GEARS, GEAR PRODUCTION AND MEASUREMENT

AN INTRODUCTION TO GEARING WORK INCLUDING SPECIAL REFERENCE TO INSPECTION METHODS

BY

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PREFACE

'HIS book, designedly "popular" in form, has been written in the ope of benefiting those in the trade who have gear problems but who ave little or no background of basic knowledge of modern gear forms, nd of the range of manufacturing and measuring methods. At the utset we disclaim any intention to cram a manual of gear design and omplete guides to gear production and performance between the overs of a single book. These and other sections of the subject have letailed, far-reaching ramifications into which research continues in his country and in others. If this book provides a beginner with a oundation of leading principles upon which he can usefully build his tudy of specialized individual sections of the subject we shall have 'ulfilled our purpose.

In writing this book, and putting the fruits of long practical experience into words, the authors have had the advantage of access to a range of established reference books on gearing and to a considerable number of specialized articles which have appeared during some years past in the engineering press and in the journals of engineering institutions, as well as to the data courteously provided by well-known gearing manufacturers—notably Messrs. David Brown & Sons (Huddersfield), Ltd.—to whom acknowledgments are made in the text.

Not only in connection with this book but also in relation to their experience of gearing work over a period of years the authors especially desire to acknowledge indebtedness to the writings of Dr. H. E. Merritt, one of the foremost British experts on gears, who has summarized the results of wide experience in his classic book *Gears* (Pitman). The same applies to the extensive writings of Professor Earle Buckingham, the well-known American expert whose three-volumed *Manual of Gear Design* (Machinery Publishing Co.) is referred to no less frequently in Great Britain than in the U.S.A. In tackling day-by-day gearing problems we find Machinery's ubiquitous *Handbook* a compact and reliable guide to many phases of the work. In compiling this book the authors have referred to the foregoing and other well-known books as well as to the interesting descriptive articles on modern gears from the able pen of Dr. W. A. Tuplin which have appeared in various periodicals and books published by Messrs. George Newnes, Ltd., and Odhams Press, Ltd., during several years past.

Thanks are tendered to a number of firms for their helpful cooperation in loaning photographs or stereos, or in supplying data in relation to the large range of gearing matters discussed. Among these are Messrs. Adam Hilger, Ltd., Bakelite, Ltd., J. E. Baty & Co., Ltd., Birmabright, Ltd., Brown & Sharpe Mfg. Co., Buck & Hickman, Ltd., David Brown & Sons (Huddersfield), Ltd., Burton, Griffiths & Co., Ltd., Charles Churchill & Co., Ltd., Drummond Bros., Ltd., Delta Metal Co., Ltd., Dowding & Doll, Ltd., Fellows Gear Shaper Co., Thos. Firth & John Brown, Ltd., Alfred Herbert, Ltd., E. H. Jones

(Machine Tools), Ltd., Lees Bradner Co., Power Plant Co., Ltd., J. Parkinson & Son, John Rigby & Sons, Ltd., W. E. Sykes, Ltd., to the Publishers, Sir Isaac Pitman & Sons, Ltd., for the use of certain standard and copyright blocks, to the British Standards Institution for permission to make extracts from a range of British Standard Specifications, a complete list of which is obtainable from the B.S.I., 28 Victoria Street, London, S.W.1, and finally to Mr. B. J. Jeffery, B.Sc. (Eng.) (Hons.), who has been actively co-operative during the preparation of the MS. and has made many helpful suggestions.

Gearing nowadays assumes an immense importance in practically every branch of the engineering trade, an importance which grew at a rapid rate under the stimulus of demands from the automobile and aeronautical industries—accentuated later by requirements arising out of the exigencies of war. Increased production of all kinds of gears to finer limits has led to many developments in the technique of measurement and inspection, a section of the work to which we have devoted a considerable amount of space. Much of the material in the preceding chapters has been provided to form a useful background for those engaged on measuring and inspection.

It is no doubt a vain hope that a book with so many and diverse technical references and diagrams is entirely free from errors and we shall be glad if any such are pointed out to us. Letters addressed to the authors, c/o Sir Isaac Pitman & Sons, Ltd., Parker Street, Kingsway, W.C.2, will be promptly forwarded.

A. C. P. AND W. H. D.

CONTENTS

CONTENTS

CONTENTS

CONTENTS

CHAPTER XII

PART II

CHAPTER XIII

CHAPTER XIV

CHAPTER XV

CONTENTS

CHAPTER XVI

CHAPTER XVII

TABLES

PART I

CHAPTER I

INTRODUCTION AND CLASSIFICATION

Toothed gears are used where power is to be transmitted from shaft to shaft *positively*, i.e. without slip, and so that the connected shafts rotate at *constant*, i.e. *unvarying*, *velocity ratio*. Generally speaking, a toothed gear drive is compact, simple, and efficient.

Friction drives, on the other hand, are always characterized by slip, so that they are called *non-positive*. Such drives are suitable in general engineering practice when the amount of power to be transmitted is low, say up to 10 h.p., and in cases where the connected shafts need not necessarily rotate at a constant velocity ratio. Among the advantages of friction drives are their comparative cheapness, silence, smoothness, and adaptability to rapid changes of speed when under load.

Cog-like gear teeth of primitive form have been used for many centuries, but the ordered investigation into the problem of using *gears for obtaining a uniform velocity ratio* is said to date from researches by a number of geometricians in different lands in medieval times. Later, in 1674, Olaf Roemer, a Danish astronomer, proposed the cycloidal system for the purpose. Later, in more recent times, Professor Robert Willis, of the University of Cambridge, did successful work in devising practical methods of applying the cycloidal curves to a system of interchangeable gears. Willis also devoted time to fashioning the theories of involute tooth form into practical usefulness. He devised the Willis *Odontograph* for laying out tooth profiles on patterns.

Toothed gears are nowadays made in a great variety of forms and sizes, varying from the tiny pieces used in a wrist-watch to the 20-ft. diameter monsters used aboard ship for reducing the high speed of the turbine shaft to the low speed of the propeller shaft. Typical of gears of "handier" size are those used in automobile gear-boxes. A large proportion of the gears used to-day are designed for **speed reduction drives.** One we have already referred to, viz. turbine to propeller. Another common example is in connection with electric motor drives. Most machine-tools, for instance, have motorized drives, and compact *reduction gearing* is essential to enable the transmission of motion from a high-speed motor shaft to a machine-tool spindle required to rotate at a considerably lower speed.

Early gears were crude, inefficient, slow-moving, and often noisy. Windlasses seen at old seaside jetties are typical examples of rough-and-ready cast gears. It must not be assumed that **cast gears** and machine-moulded gears are superseded altogether. They are still used for slow-running drives where accuracy is unimportant and noise can be tolerated. Where a number of identical cast gears are required they can be produced cheaply—for one reason because pattern cost per gear is low. Teeth cast in sand moulds are only suitable for low speeds because of inaccuracies of tooth form and tooth spacing, which together cause local and intermittent concentrations of load. It is clearly impossible to give any authoritative and generally applicable load capacity formulae for roughly-cast gears. **Mortise gears** are cast-iron wheels fitted with wooden "cogs" or inserted teeth which fit into mortises or slots formed in the cast rims. They are still used in some millwork. Hornbeam, maple wood, and beech are often used for the "cogs" for, being relatively elastic, they absorb shocks, reduce noise, and save wear on the machine-cut or cast metallic teeth of the companion gear. Interesting examples of old-time gears produced under difficulties are those made by James Watt for his beam engines—some of which are exhibited at the Science Museum, South Kensington.

Nowadays, with an abundance of modern engineering materials, special machine-tools, and accumulated research experience, firms specializing in gear production turn out gears capable of transmitting powers and speeds in tune with modern requirements, however exacting. Nevertheless, finality in gear design has not yet been reached. Much progress has been made in discovering, producing, and utilizing more efficient gear materials, in designing and manufacturing wonderful gear-cutting machines and cutters, and in designing ingenious measuring methods and appliances. These together have enabled gear manufacturing specialists to turn out gears of great accuracy in pitch and profile, and capable of amazing performance. Much research, however, remains to be done on such matters as the behaviour of gear teeth under load.

