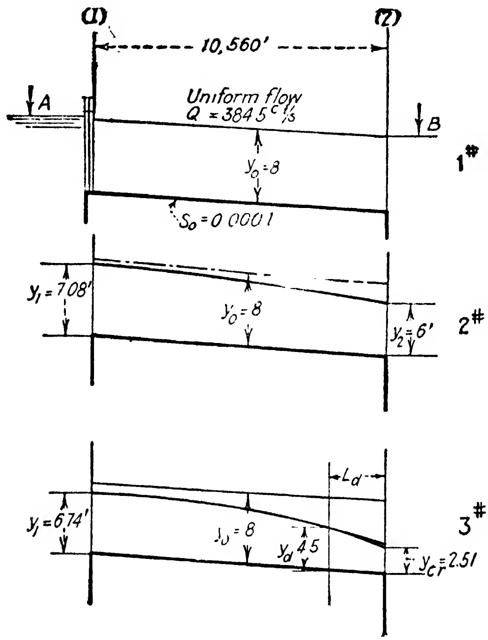

FIG. 93.—Relating to Example 11.

second. In the following, the discharge will be assumed to remain permanent, while the stages y_1 and y_2 will be subject to lowering below $y_0 = 8$ ft. The result will be flow with a falling surface curve of the M_2 type. Assuming that the lowering will not extend much beyond $y = 3$ ft., we determine the hydraulic exponent for the range between $y = 3$ ft. and $y = 8$ as

$$n = 2\frac{Lg\dfrac{\mathfrak{K}(8)}{\mathfrak{K}(3)}}{Lg\,8/3} = 2\frac{Lg\dfrac{384.5}{58.2}}{Lg\,8/3} = 2\frac{0.7924}{0.4260} = \sim 3.70$$

In applying the varied-flow-function tables, the middle value between table lines for $n = 3.6$ and $n = 3.8$ will be used. In determining the value of 1-β, we have for the range of depths between $y = 3$ ft. and $y = 8$ ft.

$$y = 8; \sigma = 56.5^{0}\!/\!_{00}; \beta = s_0/\sigma = 1/56.5 = 0.0177; 1\text{-}\beta = 0.982$$
$$y = 3; \sigma = 79.4^{0}\!/\!_{00}; \beta = s_0/\sigma = 1/79.4 = 0.0126; 1\text{-}\beta = 0.987$$

An average value of $1\text{-}\beta = 0.985$ will be used. We further have:

$$Ls_0 = 10{,}560 \times 1 \cdot 10^{-4} = 1.056; Ls_0/y_0 = 1.056/8 = 0.132$$

Question 1 (Fig. 93/2). Assume level B lowered to make $y_2 = 6$ ft. Determine the corresponding stage y_1.

We have for stage 2:

$$y_2 = 6 \text{ ft.}; \eta_2 = 6/8 = 0.75$$

$B(\eta_2)$ (middle value between $B = 0.823$ and $B = 0.815$) $= 0.819$; $\Pi(\eta_2) = \eta_2 - (1 - \beta)B(\eta_2) = 0.750 - 0.985 \times 0.819 = -0.056$

For determining stage 1, we have

$$\Pi(\eta_1) = \Pi(\eta_2) - \frac{Ls_0}{y_0} = -0.056 - 0.132 = -0.188$$

The corresponding value of η_1, which is to satisfy

$$\Pi(\eta_1) = \eta_1 - 0.985B(\eta_1) = -0.188$$

is found as follows:

We have, with $n = 3.70$:

$$B(0.88) = 1.0725; \Pi = -0.178$$
$$B(0.89) = 1.0105; \Pi = -0.198$$

Interpolating we have $\eta_1 = 0.885$, which makes $y_1 = 0.885 \times 8 = 7.08$ ft.

Question 2. Assume that stage 1 has been lowered from $y_1 = 8$ ft. to $y_1 = 7.6$ ft. Determine to what level stage 2 should be lowered in such case, in order to continue to draw water at the rate of $Q = 384.5$ cu. ft. per second.

We have for section 1

$$\eta_1 = \frac{7.6}{8} = 0.95; B(\eta_1) = 1.337; \Pi(\eta_1) = 0.95 - 0.985 \times 1.337 = -0.367$$

Stage y_2 is determined through

$$\Pi(\eta_2) = \eta_2 - 0.985B(\eta_2) = \Pi(\eta_1) + \frac{Ls_0}{y_0} = -0.367 + 0.132 = -0.235$$

To determine η_2, we have from the tables, with sufficient approximation,

$$\eta_2 = 0.91;\ B(0.91) = 1.1625;\ \Pi(0.91) = 0.91 - 0.985 \times 1.1625 = -0.237$$

Thus

$$y_2 = 0.91 \times 8 = 7.28 \text{ ft.}$$

Question 3 (Fig. 93/3). Determine the lowest position of the drop curve M_2, at which a flow of $Q = 384.5$ cu. ft. per second is still possible.

The lowest possible stage y_2 at section 2 is the critical depth. As explained in Art. 17, the depth y_2 cannot drop below $y_2 = y_{cr}$, and having once reached that stage will remain there, irrespective of any further lowering of level B.

To determine y_{cr} we have

$$\mathfrak{M}_{cr} = (a^3/b)_{cr} = Q/\sqrt{g} = 384.5/5.67 = 67.8$$

From Plate V, we find accordingly $y_{cr} = 2.51$. For section 2 with $y_2 = y_{cr} = 2.51$, we have

$$\eta_2 = 2.51/8 = 0.314;\ B(\eta_2) = 0.315;\ \Pi(\eta_2) = 0.314 - 0.985 \times 0.315 = 0.004$$

The corresponding stage 1 is determined by

$$\Pi(\eta_1) = \eta_1 - 0.985B(\eta_1) = 0.004 - 0.132 = -0.128$$

To find η_1, we have from the tables

η	$B(\eta)$	$\Pi(\eta)$
0.85	1.0000	−0.135
0.84	0.9785	−0.124

Interpolating we get $\eta_1 = 0.844$ and, accordingly the lowest possible y_1 compatible with a discharge of 384.5 cu. ft. per second $= 0.844 \times 8 = 6.74$ ft.

Question 4. Assuming that stage y_2 varies between $y_0 = 8$ ft. and $y_{cr} = 2.51$ ft., determine the effect produced on stage 1 by such variation of y_2.

This problem is similar to that of Question 2, Example 9. The limits of the change of stage y_1 are: $y_0 = 8$ ft. and $y_{1min} = 6.74$ ft. To find the intermediary points apply the procedure used in Question 1 to a series of stages.

The computation is assembled in the following table

TABLE XV

y_2	$\eta_2 = y_2/8$	$B(\eta_2)$	$0.985B(\eta_2)$	$\Pi(\eta_2)$	$\Pi(\eta_1) = \Pi(\eta_2) - 0.132$	η_1	y_1
8	Uniform flow						8
7.20	0.900	1.1300	1.113	−0.213	−0.345	0.945	7.56
6.00	0.750	0.8190	0.806	−0.056	−0.188	0.885	7.08
4.00	0.500	0.5085	0.501	−0.001	−0.133	0.848	6.79
3.20	0.400	0.4030	0.397	+0.003	−0.129	0.845	6.75
2.51	0.314	0.3150	0.310	+0.004	−0.128	0.844	6.74

The curve is drawn in Fig. 94.

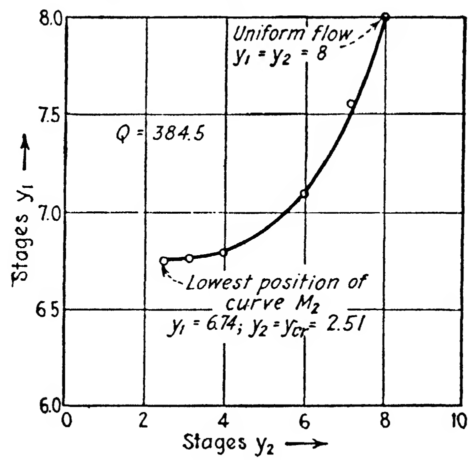

FIG. 94.

Question 5. On a lowering curve, with y diminishing stream downward, the velocity increases. In a canal with walls in natural condition, this increasing velocity may result in dangerous erosion. In fact, in the above case the velocity at y_{cr} is

$$v = Q/a_{cr} = 384.5/46.9 = 8.2 \text{ ft. per second.}$$

Assume that, for gravelly soil in which the canal is supposed to be laid, the dangerous limit of velocity is reached at $v = 4$ ft. With $Q = 384.5$, this corresponds to a cross-sectional area of $a = 96.1$ sq. ft. and a depth (Plate V) slightly below $y = 4.5$ ft.

Assume that, over all the length of the canal where the velocity may exceed 4 ft., the canal walls are to be protected. Determine on what length L_D (Fig. 93/3) such protection has to be put into effect.

The lowest position of the curve being given in Fig. 93/3, assume that protection is to be given on the whole stretch where the depth may be < 4.5 ft. The problem is thus to determine the length L_D between $y_{cr} = 2.51$ and $y_{cr} = 4.5$ ft.

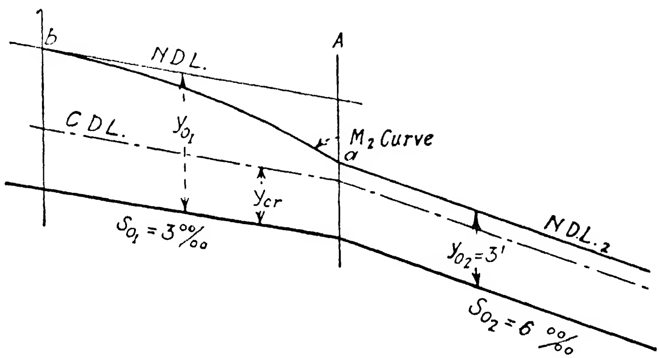

FIG. 95.—Canal with a break in the bottom slope featuring an M_2 curve (Exercise 2, Example 11).

From Table XV, for $y = 2.51$; $\Pi(\eta_2) = 0.004$. For $y_1 = 4.5$ ft.; $\eta_1 = 4.5/8 = 0.562$. With $n = 3.70$:

$B(0.56) = 0.575$	$B(0.562) = 0.577;$
$B(0.58) = 0.5975$	$\Pi_1 = 0.562 - 0.985 \times 0.577 = -0.006$

Hence

$$L_D = \frac{y_0}{s_0}[\Pi(\eta_2) - \Pi(\eta_1)] = 8 \cdot 10^4[0.004 - (-0.006)] = 800 \text{ ft.}$$

Exercise:

1. Assume a layout as in Fig. 93 with canal Type A; $s_0 = 4 \times 10^{-4}$. The constant discharge corresponds to $y_0 = 6$ ft. Determine the lowest possible value of y_1 with the length of the canal respectively: $L = 5{,}000$ ft.; $L = 10{,}000$ ft.; $L = 15{,}000$ ft.

2. The canal (Fig. 90) is laid with the bottom slopes in reversed order, namely, Fig. 95: $s_{01} = 3^0/_{00}$ and $s_{02} = 6^0/_{00}$. Determine the surface profile, assuming that the discharge corresponds to $y_{02} = 3$ ft.

Indication.—A process of reasoning, similar to that of the exercise in Example 9, will show that the transition section must be a lowering M_2 curve, located wholly within the upper section, below y_{01}. The depth in section A will be $y_a = y_{02}$.

38. The M_3 Curve.

Example 12

Assume in a layout (Fig. 96) water flowing from under a sluice into a rectangular canal, Type C (Plate IV). The canal

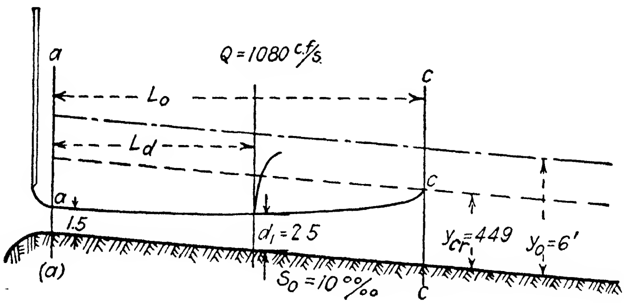

FIG. 96.—M_3 rising curve in a mild-sloped canal below a regulating sluice (Example 12).

is laid with $s_0 = 10^0/_{00}$; G.K. $n = 0.013$. The sluice is regulated to discharge $Q = 1{,}080$ cu. ft. per second, with a depth in the *vena contracta*, $y_a = 1.5$ ft.

Question 1. Determine the surface curve.

The normal depth

$$\mathfrak{K}_0 = Q/\sqrt{s_0} = 1{,}080/\sqrt{10} \cdot 10^{-2} = 329 \times 10^2; \text{ from Plate IV, } y_0 = 6 \text{ ft.}$$

The critical depth

$$q = 1{,}080/20 = 54 \text{ cu. ft. per second; } y_{cr} = \sqrt[3]{q^2/g} = \sqrt[3]{54^2/32.2} = 4.49 \text{ ft.}$$

With $y_0 > y_{cr}$ and $y_a = 1.5$ ft. $< y_{cr}$, the curve is of the M_3 type.

The elements of flow in the *vena contracta* are:

$v_a = 54/1.5 = 36$ ft. per second; $v_a^2/2g = 20.1$ ft.; $\epsilon_a = 1.5 + 20.1 = 21.6$ ft.; $\lambda = 2\frac{20.1}{1.5} = 26.7$

The 1-β *Curve* (Plate IV).
For

$y_0 = 6$ ft., $\sigma_0 = 25.8^0/_{00}$; $\beta_0 = 10/25.8 = 0.388$; $1 - \beta_0 = 0.612$

For other depths:

y	1 5	2.0	3.0	4.0	5.0
σ_0 in $^0/_{00}$	25.3	24.4	23.5	24.0	24.9
β	0.395	0.410	0.425	0.416	0.401
1-β	0.605	0.590	0.575	0.584	0.599

The 1-β curve is traced in Fig. 97.

For integration the average constant 1-β values are adopted as follows: for the depth range 2.1 − 4.2, $1 - \beta = 0.58$; for $y < 2.1$, $1 - \beta = 0.595$; for $y > 4.2$, $1 - \beta = 0.59$.

The hydraulic exponent for the range 1.5 to 6 ft.:

$$n = 2\frac{Lg\frac{\mathfrak{K}(6)}{\mathfrak{K}(1.5)}}{Lg\frac{6.0}{1.5}} = 2\frac{Lg\frac{329}{41.5}}{Lg4} = 2.99.$$

We shall figure with $n = 3$. The limits of the curve are

$$y_{cr} = 4.49 \text{ with } \eta_{cr} = 4.49/6 = 0.749$$
$$y_a = 1.5 \text{ with } \eta_a = 1.5/6 = 0.250$$
$$y_0/s_0 = 6/10 \times 10^4 = 6{,}000$$

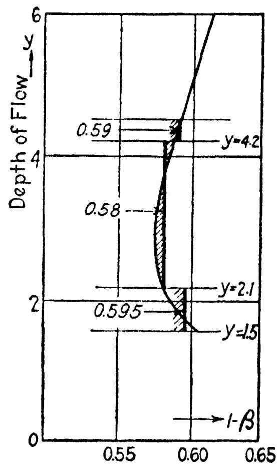

FIG. 97.—Selecting average constant values of $1 - \beta$ for depth intervals. Example 12.

The range of depths is divided into intervals; the partial distances being

$$l = \frac{y_0}{s_0}[\Delta\eta - (1 - \beta)\Delta B] = y_0/s_0\Delta\Pi$$

The distances L are figured from the critical depth stream up.

TABLE XVI

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
y	$\eta = y/6$	$(1-\beta)$	$B(\eta)$ $n = 3$	$\Delta\eta$	ΔB	$(1 - \beta)\Delta B$	ΔH	l, feet	L
4.49	0.749	↑	0.855	.....	.....	....	....	..	C
				0.009	0.015	0.00885	0 00015	1	
4.44	0.740			0.840	.....	.....	....	....	...	1
		0.59	.....	0.020	0 033	0.0195	0.0005	3	
4.32	0.720			0.807	.....	.....	....	....	..	4
		↓		0.020	0.031	0.0183	0.0017	10	
4.20	0.700	—	0.776	.....	.....	....	....	...	14
		↑		0.050	0.073	0 0424	0.0076	46	
3.90	0.650			0.703	.....	.....	....	....	...	6C
		0.58	.....	0.050	0.066	0.0383	0.0117	70	
3.60	0.600			0.637	.....	.....	....	....	...	13C
					0.100	0.120	0.0696	0.0304	182	
3.00	0.500			0.517	.....	.....	....	....	...	312
		↓		0.140	0.153	0.0888	0 0512	307	
2.16	0.360	—	0.364	.....	.....	....	....	...	619
		0.595	.....	0.110	0.113	0.0672	0.0128	257	
1.50	0.250	↓	0.251	.....	.....	....	....	...	876

The curve is traced in Fig. 98.

In the above example the total length of the curve between the *vena contracta* and y_{cr} was found to be $L = 876$ ft. If the canal is shorter than 876 ft. and the efflux is unobstructed, the stream will flow out freely as in Fig. 54. Usually, however, flow from under a sluice with an M_3 curve leads to a jump.

Question 2. Assuming under circumstances as above, that the jump starts at a depth $d_1 = 2.5$, determine the distance L_d Fig. 99 from the *vena contracta* to the foot of the jump.

The problem is answered by finding the distance between the depths $y_a = 1.5$ and $d_1 = 2.5$.

The depth 2.5 lies within the interval with $1 - \beta = 0.58$. Accordingly, following the procedure of Fig. 85, we shall determine the distance of section 2.5 from section 2.16, which (see Table XVI) lies 257 ft. from a.

For the interval 2.16 ft. to 2.50 ft., we have:

$$y_2 = 2.50;\ \eta_2 = 2.50/6 = 0.417;\ B(\eta) = 0.425$$
$$y_1 = 2.16;\ \eta_1 = 2.16/6 = 0.360;\ B(\eta) = 0.364;\ 1 - \beta = 0.58$$

$\Delta\eta = 0.057;\ \Delta B = 0.061;\ \Delta B \times 0.58 = 0.0354$
$\Delta\Pi = \Delta\eta - (1 - \beta)\Delta B = 0.057 - 0.0354 = 0.0216$
$l_{2,1} = 0.0216 \times 6{,}000 = 130$ ft.

The full distance from *vena contracta* is

$$L_d = 257 + 130 = 387 \text{ ft.}$$

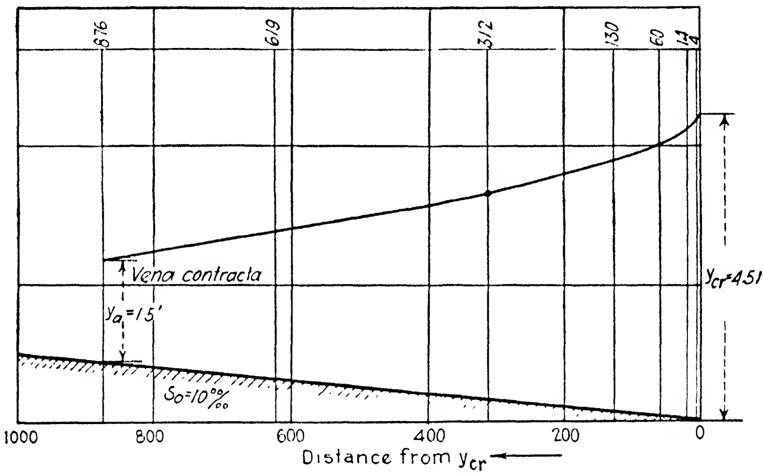

FIG. 98.—Tracing of the M_3 curve for Fig. 96, Example 12.

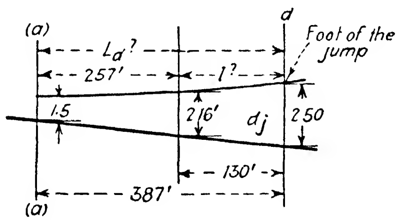

FIG. 99.—Determining the location of the hydraulic jump, Example 12, Question 2.

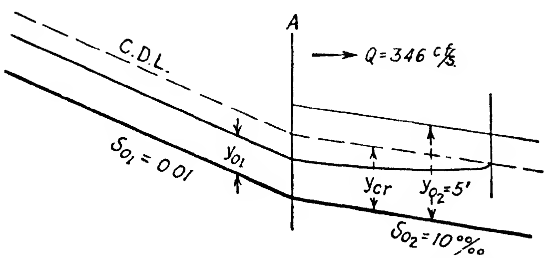

FIG. 100.—Canal with a break in the bottom slope from steep to mild, featuring an M_3 curve (exercise in Example 12).

Exercise:

Assume canal, Type *B* (Fig. 14), laid as in Fig. 100 with $s_{01} = 0.01$ and $s_{02} = 10^0\!/_{00}$.

Assume $Q = 346$ cu. ft. per second, corresponding to $y_{02} = 5$ ft. (G.K. $n = 0.013$). For the $\mathfrak{K}$ curve with $s_0 = 0.01$, use G.K. curve for $s_0 = 10^0\!/_{00}$.

a. Assuming the length of section 2 to be 500 ft., determine the longitudinal surface profile.

b. Determine the maximum theoretical length of section 2, which will allow free efflux without a jump.

Indication.—Determine y_{01} and y_{cr}. As y_{01} is less than y_{cr} and y_{02} is greater than y_{cr}, there will be an M_3 curve, commencing in section *a* with $y_a = y_{01}$.

39. The S_1 Curve.

Example 13

Assume a rectangular flume, wide enough to be considered as of type, Fig. 73, laid with $s_0 = 50^0\!/_{00}$ and emptying into a

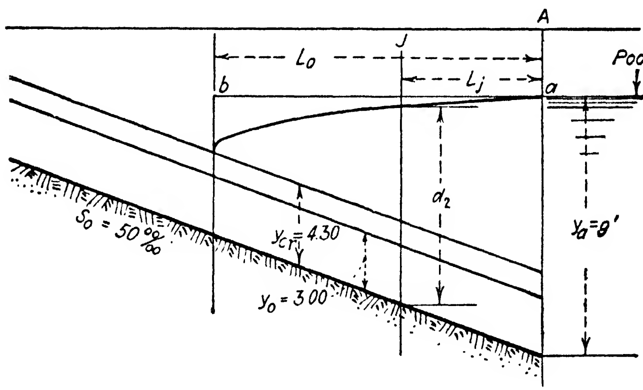

FIG. 101.—S_1 rising curve in a steep-sloped flume emptying into a pool (Example 13).

pool (Fig. 101). Use G.K. values for *C* corresponding to $s_0 = 10^0\!/_{00}$ and $n = 0.013$. Assume $y_0 = 3$ ft., which gives, for a unit width, $\mathfrak{K} = C \cdot y^{3/2} = 138 \times 3^{3/2} = 7.18 \times 10^2$, and $q = 7.18\sqrt{50} = 50.8$ cu. ft. per second.

Question 1. Assuming that the depth in section *a* is $y_a = 9$ ft., determine and trace the surface curve.

The critical depth

$$y_{cr} = \sqrt[3]{q^2/g} = \sqrt[3]{50.8^2/32.2} = 4.30 \text{ ft.}$$

With $y_0 = 3$ ft. $< y_{cr}$, the flow is in the S class.

With $y_a = 9$ ft. $> y_{cr}$, the curve is of the S_1 type. The integration limits are: $y_b = y_{cr} = 4.30$ ft. and $y_a = 9$ ft.

Hydraulic Exponent.—Take as an average value $n = 3.20$.

$$y_0/s_0 = 3/50 \cdot 10^{-4} = 600 \text{ ft.}$$

The 1-β *Curve.*—For a rectangular profile of great width the critical slope is $\sigma' = g/C^2$ (Fig. 40).

For the different depths the values of σ' and 1-β are:

y	C	σ' ‰	$\beta = s_0/\sigma'$	1-β
10.0	155	13.4	3.73	−2.73
8.0	152	14.0	3.57	2.57
6.0	149	14.8	3.38	2.38
5.0	146	15.3	3.26	2.26
4.5	144.5	15.6	3.21	2 21
4.0	142.5	15.9	3.14	2.14
3.5	140.5	16.3	3.06	2.06
3.0	138	16.9	2.96	1.96
2.5	134	17.9	2.79	1.79
2.0	130	19.0	2.63	1.63
1.5	124	21.0	2.38	1.38
1.0	116	24.0	2.08	1.08

The 1-β curve is traced in Fig. 102.

With $s_0 > \sigma_0$, 1-β is negative. Moreover, the values of 1-β are quite substantial. Under such circumstances it will be expedient to take a separate average value of 1-β for each interval.

The computations are assembled in Table XVII, page 125.

The values of 1-β (Col. 6) are taken for each depth from curve (Fig. 102). The value of 1-β for an interval (Col. 7) is the arithmetical average of the adjoining values in Col. 6. In column 9, $\Delta\Pi = \Delta\eta - (1 - \beta)\Delta B$. The partial distances l (Col. 10) are $l = \Delta\Pi \times 600$ ft. The distances L are measured from section a.

The curve is plotted in Fig. 103.

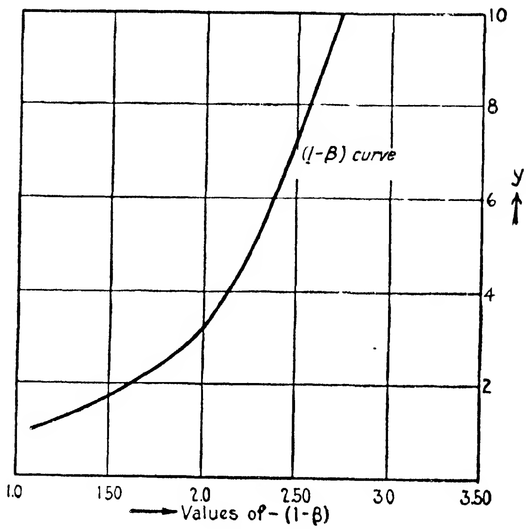

FIG. 102.

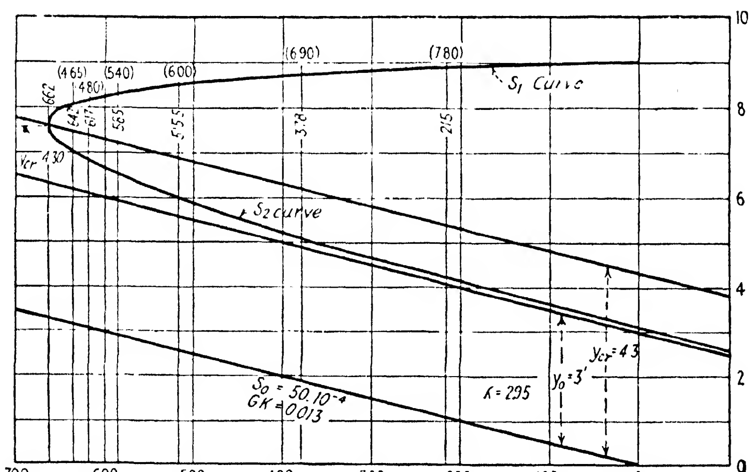

FIG. 103.—Upper curve, tracing of the S_1 curve for Fig. 101, Example 13. Lower curve, tracing of the S_2 curve for Fig. 105a Example 14.

Question 2. The surface curve, as traced above, over the whole range of depths until y_{cr} is a theoretical profile. Usually the S_1 curve is a stretch of gradually varied flow succeeding a jump (Fig. 6). Assuming that the depth d_2 after the jump is known the location of the jump is found by determining the distance L_j from A to the section J with $y = d_2$ (Fig. 101).

TABLE XVII

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
y	η	$\Delta\eta$	$B(\eta)$	$\Delta B(\eta)$	1-β	1-β for interval	(1-β)ΔB	ΔII	l	L
9.00	3.00		0.041	—	2.66	—				0
		0.40		0.016		2.61	0.0418	0.358	215	
7.80	2.60		0.057		2.56					215
		0.30		0.018		2.52	0.0454	0.255	153	
6.90	2.30		0.075		2.47					378
		0.30		0.029		2.43	0.0706	0.229	137.5	
6.00	2.00		0.104		2.38					515.5
		0.20		0.029		2.35	0.0682	0.132	79.3	
5.40	1.80		0.133		2.32					585
		0.10		0.020		2.30	0.0460	0.054	32	
5.10	1.70		0.153		2.28					617
		0.10		0.026		2.27	0.0591	0.041	24.6	
4.80	1.60		0.179		2.25					642
		0.05		0.015		2.24	0.0336	0.0164	9.85	
4.65	1.55		0.194		2.22					652
		0.05		0.018		2.21	0.0398	0.0102	6.1	
4.50	1.50		0.212		2.20					658
		0.04		0.016		2.19	0.0351	0.0049	3	
4.38	1.46		0.228		2.18					661
		0.027		0.0115		2.175	0.0250	0.0020	1.2	
4.30	1.433		0.2395		2.17					662

Assume, in our case, $d_2 = 5.60$ ft. To locate the jump in accordance with Table XVII, we may find the distance of section 5.6 to section 6 for which $L = 515.5$. Using for the interval 5.60–6.00 an average value of $(1 - \beta) = -2.36$, we have:

$$\text{For } y_2 = 6;\ \eta_2 = 2.00;\ B(\eta_2) = 0.104$$
$$\text{For } y_1 = 5.6;\ \eta_1 = 1.866;\ B(\eta_1) = 0.122$$

$$\Delta\eta = 0.134;\ \Delta B = -0.018;\ (1 - \beta)\Delta B = -2.36 \times (-0.018) = +0.0425$$

$$\Delta\Pi = \Delta\eta - (1 - \beta)\Delta B = 0.134 - 0.0425 = 0.0915$$

$$l_{6-5,6} = \frac{y_0}{s_0}\Delta\Pi = 600 \times 0.0915 = 55 \text{ ft.}$$

The total distance to the jump from section 9 is

$$L_j = 515.5 + 55 = \sim 570 \text{ ft.}$$

Exercise:

1. In a layout (Fig. 101), assume a canal, Type B (Fig. 15), laid with $s_{01} = 40^0/_{00}$; take $y_{01} = 2$ ft.; $y_a = 8$ ft.

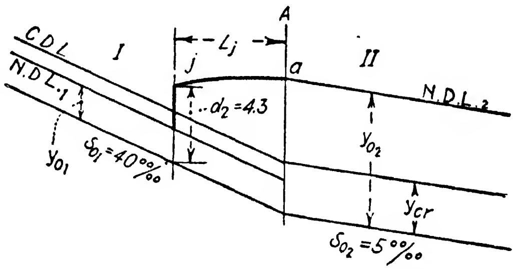

FIG. 104.—Canal with a break in the bottom slope from *steep* to *mild*, featuring an S_1 curve (Exercise 2 in Example 13).

Use Bazin coefficients and $n = 3.70$. Determine the S_1 surface curve.

2. A canal (Fig. 104), Type B, is laid with a break in the bottom slope, from $s_{01} = 40^0/_{00}$ to $s_{02} = 5^0/_{00}$; $Q = 300$ cu. ft. per second.

It is known that rapid flow in uniform condition with $s_{01} = 40^0/_{00}$ will result in a jump, the depth d_2 after which is 4.30ft. Determine the surface curve. Use Bazin coefficients and $n = 3.70$.

Remark: This is a case, when s_{01} is steep, while s_{02} is mild and $y_{02} > d_2$. The surface curve ja is an S_1 curve with $y_a = y_{02}$.

40. The S_2 Curve.

Example 14

Figure 105*a* represents the entrance to the flume, treated in Example 13 (Fig. 101). There is a "drop" over section A, with $y_a = y_{cr}$.

Question 1. Assuming the elements of flow as in Example 13, determine and trace the surface curve *a-b*.

The curve in question begins at $y_a = 4.30$ ft. and ends at a depth y_b, which we shall assume to be $y_b = 1.001y_0$. The 1-β values are to be taken from Fig. 102. The figuring of the curve on the whole is similar to that of Table XVII. The exponent value $n = 3.20$ is used.

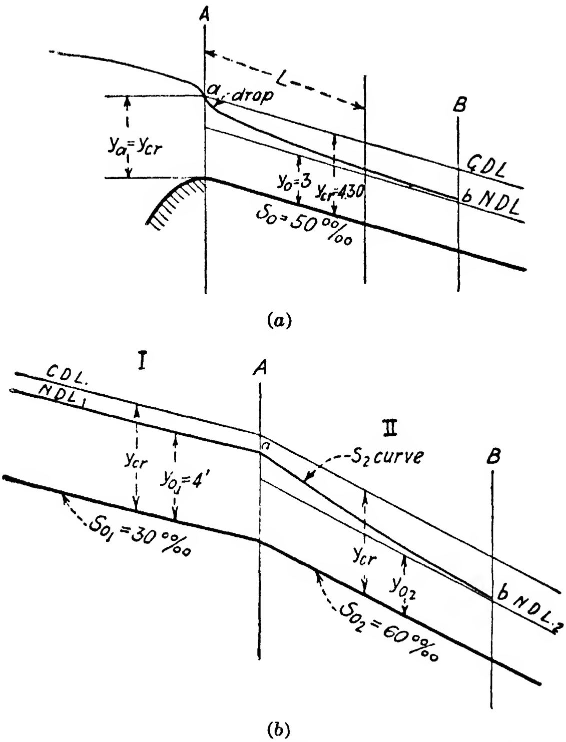

FIG. 105.—(a) An S_2 curve at the entrance of a steep-sloped flume (Example 14). (b) An S_2 curve in a canal with a break in the bottom slope (exercise in Example 14).

The distance L in this case is measured stream downwards, the initial section being section A, with $y_a = y_{cr}$. Accordingly, Col. 11 in Table XVIII is obtained by adding the lengths of the reaches l (Col. 10) from below upwards. The curve is traced in Fig. 103.

Exercise:

A canal, Type B (Figs. 14 and 15), is laid (Fig. 105/b) with a break in the bottom slope from $s_{01} = 30$‰ to $s_{02} = 60$‰. $y_{01} = 4$ ft. Use Bazin coefficients; $n = 3.70$. Determine the surface profile.

Remark: The example features the case of a canal, when both s_{01} and s_{02} are steep; the slope s_{01}, in particular being $< s_{02}$. The transition curve which lies wholly in the lower section, is an S_2 curve, beginning in section A with $y_a = y_{01}$ and asymptotically tangent to the uniform flow line y_{02}.

TABLE XVIII

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
y	η	$\Delta\eta$	$B(\eta)$ $n = 3.2$	$\Delta B(\eta)$	1-β for depth	1-β for interval	(1-β)ΔB	ΔH	l	L
3 003	1.001	--	2 008		1 96	–				1,848
		0 009		0 717		1 96	1 4050	1 3960	838.00	
3.030	1 010		1 291							1,010
		0 010		0 213		1 97	0 4200	0 4100	246.00	
3.060	1 020		1 078		1 97					764
		0 030		0 276		1 98	0 5460	0 5160	310.00	
3 150	1.050		0 802		1 99					454
		0 050		0 201		2 01	0 4040	0.3540	213.00	
3.300	1 100		0 601		2 03					241
		0 100		0 185		2 05	0 3590	0.2590	155.00	
3.600	1.200		0.416		2 07					86
		0.050		0 055		2 09	0 1150	0 0650	39 00	
3 750	1 250		0 361		2 10					47
		0.050		0 043		2 12	0 0913	0.0413	24.80	
3.900	1 300		0 318		2 13					22
		0 050		0 034		2 14	0 0727	0.0227	13.60	
4.050	1.350		0 284		2 15					8
		0 030		0.018		2.15	0 0387	0.0087	5 20	
4 140	1.380		0 266		2 16					3
		0.020		0 010		2 17	0.0217	0.0017	1.04	
4.200	1.400		0 256		2.17					2
		0 020		0 010		2 18	0 0218	0 0018	1 08	
4.260	1 420		0 246		2 19					1
		0 013		0 0065		2 20	0 0143	0.0013	0.78	
4.300	1.433		0.2395		2.21					0

41. The S_3 Curve.

Example 15

Assume, that the flume of Examples 13 and 14 is fed by means of a sluice as in Fig. 106. Section A is the *vena contracta* with $y_a = 1.5$ ft. Section B, taken as the end of the S_3 curve, has $\eta_b = 0.999$; $y_b = 2.997$ ft.

Question 1. Assuming the hydraulic elements as in Example 13, determine and draw the surface curve *a-b*.

The range of integration is from $y_a = 1.5$ ft., to $y_b = 2.997$ ft. The figuring, similar to that of Tables XVII and XVIII, is carried out, with $n = 3.20$.

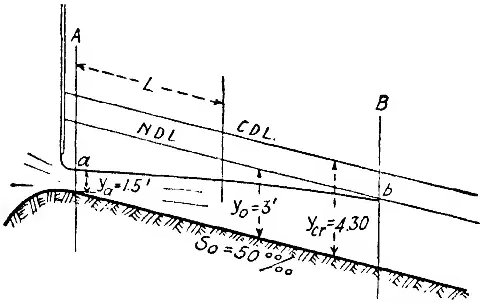

Fig. 106.—An S_3 curve in the case of outflow from under a sluice into a steep-sloped flume (Example 15).

TABLE XIX

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
y	η	$\Delta\eta$	$B(\eta)$	$\Delta B(\eta)$	1-β	1-β for interval	(1-β)ΔB	$\Delta\Pi$	l	L
2.997	0.999	.....	2.663	.....	1.96	—	—	.....	...	2,795
		0 009	.....	0.723	......	1.955	1.413	1.422	855	
2.970	0.990	. ..	1.940	.....	1.95	....	....	.....	...	1,940
		0 020	... .	0.380	......	1.920	0.738	0.758	455	
2.910	0.970	.. .	1 560	.....	1.93	....	....	.....	...	1,485
		0.030	.....	0.197	......	1.920	0.378	0.408	345	
2.820	0 940	....	1.363	.....	1.91	....	....	.....	...	1,140
		0.040	....	0.174	......	1.890	0.330	0 370	222	
2.700	0.900	.....	1.189	.....	1.87	......	....	....	...	918
		0.050	....	0.146	......	1.850	0.270	0.320	192	
2.550	0.850	.....	1 043	.....	1.82	....	....	.....	...	726
		0.050	.....	0 109	......	1.800	0.196	0.246	148	
2.400	0.800	.....	0.934	.....	1.78	....	....	.....	...	578
		0.100	...	0 168	......	1.730	0.281	0.381	229	
2.100	0.700	.....	0.766	.....	1.68	....	....	.....	...	349
		0.100	.....	0.135	......	1.610	0.212	0.312	187	
1.800	0.600	.....	0.631	.....	1.55	....	....	.....	...	162
		0.100	.....	0.117	......	1.460	0.171	0.271	162	
1.500	0.500	.....	0.514	.....	1.38	....	....	.....	...	0

Again, the distances L are measured stream downwards with the zero section at the *vena contracta, a.*

The curve, traced in Fig. 107, is referred to a horizontal line, as the drawing of the incline in the adopted scale would disfigure the presentation.

Exercise:

Assume (Fig. 108) that the sequence of the bottom slopes in Fig. 105*b* (exercise in Example 14) is reversed. Assume $y_{02} = 4$ ft. Determine and trace the surface profile.

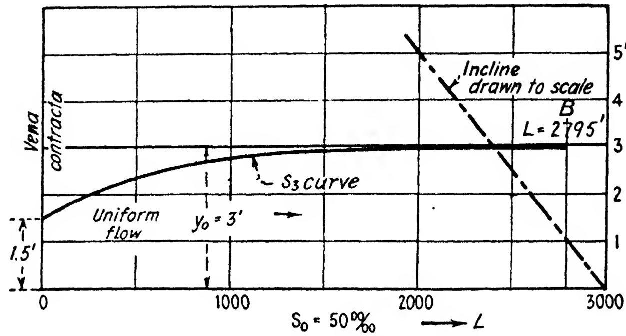

FIG. 107.—Tracing of the S_3 curve, Fig. 106, Example 15.

Remark: Both slopes are again steep, but in this case s_{01} is greater than s_{02}. The transition curve is an S_3 curve *a-b*, lying wholly within the lower section. The depth $y_a = y_{01}$.

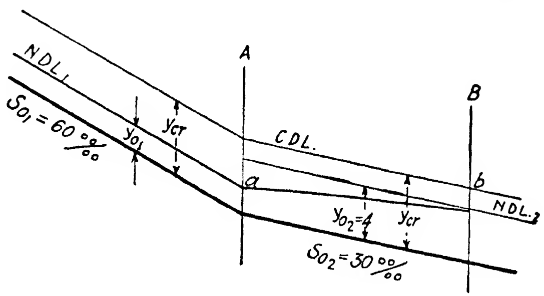

FIG. 108.—Featuring an S_3 curve in a canal with a break in the bottom slope (exercise in Example 15).

42. General Remarks.—In the light of the preceding, certain deductions of general character regarding the influence of different factors and the precision of computations seem to be in place.

Equations (86), and (91), which determine the length of a reach for a certain depth interval, give such length as the product of the factor y_0/s_0 by the parenthesis value of $(\Phi_2 - \Phi_1)$ or $(\Pi_2 - \Pi_1)$.

Other things being equal, the length is thus proportional to y_0/s_0, the length of a horizontal line, drawn through y_0 to intersect the bottom line.

Curves are longer or shorter, proportionately to the normal depth and in inverse proportion to the bottom-slope value s_0.

Outside of y_0/s_0, which is to be taken as a parameter of flow, the length elements of a curve depend on the value of $\Delta\Phi$ or $\Delta\Pi$, which for a certain interval of $\Delta\eta$ depend on the value of the hydraulic exponent n. Generally speaking shape, size, and roughness of the walls are united in a combined effect, expressed in the particular value of the hydraulic exponent.

In general, the higher the exponent, the shorter the curve. That is proven from following the tables, and comparing the values of the increments $\Delta\Phi$ or ΔB, taken for the same interval $\Delta\eta$ with different n's.

Plate VI shows the values of $\Delta\Phi = \Phi(\eta_2) - \Phi(\eta_1)$ and of ΔB, respectively, for different intervals of η. The values $\Delta\Phi$ and ΔB decrease with an increase of n.

Precision of Computations.—The $\Delta\Phi$ and ΔB curves, traced in Plate VI, are also useful in throwing light on the magnitude of errors, incurred through an assumption as to the approximate value of the hydraulic exponent.

The *incline* of the curves being small, a deviation in the exponent value of 0.1, which is one-half of the table range has a maximum effect on the value of $\Delta\Phi$ and ΔB of not over 3 to 4 per cent, and that only in the region near $\eta = 1$ and for exponents near $n = 3$. This justifies the convenient procedure of adopting one average value of n over a depth range without the detail of further subdivision. The outline of the curves, the curvature of which is slight, indicates further that arithmetical interpolation, *i.e.*, simply taking proportionate parts for Φ and B between table lines, is a procedure wholly justified under usual circumstances.

In other words, in most cases no recourse need be taken to the more cumbersome methods of graphical or analytical interpolation.

Influence of Resistances.—A major influence in the whole domain of varied flow is that of resistances. The basic premise that losses in varied flow are identical with the resistances in uniform flow at the same depth is an approximation, particularly incorrect in those special cases of retarded movement, where the restoration of kinetic energy plays a decisive part. Analytically a *corrected*

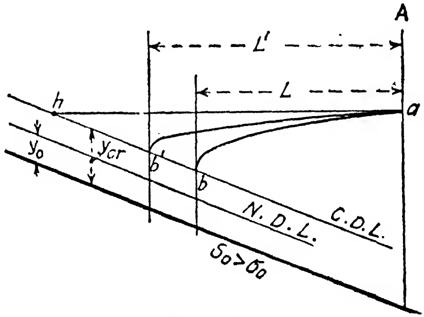

FIG. 109.

resistance factor C would lead to a change in the conveyance value $\mathfrak{K}$, as well as in $\sigma = \frac{g}{C^2}\frac{p}{b}$ and thus in 1-β. The 1-β values are particularly influential in case of $s_0 > \sigma$. For this reason the S_1, S_3, and M_3 curves which are also figured at present with no special regard for increased losses in divergent flow must be considered as the least accurate.

Nevertheless, even with such limitations, results obtained from the study of surface curves are most useful. For example, in Fig. 109, assuming that *a-b* is an S_1 curve, figured as in Art. 39, an eventual correction for increased resistances would result in a longer curve, something like *a-b'*. The curve, however, under all circumstances would be convex and would always lie below the horizontal line *a-h*. Thus the limits *a-h* and *a-b* determine the boundaries, within which the actual curve will unfold.

Again in Fig. 110, assuming that *a-c* and *a-b* are the M_3 and the S_3 curves, respectively, determined as in Arts. 38 and 41, then, if the resistances were higher, the original store of energy contained in the flow at high kineticity in *a* would be consumed more rapidly. The curves, therefore, would be shorter, like *a-c′* and *a-b′*. In other words, the curves *a-c* and *a-b* represent curves of extreme possible length.

Suppose, for example, there were to be a jump in section d_1 (Fig. 110) and the portion preceding the jump was to be protected against erosion. The length L_d determined in accordance with Question 2, Example 12, would give a margin of safety, as the jump would occur probably somewhat nearer.

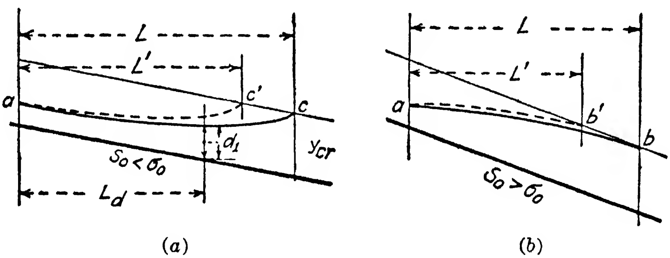

FIG. 110.

In the case of falling curves, where the potential head is transformed into kinetic energy, the precision is more satisfactory. However, here as everywhere else one should always bear in mind the general degree of accuracy which is inherent in engineering computations in the presence of the uncertainty of friction coefficients and other complicating circumstances accompanying flow in actual structures.

Effect of Curvature near y_{cr}.—In Art. 13 it was emphasized that the varied flow equations are applicable only when streaming complies with the Bélanger requirements of parallel movement.

These conditions obviously are not met in the neighborhood of y_{cr}, where the curvature of the surface is pronounced. Therefore, the curves, as obtained in the preceding paragraphs, are inaccurate in the vicinity of y_{cr}.

Such being the fact, one should not fail to realize, however, that the curves, as traced, for example in Fig. 103, are drawn on a much exaggerated vertical scale, and that the curvature becomes really pronounced only in the *immediate vicinity of* y_{cr}. Thus, the inaccuracy from this source is limited only to a short stretch, usually representing only a very small fraction of the total length of a surface curve.

With this in mind, we may continue to apply equations over the whole range of depths, as this gives a valuable general view of the movement and often furnishes, at the point of the critical depth, a convenient zero section from which distances are to be measured.

CHAPTER X

CHANNELS WITH HORIZONTAL BOTTOM

43. Equation of Flow.—In a canal with a horizontal bottom, with $s_0 = 0$ (Fig. 111), the normal depth is infinite, so y_0 cannot be used as a parameter. A simple solution, however, is reached by referring the movement to the critical depth. In fact, with $s_0 = 0$, Eq. (17) through Eq. (20) becomes

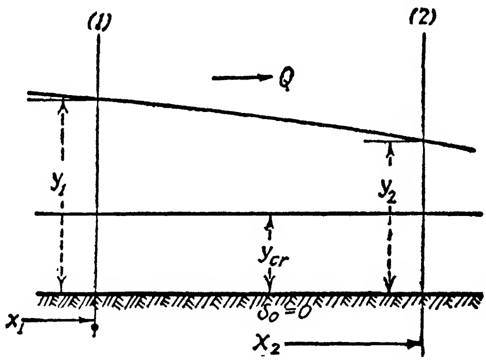

FIG. 111.—Flow in a canal with a horizontal bottom.

$$s = -\frac{dy}{dx} = \frac{Q^2}{\mathfrak{K}^2} - \frac{Q^2}{g}\frac{b}{a^3}\cdot\frac{dy}{dx} \tag{93}$$

Substitute

$$Q^2 = \mathfrak{K}^2{}_{cr}\sigma_{cr}$$

where σ_{cr} is the critical slope at the critical depth, which by definition (Art. 19) makes flow of Q at y_{cr} uniform; also, from Eq. 42,

$$a^3/b = \mathfrak{M}^2(y) = \sigma \cdot \mathfrak{K}^2/g$$

We obtain, instead of Eq. (93),

$$\frac{dy}{dx}\left(1 - \frac{\mathfrak{K}^2{}_{cr}}{\mathfrak{K}^2}\cdot\frac{\sigma_{cr}}{\sigma}\right) = -\sigma_{cr}\frac{\mathfrak{K}^2{}_{cr}}{\mathfrak{K}^2} \tag{94}$$

Multiplying by $\mathfrak{K}^2/\mathfrak{K}^2_{cr}$, and separating the variables,

$$dx \cdot \sigma_{cr} = dy\left(\frac{\sigma_{cr}}{\sigma} - \frac{\mathfrak{K}^2}{\mathfrak{K}^2_{cr}}\right) \tag{95}$$

Introducing the hydraulic exponent, so that

$$(\mathfrak{K}/\mathfrak{K}_{cr})^2 = (y/y_{cr})^n \tag{96}$$

and designating

$$\frac{\sigma_{cr}}{\sigma} = \delta \tag{97}$$

Equation (95) acquires the form

$$dx \cdot \sigma_{cr} = [\delta - (y/y_{cr})^n]dy \tag{98}$$

Introducing, further, similar to (Eq. [80]),

$$y/y_{cr} = \tau;\ dy = y_{cr}d\tau \tag{99}$$

we obtain, by separating the variables,

$$dx = \frac{y_{cr}}{\sigma_{cr}}[\delta \cdot d\tau - \tau^n d\tau] \tag{100}$$

Applied to an interval of depths between y_1 and y_2 (Fig. 111), which correspond to $\tau_1 = y_1/y_{cr}$ and $\tau_2 = y_2/y_{cr}$, and assuming that $\delta_{1.2}$ is an average value of δ within the interval, so that $\delta_{1.2}(\tau_2 - \tau_1) = \int_{\tau_1}^{\tau_2}\delta \cdot d\tau$, the respective length of the reach is

$$l_{2,1} = x_2 - x_1 = \frac{y_{cr}}{\sigma_{cr}}\left[\delta_{1.2}(\tau_2 - \tau_1) - \frac{\tau_2^{n+1} - \tau_1^{n+1}}{n+1}\right] \tag{101}$$

Designating

$$\left(\delta \cdot \tau - \frac{\tau^{n+1}}{n+1}\right) = T(y) \tag{102}$$

we have, instead of Eq. (101),

$$l_{2,1} = x_2 - x_1 = \frac{y_{cr}}{\sigma_{cr}}[T(y_2) - T(y_1)] \tag{103}$$

For a channel with a horizontal bottom, Eqs. (101) to (103) play a rôle analogous to that of Eqs. (84) to (86) in the general case of varied flow; only that instead of using y_0 as a parameter, the whole movement is referred to the critical depth. In particular, δ, as defined in Eq. (97), is the relation of the parametric value σ_{cr} to the variable σ, and replaces β; τ has replaced η; y_{cr}/σ_{cr} comes instead of y_0/σ_0. The integration, on the other hand, is a simple quadration. No tables are required and the computations may be performed with the aid of usual logarithms.

Example 16

An equalizing canal, Type B (Fig. 14), 5,000 ft. long, is laid with a horizontal bottom between two storage reservoirs A and B (Fig. 112).

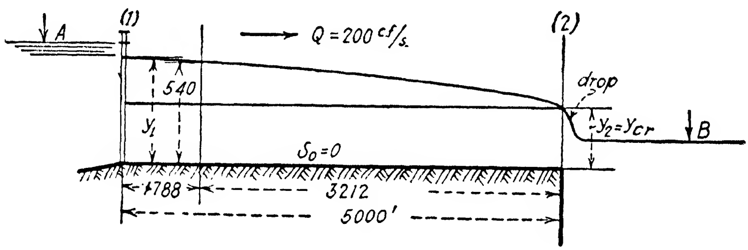

FIG. 112.—Canal layout for Example 16.

Question 1. Assuming that, when level B is at its lowest, the efflux from the canal is unobstructed and that 200 cu. ft. per second are flowing from A to B with a hydraulic drop in section 2, determine the surface curve. Use Bazin coefficients.

The Critical Depth. $\mathfrak{M}_c = 200/5.67 = 35.3$; to which there corresponds (see Fig. 15) $y_{cr} = y_2 \sim 3$ ft. From Table V, $\sigma_{cr} = 23.2^0\!/_{00}$. The hydraulic exponent for the range of y from 3 to 6 ft.,

$$n = 2\,\frac{\text{Log}\,\dfrac{15.500}{4.165}}{\text{Log}\,6/3} = \sim 3.80$$

The values of $\delta = \sigma_{cr}/\sigma$ for different depths (see Table V and Fig. 15)

$y =$	3	3.15	3.50	4.0	5.0	6.0
$\sigma =$	23.2	23.	22.8	22.7	22.8	22.8
$\delta =$	1	1.01	1.02			

$$y_{cr}/\sigma_{cr} = 3/23.2 \cdot 10^{-4} = 1{,}293 \text{ ft.}$$

Following the usual procedure, the integration range is divided into intervals, going by the depth. Equation (101), applied to an interval, gives

$$l_{2,1} = 1{,}293\left[\delta(\tau_2 - \tau_1) - \frac{\tau_2^{4.8} - \tau_1^{4.8}}{4.8}\right] = 1{,}293\left[\delta \cdot \Delta\tau - \Delta\frac{\tau^{4.8}}{4.8}\right]$$
$$= 1{,}293\Delta T$$

The values of $\tau^{4.8}/4.8$ are figured by means of four-place logarithms. The figuring is assembled in Table XX.

