ADVANCED EXPERIMENTS IN PRACTICAL PHYSICS

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SECOND EDITION

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PREFACE

It is no easy matter to write a laboratory handbook of advanced experiments in physics which shall make a wide appeal—it is commonly assumed that such exercises demand specialised instruments, and that any one laboratory should prepare for its own use schedules of experiments written round its own apparatus.

Mr. Calthrop's book demonstrates very clearly the practicability of providing exercises, some of which shall at once illustrate interesting points in advanced physical theory and be capable of giving reasonably accurate results with simple equipment. Many of the exercises do demand the use of instruments of precision, and in these exercises Mr. Calthrop has very wisely restricted himself to the description of such interferometers, bridges and the like as no reasonably well-equipped laboratory is likely to lack.

Some of the experiments are of the author's own devising; some are taken from the pages of journals not readily accessible; some are old friends. All are interesting, and the book, which should prove very helpful to those students who have to make the most of limited resources, may be regarded, even in these days when books pour from the world's presses in an ever-increasing flood, as fulfilling a very definite need.

ALLAN FERGUSON.

Queen Mary College,
March, 1938.

PREFACE TO THE REVISED EDITION

In this revised edition of the late Mr. Calthrop's book the main alterations and additions are in the section on Electricity and Magnetism. There are new experiments on the use of the triode valve as a voltmeter and on the determination of the dielectric constant and conductivity of materials by the resonance method. Quincke's method for the susceptibility of a solution is described. Two radioactivity experiments, on the stopping power of aluminium for α particles and on the decay of actinium are included. The use of the cathode-ray oscillograph to measure phase differences in a.c. circuits and the techniques of chemical silvering and evaporation are described. In the Light section are two new exercises on thick lenses. Other experiments have been added to or partially re-written and a few minor corrections made in the text.

I should like to acknowledge the helpful suggestions I have received from my colleagues at Queen Mary College. My thanks are also due to Miss G. J. Brown, B.A., for typing the manuscript so efficiently.

J. A. PRYDE.

Queen Mary College,
January, 1952.

INTRODUCTION

THE following course of practical experiments is the result of some years' experience in teaching Honours students in London University, and the modest hope of the author is that some of the work which has been found by him to be interesting and useful may be of value to other teachers and students of the very wide subject of physics. In these days of rapidly growing knowledge and highly specialised technique it becomes important to enquire what our students are to be taught and how much they can understand and use. It is hoped that success has been achieved in striking a balance between the older classical work and the new physics of the "electron" era. An attempt has been made to give a bird's-eye view of the whole subject, to give practice in the techniques of its several different branches, to encourage the use and design of simple and inexpensive apparatus, and to teach the student to make full use of graphical methods for the representation of his results and to draw appropriate deductions therefrom. The requisite theory has been kept to a minimum.

The experiments are grouped in three sections :

I. Properties of Matter and Heat,
II. Light,
III. Electricity.

In Section I there is to be found a selection of exercises, some of which are new, upon elastic constants, surface-tension, resonance, the use of an air-pump and specific heats, etc. Some of these involve practice in elementary glass-work, soldering and the simpler laboratory arts.

In Section II, the student is taught to determine the principal constants of a lens system, and to gain some idea of aberration and its measurement. Interferometry is fairly fully treated. Michelson's famous distant-slit method is described and the photo-electric cell is introduced in order to make clearer the phenomenon of elliptical polarisation. An exercise upon the photography of spectra is added, with some explanation of the calculation of the wave-lengths of unknown lines.

In Section III there are a few experiments of a classical type :

then valve curves, and the applications of a valve as a magnetron and as an oscillator are given. A method for the determination of critical potentials is described together with a few alternating-current-bridge measurements and one or two experiments on electrical oscillations. Experiments on β and γ rays are mentioned and the use of the quadrant electrometer for the measurement of an ionisation-current explained. A number of experiments with a photo-electric cell, with descriptions of suitable circuits, and experiments with an X-ray-spectrometer and a cathode-ray oscillograph bring the course to an end.

It is hoped that students who work through the major part of the course will not be altogether ignorant of modern trends and will also have a grasp of the subject as a whole.

The book concludes with a collection of additional problems and a list of useful works upon the practical teaching of physics.

ACKNOWLEDGMENTS

WHILE taking full responsibility for any possible errors of fact or method, I should like to acknowledge with grateful thanks the interest and help of a number of friends.

I am much indebted to Professor H. R. Robinson, in whose laboratory many of the experiments have been tested, to Professor A. Ferguson for permission to include new methods in the measurement of surface-tension and specific heat and for reading the manuscript, and to Mr. H. Thirkill, of Clare College, Cambridge, for permission to include an experiment with an étalon.

An expression of very special gratitude is due to Dr. F. Sherwood Taylor for his enthusiastic help. Thanks are also due to the Editors of the *School Science Review* and the *American Physics Teacher* for permission to use articles which have appeared in their journals. Finally, may I offer my thanks to all old students who have helped to test these exercises over a period of years.

J. E. C.

QUEEN MARY COLLEGE,
1938.

INSTRUCTIONS TO STUDENTS

STUDENTS should distinguish clearly between *doing* experiments in a perfunctory manner and *experimenting* with a view to the formation of habits of patience and precision. When examinations are important there is sometimes a tendency to collect results over a wide field without making the most of any one investigation. Many observations should be made, and nothing less than a careful and accurate piece of work should be accepted as adequate.

Apart from reading instruments, taking measurements and getting results, there is much to be learned in the technique of experimenting, in the improvising of simple apparatus and in performing mechanical operations. Such skill and acquirement are not easily tested by examinations, but are essential if further research work is to be carried on in the subject. The object of the present volume is to give the student exercises in the various branches of physics with some opportunities for practice in manipulation. Some advice is therefore offered upon necessary equipment, upon the performance of an experiment and upon the treatment of results.

Materials and Equipment

The student should provide himself with the following material and equipment :—

1. A few simple tools, such as a pocket screwdriver, safety-razor blade, soldering-outfit, etc.
2. A few simple materials, such as sealing-wax, plasticine, shellac, cardboard, glass tubes, etc.
3. A good variety of tools for wood and metal work. Access to a lathe is desirable.

Laboratory Arts

An attempt should be made to learn the rudiments of glass-blowing, but this is best done by watching a demonstrator or glass-blower in operation and then trying to imitate the manipulations observed. The student should learn to join tubes of the same diameter, to join those of very different diameters, to make a T-piece, to bend tubes, to make a manometer-tube, to blow a bulb, etc. Care should be taken with apparatus. Simple

repairs and re-adjustments of instruments should be done by the student, and not left to a laboratory steward. Apparatus, particularly glassware, should be kept clean. A good cleaning solution is made by adding concentrated sulphuric acid to a solution of potassium dichromate. Clocks and watches should be tested from time to time against a standard chronometer: cheap laboratory clocks are usually quite inaccurate.

The Experimental Course

With regard to the order of the experiments chosen, it is as wise to do a few exercises in one branch of the subject and then a few in another. In this way one can keep in practice without becoming stereotyped. In interferometry in particular it is well to do one or two exercises, then to rest the eyes, and do a few more perhaps a month later. In the same way one may do a little glass-blowing, and then a month afterwards do some more and see how much improvement has been made in the interval.

Performance of the Experiment

The student should possess a rough book having pages with squared paper on one side, and should enter therein *all* measurements at the time they are made. The following hints may be useful.

1. Arrange the apparatus so that observation will be as comfortable as possible, for the more comfortable the observer the more reliable will the results tend to be. Indeed, one professor has said, "I first put down the microscope in a comfortable position and then build the rest of the apparatus round it." It is not always possible to follow this example literally, but at least an attempt may be made to approach it.

2. Do not always rest content with the apparatus provided. Sometimes parts are missing, and at other times the student can suggest better ways of carrying out a piece of work. Do not look upon an experiment as static, but rather as the start of a train of thought which may very well lead to another investigation.

3. It is frequently wise to do a preliminary rough experiment to make sure that all the apparatus is functioning properly and to accustom oneself to the best way of taking the observations.

4. Record all the readings taken, even if they have not an immediate bearing on the results. Do not subtract two measurements mentally and record only the result as an observation. Temperature and pressure are sometimes unrecorded and are

found in the end to have a vital influence on the result. Times of exposure, times of development, etc., should be noted, and may be useful for future reference.

5. Take a large number of observations, spread over as wide a range as possible of the variables concerned, so that a mean may be calculated or the results employed to draw a suitable graph.

6. Do not be afraid of obtaining disappointing results, for they are often more interesting than the expected result for a conventional experiment.

Treatment of Results

Students should have a record book, having pages with squared paper on one side, for the purpose of keeping a final and permanent record of the work done. Some students use the loose-leaf system, but in the opinion of the writer a well-bound book is to be preferred. If the former system is used, experiments may be re-arranged under subjects. In the book enter the title of the experiment, a short description of the aim and method, illustrated by good clear diagrams. Record the measurements made, and whenever possible tabulate the data and results so that they can be read with ease. Whenever possible, represent the results graphically and try to obtain all possible information from the curve drawn.

In the present book, exercises are given in which the following curves are encountered :—

(1) $y = mx + c$ (2) $y = Ax^n$ (3) $y = A + B/x^2$
(4) $y = Ae^{-kx}$ or $\log y = \log A - k \log x$. (5) $y = ax^n - b$.

Hints are given on the derivation of the constants entering into these equations.

Assess the accuracy of the final result, and do not be over optimistic. For example, if

$$P = \frac{xy}{z}, \text{ then}$$

$$\partial P = \pm \frac{y}{z} . \partial x \pm \frac{x}{z} . \partial y \mp \frac{xy}{z^2} . \partial z.$$

Dividing by P,

$$\frac{\partial P}{P} = \pm \frac{\partial x}{x} \pm \frac{\partial y}{y} \mp \frac{\partial z}{z}.$$

The + and − signs indicate that the errors may be either plus

or minus, and thus in the present example the percentage error in P may be the sum of the percentage errors in x, y and z. Sometimes the errors in two variables just about cancel and give a false impression of one's skill as an experimenter. If a large number of results are obtained, application may be made of the theory of errors. For this part of the subject the student is recommended to consult W. N. Bond's *Probability and Random Errors* (Arnold and Co.).

Finally, pay attention to the units and dimensions of the result. Consider whether the result is a pure number or whether it has units and, if so, what these are. Do not be content to put C.G.S. in order to save a little thought.

The best students teach themselves to a large extent, but the above advice may help the good experimenter to become a better one.

The Drawing of Graphs

In some experiments it is essential to draw a good graph by means of a spline or other device for getting a smooth curve. The spline may consist of a strip of wood cut from an old ruler and shaved into wedge-form so that it may be bent into a non-circular curve. An artists' supply shop can also provide a device in the form of a rubber tube containing lead, which will retain the shape of any curve into which it is bent. If the tangents to a curve have to be drawn it will not be accurate enough simply to place a scale on the curve and to draw a line to touch the curve at what is apparently the point of contact. Two methods are here mentioned.

In Fig. 1 is represented a curve $A\ C\ D\ B$ for which a number

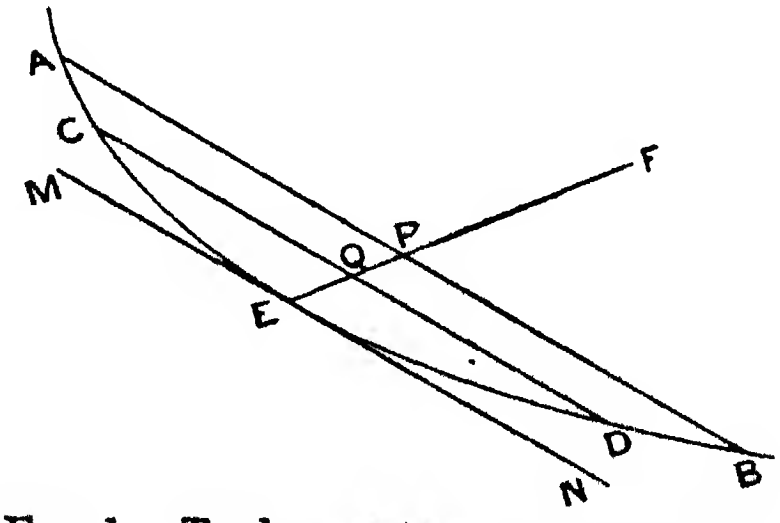


Fig. 1. To draw a tangent: method 1.

of tangents have to be drawn. Two parallel chords $A\ B$ and $C\ D$ are drawn and these are bisected at P and Q respectively.

SECTION II. LIGHT

SECTION III. ELECTRICITY AND MAGNETISM

ADVANCED EXPERIMENTS IN PRACTICAL PHYSICS

SECTION I. PROPERTIES OF MATTER AND HEAT

1. Determination of Young's modulus by 's Gravesande's method

A wire about a metre in length is securely fastened at the ends by two rigid clamps A and B as shown in Fig. 4. To the centre of the wire, which is initially horizontal, is attached a scale-pan

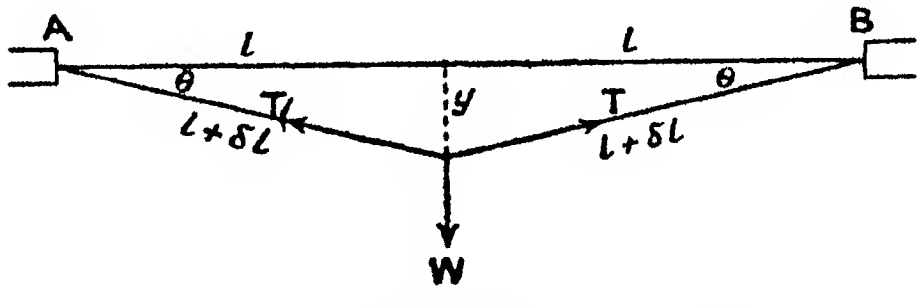


Fig. 4. 's Gravesande's method.

in which is placed a weight W. From the depressions y of the centre of the wire as measured by means of a microscope with a vertical traverse, for different loads W, it is possible to determine the value of Young's modulus for the material of the wire.

The theory is as follows :—

Let $2l$ be the initial length of the wire under a tension T_0, and $2(l + \delta l)$ be the length when a tension T is produced in the wire owing to the presence of a weight W in the scale-pan. Let a be the cross-section of the wire and q be Young's modulus for the material. θ is the small angle made by the wire with the horizontal at A and B.

$$\text{Then} \quad W = 2T \sin\theta \; ; \; q \therefore \frac{\delta l}{l} = \frac{T - T_0}{a}$$

$$\sec\theta = \frac{l + \delta l}{l} = 1 + \frac{\delta l}{l} = 1 + \frac{T - T_0}{qa}$$

$$\therefore \; T = T_0 + qa\,(\sec\theta - 1)$$

$$\therefore\ 2\sin\theta(T_0 + qa\sqrt{\sec\theta - 1}) = W$$

$$\sec\theta = (1+\tan\theta)^{\frac{1}{2}} = 1 + \frac{y^2}{2l^2}(\text{neglecting } \frac{y^4}{l^4}, \text{ etc.})$$

and to the same order

$$\sin\theta = \frac{y}{l}\left(1 - \frac{y^2}{2l^2}\right)$$

$$\therefore\ W = \frac{2y}{l}\left(1 - \frac{y^2}{2l^2}\right)\left(T_0 + qa\,\frac{y^2}{2l^2}\right)$$

$$\therefore\ \frac{Wl}{2y} = \frac{qay^2}{2l^2} + T_0\left(1 - \frac{y^2}{2l^2}\right)(\text{neglecting } \frac{y^4}{l^4}, \text{ etc.}).$$

If $L = 2l$

$$\frac{W}{y} = y^2\left(\frac{8qa}{L^3} - \frac{8T_0}{L^3}\right) + \frac{4T_0}{L}.$$

Let $y^2 = X$ and $\frac{W}{y} = Y$ and then

$Y = AX + B$ represents a straight line for which

$$A = \frac{8qa}{L^3} - \frac{8T_0}{L^3} \text{ and } B = \frac{4T_0}{L}.$$

Thus if A and B are determined from the appropriate straight-line plot, then

$$T_0 = \frac{BL}{4} \text{ and } q = \frac{AL^3 + 2BL}{8a}.$$

The wire may be of a diameter 0·5 mm. and its mean section may be found by weighing say 100 cm. in air and water, and hence finding the volume and a. Hence q may be calculated.

As an additional exercise an electric current may be passed through the wire and the loading operations repeated. The resistance of the wire of known temperature coefficient of resistance may be measured at air temperature and when electrically heated. From these results the effect of temperature upon Young's modulus may be calculated.

2. To find Young's modulus and Poisson's ratio for a specimen of glass by Cornu's method

A strip of plate-glass about 12 inches long and 2 inches wide is placed on two parallel glass rods some 2 or 3 inches apart

(Fig. 5). At the ends C and D of the plate is attached a wooden bar E F upon which may be placed various weights W. Upon and above the centre of the plate is placed a small plane piece of good plate-glass G. The air-film between the two glass surfaces

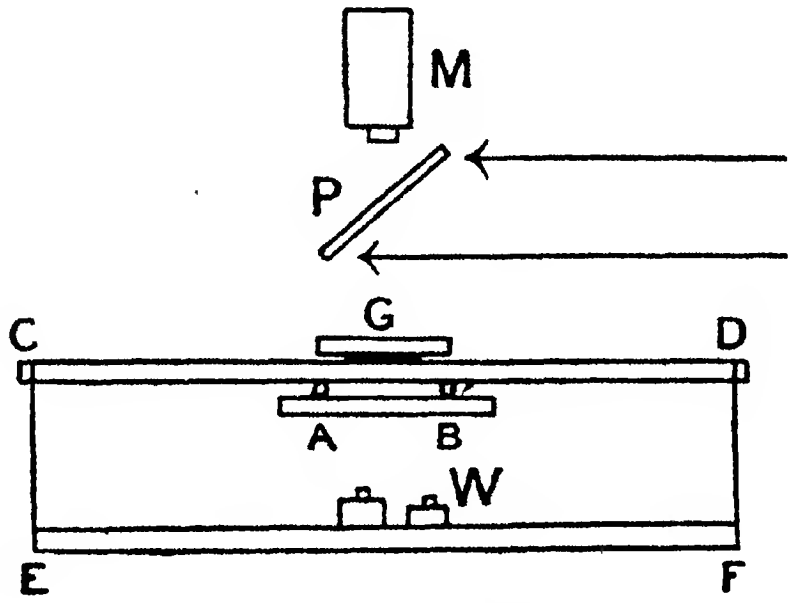


FIG. 5. Cornu's method.

is illuminated by sodium- or mercury-light reflected from a glass plate P, inclined at 45° to the horizontal as in the usual way

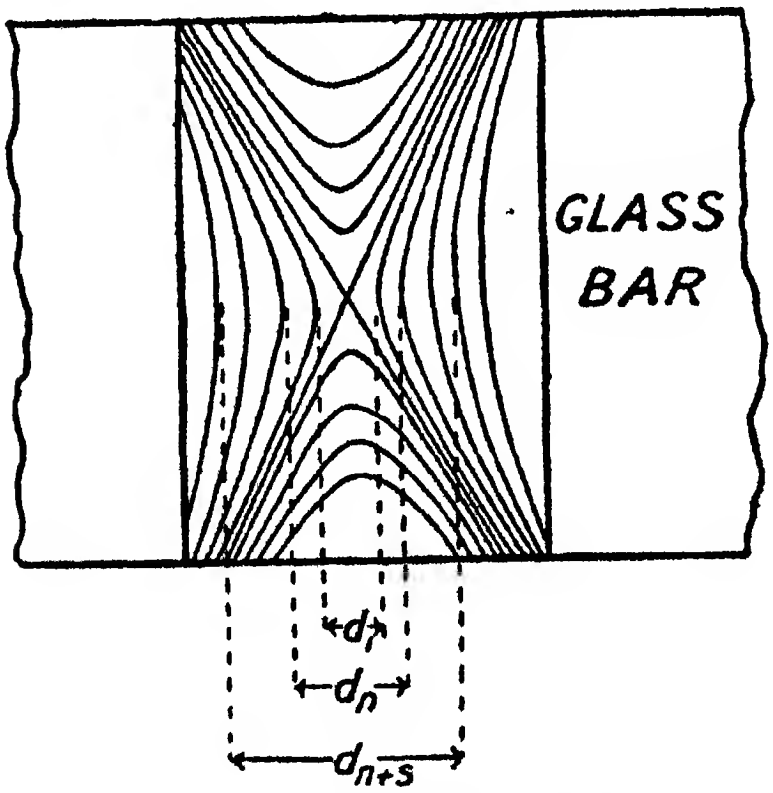


FIG. 6. Cornu fringes.

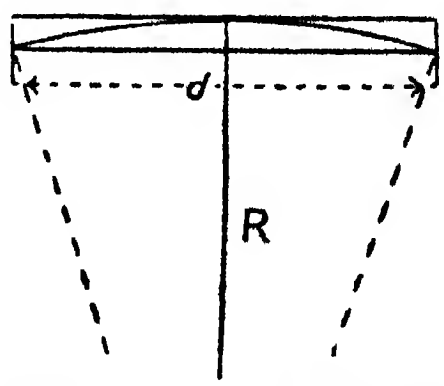


FIG. 7. The radius of curvature.

of observing Newton's rings. A vertical microscope M is employed to view the fringes, which in this experiment are hyperbolic, due to anti-clastic bending as the load W is applied. The appearance of the fringes is indicated in Fig. 6. The theory of the fringe-formation is similar to that for Newton's rings.

If in Fig. 6 d_{n+s} and d_n are the distances apart of the two arms

of two hyperbolas respectively and R in Fig. 7 is the corresponding radius of curvature of the plate, then for a bright fringe $R = \frac{d_{n+s}^2 - d_n^2}{4\lambda s}$ where λ is the wave-length of the light employed. s may be taken as some convenient number, say 10 (although in the diagram, for simplicity, n is 2 and $n + s$ is 4), and the readings grouped so as to obtain a number of sets of 10. The mean of a series of values for $d_{n+s}^2 - d_n^2$ is found and hence R and $\frac{1}{R}$ may be calculated for any load.

The bending moment for the plate is given by $M = \frac{qAk^2}{R}$ where q is Young's modulus of the material, Ak^2 is the moment of inertia of the cross-section of the plate about the trace of the neutral section, and R is the radius of curvature as above.

There is a small initial curvature owing to the weight of the plate itself. An appropriate way of dealing with the results is to plot $\frac{1}{R}$ against the load W of moment M as shown in Fig. 8. Then if W_1 and W_2 are two loads giving moments M_1 and M_2 and R_1 and R_2 the corresponding curvatures, the relation

$$M_1 - M_2 = qAk^2 \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

enables q to be found. In other words tan θ_1 in Fig. 8

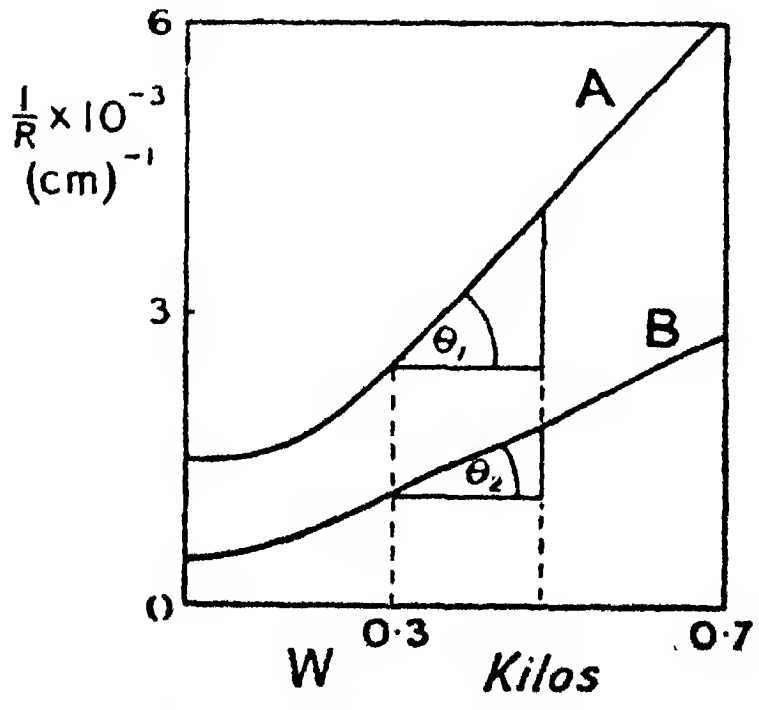


FIG. 8. Graphical method.

leads to q. Similarly the other system of hyperbolas may be measured and the results represented by Curve B. It is shown in works on elasticity, such as Thomson and Tait's *Natural*

Philosophy, that Poisson's ratio σ is given by the ratio of the curvatures in perpendicular planes. Thus for the same values M_1, M_2

$$\sigma = \frac{\frac{1}{R_1'} - \frac{1}{R_2'}}{\frac{1}{R_1} - \frac{1}{R_2}} = \frac{\tan\theta_2}{\tan\theta_1}$$

Instead of using a plane glass G a convex lens of large radius may be substituted and the fringes are then ellipses. The calculation in this method is left as an exercise for the student.

REFERENCES

CORNU : *C. R.*, p. 333, 1869.
THOMSON and TAIT : *Natural Philosophy*, pp. 263–264.
JESSOP : *Phil. Mag.* (6), **42**, p. 551, 1921.

3. THE VARIATION WITH TEMPERATURE OF YOUNG'S MODULUS FOR A METAL

The metal to be investigated is taken in the form of a wire about 0·15 cm. in diameter, and is made into a ring of some 25 cm. in diameter (Fig. 9). The ring is split and the ends joined to insulating cylinders C of bakelite, so that it can be heated electrically. The mean temperature of the wire may be determined from the variation of the resistance as found from the current and from the potential difference between the terminals A B. Alternatively, the resistance of the wire may be measured by means of a Callendar and Griffiths bridge. Appropriate loads M may be applied to the lowest point of the ring.

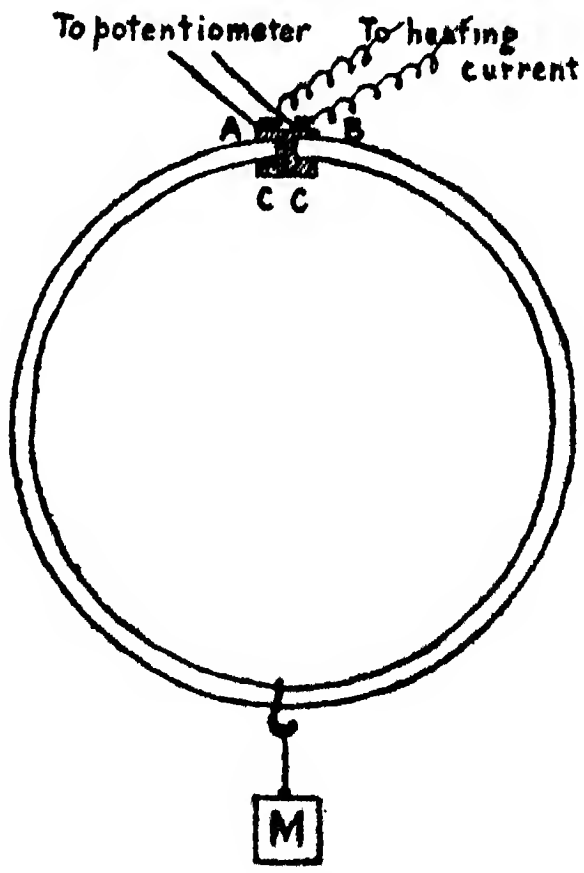


FIG. 9. Loaded ring.

Let Mg be the load in dynes ; r, the radius of the ring ; a, the radius of the wire ; d, the depression of the lowest point of the ring ; Ak^2, the moment of inertia of the wire section about a diameter ; and q, Young's modulus of the material. Then

$$q = 0{\cdot}298\ Mg\ r^3/2dAk^2.$$
$$= 0{\cdot}19\ Mg\ r^3/da^4.$$

For the main purpose it is noticed that q varies inversely as d. From a series of depression-load curves obtained for different temperatures of the ring, all the required information may be deduced.

In Figs. 10 and 11 are shown typical results for a nichrome

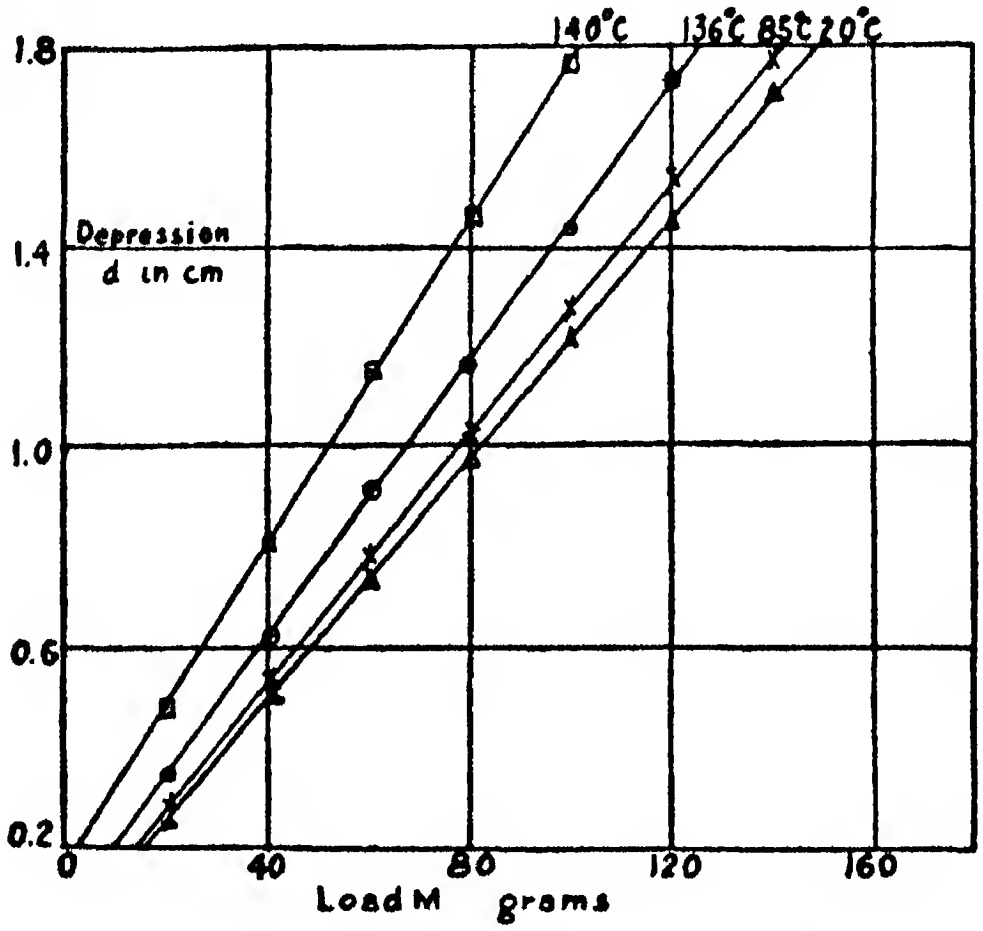


FIG. 10. Depression-load curves.

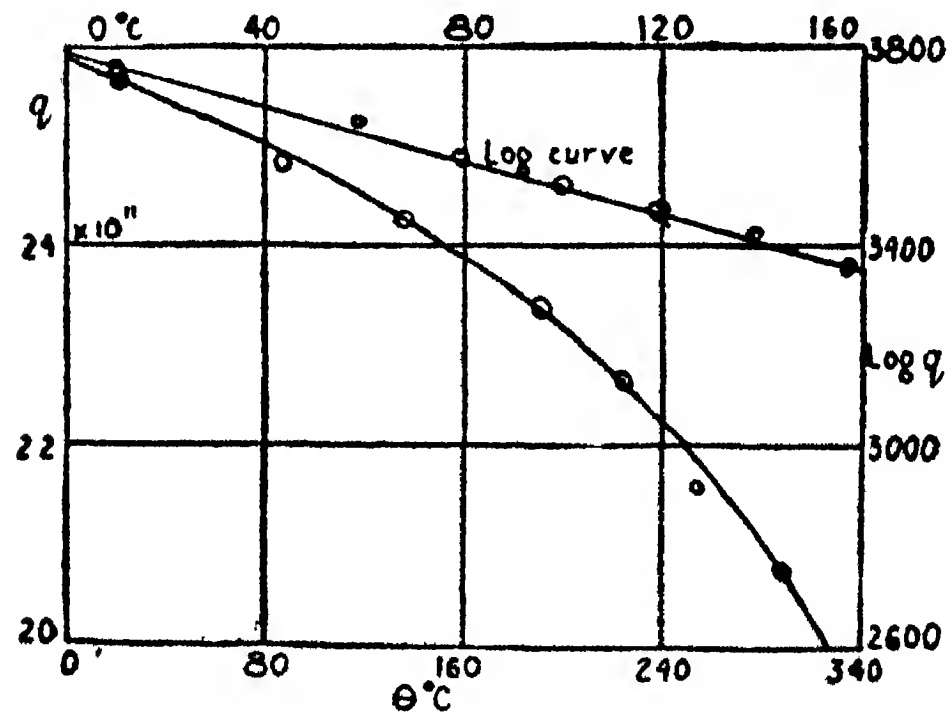


FIG. 11. Graphical representation.

specimen, for which r and a were 24·8 and 0·075 cm. respectively. If it is assumed that q is equal to $q_0e^{-K\theta}$ where K is a constant

and θ is the temperature, then on plotting $\log E$ against θ, a linear relation should be obtained. Also $-\frac{1}{q}.\frac{dq}{d\theta}$ should be constant. In Fig. 11 it will be noticed that a reasonably straight line is obtained.

The following values are obtained for K : nichrome, 49 ; brass, 34 ; steel, 25 ; aluminium, 19 ; phosphor-bronze, 34 ; tungsten, 8·6 ; manganin, 19 $(°C)^{-1}$, (all $\times 10^{-5}$). The change of dimensions due to expansion may be neglected, for this is small, except for aluminium.

REFERENCE

CALTHROP and MILLER : *Am. Phys. Teacher*, **3**, pp. 131–132, 1935.

4. TO DETERMINE POISSON'S RATIO FOR RUBBER

A piece of rubber cycle-tyre tube about 1 metre long is taken and closed securely at the lower end A by means of a rubber stopper and seccotine (Fig. 12). The upper end B also has a rubber stopper, but this is bored to take a piece of glass tube about 40 cm. long and 1 cm. in diameter. The rubber tube is filled with water so that a meniscus may be seen at the top of the glass tube, 30 cm. above the cork. Care should be taken that the glass tube does not project far below the stopper, for otherwise air bubbles may be trapped and spoil the accuracy of the observations. A weight W is applied at the lower end of the tube, and this has the effect of increasing the length of the tube and the internal volume. From the fall of the water in the glass tube, which may be calibrated, the change in volume may be found. Poisson's ratio may then be calculated from the measurements as follows :—

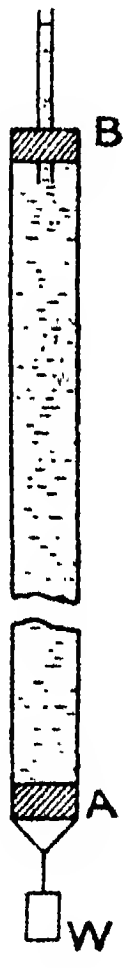


FIG. 12. Stretching a rubber tube.

Let V be the volume of the rubber tube, L its length, D its diameter and A its cross-sectional area. Then $V = LA$.

For a small change of volume δV accompanying an increase of length δL

$$V + \delta V = (L + \delta L)(A - \delta A)$$

$$\therefore\ \delta V = A\delta L - L\delta A \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Poisson's ratio $$\sigma = -\frac{\delta D}{D} \Big/ \frac{\delta L}{L} = -\frac{L}{D}.\frac{\delta D}{\delta L} \quad . \quad . \quad . \quad (2)$$

where the minus sign allows for the fact that D decreases as L increases.

Since $A = \frac{\pi}{4} D^2$, $\delta A = \frac{\pi}{2} . D . \delta D$ and $\frac{\delta A}{A} = 2 \frac{\delta D}{D}$.

$$\text{or } \frac{\delta D}{D} = \frac{1}{2} \cdot \frac{\delta A}{A}$$

$$\therefore \text{ From (2) } \sigma = -\frac{1}{2} \cdot \frac{\delta A}{A} \cdot \frac{L}{\delta L} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the ideal case of change of shape without change of volume

$$\delta V = 0, \ \frac{\delta A}{A} = \frac{\delta L}{L} \text{ and}$$

$$\sigma = -\frac{1}{2} \cdot \frac{\delta L}{L} \cdot \frac{L}{\delta L} = -\frac{1}{2}.$$

Putting $\delta A = \frac{A\delta L - \delta V}{L}$ from equation (1)

$$\sigma = -\frac{1}{2} \cdot \frac{\delta A}{A} \cdot \frac{L}{\delta L} = -\frac{(A\delta L - \delta V)}{2AL} \cdot \frac{L}{\delta L}$$

$$\therefore \quad \sigma = -\frac{1}{2} + \frac{1}{2A} \cdot \frac{\delta V}{\delta L} = -\frac{1}{2}\left(1 - \frac{1}{A} \cdot \frac{\delta V}{\delta L}\right) . \quad . \quad (4)$$

The second term shows the departure from the ideal case. Determine the values of δV and δL for various loads δW. Plot graphs for δV and δW and δL and δW. Find $\frac{\delta V}{\delta W}$ and $\frac{\delta L}{\delta W}$ and hence $\frac{\delta V}{\delta L}$. For rubber and other substances $\frac{\delta V}{\delta L}$ is positive, and therefore Poisson's ratio has a value less than $\frac{1}{2}$. In a typical experiment $L = 180$ cm., $A = 5{\cdot}3$ sq. cm., $V = 950$ c.c. A value of $\delta L = 15$ cm. corresponded to a volume change $\delta V = 22{\cdot}5$ c.c., giving a value for σ of 0·36. It is possible but more difficult to perform the experiment with a glass tube, when a fine capillary tube must be used to determine the change of volume.

5. To find the coefficient of viscosity and the critical velocity for water

The apparatus is shown in Fig. 13. A glass tube about 30 cm. long and 3 mm. bore is taken and two small holes A and

B are bored in it about 25 cm. apart. Glass T-pieces are placed over the tube and fixed by means of sealing-wax:

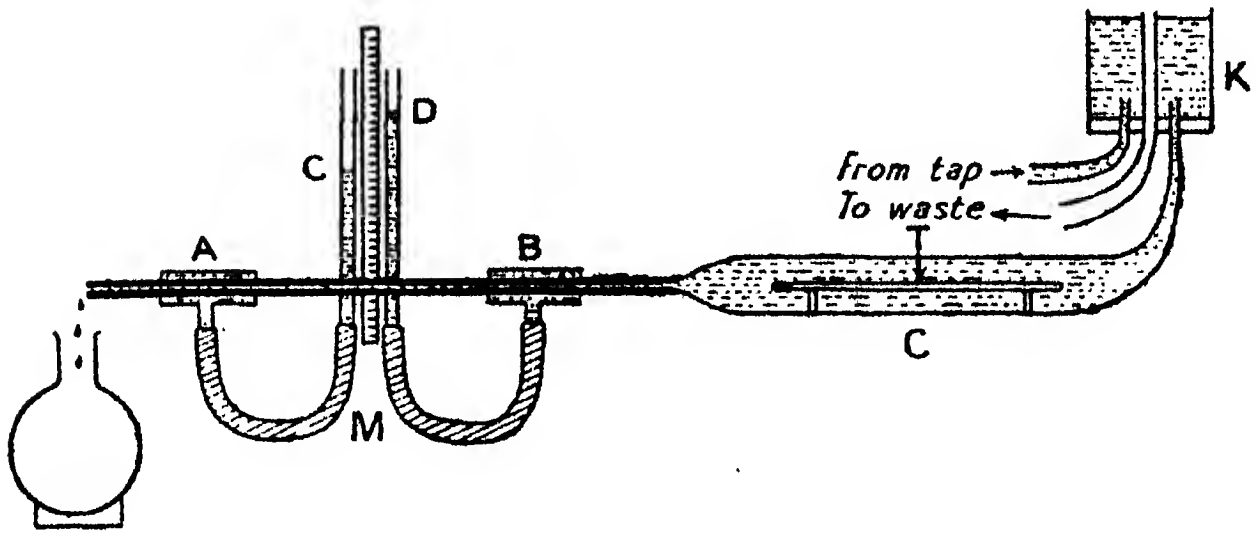


FIG. 13. Apparatus for measurement of viscosity.

rubber tubing leads from these to the arms *C* and *D* forming a manometer *M*. Water from a steady pressure-head arrangement *K* is passed through a wide tube *C* containing a thermometer *T*. It is important that the temperature be reasonably constant or, if not, that temperature corrections be made.

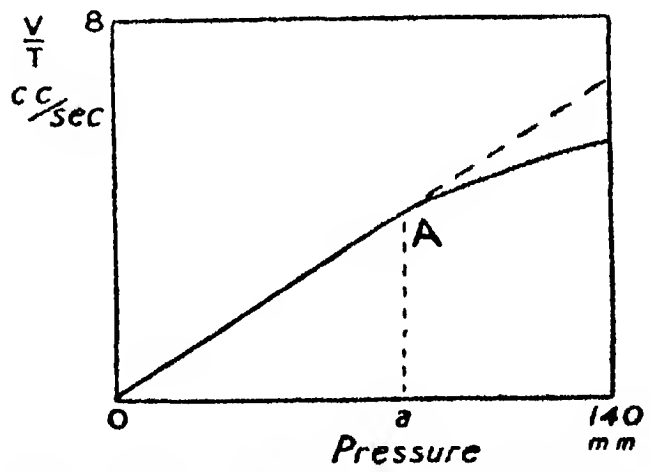


FIG. 14. Graphical representation.

The time for, say, 250 c.c. to flow through the tube is found for gradually increasing pressure-heads as read on the manometer. The volume flowing per second or per minute is then plotted against the pressure-head, as shown in Fig. 14. At a certain pressure (*a* in the diagram) the curve ceases to be rectilinear and the linear velocity corresponding to this departure from the straight line may be found.

Recent determinations show that the product of this critical velocity and the radius of the tube is about $1160 \times \frac{\text{viscosity}}{\text{density}}$.

The viscosity of water is calculated from the expression $\frac{V}{T} = \frac{\pi p a^4}{8\eta l}$ where p is the pressure, a the radius, l the length of the tube, η the viscosity and $\frac{V}{T}$ the flow in c.c. per second. As it is difficult to obtain tubes of sufficiently uniform bore, and as the fourth power of the radius occurs, it is advisable to break

up the tube into about ten sections and measure l_1, a_1; l_2, a_2, etc., for each part. Then find $\Sigma \frac{l}{a^4}$ and insert in the expression $p = \frac{8\eta}{\pi} \cdot \frac{V}{T} \Sigma \frac{l}{a^4}$, giving the total pressure difference. η may then be calculated. Determine also the dimensionless quantity called the Reynolds Number which is given as 1160 above.

6. To investigate the performance of a vacuum-pump and to test the validity of Gaede's equation

A vacuum-pump is characterised by its velocity of extraction, *i.e.*, the volume of gas the pump will take out per second under a given pressure, the volume being measured at that pressure.

Let V_0 be the volume of an enclosure containing a gas at pressure P. Let the volume become $V_0 + dV$ at pressure $P - dP$. Then by Boyle's Law

$$dV = \frac{V_0 dP}{P} \text{ at constant temperature.}$$

$$\therefore \qquad V = V_0 \log P + C.$$

If $V = 0$ when $P = P_0$, $C = -V_0 \log P_0$

$\therefore$ $V = V_0 (\log P - \log P_0)$.

Write $V = v \times t$, where

v = volume extracted per sec.

and t = time in secs.

Then $v = \frac{V_0}{t} \log P/P_0$ where P and P_0 are the pressures at two instants differing by t secs.

This is known as Gaedè's equation.

In the experiment a large flask A of some 5 litres capacity is

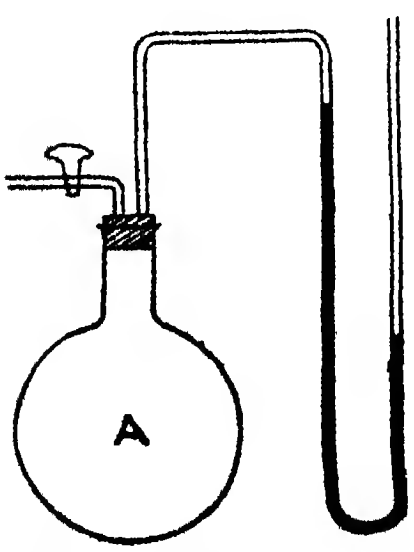


Fig. 15. Performance of an air-pump.

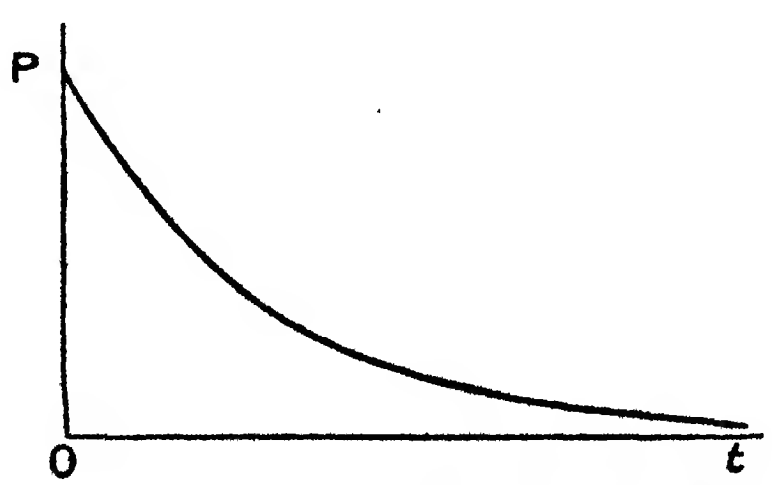


Fig. 16. Variation of pressure with time.

connected to the air-pump, which may be a glass or metal pump of the Sprengel type or a Hyvac oil-pump. The flask is also connected to a mercury manometer (Fig. 15). The pump is set to work and pressure readings are taken at convenient intervals of time: these are plotted as in Fig. 16. From any two near pressure readings it is possible to calculate the volume of air removed at this mean pressure and the time required is obtainable from the graph in Fig. 16. Thus for a series of values of vt the corresponding values of $(\log P - \log P_0)$ may be found. Hence Gaede's equation may be tested.

It is also instructive to allow the pump to exhaust the flask through capillary tubes of various diameters and to note the very large decrease in the velocity of extraction if the air has to pass through a tube of very fine bore.

7. The construction of a manometer to measure small differences in pressure and its use for the determination of the surface-tension of a soap solution

The manometer may consist of a large tube or vessel of sectional area A, joined to a finer tube of section a which is inclined at a small angle to the horizontal (Fig. 17).

If δp is the excess pressure on a liquid contained in the large

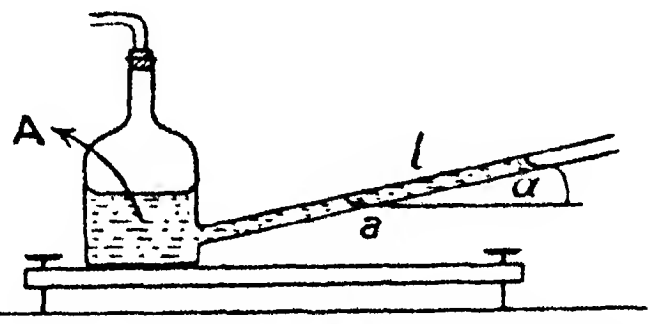


Fig. 17. Principle of sensitive manometer.

vessel, the liquid in the adjoining tube is displaced through a distance l, such that the depression in the large vessel is $\frac{la}{A}$. Thus if g is the acceleration due to gravity, d is the density of the liquid, and α is the angle of inclination of the tube,

$$\delta p = g\rho l\left(\frac{a}{A} + \sin\alpha\right).$$

To make l large, keep α small. If the tube is about 0·4 cm. in diameter and the large vessel is 4 cm. in diameter, a/A may be neglected for an accuracy of 1%, and $\delta p = lg\rho\alpha$ if α is small and is in radians.

The experiment following is an application of this manometer principle and gives a little practice in simple glass-blowing.

To measure the surface-tension of a soap-bubble, a tube A is drawn out to a fine point and a manometer tube B, 1 or 2 cm.

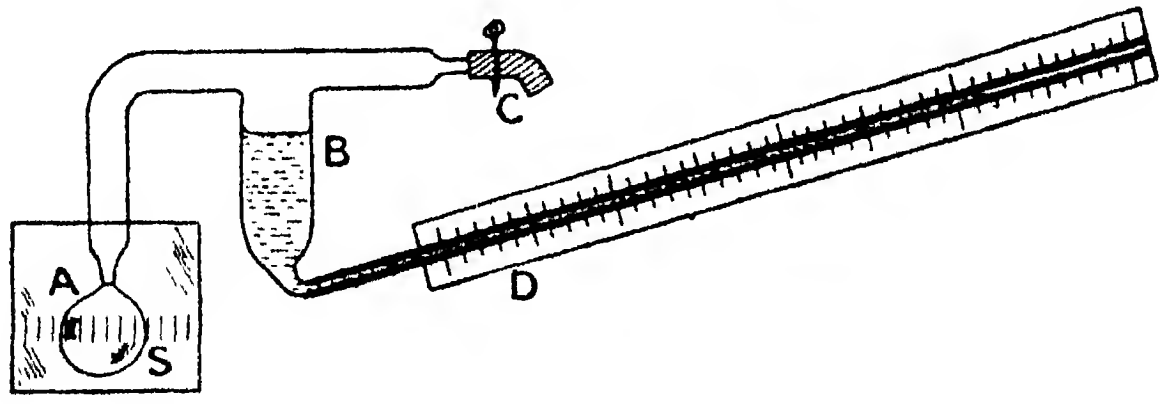


FIG. 18. Surface-tension of a soap-bubble.

in diameter, is sealed on to it as shown in Fig. 18. D is a fine manometer tube sealed on the end of B and bent as shown. A piece of rubber tube and a clip C enable adjustments of the manometer level to be made. D, the fine manometer tube, contains water, as described above, and if the diameter of the tube B is not sufficiently large for a/A to be neglected a suitable correction must be made. Various soap-bubbles of diameters ranging from 1 to 3 cm. are blown inside a glass-sided cube (to minimise air disturbances) and the diameters are measured by means of a mirror scale, S. The corresponding manometer readings are also recorded.

Since the pressure excess $p = \dfrac{4\gamma}{\text{radius}} = \dfrac{8\gamma}{\text{diameter}}$ where γ is the surface-tension, plot p against $\dfrac{1}{D}$. The resulting straight line enables γ to be determined from its slope, but it may not go through the origin owing to part of the pressure difference being due to the surface-tension of the manometer liquid of amount $2\gamma_1/r$ where γ_1 is the surface-tension of the manometer liquid and r is the radius of the tube. For accurate measurement, it is best to ensure that the bubbles are dry, that is, they should not have a drop of solution hanging on the lower halves.

8. CAPILLARY TUBE METHOD FOR SURFACE-TENSION OF A LIQUID

In Fig. 19 is depicted an arrangement of apparatus which enables the pressure in an experimental capillary tube to be so

adjusted that the meniscus formed at the air-liquid surface may be forced down so as to be level with the plane end of the capillary

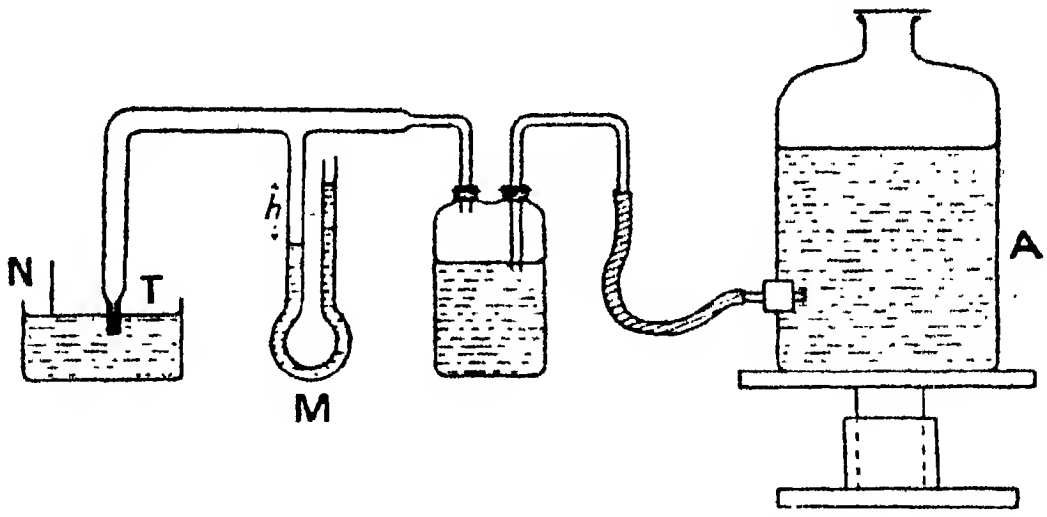


FIG. 19. Surface-tension by a capillary tube method.

tube employed. T is the capillary tube of suitable diameter, dipping in the liquid to be investigated. By means of the bottle A, placed on a turntable and capable of fine adjustment, the meniscus is forced down and the pressure required is determined from the reading of the aniline manometer shown at M.

N is a needle point just touching the horizontal surface of the given liquid and the vertical distance h_1 between this point and the end of the tube is measured by means of a microscope with a vertical traverse. The radius of the end of the tube may be determined by means of a micrometer eyepiece.

It was shown by Ferguson and Hakes that if ρ is the density of the manometer liquid, h the manometer reading, ρ_1 the density of the liquid under test, r the radius of the tube and γ the surface-tension, then

$$\rho h = \rho_1\left(h_1 - \frac{r}{3}\right) + 2\gamma/gr.$$

It follows therefore that a plot of ρh against $(h_1 - r/3)$ should give a straight line. The slope of this line gives ρ_1 and the intercept gives $2\gamma/gr$ and hence γ.

The method described above may also be used for the variation of surface-tension with temperature if the liquid is heated, allowed to cool and readings of h taken at various temperatures with h_1 constant. The temperature of the liquid may be measured by means of a mercury thermometer or thermo-couple.

An alternative method for the measurement of the surface-tension of a small quantity of liquid has been described by Ferguson and Kennedy.

The small quantity of liquid is contained in a horizontal tube,

of bore not greater than 1 mm., the end of which has been very carefully ground. A small filament lamp δ is used to illuminate the meniscus as shown in Fig. 20, and the end of the tube is viewed

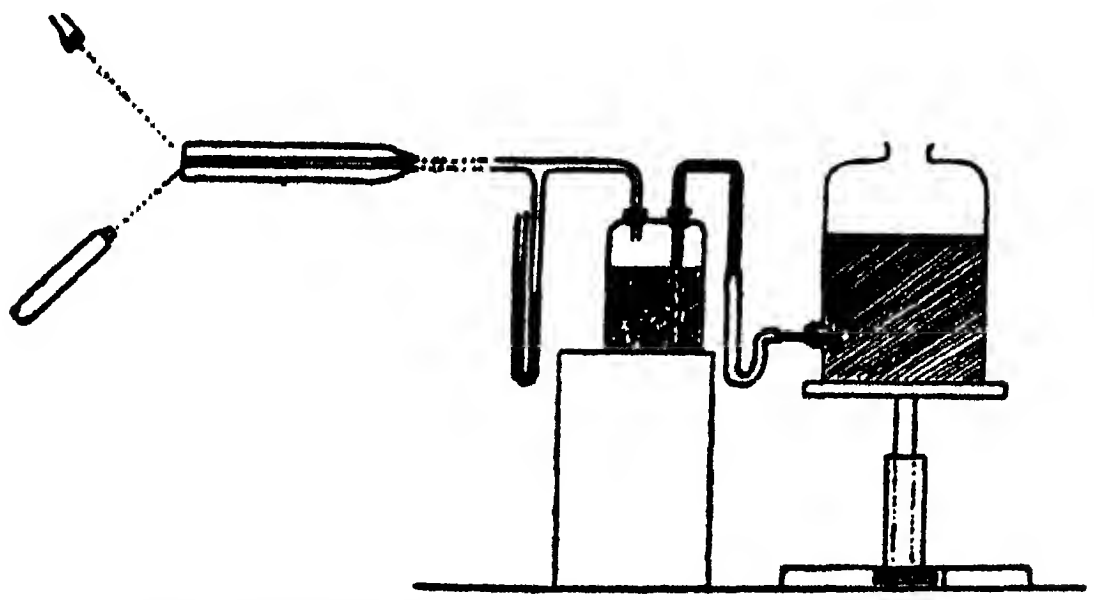


FIG. 20. Ferguson's method (from a block kindly lent by the Cambridge University Press, by permission of the Physical Society).

by means of a microscope. It is possible to alter the pressure so as to change the meniscus from concave to convex and by observing the images of the filament to decide when the meniscus is plane.

If the meniscus is convex or concave an "image" of the filament is seen, but for the plane position the surface appears to be uniformly illuminated.

The surface-tension is then given by $2\gamma/gr = \rho_1 h_1$ where h_1 is the pressure-head.

REFERENCES

FERGUSON and HAKES: *Proc. Phys. Soc.*, **41**, pp. 214–222, 1929.
FERGUSON and KENNEDY: *Proc. Phys. Soc.*, **44**, pp. 511–519, 1932.

9. AN EXPERIMENT WITH A RESONANCE-PENDULUM

The experiment to be described provides a study of resonance and determines the damping coefficient in the appropriate equation of motion.

The apparatus consists of a large solid pendulum S, some 2 or 3 metres in length, to which may be attached a small rod or gallows G in the position shown in Fig. 21. To the gallows is tied the string of a simple pendulum. The bob B of this pendulum consists of a cylindrical bakelite case such as is used for holding a shaving-stick. Various weights may be placed inside the box without altering its outward shape or dimensions.

A study of the relation between the amplitude of the simple

pendulum when forced by the solid one and the mass inside the bob enables the student to verify the solution of the equation for the motion involved and to determine the coefficient of resistance and the amplitude of the gallows.

The theory is slightly complicated by the fact that the motion

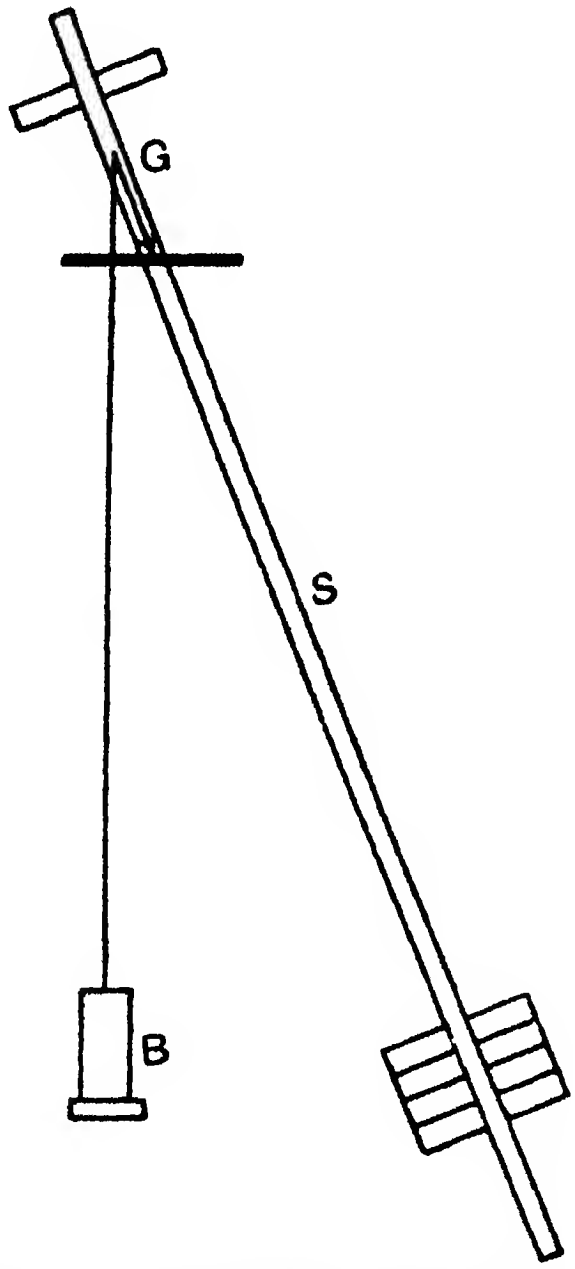


FIG. 21. Resonance-pendulum.