CLASSIFICATION OF GEARS. Before going further it will be helpful to attempt to arrange gear types into some orderly grouping—but it is not a simple matter to do so. The same gear may be described in different terms according to the manner in which it is used, and the specialist would certainly desire to see more detailed classifications than a simple table could yield. However, a fairly satisfactory preliminary classification of gears can be made on the "connected shaft basis" as follows—

(*a*) *Gears which connect parallel shafts lying in the same plane*, comprising spur helical, double-helical, treble-helical, internal. Teeth and shaft axes are parallel in the case of straight-toothed spur and internal gears.

(*b*) *Gears connecting shafts which, if produced, would intersect at some angle*, comprising bevel and angle gears. Generally the shafts lie in the same plane.

(*c*) *Gears connecting "skew" shafts, i.e. shafts which are non-parallel and non-intersecting*, comprising worms and wormwheels, spiral and skew gears. This group is, ordinarily, the least efficient, but modern gear specialists, such as Messrs. David Brown & Sons, of Huddersfield, produce multi-start worm gearing of very high efficiency.

It should be remembered that all these gears, if properly designed, cut and mounted, are capable of transmitting uniform angular velocity between the connected shafts. We shall deal with all in turn but shall no doubt find it best at this stage to deal fairly fully with spur gears—these being the simplest and most commonly used. Then, too, in describing spur gears we can introduce a range of basically important definitions and discuss certain general principles, many of which are equally applicable to other types of gears, but are more simply explained and probably more readily understood by beginners in connection with simple spur gears.

GREEK LETTERS EMPLOYED IN GEARING NOTATION

Small Greek Letter	Name	Sound	Small Greek Letter	Name	Sound
α	alpha	al′-fȧ	μ	mu	as *mew* in English
β	beta	bē′-tȧ	π	pi	as *pie* in English
γ	gamma	gam′-mȧ	ρ	rho	as *row* in English
ε	epsilon	ĕ (as in *met*)	σ	sigma	sig′-mȧ
ζ	zeta	z′ē-tȧ	φ	phi	as *fie* in English
θ	theta	thē′-tȧ	ψ	psi	sound *ps-si*
λ	lambda	lamb′-dȧ	ω	omega	o′-me-gȧ

Note: Σ is the capital letter *sigma*. Δ is the capital letter *delta*, the small letter *delta* being δ. The single dot over ȧ in the sound column is a simple attempt to illustrate the pronunciation of the vowel *a*, as in *speakable* (pronounced speek′ abl), as distinct from its pronunciation in *father* (pronounced fä′ther).

CHAPTER II

SPUR GEARS AND ELEMENTARY GEOMETRY OF THE INVOLUTE

INTRODUCTION TO SPUR GEARING. Spur gears are cylindrical gears used to connect parallel shafts. Teeth are cut straight across the rims parallel to the shaft axes. This is the simplest and most commonly-used type of gearing, and study of the basic principles applying to the design and generation of spur gears enables the beginner to understand other types of gears more readily.

FRICTION GEARS. In Fig. 1 we show two parallel shafts, on each of which is mounted a cylindrical disc. Then, if the discs are kept tightly pressed together, as indicated by arrows representing forces P and P_1, one disc will drive the other in virtue of the friction between their mating surfaces. The enemy of any friction drive is "slip," especially at starting. In bearings and in vast numbers of other engineering applications the constant aim is to *reduce* friction—which a celebrated engineer described as "a tax which Nature imposes on all moving parts." In friction drives, however, it becomes necessary to *increase* friction, and this may be done by facing one of the discs with a "friction-creating" material such as leather, wood, paper, woven material such as "Ferodo," etc. The driving utility of a pair of "friction gears" is dependent on the coefficient of friction (μ) of the driver against the driven gear.

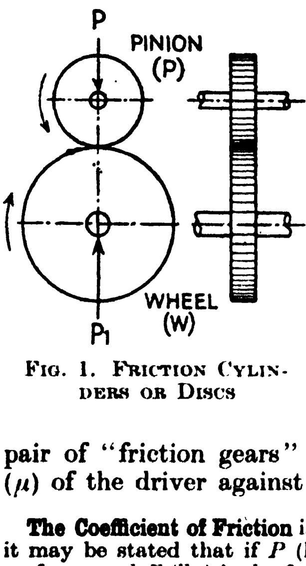


FIG. 1. FRICTION CYLINDERS OR DISCS

The Coefficient of Friction is explained in books on Applied Mechanics. Briefly it may be stated that if P (lb.) is the normal pressure between the contacting surfaces, and F (lb.) is the force of friction acting on either of the bodies, F/P is a constant $= \mu$. Thus $F = \mu P$. The coefficient of friction (μ) is constant only for a given pair of surfaces, its actual value being dependent upon the materials of such contacting surfaces and their respective degrees of smoothness. If two bodies are in relative motion, friction leads to waste of energy. Engineers are usually concerned with reducing this waste by copious use of lubricants and by using special types of bearing surfaces. This is instanced by the use of anti-friction metal in high-speed bearings. However, in "friction drives," including friction gears, friction clutches, belt and rope drives, etc., every endeavour is made to prevent or reduce slip and to increase friction by (1) the use of surfaces having a high coefficient of friction, (2) increasing the normal pressure between the surfaces.

Friction drives are only suitable for light duty and high speeds, and where the motion of the shafts is necessarily of an intermittent character. The discs may be "thrown out of gear" by separating them by a mere fraction of an inch.

TOOTHED GEARS. Where power has to be transmitted positively (i.e. without slip) and at a constant velocity ratio, the smooth rolling cylinders shown in Fig. 1 may be converted into "toothed gears" by cutting grooves in their curved rubbing surfaces and providing pro-

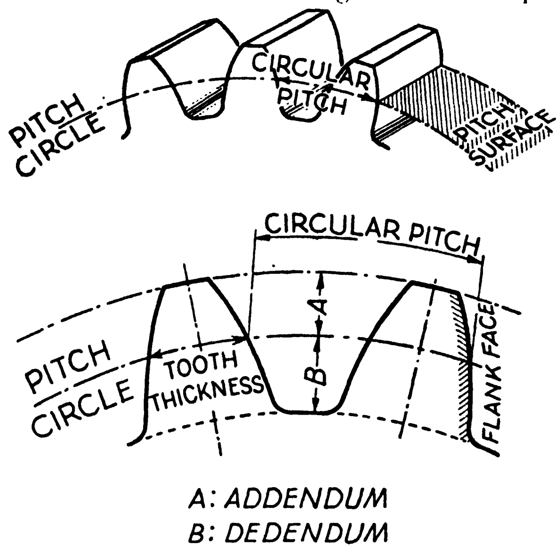


FIG. 2. TEETH ARRANGED ABOVE AND BELOW THE PITCH SURFACE

jections between the grooves, so forming teeth which lie partly above and partly below those curved surfaces.

The design of the teeth is a matter of great importance and when properly accomplished the result is the same as if *ideal*, or *slipless*, friction discs were employed, i.e. a constant velocity ratio between the shafts is obtained.

PITCH SURFACES. The curved cylindrical surfaces of the friction discs correspond to what are called the pitch surfaces of the corresponding spur gears. As will be seen from Fig. 2 we take measurements on and from the **pitch circle,** i.e. the "edge view" of the cylindrical pitch surface. No such circle is actually marked on a spur gear; it is a purely imaginary circle useful in calculations. For instance, if D is the pitch diameter of a spur gear with n teeth of p in. circular pitch, the circumference of its pitch circle is πD in. It is also np in. Thus $\pi \times D = np$.