TABLE XX

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
y	τ	δ	$\Delta\tau$	$\delta \cdot \Delta\tau$	$\frac{\tau^{4.8}}{4.8}$	$\Delta\frac{\tau^{4.8}}{4.8}$	ΔT	l	L
3.00	1.00	1.00	–	–	0.2083	–	+	....	0
			0.05	0.0502	... .	0.0551	0.0049	6.3	
3.15	1.05	1.01	.....	....	0.2634	....	....	....	6.3
			0.10	0.1015	......	0.1445	0.0430	55.5	
3.45	1.15	1.02	.....	....	0.4079	....	....	....	62.0
			0.15	0.1530	......	0.3062	0.1532	198.0	
3.90	1.30	1.02	.....	....	0.7141	....	....	....	260.0
			0.15	0.1530	......	0.5269	0.3739	483.0	
4.35	1.45	1.02	.....	....	1.2410	....	....	....	743.0
			0.15	0.1530	......	0.7480	0.5950	771.0	
4.80	1.60	1.02	.....	....	1.9890	....	....	....	1,514.0
			0.10	0.1020	......	0.6990	0.5970	767.0	
5.10	1.70	1.02	.....	....	2.6840	....	....	....	2,281.0
			0.10	0.1020	......	0.8220	0.720	931.0	
5.40	1.80	1.02	.....	....	3.5060	....	....	....	3,212.0
			0.20	0.2040	......	2.3130	2.109	2,725.0	
6.00	2.00	1.02	.....	....	5.8190	....	....	....	5,937.0

The distances in the last column are measured from the drop section 2. The surface curve is plotted in Fig. 113.

Question 2. Determine the depth in a section immediately below the sluice in Fig. 112, at a distance 5,000 ft. from section 2. The reach length between section 5.40 and (1) is 5,000 − 3,212 = 1,788 ft. Hence

$$1{,}788 = 1{,}293[T(5.40) - T(y_1)]$$

From Table XX

$$T(5.40) = \delta \cdot \tau - \frac{\tau^{4.8}}{4.8} = 1.02 \times 1.80 - 3.506 = -1.67$$

The solution lies in finding the value of τ, which would satisfy

$$T(y_1) = T(5.40) - \frac{1{,}788}{1{,}293} = -1.67 - 1.38 = -3.05$$

The solution of the equation

$$T(y_1) = 1.02\tau - \frac{\tau^{4.8}}{4.8} = -3.05$$

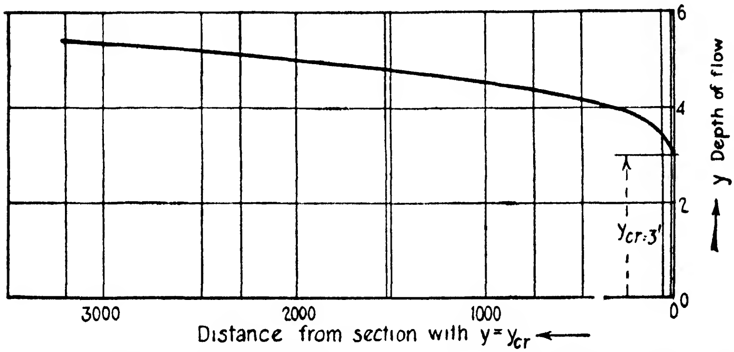

FIG. 113.—Tracing of the M_2 curve for the canal with horizontal bottom, Fig. 112, Example 16.

is found by tentative probes; we have:

τ	1.02τ	$\tau^{4.8}/4.8$	T
1.95	1.989	5.126	−3.137
1.94	1.978	4.998	−3.020

$\tau = 1.943$ gives $y_1 = 3 \times 1.943 = 5.83$ ft.

Exercise:

Assuming a canal, Type *D*, laid with a horizontal bottom at a length of 10,000 ft., determine the surface curve corresponding to a flow of 670 cu. ft. per second. Use G.K. coefficients for $s_0 = 0.0001$ with $n = 0.025$. Assume hydr. exponent $n = 3.8$.

PART II

PRACTICAL APPLICATIONS

The methods and means developed in Part I are to be applied to different practical cases of hydraulic engineering. Foremost are problems relating to the design of canals. It will be found that design based solely on notions of uniform flow is inadequate and may lead to unexpected and, at times, unremediable consequences. Only when the designing is approached in terms of varied flow, may one feel positive how a canal will stand up and function in actual practice, especially when the discharge or the levels deviate from what was assumed in theoretical computations; or, again, when the friction coefficients are not quite what they were taken to be in the figuring.

Chapters XI to XIV deal in particular with canals of mild bottom slope ($s_0 < \sigma_0$). Canals of steep slope ($s_0 > \sigma_0$) are treated separately in Chap. XV. The last chapter is devoted to backwater curves in natural watercourses.

It is assumed in the following that the reader has familiarized himself sufficiently with the basic conceptions and elementary methods developed in Part I.

CHAPTER XI

DELIVERY OF A CANAL

44. Definitions. Examples.—A canal of given cross-sectional forms is laid at a length L between section 1 and 2 with a bottom slope s_0 (Fig. 114). The levels, and accordingly the depths y_2 and y_1, in the different sections

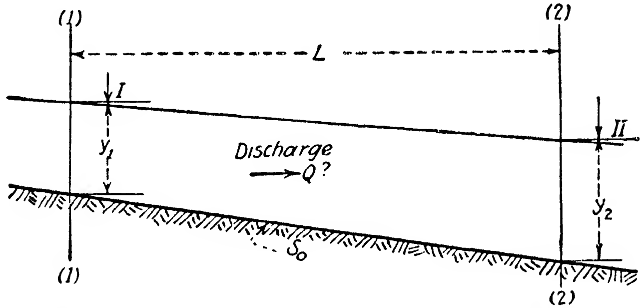

FIG. 114.—The problem of *delivery*. Determining the discharge Q under given stages of flow y_1 and y_2.

vary. The problem is to determine the discharge which the canal will *deliver* at each particular combination of stages y_1 and y_2. Accordingly the *delivery* of a canal, as it is called, is the yield in discharge of a canal under varying conditions of level or, which is identical, under varying depths in certain given sections, usually at the extremities of the structure.

To illustrate, we have in Fig. 115 a canal connecting two reservoirs A and B with varying levels, the problem being to determine the discharge which will flow from reservoir A to B under different conditions of levels. Another instance (Fig. 116) is that of a storage reservoir S placed between two canals, the first of which (a) draws water from a source of supply A, while the other (b) feeds water to the locus of usage *B*. Depending on the position of the various levels, the storage reservoir will either

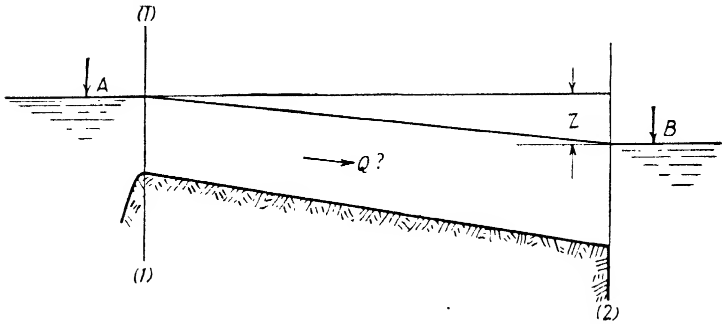

FIG. 115.—Varying delivery Q in a canal, connecting reservoirs with varying levels A and B.

accumulate water or supply the difference between the deliveries Q_b and Q_a.

These are but instances to illustrate the nature of the problems. Actually any canal called on to carry varying

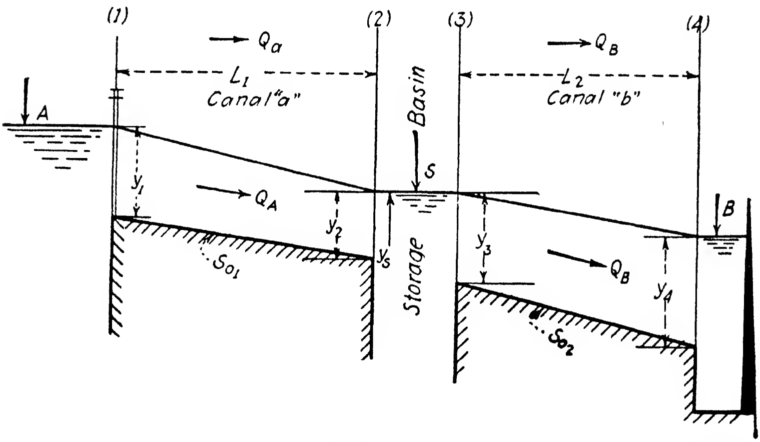

FIG. 116.—Varying delivery in a system with a storage basin placed between two canals.

quantities of water or subject to fluctuation of levels at its extremities will be working under conditions of varied flow; and practically in all such cases knowing the *delivery* of the canal under the different possible combinations of levels is precedent to the answering of questions, relating to the functioning of such canals. The problem of delivery is therefore basic. It is analogous to discharge in the field of uniform movement, and plays the same rôle in computations concerned with varied flow.

45. Case of y_1 Constant.—The simplest case is when the level at one of the extremities of the canal does not change. For example, in Fig. 117/I, the depth y_1 is assumed to remain constant, while y_2, determined by level B, fluctuates.

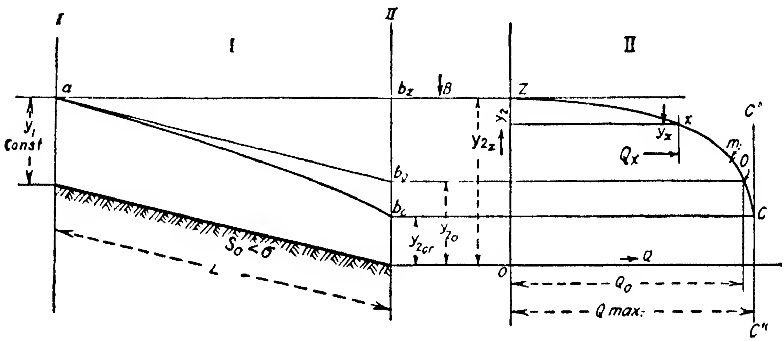

FIG. 117.—Delivery curve $Q = f(y_2)$ in the case of y_1 = constant.

The delivery curve $Q = f(y_2)$ has the outline as in Fig. 117/II. The curve has the following characteristic points:

1. Point Z: The surface line ab_z is level; $y_{2z} = y_1 + s_0L$; the delivery obviously is $Q = o$.

2. Point O: The surface line ab_0 is parallel to the bottom; $y_2 = y_1$; flow is uniform with $Q_0 = \mathfrak{K}_0\sqrt{s_0}$, where $\mathfrak{K}_0$ is the conveyance of the cross-section at the depth y_1.

3. Point C: Corresponding to the maximum possible discharge Q_{max}. The depth y_{2c} is the critical depth, connected with the maximum delivery through $Q^2_{max}/g = (a^3/b)_{y=y_c}$. The curve ab_c is the lowest possible surface curve of the M_2 type, compatible with the given depth y_1. A further lowering of level B in the reservoir below $y_2 = y_c$ will not affect flow in the canal and will not increase the discharge. The depth at the extremity of the canal will

remain y_{cr}; the outflow into the pool B will take place by means of a hydraulic drop.

Intermediary points, relating to the portion z-o of the delivery curve, will correspond to a rising curve of the M_1 type, with a delivery $Q \gtrless {}^{O}_{Q_o}$. To determine such curve follow the procedure of Example 9, Art. 36. Briefly, for a selected value Q_x, determine the normal depth y_{0x}.* Then with $\eta_{1x} = y_1/y_{0x}$ in section 1, determine $\eta_{2x} = y_2/y_{0x}$ and thus y_2, by means of Eq. (91).

$$\Phi(\eta_2) = \frac{Ls_0}{y_0} + \Phi(\eta_1) \tag{104}$$

The points between O and C will relate to a falling curve M_2 with $Q \begin{smallmatrix} > Q_0 \\ \leqslant Q_{max} \end{smallmatrix}$. The procedure to be applied is that of Example 11 Art. 37. It is similar in general to that used for the z-o portion. Only instead of Eq. (104), one should use Eq. (86),

$$\left.\begin{aligned} \Pi(\eta_2) &= \frac{Ls_0}{y_0} + \Pi(\eta_1) \\ \text{where} \qquad \Pi(\eta) &= \eta - (1 - \beta)B(\eta) \end{aligned}\right\} \tag{105}$$

which equations take into account the change of kinetic energy in accelerated flow.

It will be found in most practical cases, except when the canal is very short or the bottom slope exceptionally small, that the o-c portion of the delivery curve (Fig. 117/II), between the normal discharge Q_0 and Q_{max}, is very steep, so that Q_{max} exceeds Q_0 only by a very small amount. This is a point of paramount importance. It should be accepted as a practical rule; a generalization which ripens from accumulated experience in actual computation. The mean-

* The depth y_{0x} may be either determined through $\mathfrak{K}_{0x} = Q_x/\sqrt{s_0}$, by means of the $\mathfrak{K} = f(y)$ curve, or taken directly from a normal discharge curve (Art. 9), which curve will always be found a useful auxiliary when handling problems of this nature.

ing is that in most practical cases the discharge Q_0, corresponding to uniform movement, is actually very close to the maximum possible discharge. As a consequence, for example, the lowering of level B at the lower extremity of the canal and an increase in the surface slope do not result in any measurable addition to the discharge, and thus in most instances should be discarded as a means to increase the delivery, to "draw" additional water.

The reason lies in the fact, that the falling curve M_2 is comparatively short. In connection therewith, Fig. 118

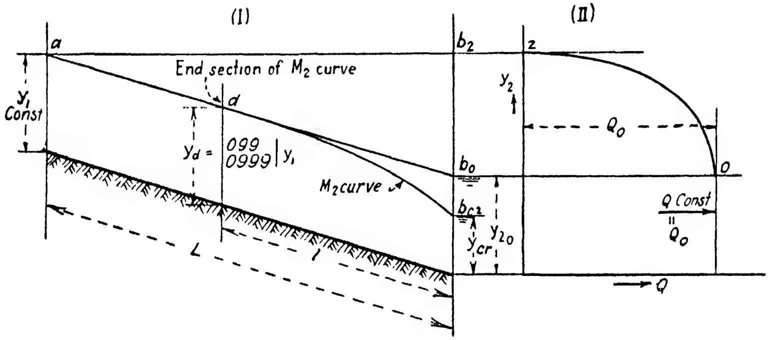

FIG. 118.—Delivery curve in a canal, the length of which exceeds the length of the falling M_2 curve.

pictures the rather extreme case when the total length l of the curve db_c, between the critical depth and the depth $y_d = \frac{0.99}{0.999} y_1$ is less than the length of the canal L. In such a case, which occurs rather frequently, the falling curve under all circumstances ends below section a, which means that the reduction of level in B below $b_0 (y_2 = y_1)$ never extends beyond d and, therefore, has no effect on the flow in the upper portion of the canal. The normal discharge Q_0 in this case is the maximum discharge. The fluctuation of level B below b_0 does not affect the delivery at all. The $Q = f(y_2)$ curve in this case has the outline as given in Fig. 118/II.

Example 17

Assume a canal, Type *A* (Plate III), laid at a length of 2 miles (10,560 ft.). Assume y_1 maintained constant and equal to $y_1 = 6$ ft. (Fig. 119); $s_0 = 4^0\!/_{00}$; the drop of the canal bottom $Ls_0 = 10{,}560 \text{ ft.} \times 4 \cdot 10^{-4} = 4.224$ ft.

Question 1. Assuming that the stage *B* varies from level conditions (*az*) downwards, determine the delivery as $Q = f(y_2)$.

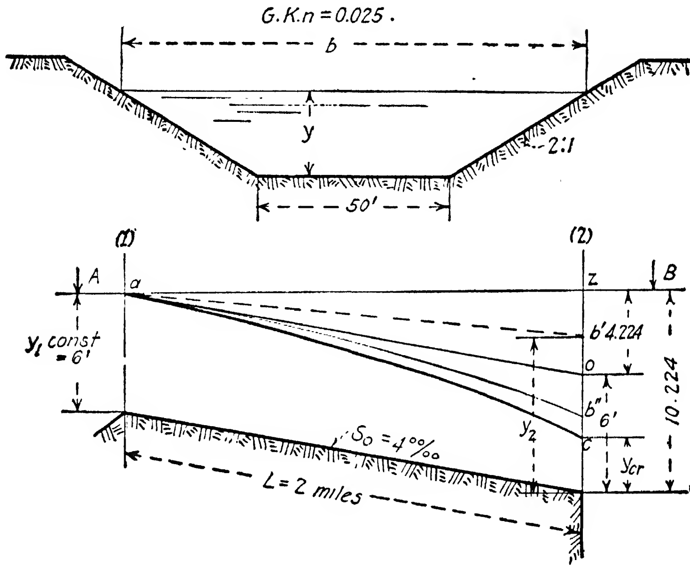

FIG. 119.—Canal layout, Example 17.

Portion of the curve z-o (Fig. 117) corresponding to a rising curve *ab′* in Fig. 119 of the M_1 type:

1. Zero delivery. $Q = 0$; $y_{2z} = 6 + 4.224 = 10.224$.
2. Uniform movement: $y_2 = y_1 = 6$ ft.; $\mathfrak{K}_0 = 639 \times 10^2$ (see Plate III);

$$Q_0 = 639.10^2 \times \sqrt{4} \cdot 10^{-2} = 1{,}278 \text{ cu. ft. per second.}$$

3. Intermediate points between *O* and *Z* with $y_2 \begin{matrix} > 6 \text{ ft.} \\ < 10.224 \end{matrix}$ and $O < Q < Q_0 = 1{,}278$ cu. ft. per second; in selecting the intermediate discharges, for which y_2 is to be determined, go by

normal depths, selecting a series of y_0 values between 6 ft. and zero.

As an example make $y_0 = 5$ ft.; the conveyance at $y_0 = 5$ (Plate III) is $\mathfrak{K}_0 = 465 \times 10^2$; the discharge $Q = 465 \cdot \sqrt{4} = 930$ cu. ft. per second.

With the hydraulic exponent value at $n = 3.6$, we have for section 1:

$$\eta_1 = y_1/y_0 = 6/5 = 1.20$$

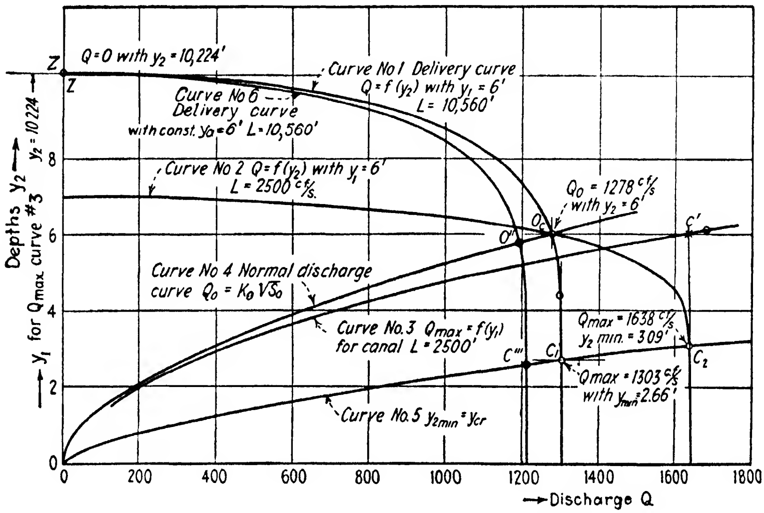

FIG. 120.—Delivery curves $Q = f(y_2)$ for canal layout, Fig. 119

and from the tables $\Phi(1.20) = 0.880$. With

$$s_0L/y_0 = 4.224/5 = 0.845$$

we have (Eq. 104)

$$\Phi(\eta_2) = 0.845 + 0.880 = 1.725$$

In determining the corresponding η_2, take proportionate parts between table values for $\eta_2 = 1.80$ and $\eta_2 = 1.85$, obtaining

$$\eta_2 = 1.80 + 0.05\frac{1.725 - 1.712}{1.768 - 1.712} = 1.80 + 0.0116 = 1.812$$

which gives

$$y_2 = \eta_2 y_0 = 1.812 \times 5 = 9.06 \text{ ft.}$$

The computation elements for other points are assembled in Table XXI.

TABLE XXI

y_0	$\mathcal{K}_0$ in 10^2 units	Q_0	η_1	$\Phi(\eta_1)$	Ls_0/y_0	$\Phi(\eta_2)$	η_2	y_2
6.0	639.00	1,278	Uniform movement					6.00
5.8	600.00	1,200.0	1.034	0.273	0.728	1.001	1.262	7.330
5.4	530.00	1,060.0	1.110	0.653	0.783	1.436	1.568	8.470
5.0	465.00	930.0	1.200	0.880	0.845	1.725	1.812	9.060
4.0	313.00	626.0	1.500	1.351	1.056	2.407	2.440	9.790
3.0	190.00	380.0	2.000	1.934	1.410	3.344	3.360	10.100
2.0	93.70	187.4	3.000	2.978	2.112	5.090	5.096	10.190
1.0	27.35	54.7	6.000	5.996	4.224	10.220	10.221	10.221
0	Level conditions							10.224

The delivery curve $Q = f(y_2)$, figured in part as above, is plotted in Fig. 120 as curve 1.

Portion of the curve o-c, correspondng to a falling curve ab'' in Fig. 119 of the M_2 type.

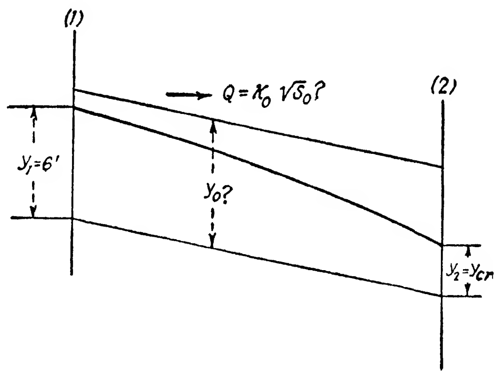

FIG. 121.

For this portion

$$y_2 \begin{matrix} < y_0 = 6 \text{ ft.} \\ > y_{2min} = y_{cr} \end{matrix}; \quad Q \begin{matrix} > Q_0 = 1{,}278 \text{ cu. ft. per second} \\ \leqslant Q_{max} \end{matrix}$$

Maximum Discharge.—Naturally one should first determine the limiting value of Q_{max} and the respective y_{2min}. The depth y_{2min} is the critical depth, corresponding to Q_{max}. In other words, Q_{max} is the discharge, which on an M_2 curve with $y_2 = y_{cr}$ will make y_1 in section (1) equal to 6 ft. To find Q_{max} take a series of Q's, beginning from $Q_0 = 1{,}278$ cu. ft. per second upwards; then, making in each case $y_2 = y_{cr}$, determine the corresponding y_1.

The discharge, which will make $y_1 = 6$ ft., is the Q_{max} sought (Fig. 121). In computing follow in general the procedure as given in Example 11. In selecting the series of Q's, proceed by the normal depths.

The basic hydraulical elements, required for the computation are assembled in Table XXII.

TABLE XXII

(1)	(2)	(3)	(4)	(5)
y_0	$\mathfrak{K}_0$	$Q = \mathfrak{K}_0\sqrt{s_0}$	$\mathfrak{M}_{cr} = Q/\sqrt{g}$	y_{cr}
5.80	600×10^2	1,200	213.0	2.54
6.00	639×10^2	1,278	225.5	2.63
6.10	653×10^2	1,316	232.0	2.68
6.20	678×10^2	1,356	239.0	2.73

The $\mathfrak{K}_0$ values in the above table are taken from Plate III. The $\mathfrak{M}(y)$ values (Col. 4) correspond to critical flow of the discharge as given in Col. 3. The critical depths (Col. 5) are taken from the $\mathfrak{M} = a\sqrt{a/b}$ curve of Plate III.

The values of 1-β, within the range of depths $y_0 - y_{cr}$ are:
$y = 6$ ft.: $\sigma = 55 \times 10^{-4}$; $\beta = s_0/\sigma = 4/55 = 0.073$; $1 - \beta = 0.927$
$y = 2.6$ ft.: $\sigma = 72 \times 10^{-4}$; $\beta = s_0/\sigma = 4/72 = 0.056$;
$1 - \beta = 0.944$

An average value of $1 - \beta = 0.935$ will be used. The procedure of computation will be made clear by figuring out a point in detail. Take $y_0 = 6.10$ ft., corresponding to $Q = 1{,}316$ cu. ft. per second and $y_2 = y_{cr} = 2.68$ ft.

For section 2 we have: $\eta_2 = 2.68/6.10 = 0.439$; with $n = 3.6$, $B(\eta_2) = 0.444$; $0.935 \times 0.444 = 0.415$; $\Pi(\eta_2) = 0.439 - 0.415 = 0.024$.

$$Ls_0/y_0 = 4.224/6.10 = 0.695$$

Accordingly

$$\Pi(\eta_1) = \Pi(\eta_2) - \frac{Ls_0}{y_0} = 0.024 - 0.695 = -0.671$$

To facilitate the finding of η_1 for the particular point in question as well as for other points, we have drawn a curve (Fig. 122)

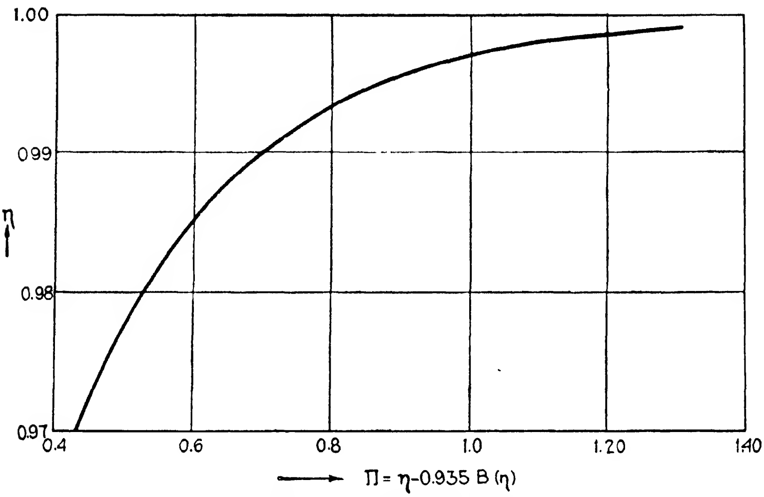

FIG. 122.—Auxiliary curve, for solving equation $\Pi = \eta - 0.935B(\eta)$.

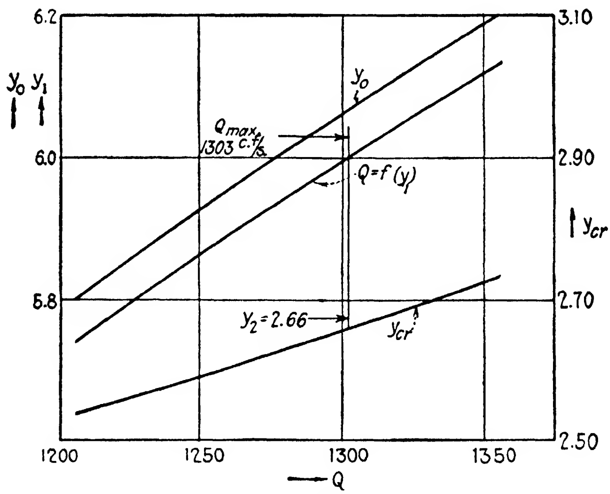

FIG. 123.—Auxiliary curve for determining the maximum discharge in canal layout Fig. 119. For a chosen discharge (abscissae) the y_{cr} curve gives the depth $y_2 = y_{cr}$ in section 2 while the curve $Q = f(y_1)$ gives the corresponding stage in section 1.

of $\Pi(\eta_1) = \eta_1 - 0.935B(\eta_1)$ in accordance with the following auxiliary table.

η	$B(\eta)$ with $n = 3.6$	$0.935B(\eta)$	$\Pi(\eta) = \eta - 0.935B(\eta)$
			—
0.970	1.501	1.402	0.432
0.975	1.554	1.451	0.476
0.980	1.617	1.510	0.530
0.985	1.699	1.587	0.602
0.990	1.814	1.696	0.706
0.995	2.008	1.870	0.875
0.999	2.457	2.307	1.308

For $y_0 = 6.10$, the value of η_1 corresponding to $\Pi(\eta_1) = -0.671$, is $\eta_1 = 0.9887$.

The figuring for the other points is assembled in Table XXIII.

TABLE XXIII

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
y_0	Q	$y_2 = y_{cr}$	η_2	$B(\eta_2)$	$0.935B(\eta_2)$	$\Pi(\eta_2)$	$\frac{L_{80}}{y_0}$	Π_1	η_1	y_1
								—		
5.80	1,200	2.54	0.438	0.443	0.414	0.024	0.731	0.707	0.9901	5.74
6.00	1,278	2.63	0.438	0.443	0.414	0.024	0.707	0.683	0.9893	5.94
6.10	1,316	2 68	0.439	0.444	0.415	0.024	0.695	0.671	0.9887	6.03
6.20	1,356	2.73	0.440	0.445	0.416	0.024	0.683	0.659	0.9881	6.13

The Q, y_1 and $y_{cr} = y_2$ values (Cols. 2, 3, and 11) are plotted in curve form $Q = f(y_1)$ in Fig. 123. The discharge, corresponding to $y_1 = 6$ ft., is found to be $Q_{max} = 1{,}303$ cu. ft. per second with $y_2 = y_{cr} = 2.66$ ft.

Intermediary Points between Q_0 and Q_{max}.—Inasmuch as $Q_{max} = 1{,}303$ is rather close to $Q_0 = 1{,}278$ (difference about 2 per cent), there is little room for finding additional points. One point at least should be established nevertheless, say that corresponding to $y_0 = 6.06$ ft., with $Q = 1{,}300$ cu. ft. per second. For this point, the elements in section 1 are:

$\eta_1 = 6/6.06 = 0.99; \Pi(\eta_1) = -0.706; Ls_0/y_0 = 4.224/6.06 =$ 0.696

Accordingly

$$\Pi(\eta_2) = 0.696 + (-0.706) = -0.010$$

This equation is satisfied with $\eta_2 = 0.725$ and $y_2 = 0.725 \times 6.06 = 4.40$ ft. In fact, the respective table values give:

η	$B(\eta)$	$0.935B(\eta)$	$\Pi(\eta)$
0.72	0.779	0.728	−0.008
0.73	0.793	0.742	−0.012

By plotting this point and the Q_{max} point into Fig. 120, we obtain the $Q = f(y_2)$ delivery curve (1) in complete form.

46. Long and Short Canals. Effect of Bottom Slope.—The delivery curve, as computed above, presented in Fig.

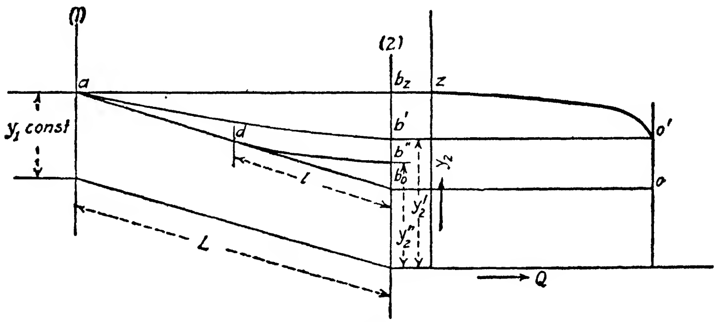

FIG. 124.—Delivery curve in a very long canal.

120 (curve 1) is typical of canals of *medium* length. The maximum discharge is, even if slightly, still greater than Q_0; on the other hand, within the whole range *o-z* of the rising backwater curve, the fluctuations of y_2 have a marked effect on the delivery.

As the length of the canal increases, Q_{max} and Q_0 become closer and closer to each other; and finally, as in Fig. 118, they coincide. As the canal becomes longer, its length L may become greater (Fig. 124) than the length of the rising curve l, corresponding to a depth y_2''. Obviously flow in section

1 will begin to become affected by the variations of level B, only when, and after the depth y_2 will reach and exceed a certain depth y'_2, which makes the curve $b'a$ equal to the length of the canal. Naturally, for all $y_2 < y'_2$, the delivery remains constant and equal to Q_0. The respective portion of the $Q = f(y_2)$ diagram (Fig. 124) is a vertical line o-o'. To determine y'_2, make y_1 in section 1 equal to 1.01 or $1.001y_0$, and figure the corresponding stage in section 2.

As to *short* canals, it is evident that the smaller the length L, the greater the excess of Q_{max} over Q_0. The difference in the character of the delivery curve will be made clear from the following example which refers to a canal identical with that of Example 17 but of a shorter length.

Example 18

Determine $Q = f(y_2)$ for the canal, Example 17, with a length $L = 2{,}500$ ft. (Fig. 125).

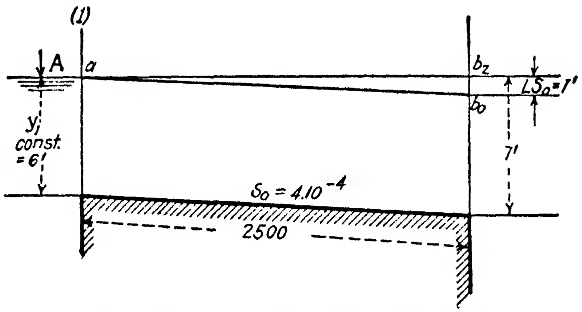

Fig. 125.—Canal layout for Example 18.

$$L \cdot s_0 = 2{,}500 \times 4 \cdot 10^{-4} = 1 \text{ ft.}$$

accordingly y_{2z}, corresponding to level conditions and $Q = 0$, is

$$y_{2z} = 6 + 1 = 7 \text{ ft.}$$

The elements of the o-z portion of the curve are given in Table XXIV which in its first three columns is identical with Table XXI.

TABLE XXIV

y_0	Q_0	$\Phi(\eta_1)$	Ls_0/y_0	$\Phi(\eta_2)$	η_2	y_2
6.0	1,278.0	Uniform movement				6
5.4	1,060.0	0.653	0.185	0.838	1.180	6.380
5.0	930.0	0.880	0.200	1.080	1.310	6.550
4.0	626.0	1.351	0.250	1.601	1.705	6.820
3.0	370.0	1.934	0.333	2.267	2.312	6.936
2.0	187.4	2.978	0.500	3.478	3.493	6.986
1.0	54.7	5.996	1.000	6.996	6.998	6.998
0.0	0.0	Level conditions				7

In figuring out the *o-c* portion of the curve, corresponding to falling curves M_2, we assume once more the average value of 1-β to be 0.935. The figuring, in accordance with formula

$$\Pi(\eta_2) = \frac{Ls_0}{y_0} + \Pi(\eta_1)$$

is carried out in Table XXV.

TABLE XXV

y_0	Q	η_1	$B(\eta_1)$	$0.935B(\eta_1)$	$\Pi(\eta_1)$	Ls_0/y_0	$\Pi(\eta_2)$	η_2	y_2
6.0	1,278	Uniform flow							6
6.2	1,356	0.968	1.485	1.389	−0.421	0.161	−0.260	0.937	5.81
6.5	1,474	0.923	1.222	1.142	0.219	0.154	0.065	0.821	5.35
6.8	1,610	0.883	1.090	1.019	0.136	0.147	+0.011	0.650	4.42
7.0	1,684	0.858	1.026	0.960	0.102	0.143	0.041		

The last row (y_0 = 7 ft.) carries no figures for η_2 and y_2. The reason is that, as the reader may easily ascertain, no table values of η_2 may be found which would satisfy the relation: $\Pi(\eta) = \eta - 0.935B(\eta) = +0.041$.

The biggest *positive* value that $\Pi(\eta)$ may reach is +0.024. The physical meaning of this is that the drop curve which would correspond to y_0 = 7 ft. and Q = 1,684 cu. ft. per second is shorter than the length of the canal. In other words, the delivery Q = 1,684 is in excess of Q_{max}.

To find the value of the maximum delivery, which obviously lies between 1,610 cu. ft. per second and 1,684 cu. ft. per second, we may apply the procedure of the preceding paragraph:

TABLE XXVI

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
y_0	Q	$\mathfrak{M}_c = Q/\sqrt{g}$	y_{cr}	$\eta_2 = y_{cr}/y_0$	$B(\eta_2)$	$0.935B(\eta_2)$	$\Pi(\eta_2)$	Ls_0/y_0	$\Pi(\eta_1)$	η_1	y_1
6.80	1,610	284	3.06	0.45				0.147	−0.123	0.874	5.94
6.90	1,646	290	3.10	0.45	0.4555	0 426	0.024	0.145	0.121	0.873	6.01
7.00	1,684	297.5	3.15	0.45				0.143	0.119	0.872	6.10

The y_{cr} values in Col. 4 are taken from the $\mathfrak{M}(y)$ curve of Plate III. Characteristic of the situation is the identity for all the critical depths of the numerical value of $\Pi(\eta_2) = \Pi(\eta_{cr})$, which in Table XXVI as well as in Table XXIII is equal to 0.024. As mentioned before, this is the maximum numerical value that the expression $\Pi(\eta) = \eta - 0.935B(\eta)$ may attain for the selected hydraulical exponent and the assumed 1-β. This feature of

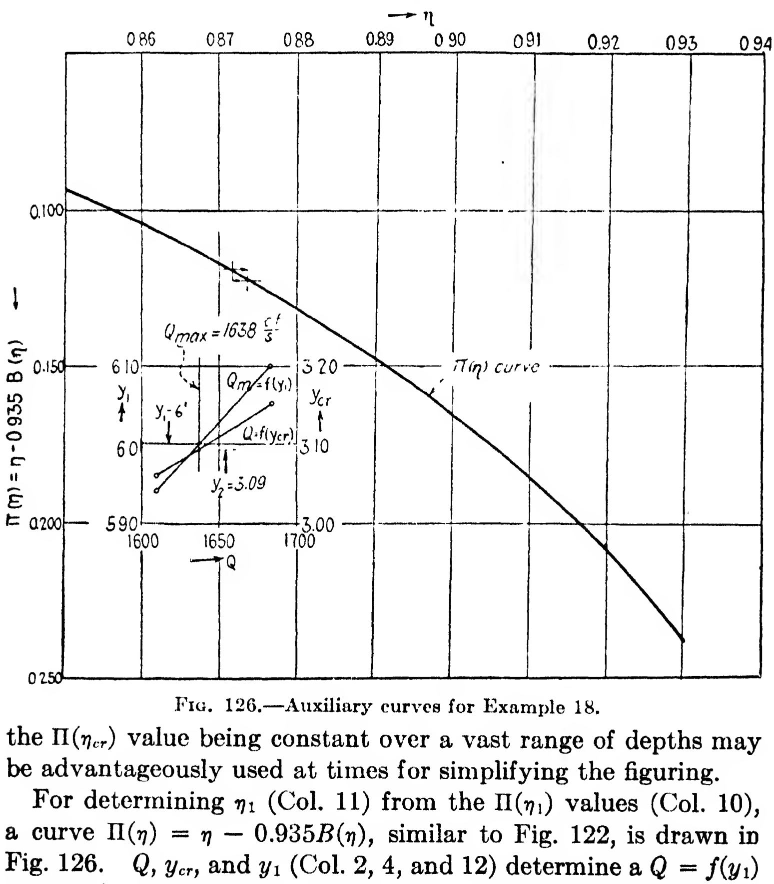

FIG. 126.—Auxiliary curves for Example 18.

the $\Pi(\eta_{cr})$ value being constant over a vast range of depths may be advantageously used at times for simplifying the figuring.

For determining η_1 (Col. 11) from the $\Pi(\eta_1)$ values (Col. 10), a curve $\Pi(\eta) = \eta - 0.935B(\eta)$, similar to Fig. 122, is drawn in Fig. 126. Q, y_{cr}, and y_1 (Col. 2, 4, and 12) determine a $Q = f(y_1)$ curve,* from which we determine $Q_{max} = 1{,}638$ cu. ft. per second as Q for $y_1 = 6$ ft. and the corresponding $y_2 = y_{cr}$ as 3.09 ft.

The delivery curve itself is plotted in Fig. 120 as curve 2. The two $Q = f(y_2)$ curves, one for a canal length of 10,224 ft., and

* Drawn in the quadrangle inside Fig. 126.

the other for $L = 2{,}500$ ft., bear comparison. In the long canal, Q_{max} exceeds Q_0 only by

$$\frac{1{,}303 - 1{,}278}{1{,}278} = \sim 2 \text{ per cent}$$

while in the short canal, the increase in delivery is

$$\frac{1{,}638 - 1{,}278}{1{,}278} = \sim 28 \text{ per cent}$$

Effect of Bottom Slope.—The outline of the delivery curve depends obviously on the relation of the canal length L to the longitudinal elements of the surface curves. As elucidated in Art. 42, other factors being equal, the longitudinal elements of the surface curves change inversely to s_0. The smaller the slope, the longer the curves and *vice versa.*

For that reason, reducing the bottom slope will have an effect similar to that of making the canal shorter.

47. The Q Maximum Curve.—The procedure, used in the preceding examples in determining Q_{max} for a given y_1 may be extended to build a Q_{max} curve, covering a wider range of conditions and showing in a general way the possible maximum discharge which may flow in a canal under the varying stage y_1. A curve of this kind will be found to be helpful in many instances. The computation procedure is that of Tables XXIII and XXVI applied to a wider range of depths. For a series of y_0 and Q values respectively, determine y_{1c} in section 1, assuming in each case y_2 to be the critical depth, corresponding to Q.

Example 19

For a canal, as in Example 18 (Fig. 125), determine the $Q_{max} = f(y_1)$ curve for a range of discharges between 0 and 2,000 cu. ft. per second.

In the table on p. 160, the value of the critical depth (Col. 4) is taken from the $\mathfrak{M}(y)$ curve in Plate III. For the points $y_0 \geqq 5$ ft., the average value of $1 - \beta = 0.935$ was accepted, as in the preceding examples, which results in an identical value of $\Pi(\eta_{cr}) = 0.024$ for all the respective points and permits using the $\Pi(\eta_1)$ curve of Fig. 126 for determining η_1.

TABLE XXVII

COMPUTATIONS RELATING TO a $Q_{max} = f(y_1)$ CURVE FOR A CANAL, FIG. 125, EXAMPLE 19

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
y_0	Q	$\mathfrak{M}_{cr} = Q/\sqrt{g}$	$y_{cr} = y_2$	η_2	$B(\eta_2)$	$1-\beta$	$(1 - \beta) \times B(\eta_2)$	$\Pi(\eta_2)$	s_0L_0/y_0	$\Pi(\eta_1)$	η_1	y_1
2	187.4	33.10	0.77	0.385	0.3880	0.960	0.372	0.013	0.500	0.487	0.973	1.95
3	380	67.15	1.20	0.400	0.4030	0.950	0.383	0.017	0.333	0.366	0.943	2.83
4	626	110.20	1.64	0.410	0.4135	0.945	0.390	0.020	0.250	0.230	0.922	3.69
5	930	163.90	2.15	0.430	0.4345				0.200	0.176	0.905	4.52
6	1,278	225.50	2.63	0.438	0.4430	0.935		0.024	0.167	0.143	0.887	5.32
7	1,684	296.70	3.15	0.448	0.4536				0.143	0.119	0.872	6.10
8	2,136	378.00	3.62	0.453	0.4587				0.125	0.101	0.857	6.86

For the points below $y_0 = 5$ ft., the average 1-β values were determined by taking in each case the average critical slope σ from Plate III for the range between y_0 and y_{cr}, and figuring

$$1 - \beta = 1 - \frac{4 \times 10^{-4}}{\sigma}$$

The table values of Q (Col. 2) and y_1 (Col. 13) determine the maximum discharge curve $Q_{max} = f(y_1)$ which is plotted as

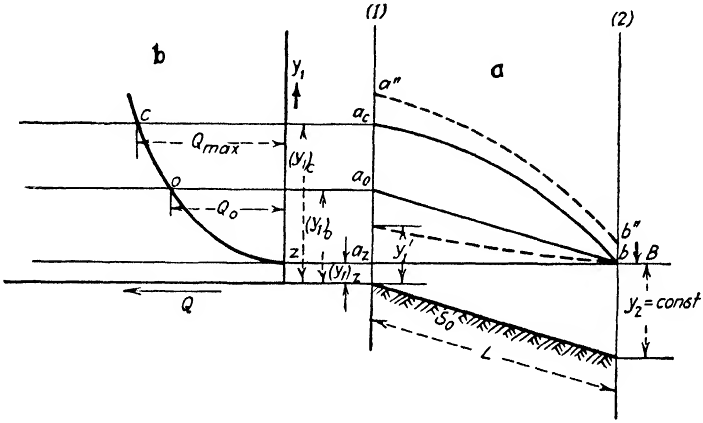

FIG. 127.—Delivery curve $Q = f(y_1)$ in the case of y_2 constant.

curve 3 in Fig. 120. The curve follows rather closely the normal discharge curve 4, corresponding to uniform flow, once more showing that even in a rather short canal the uniform-flow delivery is not far from Q_{max}.

Point C' on the Q_{max} curve corresponding to $y_1 = 6$ ft. is identical with point C_2 on the $Q = f(y_2)$ delivery curve. To complete the picture there is also plotted the curve of $y_{cr} = y_{2min} = f(Q_{max})$. Obviously this curve, if viewed as $y_{2min} = f(y_1)$, indicates the lowest possible position to which level in section 2 may sink with the given initial depth y_1 in section 1. Obviously the y_{2min} curve intersects the delivery curves $Q = f(y_2)$ at points C_1 and C_2.

48. Case of y_2 Constant.—It is assumed now (Fig. 127) that level B and thus the depth y_2 remain constant. The fluctuating stage is stage y_1 in section 1. Accordingly, the deliveries vary with the depth y_1, the curve becoming $Q = f(y_1)_{y_2 = const.}$

The characteristic points are:

1. Level condition b—a_z; with $Q = 0$ and $(y_1)_z = y_2 - s_0L$.

2. Uniform flow b—a_o; with $Q = \mathfrak{K}_2\sqrt{s_o}$, where $\mathfrak{K}_2$ is the conveyance at $y_o = y_2$.

3. Maximum discharge curve b—a_c, compatible with the given y_2; Q_{max} in this case is the critical discharge corresponding to the given y_2. Hence $Q_{max} = \mathfrak{M}(y_2)\sqrt{g}$, where $\mathfrak{M}(y_2)$ is the value of $a\sqrt{a/b}$ at $y = y_2$.

The depth y_{1c}, corresponding to Q_{max}, is the particular point on the maximum discharge curve $Q_{max} = f(y_1)$ of the preceding article. Obviously, y_{1c} is the highest possible stage in section 1 which is compatible with the given constant depth y_2. In case y_1 were to increase beyond y_{1c}, the depth in section 2 would have to be greater than the given y_2. If, under such conditions, level B in the reservoir were to be maintained at the same stage, flow would emerge from the canal by means of a drop (see dotted curve $a''b''$).

The delivery curve $Q = f(y_2)$ has the outline, as given in Fig. 127*b*. The portion *z-o*, with

$$y_1 \begin{matrix} > y_{1z} \\ < y_{10} \end{matrix} \text{ and } Q \begin{matrix} > O \\ < Q_0 \end{matrix}$$

corresponds to rising-surface curves of the M_1 type. The portion *o-c*, with

$$y_1 \begin{matrix} > y_{10} \\ \leqslant y_{1c} \end{matrix} \text{ and } Q \begin{matrix} > Q_0 \\ \leqslant Q_{max} \end{matrix}$$

corresponds to drop curves of the M_2 type.

Example 20

Assume (Fig. 128) a layout similar to that of Example 17; assume, further, that the stage in section 2 is maintained constant at a depth $y_2 = 6$ ft.

Compute and draw the delivery curve $Q = f(y_1)$.

The characteristic points are:

1. Level conditions, with $Q = 0$ at $y_{1z} = 6$ ft. $- 4.224 = 1.776$ ft.

2. Uniform flow, with $Q_0 = 1{,}278$ cu. ft. per second at $y_1 = 6$ ft.

3. Maximum discharge.

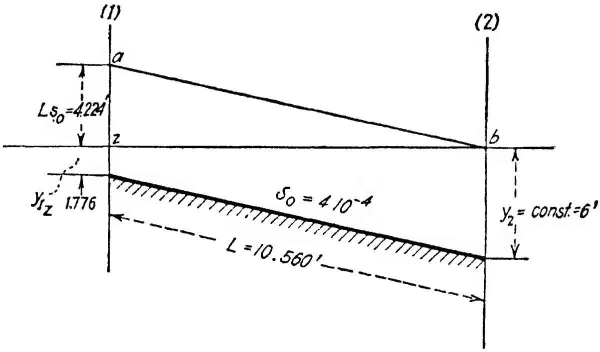

FIG. 128.—Layout for Example 20.

From Plate III we have $\mathfrak{M}(y = 6$ ft.$) = 834$; accordingly $Q_{max} = 834\sqrt{g} = 4{,}720$ cu. ft. per second. To find the corresponding normal depth:

$$\mathfrak{K}_0 = Q_{max}/\sqrt{s_0} = 4{,}720/\sqrt{4} \cdot 10^{-2} = 2{,}360 \cdot 10^2$$

to which there corresponds in Plate III, $y_0 = 12.43$ ft.

Depth y_{1c}.—The average value of σ between $y_2 = 6$ ft. and $y_{1c} = 12.43$ ft. is $\sigma = \sim 50 \cdot 10^{-4}$. Hence

$$\beta = 4/50 = 0.08 \text{ and } 1\text{-}\beta = 0.92$$

For section 2, we have:

$$\eta_2 = 6/12.43 = 0.483;\ B(\eta_2) = 0.491;\ \Pi(\eta_2) = 0.483 - 0.92 \times 0.491 = 0.031$$

With

$$L_0 s_0/y_0 = 4.224/12.43 = 0.34$$
$$\Pi(\eta_1) = 0.031 - 0.340 = -0.309$$

which is satisfied by $\eta_1 = 0.954$, making $y_{1c} = 0.954 \times 12.43 = 11.87$ ft.

Regarding intermediary points, we shall compute one point in each of the *o-z* and *o-c* portions of the curve:

ZO Portion; M_1 Curve: Take $y_0 = 4$ ft.; $Q = 626$ cu. ft. per second. With $y_2 = 6$ ft.; $\eta_2 = 6/4 = 1.5$; $\Phi(\eta_2) = 1.351$; $Ls_0/y_0 = 1.056$; hence

$$\Phi(\eta_1) = \Phi(\eta_2) - \frac{Ls_0}{y_0} = 1.351 - 1.056 = 0.295$$

which corresponds to $\eta_1 = 1.0385$; and $y_1 = 4.15$ ft.

OC Portion; M_2 Curve: Take $y_0 = 8$ ft.; $Q = 2{,}140$ cu. ft. per second.

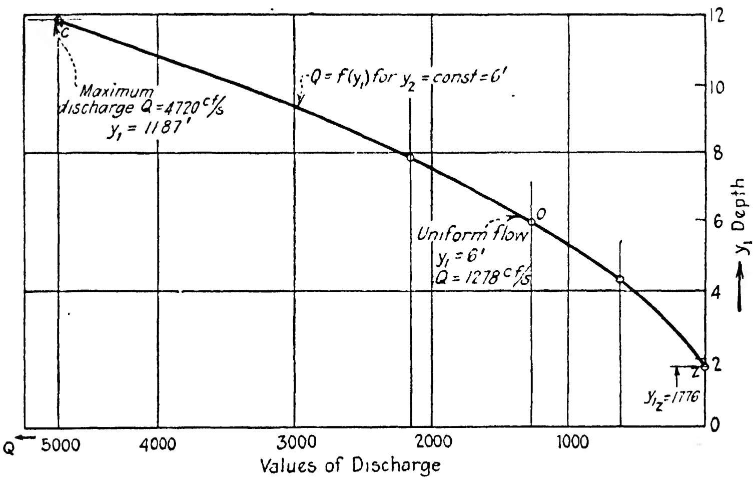

FIG. 129.—The $Q = f(y_1)_{y_2 = \text{const}}$ delivery curve for Example 20.

$\eta_2 = 6/8 = 0.75$; $B(\eta_2) = 0.823$; $\Pi(\eta_2) = 0.75 - 0.92 \times 0.823 = -0.008$

With

$$Ls_0/y_0 = 4.224/8 = 0.528$$

$$\Pi(\eta_1) = \Pi(\eta_2) - \frac{Ls_0}{y_0} = -0.008 - 0.528 = -0.536$$

This corresponds to $\eta_1 = 0.983$ and $y_1 = 7.86$ ft.

The curve is plotted in Fig. 129.

49. $Q = f(y_1, y_2)$. The Q Constant Curve.—In the general case, when both y_1 and y_2 vary, the problem in its broadest sense is to determine the delivery which takes place under any possible combination of depths y_1 and y_2.

Assuming that a certain pair of depths y_1 and y_2 is given, the question is answered by computing and tracing for that or the other of the given depths a portion of the $Q = f(y_2)$ or of the $Q = f(y_1)$ delivery curves, suggested in the preceding paragraphs. In fact, suppose y_2, as given, were to be $> y_1$; the case will be that of a rising curve

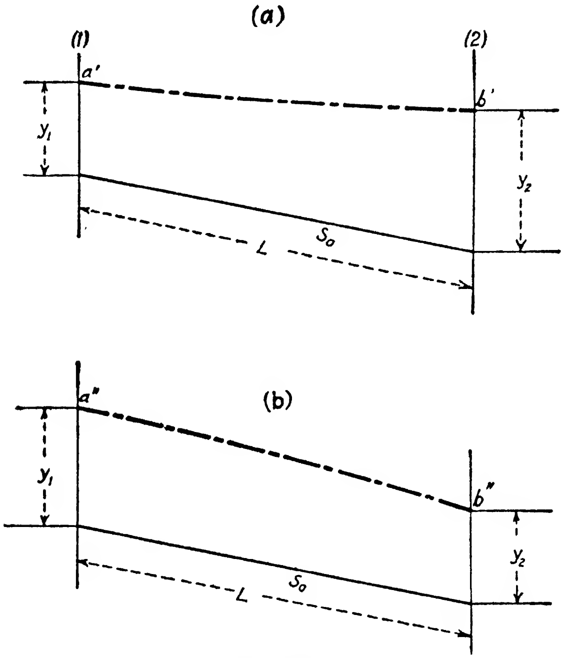

FIG. 130.