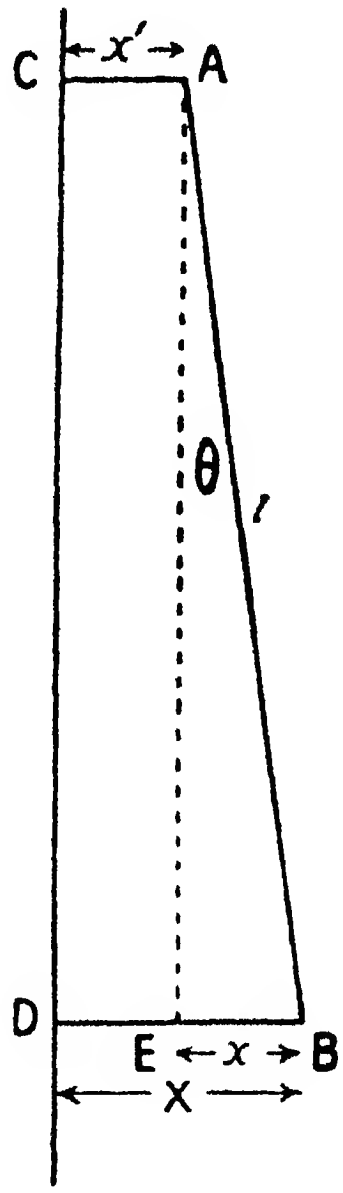


FIG. 22. Displacement diagram.

of the gallows has to be considered. In Fig. 22 let x be the amplitude of the simple pendulum about its own point of support, X be the amplitude as measured from a vertical through the knife edge of the solid pendulum, and let $x' = a \cos pt$ be the displacement of the support, then

$$X = x + x' \text{ and } \ddot{X} = \ddot{x} + \ddot{x}'.$$

If m is the mass of the pendulum bob, the force on the bob = $m \times$ acceleration $= -mg \sin\theta = -mg\theta = -\frac{mg}{l}(X - x') = mn^2(X - x')$ where $n^2 = g/l$ for the simple pendulum. Thus in the absence of resistance, acceleration $= \frac{\text{Force}}{\text{Mass}} = \ddot{X} = -n^2(X - x')$. (1)

If resistance, proportional to velocity, is allowed for and if k is the resistance coefficient per gram, then

$$\ddot{X} = -[n^2(X - x') + k\dot{X}] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus
$$\ddot{X} + k\dot{X} + n^2X = an^2 \cos pt \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which equation has for its solution

$$X = A \cos (pt - \delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where
$$\frac{a^2n^4}{A^2} = (n^2 - p^2)^2 + k^2p^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Put $k = \frac{r}{m}$ where r is the damping coefficient for the simple pendulum.[1]

Substituting for k we get

$$\frac{a^2n^4}{A^2} = (n^2 - p^2)^2 + \frac{r^2}{m^2} \cdot p^2 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or
$$\frac{1}{A^2} = \frac{(n^2 - p^2)^2}{a^2n^4} + \frac{r^2p^2}{a^2n^4m^2} \cdot \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In the experiment the maximum amplitude A is measured for

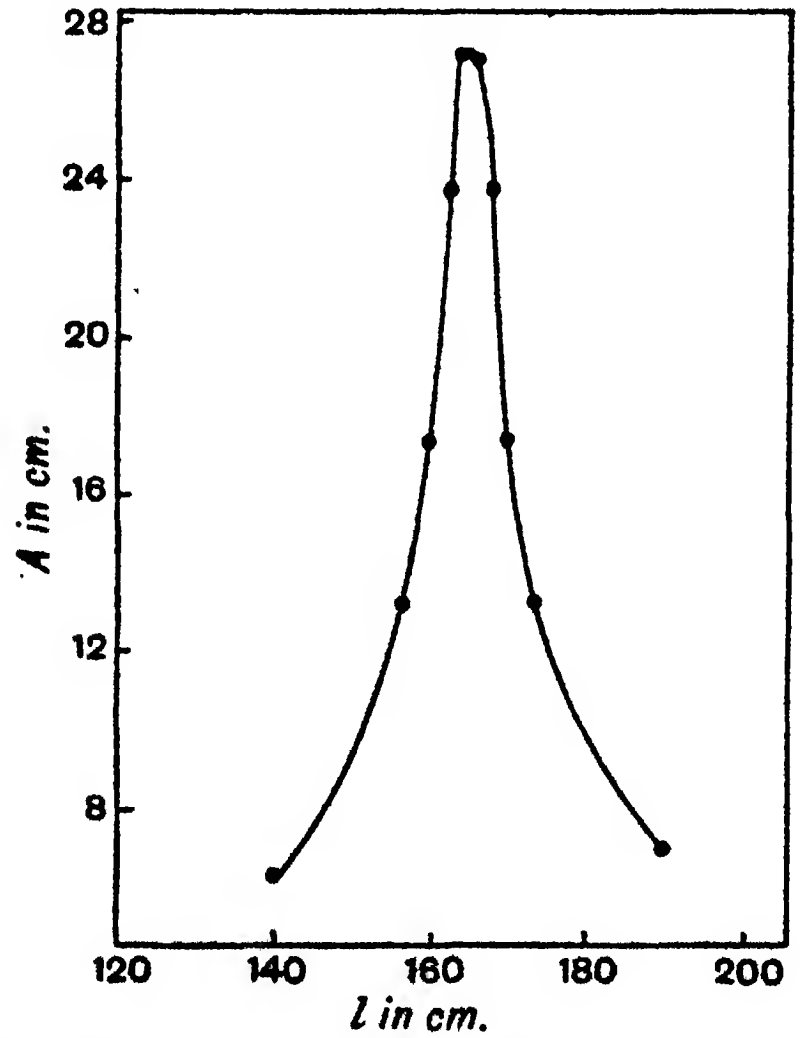


FIG. 23. Resonance curve. (Block kindly lent by John Murray.)

[1] It may be remarked that an experiment suggested by Dr. Brinkworth, which consists in measuring the log decrement of a simple pendulum for various loads inside the bob, proves the constancy of r and enables its value to be found.

various values of m. n and p are determined experimentally and $\frac{1}{A^2}$ is plotted against $\frac{1}{m^2}$.

The resulting straight line gives a value for r and a value for a, the maximum displacement of the gallows. In Fig. 23 is exhibited

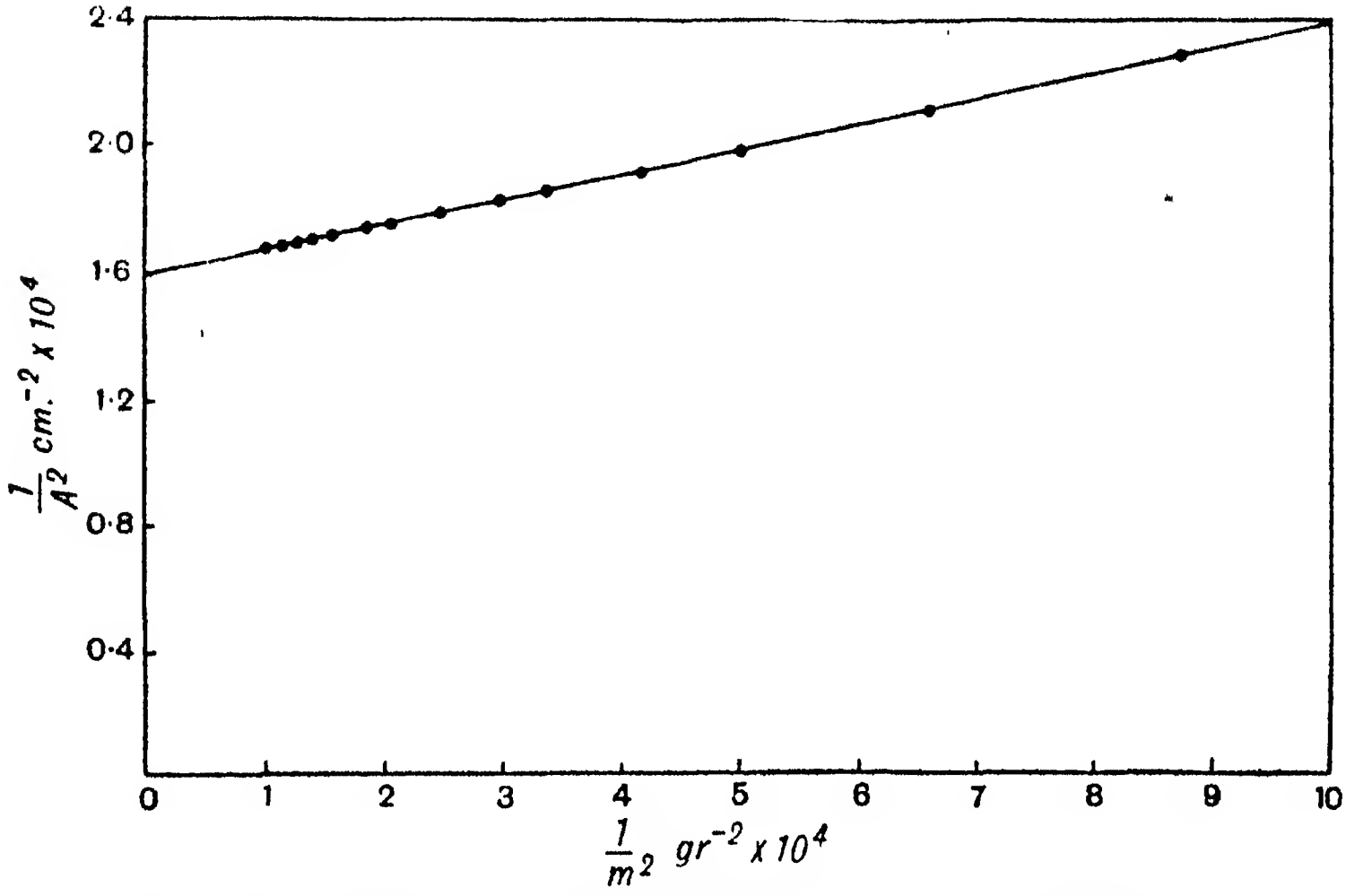


FIG. 24. Linear plot. (Block kindly lent by John Murray.)

a typical resonance curve, and in Fig. 24 a straight-line graph.

n^2 was 5·934 sec^{-2}, p^2 was 5·977 sec^{-2}. a determined from the intercept was 1·49 cm., and r from the slope was 0·36 dynes per unit velocity.

REFERENCE

CALTHROP and WOODALL: *Sch. Sci. Rev.*, Univ. Section, pp. 576–578, 1937.

10. TO DETERMINE THE SPECIFIC HEAT OF WATER BY MEANS OF CALLENDAR'S CONTINUOUS-FLOW CALORIMETER

In this experiment heat is supplied to flowing water by means of a long coil of wire carrying a current. Assuming the value of Joule's equivalent and measuring the rise in temperature of a measured quantity of water flowing at a suitable rate, it is possible to calculate the mean specific heat of water over the range of temperatures selected.

A spiral of thick wire S is wound inside a long glass tube

which itself is enclosed in an exhausted envelope. Water under a constant pressure-head is allowed to flow through the tube and current is supplied to the wire from the 240-volt mains with

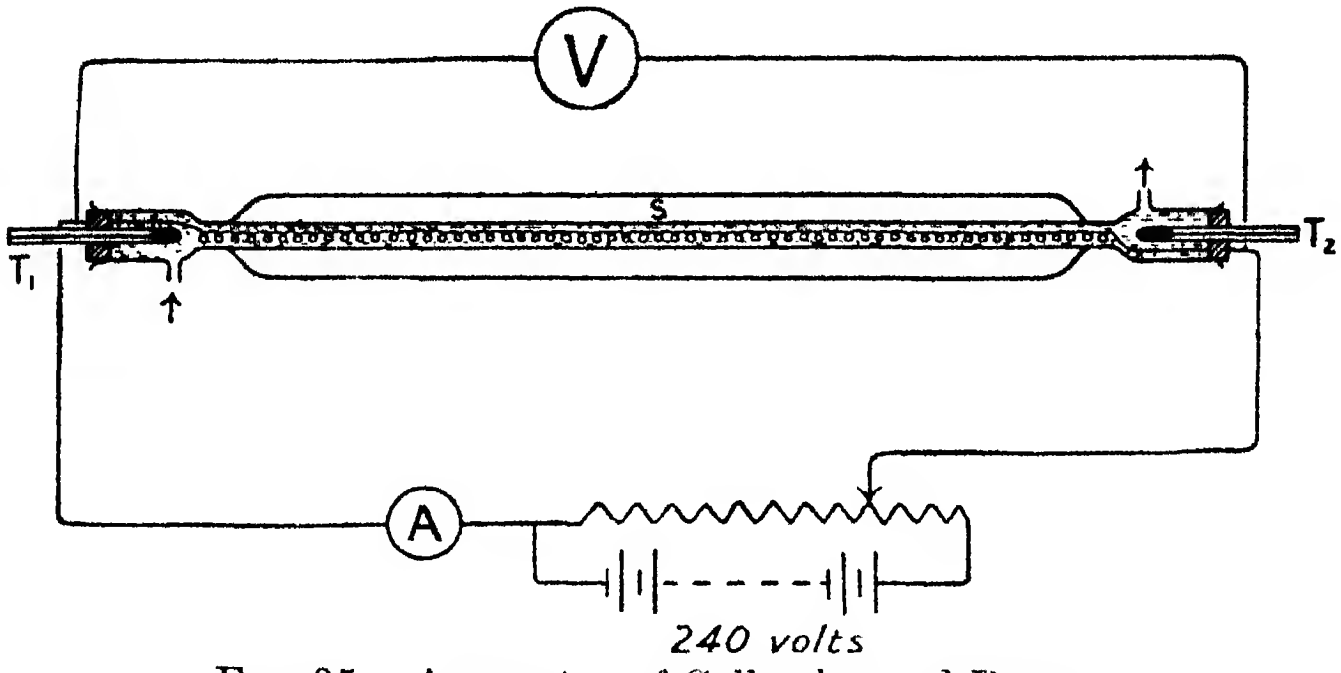


FIG. 25. Apparatus of Callendar and Barnes.

suitable resistances (Fig. 25). The temperatures of the water before and after heating are read on the thermometers T_1 and T_2. The current is measured by an ammeter A and the potential-drop across the wire by a voltmeter V. The pressure of the water and the current should be adjusted so that the temperature difference is about 10° C. The rate of flow of the water is measured by weighing the water collected in, say, five minutes.

Let m_1 be the mass of liquid passing in t_1 seconds, s the specific heat, T_1 and T_2 the initial and final temperatures of the water, A_1 the ammeter reading, and V_1 the voltmeter reading.

Then
$$A_1V_1 = \frac{J \,.\, m_1(T_2 - T_1)S}{t_1} + E$$

where E is the radiation loss per second at a mean temperature of $\frac{T_1 + T_2}{2}$ and J is Joule's equivalent.

Now repeat the experiment, altering the pressure of the water and the current but keeping T_1 and T_2 as nearly as possible equal to their previous values. If m_2, A_2, V_2, t_2, T_1' and T_2' are the new values,

$$A_2V_2 = J \,.\, \frac{m_2(T_2' - T_1')S}{t_2} + E.$$

Subtracting

$$S = \frac{0{\cdot}24(A_2V_2 - A_1V_1)}{\dfrac{m_2(T_2' - T_1')}{t_2} - \dfrac{m_1(T_2 - T_1)}{t_1}}.$$

There is a serious danger of fatal electric shock if wet hands come in contact accidentally with the mains. In the interests

of safety and accuracy the experiment is best performed with a bank of low-voltage heavy duty accumulators.

11. To find the specific heat of a liquid by Ferguson's cooling method

The principle of this method is to supply heat electrically to a known quantity of the liquid, contained in a calorimeter, at such a rate as to maintain the temperature of the liquid at 5° C., 10° C., 15° C., etc., above that of the surrounding medium. When the temperature excess reaches some 50° C., the power supply is switched off and the liquid is allowed to cool. From

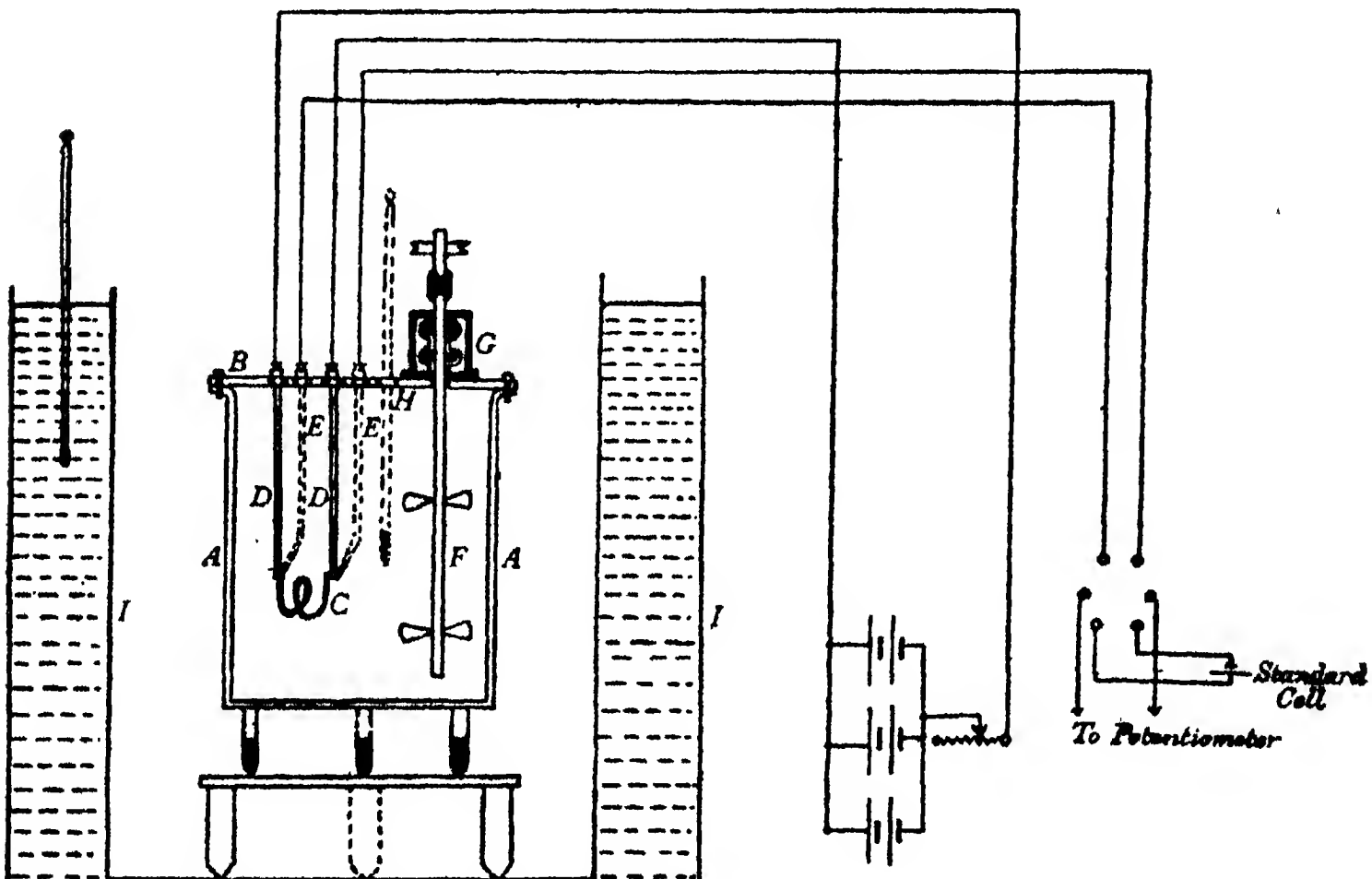


Fig. 26. Ferguson's cooling method. (Block kindly lent by the Cambridge University Press, with the permission of the Physical Society.)

the electrical measurements and the final cooling-curve the specific heat of the liquid may be deduced.

The theory is approximately as follows :—

Let M be the mass of liquid, S its specific heat, W the water-equivalent of the calorimeter, E the voltage for the heating current supply and R the resistance of the heating-coil. If J is Joule's equivalent, θ is the excess temperature, and t is the time :

$$-(MS + W)\frac{d\theta}{dt} = \frac{E^2}{JR} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If $-\frac{d\theta}{dt} = K\theta^n$ where K and n are constants,

$$-(MS + W)K\frac{d\theta}{dt} = E^2/RJ \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is found that the rate of fall of temperature,

$$-\frac{d\theta}{dt} = K\theta^{5/4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\therefore\ (MS + W)K\theta^{5/4} = E^2/JR \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Integrating (3), $\theta^{-\frac{1}{4}} - \theta_0^{-\frac{1}{4}} = \frac{1}{4}Kt$ . . . . . (5)

From the observations for the cooling process a graph may be drawn to show the relation between $\theta^{-\frac{1}{4}}$ and t. This is found to be a straight line from which K may be determined. Thus everything in equation (4) is determined except S the specific heat required.

The apparatus is depicted in Fig. 26, in which A represents a

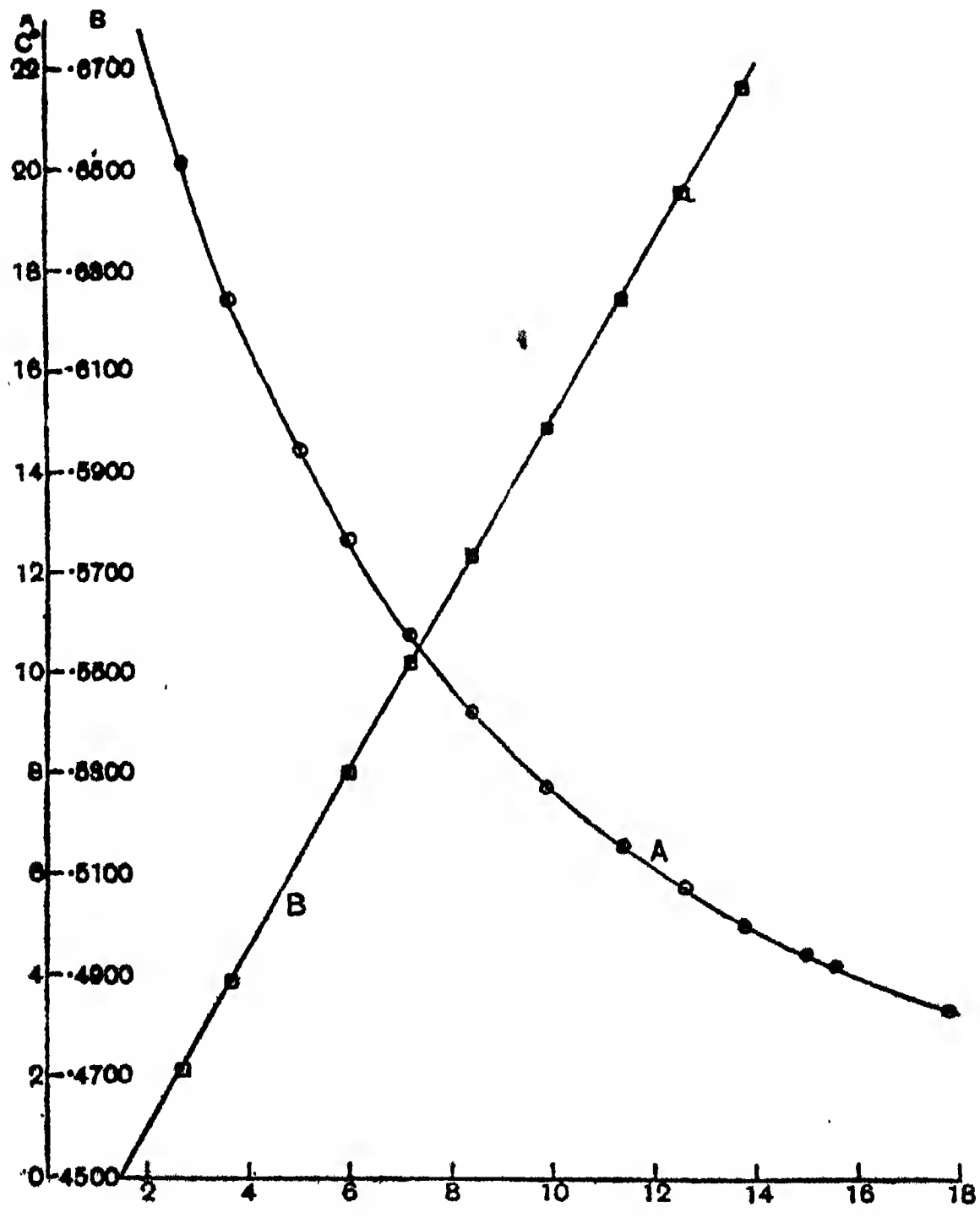


FIG. 27. Linear plot. (Block kindly lent by the Cambridge University Press, with the permission of the Physical Society.)

copper calorimeter having an internal diameter of 10 cm. To the lid *B* are attached current leads *D* and potential leads *E* and a rotating stirrer *F*. A thin film of grease is placed on the flange of the calorimeter before the lid is screwed on. A thermometer is inserted at *H*. The measuring and heating arrangement is shown on the right of the diagram. For accurate work a double-walled tank *I* containing water at room-temperature serves to define the temperature of the surroundings. In Fig. 27 is shown a typical cooling-curve and the corresponding plot of $\theta^{-\frac{1}{4}}$ against *t*.

Water, aniline and benzene are suitable liquids for the experiment described.

REFERENCES

FERGUSON and MILLER: *Phys. Soc. Proc.*, **45**, pp. 194–204, 1933.
J. LEECH: *Phys. Soc. Proc.* B, **62**, p. 390, 1949.

12. THERMAL CONDUCTIVITY BY FORBES'S BAR METHOD

This method is not one which gives results of the highest precision, but as an exercise it is very instructive.

In the experiment a long copper bar *G* (Fig. 28) about 1 metre long and 1 cm. in diameter has one end soldered to a metal

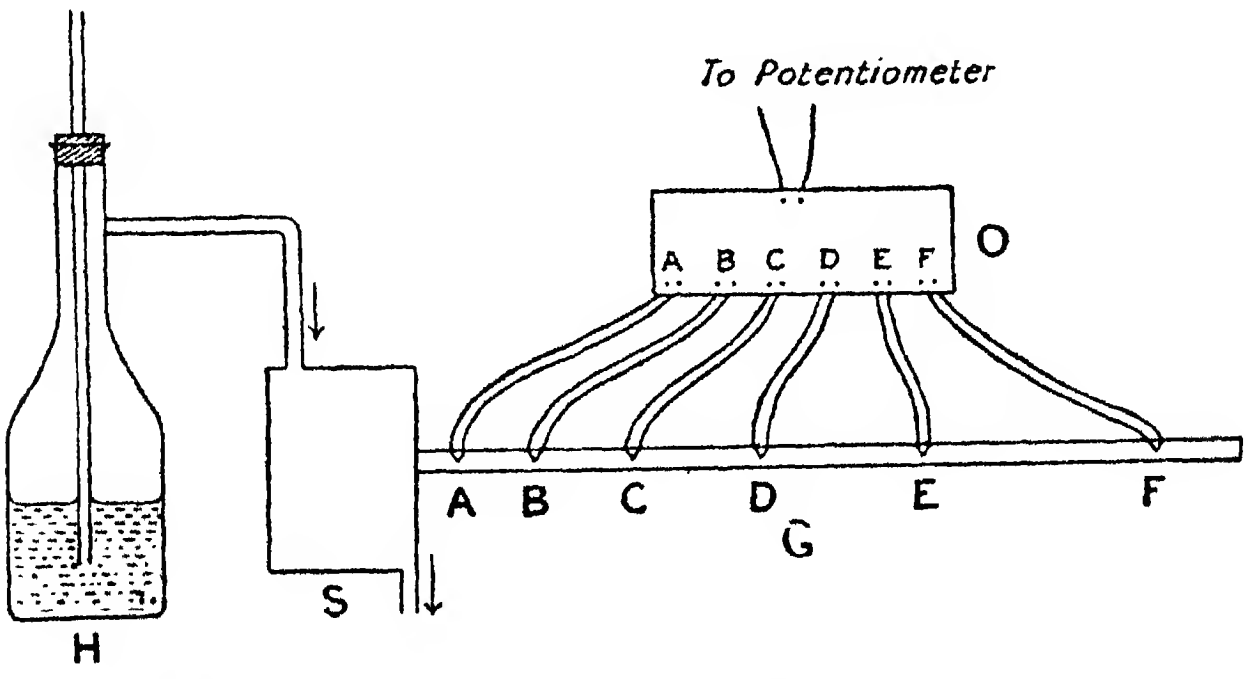


FIG. 28. Thermal conductivity by Forbes' bar method.

steam chamber *S*. Steam may be passed through from the steam heater *H*. At about six points of the bar, such as *A*, *B*, etc., are soldered, copper-eureka couples, and the leads may be connected to a terminal board *O* so that each couple may be put into the circuit in turn. A similar junction kept in melting ice will serve as the cold junction for each couple. The experiment consists in measuring the temperatures at

various points on the wire and for this purpose a potentiometer arrangement must be set up as in Fig. 29.

A B is a uniform wire having a resistance rather greater than

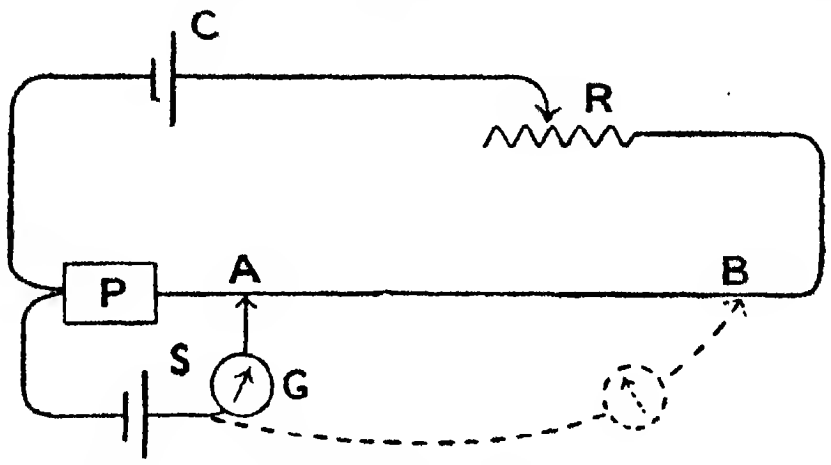


FIG. 29. Calibration of potentiometer.

1 ohm, and this is joined to a P.O. box *P*, a cell *C* and a variable resistance *R* of 1,000 ohms.

It may be arranged that the E.M.F. of a standard cell *S* is balanced at a point *A* with say 500 ohms out of the box and at *B*

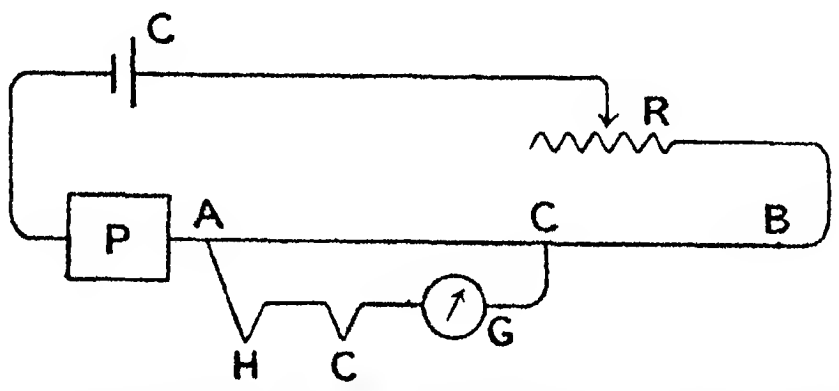


FIG. 30. Method of measuring thermo-electric E.M.F.'s.

with (500 — 1) ohms out, so that the length *AB* may be taken as 1 ohm. If the cell *S* has an E.M.F. of 1·018 volts then this length is equivalent to a p.d. of $\frac{1{\cdot}018}{500}$ volts so that, if *AB* is about 100 cms., the sensitivity of the arrangement is of the order of 20 microvolts per cm. A couple *H C* may be connected as shown in Fig. 30, and may be calibrated conveniently against a mercury thermometer reading 0–100° C. Now each couple on the bar is

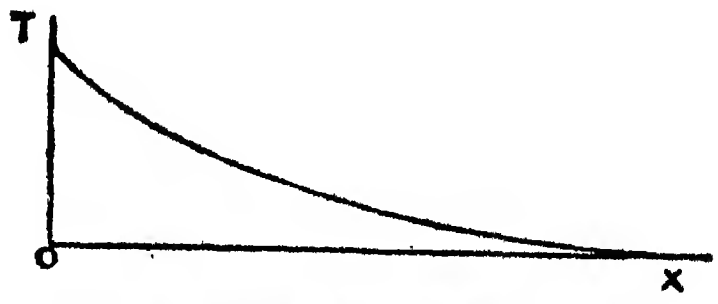


FIG. 31. Variation of temperature along the bar.

connected in turn to the circuit containing the cold junction, and from the potentiometer readings and the previous calibration it is possible to plot the temperature T at a point against x, the distance from the hot end, as in Fig. 31. It is now necessary to determine the rate of cooling curve, and for this purpose a short

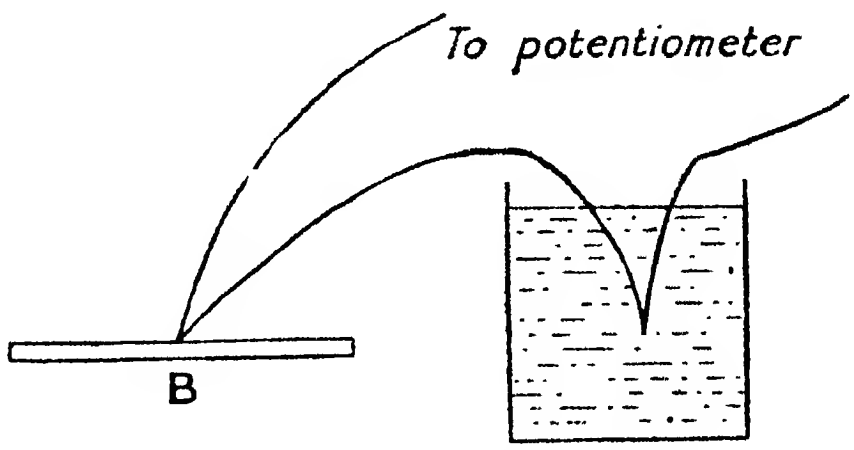


FIG. 32. Arrangement to determine the cooling curve.

length of bar B in Fig. 32, some 20 cm. long, is taken and a thermocouple is soldered to the centre. This portion is heated in a bath to 100° C. and then taken out and allowed to cool in air. Potentiometer readings at various times t enable a graph between T and t to be plotted as in Fig. 33.

Another graph between $\frac{dT}{dt}$ and x may be constructed as in Fig. 34. (See Instructions to Students on the drawing of tangents.)

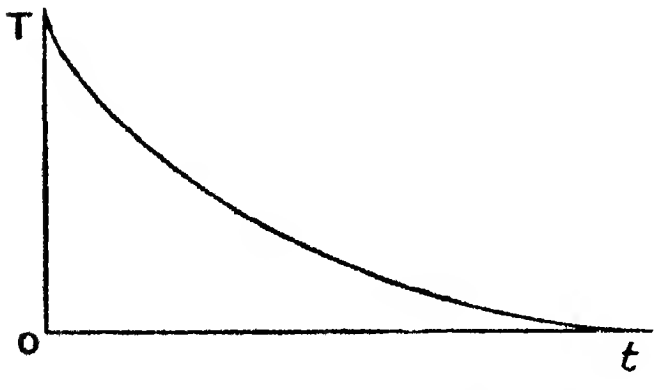


FIG. 33. Cooling curve.

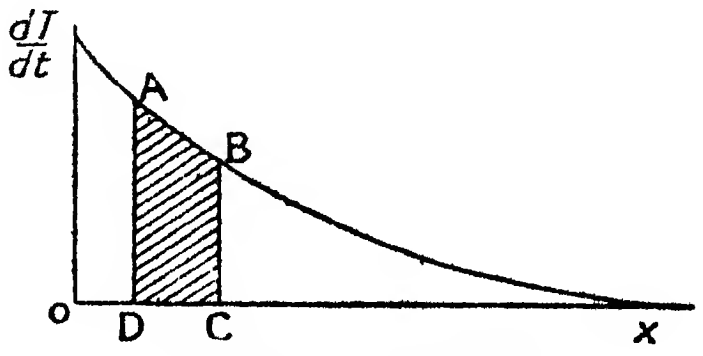


FIG. 34. Integration.

The calculation of the thermal conductivity is made as follows. If K is the conductivity, ρ the density, S the specific heat of the material and A the sectional area of the bar, then for a portion of the bar, distant x from the end, the equation equating the heat transmitted to the heat lost is

$$-KA\frac{dT}{dx} = SA\rho \cdot \int_x^\infty \frac{dT}{dt}\,dx.$$

Integrate between two limits x_1 and x_2 and obtain

$$K\left[\left(\frac{dT}{dt}\right)_1 - \left(\frac{dT}{dx}\right)_2\right] = \rho S \int_{x_1}^{x_2} \left(\frac{dT}{dt}\right) dx.$$

Thus $\left(\frac{dT}{dx}\right)_1$ $\left(\frac{dT}{dx}\right)_2$ are the gradients at two suitable points x_1 and x_2 and the integral $\int_{x_1}^{x_2}\left(\frac{dT}{dt}\right).\ dx$ required is shown by the shaded area *ABCD* in Fig. 34, and may be found by counting the squares and fractions of squares on the graph paper.

Thus K may be evaluated. Care should be taken to express the units correctly.

13. To find the emissivity of a metallic surface by an electrical method

By emissivity in this experiment is meant the total loss of heat per square centimetre per second by convection and radiation from a surface.

To determine the emissivity of the given metal, the material is taken in the form of a stiff wire of some 2 mm. diameter and some 30 cm. long. First it is necessary to determine the coefficient of expansion by finding, by means of a suitable travelling microscope, the change in the distance between two marks at the ends of a bar for a change in the temperature of the bar of 100° C.

If preferred, any other well-known method may be employed.

A known steady current for a known potential difference is now passed through the wire and the increase in the distance between the two marks is determined as before. Assuming the metal to have a known temperature coefficient of expansion α, the emissivity of the surface may be found as follows :—

Let C be the current, E the fall of potential down the portion l of the bar between the two marks, p be the perimeter of the bar, θ the temperature excess over the air temperature and e the emissivity.

Then in the steady state

$$0{\cdot}24\ EC = ep\theta l.$$

But
$$\delta l = \alpha l \theta$$

where δl is the increase in l for a temperature excess θ.

$$\therefore \quad 0{\cdot}24\ EC\alpha = \delta l\ .\ e\ .\ p$$

$$\therefore \quad e = \frac{0{\cdot}24\ EC\alpha}{p\ .\ \delta l}.$$

Vary the current and find δl for each value of EC.

Plot a graph between EC and δl, and from the slope of the straight line deduce a value for e.

Aluminium which has a coefficient of expansion of $25{\cdot}5 \times 10^{-6}$ per ° C. is a suitable metal for the exercise.

14. Stefan's Law

In Fig. 35 there is illustrated a Wheatstone bridge arrangement in which L represents a tantalum or other filament lamp

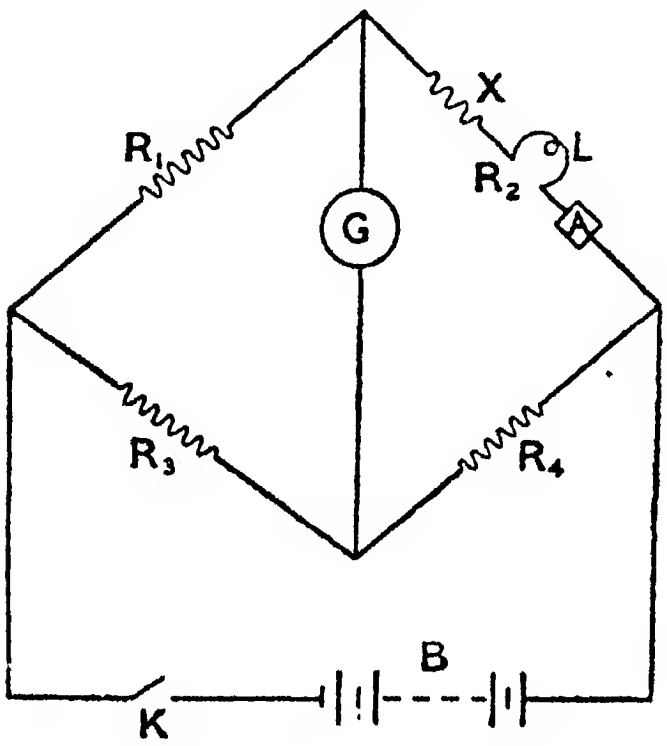


Fig. 35. The lamp-circuit.

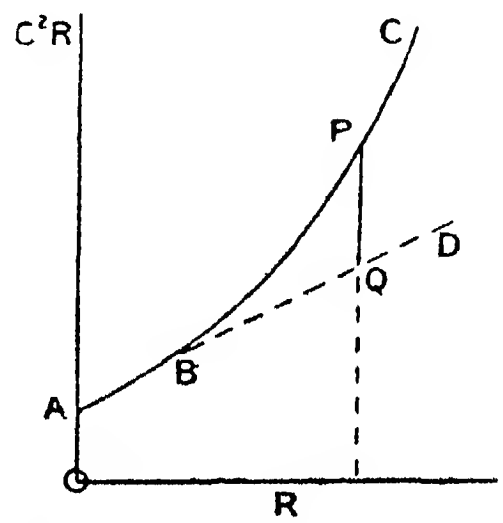


Fig. 36. Plot of energy against resistance.

together with an ammeter A for reading the current through it. An additional resistance X may be included in the same arm, if necessary, to give a total resistance R_2. R_1, R_3, R_4 are resistances suitably chosen for the other arms, G being a shunted galvanometer and B a battery of sufficient voltage to illuminate the lamp filament. The resistance of the filament R is measured for gradually increasing values of the current C. The electrical energy supplied per sec. is calculated from the expression C^2R, and this is plotted against R as shown in Fig. 36. (Another method is to determine R from ammeter and voltmeter readings.) It is found that a straight

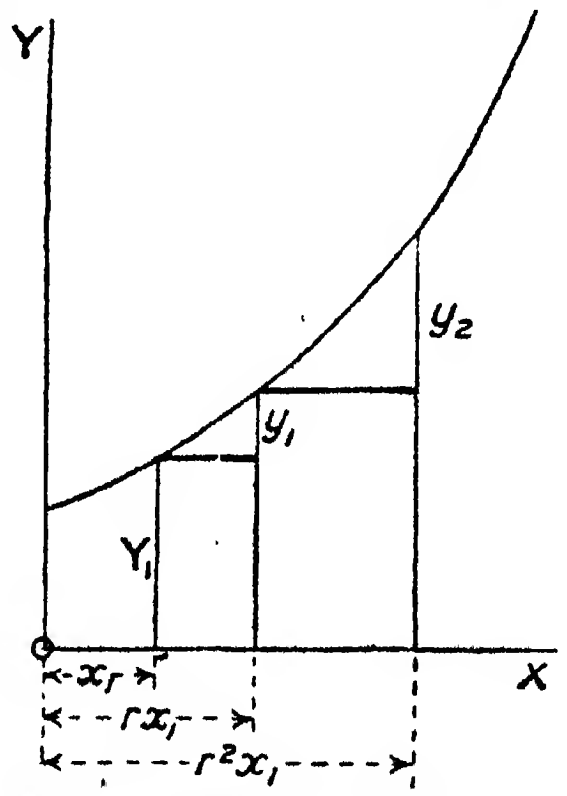


Fig. 37. Method of solution.

portion is first obtained for the curve indicating the predominance of conduction, but as the temperature of the filament rises there is a considerable departure from the straight line, indicating the predominance of radiation. The straight line AB is produced to D and the difference in the ordinates for the two curves such as PQ is found for a number of values of R. Fig. 37 represents a new plot of the radiation loss Y against resistance represented by x. The energy supplied per sec. equals the sum of the losses by conduction and radiation.

Thus $$C^2R = A(T - T_0) + B(T^n - T_0{}^n) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

A and B are constants, T_0 is the absolute temperature of the envelope, and T the absolute temperature of the filament. n may be treated as an unknown power to be found from the experiment. It is assumed that resistance R is proportional to the absolute temperature, since the temperature coefficient of tantalum is 0·0033 per ° C. Thus write

$$C^2R = A'(R - R_0) + B'(R^n - R_0{}^n) \quad . \quad . \quad (2)$$

From the treatment in Fig. 36 the conduction loss has been eliminated, and the radiation loss may be written

$$Y = B'R^n - B'R_0{}^n.$$

Thus the problem is to determine the power n from a relation of the type

$$Y = ax^n - b$$

where a and b are constants. n may be found as follows :—

In Fig. 37 some convenient value of x such as x_1 is chosen with $x_2 = rx_1$, $x_3 = r^2x_1$, etc., in a suitable geometrical progression with r as a convenient common ratio. Then, as in Fig. 37,

$$Y_1 = ax_1{}^n - b \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$(Y_1 + y_1) = a(rx_1)^n - b \quad . \quad . \quad . \quad . \quad (4)$$

Subtracting $$y_1 = a(r^n - 1)x_1{}^n.$$

Thus $$\log y_1 = n \log x_1 + \log a(r^n - 1).$$

Another graph may then be drawn between $\log y_1$ and $\log x_1$ where x_1, y_1 may now be thought of as variables, and the resulting straight line enables n to be found from the slope. It will be noted that y_1 corresponds to the previous abscissa x_1, y_2 to $x_2 = rx_1$, y_3 to $x_3 = r^2x_1$. Some care should be taken to draw a good curve through the observations on a large sheet of graph paper by means of a spline, and it is then possible to get a value for n in good agreement with Stefan's value of 4, thus verifying

that heat energy is being radiated according to the fourth power of the absolute temperature. In the typical result illustrated in

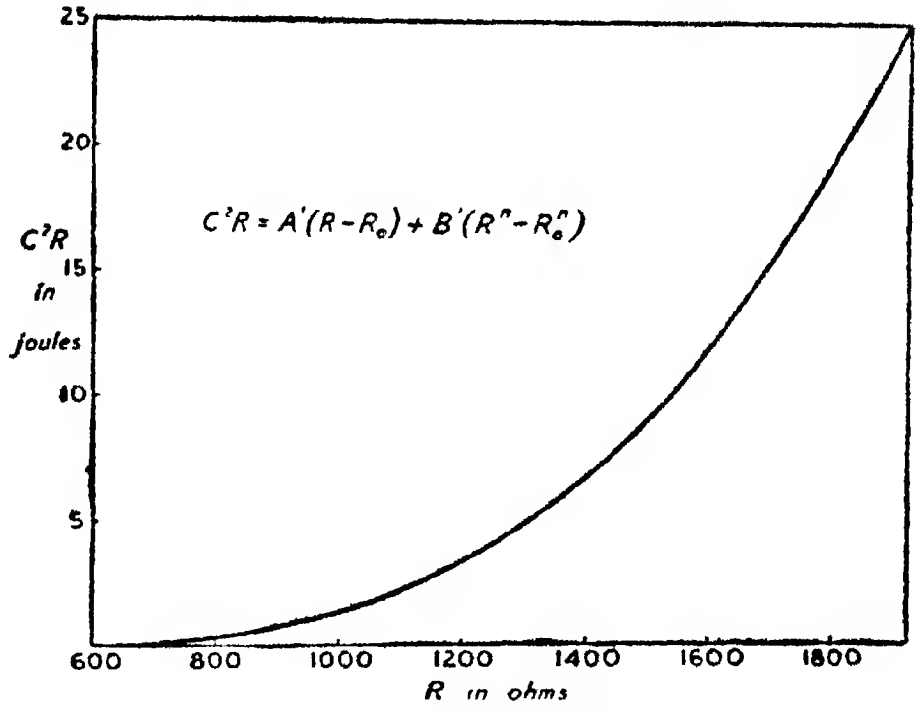


FIG. 38. Typical experimental curve.

Fig. 38 the power n was found to be $4{\cdot}0_3$ for a 25-watt lamp working on 240 volts. The conduction effect was very small.

15. TO ESTIMATE THE TEMPERATURE OF A FLAME

This exercise is a useful application of the well-known experiment of Kirchhoff and Bunsen, who demonstrated the reversal of the sodium lines. In Fig. 39, F is a 500-watt tungsten filament lamp fed by the 240-volt mains. A voltmeter V enables the fall

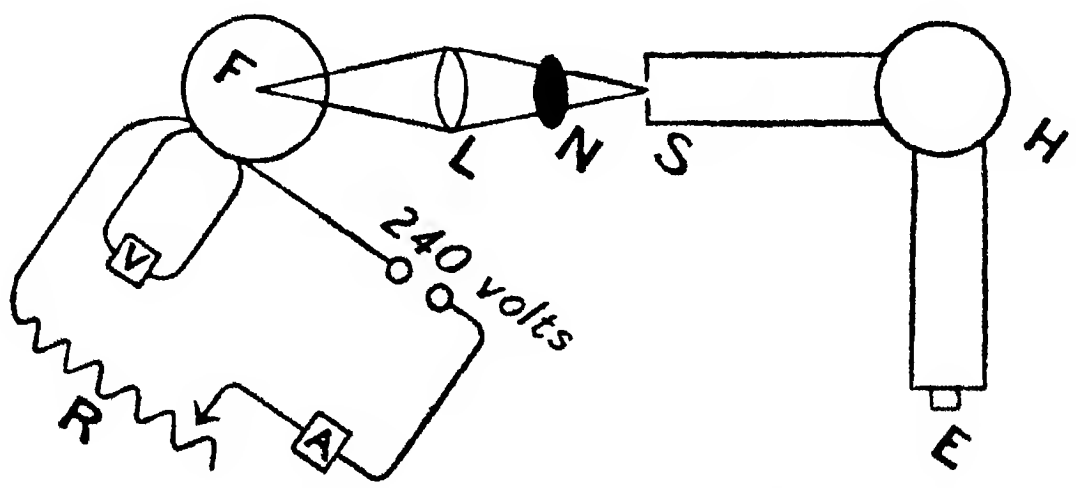


FIG. 39. Experiment of Kirchhoff and Bunsen.

of potential across the lamp to be read and a variable resistance R and an ammeter A are placed in series. A lens L enables an image of the filament to be thrown on the slit S of a Hilger or other spectrometer H, but a large flame, obtained by placing a block of salt in a bunsen flame, is interposed at N. The eyepiece

E enables the spectrum to be viewed with the wavelength drum of the instrument set near 5893A. The current in the filament is gradually increased. At first the sodium flame, being hotter than the filament, will show the two bright sodium D lines superposed on a faint spectrum, but as the temperature of the filament rises a stage is reached at which the sodium lines appear dark superposed upon a very bright spectrum. The sodium flame is then absorbing the radiation of the same wavelength.

The experiment consists in judging when the reversal occurs and in measuring the corresponding potential difference and current for the lamp. The resistance of the filament when hot may then be calculated. The resistance of the filament at air-temperature is measured by means of a P.O. box.

For the purposes of the exercise it will be sufficient to take the temperature coefficient of resistance of tungsten near 1,000° C. to be $\alpha = 52 \times 10^{-4}$ per ° C., and to assume a relation of the type $R_t = R_0(1 + \alpha t)$ for the purpose of calculating the high temperature of the filament and flame. A temperature of 1,500–1,700° C. will be obtained, but the temperature varies considerably with the quality of the gas, air supply, etc. Use could be made of an optical pyrometer for the measurement of the filament temperature. Stefan's Law $E = \sigma T^4$ would then be employed where E is the energy supplied per unit area per sec. Since the filament does not radiate as a black body, the result may be 20% too high. It is assumed that the flame is not luminescent and the emissivity for the sodium particles is taken as unity.

The result may be taken to be a reasonable estimate without unduly stressing the accuracy of the determination.

SECTION II. LIGHT

16. To determine the cardinal points of a thick lens

The cardinal points of a thick lens include the focal points, the principal points and the nodal points. These points have the following properties :—

(*a*) *Focal points.* A pencil of rays passing through the lens parallel to the axis will converge to or diverge from a point. This is one of the principal foci of the lens. The other corresponds to rays passing through the lens in the opposite direction.

(*b*) *Principal points.* The principal points are two conjugate foci such that an object placed at one forms an image at the other of the same size and erect.

(*c*) *Nodal points.* These are a pair of conjugate foci such that a ray incident on one emerges in a parallel direction from the other.

The focal, principal and nodal planes are the planes perpendicular to the respective cardinal points.

The focal lengths of a thick lens are the distances between the focal points and the corresponding principal points. When the media on each side of the lens are the same the principal points coincide with the nodal points, and in addition, the two focal lengths become equal.

The properties of the cardinal points enable ray diagrams to be drawn for thick lens systems when their position is known. In addition many of the formulæ for thin lenses can be used for thick lenses provided that object and image distances are measured from the appropriate principal planes.

16*a*. To determine the principal points and focal lengths of a thick convex lens by Newton's method

The position of the focal points F_1, F_2 with respect to the faces AB of the lens are first found using a plane mirror.

On an optical bench set up the thick lens, a plane mirror M and an object pin P as in Fig. 40.

Adjust the position of P until no parallax is observed between

it and its image formed by reflection in the mirror, and measure the distance between the pin and the nearest surface of the lens. Repeat for the other focal point F_2 using a pin Q on the other side of the lens. Let these distances be F_1A and F_2B.

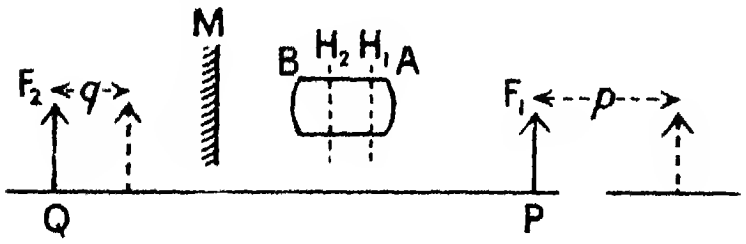


FIG. 40. Newton's method for the principal points.

Now remove the mirror and displace the pin P through a distance p. Adjust Q till the image of P again coincides with it. Measure the distance q through which Q has been moved. The focal length of the lens is given by

$$pq = f^2 \text{ numerically.}$$

Calculate f from the mean of several values of p and q.

The distances of the principal planes H_1, H_2 from the faces of the lens are $f - F_1A$ and $f - F_1B$.

16*b*. TO FIND THE FOCAL LENGTH AND THE POSITIONS OF THE CARDINAL POINTS OF A LENS SYSTEM

The following method is described for a system of two separated convergent lenses in air. The points required are the foci and the principal points. If u and v are the distances of the object O

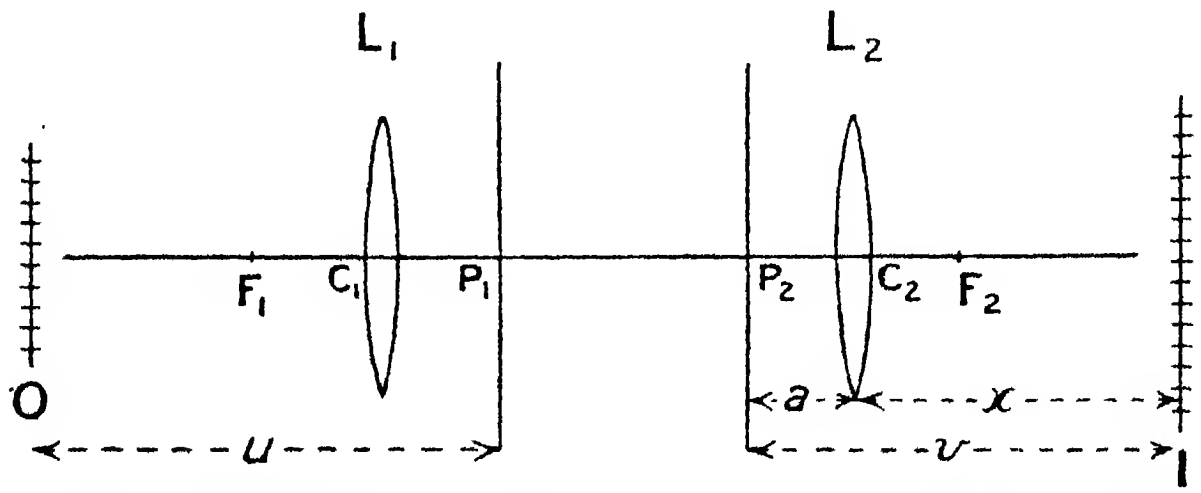


FIG. 40*a*. Magnification method for the focal length of a lens system.

and image I in Fig. 40*a*, and if f is the focal length of the system all measured from the appropriate principal point, then using the convention that real objects and images are positive and virtual negative,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}.$$

The magnification m is given by

$$1 + m = \frac{v}{f}$$

but the distance v is unknown.

Let u be constant and let $v = x + a$ where x is the distance to a point on the front of the lens L_2 and a is the distance of the principal point or plane from the same point on the lens.

Then $$m + 1 = \frac{x}{f} + \frac{a}{f}.$$

Plot $(m + 1)$ as y against x and there is a straight line relation of the type $y = bx + c$ (Fig. 40*b*).

The slope of the line $b = \frac{1}{f}$ gives the focal length f and the intercept of x when $y = (m + 1) = 0$ is $-a$, the distance required to fix the principal point P_2.

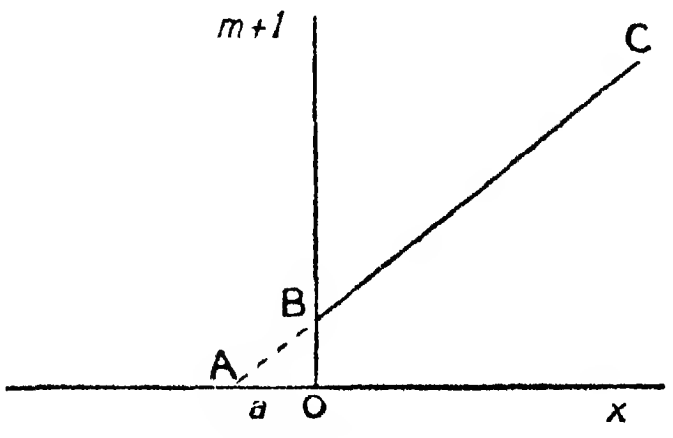


FIG. 40*b*. Linear plot.

In the experiment a glass millimetre scale illuminated by a pearl lamp serves as the object and the image may be caught on a graduated screen, or observed in a Ramsden eyepiece which has been suitably calibrated.

Obviously a similar graph will be obtained if $\left(1 + \frac{1}{m}\right)$ is plotted against the corresponding object distances as measured from L_1 and, as before, f and the position of P_1 may be found.