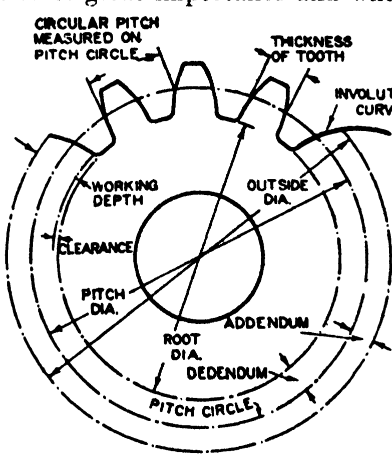


FIG. 3. TOOTHED GEARING TERMS

$$\therefore D = \frac{np}{\pi}; \text{ and } p = \frac{\pi D}{n}.$$

An examination of Figs. 2 and 3 will show that *circular pitch* is a measurement made *round the pitch circle*. This is explained in greater detail in succeeding pages. *Tooth thickness* was defined by the B.S.I. as the *length of the arc of the pitch circle between opposite faces of the same tooth*. Of course, the shortest distance between two points on an arc is the *chord* joining the points. Thus, tooth thickness may be *measured directly* as *chordal thickness*. This is explained more fully in Chapter XV. Two important measurements or dimensions taken *from* the pitch circle are (1) the *addendum*, (2) the *dedendum*. Both these terms are defined carefully on page 18 and are shown as dimensions in Figs. 2 and 3.

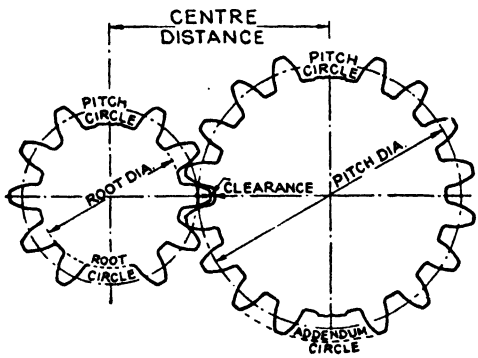


FIG. 4. A WHEEL AND PINION

Important Circles. In Fig. 3 three important circles are shown, viz. *addendum circle*, *pitch circle*, *dedendum circle*. Their relative positions should be noted. There is a fourth circle of equal importance shown

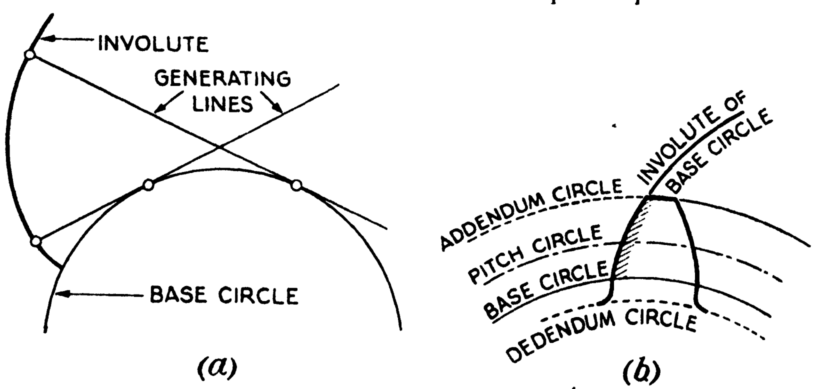


FIG. 5. THE INVOLUTE OF A CIRCLE
(*a*) How generated. (*b*) The base circle in relation to other circles.

in subsequent illustrations and described as the **base circle.** We shall explain something of its significance in the following notes.

THE INVOLUTE OF A CIRCLE. If a straight line rolls round a circle (without slipping) the locus or path of any point on the line is a spiral curve called the involute of the circle. See Fig. 5 (*a*). The circle around which we roll the line enables us to draw an involute curve in respect of which the circle is called the base circle. We can draw the involute of *any* circle. An *involute tooth* is shown in Fig. 5 (*b*). Those parts of its profile beyond the base circle are "involute." Below the base circle the curve has nothing to do with the involute.

To Draw the Involute of a Circle. (Fig. 6.) When drawing an involute curve for the first time, it is advisable to commence with a circle of fairly large diameter, say, $2\frac{1}{2}$ in. Having drawn the circle, divide its circumference into an even number of equal parts, say, twelve, for this will enable you to draw the twelve diameters P–6, 11–5, 10–4, etc., by means of a 30°–60° set-square. It will simply entail drawing lines at 30° and 60° to the edge of the tee-square, all passing through the centre of the circle. Diameters P–6 and 3–9 are at right-angles, 3–9 being parallel to the edge of the tee-square. From P draw a horizontal tangent PC equal in length to the circumference of the circle. Divide PC into the same number of equal parts as was selected when dividing the circumference of the circle into equal parts—in this case twelve. From the end of every radius draw a *tangent*, bearing in mind that *a tangent to a circle at any point in its circumference is at right angles to the radius drawn to that point*. Then,

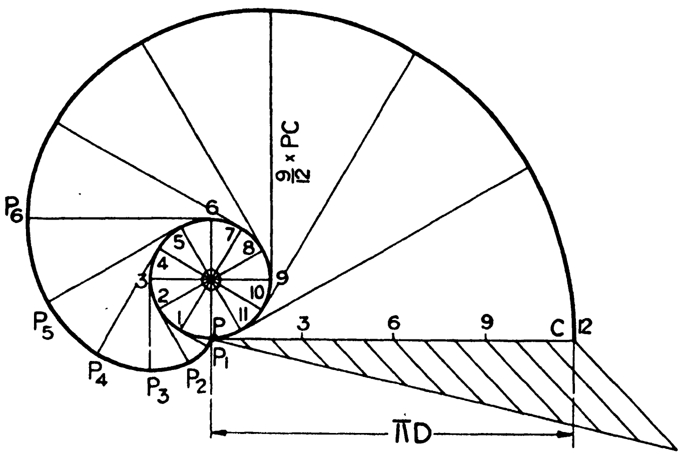


FIG. 6. THE INVOLUTE OF A CIRCLE—DRAWING EXERCISE

using PC as a scale, mark off the lengths of the tangents, so that $1\text{-}P_1 = \frac{1}{12}CP$; $2\text{-}P_2 = \frac{2}{12}CP$; $3\text{-}P_3 = \frac{3}{12}CP$, etc. Note that the lengths of the tangents increase in *arithmetical progression*, the *common difference* between any two successively-placed tangents being $\frac{1}{12}$ of PC. A graceful curve is then sketched through points P, P_1, P_2, P_3, etc. The curve can, if desired, be finished by careful use of a French curve.

In toothed gearing work only a small part of the involute curve is required, so that the principles explained in the foregoing exercise can be applied to drawing the involute of a short circular arc, as shown in Fig. 7.

Note that the curve could extend outwards indefinitely because *the involute is a spiral*.

Tangents to the Circle are Normals to the Involute. Thus consider point P_6 on the involute (Fig. 6). The line P_6–6 is a normal to the involute, but a tangent to the circle. Normals through points P_5, P_4, P_3, etc., on the involute touch the circle in points 5, 4, 3, etc., respectively, such points being called *centres of curvature*. The curve (in this case a circle) passing through these centres is called the *evolute of the involute*. We see in this particular case that the evolute of the involute is a circle. It is known as the **base circle.** It can be shown that corresponding to *any* curve there is only one evolute, but any number of involutes.

To Draw the Involute of a Circular Arc Subtending an Angle α at the Centre of the Circle. (Fig. 7.) For most purposes an angle α of about 70° or less would suffice, but we have selected an angle of 90° to keep the diagram "open" and clear. Commence by drawing the *sector* AOE having an angle of 90°. Divide the arc AE into (say) four equal parts and draw the radii OB, OC, OD. From E draw the tangent EJ equal in length to the *arc* AE. Suppose D_0 is the diameter of the base circle and α is the angle at the centre subtending the arc of which the involute is required, then $JE = \frac{\alpha}{360} \times \pi D_0$. In this case, if the radius R_0

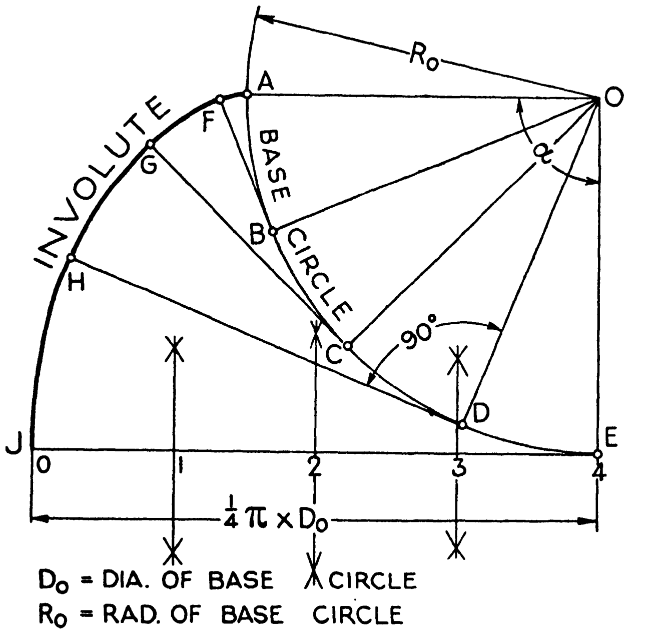


FIG. 7. THE INVOLUTE OF A CIRCULAR ARC

$= 4$ in., it is clear that $JE = \frac{90}{360} \times \pi \times 8 = 6{\cdot}28$ in. Divide JE into four equal parts.