(Fig. 130*a*); the delivery Q is $< Q_0$, which would correspond to $y_0 = y_1$; the value of the actual discharge is determined either from a $Q = f(y_2)$ curve as in Art. 45, built with the given y_1 as constant; or from a $Q = f(y_1)$ curve, as in Art. 48, built with the given y_2 as permanent.

When y_2 is $< y_1$ (Fig. 130*b*) the delivery will be between the normal discharge Q_0 corresponding to $y_1 = y_0$ and Q_{max} corresponding to $y_2 = y_{cr}$. Start by trying out an M_2 falling curve, assuming the given y_1 to be 0.999 or 0.99 of y_0. Determine the corresponding y'_2. If the y'_2 so determined, is $< y_2$ as given, Q_0 is actually the maximum discharge. If y'_2 is $> y_2$ as given, build the *o-c* portion of the $Q = f(y_1)$ curve as in Fig. 129 for the given y_2.

The Q Constant Curve.—A very useful chart, picturing the delivery under all possible circumstances, is obtained by means of what we call the *Q constant curve.*

Assume a discharge Q, flowing in a canal (Fig. 131). The same volume of flow may be delivered by the canal in an infinite number of ways; each way of flowing being featured by a pair of depths y_2 and y_1 in the given sections 2 and 1, usually at the extremities of the structure. Obviously the depths y_2 and y_1 are mutually interdependent.

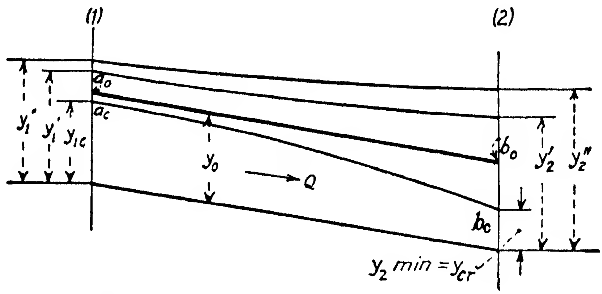

FIG. 131.—Reciprocal depths for a constant Q.

To each depth y_2, there corresponds one and only one depth y_1 and *vice versa;* so that each possible pair of depths may be said to constitute a pair of "reciprocal" depths.

One of the possible pairs of the reciprocal depths is $y_1 = y_2 = y_0$, corresponding to uniform flow of the given discharge Q. Above the line of uniform flow a_0b_0, we have y'_2 reciprocal to y'_1; y''_2 to y''_1, etc., all on M_1 curves. Below a_0b_0, the reciprocal depths correspond to points on an M_2 curve. Curve a_cb_c is the lowest possible position of the M_2 curve, the depth y_{2min} being the critical depth for the given discharge; y_{1c} is the ordinate of the Q_{max} curve (see Art. 47) corresponding to the given Q. Together and separately y_{2min} and y_{1c} are the lowest possible stages in their respective sections, at which the given delivery Q may flow.

As to the upper limit of the surface curve, it is evident that, as the depths y_1 and y_2 grow, the curve *a-b* tends to become a horizontal line with lim. $(y_2 - y_1)_{y=\sim} = s_0L$.

The functional relation between y_1 and y_2 may be represented by tracing a curve (Fig. 132), each point of which is determined by a pair of reciprocal values of y_1 and y_2. The resulting curve is the *Q constant curve.*

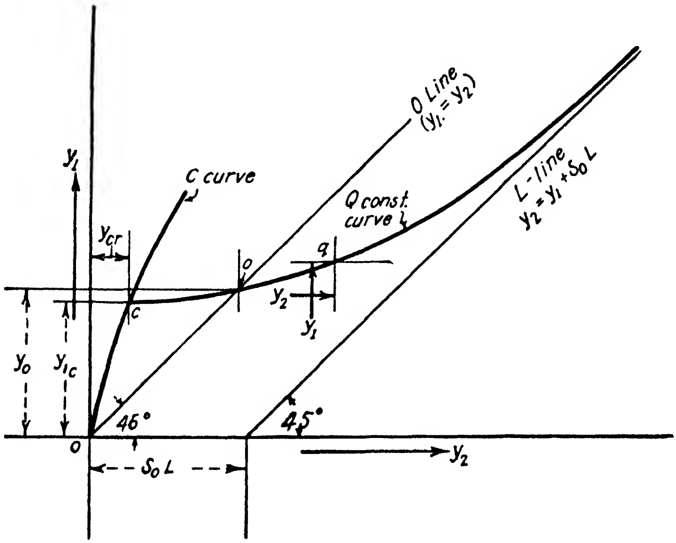

FIG. 132.—The Q constant curve.

The characteristic points and features of the curve are as follows:

1. Point O, with $y_1 = y_2$, corresponding to uniform flow, lies on a straight line drawn from the origin of the coordinates, at an angle of 45 deg. This line is the locus of the normal depths for all discharges.

2. Point C, determined for each given discharge by $y_2 = y_{cr}$ and $y_1 = y_{1c}$. Point C is the lowest possible limit of the curve for the discharge as given. By plotting the C points for the different Q's, one obtains the C line which is the locus of critical depths, so far as the abscissas y_2 are concerned. The ordinates of the C curve are the y_{1c} in the sense of Art. 47.

3. The upper limit, determined by $y_2 - y_1 = s_0L$ is a straight line (L line) drawn at 45 deg. from a point at a distance s_0L from zero. The limiting character of the L line is exemplified by the fact, that the Q constant curve lies to the left of such a line, to which it is asymptotically tangent.

By plotting a series of Q_{const} curves for different discharges, a universal chart is obtained giving a summarized picture of possible flow in the given canal under all possible combinations of levels (Fig. 133). The curves drawn for different Q's are congruent but do not intersect each other. Each point of the $Q = f(y_1, y_2)$ plane (Fig. 133) determines one and only one discharge, featured by the particular Q curve which happens to pass through such point.

Example 21

Question 1. Assume a canal layout, as of Example 18 (canal, Type A; length 2,500 ft.); determine the Q_{const} curve for Q = 1,278 cu. ft. per second, corresponding to y_0 = 6 ft.

For point O we have $y_1 = y_2 = 6$ ft. For point C (see Table XXVII): $y_2 = y_{cr} = 2.63$ ft.; $y_1 = y_{1c} = 5.32$ ft.

For the rising portion of the curve, use formula

$$\Phi(\eta_1) = \Phi(\eta_2) - \frac{Ls_0}{y_0} = \Phi(\eta_2) - 0.167$$

For the falling part of the curve, use formula

$$\Pi(\eta_1) = \Pi(\eta_2) - 0.167$$

The figuring, with $n = 3.6$ and $1 - \beta = 0.935$, is given in Table XXVIII.

TABLE XXVIII

(1) RISING CURVE M_1. ($y_2 > 6$ ft.)

η_2	y_2	$\Phi(\eta_2)$	$\Phi(\eta_1)$	η_1	y_1
1.00	Uniform flow				6.00
1.05	6.30	0.394	0.227	1.029	6.17
1.10	6.60	0.620	0.453	1.060	6.36
1.20	7.20	0.880	0.713	1.130	6.78
1.35	8.10	1.141	0.974	1.248	7.49
1.50	9.00	1.351	1.184	1.379	8.27
1.75	10.50	1.655	1.488	1.609	9.65
2.00	12.00	1.934	1.767	1.849	11.09

TABLE XXVIII.—(*Continued*)

(2) DROPPING CURVE M_2. ($y_2 < 6$ ft.)

η_2	y_2	$B(\eta_2)$	$0.935B(\eta_2)$	$\Pi(\eta_2)$	$\Pi(\eta_1)$	η_1	y_1
0.9	5.4	1.140	1.066	−0.166	−0.333	0.954	5.72
0.8	4.8	0.907	0.848	−0.048	−0.215	0.922	5.53
0.6	3.6	0.623	0.582	+0.018	−0.149	0.8905	5.34

The curve is plotted in Fig. 133 as curve 1.

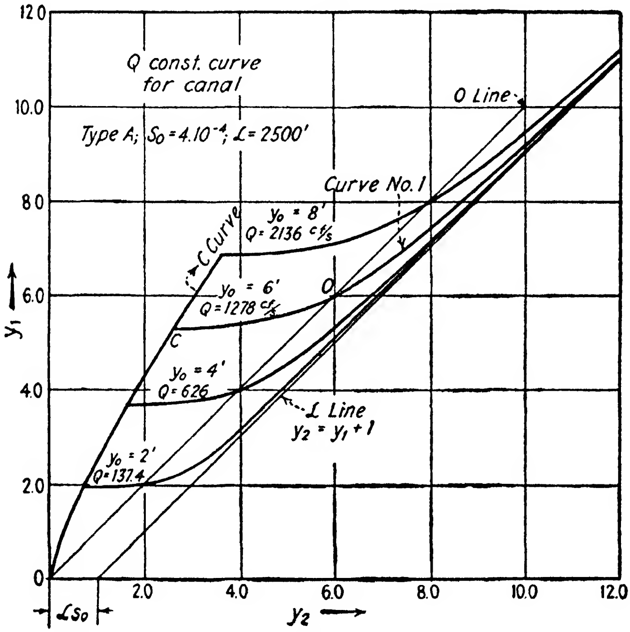

FIG. 133.—The Q constant curve in Example 21.

The L line, parallel to the O line, is drawn at a distance of $Ls_0 = 1$ ft.

Question 2. In addition to the above, figure and trace $Q_{const.}$ curves for the following:

y_0 = 8 ft., corresponding to Q = 2,136 cu. ft. per second
y_0 = 4 ft., corresponding to Q = 626 cu. ft. per second
y_0 = 2 ft., corresponding to Q = 137.4 cu. ft. per second

The points on the O line are $y_1 = y_2 = y_0$.

The points for the C curve may be taken from Table XXVII.

$$y_0 = 8 \text{ ft.}; \quad y_2 = y_{cr} = 3.62; \quad y_1 = y_c = 6.86$$
$$y_0 = 4 \text{ ft.}; \quad y_2 = y_{cr} = 1.64; \quad y_1 = y_c = 3.69$$
$$y_0 = 2 \text{ ft.}; \quad y_2 = y_{cr} = 0.77; \quad y_1 = y_c = 1.95$$

In figuring the rising portion of the curve with $y_2 > y_1$, we may use the same series of η_2's and $\Phi(\eta_2)$'s as in Table XXVIII, determining the $\Phi(\eta_1)$'s by subtracting in each case from $\Phi(\eta_2)$ the value of Ls_0/y_0, corresponding to the different y_0's. The figuring is clear from Table XXIX.

TABLE XXIX

η_2	$\Phi(\eta_2)$	$y_0 = 8$; $Ls_0/y_0 = 0.125$				$y_0 = 4$ ft.; $Ls_0/y_0 = 0.250$				$y_0 = 2$ ft.; $Ls_0/y_0 = 0.500$			
		y_2	$\Phi(\eta_1)$	η_1	y_1	y_2	$\Phi(\eta_1)$	η_1	y_1	y_2	$\Phi(\eta_1)$	η_1	y_1
1.10	0.620	8.80	0.495	1.069	8.55	4.40	0.370	1.045	4.18	2.20	0.120	1.020	2.04
1.20	0.880	9.60	0.755	1.146	9.17	4.80	0.630	1.103	4.41	2 40	0.380	1.048	2.09
1.35	1.141	10.80	1.016	1.272	10.18	5 40	0.891	1.205	4.82	2.70	0.641	1.106	2.21
1.50	1.351	12.00	1.226	1.408	11.26	6.00	1.101	1.324	5.29	3.00	0.851	1.186	2.07
2.00	1.934	.....	.....	.....	.....	8.00	1.684	1.775	7.10	4.00	1.434	1.565	3.13
3.00	2.978	.....	.....	.....	.....	12.00	2.728	2.756	11.02	6.00	2.478	2.513	5.03
5.00	4.994	.....	.....	.....	.....	.....	.....	.....	.....	10.00	4.494	4.502	9.00

The falling portion of the curve is figured in Table XXX.

The curves are plotted in Fig. 133, which thus gives an example of the universal chart mentioned at the beginning of the article.

Q Constant Curves for Long and Short Canals.—The difference in the outline of the curves is best exhibited by means of a numerical example; namely by comparing curves, computed for the same cross-section and the same delivery, but for canals of different lengths.

Example 22

Determine the Q_{const} curve for $y_0 = 6$ ft. for the canal, as of Example 21, but with a length $L = 2$ miles (10,560 ft.). The curve, designated as curve 2, is plotted in Fig. 134, while the respective curve for $L = 2{,}500$ ft. is traced in as curve 1. The O points of the two curves coincide. The C point for curve 2 is taken from Table XXIII, namely,

$$y_{2c} = y_{cr} = 2.63 \text{ ft.}; \; y_1 = y_{1c} = 5.94 \text{ ft.}$$

TABLE XXX

The Average Values of $1-\beta$ are taken from Table XXVII; for η_2 and $B(\eta_2)$ values see Table XXVIII

η_2	$B(\eta_2)$	$y_0 = 8$; $Ls_0/y_0 = 0.125$; $1-\beta = 0.935$						$y_0 = 4$ ft.; $Ls_0/y_0 = 0.250$; $1-\beta = 0.945$						$y_0 = 2$ ft.; $Ls_0/y_0 = 0.500$; $1-\beta = 0.96$					
		y_2	$0.935 \times B(\eta_2)$	$\Pi(\eta_2)$	$\Pi(\eta_1)$	η_1	y_1	y_2	$0.945 \times B(\eta_2)$	$\Pi(\eta_2)$	$\Pi(\eta_1)$	η_1	y_1	y_2	$0.96 \times B(\eta_2)$	$\Pi(\eta_2)$	$\Pi(\eta_1)$	η_1	y_1
				−	−						−						−		
0.9	1.140	7.2	1.066	0.166	0.291	0.945	7.56	3.6	1.078	0.178	0.428	0.968	3.37	1.8	1.092	0.192	0.692	0.987	1.97
0.8	0.907	6.4	0.848	0.048	0.173	0.903	7.22	3.2	0.857	0.057	0.307	0.945	3.78	1.6	0.870	0.070	0.570	0.980	1.96
				+															
0.6	0.623	4.8	0.582	0.018	0.107	0.862	6.90	2.4	0.589	0.011	0.239	0.926	3.70	1.2	0.598	0.002	0.488	0.9735	1.95
C point (Table XXVII)		3.62					6.86	1.64					3.69	0.77					1.95

In view of the closeness of the y_{1c} depth to y_0 no further points are computed for the O-C part of the curve.

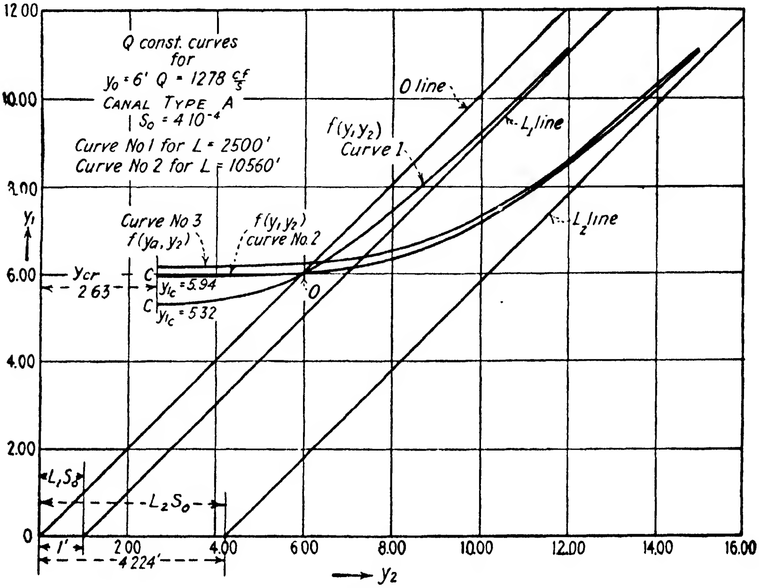

FIG. 134.—Q constant curve for similar canals of different lengths (Example 22).

The O-L portion, computed with

$$\Phi(\eta_1) = \Phi(\eta_2) - \frac{4.224}{6} = \Phi(\eta_2) - 0.704$$

is tabulated in Table XXXI.

TABLE XXXI

η_2	y_2	$\Phi(\eta_2)$	$\Phi(\eta_1)$	η_1	y_1
1.10	6.6	0.620	−0.084	1.010	6.06
1.20	7.2	0.880	+0.176	1.025	6.15
1.35	8.1	1.141	0.437	1.059	6.36
1.50	9.0	1.351	0.647	1.108	6.64
1.70	10.2	1.597	0.893	1.205	7.24
2.00	12.0	1.934	1.230	1.410	8.47
2.50	15.0	2.464	1.760	1.843	11.06

CHAPTER XII

ENTRANCE CONDITIONS

50. Local Phenomena at the Extremities of the Canal.—In the preceding chapter, the relations between the delivery Q and the depths y_1 and y_2 at the extremities of a canal were established without taking into account the phenomena which accompany the inflow or the outflow of water, as it enters or leaves the canal.

Outflow.—When a canal, as in Fig. 135, empties into a reservoir, one may anticipate a certain "gain" of level $\Delta y_B = y_B - y_2$, caused by the "restoration" of at least a part of the velocity head $v_2^2/2g$ carried by the water as it leaves the structure. Observation, however, shows that such restoration, if any, is inconsequential; in other words, under usual circumstances the kinetic energy is actually wholly dissipated in eddies and whirls. Therefore, in practical computation the gain Δy_B may be safely neglected and the stage y_2 at the end of a canal simply taken as being equal to y_B (Fig. 136).*

Entrance Regulated by a Sluice.—Figure 138 exemplifies a sluice or some other device, by means of which the depth y_1 may be established and regulated at will. In this case, the depth y_1, as considered in the preceding chapter, is simply taken as being the depth below the regulator. The length of the canal L, which figures in the computations, should be figured from the sluice section downwards, disregarding the short stretch, where the movement establishes itself. There will be a difference of level $\Delta y_A =$

* This simple rule will apply in all cases, except where the outflow is accompanied by a hydraulic drop (Fig. 137) with $y_2 = y_{cr}$ and $y_B < y_{cr}$. In this case as mentioned before, the depth $y_2 = y_{cr}$ will continue to remain as the lowest possible depth of outflow, irrespective of the position of level B.

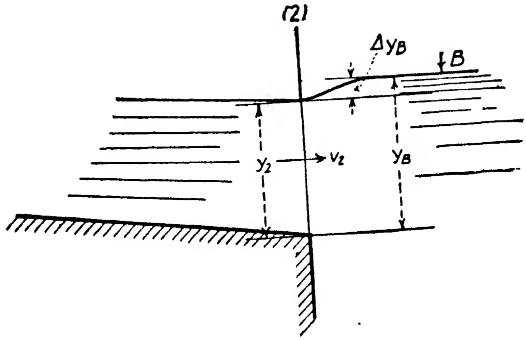

FIG. 135.

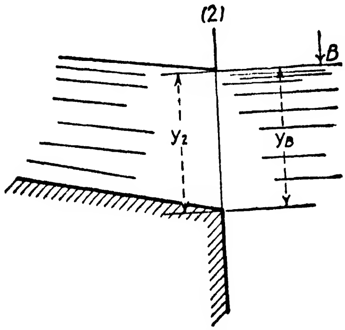

FIG. 136.

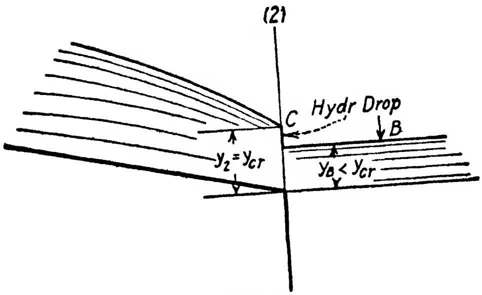

FIG. 137.

$y_A - y_1$, dependent on the discharge and the opening of the sluice, but this Δy_A which may be maintained at will has no organic connection with the varied flow in the canal as such.

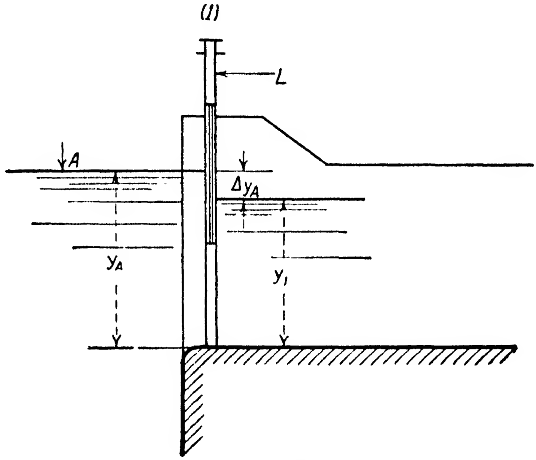

FIG. 138.

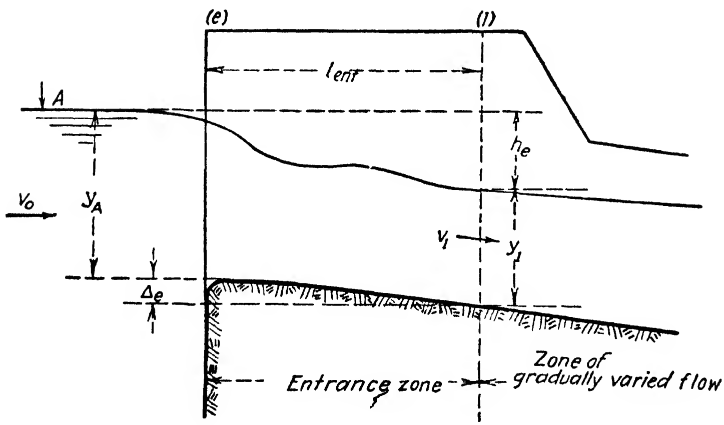

FIG. 139.—Free inflow into a canal.

Free Influx.—A most important case of entrance conditions is illustrated in Fig. 139. In this instance, water is supposed to flow into the canal *freely*. The resistances and losses, determined by the shape of the entrance walls and eventually by intermediate pillars, are comparatively small, so that one may designate the flow as "unobstructed." The greater part of the entrance head h_e is used to produce the velocity head $v_1^2/2g$.

In this case, flow in the canal and the entrance phenomena are organically connected. To each discharge Q, as determined by the reciprocal depths y_1 and y_2 (Fig. 140), there corresponds a definite value of an entrance head h_e and thus a stage $y_a = y_1 + h_e$. So the depths y_a, y_1, and y_2

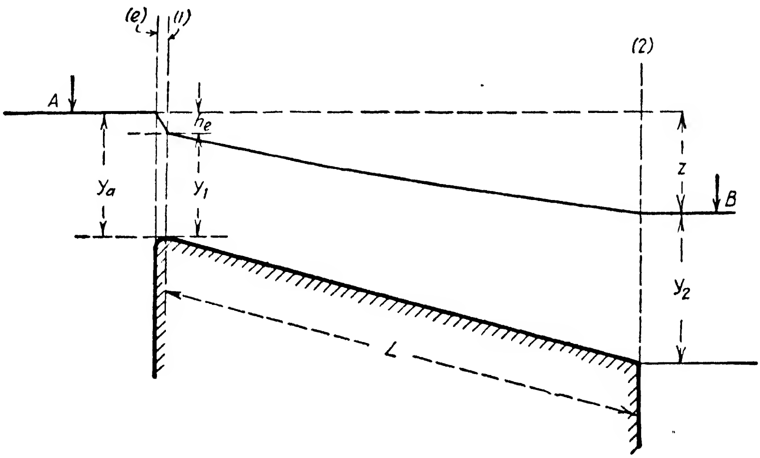

FIG. 140.—The entrance head h_e.

become functionally connected, representing features of one single phenomenon.

In most practical cases, it is the depth y_a, determined by level A above the intake, which is given. For example, with reference to the problem handled in Example 17, it usually would be level A in the feeding reservoir and thus the depth y_a which would remain constant, the problem being to determine the delivery in the system as a function of the varying level B.

Whenever the velocity in the canal is substantial and the length of the canal not too great, the entrance head may constitute a substantial part of the total fall of level Z (Fig. 140) and should by no means be left out of consideration.

Free inflow, functionally connected with varied flow in the canal, constitutes specifically the content of the following articles.

51. Evaluation of the Entrance Head. Entrance Zone.—The inflow into a canal of mild slope features an *entrance zone* (Fig. 139), marked by an undulated surface. Section 1 with the depth y_1 and a velocity v_1 is assumed schematically as a demarcation between the entrance zone and the zone of gradually varied (parallel) flow. In the entrance zone, flow may be summarily evaluated by the simple relation

$$\left.\begin{aligned} v_1 &= \varphi\sqrt{2g\left(h_e + \frac{v_0^2}{2g}\right)} \\ Q &= v_1 a_1 = a_1\varphi\sqrt{2g\left(h_e + \frac{v_0^2}{2g}\right)} \end{aligned}\right\} \qquad (106)$$

where v_0 is the velocity of approach and φ a velocity coefficient which takes into account the losses between A and section 1; a_1 in Eq. (106) is the canal cross-section in section 1, corresponding to the depth y_1.

Neglecting $v_0^2/2g$, which is usually small, the relation may be presented as

$$\left.\begin{aligned} h_e &= \frac{1}{\varphi^2}\frac{v_1^2}{2g} = \frac{1}{\varphi^2}\frac{Q^2}{2ga_1^2} \\ \text{or} \qquad h_e &= (1 + \zeta)\frac{v_1^2}{2g} = (1 + \zeta)\frac{Q^2}{2ga_1^2} \end{aligned}\right\} \qquad (107)$$

where $\zeta = \dfrac{1}{\varphi^2} - 1$ is the "resistance" coefficient.

The values of φ and ζ depend on the configuration and the size of the entrance structure and are treated in hydraulics under the captions relating to *flow through orifices.** The relation between the velocity coefficient φ and the resistance factor $1 + \zeta = 1/\varphi^2$ is as follows:

$\varphi = 1$	0.95	0.925	0.90	0.875	0.85	0.815
$1 + \zeta = 1$	1.11	1.17	1.23	1.31	1.39	1.50

* Most valuable data on the losses in entrance structures are given by Hinds, *Trans.* A.S.C.E., Vol. 92, p. 1422, 1928.

In the examples to follow, we shall assume an average value corresponding to well-rounded entrance forms of $\frac{1}{\varphi^2} = 1 + \zeta = 1.25$. A curve of $h_e = 1.25\frac{v_1^2}{2g}$ values is given in Plate VII.

The Inflow Discharge Curve.—For a given canal (Fig. 141), assuming that y_a remains constant and neglecting the possible small drop of the canal bottom Δ_{ent} within the entrance zone (Fig. 139), so that $y_1 = y_a - h_e$, we have

$$Q = a_1 v_1 = a_1 \varphi \sqrt{2gh_e} = a_1 \varphi \sqrt{2g(y_a - y_1)} \qquad (108)$$

By plotting $Q = f(y_1)$ in Eq. (108) as a curve, we obtain the *inflow discharge curve,* a curve which is basic in the handling of problems of varied flow, where entrance circumstances are to be taken into consideration.

Example 23

Compute and draw the inflow discharge curve for a canal, Type *A*, with $y_a = 6$ ft. using $1/\varphi^2 = 1.25$.

TABLE XXXII

y_1	$h_e = 6 - y_1$	a_1	v_1	Q
5.99	0.01	371.26	0.72	267
5.98	0.02	370.52	1.02	378
5.95	0.05	368.30	1.61	594
5.90	0.10	364.62	2.62	826
5.80	0.20	357.28	3.21	1,147
5.60	0.40	342.72	4.54	1,560
5.40	0.60	328.32	5.54	1,820
5.20	0.80	314.08	6.40	2,010
5.00	1.00	300.00	7.18	2,154

The velocities in Col. 4 are taken from Plate VII.

The Q_{ent} curve is plotted in Fig. 142 as No. 1.

52. Uniform Movement (Fig. 143).—Given y_a, determine the depth y_1 and the discharge in uniform movement. In uniform flow, obviously, $y_1 = y_0$. The discharge Q_{ent}, corresponding to y_1 (Eq. [108]) must be equal to the discharge $Q_0 = \mathfrak{K}_1\sqrt{s_0}$, to take place in uniform flow under

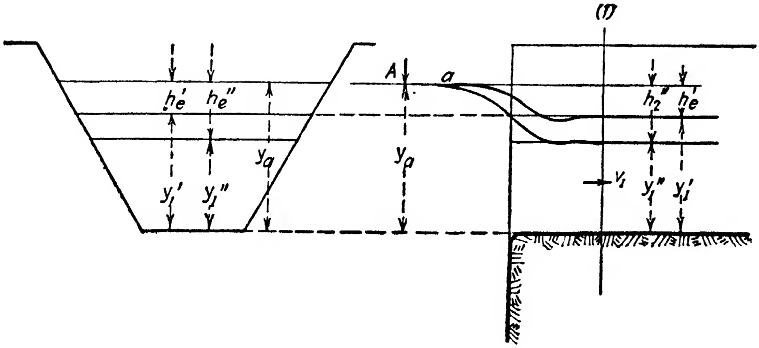

Fig. 141.

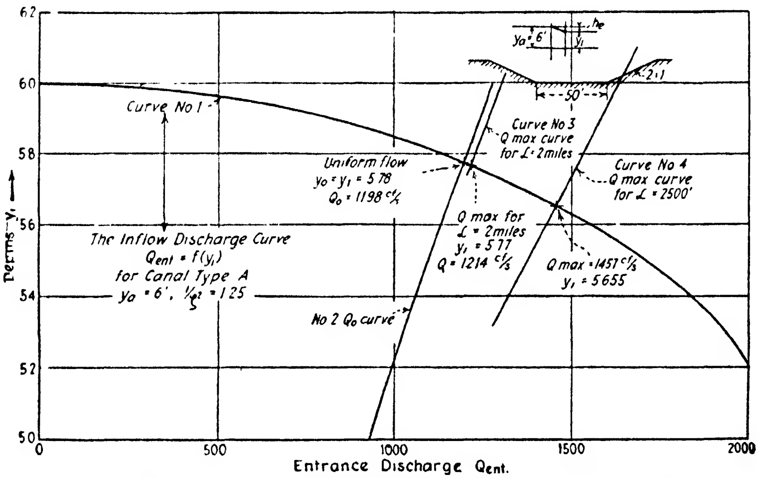

Fig. 142.—Q entrance curve for canal Type A, Example 23.

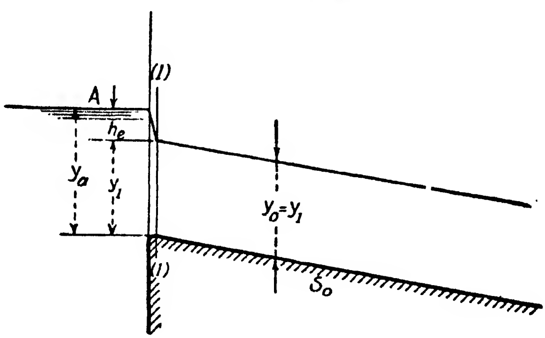

Fig. 143.

the normal depth y_1. The problem is solved by drawing into one diagram (Fig. 144) as functions of y_1 the Q_{ent} curve and the normal discharge curve $Q_0 = \mathfrak{K}_0\sqrt{s_0}$. The point of intersection of the curves determines the discharge sought.

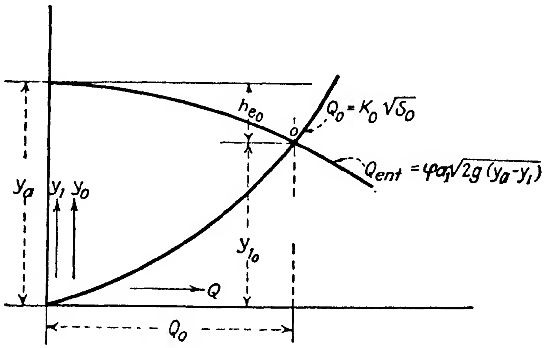

FIG. 144.—Determining normal flow in a canal with free inflow.

Example 24

Determine the discharge Q_0 and the normal depth in uniform flow in a canal, Type *A* (Plate III), assuming that y_a = 6 ft., and that the entrance conditions are as of Example 23.

To solve the problem, draw into Fig. 142 the portion of the $Q_0 = \mathfrak{K}_0\sqrt{s_0}$ curve (curve 2), beginning with y_0 = 6 ft. down. Use, for this, data of Table XXI, namely,

y_0	6.0	5.8	5.4	5.0
Q_0	1,278	1,206	1,060	930

The point of intersection gives $y_1 = y_0$ = 5.78 ft.; h_e = 0.22 ft. The normal discharge Q_0 = 1,198 cu. ft. per second.

The Maximum Discharge.—The same procedure applies in determining the maximum discharge:

Example 25

Assume once more a canal, Type *A*, with y_a = const. = 6 ft. and $1/\varphi^2 = 1.25$.

Determine the maximum possible discharge for a canal length L = 2 miles, and L = 2,500 ft., respectively. The problem is solved by plotting into Fig. 142 the respective portions of the Q_{max} curve. Take the required elements of the $Q_{max} = f(y_1)$ curve, with $L = 2$ miles, from Table XXIII. The curve is designated in Fig. 142 as curve 3.

For curve, relating to $L = 2{,}500$ ft., use the figures of Table XXVII; the curve is designated as curve 4.

The points of intersection give:

Length $L = 2$ miles,

$Q_{max} = 1{,}214$ cu. ft. per second; $y_1 = 5.77$ ft.; $h_e = 0.23$; $y_2 = y_{cr} = 2.51$.

Length $L = 2{,}500$ ft.

$Q_{max} = 1{,}457$ cu. ft. per second; $y_1 = 5.655$ ft.; $h_e = 0.345$; $y_2 = y_{cr} = 2.84$.

53. The $Q = f(y_2)$ Delivery Curve.—In the case of y_a constant, the characteristic points of the $Q = f(y_2)_{y_a = const}$ curve (compare with Art. 45 and Fig. 117) are:

1. Point Z with $Q = 0$, and $y_{2_z} = y_a + Ls_0$.

2. Point O and point C, corresponding to uniform flow and to Q_{max}, respectively, determined as in Fig. 142. For figuring out the intermediate points on the o–c or the o–z portion of the curve, the procedure in general is analogous to that of Art. 45, except that instead of using a constant value of y_1 in determining $\eta_1 = y_1/y_0$ in Col. 4, Table XXI, the values of y_1 will vary. For each discharge Q (corresponding to the selected y_0) the respective y_1 is to be taken from the inflow discharge curve (Art. 51); with this, the corresponding η_2 and $y_2 = \eta_2 y_0$ are found in the same manner as in Example 17.

Example 26

In the layout of Example 17, assume the constant depth to be $y_a = 6$ ft. Determine the delivery curve $Q = f(y_2)$.

The characteristic points are:

1. Zero point: $Q = 0$; $y_2 = 6 + 4.224 = 10.224$.
2. O point (uniform movement) from Example 24.

$$Q = 1{,}198;\ y_2 = 5.78 \text{ ft.}$$

3. C point (maximum discharge) from Example 25.

$$Q = 1{,}214;\ y_2 = y_{cr} = 2.51.$$

As an example of determining an intermediary point take $y_0 = 5$ ft. with $Q = 930$ cu. ft. per second (see Table XXI).

From the Q_{ent} curve (Fig. 142) for $Q = 930$, $y_1 = 5.875$. Thus for section 1:

$$\eta_1 = 5.875/5 = 1.175;\ \Phi(\eta_1) = 0.8255$$

With $Ls_0/y_0 = 0.845$,

$$\Phi(\eta_2) = \Phi(\eta_1) + \frac{Ls_0}{y_0} = 0.8255 + 0.845 = 1.6705$$

hence

$$\eta_2 = 1.764 \text{ and } y_2 = 1.764 \times 5 = 3.82$$

Computations relating to the other points are gathered in Table XXIII

TABLE XXXIII

y_0	Q_0	y_1	η_1	$\Phi(\eta_1)$	Ls_0/y_0	$\Phi(\eta_2)$	η_2	y_2
5.78	1,198			Uniform flow				5.780
5.40	1,060	5.830	1.079	0.5410	0.783	1.3240	1.480	8.000
5.00	930	5.875	1.175	0.8255	0.845	1.6705	1.764	8.820
4.00	626	5.945	1.486	1.3320	1.056	2.3880	2.427	9.708
3.00	380	5.980	1.993	1.9265	1.410	3.3365	3.353	10.059
2.00	187.4	5.990	2.995	2.9730	2.112	5.0850	5.091	10.180

The curve is plotted in Fig. 120 as curve 6.

Case of y_2 Constant.—This case is particularly simple. Suppose a delivery curve $Q = f(y_1)y_{2,const}$ has been computed as in Fig. 127. To determine the value of y_a for any discharge, one simply has to add to the respective value of y_1 the corresponding entrance head

$$h_e = \frac{1}{\varphi^2}\frac{Q^2}{2ga_1^2}$$

where a_1 is the cross-sectional area, corresponding to y_1.

Q Constant Curve.—The same applies to the case of a Q_{const} curve. If C_1-O-L (Fig. 145) is the Q_{const} curve, similar to that of Fig. 132 and showing the relation between y_2 and y_1, the $f(y_2, y_a)$ curve is obtained by adding to the y_1 ordinate of the $f(y_2, y_1)$ curve the value of the entrance head $\frac{1}{\varphi^2}\frac{Q^2}{2ga_1^2}$.

Example 27

With reference to Example 22 and in addition to the Q_{const} curve $f(y_1, y_2)$ for $Q = 1{,}278$ cu. ft. per second ($y_0 = 6$ ft., traced in Fig. 134 as curve 2) compute and trace the Q_{const} curve as $f(y_a, y_2)$ assuming $1/\varphi^2 = 1.25$. In Table XXXIV the values of y_2 and y_1 are taken from Table XXXI. The velocities $v_1 = 1.278/a_1$ (Col. 3) are obtained by dividing the constant discharge by the value of a_1, corresponding to y_1. The h_e values are taken from Plate VII.

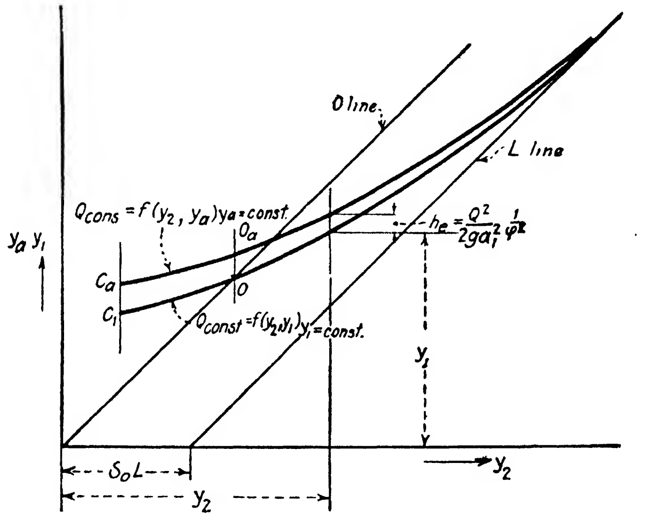

Fig. 145.

The curve is plotted in Fig. 134 (curve 3) above the $f(y_1, y_2)$ curve.

Table XXXIV

y_2	y_1	v_1	h_e	$y_a = y_1 + h_e$
2.63	5.94	3.48	0.235	6.17
6.00	6.00	3.44	0.230	6.23
7.20	6.15	3.34	0.215	6.36
8.10	6.36	3.26	0.200	6.56
9.00	6.64	3.05	0.180	6.82
10.20	7.24	2.74	0.148	7.39
12.00	8.47	2.27	0.100	8.57
15.00	11.06	1.60	0.049	11.11

CHAPTER XIII

DELIVERY OF A CANAL WITH A HORIZONTAL BOTTOM

54. Computation Procedure.—The reasoning, developed in the preceding chapters is applicable towards estimating the delivery of a canal with a horizontal bottom ($s_0 = 0$). The equations, used in such case, should be the ones developed in Chap. X. Referring the movement for each Q to the respective critical depth, and with reference to Fig. 146, the equations in the case (see Eqs. [99] to [103]) are:

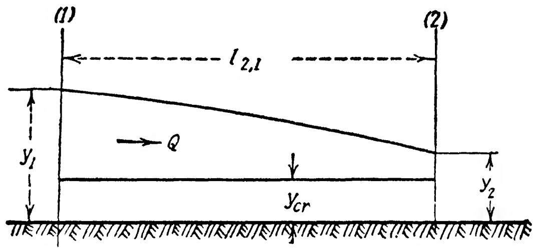

FIG. 146.

$$l_{2,1} = \frac{y_{cr}}{\sigma_{cr}}\left[\delta_{2,1}(\tau_2 - \tau_1) - \frac{\tau_2^{n+1} - \tau_1^{n+1}}{n+1}\right] \tag{109}$$

where

$$\tau = y/y_{cr} \text{ and } \delta = \sigma_{cr}/\sigma$$

Further with

$$T(\tau) = \delta \cdot \tau - \frac{\tau^{n+1}}{n+1} \tag{110}$$

Equation (109) becomes

$$l_{2,1} = \frac{y_{cr}}{\sigma_{cr}}[T(\tau_2) - T(\tau_1)] \tag{111}$$

Equation (111), of simpler form, may be advantageously used when the canal cross-section happens to be such that the value of σ remains substantially constant throughout the range of depths. In such case (see Example 16), a single average value of δ may be used throughout the computations, which become comparatively simple.

Whenever, on the other hand, δ varies substantially, Eq. (109) will have to be used. It will be found, that the member $\delta_{2,1}(\tau_2 - \tau_1)$ exercises a considerable effect, particularly in the vicinity of depths near to y_{cr}. So appropriate care will have to be exercised, to evaluate the value of $\delta_{2,1}$ with sufficient precision.

As pointed out in Art. 43 it will usually be sufficient to accept as the $\delta_{2,1}$ value the arithmetical average

$$\left(\delta_{2,1} = \frac{\delta_1 + \delta_2}{2}\right)$$

for the two limiting depths. But in solving Eq. (109), one should bear in mind that δ_1 and δ_2 change with τ_1 and τ_2, and that such a change will usually have to be taken into account.

Equation (109) is solved by successive probes. The computations will be simpler if the figuring is so organized, that the solving of the problem will consist in determining the distance between the two assumed depths. The matter will be best made clear by means of a practical example.

Example 28

Assume (Fig. 147) an equalizing canal, laid with a horizontal bottom between two storage reservoirs at a distance of $L = 5{,}000$ ft. The cross-section and the other elements of the canal are of Type *D* (Plate V). As values for *C*, we shall take G.K. coefficients with $n = 0.025$ and $s_0 = 0.0001$. Depending on circumstances, flow will be from *A* to *B* or in the opposite direction. Assume that the highest possible stage in either of the reservoirs is $y = 8$ ft., and that the lowest level corresponds to 3 ft. The entrance arrangements are featured by a value of $1/\varphi^2 = 1.25$.

Question: Determine the delivery curve $Q = f(y_2)$, assuming that y_a is maintained constant and equal to 8 ft.

The delivery curve (Fig. 148) has only two characteristic points: (1) The zero point Z with $y_2 = y_a$, and (2) The C point of maximum discharge with $y_{2c} = y_{cr}$. The lowering of y_B below y_{2c} will have no further effect on the flow in the canal.

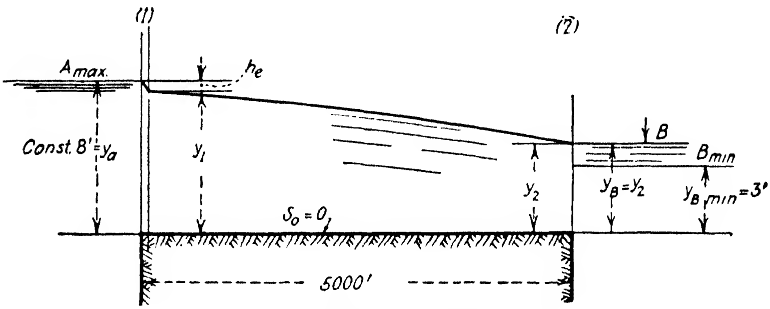

FIG. 147.—Layout for Example 28.

Curve a-c (Fig. 148a) corresponds thus to the lowest possible position of the surface curve. All intermediary points between Z and C correspond to flow with a dropping curve of the M_2 type.

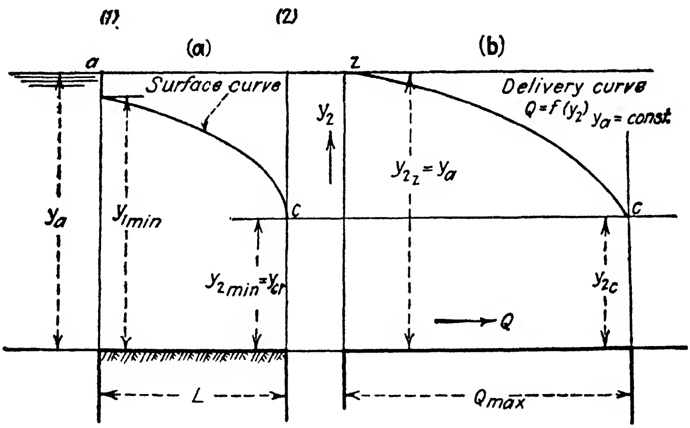

FIG. 148.—Delivery curve for a canal with horizontal bottom.

Preliminary.—As auxiliaries we compute:

1. The inflow discharge curve for canal, Type D, $Q_{ent} = f(y_1)$, with y_a = const. = 8 ft. The elements of the curve drawn in Fig. 149 as curve 1 are as follows:

h_e	d_1	v_1	a_1	$Q = a_1v_1$
0.01	7.99	0.72	215.6	155.0
0.02	7.98	1.02	215.3	219.5
0.04	7.96	1.45	214.4	311.0
0.07	7.93	1.89	213.4	404.0
0.10	7.90	2.26	212.1	480.0
0.15	7.85	2.79	210.2	586.0
0.20	7.80	3.21	208.3	671.0
0.25	7.75	3.59	206.3	741.0
0.30	7.70	3.94	204.4	807.0
0.40	7.60	4.54	201.6	915.0
0.50	7.50	5.08	196.9	1002.0

2. The critical discharge curve $Q_{cr} = f(y_{cr})$ drawn in Fig. 149 as curve 2.

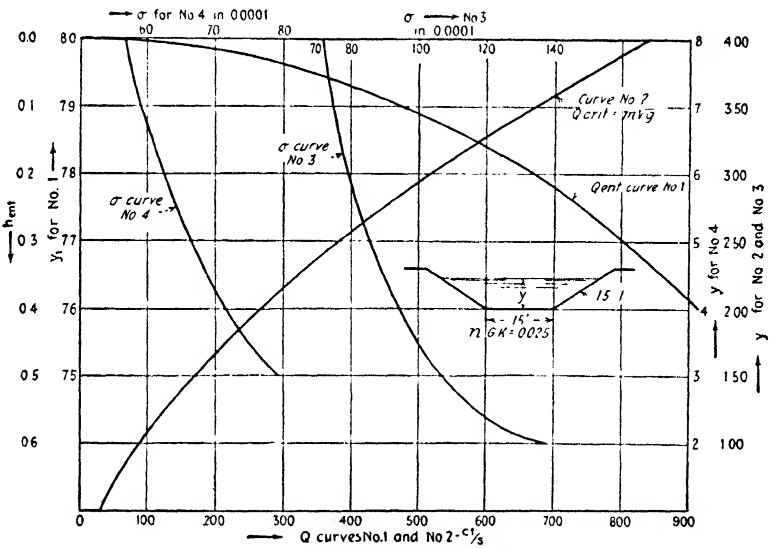

FIG. 149.—Auxiliary curves for Example 28.

3. The critical slope curve $\sigma = f(y)$; curves 3 and 4 (Fig. 149). The elements of the curves are taken from Plate V with a few additions.

For convenience sake, the σ curve for the different regions of depths has been drawn in two different scales. The hydraulic exponent is taken at $n = 3.80$.

The Maximum Discharge.—In determining point C, with $Q = Q_{max}$, and $y_{2c} = y_{cr}$, take a series of $y_2 = y_{cr}$ values, to each of which there corresponds the critical discharge $Q = \mathfrak{M}_{cr}\sqrt{g}$. From the inflow curve, determine for each such discharge the depth $y_1 = y_a - h_e$. Applying Eq. (109) or (111) between the depths y_1 and y_2 determine the length $l_{1,2}$. Draw a curve of lengths, so determined, as a function of Q or of its equivalent y_{cr}. The point of the $l = f(Q)$ curve so drawn, which corresponds to the given canal length, solves the problem.

For clarity, we shall carry out in detail the computation of a point, corresponding to $y_2 = y_{cr} = 4$ ft.:

For this point from Fig. 149, curve 2, we have

$$Q_{cr} = 842 \text{ cu. ft. per second}; \sigma_{cr} = 71.4^0/_{00}$$

The entrance head for $Q = 842$ (curve 1) is $h_e = 0.33$, so that

$$y_1 = 8 - 0.33 = 7.67$$

We have thus for section 1:

$$\tau_1 = 7.67/4 = 1.92; \tau_1^{n+1}/n + 1 = (1.92)^{4.8}/4.8 = 4.774$$

Further, from curve 4, σ_1 for $y_1 = 7.67$ is $57.2^0/_{00}$ and thus

$$\delta_1 = \sigma_{cr}/\sigma_1 = 71.4/57.2 = 1.245$$

Similarly, for section 2,

$$y_{cr} = y_2 = 4 \text{ ft.}; \tau_2 = 1; \sigma_2 = \sigma_{cr} = 71.4$$

hence

$$\frac{\tau_2^{n+1}}{n+1} = \frac{1}{4.8} = 0.208, \text{ and } \delta_2 = 1$$

The average value of

$$\delta_{1,2} = \frac{1.245 + 1}{2} = 1.123$$

The value of

$$y_{cr}/\sigma_{cr} = 4/71.4 \cdot 10^{-4} = 559 \text{ ft.}$$

The distance $l_{2,1}$ (Eq. [109])

$$l_{2,1} = 559[1.123(1 - 1.92) - (0.208 - 4.774)] = 559[-1.032 + 4.566] = 1{,}980 \text{ ft.}$$

This length is substantially shorter than the canal length $L = 5{,}000$ ft., which indicates that the discharge, $Q = 842$ cu. ft. per second, is in excess of the value of Q_{max}.

Similar computations, relating to other points are gathered in Table XXXV. Obviously for all points

$$\tau_2 = 1, \frac{\tau_2{}^{4.8}}{4.8} = 0.208, \text{ and } \delta_2 = 1$$

TABLE XXXV

$y_2 = y_{cr}$	Q	$\sigma_{cr}{}^0/_{00}$	y_{cr}/σ_{cr}	y_1	$\tau_1 = y_1/y_{cr}$	$\tau_1{}^{4.8}/4.8$	$\sigma_1{}^0/_{00}$	$\delta_1 = \sigma_{cr}/\sigma_1$	δ_{av}	$l_{1,2}$
4.00	842	71.4	559	7.67	1.920	4.774	57.2	1.245	1.123	1,980
3.50	670	75.0	466	7.80	2.226	9.700	57.0	1.315	1.158	3,850
3.30	608	76.6	431	7.837	2.373	13.190	56.9	1.344	1.172	4,870
3.25	592	77.0	422	7.846	2.410	14.200	56.9	1.352	1.176	5,220
3.00	519	79.4	378	7.881	2.627	21.120	56.8	1.395	1.198	7,170

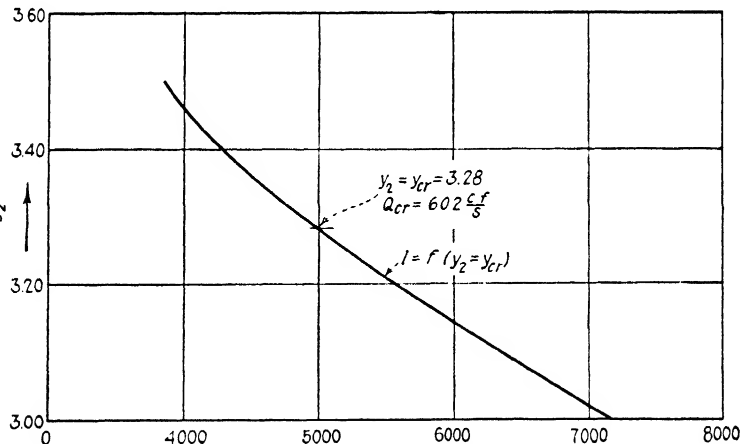

FIG. 150.—Auxiliary for Example 28.

The $y_{cr} = f(l)$ curve is traced in Fig. 150; the solution for $L = 5{,}000$ ft. is $y_2 = y_{cr} = 3.28$ ft.; and the corresponding discharge and y_1 from curves 2 and 1 (Fig. 149) are $Q_{max} = 602$ cu. ft. per second; $y_1 = 7.84$, and $h_e = 0.16$.

The Z point and the C point determine the extreme points of the delivery curve (Fig. 151).

We shall bring forth now in detail the computations relating to an intermediary point:

Take $y_{cr} = 2.5$ ft., corresponding to

$$Q = 383 \text{ cu. ft. per second}; \sigma_{cr} = 86 \cdot 10^{-4}; y_{cr}/\sigma_{cr} = 290.5$$

For section 1, we have:

$$y_1 = 7.937; \tau_1 = 7.937/2.5 = 3.177; \tau_1^{4.8}/4.8 = 53.5$$

$$\sigma_1 = 56.7\,^0\!/_{00}; \delta_1 = 86/56.7 = 1.532$$

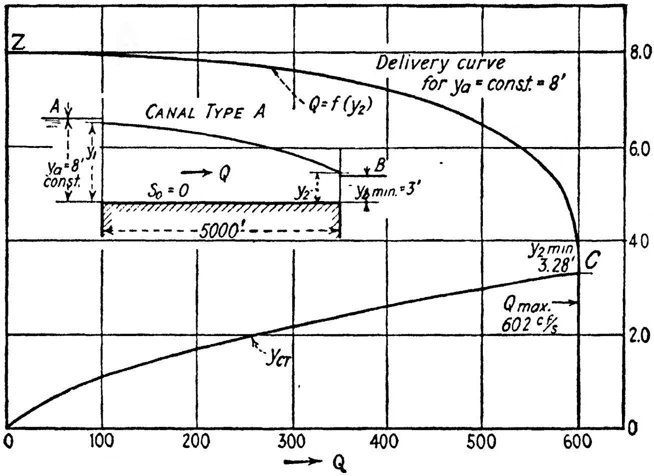

FIG. 151.—Delivery curve for canal with a horizontal bottom, Example 28.