16*c*. TO DETERMINE THE NODAL POINTS BY THE NODAL SLIDE

The nodal slide is a carriage to hold the thick lens which can be moved along the optical bench and is capable of rotation about a vertical axis. By an adjustment any point on the longitudinal axis of the carriage can be brought over this vertical axis and made to rotate about it.

It is first necessary to set the axis of the lens vertically above the axis of rotation. Arrange the lens and nodal slide on the optical bench so that a distant object may be viewed as in Fig. 41.

Let SO be the line from the object through the axis of rotation and let N_1N_2 be the nodal points of the lens. View the distant

object through the lens and turn the slide till the image is not displaced laterally by a further *slight* rotation. This indicates that a line from the nodal point N_2 to O is perpendicular to SO. Fix the position of the image with a pin P, rotate the slide through 180° and move the carriage till an image is again formed in the plane of the image pin at P_1 and does not move laterally for a slight rotation. The line N_1O is now perpendicular to SO. It is clear that the lens must be displaced through a distance $\frac{1}{2}PP_1$ to bring its axis above SO.

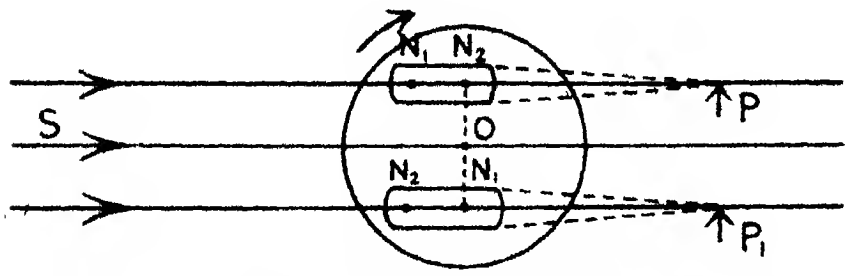


FIG. 41. Alignment of lens along SO.

The lens may now be adjusted parallel to SO till the image is not displaced for a *large* rotation of the slide.

This indicates that a nodal point N_2 is above the axis of rotation O. Rotate through 180° and move the carriage parallel to SO till the other nodal point N_1 lies above O. The distance moved gives the distance between the nodal points. Also the focal length of the lens is the distance between the marker pin and the axis of rotation.

Alternatively a plane mirror may be used to reflect back the image of a pin at the focal point.

If the proper nodal point is on the axis of rotation the image will not be displaced.

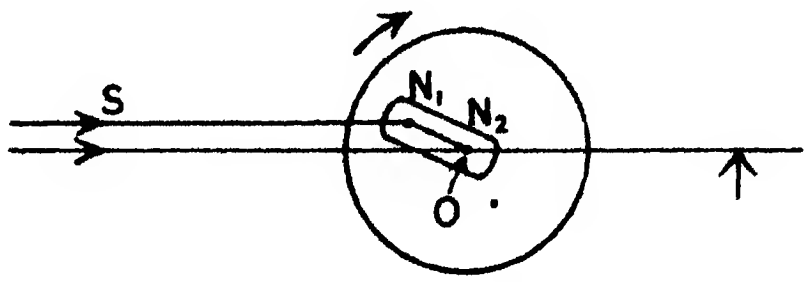


FIG. 41*a*. Determination of nodal point.

17. THE INVESTIGATION OF THE LONGITUDINAL SPHERICAL ABERRATION OF A LENS

A large plano-convex lens AB in Fig. 42 is fitted with a series of circular metal plates, containing a circular hole as in Fig. 43 or a series of circular zones as shown in Fig. 44. The lens is illuminated by a powerful distant sodium-lamp or by white light with a colour-screen, and if the smallest aperture CD is used, the position of the principal focus F of the lens may be obtained. If a screen containing a zone such as AB is substituted the rays will come to a focus nearer the lens at f. The distance Ff is the longitudinal aberration S of the lens corresponding to a mean radius r of the

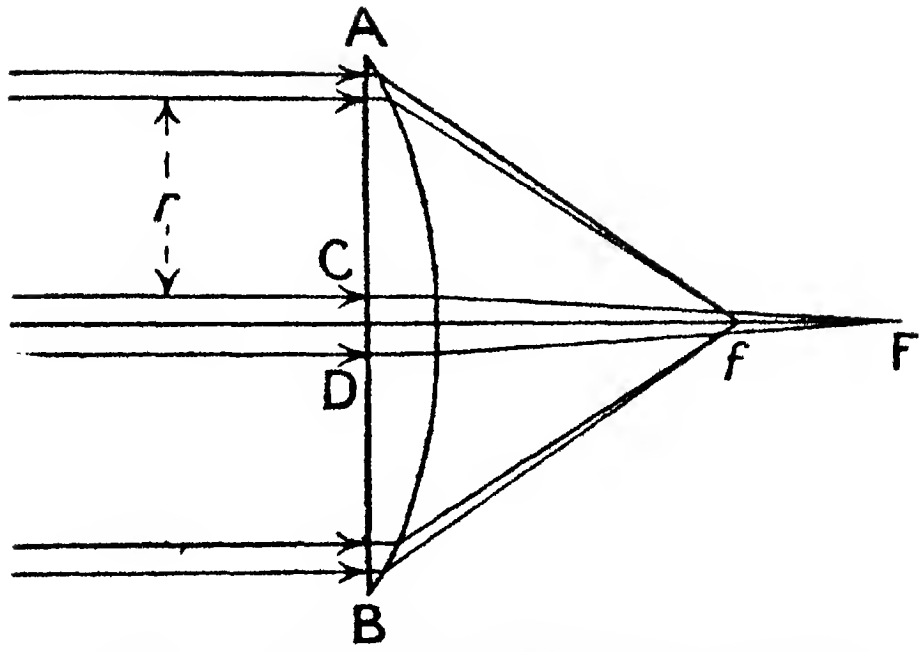
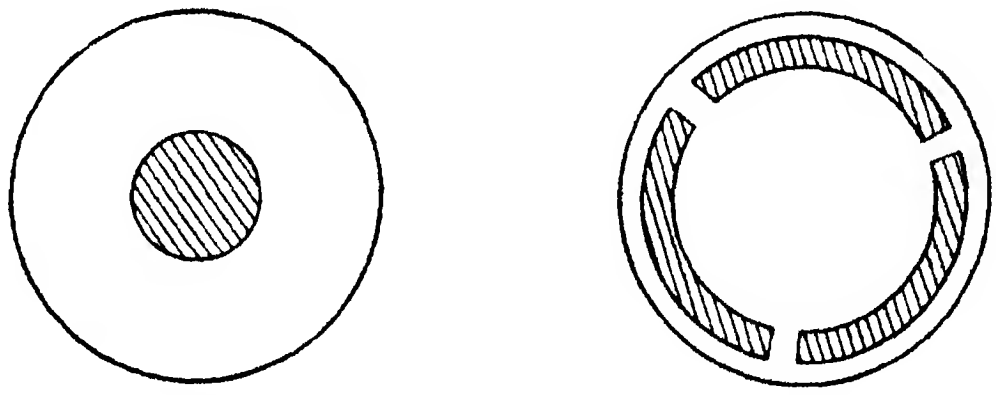


FIG. 42. Longitudinal aberration of a lens.

circular zone employed. Take observations with the six or seven zones available. Since a relation of the form $S = cr^n$ is expected,



FIGS. 43 and 44. Diaphragms.

plot the log of S against the log of r.

$$\log S = \log c + n \log r.$$

A straight line should therefore be obtained as in Fig. 45 and the power n may be determined from the slope. Now reverse the lens, repeat the observations and find out which position gives the smallest amount of aberration.

An alternative method is described in the experiment which follows.

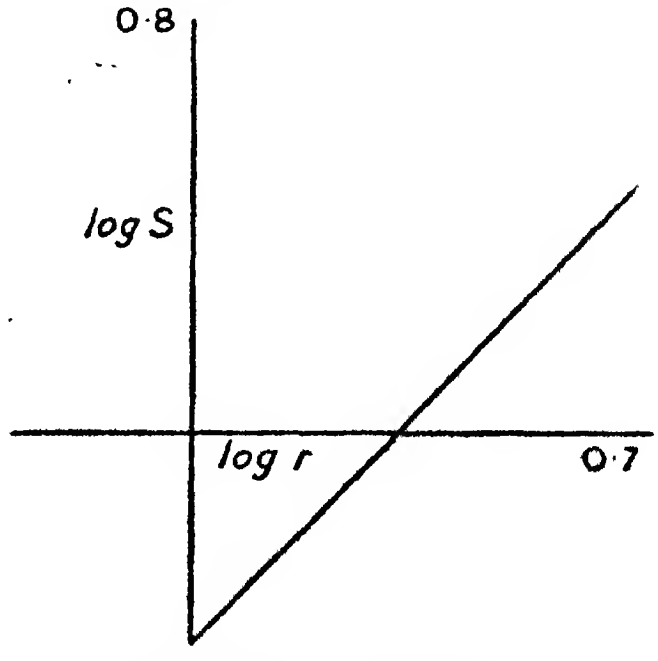


FIG. 45. Linear plot.

18. The investigation of the spherical aberration of a lens by means of shadows

In the method of shadows a real image I of a small bright source O is formed by a convex lens and a coarse grating made of wire or string is placed with the lines vertical in the neighbourhood of the focussed image at G or G' in Fig. 46. The shadow effects are examined on a screen placed some 2 or 3 metres away, and it is found that for position G' the shadow of

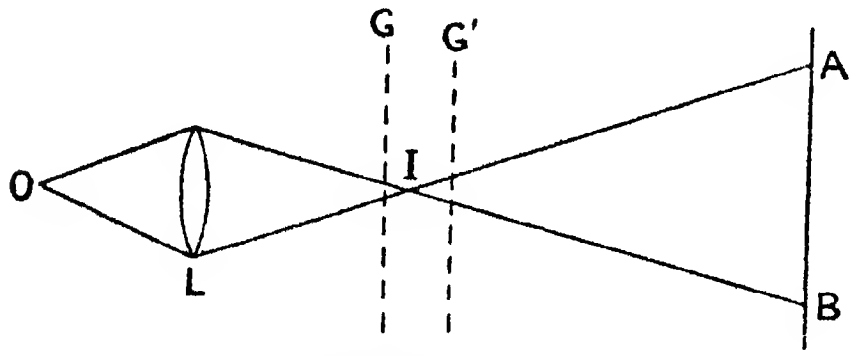


Fig. 46. Grating near the real focus.

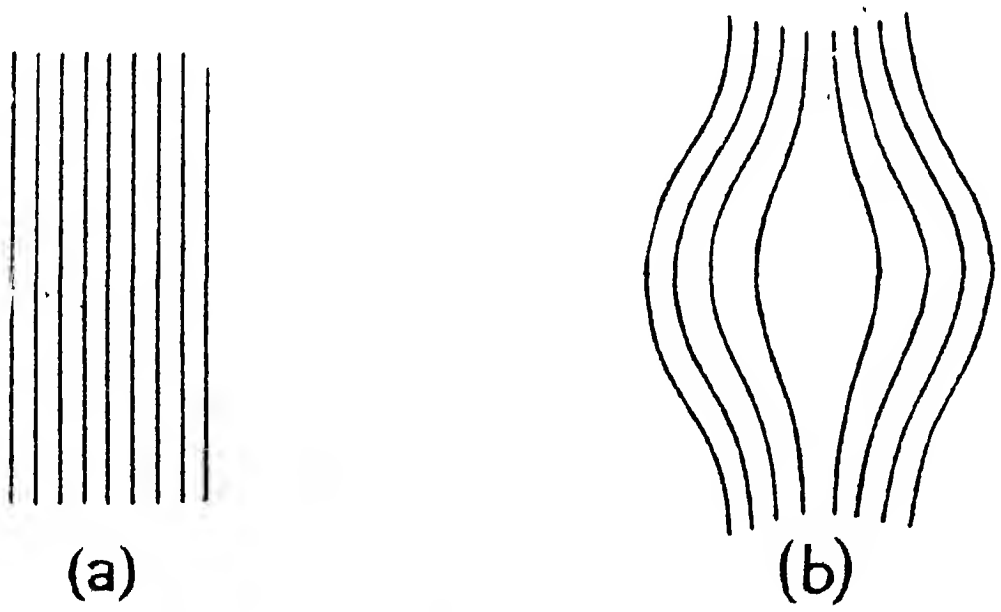


Fig. 47. Image of a grating.

a grating (a) in Fig. 47 is distorted into the appearance of (b) (Fig. 47). This is due to the fact that the rays of light do not

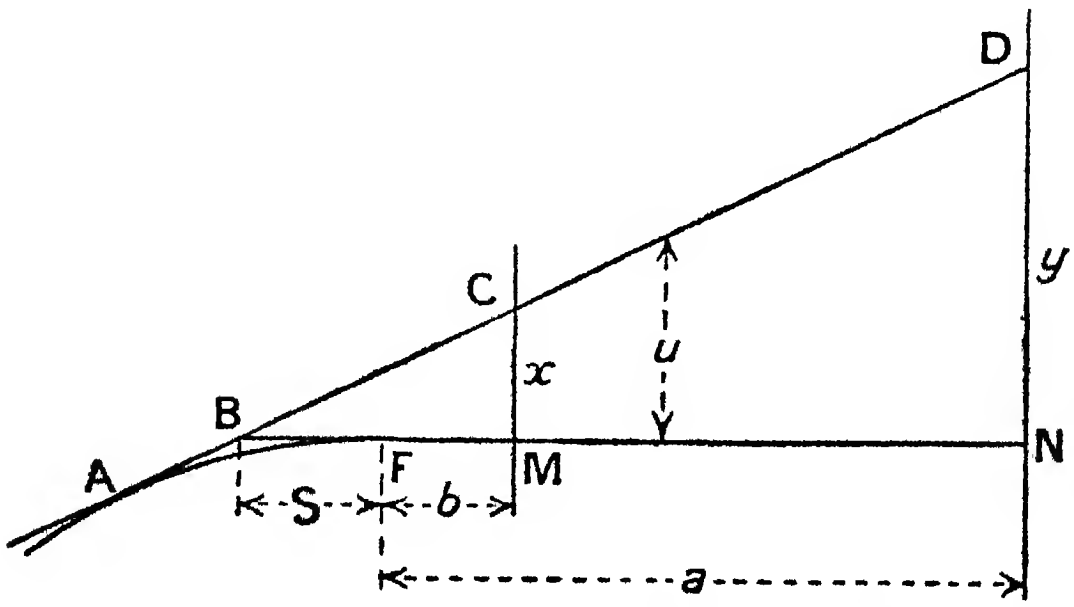


Fig. 48. A tangent to the caustic curve.

come from a single focus but from various points of the caustic curve in the neighbourhood of the focus.

Thus in Fig. 48 let F be the focus or the cusp of the caustic curve AF. Consider a ray $ABCD$ which is tangential to the caustic at A, which cuts the axis at B, the grating at C and the screen at D, and which makes an angle u with the principal axis of the lens. Let x be the distance CM to a ruling on the grating and $y = ND$ the corresponding distance to the shadow image. Then with a for the distance between the real focus and the screen, b for the distance between the real focus and the grating, and S the longitudinal aberration,

$$\tan u = \frac{y - x}{a - b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Also } \frac{b + S}{x} = \operatorname{cotan} u = \frac{a - b}{y - x}$$

$$\therefore \qquad S = \frac{x(a - b)}{(y - x)} - b \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus if x and y can be measured and the distances a and b are known, it is possible to calculate the longitudinal aberration S in terms of tan u.

In the experiment a mercury-lamp with a green filter is a suitable means of illumination, and a grating with about 4 or 5 lines per cm. may be used. The student is recommended to make a series of measurements to test the validity of the relation $S = A \tan^n u$ where A is a constant and n is a power to be determined.

The method here described is worthy of further experiments of a qualitative nature, and the shadows of a line, a square grating or gauze, etc., are very beautiful if white light is used, owing to the effect of chromatic aberration.

REFERENCES

Silvanus Thompson : *Photograph. Journal*, **48**, p. 383, 1901.
Bennett : *Phys. Soc. Proc.*, **19**, p. 205, 1904.
Jentzsch : *Phys. Zeits.*, **29**, pp. 66–72, 1928.

19. To determine the thickness of a soap-film by means of the Jamin interferometer

The plates A, B of a Jamin interferometer are illuminated by means of a sodium flame S (Fig. 49) and a convex lens L. By suit-

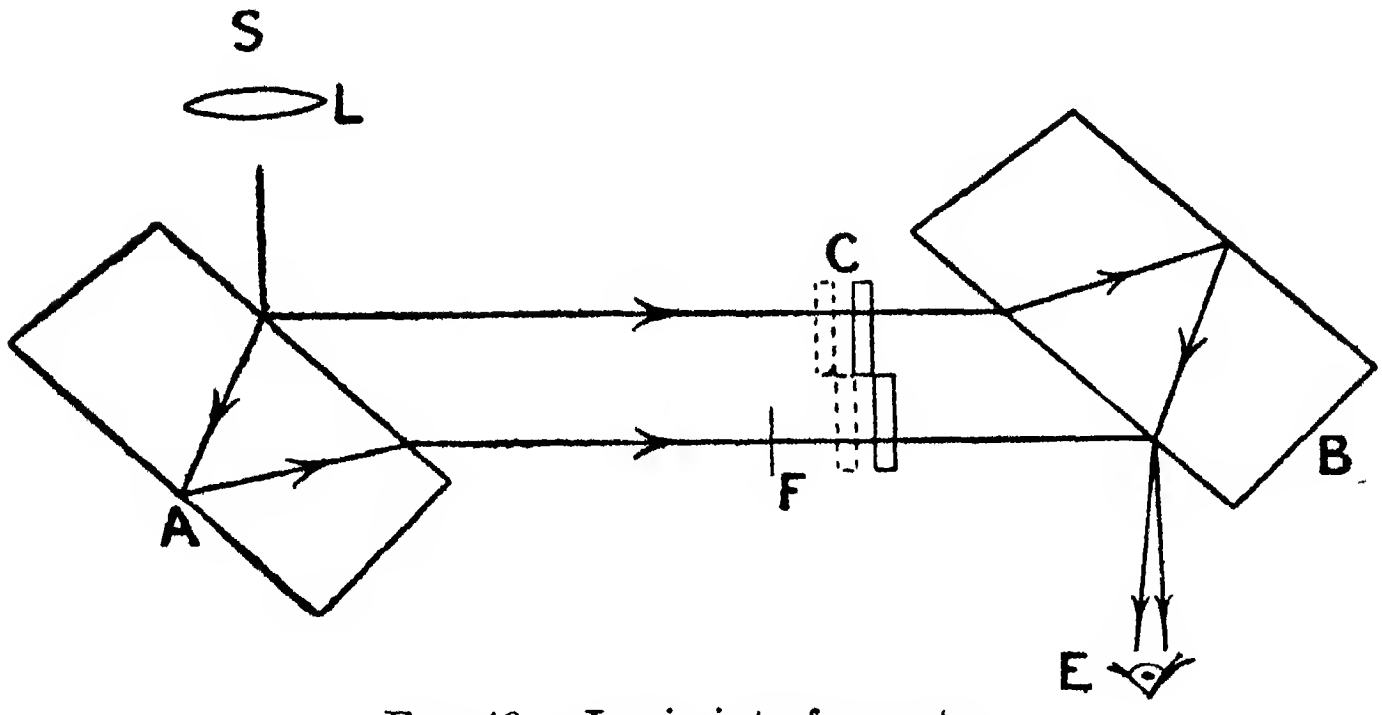


FIG. 49. Jamin interferometer.

able manipulation, which is best explained by a demonstrator, it is possible to view interference fringes by means of the eye *E* or by a telescope focussed for a distance of a metre or two (Fig. 50).

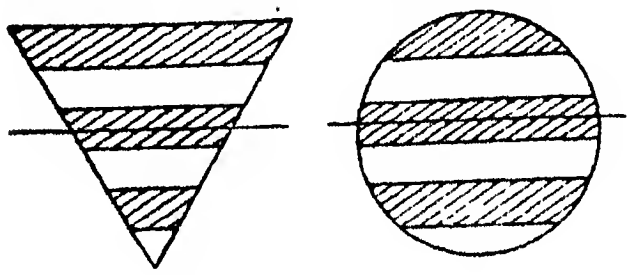


FIG. 50. Interference fringes.

C is a compensator formed of two similar glass plates slightly inclined at an angle and a rotation of this compensator shifts the fringes. A scale graduated in degrees is attached and with fairly wide sodium fringes a relation between the fringe-shift and angle may be obtained.

A white bat's-wing burner flame is now substituted for the

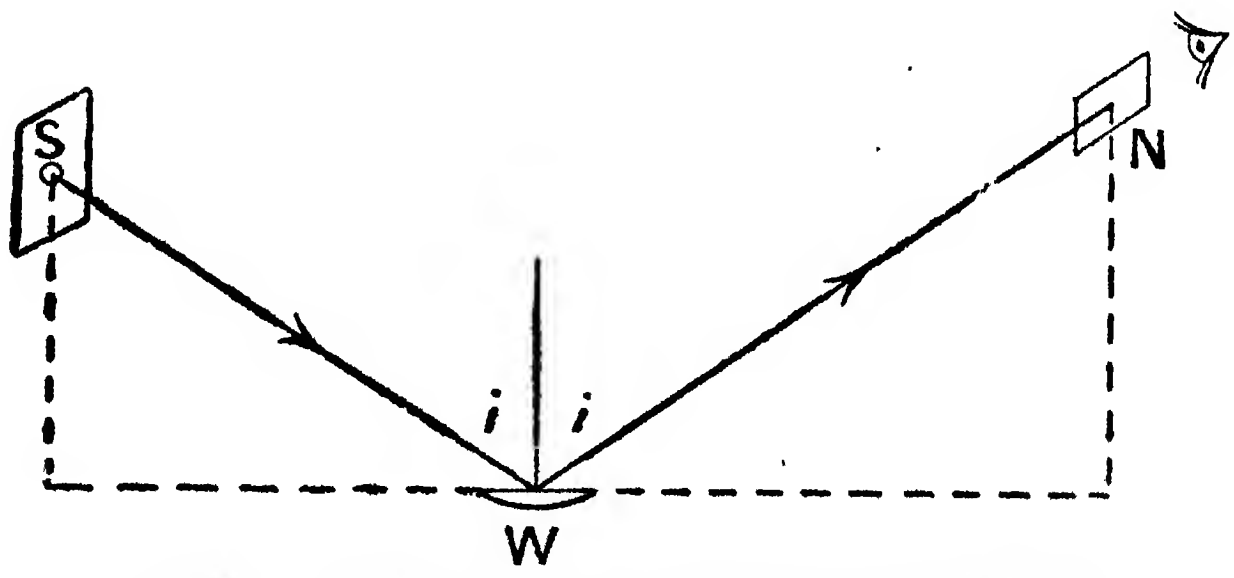


FIG. 51. Refractive index by the polarising angle.

sodium flame, and with care it is possible to find a series of coloured fringes with a white central fringe, which serves to mark out the position of zero path difference. It is arranged that the cross wire of the telescope eyepiece lies centrally across the

central white band. Now a small wire frame should be used to hold the soap film which is inserted in one of the interfering beams at F. The change of path difference produced shifts the system of coloured fringes, but by suitably turning the compensator the central white fringe may be restored to its original position in relation to the cross wire. (This is not quite true, because of dispersion, but since the thickness is small there is little error.) From the previous calibration of the compensator, the total shift in terms of sodium fringes may be found.

The refractive index of the soap solution is now required. This may be found by placing a small quantity of the solution in a watch glass W as shown in Fig. 51 and covering the solution except for a small area. S is a distant illuminated aperture, suitably placed, and N is a nicol prism, which when suitably placed and rotated about the line WN as axis will cut off the light reflected from the film. From the positions of the slit, solution and nicol, the polarising angle may be found. The refractive index μ is given by the tangent of this angle, by Brewster's Law. A value near that for water will be obtained.

To find the thickness t of the soap-film we have then $(\mu - 1)t = n\lambda$ where n is the number of the fringe-shift and λ is the wavelength employed. There is a tendency for the film to drain, but a value equal to a few wavelengths will be found.

20. To find the thickness of a Fabry and Perot étalon

The étalon to be used may be made by means of two triangular pieces ABC of hard wood cut with holes to hold two good glass flats

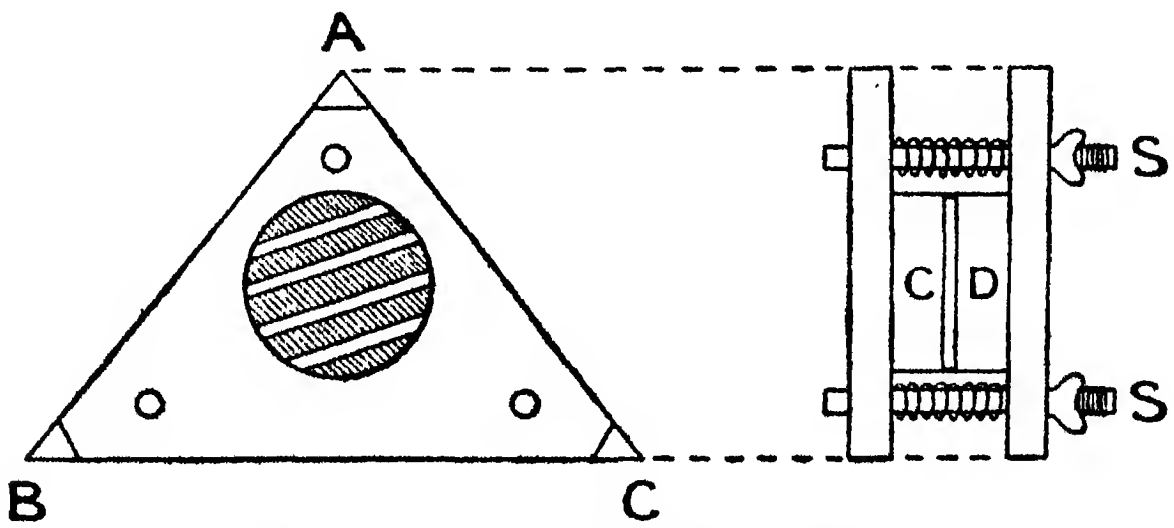


Fig. 52. Fabry and Perot étalon.

C and D (Fig. 52). By means of suitable screws S and a small thickness of mica the glass flats after being semi-silvered by the

method described at the end of this book may be pressed so as to form an air-film in the form of a wedge of variable angle. The effect of the silvering is to produce finer fringes of better visibility than are obtainable with plain glass. If such an air-film is viewed in reflected sodium light, fringes are usually seen, but by careful adjustment of the screws, the number of the fringes may be reduced to two or three, as seen in Fig. 53.

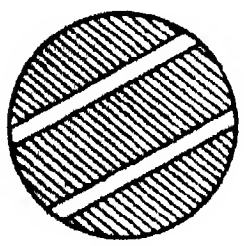


FIG. 53. Interference fringes.

The étalon is now ready for use, and its thickness may be determined as follows. If strongly convergent light is allowed to fall on the étalon E, as shown in Fig. 54, there

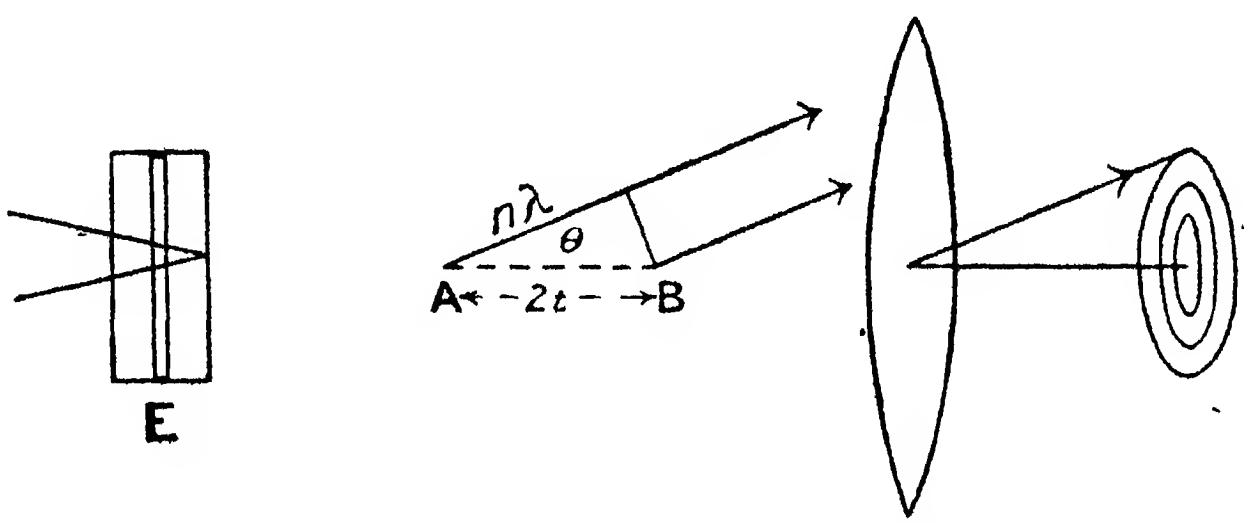


FIG. 54. Fringes of equal inclination.

are "to-and-fro" reflections in the air-film but the fringes obtained may be explained by the assumption of interference between two point sources which are a distance apart equal to twice the thickness t of the film.

For the normal direction AB, $2t = n\lambda + f_\lambda$ where n is an

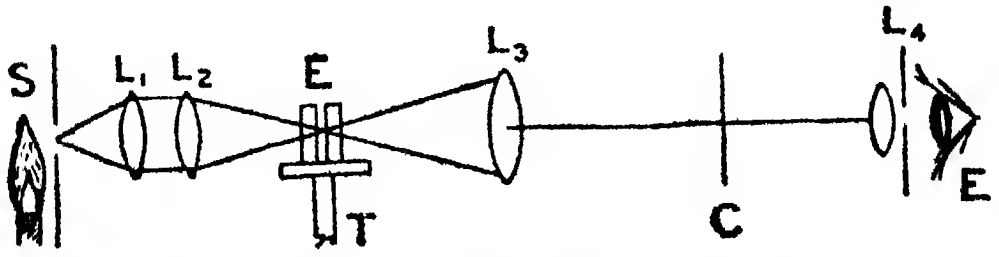


FIG. 55. Method of observation.

integer and f_λ is a fraction of a wavelength. For an angle θ, $2t \cos \theta = n\lambda$.

If therefore light from the sources A and B is collected by a convex lens L there is formed in the focal plane of the lens a system of bright and dark circular fringes which are known as *fringes of equal inclination*. The aim of the experiment is to measure the angular diameters of these rings and from the results

deduce a value for t the thickness of the film. The experimental arrangement is shown in Fig. 55. A small circular hole in a metal screen S is illuminated by means of sodium light and by means of lenses L_1 L_2 a fine cone of rays is focussed a little beyond the air-film in the étalon E, which is placed on the turntable T of an ordinary spectrometer. L_3 is a lens of fairly long focal length, say 30 cm., which produces a system of circular fringes at C where a silk or cotton fibre or a pin is used for purposes of reference. Another lens L_4 may be set up as an eyepiece and focussed on C, or a pin hole in a piece of card may be used to enable the observer to know where to place his eye. Fig. 56 represents two fringes with a cross wire at the ends of a diameter. In practice, by rotating the turntable it is possible to measure the angular diameter 2θ for any ring, with the cross wire fixed.

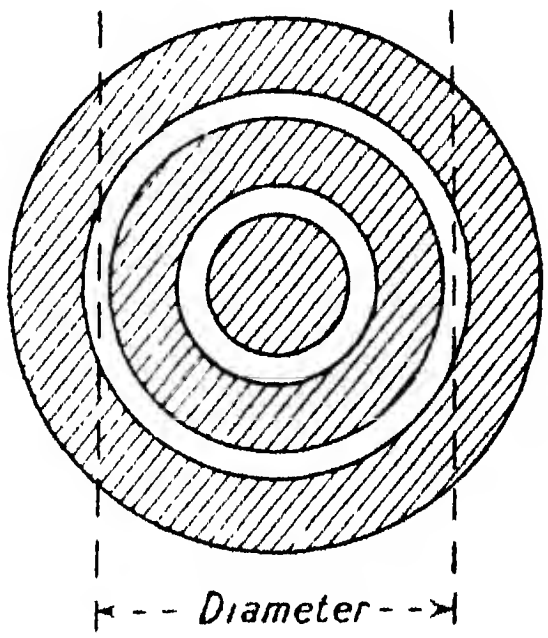


FIG. 56. Circular fringe with cross wire at ends of a diameter.

This is repeated for a number of rings, and if the rings appear almost equally spaced they may be taken in fives. Students have measured angles for numbers up to 30 or 50 rings. The results are treated as follows, λ being the wavelength.

For the centre of the system

$$2t = n\lambda + f_\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For 1st ring $\quad 2t \cos \theta_1 = n\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$

For 2nd ring $\quad 2t \cos \theta_2 = (n - 1)\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$

For sth ring $\quad 2t \cos \theta_s = (n - \overline{s - 1})\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

$$\therefore \quad \frac{2t}{\lambda} . \cos \theta_s = \overline{n + 1} - s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Plot $\cos \theta_s$ for a ring s against its number s and a straight line is obtained. The slope of the line gives $2t/\lambda$, and hence t if λ is known.

Hence n and the fraction f_λ may also be found.

An interesting method used by Fabry and Perot to find the fraction is to use the diameter of the first ring. From equations (1) and (2) above we have

$$2t(1 - \cos \theta_1) = f_\lambda.$$

Thus is determined the thickness of the film, the order of the fringes at the centre, and the fractional part, if any.

21. To find the length of a Fabry and Perot étalon by the method of fractions

The maker of an étalon furnishes the approximate length or the distance between the plates of the étalon, but the aim of the present exercise is to obtain by an optical method an accuracy very much greater than that assumed by the maker.

In Fig. 57 light is shown converging on an étalon E and the resulting beams are focussed by means of a good achromatic lens, A_1, upon SS', which represents the slit of a spectrometer. A bright ring of order n, and angle θ, will have a radius in the focal plane of the lens A_1 of $r_1 = F_1\theta$ if F_1 is taken as the focal length.

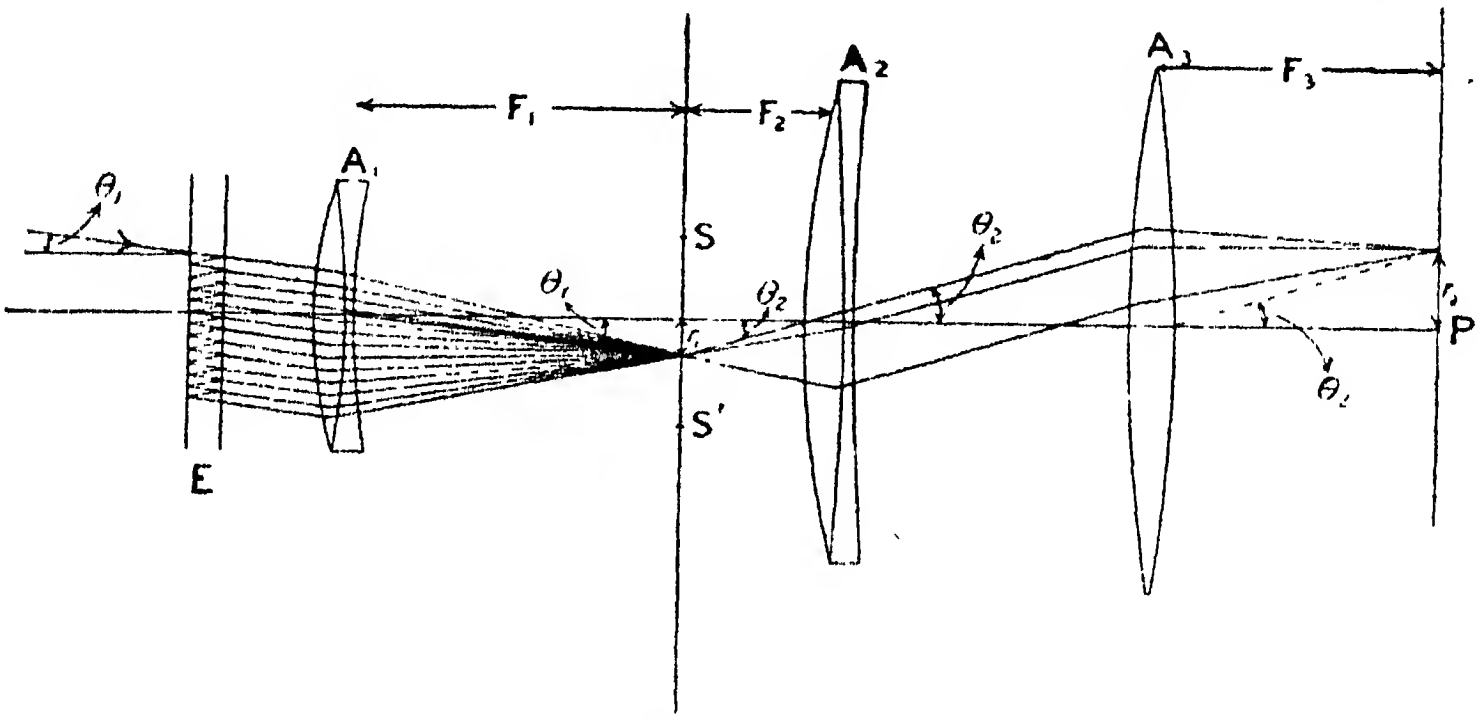


Fig. 57. Formation of circular fringes.

Light from the ring r_1 now passes through the lens A_2 of the collimator so as to form a beam of light making an angle θ_2 with the principal axis of the lens. After passing through the prism of a spectrometer (omitted in the diagram) the light passes through A_3, the objective of the telescope or camera, to form an image of the ring in the focal plane P. The radius of the ring r_2 in the focal plane or on the photographic plate is given by $r_2 = F_3\theta_2$ where F_3 is the focal length of the objective. Thus if the linear magnification of the spectrograph has been previously determined, and if r_2 is measured on a photographic plate or by means of a micrometer eyepiece, r_1 may be calculated. If the focal length F_1 of the achromatic lens is also known $\theta = \frac{r_1}{F_1}$ for the ring may be found. As previously described, $2t = n\lambda + f_\lambda$ where t is the thickness of the étalon and f_λ is the fractional part of the order.

$$2t \cos \theta_1 = n\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and
$$2t(1 - \cos \theta_1) = f_\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Thus from the determination of θ_1 described above it is possible to determine the fraction f_λ. In practice it is more convenient to put $n = n_0 \cos \theta = n \cos \varphi/2$ where n_0 is not necessarily an integer and φ is the angular diameter of a ring. Expanding the cosine as a series and only retaining squares of φ, it is found that

$$n = n_0(1 - \varphi^2/8) \text{ or } \varphi = \sqrt{\frac{8}{n_0}} \cdot \sqrt{n_0 - n}.$$

For the first bright ring $n_1 = n_0 - f_\lambda$, so that

$$\varphi_1 = \sqrt{\frac{8}{n_0}} \cdot \sqrt{f_\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and for the p th ring

$$\varphi_p = \sqrt{\frac{8}{n_0}} \cdot \sqrt{f_\lambda + (p - 1)} \quad . \quad . \quad . \quad (5)$$

If the angular diameter φ_1 of the first ring has been determined, and if n_0 is known approximately from the maker's value of t, f_λ may be calculated from relation (4). To avoid ambiguity, the fractions are determined for three or four wavelengths. An example will make the method clear.

For a red radiation λ_1, $n_0\lambda_1$ was known to be near 31,050. Then the order for any other wavelength λ_2 was given by $\frac{\lambda_1}{\lambda_2} \times n_0\lambda_1$. The following table was drawn up :—

n_0 for Red.	Green.	Blue.	Violet.
31,048·82	39,306·70	41,648·06	42,732·04
⋮	⋮	⋮	⋮
31,053·82	39,313·03	41,654·77	42,738·92
⋮	⋮	⋮	⋮
31,059·82	39,320·63	41,662·81	42,747·18

The fractions measured for the red, green, blue and violet radiations were 0·82, 0·00, 0·79 and 0·93 respectively, leaving no doubt that the central row of the table was the one required. Thus n_0 for cadmium red was 31,053·82, giving an accuracy of the order of one part in a million. Williams recommends that a ring other than the first be measured owing to the fact that the dispersion $d\theta/d\lambda$ increases very rapidly near the centre.

To perform the experiment the étalon *E* in Fig. 58 is placed upon a special stand supplied by Messrs. Hilger, and having the achromatic lens *A* capable of slight adjustment but placed in a holder attached to the stand. Light from a neon tube passes through the lens *L* to focus a converging beam in the usual way. A system of rings is focussed upon the fairly wide slit *S* of the

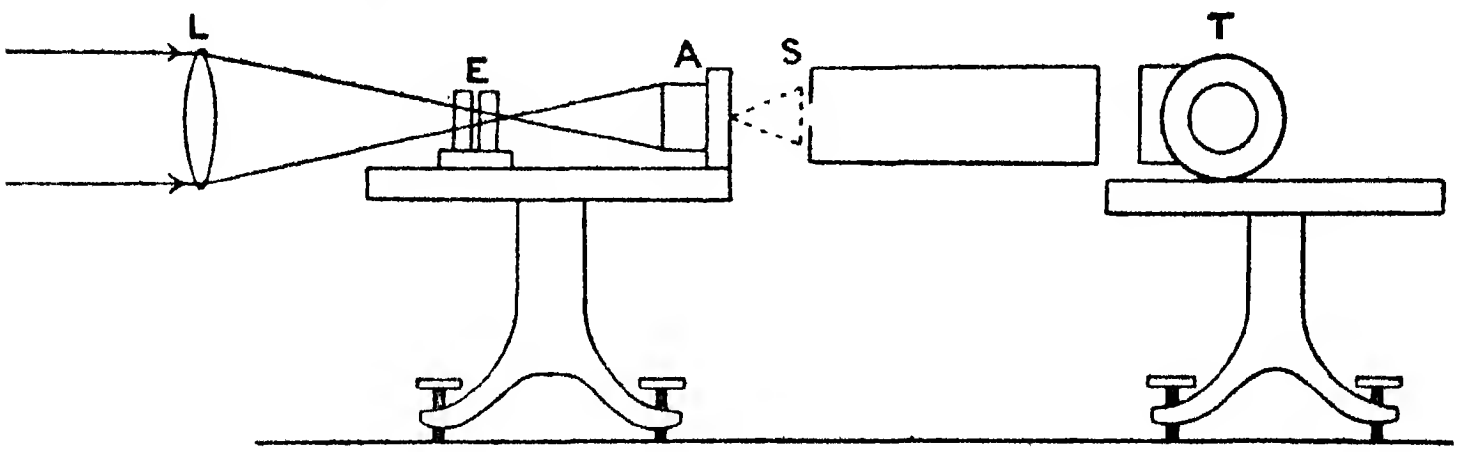


FIG. 58. Formation and observation of circular fringes.

spectrometer and the ring-images may be viewed in the eye-piece of the telescope *T* or may be photographed by means of a camera attachment. If a number of radiations are present each radiation forms its own system of rings. Owing to the large width of the slit, sections of the rings are observed as shown in Fig. 59, the rings at the red end of the spectrum appearing more widely separated than those for the violet end of the spectrum. The linear diameters of the ring-images may be measured as described above. For neon suitable lines to use are 6096A, 5852A and 5015A. For further details the student might well consult *Applications of Interferometry*, by W. E. Williams (Methuen).

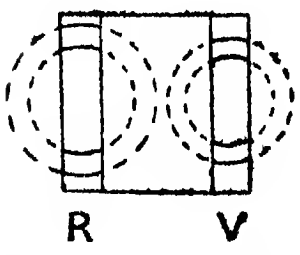


FIG. 59. Sections of circular fringes.

22. TO FIND THE THICKNESS OF AN AIR-FILM

The thickness of an air-film formed between two parallel semi-silvered glass plates, such as is obtained in the étalon used in experiment 20, may also be determined by the following method.

If parallel white light is incident normally on the glass plate, interference occurs between a beam of light which goes straight through the film and one which is twice reflected in the film at the semi-silvered surfaces of the glass. If light emerging from the film is examined by means of a spectroscope, the white light spectrum will appear to be crossed by a number of dark bands

as indicated in Fig. 60. The distance between two neighbouring bands decreases from the red to the violet end of the spectrum.

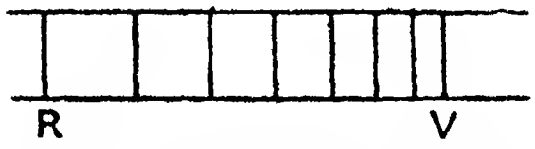


FIG. 60. Banded spectrum.

If t is the thickness of the film and λ_r is some wavelength in the red, the condition for a dark band is given by

$$2t = N\lambda_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where N is an integer and a half.

Similarly for some wavelength λ_v in the violet, a dark band will be obtained if

$$2t = (N + n)\lambda_v \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where n is an integer, giving the number of the band, counting from the band chosen in the red.

Thus

$$2t\left(\frac{1}{\lambda_v} - \frac{1}{\lambda_r}\right) = n \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here, therefore, is a method for finding t.

In the arrangement depicted in Fig. 61 L is a powerful white light source placed at a distance from the slit of a spectrometer

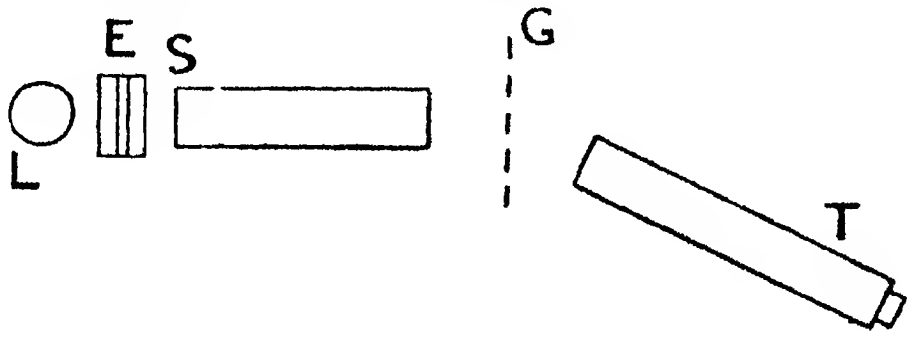


FIG. 61. Method of observation.

sufficiently large to allow of the étalon E being placed next to the slit S. A diffraction grating G of known grating interval is used to produce the spectrum which is observed by means of the telescope T. For the first order spectrum, $\lambda = (a+b) \sin \theta$ where $(a + b)$ is the grating interval.

If $N = \dfrac{1}{a+b}$ is the number of lines per cm., $\dfrac{1}{\lambda_v} = N \operatorname{cosec} \theta_v$.

Substituting in (3)

$$2tN \,.\, (\operatorname{cosec} \theta_v - \operatorname{cosec} \theta_r) = n \quad . \quad . \quad . \quad . \quad (4)$$

Thus, measure θ for a number of bands for which n has the

values 1, 2, 3, etc., counting from the band for the arbitrary wavelength λ_r.

Plot a graph between cosec θ_v and n and from the slope of the resulting straight line determine t,-the thickness of the film.

The film may also be used to calibrate a spectrum by the method of Edser and Butler.

Suppose that λ_v and λ_r are two known wavelengths and that λ' is some wavelength for which the number of the band is n'.

Then $$\frac{\frac{1}{\lambda'} - \frac{1}{\lambda_r}}{\frac{1}{\lambda_v} - \frac{1}{\lambda_r}} = \frac{n'}{n} \text{ or } \frac{1}{\lambda'} = \frac{n'}{\lambda_v} + \frac{(n - n')}{\lambda_r} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus a table may be constructed showing the wave-numbers of all the dark bands in the spectrum, and the corresponding angles on the scale of the spectrometer.

In this way the whole of the spectrum may be calibrated.

23. To find the refractive index of air by means of a Fabry and Perot étalon

For the interference rings produced by a Fabry and Perot étalon the relation between the order of interference n and the length of the étalon t is given by $2\mu t \cos\theta = n\lambda$ or

$$2\mu t = n\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

if θ be constant, and so small that $\cos\theta \longrightarrow 1$.

μ is the refractive index of the air between the plates. If the Gladstone and Dale relation be assumed,

$$(\mu - 1) = kp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where p is the pressure of the air and k is a constant. Thus

$$\frac{d\mu}{dp} = k = \frac{(\mu_0 - 1)}{p_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where μ_0 is the refractive index at normal pressure p_0. But from (1), for a particular ring,

$$2td\mu = \lambda dn \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and $$\frac{d\mu}{dn} = \frac{d\mu}{dp} \cdot \frac{dp}{dn} = \frac{\lambda}{2t}$$

$$\therefore \frac{d\mu}{dp} = \frac{\mu_0 - 1}{p_0} = \frac{\lambda}{2t} \cdot \frac{dn}{dp}$$

or $$\mu_0 - 1 = \frac{\lambda \cdot p_0}{2t} \cdot \frac{dn}{dp} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus if the pressure of the air is varied, the rings may be made to move, and for a change in order dn, the pressure-change dp may be observed.

In Fig. 62 the étalon E is shown in a bell-jar B, which may be

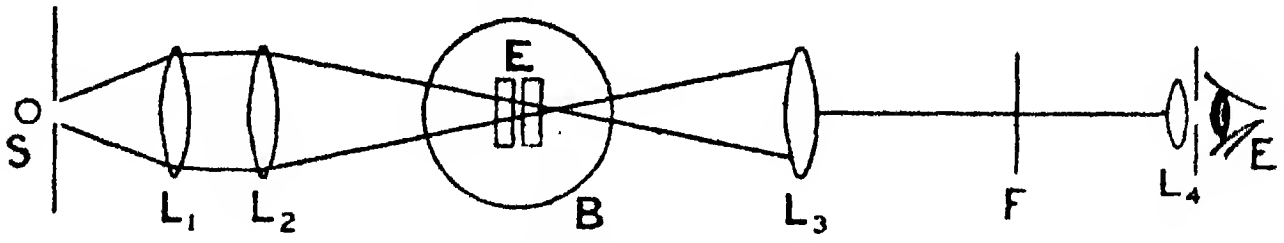


FIG. 62. Formation of circular fringes.

exhausted by means of a Hyvac air-pump. S represents the tube of a neon lamp working from a transformer. The interference rings are formed in the manner previously described, L_4 being a small magnifying-lens near a small hole in a screen

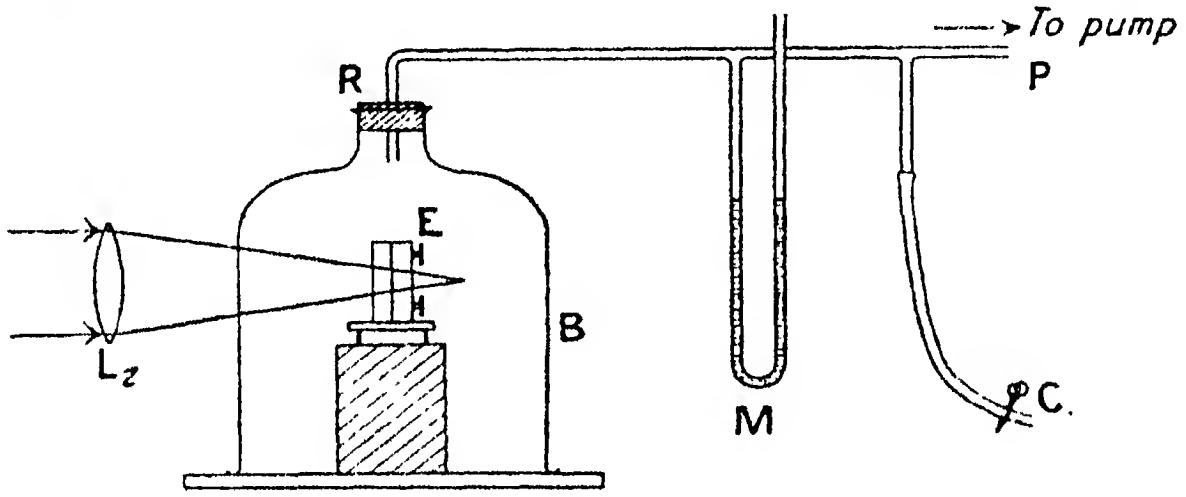


FIG. 63. Measurement of pressure.

focussed on a cross wire at F to view the rings. In Fig. 63 the étalon is shown mounted on a block of wood placed on a very thick sheet of plate-glass (a thin sheet will crack very easily when the bell-jar is exhausted). The bell-jar should be fitted on the glass plate by means of vacuum-grease or candle-grease. Through a rubber cork R at the top of the bell-jar there is a glass tube leading to a mercury manometer M, to the Hyvac pump P, and having attached to it a long piece of rubber tube closed with a clip C. It is best to exhaust the bell-jar as completely as possible and then, by manipulating the clip C, to let in air at a suitable rate.

Owing to the length of the rubber tube the eye may be kept near the eyepiece for viewing the fringes and near the manometer scale for reading the pressure difference. Two observers may share the work if necessary. Record the pressure difference for a change of one ring into the position of the next and repeat until

the pressure in the bell-jar again reaches the atmospheric pressure. Plot a graph between n, the number of rings passing the cross

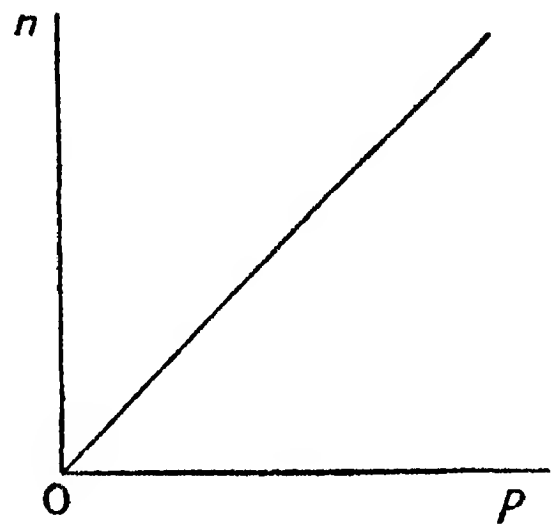


FIG. 64. Number of fringes and pressure.

wire, and p as in Fig. 64. Find dn/dp from the slope of the straight line and calculate $\mu_0 - 1$ from equation (5). Correct for temperature by means of Charles's Law.

Example

For an étalon for which $t = 1{\cdot}017$ cm., the maximum change in order for neon 6402A for complete exhaustion is approximately $\frac{2 \times 0{\cdot}0003}{6{,}400 \times 10^{-8}} = 10$. At a temperature of 18·5° C., a change of pressure of 58 cm. of mercury produced a shift of 7 fringes, thus $\frac{dn}{dp} = 0{\cdot}121$.

Thus $\mu_0 = 1{\cdot}00029_4$ at 0° C. and 760 mm. pressure.

24. TO FIND $\frac{\lambda}{d\lambda}$ FOR THE SODIUM D LINES BY THE MICHELSON INTERFEROMETER

The Michelson interferometer consists of two mirrors A and B placed at right angles so that B is fixed except for alterations in tilt and A may be moved parallel to itself by means of a long and accurately made screw (Fig. 65). D is a plate of glass semi-silvered on the side nearest to the observer at O, and C is a similar plate called the compensator, put in to equalise the two paths in glass of the interfering beams. These are divided as follows: a beam from a source S is refracted into plate D, partially reflected internally at the semi-silvering and refracted out towards the mirror A and back again through the plate D.

The other part of the beam is refracted out of D and goes towards the mirror B and back and is reflected to O. Thus one beam has had three paths in glass and the other only one. The

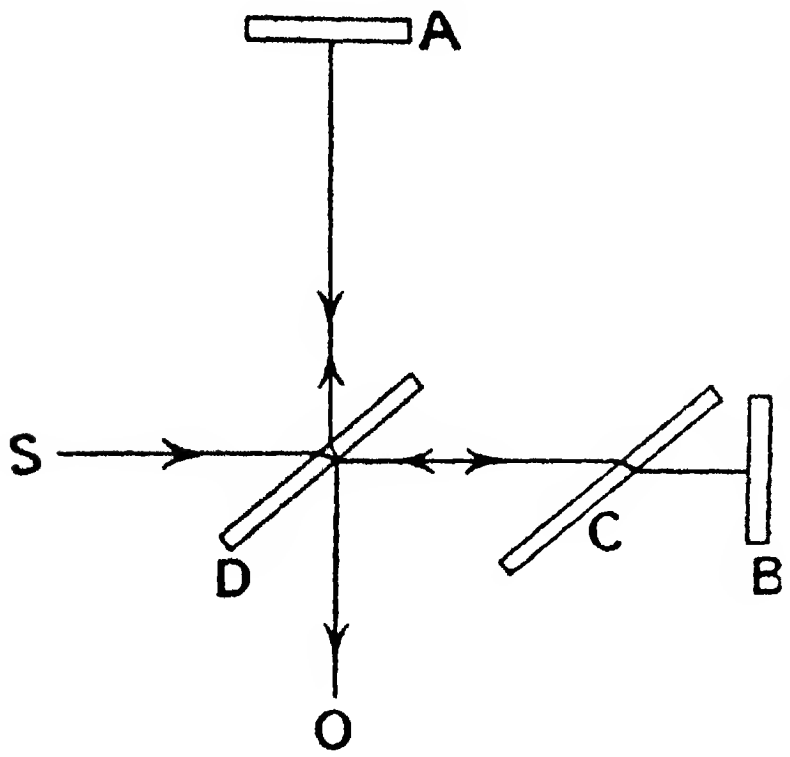


FIG. 65. Michelson interferometer.

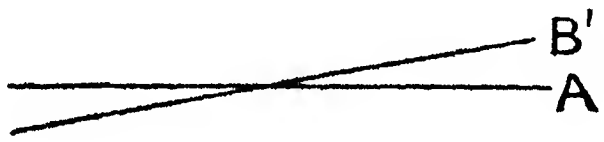


FIG. 66. Fringes of equal thickness.

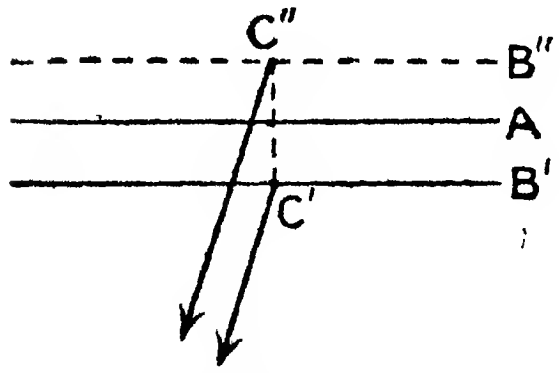


FIG. 67. The formation of "parallel plate" fringes.

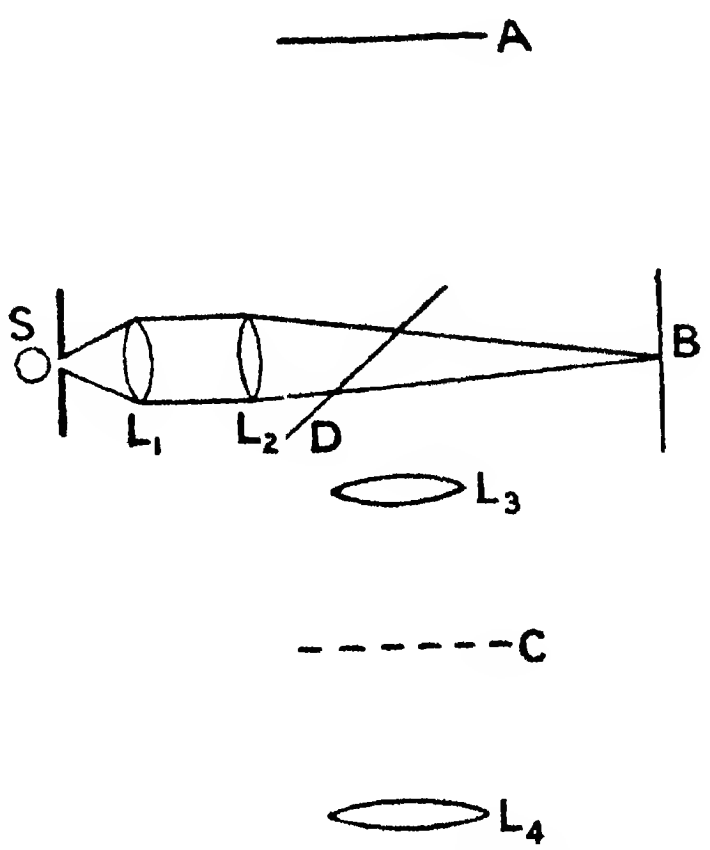


FIG. 68. Procedure for circular fringes.