From points B, C, D draw tangents, measuring off $FB = \frac{1}{4}JE$, $GC = \frac{1}{2}JE$, $HD = \frac{3}{4}JE$. Sketch a graceful curve through the points $AFGHJ$. Of course, the greater the number of measured tangents, the greater the accuracy of the curve-shape, provided, of course, that care is taken in draughtsmanship.

To Draw an Approximately Accurate Involute by Means of Circular Arcs. (Fig. 8.)

Method. Along the base circle set off any number of equal spaces AB, BC, CD, etc. Join points A to F to the centre, so obtaining radii shown. From each of the points B to F, draw tangents, each being at right angles to the radius drawn to the point from which the particular tangent is to be drawn. Now, with centre B and radius BA, describe an arc from the point A (on the base circle) to point B_1 (on the tangent drawn from B). To describe the next arc (B_1–C_1), use centre C and radius C–B_1. The next arc is evidently C_1–D_1, and this is described from centre D with radius D–C_1. These arcs, joining one another and running from tangent to tangent, make up a curve which forms a good approximation to the involute. Naturally, the finer the spacing round the circle, the more nearly accurate is the result.

The Unwound String Conception of the Involute. The involute of a circle is sometimes defined as the spiral curve traced by a point on a string (imagined as inextensible) as it is unwound from the circumference of a circle. Thus in Fig. 6 imagine the string tightly wound round the base circle in the first instance. Next, suppose that one end of it is unwound, but kept taut meanwhile, so that when one-quarter is unwound it takes up the position P_3–3–6–9–P. When one-half is unwound, the string takes up the position P_6–6–9–P. Points P, P_1, P_2, P_3, etc., lie on the *locus* (or *path*) of the free end of the string as it is unwound from the circle. The locus of such points is evidently the involute curve shown. This *unwound string conception* of the involute should make the preceding exercise more readily understood. The closer the spacing of the radii, the closer

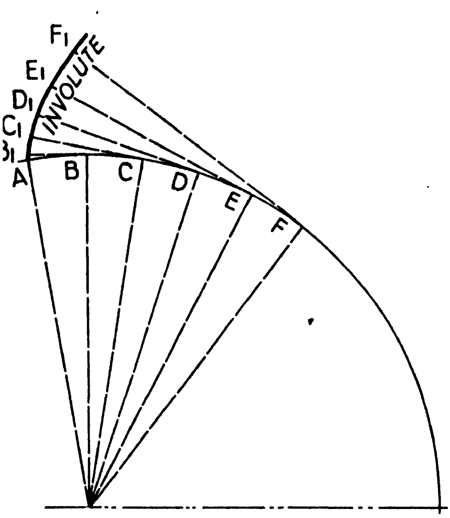


FIG. 8. USING COMPASSES TO DRAW AN APPROXIMATE INVOLUTE

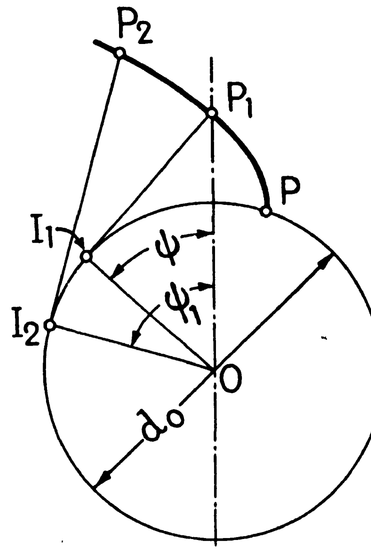


FIG. 9. NORMALS TO INVOLUTE

are the centres of curvature and the more nearly does the compass-drawn involute approximate to a true involute curve.

Suppose we have a base circle of centre O and radius OP, as shown in Fig. 9. Imagine that the inextensible piece of string is being unwound from the base circle, commencing from point P. The end of the string will follow an involute path on which we may mark points P_1, P_2, etc. The circular arc I_1P will be equal in length to the tangent I_1P_1. Similarly the circular arc I_2P will be equal in length to the tangent I_2P_2.

Next, imagine a very tiny motion of the end of the string, say, at the point P_2. The point I_2 (on the base circle) is called the *instantaneous centre* from which the involute at P_2 is being described, i.e. I_2 is the *centre of curvature* at the moment considered, and I_2P_2 is a radius and therefore normal to the involute at P_2. Thus, as mentioned previously, **a normal to an involute at any point is a tangent to the base circle.**

The larger the radius of the base circle, the "more flat" is a short part of an involute springing away from the circumference. The involute of a base circle of infinite diameter is a straight line. This is referred to when dealing with racks and rack cutters.

How is the Base Circle Fixed or Selected for any Gear? It can be shown that the diameter of the base circle for any gear depends on what is called the **pressure angle,** which may be chosen arbitrarily by the designer from a small range of angles which in years of experience engineers have found suitable. For many years a pressure angle of 14½° was almost universally employed, but the tendency now is to use larger angles. The B.S.I. recommend an angle of 20° and the employment of the symbol ψ to represent it.

Fig. 10. Suppose O and O_1 are the centres of two mating spur gears. Their pitch circles pass through the point P which lies on both and also lies on the line of centres OO_1. It is called the **pitch point.** The pitch point always lies on the line of centres. Through P we have

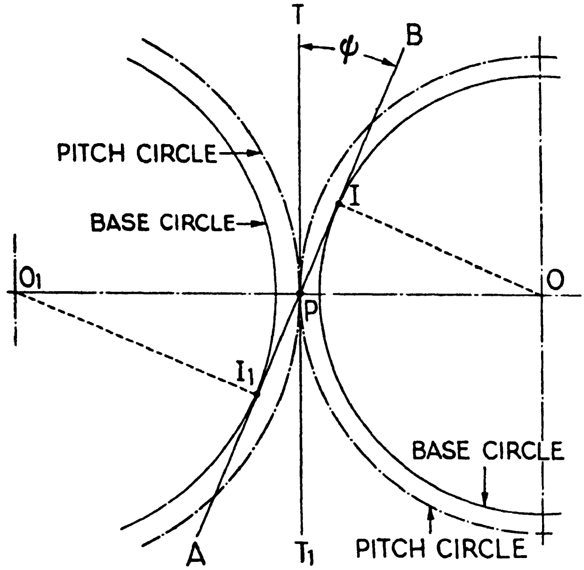


FIG. 10. OBTAINING THE BASE CIRCLES BY GEOMETRICAL CONSTRUCTION

drawn TT_1 perpendicular to OO_1. It is known as the **common tangent to the pitch circles.** Through P we have drawn a line AB, usually called the **path of contact** or, sometimes, **the line of action.** The angle between the common tangent (TT_1) and the path of contact (AB) is called the **pressure angle** or **angle of obliquity.** By convention it is represented by the Greek sign ψ. Nowadays it is usually taken as 20° on machine-cut gears.

Using centres O and O_1 we have described circles concentric with the pitch circles and tangential to AB. These are the **base circles** of the respective gears. The involutes of these base circles give us the "profiles" of the involute teeth for each gear.