To determine y_2, we have to find the value of $\tau_2 = y_2/2.5$ which will satisfy the equation:

$$\frac{\tau_2^{n+1}}{n+1} - \delta_{1.2}(\tau_2 - \tau_1) = \frac{\tau_1^{n+1}}{n+1} - l_{1.2}\frac{\sigma_{cr}}{y_{cr}} \qquad (112)$$

which, in our case, becomes

$$\frac{\tau_2^{4.8}}{4.8} - \delta_{1.2}(\tau_2 - 3.18) = 53.5 - \frac{5{,}000}{290.5} = 36.30$$

In successive approximation, make first

$$\frac{(\tau'_2)^{4.8}}{4.8} = 36.3$$

which, by using ordinary logarithms, gives $\tau'_2 = 2.93$, and $y'_2 = 2.93 \times 2.5 = 7.33$ ft.

By figuring the more exact value of $\phi(\tau_2) = \frac{\tau_2^{4.8}}{4.8} - \delta_{1.2}(\tau_2 - 3.18)$

for $y_2 = 7.33$ and that for a neighboring value of $y_2 = 7.25$, we obtain

y_2	τ_2	$\frac{\tau_2^{4.8}}{4.8}$	$\sigma_2{}^0\!/\!_{00}$	$\delta_2 = \frac{86}{\sigma_2} \cdot 10^{-4}$	$\delta_{1.2} = \frac{\delta_2 + 1.532}{2}$	$\delta_{1.2}(\tau_2 - 3.18)$	$\phi(\tau_2)$
7.33	2.93	36.30	58.2	1.48	1.506	−0.376	36.68
7.25	2.90	34.54	58.4	1.47	1.501	−0.421	34.96

Interpolating between these points, we obtain for $\varphi(\tau_2) = 36.3$, the value of $y_2 = 7.31$.

In this particular instance, the approximate value of $y_2' = 7.33$ obtained by neglecting the member $\delta_{1.2}(\tau_2 - \tau_1)$ in Eq. (112), proved to be very close to the final value of $y_2 = 7.31$.

Obviously the approximation will be still more satisfactory for smaller deliveries.

In fact for point $y_{cr} = 2$, we have

$Q = 267$ cu. ft. per second; $\sigma_{cr} = 94.3{}^0\!/\!_{00}$; $y_{cr}/\sigma_{cr} = 212$;

$$L\sigma_{cr}/y_{cr} = 23.6$$

$$y_1 = 7.972;\ \tau_1 = 3.986;\ \tau_1^{4.8}/4.8 = 158.9$$

$$\sigma_1 = 56.6{}^0\!/\!_{00};\ \delta_1 = 94.3/56.6 = 1.67$$

The equation is:

$$\phi(\tau_2) = \frac{\tau_2^{4.8}}{4.8} - \delta_{1.2}(\tau_2 - 3.99) = 158.9 - 23.6 = 135.3$$

In first approximation, making $(\tau'_2)^{4.8}/4.8 = 135.3$, we have $\tau_2 = 3.846$ and $y_2 = 7.692$. In computing a more precise value of τ_2, we have

y_2	τ_2	$\tau_2^{4.8}/4.8$	$\sigma_2{}^0\!/\!_{00}$	δ_2	$\delta_{1.2}$	$\delta_{1.2}(\tau_2 - 3.99)$	$\phi(\tau_2)$
7.692	3.846	135.30	57.4	1.64	1.655	−0.238	135.54
7.650	3.825	130.24				−0.274	130.51

The final value of $y_2 = 7.69$ is practically identical with $y'_2 = 7.692$.

In figuring the two following points, we simply omit the member $\delta_{1.2}(\tau_2 - \tau_1)$, making simply

$$\frac{\tau_2^{4.8}}{4.8} = \frac{\tau_1^{4.8}}{4.8} - \frac{L\sigma_{cr}}{y_{cr}}$$

and obtain:

y_{cr}	Q	y_1	$\sigma_{cr}{}^0\%_0$	$5{,}000 \cdot \sigma_{cr}/y_{cr}$	τ_1	$\tau_1{}^{4.8}/4.8$	$\tau_2{}^{4.8}/4.8$	τ_2	y_2
1.5	169.2	7.99	107	35.7	5.33	638.1	602.4	5.266	7.90
1.0	89.8	7.997	138	69.0	7.997	4506.0	4437.0	7.961	7.96

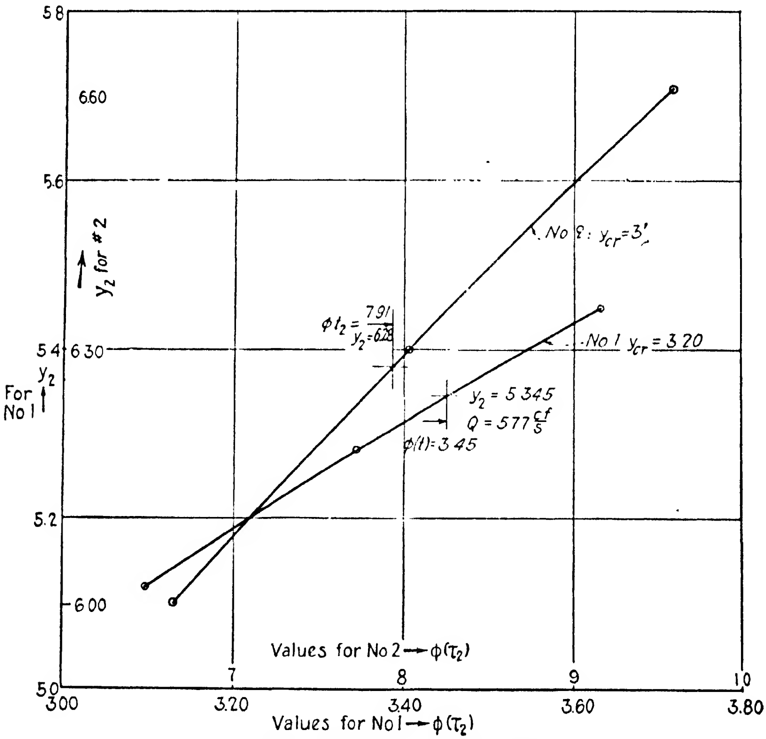

FIG. 152.—Auxiliary for Example 28.

This simplification, on the other hand, would be quite out of order for larger Q's when the discharge approaches Q_{max}. For example for point $y_{cr} = 3.20$, with

$$Q = 577 \text{ cu. ft. per second; } \sigma_{cr} = 77.5^0\%_0;\ L\sigma_{cr}/y_{cr} = 12.1$$

we have in section 1:

$$y_1 = 7.854;\ \tau_1 = 7.854/3.20 = 2.456;\ 2.456^{4.8}/4.8 = 15.55$$
$$\sigma_1 = 56.9^0\%_0;\ \delta_1 = 77.5/56.9 = 1.36$$

Hence, for section 2,

$$\phi(\tau_2) = \frac{\tau_2^{4.8}}{4.8} - \delta_{1.2}(\tau_2 - 2.46) = 15.55 - 12.10 = 3.45$$

Going by successive tests, we try

τ_2	y_2	$\tau_2^{4.8}/4.8$	$\sigma_2{}^0\%_0$	δ_2	$\delta_{1.2}$	$\delta_{1.2}(\tau_2 - 2.46)$	$\phi(\tau_2)$
1.70	5.44	2.659	64.6	1.200	1.280	−0.974	3.633
1.65	5.28	2.305	65.2	1.188	1.286	1.041	3.346
1.60	5.12	1.988	65.9	1.176	1.292	1.112	3.100

From the graph $\phi(\tau_2) = f(y_2)$, curve 1 (Fig. 152), we obtain the sought $y_2 = 5.345$.

Repeating the procedure for point $y_{cr} = y_2 = 3$ ft., we have $Q = 519$ cu. ft. per second; $\sigma_{cr} = 79.4{}^0\%_0$; $L\sigma_{cr}/y_{cr} = 13.21$

For section 1

$$y_1 = 7.882; \tau_1 = 2.63; \tau_1^{4.8}/4.8 = 21.12$$
$$\sigma_1 = 56.9{}^0\%_0; \delta_1 = 1.395$$

Hence

$$\phi(\tau_2) = \frac{\tau_2^{4.8}}{4.8} - \delta_{1.2}(\tau_2 - 2.63) = 21.12 - 13.21 = 7.91$$

The equation is solved by means of:

τ_2	y_2	$\tau_2^{4.8}/4.8$	$\sigma_2{}^0\%_0$	δ_2	$\delta_{1.2}$	$\delta_{1.2}(\tau_2 - 2.63)$	$\phi(\tau_2)$
2.00	6.00	5.819	62.5	1.27	1.333	−0.841	6.660
2.10	6.30	7.318	61.4	1.296	1.345	0.713	8.031
2.20	6.60	8.960	60.3	1.316	1.356	0.583	9.543

Graph $\phi(\tau_2) = f(y_2)$, curve 2 (Fig. 152), gives $y_2 = 6.28$.

Gathering all of the points, as figured above, we obtain

TABLE XXXVI

y_{cr}	Q	y_2	y_1
1.00	89.8	7.960	7.997
1.50	169.2	7.900	7.990
2.00	267.0	7.690	7.972
2.50	383.0	7.310	7.937
3.00	519.0	6.280	7.882
3.20	577.0	5.345	7.854
3.28	602.0	3.280	7.840

The $Q = f(y_2)$ curve is traced in Fig. 151.

CHAPTER XIV

DESIGNING CANALS

The delivery curves, as already developed, find useful application in the designing of canals.

55. Increasing the Delivery.—A canal is usually designed for a certain volume of flow Q, which the canal is supposed to deliver per unit of time in uniform movement. The design consists in determining the cross-sectional dimensions and the bottom slope s_0, which will cause the discharge $Q = \mathfrak{K}_0\sqrt{s_0}$ to flow under the depth y_0.

In actual practice the friction factor often exceeds what was assumed; or the canal may be called to deliver a larger volume of water. In both cases the canal, as designed, will be short of requirements and the delivery will have to be increased.

The problem should be considered in the light of the delivery curves as featured in Fig. 117 and illustrated by practical examples in Fig. 120. While in a short canal, or in a canal with an unusually flat-bottom slope, an increase of delivery beyond that corresponding to uniform flow can be brought about by lowering the stage y_2 at the end of the canal below the normal depth y_0 (curve 2, Fig. 120), such means, under ordinary circumstances, are totally ineffectual (curves 1 and 6, Fig. 120). In other words, one should firmly bear in mind, that outside of exceptional conditions, uniform flow with $y_2 = y_1$ is for all practical purposes the maximum discharge which a canal is able to deliver with the initial stage of flow y_1.

The designing engineer may, therefore, often do well to keep away from the dangerous limit of a canal designed to be of just sufficient capacity to carry the required Q in uniform movement; money and trouble may eventually be saved in laying out the structure so that a reasonable margin of possible increase in delivery is left to meet the uncertainties and urgencies of a particular case.

Under normal conditions, with the delivery curve as curve 1 (Fig. 120), the only effective way to increase the delivery is to increase the depth y_1 at the beginning of the canal. A possible requirement of this kind should be

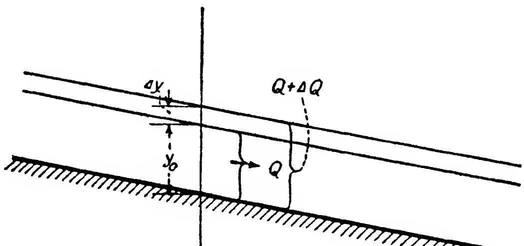

FIG. 153.

anticipated at the time of the original design. Assuming that uniform flow is close to maximum flow, the raise Δy of the stage y_0 (Fig. 153) required to increase the delivery by ΔQ, is determined as follows:

For normal flow we have

$$Q = \mathfrak{K}\sqrt{s_0} \text{ and } \mathfrak{K}^2 = \text{const. } y^n$$

where n is the hydraulic exponent. Thus, for s_0 given,

$$Q = \text{const. } y_0^{n/2}$$

The increase in delivery ΔQ caused by Δy_0

$$\Delta Q = \text{const. } \frac{n}{2} \cdot y_0^{\left(\frac{n}{2}-1\right)} \cdot \Delta y_0$$

Dividing by Q, we obtain

$$\frac{\Delta Q}{Q} = \frac{n}{2}\frac{\Delta y_0}{y_0} \tag{113}$$

Thus the relative change in discharge is $n/2$ times the relative change in depth.

As an example, for canal (Fig. 119), with $n = 3.6$, a margin of 10 per cent in delivery over $Q = 1{,}280$ cu. ft. per second, corresponding to $y_0 = 6$ ft. would require an increase in depth

$$\Delta y = 2/n \cdot y \cdot \Delta Q/Q = 1/1.8 \times 6 \times 0.1 = \tfrac{1}{3} \text{ ft.}$$

By raising y_1 to $y_1 + \Delta y$, the delivery will change by ΔQ, irrespective of what the stage y_2 is, provided only that y_2 is $\leq y_1 + \Delta y$.

56. Variable Delivery.—In engineering practice, the volume of water, called for, is oftentimes subject to considerable variation. Moreover, the fluctuations in the required discharge may be rapid and do not always follow predetermined schedules. Changes in delivery may usually be effected by means of regulating sluices. However,

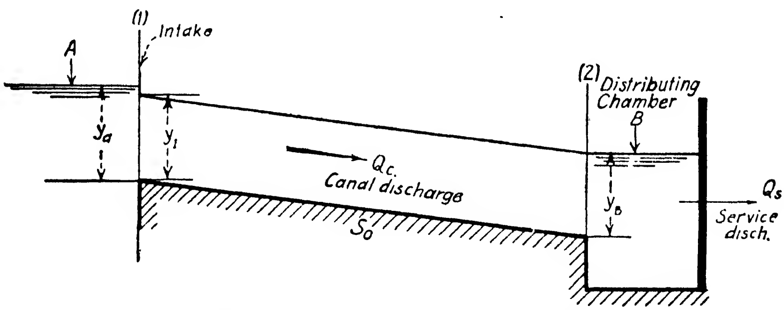

FIG. 154.—The *service* and the *canal* discharge.

operating such sluices is at times a clumsy procedure, so there is yet a large field open to the ingenuity of the designing engineer for meeting circumstances through judicious application of the properties of varied flow.

With reference to Fig. 154, B designates a distributing chamber, from which there is drawn the *service discharge*, Q_s. The service discharge Q_s may eventually differ from the discharge in the canal Q_c, drawn through the intake from the reservoir A. The difference between Q_s and Q_c is supplied or absorbed by pondage in B.

Two basic cases will be considered:

1. Q_s—*Variable;* Q_c—*Constant;* $Q_s \gtrless Q_c$.—This case features conditions when the volume of the water to be drawn from the intake is limited to a certain prescribed quantity. The fluctuations of the service discharge are to be covered entirely by the storage capacity of the distributing basin,

the average of Q_s over a period of time T being equal to the canal delivery Q_c (Fig. 155).

Assuming level A permanent and Fig. 156 to be a delivery curve $Q = f(y_2)_{y_a,const}$, similar to Fig. 120, the problem is conveniently solved by arranging levels in basin B to fluctuate within those stages (between B''_{max} and B'_{min}), where the delivery curve is steep and the change of the depth y_2 does not affect materially the delivery of the canal. In certain cases, when s_0L is sufficiently great, the canal may be made to work without substantial fluctuation of Q_c not only in the portion of the falling curve o-b', but also on a part of the rising curve o-b''. On the whole, this is a very simple and fully automatic solution.

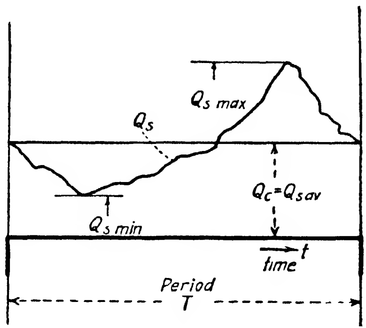

FIG. 155.—Periodical fluctuation of the service discharge Q_s.

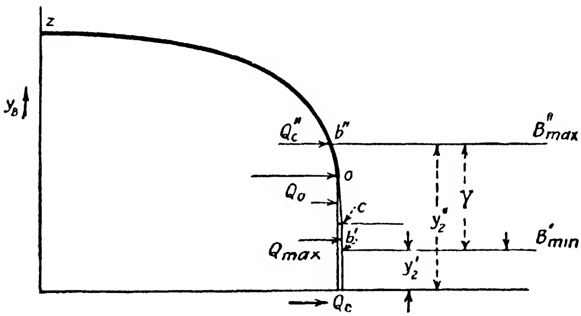

FIG. 156.—Using the properties of delivery curves for a layout, where the canal deliveries remain practically unchanged irrespective of the fluctuations of level in the distributing basin.

As an illustration, Fig. 157 shows a delivery curve for a canal, Type B, laid with a bottom slope of $5^0\%_0$, at a length $L = 3$ miles and $y_1 =$ const. $= 5$ ft. A change of level B by an amplitude of $Y = 6$ ft. from $y'_{2min} = 2$ ft. to $y''_{2max} = 8$ ft. would fluctuate the discharge Q_c from 244 to 241 cu. ft. per second, or less than 1½ per cent.

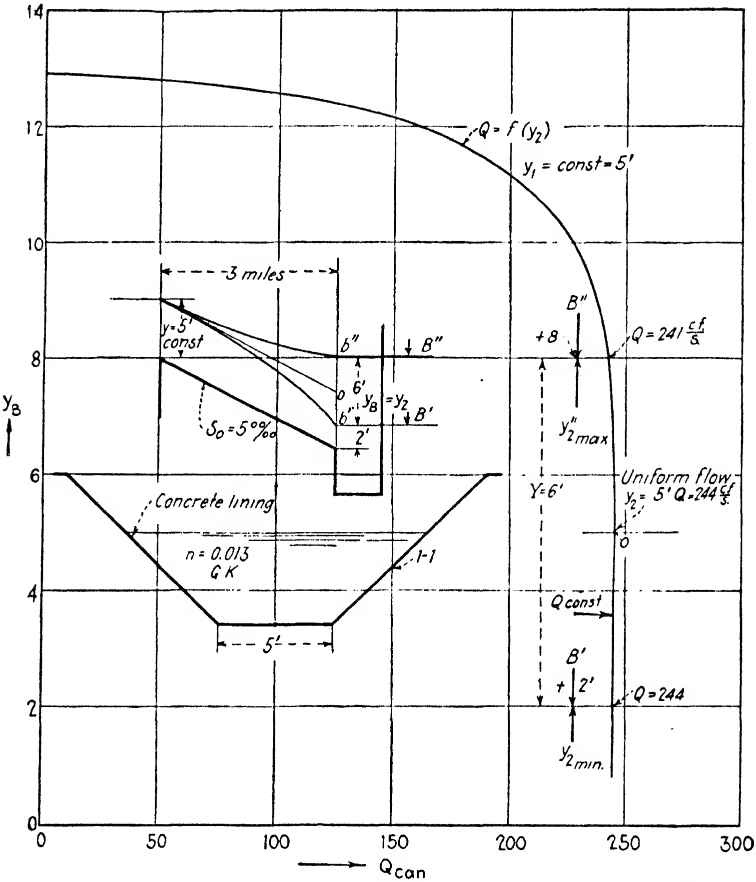

FIG. 157.—A layout with a variable service discharge and a practically constant Q_c.

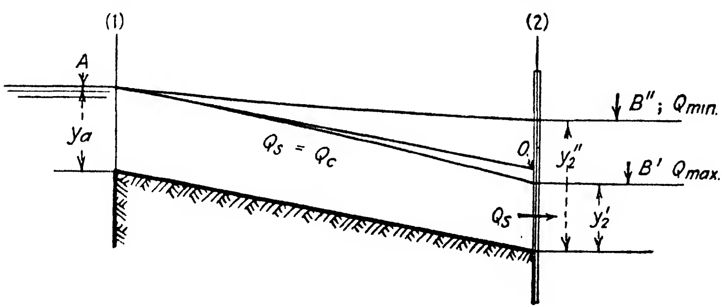

FIG. 158.

2. $Q_s = Q_c$; Q_c—*Variable.*—This case is the opposite to the one considered above. Storage possibilities in B are totally disregarded. The service discharge Q_s, when and as required, is to be drawn directly from the reservoir A.

With reference to Figs. 155, 158 and 159, assuming that

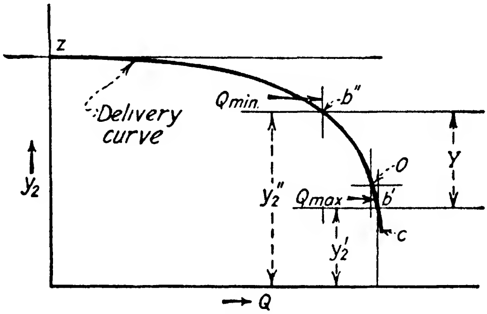

FIG. 159.

the intake is free and that level A does not change, the limiting stages at the end of the canal will be the depths y''_2 and y'_2, corresponding respectively to Q_{min} and Q_{max}.

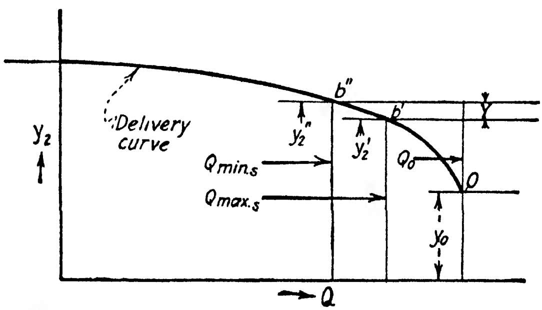

FIG. 160.—Using the properties of delivery curves for securing considerable fluctuations of discharge in a canal with comparatively slight variations of level at the lower extremity.

The stage y_2 and, thus, the flow in the canal will adapt itself automatically to the delivery, as called for. The drawback of this layout is that the fluctuation of the stages in the lower reaches may be quite substantial and present structural inconveniences.

The level fluctuations may be reduced by avoiding the steep portion of the delivery curve and having the canal work between points b' and b'' as in Fig. 160. This is caused by increasing the bottom slope of the canal and making Q_0 (point O) larger than the Q_{max} required. The matter will be made more clear through a practical example.

Example 29

Assume, with the outlay of Fig. 119 and y_a = const. = 6 ft., that the required delivery is to fluctuate regularly between Q''_s = 1,100 cu. ft. per second and Q'_s = 700 cu. ft. per second, with extreme cases ranging between 1200 and 600 cu. ft. per second.

From curve 6 (Fig. 120), we find the corresponding stages to be

Q =	1,200	1,100	700	600
y_2 =	5.5	7.6	9.55	9.75

For usual conditions, the variations of stage will be $Y = 9.55 - 7.60 = 1.95$ ft. The highest and lowest possible stages will differ by $9.75 - 5.50 = 4.25$ ft.

To reduce variation, we shall now increase the bottom slope. Assume the same canal laid with $s_0 = 6^{00}\!/_{00}$, making the fall of the bottom $s_0L = 6.37$ ft. Assuming y_1 constant and equal to 6 ft., find the depths y_2, corresponding to the discharges, as above stated. To make the procedure clear, take $Q = 1,200$ cu. ft. per second. For such discharge

$$\mathfrak{K}_0 = 1,200/\sqrt{6} \cdot 10^{-2} = 490 \cdot 10^2;\ y_0 = 5.15 \text{ ft.}$$

Accordingly

$$\eta_1 = 6/5.15 = 1.165;\ \Phi(\eta_1)_{n=3.6} = 0.803;\ s_0L/y_0 = 6.37/5.15 = 1.237$$

and

$$\Phi(\eta_2) = \Phi(\eta_1) + \frac{Ls_0}{y_0} = 1.237 + 0.803 = 2.04$$

to which there corresponds

$$\eta_2 = 2.098, \text{ and } y_2 = 2.098 \times 5.15 = 10.80$$

Applying the procedures to other points, we obtain

TABLE XXXVII

	Q, cu. ft. per second			
	1,200	1,100	700	600
$\mathfrak{K}_0$	490×10^2	449×10^2	285×10^2	244×10^2
y_0	5.150	4.900	3.800	3.480
η_1	1.165	1.223	1.580	1.720
$\Phi(\eta_1)$	0.803	0 927	1.451	1.621
s_0L/y_0	1.237	1.300	1.677	1.830
$\Phi(\eta_2)$	2.040	2.227	3.128	3.451
η_2	2.098	2.273	3.138	3.467
y_2	10.800	11.120	11.910	12.000

As we see by comparison with case $s_0 = 4^0/_{00}$, the amplitude of variation of the level at B is reduced from 4.25 to 1.20 ft. The result, however, is obtained at the expense of substantially increasing the depths of flow.

57. Pondage. Slowly Variable Flow.—Problems of variable delivery bear direct relation to those of pondage. In fact, chamber B (Fig. 154) was spoken of as taking care of the fluctuations of the variable service discharge Q_s, while a constant delivery Q_c was being drawn from the intake A. Also, with reference to Fig. 158, there might be a distributing chamber at the lower extremity of the canal to cover at least some of the fluctuations of Q_s, in which case, as it usually happens, both Q_s and Q_c would be variable.

Finally, in Fig. 116, we have a layout with two canals, possibly of different cross-sectional dimensions and bottom slopes with a storage reservoir S between them. In general, all the three levels A, B, and S may vary, each of the three basins contributing to pondage. The variation of level in that or any other reservoir depends on the difference in volume of the inflowing and outflowing water. Thus, for example, the change of level in the intermediary basin S for a lapse of time Δt would be

$$\Delta y_s \cdot A_s = (Q_A - Q_B)\Delta t \tag{114}$$

where A_s is the respective surface area of the basin at the depth y_s, and Q_A and Q_B are the respective discharges in canals I and II. Similarly, for basin B, we would have

$$\Delta y_B \cdot A_B = (Q_B - Q_s)\Delta t$$

As to the discharge values in that or any other canal, they depend primarily on the value of the respective depths at the extremities of the canal, so that

$$Q_A = f(y_1, y_2);\ Q_B = f(y_3, y_4), \text{ etc.} \qquad (115)$$

We have said *primarily,* for the reason that flow actuated by stages which *change in time,* is no more permanent; so we are dealing not with *varied* flow, but with *variable* flow, *i.e.*, with flow in which the different elements are no more stationary. As a result, the equations of varied flow which were developed for stationary movement do not apply strictly and should be used only within certain limitations.

The general equation of variable-varied flow, *i.e.*, movement which is non-uniform from section to section and simultaneously changes in time, is obtained from Eq. (17) by adding a member which registers the rate of change of the momentum of flow in time. The equation reads

$$s = \frac{v^2}{c^2R} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) + \frac{1}{g}\frac{\delta v}{\delta t} \qquad (116)$$

In the present state of the art, there is no general method by which problems of variable flow can be handled in a manner adaptable to engineering practice. Only a few particular cases lend themselves to comparatively simple treatment. Such for example are the so-called *translation waves* or surges produced by sudden changes in delivery.

Another case is when the variation of flow in time takes place very slowly, as would be the case if the surface areas of the reservoirs were to be large, so that the change of the stages, and thus of the deliveries, would become noticeable only over a considerable period. Such movements may be properly designated as *slowly variable.* The value of the derivative per time $\delta v/\delta t$ in Eq. (116) in such case becomes negligible and may be ignored.

That will mean (Fig. 161) that at a certain moment, when the stages at the extremities of the canal happen to be y_1 and y_2, flow in slowly variable movement will differ only infinitesimally from flow in varied permanent movement under the same depths. In other words, at the given moment both the delivery Q and the surface curve will be assumed to be identical with what would be the case if the canal was operating under the depths y_1 and y_2 in permanent non-uniform flow. This approximation is permissible in many engineering problems, where cases of pondage are handled. In fact, the very purpose of pondage often is to regulate fluctuations over periods of many hours and eventually days.

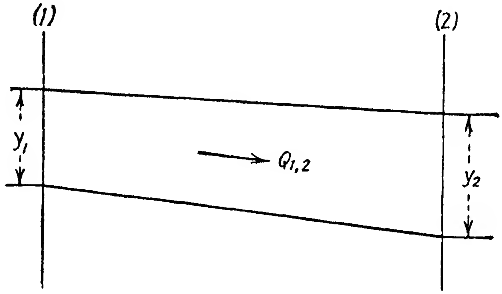

FIG. 161.—Symbolizing slowly variable flow

On the other hand, there are many cases, for example in hydroelectric installations with sudden bumps in the load, when the forebay is in no position to prevent rapid changes. In such cases, slowly variable flow does not prevail and it will be found necessary to introduce treatment which takes into account the phenomena of swells and surges.

At this time we shall deal only with problems of slowly variable motion. As said above, flow will be assumed to be identical with that which would be under the depths y_1 and y_2 in permanent varied flow. This means that for any combination of stages the delivery curves as developed in the preceding chapters and illustrated in Figs. 117, 127, and 132 will be directly applicable.

Pondage problems will be solved by dividing the process of filling or emptying of a reservoir into small periods, going by the increments of the stage Δy. For each such increment, an equation of the type 114 may be established covering a certain lapse of time Δt. Thus, step by step, a complete picture of the process of flow will be built up.

The procedure will be illustrated by a practical example. In general, each problem will require its own particular

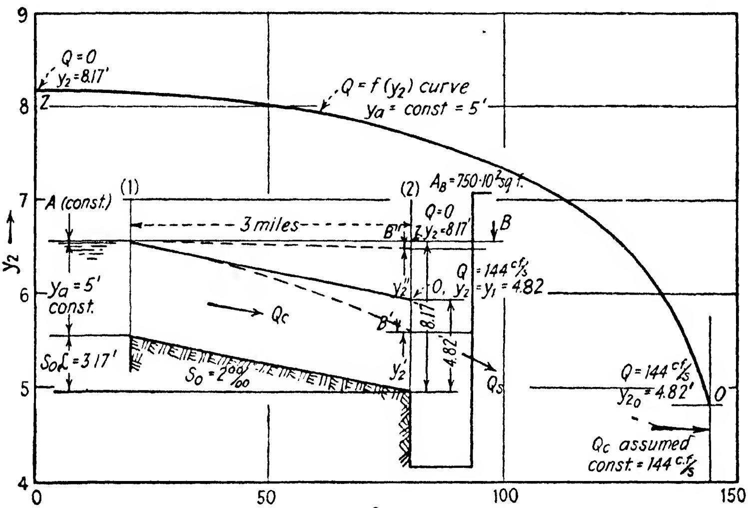

FIG. 162.—Outlay and delivery curve for Example 30.

method of approach, the choice of means always being left to the resourcefulness of the designing engineer.

Example 30

Assume (Fig. 162) a canal, Type B (Fig. 15), laid with a bottom slope of $2^0/_{00}$ at the length of $L = 3$ miles. The level in reservoir A is maintained constant with $y_a = 5$ ft. The intake is free ($1 + \zeta = 1.25$). The delivery curve $Q = f(y_2)_{y_a = 5 \text{ ft.}}$ is computed in accordance with Art. 53. The normal discharge is $Q_0 = 144$ cu. ft. per second, with $y_2 = y_1 = 4.82$ ft. The effect of the drop curve on the increase of delivery is neglected, so that for all depths $y_2 < 4.82$ ft., Q_c is assumed to be constant and equal to 144 cu. ft. per second.

The time diagram of the service discharge, to be drawn from basin B, is given in Fig. 163*a*. The forebay B has a surface area of 750×10^3 sq. ft.

1. Determine the limits of the fluctuation of level $y_B = y_2$ in the forebay and, with that, the limits of variation of the canal discharge Q_c.

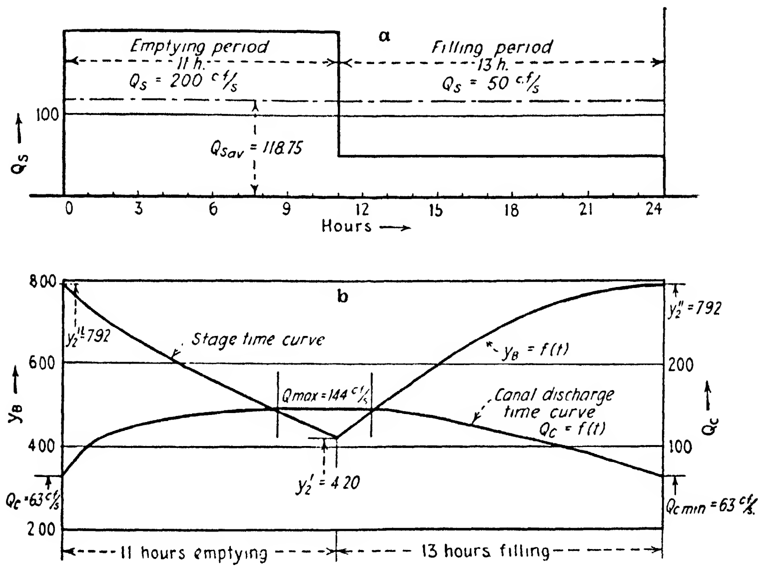

FIG. 163.—(*a*) Service discharge diagram for Example 30. (*b*) Canal discharge and stage time curve for Example 30.

2. Draw a time diagram which will picture the variation of y_B and Q_c in time.

In Fig. 162, y''_2 and y'_2 designate, respectively, the highest and the lowest stage in reservoir B, when working on the time-service diagram (Fig. 163). Evidently y''_2 will be reached at the end of the filling period, the duration of which is 13 hr. with $Q_s = 50$ cu. ft. per second. The lowest level y'_2 will be at the end of the emptying period, the duration of which is 11 hr. with $Q_s = 200$ cu. ft. per second. It is further evident that at no time may the canal discharge fall below the minimum service discharge, to which there corresponds on the delivery curve

$y_2 = 8.01$ ft. Thus $y_B = 8.01$ ft. is the highest conceivable stage at B.

Emptying Period.—The time-stage curve will answer the relation

$$\Delta y_B \times 750 \cdot 10^3 = (200 - Q_c)\Delta t$$

or if, as convenient in such cases, time is measured in minutes:

$$\Delta t = \frac{750 \cdot 10^3}{60} \times \frac{\Delta y_B}{200 - Q_c} = 1.25 \frac{\Delta y_B}{200 - Q_c} \cdot 10^4 \text{ min.}$$

We may now compute the stage time curve downward, commencing with $y_B = 8.01$ ft. We start with a depth interval of $\Delta y_B = 0.21$ ft., between $y_B = 8.01$ and $y_B = 7.80$. From the delivery curve, we take the discharges corresponding to the stages at the beginning and the end of the depth interval to be: $Q_c = 50$ cu. ft. per second and 72 cu. ft. per second, with an average value of $Q_c = 61$ cu. ft. per second for the interval. The time in which the level will lower from 8.01 to 7.80 will be

$$\Delta t = 1.25 \frac{0.21}{(200 - 61)} \cdot 10^4 \text{ min.} = 18.9 \text{ min.}$$

Applying the procedure to successive depth intervals, we obtain

TABLE XXXVIII

y_B	Q_c	$200 - Q_c$	$200 - Q_c$ average for interval	Δy_B	Δt_{min}	t_{min}
8.01	50.0	150.0	......	....	.....	0.0
			139.00	0.21	18.9	
7.80	72.0	128.0	......	....	.....	18.9
			121.00	0.20	20.6	
7.60	86.0	114.0	......	....	.....	39.5
			108.50	0.20	23.0	
7.40	97.0	103.0	......	....	.....	62.5
			96.75	0.30	38.8	
7.10	109.5	90.5	......	....	.....	101.3
			85.75	0.30	43.7	
6.80	119.0	81.0	......	....	.....	145.0
			76.50	0.40	65.3	
6.40	128.0	72.0	......	.....	.....	210.3
			69.00	0.40	72.5	
6.00	134.0	66.0	......	....	.....	282.8
			63.00	0.50	99.3	
5.50	140.0	60.0	......	....	.....	382.1
			58.00	0.68	146.5	
4.82	144.0	56.0	......	....	.....	528.6

The last column, which is the sum total of the preceding time intervals, gives the time required to lower the level from the original level stage $y_B = 8.01$ ft. to the respective stage presented in the first column.

The table extends to the depth of uniform flow $y_2 = 4.82$ ft. Below, with Q_c = const. = 144 cu. ft. per second, the emptying will take place at a uniform rate of

$$\frac{\Delta y}{\Delta t} = \frac{200 - 144}{1.25 \times 10^4} = 0.447 \cdot 10^{-2} \text{ ft. per minute}$$

The curve $f(t, y_B)$ is plotted (curve 1) in Fig. 164.

Filling Curve.—The equation of the filling curve is

$$\Delta t = 1.25 \frac{\Delta y_B}{Q_c - 50} \cdot 10^4 \text{ min.}$$

The elements of computation are assembled in Table XXXIX.

TABLE XXXIX

y_B	Q_c	$Q_c - 50$	$Q_c - 50$ average for interval	Δy_B	Δt	t
8.01	50.0	0.0	.....	....	.....	0.0
			7.00	0.11	196.0	
7.90	64.0	14.0	.....	....	.....	196.0
			18.00	0.10	69.5	
7.80	72.0	22.0	.....	....	.....	265.5
			29.00	0.20	86.3	
7.60	86.0	36.0	.....	....	.....	351.8
			41.50	0.20	60.3	
7.40	97.0	47.0	.....	....	.....	412.1
			53.25	0.30	70.5	
7.10	109.5	59.5	.....	....	.....	482.6
			64.25	0.30	58.4	
6.80	119.0	69.0	.....	....	.....	541.0
			73.50	0.40	68.1	
6.40	128.0	78.0	.....	....	.....	609.1
			81.00	0.40	61.7	
6.00	134.0	84.0	.....	....	.....	670.8
			87.00	0.50	71.8	
5.50	140.0	90.0	.....	....	.....	742.6
			92.00	0.68	92.4	
4.82	144.0	94.0	.....	....	.....	835.0

One should notice that the table is built in a direction opposite to the actual movement of the level, so that the t values in the last column indicate the time, which it will require to have the level reach the final stage $y_B = 8.01$ ft., from the respective depth as given in the first column. Below the normal depth 4.82 ft., the filling will take place at the uniform rate of

$$\frac{\Delta y_B}{\Delta t} = \frac{144 - 50}{1.25} \cdot 10^{-4} = 0.752 \cdot 10^{-2} \text{ ft. per minute}$$

The curve is plotted as curve 2 in Fig. 164.

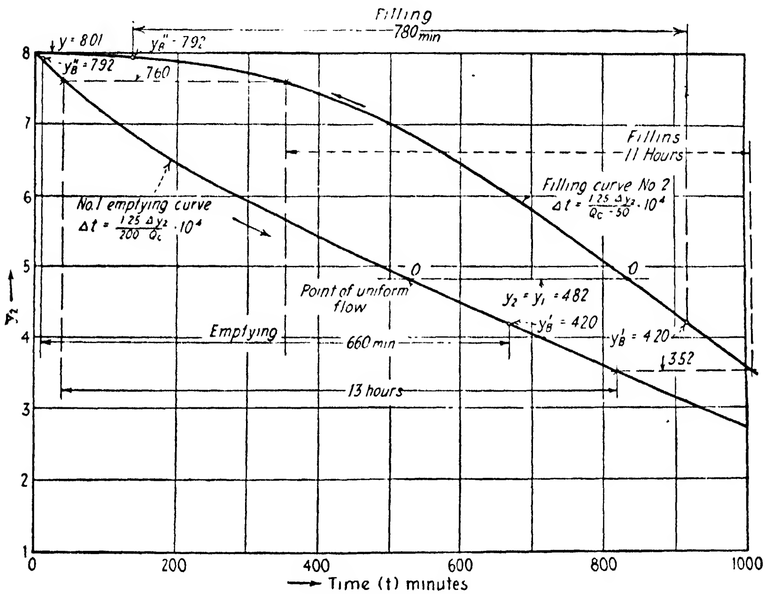

FIG. 164.—The filling- and emptying-time curve for Example 30.

The Limits of Fluctuation of Level.—Assume, in the first probe, y''_B to be 8.01 ft. In 11 hr. of emptying (see diagram 163a) the level would drop in accordance with curve 1 (Fig. 164) from 8.01 to $y'_B = 4.24$ ft.

On the filling curve the time distance from stage 4.24 back to 8.01 is 914 min., which is in excess of the time of $13 \times 60 = 780$ min. allowed for the filling in diagram, Fig. 163a. That means the initial 8.01-ft. stage was assumed too high.

Assume in second probe $y''_B = 7.80$ ft. Eleven hours on the emptying curve would bring the level from 7.80 down to 4.15

ft. The time on the filling curve between 4.15 and 7.80 ft. is 660 min., which is less than the schedule time of 780 min. Notice that a slight variation in the initial stage y''_B has a pronounced effect on the time of refilling, the reason being that in the neighborhood of $y_2 = 8.01$ ft. the difference between Q_r and Q_s is very small and the curve excessively flat. Obviously, the actual stage y''_B lies between 8.01 and 7.80. The problem is solved by finding such a pair of y''_B and y'_B, the time distance for which on the emptying and filling curve will be, respectively, 660 and 780 min. In our particular instance, as shown in Fig. 164, the condition is satisfied with sufficient approximation by $y''_B = 7.92$ and $y'_B = 4.20$ ft.

Thus, the total fluctuation of the level is $7.92 - 4.20 = 3.72$ ft., with a variation of the service discharge between a maximum of 144 cu. ft. per second and a minimum of 63 cu. ft. per second. Once the initial and terminal points are determined, one may draw, point by point, with the assistance of the curves of Fig. 164 the actual time diagrams for y_B and Q_r, which in conjunction with the service diagram give a complete picture of the functioning of the installation. This is done in Fig. 163*b*.

Obviously, the curves in Fig. 164 may be used for any other time set up of the service diagram, provided the emptying and filling period have the same service discharges.

For example, for a diagram in which the emptying period was to last 13 hr. and the filling period would be limited to 11 hr. the level would fluctuate between $y''_B = 7.60$ and $y'_B = 3.52$ ft. (see Fig. 164, dotted lines).

CHAPTER XV

CANALS WITH STEEP BOTTOM SLOPE

Channels with a bottom slope above the critical ($s_0 > \sigma$), which by the very nature of things are mostly of short length, are used in raft and log chutes, as discharge flumes for spillways and in other similar structures.

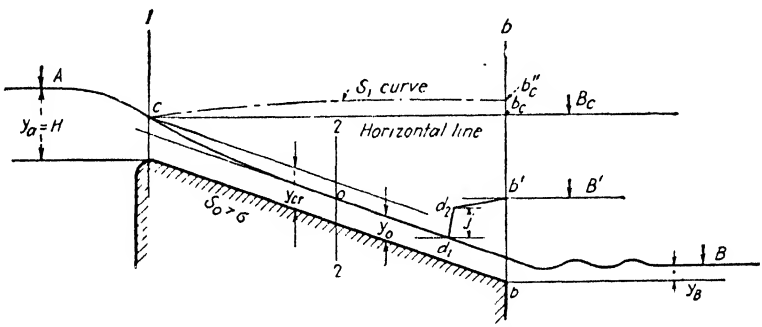

FIG. 165.—Flume with a steep bottom slope.

58. Delivery and Entrance Conditions.—In Fig. 165, y_a is the stage in reservoir A above the entrance sill; y_0 the depth of uniform flow in the channel. With $s_0 > \sigma$, y_0 is $< y_{cr}$. The surface curve crosses the critical depth near point C and approaches the normal depth by means of a falling curve of the S_2 type. As previously indicated, the transition zone shows no undulations. The surface curve passes from convex to concave form with an inversion point at the critical depth.

When the inflow is free, that is, when it is unaffected by tail-water, the entrance phenomena feature unobstructed *flow over a weir.* The flow in the flume itself with $y < y_{cr}$ does not affect the inflow. So the delivery of the canal is fully conditioned by the *entrance discharge,* which is simply the weir discharge

$$Q_c = Q_{ent} = b \cdot m \cdot \sqrt{2g} \cdot \left(y_a + \frac{v_0^2}{2g}\right)^{3/2} \qquad (117)$$

where b is the width of the weir, and v the velocity of approach.*

Another feature, characteristic of flow in steep channels, is the comparatively short length of the drop curve *c-o*. The latter may be considered as an extended transitory zone through which the depth rapidly reaches the uniform depth y_0. y_0 is the lowest possible stage. Thus, when designing flumes or chutes with steep bottom slopes, the engineer without further complications may simply operate with the normal depth as the depth, which will be the lowest for passing floating craft; which will give the greatest scouring effect, etc.

Tail-water Effect.—With tail-water rising (B' in Fig. 165) there will be a jump j between d_1 and d_2, with a connecting reach of an S_1 curve between d_2 and b'. Flow above the jump (to the left of d_1) will not be affected by what takes place below.

As the level rises, the jump will move upwards maintaining its height and form, while in the zone of uniform movement, that is, until its foot reaches point o (section 2). From there on, the jump will be moving upward on the curve *o-c*, gradually diminishing in height. As explained before, the height of the jump at the critical depth is infinitely small ($j = 0$). Stage b_c'', which determines an S_1 curve ending at c is the theoretical† limit, beyond which the entrance flow becomes one over a submerged weir. In submerged flow, tail-water has a direct effect on the entrance discharge.

59. Transition to Tail-water.—An interesting problem in hydraulical design is that of establishing proper transition between water flowing down a chute and the surface in the tail-water pool. In discharge flumes, leading from

* For values of m see, for example, Horton's "Weir Experiments and Formula," *U. S. Water Supply Paper* 200.

† In practical calculations, neglecting the convexity of the cb_c'' curve and adding to the margin of safety, it may be assumed that the practical limit of tail-water stage, which does not affect entrance conditions, is level B_c determined by a horizontal line passing through c.

spillways, the object is usually to annihilate the scoring effect. This case is similar to that of a toe roll at the foot of a dam (see Chap. XXI). A particular feature is present when the steep channel is part of a raft chute or some other structure, intended to lower floating craft from the upper down to the lower pool. When tail-water is sufficiently low, B in Fig. 165, the situation takes care of itself. There may be eventually more or less violent undulations, but these can be attenuated by inserting a hinged movable link.

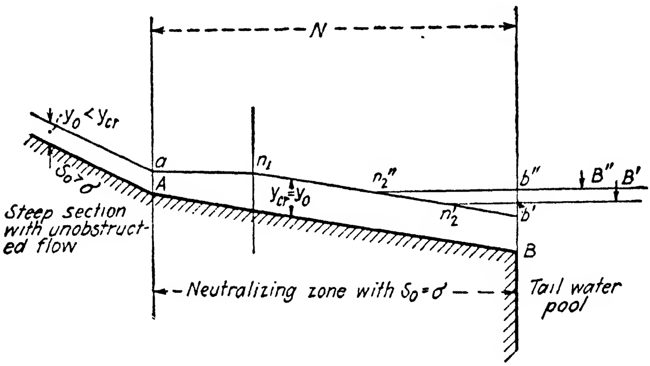

FIG. 166.

Transition conditions, on the other hand, become quite unsatisfactory when the rising level moves upwards and a jump is formed. The whirling roll of water at the foot of the jump may constitute at times a barrier totally impassable for floating craft and which under all circumstances presents a dangerous obstacle.

The writer has found it practical in certain cases to insert, between the steep slope section of the chute where flow should remain unobstructed and the tail-water pool, a *neutralizing* zone in the form of a reach (N in Fig. 166), laid with $s_0 = \sigma_0$. The neutralizing effect is caused by water flowing over N in critical flow with a possible minimum content of energy, under which circumstances no jumps can be formed. In accordance with Art. 30 (curves, Class C), the theoretical surface curve a-n_1, between y_0 and y_{cr}, as well as n_2b between y_{cr} and the tail-water pool will be horizontal lines. As the level B rises, the horizontal line n_2b will move up the reach, without any appreciable disturbance being formed at the junction point n_2, where theoretically there will be a jump of zero height.

Example 31

A raft chute is to be built between level A and B (Fig. 167). Level A is maintained constant, +30 ft. over the lowest stage of

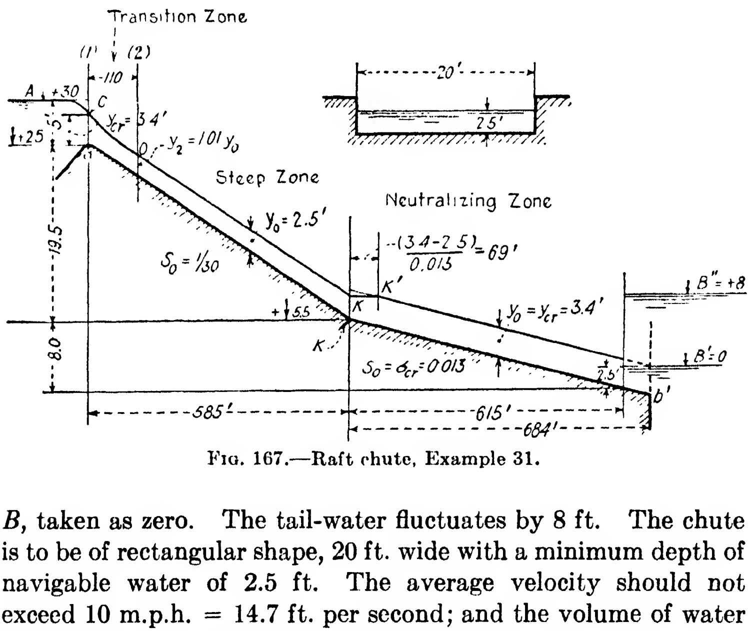

FIG. 167.—Raft chute, Example 31.

B, taken as zero. The tail-water fluctuates by 8 ft. The chute is to be of rectangular shape, 20 ft. wide with a minimum depth of navigable water of 2.5 ft. The average velocity should not exceed 10 m.p.h. = 14.7 ft. per second; and the volume of water used in the chute should be kept below 750 cu. ft. per second.

To economize in length and reduce velocity, the roughness of the channel is increased by lining the bottom and the walls with rough rubble which is supposed to raise the G.K. friction factor to $n = 0.030$.

In uniform flow we have $a = 20 \times 2.5 = 50$ sq. ft.; $p = 25$ ft.; $R = 2$ ft.; C, with $s_0 = 0.01 = \sim 55$.

With v_{max} given, we determine

$$s_0 = v^2/C^2R = (14.7)^2/55^2 \cdot 2 = 0.0356$$

Take for safety $s_0 = 1/30 = 0.0333$, which makes the actual velocity

$$v = 55\sqrt{2 \cdot 0.0333} = 14.2 \text{ ft. per second}$$

$$Q = 14.2 \times 2.5 \times 20 = 710 \text{ cu. ft. per second; } q = 35.5$$

Entrance Conditions.—Assuming the weir coefficient $m = 0.40$, and neglecting the velocity of approach we have

$$y_a^{3/2} = q/m\sqrt{2g} = 35.5/0.4 \cdot 8.02 = 11.08 \text{ and } y_a = 4.97 \text{ ft.}$$

We make $y_a = 5$ ft.

Length of Transition Curve (c-o in Fig. 167).—The critical depth

$$y_{cr} = \sqrt[3]{q^2/g} = \sqrt[3]{(35.5)^2/32.2} = 3.40 \text{ ft.}$$

The hydraulic elements in critical flow

$$a_{cr} = 20 \times 3.4 = 67.8$$
$$p = 20 + 2 \times 3.40 = 26.8$$
$$R = 67.8/26.8 = 2.54;\ C = \sim 57$$
$$\sigma_{cr} = g/C^2 \cdot p/b = 32.2/(57)^2 \times 26.8/20 = 0.013$$

The critical slope at $y_0 = 2.5$

$$\sigma_0 = 32.2/55^2 \times 25/20 = 0.0133$$

The average value of

$$\beta = s_0/\sigma = 0.0333/0.0131 = 2.53$$

To determine the hydraulic exponent, we have

$$(y_{cr}/y_0)^n = (\mathfrak{K}_{cr}/\mathfrak{K}_0)^2 = s_0/\sigma_{cr} = 2.53$$

with

$$\frac{y_{cr}}{y_0} = \frac{3.4}{2.5} = 1.36$$

we obtain

$$n = \frac{\log 2.53}{\log 1.36} = 3.04$$

Using the table values with $n = 3$, we have for the curve between $\eta_1 = y_{cr}/y_0 = 1.36$ and $\eta_2 = 1.01$:

$$l = \frac{y_0}{s_0}[(1.01 - 1.36) - (1 - 2.53)(1.419 - 0.329)] =$$

$$1.47\frac{y_0}{s_0} = 1.47\frac{2.5}{0.033} \doteq 110 \text{ ft.}$$

Neutralizing Section.—The elements are $y_{cr} = y_0 = 3.4$ ft.; $s_0 = \sigma_{cr} = 0.013$; $v_{cr} = \dfrac{35.5}{3.4} = 10.4$ ft. per second.

Theoretical minimum length of the neutralizing section

$$l_n = \frac{8}{0.013} + \frac{(3.4 - 2.5)}{0.013} = 615 + 69 = 684 \text{ ft.}$$

Length of the steep section

$$l_s = \frac{19.5}{1/30} = 585 \text{ ft.}$$

The theoretical profile is given in Fig. 167.