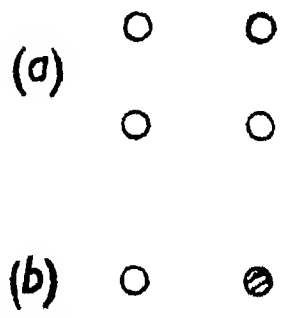


FIG. 69.

insertion of plate C equalises the glass paths. The two beams proceeding towards O may produce interference fringes of either of two kinds. If by means of the screws at the back of B the mirror is so inclined that its image B' in the semi-silvered plate makes an angle with the mirror A as shown in Fig. 66, then the

fringes observed are such as would be seen with a wedge-shaped air-film, and they are known as fringes of *equal thickness*.

If the image of B is parallel with A and a small converging beam of rays is used for illuminating B it may be imagined that the image point C' in B' interferes with its image C'' formed in A. Thus we have fringes of equal inclination as in the Fabry and Perot étalon (Fig. 67). The fringes may be obtained as follows :—

Make the paths A to D and B to D approximately equal by measurement with a scale. Illuminate a $\frac{1}{8}$-inch hole in a screen by means of a sodium flame S in Fig. 68. In general, four images will be seen, as in Fig. 69 (a), but a movement of the screws or fine adjustment on B will enable them to be reduced to two, as in Fig. 69 (b). Small straight fringes may usually be seen in one image, but if not, a slight turn of the large screw controlling A will bring them into view. The previous position may have been one of discordance for the sodium fringes, so that they are only seen with difficulty, if at all.

Now collect the light by means of a lens L_1 and allow a parallel beam to illuminate the whole of the mirror B. The fringes will be easily seen, and if too numerous should be reduced to two or three in number by means of the adjusting screws. The image of B will then be almost parallel to A. Now introduce a convex lens L_2 of some 20 cm. focal length and focus a small image on B. Insert a similar lens L_3 and circular fringes should be seen in its focal plane C. These may be observed by means of a lens L_4 used as an eyepiece, or by means of a small pin-hole in a piece of cardboard. Movement of the mirror screw will alter the thickness of the air-film between A and the image of B. Thus the circles increase or decrease in number and near the position of zero path difference they become very few and large ; as the image of B passes through A there is distortion of the fringes with circles again appearing on the other side of the zero position.

For sodium light with two near wavelengths there are positions for which the circular fringes are very bright, that is, for coincidences of the wavelengths, and positions of the mirror A for which the fringes disappear, known as discordances. From measurements of the coincidence or discordance length, *i.e.*, the distance between two adjoining coincidences or discordances, it is possible to determine $\lambda/d\lambda$ for the lines used. Thus if t is the coincidence or discordance length,

$$2t = N(\lambda + d\lambda) \text{ where } N \text{ is an integer}$$

and

$$2t = (N + 1)\lambda$$

$$\therefore \qquad 2t\left(\frac{1}{\lambda} - \frac{1}{\lambda + d\lambda}\right) = 1$$

$$\therefore \qquad \frac{2t \,.\, d\lambda}{\lambda^2} = 1$$

and
$$\frac{\lambda}{d\lambda} = \frac{2t}{\lambda}.$$

Thus in the experiment a large number of observations of the disappearance of the fringes is made and the distances measured on a scale connected to the screw moving the mirror A. By taking the observations in sets, say 20 — 10, 19 — 9, etc., the mean value of t may be found and hence $\frac{\lambda}{d\lambda}$ may be calculated.

It is sometimes useful and necessary to find the position of zero path difference. This may be done by observing when the circular fringes become very few and large, and then removing all lenses and screen and substituting a bat's-wing burner for the sodium flame.

By turning the A screw very slowly, coloured fringes should be brought into view near the position indicated by the circular fringes as described above.

Here the central fringe is black because one beam has undergone a reflection in glass and the other a reflection in air, so that the usual condition for an achromatic fringe is reversed.

25. To find the diameter of lycopodium particles

Observation shows that the curve representing the intensity in the diffraction pattern of a small circular aperture or

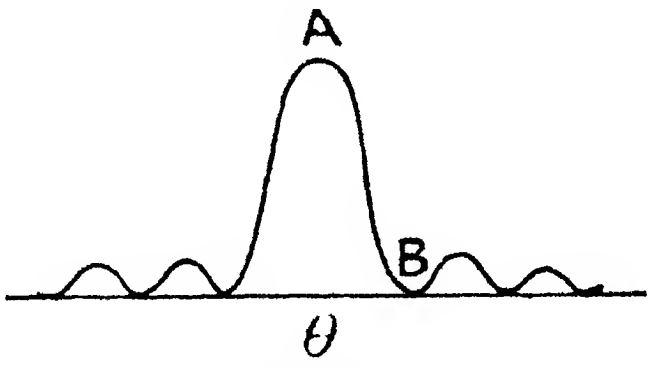


Fig. 70. Angular distribution of intensity in a diffraction pattern.

obstacle is somewhat as depicted in Fig. 70. The angle between the directions of the principal maximum A and the first minimum B is given by $1{\cdot}22\frac{\lambda}{d}$ where d is the diameter of

the aperture or obstacle and λ is the wavelength employed. Similarly the angle for the second minimum is given by $2{\cdot}23\,\frac{\lambda}{d}$.

If these angles can be measured by some means it is possible to calculate the diameter of a particle.

A piece of cardboard A in Fig. 71 has a small hole O about

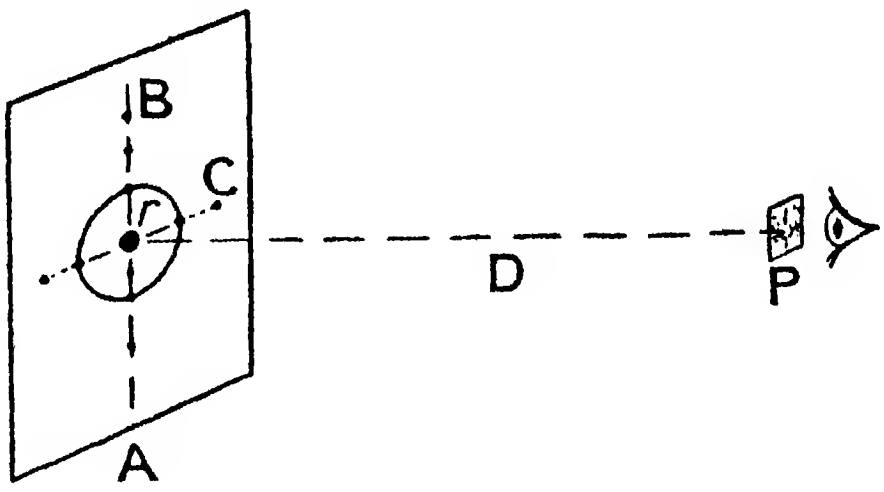


FIG. 71. Corona experiment.

$\frac{1}{8}$ inch diameter bored in the centre and along two axes at right angles OC and OB a number of pin-holes are made at different distances from the centre O.

A small quantity of lycopodium powder is sprinkled on a small glass plate P, and the hole in the card, illuminated with a

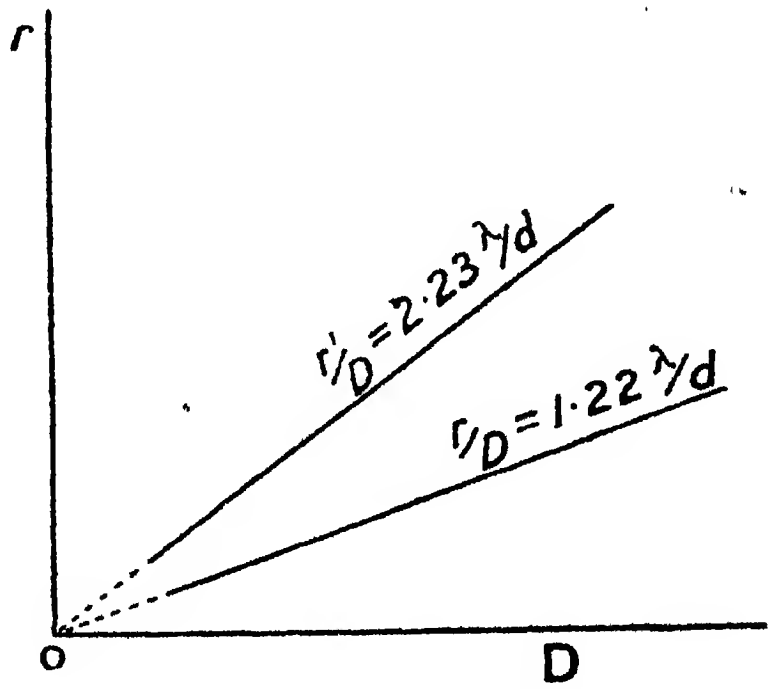


FIG. 72. Linear plot.

sodium light, is viewed by an eye looking through the plate. A number of bright and dark rings are seen surrounding the central hole, and by suitably adjusting the distance D between the glass plate and the cardboard screen, it is possible to fit the first dark ring to a pin-hole distant r from the centre. Then $\theta_1 = r/D = 1{\cdot}22\,\frac{\lambda}{d}$.

Similarly for the second dark ring

$$\theta_2 = \frac{r'}{D} = 2{\cdot}23\,\frac{\lambda}{d}.$$

Take a series of different values of r and D and plot a graph between r and D, and r' and D respectively (Fig. 72). From the slopes of the graph calculate d if λ is assumed. Then place 10–20 particles on the stage of a good microscope and measure the diameters of these particles for two directions at right angles. The particles will be found to be almost circular, and the mean value of the measured diameter should agree fairly well with the optical value. Then substitute a small electric bulb for the sodium flame, and although white light is being used a beautiful system of rings, with well-defined minima, will be observed.

Repeat the observations above, assume the value obtained for d and calculate the value of λ', the wavelength in white light for which the eye is most sensitive.

The powdered plate may be placed on the table of an ordinary spectrometer and angular measurements made, but the angles are very small, of the order of two or three degrees.

26. Michelson's method for the determination of the width of a distant slit

This experiment is of interest because it illustrates the principle of the method employed in recent times for the measurement of the diameters of giant stars. The essence of the method is to place a double slit over the objective of the telescope and view a distant slit of finite breadth through them. In general, interference fringes may be seen by means of the eyepiece, but by altering the distance between the slits it is possible to make the fringes disappear. If λ is the wavelength used, d the distance between the slits for the first disappearance of the fringes, D the distance between the fixed and adjustable slits, b the width of the distant slit, then the angular width of a distant slit is given by $\frac{\lambda}{b}$ or of a circular aperture by $1{\cdot}22\,\frac{\lambda}{b} = \frac{d}{D}$. Increasing the distance between the slits will make the fringes re-appear, to disappear again at values of d which are multiples of the smallest separation. In astronomy the two slits have to be replaced by small mirrors on the arms of a giant interferometer having a separation of 20 feet.

In the laboratory the method may be tested and the fringes

made to disappear by altering the width of a good spectrometer slit suitably illuminated as the source. A convex spectacle lens of some 30 cm. focal length and a piece of cardboard in which are cut two parallel slits each about 1 cm. long and a few mm. wide are also required. The distance from the source to the objective may then be about 1 or 2 metres. The fringes are viewed with a good eyepiece. Perhaps a more satisfactory test of the method may be made by using a fine slit at a distance of

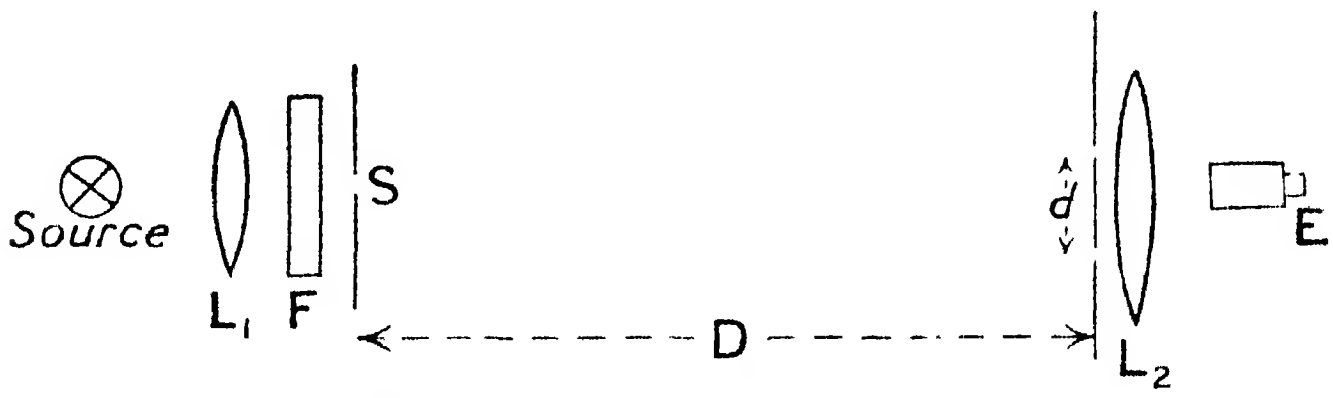


FIG. 73. Michelson method for width of distant slit.

some 20–24 metres and using a specially constructed adjustable double slit.

Set up at one end of a long laboratory a Pointolite lamp and a condensing lens L_1 to focus the light on the slit, and introduce a potassium bichromate filter F to isolate a region of wavelength in the yellow, which range must be determined by means of a spectrometer (Fig. 73). Use a spectrometer slit S of the order of 0·03 cm. and arrange that a travelling microscope may be placed nearby so that the width of the slit may be checked without disturbing the adjustment of the light source arrangement. About 24 metres away set up an object glass of some 1–2 metres focal length and 8–15 cm. in diameter. Use a good eyepiece at E and first focus on the image of the slit with the adjustable slits removed. Then place the two slits in front of the lens, and by adjusting S parallel to them, fringes will usually be seen.

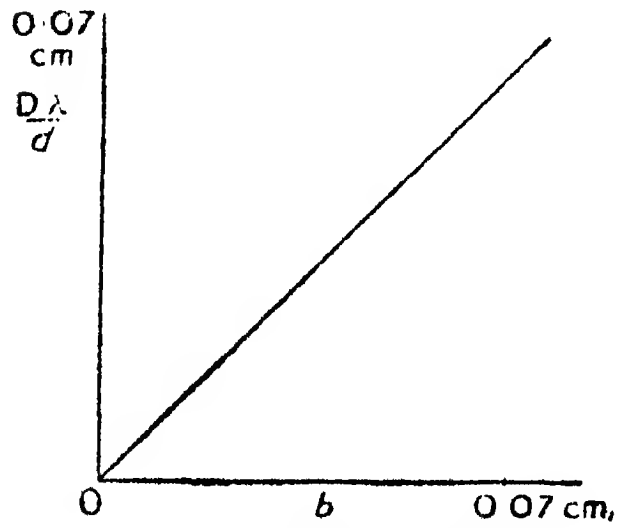


FIG. 74. Linear plot.

Alter the distance apart d of the variable slits to the smallest value which will make the fringes disappear. Calculate the width of the distant slit b from the expression $b = \frac{D\lambda}{d}$. Check

$$I = a^2 \cos^2\theta + b^2 \sin^2\theta + 2ab \sin\theta \cos\theta \cos\frac{\pi}{2}.$$

$$= a^2 \cos^2\theta + b^2 \sin^2\theta$$
$$= (a^2 - b^2)\cos^2\theta + b^2.$$

Thus for $\theta = 0 \quad I_{\text{max.}} = a^2$

$$\theta = \frac{\pi}{2} \quad I_{\text{min.}} = b^2.$$

A plot of I against $\cos^2\theta$ should give a straight line, the slope of which gives $a^2 - b^2$. Thus a^2 may be found most accurately, and hence b^2 and the ratio a/b.

The experimental arrangement is shown in Fig. 80. S is a 100-watt opal electric bulb, L is a lens. N_1 and N_2 are nicols

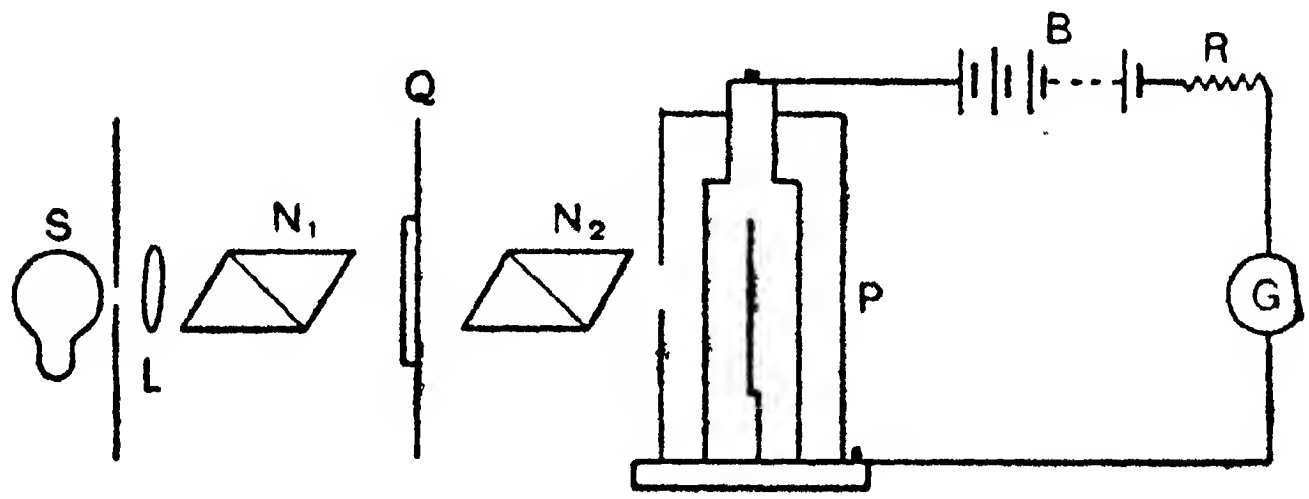


FIG. 80. Experimental arrangement.

and Q is the quarter-wave plate. P is the photo-electric cell, type KMV6, supplied by the General Electric Co., and most sensitive about the middle of the visible spectrum. It is placed in the usual circuit, which includes a battery B of 100 volts, a fairly high resistance R and a galvanometer of sensitivity of the order of 10^{-8} amperes per mm. at a scale distance of a metre. Fig. 81 represents a plot of the measured intensity in arbitrary units for various angles of the nicol N_2 throughout a complete rotation. The maximum corresponds to a deflection of about 10 cm. Fig. 82 shows the straight-line plot for which

$$I_{\text{max.}}\ (\theta = 0) = 9{\cdot}75.$$

From the slope $\quad a^2 - b^2 = 7{\cdot}9$

$$\therefore\ b^2 = 1{\cdot}85 \text{ and } \frac{a}{b} = 2{\cdot}3.$$

The maximum intensities separated by a rotation of 180° are not quite equal, but with the limitations of the quarter-wave

plate and the use of white light, the experiment does not seem to be too unsuccessful.

The sense of the vibration and the alteration of the ellipse by a suitable adjustment of the quarter-wave plate are not

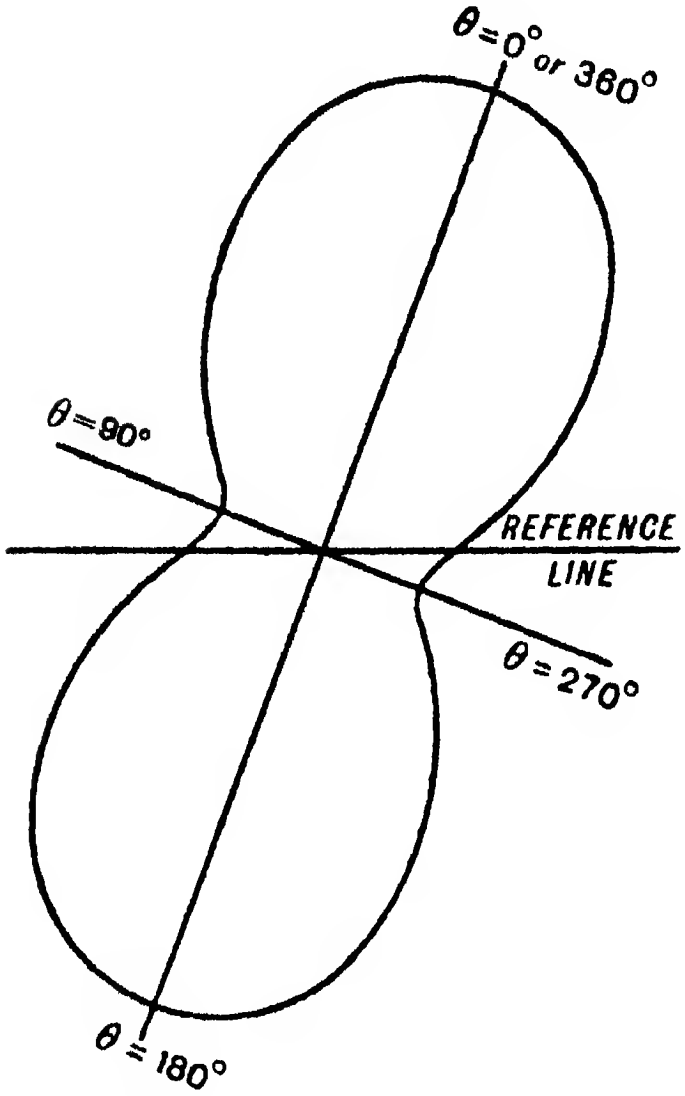


FIG. 81. Experimental curve.

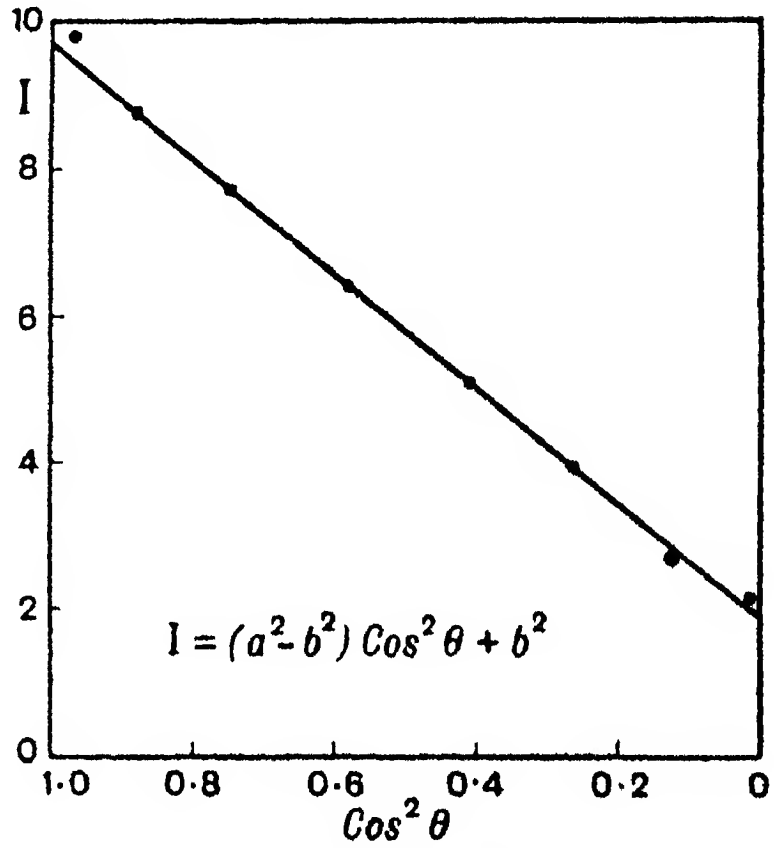


FIG. 82. Linear plot.

discussed, but we may deduce from the above that if $b = 0$, we have $I = a^2 \cos^2 \theta$, representing linearly polarised light with an intensity of a^2.

REFERENCE

CALTHROP : *Sch. Sci. Rev.*, Univ. Section, pp. 436–438, 1936.

29. TO INVESTIGATE THE ROTATORY DISPERSION OF QUARTZ

In this experiment use is made of the fact that a piece of quartz cut with faces perpendicular to the optical axis of the crystal will (for normal incidence) rotate the plane of polarisation by an amount of 21·724° per mm. of path for sodium light at 20° C. Thus if a quartz block of length 60–80 mm. be taken it is possible to secure a rotation of several complete turns. The effect of rotatory dispersion may then be observed and a curve showing the relation between rotation and wavelength may be

obtained. The experimental arrangement is shown in Fig. 83. *A* is a powerful white light behind a small hole in a screen. By

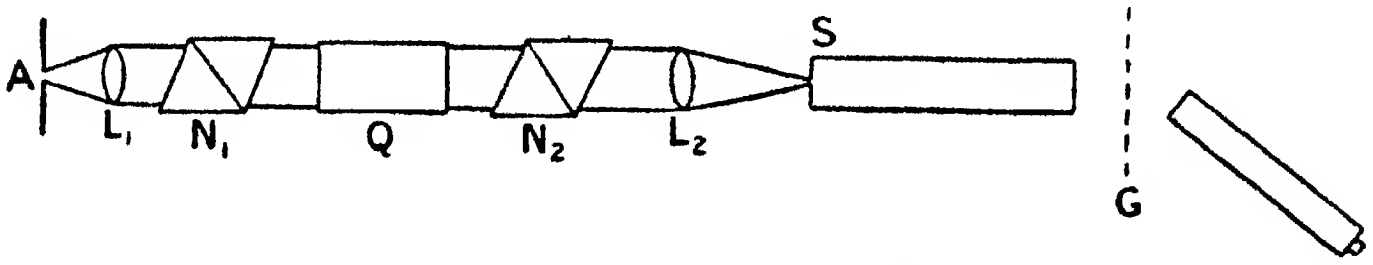


FIG. 83. Experimental arrangement.

means of a lens, L_1, a parallel beam is produced and polarised by a nicol prism N_1. A second nicol N_2 serves to cut out the radiation or transmit it as required, and L_2 is a lens for focussing

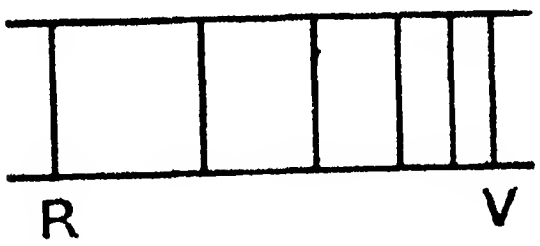


FIG. 84. Banded spectrum.

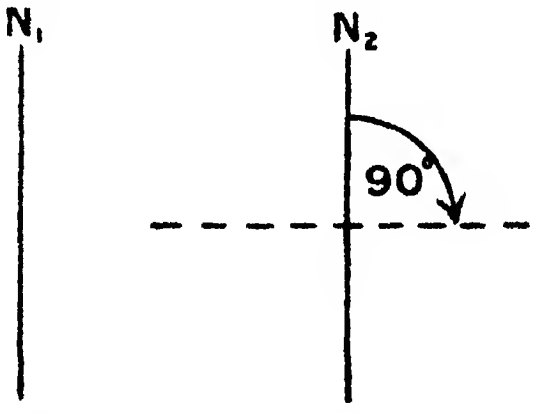


FIG. 85. Nicols parallel.

an image of the hole upon the slit of a spectrometer *S*, which is provided with a diffraction grating *G*.

With the spectrometer in correct adjustment, and with

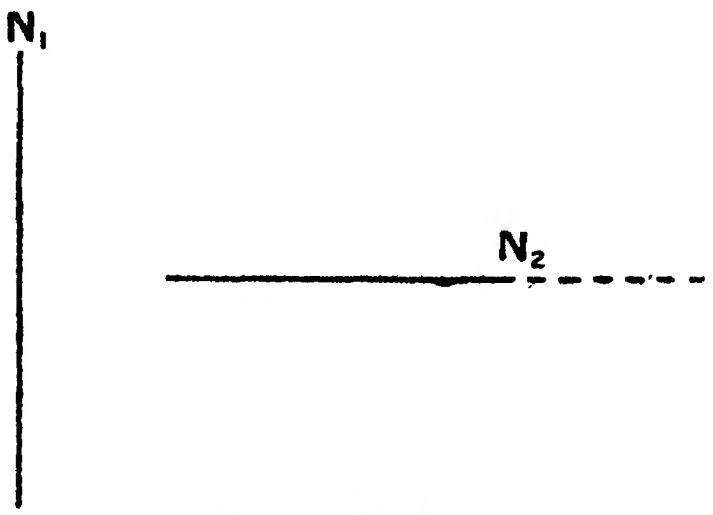


FIG. 86. Nicols crossed.

the nicols parallel, a good spectrum is obtained. Then the quartz block Q is put into the beam and it will be found that the spectrum is crossed by a number of dark bands as shown in Fig. 84. The reason for this is as follows :—

Suppose the nicols are originally parallel. The various radiations are rotated differently and some wavelength λ will be rotated

through some multiple of 90°; radiation of this wavelength will be stopped by the second nicol and a dark band will be shown in the spectrum (Fig. 85).

Thus the condition for a dark band is that the rotation $\rho = (2n + 1) \cdot \frac{\pi}{2}$. If the nicols are now crossed a second set of dark bands will be obtained due to a rotation of 180° or multiples of it (Fig. 86). Thus $\rho' = 2n\frac{\pi}{2}$. The wavelengths corresponding to the dark bands are measured. Since ρ_D for the sodium radiation has been assumed it is possible to determine the integer n so as to

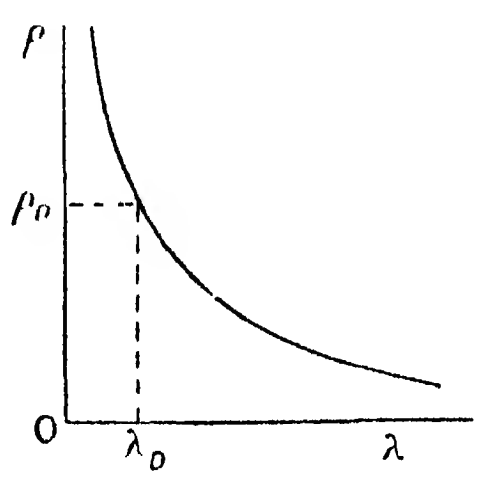


FIG. 87. Rotation and wavelength.

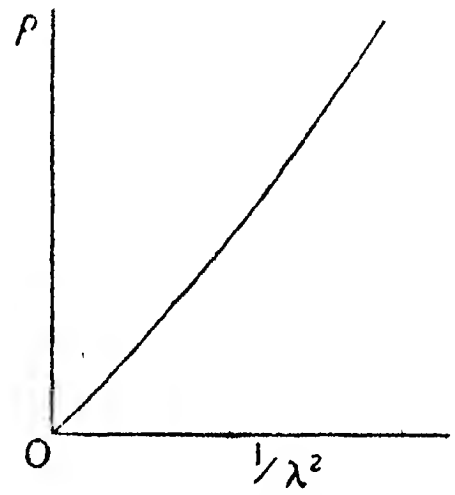


FIG. 88. Rotation and inverse square of wavelength.

give the wavelengths on either side of the sodium line. In Fig. 87 is shown a typical plot of ρ and λ. If ρ and $\frac{1}{\lambda^2}$ are plotted as shown in Fig. 88, the curve is almost a straight line. The equation is $\rho = A + B/\lambda^2$ for ρ in terms of λ.

Strictly speaking, a more complicated formula is required to represent the rotatory dispersion of quartz; the graphs indicated will serve to show the closeness of the approximation obtained.

30. ANALYSIS OF A SPECTRUM

A brief mention will be made of several types of spectrometer available for the analysis of spectra. In Fig. 89 is illustrated the usual type of instrument common to most teaching laboratories. It consists of a collimator C with a variable slit S in front of which is placed the source of radiation R. P is a triangular prism set for minimum deviation for a wavelength in about the middle of the spectrum to be investigated. T the telescope with the eye-

piece *E* is the means of observing the spectral lines, so that angular measurements are made. As this type of instrument is

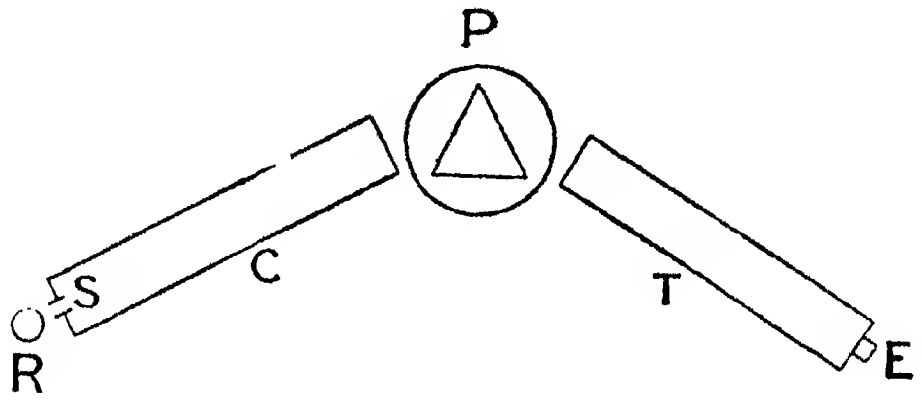


FIG. 89. Prism spectrometer.

fully described in the text-books, it will not be described in further detail.

In Figs. 90 and 91 is illustrated a Hilger constant-deviation spectrometer.

In Fig. 90 let the prism have the angles as shown and let *E F G H I* be a ray which has been refracted at *F*, reflected at *G* and refracted again at *H*. For a ray which is reflected at an

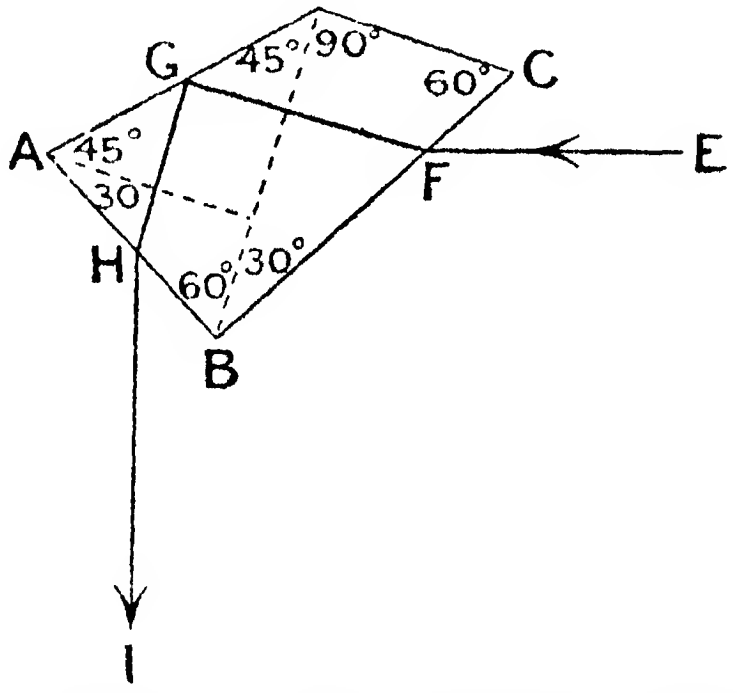


FIG. 90. Prism in constant-deviation spectrometer.

angle of 45° at the face *AD*, it may be shown that the angle of emergence from the face *AB* is equal to the angle of incidence on the face *BC*. Thus for this ray the prism is in the position of minimum deviation with the exception that the reflection at *AD* has produced an additional deviation of 90°. If *EF* corresponds to the fixed axis of a collimator, *HI* may correspond approximately to the axis of a telescope, placed at 90° to the collimator. If the prism is rotated each radiation in turn may be brought into this position of minimum deviation as the

corresponding spectral line passes the cross wire of the eyepiece. Thus the table carrying the prism may be turned by means of a screw attached to a drum, which may be graduated directly in wavelengths.

In Fig. 91, *C* represents the collimator with slit *S*. *P* is the

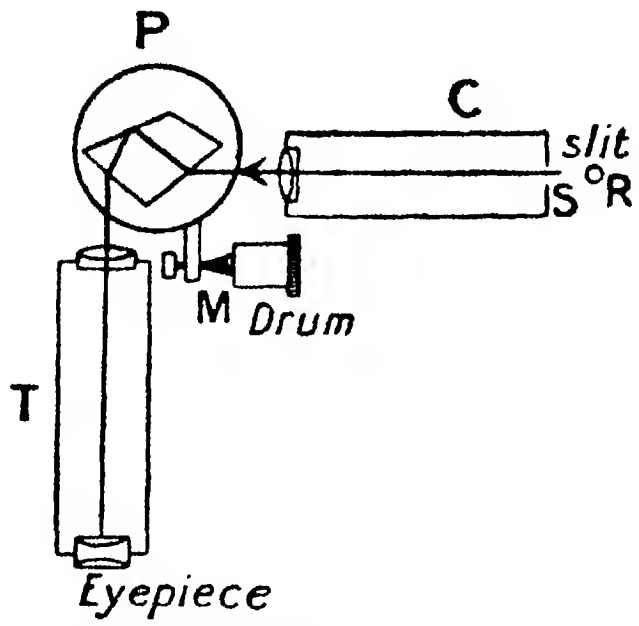


FIG. 91. Hilger spectrometer.

prism, *T* is the telescope and *M* represents the screw-and-drum arrangement. For the photography of a spectrum, the

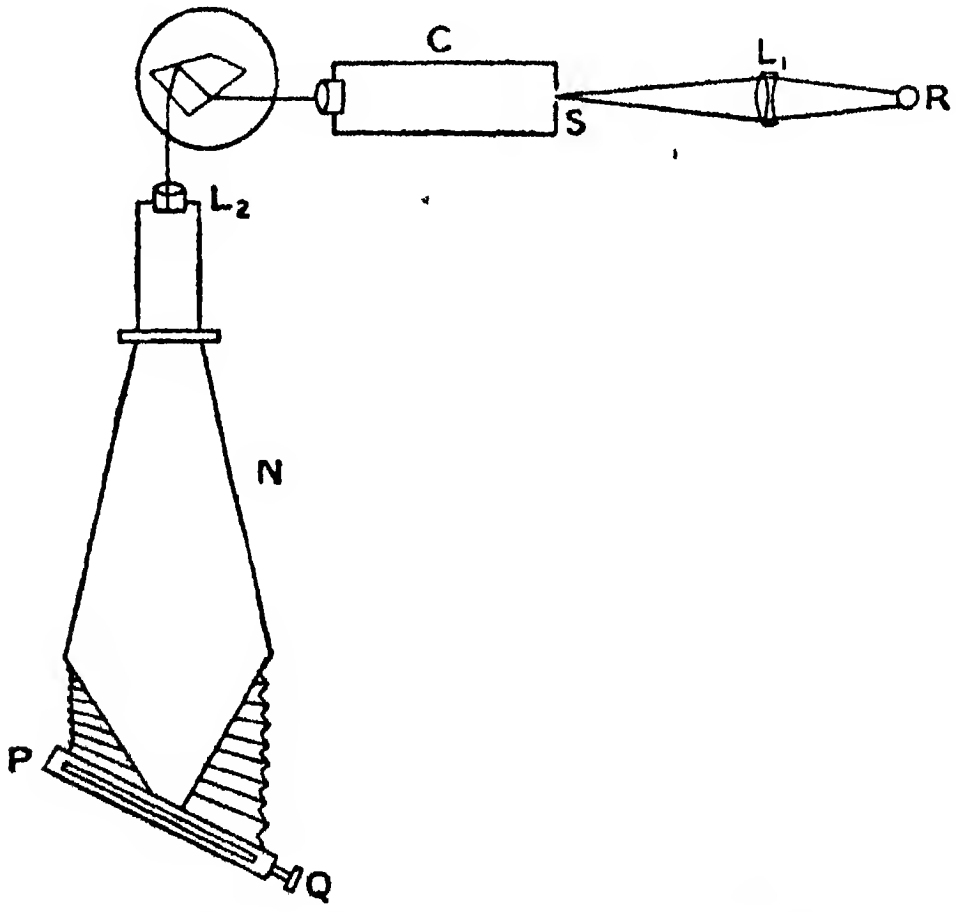


FIG. 92. Hilger spectrometer with camera attachment.

telescope may be removed and replaced by a lens-and-camera, as shown in Fig. 92. *R* is the source of radiation. Light is focussed by means of a lens L_1, upon the slit of the collimator. L_2 is now the camera lens and *N* is the camera fitted with a plate holder *P*.

In the holder is a horizontal slit S as in Fig. 93, through which the radiation may come so as to give a focussed image upon a photographic plate or film F, which may be given a vertical traverse by means of the screw Q. Sources which are usually obtainable with ease are the sodium flame, a neon lamp and a mercury lamp, with the last of which care should be taken to make the electrical connections correctly. Tubes containing other gases than neon and worked conveniently from the mains and a transformer may now be obtained. Ilford hypersensitive, panchromatic plates and films may be used.

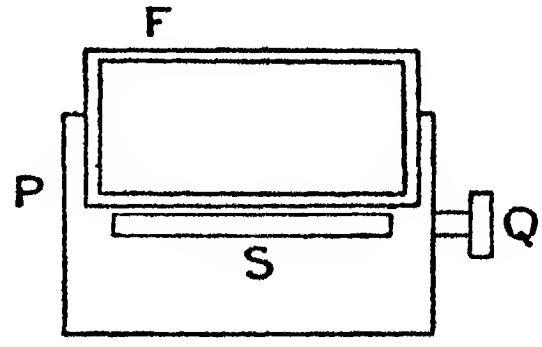


FIG. 93. Plate holder.

One method of proceeding is to have a small 45° prism placed across the slit of the spectrometer with the two sources S_1 and S_2 placed as shown in Fig. 94. The spectrum of S_1 may then be photographed, bordered by that of S_2. Another method is to

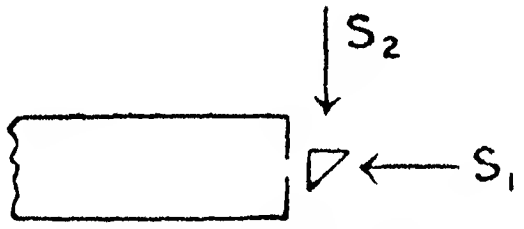


FIG. 94. 45° prism.

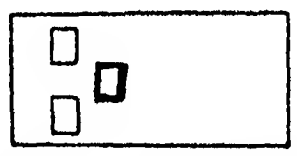


FIG. 95. Slit arrangement.

place over the slit a sliding shutter containing three apertures as shown in Fig. 95. The two outer apertures may be used for the comparison spectrum and the inner one for the unknown spectrum. Only two exposures are required in each method, but students may have to take a few photographs before obtaining a suitable time of exposure.

The following exercise is suggested.

Let R in Fig. 92 be a helium tube. By means of the lens L_1 focus a small image of the tube on the slit of the spectrometer. Place in the camera a plate or film and find by trial a time of exposure suitable to give a photograph of some of the principal lines in the helium spectrum as indicated in the upper line in Fig. 96. Turn the screw controlling the movement of the plate and re-photograph the spectrum in a slightly lower position as indicated in the second line of Fig. 96. Now substitute a mercury lamp for the helium source and photograph the mercury spectrum on the second line. Then turn the screw of the camera to move

the plate so as to photograph the mercury spectrum alone on line 3 in Fig. 96. It is thus possible to have both spectra on one

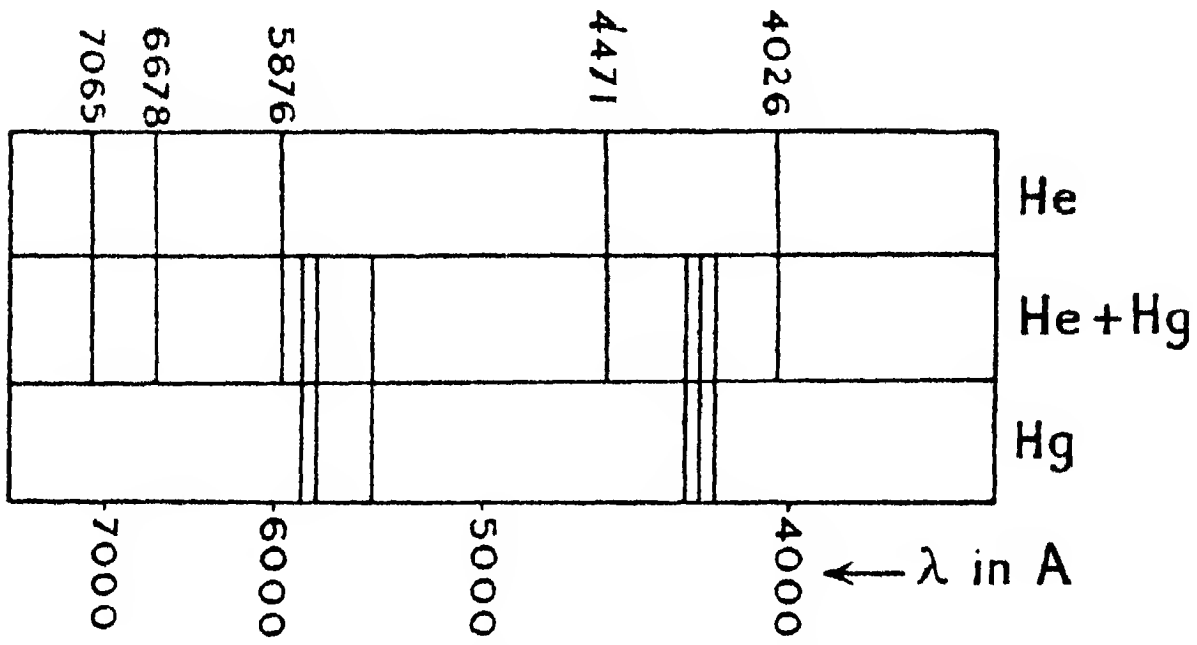


FIG. 96. Spectra of helium and mercury.

line, the upper and lower lines serving to identify the separate spectra without confusion. With a good travelling microscope measure the positions of three prominent helium lines, say x_1, x_2, x_3 for λ_1, λ_2 and λ_3. Assume Hartmann's formula

$$\lambda = \lambda_0 + \frac{k}{(x - x_0)}$$

or

$$(\lambda - \lambda_0)(x - x_0) = k.$$

From the three measured values calculate the values of the constants λ_0, x_0 and k. Measure the prominent mercury lines and determine the wavelengths from the readings taken. Compare with the wavelengths given in some work of reference. If another three helium wavelengths had been chosen the calculated constants might be slightly different. Thus when a set of constants has been calculated, the wavelengths of other lines in the helium spectrum may be found, and since these are assumed known, a table of differences may be drawn up for the whole range of the spectrum. The differences may be represented on a correction graph and these small corrections applied to the calculated values of the mercury lines.

In Plate I. are shown above and below the well-known Balmer lines of hydrogen.

The wavelengths in a vacuum are

$H_\alpha = 6564{\cdot}66\text{A.}$
$H_\beta = 4862{\cdot}71\text{A.}$
$H_\gamma = 4341{\cdot}71\text{A.}$
$H_\delta = 4102{\cdot}20\text{A.}$

These lines fit the Bohr formula $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ where $n = 3, 4, 5$ and 6 and R is the Rydberg constant, 109,677·7 cm.$^{-1}$, or approximately 110,000 cm.$^{-1}$.

Between the two spectra of hydrogen is the spectrum of helium.

Similarly in Plate II. is depicted the spectrum of hydrogen with some of the principal lines in the neon spectrum.

The Zeeman Effect

If a source of light be placed in a magnetic field, a spectral line which gives the normal Zeeman effect becomes a doublet when viewed along the field and a triplet when viewed perpendicularly to the field. Also for the triplet the central component has its

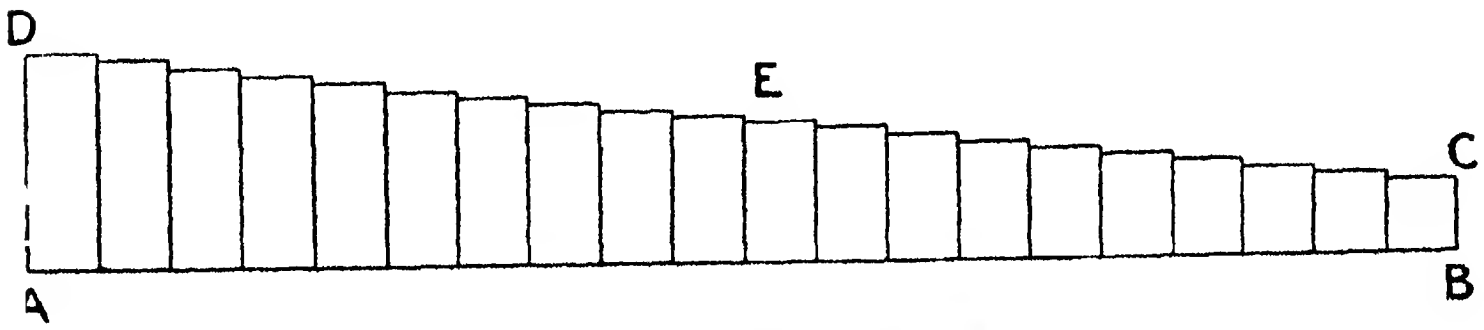


FIG. 97. Echelon grating.

frequency unaltered and the corresponding vibration is along the direction of the field. For the displaced components, one frequency is less and the other larger than the undisturbed frequency and both show a linear vibration perpendicular to the field.

If $\delta\lambda$ is normal separation produced in a wavelength λ by a magnetic field H, and if c is the velocity of light, e/m for the electron is given by the relation

$$\frac{e}{m} = \frac{4\pi c}{H} \cdot \frac{\delta\lambda}{\lambda^2}.$$

For the measurement of $\delta\lambda$ it is convenient to polarise the light so as to cut out the central component when viewed perpendicularly to the field, so that twice the normal separation may be measured. The optical measurement may be made with an echelon grating, which consists of some 20–30 plates of glass, arranged in the form of steps as shown in Fig. 97. Each plate is about 1 cm. in thickness and the step is 1 mm.

The arrangement of the apparatus is shown in Fig. 98. T is a neon tube placed between the poles of an electromagnet and N is a nicol-prism rotated so as to cut out the vibration parallel to

the field. The polarised light is allowed to fall upon the collimator C of a Hilger-spectrometer, which has the echelon interposed between the collimator and the prism. The echelon is placed with AB horizontal and the face AD (Fig. 97) vertical. If the spectrum be viewed or photographed it is seen that each line shows a few orders such as L, M, N, in Fig. 99. By means of a camera Q the line to be investigated must be photographed before the field is applied. When the field is applied line M will

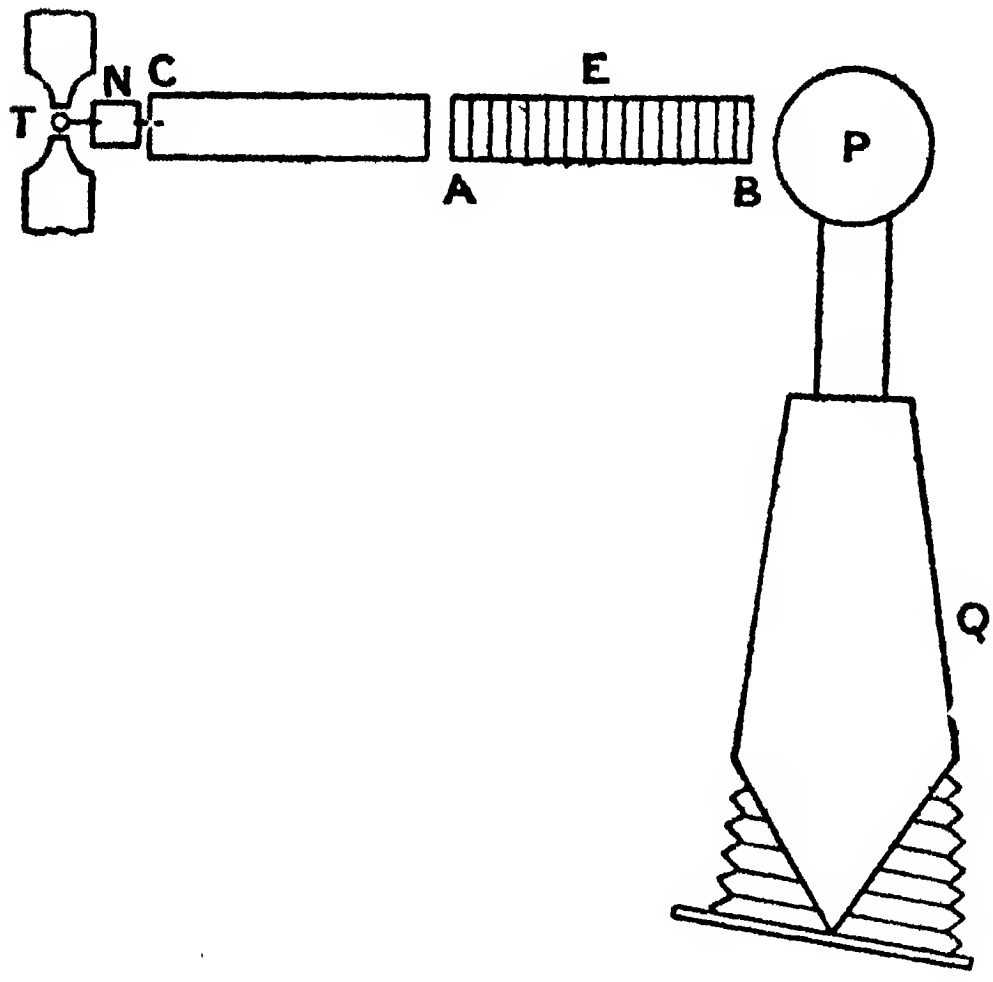


FIG. 98. Zeeman effect. Experimental arrangement.

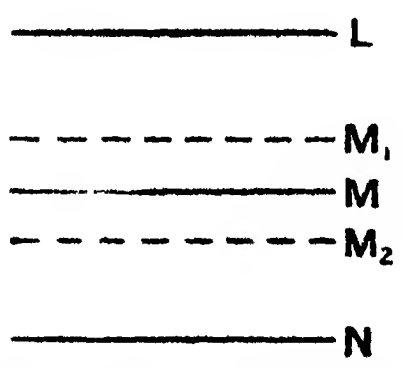


FIG. 99. Zeeman effect. Appearance of orders.

not appear because of the orientation of the nicol, but two lines M_1, M_2, separated by an amount equal to twice the normal Zeeman separation will be seen. The Zeeman spectrum is then photographed. From the measurement of the distance between two near orders, and the distance between the two separated lines M_1M_2 it is possible to calculate $\delta\lambda$. It may be shown that the increment $\Delta\lambda$ between λ and $\lambda + \Delta\lambda$ corresponding to the distance between two near orders for λ is given by $\Delta\lambda = \dfrac{\lambda^2}{t\left(\mu - 1 - \lambda\dfrac{d\mu}{d\lambda}\right)}$ where t is thickness of a plate, μ is the refractive index, and $d\mu$ is the change in μ for a change $d\lambda$ in λ. The corresponding distance on the photographic plate will serve as a

unit with which to compare the distance $M_1M_2 = 2\delta\lambda$, and hence to determine $\delta\lambda$. The magnetic field H must be carefully measured by means of a fluxmeter. A typical result obtained by a student will make the method clear and will indicate the way in which a value for e/m may be calculated. The neon line 5852·5*A* gives a normal triplet and is therefore convenient to use.

Particulars of the echelon. Number of plates = 20.
Thickness of each plate = 1·0466 cm.
Length of step = 1 mm.

Line used = Neon 5852·5*A* at 15° C., 760 mm.

$\mu = 1{\cdot}5752$ $\qquad \Delta\lambda \equiv 0{\cdot}00284$ cm.

$-\lambda \cdot \dfrac{d\mu}{d\lambda} = 0{\cdot}0468$ $\qquad 2\delta\lambda \equiv 0{\cdot}00142$ cm.

$\delta\lambda \equiv 0{\cdot}00071$ cm.

$H = 8350$ oersted.

$$e/m = \frac{4\pi c}{H} \cdot \frac{\delta\lambda}{\lambda^2} = 1{\cdot}74 \times 10^7 \text{ E.M.U./gr.}$$

SECTION III. ELECTRICITY AND MAGNETISM

31. Experiments with an electromagnetic pendulum

In these experiments is investigated the mutual behaviour of two coils carrying currents. Simple methods are deduced for the absolute measurement of a current in terms of a mass, a

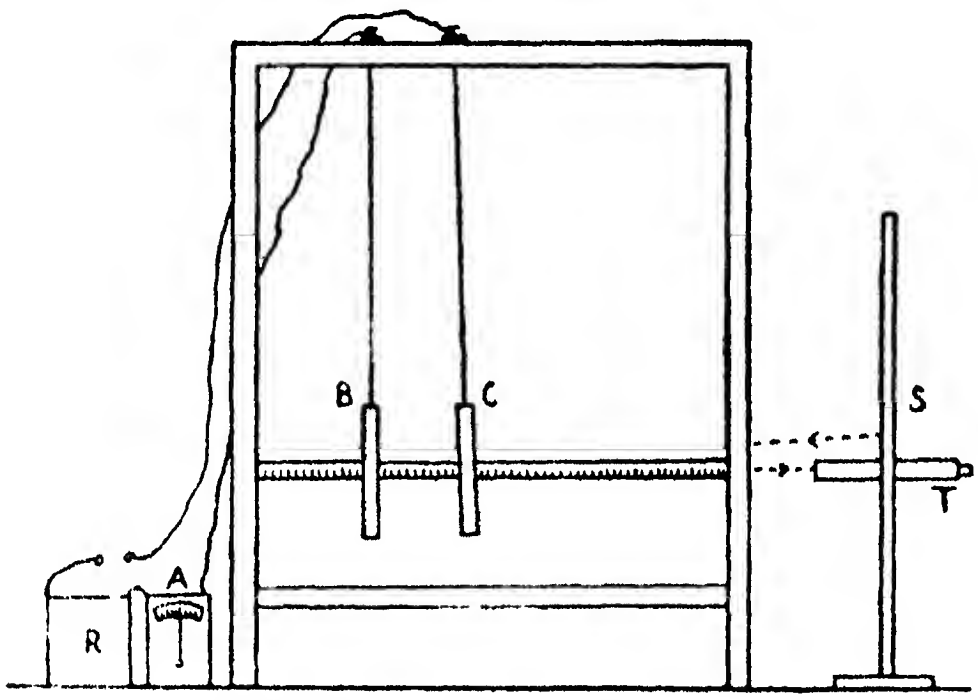


Fig. 100. Electromagnetic pendulum.

length and a time. The apparatus (Fig. 100) consists of two coils of wire, B and C, each suspended in a vertical plane by means of two wires in the manner of a bifilar suspension. The wires also serve to lead current to the coils. The circuit includes an ammeter A and a variable resistance R.