Thus, to find the base circle of a gear, we can draw its pitch circle; select any point on this circle and call it the pitch point (P); draw a tangent (TT_1) to the pitch circle through P; draw the line of action (AB) passing through the pitch point and at an angle (ψ) equal to the chosen pressure angle of the gear; and finally draw a circle tangential to AB and concentric with the pitch circle.

By calculation: **Diameter of base circle** (d_o) = **diameter of pitch circle** (d) × **cosine of pressure angle** (ψ). This is represented by $d_o = d \cos \psi$.

EXERCISES

(1) In a manner similar to Fig. 10, draw tangential pitch circles representing a wheel and pinion with 40 and 20 teeth respectively, ½ in. circular pitch. Assuming a pressure angle of 14½°, draw the base circles. Calculate (1) the diameters of the base circles, (2) the centre distance.

Answers: Base Circle Diameters: 6·163 in., 3·082 in.; Centre Distance: 4·775 in.

INTERFERENCE POINTS. Note in Fig. 10 that the base circles are respectively tangential to the path of contact in points I and I_1, known as *interference points*. True involute contact cannot extend beyond them. This will be simpler to understand after reading the following notes.

THE PATH OF CONTACT.[1] Contact between involute teeth always takes place along the common tangent to the base circles, i.e. along the **line of action.** The actual path of contact cannot in fact extend beyond the interference points I and I_1, indeed, its length is usually less than the distance II_1. Actually it is that part of the common tangent to the base circles intersected by the addendum circles. Thus, in Fig. 11 the path of contact is bounded by the addendum circles of the gears at points a and b, and is obviously shorter than the line I_1I_2.

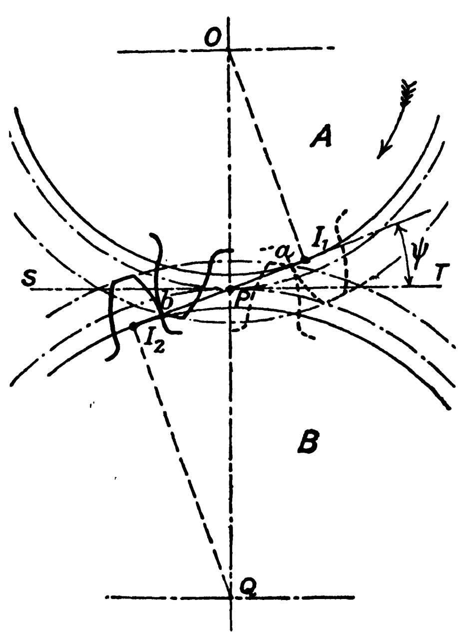


FIG. 11. PATH OF CONTACT OF INVOLUTE GEARS

Approach and Recess. (See Fig. 11.) Suppose gear A is the driver and that it moves clockwise as shown. Contact between the teeth begins at point a and the *point of contact* moves along the straight line towards P. This phase of motion is called "approach." During "approach" the flank of the tooth on A (indicated by broken lines in the drawing) engages with the face of the tooth on B. When the point of contact has passed through P it continues to travel along the straight line until it reaches b. This phase of motion is called "recess."

Common Normal to the Tooth Profiles. Again examine Fig. 11. I_1I_2 is the common tangent to the base circles. The path of contact

[1] See B.S. definitions of *line of action* and *path of contact* on page 19.

lies along it. The latter is also a common normal to the profiles of any two mating teeth. If we have properly shaped mating involute teeth of the same pressure angle it is an accepted principle that the angular velocity ratio between the gears will be constant and that **the common normal to the tooth profiles will pass through the pitch point.**

Now see Fig. 12, which contains less detail and will be clearer to follow. The gears rotate about centres O and Q respectively. P is the pitch point. Teeth make contact at the point c. Then from c is drawn a *common normal* to the two profiles, i.e. a line perpendicular to both. The fundamentally important condition is that the common normal (shown as cP in Fig. 12) must pass through the pitch point. If this condition is satisfied *at every instant*, the mating tooth profiles are said to be **conjugate.**

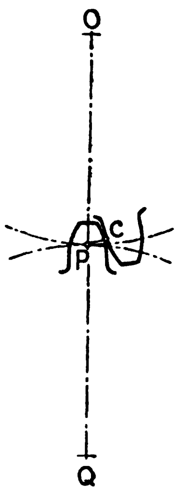


FIG. 12.
THE COMMON NORMAL TO THE TOOTH PROFILES

Further Notes on Pressure Angles. If we consider a straight-sided involute rack (see Chapter III) we see that linear pitch is the same if measured on any lines parallel to the tops or roots of its teeth. The pressure angle, too, is the same at all points along the sloping lines representing the sides of the tooth profile.

When the term pressure angle is applied to involute gears, it really means the pressure angle at a certain point between the base circle and the tip circle; in fact, usually at the point through which a circle can be described, so that the tooth thickness arcs equal the tooth space arcs (neglecting backlash). That particular circle can properly be described as the *pitch circle of generation* on which the pressure angle and circular pitch are equal to the pressure angle and circular pitch of the rack cutter which could be used to generate the teeth. As shown in Fig. 13, the pressure angle increases at points nearer to the tips of the teeth of a spur gear. At the base circle its value is zero. The pressure angle at the point P_2 in Fig. 13 is the angle between the tangent to the tooth profile at P_2 and the line joining P_2 to the centre of the gear. On a spur gear it is assumed that this angle lies in a plane at right angles to the axis of the gear.

Tangential Involutes. See Fig. 14, in which base circles are shown having diameters d_o and D_o respectively. Their centres are O and S, their centre-distance is C. Involute curves are drawn from points A_1 and A_2 on the base circles, these curves being the outlines of mating teeth. The line I_1I_2 is tangential to both base circles and is normal to both involutes at the point of contact Q. Tooth contact occurs along this line which is generally called the **line of action.** The **path of contact** is the term usually applied to that part of the line of action on which tooth contact takes place.

The pitch point P is situated on both the line of action and the line of centres. Evidently the ratio of the distances OP, PS is related to the base circle diameters. Thus $OP/PS = d_o/D_o$. It can be shown that $d_o + D_o = 2C \cos \psi$.

For given base circles the angle ψ changes if the centre-distance is altered.

If the centre-distance of involute gears is altered, it does not prevent the transmission of motion at constant angular velocity. That is one of the advantages of the system. It will be clear that, strictly speaking, an involute gear

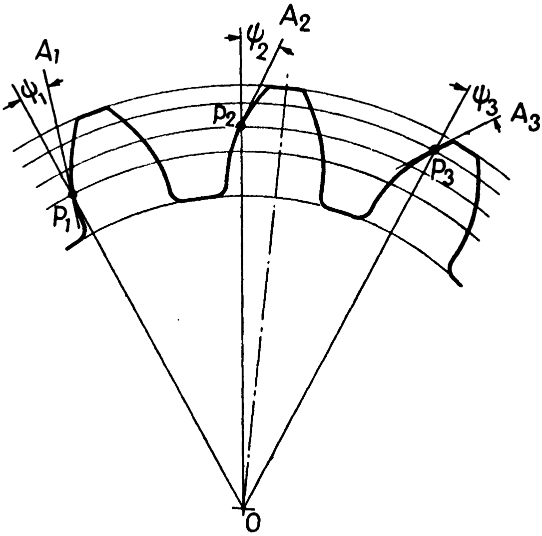


Fig. 13. Pressure Angles at Different Points on Profile

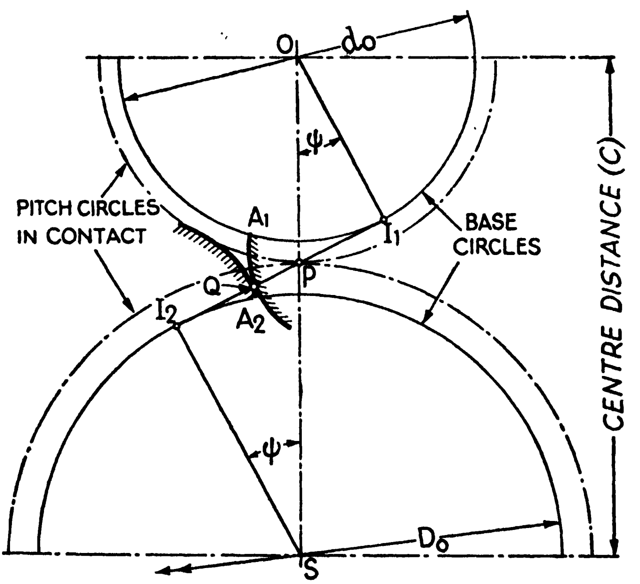


Fig. 14. Tangential Involutes

cannot be said to have a definite pressure angle or pitch diameter until actually assembled in mesh with its mating gear. The pressure angle has relationship to the *actual* centre-distance.