CHAPTER XVI

BACKWATER CURVES IN NATURAL WATERCOURSES

60. Generalities.—This is the traditional problem of varied flow, formerly the only one dealt with in most treatises on hydraulics. In Fig. 168, *H-H* being a datum line, the water level in section *D* is lifted by a dam from elevation *d* to *d′*. The surface profile in natural conditions

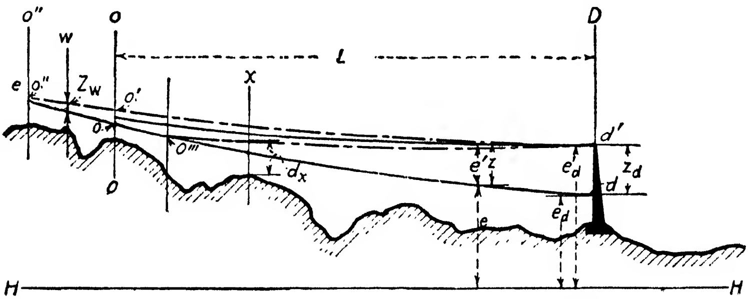

FIG. 168.—Backwater curve in a river: *d′-o″* upper limiting curve; *d′-o‴* lower limiting curve.

with a discharge *Q* is *d-o*, and the problem is to determine the backwater curve *d′-o′* produced by the dam.

Speaking generally, in view of the irregularity of flow, any solution arrived at should be considered as a rough approximation. In fact, figuring out backwater curves belongs essentially to the class of what may be termed *control computations*, the purpose of such computations usually being to make certain that given limiting conditions are not transgressed.

In problems of this character the specific object of the computation should be always borne in mind and the figuring carried out under appropriate assumptions. The course will be made clear by comparing the two extreme cases, leading to what we term the *upper* and the *lower* limiting backwater curves.

Upper Limiting Curve.—Assume that the dam in Fig. 168 is intended for hydroelectric work. Naturally the lift Z_d should be given its maximum possible value. The limiting condition usually is that the backwater curve should not extend beyond a certain given location o'', or that in a certain determined section w, the rise of the level should not exceed a certain given value Z_w.

In such a case, the premises underlying the figuring should be assumed to be such as to provide for a reasonable margin of safety, leading to a curve of the greatest possible length and with the highest possible rise d'-o''.

Lower Limiting Curve.—Suppose, on the contrary, that the dam is a part of a scheme intended for improving navigable conditions. In this case level d' is often conditioned by the requirement that in a certain section x the navigable depth should not be lower than a certain minimum value of d_x. The backwater curve in this case should be computed under premises and assumptions which will result in the lowest possible curve d'-o'''. The actual curve d'-o' will probably lie in between.*

Rivers and Torrents (see Art. 23).—It should be remembered that backwater curves of the type shown in Fig. 168 will take place only in "rivers," that is in watercourses where the flow under natural conditions is *tranquil* and where the slope is below the critical ($s < \sigma$). In a "torrent," that is in a watercourse where natural flow is *rapid* and the slope is above the critical ($s > \sigma$), the backwater curve d'-j (Fig. 169) will be a convex curve of the S_1 type, ending with a jump j. In actual practice it will be seldom necessary to determine the exact profile of such surface curves. The important fact is that the whole curve and the jump lie below the horizontal line d'-o. This level line, therefore, should be simply assumed as the outward limit of all possible backwater curves. In fact, it will be

* Regarding the *length* and the *end* of a backwater curve, see Art. 33.

found that in practice the actual surface curve will differ only slightly from this horizontal line, the reason being that usually the slope of a torrential watercourse is not much

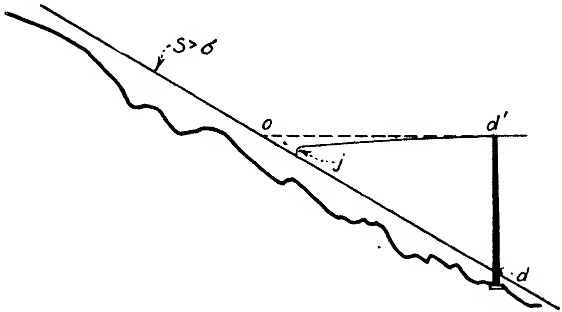

FIG. 169.—Backwater curve in a *torrent*.

in excess of the critical slope, so that the convexity of the S_1 curve is small.

Another feature, characteristic of natural watercourses, is that the jump j is not as conspicuous as, for example, the

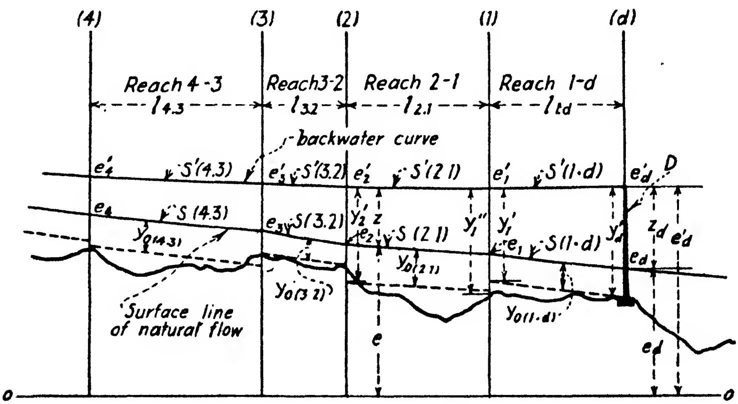

FIG. 170.—Dividing a watercourse into reaches.

jump below a sluice or at the foot of a weir. As will be made clear in the next chapters, natural watercourses, because of low kineticity, produce mostly jumps presenting a series of undulations.

61. Practical Procedures.—The usual method used in computing backwater curves is to divide the longitudinal profile (Fig. 170) into *reaches*, making the division so that each reach should be featured by a more or less homogeneous surface slope s, a more or less equal surface width b, and, in general, by more or less similar hydraulic characteristics. Assuming that, within the reach in question, certain average conditions prevail, the surface drop of the backwater curve between section $n+1$ and section n is taken to be

$$e'_{n+1} - e'_n = (s' \times l)_{n+1,n} = \left(\frac{v^2}{C^2R} \times l\right)_{n+1,n} + \frac{v^2_n - v^2_{n+1}}{2g}$$

The first member in the above represents the friction head lost over the reach, while the second member is the head gained through restoration of kinetic energy. In a rising curve the latter should be neglected, so that

$$(\Delta e')_{n+1,n} = e'_{n+1} - e'_n = (v^2/C^2R \times l)_{n+1,n} \quad (118)$$

Assuming that adequate survey material is available, one may subdivide the watercourse into any number of reaches and, by applying Eq. (118) to each consecutive reach starting from section d, in which the elevation e'_d is given as the lift caused by the dam, determine consecutively the elevations of the water surface in each section and thus trace the backwater curve. Naturally, in the handling of the problem, a vast field is left to the choice and flair of the designing engineer. Moreover, under all circumstances, the procedure is tedious and clumsy.

Equivalent Profiles.—To simplify calculations, first Dupuit and later other authors suggested replacing the naturally varied and irregular watercourse bed by *equivalent* cross-sections of a regular and simple form. Thus, Fig. 171*b* gives the equivalent parabolic profile recommended by Tolkmitt; and Fig. 171*a* pictures a rectangular profile, used by Dupuit, Rühlmann, Bresse, Schaffernack, and others.

In figuring out an equivalent profile, one usually retains the natural surface width b, while the average depth y_0 for the reach in question is determined by assuming that the discharge will be carried by the equivalent profile in uniform flow with a slope equal to the average surface slope of the respective reach. In terms adopted in this book, the equivalent profile for a reach will be one which will provide for a conveyance $\mathfrak{K} = Q/(\sqrt{\bar{s}})_{n+1,n}$. With reference to Chap. VIII, it may be further remarked that it was specifically the problem of computing backwater curves by means of equivalent profiles, which in the past prompted the working out of the tables for the specific cases referred to in Art. 31.

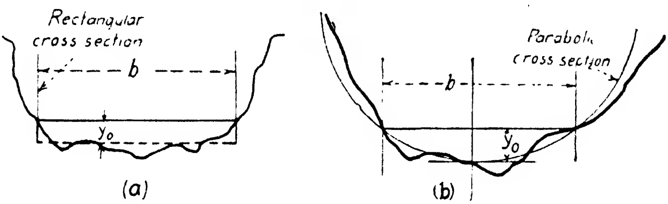

FIG. 171.—Equivalent profiles.

The result of substituting equivalent cross-sections will be that in Fig. 170 the irregular bottom is replaced in every reach by an ideal straight bottom line, shown dotted and drawn parallel to the surface at the respective equivalent depth $(y_0)_{n+1,n}$ of uniform flow. One may now apply, in calculating the backwater curve, Eq. (91) which for a certain reach will read

$$(s \cdot l/y_0)_{n+1,n} = \Phi(\eta)_n - \Phi(\eta)_{n+1} \tag{119}$$

Starting with the reach adjoining the dam, and with the initial rise z_d and the depth y'_d given, Eq. (119) will determine the depth y'_1 and thus the elevation e'_1 in section 1. Passing now to reach 2–1, and, knowing the depth y''_1, we may determine y'_2, elevation e'_2, and so on.

62. Generalized Method.—In the light of the method of calculating surface curves, as developed in Art. 33, a closer and seemingly more comprehensive approach towards the backwater problem is possible. In fact, in using Eq. (119) there is no more necessity of holding on to some ideal equivalent cross-section of a definite geometric form. All that is necessary is to know the particular value of the hydraulic exponent n, which features within that or other reach the change of the conveyance $\mathfrak{K} = f(y)$ with the depth of flow. In other words, instead of squeezing the varying cross-sectional profiles of the natural watercourse

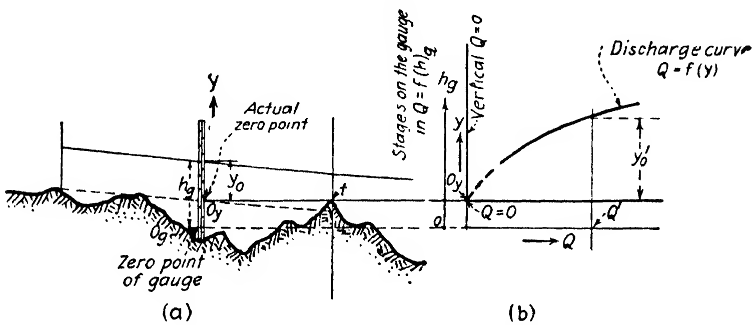

FIG. 172.—Establishing the zero point.

into prescribed stiff geometric forms, one has simply to determine, by means of hydrographic data available, the closest average value of the hydraulic exponent, which governs flow within the given reach.

If a gauge and a discharge curve $Q = f(h)_g$ are available for a certain section (Fig. 172a), the discharge curve may be assumed directly proportional to the $\mathfrak{K}$ curve, the deviation if any being due to the fact that the surface slope changes somewhat at different stages. A logarithmic plotting of the discharge curve in this case would directly give the value of $n/2$ for the corresponding reach. To carry out the logarithmic plotting it is imperative first to know the position of the actual zero point o_y, from which the stages in equation $Q^2 = \text{const} \cdot y^n$ are to be measured. The zero point may be determined by extrapolating the discharge curve until the point o_y of the intersection of the Q curve with the vertical $Q = 0$. A simple hint, which has proved to be useful in many instances, is to draw a horizontal line *t–t*, starting from the highest bottom point *t* below the gauge. The intersection point of such a horizontal, with the gauge vertical, gives the o_y zero point at which $Q = 0$. It is further obvious that the zero point directly determines the respective equivalent uniform depth. In fact, for any discharge Q' the corresponding normal depth to be taken from the discharge curve will be y'_0.

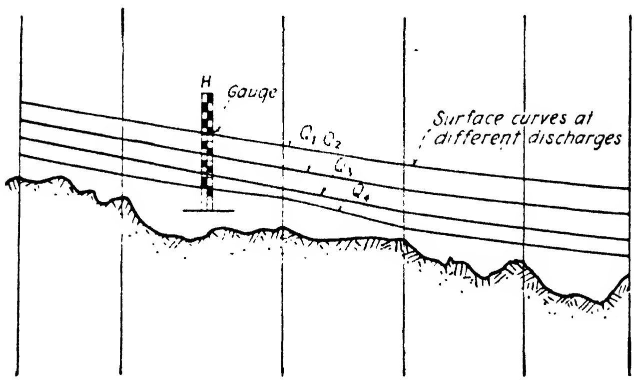

FIG. 173.

A thorough hydrographic survey usually gives surface profiles (Fig. 173) for a series of discharges from low to high. Those profiles, together with a discharge curve referred to a gauge, are material enough to enable logarithmic plotting of the $Q^2 = \text{const} \cdot y^n$ relation for each and every reach. In other words, with proper hydrographic material on hand, the values of the hydraulic exponent, as well as the positions of the zero points, may be determined for the whole watercourse in question. With n known, Eq. (119) with the $\Phi(\eta)$ values corresponding to the hydraulic exponent as determined, may be directly used.

Upper and Lower Limiting Curves.—Always remember the rule: The smaller the value of n, the longer the curve. Hence, whenever a certain average value of the hydraulic exponent is to be selected out of a group of differing figures, use the *smaller optional* value in case the computation demands knowing the upper limiting backwater curve; while, on the contrary, the *highest optional value of n should* be used, when the lower limiting backwater curve is under consideration. Similar reasoning should govern when the nearest table line of n is to be used instead of the intermediary values of the exponent, as actually determined.

Again, with regard to the bottom line, the greater the y_0 value, the longer the curve for a given lift Z. Hence, in determining the zero point, the process of extrapolating should be also guided by the specific purpose of the design.

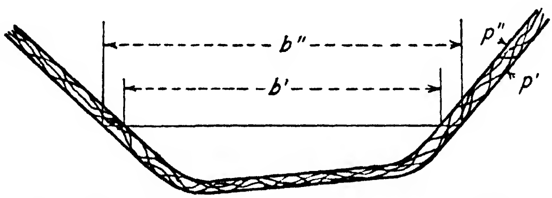

Fig. 174.—The inner and outer enveloping profiles.

When sufficient direct hydrographic data are not available, a series of successive cross-sectional profiles may be traced as in Fig. 174, and an *inner* and an *outer* enveloping profile p' and p'' drawn. The inner profile will result in a longer and higher surface curve; the outer profile, on the contrary, will show shorter curves and a smaller rise of the level. Finally, if a friction factor is to be selected, remember that a higher friction factor will result in longer and higher curves.

Approximate Method for Brief Calculations.—A quick approximate result may finally be obtained, when bearing in mind that practically all actual values of the hydraulic exponent will be found to be intermediary between $n = 3.2$ and 4.8, and that further only in very rare cases will the exponent be found outside the limits of 3.4 and 4.4. Under such circumstances, wherever a preliminary control curve is sought, the upper or the lower limit of the hydraulic exponent, as above given, may be used, without any further detailed investigation.

The reader will find that the curves, obtained by this simple and brief procedure, will not differ substantially from results obtained by more elaborate means.

PART III

THE HYDRAULIC JUMP

CHAPTER XVII

THEORY OF THE JUMP

63. Introductory.—The hydraulic jump, as previously defined, is a local phenomenon by means of which flow passes in an abrupt manner from a *rapid* to a *tranquil* state. Accordingly, in Fig. 175, where the flow is referred

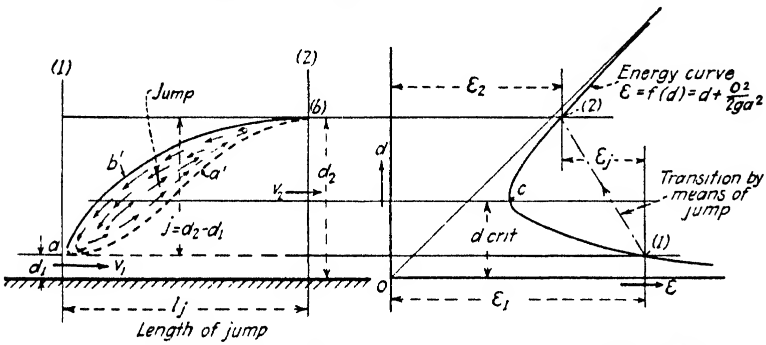

FIG. 175.—The hydraulic jump referred to the specific energy diagram.

to a specific energy diagram, the lower stage d_1 before the jump and the upper stage d_2 after the jump correspond to the points 1 and 2 which respectively lie on the lower and the upper branches of the energy curve.

Sections 1 and 2 demarcate the jump from the adjacent regions, where the movement is gradually varied and the flow *parallel*. The energy in sections 1 and 2 is respectively:

$$\epsilon_1 = d_1 + \frac{Q^2}{2ga_1^2}$$
$$\epsilon_2 = d_2 + \frac{Q^2}{2ga_2^2} \qquad (120)$$

The difference

$$\epsilon_j = \epsilon_1 - \epsilon_2 \qquad (121)$$

represents the energy head lost in the jump. The energy losses inherent in the jump are mostly of the *impact* type, meaning losses which usually accompany rapid and abrupt change of movement. By analogy with other impact phenomena, one may expect that these losses are large in comparison with the usual friction losses in uniform or gradually varied flow.

The depths d_1 and d_2, before and after the jump, will be called the *conjugated depths*. The vertical distance between the stages $j = d_2 - d_1$ is the *height* of the jump.

The purpose of the theory is to determine the relation between the conjugated depths. Given, the form of the canal, the discharge Q, and one of the two conjugated depths, the problem is to determine the other unknown depth. The energy principle offers a lucid explanation of the physical essence of the phenomenon, but cannot serve as a basis for a theory, for the reason that there exists no direct way to evaluate the energy losses in the jump. On the other hand, a most satisfactory solution is obtained by applying the momentum principle. It was first used to study the jump in the forties of the last century by Bélanger. It yields results, fully congruent with experimental observations. In this connection, it may not be out of place to call attention to the fact that the momentum principle is commonly used in rigid dynamics for studying impact between bodies. In hydraulics it is also applied to determine losses caused by an abrupt change of form of flow in closed conduits (the Borda theorem).

Forms of the Jump.—There are two distinct forms in which the jump may occur: The *direct* form (Fig. 176), and the *undular* form (Fig. 177).

In the *direct* jump the upper stage is reached practically by one continuous rise of the surface. Observed in a flume with glass walls, the direct jump features an underlying portion of expanding live stream covered by a surface roll within which the particles are engaged in circuitous movement and do not participate in the translatory movement of the liquid from section 1 to section 2.*

The *direct* form is typical of jumps of comparatively large height. It is usually present in jumps accompanying flow through hydraulic structures.

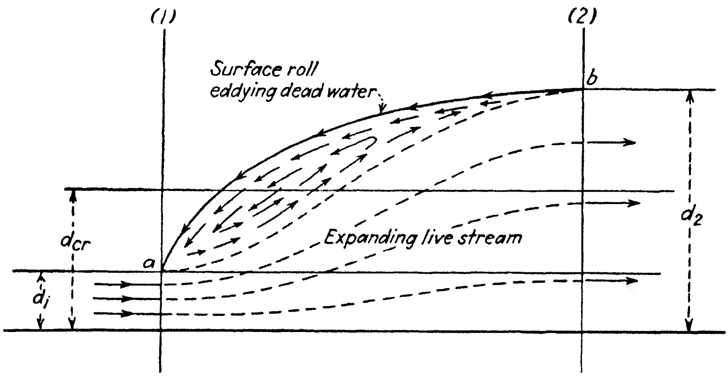

Fig. 176.—The *direct* jump.

The *undular* form is characteristic of jumps of comparatively low height. It is mostly observed in natural watercourses with moderately steep bottom slopes. The transition from the lower to the upper stage features a

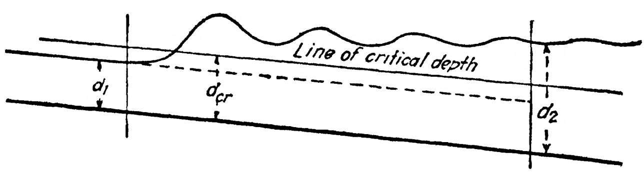

Fig. 177.—The *undular* jump.

series of undulations of gradually diminishing size. In cases where the jump is particularly small, the surface may be continuous all the way through, as shown in Fig. 177. In other cases (Fig. 178), local surface rolls may take place on the first or on several consecutive waves.

* These physical aspects of the jump are masterfully elucidated by Professor Rehbock: *Handbuch der Ingenieur-Wissenschaften:* "Stauwerke", "Versuche über Abfluss, etc."

It should be generally remembered that a jump, by its very nature, presents a disruption of what otherwise would be smooth and continuous flow. Therefore, the features of the jump are to be considered permanent only in the sense of presenting a stable average over a certain

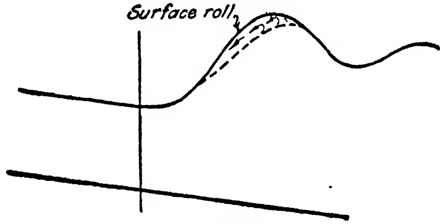

Fig. 178.

period of time. Round these averages, the phenomenon is in a state of incessant pulsation. This refers in the first place to the toe of the jump, so that the foot of the jump,

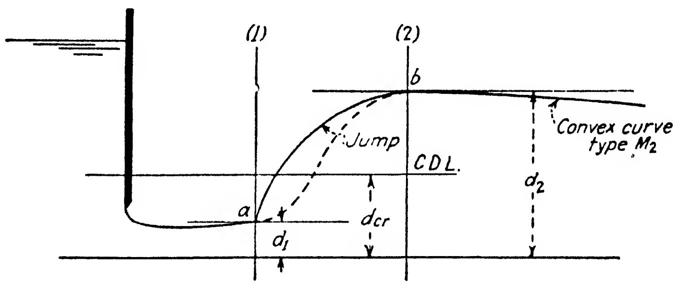

Fig. 179.

marked by point *a* (Fig. 176), oscillates in the direction of the axis of flow round a certain average location marked by section 1. In the end section 2, point *b*, the oscillations are both horizontal and vertical.

Under such circumstances it may not always be simple to define with precision the *beginning* or the *end* of the jump. In fact, the process of defining greatly depends on the type of the jump and on other circumstances which surround the phenomenon.

For example, in the direct jump, it is easy to define the beginning of the jump, for there is an unmistakably observable line of demarcation between the smooth undisturbed surface before the jump and the surface roll. Again, in the case as pictured in Fig. 179, flow after the jump is in the form of a falling curve of the M_2 type, the end of the jump and the demarcation section 2 being thus determined by the point of maximum depth d_2.

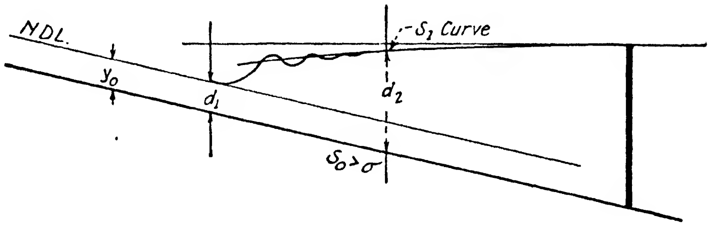

Fig. 180.

With regard to the undular jump, in a case like Fig. 180, where rapid flow before the jump is uniform flow, the depth d_1 is y_0 and therefore is easily measurable. On the other hand, with the upper stage in the form of a convex rising

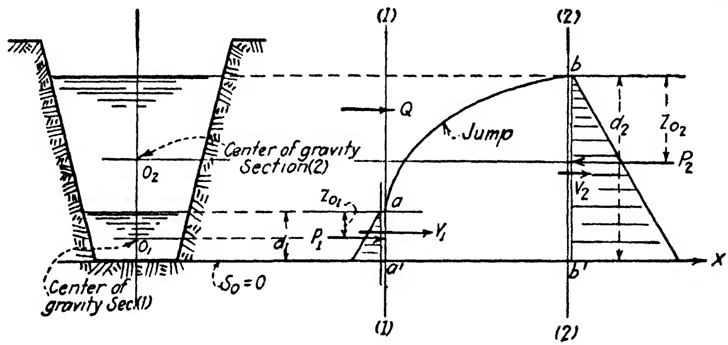

Fig. 181.—Applying the momentum principle.

curve of the S_1 type, it is practically impossible to demarcate the end of the jump amidst the greatly extended surface waves of small curvature.

All such circumstances affect substantially the precision of observations and should be given serious consideration in experimental work.

One more point must be made clear before closing these introductory remarks. Generally speaking, up until now, theory and observation have dealt mostly with the *vertical* elements of the jump, that is, with the depths d_1 and d_2. On the other hand, little, if any, material is available which would enable the engineer to foretell the longitudinal elements, such as the length of the jump, the more or less exact form of the surface, etc. Also, little is known regarding the inner mechanism of the phenomenon, such as the distribution of velocities and pressures; the nature and character of the losses; etc. Thus a fruitful and fascinating field still lies open to eventual research.

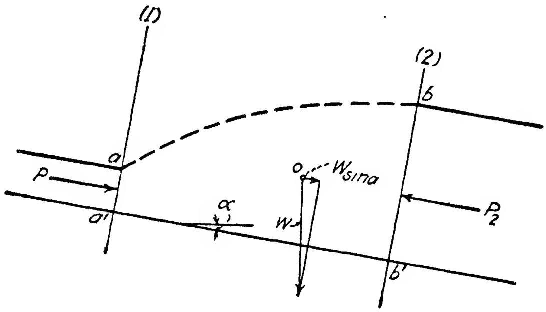

FIG. 182.

64. The Momentum Equation.—Consider a discharge Q, flowing in a prismatic channel of given form (Fig. 181), laid with a horizontal bottom. We shall apply the momentum principle towards the body of liquid $aa'b'b$, enclosed between sections 1 and 2. Flow being permanent, the change of momentum in the direction of the X-axis per unit of time is the difference of the momentum, contained in the liquid leaving the body under consideration through section 2 and the momentum carried by the liquid as it enters the body at section 1. The mass of the outflowing and inflowing liquid is identical and equal to $\Delta Q/g$. Thus the change of the momentum per unit of time is

$$\frac{\Delta Q}{g}(v_2 - v_1) \qquad (122)$$

This change is equal to the time-impulse of the components of all forces acting on the liquid body in the direction of the X-axis. As the movement is stationary, the time impulse per unit of time is the sum total of all forces acting on or within the liquid body in the direction of X. We shall now evaluate such forces.

The effect of gravity, because $s_0 = 0$, is eliminated.

Remark: The advantage of considering the jump in a canal with a horizontal bottom lies specifically in the fact that the effect of gravity is eliminated. In case the bottom were not horizontal, as in Fig. 182, one would have to add to the forces contributing to the change of the momentum the gravity component $w \sin \alpha = ws_0$, where w is the weight of the liquid body $aa'bb'$. That would require knowing the length and the form of the jump, unless, as done by many authors in the past, the effect of the weight component is simply neglected. Such an approximation, however, is scarcely permissible and, as observations have shown, leads to serious incongruencies.

Further, and therein generally lies the particular advantage of using the momentum principle, the effect of all and every *internal* force, whether pressure or friction, is eliminated, for the simple reason that for any force which acts on a certain particle a from the adjoining particle b, there is a force of equal size but opposite in direction, which acts on particle b from particle a. In the summing up, these pairs of mutually equal, but directly opposed, internal forces nullify each other, so that the sum total of all forces is reduced to the sum of the external forces only. Such external forces in our case are:

1. The resultants P_1 and P_2 of the hydrodynamic pressures acting over sections a_1 and a_2.

2. The *external* forces of friction Σf_w acting in the direction opposed to the flow on the contour surface of the liquid body, the seat of such forces being in the liquid boundary layer adjacent to the solid walls of the canal.

As flow in sections 1 and 2 is taken to be parallel, the distribution of hydrodynamic pressure across the sections is taken to conform with the hydrostatic law. Thus P_1 and P_2 are, respectively, equal to $\Delta a_1 z_{01}$ and $\Delta a_2 z_{02}$, where a_1 and a_2 are the cross-sectional areas, while z_{01} and z_{02} are the distances of the centers of gravity of the respective sections below the surface of flow. The momentum equation will thus read

$$\frac{\Delta Q}{g}(v_2 - v_1) = P_1 - P_2 - \Sigma f_w$$

The only undetermined element is the external friction component Σf_w. The customary assumption is that, because of the comparatively small length of the jump, the external friction forces are small when compared to the internal forces responsible for the major losses of energy in the jump, and that therefore these external forces may be simply neglected. This assumption is substantiated by experiments which, as will later be seen, show that the height of the jump determined by the momentum equation with Σf_w omitted is only slightly larger than the height, actually observed in properly conducted experiments.

Omitting Σf_w and substituting Q/a for v, we obtain

$$\frac{\Delta Q}{g}\left(\frac{Q}{a_2} - \frac{Q}{a_1}\right) = \Delta(a_1 z_{01} - a_2 z_{02})$$

which can be presented as

$$\frac{Q^2}{ga_2} + a_2 z_{02} = \frac{Q^2}{ga_1} + a_1 z_{01} \tag{123}$$

The equation carries on both sides an analogous expression which suggests that the two conjugated depths d_2 and d_1 correspond to two equal values of a certain single function, to be designated as

$$M(d) = \frac{Q^2}{ag} + a z_0 \tag{124}$$

In other words Eq. (123) may be presented as

$$M(d_1) = M(d_2) \tag{125}$$

Evidently the M curve has two branches. It is further easily shown that, similar to the energy curve $\epsilon = f(d)$, the M curve passes through a minimum at the critical depth. In fact

$$\frac{\delta M}{\delta d} = -\frac{Q^2}{ga^2} \cdot \frac{\delta a}{\delta d} + \frac{\delta}{\delta d}(az_0) \tag{126}$$

The first member, by virtue of Eq. (19), is equal to Q^2b/ga^2. The second is the derivative of the statical moment of the cross-section by the depth d. The value of this derivative is gained by considering Fig. 183. Let the statical moment of the area a, corresponding to the depth d, with relation to the surface line b–b, be az_0. With a change of stage by δd, the moment with regard to the new surface line b'–b' is

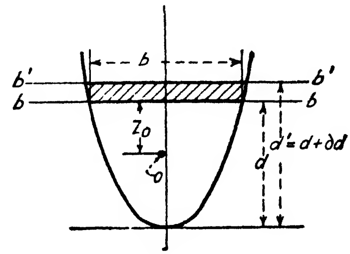

FIG. 183.

$$a(z_0 + \delta d) + b\frac{(\delta d)^2}{2}$$

Omitting the second member as one of a higher degree of smallness, the change of the statical moment is

$$\delta(az_0) = a(z_0 + \delta d) - az_0 = a\delta d$$

so that

$$\delta(az_0)/\delta d = a \tag{127}$$

by inserting which into Eq. (126), we obtain

$$\frac{\delta M}{\delta d} = -\frac{Q^2 b}{ga^2} + a$$

The minimum value of $M(d)$ corresponds to

$$\frac{\delta M}{\delta d} = a\left(1 - \frac{Q^2 b}{ga^3}\right) = 0$$

The expression in the brackets is identical with $\delta\epsilon/\delta d$ (Eq. [26]), which shows that the $M(d)$ function passes through a minimum at the same stage as the energy curve, namely that of the critical depth.

When the form of the channel and the discharge Q are given, the M curve may be calculated and traced from point to point resulting in a chart (Fig. 184). Any vertical V which intersects the M curve at the two points 1_M and 2_M

determines two conjugated depths d_1 and d_2. Obviously, there is an infinite number of possible conjugated depths,

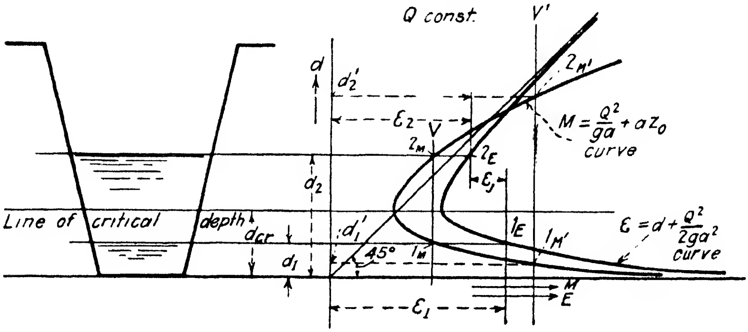

FIG. 184.—The M (d) curve in conjunction with the energy diagram.

each pair corresponding to a possible vertical. However for each vertical, that is for each and every point 1_M on

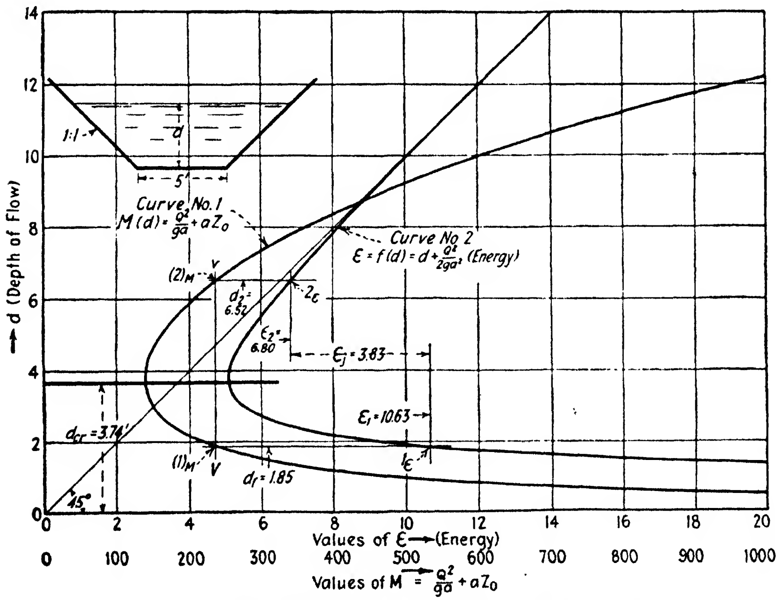

FIG. 185.—The $M(d)$ and the $\epsilon(d)$ curve for Example 32.

the lower branch of the M curve there is one and only one conjugated point 2_M. So to each value of d_1 there corresponds one and only one conjugated depth d_2, and *vice versa.*

If, in addition to the $M(d)$ curve, we were to trace the energy curve computed for the same volume of flow Q, the combined diagram would allow us to determine in each instance the respective loss of energy. In fact, by drawing horizontal lines through points 1_M and 2_M to intersect the energy curve at 1_ϵ and 2_ϵ, we determine the respective values of the specific energies carried by the flow at the depths d_1 and d_2, and thus the energy loss in the jump $\epsilon_j = \epsilon_1 - \epsilon_2$.

Example 32

A discharge of $Q = 300$ cu. ft. per second is flowing in a canal, Type B (Fig. 185).

Question 1. Compute and trace the $M(d)$ curve.

With reference to Example 3, the critical depth for our case is $d_{cr} = 3.74$ ft. For computing $M = \frac{Q^2}{ag} + az_0$, we have

$$Q^2/ag = 300^2/a \cdot 32.16 = 2{,}795/a$$

$$z_0 = \frac{d\left(2.5 + \frac{d}{3}\right)}{5 + d}$$

The computation is assembled in Table XL.

The curve is plotted as curve 1 (Fig. 185), together with the energy curve (curve 2), the elements for which are taken from Table IV.

Question 2. For the circumstances of flow given above, find the depth d_2 conjugated to $d_1 = 1.85$. Find also the loss of energy in the jump.

In Fig. 185, draw a vertical through point 1_M of the M curve corresponding to $d_1 = 1.85$ ft. The intersection with the upper branch at 2_M determines the conjugated depth $d_2 = 6.52$ ft. The height of the jump is $j = 6.52 - 1.85 = 4.67$ ft.

By drawing horizontals through 1_M and 2_M, we find

$$\epsilon_1 = 10.63 \text{ and } \epsilon_2 = 6.80 \text{ ft.}$$

The energy lost in the jump is $\epsilon_1 = 10.63 - 6.80 = 3.83$ ft. The quotient

$$\epsilon_2/\epsilon_1 = 6.80/10.63 = 0.64$$

features the part of the original energy remaining in the flowing

TABLE XL

d	a	v	$2{,}795/a$	z_0	az_0	$M(d)$
0.50	2.75	109.00	1,017.0	0.242	0.66	1,018.0
0.75	4.31	69.60	648.0	0.360	1.55	649.5
1.00	6.00	50.00	466.0	0.471	2.83	469.0
1.25	7.81	38.40	358.0	0.585	4.56	363.0
1.50	9.75	30.80	287.0	0.693	6.75	294.0
2.00	14.00	21.40	200.0	0.905	12.70	213.0
2.50	18.75	16.00	149.0	1.110	20.60	170.0
3.00	24.00	12.50	116.0	1.315	31.50	147.5
3.25	26.81	11.17	104.0	1.410	37.80	142.0
3.50	29.75	10.11	94.0	1.510	44.90	139.0
3.60	30.96	9.69	90.4	1.550	48.00	138.4
3.70	32.19	9.32	87.0	1.590	51.20	138.2
3.74	32.69	9.08	85.5	1.600	52.30	137.8
3.80	33.44	8.96	83.7	1.630	54.50	138.2
3.90	34.70	8.64	80.7	1.670	57.90	138.6
4.00	36.00	8.33	77.7	1.710	61.50	139.2
4.25	39.31	7.63	71.1	1.800	70.80	141.9
4.50	42.75	7.02	65.4	1.900	81.30	146.7
5.00	50.00	6.00	55.9	2.080	104.00	160.0
5.50	57.75	5.20	48.4	2.260	130.50	179.0
6.00	66.00	4.55	42.3	2.460	162.30	205.0
6.50	74.75	4.01	37.4	2.640	197.00	234.0
7.00	84.00	3.57	33.25	2.820	237.00	270.0
8.00	104.00	2.88	26.9	3.180	331.00	358.0
9.00	126.00	2.38	22.2	3.540	446.00	468.0
10.00	150.00	2.00	18.6	3.890	584.00	603.0
11.00	176.00	1.71	15.9	4.220	743.00	759.0
12.00	204.00	1.47	13.7	4.580	945.00	959.0

liquid after the jump. Obviously $1 - \frac{\epsilon_2}{\epsilon_1} = 0.36$ shows the rate of initial energy lost in the eddies and vortices accompanying the rapid change of form.

Question 3. Assuming that the stage d_2 after the jump is $d_2 = 6$ ft., determine the depth d_1 required to sustain such upper stage by means of a jump.

This is the reverse problem. A vertical (Fig. 185) drawn through the point corresponding to $d_2 = 6$, intersects the lower branch at a point corresponding to $d_1 = 2.11$ ft.

65. The $Q_{const.}$ Characteristics of the Jump.—Applying the procedure as outlined in the above example to a series of verticals, one may summarize the features of all and every jump which may occur in the canal as given, with the given discharge Q in a set of curves which may be properly called the Q_{const} *characteristics.*

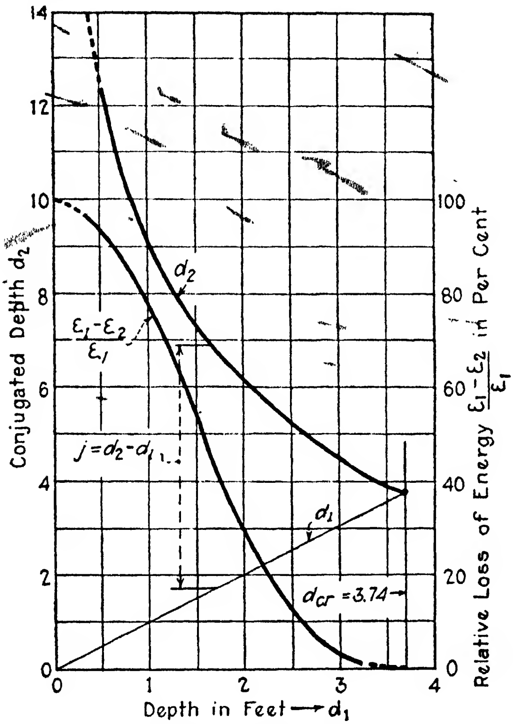

FIG. 186.—The Q constant characteristics of a jump for canal Type B and $Q = 300$ cu. ft. per second, Example 32.

With reference to Fig. 185, the elements of the characteristics for $Q_{const} = 300$ cu. ft. per second are assembled in

TABLE XLI

d_1	d_2	ϵ_1	ϵ_2	$j = d_2 - d_1$	j/d_1	$\epsilon_1 - \epsilon_2$	ϵ_2/ϵ_1	$\frac{\epsilon_1 - \epsilon_2}{\epsilon_1}$ per cent
0.5	12.25	185.50	12.26	11.75	23.50	173.20	0.07	93
1.0	9.00	39.90	9.10	8.00	8.00	30.80	0.23	77
1.5	7.28	16.20	7.46	5.78	3.85	8.74	0.46	54
2.0	6.15	9.14	6.43	4.15	2.07	2.77	0.70	30
2.5	5.26	6.48	5.73	2.76	1.10	0.75	0.88	12
3.0	4.54	5.43	5.27	1.54	0.50	0.16	0.97	3

They are traced in chart form in Fig. 186.

CHAPTER XVIII

THE JUMP IN A RECTANGULAR CHANNEL

66. Fundamental Relations.—The grapho-analytical method, developed in the preceding chapter, is quite general and can be applied to jumps in a prismatic channel of any form. In certain particular cases, however, the subject can be treated by purely analytical methods. Most important is the case of a rectangular channel.

Consider a discharge Q flowing in a rectangular channel of the width b. Once more assume the bottom to be horizontal.

For a rectangular channel with reference to Eq. (124) we have

$$a = bd;\ z = d/2;\ Q = qb;\ d^3_{cr} = q^2/g$$

Thus

$$M(d) = \frac{Q^2}{gbd} + \frac{bd^2}{2} = b\left(\frac{q^2}{gd} + \frac{d^2}{2}\right).$$

Eq. (125) becomes

$$\frac{q^2}{gd_1} + \frac{d_1^2}{2} = \frac{q^2}{gd_2} + \frac{d_2^2}{2}$$

wherefrom

$$2q^2/g = d_1d_2(d_1 + d_2) \tag{128}$$

The solution of this symmetrical equation is

$$\left.\begin{aligned} d_2 &= \frac{d_1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gd_1^3}}\right] \\ d_1 &= \frac{d_2}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{gd_2^3}}\right] \end{aligned}\right\} \tag{129}$$

Further, by substituting q^2/g out of Eq. (128) into

$$\epsilon_j = \epsilon_1 - \epsilon_2 = (d_1 - d_2) + \frac{q^2}{2g}\left(\frac{1}{d_1^2} - \frac{1}{d_2^2}\right)$$

and properly transforming, one obtains the loss of energy in the jump

$$\epsilon_j = (d_2 - d_1)^3/4d_1d_2 \tag{130}$$

Finally, replacing in Eq. (129) q^2/g by d^3_{cr}, we obtain the equations in form

$$\left.\begin{aligned} d_2 &= \frac{d_1}{2}\left[-1 + \sqrt{1 + 8\left(\frac{d_{cr}}{d_1}\right)^3}\right] \\ d_1 &= \frac{d_2}{2}\left[-1 + \sqrt{1 + 8\left(\frac{d_{cr}}{d_2}\right)^3}\right] \end{aligned}\right\} \tag{131}$$

67. Generalized Form of Equation.—In Art. 27, we introduced the notion of *kineticity of flow*, as measured by a kinetic flow factor

$$\lambda = 2\frac{v^2/2g}{d}$$

For a rectangular channel

$$\lambda = d^3_{cr}/d^3 = q^2/gd^3$$

Accordingly Eq. (129) may be presented as

$$\left.\begin{aligned} d_2/d_1 &= \tfrac{1}{2}[-1 + \sqrt{1 + 8\lambda_1}] \\ d_1/d_2 &= \tfrac{1}{2}[-1 + \sqrt{1 + 8\lambda_2}] \end{aligned}\right\} \tag{132}$$

while in reversed form, the relations between the kineticity before and after the jump are

$$\left.\begin{aligned} \lambda_1 &= \frac{1}{2}\frac{d_2}{d_1}\left(\frac{d_2}{d_1} + 1\right) \\ \lambda_2 &= \frac{1}{2}\frac{d_1}{d_2}\left(\frac{d_1}{d_2} + 1\right) \end{aligned}\right\} \tag{133}$$

The Conjugated Values of λ_1 *and* λ_2.—Substituting (d_2/d_1) from Eq. (132) into

$$\lambda_2 = (d_{cr}/d_2)^3 = (d_{cr}/d_1)^3 \cdot (d_1/d_2)^3 = \lambda_1 \cdot \left(\frac{1}{d_2/d_1}\right)^3$$

and repeating the procedure for $\lambda_1 = \lambda_2 \cdot \left(\frac{1}{d_1/d_2}\right)^3$, we obtain the symmetrical relations

$$\left.\begin{aligned} \lambda_1 &= 8\lambda_2/(-1 + \sqrt{1 + 8\lambda_2})^3 \\ \lambda_2 &= 8\lambda_1/(-1 + \sqrt{1 + 8\lambda_1})^3 \end{aligned}\right\} \tag{134}$$

in which the kinetic flow factors λ_1 and λ_2 represent a pair of conjugated values, corresponding to a pair of conjugated stages d_1 and d_2.

Efficiency of the Jump.—The value of ϵ_2/ϵ_1 features what may be termed the *efficiency of the jump*. In terms of kineticity of flow it may be expressed as follows. We have

$$\epsilon_2/\epsilon_1 = \epsilon_2/d_1 \cdot d_1/\epsilon_1 = \epsilon_2/d_2 \cdot d_2/d_1 \cdot d_1/\epsilon_1$$

Substituting d_2/d_1 from Eq. (132) and taking into account that $\epsilon_1 = d_1\left(1 + \frac{\lambda_1}{2}\right)$, so that $\frac{d_1}{\epsilon_1} = \frac{1}{1 + \frac{\lambda_1}{2}}$, and that, on the other hand,

$$\frac{\epsilon_2}{d_2} = 1 + \frac{\lambda_2}{2} = 1 + \frac{1}{2}\left[\frac{8\lambda_1}{(-1 + \sqrt{1 + 8\lambda_1})^3}\right]$$

we obtain

$$\frac{\epsilon_2}{\epsilon_1} = \left[1 + \frac{4\lambda_1}{(-1 + \sqrt{1 + 8\lambda_1})^3}\right] \left[\frac{1}{2}(-1 + \sqrt{1 + 8\lambda_1})\right]\left[\frac{1}{1 + \frac{\lambda_1}{2}}\right]$$

which after suitable transformations gives

$$\frac{\epsilon_2}{\epsilon_1} = \frac{\frac{1}{2}(-1 + \sqrt{1 + 8\lambda_1}) + \frac{2\lambda_1}{(-1 + \sqrt{1 + 8\lambda_1})^2}}{1 + \frac{1}{2}\lambda_1} \tag{135}$$

The relative loss is $1 - \frac{\epsilon_2}{\epsilon_1}$, where $\frac{\epsilon_2}{\epsilon_1}$ is to be taken from Eq. (135).

Equations (132) to (135) are not limited to any particular circumstances of flow. They are dimensionless equations which apply to jumps in rectangular canals in general, expressing the basic relation between the elements in terms of a generalized dimensionless coordinate, the kinetic flow factor λ. Traced in chart form (Fig. 187) they feature characteristics covering jumps in rectangular canals under all possible conditions.

In the following, the chart (Fig. 187) is used to solve different practical problems.

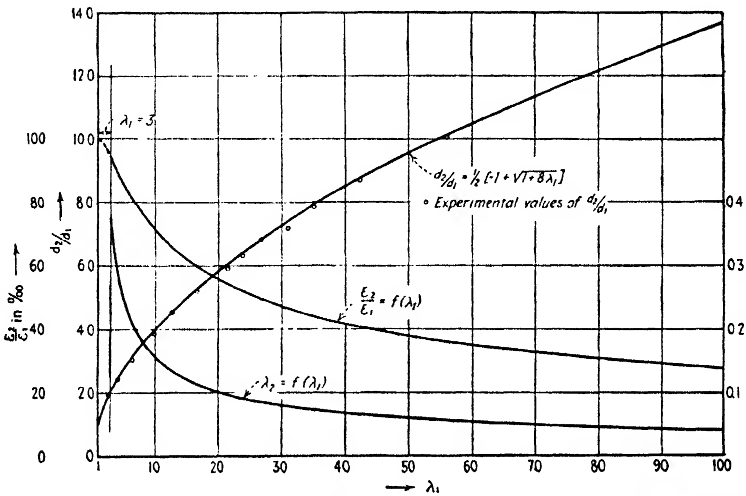

FIG. 187—Generalized characteristics for a jump in rectangular canals in terms of the kinetic flow factor λ_1. Points marked by circles represent experimental values of d_2/d_1 obtained by the author.

Example 33

Assume a flow of 500 cu. ft. per second in a rectangular channel 20 ft. wide.

Question 1. Given $d_1 = 0.8$ ft., determine the conjugated depth d_2 and the relative loss in the jump.

We have

$q = 500/20 = 25$ cu. ft. per second; $y_{cr} = \sqrt[3]{25^2/32.2} = 2.69$ ft. The kinetic flow factor at $d_1 = 0.8$ ft. is

$$\lambda_1 = (2.69/0.8)^3 = 38$$

From the d_2/d_1 curve (Fig. 187), we obtain for $\lambda_1 = 38$ the value of d_2/d_1 as 8.2 and $\epsilon_2/\epsilon_1 = 0.425$. Accordingly we get: $d_2 = 0.8 \times 8.2 = 6.56$ ft. The initial energy $\epsilon_1 = d_1\left(1 + \frac{\lambda_1}{2}\right) = 0.8\left(1 + \frac{38}{2}\right) = 16$ ft. The energy after the jump $\epsilon_2 = 0.425 \times$

16 = 6.8 ft. The loss of energy $\epsilon_j = 16 - 6.8 = 9.2$ ft. or 57.5 per cent of ϵ_1.

Question 2. Determine the depth d_1 which will sustain an upper stage of $d_2 = 5$ ft. We have

$$\lambda_2 = (d_{cr}/d_2)^3 = (2.69/5)^3 = 0.156$$

The corresponding value of $d_2/d_1 = 4.07$; so that

$$d_1 = 5/4.07 = 1.23 \text{ ft.}$$

68. The ϵ_1 Constant Characteristics.—Important features are revealed by determining the characteristics of a jump under conditions that the energy ϵ_1 in the section before the jump remains constant. A comprehensive

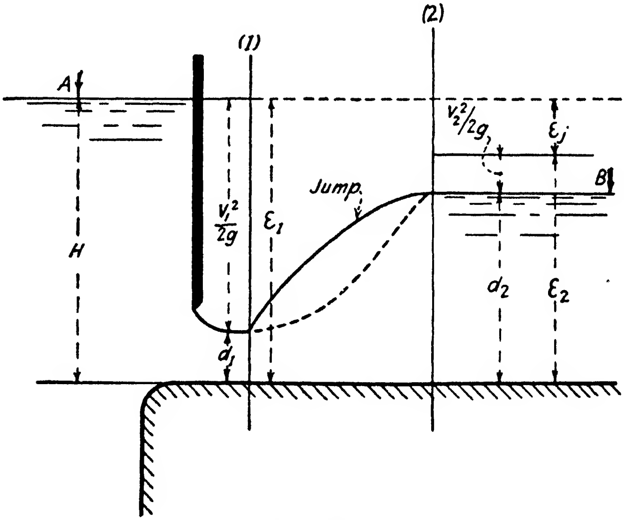

FIG. 188.

approach to this case is gained by considering a jump, sustained by a vein flowing from under a sluice (Fig. 188) under the assumption that level A above the sluice and thus the head H remain constant. If further we were to disregard the friction losses between pool A and section 1, then the energy ϵ_1 in section 1 will remain permanent and equal to $H = d_1 + \dfrac{v_1^2}{2g}$.

By raising or lowering the sluice, we change d_1 and with this the other features of flow. For example, as the sluice is lifted, the velocity v_1 is diminished and the flow becomes less rapid, while the discharge q increases. When d_1 becomes $\frac{2}{3}H$ the limiting condition is reached, corresponding to free efflux over a broad-crested weir (Fig. 189). At such flow $d_1 = d_{cr}$ and the discharge is maximum. Obviously, for $d_1 < \frac{2}{3}H$, flow will be in rapid state.

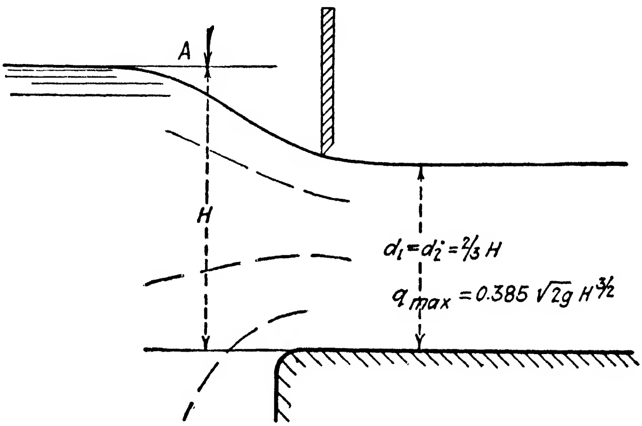

FIG. 189.

Accordingly, for each value of d_1, there is a conjugated depth d_2, to be determined by Eq. (129) or (132). This depth gives the position of the upper stage level B which the outflowing vein is able to sustain by means of a jump of the height $j = d_2 - d_1$.

All the features characterizing the flow may be presented as functions of d_1. Namely

$$\left.\begin{aligned} v_1 &= \sqrt{2g(H - d_1)} = \sqrt{2gH\left(1 - \frac{d_1}{H}\right)} \\ q &= v_1 d_1 = d_1\sqrt{2gH\left(1 - \frac{d_1}{H_1}\right)} \\ \lambda_1 &= 2\frac{v_1^2/2g}{d_1} = 2\frac{H - d_1}{d_1} = 2\frac{1 - \frac{d_1}{H_1}}{d_1/H} \end{aligned}\right\} \quad (136)$$

$$\left.\begin{aligned} d_2 &= \frac{d_1}{2}[-1+\sqrt{1+8\lambda_1}] = \frac{d_1}{2}\left[-1+\sqrt{16\frac{H}{d_1}-15}\right] \\ d_{cr} &= d_1\sqrt[3]{\lambda_1} = d_1\sqrt[3]{2\frac{1-d_1/H}{d_1/H}} \end{aligned}\right\} \quad (136)$$

With d_2 known, we determine $v_2 = q/d_2$ and thus the energy after the jump $\epsilon_2 = d_2 + \frac{v_2^2}{2g}$. The loss of energy $\epsilon_j = \epsilon_1 - \epsilon_2$, is $\epsilon_j = H - \epsilon_2$.