Experiment 1

One of the coils, B, is clamped in a vertical plane and the other, C, is free to move and carries a strip of mirror placed horizontally along a diameter. At a distance of about 2 m. is placed a scale S and a telescope T, so that angular deviations of the coil C from the vertical may be determined from changes in the scale-reading as seen in the telescope. Various currents are passed through both coils in opposite directions so that repulsion occurs and the angular displacements of C are found for the corresponding currents.

The approximate theory is as follows: in the equilibrium position, the mechanical couple is $mgl\theta$, where m is the mass of the coil C and l the distance from the point of suspension to the centre of the coil is taken as the length of the equivalent simple pendulum. This couple is balanced by the electromagnetic couple, an expression for which may be found in the following way.

If the distance x between the two coils is small compared with their equal radii a, the mutual inductance M for one turn in each coil is given by $4\pi a(\log 8a - \log x - 2)$. The repulsive force on the moving coil is therefore $-\partial M/\partial x = 4\pi a/x = 4\pi a/(x_0 + l\theta)$, where x_0 is the distance between the coils when no current is passing.

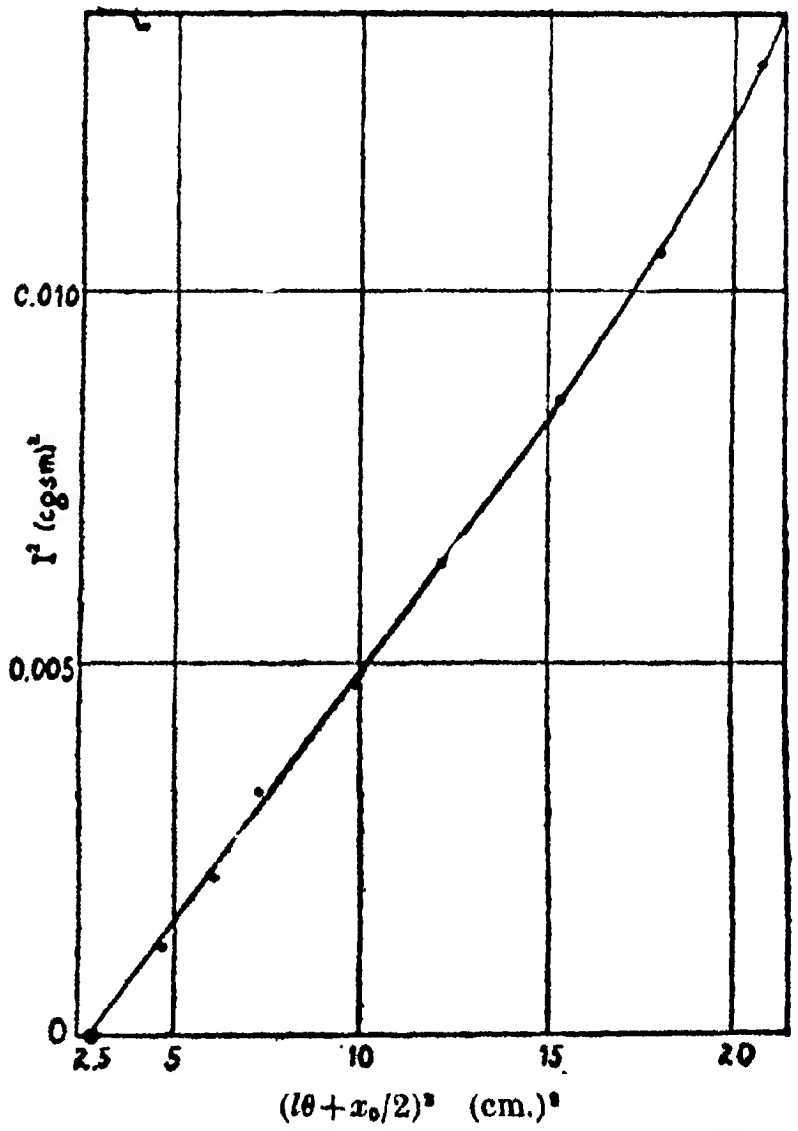


FIG. 101. Plot of I^2 and $\left(l\theta + \frac{x_0}{2}\right)^2$.

For a current I in each coil and n turns in each, we have for equilibrium $4\pi n^2 alI^2/(x_0 + l\theta) = mgl\theta$, or $4\pi n^2 alI^2 = mglx_0 + mgl^2\theta^2 = mg(l\theta + x_0/2)^2 - mgx_0^2/4$. Thus, $I^2 = (mg/4\pi n^2 al)(l\theta + x_0/2)^2$—const. This suggests plotting I^2 against $(l\theta + x_0/2)^2$ in order to test the validity of the relation obtained. For a case where $l = 74{\cdot}8$ cm., $a = 9{\cdot}5$ cm., $n = 250$, $m = 387$ gram and $x_0 = 3{\cdot}3$ cm., the results are plotted in Fig. 101 and give a slope of 0·000685 instead of 0·000680 by calculation from the theory. The current I is seen to be expressed in terms of mass, length and time.

Experiment 2

One of the coils B is fixed as before and the other, C, is allowed to oscillate as a simple pendulum. It is found that damping may be neglected and that the mass of the moving coil may be assumed to be concentrated at the centre.

The applicable theory is as follows : alter and supplement the foregoing nomenclature ; let x_0 = distance between coils in the rest position, with current on ; θ_0 = angular displacement from the vertical in the rest position with current on ; θ = additional angular displacement when the coil is oscillating ; T_0 = period of oscillation without current ; T = period of oscillation with current on. When the current is off, we put $\theta_0 = 0$ and the equation of motion is $ml^2d^2\theta/dt^2 + mgl\theta = 0$; therefore

$$1/T_0^2 = g/4\pi^2 l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When a current is flowing through both coils in opposite direc-

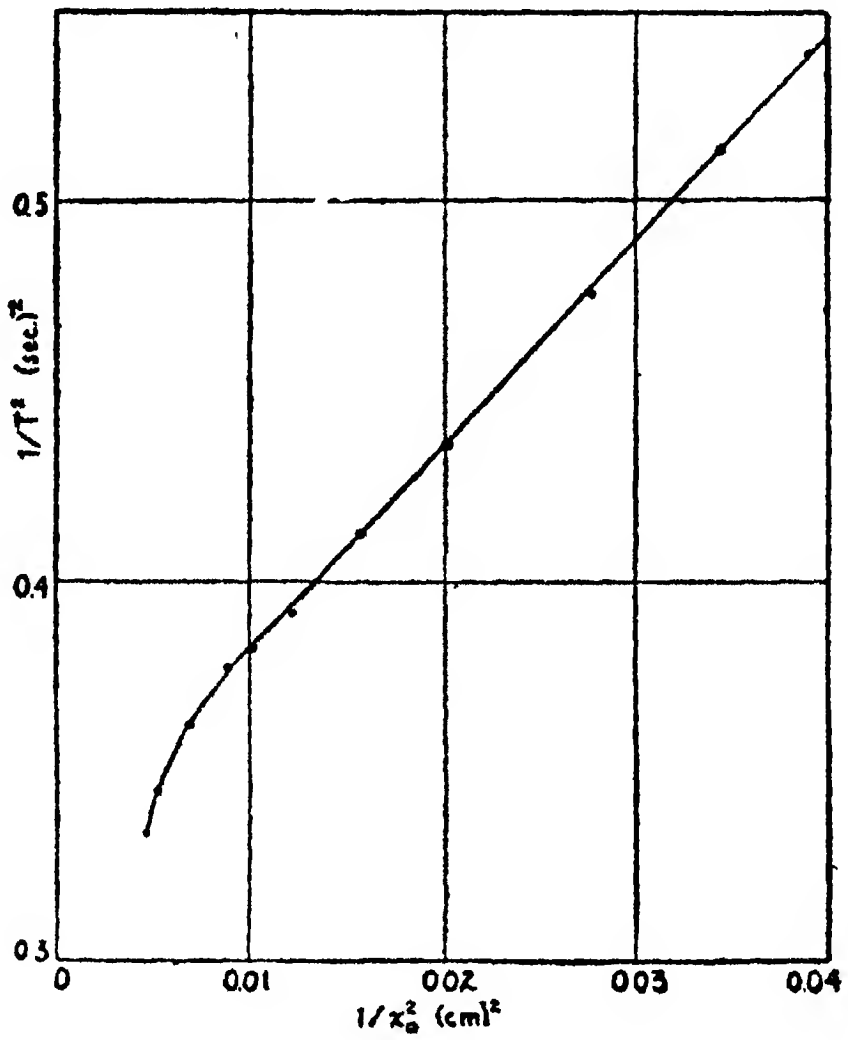


FIG. 102. Plot of $\frac{1}{T^2}$ and $\frac{1}{{x_0}^2}$.

tions the effect of the mutual inductance introduces a force $4\pi a/(x_0 + l\theta)$ as before. Consider $l\theta$ small compared with x_0 ; expand by the binomial theorem, and retain only the first power of $1/x_0$; then

$$-\partial M/\partial x = (4\pi a/x_0)(1 - l\theta/x_0).$$

Allowing for n turns and a current I in each coil, the additional couple due to electromagnetic causes is $(4\pi aln^2I^2/x_0)(1 - l\theta/x_0)$. Thus the equation of motion becomes

$$ml^2\frac{d^2\theta}{dt^2} + mgl(\theta + \theta_0) + \frac{4\pi al^2n^2I^2\theta}{{x_0}^2} - \frac{4\pi aln^2I^2}{x_0} = 0.$$

The new period of oscillation is then given by

$$T^2 = \frac{4\pi^2 ml^2}{mgl + 4\pi al^2 n^2 I^2/x_0^2}$$

or

$$1/T^2 = 1/T_0^2 + (an^2/\pi m)(I^2/x_0^2) \quad . \quad . \quad . \quad . \quad (2)$$

This result suggests two possible experiments : (*a*) the current may be kept constant and the period found for various values of x_0 ; (*b*) the period may be found for various values of the current with x_0 fixed.

Results exhibited in Fig. 102 are for a constant current of

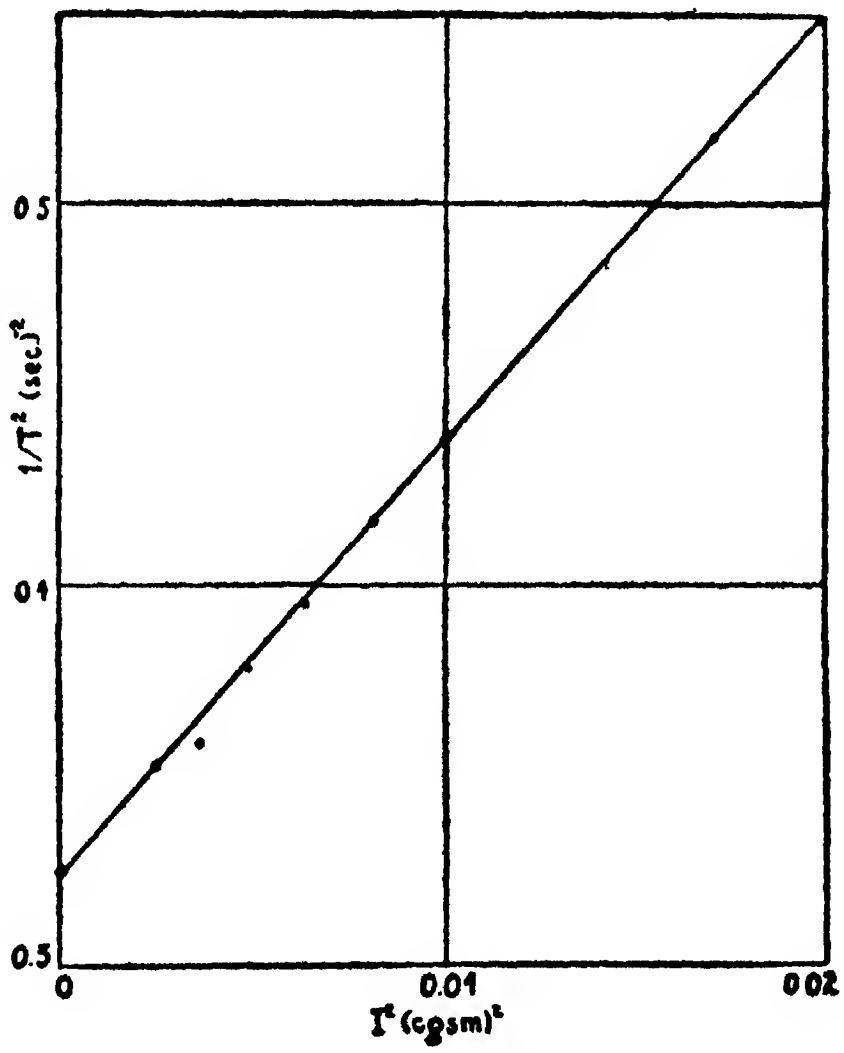


FIG. 103. Plot of $\frac{1}{T^2}$ and I^2.

0·104 e.m.u. It will be noticed that the curve becomes a straight line for the smaller values of x_0, with the tangent of the angle of slope 5·4 ; by calculation it is 5·3 in appropriate units. (*b*) Results for a constant distance $x_0 = 6{\cdot}5$ cm. appear in Fig. 103. The graph is a straight line, for which the tangent of slope is 114 instead of 116 by the formula.

When it is considered that the formula is an approximation, the agreement is quite satisfactory. If necessary, the amplitudes may be kept small by observing the oscillations of a spot of light produced by reflection from a mirror attached to the moving coil.

The relation $\nu^2 = \nu_0^2 + KI^2$, where ν, ν_0 are frequencies and K is a constant for a fixed distance x_0, enables a current to be determined in terms of grams, centimetres and seconds.

REFERENCE

CALTHROP : *Am. Phys. Teacher*, **3**, pp. 32–34, 1935.

32. THE DETERMINATION OF GALVANOMETER CONSTANTS

The equation for the angular motion of a galvanometer needle or coil may be written

$$I\ddot{\theta} + f\dot{\theta} + C\theta = ki \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where I is the moment of inertia of the moving system about the axis of rotation, f is the damping-coefficient, C is the torsional coefficient and k is a constant depending upon the magnetic field and upon the effective area of the coil for a moving-

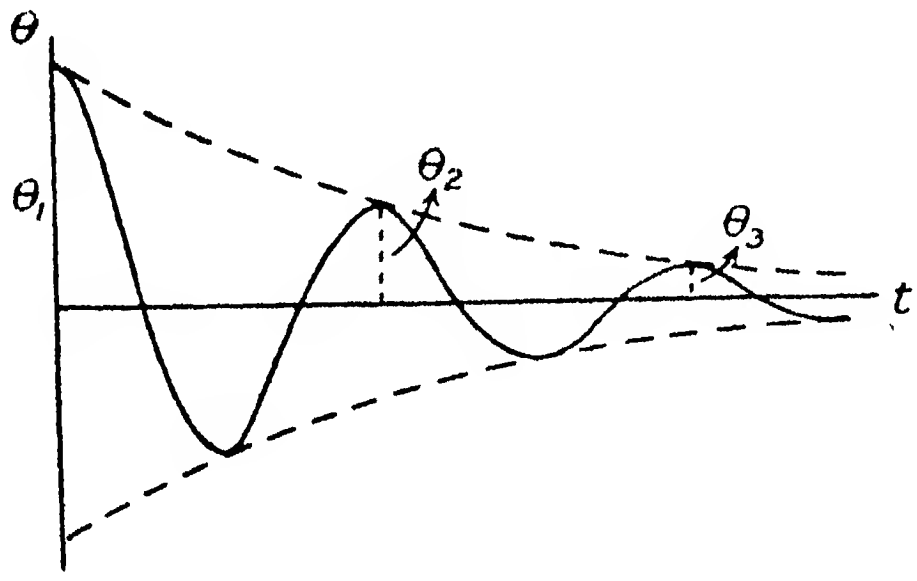


FIG. 104. Damped oscillation.

coil type of instrument ; i is the current. It may be shown that if $f^2 - 4IC < 0$, damped oscillations of the type

$$\theta = \theta_0 e^{-\lambda t} \cos(pt - \varphi) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

are obtained where

$$\lambda = \frac{f}{2I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$p = \sqrt{\frac{4IC - f^2}{2I}} = \sqrt{\frac{C}{I}\left(1 - \frac{f^2}{4IC}\right)} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This motion is shown in Fig. 104.

If $T = 2\pi/p$, the period, and if $\delta = \lambda T$

$$\theta = \theta_0 e^{-\frac{\delta t}{T}} \cos(pt - \varphi) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

T may be easily measured and $\delta = \log \frac{\theta_0}{\theta_1} = \log \frac{\theta_1}{\theta_2}$, etc., where θ_1 and θ_2 are deflections on the same side of the zero. Thus δ may also be found. Better still, if θ_n is the nth deflection $(n-1)\delta = \log \theta_1 - \log \theta_n$.

Then $\frac{\delta}{T} = \frac{f}{2I}$ which may be substituted in (4)

and $$\frac{C}{I} = p^2 + \frac{f^2}{4I^2} = \frac{4\pi^2}{T^2} + \frac{f^2}{4I^2}.$$

Thus $$\frac{C}{I} = \frac{4\pi^2}{T^2} + \frac{\delta^2}{T^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

may be round.

If the damping is small, $f = 0$, $\lambda = \delta = 0$, and if T_0 is the period in the absence of damping,

$$T = T_0 \sqrt{1 + \frac{\delta^2}{4\pi^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The moment of inertia I may be determined by adding a small rider of known moment of inertia to the needle or coil and re-determining the period of oscillation. Hence I, f and C may be found.

For a galvanometer of the moving-coil type, the equation of motion becomes

$$I\ddot{\theta} + f\dot{\theta} + C\theta = ki$$

where i now includes a current due to an induced E.M.F., produced by the motion of the coil in the magnetic field. Considering the current as made up of a part i_0 due to a steady E.M.F. and i due to electromagnetic causes, it is found that, because the rate of change of flux is proportional to $\dot{\theta}$, the induced current is $-\frac{k}{R}\dot{\theta}$, giving, when the two terms in $\dot{\theta}$ are grouped together,

$$I\ddot{\theta} + \left(f + \frac{k^2}{R}\right)\dot{\theta} + C\theta = ki_0.$$

Thus the damping term is found to consist of two terms, one for mechanical damping and one for electromagnetic damping.

For the most rapid return to zero after deflection, or what is known as critical damping,

$$f + \frac{k^2}{R_c} = 2\sqrt{IC}$$

where R_c is the critical resistance. If the total resistance of the circuit exceeds R_c the galvanometer coil oscillates, and if less than R_c the motion is aperiodic as shown in Fig. 105.

Determine $f/2I$ and C/I from observations on open circuit.

Pass a steady current i such that

$$C\alpha = ki$$

where α is the steady deflection,

and

$$\therefore \frac{\alpha}{i} = \frac{k}{C}.$$

Hence find k/I.

Then the critical resistance having been determined by experi-

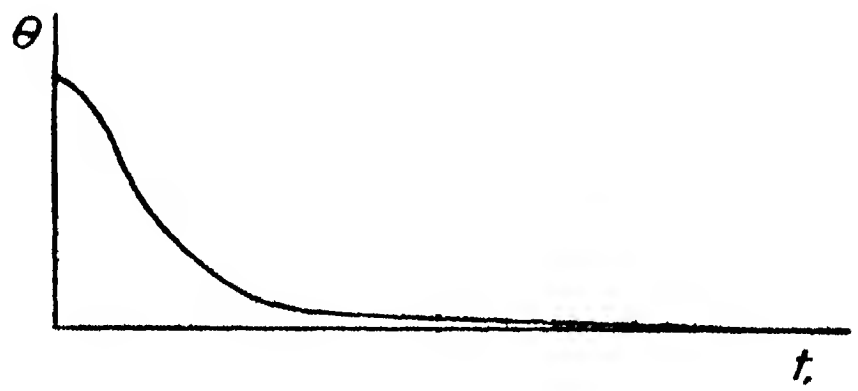


Fig. 105. Aperiodic motion.

ment, it may be written in the following equation :—

$$\frac{f}{2I} + \frac{k}{I} \cdot \frac{k}{2R_C} = \sqrt{\frac{C}{I}}.$$

Thus k/R_c and hence k may be found.

Thus K, I, C, and f are determined.

For a graphic method of solution students are referred to a paper by Ferguson and Irons.

As an example of this method an experiment with a fluxmeter is described.

Experimental study of a Fluxmeter.

A fluxmeter is essentially a ballistic galvanometer with negligible torsional constant c, and a radial field. Also f is small, but if it is not negligible the equation of the instrument may be written

$$\left(f + \frac{k^2}{R}\right)(\theta_1 - \theta_0) = \frac{k}{R} \cdot \varphi \quad . \quad . \quad . \quad . \quad (1)$$

In these circumstances the deflection $(\theta_1 - \theta_0)$, where θ_0 is the

reading in the undeflected position, is proportional to the total flux φ.

Rewrite in the form

$$(Rf + k^2)\alpha = k\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where α is the total deflection.

Thus two experiments are suggested :—

1. Keep R constant and discharge various fluxes through the instrument and test the validity of the relation Deflection = constant $\times$ flux. A suitable current in a solenoid may be used to provide a variable flux.

2. Keep the flux constant, but vary R.

Plot $\frac{1}{\alpha}$ against R.

Then $$R = -\frac{k^2}{f} + \frac{k\varphi}{f\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Test the straight line relation. Determine k/f from the slope, and $\frac{k^2}{f}$ from the intercept. Hence determine k and f separately.

Additional exercise on the Fluxmeter.

Use the instrument to calibrate the field between the poles of an electro-magnet and find

(*a*) the relation between the field in the centre of the gap and the current in the field-coils ;

(*b*) the relation between field and pole-distance for a constant current in the field-coils.

REFERENCE

FERGUSON and IRONS : *Proc. Phys. Soc.*, **40**, pp. 95–105, 1928.

33. TO DETERMINE THE MOBILITY OF AN ION IN SOLUTION

The following exercise enables the student to realise the motion of the ions in an electrolyte and to determine a mobility directly.

Take a glass tube about 0·5 cm. in diameter and bend into a U as shown in Fig. 106. Put deci-normal KCl in one limb and deci-normal KI in the other. Make the surface of separation visible by dissolving a little mercuric chloride in the KCl. There is formed a yellow disc T of mercuric iodide, soluble in excess of KI, for which reason the disc does not thicken. Use a milli-ammeter A to indicate the current provided by a battery giving

100 volts. Shunt a voltmeter across the tube. *R* is the rheostat to adjust the current.

Close the circuit and read the position of the disc at intervals of about half a minute. Plot a graph between the distance moved

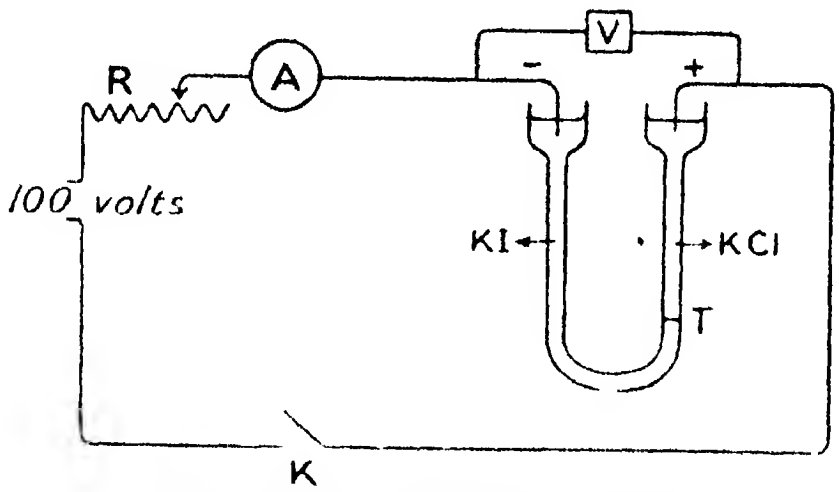


FIG. 106. Mobility of an ion.

in cm., and the time. With the polarity shown determine the potential gradient and hence the mobility of the I^- ion in cm./sec. per volt/cm. Keep the tube in a constant temperature bath to obtain the best results. The resistivity must be approximately uniform, otherwise the potential gradient is unknown. For further information on this subject consult Starling's *Electricity and Magnetism*, pp. 181 and 182.

34. TO FIND THE CAPACITANCE OF A DOUBLE-CABLE BY MEANS OF A NEON-LAMP

The period of flashing of a neon-lamp when placed in a suitable circuit of which the capacity or resistance or both may be varied

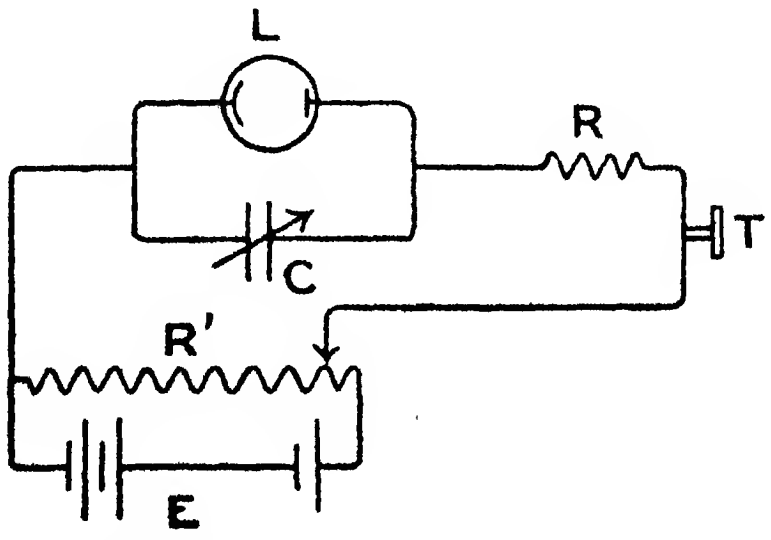


FIG. 107. Neon-lamp circuit.

is investigated as a preliminary experiment. In Fig. 107 a neon-lamp *L* is shown in parallel with a variable capacitance *C*, a resistance *R* of the order of 1 megohm being in series with both.

The 240-volt mains are connected to another resistance R' so as to form a potentiometer device for applying a potential difference of some 220 volts to the circuit. If the voltage is too low the lamp will not flash, but above a certain value the lamp flashes quite regularly with a period which depends on the values of the capacitance and resistance.

It is an instructive theoretical exercise to find an expression for the period T on the assumption that Ohm's law is obeyed and that an electrode charges up to a certain " flashing potential " V_F and then discharges until a certain lower " quenching potential " V_Q is reached. If V_0 is the applied voltage the charging time is given by

$$t = RC \log_e \frac{V_0 - V_Q}{V_0 - V_F}.$$

Similarly the discharge time

$$t' = R'C \log_e \frac{V_F}{V_Q}$$

where R' is the resistance of the lamp, of the order of 2,000 ohms. Thus if R is 1 megohm the discharge time is very short compared with the charging time.

The following experiments are instructive :—

1. Make the capacitance C of the order 0·2 to 5 or 10 mfd. ; the period of flashing may then be obtained by visual counting.

2. Take as the capacitance C a standard variable air-condenser going to 2,400 $\mu\mu$fd. The oscillations are then of such a high frequency that a telephone or loud speaker T must be included in the circuit as shown in Fig. 107 and the audible frequencies measured by tuning with a sonometer-wire and tuning-fork.

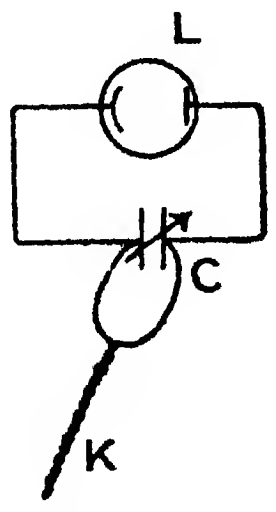


FIG. 108. To find the capacitance of a double-cable.

3. Keep C constant and vary R.

In experiments (1) and (2) plot the period T against the value of the capacitance C and verify that the relation is very approximately linear.

In experiment (3) plot a graph between T and R and test the relation suggested by the above equations.

4. A useful practical application is to find the capacitance per metre of a double or twisted cable such as is used for electric light circuits. Join the ends of about 5–10 metres of the cable K to the armatures of the air-condenser C as in Fig. 108. Vary the

capacitance C and re-determine the period-capacitance curve, which will be slightly displaced from the one drawn for experiment 2.

The displacement will not be found to be quite the same for all frequencies, but a mean value of the cable-capacitance may be obtained for the range chosen, and hence the capacitance per metre calculated. It is to be remembered that in the theory giving the above expressions, Ohm's law for steady currents has been assumed.

REFERENCE

PEARSON and ANSON : *Proc. Phys. Soc.*, **34**, pp. 175–176, 1921–22.

35. VALVE-CURVES AND CHARACTERISTICS

1. *The Diode*

The diode is formed by joining the grid and the plate of an ordinary triode, and it is desired to find the relation between plate-current and applied voltage for several values of the filament current. The apparatus is depicted in Fig. 109. The filament F is heated by the current from a 2- or 4-volt battery B in a circuit

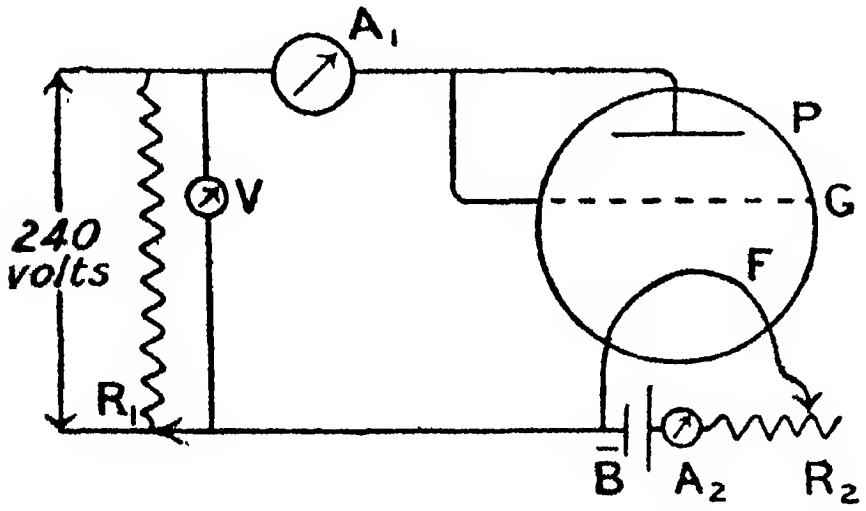


FIG. 109. The diode.

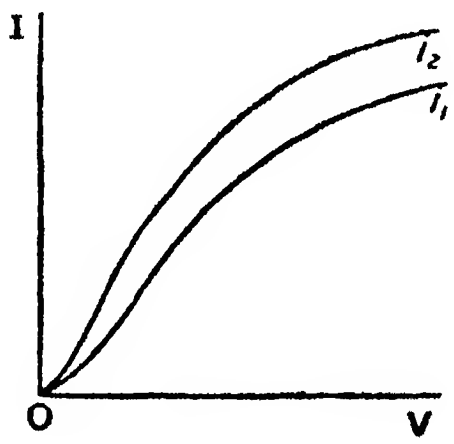


FIG. 110. Relation between plate-current and plate-voltage.

which includes an ammeter and a variable resistance. The 240-volt mains or other voltage supply is used with a potentiometer device so that a variable voltage may be applied between the negative end of the filament and the plate. A voltmeter V serves to measure the potential difference between plate and filament, and A_1 is a milli-ammeter, a shunted micro-ammeter or other instrument suitable for the measurement of a current of the order of milli-amperes. Read the plate-current for various values of the increasing voltage and plot a graph which will be of the type shown in Fig. 110. There may be a small current for zero voltage owing to some of the electrons from the hot filament

having sufficient kinetic energy to reach the plate without assistance. For the higher voltages the current tends to a limiting value, which may only be increased by using a larger filament current. Plot log I against log V and test if there is a straight-line relation. Theory gives an expression of the type $I = kV^n$ for the initial part of the curve. Test this relation and if possible determine the value of the power n.

2. *The Triode*

To determine the characteristics of a triode a voltage of 60–120 volts is applied to the plate as before, but now a positive or negative voltage of 9–12 volts is applied to the grid as shown

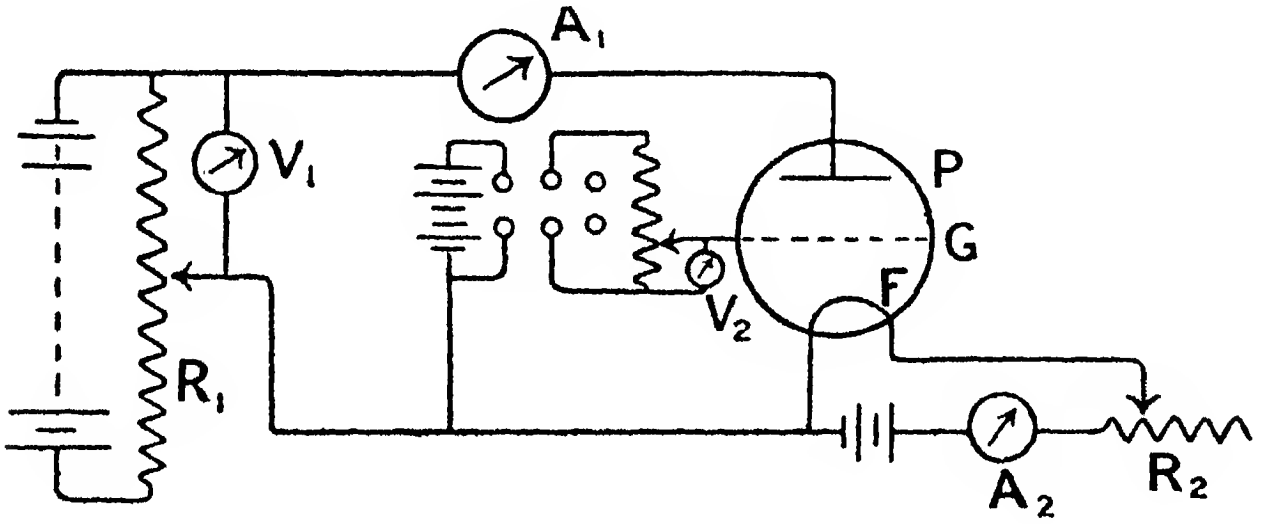


Fig. 111. The triode.

in Fig. 111. Maintaining the filament current constant at the value given by the makers of the valve plot curves of:

(*a*) The plate current I_p (ordinate) against the plate voltage V_p for various fixed grid voltages. These curves are the plate characteristics of the valve.

(*b*) The plate current (ordinate) against the grid voltage V_g, for various fixed plate voltages. These are the mutual characteristics of the valve.

(*c*) The plate voltage (ordinate) against the grid voltage, maintaining the plate current constant.

Begin with the maximum negative grid potential in each case and be careful not to exceed the maximum plate power dissipation (equal to V_pI_p) given by the makers.

The general nature of the curves is shown in Fig. 112. The slopes of the curves over the straight parts are $\left(\frac{\partial V_p}{\partial I_p}\right)_{V_g}$ $\left(\frac{\partial I_p}{\partial V_g}\right)_{V_p}$ and $\left(\frac{\partial V_p}{\partial V_g}\right)_{I_p}$ where the suffix indicates the quantity which is kept constant in each experiment. These differential coefficients

are called the plate resistance r_p, the mutual conductance g_m and the amplification factor μ, respectively, and are important constants of the valve. The value of r_p is expressed in ohms, that of g_m in milliamperes per volt or micromhos while μ is a pure number. From their definitions it follows that

$$\mu = -g_m r_a \quad (g_m \text{ in mhos})$$

which may be verified from the experimental results.

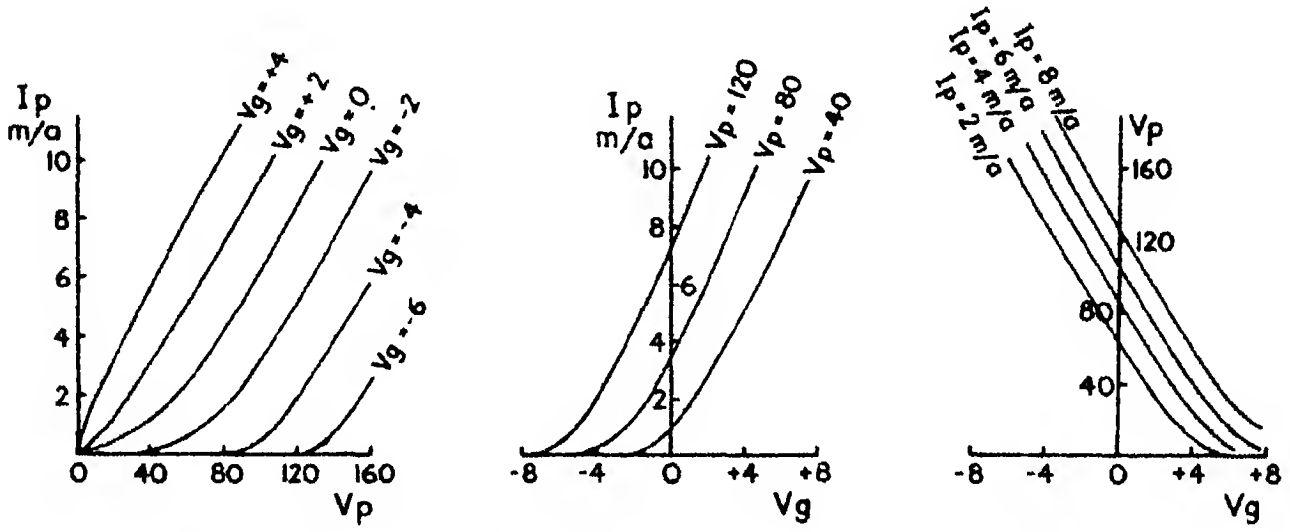


FIG. 112. Characteristic curves of triode valve.

If a high resistance is inserted in the plate circuit the dynamic characteristics of the valve may be obtained. These differ from the above static characteristics because the plate potential V_p depends on the voltage drop in the resistance and so on I_p. This matter is well treated in Prof. E. V. Appleton's book *Thermionic Vacuum Tubes*.

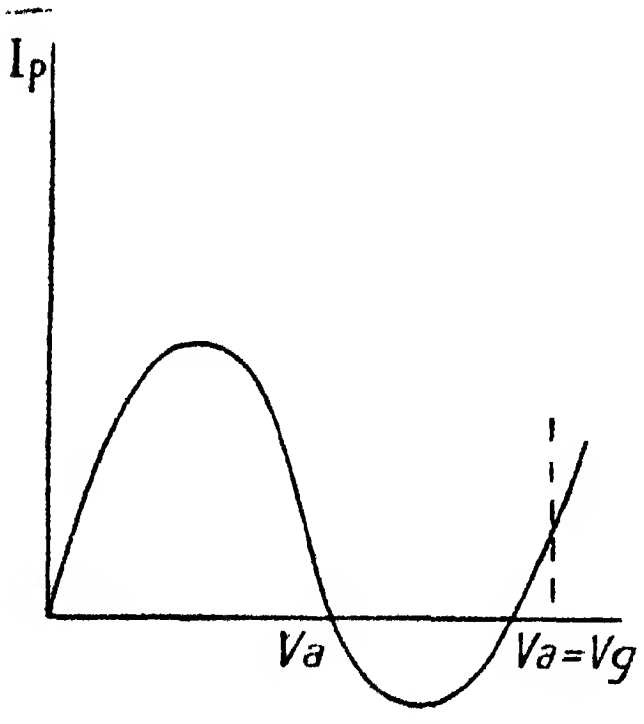


FIG. 113. The curve of a dynatron.

If an additional exercise is required the valve may be used as a dynatron. Put a fixed positive voltage on the grid of some 100 volts. Plot the anode current against anode voltage up to about 200 volts. The characteristic curve under these conditions is shown in Fig. 113, and the shape is due to secondary emission from the anode produced by electron bombardment. There is a region of negative resistance, so that the valve acts as a source of power. Such a device is called a dynatron.

REFERENCE

APPLETON, E. V.: *Thermionic Vacuum Tubes*. Methuen.

36. The determination of e/m by means of a magnetron

In Fig. 114, P_1 and P_2 represent the filament and cylinder of a triode valve and a magnetic field H is applied parallel to the filament. It has been shown that for a certain critical field given by the relation $e/m = \frac{8V}{r^2H^2}$ where V is the voltage and r is the radius of the cylinder, no electrons will reach the cold electrode (see Greinacher: *Verhandlg. d. Deutsch. Physik. Ges.*, **14**, p. 836, 1912, or Page and Adams: *Principles of Electricity*, p. 305). The apparatus is seen arranged in Figs. 115 and 116. The grid of the valve is joined to earth and by means of a potentiometer device an accelerating potential may be applied between the filament and the grid. Join the

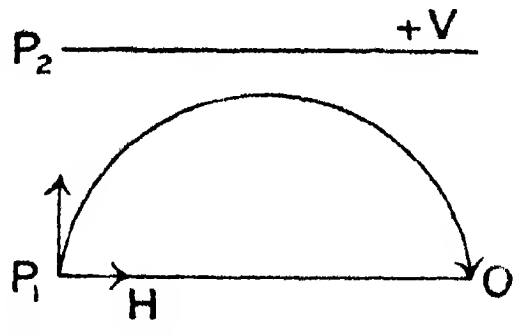


Fig. 114. The magnetron.

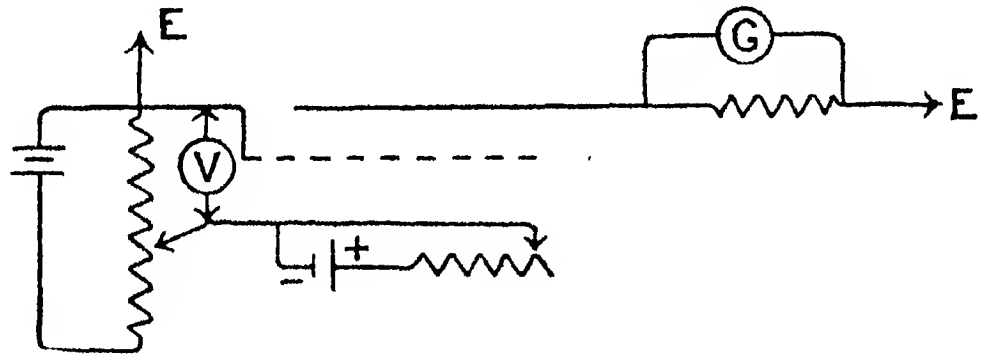


Fig. 115. The circuit.

plate of the valve to earth through a shunted galvanometer. Apply a magnetic field either as shown in Fig. 116 (*a*) by placing the valve V between the poles of an electromagnet or as in Fig. 116 (*b*), where the field is created by the current

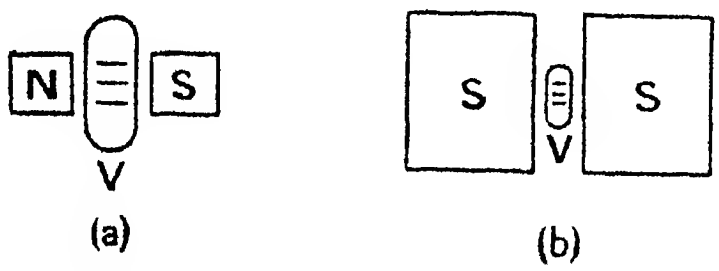


Fig. 116. The magnetic field.

in a large solenoid S. The coil may be conveniently made in two halves, shown slightly separated in the diagram. Keep the accelerating potential constant. Observe that a small magnetic field hardly affects the current through the galvanometer, but that by increasing the current in the field coils or

solenoid a field may be obtained which is sufficient to reduce the electron current to zero. Read the currents in the field coils and the corresponding galvanometer currents. By means of a magnetic needle or a search-coil determine the field produced by a current of 1 ampere in the field coils. Plot a graph between galvanometer-current and field-current for two or three values of the accelerating potential. Calculate the fields corresponding to the field-currents and plot another set of graphs showing the relation between galvanometer-current and field. A typical curve is shown in Fig. 117, where the drop in the current is seen to be fairly sharp. For the crucial field and the known value of V calculate the value of e/m.

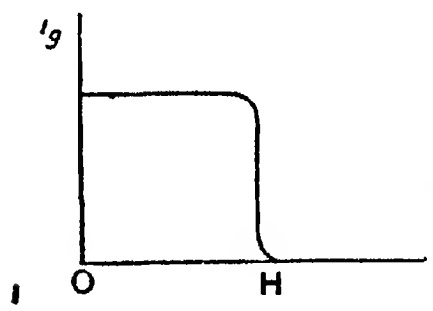


FIG. 117. Action of a magnetron.

The following is a typical example which will give the student some notion of the appropriate magnitudes.

Accelerating potential = 6 volts.
Solenoid current = 4·5 amp.

1 amp. in solenoid gave a field of 24·3 oersted. Radius of cylinder (measured by kathetometer) = 0·153 cm.

$$\therefore\ e/m = \frac{8V}{r^2H^2} = \frac{8\,.\,6\,.\,10^8}{(0{\cdot}153)^2(4{\cdot}5)^2(24{\cdot}3)^2} = 1{\cdot}72 \times 10^7\ EMU/gr.$$

The poles of the filament battery should be joined so that the electrons most accelerated leave the filament at the end nearest the voltmeter, *i.e.*, from the left in Fig. 115. Many valve electrodes contain a trace of nickel which will prevent the behaviour required, so that a valve suitable for the purpose must be selected.

In an alternative circuit a potential difference is maintained between the filament and the plate, and the grid is maintained at a sufficiently high potential to ensure an even flow of plate-current. The calculation is the same as that given above.

The magnetic field of the earth should be taken into consideration, and the total magnetic field found.

REFERENCE

GREINACHER : *Verhandlung. d. Deutsch. Physik. Gesell.*, **14**, p. 836, 1912.

37. TO DETERMINE THE CRITICAL POTENTIALS OF MERCURY

Two convenient methods are available for use in the laboratory. One method uses a mercury-rectifier as a diode and requires the determination of the plate-current for various values of the plate-voltage as described in experiment 35. It is assumed that

$I = kV^{3/2}$ or that $I^{2/3} = k^1V$, and thus a plot of $I^{2/3}$ against V gives a straight line, which, however, is departed from when ionisation of the mercury vapour occurs. Straight lines are obtained when the accelerating voltage is applied at the positive and negative ends of the filament.

Allowing for the intercepts of the straight lines on the axis of V, it is possible to obtain two values for the ionisation potential for which the mean value is then found. While the experiment is a useful and simple approach to the subject, it cannot be claimed that the determination is very precise.

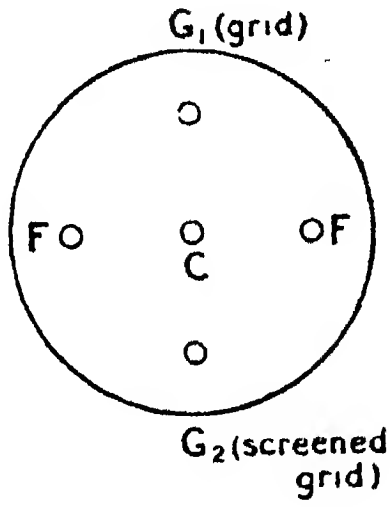


FIG. 118. Valve connections.

The method to be recommended is the method of Davis and Goucher as described by Arnot, who used an ordinary screened grid valve * containing a little mercury.

The valve has a heating filament surrounded by the oxide cathode, which is connected to a central pin in the valve holder. The plan of the valve connections is seen in Fig. 118 and the diagram of the circuit in Fig. 119. *FF* represent the filament

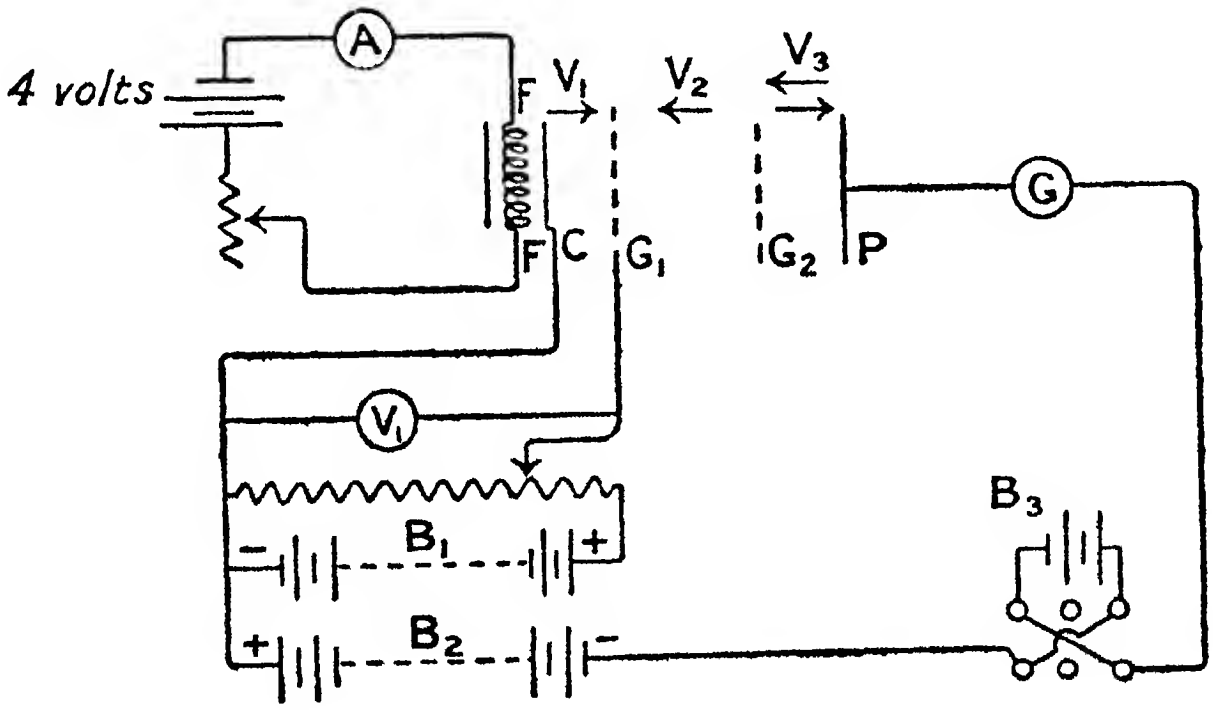


FIG. 119. The circuit.

connections, C that of the cathode and G_1 and G_2 the grid and the screened grid respectively. The plate connection is made at the top of the valve. The filament is heated by a battery of 4 volts. Between the cathode C and the grid G_1 an

* Supplied by the General Electric Company.

accelerating voltage V_1 is applied by a potentiometer device which makes use of battery B_1.

It should be possible to increase the voltage gradually up to about 15–20 volts and the voltmeter V_1 should read in tenths of a volt. B_2 is an opposing voltage of about 10 volts, which is fixed, and G_2, the screened grid, is made negative. To the plate P may be applied a small reversible voltage of about 4 volts. G is a sensitive galvanometer used to detect the plate-current. The student is referred to the modern works on physics for a description of the methods of determining critical potentials, but briefly the nature of the present experiment is as follows : The voltage V_1 accelerates the electrons from the filament or cathode

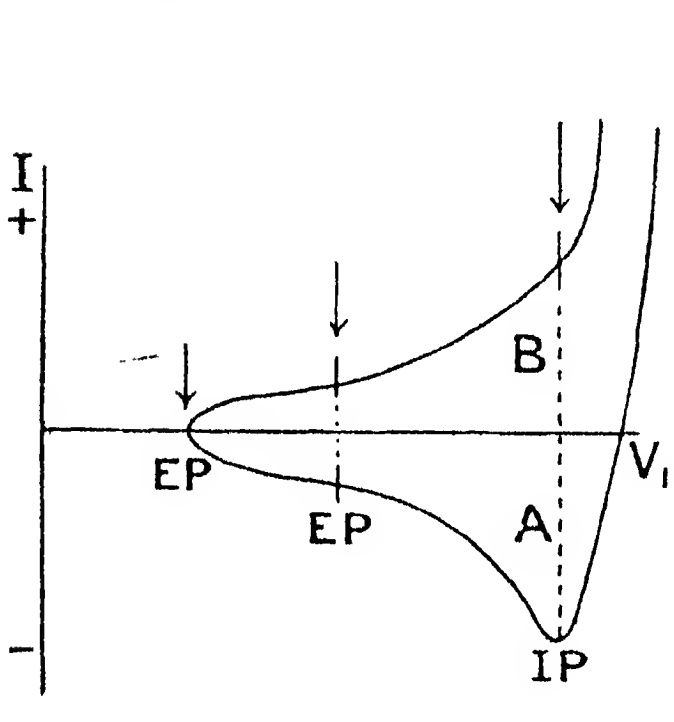


FIG. 120. Plot of I.

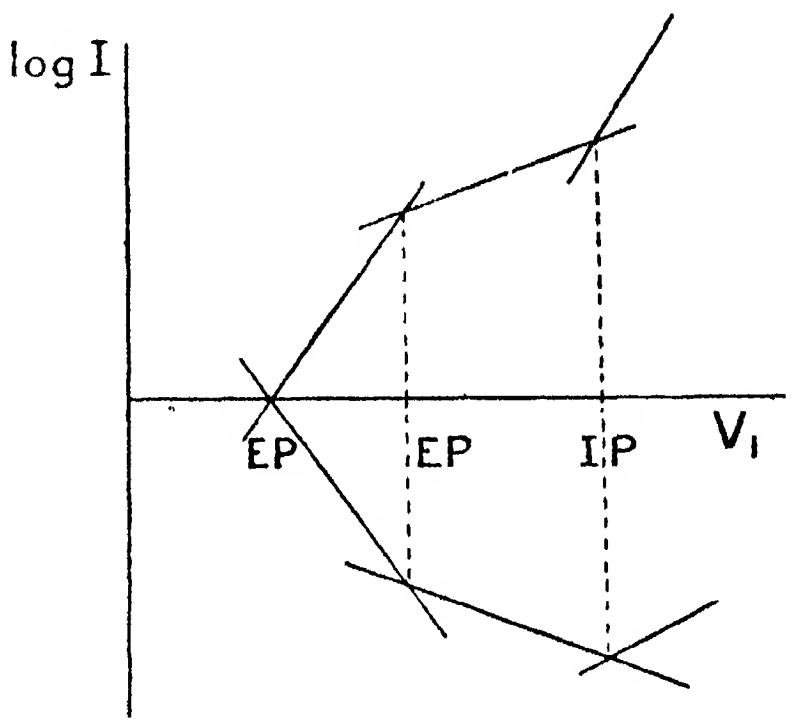


FIG. 121. Plot of log I.

to the grid G_1, and after passing through the potential V_2 they are retarded so that no electrons reach G_2.

Suppose V_3 is first set to make the plate P positive with respect to G_2 and that V_1 is gradually increased. When V_1 becomes equal to an excitation potential, atoms are excited near G_1. The radiation produced liberates photo-electrons from G_2 and P. Those from P cannot escape because of V_3, but those from G_2 are attracted to P. Therefore a negative current flows to the galvanometer. As V_1 is increased the electrons penetrate further into the space between G_1 and G_2 and so the probability of collision increases and consequently the current I increases. When V_1 becomes equal to an ionisation potential, positive ions are formed near G_1 and these are accelerated to G_2 where they have sufficient energy to overcome V_3 and to reach P, where they produce a positive current in the galvanometer. Thus for an ionisation potential I changes sign.

The curve A in Fig. 120 represents these effects obtained with the plate P positive with respect to G_2.

Now reverse V_3. Photo-electrons from G_2 are prevented from reaching P, while those set free from P are accelerated to G_2 and so I is positive. For an ionisation potential the positive current rapidly increases on account of the arrival of positive ions at P. These effects are shown by curve B in Fig. 120. From the graphs determine the points of inflexion from which two excitation and one ionisation potential should be obtained. If another graph between log I and V is plotted as in Fig. 121, the required points can be determined with good accuracy.

The advantage of the above method in separating photo-electric from ionisation-effects will be apparent.

REFERENCE

ARNOT : *Collision Processes in Gases.* Methuen.

38. ALTERNATING-CURRENT-BRIDGE MEASUREMENTS

As a source of alternating current a valve oscillator may be used. A simple type is shown in Fig. 122.

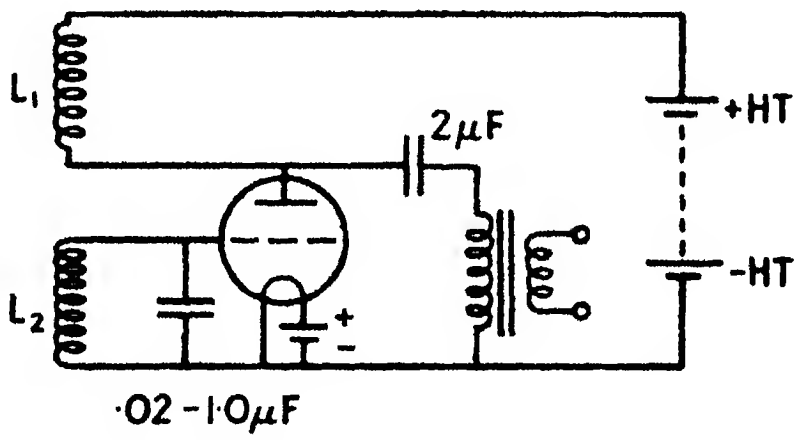


FIG. 122. Valve oscillator.

The inductances L_1 and L_2 should be of the order of 100 millihenries or more coupled magnetically. Fixed condensers in the tuned circuit ranging from say 0·02 to 1·0 μF permit an adjustment of frequency. The anode of the valve is connected via the blocking condenser of 2 μF capacity to the primary of an iron-cored transformer and the output is taken from the secondary. For the valve a triode such as is used in the power output stage of a radio may be used. To secure oscillations care should be taken that the connections of the coils are made correctly so that the current in one assists that in the other.

The above circuit has the disadvantage of a limited frequency range. Also harmonics of the fundamental frequency will be

present in the output and if the balance of a bridge depends on the frequency or if the inductances, etc., in the bridge vary with the frequency the balance point may be difficult to locate precisely. For accurate work a resistance-capacity tuned oscillator may be constructed. This will provide a virtually constant output of good waveform and frequency stability over a large frequency range. For the practical details of such an oscillator see F. E. Terman, *Radio Engineers' Handbook*, McGraw-Hill Book Company, New York and London, 1943.

The Wagner earth

Errors arise in the measurement of small impedances by a.c. bridges because of the stray capacities to earth of the impedances in the bridge arms. These shunt the bridge impedances and alter their effective value. In addition stray capacities between the observer's earphones and his head may give a note in the phones when the bridge is balanced and thus the exact location of the balance point is difficult. These troubles are avoided if a Wagner earth device is employed.