PITCH OF TEETH

There are three different methods of specifying pitch, as follows—(1) **Circular Pitch**; (2) **Diametral Pitch**; (3) **Module.** In the following paragraphs B.S.I. definitions are printed in italics.

(1) **Circular Pitch** (p) *of a gear is the length of an arc of the pitch circle between similar faces on successive teeth.*

Thus circular pitch is "one tooth and one space" measured round the pitch circle. Clearly, therefore,

$$\text{Circular pitch } (p) = \frac{\text{Circumference of pitch circle}}{\text{No. of teeth}} = \frac{\text{Pitch dia.} \times \pi}{\text{No. of teeth}}$$

Note that circular pitch is expressed in linear units, e.g. in inches or millimetres, according to the unit chosen when measuring the pitch diameter. This method of specifying "tooth spacing" is probably the oldest, and was no doubt most useful in days when tooth shapes were "marked out." It assists in the direct formation of a clear mental picture of size. We shall show later that the quantity π has to be used in calculations of pitch diameter and centre distances. When cutting teeth on a spur gear blank, the spacing is accomplished by angular rotation of the blank, so that the machinist has no direct concern with circular pitch as a linear dimension.

(2) **Diametral Pitch** (P) *of a gear is the number of teeth divided by the pitch diameter.*

$$\text{Diametral Pitch } (P) = \frac{\text{No. of teeth}}{\text{Pitch dia.}}$$

i.e. "the number of teeth per inch of the pitch circle diameter." Or, as shown later,

$$P = \frac{\pi}{p}$$

For many purposes it is possible to use a diametral pitch which is a "useful" whole number, such as 1, 2, 4, 8; but pitches expressed as other integers, such as 3, 6, etc., often introduce recurring decimals in the centre-distance. Thus, vulgar—or decimal—fractional values of diametral pitch are often used. The use of fractional circular and module pitches is very common. Diametral pitch is nowadays usually given on drawings of machined gears; but if gear teeth are larger than about one diametral pitch, it is common practice to state the circular pitch. The circular pitch is usually given for cast gears, and frequently for worm gears and racks.

Any gear must have a whole number of teeth, and an increase in pitch diameter *per tooth increase* varies directly as the number of teeth. Thus, suppose a gear has 50 teeth of $10P$, its pitch diameter will be 5 in.

$$\text{Pitch diameter} = \frac{\text{No. of teeth}}{\text{Diametral pitch}}$$

$$\text{No. of teeth} = \text{Pitch diameter} \times \text{Diametral pitch}$$

If the gear had 51 teeth of $10P$, its pitch diameter would be 5·1 in.; if it had 52 teeth, its pitch diameter would be 5·2 in., and so on. There is a regular increase of 0·1 in. in pitch diameter per additional tooth of $10P$. A little thought will show that if the diametral pitch is 2, the increase in diameter per additional tooth is ½ in.

How Circular Pitch and Diametral Pitch are Related.

$$p = \frac{\text{Pitch dia.} \times \pi}{\text{No. of teeth}}; \qquad P = \frac{\text{No. of teeth}}{\text{Pitch dia.}}$$

$$\therefore p \times P = \frac{\text{Pitch dia.} \times \pi \times \text{No. of teeth}}{\text{No. of teeth} \times \text{Pitch dia.}} = \pi$$

$$\therefore \boldsymbol{p} = \frac{\pi}{\boldsymbol{P}}, \text{ and } \boldsymbol{P} = \frac{\pi}{\boldsymbol{p}}$$

EXERCISES

Convert the following circular pitches (p) into diametral pitches (P), (a) 0·628 in., (b) 0·7854 in., (c) 1·047 in.

Answer: (a) $P = \frac{\pi}{p}$

$= \frac{\pi}{0 \cdot 628} = 5$

(b) $P = \frac{\pi}{0 \cdot 7854} = 4$

(c) $P = \frac{\pi}{1 \cdot 047} = 3$

Thus corresponding diametral pitches are 5, 4, 3 respectively

2. Convert the following diametral pitches (P) into circular pitches (p): 10, 6, 2·5.

Answer: (a) $p = \frac{\pi}{P}$

$= \frac{\pi}{10} = 0 \cdot 314$ in.

(b) $p = \frac{\pi}{6} = 0 \cdot 524$ in.

(c) $p = \frac{\pi}{2 \cdot 5} = 1 \cdot 257$ in.

Thus corresponding circular pitches are 0·314 in., 0·524 in., 1·257 in. respectively

3. A gear has 48 teeth of $\frac{3}{4}$ in. circular pitch. Calculate its pitch diameter.

Answer: Pitch diameter $= \frac{48 \times \frac{3}{4}}{\pi} = \frac{48 \times 3}{4\pi} = 11 \cdot 459$ in.

Note: This pitch diameter is not expressed exactly, but correct to five significant figures. Similarly, as the centre distance between two spur gears equals the sum of their pitch radii, it follows that it cannot be expressed exactly by using the foregoing data involving the use of π.

4. A gear has 48 teeth of 4 diametral pitch. Find its pitch diameter.

Answer: Pitch diameter $= \frac{\text{No. of teeth}}{\text{Diametral pitch}} = \frac{48}{4} = 12$ in.

Note: This is an exact centre distance, obtained by employing a "useful" whole number, viz. 4, for diametral pitch.

Circular Pitch for Given Centre-distance. It may be necessary to calculate a circular pitch when a special centre-distance is given.

EXAMPLE. Gear has 48 teeth, pinion has 15 teeth, centre-distance is 10·027 in. A circular pitch equivalent to about 3 diametral pitch is desired.

Rule—

$$\textbf{Circular pitch} = \frac{\textbf{Centre-distance} \times 2\pi}{\textbf{No. of teeth in gear} + \textbf{No. of teeth in pinion}}$$

$$= \frac{10 \cdot 027 \times 2\pi}{63}$$

$$= \frac{62 \cdot 9696}{63} = 1 \text{ in. approx.}$$

If $3P$ had been used, the centre-distance would have been 10·5 in. If $3\frac{1}{4}P$ had been used, the centre-distance would have been 9·692 in.

Numbers of Teeth for Given Centre-distance and Diametral Pitch.

EXAMPLE. Centre-distance is $10\frac{1}{2}$ in., diametral pitch is 3. Speeds of driving and driven shafts are respectively 240 and 75 r.p.m. Required: the numbers of teeth.

Rules—

(1) Teeth in driver + Teeth in driven = 2 (centre-distance) × P

(2) $$\textbf{Teeth in driver} = \frac{\textbf{Teeth in driver + Teeth in driven}}{\dfrac{\textbf{R.p.m. of driver}}{\textbf{R.p.m. of driven}} + 1}$$

To find total number of teeth (N), use rule (1), whence

$$N = 2\,(\text{centre-distance}) \times P$$
$$= 21 \times 3 = 63$$

To find number of teeth in driver (n), use rule (2), whence

$$n = \frac{63}{\dfrac{240}{75} + 1} = 15.$$

Thus numbers of teeth are 15 (driver) and 48 (driven).

(3) **Module** (m) *of a gear is the pitch diameter divided by the number of teeth. It is the reciprocal of the diametral pitch.*

Whilst the module as a means of defining pitch is usually associated with Continental practice, and therefore with the metric system, it is well to emphasize that *the module can be expressed in any linear units.* When no units are stated, however, it is usually understood to be in millimetres. Module is an actual dimension, whereas diametral pitch is a *ratio* (using the term as customarily employed).