The above relations may be presented in a more general and useful form by introducing what is to be designated as the *reduced values* of the entering parameters. The reduced value of d_1, which is to be the principal coordinate, is the quotient value of $d'_1 = d_1/H$. The reduced values of the other factors may be obtained either directly from Eq. (136) or may be determined by reason of dimensional homogeneity. Thus the reduced value of the velocity is $v'_1 = v_1/\sqrt{H}$; that of the discharge $q' = q/H\sqrt{H}$. The reduced value of the energy $\epsilon'_1 = \epsilon_1/H$ is unity. In terms of $d'_1 = d_1/H$, the reduced values of the different factors, obtained from Eq. (136) are

$$\left.\begin{aligned} v'_1 &= v_1/\sqrt{H} = \sqrt{2g(1-d'_1)} \\ q' &= q/H\sqrt{H} = d'_1\sqrt{2g(1-d'_1)} \\ \lambda_1 &= 2\frac{1-d'_1}{d'_1} \\ d'_{cr} &= \frac{d_{cr}}{H} = d'_1\sqrt[3]{2\frac{1-d'_1}{d'_1}} \\ d'_2 &= \frac{d_2}{H} = \frac{d'_1}{2}\left[-1+\sqrt{\frac{16}{d'_1}-15}\right] \end{aligned}\right\} \quad (137)$$

On the other hand

$$v'_2 = v'_1(d_1/d_2)'$$

$$\epsilon'_2 = \frac{\epsilon_2}{H} = d'_2 + \frac{(v'_2)^2}{2g}.$$

$$\epsilon'_j = \frac{\epsilon_1 - \epsilon_2}{H} = 1 - \epsilon'_2$$

Obviously, the reduced values represent the values of the respective factor at $H = 1$. The following table contains the respective numerical figures and may prove to be useful.

TABLE XLII

d'_1	v'_1	q'	d'_{cr}	λ_1	d'_2	$(v_2^2/2g)'$	ϵ'_2	$1 - \epsilon'_2$
0.010	7.98	0.0798	0.058	198.00	0 195	0 0025	0.197	0 803
0.025	7.91	0.1980	0.107	78 00	0.300	0.007	0.307	0.693
0.050	7.81	0.3900	0.168	38.00	0.411	0.014	0.425	0.575
0.075	7.71	0.5790	0.218	24 70	0.491	0.022	0.513	0 487
0.100	7.61	0.7610	0 262	18 00	0.552	0 029	0.581	0.419
0.150	7.40	1.1100	0.336	11.30	0.642	0.046	0.688	0 312
0.200	7.18	1 4400	0.400	8.00	0.706	0 064	0.770	0.230
0.250	6.94	1.7300	0.455	6.00	0.750	0.083	0.833	0.167
0.300	6.71	2.0200	0.501	4.66	0.780	0.104	0.884	0.116
0.350	6.46	2.2600	0.542	3.73	0.797	0.125	0.922	0.078
0.400	6.22	2.4900	0.577	3.00	0.800	0.150	0.950	0.050
0.450	5.94	2.6800	0 607	2.44	0.797			
0.500	5.67	2.8300	0.630	2.00	0.780			
0.550	5.38	2.9600	0.648	1.64	0.760			
0.600	5.07	3.0400	0.660	1.33	0 726			
0.650	4.74	3.0800	0.665	1.08	0.683			
0 666	4.63	3.0950	0 666	1.00	0.666			

Finally, in Fig. 190, the relations (Eq. [137]) are presented in graphical form. The chart constitutes what we call the *constant* ϵ_1 *characteristics of the jump*. These characteristics will be found useful in many practical applications. Moreover, they reveal certain general properties of the jump. Obviously, Eq. (137) and the chart (Fig. 190) are not limited to the specific way by which the jump was produced in Fig. 188. Once more, and similar to Fig. 187, chart (Fig. 190) is a general characteristic of the jump, presenting the fundamental features of the phenomenon as a function of the rate

$$d'_1 = \frac{\text{potential energy}}{\text{total energy}}$$

of flow in the section before the jump. On the other hand,

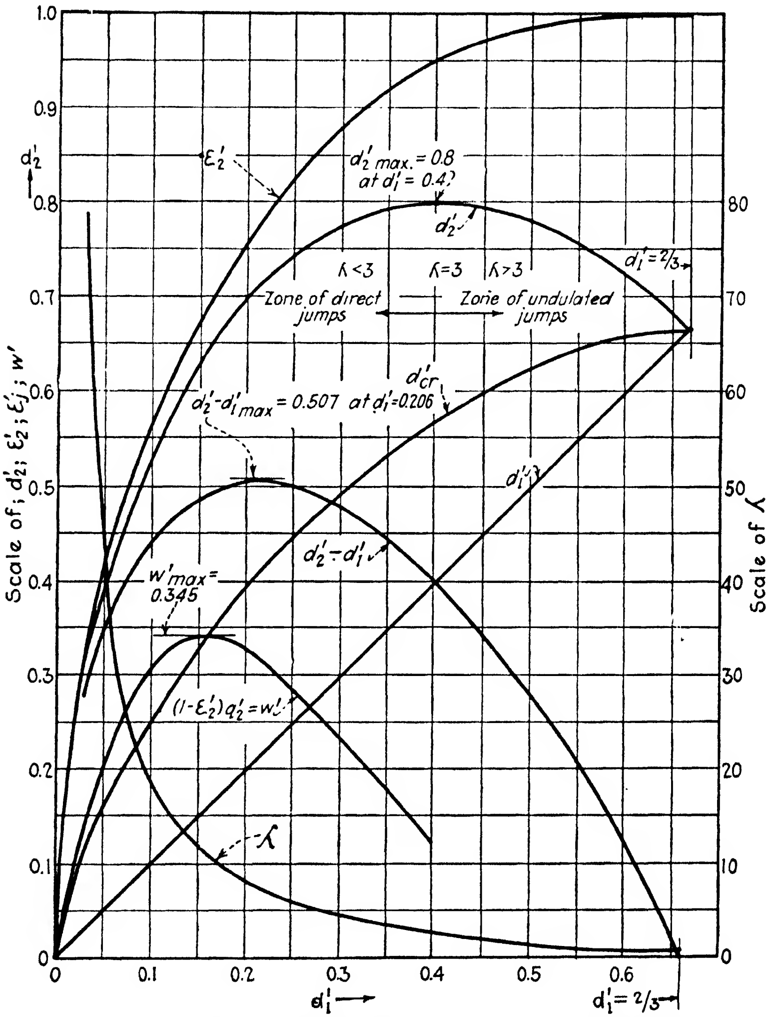

FIG. 190.—The ϵ constant characteristics of a jump in a rectangular canal.

the d'_2 curve, which represents the reduced value of the depth d_2 conjugated to d_1, pictures the rate of

$$\frac{\text{potential energy}}{\text{total energy}}$$

after the jump.

The outline of the d'_2 curve shows that the highest value of $d'_2 = 0.8$ and thus the highest possible position of the upper stage after the jump is reached at $d'_1 = 0.4$. To this, there corresponds $\lambda_1 = 3$ and $\epsilon'_2 = 0.95$. The loss of energy is 5 per cent. For values of d'_1 above 0.4, d'_2 declines and the derivative $\delta d_2/\delta d_1$ becomes negative.

The point of maximum d'_2, corresponding to $d'_1 = 0.4$ is of great physical importance. Remembering that the kineticity at such point is $\lambda_1 = 3$, we shall consider the point $d'_1 = 0.4$ as one which divides all possible conditions under which the jump may take place, into two zones.

The region with $d'_1 \leqslant 0.4$ and $\lambda_1 \geqslant 3$, we shall call the zone of *high* kineticities; and the region with $2/3 > d_1 > 0.4$ and with kineticities $\lambda_1 < 3$ will be that of *low* kineticities.

The distinction introduced seems to have, for example, a direct bearing on the form in which the jump takes place. In fact, experiments carried out by the author show that in the region of high kineticities ($\lambda_1 \geqslant 3$) the jump occurs in the direct form. Moreover, the phenomenon is stable and the conjugated depths observed comply exceedingly well with the theoretical values. On the contrary, in the region of low kineticities ($\lambda_1 < 3$) the jump acquires undulated features. The waves increase as λ decreases. Moreover, as the kineticity is reduced, the phenomenon seems to lose stability. The surface of the outflowing vein becomes pendulating.

69. Experiments on the Jump.—We shall refrain from any detailed description of experiments on the jump, referring the reader to the original monographs.* Speaking

* See bibliographical notes in Appendix.

generally, experiments made on "direct" jumps with high kineticity give results in good agreement with the momentum formula and thus fully justify the approach suggested by Bélanger. In Fig. 187 there are plotted certain results of experiments carried out by the author.* The experimental points follow the theoretical curve most closely. The principal data obtained were as follows:

q, liters per second	d_1 mm.	λ_1	Observed		Calculated d_2/d_1	Deviation, per cent
			d_2	d_2/d_1		
15.60	83.0	4.26	200.0	2.415	2 46	−1.83
12.60	62.5	6.50	189.0	3.03	3.13	−3.19
9.26	44.0	10.00	170.5	3.88	4.00	−3.00
6.64	32.5	12.80	149.0	4.58	4.59	−0.22
5.42	26.0	16.60	135.0	5.20	5.31	−2.08
4.46	21.0	21.50	123.5	5.90	6.08	−2.96
3.32	16.0	26.80	109.0	6.81	6.82	−0.14
3.08	14.5	31 05	104.0	7.18	7.40	−3.14
2.62	12.5	35.10	97.0	7.77	7.89	−1.52
2.06	10.0	42.50	87.0	8.70	8.74	−0.46
1.70	8.0	56.10	80.5	10.05	10.12	−0.70

The kineticity observed was as high as 56.1 resulting in d_2/d_1 of over 10.

The conjugated depths d_2, observed, are somewhat smaller than those derived from the momentum formula; which should be the case, as the reasoning leading to the formula omitted to take into account the outward friction resistance. Results in agreement with the theory were also obtained by Koch, Rehbock, Gibson, and others. One may state that the congruency of experiments and theory for jumps at high kineticity is well established. The author's experiments (as well as most of the more recent work) were made with a jump produced by a vein flowing from under a sluice (Fig. 188) in a canal with horizontal bottom. The author feels confident in stating

* For detailed description see *Ann., Polytech. Inst.*, St. Petersburg, 1912. The flume measured 300 × 100 mm.; q per decimeter width varied from 1.7 to 15.6 liters per second.

that this is by far the best layout for experimenting with the jump. In fact, with the gravity effect eliminated the indeterminate factors are reduced to outward friction only. On the other hand, it is possible to produce flow of any desired rapidity.

As to jumps of small height, obtained under conditions of low kineticities ($\lambda_1 < 3$), the experimental results, so far available, are less satisfactory. In a number of cases, for example, the conjugated depths, reported as observed, proved to be in excess of the theoretical values. The author believes that this was mostly due to defects, inherent in the very method of experimentation. As a matter of fact, in earlier experiments, jumps were usually produced by inserting a barrier into a channel of steep slope, rapid uniform flow being produced by the incline of the flume. Under such circumstances, the kineticity is usually low ($\lambda < 3$) and the jump is of undulated form. The difficulties of measurement under such conditions have already been mentioned. The principal source of error, however, in the author's opinion lies in neglecting the effect of the gravity component (see Fig. 182). The length of the jump at low kineticities is comparatively great. On the other hand, the losses $\epsilon_1 - \epsilon_2$, as shown in Fig. 190 are relatively small; less than 5 per cent. No wonder that the neglected gravity component may outweigh the losses, and lead to observed values of d_2, in excess of theoretical values obtained from the momentum formula.

The small losses inherent in jumps of low kineticity, as evidenced by the chart (Fig. 190), explain also why Bidone (1820) and also Bélanger in his earlier work (1828) felt justified in determining the relation between the conjugated depths simply by using the Bernoulli equation. That meant that the losses in the jump were neglected and ϵ_2 taken to be equal to ϵ_1. Only later, when observations on jumps of greater height evidenced discrepancies of a more substantial character, was Bélanger led to suggest the use of the momentum equation.

To elucidate the effect of the bottom slope the author carried out a special series of experiments, the main results of which were as follows:

s_0 (Bottom slope)	d_1 mm.	d_2 mm.	h_j mm.
0.000	48.5	175.5	127.0
0.002	48.5	177.0	126.8
0.004	48.5	178.5	126.6
0 007	48.5	180.5	126.0
0.010	48.5	183.5	126.7
0.020	48.5	190.5	125.0

In these experiments, the depth d_1 and thus the initial kineticity were kept constant. As the flume was tilted

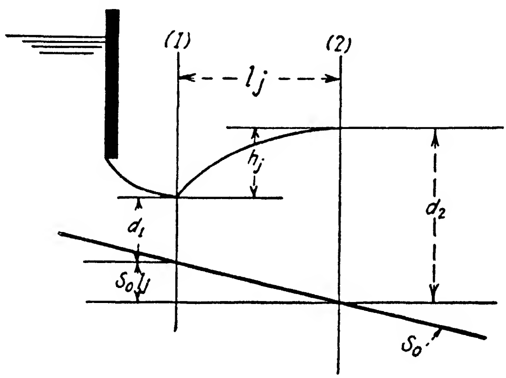

FIG. 191.

and the slope increased, the observed depth d_2 became greater and greater. The interesting part was, on the other hand, that the height of the jump h_j Fig. 191, measured as the vertical distance between the upper and lower level, remained practically without change.

CHAPTER XIX

LOCATION OF THE JUMP

70. The Jump as a Standing Wave.—In Chaps. XVII and XVIII there was determined the relation between the upper and lower stage d_1 and d_2 of the jump. It is incumbent now to establish the location within a stream where the jump

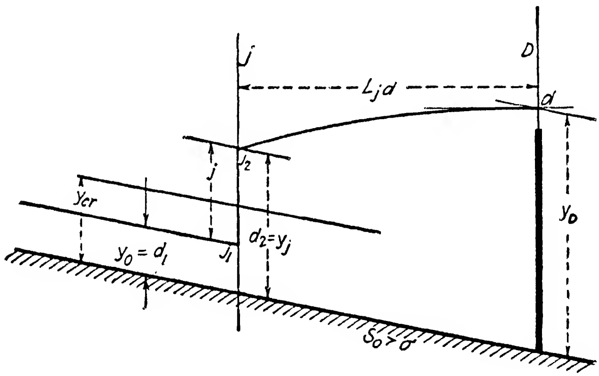

FIG. 192.

actually takes place. For example, in a watercourse with a steep bottom slope (Fig. 192) with a barrier at D, the aim is to determine the distance L_{jd} from the dam to the end of the jump.

Remark: The length of the jump l_i (see Fig. 175) is usually small compared to the length of the adjacent surface curves in gradually varied flow. In fact, the length of the jump will scarcely be noticeable when the longitudinal surface profile will be drawn in the customary reduced scale. Therefore, in Fig. 192 and in the following figures the jump will be schematically indicated by a heavy vertical line, the length of the jump thus being simply neglected.

Another illustrative case is that of a flume (Fig. 193) with a break in the bottom slope; the bottom slope changing in section o from steep ($s_{o1} > \sigma$) to mild ($s_{o2} < \sigma$). With $y_{o1} < y_{cr}$ and $y_{o2} > y_{cr}$ as the respective depths of uniform flow, the transition from rapid flow to tranquil movement must include a jump. It remains to be determined, however, what will actually be the height of the jump and, furthermore, in which part of the flume and in what exact location the jump will take place. In fact the

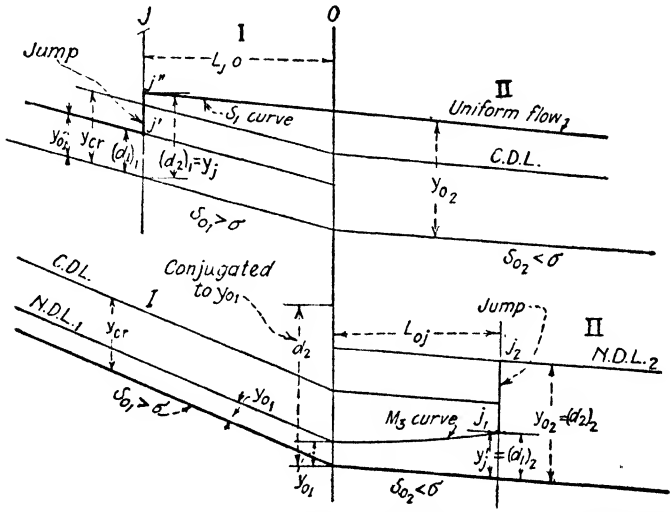

FIG. 193.—Jump in a canal with a break in the bottom slope from steep to mild; (*a*, above) jump in the steep region; (*b*, below) jump in the mild region.

transition may unfold either as shown in Fig. 193*a*, with the jump occurring within the steep section, and with tranquil flow in the form of an S_1 curve extending into the steep section over a distance L_{jo}; or, the alternative is for the jump to take place in the mild section (Fig. 193*b*), in which case rapid flow in the form of an M_3 curve will extend for a distance L_{oj} into the mild portion.

In both cases, locating the jump will mean determining the length L from section 0 to the respective extremity of the jump.

Problems of this nature are greatly facilitated by considering the jump as a *stopped translation wave.* This manner of approach was used by Bazin, although it actually goes back to the earliest experiments of Bidone (1820) who, as mentioned before, produced a jump by inserting a barrier *D* into a channel with previously unrestricted uniform flow in rapid state (Fig. 194). The impediment, lowered into the stream, produces a *surge* in the form of a wave with a steep front, moving upstream. In measure, as the basin back of the barrier fills, the surge moves away from *D*. In such movement upstream, the height of the surge and the velocity w* with which the surge moves relative to the

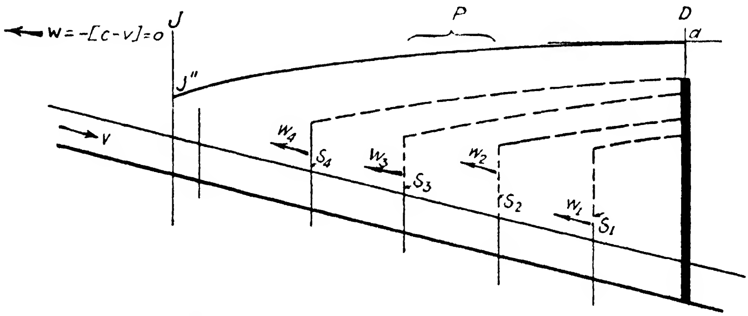

FIG. 194.—The jump as a stopped translation wave.

embankment gradually diminish. In Fig. 194 this is illustrated by the successive positions of the surge. A position of equilibrium is finally reached, at which the volume of water flowing over the barrier is equal to the water flowing into the pool *P* through the face of the surge. Also, the surge in its movement upward has been stopped. The velocity v of the water, flowing down the flume in unrestricted state, counteracts the natural tendency of the surge to move upstream. The surge has been transformed into a *standing wave.* A hydraulic jump has been formed.

71. Celerity of Propagation of a Translatory Surface Wave.—It is essential first to determine the *celerity* with which a surge (Fig. 195*a*) or a solitary wave (Fig. 195*b*) will move along the surface of a resting liquid. May it

* w is negative as the movement is upstream against the positive direction of the X-axis.

be noted that the word "celerity" is purposely used in order to differentiate the speed with which the wave travels in relation to the surface, from the actual velocity with which the liquid particles are animated in their flow through a cross-section.

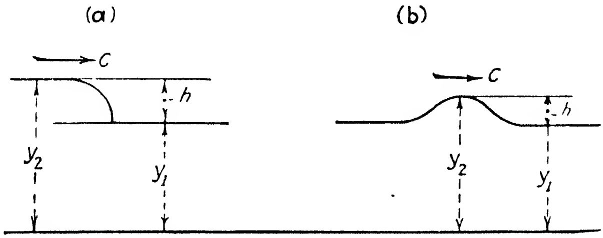

FIG. 195.—(a) A surge. (b) A solitary translation wave.

A most simple and yet elegant method for treating the subject was suggested by St. Vénant (1870), which reasoning we shall apply to a prismatic channel of arbitrary cross-sectional form. With reference to Fig. 196, assume that liquid is at rest in the canal as given, at a depth y_1. Imagine

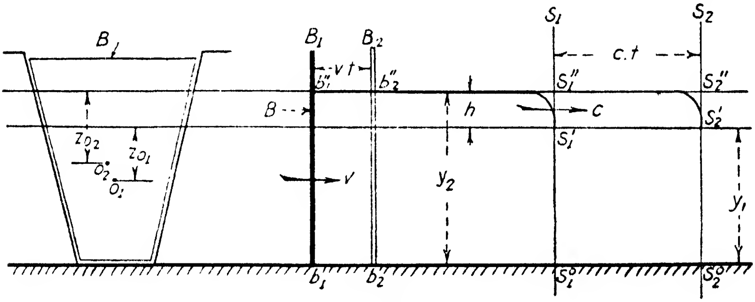

FIG. 196.

further that a water-tight barrier B may be moved canal-lengthwise. The barrier evidently will act as a piston, displacing the liquid in front of it. The displacement of the liquid, actuated by the movement of the barrier, will be accompanied by a formation of a surge of the height h which will be propagated over the surface of the water with a celerity c, different and generally much larger than the velocity of the barrier v, assumed in this analysis to be uniform. Assume, now, that at a certain moment the position of the barrier is B_1, while the toe of the surge is in section S_1. To the right of S_1, the liquid is still undisturbed, while back of the surge the water has been set into motion and is moving together with the piston at a velocity v at a depth $y_2 = y_1 + h$. In fact, the function of the surge, as it is propagated over the resting liquid, is to set such liquid into motion. In a lapse of time t, the barrier will have moved from B_1 to B_2, by a distance $v \cdot t$. At the same time the surge has rolled from S_1 to S_2 over a distance $c \cdot t$. In so doing, it has set into motion with the velocity v, the volume of liquid $s_1^0\, s_1'\, s_2'\, s_2^0$. The relation between the elements of the movement is gained from the following reasoning:

In the first place, the volume of water $b'_1\, b''_1\, b''_2\, b'_2$, displaced by the barrier, is obviously equal to $s_1'\, s_1''\, s_2''\, s_2'$, which is the increase of volume between sections S_1 and S_2 caused by the raising of the level by h. By designating the cross-sectional areas corresponding to the depths y_1 and y_2 by a_1 and by a_2, respectively, we get

$$a_2 \cdot v \cdot t = (a_2 - a_1)c \cdot t$$

wherefrom

$$c = v \cdot \frac{a_2}{a_2 - a_1} \tag{138}$$

Another relation is obtained from the momentum equation. The setting into motion of the volume of liquid $s_1^0\, s_1'\, s_2'\, s_2^0$, equal to $a_1 \cdot c \cdot t$ from a condition of rest into uniform movement with v, corresponds to an increase of momentum equal to $\frac{\Delta}{g} a_1 \cdot c \cdot t \cdot v$. This momentum is produced by the time impulse over a period t of the difference of the hydrodynamic pressures acting across the cross-sections a_1 and a_2 respectively, before and after the surge.

Designating similarly, to Fig. 181, the distances of the respective centers of gravity below the surface of the liquid by z_{01} and z_{02}, and omitting the forces of external friction acting between the walls and the liquid boundary layer, we obtain the difference of the hydrodynamic pressures as $\Delta(a_2 z_{2\,0} - a_1 z_{1\,0})$. The momentum equation reads now:

$$\Delta(a_2 z_{0\,2} - a_1 z_{0\,1})t = \frac{\Delta}{g} a_1 \cdot c \cdot v \cdot t$$

from which we obtain

$$v \cdot c = g \frac{a_2 z_{0\,2} - a_1 z_{0\,1}}{a_1} \tag{139}$$

Eliminating v between Eqs. (139) and (138) we get

$$\frac{c^2}{g} = \frac{a_2(a_2 z_{0\,2} - a_1 z_{0\,1})}{a_1(a_2 - a_1)} \tag{140}$$

Equation (140) determines the celerity of propagation of the surge in a prismatic channel in terms of the initial depth y_1 and the height of the surge $h = y_2 - y_1$.

Celerity c for a Channel of Rectangular Form.—In this instance $a = by$ and $z_0 = y/2$; Eq. (140) becomes

$$\frac{c^2}{g} = \frac{y_2\left(\frac{y_2^2 - y_1^2}{2}\right)}{y_1(y_2 - y_1)} = \frac{y_2}{2y_1}(y_2 + y_1)$$

which after substituting $y_2 = y_1 + h$ becomes

$$\frac{c^2}{g} = \frac{y_1 + h}{2y_1}(2y_1 + h)$$

from which we get

$$c = \sqrt{gy_1}\sqrt{1 + \frac{3}{2}\frac{h}{y_1} + \frac{1}{2}\left(\frac{h}{y_1}\right)^2} \tag{141}$$

In most cases, where the height of the surge is not too large, one may simply put

$$C = \sim\sqrt{gy_1}\sqrt{1 + \frac{3}{2}\frac{h}{y_1}} \tag{142}$$

or with further approximation what we shall call the St. Vénant formula

$$C = \sqrt{gy_1}\left(1 + \frac{3}{4}\frac{h}{y_1}\right) \tag{142a}$$

When the waves are of very small height, so that h/y_1 is small and can be neglected, the St. Vénant expression becomes identical with the well-known *Lagrange formula*

$$c = \sqrt{gy_1} \tag{143}$$

which gives the celerity of propagation of disturbances of small height (ripples) over a deep fluid at rest.

Simplified Formula for Prismatic Channels of Any Form.—In channels other than rectangular, approximate formulas of the type, Eq. (142), may be obtained, applicable in cases when the relative height of the surge h/y is not too large. In fact, with reference to Fig. 197, we may put in such case with sufficient approximation

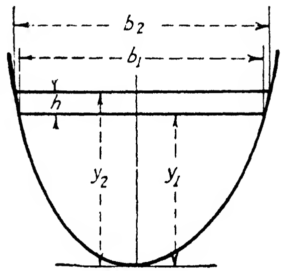

FIG. 197.

$$a_2 = a_1 + b_1h$$

$$a_2z_{20} = a_1z_{10} + a_1h + b\frac{h^2}{2}$$

Substituting into Eq. (140) and developing, we obtain

$$\frac{c^2}{g} = \frac{a}{b} + \frac{3}{2}h + \frac{b}{2a}h^2 = \frac{a}{b}\left(1 + \frac{3}{2}\frac{h}{a/b} + \frac{1}{2}\frac{h^2}{(a/b)^2}\right)$$

The value of a/b is the average depth of a canal (see Eq. [41]). Substituting, we obtain

$$c = \sqrt{g\delta} \cdot \sqrt{1 + \frac{3}{2}\frac{h}{\delta} + \frac{1}{2}\left(\frac{h}{\delta}\right)^2} \tag{144}$$

$$c = \sim\sqrt{g\delta} \cdot \left(1 + \frac{3}{4}\frac{h}{\delta}\right) \tag{145}$$

which for small values of h/δ gives

$$c = \sqrt{g\delta} \tag{146}$$

These expressions are analogous to Eqs. (141) to (143). In fact, for a rectangular canal, the celerity is simply obtained from Eqs. (144) to (146) by making $\delta = a/b = y$. Inasmuch as in a trapezoidal or any other open canal other than rectangular, $\delta = a/b$ is always less than y, the disturbances will be propagated with a celerity smaller than in a rectangular canal of equal depth.

Example 34

Question 1. In a rectangular canal, assume the depth $y_1 = 5$ ft., and the height of the surge to be $h = 0.5$; $h = 1.0$; and $h = 2.5$, respectively.

Compute the celerity by Eq. (141) and compare the results obtained by using the approximate relations Eq. (142).

The basic celerity in accordance with the Lagrange formula is $c = \sqrt{32.2 \times 5} = 12.7$ ft. The value of the multiplier in the different formulas is as follows:

h	$\frac{h}{y}$	$\sqrt{1+\frac{3}{2}\frac{h}{y}+\frac{1}{2}\left(\frac{h}{y}\right)^2}$	$\sqrt{1+\frac{3}{2}\frac{h}{y}}$	$1+\frac{3}{4}\frac{h}{y}$
0.5	0.1	1.073	1.072	1.075
1.0	0.2	1.149	1.140	1.150
2.5	0.5	1.370	1.322	1.375

We see that the St. Vénant formula (Eq. [142]) gives results in best accordance with the more exact Eq. (141).

Question 2. In a trapezoidal canal, Type *B*, with $y_1 = 5$ ft., determine the celerities for a surge height of $h = 0.5$; $h = 1$; and $h = 2.5$. Use Eq. (145). The average depth at $y_1 = 5$ is $\delta = a/b = {}^{50}\!/_{15} = 3.33$. The basic celerity $c = \sqrt{g \times 3.33} = 10.37$ ft.

The value of the multiplier is

h	$\frac{h}{\delta}$	$1+\frac{3}{4}\frac{h}{\delta}$
0.5	0.15	1.125
1.0	0.30	1.225
2.5	0.75	1.560

Question 3. Assume that a canal (Fig. 198) of the cross-section as given connects two reservoirs at a distance of 3 miles. The bottom is horizontal and the water is at rest with $y = 8$ ft. Suppose, at a certain moment, water is begun to be drawn from reservoir *A*, resulting in the lowering of the level. Determine the time it will take for a depression of level *A* to reach the mouth of the canal at *B*, and thus cause water to begin to flow from reservoir *B* into the canal.

A small depression of level in A will be propagated over the canal with the Lagrange velocity of $c = \sqrt{g\delta}$.

We have $a = 8(30 + 16) = 368$; $b = 62$ ft.; $\delta = {}^{368}\!/_{62} = 5.94$ ft., so that

$$c = \sqrt{g \times 5.94} = 13.8 \text{ feet per second}$$

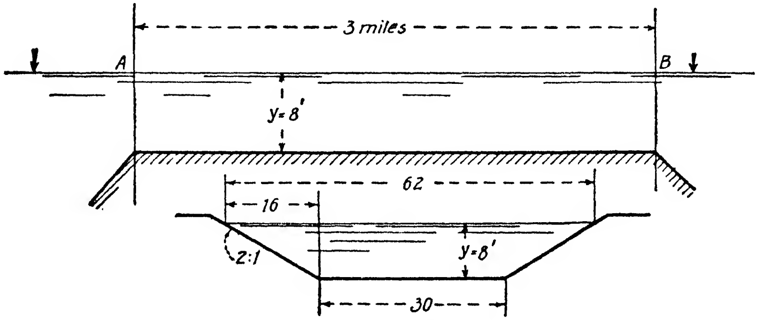

FIG. 198.—Relating to Example 34.

The time in minutes for a disturbance to travel the length of 3 miles

$$T = 3 \times 5{,}280/13.8 \cdot 60 = 19.1 \text{ min.}$$

72. Stopping a Translation Wave.—Assume now, that a surge is moving upstream against the current which flows

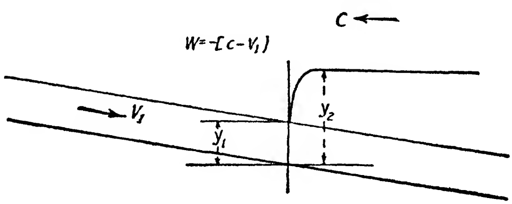

FIG. 199.—Stopping a surge.

in its turn with a velocity v_1 (Fig. 199). The rate w at which the surge will move relatively to the embankment is taken to be the difference between the celerity c and the translatory velocity v_1 of the current. In other words,

$$w = -[c - v_1] \tag{147}$$

As long as the celerity is greater than the velocity of the current, the surge will be moving upward, so that with regard to the X-axis, w will be negative. Should, on the contrary, v_1 be greater than c, w will become positive and the surge will be carried down by the stream. In the event that c and v_1 are equal, the resultant velocity w will be zero. The surge will stop and will form a jump. Using for c Eq. (140), we find that a wave will be stopped when

$$\frac{v_1^2}{g} = \frac{c^2}{g} = \frac{a_2(a_2z_{2\,0} - a_1z_{1\,0})}{a_1(a_2 - a_1)} \tag{148}$$

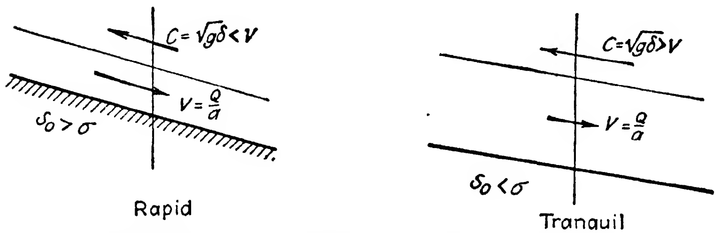

FIG. 200.—Distinction between *tranquil* and *rapid* flow in terms of *celerity* of propagation of surface disturbances.

Multiplying both sides by a_1^2 and remembering that $v_1^2a_1^2 = Q^2$, we obtain

$$\frac{v_1^2a_1^2}{g} = \frac{Q^2}{g} = a_1a_2\frac{a_2z_{2\,0} - a_1z_{1\,0}}{a_2 - a_1}$$

which through proper rearrangement gives

$$\frac{Q^2}{ga_1} + a_1z_{1\,0} = \frac{Q^2}{ga_2} + a_2z_{2\,0} \tag{149}$$

a relation which is identical with the momentum equation (Eq. [123]) determining the conjugated depths before and after the jump. Thus the relations between the hydraulic elements in a stopped translation wave are identical with those which prevail in a jump. The two phenomena are hydraulically equivalent.

Distinction between Rapid and Tranquil Flow in Terms of Celerity (Fig. 200).—We may now introduce, in addition to what was developed in Art. 24, another physical distinction between *rapid* and *tranquil* flow. In fact, a surge or a translation wave will be moving upstream, as long as the celerity is greater than the velocity of the current. The celerity, on the other hand, depends on the height of the wave. The smallest possible celerity in a canal of given form, the *basic celerity*, is the Lagrange celerity $c = \sqrt{g\delta}$.

Let the velocity of the current v be less than $c = \sqrt{g\delta}$. In this case every disturbance, no matter how small, will travel upstream, until in actual movement it will be absorbed by frictional losses. If the velocity of the stream v were, on the contrary, to be larger than $c = \sqrt{g\delta}$, the phenomenon could unfold in two ways. Either, if the surge is of small height, the disturbance will be carried down by the stream; or, if the wave is high enough to make $c = \sqrt{g\delta}\left(1 + \frac{3}{4}\frac{h}{\delta}\right) > v$, the surge will travel upstream with diminishing height, until a point is reached where the celerity will become equal to v and where for this reason the wave will be stopped and a jump formed. In the light of the above, a velocity $[v]$ equal to the Lagrange celerity

$$[v] = c = \sqrt{g\delta} \tag{150}$$

divides the possible phenomena into two classes. By comparing Eq. (150) with Eq. (38), we see that the dividing velocity $[v]$ is the critical velocity, corresponding to critical flow.

Accordingly, the following distinction between *rapid* and *tranquil* flow may be made (Boussinesq):

In *tranquil* movement, with $v < [v] = \sqrt{g\delta}$, the celerity is always greater than the velocity of the current, so an intumescence, no matter how small, will travel upstream indefinitely.

In *rapid* movement, with $v > [v] = \sqrt{g\delta}$, an intumescence, if of sufficient height, will be stopped and will form a jump. Otherwise, if the intumescence is not sufficiently high, it will be carried down by the stream.

The process underlying the formation of backwater curves in watercourses becomes explainable. In a canal of mild slope, with normal flow in tranquil state, the surge created by a dam through the piling up of the water back of the barrier will travel upward indefinitely, progressively diminishing in height and connecting asymptotically with the level of the unimpaired watercourse. In the case of a channel with steep slope, with uniform flow in rapid state, the surge produced by a barrier of sufficient height will proceed upward until a point where, by reason of diminished height, it will be stopped forming a jump.

Relation between the Celerity of Propagation and the Kineticity of Flow.—An interesting relation prevails between the celerity of propagation and the kinetic flow factor. We shall limit ourselves to the case of a rectangular canal.

In this case the celerity for a small intumescence is $c = \sqrt{gy}$, while the kinetic flow factor is

$$\lambda = 2\frac{v^2/2g}{y}$$

Eliminating y between the above, we obtain:

$$\lambda = v^2/c^2 \tag{151}$$

Thus the kineticity is the square of the quotient of the average velocity of flow by the celerity of propagation of a small intumescence. In critical state, with $[v] = c$, Eq. (150) gives $\lambda = 1$; in tranquil movement, with $[v] < c$, the kineticity is $\lambda < 1$. The speed with which a small intumescence will travel, relative to the embankment, is

$$w = -[c - v] = -c[1 - \sqrt{\lambda}] = -v\left[\frac{1}{\sqrt{\lambda}} - 1\right] \tag{152}$$

73. Locating the Jump.—With these conceptions in mind, it will be easy to locate the jump. The process of reasoning will be best shown in connection with concrete cases.

A. Weir in a Torrential Watercourse (Fig. 192).—Assume that the discharge is Q, and that the depth of uniform movement in rapid flow is y_0. A weir is located in D, raising the level to a stage represented by y_D.

The problem is to establish the type of the phenomenon and, in the event that a jump is to take place, to determine its location and its height.

Proceed, first, by computing the conjugated depth d_2, corresponding to $d_1 = y_0$.

1. $d_2 < y_D$.—Suppose that d_2 as determined is less than y_D, which is the usual case. This means that the discharge as given, when flowing at a normal depth y_0, may stop a surge of the depth d_2, which depth is smaller than the depth corresponding to the level y_D caused by the dam. Consequently, the level caused by the dam will move upstream with depth diminishing until section J is reached, in which the depth y_j on the surface curve of the S_1 type will be equal to d_2, as above determined. y_j being equal to d_2 means that the celerity at the upper stage, corresponding to y_j, is equal to the velocity of the current and that the translation wave will be stopped.

The location of the jump will be determined by finding the position of section J with $y_j = d_2$. This is done by computing the length of the curve L_{jD} between the stages y_D and y_j. The procedure is given in Example 13, Question 2.

2. $d_2 > y_D$.—Suppose now that d_2 as determined proves to be greater than y_D. This will mean that the current in uniform flow is able to sustain a wave of greater height than the one caused by the barrier. In other words, the celerity of the surge is smaller than the velocity of the current. The rise produced by the dam cannot proceed upstream. The intumescence caused by the barrier will be swept downstream, with the current crossing the weir by means of a *standing swell*, as described in Art. 25.

B. Channel with a Break in the Bottom Slope.—With reference to Fig. 193, the question, as stated before, is to determine in the first place, as to whether the jump will take place within the steep or within the mild section. In the following, it is assumed that the lengths of the steep as well as of the mild section are sufficiently long for uniform movement to install themselves. The *normal* depths are respectively y_{01} and y_{02}.

To solve the problem, start by determining the depth d_2 conjugated to y_{01}. In other words find the upper stage of the standing wave which the rapid current flowing in the steep section with a depth $y_{01} = d_1$ is able to sustain. Compare, then, the conjugated depth d_2 as found, with the depth of uniform flow y_{02} in the mild section. Two cases are possible.

1. $d_2 > y_{02}$ (Fig. 193*b*).—When d_2 proves to be greater than y_{02}, it means that the velocity of the rapid movement in the steep section exceeds the celerity of the surge, corresponding to the uniform depth y_{02}. So, in the dividing section 0, the level y_{02} will not be able to counteract the current and will be repelled. The rapid flow will extend into the mild section, by means of an M_3 type curve, until in section J, the depth y'_j is reached, equal to the depth $(d_1)_2$ conjugated to the uniform depth $y_{02} = (d_2)_2$. To locate the jump, determine the depth $(d_1)_2$ conjugated to the given y_{02}, and then compute the length L_{0j} of an M_3 curve between the stages y_{01} and $y'_j = (d_1)_2$.

2. $d_2 < y_{02}$ (Fig. 193*a*).—This means that the depth y_{02}, which confronts the rapid current in section 0 is greater than the stage d_2, which the current flowing in rapid state at y_{01} is able to sustain. The level of tranquil flow in reach II will move in such case upstream, extending into the steep section until in section J the depth y_j is reached, equal to d_2, conjugated to $d_1 = y_{01}$.

To locate the jump determine $(d_2)_1$ conjugated to $(d_1)_1 = y_{01}$. Then compute the length L_{j0} of the S_1 curve between the stages y_j and y_{02}.

Example 35

A canal, Type *B*, is laid with a break in the bottom slope (Fig. 201). The bottom slope in the mild section is $s_{02} = 8^0\%_0$. The slope of the steep section is $s_{01} = 50^0\%_0$ and $s_{01} = 200^0\%_0$, respectively. The discharge is $Q = 300$ cu. ft. per second. Use Bazin coefficients.

Question 1. Determine in both cases the type of movement and the locus of the jump.

With $Q = 300$ cu. ft. per second, we have for uniform flow (see Fig. 15),

$s_0 = 8\,^0\!/_{00}$; $\mathfrak{K} = Q/\sqrt{s} = 106 \times 10^2$ cu. ft. per second; and $y_{02} = 4.95$.

$s_0 = 50\,^0\!/_{00}$; $\mathfrak{K} = Q/\sqrt{s} = 42.4 \times 10^2$ cu. ft. per second; and $y_{01} = 3.05$.

$s_0 = 200\,^0\!/_{00}$; $\mathfrak{K} = Q/\sqrt{s} = 21.2 \times 10^2$ cu. ft. per second; and $y_{01} = 2.05$.

The critical depth is $y_{cr} = 3.74$ ft.

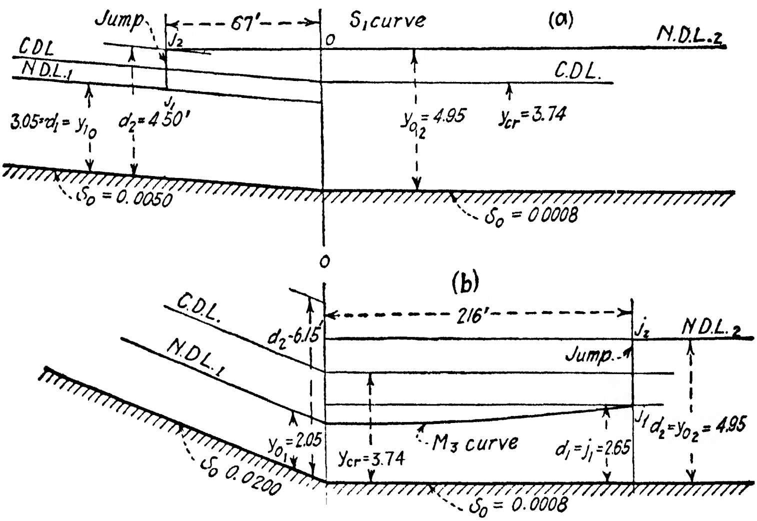

FIG. 201.—Relating to Example 35.

From the $M(d)$ curve (Fig. 185 or 186), we find the conjugated depths d_2, corresponding to the respective uniform depths in the steep section:

Case I (Fig. 201*a*): $s_{01} = 50\,^0\!/_{00}$; $y_{01} = d_1 = 3.05$ ft.; conjugated depth $d_2 = 4.50$ ft.

Case II (Fig. 201*b*): $s_{01} = 200\,^0\!/_{00}$; $y_{01} = d_1 = 2.05$; conjugated depth $d_2 = 6.15$ ft.

The depth $(d_1)_2$ conjugated to $y_{02} = (d_2)_2 = 4.95$ is $d_1 = 2.65$.

In Case I, the conjugated depth $d_2 = 4.50$ is less than $y_{02} = 4.95$ ft. Hence, the jump will be within the steep section as in Fig. 193*a*.

In Case II, $d_2 = 6.15$ is greater than $y_{02} = 4.95$ ft. Hence, the jump will be in the mild section as in Fig. 193*b*.

To locate the jump in the case when $s_{01} = 50^0\!/_{00}$, L_{j0} (Fig. 193*a*) is the length of an S_1 curve between $d_2 = 4.50$ and $y_{02} = 4.95$ ft.

For the *hydraulic exponent* for the region $y = 4.5$ to 5 ft. with $y_0 = 3.05$ we have from Table I

$$\mathfrak{K}(5)/\mathfrak{K}(3) = 108.8/41.65 = (5/3)^{n/2}$$

from which

$$n = 2\frac{Lg2.61}{Lg1.666} = 3.74$$

An identical value corresponds to

$$\mathfrak{K}(4.5)/\mathfrak{K}(3.0) = (4.5/3)^{n/2}$$

$1 - \beta$ *Value.*—The average value of σ for the range of depths is $\sigma = 22.7^0\!/_{00}$. Thus,

$$\beta = s_0/\sigma = 50/22.7 = 2.2;\ 1 - \beta = -1.2$$

Interpolating the values of $B(\eta)$ between the table lines for $n = 3.6$ and $n = 3.8$, respectively, we have with $n = 3.74$:

$$\eta_2 = 4.95/3.05 = 1.623;\ B(\eta_2) = 0.1047;\ \Pi_2 = 1.623 - (-1.2) \times 0.1047 = 1.7488$$

$$\eta_1 = 4.50/3.05 = 1.470;\ B(\eta_2) = 0.146;\ \Pi_1 = 1.470 - (-1.2)0.146 = 1.645$$

The distance L_{j0}, from section 0 to the end of the jump at $y_j = d_2 = 4.50$ is

$$L_{j0} = 3.05/50 \cdot 10^{-4}[1.7488 - 1.6450] = 610 \times 0.1038 = 67 \text{ ft.}$$

To locate the jump in the case of $s_{02} = 200^0\!/_{00}$ *we have:* L_{0j} (Fig. 193*b*) is the length of an M_3 curve between $y_{01} = 2.05$ and $y_j = (d_1)_2 = 2.65$ ft.

Hydraulic Exponent.—For the range of depth between $y = 2.05$ and 2.65 with $y_0 = 4.95$, as an average value,

$$n = 2\frac{Lg\dfrac{\mathfrak{K}(5)}{\mathfrak{K}(2.5)}}{Lg\dfrac{5}{2.5}} = 3.70$$

$1 - \beta$ *Value.* σ for the range $= 23.5^0\!/_{00}$; $\beta = 8/23.5 = 0.34$; $1 - \beta = 0.66$. Interpolating table values of $B(\eta)$ we have with $n = 3.70$

$\eta_2 = 2.65/4.95 = 0.535$; $B(\eta) = 0.547$; $\Pi_2 = 0.535 - 0.66 \times 0.547 = 0.174$

$\eta_1 = 2.05/4.95 = 0.415$; $B(\eta) = 0.418$; $\Pi_1 = 0.415 - 0.66 \times 0.418 = 0.139$

The distance L_{0j} from section 0 to the toe of the jump

$$L_{0j} = \frac{4.95}{8 \cdot 10^{-4}}[0.174 - 0.139] = 6{,}190 \times 0.035 = 216 \text{ ft.}$$

CHAPTER XX

THE JUMP BELOW A REGULATING SLUICE

There are a number of important cases in engineering practice where chart (Fig. 190) and the Eq. (137) may be usefully applied.

74. The Effective Head.—When using the chart and computing the *reduced values* it will be necessary to refer the depth d_1 and the other elements of flow to the *effective*

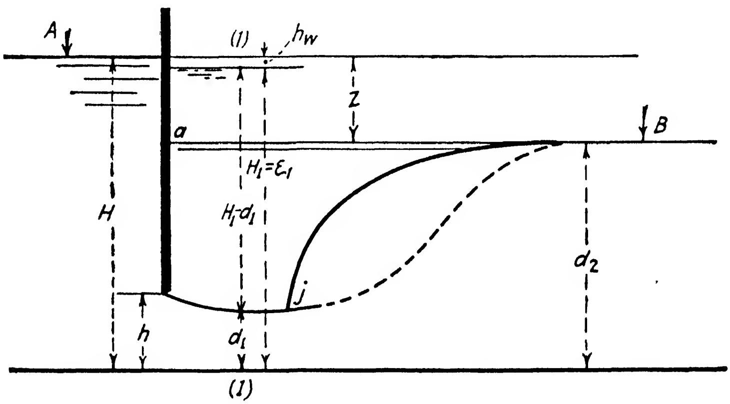

FIG. 202.—The *effective* head.

head which is the total energy head H_1 in section 1 of the outflowing vein (Fig. 202). This effective head is less than the head H by the loss h_w.

With φ, the velocity coefficient, determining $v_1 = \varphi\sqrt{2g(H - d_1)}$, the effective head is

$$H_1 = d_1 + \frac{v_1^2}{2g} = d_1 + \varphi^2(H - d_1) = H\left[\varphi^2 + \frac{d_1}{H}(1 - \varphi^2)\right] = \vartheta H \quad (153)$$

and the loss

$$h_w = H - H_1 = (1 - \varphi^2)(H - d_1) = (1 - \vartheta)H$$

For convenience sake, the value of

$$\vartheta = \varphi^2 + \frac{d_1}{H}(1 - \varphi^2) \tag{154}$$

is given in chart form (Fig. 203).

75. Free or Submerged Efflux.—In Fig. 202 assume as given: the position of level A above, and that of level B below the sluice; the opening of the gate h and a contraction coefficient α which determines the depth of the outflowing vein, $d_1 = \alpha h$.

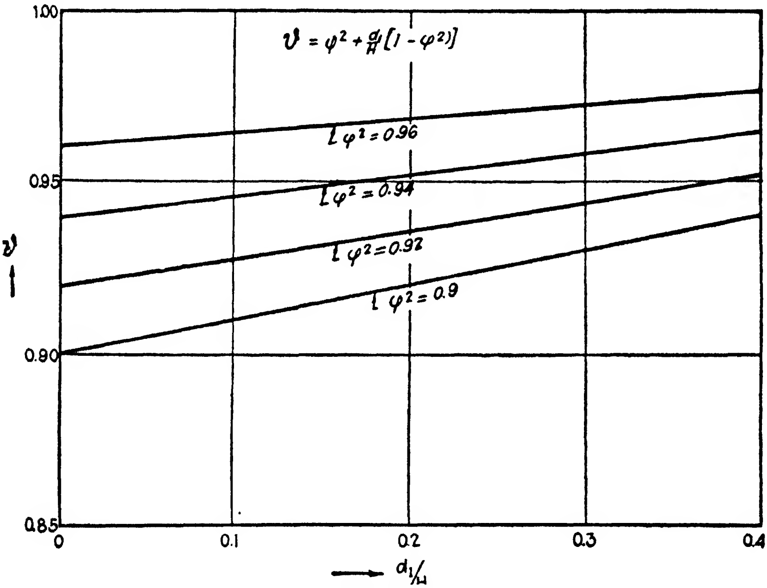

FIG. 203.

Depending on circumstances, the efflux may either be "submerged," with the tail-water level extending back to the sluice and drowning the outflowing vein ($B - a$ in Fig. 202). Or, if level B is sufficiently low, the outflowing stream may repel the tail-water, making the efflux free. The freely outflowing vein in this case will connect with level B by means of a jump j-B.

In submerged flow, the discharge will be actuated by the step Z, which is the vertical distance between levels A and B.

In case of free outflow, the discharge will be actuated by the head $H - d_1$; it will be considerably greater. Also the velocities and the erosive action of the vein will substantially increase.

To establish the type of flow, first determine the depth d_2 conjugated to the depth d_1, assuming that the outflow is free.

1. $d_2 < y_B$.—If the conjugated depth d_2, as determined, is smaller than the depth y_B, corresponding to the position of level B, the flow will be submerged.

2. $d_2 > y_B$.—If the conjugated depth d_2, as determined, is greater than y_B, a jump will be formed and the flow will be free. Usually the jump will be of maximum possible height when $y_B = d_2$, that is, when the toe of the jump is located at the *vena contracta*. In case $d_2 > y_B$ the jump will be repelled stream downwards.

Example 36

Assume a rectangular sluice opening with $H = 10$ ft. and $h = 4$ ft. (Fig. 204). Let the contraction coefficient be $\alpha = 0.62$ and $\varphi^2 = 0.92$.

Question 1. Assume stage B to be $y_B = 7.5$ ft. and $y_B = 6.5$ ft., respectively. Determine in each case the type of efflux and the discharge.

The depth $d_1 = \alpha h = 0.62 \times 4 = 2.48$ ft.; $d_1/H = 2.48/10 = \sim 0.25$. Accordingly, from Fig. 203, with $\varphi^2 = 0.92$, $\vartheta = 0.94$, and the effective head $H_1 = 9.4$ ft. The reduced value of d_1, referred to H_1 is $d'_1 = 2.48/9.4 = 0.264$. From chart (Fig. 190), the conjugated depth $d'_2 = 0.755$. Accordingly $d_2 = 0.755 \times 9.4 = 7.1$ ft.

With $y_B = 7.5$ ft., we thus have $y_B > d_2$. The flow is submerged. The step

$$Z = 10 - 7.5 = 2.5; q = h \cdot \alpha \cdot \varphi \cdot \sqrt{2gZ} = 0.6 \times 4 \times \sqrt{2g \cdot 2.5} = 30.4 \text{ cu. ft. per second.}$$

In case $y_B = 6.5$ ft., $y_B < d_2$. The flow is free. The head actuating the discharge is $H - d_1 = 10 - 2.48 = 7.52$.

$$q = \varphi \cdot d_1 \cdot \sqrt{2g(H - d_1)} = 0.96 \times 2.48 \times \sqrt{2g \times 7.52} = 52.8 \text{ cu. ft. per second}$$

Question 2. Determine the highest possible stage, consistent with free efflux.

The maximum stage is $y_B = d_2 = 7.1$ ft. as above determined. For $y_B \leqslant 7.1$ ft., the discharge will remain constant and equal to 52.8 cu. ft. per second.