Z_1, Z_2, Z_3, Z_4 are the four arms of a bridge with the stray capacities shown dotted. The procedure adopted is to bring the points A and D to earth potential. To do this, a pair of impedances Z_5, Z_6 which are copies of the impedances Z_1, Z_2 are connected as shown and their junction at E earthed to a water pipe. The detector can be connected either to E or to A and alternate balances are sought in these two positions till the minimum output is obtained. When this is achieved A and D are at earth potential and therefore the effects of C_A and C_D are eliminated. Also C_B and C_C are reduced merely to shunts across Z_5 and Z_6 and do not

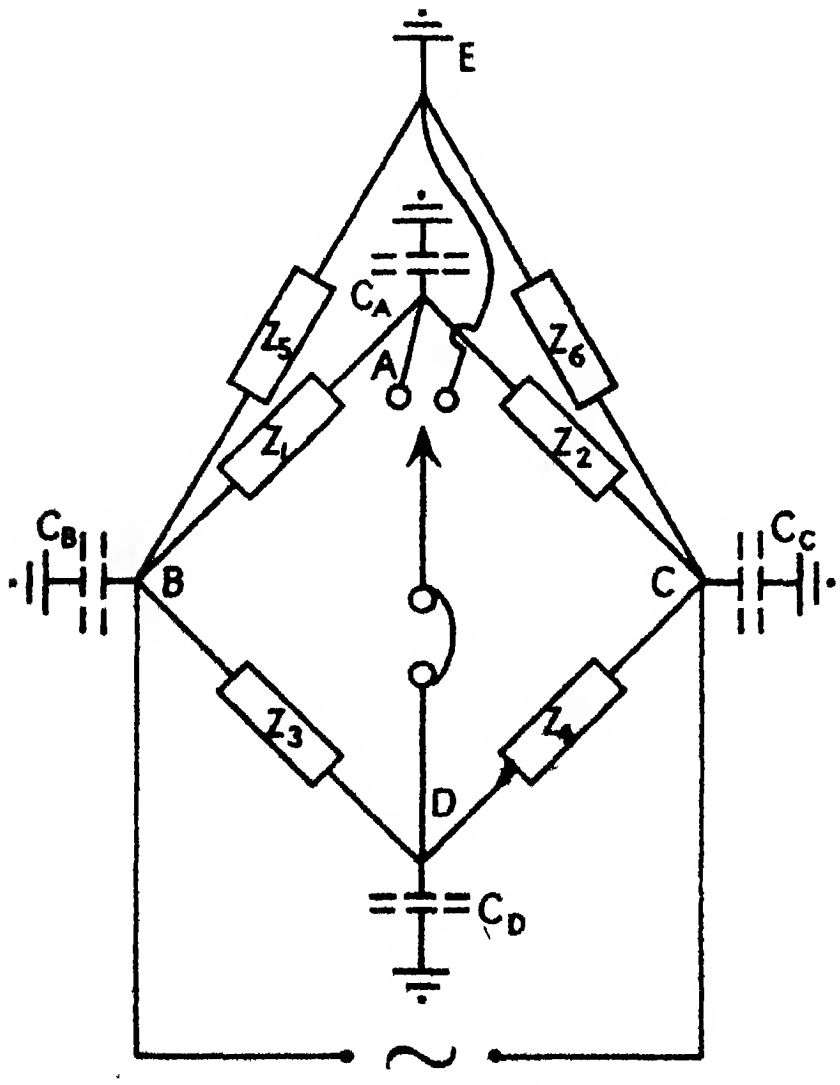


FIG. 122*a*. The Wagner earth.

cause errors by shunting the bridge impedances. The direct earthing of A or D would not avoid this last source of error.

It is only possible to choose a few of the large number of bridge measurements available, and those selected are Owen's method and Anderson's method for a self-inductance.

Owen's method for a self-inductance

In the Wheatstone bridge shown in Fig. 123 L_1 is the inductance to be measured and R_1 represents the resistance in the arm AB consisting of a P.O. box and the resistance of the coil L_1. R_2

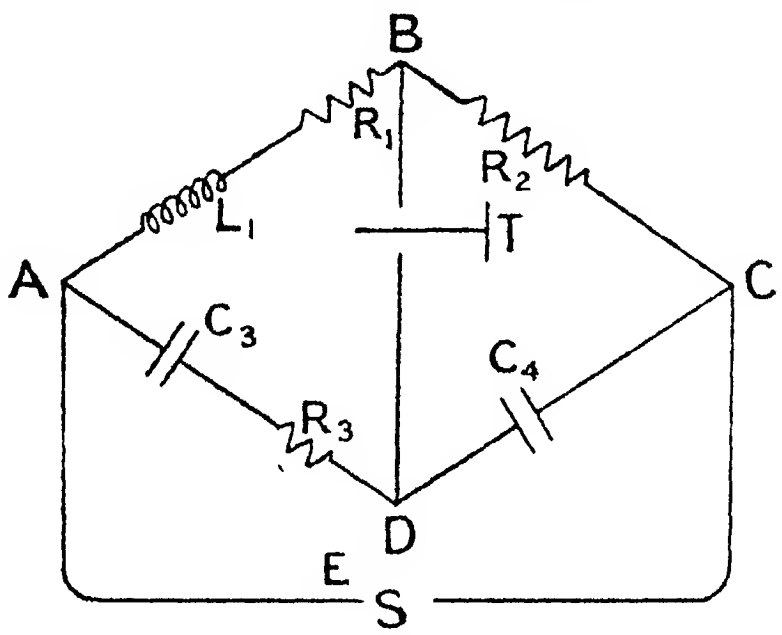


FIG. 123. Owen's method for a self-inductance.

and R_3 may be P.O. boxes. C_3 and C_4 are capacitances of the same order of 0·1 μfd. T represents a telephone joining B and D and to A and C are joined the leads from the transformer of the valve oscillator.

If i without a suffix stands for $\sqrt{-1}$, the condition for a balance of the bridge is

$$\frac{R_1 + iL_1p}{R_2} = \frac{R_3 - i\dfrac{1}{C_3p}}{-i\dfrac{1}{C_4p}}$$

where $\frac{p}{2\pi}$ is the frequency of the alternating current.

This gives two conditions for a balance,

$$\frac{R_1}{R_2} = \frac{C_4}{C_3} \text{ and } \frac{L_1}{R_2} = C_4R_3.$$

The first condition does not represent a direct current balance because no direct current flows through the condenser, but the balance condition may be made by obtaining zero ballistic throw in a galvanometer, or with telephones first adjust R_2 and then R_3 until no sound is heard.

Since L_1 is proportional to R_3 the method is useful for inductances of any value.

For a wireless coil C_4 may be $0{\cdot}1 - 0{\cdot}25\ \mu fd.$ R_1, R_2, R_3, 200–2,000 ohm. L might be 130 millihenries.

Anderson's L and C bridge

In Fig. 124 let L_1 be the inductance to be measured. Let i_1, i_2, i_3, i_4 be the currents in the arms of the bridge and i_R and i_C

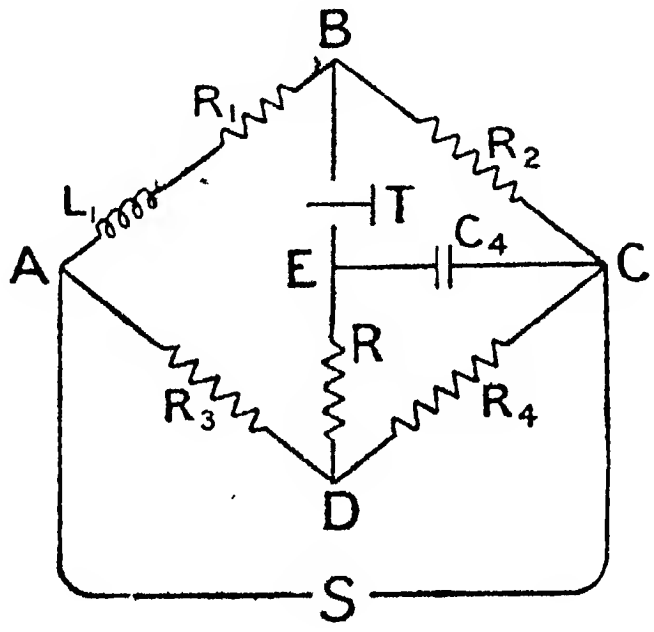


FIG. 124. Anderson's method for a self-inductance.

be the currents for the resistance R and the condenser C as shown. Apply Kirchhoff's Laws. Then :—

$$\left.\begin{aligned}(R_1 + iL_1p)i_1 &= R_3i_3 + Ri_R \text{ for circuit } ABD\\ R_2i_2 &= -i\frac{1}{C_4p}.\,i_C \text{ for circuit } BCE\end{aligned}\right\} \quad . \quad . \quad (1)$$

For a balance $i_1 = i_2$, $i_R = i_C$.
Divide and obtain

$$\frac{R_1 + iL_1p}{R_2} = \frac{R_3\left(\frac{i_3}{i_C}\right) + R}{-i\frac{1}{C_4p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

To find i_3/i_C note that for the circuit ECD

$$R\,.\,i_R - i\,.\,\frac{1}{C_4p}.\,i_C = R_4i_4$$

and $$i_3 = i_R + i_4 \text{ for a balance.}$$

Substitute $i_R = i_C$

and $$\left(R - i\frac{1}{C_4 p}\right) i_C = R_4 i_4$$

$$i_3 = i_C + i_4.$$

$$\therefore \quad \frac{i_3}{i_C} = \frac{R_4 + R - i\frac{1}{C_4 p}}{R_4} \quad . \quad . \quad . \quad . \quad (3)$$

Insert in (2) and

$$\frac{R_1 + iL_1 p}{R_2} = \frac{\frac{R_3}{R_4}\left(R_4 + R - i\frac{1}{C_4 p}\right) + R}{-i \cdot \frac{1}{C_4 p}}.$$

The balance conditions are therefore

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \text{ and } \frac{L_1}{R_2} = C_4\left[\frac{R_3}{R_4}(R_4 + R) + R\right] \quad . \quad . \quad . \quad (4)$$

The first condition may be established by means of a galvanometer and then the AC balance by varying R and finding the point of balance with the aid of a telephone placed in the arm BE. R_3 and R_4 should usually be of the same order of magnitude. If they are equal,

$$L_1 = C_4 R_2(2R + R_4).$$

This is a very good method for a large range of values. For a typical wireless coil $R_3 = R_4 = 1{,}000$ ohm. $C = 1$ $\mu fd.$ $R_3 = 3{,}000$ ohm. $R = 1{,}715$ ohm. $L = 133$ millihenries.

To compare a mutual inductance with a capacitance

In Fig. 125 L_1 and L_2 form the mutual inductance M to be determined. Arrange L_1 with R_1, which may be a few thousand

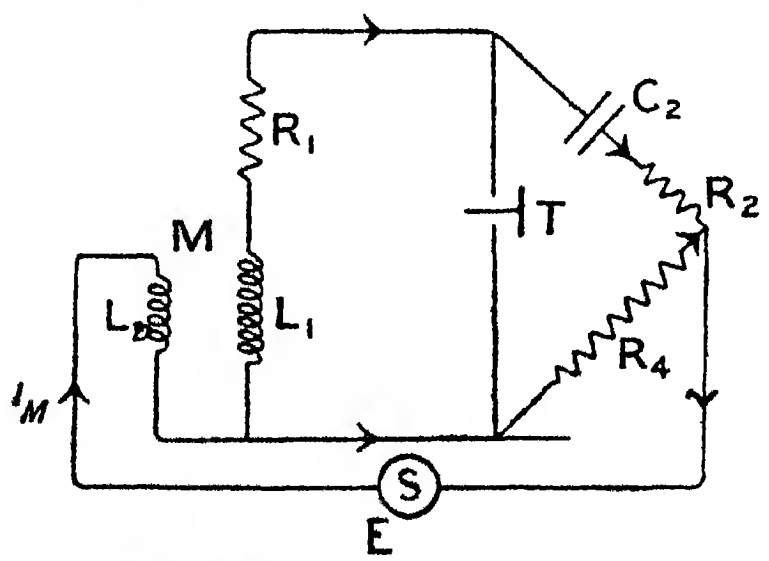


Fig. 125. Method for mutual inductance.

ohms with C_2, about 0·1 of a μfd and R_4 100 ohms. Join L_2 as shown. The conditions for a balance are :—

$$(R_1 + iL_1p)i_1 + iMpi_M = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\left(R_2 - i\,\frac{1}{C_2p}\right)i_2 = R_4i_4 \quad . \quad . \quad . \quad . \quad (2)$$

together with the equations

$$i_1 = i_2\,;\; i_1 + i_4 = i_M \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Eliminate i_M and use (2)

$$\frac{(R_1 + iL_1p) + iMp}{R_2 - i\,\dfrac{1}{Cp}} = -\frac{iMp}{R_4}$$

∴ The equations for a balance are

$$\left.\begin{aligned} M &= -L_1\,\frac{R_4}{R_2+R_4} \\ M &= -C_2R_1R_4 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Adjust R_1 and R_2 alternately, since each appears in only one of the balance equations. L_1 must be $> M$ so that it may be necessary to place an additional self in series with the coil L_1.

A typical result for two wireless coils used as a mutual gave $C_2 = 0{\cdot}1\ \mu fd$. $R_4 = 100$ ohm. $L_1 = 132$ millihenries. $R_1 = 2{,}780$ ohm. $R_2 = 390$ ohm. $M = 27{\cdot}8$ millihenries.

As an additional exercise use Anderson's method with a self-inductance (previously determined) to calibrate a variable condenser having, for example, a capacitance $1 - 5\ \mu fd$.

REFERENCES

VIGOUREUX and WEBB : *Electrical and Magnetic Measurements.* Blackie.
PAGE and ADAMS : *Principles of Electricity.* Chapman and Hall.
OWEN : *Alternating Current Measurements.* Methuen.

39. EXPERIMENTS WITH A WAVE-METER

One type of wave-meter, which may be used either as a receiver or as a sender of electromagnetic impulses, consists of an L and C circuit, which may either be connected to a crystal-rectifier and telephones for receiving, or to a battery and a buzzer for use as an emitter (Fig. 126). Various coils are provided to be substituted for L so as to obtain different ranges of wavelength. Calibration curves are provided giving the relation between wavelength and the condenser-setting.

To determine the self-inductance and capacitance of a coil

Arrange to use the wave-meter in Fig. 127 as an emitter. Make up another circuit consisting of the coil A of which the L and C are to be determined, a variable air-condenser B of about 0·002 μfd

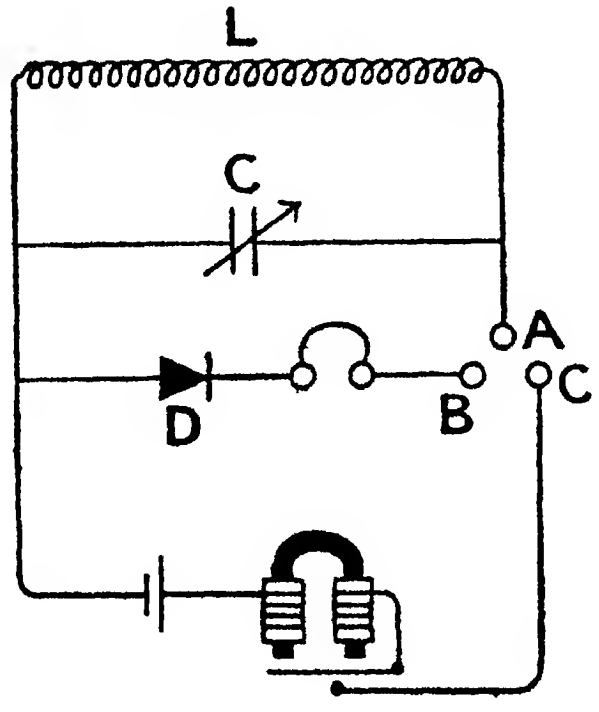


FIG. 126. Wave-meter.

in parallel with the coil A, and this parallel circuit joined to a sensitive galvanometer G shunted by a capacitance E of about 0·25 μfd. A crystal detector D is placed between the parallel

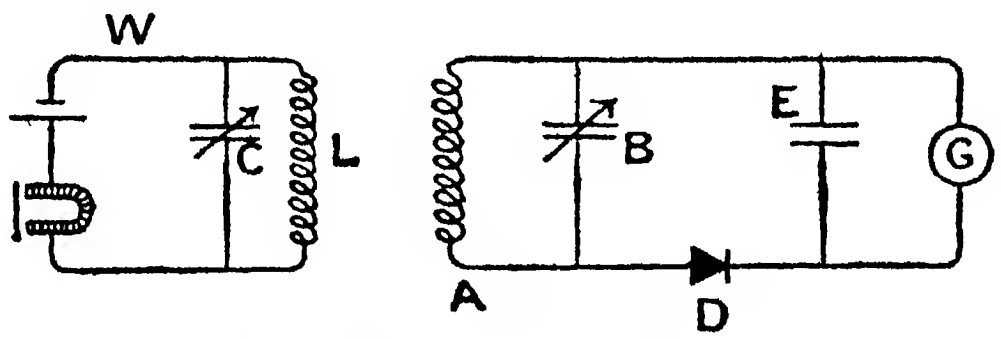


FIG. 127. Coupled circuits.

circuit and the galvanometer. The variable condenser B may either be a standard graduated condenser or else an air-condenser with a 180° scale for which the calibration curve is known. It is first necessary to find the wave-meter coil which gives a suitable range of wavelength when the capacitance B is varied. To do this join telephones across B, with G, E and D disconnected. Set the condenser B at a definite reading, try each coil in turn and turn the condenser dial. Choose the coil which gives a tuning effect.

Now place the coil A of the second circuit about 30 cm. from that of the wave-meter. With the appropriate coil in the wave-meter, re-join the circuit and set at 10° the condenser B, which

is assumed to have a scale in degrees. Then turn condenser dial C until the galvanometer shows the maximum current. Repeat for 20°, etc. Convert the readings in degrees into capacities by using the calibration curve. Determine λ for the reading of C from the wave-meter calibration and plot a graph between λ^2 and K where K represents the capacitance of B.

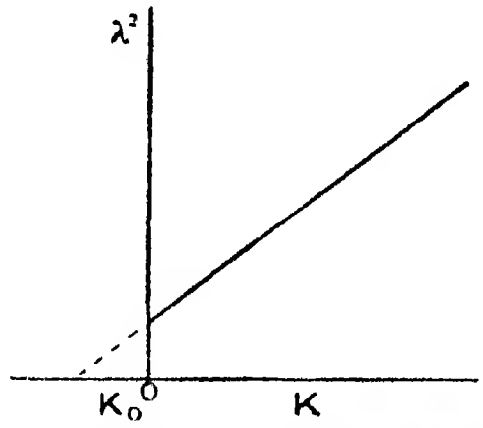


FIG. 128. Plot of λ^2 and capacitance.

Since $\lambda = 1{,}885\sqrt{L(K + K_0)}$ if λ is in metres, L in microhenries and K, K_0 the self-capacitance of the coil are in μfd, a plot of λ^2 against K should give a straight line (Fig. 128). From the slope it is possible to determine L and from the intercept for $\lambda^2 = 0$ it is possible to determine K_0, the self-capacitance of the coil. The latter may be about 100 $\mu\mu fd$.

To determine the dielectric constant of a liquid

As an additional exercise it is useful to place a small variable

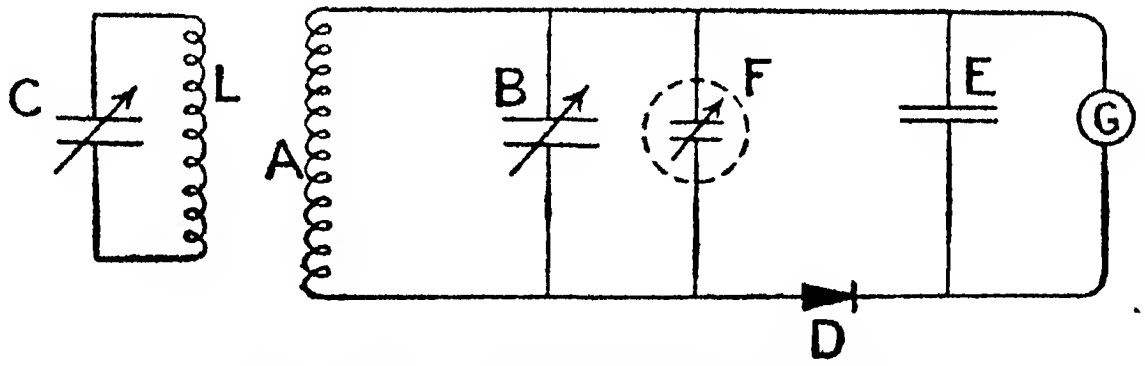


FIG. 129. Condenser in parallel.

air-condenser F in parallel with condenser B (Fig. 129) and repeat the observations as shown in Fig. 130 by curve B. If A represents the original curve and K_1 the new intercept, $K_1 - K_0$ gives the capacitance of the small condenser F. F is now placed in a cylindrical glass jar containing, say, paraffin oil. The observations are repeated and result in the curve C in Fig. 130. If K_2 is the new intercept $K_2 - K_0$ gives the capacitance of the liquid condenser and the dielectric constant of the liquid is given by $S = \dfrac{K_2 - K_0}{K_1 - K_0}$.

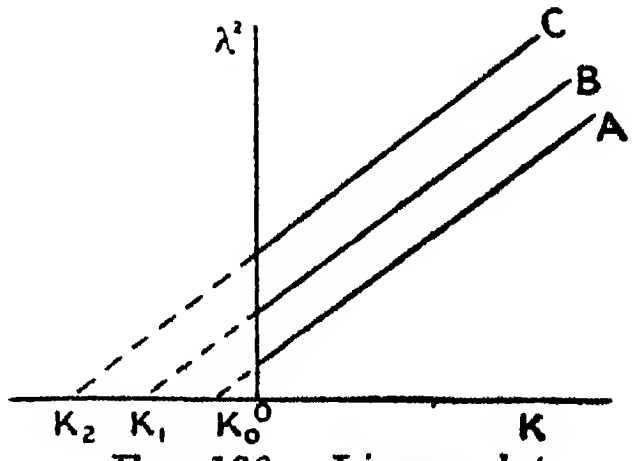


FIG. 130. Linear plot.

Paraffin gives a value near 2.

The student is left to investigate the question of the variation of dielectric constant with wavelength for the range of the wave-meter used.

40. An experiment on resonance

In Fig. 131, I represents the wave-meter used as a transmitter, II represents the resonance circuit formed of a small inductance-

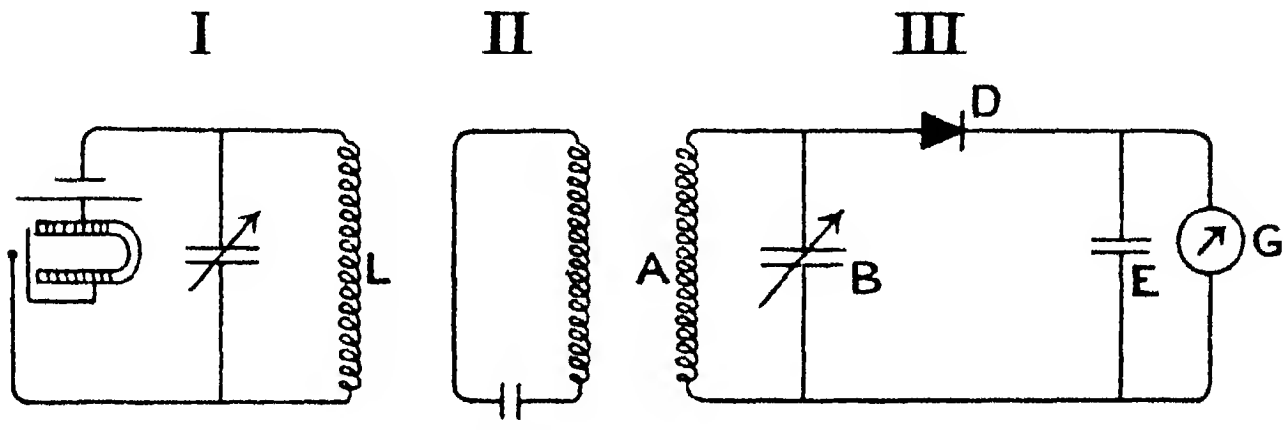


Fig. 131. Circuit for resonance experiment.

coil and a capacitance and III represents the detector-circuit as previously described. The wavelength of the transmitter may be varied over a wide range and for each wavelength the galvanometer deflection may be read for the detector-circuit. The resonance curve is obtained by plotting the detector current against the wavelength, and the type of curve obtained depends upon the nature of the coupling between the coils in the two circuits.

For loose coupling n, the frequency of the circuit acting singly is equal to $\frac{1}{2\pi\sqrt{LC}}$ where L is the self-inductance and C is the capacitance. For close coupling there are two frequencies, $n_1 = \frac{1}{2\pi\sqrt{(L+M)C}}$ and $n_2 = \frac{1}{2\pi\sqrt{(L-M)C}}$ where M equals $k\sqrt{L_1 L_2}$; L_1, L_2 are the inductances of the coupled coils and k is the coefficient of coupling.

Then $\lambda_1 = \lambda\sqrt{1+k}$ and $\lambda_2 = \lambda\sqrt{1-k}$ where λ is the wavelength for loose coupling and λ_1 and λ_2 are the wavelengths for close coupling. A and B in Fig. 132 are typical curves for loose and close coupling. From the above

$$\frac{\lambda_1^2}{\lambda^2} = 1 + k \text{ and } \frac{\lambda_2^2}{\lambda^2} = 1 - k.$$

Thus if $$\lambda = \frac{\lambda_1 + \lambda_2}{2}, \; k = \frac{\lambda_1 - \lambda_2}{\lambda}.$$

The experiment is to obtain resonance curves for loose and close coupling, and to determine the coefficient of coupling.

The apparatus is arranged so that the coils are parallel with about 30 cm. between I and II and a similar distance between II and III. Set the condenser in I at 10° and read the deflection produced in the galvanometer, repeat for 20°, etc. Convert the condenser readings into wavelengths by means of the calibration curves provided with the instrument and repeat the determinations with other coils in the wavemeter if necessary.

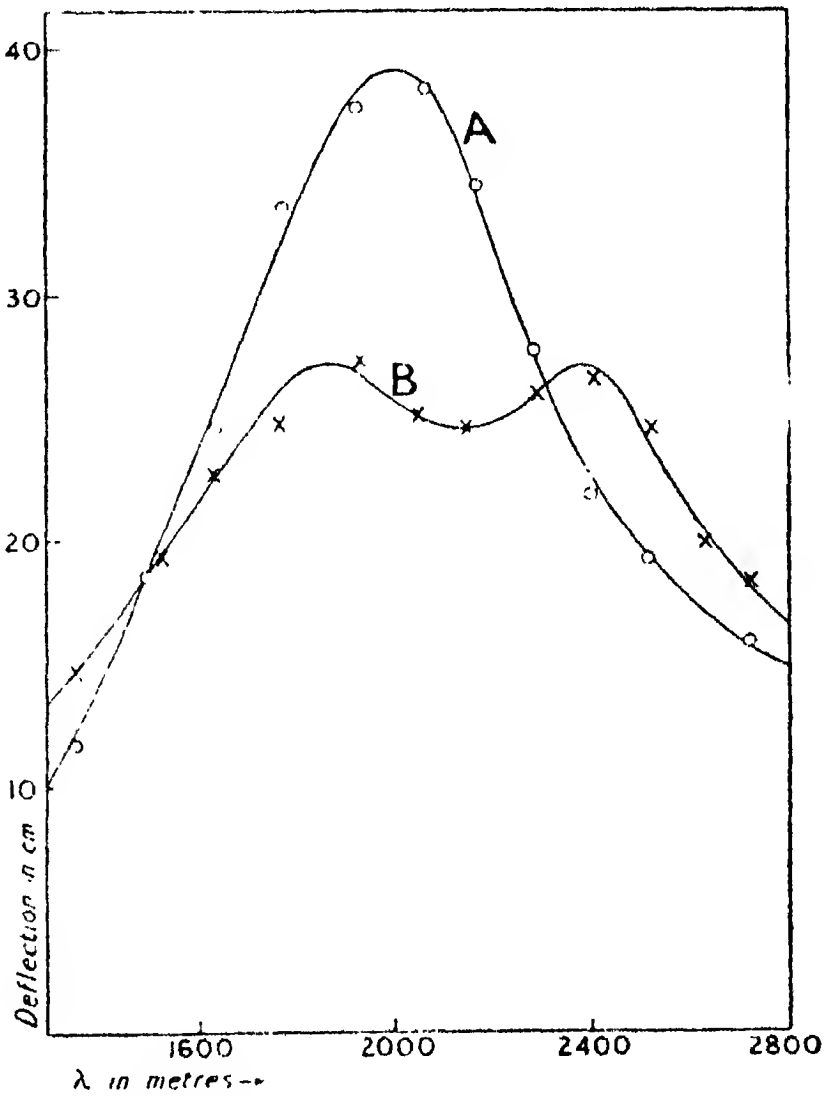


FIG. 132. Resonance curve, etc.

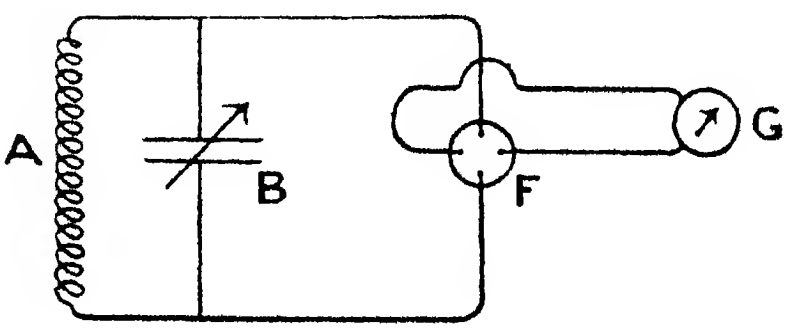


FIG. 133. Thermo-element.

Obtain a curve such as *A* in Fig. 132. The coils are now placed as close as possible and the deflection in the galvanometer will be much larger. If necessary the crystal detector circuit may be replaced by a thermo-element *F*, in which the current in the circuit heats a thermo-electric junction and the thermo-electric current is measured in the galvanometer circuit (Fig. 133). For close coupling obtain a curve such as *B* in Fig. 132, for which a thermo-element was employed.

For the given curves $\lambda_1 = 2{,}380$ m, $\lambda_2 = 1{,}850$ m, $\lambda = 2{,}000$ m and $k = 0{\cdot}26$, or the degree of coupling was 26%. A large coefficient corresponds to *close* or *tight* coupling.

41. The valve voltmeter

The simple wave-meter described in the preceding experiments is unsuitable for accurate measurements at radio frequencies. It is relatively insensitive and may absorb a considerable amount of energy from the circuit under measurement. A valve voltmeter does not suffer from these disadvantages.

In the triode valve voltmeter the voltage to be measured is applied to the grid of a triode valve and causes a change in anode current. This change is measured on a meter in the anode circuit

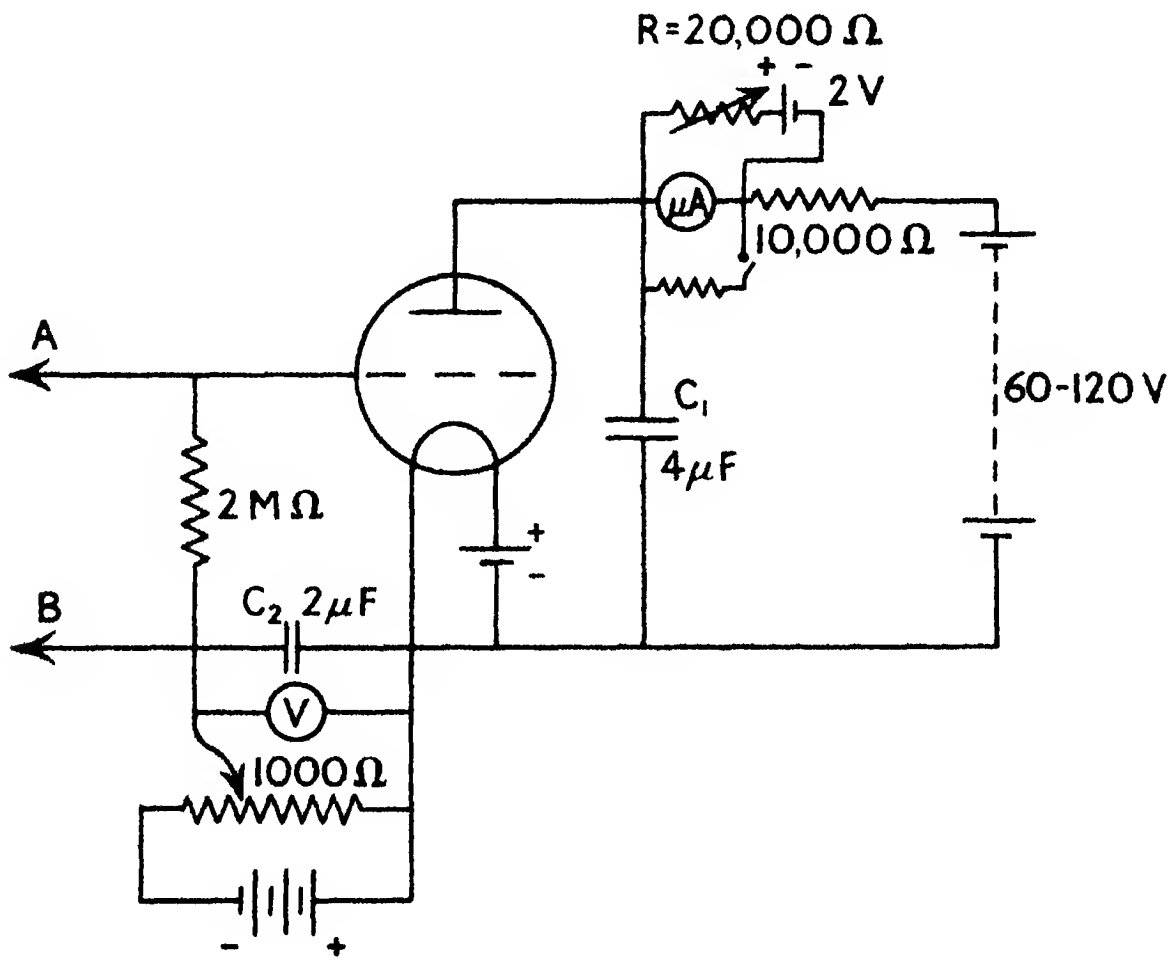


Fig. 134. Valve voltmeter.

of the valve. The instrument requires calibration but the indications are substantially independent of frequency except at high radio frequencies.

A simple circuit is shown in Fig. 134. While this circuit does not possess all the advantages of a properly designed instrument, particularly the very high input impedance, it serves to measure resonance curves on tuned circuits and in similar applications.

The meter in the anode circuit is a sensitive microammeter with protective shunts for the initial adjustments, and the variable resistance R is arranged to balance out the steady anode current which flows when the terminals A and B are connected together. The instrument should be totally enclosed in an earthed metal box.

The indications of the instrument depend on the value of the grid bias voltage. If the bias is such that the operating point of the valve is on the lower bend of the mutual characteristic (point *A* in Fig. 135) so that anode current flows over the whole cycle of the applied voltage, the anode current is approximately proportional to the square of the applied voltage provided this does not exceed about 1 volt peak. If the bias is adjusted to the cut-off value (point *B*) the change in anode current is approximately proportional to the square of the positive half cycles of the applied voltage. If the valve is biased beyond cut-off (point *C*) the indications are determined chiefly by the peak value of the applied voltage.

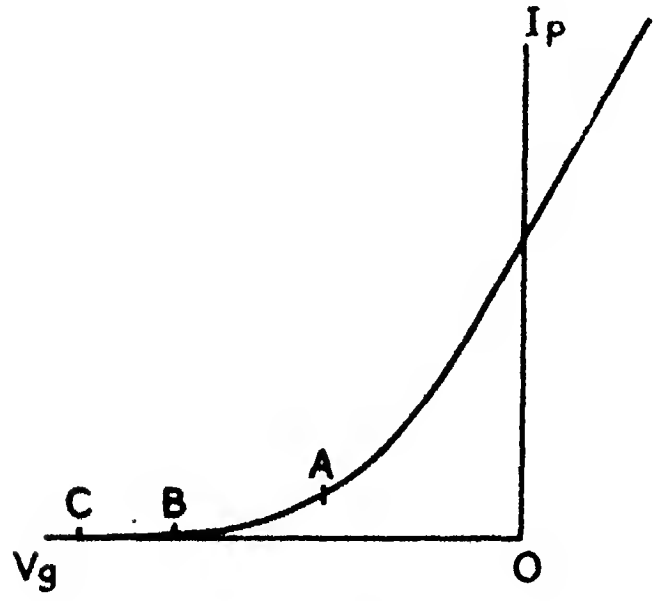


FIG. 135. Operating points on mutual characteristic.

Calibrate the valve voltmeter of Fig. 134, using an input derived from the a.c. mains and a voltage divider, for various values of the grid bias. The valve may be an ordinary or miniature type pentode connected as a triode. The condensers C_1 and C_2 are by-passes to the alternating currents in the circuit. These allow the calibration at 50 c/s to be used at radio frequencies, up to about 1 megacycle per second without serious error.

The simple valve voltmeter described above may require frequent recalibration. If the laboratory does not possess a commercial valve voltmeter for permanent use, a stable and accurate multi-range instrument may be constructed cheaply from a design in *Radio Designers' Handbook*, edited by F. Langford Smith, Iliffe and Sons Ltd., London.

42. DIELECTRIC CONSTANT AND CONDUCTIVITY BY THE RESONANCE METHOD

The dielectric constant and conductivity of a solid or liquid at a single frequency may be found by measurements on a tuned circuit. The material forms the dielectric of a condenser in the circuit and its presence modifies the shape and position of the resonance curve.

Fig. 136 shows the experimental arrangement.

C_1 is a measuring condenser with an air dielectric calibrated in micro-microfarads and C_2 contains the dielectric. An oscillator of known frequency is loosely coupled inductively to the tuned circuit by the coil L_2 and the voltage developed across the tuned circuit measured on a valve voltmeter V.

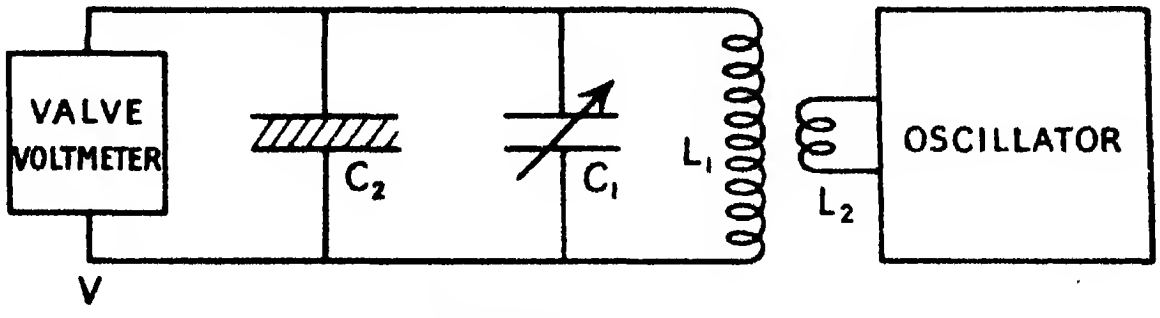


FIG. 136. Experimental arrangement.

If a solid dielectric is to be used C_2 may be constructed from two circular brass discs of about 10 cm. radius mounted parallel to each other, between which the dielectric fits closely. The dielectric is in sheet form and should be larger than the condenser plates. If a liquid is to be used C_2 may be an air condenser mounted in a suitable container.

The capacity of C_2 with dielectric must be somewhat less than the range covered by C_1 and a rough estimate of C_2 should be made from its dimensions.

It is useful to remember that

$$1\ \mu\mu\text{F} = 0{\cdot}9 \text{ e.s.u. of capacity.}$$

Method. With C_2 in circuit but without the dielectric adjust C_1 to a point near the maximum of the scale. Vary the oscillator frequency slowly till the valve voltmeter shows a resonant rise in voltage. Adjust the coupling coil L_2 till the valve voltmeter reads nearly full scale on the lowest range. The coupling between L_2 and L_1 must be loose to avoid a flat-topped resonance curve.

Vary C_1 and plot a resonance curve of voltage against C_1. Now insert the dielectric and repeat, keeping the frequency and the coupling of the oscillator the same. The curves will have the shape of Fig. 137. The curve in the second experiment will be lower and broader than in the first.

Theory. The condenser C_2 with dielectric may be regarded as a perfect condenser C_2' in parallel with a resistance R'. The induced voltage is in series with L_1 and the circuit may be re-drawn as in Fig. 138.

E is the induced e.m.f. and R is the resistance of the coil L_1. Let $C_1 + C_2' = C$ and let $G = 1/R'$. The impedance of the

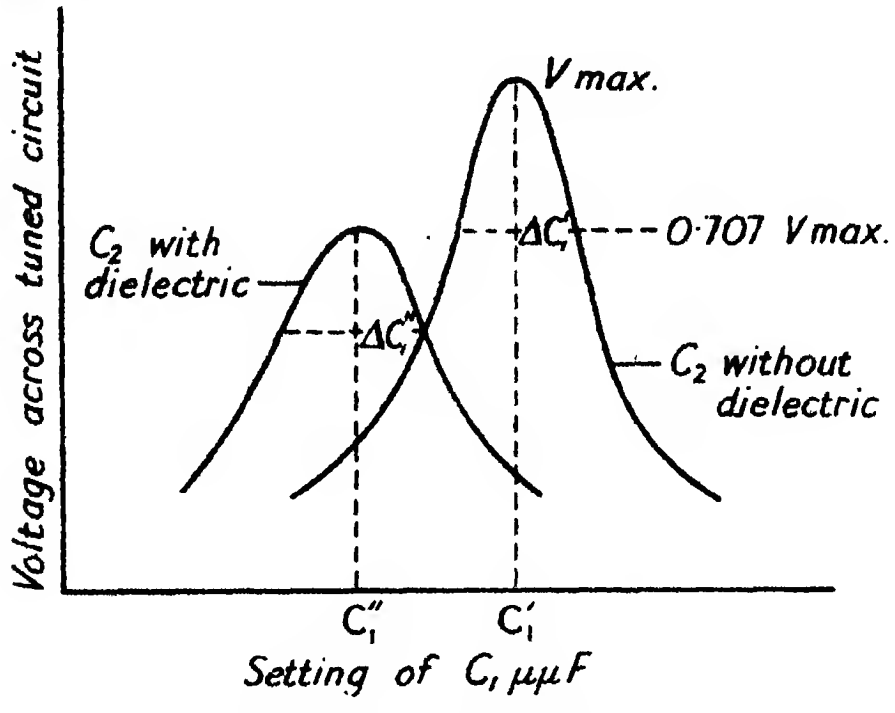


FIG. 137. Resonance curves for tuned circuit.

whole circuit consists of L_1 and R in series and C and R' in parallel. The impedance of C and R' in parallel is $1/(j\omega C + G)$

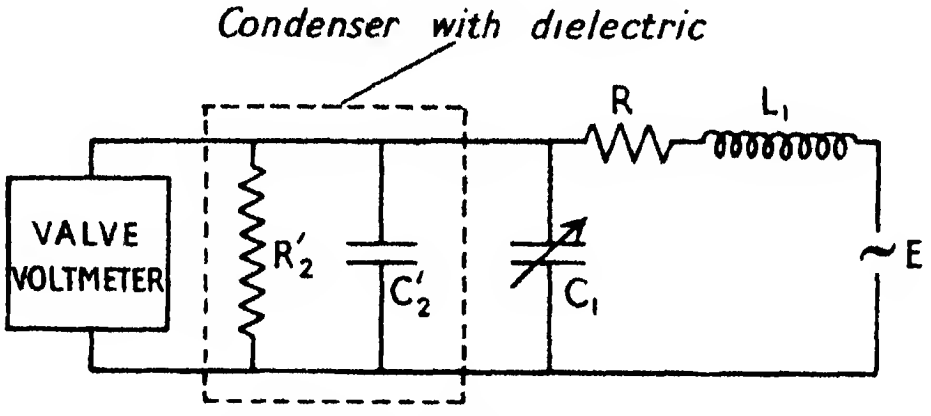


FIG. 138. Circuit equivalent to Fig. 136.

and that of the coil is $R + j\omega L_1$. The vector current through L_1 is therefore

$$I = \frac{E}{R + j\omega L_1 + \dfrac{1}{j\omega C + G}}$$

and the vector voltage across C and R' in parallel is

$$V = \frac{I}{(G + j\omega C)}$$

that is

$$V = \frac{E}{\{R + j\omega L_1\}\left\{ G + \dfrac{R}{Z^2} + j\omega\left(C - \dfrac{L_1}{Z^2}\right)\right\}}$$

where $Z^2 = R^2 + \omega^2 L_1^2$, the impedance of the coil. Collecting real and imaginary terms and taking the modulus of each bracket the square of the measured voltage is

$$V^2 = \frac{E^2}{Z^2\left\{\left(G + \frac{R}{Z^2}\right)^2 + \omega^2\left(C - \frac{L_1}{Z^2}\right)^2\right\}}.$$

As only C is variable the maximum value of V occurs when $C = L_1/Z^2 = C_0$ say and is

$$V_{\text{max.}} = \frac{E}{Z\left\{G + \frac{R}{Z^2}\right\}},$$

hence

$$\left(\frac{V}{V_{\text{max.}}}\right)^2 = \frac{\left(G + \frac{R}{Z^2}\right)^2}{\left(G + \frac{R}{Z^2}\right)^2 + \omega^2(C - C_0)^2}.$$

As C occurs in a squared term the resonance curve is symmetrical about the maximum ordinate.

At the two points on the curve where $\left(\frac{V}{V_{\text{max.}}}\right)^2 = \frac{1}{2}$ the above equation gives

$$\omega(C - C_0) = G + \frac{R}{Z^2}.$$

Let $C - C_0 = \Delta C_1$, that is ΔC_1 is the change in capacity of C_1 required to reduce the voltage to $1/\sqrt{2}$ or 70·7% of the maximum (see Fig. 137). Let $\Delta C_1'$ and $\Delta C_1''$ be the values of ΔC_1 when the condenser is empty and when it contains the dielectric. In the first case G may be considered to be zero so that

$$\frac{R}{Z^2} = \omega \Delta C_1',$$

with dielectric

$$G + \frac{R}{Z^2} = \omega \Delta C_1''.$$

So that

$$G = \omega(\Delta C_1'' - \Delta C_1'),$$

from which the conductivity may be calculated.

To determine the dielectric constant K let C_2' be the capacity of the condenser C_2 without dielectric and $k.C_2'$ its capacity with. Let C_1' and C_1'' be the settings of the measuring con-

denser in each case, at resonance. Then as the total capacity C is the same in each case, being equal to L_1/Z^2,

$$C_2' + C_1' = C$$
$$kC_2' + C_1'' = C.$$
$$\therefore \qquad k - l = \frac{C_1' - C_1''}{C_2'}.$$

The capacity of the condenser C_2 without dielectric is thus required and is determined in a separate experiment.

By using coils of different inductances the conductivity and dielectric constant may be measured over a range of frequencies. For accurate results the resistance of the coil should be small compared with its reactance, that is it should have a high " Q " factor.

43. To find the absorption coefficient of a metal for the γ-rays from radium C : absorption of β-rays

For this experiment a small container holding 1 milligram of radium bromide is required and a γ-ray electroscope which may be very cheaply made as follows :—

A is a closed tin cubical box (Fig. 139) of side about 12 cm. In the centre of the top is bored a hole of about 1·5 cm. diameter which holds a sulphur bead B. Through B a stout metal wire

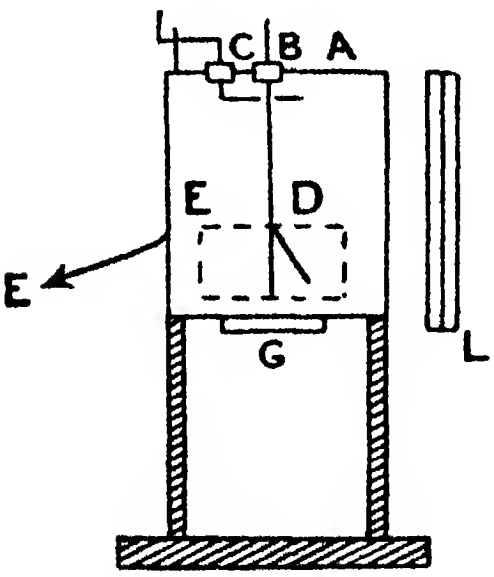


Fig. 139. Gamma-ray electroscope.

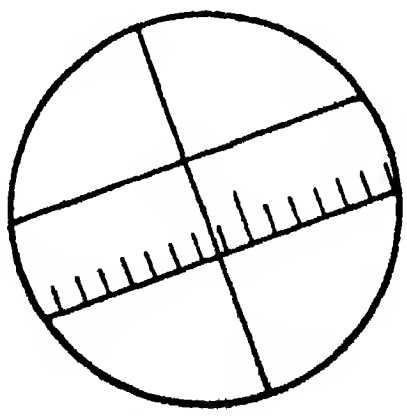


Fig. 140. Eyepiece scale.

is fixed, and this serves to carry the gold leaf D. Another small hole in the top shown at C is provided with another sulphur bead and serves to take a wire charging handle. In the back and front of the tin is cut out a rectangular window E which is covered by means of a glass plate so that the electroscope is airtight. A micro-telescope focusing at a distance of some 12 cm. and

provided with a scale of 100 divisions is used to view the scale as depicted in Fig. 140. The sulphur bead may be prepared by just melting, but not overheating, some sulphur and pouring it into a glass tube of a suitable diameter. When the sulphur cools, it may be pushed out of the tube as a rod and cut up into suitable lengths. The end of the wire which is to carry the leaf is heated and pushed into the sulphur bead, where it will be held tightly when the sulphur cools. The bead should be well scraped before use to ensure that it is free from grease.

To prepare the leaf, a thin sheet of either gold or aluminium is placed between two sheets of paper and with scissors or a safety-razor blade a thin strip is cut about 7 cm. long and 2 or 3 mm. wide. The rod is moistened with the tongue and touched on to the leaf, which will adhere. No attempt should be made to touch the leaf with the fingers. The case of the instrument should be earthed and when the leaf system has been charged up by means of an ebonite rod rubbed with flannel, the charging handle *C* should be turned to touch a wire attached to the case and so to be earthed. *G* at the bottom of the case is a projecting rim which enables various foils to be placed at the bottom. The instrument may then be used as a γ-ray electroscope The whole of the case is covered with lead plates about 0·5 cm. in thickness. The absorption coefficient of, say, lead for the penetrating radiation from radium C is found as follows :—

Charge the leaf system so that the image of the leaf appears to be quite fine as seen on the microscope scale shown in Fig. 140. Determine the time for the leaf to fall between two wide readings on the scale, say sixty to eighty divisions in all. Repeat a few times and verify that the behaviour is quite regular and that the natural leak as measured in divisions per minute is reasonably small.

Now in a V-shaped piece of cardboard shown at *F* in Fig. 139 place the radium source and re-determine the rate of fall over the same range of the scale. Insert a sheet of lead of known thickness at *L* and repeat the observations. The rate of fall will have decreased. Introduce one by one a number of sheets up to perhaps ten in number. The first few sheets serve to isolate the homogeneous radiation and only after that can it be expected that an exponential absorption curve will be obtained as shown in Fig. 141. Corrections must be made to the measured rates of fall by subtracting the natural leak in each case.

For homogeneous rays $I = I_0 e^{-\mu d}$ where I_0 is the incident

intensity, I is the intensity of radiation transmitted by a sheet of thickness d and λ is the linear absorption coefficient. If it is supposed that $R = R_0 e^{-\lambda d}$ where R_0 and R_1 are rates of fall, then $\log R_0 - \log R = \lambda d$.

Thus a plot of $\log R$ against d should give a straight line as shown in Fig. 142. Determine λ from the slope, or find the thick-

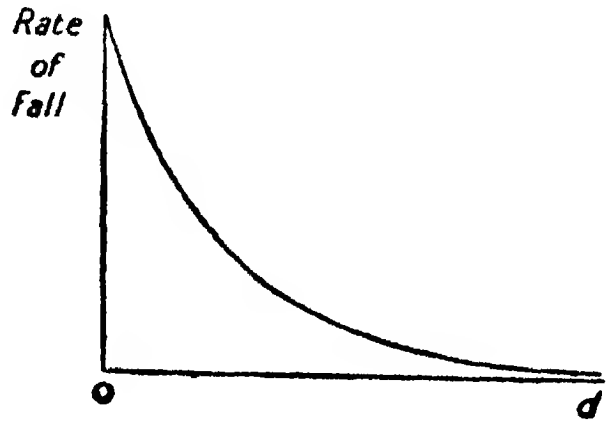


FIG. 141. Plot of rate and thickness.

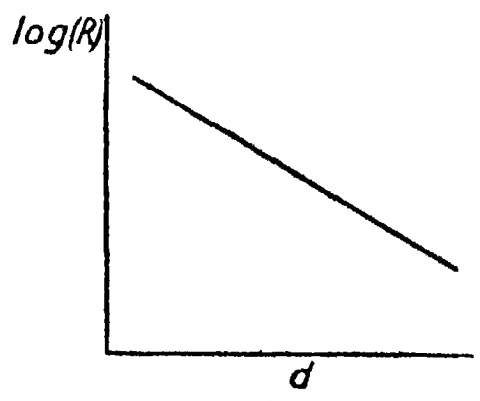


FIG. 142. Linear plot.

ness d_1, required to reduce the intensity to one half its initial value. Then since $\log_e 2 = 0{\cdot}693$, $\lambda d_1 = 0{\cdot}693$, which enables λ to be calculated. β-rays produce some effect, but about 2 mm. of lead is sufficient to stop them. Place the lead sheets near the electroscope and away from the source and then near the source and further away from the electroscope. See if there is any difference in the rates of fall in the two cases.

To compare two radium sources

Place each source in turn in the same position in the V-shaped holder, determine the rate of fall of the leaf for each specimen and take the ratio of the two radium contents to be in the ratio of the rates of fall.

To investigate the absorption of β-radiation in a metal

For this purpose finely powdered pitchblende may be used as the source when spread uniformly on a shallow tin of some 5 cm. diameter. Aluminium or copper foil of about 0·05 mm. thickness is suitable for the production of the absorption. The thickness is best found by taking a large sheet of the foil, weighing it, and calculating from the measured area and the known density of the metal. The plate of pitchblende is placed under G, which is now covered with a very thin sheet of foil. The rate of fall of the leaf is determined as before and then when successive sheets of the thin foil are placed over the source. A plot of the rate of

fall against the thickness will give a curve somewhat resembling an exponential curve. The first effect will include the ionisation produced by α-rays.

·44. To measure the stopping power of aluminium for α-rays

For this experiment an α-ray electroscope is required. It differs from the γ-ray electroscope in that it must allow α-rays to enter without absorbing them. Since α-rays are absorbed completely by $\frac{1}{10}$ mm. of aluminium or about 10 cm. of air the bottom of the electroscope, though which the α-rays are to enter, is covered by an extremely thin aluminium foil. In other respects the instrument is similar to the γ-ray electroscope. In any radioactive experiment an α-ray electroscope measures the total ionisation due to α-, β- and γ-rays. The ionisation of α-rays greatly exceeds the ionisation due to the β- and γ-rays.

The most suitable source for this experiment is polonium (radium F) which emits only α-rays. The α-rays are first canalised by placing thick sheets of aluminium pierced with a small hole over the source so that the emerging beam lies within a small solid angle, and all particles can enter the electroscope. Place the source on a table with an adjustable vertical movement beneath the electroscope and arrange a travelling microscope to measure the height of the source. After testing the natural leak of the instrument, plot a graph of the rate of fall of the leaf against the height of the source, measured from an arbitrary level. A straight line should be obtained over a range of a centimetre or two. Cover the source with a thin aluminium foil and plot a second graph measuring from the same level. This will be another straight line parallel to the first but displaced along the horizontal axis as in Fig. 143.

The horizontal distance between the lines is the equivalent air thickness of the aluminium foil. Let this be x cm. This equivalent air thickness depends on the mass of air traversed and so on the temperature and pressure. x is reduced to standard conditions (taken as 15° C. and 76 cm.) by the equation

$$x' = x.\frac{288.p}{T.76}$$

where x' is the corrected air thickness, p is the barometric pressure in cm. of mercury and T is the absolute temperature.

Calculate the thickness of the foil by weighing it on a good

balance and measuring its area. The density of aluminium is 2·7 gm. per c.c. From the result calculate the thickness d cm. of aluminium which would produce the same absorption as

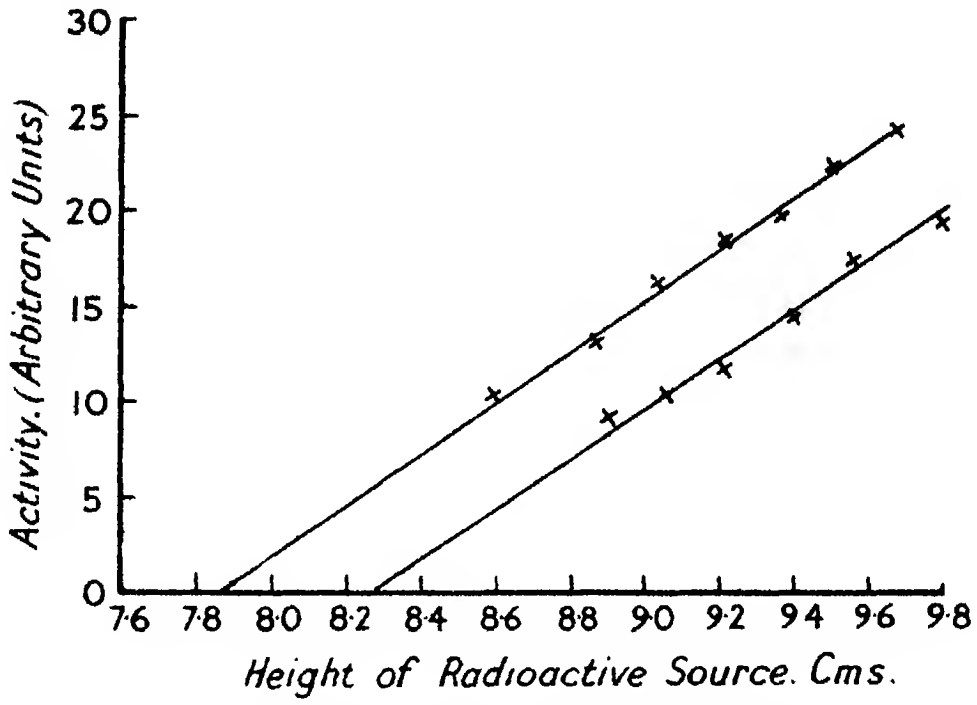


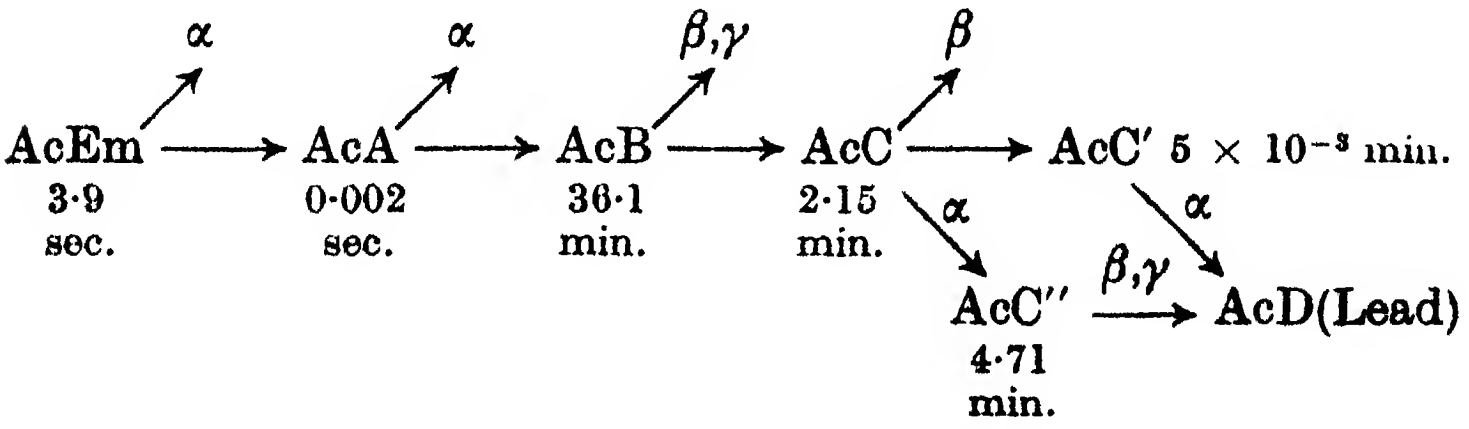
FIG. 143. Absorption of α-particles in aluminium.

one cm. of air under standard conditions. The stopping power of aluminium is given by

$$\frac{\text{Mass of 1 c.c. of air under standard conditions}}{\text{Mass of 1 sq. cm. of aluminium of thickness } d} = \frac{\text{density of air}}{\text{density of aluminium} \times d}.$$

45. TO DETERMINE THE DECAY CONSTANTS OF ACTINIUM B AND C

The element actinium emits a gas called actinium emanation which decomposes into successive radioactive elements according to the following scheme :—



The nature of the radiation emitted in each transition is shown and also the half-life of each element. Actinium C decomposes

in two ways, but the end product, actinium D, an isotope of lead, is the same in each case.