One advantage of defining pitch in this way is that the pitch diameter of a gear (in millimetres) is obtained simply by multiplying module (in millimetres) by the number of teeth. It will be shown later that on many "uncorrected" gears, addendum = module.

$$\textbf{Metric Module } (m) = \frac{\text{Pitch dia. of gear (in mm.)}}{\text{No. of teeth}}$$

(i.e. "pitch diameter per tooth")

$$= \frac{\text{Circular pitch (in mm.)}}{\pi} = \frac{p}{\pi}$$

$$= \frac{1}{\text{Diametral pitch}} = \frac{1}{P}$$

The module method is handy when one starts with two fixed quantities, viz. pitch diameter and number of teeth.

$$\text{Module} = \frac{\text{Pitch dia.}}{\text{No. of teeth}}; \quad \text{Diametral pitch} = \frac{\text{No. of teeth}}{\text{Pitch dia.}}$$

1 D.P. is the equivalent of 25·400 mm. module, or of 1 in. module.

To find the metric module equivalent to a given diametral pitch, divide 25·4 by the diametral pitch. To find the diametral pitch equivalent to a given module, divide 25·4 by the module. (25·4 is approximately the number of millimetres per inch.)

EXAMPLE. A gear has 48 teeth of module 6 mm. Find the pitch diameter.

Answer. Pitch dia. = Module × No. of teeth

= 6 × 48 = 288 mm. = 28·8 cm.

Corresponding circular pitch = Module × π

= 6 × π = 18·85 mm. = 0·7421 in.

Corresponding diametral pitch = 25·4 ÷ 6 = 4·2333

Parallel Involutes and Base Pitch. Suppose in Fig. 15 a cord is being unwound from a base circle. Points A_1 and A_2 are marked on the cord while it lies on the circle. The paths of these points are shown as parallel involutes. Let these represent profiles of adjacent gear teeth making contact, at points Q_1 and Q_2, with parallel straight-sided teeth aa_1 and bb_1 of a rack. Points Q_1 and Q_2 lie along I_1I_2 (the path of contact) which is simultaneously normal to both involutes. The lengths Q_1Q_2 (straight) and A_1A_2 (curved) are the same, for pairs of

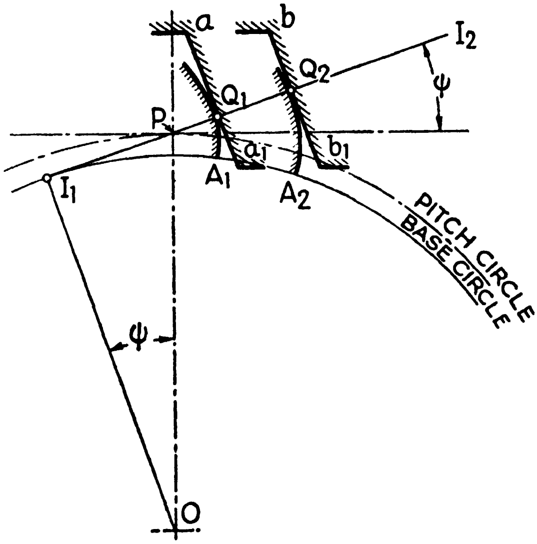


FIG. 15. PARALLEL INVOLUTES

points A and Q are in fixed positions on the supposedly inextensible cord. The *normal distance* Q_1Q_2 is known as the **base pitch.**[1]

$$\textbf{Base pitch } (p_o) = \textbf{Circular pitch } (p) \times \textbf{Cos } \psi$$
$$\textbf{Circular pitch } (p) = \textbf{Base pitch } (p_o) \times \textbf{Secant } \psi$$

If there are N teeth in the gear, then $N \times A_1A_2 = 2\pi \times$ radius of base circle, i.e.

$$\text{Base pitch } (p_o) = \frac{2\pi \times \text{Radius of base circle}}{\text{No. of teeth}}$$

and $$\text{Radius of base circle} = \text{Radius of pitch circle} \times \cos \psi$$

also, $$\text{Base pitch } (p_o) = \frac{\pi \cos \psi}{\text{Diametral pitch}}$$

For module gears, Base pitch $(p_o) = \pi m \cos \psi$

Comparative Significance of Base Pitch and Circular Pitch. If two involute gears are to drive each other satisfactorily one essential requirement is that their base pitches must be equal. Granted that base pitches are equal, and that the tooth profiles are accurately

[1] In older reference books it is described, perhaps not inaptly, as the normal pitch.

shaped involutes derived from base circles having correct diameters, the gears will transmit motion with uniform velocity, even if the centre-distance is slightly altered.

It has previously been stated that, considered alone, i.e. out of engagement with a mating gear, an involute gear really has no special pitch circle, i.e. pitch circle of engagement, although inasmuch as it may probably have been generated by a rack cutter, or by some cutter derived directly or indirectly from the rack cutter, it has a pitch circle of generation. In the ordinary way we assume that the pitch circle of engagement equals the pitch circle of generation. If that be accepted in the case of a pair of mating gears, we have pitch circles equally definite and equally as important as the base circles.

SPUR GEAR DEFINITIONS

(All the following, except those marked, are British Standard Definitions, reprinted by permission from B.S. No. 436—1940).*

The **addendum** (A) of a gear is the height from the pitch circle to the tip of the tooth.

The **dedendum** (B) of a gear is the depth of tooth space below the pitch circle to the root of the tooth. It equals "addendum plus clearance."

The **working depth of a gear** is the depth in the tooth space to which the tooth of the mating gear extends, and is equal to the sum of the addenda of the two gears.

The **bottom clearance** (c) is the shortest distance between the top of the tooth and the bottom of its mating space.

The **whole depth of tooth space** is the radial distance from the tips of the teeth to the circle passing through the bottom of the tooth spaces. It equals "working depth plus clearance."

The **circular pitch** (p) is the length of arc of the pitch circle between similar faces of successive teeth. Where the term "pitch" is used without qualification, circular pitch is always implied.

The **diametral pitch** (P) is the number of teeth divided by the pitch diameter. As the diametral pitch is commonly used with English units, it is understood that the pitch diameter is measured in inches unless otherwise stated.

The **module** (m) is the pitch diameter divided by the number of teeth; it is the reciprocal of the diametral pitch. The module may be expressed in any unit of length and the unit must therefore be stated.

The **outside diameter of a gear** (J) is the diameter at the tips of the teeth.

The **root diameter of a gear** (I) is the diameter at the bottom of the tooth spaces.

The **pitch diameter of a gear** (D) is the diameter of its pitch circle.

The **pitch point** of a pair of gears is the point of tangency of the pitch lines or circles.

The **pitch cylinders** of a pair of gears are those cylinders co-axial with the gears, and in peripheral contact, which will roll together without slip.

The **pitch circle** of a gear is any transverse section of a pitch cylinder normal to the axis.

The **tooth thickness** is the length of arc of the pitch circle between opposite faces of the same tooth. (If the thickness of a tooth is measured as a chord, a correction must be applied to obtain the length of arc. See page 201.)

The **face*** is that part of the tooth surface above the pitch circle.

The **flank*** is that part of the tooth surface below the pitch circle.

The **crest*** or *land* is that part of the original periphery of the blank left after the teeth have been cut.

The **tip*** is the edge in which the face meets the crest. (See Fig. 16.)

The **easing*** (ε) is sometimes called *tip relief*. It is a modification of a tooth profile in which a small amount of metal is removed from the part bounded by the face and the crest. (See Fig. 22 and the B.S. racks in Chapter III.)

The **chordal thickness*** is the distance between points on opposite sides of a tooth, such points being at the intersection of the pitch circle and the tooth profiles. The distance is measured along a chord, and not round the pitch circle.

The **arc of approach** is the arc of the pitch circle through which a tooth travels from the time it is in contact with a mating tooth until it is in contact at the line of centres.

The **arc of recession** is the arc of the pitch circle through which a tooth travels from the time it is in contact with a mating tooth at the line of centres until contact ceases.