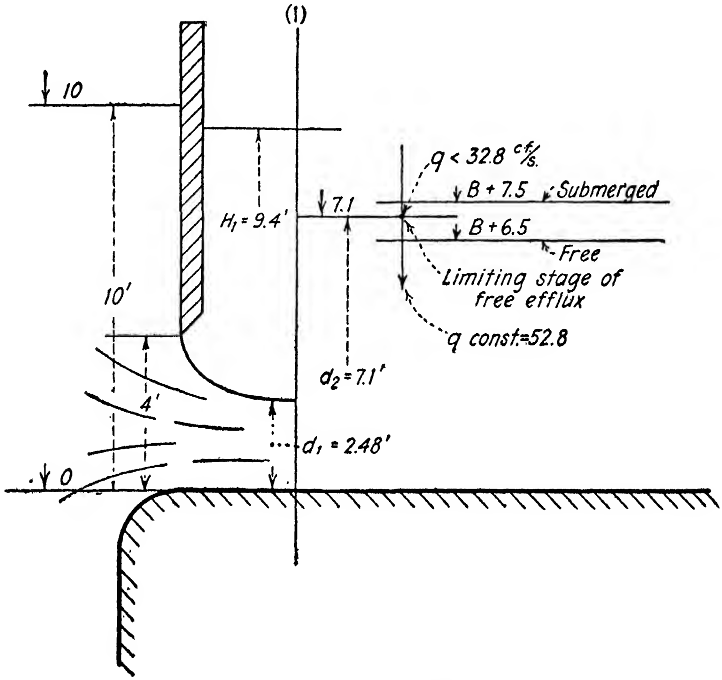

FIG. 204.—Relating to Example 36.

As soon as level B overreaches $y_B = 7.1$, the tail-water will drown the outflowing vein. The discharge will drop abruptly to below the limiting figure of

$$q = 0.6 \times 4\sqrt{2g(10 - 7.1)} = 32.8 \text{ cu. ft. per second.}$$

76. Flow in a Canal below a Regulating Sluice.—In the preceding, the tail-water stage B was assumed as given. Such, for example, would be the case if the sluice were to be located between two reservoirs (Fig. 205) with only a short connecting flume between the two bodies of water. In this case the natural stage below the sluice would be the depth, corresponding to the level in reservoir B.

Whenever the canal below the sluice is of appreciable length, the stage below the sluice is influenced by the circumstances of the flow in the canal. Evidently, in such case the efflux from under the sluice and the gradually varied movement in the canal become organically interconnected.

The reasoning to be applied in problems of this kind is here to be illustrated by considering the case of a canal of mild bottom slope ending with a fall (Fig. 206). In the following, it is assumed that the streaming over the fall is unhampered. Generally speaking, the type of movement

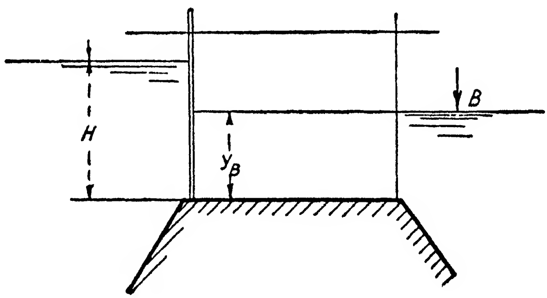

FIG. 205.

will depend on the length of the canal and on its slope. For example, in a short canal (Fig. 206/I) the streaming may continue to be rapid over the entire course of the flume, the surface 1–*f* being a rising curve of the M_3 type, which reaches the crest of the fall and leaps over it before having attained the critical depth. In Fig. 206/II, on the other hand, the canal is long enough for uniform movement to establish itself. The picture as drawn corresponds to free efflux. The stream is rapid until section *J* connects by means of a jump with the uniform movement stage $y_0 = d_2$.

Figure 206/III represents an intermediary case. The flow between *A* and *J* is rapid. Beyond the jump the surface is a falling curve j''–*c* of the M_2 type.

In all the above cases the efflux was pictured to be free. Now, in Figs. 206/II and 206/III, the flow could be submerged, the level of tranquil flow reaching back to the sluice and drowning the vein (shown in dotted lines). To establish the type of efflux, assume the outflow free with a

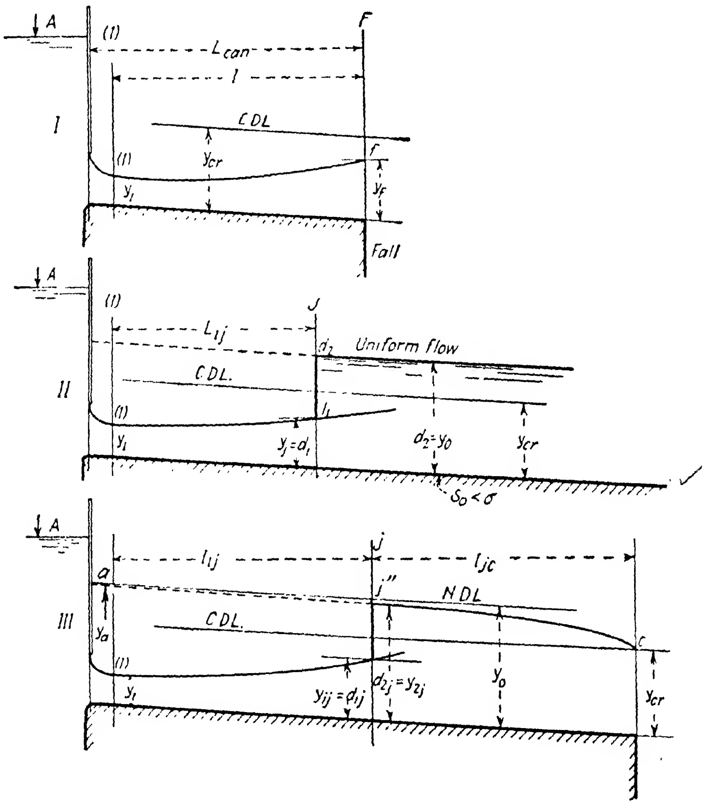

FIG. 206.—Flow in a canal below a regulating sluice.

discharge Q_f and determine the depth d_2 conjugated to the depth y_1 of the outflowing vein. Next, compute the depth y_a in the section immediately below the sluice, for the free discharge Q_f, assuming that the depth over the fall is y_{cr} (Fig. 206/III). If y_a, thus determined, is less than d_2, the efflux will be free. If, on the contrary, y_a is greater than d_2, the outflow will be submerged.

Example 37

A sluice opens into a cement-lined canal of rectangular shape (Fig. 207) Type C (see Plate IV). $H = 10$ ft.; the opening of the sluice is $h = 5$ ft.; $\alpha = 0.62$; $\varphi^2 = 0.90$ ($\varphi = 0.95$); $d_1 = 5 \times 0.62 = 3.10$; $q = 0.95 \times 3.10 \times \sqrt{2g(10 - 3.1)} = 62$ cu. ft. per second; $Q = 62 \times 20 = 1{,}240$ cu. ft. per second.

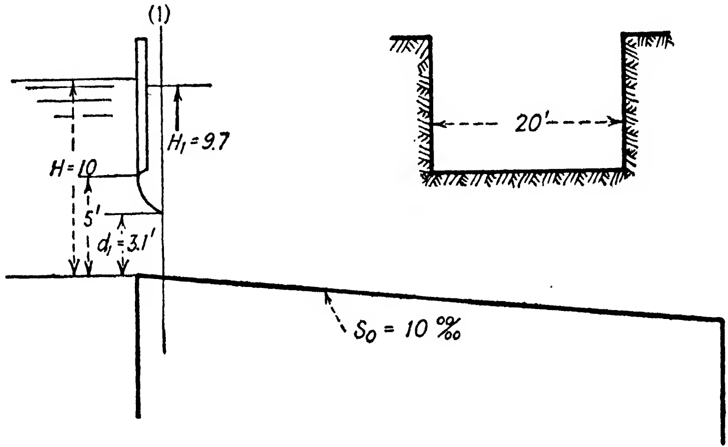

FIG. 207.

Question 1. Given the length of the canal $L = 300$ ft., and $s_0 = 10$ ‰, determine the type of flow.

The canal being short, the movement may be as shown in Fig. 206/I. To establish the type of flow, determine:

The normal depth y_0.

$$\mathfrak{K}_0 = Q/\sqrt{s_0} = 1{,}240/\sqrt{10} \cdot 10^{-2} = 392 \times 10^2 \text{ cu. ft. per second}$$

and $y_0 = 6.8$ ft.

The critical depth

$$y_{cr} = \sqrt[3]{q^2/g} = \sqrt[3]{62^2/g} = 4.93$$

To find, whether flow is rapid over the whole course of the canal, determine (Fig. 208) the length of an M_3 curve between the depths $y_1 = 3.1$ ft. and $y_{cr} = 4.93$ ft.

We shall use the varied-flow-function tables with $n = 2.8$. The critical slope, for $y = 5$ ft. and $y = 3$ ft., is respectively (see Plate IV) $\sigma = 24.9$ ‰ and 23.5 ‰. The values of $\beta =$

10/24.9 = 0.401 and 10/23.5 = 0.425; $1 - \beta = 0.599$ and 0.575, with an assumed average $1 - \beta = 0.59$. We have now:

$$\eta_1 = 3.10/6.8 = 0.456; B(\eta_1) = 0.4700; \Pi_1 = 0.456 - 0.59 \times 0.470 = 0.178$$

$$\eta_2 = 4.93/6.8 = 0.725;\ B(\eta_2) = 0.8285;\ \Pi_2 = 0.725 - 0.59 \times 0.8285 = 0.237$$

The length of the curve

$$L = \frac{6.80}{0.001}[0.237 - 0.178] = 6{,}800 \times 0.059 = 401 \text{ ft.}$$

This is substantially longer than the given length of the canal, 300 ft. So, the streaming is of the type, Fig. 206/I as assumed.

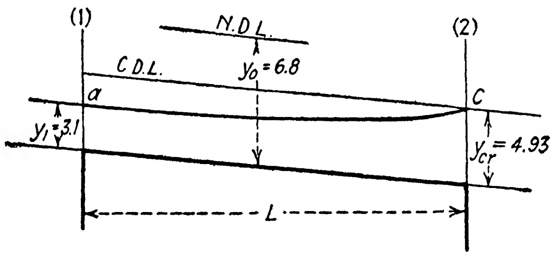

FIG. 208.—Curve for Example 37, question 1.

Question 2. Determine the type of flow and the locus of the jump if any, in case the canal were 5,000 ft. long.

The canal appears to be long enough to make the tail-water stage below the sluice close to the normal depth, so that in Fig.

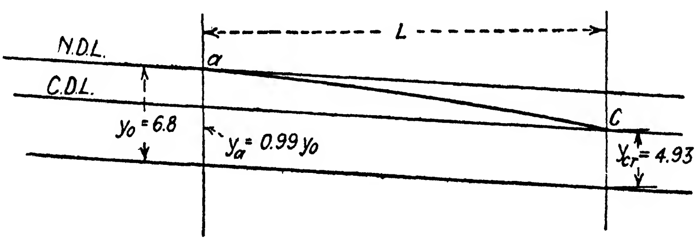

FIG. 209.

206 y_a is y_0. To prove whether the assumption is correct find the length of an M_2 drop curve (Fig. 209) between the critical depth $y_2 = y_{cr} = 4.93$, and the end of the curve assumed to be at $y_a = 0.99 \times y_0 = 6.73$ ft.

For the $1 - \beta$ values in the depth interval we have:

$$y = 5 \text{ ft.}; \sigma = 24.9; \beta = 0.401; 1 - \beta = 0.599$$
$$y = 7 \text{ ft.}; \sigma = 26.6; \beta = 0.376; 1 - \beta = 0.624$$

An average value of $1 - \beta = 0.615$ will be assumed.

To figure the length of the curve with $n = 2.8$:

$$\eta_2 = 4.93/6.80 = 0.725; B(\eta_2) = 0.8285; \Pi_2 = 0.725 - 0.615 \times 0.8285 = 0.216$$

$$\eta_1 = 0.99; B(\eta_1) = 2.106; \Pi_1 = 0.99 - 0.615 \times 2.106 = -0.307$$
$$L = 6{,}800\ [0.216 - (-0.307)] = 6{,}800 \times 0.523 = 3{,}560 \text{ ft.}$$

This length is less than $L_{can} = 5{,}000$ ft.; thus $y_a \sim y_0 = 6.8$ ft.

To determine the type of flow, compute the depth d_2, conjugated to $d_1 = 3.1$ in the *vena contracta*.

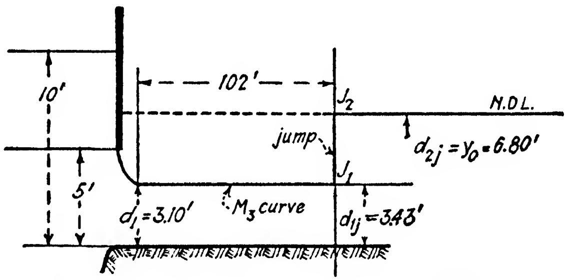

FIG. 210.—M_3 curve in Example 37, question 2.

With $\varphi^2 = 0.9$ and $d_1/H = 0.31$, we have (Fig. 203) $\vartheta = 0.93$, and thus $H_1 = 9.3$ ft.

The reduced value of $d'_1 = 3.1/9.3 = 0.333$. The reduced conjugated depth (Fig. 190) $d'_2 = 0.785$, so that $d_2 = 0.785 \times 9.3 = 7.3$ ft. With $d_2 > y_a = 6.8$ ft., the flow is free with a repelled jump.

To locate the jump (Fig. 210), determine first the depth $y_j = d_{1j}$, conjugated through the jump to $d_{2,j} = y_0 = 6.8$. Applying Eq. (131) we have

$$d_1 = \frac{d_2}{2}\left[-1 + \sqrt{1 + 8\left(\frac{d_{cr}}{d_2}\right)^3}\right] = 3.4\left[-1 + \sqrt{1 + 8\left(\frac{4.93}{6.80}\right)^3}\right] = 3.43$$

The locus of the jump is determined by the length of the M_3 curve (Fig. 206/II) between $y_1 = 3.1$ and $y_{1j} = 3.43$.

Taking $1 - \beta = 0.58$, we have, with $n = 2.8$:

$\eta_1 = 3.10/6.80 = 0.456$; $B(\eta_1) = 0.470$; $\Pi_1 = 0.456 - 0.58 \times 0.470 = 0.184$

$\eta_2 = 3.43/6.80 = 0.505$; $B(\eta_2) = 0.527$; $\Pi_2 = 0.505 - 0.58 \times 0.527 = 0.199$

$$L_{1j} = 6{,}800(0.199 - 0.184) = 102 \text{ ft. (Fig. 210).}$$

Question 3. Determine the type of flow and the locus of the jump in case the canal length were $L = 1{,}000$ ft. With the canal

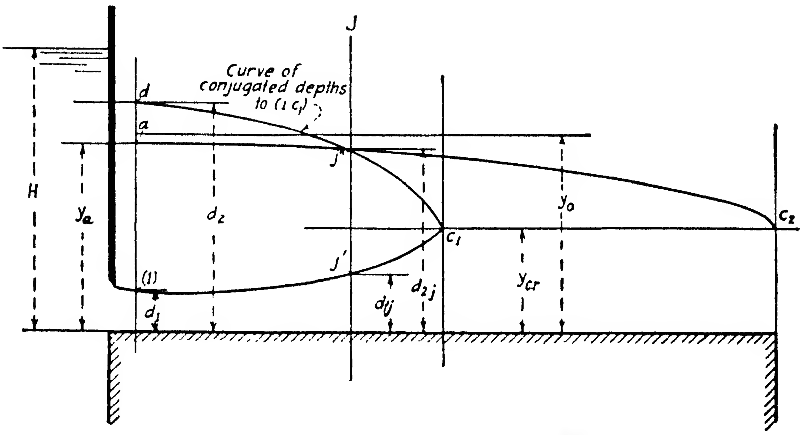

FIG. 211.—Locating the jump in the case of Fig. 206, III.

length intermediary between 401 ft. (Question 1) and 3,560 ft. (Question 2), the flow will be of the type as shown in Fig. 206/III. To locate the jump proceed in general as follows (Fig. 211).

Draw the M_3 curve (shown as $1\text{-}c_1$) corresponding to free rapid flow from d_1 to y_{cr} in section C_1. Draw the M_2 drop curve (shown as $c_2\text{-}a$) from y_{cr} in section C_2 over the fall back to y_a below the sluice.

Using Eq. (129) or (131), compute and draw the curve $c_1\text{-}d$ of the conjugated depths, corresponding to the ordinates of the M_3 curve $(1\text{-}c_1)$. Obviously, in the *vena contracta* the ordinate of this curve is d_2, conjugated to d_1, while in C_1, $d_2 = d_1 = y_{cr}$.

The point of intersection j'' of the $c_1\text{-}d$ curve with the M_2 curve determines the position of the jump, as well as the respective stages d_{2j} and d_{1j}. The uncertainty of the solution lies in neglecting the length of the jump. However, if and when section

J happens to be in the flat portion of the drop curve, the error will not be substantial.

It will also be usually expedient to omit from consideration the distance between the sluice and the *vena contracta*, assuming simply that the length between section 1 and the fall is equal to the total length of the canal.

With reference to the numerical example:

The M_3 Curve.—The total length, as determined in Question 1, is 401 ft. For the *vena contracta* we have $y_1 = d_1 = 3.10$ ft.

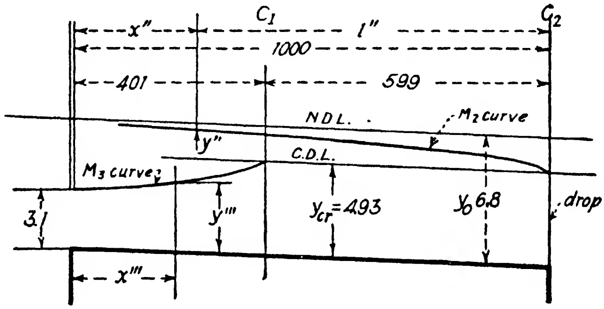

Fig. 212.

and $\Pi(\eta_1) = 0.178$. The coordinate x''' (Fig. 212) for any depth y''' is $x''' = 6{,}800\ [\Pi(\eta_2) - 0.178]$ where $\Pi(\eta_2) = \eta_2 - 0.59B(\eta_2)$, with $\eta_2 = y'''/6.8$ for the chosen y'''.

Accordingly we have, with $n = 2.8$:

TABLE XLIII

(1) y'''	(2) η_2	(3) $B(\eta_2)$	(4) $\Pi(\eta_2)$	(5) $\Pi(\eta_2) - 0.178$	(6) x'''	(7) d_2
3.10	.....	.....	.....	.....	0	7.30
3.40	0.500	0.521	0.192	0.014	95	6.87
3.67	0.540	0.568	0.204	0.026	177	6.47
4.08	0.600	0.644	0.220	0.042	286	5.91
4.76	0.700	0.787	0.236	0.058	394	5.12
4.93	.....	.....	0.237	.....	401	4.93

The curve 1-c_1 is traced in appropriate scale in Fig. 213.

The Conjugated Depth d_2 Curve.

For $d_1 = 3.10$ we have, as above determined in Question 2, $d_2 = 7.3$ ft. For intermediary points

$$d_2 = \frac{y'''}{2}\left[-1 + \sqrt{1 + 8\left(\frac{(4.93)}{y'''}\right)^3}\right]$$

The computed values are given in the last column of the above table, as corresponding to the preceding abscissae value of x. The curve a-c_1 is traced in Fig. 213 (upper curve).

The Drop Curve M_2.—We have (Question 2) for section C_2 over the fall: $y_2 = y_{cr} = 4.93$; and with $1 - \beta = 0.615$, $\Pi(\eta_{cr}) = 0.216$. The coordinate x'' (Fig. 212) for a depth y'' will be $x'' = 1{,}000 - l'' = 1{,}000 - 6{,}800\ [0.216 - \Pi(\eta)]$.

Accordingly, we obtain

y''	η	$B(\eta)$	$\Pi(\eta)$	$0.216 - \Pi(\eta)$	l''	x''
6.324	0.93	1.391	0.074	0.142	966	34
6.256	0.92	1.340	0.096	0.120	816	184
6.188	0.91	1.294	0.113	0.103	700	300
6.120	0.90	1.253	0.128	0.088	598	402

The curve is drawn into Fig. 213; it intersects the d_2 curve at $x_j = 226$ ft. This locates the jump. The depths before and after the jump are, respectively, $d_{2j} = 6.23$; $d_{1j} = 3.82$ ft.

The control by the formula

$$d_{1j} = \frac{6.23}{2}\left[-1 + \sqrt{1 + 8\left(\frac{4.93}{6.23}\right)^3}\right]$$

confirms the result obtained.

77. The Saugey Fall Increaser.—An interesting example, where chart (Fig. 190) is directly applicable, is the fall increaser suggested by Saugey.* The device is intended to increase the effective head in a water-power plant during periods of flood. Figure 214 represents schematically a power house, the draft tubes of which discharge into a tail race located below the sluice S. At periods of high water the head is reduced to Z. An appropriate stream from under the sluice may repel the tail-water and increase the head available immediately below the sluice to Z_{art}. This increased head may be used to advantage to maintain the output of the turbines T, reduced through flood water. The operation of the device is obviously conditioned on the discharge through the wheels being small, compared to the water flowing under the sluiceway.

* *Zeits. d. Ver. Deutsch. Ing.*, 1906.

The hydraulic features are easily handled by means of chart (Fig. 190) which is best illustrated by a practical example.

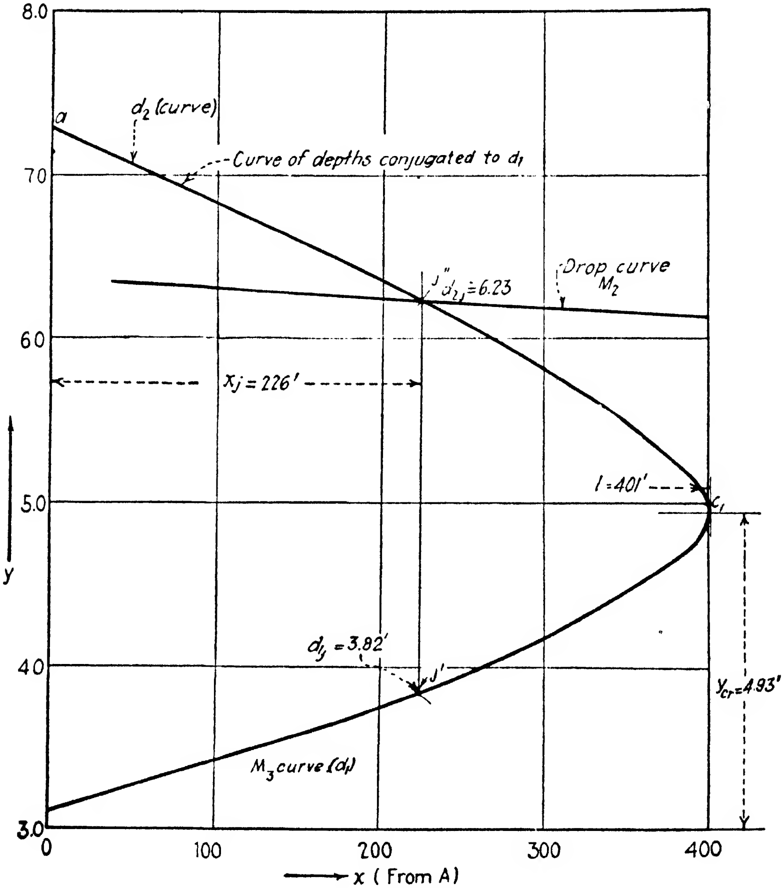

FIG. 213.—Locating the jump in question 3, Example 37.

Example 38

Assume in Fig. 215a $H = 10$ ft., $d_b = 6.5$ ft.; $Z = 3.5$ ft. Take $\alpha = 0.62$; $\varphi^2 = 0.9$. Determine the possible increase of head.

To apply chart (Fig. 190), we must know the effective head, which because of Eq. (154) depends on the opening of the sluice, etc. Assume, subject to later correction, ϑ in Eq. (154) = 0.92, making H' = 9.20 ft. Assuming the jump to be near the *vena contracta*, we take the conjugated depth d_2 to be $d_2 = d_b = 6.5$ and thus $d'_2 = 6.5/9.2 = 0.707$. Accordingly for the depth

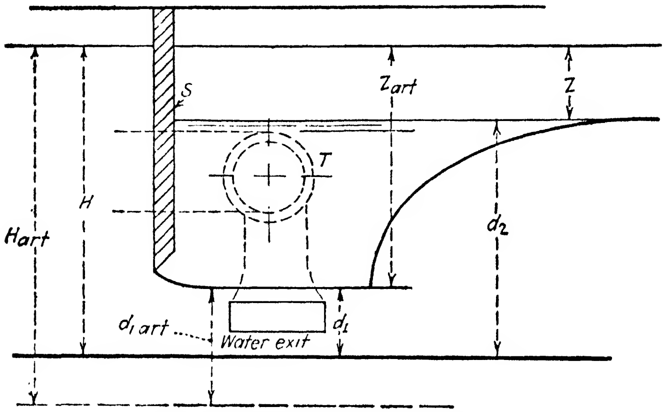

FIG. 214.—The Saugey fall increaser.

before the jump we obtain $d'_1 = 0.21$ which gives $d_1 = 0.21 \times 9.2 = \sim 1.95$ ft., and the opening of the sluice $h = 1.95/0.62 = 3$ ft.

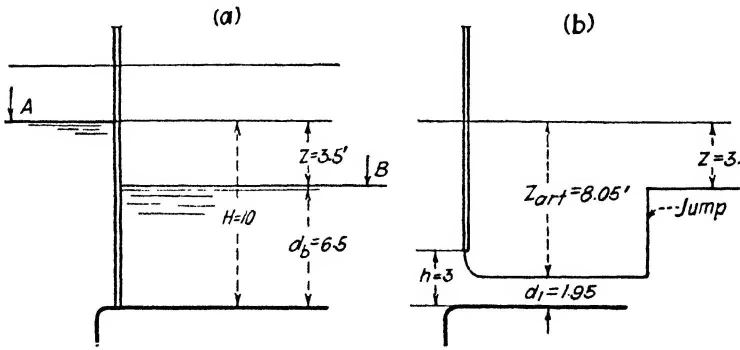

FIG. 215.

The head is increased (Fig. 215*b*) by

$$j = 6.5 - 1.95 = 4.55 \text{ ft.}$$
$$Z_{art} = 3.5 + 4.55 = 8.05 \text{ ft.}$$

The relative increase of head is

$$Z_{art}/Z = 8.05/3.50 = 2.3$$

As seen from the $j' = d'_2 - d'_1$ curve in chart (Fig. 190), with $d'_1 = 0.21$, the structure is working near the point where the reduced height of the jump is maximum. If circumstances were different, one would be able to have the layout approach conditions of optimum work by changing H, that is, by lowering or lifting the sill. In other words, with Z given, one may always bring the flow to any desired point of the diagram by appropriately designating the position of the sluice sill.

78. The Jump as an Annihilator of Energy.—In problems of flood control, when designing spillways and similar structures, it becomes necessary to pass a given quantity of water from a certain given upper stage A to a lower stage B, with a vertical difference of level Z. In lowering the water, the particular object in many cases is to annihilate as great a part as possible of the energy stored at the upper stage and thus to reduce erosion and other destructive effects in the tail race of the lower stage. The jump constitutes a most efficient *annihilator of energy* and has often been used for this purpose.* In Fig. 190, the reduced loss of energy $\epsilon'_j = 1 - \epsilon'_2$ is the distance from the upper border of the chart to the ϵ'_2 curve. The smaller the value of d'_1, the larger the loss. For example, one may dispose of 60 per cent and even more of the initial energy by causing a jump to take place with $d'_1 = 0.04$ and less. However, under these circumstances the sluice opening h will be very small and the passing of a given discharge may require a structure of excessive length.

The optimum conditions will be those at which the maximum *quantity* of energy will be annihilated per unit length of the structure.

For a certain d'_1, the reduced value of the quantity of energy dispersed in the jump will be

$$w' = q'(1 - \epsilon'_2) \tag{155}$$

By using the data of Table XLII, one may compute the w' curve. The curve drawn into chart (Fig. 190) has a

* *Tech. Repts. Miami Conservancy District, Pt.* III, 1917; Rehbock, *Internat. Cong. Appl. Mech.*, Delft, 1925; *Bauingenieur*, 1923; etc.

maximum of $w' = 0.345$, reached at $d'_1 = 0.15$ and $d'_2 = 0.63$. The maximum possible dissipation of energy per unit width attainable under these conditions is

$$w_{max} = 0.345 \times \Delta \times H^{3/2} \times H \tag{156}$$

For water, with $\Delta = 62.4$ lb. per cubic foot, we have

$$w_{max} = 21.5 \times H^{5/2} \text{ lb.} \times \text{ft. per second} \tag{157}$$

Example 39

Assume in Fig. 216*a*, $Q = 1{,}000$ cu. ft. per second, corresponding to an energy output per second of $6 \times 1{,}000 \times 62.4 = 374 \cdot 10^3$ lb. $\times$ ft. per second.

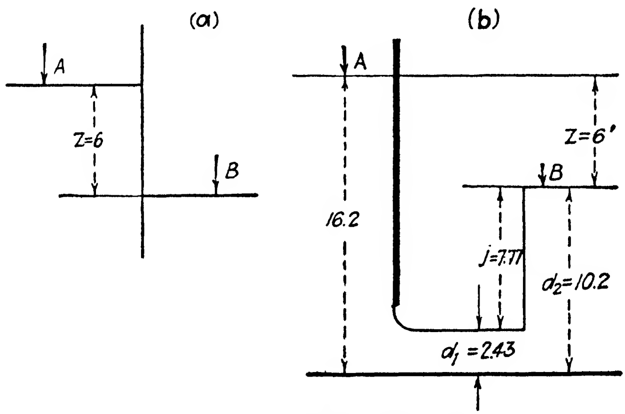

FIG. 216.—Relating to Example 39.

Neglecting losses in the efflux, we have (Fig. 216 *b*) for optimum conditions: $Z/H = 1 - d'_2 = 0.37$; $H = 6/0.37 = 16.2$; $d_1 = 0.15 \times 16.2 = 2.43$. Accordingly $d_2 = 0.63 \times 16.2 = 10.2$ with $j = 10.2 - 2.43 = 7.77$ ft.

The quantity of energy destroyed per foot of sluice length is $w = 21.5 \times 16.2^{5/2} = 22.2 \times 10^3$. The required net width of the jump $b_j = 374/22.2 = 17$ ft. The resulting layout is presented in Fig. 216*b*.

CHAPTER XXI

THE JUMP AT THE FOOT OF A WEIR

Figure 217 refers to the important case of a jump at the foot of a weir. Depending on the stage of the tail-water, the vein falling over the dam may either be covered by level $B(B'b')$; or, if level B'' is not high enough, the tail-water may be repelled; in this case the vein at the foot of

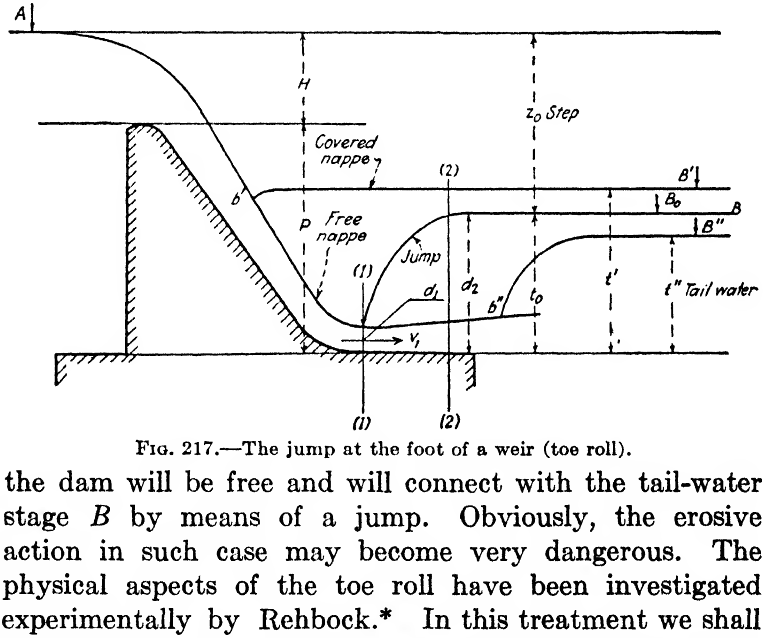

FIG. 217.—The jump at the foot of a weir (toe roll).

the dam will be free and will connect with the tail-water stage B by means of a jump. Obviously, the erosive action in such case may become very dangerous. The physical aspects of the toe roll have been investigated experimentally by Rehbock.* In this treatment we shall limit ourselves to determining the circumstances which cause flow to be covered or leave it to be free.

79. Bazin's Experiments.—The problem is practically identical with that investigated by Bazin, who in his classical work on the flow of water over weirs made the

* "Versuche über Abfluss, etc.," and other works.

distinction between *nappes covered by tail-water,* and *free nappes with a repelled jump.*

Guided merely by experimental observations, the eminent hydraulician suggested that the presence of that or the other form depends on the value of the *relative step* Z/P, which is the quotient of the vertical difference of the levels above and below the weir to the height of the structure. Bazin summarized his observations by stating that the value of the relative step, which demarcates between the two possible forms, is a constant average value of $(Z/P)_0 = 0.75$. Steps with $Z/P > 0.75$ produce free nappes, independent of whether the weir is submerged or not in the customary sense of the word. On the contrary, whenever $Z < 0.75P$, the nappe will be covered.

80. Theory of the Phenomenon.—The problem lends itself easily to theoretical treatment. We shall consider the weir crest and the tail race below the dam being of equal length, so that the discharge per unit width q over the weir and at the toe of the structure is identical. Assume in (Fig. 217) the head over the weir, corrected for the velocity of the approach, to be H, the depth at the toe in free flow being d_1. The height of the structure, to be measured on the lower side from the floor of the adjacent tail race section, to be P.

We have for section 1

$$v_1 = \varphi\sqrt{2g(P + H - d_1)}$$
$$q = \varphi d_1\sqrt{2g(P + H - d_1)}$$

where φ is a velocity coefficient, accounting for all losses between the pool A and the toe section 1.

On the other hand, the discharge over the weir with a weir coefficient m is $q = m\sqrt{2g}H^{3/2}$. Eliminating q, between the above we obtain

$$m^2H^3 = \varphi^2d_1^2(P + H - d_1) \tag{158}$$

Designating

$$x = H/P;\ y = d_1/P \tag{159}$$

we make Eq. (158)

$$m^2x^3 = \varphi^2y^2(x - y + 1) \tag{160}$$

which, for a given P, H, and m, determines the depth at the toe d_1.

Now, flow will be *submerged* or it will be *free*, depending on whether the depth t, caused by level B, is greater or smaller than the depth d_2 conjugated to d_1. The lower the tail-water (B'' in Fig. 217) the farther away the repelled jump. As the depth t increases, the jump moves towards the dam until with $t_0 = d_2$, the limiting condition distinguishing between the two forms of the phenomenon is reached.

To determine $d_2 = t_0$, we have the kineticity in section 1

$$\lambda_1 = 2\frac{v^2/2g}{d_1} = 2\varphi^2\frac{P + H - d_1}{d_1} = \frac{2\varphi^2}{y}(x - y + 1) \tag{161}$$

and thus

$$d_2 = t_0 = \frac{d_1}{2}[-1 + \sqrt{1 + 8\lambda_1}] =$$

$$\frac{d_1}{2}\left[-1 + \sqrt{1 + \frac{16\varphi^2}{y}(x - y + 1)}\right] \tag{162}$$

The value of $t_0 = d_2$ corresponds to the demarcating value of

$$Z_0 = H + P - t_0 \tag{163}$$

which after division by P and in view of Eqs. (162) and (159) becomes,

$$\left(\frac{Z}{P}\right)_0 = x + 1 - \frac{y}{2}\left[-1 + \sqrt{1 + 16\varphi^2\frac{x - y + 1}{y}}\right] \tag{164}$$

Equation (164) together with Eq. (160) solves the problem. For any given P and H, one first determines d_1 from Eq. (160), and then the demarcating value of $(Z/P)_0$ from Eq. (164).

In order to facilitate calculations, appropriate curves have been computed and traced (Figs. 218 to 220). Figures 218 and 219 give the $(Z/P)_0$ and the $y = d_1/P$ values in terms of $x = H/P$, for values of the weir coefficient $m = 0.42$, $m = 0.45$, and $m = 0.48$. These coefficients embrace the range from thin-crested weirs to dams with well-rounded ogee forms. There is a set of curves for the ideal case of φ^2

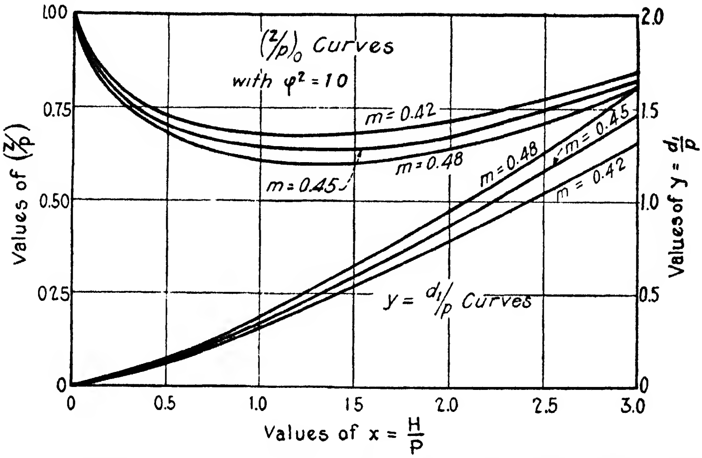

FIG. 218.—Curves for determining the value of the relative step $(Z/P)_0$ demarcating between *free* and *covered* toe rolls.

= 1; and a set for an average value of $\varphi^2 = 0.9$. It may be mentioned that scarcely any experimental material is

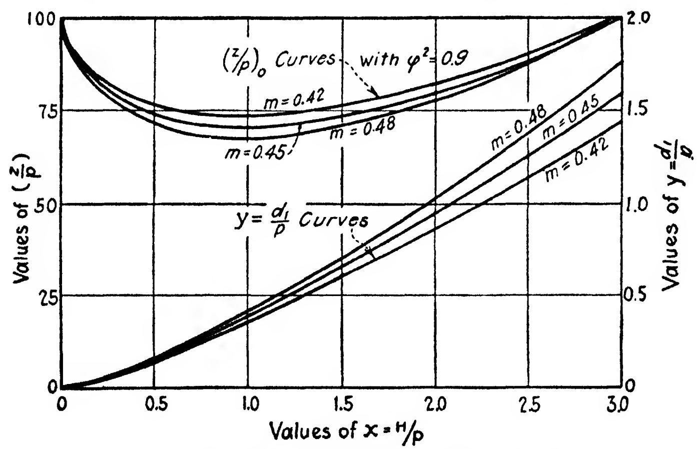

FIG. 219.—Same as for Fig. 218 but with $\varphi^2 = 0.9$.

available regarding the coefficient of velocity, so experimental work in this line is badly needed.* Finally, Fig.

* In computing the curves, an expedient procedure, obviating the necessity of solving cubic equations, is to put $\xi = x/y$; $\eta = 1/y$. This makes Eq. (158)

220 contains a set of curves for weirs of the broad-crested type. The resistance coefficients, corresponding to the discharge coefficients, are taken somewhat in excess of those determined in Art. 17.

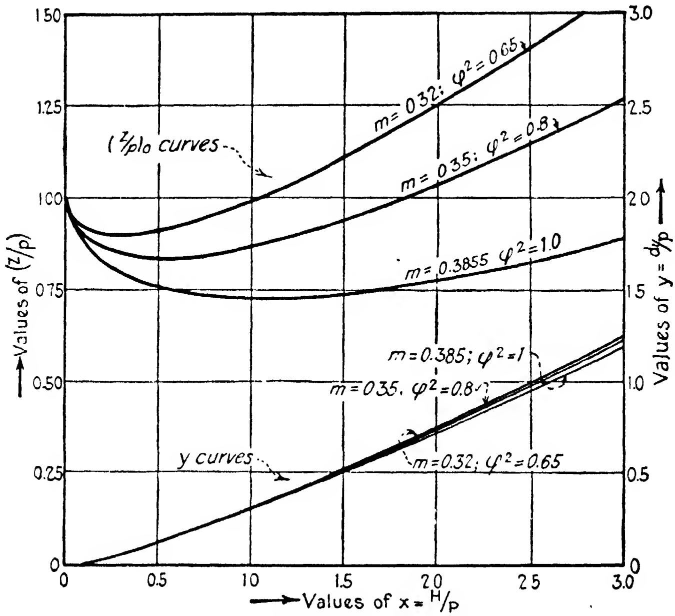

FIG. 220.—Same as Fig. 218 for broad-crested weirs.

Comparison with Bazin's Experiments.—It is interesting to compare the theoretical results with the observations of Bazin. The latter experimented on thin-crested weirs of P = 1.24 and 0.75 meters respectively. The range of relative heads H/P was between 0.37 and 1.74. The discharge coefficient in this case was on the average close to 0.42. With $\varphi^2 = 0.9$ (Fig. 219) the theoretical curve for

---

$$\eta = \frac{m^2}{\varphi^2}\xi^3 - \xi + 1 \qquad (a)$$

and Eq. (164)

$$(Z/P)_0 = y[\xi + \eta + \tfrac{1}{2} - \tfrac{1}{2}\sqrt{1 + 16\varphi^2(\xi + \eta - 1)}] \qquad (b)$$

For a chosen value of ξ, Eq. (a), determines η. Hence $y = 1/\eta$ and $x = \xi/\eta$. With this the figuring of $(Z/P)_0$ from Eq. (b) proves to be a rapid and simple procedure.

$m = 0.42$ gives the $(Z/P)_0$ values between the limits of 0.73 and 0.78. The comparison with Bazin's average value of 0.75 is very gratifying. One should further notice that the $(Z/P)_0$ curve within the Bazin range of H/P is rather flat, which explains why Bazin should have been prompted to accept one constant value of $(Z/P)_0$ as demarcating the type of the phenomenon. From the outline of the curves, however, it is seen that for small values of H/P, meaning very high dams; or in the opposite case of a relatively large H/p (that is for very low dams) the demarcating value of $(Z/P)_0$ is considerably larger.

Example 40

A watercourse is crossed (Fig. 221) by a dam 20 ft. high, with a rounded top, corresponding to a discharge coefficient $m = 0.45$. The head over the weir is $H = 5$ ft.; the tail-water depth $t = 8$ ft. Take $\varphi^2 = 0.9$. Determine the type of flow at the foot of the dam.

We have $x = H/P = 5/20 = 0.25$. The relative step $Z/P = 17/20 = 0.85$. From curve (Fig. 219) with $m = 0.45$ and $H/P = 0.25$, the demarcating value of $(Z/P)_0$, which will repel the tail-water will be $(Z/P)_0 = 0.8$.

As $Z/P = 0.85$ in our case is larger than $(Z/P)_0 = 0.8$, the flow at the toe will be free with a repelled jump.

To make the nappe covered, the value of (Z/P) will have to be reduced. There are two methods by which this can be achieved.

1. *A Toe Basin* (Fig. 221*b*).—The floor below the dam is excavated to form a basin. The height P below the dam is thus increased to $P' = P + \Delta P$, and the relative step $\frac{Z}{P + \Delta P}$ proportionately reduced.

With $Z = 17$ ft. given, in order to make $Z/P' = 0.8$, we must have $P' = 17/0.8 = 21.3$. The basin must be at least 1.3 ft. deep. To add a margin of safety, we make $\Delta P = 3$ ft. and $P' = 23$ ft. We have now: $H/P' = 5/23 = 0.22$; $Z/P' = 17/23 = 0.74$, while from the curve $(Z/P)_0$ (Fig. 219), with $x = 0.22$; $(Z/P)_0 = 0.82$.

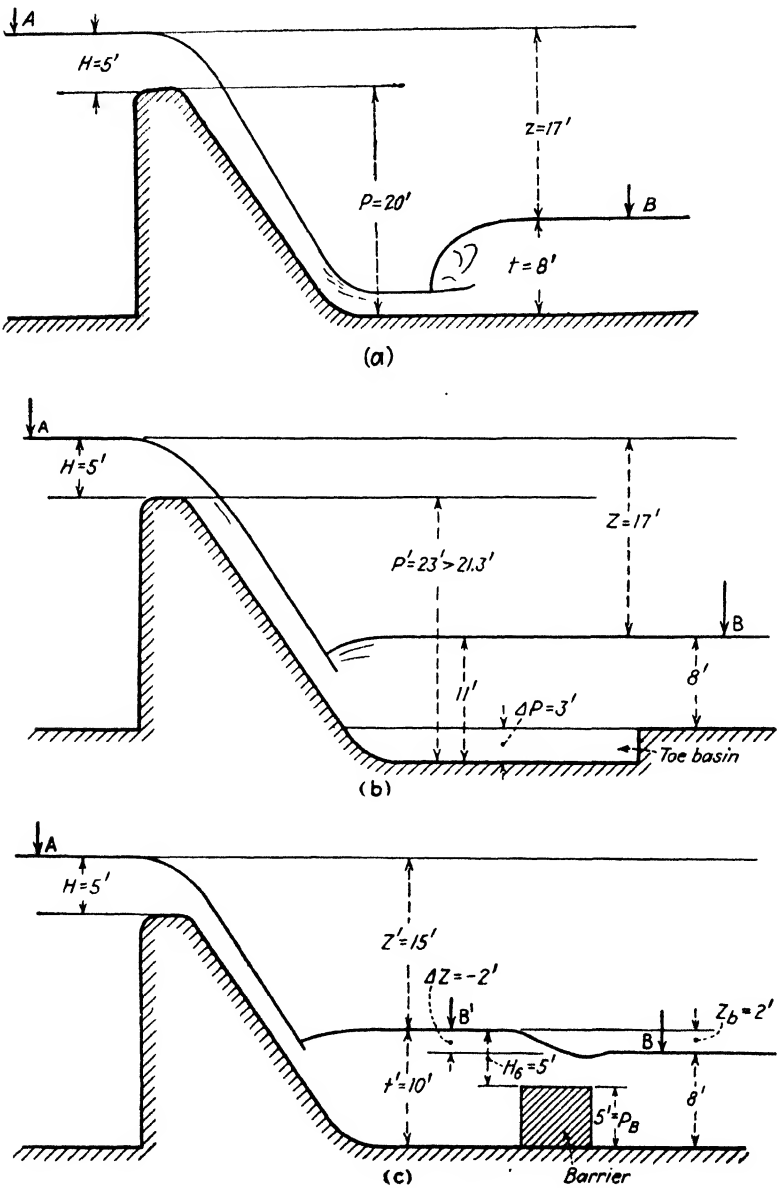

Fig. 221.—Referring to Example 40.

2. *A Barrier* (Fig. 221*c*).—An additional barrier to be located at an appropriate distance below the dam providing for a pool with a raised tail-water level over the toe of the structure. The step becomes $Z' = Z - \Delta Z$, and proportionately lowers the value of Z'/P.

In our case to make Z'/P equal to $(Z/P)_0 = 0.8$, we must reduce the step to $Z' = 0.8 \times 20 = 16$ ft. For margin's sake, make $\Delta Z' = 2$ ft. and $Z' = 15$ ft. We have now

$$Z'/P = 15/20 = 0.75$$

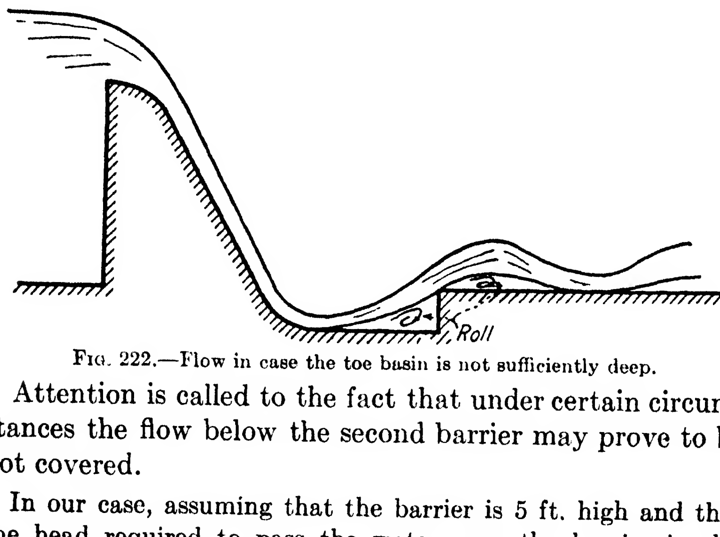

Fig. 222.—Flow in case the toe basin is not sufficiently deep.

Attention is called to the fact that under certain circumstances the flow below the second barrier may prove to be not covered.

In our case, assuming that the barrier is 5 ft. high and that the head required to pass the water over the barrier is also $H_b = 5$ ft., we have, using the same $(Z/P)_0$ curve

$$H_B/P_B = 5/5 = 1;\ Z_B/P_B = 2/5 = 0.4.$$

while from the curve $(Z/P)_0 = 0.70$. The margin providing for a covered nappe is ample.

Attention is drawn further to what happens if the barrier in the layout (Fig. 221*c*) is not high enough, or when the basin (Fig. 221*b*) is not sufficiently deep. Figures 222 and 223 give an idea of the form which flow may assume in such case, with the live vein, undular in form, springing over the barrier or out of the basin, with local rolls of eddying water under the wave tops and in the pockets. Obviously, the danger of erosion in this case is only increased.

Another feature to be remembered is that the formula developed above permits us to determine the vertical elements of the layout only.

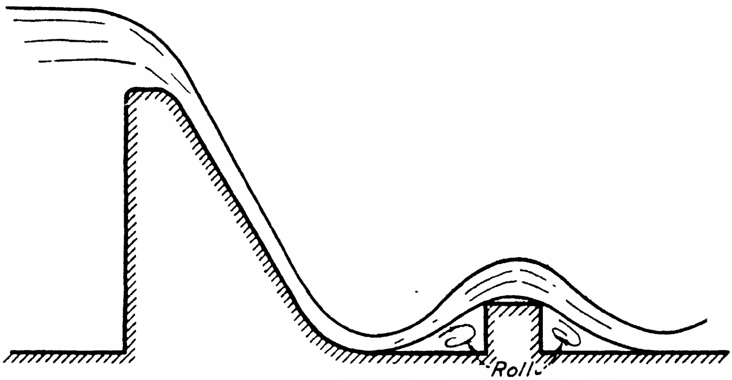

FIG. 223.—Flow in case the barrier is not sufficiently high.

Nothing positive at this time may be said about the required length of the toe basin, the required minimum distance of the barrier from the toe of the dam, etc.

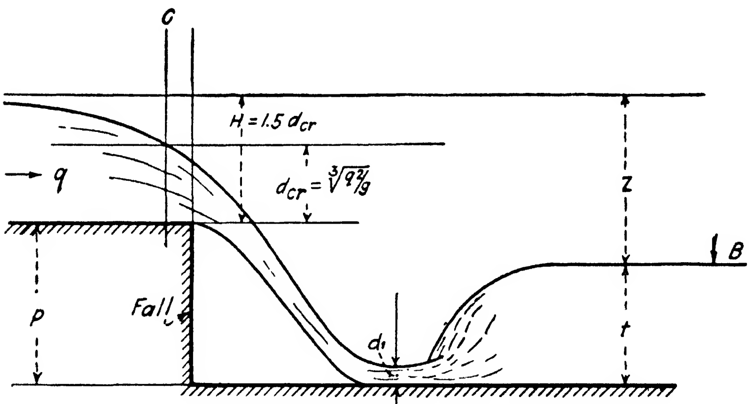

FIG. 224.—The jump below a *fall.*

As remarked in Chap. XVIII, these problems remain open to further research.

81. Jump below a Fall.—Figure 224 refers to a *fall* in a canal, a device which is often used, particularly in irrigation practice for lowering water from one level to another. Here again, depending on the relative position of level B, the falling vein may either be free as shown in the drawing, or be covered.

Usually the slope of the canal bottom is mild, so the depth of flow near the edge of the fall passes through the critical depth. Assume the flume to be rectangular. The specific energy in such case will be $\epsilon_{cr} = 1.5d_{cr}$. The discharge q, flowing over the fall, may be visualized as the discharge over a broad-crested weir in the ideal case of $\varphi = 1$,

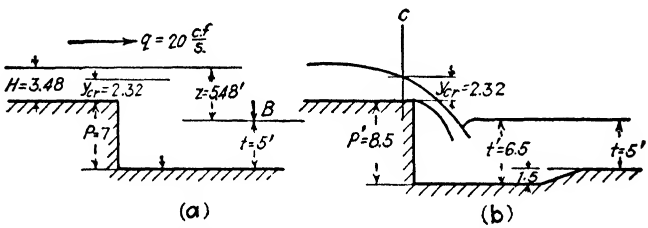

FIG. 225.—Referring to Example 41.

with a theoretical discharge coefficient $m = 0.385$ and $H = \epsilon_{cr}$. In other words, we may put

$$q = 0.385\sqrt{2g}H^{3/2} \tag{165}$$

with the head

$$H = 1.5d_{cr} = 1.5\sqrt{q^2/g} \tag{166}$$

This analogy permits us to apply the method developed above without further change. In fact, the elements of flow at the foot of the fall will be determined by Eqs. (158) and (164), provided one makes $m = 0.385$ and $\varphi^2 = 1$. With the discharge $Q = qb$ given, H is to be taken as $1.5\sqrt{q^2/g}$. The respective $(Z/P)_0$ and d_1/P curves are given in Fig. 220.

Example 41

Assume (Fig. 225a) the fall to be of rectangular form with $P = 7$ ft. The discharge per unit width is $q = 20$ cu. ft. per second. The depth of the tail-water $t = 5$ ft.