Actinium A may be prepared from the emanation by the method described below and the radioactivity measured as a function of the time in an α-ray electroscope. In interpreting the experimental curve only the intense ionisation due to α-particles need be considered. Also actinium A disappears rapidly so that after the first fraction of a second the activity at any instant is determined by the α-rays from actinium C and C′. It is effectively proportional to the amount of C present, since the lifetime of C′ is so short.

The quantity of actinium C present depends on the rate at which it is formed from actinium B, and also on the rate of decay of C. It is possible to obtain the decay constants of both B and C from a single activity-time curve by a suitable procedure.

Preparation of the active deposit

The active deposit formed by the decomposition of the emanation is positively charged and may be collected on a metal plate maintained at a negative potential of about 100 volts.

The actinium salt is contained in a small metallic cup with an airtight insulating cover. On the underside of the cover, insulated from the cup, is a small copper or platinum disc held

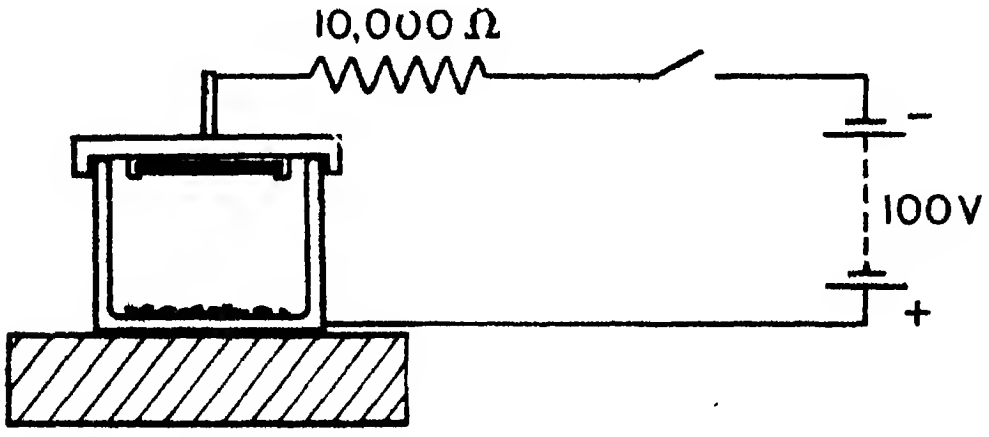


Fig. 144. Emanation cup.

in clips which is connected to a source of negative potential through a protective resistance of say 10,000 ohms. The disc should be one or two centimetres in diameter.

The actinium salt contained in the cup is moistened with a drop of distilled water to help free the emanation, which tends to be occluded by the dry salt. The disc, previously cleaned with emery paper and tested for activity by the electroscope,

is placed in the apparatus and a stop watch is started at the instant the potential is applied. This is the zero time on the decay curve. After an exposure of twenty seconds the voltage is switched off, the disc removed with tweezers and transferred to the electroscope as rapidly as possible.

The time taken for the leaf of the electroscope to cross two selected eyepiece divisions may be taken as a measure of the activity throughout the main part of the experiment. During the first few minutes, however, the activity is changing so rapidly that the leaf should be timed as it crosses the whole scale, and the values reduced to the equivalent time for the selected divisions.

Readings should be continued for at least an hour but may be made at less frequent intervals after the first twenty minutes. Throughout the experiment the natural leak of the instrument must be tested and allowed for.

Subtract the leak from the observed activity and plot the corrected activity against the time. A curve similar to that of (1) in Fig. 145 should be obtained. The form of the curve is accounted for as follows :—

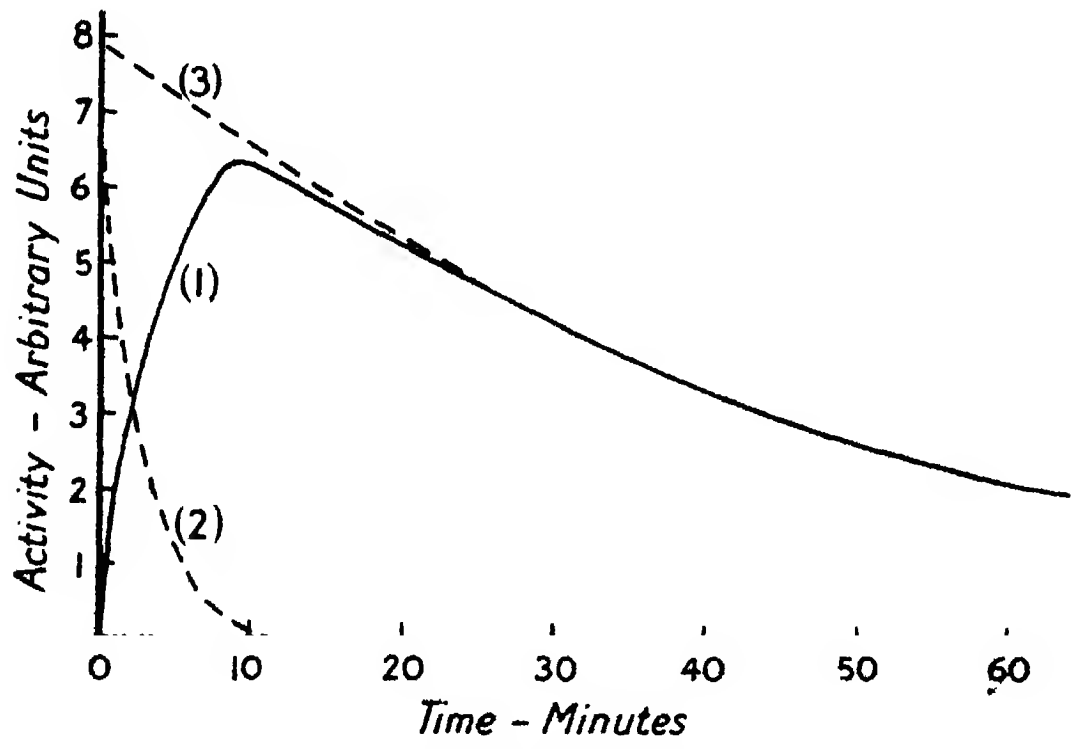


FIG. 145. Activity curve for actinium B ⟶ C.

At the beginning of the experiment no actinium C is present, but is gradually formed by the decay of actinium B. The activity therefore at first rises. But C is itself decaying and its rate of formation from B slows down as B is used up so that the total quantity of C present eventually begins to decrease. After a long time a steady state is reached in which the quantity

of C is in a fixed proportion to the quantity of B. The tail of the curve follows the exponential decay of the longer lived element, B.

Transformation theory

Let B be the amount of actinium B present at any time and C the amount of actinium C. Let λ_B and λ_C be the respective decay constants. At the beginning of the experiment only B is present, let the initial quantity be B_0.

Then
$$\frac{dB}{dt} = -\lambda_B B \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
represents the decay of B.

The rate of change of the quantity of C present is equal to its rate of formation from B less its rate of decay.

$$\frac{dC}{dt} = \lambda_B B - \lambda_C C \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The solution of (1) is

$$B = B_0 e^{-\lambda_B t} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and of (2) is

$$C = \frac{\lambda_B B_0}{\lambda_C - \lambda_B}(e^{-\lambda_B t} - e^{-\lambda_C t}). \quad . \quad . \quad . \quad (4)$$

This is the equation of the experimental curve. It is formed by subtracting the two simple exponential curves (2) and (3) which are shown dotted in Fig. 145.

Plot the logarithm of the corrected activity against the time. As λ_B is very small compared with λ_C the second term $e^{-\lambda_C t}$ in equation 4 rapidly becomes negligible and, after about 20 minutes, a linear graph is obtained. From the slope of this graph the value of λ_B is calculated. This value of λ_B is used to extrapolate the curve (3) in Fig. 145 backwards. The curve (2) is then formed by subtraction of the experimental curve from (3). Select suitably spaced ordinates on curve (2) and again plot logarithms of the activity against the time. Deduce the value of λ_C.

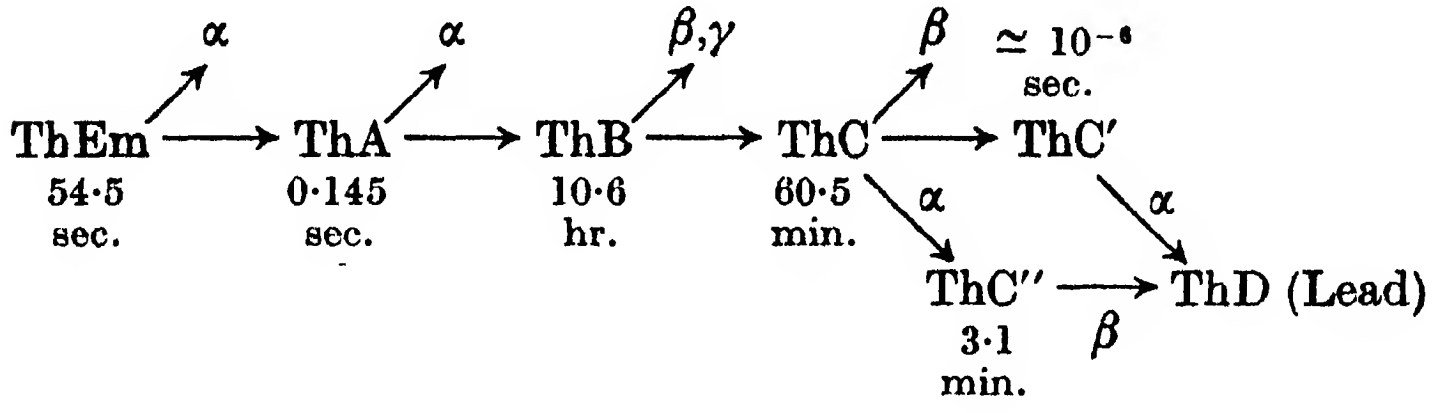
The half-lives of B and C may be calculated from the equation $T = \frac{0{\cdot}693}{\lambda}$ and compared with the values given in the transformation scheme.

NOTES: A more accurate value of λ_B may be found in a separate experiment by giving the disc a longer exposure, say one of a minute and a half. The value of λ_C is affected by errors in λ_B.

The electroscope should be kept in a separate room from the radioactive materials and emanation cup, otherwise emanation gas may add greatly to the natural leak of the electroscope.

The Decay Constants of Thorium B and C

The procedure is similar to the above. The disintegration scheme is :—



The maximum in the experimental curve is not reached till after about 2 hours and observations should be continued over a period of 30 hours. Thorium hydroxide evolves the emanation freely and is a suitable salt to use.

REFERENCES

W. MAKOWER and H. GEIGER: *Practical Measurements in Radioactivity.* Longmans, Green and Co., 1912.

J. B. HOAG: *Electron and Nuclear Physics.* Chapman and Hall Ltd.

46. THE DOLEZALEK ELECTROMETER

The electrometer consists of a light metal needle C suspended between two pairs of metal quadrants shown at AA, BB in Fig. 146. A mirror on the needle-suspension enables a spot of light to be observed on a distant scale, so that deflections of the needle system may be observed. The instrument must first be adjusted as follows :—

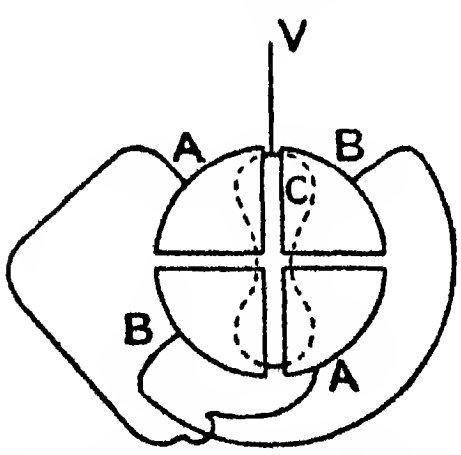


FIG. 146. Quadrant electrometer.

Adjustments

By means of the screw-head at the

top of the electrometer arrange that the needle shall have its long axis symmetrically between the quadrants. By means of the three levelling-screws, arrange that the electrometer shall be level, so that when the needle turns, it turns in a plane which is parallel to that of the quadrants. To test the adjustment earth the quadrants and the needle, and read the position of the image of the cross wire on the scale. Now charge the needle and if the position of the image of the cross wire has

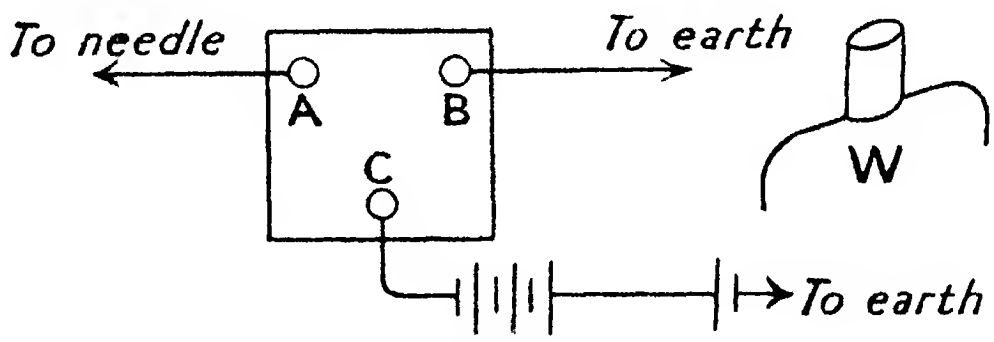


FIG. 147. Charging and earthing key.

altered, bring it back by turning one of the levelling-screws slightly. Repeat the process until the spot on the scale does not move on charging or earthing the needle.

In order to charge or discharge the needle rapidly, construct a paraffin key, consisting of a block of paraffin with three holes bored in it as shown in Fig. 147, and filled with mercury. A bent piece of wire *W* with a sealing-wax handle serves as a connector, so that the needle may be connected to a battery or to earth. Care must be taken that no short circuits occur, for there is the possibility of burning up the needle and its suspension and of spoiling cells used in electrostatic work. To guard against damage a high resistance formed of water contained in a capillary U-tube is joined in series with the cells employed. The tube may be fixed on a paraffin block as shown in Fig. 148.

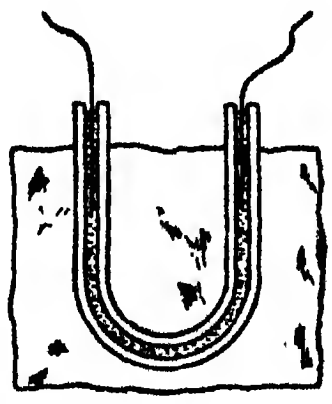


FIG. 148. High resistance.

As the insulated quadrant must be shielded from all external electric fields, the case of the instrument must be earthed. Any wires connecting to the insulated electrodes should pass through earthed shields, which may consist of glass tubes surrounded by lead foil.

The Sensitivity of the Electrometer

If V_1 and V_2 are the potentials of the quadrants, V the potential of the needle, θ the deflection and k a constant for the instruments, then elementary theory shows that

$$k\theta = (V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right).$$

V_2 is usually made zero by joining one pair of quadrants to the case and the equation becomes

$$k\theta = \frac{V_1}{2}(2V - V_1).$$

Usually V is of the order 100 to 400 volts and V_1 of the order of 1 volt, so that in these circumstances the deflection per volt θ/V_1 should be proportional to the voltage on the needle as shown by curve A in Fig. 149. This is found not to be the case, and fuller

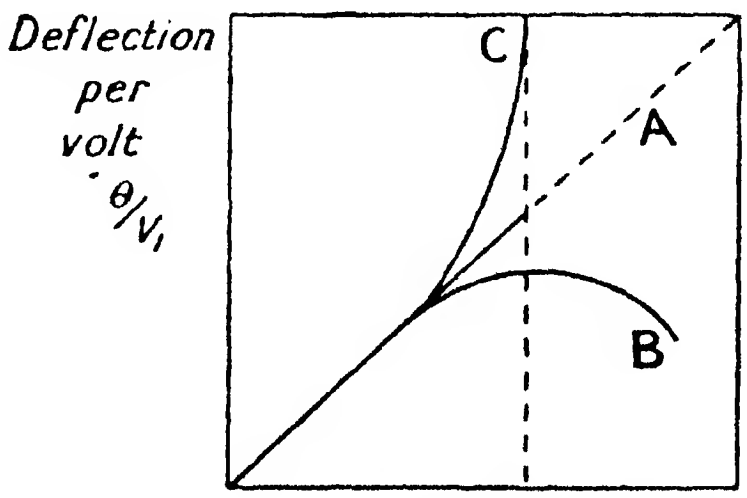


FIG. 149. Variation of sensitivity with voltage on the needle.

theory shows that instruments with a positive electrostatic control reach a maximum sensitivity as shown by curve B, and those with a negative electrostatic control attain a very high sensitivity as shown by curve C. The C. T. R. Wilson tilted electroscope is an example of the second type, and the Compton electrometer can be given either a positive or a negative control. Briefly, the reason for this behaviour is an alteration of the capacities due to the movement of the needle. This the elementary treatment does not consider. Allowing for changes of the capacity coefficients with angle, an expression of the form

$$k\theta = \frac{V_1 V}{A \pm BV^2} \text{ is obtained.}$$

The positive sign corresponds to the positive control and gives a maximum sensitivity for $V = \sqrt{\frac{A}{B}}$. The negative sign corresponds to the negative control and gives very high sensitivity for the same value of V. (See reference on p. 114.) It is an

instructive exercise to study the sensitivity of the electrometer, at least for the range over which the instrument is to be used. By means of a suitable battery apply voltages on the needle from 50 to 300 or 400 in increments of 10 or 20 volts with $V_1 = 1$ volt. Plot a graph between θ and V and it will be

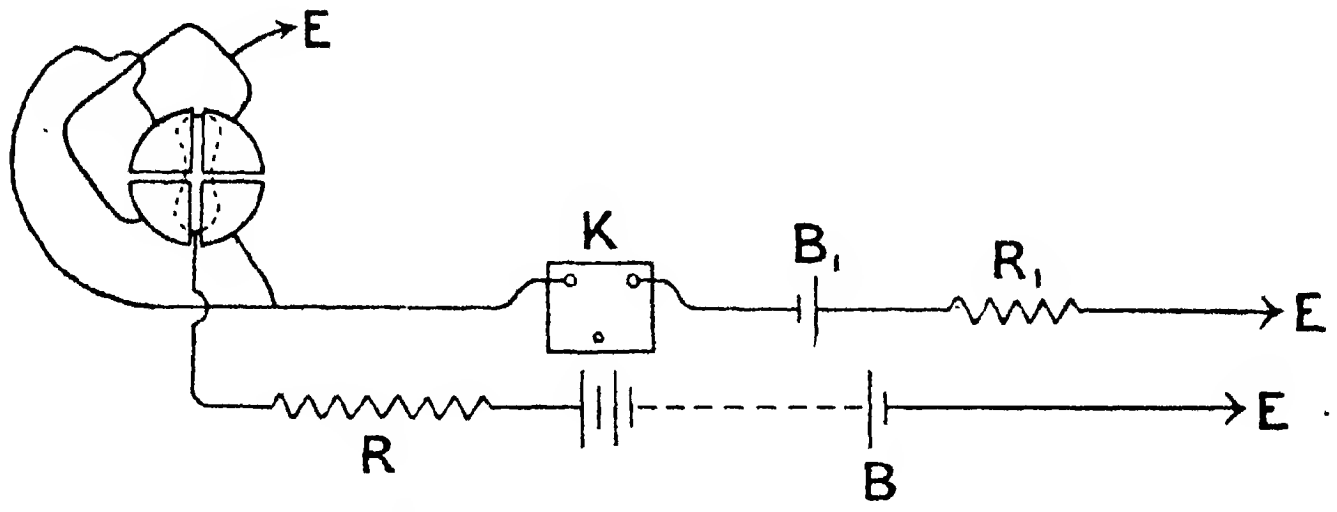


FIG. 150. Arrangement for the determination of the sensitivity curve.

found that the relation is not strictly linear, but the sensitivity, *i.e.*, the deflection per volt, tends to a maximum value. The connections are shown in Fig. 150, in which K is the key, B_1 on one pair of quadrants may be a Weston-cell and B is the variable high potential. R and R_1 are high resistances.

Measurement of Capacitance

It is sometimes necessary to know the capacitance of the instrument, and if a standard condenser is available the determination may be made as follows :—

Use the electrometer with the needle at such a potential that the sensitivity is about 100 divisions per volt. Charge the electrometer to this potential and then insulate the quadrant. Join the standard capacitance in parallel with the electrometer and note the new deflection. From these data the capacitance of the instrument may be easily calculated. If C is the capacitance of the electrometer, C' the known capacitance, V the potential at which the electrometer is charged initially, V' the final potential, then a charge $Q = CV$ is given to the electrometer. After connection the capacitance is $C + C'$, and if Q remains the same,

$$Q = (C + C')V'$$

$$\therefore \quad \frac{C + C'}{C} = \frac{V}{V'} = \frac{\theta}{\theta'}$$

$$\therefore \qquad \frac{C'}{C} = \frac{\theta - \theta'}{\theta'}.$$

V and V' may be taken as proportional to the deflections θ and θ', if these are not too large. A convenient condenser for the purpose may be constructed by pasting tin-foil on glass coated with shellac, or if the dielectric constant of glass is uncertain, a larger air-condenser may be made of metallic plates separated by small pieces of an insulator.

The condenser should be carefully shielded by enclosure in an

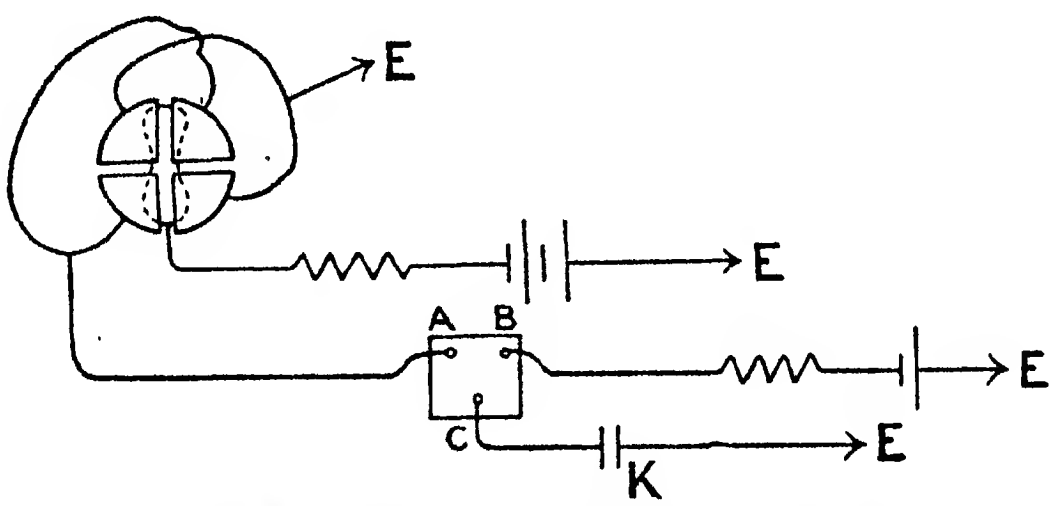


FIG. 151. Measurement of capacitance.

earthed box. The arrangement is shown in Fig. 151. To charge the electrometer, connect A—B. Insulate by breaking contact. Connect condenser K by joining A—C.

Measurement of a High Resistance

The electrometer may be usefully employed for the measurement of high resistances of the order of 10^9 to 10^{12} ohm. The method employed is the well-known one of the leaking condenser. From the potentials V_1, V_2, before and after a time T seconds, with a capacitance C and a resistance R it may be shown that

$$\log_e \frac{V_1}{V_2} = \frac{T}{CR} = \log_e \frac{\theta_1}{\theta_2}.$$

In Fig. 151 the capacity K may be replaced by the high resistance, which may consist of an Indian ink or graphite line on a sheet of paraffin paper or of a glass tube filled with wet paraffin wax and furnished with wire electrodes. The electrometer is charged as before and allowed to discharge for a given time through the high resistance. If θ_2 and T are varied, then

$$\log \theta_1 - \log \theta_2 = \frac{T}{CR}.$$

Plot $\log \theta_2$ against T and if Ohm's law is obeyed a straight line

should be obtained. In this way test the validity of Ohm's law for the resistance and calculate the value of R.

Measurement of ionisation currents

A cylindrical tin A is taken and a layer of uranium oxide U emitting α-rays is placed on the bottom (Fig. 152). Through a sulphur stopper S passes a metal rod holding a circular electrode C. A wire soldered to the outside of the tin allows of connection to a

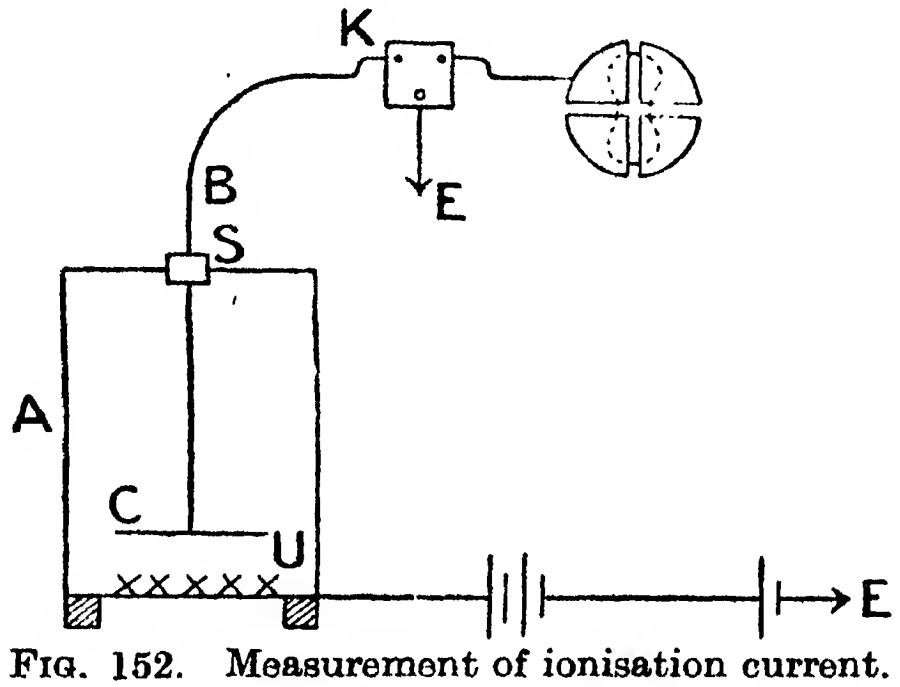


FIG. 152. Measurement of ionisation current.

battery of some 160 volts, which has the other terminal earthed. The electrode C is joined to a key K, which enables connection to be made to earth or to one pair of quadrants of the electrometer. The whole apparatus should be insulated on blocks of paraffin.

When the earth connection is broken, the ions formed by the oxide will be directed by the electric field to one or other of the electrodes according to sign. Thus an ionisation current i will be produced, and since

$$i = \frac{dQ}{dt} = C \cdot \frac{dV}{dt} = k \cdot \frac{d\theta}{dt},$$

the rate of deflection $\frac{d\theta}{dt}$ of the needle may be taken as a measure of the current after the earth connection has been broken. The rate of movement of the spot of light should be found for various values of the applied voltage and the curve showing the relation between i and V will be somewhat as depicted in Fig. 153. The saturation stage, for which a steady current is obtained, may be indicated by the graphs.

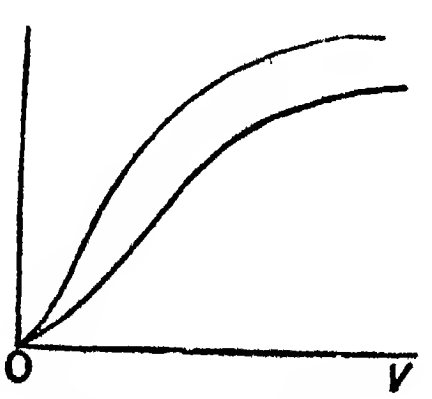


FIG. 153. Ionisation curves.

The following exercise is suggested :—

1. With a fixed distance between the electrodes plot a curve showing the relation between the electric field and the current.

2. Repeat for various distances between the electrodes.

3. Verify that the saturation-current varies as the distance apart of the electrodes.

A better null-method of observation is to join an earth-connected high resistance to the quadrants so that as fast as charge reaches them it leaks away through the high resistance. The steady deflection then obtained is a measure of the ionisation current, but for its absolute determination the resistance must be known.

REFERENCE

BEATTIE : *The Electrician*, **65**, p. 729, 1910 ; **69**, p. 233, 1912.

47. EXPERIMENTS WITH A PHOTO-ELECTRIC CELL

A few circuits of use in photo-electric work will first be described.

If a sensitive galvanometer is available, if the powers of the light sources are high, and if the greatest accuracy is not required,

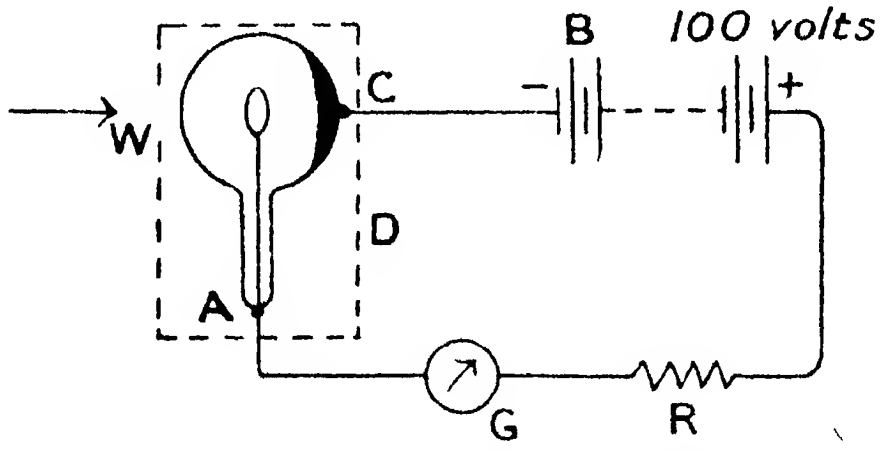


FIG. 154. Photo-electric cell with galvanometer.

the simplest arrangement is shown in Fig. 154. *C* represents the cathode of the photo-electric cell, joined to the negative pole of a dry battery *B* of 100–120 volts. The anode shown as a ring in the diagram is joined to *A*, the anode terminal, to which is joined one lead of the galvanometer *G*. The other lead from the galvanometer goes to a high resistance of about 50,000 ohms (to protect the instrument) and then to the positive pole of the battery. *D* represents the case of the cell with a window at *W* for the admission of the light in the direction indicated.

If the galvanometer available is not so sensitive the following

very good method, due to Professor J. A. Crowther, is useful. The circuit is shown in Fig. 155. It is arranged that the charge on the cathode C of the cell shall charge up a condenser S of capacitance about 1 μfd when the shorting key K_1 is open and key K_2 is depressed so that the circuit is closed at M. After a time of some seconds, which must be accurately measured, the key K_2 is

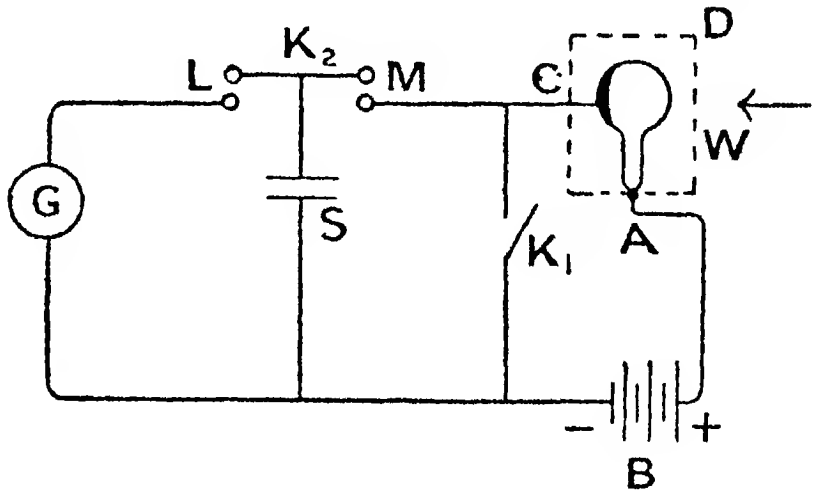


FIG. 155. Photo-electric cell with ballistic galvanometer.

depressed so that contact is made at L and the quantity which has accumulated on the condenser is discharged through a ballistic galvanometer G. By suitably choosing the constant charging time

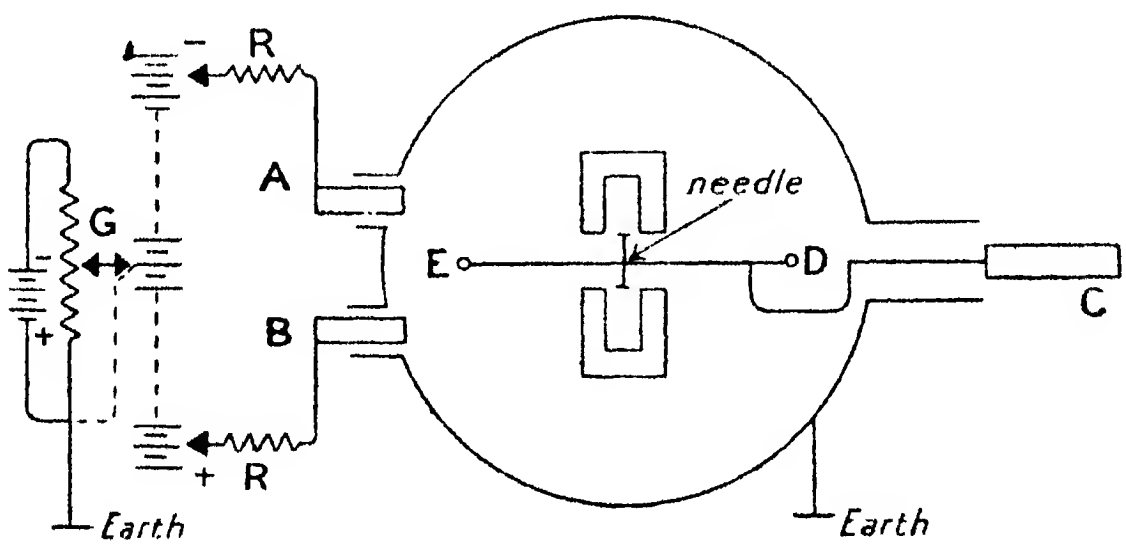


FIG. 156. Lindemann electrometer.

a galvanometer of low sensitivity may be used to measure small photo-electric currents. For work of the highest accuracy some form of electrometer is used, and for this purpose the Lindemann electrometer is much used. The instrument is shown diagrammatically in Fig. 156, in which A and B represent the terminals to the quadrants, C the terminal for the needle which is suspended at the centre of a silvered quartz fibre ED. At the end of the needle is a small rod, and the deflection of the needle is observed

by focussing on this small rod with a microscope with a ½-inch objective and an eyepiece with a scale giving a magnification about 8. There is also a small chamber in the instrument for containing drying material, but this is not shown in the diagram. In Fig. 156 the quadrants are shown joined to the terminals of a high-tension battery with 3-volt tappings, such as is used in wireless work. The centre of the battery *G* is earthed or may be joined to a potentiometer device, so that the potential of the centre of the battery may be slightly varied as required. *RR* are safety-resistances of the order of 25,000 ohms. The voltage on the needle is usually about 1 volt.

For example :—

Total voltage.	Voltage on *A*.	Voltage on *B*.	Voltage on needle.	Deflection in divisions of scale.
45	22	23	1·0	48

For photo-electric use a suitable arrangement is shown in Fig. 157.

The needle is joined to the anode *A* as shown, and *R* is a high

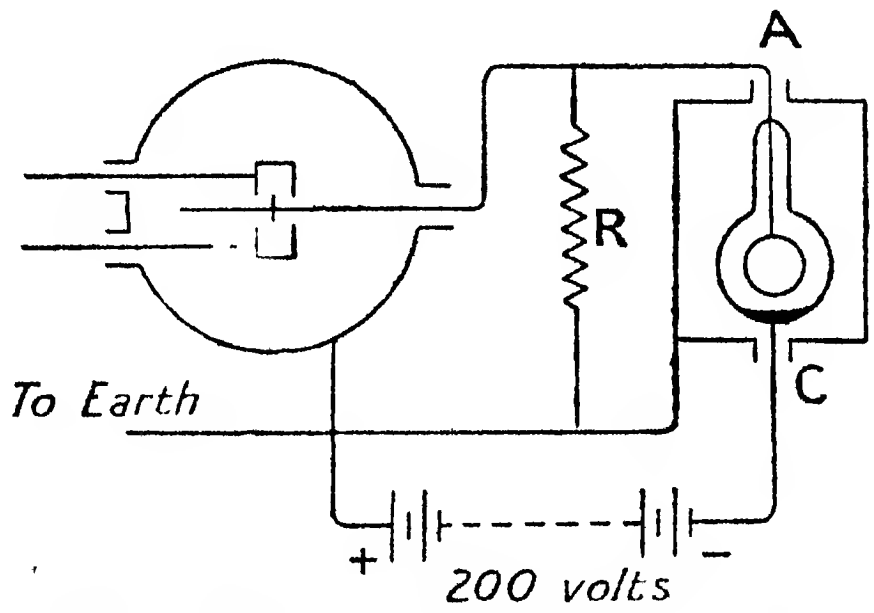


FIG. 157. Electrometer with photo-electric cell.

resistance of 10^{10} ohm. *C* represents the cathode as before. For further details about the construction and use of the instrument the reader is referred to a paper by A. F. Lindemann and T. C. Keeley, *Phil. Mag.*, **47**, 1924.

The following experiments are instructive :—

1. With one of the devices described, preferably a galvano-

meter arrangement for the beginner, find the relation between photo-electric current and voltage on the photo-electric cell. Note that the current tends to a maximum but do not use too high a voltage on the cell. A working rule is to stop when a change of 10 volts doubles the initial value of the gradient dI/dV. Keeping the applied voltage constant, vary the distance of an electric lamp from the window of the photo-electric cell and plot a graph between photo-electric current and the reciprocal of the square of the distance. Hence verify that the inverse square law is applicable.

2. Allow the radiation from a powerful source to go through two nicol prisms and measure the photo-electric current for various angles between the principal planes of the nicols. Hence test the relation that $I = I_0 \cos^2 \alpha$ where I_0 is the incident intensity and α is the angle between the principal planes of the nicols. This illustrates the principle of photo-electric photometry.

3. To find the variation of the density of a photographic plate with the time of exposure.

Use one of the methods described above. Expose about six photographic plates to a constant source of light for times of 1, 2 . . . 6 secs. Measure the incident intensity and then insert in the beam each photographic plate filter in turn, and measure the reduced intensity from determinations of the photo-electric currents. Plot a graph between transmitted intensity and time of exposure.

If the density of the plate D is defined by the relation $I = I_0 e^{-D}$ where I_0 is the incident intensity and I is the transmitted intensity, then $D = \log_e I_0/I$.

According to the law of Schwarzchild, D is proportional to Et^p where E is the incident energy, assumed constant in the present experiment, and p is a little less than unity. Thus determine D for each plate and plot it against t the time of exposure. Hence test the Schwarzchild relation.

Note.—Students interested in the theory of the Lindemann electrometer and the meaning of the electrostatic control should consult a very instructive paper by McHenry : *Journ. Sci. Inst.*, **10**, pp. 305–310, 1933.

REFERENCES

THOMSON : *Phil. Mag.*, VII series, **8**, p. 990, 1929.
LINDEMANN and KEELEY : *Phil. Mag.*, **47**, p. 577, 1924.

48. To investigate the period of the transverse oscillations of flexible rods

A uniform bar of length l and of linear density m is rigidly clamped at one end and has a mass M suspended at the other. If I is the moment of inertia of the section about the trace of the neutral section and q is Young's modulus of the material the period of oscillation T is given by

$$T = 2\pi\sqrt{\frac{(M + 0{\cdot}24ml)l^3}{3qI}}$$

or

$$T^2 = \frac{4\pi^2 Ml^3}{3qI} + \frac{0{\cdot}32\pi^2 ml^4}{qI}.$$

To verify this relation experimentally two experiments may be performed :—

1. Take $M = 0$, vary l and plot T^2 against l^4.
2. Take l constant, vary M and plot T^2 against M.

The slopes of the resulting straight lines enable q to be calculated. Accurately to measure the period of oscillation T,

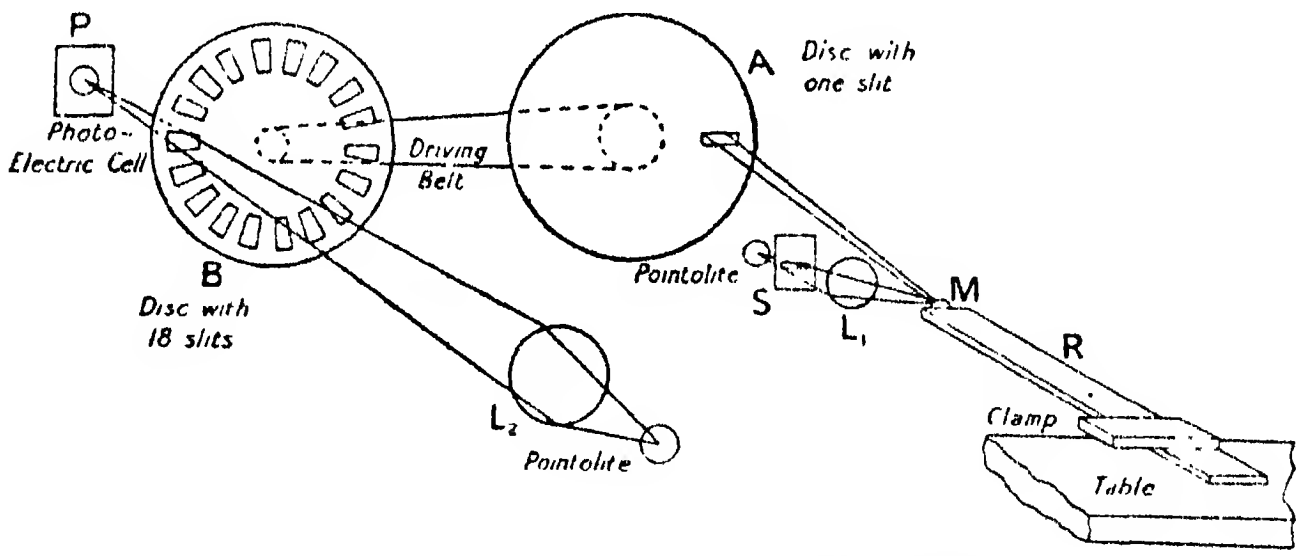


Fig. 158. Arrangement of apparatus.

the method illustrated in Fig. 158 may be employed. A slit, S, is illuminated by a pointolite-lamp and by means of a lens, L_1, light is focussed upon a small concave mirror, M, attached to the end of a vibrating rod, R, so as to give a sharp image on a distant ground-glass screen (not shown in Fig. 158). In front of the screen is a rotating cardboard disc, A, provided with a single slit, so that the image seen on the screen first appears to oscillate about a mean position, but when the period of the disc equals the period of the rod or some simple fraction of it, the image moves steadily into the central position. The rotating

disc is geared to the axle of an electric motor provided with another disc, B, having eighteen slits.

Light from a pointolite-lamp is focussed through these slits by a lens, L_2, so as to illuminate a photo-electric cell, P, intermittently. The cell, P, is joined to a 2- or 3-stage valve-amplifier

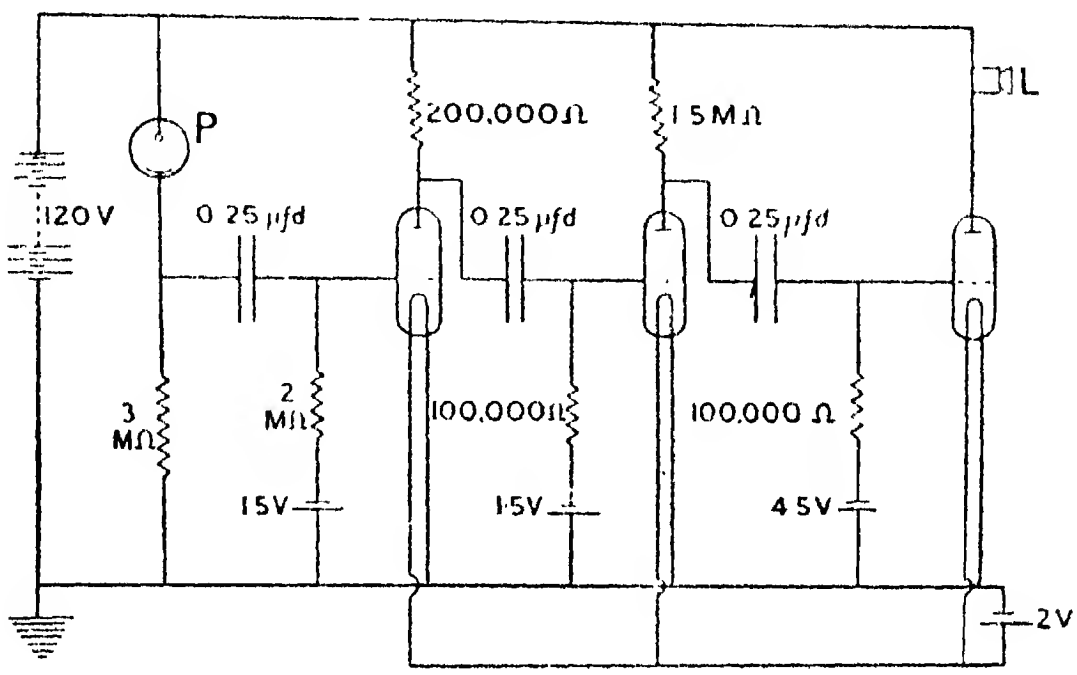


Fig. 159. Three-stage valve amplifier.

so as to produce in telephones or a loudspeaker, L, an audible note, the frequency of which may be determined by obtaining beats with a sonometer wire (see Fig. 159).

The gearing-ratio may be found by replacing the disc with a single slit by one with thirty-six slits, illuminated so as to give an audible note. Thus if n is the frequency of the motor and x is the gearing-ratio, the first disc gives a note of $18n$ and the second a note of $\frac{36n}{x}$. From the ratio of the corresponding lengths of the sonometer wire the ratio of these frequencies and hence x may be found.

Typical result for steel

Length of rod = 88·2 cm.
Breadth of rod = 1·915 cm.
Thickness of rod = 0·325 cm.
Linear density = 5·34 gr. per cm.
Frequency of motor = n.
Gearing-ratio = x.
Number of slits in disc = 18.
Number of slits in disc (for gearing-ratio) = 36.
Length of sonometer wire for frequency $18n$ = 18·55 cm.

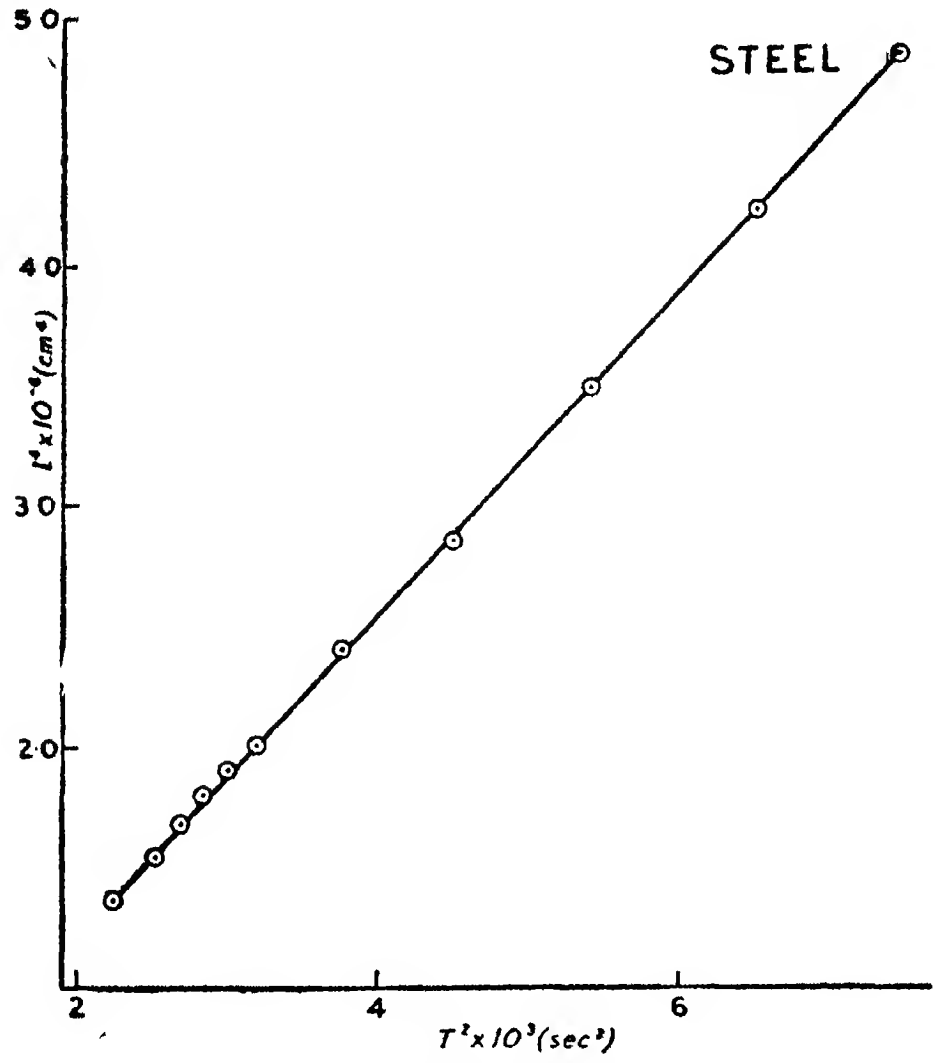


FIG. 160. Plot of l^4 and T^2.

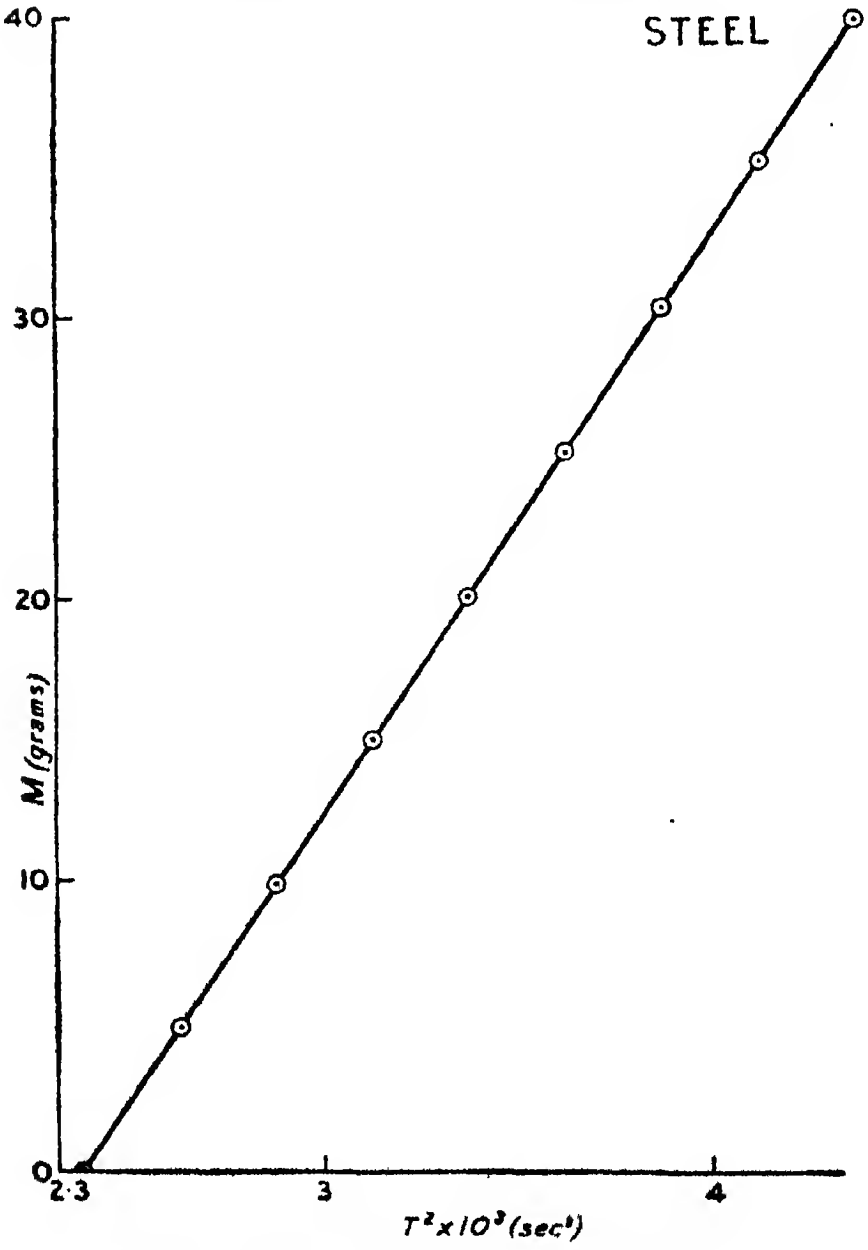


FIG. 161. Plot of M and T^2.

Length of sonometer wire for frequency $\frac{30n}{x} = 45{\cdot}2$ cm.

$$\therefore \qquad \text{Gearing-ratio } x = \frac{45{\cdot}2}{18{\cdot}55} = 2{\cdot}44.$$

Fig. 160 shows a plot of l^4 and T^2, and the resulting straight line gave a value for Young's modulus of $2{\cdot}09 \times 10^{12}$ dynes/sq. cm.

Fig. 161 shows a plot of M and T^2, and the corresponding value of q was $2{\cdot}07(5) \times 10^{12}$ dynes/sq. cm.

A glass rod may be used in a similar manner.

49. The X-ray tube

There are two types of X-ray tube usually to be found in physical laboratories. The older type, a gas-tube depicted in Fig. 162, possesses a cathode C, an anti-cathode A, and an anode D,

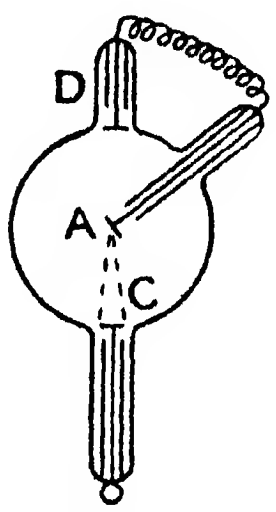


Fig. 162. X-ray tube (older pattern).

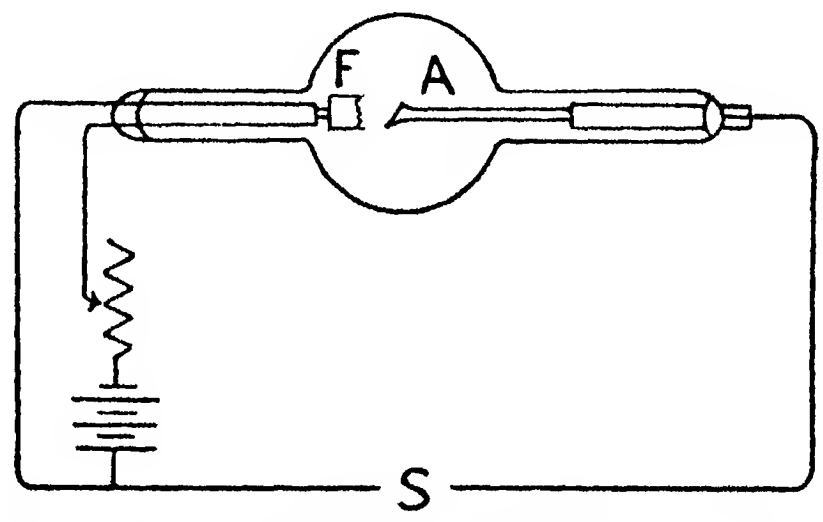


Fig. 163. X-ray tube (Coolidge pattern).

placed at the side of the anti-cathode. The more modern type of tube is of the Coolidge pattern shown in Fig. 163. The cathode here consists of a heated filament F and A is the anti-cathode. The anode is dispensed with.

The high voltage required to run a tube may be provided by the secondary of a transformer or induction coil. With the latter the primary current is rapidly interrupted by means of a mechanical break which is not very efficient, or by a mercury or electrolytic interrupter.

In the mercury interrupter, mercury rises in a vessel which is rapidly rotated and a stream of mercury is flung past an electrode once in each rotation. The mercury stream thus makes contact for a very short interval of time. The Wehnelt electrolytic interrupter consists of a lead plate acting as the cathode and a

platinum point in a porcelain holder serving as the anode. The containing vessel is filled with a solution of sulphuric acid of specific gravity 1·06, that is, about a 10% solution. Bubbles of gas leave the platinum point and make the interruption rather noisy. A rheostat in the primary circuit serves to control the current and the frequency of the interruptions.

As the gas-filled tube may be found in many laboratories, the following circuit may be used for demonstration purposes or on occasions when the tube has not to be run for a long time.

In Fig. 164 an X-ray tube is shown in series with the secondary of a large induction coil. *M* is a milli-ammeter placed in the middle of the secondary winding if the coil has terminals which admit of this. *A* is a simple rectifier formed of a metallic point and

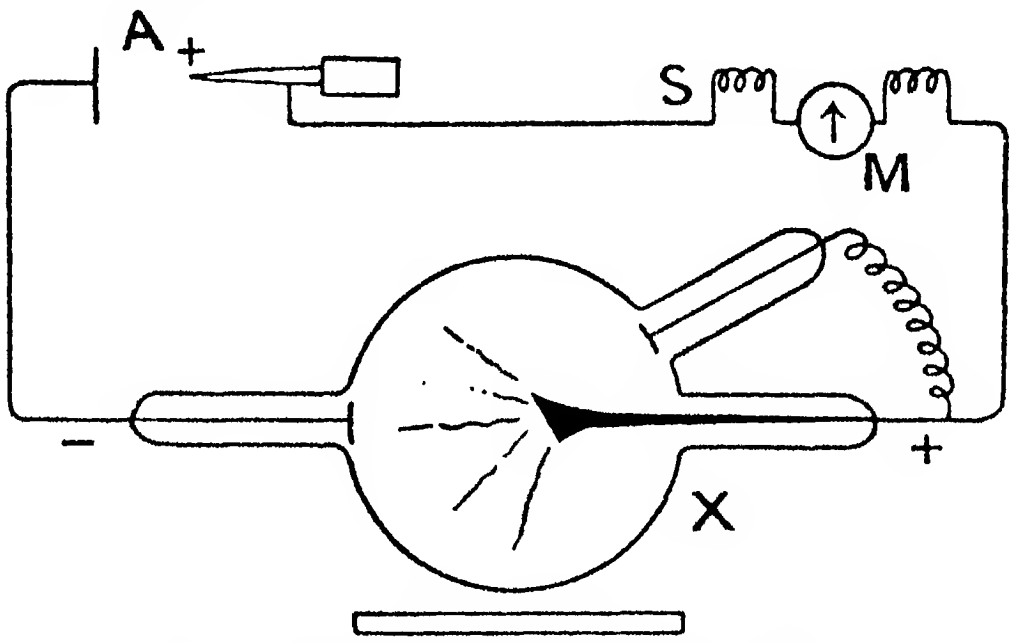


FIG. 164. X-ray tube in operation.

plane. It has been found that for electricity to pass between the point and the plane, the potential of the point must exceed that of the plane by a certain minimum value, which for short distances increases with distance. When the minimum potential is exceeded a current passes to the plane, but rapidly diminishes as the distance of the point from the plane increases.

The end of the spark produced when a current flows across the gap should be near the middle of the plate. If it is near the edge, reverse the connections of the primary coil and the point will then be positive. If the tube is in order and is properly connected up, the whole of the bulb between the plane of the anti-cathode and the cathode should appear to be filled evenly with a green fluorescence, while if the connections are wrong the part of the bulb behind the tube is green in patches and rings. If this is so the connections should be reversed immediately so that the tube may not be damaged. On first sending current through the tube the spark gap on the coil should be reduced to

5 cm., and the nature of the discharge through the tube, if any, be noted. If there is no discharge increase the spark gap on the coil to 6 cm., increase the primary current if necessary and repeat the observation. If no discharge passes continue to increase the spark gap up to 20 cm., if necessary, noting the current supplied to the primary and the current in the secondary in each case. A tube which transmits the discharge when the equivalent spark gap is 5 cm. may be considered as very soft, when 8 cm. as soft, when 10–15 cm. as medium and when 20 cm. as hard. A tube tends to get harder with use as the residual gas is forced into the glass of the tube. A hard tube may be softened by admitting a little more gas, generally by heating a bundle of mica plates in a side tube *R*, Fig. 165, through which a discharge may be passed for a few seconds by bringing the movable wires *A* and *B* near the terminals of the tube. As a tube gets harder the current sent through it by a constant E.M.F. decreases and this decrease may be observed on the milli-ammeter. As the tube is softened the current increases.

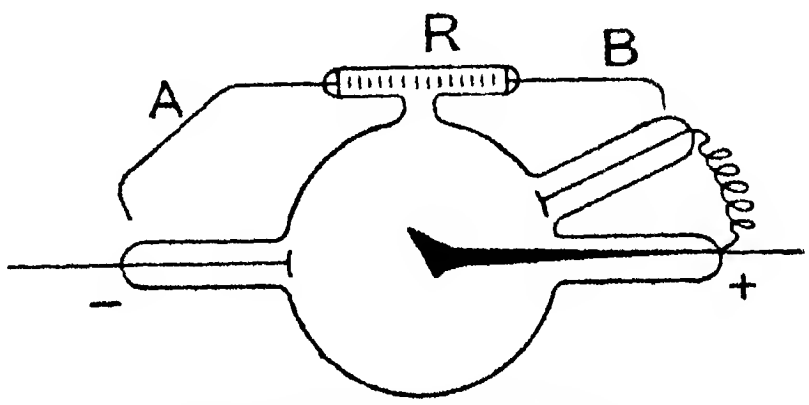


FIG. 165. Softening device.

The following precautions are worthy of notice :—

1. Dust and dry the tube carefully before use and do not have metallic parts too near one another, otherwise sparking may occur.

2. If the terminals of the coil are wrongly joined, inverse current may cause a puncture of the gas-regulator or the tube itself in the region of the anti-cathode.

3. A tube sometimes appears hard at first but settles down when warmed up. Such a tube shows a metallic deposit on the glass facing the anti-cathode. There is a brown stain due to inverse current.

4. If the anti-cathode is a thin sheet of platinum there may be a metallic deposit on the walls of the tube due to the piercing of the anti-cathode by inverse current.

5. "Flickering" is generally due to sudden variations in current supply or a bad condition of the interrupter.

6. See that the interrupter is clean and is working smoothly and has clean contacts.

To use air-regulator

Adjust the regulator wires to the metal caps, alter the value of the primary resistance to give 1 milliampere, switch on the current for a few seconds and then test the tube. If it is still too hard repeat as required. Warming up the tube is sometimes sufficient. If the tube is over-softened make it harder by passing a small current for 10–20 minutes when the tube has cooled down. Cool by blowing an air current in front of the tube. Briefly, cleanliness, absence of points, warmth and good interruption are essential to smooth working.

The secondary of a transformer *S* may also be joined to the X-ray tube with a rectifier *R*, which consists of a heated filament and electrode, so that electrons can only pass one way as

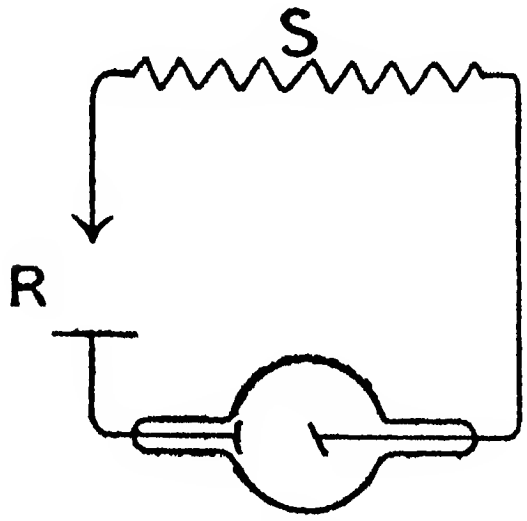


FIG. 166. X-ray tube with transformer and rectifier.

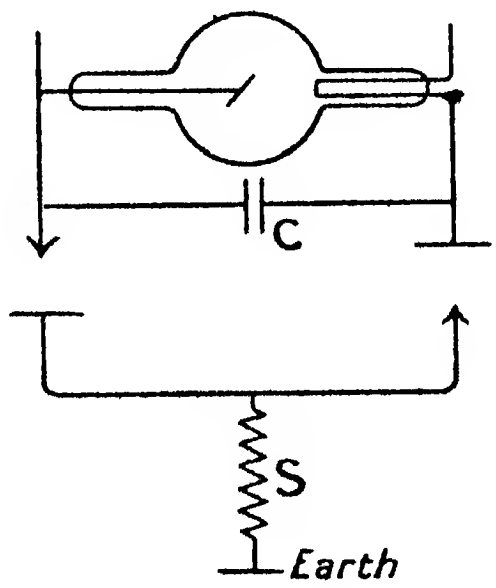


FIG. 167. Two rectifiers.

indicated in Fig. 166. A half period of the alternating current is therefore suppressed. If both half periods of the alternating current are to produce a direct current one end of the secondary *S* is earthed and the other is joined with two rectifiers as shown in Fig. 167 with a capacity *C* placed across the tube. The capacity must be capable of standing a high voltage. The current in the tube only alters slightly during a period. If the upper end of the transformer coil is positive, the left-hand plate of the condenser becomes positive and no current can flow through the right-hand side. If the upper end of the coil is negative, the right-hand plate of the condenser becomes negative and no current can flow on the left. The X-ray tube is therefore fed by slightly pulsating direct current. If necessary, a choke-coil may be inserted between the condenser and the tube to suppress the pulsation of direct current.

For purposes of demonstration and instruction, the simpler

the circuit the better. The more advanced circuits involve possible dangers of which the beginner would not be aware.

Protection against X-rays

The student may very easily underestimate the danger from X-rays. With the apparatus of Fig. 163, for example, an exposure of five minutes is exceedingly dangerous. An X-ray tube should be shielded by the following thicknesses of lead specified by the Third International Congress on X-ray and Radium Protection 1931.

Peak Kilovolts across tube	Thickness of lead mm.
75	1·0
100	1·5
125	2·0
150	2·5
175	3·0
200	4·0
250	6·0
300	9·0
350	12·0
400	15·0
500	22·0
600	34·0

For a gas-filled tube operated by an induction coil 2 mm. of lead should be used. In setting up X-ray apparatus, remember that walls, floor and ceiling do not adequately protect people in surrounding rooms.

For additional information on protection and the biological effects of X-rays, consult G. L. Clarke, *Applied X-Rays*, McGraw-Hill Company Inc., 1940.

50. The diffraction of X-rays

1. *The von Laue-Friedrich-Knipping experiment*

In 1912 von Laue suggested that a crystal might be used as a three-dimensional grating, and Friedrich and Knipping carried out the experiment, using a zinc-blende crystal with the result which is now well known. A Laue-photograph may be made with the simple apparatus illustrated in Fig. 168. T is an X-ray tube, S_1, S_2 two lead screens, each having a small hole 0·5 to 1 mm.

in diameter bored in it. The screens serve as a shield for scattered radiation from the tube and prevent such radiation reaching the plate *P*. They also serve to limit the X-ray beam to a narrow cylindrical pencil. *C* is the crystal.

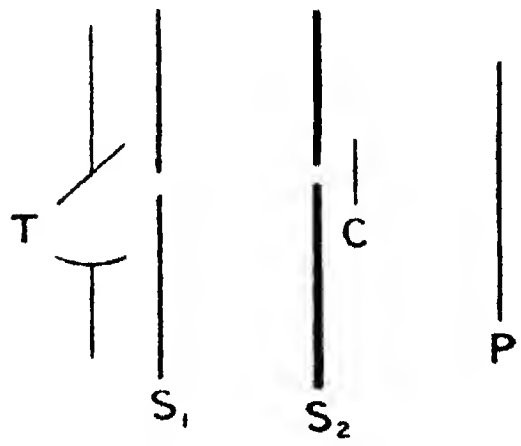


FIG. 168. Arrangement for von Laue-photograph.

The X-ray tube and the screens are adjusted to give the maximum X-ray intensity as shown by the fluorescence of a barium platinocyanide screen. The crystal may be mounted in the beam on a fine glass fibre, to which it is attached with shellac, or it may be fixed directly over the hole in the screen S_2.

The crystal should have well-developed faces. For example, rock salt is placed in the beam so that the X-rays strike one of the crystal faces perpendicularly. The photographic plate may now be placed in position about 3 to 5 cm. from the specimen. A small circular stop should be attached to the cover of the photographic plate to prevent the direct beam from striking it. The time of exposure for a good photograph should be 30 to 60 minutes.

The interpretation of these photographs is relatively difficult, and for further information the student is referred to such standard works as Bragg's *Crystalline State* or Wyckoff's *Structure of Crystals*.

In Plate III is shown a von Laue-photograph of NaCl along the cube edge. Copper was used as the X-ray anti-cathode.

2. *The determination of the unit-cell of a crystal, by the rotation photographic method*

For the rotation method a crystal is mounted as shown at *C*

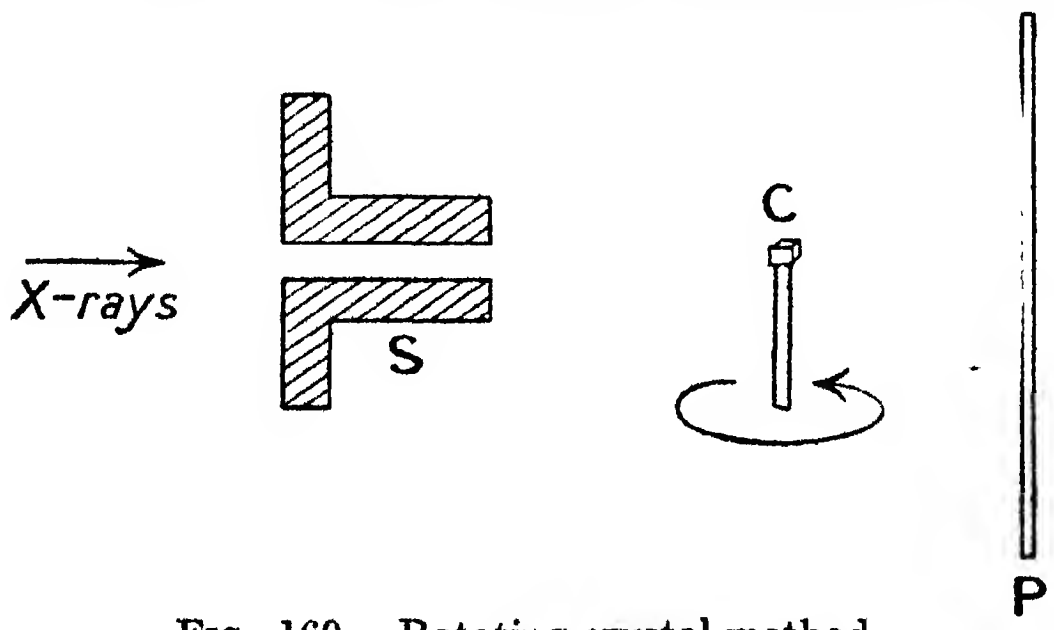


FIG. 169. Rotating crystal method.

in Fig. 169 and is made to revolve about a vertical axis. A horizontal pencil of monochromatic X-rays, limited by a fine cylindrical aperture, is allowed to fall on the crystal. P is a photographic plate which receives the diffracted rays. As the crystal revolves one set of planes after another is brought into such a position that it makes an angle θ with the incident beam, such that the Bragg law $n\lambda = 2d \sin \theta$ is obeyed. Here n is the order of interference, λ the wavelength and d is the spacing of the planes. When the photographic plate is developed it shows a series of spots due to the diffraction from various planes in the crystal.

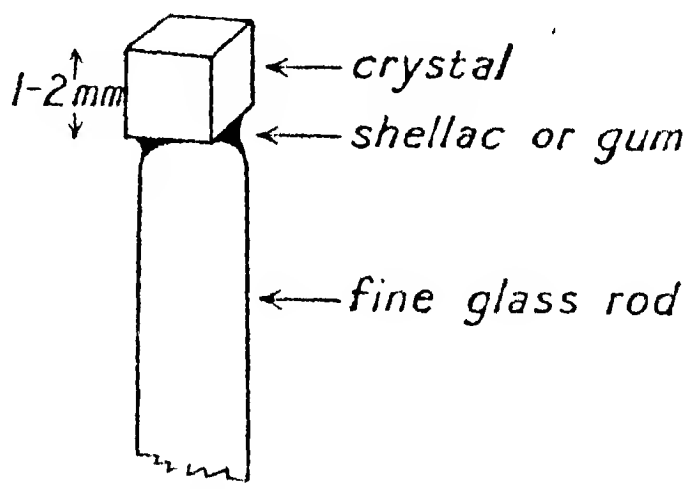


FIG. 170. Crystal mounting.

For this method of analysis a device for rotating the crystal at a uniform speed, and a goniometer head for easy adjustment of the crystal are necessary. Usually a special spectrometer is employed. Suppose that the instrument is in adjustment and consider the setting of the crystal. Select a small crystal with well-formed faces and mount it with a little gum or shellac on a fine glass rod as shown in Fig. 170, so that one of the crystal edges is as nearly as possible parallel to the axis of the rod. The latter is mounted on the goniometer head with plasticine and the height adjusted until the crystal can be observed by looking through the collimator. In general, the crystal will not be on the axis of rotation but will turn about this axis. The crystal should be rotated slowly and observed through a telescope with cross wire, placed on the opposite side of the crystal to the collimator and with its axis collinear with that of the collimator. The position of the crystal is carefully altered by moving the rod until on rotation the crystal does not move with reference to the cross wires.

The next operation is to orient the crystal so that it rotates about one of the crystalline axes. In a cubic crystal this will correspond to a cube edge.

A point source of light is placed at L (Fig. 171) about 1 m. from the crystal, which is observed through one slit of the spectrometer. If the crystal is correctly set the faces will reflect the light down the collimator as they come into the reflecting

position. If some of the faces do not reflect or reflect only weakly, the position of the crystal must be altered by means of the goniometer arcs until all the faces reflect with equal intensity. The X-ray tube is then arranged to give the strongest possible beam, and the photographic plate is placed in position and exposed for 20 to 30 minutes.

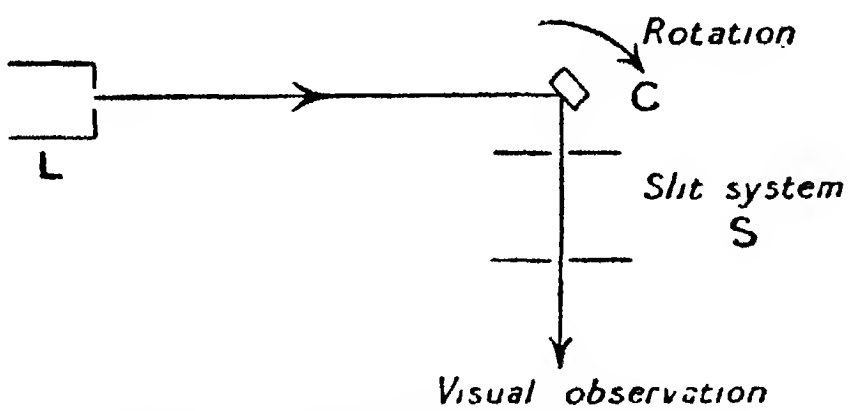


FIG. 171. Adjustment of crystal.

The spots on the developed plate will lie on a series of hyperbolas above and below the equatorial or zero layer line. From the separation of the hyperbolas the edge of the unit-cell along the direction of the axis of rotation may be calculated. In Fig. 172, QP is the incident beam, and PM is the diffracted beam giving a spot on the first layer line, vertically above the origin O.

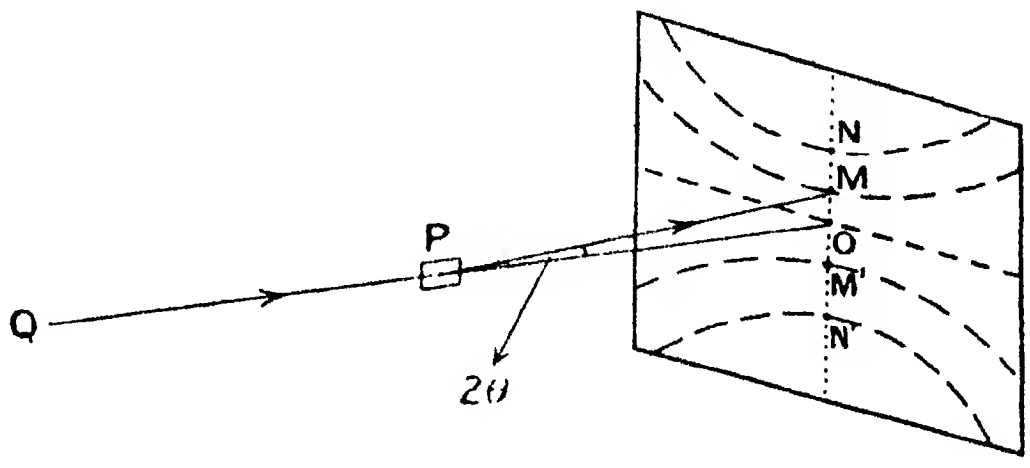


FIG. 172. X-ray pattern.

The angle MPO is 2θ where θ is the Bragg angle. This may be calculated in terms of the specimen-plate distance OP and the distance of the lowest point in the first layer line from the origin. In practice it is the distance MM' which is measured. All the variables in the Bragg relation $n\lambda = 2d \sin\theta$ are known except d, one edge of the unit-cell, which may therefore be calculated. If the crystal is not cubic the above measurements must be repeated for the other axes, if the dimensions of the unit-cell are to be completely determined.

51. The cathode-ray oscillograph

The cathode-ray tube has a wide application in oscillography, and is a valuable weapon for modern research. There are two principal types of tube, one the continuously-pumped high-voltage tube and the other the sealed low-voltage instrument, which is more likely to be found in the usual laboratory. The account given here will therefore refer to the latter type. The instrument consists of a heated cathode F shown in Fig. 173 and

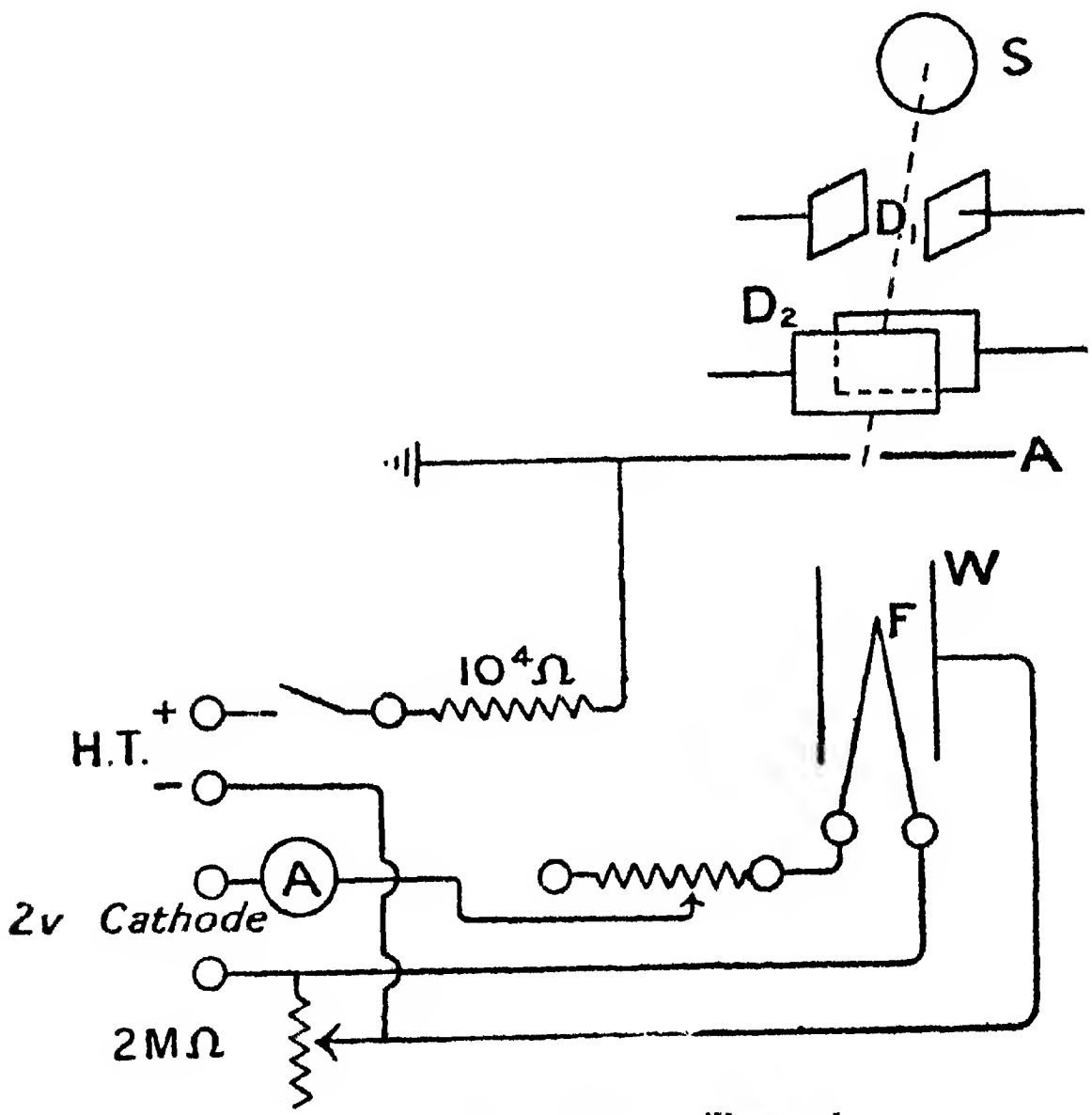


Fig. 173. Cathode-ray oscillograph.

serving as the source of electrons, an anode A to which an accelerating voltage of 300–3,000 volts may be applied, and a Wehnelt cylinder W, carrying a variable negative potential to focus the electron beam. There are also two pairs of deflecting plates D_1 and D_2, to which potentials may be applied, and a fluorescent screen S. The latter usually consists of calcium tungstate or zinc silicate burnt on to the end of the glass bulb. The theory of the deflection of an electron beam by means of

electric and magnetic fields is adequately treated in the standard text-books. One of the principal uses of the cathode-ray tube is as an oscillograph, for the instrument has a variable sensitivity over a very wide range, and a moving element which is free from inertia.

Comparison of frequency

A simple and useful application of the instrument is as a frequency comparator for a variable frequency oscillator which may be calibrated in terms of a known frequency such as the A.C.-mains giving 50 c.p.s.

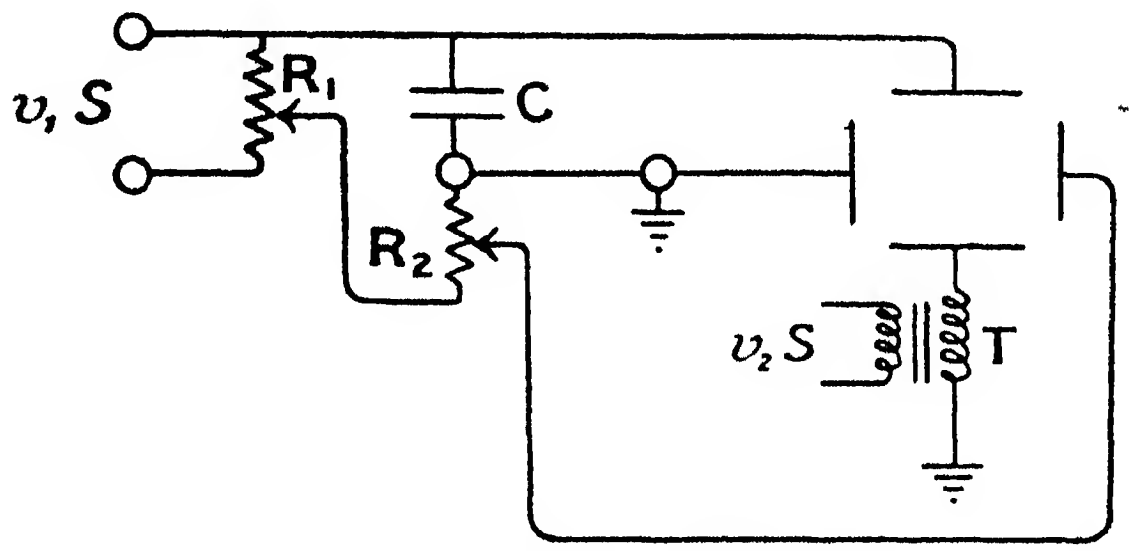


FIG. 174. Application of alternating potential differences.

The voltage of known frequency ν_1 is applied across a potential-divider consisting of a condenser C and a resistance R_2, the potentials across these two components being applied respectively to the two perpendicular pairs of deflecting plates of the cathode-tube as shown in Fig. 174.

Owing to the phase difference of $\pi/2$ introduced by the condenser, the resulting figure on the screen is an ellipse, the axes of which may be varied by altering the capacitance or resistance. If one frequency ν_1 is that of the 50 c.p.s. A.C.-mains, the time to trace the ellipse is 1/50 sec. This serves as the unit of comparison or time-base. Suppose an additional deflecting voltage of frequency ν_2, which is some integral multiple of the standard, is now applied through a transformer T to the pair of plates giving vertical deflection. The ellipse will now have superposed upon it a number of waves or ripples, and the number of peaks n

which may be counted gives the ratio of the unknown frequency to the standard frequency. The reason for this is fairly obvious. Thus Plate IV illustrates the measurement of the frequency of a valve oscillator. The standard frequency was 50 c.p.s., and since there are nine peaks the frequency of the oscillator was 450 c.p.s. The wave form may also be observed, and Fig. 175 illustrates that the oscillation is by no means simple-harmonic.

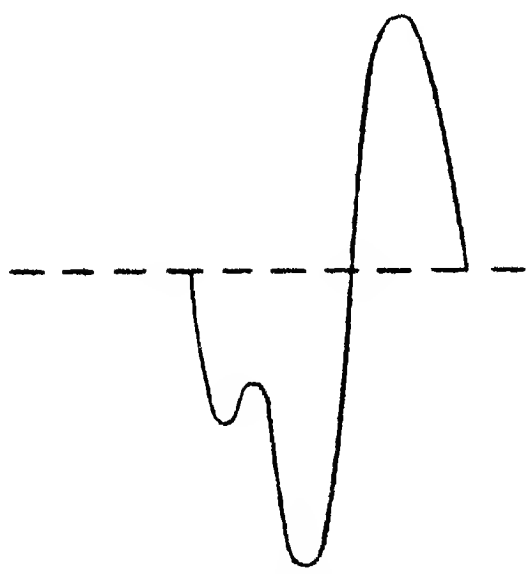


FIG. 175. Wave form.

Note.—Any instructions supplied with a tube should be carefully read and care taken that the current through the filament does not exceed the stated value. The focused or undeflected spot should not be allowed to remain stationary on the screen, otherwise the fluorescent covering will be irreparably damaged.

Measurement of phase difference

If two alternating voltages of the same frequency are applied to the X and Y plates and are exactly in phase the trace will be a straight line inclined to the axes. If the phase differs from zero the trace becomes an ellipse and the phase angle between the voltages may be found by measurements on the ellipse.

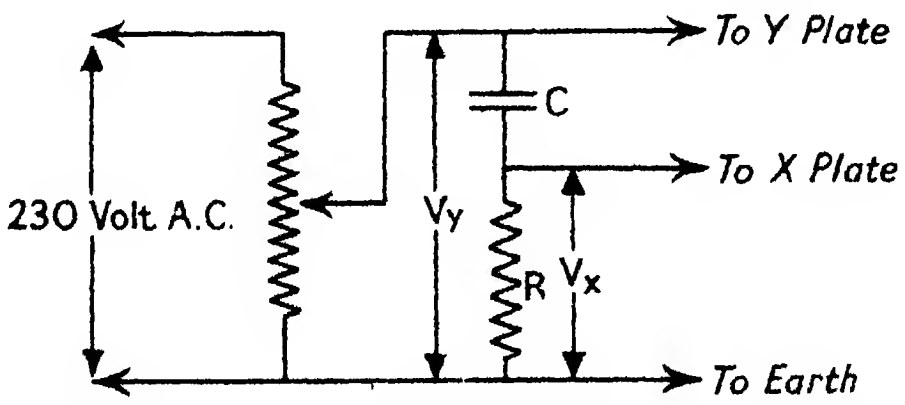


FIG. 176. Circuit for producing phase ellipse.

To illustrate the method arrange a condenser and resistance in series and apply an alternating voltage across them by a potential divider as in Fig. 176. The voltages V_y and V_x will differ in phase by an angle between 0° and 90° depending on the magnitudes of R and C. Apply V_x and V_y to the X and Y plates of the oscillograph. An ellipse like that of Fig. 177

should appear. The phase angle between V_x and V_y is given by

$$\sin\delta = \frac{OD}{OC} = \frac{OE}{OF}.$$

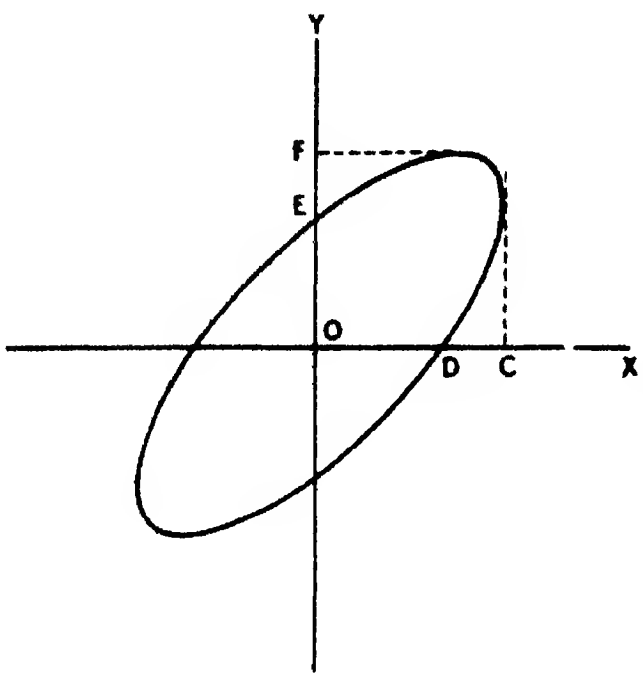


FIG. 177. Measurement of phase difference.

To see this let $V_y = A \sin \omega t$ and $V_x = B \sin(\omega t + \delta)$. The maximum value of V_x is B and this is proportional to the length OC. When $t = 0$, $V_y = 0$, $V_x = B \sin\delta$, and is proportional to the length OD. Hence $\sin\delta = OD/OC$ and similarly $\sin\delta = OE/OF$.

Measurements on the ellipse may be made on a transparent graticule covering the cathode ray tube, or the ellipse may be traced on paper. The axes of the figure are obtained by disconnecting the wire to the X and Y plates in turn.

Take the mean value of OD/OC and OE/OF and calculate δ. From the theory of alternating currents $\tan\delta = 1/2\pi fCR$, so C may be found if R is known.

If voltages derived from the mains are to be applied to the plates of the oscillograph a 1 μF condenser should be placed in each lead between the circuit and the mains to prevent accidental short circuits to earth.

52. TO DETERMINE THE SUSCEPTIBILITY OF MANGANOUS SULPHATE SOLUTION BY QUINCKE'S METHOD

This experiment requires an electromagnet capable of producing a field of the order of 10,000 oersted. The field between the pole pieces is first found in terms of the magnetising current using a search coil S, standard mutual inductance M and ballistic galvanometer arranged as in Fig. 178.

Graphs are drawn of :—

(1) Ballistic throw θ_1 on removal of the search coil from the magnet against magnetising current I_m.

(2) Ballistic throw θ_2 against current reversed in the primary of the mutual inductance I_r.

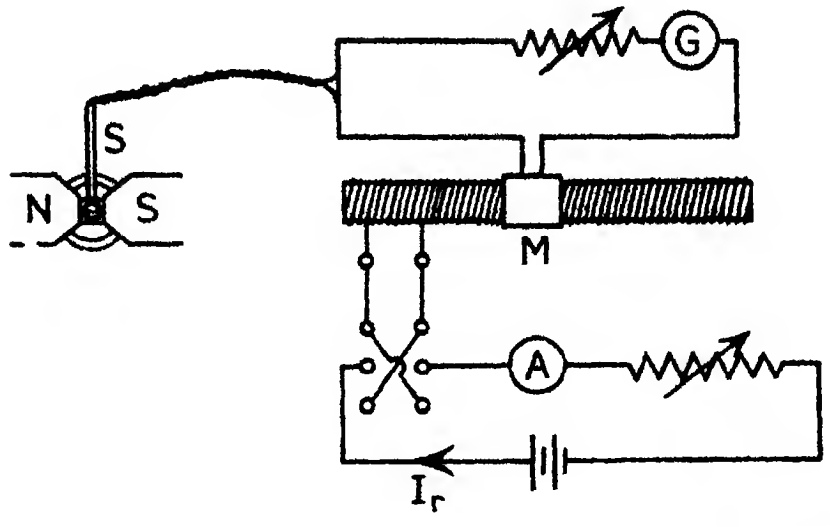


FIG 178. Circuit for measuring field of magnet.

If A is the mean area of the search coil and n the number of turns then for the first experiment

$$\frac{HAn}{R} = k\theta_1,$$

where R is the total resistance of the secondary circuit and k is an instrumental constant of the ballistic galvanometer.

For the second experiment

$$\frac{2MI_r}{10^8R} = k\theta_2.$$

So that $$H = \frac{2M}{An} \cdot \frac{I_r}{\theta_2} \cdot \theta_1 \times 10^8 \text{ oersteds.}$$

In this equation M is in henries and I_r in amperes. The value of $\frac{I_r}{\theta_2}$ is found from the slope of the second graph. The first graph then enables the value of H to be found for any magnetising current.

Note that if the area of the uniform field of the magnet is small a very small search coil will be required.

To determine the susceptibility a U-tube is first constructed, one of whose limbs is narrow and is to fit between the pole pieces of the magnet. This limb may be perhaps 0·5 cm. in diameter, depending on the extent of the region of uniform field. The other limb of the U-tube should be three or four times the diameter of the narrow tube.

The U-tube is cleaned with warm chromic acid and filled with a solution of manganous sulphate in water containing about 30 gm. of hydrated salt per 100 c.c. of solution.

The density of this solution at the temperature of the experi-

ment is required and is determined with a density bottle. The narrow limb of the U-tube is inserted between the pole pieces and is arranged so that when the magnet is energised the meniscus is in the central region of uniform field. The meniscus is viewed with a travelling microscope with a vertical traverse and the fall in the height of the meniscus on switching off the current is measured. This is done over a range of magnetising currents. The maximum elevation of the meniscus will be of the order of 1 cm.

Theory. The elevation h of the meniscus is related to the magnetic field H by the equation

$$g(\rho - \rho')h\left(1 + \frac{a}{A}\right) = \tfrac{1}{2}(K - K_0)H^2$$

where ρ is the density of the solution, ρ' the density of the air and $\frac{a}{A}$ is the ratio of the areas of the narrow and wide limbs of the U-tube. K and K_0 are the volume susceptibilities of the solution and the air. The field at the liquid surface in the wide limb may be neglected. Hence a plot of h against H^2 should give a straight line from which the value of K may be determined. The volume susceptibility of air may be neglected. Greasiness or dryness in the narrow limb will cause irregular results.

From the value of K the magnetic moment of the manganous iron may be determined in terms of the Bohr magneton. The mass susceptibility of the anhydrous salt is first required and may be found as follows :—

Calculate the mass susceptibility of the solution

$$\chi_s = \frac{K}{\rho}.$$

Let χ_c be the mass susceptibility of the anhydrous crystals of $MnSO_4$. χ_w that of water ($= -0{\cdot}72 \times 10^{-6}$ e.m.u./gm.). If there are x grams of *anhydrous* salt per 100 c.c. of solution then

$$100\chi_s = x\chi_c + (100 - x)\chi_w.$$

The value of x may be found from tables—e.g. Landolt and Börnstein, " Physikalisch Chemische Tabellen ", which tabulate this quantity against the density of the solution.

Having found χ_c calculate the molecular susceptibility

$$\chi_m = M\chi_c$$

where M is the molecular weight of manganous sulphate.

The molecular susceptibility is the sum of the ionic molecular susceptibilities of the Mn and SO_4'' ions.

$$\chi_m = \chi_{Mn} + \chi_{SO_4''}$$

where $$\chi_{SO_4''} = -33{\cdot}6 \times 10^{-6} \text{ e.m.u.}$$

Hence χ_{Mn} the ionic molecular susceptibility of the Mn ion may be found.

The magnetic moment of the manganese ion in Bohr magnetons is given by

$$p_B = \frac{1}{\beta}\sqrt{\frac{3kT\chi_{Mn}}{N_0}}$$

where β is the Bohr Magneton $= 9{\cdot}27 \times 10^{-21}$ e.m.u., k is Boltzmann's constant, N_0 Avogadro's number and T the absolute temperature.

According to the quantum theory the magnetic moment of the manganese ion, which is in the spectroscopic state ${}^6S_{5/2}$ arises from the spins of 5 " unbalanced " electrons and is given with good accuracy by the formula

$$p_B = \sqrt{4s(s+1)}$$

with $s = \frac{5}{2}$: that is $p_B = 5{\cdot}92$. Compare the experimental value with this figure.

REFERENCE

E. C. Stoner: *Magnetism.* Methuen.

53. Cathode-sputtering

Cathode-sputtering is a method of depositing thin films of metal upon solid surfaces, and may be used for making small mirrors fully-, semi-silvered, or even more lightly silvered, for application in optical work. The apparatus is shown in Fig. 179. A large bell-jar is fitted with a brass stopper A, through which is passed an aluminium rod B. Attached to the lower end of the rod by means of a screw-thread, for adjusting the distance between the electrodes, is the cathode. This consists of the disc C, of the metal to be sputtered, let into the surface of a larger aluminium disc, which is employed as a guard ring.

The small glass disc E to be sputtered is placed on the anode F, which consists of a cylindrical brass box soldered on to a brass

plate, which has an anode connection at *J*. Owing to the discolouring effect of heat on the film during sputtering, the anode is cooled by the passage of water through the brass tubes *G*, *H*, which are soldered through holes drilled in the base plate. The bell-jar is sealed to the base plate by smearing a low vapour-pressure vacuum-wax around the base, all other joints being

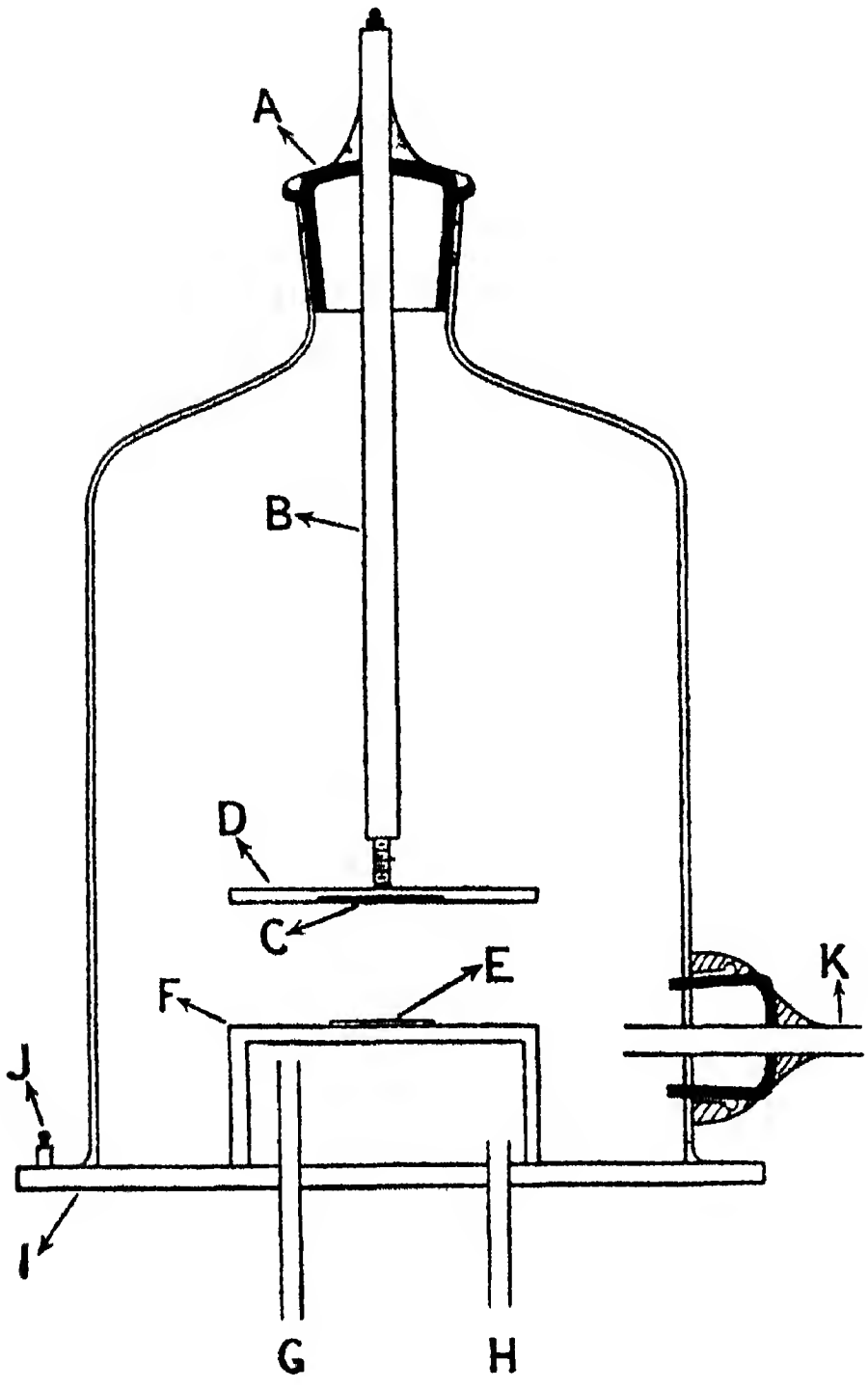


FIG. 179. Sputtering apparatus.

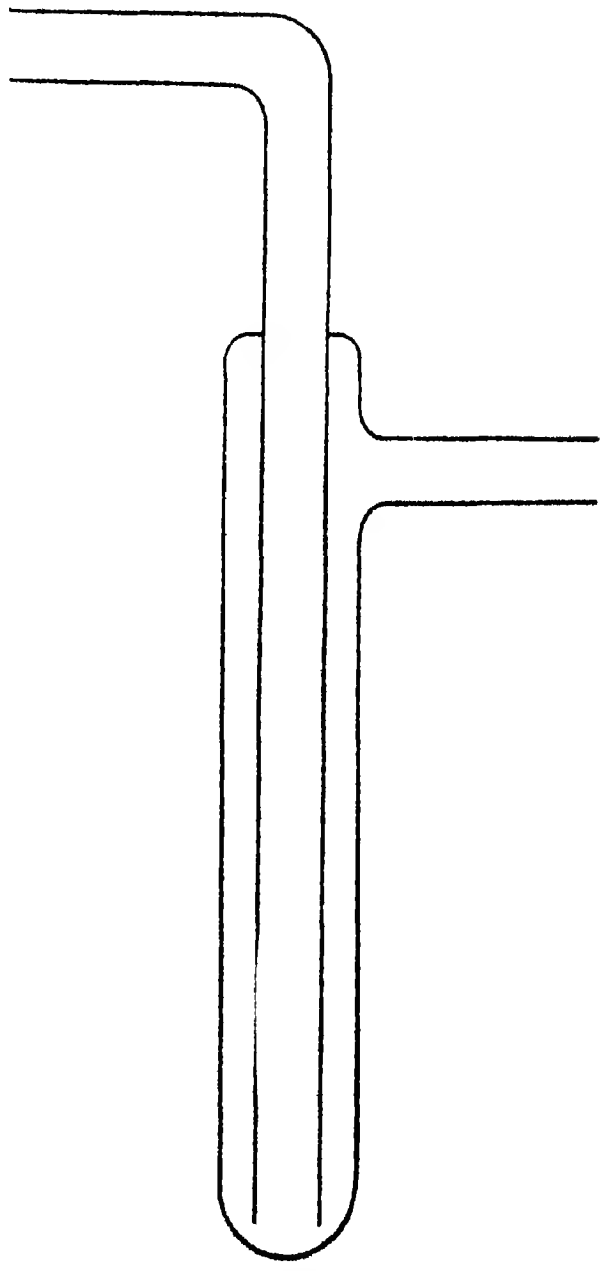


FIG. 180. Liquid-air trap.

made vacuum-tight with sealing-wax. To evacuate the apparatus a Hyvac pump is connected to the side-tube *K* and the apparatus evacuated to a pressure of about 0·1 mm. of mercury, which can be measured by a McLeod gauge. It has been found that traces of mercury vapour from the McLeod gauge enter the bell-jar and are liable to produce spots on the surface of the film. This trouble can be eliminated by inserting a liquid-air trap as shown in Fig. 180 between the gauge and the sputtering apparatus.

To obtain uniform deposition it is essential that the glass

surface to be sputtered should be perfectly clean. This can be done by placing the glass disc in warm or hot chromic acid solution. An alternative method is to put the dry glass disc into a beaker and rinse with alcohol, pouring off the excess. About 5 c.c. of concentrated nitric acid is then poured in and the beaker placed in a fume cupboard. Rinse the disc with tap-water and finally with well-distilled water.

If films of uniform thickness and colour are to be obtained, it is important that the glass disc should not be too near the edge of the dark space. A distance of 5 cm. between the electrodes and a dark space of 1·5 cm. in length has been found to give good results, using a potential difference of approximately

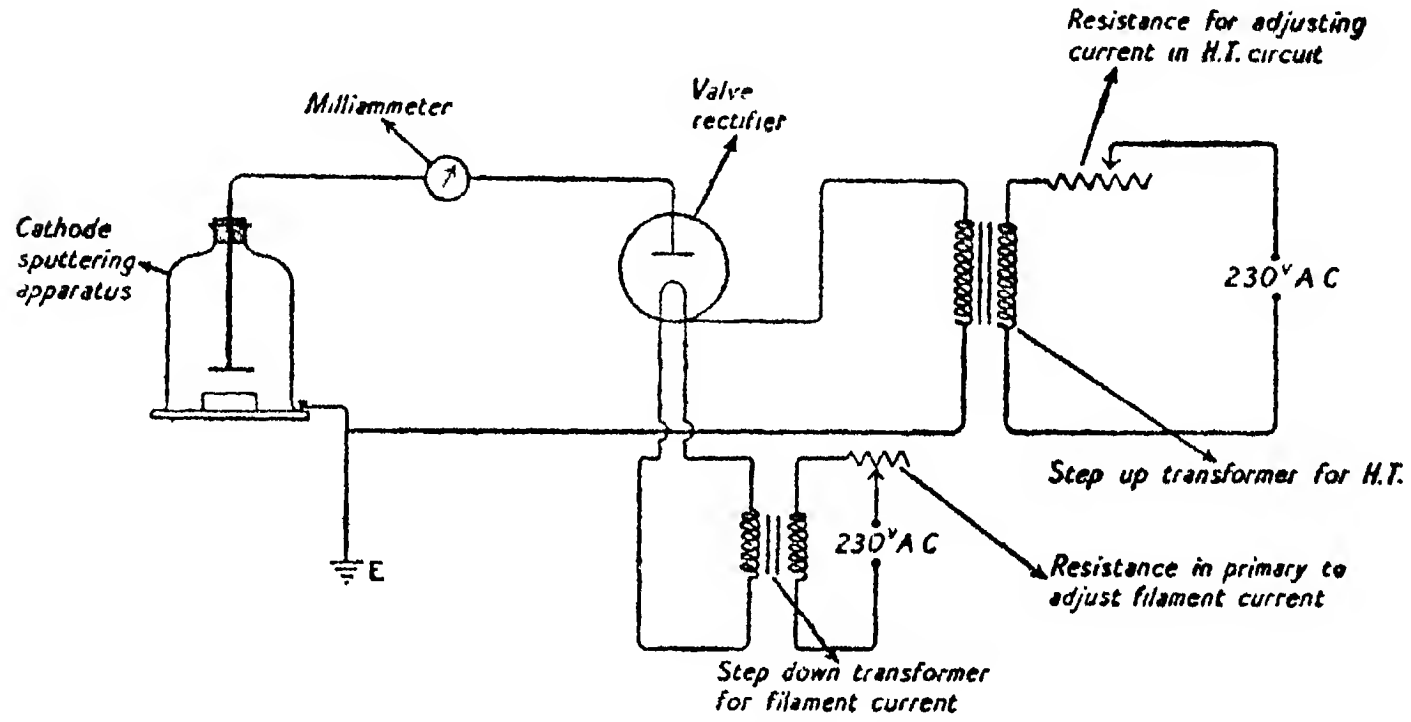


FIG. 181. Electrical circuit.

700–1,000 volts and a current of 20 m.a. from a small induction coil or A.C. transformer with rectifier. The sputtering can be stopped at any time, depending on the thickness of the film required. Experience will enable a suitable film to be obtained for the special purpose in hand.

The electrical circuit is shown in Fig. 181, where the current from the high tension transformer is rectified by a valve rectifier operated by a step-down transformer. Further explanation is unnecessary.

54. CHEMICAL SILVERING BY THE ROCHELLE SALT PROCESS

Two solutions are required, A and B. To prepare solution A dissolve 5 gm. of pure silver nitrate in 40 c.c. of distilled water. Add concentrated (specified gravity 0·880) ammonia till the precipitate which is formed is nearly redissolved, leaving only

a slight cloudiness. Filter and make up to 500 c.c. with distilled water. Solution B is prepared by dissolving 1 gm. of silver nitrate and 0·83 gm. of Rochelle salt (sodium potassium tartrate) in 500 c.c. of distilled water boiling for 20 minutes and filtering while hot. The solutions should be stored in the dark till ready for use.

To silver a glass surface successfully the surface must be chemically clean. Wash the glass in hot soapy water or in a soapless detergent to remove grease. Afterwards clean with hot chromic acid solution, wash in distilled water and leave the glass under distilled water until ready to begin silvering. The cleaned glass surface must not be allowed to dry in the air before silvering.

Use equal quantities of solutions A and B in a clean beaker and arrange the surface to be silvered to face downward. The rate of silvering is best determined by trial beforehand. It may be increased by warming the solution.

55. Evaporation

This process is chiefly used in the laboratory to deposit a reflecting coat of aluminium on glass.

The principle of the method is to heat a metal in a high

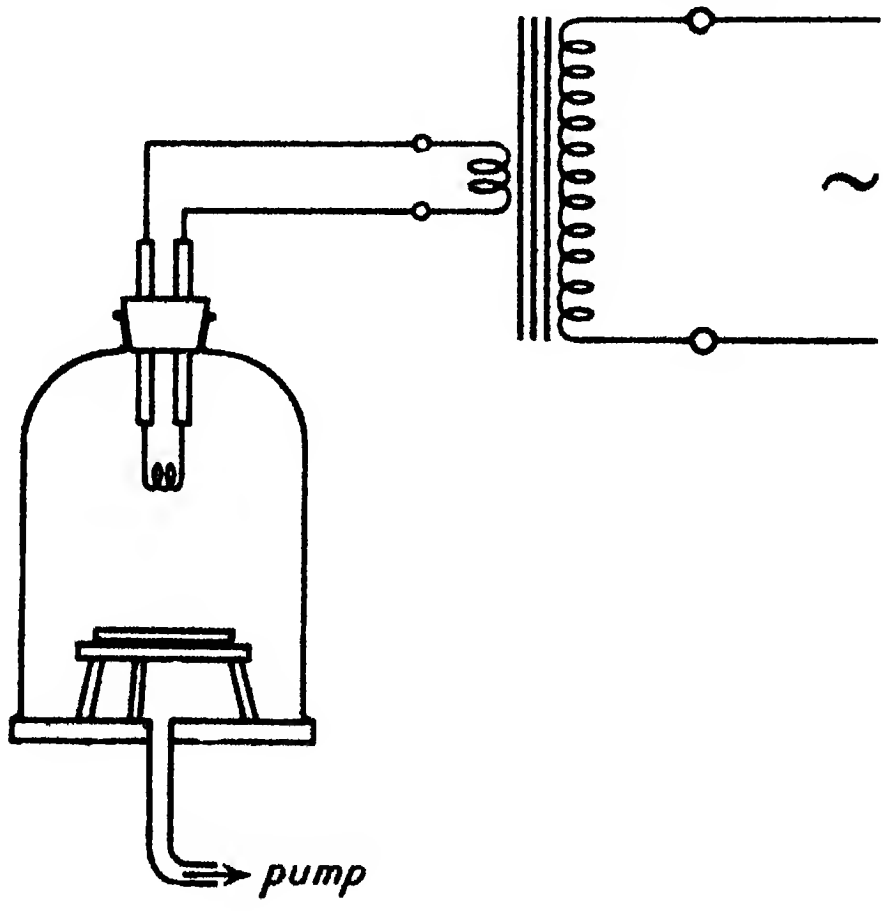


Fig. 182. Evaporation apparatus.

vacuum until its vapour pressure is of the order of 2 mm. of mercury. Molecular rays of the metal are then emitted in all

directions and adhere to form a compact coat on any surface they strike. A suitable apparatus is shown in the figure.

A bell-jar is waxed on to a metal base plate to form an airtight seal. Through a rubber bung in the top of the apparatus pass two stout copper or brass rods which support at their lower end a spiral of stout tungsten wire of about 22 S.W.G. Small pieces of pure aluminium foil are hung on the tungsten spiral. The system is evacuated to a pressure of 10^{-3} mm. of mercury or less with the aid of a rotary pump and charcoal tube and a heavy current passed through the tungsten wire so that the aluminium fuses and clings to the wire by surface tension. As soon as this happens the current is switched off.

If a glass surface is to be coated it is first chemically cleaned, as described in the section on the silvering process. It is dried on a clean cotton or linen cloth which has been previously boiled for a long time to remove as far as possible the vegetable oils in the fibres and mounted in the evaporation apparatus. After pumping out as before, the tungsten is heated to white heat. The aluminium is deposited in a time varying between a few seconds and a minute or two. Experience is the best guide.

Traces of volatile organic compounds will cause a black deposit to form. If a high voltage source is available the cleaning action of gaseous ions may be used to remove residual traces of contamination from glass surfaces. This is done by passing a gaseous discharge, at a pressure of 10^{-2} to 10^{-3} mm. between the base plate and a top electrode.

Other metals besides aluminium may be successfully evaporated, though some require slightly different techniques from that described above. For fuller information consult the references below.

REFERENCES

J. Strong: *Procedures in Experimental Physics.* Prentice-Hall Inc., New York, 1945.

J. Hoag: *Electron and Nuclear Physics.* Chapman and Hall Ltd., London, 1944.

Additional problems

1. Determine Young's modulus and Poisson's ratio σ' for a highly-polished plate of chromium-steel.

2. Investigate by the method of 's Gravesande the variation with temperature of Young's modulus for a metal in the form of a wire.

3. Determine Poisson's ratio for glass in the form of a long thin-walled tube.

4. Determine for water, the relation between the critical velocity of flow and the radius of the tube.

5. Investigate the performance of an air-pump, exhausting air from a flask of large volume through a capillary tube. How is the performance related to the diameter and length of the intervening tubes ?

6. Determine a value of Joule's equivalent, assuming a value of unity for the specific heat of water for the range of temperature 19·5–20·5° C.

7. Investigate the relation between the position of the principal points of a system consisting of two separated convex spectacle-lenses, and the separation of the lenses.

8. Use a Jamin interferometer to determine the thickness of a mica-sheet. A mean value for the refractive index may be assumed.

9. Use a Fabry and Perot étalon, or interferometer, to compare the wavelengths of two radiations.

10. Use the Michelson-interferometer to find the thickness of a mica-sheet.

11. Apply Brewster's law to determine the refractive-index of a specimen of ebonite.

12. Use some form of interferometer to find the refractive-index of a gas.

13. Use a neon-lamp circuit to determine the specific inductive capacity of castor-oil over a suitable range of frequency.

14. Use a mercury rectifying-valve to obtain a value for the ionisation-potential of mercury.

15. Investigate the relation between the mutual inductance between two coils and their distance apart.

16. Use a quadrant-electrometer and an A.C.-bridge to determine the dielectric constant of a liquid.

17. Plot a spectral intensity curve and determine the most sensitive wavelength region for a photo-electric cell.

18. Sputter a thin silver film upon a glass plate and investigate the absorption it produces for radiations throughout the visible spectrum.

19. Use a Hilger-spectrometer and a linear thermopile to investigate the distribution of radiant energy in a spectrum.

20. Construct a linear thermopile and use it to investigate the relation between the partial-polarisation of radiation reflected from a glass surface and the angle of incidence.

BOOKS FOR REFERENCE

WORSNOP and FLINT : *Advanced Practical Physics for Students.* (Methuen, 1937.)

WATSON : *A Textbook of Practical Physics.* (Longman, 1922.)

VIGOUREUX and WEBB : *Electrical and Magnetic Measurements.* (Blackie, 1936.)

HOAG : *Electron and Nuclear Physics.* (Chapman and Hall, 1938.)

HUND : *High Frequency Measurements.* (McGraw-Hill, 1933.)

HARNWELL and LIVINGOOD : *Experimental Atomic Physics.* (McGraw-Hill, 1933.)

CURTIS : *Electrical Measurements.* (McGraw-Hill, 1937.)

QUEVRON : *Le Laboratoire de Physique d'Enseignement.* (Librairie de l'Enseignement Technique, 1934.)

KOHLRAUSCH : *Lehrbuch der Praktischen Physik.* (Teubner, Berlin, 1935.)

MECKE and LAMBERTZ : *Leitfaden der Praktischen Experimentalphysik.* (Springer, Berlin, 1926.)

PERUCCA : *Guida Pratica per Esperienze Didattiche de Fisica Sperimentale.* (Zanichelli, 1937.)

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