To determine the type of flow, we have

$$y_{cr} = \sqrt[3]{20^2/g} = 2.32$$

Accordingly $H = 1.5y_{cr} = 3.48$ ft., which makes $Z = P + H - t = 5.48$ ft,

We have now, in using the $(Z/P)_0$ curve (Fig. 220) with $m = 0.385$ and $\varphi^2 = 1$,

$$H/P = 3.48/7 = \sim 0.5; \; Z/P = 5.48/7 = 0.79.$$

On the other hand, according to the curve, the demarcation step is $(Z/P)_0 = 0.755$. The flow with $Z/P = 0.79 > (Z/P)_0$ is free with a repelled jump.

The nappe may be covered by providing a toe basin 1.5 ft. deep. In fact we have in such case: $P' = 7 + 1.5 = 8.5$ ft. $H/P' = 3.5/8.5 = \sim 0.41$; $(Z/P)_0 = 0.725$, while Z/P' in the structure is now reduced to $5.5/8.5 = 0.65$.

The layout is schematically presented in Fig. 225*b*.

APPENDICES

APPENDIX I

HISTORICAL AND BIBLIOGRAPHICAL NOTES

The beginning of the theory of varied flow is usually associated with the name of J. M. Bélanger.* In fact his "Essai sur la solution numérique de quelques problémes, relatives au mouvement permanent des eaux courantes," Paris, 1828, contains the general differential equation for *parallel* non-uniform flow, deals with methods of approximate integration, discusses the nature of the jump, and, in general, covers the whole subject of varied flow in a remarkably complete and comprehensive manner.

The next important step is Coriolis' paper, "Sur l'établissement de la formule qui donne la figure du remous," *Ann. Ponts et Chaussées,* 1836. While Bélanger and his followers deduced the varied flow equation from the general Newtonian equation of motion, Coriolis made use of the principle of conservation of energy, and thus was first to suggest the reasoning which since has been followed in textbooks on hydraulics in establishing the so-called Bernoulli equation.

A very interesting account of these earlier stages is given by St. Vénant in a manuscript dating from 1876, and published posthumously in the *Ann. Ponts et Chaussées,* 1886.

With regard to the integration of the equation, the case of a rectangular canal of great width was handled by Dupuit in 1848: "Études théoriques et pratiques sur le mouvement des eaux," 2d ed., Paris, 1863. In a somewhat different manner the same problem was treated by Rühlmann, "Hydromechanik," 2d ed., Hannover, 1880. Both writers ignored the effect of the change of kinetic energy. The case was presented in complete form by Bresse, "Hydraulique," Paris, 1860, and subsequently by Grashof, "Theoretische Maschinenlehre," Vol. I. The case of a

* Earlier partial references date back to Dubuat, "Principes d'hydraulique," 1779; Venturoli, 1818, and, particularly, to Masetti 1827. Also Poncelet is believed to have established the varied flow equation about the same time as Bélanger. The foundations of hydrodynamics were laid by Euler (1755).

parabolic channel was dealt with by Tolkmitt, "Grundlagen der Wasserbaukunst," Berlin, 1898. For more recent solutions by Schaffernack, Ehrenberger, and Kozény, see Forcheimer, "Hydraulik," 2d ed., Leipzig, 1930. Other methods of approach: Baticle, *Génie Civil*, 1921; Husted, *Eng. News-Record*, 1924.

A comprehensive description and classification of the different surface curves were first given by M. Boudin, "Sur l'axe hydraulique, etc.," *Ann. des travaux publiques de la Belgique*, Vol. 20, 1861–1862. The classification of watercourses depending on the bottom slope was suggested by St. Vénant, *Ann. mines*, 1851. The distinction between *states of flow* was made clear by Boussinesq, "Essai sur la théorie des eaux courantes," Paris, 1877. This *opus magnum* presents a cornerstone in the development of mechanics of fluids, and remains a treasure of inspiring suggestions. In part it made use of the experimental material, assembled by Darcy and Bazin, "Recherches hydrauliques," Paris, 1865. Among other features, we owe to Boussinesq the term *turbulent motion*, and probably the first attempt of a "statistical" approach towards the mechanism of turbulent flow. Other works by Boussinesq, "Théorie de l'écoulement tourbillonnant et tumultueux," Paris 1897; "L'écoulement en déversoir," Mém. de l'Acad., 1907.

Excellent summaries of Boussinesq's work are given by:

FLAMANT, "Hydraulique," last edition, Paris, 1923.

FORCHEIMER, "Hydraulik," 2d ed., Leipzig, 1930.

MASONI'S, "Hydraulica," 2d ed., 1900; in Italian.

Regarding manuals in English, varied flow is discussed at some length in the well-known textbooks on hydraulics by Merriman, King, and Gibson. The subject is dealt with in detail in the *Technical Reports Miami Conservancy District.*

The most important special papers are:

KENNISON, The Hydraulic Jump in Open Channel Flow, *Trans. A.S.C.E.*, 1916.

JOHNSON, Surges in an Open Canal, *Trans. A.S.C.E.*, 1917.

HINDS, *Eng. News-Record*, Vol. 85, pages 1034–1040, 1920.

For an account of the experimental study of the physical aspects of flow, as carried out by Professor Rehbock (Karlsruhe) and his followers, see:

REHBOCK, "Stauwerke," Vol. II, *Handbuch der Ingenieur-Wissenschaften*, III, Leipzig, 1912; "Betrachtungen über Abfluss, Stau- und Walzenbildung," Berlin, 1917; articles in Hydraulical Laboratory Practice, *A.S.M.E.*, 1929.

BÖSS, Mitt. für Forschungsarbeiten, *V.D.I.*, No. 284.

Also numerous articles in the *Bauingenieur* and other publications, a detailed list of which is given in "Hydraulisches Rechnen" by Weyrauch-Strobel, 6th ed., Stuttgart, 1930.

The lifetime contribution of Professor Koch (Darmstadt) is assembled in the posthumous "Bewegung des Wassers" by Koch-Carstanjen, Berlin, 1926. Other larger contemporaneous works in German are:

FORCHEIMER, "Wasserschwall und Wassersunk," Leipzig, 1924.

SCHOKLITSH, "Der Wasserbau," Berlin, 1930.

KOZÉNY, "Wasserführung der Flüsse," Leipzig–Vienna, 1920.

A detailed list of smaller contributions is to be found in Weyrauch-Strobel (*loc. cit.*, page 355).

For an account of the recent development of the more theoretical aspects of applied hydromechanics, see:

PRANDTL–TIETJENS, "Hydro- und Aeromechanik," Berlin, 1929–1931 (a version in English to appear shortly).

KAUFMAN, "Hydromechanik," Berlin, 1931.

Handbuch der Experimentalphysik, Vol. 4.

Handbuch der Physik, Vol. 7.

Handbuch der physikalischen und technischen Mechanik, Vol. 5.

"Hydraulische Probleme," collective work, *V.D.I.*, 1926.

KARMAN AND LEVI-CIVITA, "Vorträge aus dem Gebiete der Hydro- und Aerodynamik," Berlin, 1924.

APPENDIX II

METHODS OF COMPUTATION OF THE VARIED-FLOW-FUNCTION TABLES

The numerical values of the varied flow function

$$B(\eta) = -\int_0^\eta \frac{d\eta}{\eta^n - 1} = \int_0^\eta \frac{d\eta}{1 - \eta^n}$$

were computed by means of the following methods, one or the other method being used depending on the values of the argument η.

Method 1. For values of $\eta < 1$, one may use the infinite series

$$\frac{1}{1 - \eta^n} = 1 + \eta^n + \eta^{2n} + \cdots + \eta^{(p-1)n} + r_p \qquad (a)$$

where n is the hydraulic exponent, p the number of members in the series, and

$$r_p = \eta^{pn} + \eta^{(p+1)n} + \cdots$$

By integrating, one obtains

$$\int \frac{d\eta}{1 - \eta^n} = \eta + \frac{1}{n+1}\eta^{n+1} + \frac{1}{2n+1}\eta^{2n+1} + \cdots + \frac{1}{(p-1)n+1}\eta^{(p-1)n+1} + R_p \qquad (b)$$

where

$$R_p = \int r_p d\eta = \frac{1}{pn+1}\eta^{pn+1} + \cdots = \frac{1}{pn+1}\eta^{pn+1}\left(1 + \frac{pn+1}{pn+n+1}\eta^n + \cdots\right)$$

Obviously,

$$R_p < \frac{\eta^{pn+1}}{pn+1}(1 + \eta^n + \eta^{2n} + \cdots) = \frac{\eta^{pn+1}}{pn+1} \cdot \frac{1}{1 - \eta^n} \qquad (c)$$

Equation (*c*) allows us to determine the number p of the members of the series, which are necessary to guarantee the required precision of the computation. The value of $B(\eta)$ is then computed by means of Eq. (*b*). For reasonably small values of η, the series (*a*) is rapidly convergent. The method was found to be practicable for values of $\eta \leqslant 0.70$. For larger values of η, and particularly for η close to unity, the number p of series members required becomes impracticably large.

Method 2.—For values of $\eta > 1$, a convergent series is obtained by making $\eta = 1/z^2$ and $n = k/2$; so that $\eta^n = 1/z^k$, while

$$d\eta = -2dz/z^3 \qquad (a)$$

We have

$$\frac{1}{2}\int\frac{d\eta}{1-\eta^n} = \int\frac{z^{k-3}}{1-z^k}dz = \frac{z^{k-2}}{k-2} + \frac{z^{2k-2}}{2k-2} + \cdots + \frac{z^{(p-1)k-2}}{(p-1)k-2} + R'_p \qquad (b)$$

with

$$R'_p < \frac{z^{pk-2}}{pk-2}\cdot\frac{1}{1-z^k} \qquad (c)$$

Substituting $z^k = 1/\eta^n$ from Eq. (a) into Eqs. (b) and (c), we obtain

$$2\int\frac{z^{k-3}}{1-z^k}dz = \int\frac{d\eta}{1-\eta^n} = \frac{1}{(n-1)\eta^{n-1}} + \frac{1}{(2n-1)\eta^{2n-1}} + \cdots + R_p \qquad (d)$$

where

$$R_p < \frac{1}{(pn-1)\eta^{pn-1}}\cdot\frac{\eta^n}{\eta^n-1} \qquad (e)$$

Series (d) is convergent, the rapidity of convergence increasing with the value of η. The method was found practicable for values of $\eta \geqslant 1.50$.

Method 3. For the region $0.7 < \eta < 1.50$, it was found more expedient to use the well-known Poncelet formula for approximate integration, namely

$$\int_a^b y\,dx = \frac{b-a}{2m}(2q+s) \qquad (a)$$

where $2m$ is the number of equal intervals, into which the range a-b is divided; $y_1, y_2, y_3, \ldots y_{2m}$ and y_{2m+1} are the respective values of the function $y = f(x)$, corresponding to the above intervals, while

$$\left.\begin{aligned} q &= y_2 + y_4 + \cdots y_{2m} \\ s &= \tfrac{1}{4}(y_1 + y_{2m+1} - y_2 - y_{2m}) \end{aligned}\right\} \qquad (b)$$

The error ϵ in this case is

$$\epsilon < \frac{b-a}{2m} \cdot s \qquad (c)$$

The Poncelet procedure determines the value

$$-\Delta B(\eta) = \int_a^b \frac{d\eta}{1-\eta^n}$$

which is to be added to

$$-B(a) = \int_0^a \frac{d\eta}{1-\eta^n}$$

to determine

$$-B(b) = \int_0^b \frac{d\eta}{1-\eta^n}$$

Starting with a certain table value $B(a)$, previously determined by some other method, the Poncelet formula allows us to build up the consecutive table values. For each range a-b, Eq. (c) determines the number of intervals $2m$, into which the respective range a-b has to be divided to obtain the required precision.

Method 4. For values of $\eta \gtrless 1$, in the immediate vicinity of unity, the number of intervals to be used in the Poncelet formula proves to be very large. In this case, advantage can be taken of an infinite series, obtained by making

$$1 - \eta^n = \pm z \qquad (a)$$

where the sign + corresponds to values of $\eta < 1$, and the sign − to values of $\eta > 1$.

This leads to

$$\int \frac{d\eta}{1-\eta^n} = \text{const.} + \frac{\log z}{n} \pm \frac{n-1}{n^2} z + \frac{(n-1)(2n-1)}{2n^3 \times 2!} z^2 \pm \frac{(n-1)(2n-1)(3n-1)}{3n^4 \times 3!} z^3 + \cdots R_p.$$

The rapidity of convergence of this series increases in measure with the decrease of z, that is as η approaches unity. The number p of series members, required to guarantee a desired precision, is determined by

$$R_p < \frac{z^p}{n} \cdot \frac{1}{1-z}$$

The precision of computation of all tables was to make the error.

$$\epsilon \leqslant 0.0005$$

TABLES OF THE VARIED FLOW FUNCTION

Table I. $B(\eta) = -\int_0^\eta \frac{d\eta}{\eta^n - 1}$ for $\begin{cases} \eta > 1 \\ \eta < 1 \end{cases}$

Table II. $\Phi(\eta) = \eta - B(\eta)$ for $\eta > 1$

TABLE IA.—THE VARIED FLOW FUNCTION $B(\eta)$ FOR $\eta > 1$
Function values for exponent values

η	n = 2.8	n = 3.0	n = 3.2	n = 3.4	n = 3.6	n = 3.8	n = 4.0	n = 4.2	n = 4.6	n = 5.0	n = 5.4
1.001	2.399	2.184	2.008	1.856	1.725	1.610	1.508	1.417	1.264	1.138	1.033
1.005	1.818	1.649	1.506	1.384	1.279	1.188	1.107	1.036	0.915	0.817	0.737
1.010	1.572	1.419	1.291	1.182	1.089	1.007	0.936	0.873	0.766	0.681	0.610
1.015	1.428	1.286	1.166	1.065	0.978	0.902	0.836	0.778	0.680	0.602	0.537
1.02	1.327	1.191	1.078	0.982	0.900	0.828	0.766	0.711	0.620	0.546	0.486
1.03	1.186	1.060	0.955	0.866	0.790	0.725	0.668	0.618	0.535	0.469	0.415
1.04	1.086	0.967	0.868	0.785	0.714	0.653	0.600	0.554	0.477	0.415	0.365
1.05	1.010	0.896	0.802	0.723	0.656	0.598	0.548	0.504	0.432	0.374	0.328
1.06	0.948	0.838	0.748	0.672	0.608	0.553	0.506	0.464	0.396	0.342	0.298
1.07	0.896	0.790	0.703	0.630	0.569	0.516	0.471	0.431	0.366	0.315	0.273
1.08	0.851	0.749	0.665	0.595	0.535	0.485	0.441	0.403	0.341	0.292	0 252
1.09	0.812	0.713	0.631	0.563	0.506	0.457	0.415	0.379	0.319	0.272	0.234
1.10	0.777	0.681	0.601	0.536	0.480	0.433	0.392	0.357	0.299	0 254	0 218
1.11	0.746	0.652	0.575	0.511	0.457	0.411	0.372	0.338	0.282	0.239	0 204
1.12	0.718	0.626	0.551	0.488	0.436	0.392	0.354	0.321	0.267	0.225	0.192
1.13	0.692	0.602	0.529	0.468	0.417	0.374	0.337	0.305	0.253	0.212	0.181
1.14	0.669	0.581	0.509	0.450	0 400	0.358	0.322	0.291	0.240	0.201	0.170
1.15	0.647	0.561	0.490	0.432	0.384	0.343	0.308	0.278	0.229	0.191	0.161
1.16	0.627	0.542	0.473	0.417	0.369	0.329	0.295	0.266	0.218	0.181	0.153
1.17	0.608	0.525	0.458	0.402	0.356	0.317	0.283	0.255	0.208	0.173	0 145
1.18	0.591	0.509	0.443	0.388	0.343	0.305	0.272	0.244	0.199	0.165	0.138
1.19	0.574	0.494	0.429	0.375	0.331	0.294	0.262	0.235	0.191	0.157	0.131
1.20	0.559	0.480	0.416	0.363	0.320	0.283	0.252	0.226	0.183	0.150	0.125
1.22	0.531	0.454	0.392	0.341	0.299	0.264	0.235	0.209	0.168	0.138	0.114
1.24	0.505	0.431	0.371	0.322	0.281	0.248	0.219	0.195	0.156	0.127	0.104
1.26	0.482	0.410	0.351	0.304	0.265	0.233	0.205	0.182	0.145	0.117	0.095
1.28	0.461	0.391	0.334	0.288	0.250	0.219	0.193	0.170	0.135	0.108	0.088
1.30	0.442	0.373	0.318	0.274	0.237	0.207	0.181	0.160	0.126	0.100	0.081
1.32	0.424	0.357	0.304	0.260	0.225	0.196	0.171	0.150	0.118	0.093	0.075
1.34	0.408	0.342	0.290	0.248	0.214	0.185	0.162	0.142	0.110	0.087	0.069
1.36	0.393	0.329	0.278	0.237	0.204	0.176	0.153	0.134	0.103	0.081	0.064
1.38	0.378	0.316	0.266	0.226	0.194	0.167	0.145	0.127	0.097	0.076	0.060
1.40	0 365	0.304	0.256	0.217	0.185	0.159	0.138	0.120	0.092	0.071	0.056
1.42	0.353	0.293	0.246	0.208	0.177	0.152	0.131	0.114	0.087	0.067	0.052
1.44	0.341	0.282	0.236	0.199	0.169	0.145	0.125	0.108	0.082	0.063	0.049
1.46	0.330	0.273	0.227	0.191	0.162	0.139	0.119	0.103	0.077	0.059	0.046
1.48	0.320	0.263	0.219	0.184	0.156	0.133	0.113	0.098	0.073	0.056	0.043
1.50	0.310	0.255	0.211	0.177	0.149	0.127	0.108	0.093	0.069	0.053	0.040
1.55	0.288	0.235	0.194	0.161	0.135	0.114	0.097	0.083	0.061	0.046	0.035
1.60	0.269	0.218	0.179	0.148	0.123	0.103	0.087	0.074	0.054	0.040	0.030
1.65	0.251	0.203	0.165	0.136	0.113	0.094	0.079	0.067	0.048	0.035	0.026
1.70	0.236	0.189	0.153	0.125	0.103	0.086	0.072	0.060	0.043	0.031	0.023
1.75	0.212	0.177	0.143	0.116	0.095	0.079	0.065	0.054	0.038	0.027	0.020
1.80	0.209	0.166	0.133	0.108	0.088	0.072	0.060	0.049	0.034	0.024	0.017
1.85	0.198	0.156	0.125	0.100	0.082	0.067	0.055	0.045	0.031	0.022	0.015

TABLE IA.—THE VARIED FLOW FUNCTION $B(\eta)$ FOR $\eta > 1$.—(*Continued*)

η	$n =$ 2.8	$n =$ 3.0	$n =$ 3.2	$n =$ 3.4	$n =$ 3.6	$n =$ 3.8	$n =$ 4.0	$n =$ 4.2	$n =$ 4.6	$n =$ 5.0	$n =$ 5.4
1.85	0.198	0.156	0.125	0.100	0.082	0.067	0.055	0.045	0.031	0.022	0.015
1.90	0.188	0.147	0.117	0.094	0.076	0.062	0.050	0.041	0.028	0.020	0.014
1.95	0.178	0.139	0.110	0.088	0.070	0.057	0.046	0.038	0.026	0.018	0.012
2.00	0.169	0.132	0.104	0.082	0.066	0.053	0.043	0.035	0.023	0.016	0.011
2.1	0.154	0.119	0.092	0.073	0.058	0.046	0.037	0.030	0.019	0.013	0.009
2.2	0.141	0.107	0.083	0.065	0.051	0.040	0.032	0.025	0.016	0.011	0.007
2.3	0.129	0.098	0.075	0.058	0.045	0.035	0.028	0.022	0.014	0.009	0.006
2.4	0.119	0.089	0.068	0.052	0.040	0.031	0.024	0.019	0.012	0.008	0.005
2.5	0.110	0.082	0.062	0.047	0.036	0.028	0.022	0.017	0.010	0.006	0.004
2.6	0.102	0.076	0.057	0.043	0.033	0.025	0.019	0.015	0.009	0.005	0.003
2.7	0.095	0.070	0.052	0.039	0.029	0.022	0.017	0.013	0.008	0.005	0.003
2.8	0.089	0.065	0.048	0.036	0.027	0.020	0.015	0.012	0.007	0.004	0.002
2.9	0.083	0.060	0.044	0.033	0.024	0.018	0.014	0.010	0.006	0.004	0.002
3.0	0.078	0.056	0.041	0.030	0.022	0.017	0.012	0.009	0.005	0.003	0.002
3.5	0.059	0.041	0.029	0.021	0.015	0.011	0.008	0.006	0.003	0.002	0.001
4.0	0.046	0.031	0.022	0.015	0.010	0.007	0.005	0.004	0.002	0.001	0.000
4.5	0.037	0.025	0.017	0.011	0.008	0.005	0.004	0.003	0.001	0.001	0.000
5.0	0.031	0.020	0.013	0.009	0.006	0.004	0.003	0.002	0.001	0.000	0.000
6.0	0.022	0.014	0.009	0.006	0.004	0.002	0.002	0.001	0.000	0.000	0.000
7.0	0.017	0.010	0.006	0.004	0.002	0.002	0.001	0.001			
8.0	0.013	0.008	0.005	0.003	0.002	0.001	0.001	0.000			
9.0	0.011	0.006	0.004	0.002	0.001	0.001	0.000	0.000			
10.0	0.009	0.005	0.003	0.002	0.001	0.001	0.000	0.000			
20.0	0.003	0.002	0.001	0.001	0.000	0.000	0.000	0.000			

TABLE IB.—THE VARIED FLOW FUNCTION $B(\eta)$ FOR $\eta < 1$
Function values for exponent values

η	$n =$ 2.8	$n =$ 3.0	$n =$ 3.2	$n =$ 3.4	$n =$ 3.6	$n =$ 3.8	$n =$ 4.0	$n =$ 4.2	$n =$ 4.6	$n =$ 5.0	$n =$ 5.4
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.02	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.020
0.04	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040	0.040
0.06	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060	0.060
0.08	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080
0.10	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
0.12	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120	0.120
0.14	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140	0.140
0.16	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160	0.160
0.18	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180	0.180
0.20	0.201	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
0.22	0.221	0.221	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220	0.220
0.24	0.241	0.241	0.241	0.240	0.240	0.240	0.240	0.240	0.240	0.240	0.240
0.26	0.262	0.261	0.261	0.261	0.260	0.260	0.260	0.260	0.260	0.260	0.260
0.28	0.282	0.282	0.281	0.281	0.281	0.280	0.280	0.280	0.280	0.280	0.280
0.30	0.303	0.302	0.302	0.301	0.301	0.301	0.300	0.300	0.300	0.300	0.300
0.32	0.324	0.323	0.322	0.322	0.321	0.321	0.321	0.321	0.320	0.320	0.320
0.34	0.344	0.343	0.343	0.342	0.342	0.341	0.341	0.341	0.340	0.340	0.340
0.36	0.366	0.364	0.363	0.363	0.362	0.362	0.361	0.361	0.361	0.360	0.360
0.38	0.387	0.385	0.384	0.383	0.383	0.382	0.382	0.381	0.381	0.381	0.380
0.40	0.408	0.407	0.405	0.404	0.403	0.403	0.402	0.402	0.401	0.401	0.400
0.42	0.430	0.428	0.426	0.425	0.424	0.423	0.423	0.422	0.421	0.421	0.421
0.44	0.452	0.450	0.448	0.446	0.445	0.444	0.443	0.443	0.442	0.441	0.441
0.46	0.475	0.472	0.470	0.468	0.466	0.465	0.464	0.463	0.462	0.462	0.461
0.48	0.497	0.494	0.492	0.489	0.488	0.486	0.485	0.484	0.483	0.482	0.481
0.50	0.521	0.517	0.514	0.511	0.509	0.508	0.506	0.505	0.504	0.503	0.502
0.52	0.524	0.540	0.536	0.534	0.531	0.529	0.528	0.527	0.525	0.523	0.522
0.54	0.568	0.563	0.559	0.556	0.554	0.551	0.550	0.548	0.546	0.544	0.543
0.56	0.593	0.587	0.583	0.579	0.576	0.574	0.572	0.570	0.567	0.565	0.564
0.58	0.618	0.612	0.607	0.603	0.599	0.596	0.594	0.592	0.589	0.587	0.585
0.60	0.644	0.637	0.631	0.627	0.623	0.620	0.617	0.614	0.611	0.608	0.606
0.61	0.657	0.650	0.644	0.639	0.635	0.631	0.628	0.626	0.622	0.619	0.617
0.62	0.671	0.663	0.657	0.651	0.647	0.643	0.640	0.637	0.633	0.630	0.628
0.63	0.684	0.676	0.669	0.664	0.659	0.655	0.652	0.649	0.644	0.641	0.638
0.64	0.698	0.690	0.683	0.677	0.672	0.667	0.664	0.661	0.656	0.652	0.649

TABLE IB.—THE VARIED FLOW FUNCTION $B(\eta)$ FOR $\eta < 1$.—(*Continued*)

η	$n =$ 2.8	$n =$ 3.0	$n =$ 3.2	$n =$ 3.4	$n =$ 3.6	$n =$ 3.8	$n =$ 4.0	$n =$ 4.2	$n =$ 4.6	$n =$ 5.0	$n =$ 5.4
0.64	0.698	0.690	0.683	0.677	0.672	0.667	0.664	0.661	0.656	0.652	0.649
0.65	0.712	0.703	0.696	0.689	0.684	0.680	0.676	0.673	0.667	0.663	0.660
0.66	0.727	0.717	0.709	0.703	0.697	0.692	0.688	0.685	0.679	0.675	0.672
0.67	0.742	0.731	0.723	0.716	0.710	0.705	0.701	0.697	0.691	0.686	0.683
0.68	0.757	0.746	0.737	0.729	0.723	0.718	0.713	0.709	0.703	0.698	0.694
0.69	0.772	0.761	0.751	0.743	0.737	0.731	0.726	0.722	0.715	0.710	0.706
0.70	0.787	0.776	0.766	0.757	0.750	0.744	0.739	0.735	0.727	0.722	0.717
0.71	0.804	0.791	0.781	0.772	0.764	0.758	0.752	0.748	0.740	0.734	0.729
0.72	0.820	0.807	0.796	0.786	0.779	0.772	0.766	0.761	0.752	0.746	0.741
0.73	0.837	0.823	0.811	0.802	0.793	0.786	0.780	0.774	0.765	0.759	0.753
0.74	0.854	0.840	0.827	0.817	0.808	0.800	0.794	0.788	0.779	0.771	0.766
0.75	0.872	0.857	0.844	0.833	0.823	0.815	0.808	0.802	0.792	0.784	0.778
0.76	0.890	0.874	0.861	0.849	0.839	0.830	0.823	0.817	0.806	0.798	0.791
0.77	0.909	0.892	0.878	0.866	0.855	0.846	0.838	0.831	0.820	0.811	0.804
0.78	0.929	0.911	0.896	0.883	0.872	0.862	0.854	0.847	0.834	0.825	0.817
0.79	0.949	0.930	0.914	0.901	0.889	0.879	0.870	0.862	0.849	0.839	0.831
0.80	0.970	0.950	0.934	0.919	0.907	0.896	0.887	0.878	0.865	0.854	0.845
0.81	0.992	0.971	0.954	0.938	0.925	0.914	0.904	0.895	0.881	0.869	0.860
0.82	1.015	0.993	0.974	0.958	0.945	0.932	0.922	0.913	0.897	0.885	0.875
0.83	1.039	1.016	0.996	0.979	0.965	0.952	0.940	0.931	0.914	0.901	0.890
0.84	1.064	1.040	1.019	1.001	0.985	0.972	0.960	0.949	0.932	0.918	0.906
0.85	1.091	1.065	1.043	1.024	1.007	0.993	0.980	0.969	0.950	0.935	0.923
0.86	1.119	1.092	1.068	1.048	1.031	1.015	1.002	0.990	0.970	0.954	0.940
0.87	1.149	1.120	1.095	1.074	1.055	1.039	1.025	1.012	0.990	0.973	0.959
0.88	1.181	1.151	1.124	1.101	1.081	1.064	1.049	1.035	1.012	0.994	0.978
0.89	1.216	1.183	1.155	1.131	1.110	1.091	1.075	1.060	1.035	1.015	0.999
0.90	1.253	1.218	1.189	1.163	1.140	1.120	1.103	1.087	1.060	1.039	1.021
0.91	1.294	1.257	1.225	1.197	1.173	1.152	1.133	1.116	1.088	1.064	1.045
0.92	1.340	1.300	1.266	1.236	1.210	1.187	1.166	1.148	1.117	1.092	1.072
0.93	1.391	1.348	1.311	1.279	1.251	1.226	1.204	1.184	1.151	1.123	1.101
0.94	1.449	1.403	1.363	1.328	1.297	1.270	1.246	1.225	1.188	1.158	1.134
0.95	1.518	1.467	1.423	1.385	1.352	1.322	1.296	1.272	1.232	1.199	1.172
0.96	1.601	1.545	1.497	1.454	1.417	1.385	1.355	1.329	1.285	1.248	1.217
0.97	1.707	1.644	1.590	1.543	1.501	1.464	1.431	1.402	1.351	1.310	1.275
0.975	1.773	1.707	1.649	1.598	1.554	1.514	1.479	1.447	1.393	1.348	1.311
0.980	1.855	1.783	1.720	1.666	1.617	1.575	1.536	1.502	1.443	1.395	1.354
0.985	1.959	1.880	1.812	1.752	1.699	1.652	1.610	1.573	1.508	1.454	1.409
0.990	2.106	2.017	1.940	1.873	1.814	1.761	1.714	1.671	1.598	1.537	1.487
0.995	2.355	2.250	2.159	2.079	2.008	1.945	1.889	1.838	1.751	1.678	1.617
0.999	2.931	2.788	2.663	2.554	2.457	2.370	2.293	2.223	2.102	2.002	1.917

TABLE II.—$\Phi(\eta) = \eta - B(\eta)$ FOR EXPONENT VALUES

η	$n = 2.8$	$n = 3.0$	$n = 3.2$	$n = 3.4$	$n = 3.6$	$n = 3.8$	$n = 4.0$	$n = 4.2$
1.001	−1.398	−1.183	−1.007	−0.855	−0.724	−0.609	−0.507	−0.416
1.005	0.813	0.644	0.501	0.379	0.274	0.183	0.102	0.031
1.010	0.562	0.409	0.281	0.172	0.079	+0.003	+0.074	+0.137
1.015	0.413	0.271	0.151	0.050	+0.037	0.113	0.179	0.237
1.02	0.307	0.171	0.058	+0.038	0.120	0.192	0.254	0.309
1.03	0.156	0.030	+0.075	0.164	0.240	0.305	0.362	0.412
1.04	0.046	+0.073	0.172	0.255	0.326	0.387	0.440	0.486
1.05	+0.040	0.154	0.248	0.327	0.394	0.452	0.502	0.546
1.06	0.112	0.222	0.312	0.388	0.452	0.507	0.554	0.596
1.07	0.174	0.280	0.367	0.440	0.501	0.554	0.599	0.639
1.08	0.229	0.331	0.415	0.485	0.545	0.595	0.639	0.677
1.09	0.278	0.377	0.459	0.527	0.584	0.633	0.675	0.711
1.10	0.323	0.419	0.499	0.564	0.620	0.667	0.708	0.743
1.11	0.364	0.458	0.535	0.599	0.653	0.699	0.738	0.772
1.12	0.402	0.494	0.569	0.632	0.684	0.728	0.766	0.799
1.13	0.438	0.528	0.601	0.662	0.713	0.756	0.793	0.825
1.14	0.471	0.559	0.631	0.690	0.740	0.782	0.818	0.849
1.15	0.503	0.589	0.660	0.718	0.766	0.807	0.842	0.872
1.16	0.533	0.618	0.687	0.743	0.791	0.831	0.865	0.894
1.17	0.562	0.645	0.712	0.768	0.814	0.853	0.887	0.915
1.18	0.589	0.671	0.737	0.792	0.837	0.875	0.908	0.936
1.19	0.616	0.696	0.761	0.815	0.859	0.896	0.928	0.955
1.20	0.641	0.720	0.784	0.837	0.880	0.917	0.948	0.974
1.22	0.689	0.766	0.828	0.879	0.921	0.956	0.985	1.011
1.24	0.735	0.809	0.869	0.918	0.959	0.992	1.021	1.045
1.26	0.778	0.850	0.909	0.956	0.995	1.027	1.055	1.078
1.28	0.819	0.889	0.946	0.992	1.030	1.061	1.087	1.110
1.30	0.858	0.927	0.982	1.026	1.063	1.093	1.119	1.140
1.32	0.896	0.963	1.016	1.060	1.095	1.124	1.149	1.170
1.34	0.932	0.998	1.050	1.092	1.126	1.155	1.178	1.198
1.36	0.967	1.031	1.082	1.123	1.156	1.184	1.207	1.226
1.38	1.002	1.064	1.114	1.154	1.186	1.213	1.235	1.253
1.40	1.035	1.096	1.144	1.183	1.215	1.241	1.262	1.280
1.42	1.067	1.127	1.174	1.212	1.243	1.268	1.289	1.306
1.44	1.099	1.158	1.204	1.241	1.271	1.295	1.315	1.332
1.46	1.130	1.187	1.233	1.259	1.298	1.321	1.341	1.357
1.48	1.160	1.217	1.261	1.296	1.324	1.347	1.367	1.382
1.50	1.190	1.245	1.289	1.323	1.351	1.373	1.392	1.407
1.55	1.262	1.315	1.356	1.389	1.415	1.436	1.453	1.467
1.60	1.331	1.382	1.421	1.452	1.477	1.497	1.513	1.526
1.65	1.399	1.447	1.485	1.514	1.537	1.556	1.571	1.583
1.70	1.464	1.511	1.547	1.575	1.597	1.614	1.628	1.640
1.75	1.538	1.573	1.607	1.634	1.655	1.671	1.685	1.696
1.80	1.591	1.634	1.667	1.692	1.712	1.728	1.740	1.751
1.85	1.652	1.694	1.725	1.750	1.768	1.783	1.795	1.805
1.90	1.712	1.753	1.783	1.806	1.824	1.838	1.850	1.859
1.95	1.772	1.811	1.840	1.862	1.880	1.893	1.904	1.912

TABLE II.—$\Phi(\eta) = \eta - (B)\eta$ FOR EXPONENT VALUES.—(*Continued*)

η	$n = 2.8$	$n = 3.0$	$n = 3.2$	$n = 3.4$	$n = 3.6$	$n = 3.8$	$n = 4.0$	$n = 4.2$
1.95	1.772	1.811	1.840	1.862	1.880	1.893	1.904	1.912
2.00	1.831	1.868	1.896	1.918	1.934	1.947	1.957	1.965
2.1	1.946	1 981	2.008	2.027	2.042	2.054	2.063	2.070
2.2	2.059	2.093	2.117	2.135	2.149	2.160	2.168	2.175
2.3	2.171	2 202	2.225	2.242	2.255	2.265	2.272	2.278
2.4	2.281	2.311	2.332	2.348	2.360	2.369	2.376	2.381
2.5	2.390	2.418	2.438	2.453	2.464	2.472	2.478	2.483
2.6	2.498	2.524	2.543	2.557	2.567	2.575	2.581	2.585
2.7	2.605	2.630	2.648	2.661	2.671	2.678	2.683	2.687
2.8	2.711	2.735	2.752	2.764	2.773	2.780	2.785	2.788
2.9	2.817	2.840	2.856	2.865	2.876	2.882	2.886	2.890
3.0	2.922	2.944	2.959	2.970	2.978	2.983	2.988	2.991
3.5	3.441	3.459	3.471	3.479	3.485	3.489	3.492	3.494
4.0	3.954	3.969	3.978	3.985	3.990	3.993	3.995	3.996
4.5	4.463	4.475	4.483	4.489	4.492	4.495	4.496	4.497
5.0	4.969	4.980	4.987	4.991	4.994	4.996	4.997	4.998
6.0	5.978	5 986	5.991	5.994	5.996	5.998	5.998	5.999
7.0	6.983	6.990	6.994	6.996	6.998	6.998	6.999	6.999
8.0	7.987	7.992	7.995	7.997	7.998	7.999	7.999	
9.0	8.989	8.994	8.996	8.998	8.999	8.999		
10.0	9.991	9.995	9.997	9.998	9.999	9.999		

PLATES

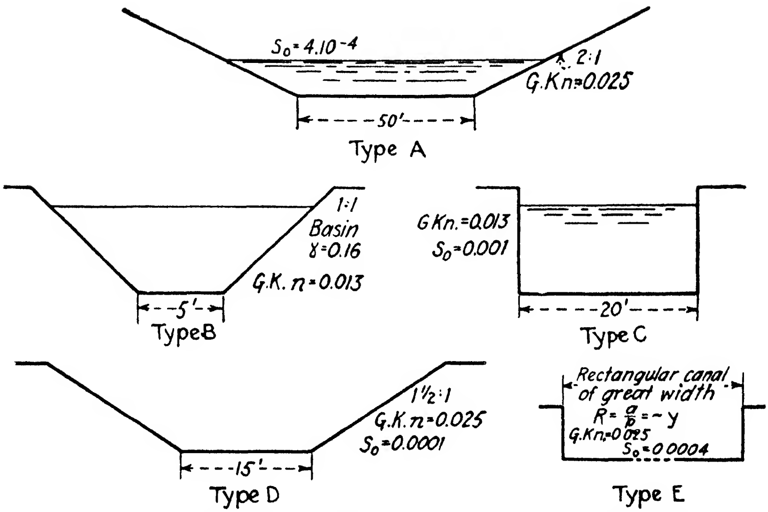

PLATE I.—Types of canal cross-sections used for practical examples.

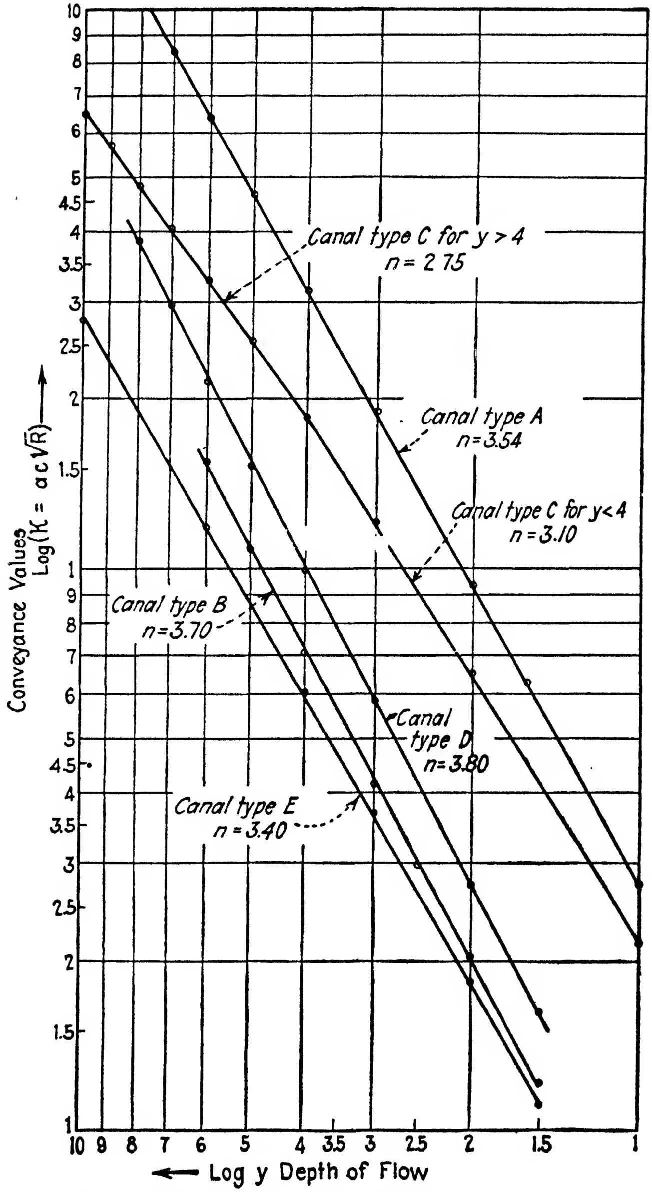

Plate II.—Logarithmic plotting of the conveyance curve $K = aC\sqrt{R}$ for canal cross-sections Plate I.

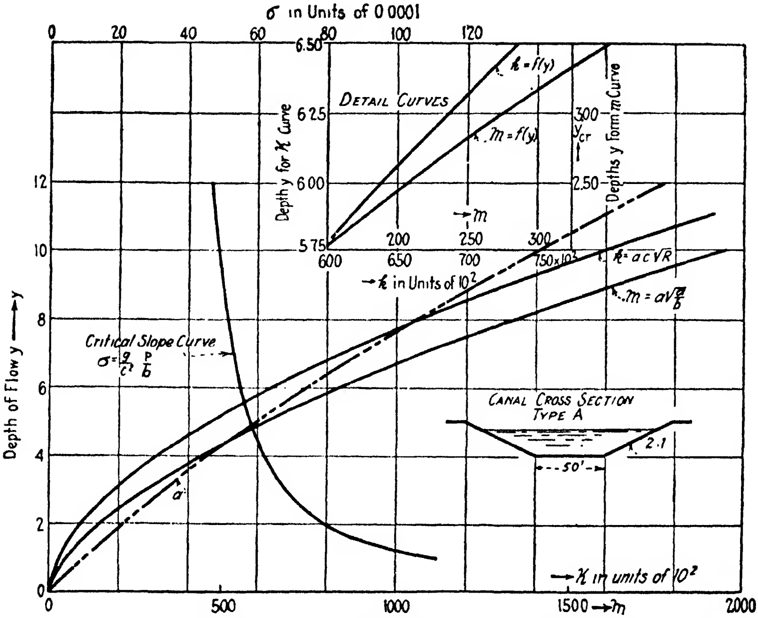

Plate III.—Characteristics for canal cross-section Type A.

y	a	b	p	R	C	$K = aC\sqrt{R}$ in units of 10^2	$M = a\sqrt{a/b}$	p/b	g/C^2 in units of $1 \cdot 10^{-4}$	σ in units of $1 \cdot 10^{-4}$
1.0	52.00	54.0	54.48	0.95	54.0	27.35	51.0	1.011	112.0	113.0
1.6	85.12	56.4	57.17	1.49	61.0	63.40	104.7	1.013	87.0	88.8
2.0	108.00	58.0	58.96	1.83	64.0	93.70	147.8	1.015	80.0	81.3
3.0	168.00	62.0	63.44	2.65	69.5	190.00	277.0	1.020	67.0	68.3
4.0	232.00	66.0	67.92	3.42	73.0	313.00	436.0	1.025	61.0	62.5
5.0	300.00	70.0	72.40	4.15	76.0	465.00	621.0			
6.0	372.00	74.0	76.88	4.85	78.0	639.00	834.0	1.039	53.0	55.1
7.0	448.00	78.0	81.36	5.51	80.0	842.00	1,073.0			
8.0	528.00	82.0	85.34	6.20	81.5	1,068.00	1,340.0	1.041	49.4	51.3
9.0	612.00	86.0	90.32	6.78	83.0	1,322.00	1,637.0			
10.0	700.00	90.0	94.80	7.40	84.0	1,600.00	1,952.0	1.053	45.5	48.0
11.0	792.00	94.0	99.30	7.98	85.0	1,903.00	2,300.0			
12.0	888.00	98.0	103.70	8.56	86.0	2,230.00	2,665.0	1.060	43.5	46.1
13.0	988.00	102.0	108.20	9.14	87.0	2,600.00	3,070.0			

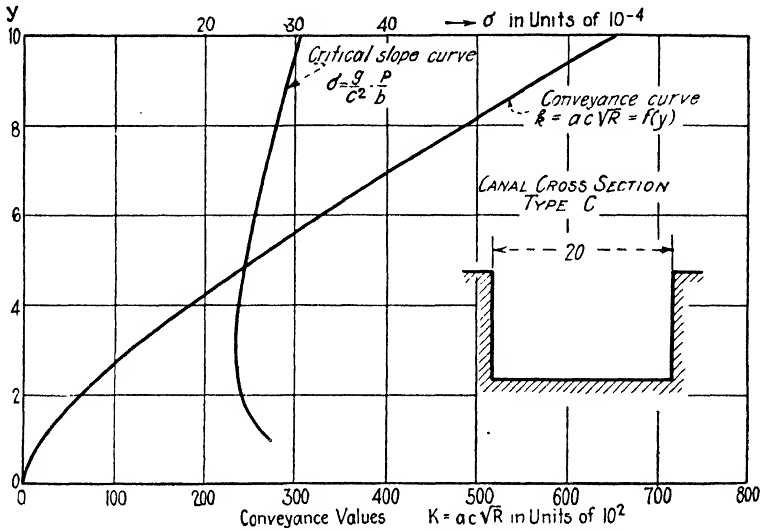

PLATE IV.—Characteristics for canal cross-section Type C.

y	a	p	R	C	$K = aC\sqrt{R}$ in units of 10^2	p/b	g/C^2 in units of $1 \cdot 10^{-4}$	σ in units of $1 \cdot 10^{-4}$
1	20	22	0.910	114.0	21.6	1.10	24.9	27 40
1.5	30	23	1.305	121.0	41.5	1.15	22.0	25.30
2	40	24	1.660	126.0	65.0	1 20	20.4	24.45
3	60	26	2.300	133.0	121.0	1.30	18 1	23.50
4	80	28	2 860	137.0	185.5	1.40	17.0	24.00
5	100	30	3.330	139.5	254.5	1 50	16.6	24.90
6	120	32	3.750	141.5	329.0	1.60	16.0	25.80
8	160	36	4.450	144.0	486.0	1.80	15.7	28.10
10	200	40	5 000	146.0	653.0	2.00	15.2	30.40

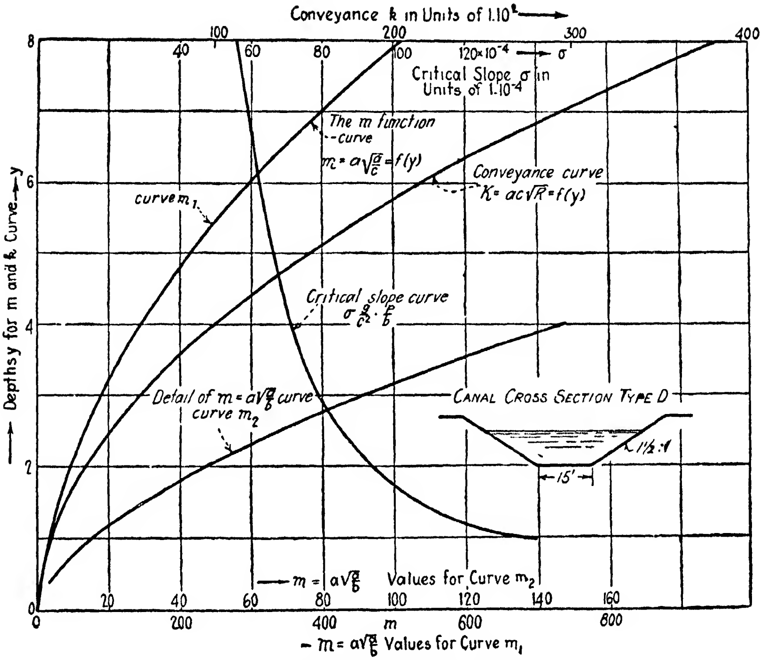

PLATE V.—Characteristics for canal cross-section Type D.

y	a	b	p	R	C	K in units of $1 \cdot 10^2$	$\mathfrak{M}$	σ in units of $1 \cdot 10^{-4}$
1.0	16.50	18.0	18.6	0.888	50.0	7.78	15.82	138.0
1.5	25.87	19.5	20.4	1.268	56.0	16.30	29.85	107.2
2.0	36.00	21.0	22.2	1.620	60.0	27.50	47.10	94.3
3.0	58.50	24.0	25.8	2.260	66.0	58.20	91.50	79.4
4.0	84.00	27.0	29.4	2.860	70.0	99.40	148.20	71.4
5.0	112.50	30.0	33.0	3.410	73.0	152.00	218.00	66.5
6.0	144.00	33.0	36.6	3.940	75.5	215.50	300.00	62.5
7.0	178.50	36.0	40.2	4.440	78.0	294.00	398.00	59.0
8.0	216.00	39.0	43.8	4.930	80.0	384.50	509.00	56.5

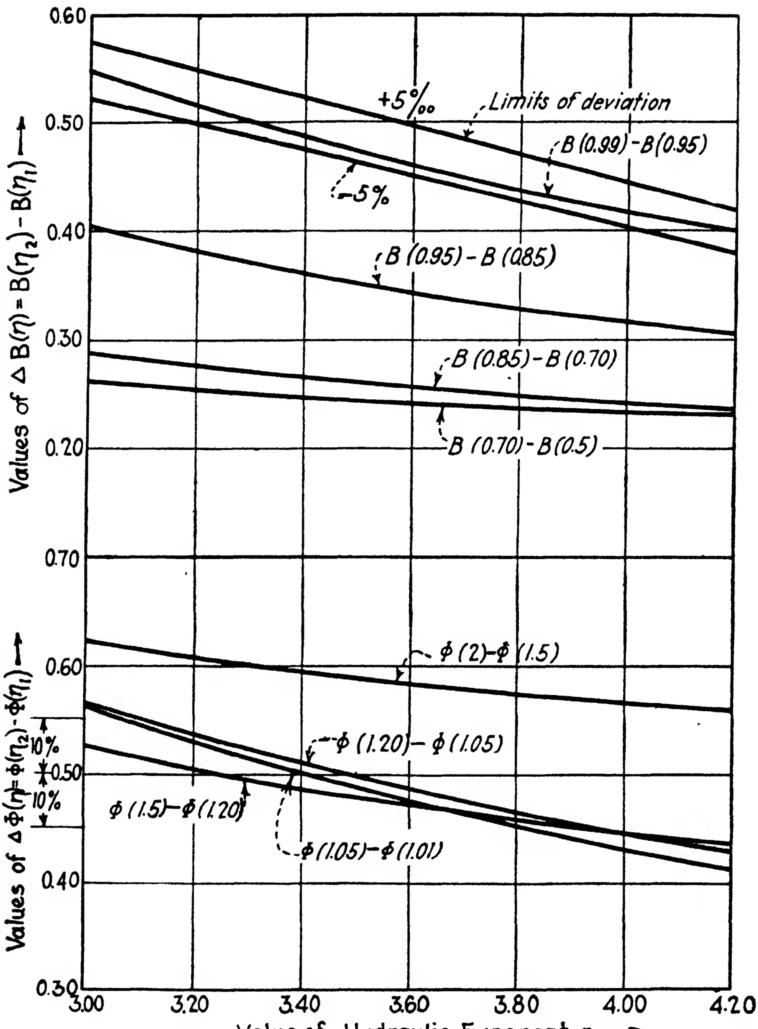

PLATE VI.—Illustrating the precision of computations by means of the *Varied Flow Function Tables.*

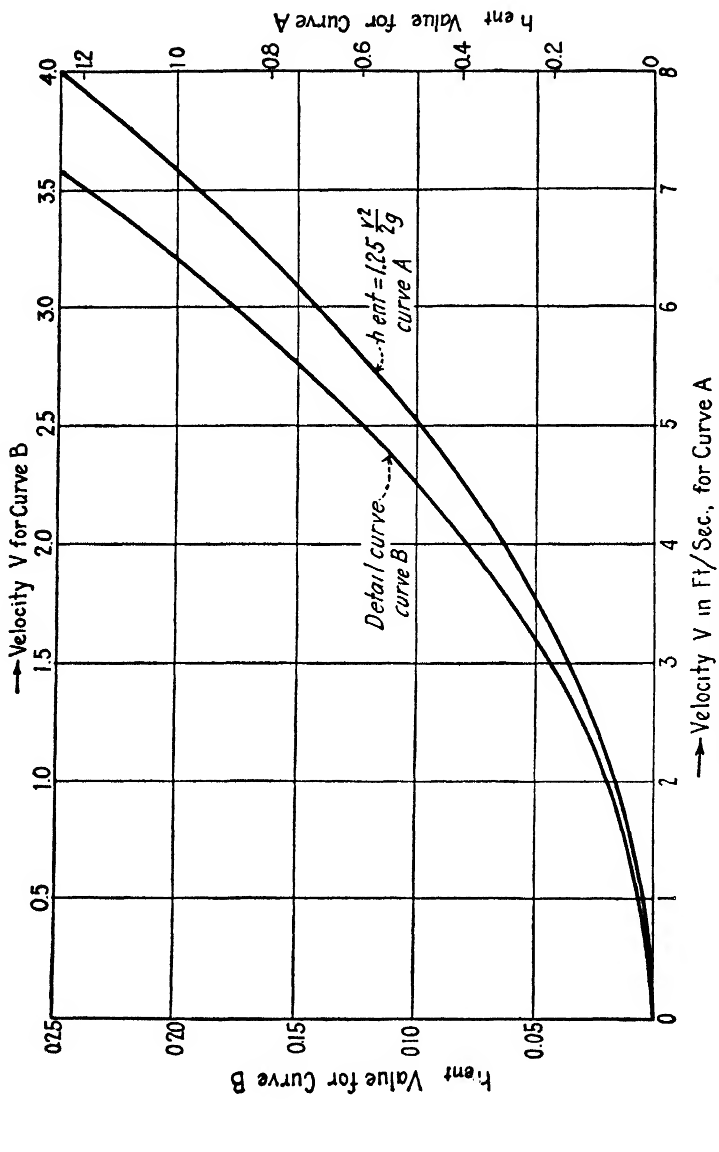

PLATE VII.—Entrance head with the value of the coefficient $1/\varphi^2 = 1.25$.

INDEX

(Names of authors are set in *italics*. For further references see "Historical and Bibliographical Notes," page 299.)