The **backlash*** is the shortest distance between non-driving surfaces of adjacent teeth in mating gears. The B.S. definition is as follows: The *backlash* is the total free movement at the pitch circle of one gear in the direction of its circumference, when the other member of the pair is fixed, and the bearing clearances are eliminated.

The **base circle** is the circle from which the involute tooth curve is developed.

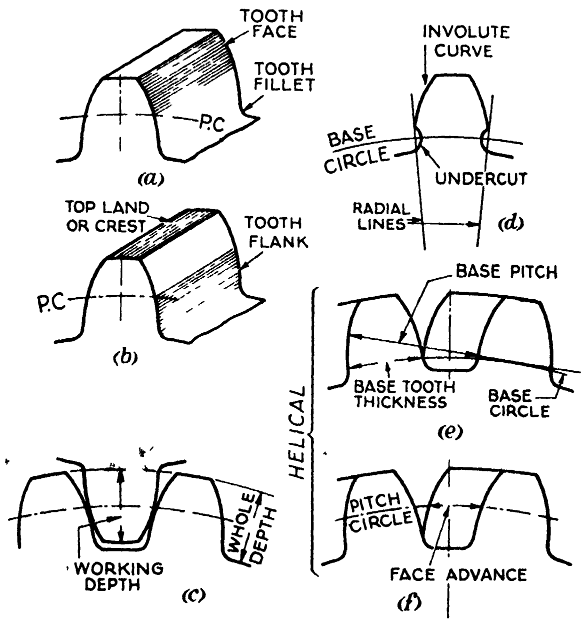


FIG. 16. ILLUSTRATING TERMS EMPLOYED

The **line of action** is the common tangent to the two base circles which passes through the pitch point of a pair of mating gears.

The **path of contact** is that portion of the line of action on which tooth contact takes place.

The **pressure angle** (ψ) is the acute angle between the line of action and the common tangent to the pitch circles at the pitch point.

The **adjacent pitch error** is the error in the circular pitch of any two successive teeth.

The **accumulated pitch error** is the algebraic sum of all the pitch errors in any arc or number of teeth under consideration.

The **profile error** is the maximum deviation of any point on the tooth surface from the designed profile passing through the point of intersection of the actual curve and the pitch line. The designed profile takes into account the easing of the teeth provided for in B.S. No. 436—1940 and shown in this book in Fig. 32 (*a*), (*b*), (*c*).

The direction of **measurement of profile error** is normal to the designed profile. If the actual profile lies outside the designed profile at any point, the error is to be regarded as positive: if inside, negative.

SIMPLE SPUR GEAR FORMULAE

Diametral pitch = P; circular pitch = p; module = m; teeth have the long "uncorrected" standardized addendum (still used to a great extent) which is a multiple of p, viz. $0{\cdot}3183p$. See Fig. 29 (*a*). Care must be exercised not to apply certain of these formulae to gears to which they are inapplicable. For instance, gears (including those with B.S. teeth) may very likely have addendum modifications. Although the applicable B.S. rack may have a fixed addendum of $0{\cdot}3183p$ the addenda of mating gears may be "corrected" or varied by the use of formulae depending on the pitch and numbers of teeth in the mating gears. These formulae are given in B.S. No. 436. Care must be taken

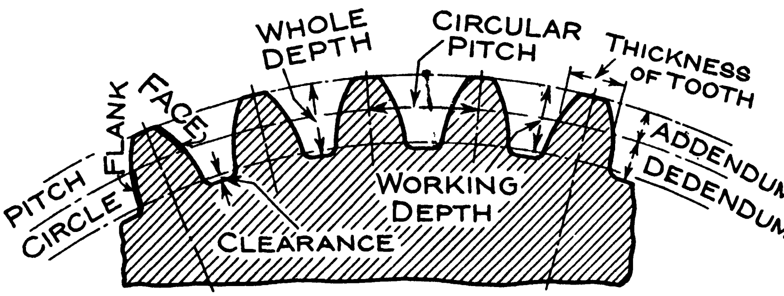


Fig. 17. Terms Used in Connection with Spur Gearing

not to apply full-depth formulae to stub tooth gear calculations, especially in regard to addendum, dedendum, whole depth, and outside diameter.

The tooth proportions are as follows, backlash being ignored—

$$\text{Addendum} = \frac{p}{\pi} = 0{\cdot}3183p = \frac{1}{P} = m$$

$$\text{Min. Dedendum} = 0{\cdot}3683p = \frac{1{\cdot}1571}{P} = 1{\cdot}1571m$$

$$\text{Working Depth} = \text{Twice Addendum} = 0{\cdot}6366p = \frac{2}{P} = 2m$$

$$\text{Min. Clearance} = \text{Dedendum} - \text{Addendum} = 0{\cdot}05p = \frac{0{\cdot}1571}{P} = 0{\cdot}1571m$$

$$\text{Min. Total Depth} = 0{\cdot}6866p = \frac{2{\cdot}1571}{P} = 2{\cdot}1571m$$

$$\text{Pitch Line Thickness} = 0{\cdot}5p = \frac{1{\cdot}5708}{P} = 1{\cdot}5708m$$

(For proportions of teeth of B.S. basic racks, see Fig. 32.)

When Diametral Pitch (P) is Given

$$\text{Diametral pitch} = \frac{\text{No. of teeth}}{\text{Dia. of pitch circle}} = \frac{\text{No. of teeth} + 2}{\text{Outside dia.}}$$

$$= \frac{\text{Sum of teeth of mating pair}}{\text{Twice the centre-distance}}$$

$$\text{Pitch diameter} = \frac{\text{No. of teeth}}{P} = \text{Outside dia.} - 2\,(\text{addendum})$$

$$\text{No. of teeth} = P \times \text{Dia. of pitch circle}$$

$$\text{Outside diameter} = \frac{\text{No. of teeth} + 2}{P}$$

$$= (\text{No. of teeth} + 2) \times (\text{addendum})$$

$$\text{Centre distance} = \frac{\text{No. of teeth in gear} + \text{No. of teeth in pinion}}{2 \times P}$$

When Circular Pitch (p) is Given

$$\text{Circular pitch} = \frac{\text{Circumference of pitch circle}}{\text{No. of teeth}} = \frac{\text{Outside dia.} \times \pi}{\text{No. of teeth} + 2}$$

$$\text{Pitch diameter} = \frac{\text{No. of teeth} \times p}{\pi} = \text{No. of teeth} \times p \times 0{\cdot}3183$$

$$= \text{No. of teeth} \times \text{Addendum (if addendum} = p \div \pi \text{ and only then)}$$

$$\text{Outside diameter} = \frac{(\text{No. of teeth} + 2) \times p}{\pi}$$

$$= (\text{No. of teeth} + 2) \times \text{Addendum (if addendum} = p \div \pi)$$

$$\text{Centre distance} = \frac{\text{Pitch diameter of wheel} + \text{Pitch dia. of pinion}}{2}$$

$$= \frac{(\text{No. of teeth in } A + \text{No. of teeth in } B) \times p}{2 \times \pi}$$

When Module (m) is Given

$$\text{Module } (m) = \frac{\text{Pitch diameter}}{\text{No. of teeth}}$$

$$\text{Tooth thickness} = \frac{p}{2} = \frac{1{\cdot}5708}{P} = 1{\cdot}5708m$$

$$\text{Whole depth} = 0{\cdot}6866p = \frac{2{\cdot}157}{P} = 2{\cdot}157m$$

$$\text{Clearance} = \frac{p}{20} = \frac{0{\cdot}157}{P} = 0{\cdot}157m$$

$$\text{Pitch diameter} = \text{No. of teeth} \times \text{Module}$$

$$\text{Outside diameter} = (\text{No. of teeth} + 2) \times m$$

$$\text{Centre distance} = \frac{\text{No. of teeth in gear} + \text{No. of teeth in pinion}}{2} \times m$$

EXERCISES

1. Gear has 48 teeth, pinion has 15 teeth, 3 diametral pitch (P). They are machine-cut gears having addendum = $0{\cdot}3183p$; dedendum = $0{\cdot}3683p$.
Find (*a*) circular pitch, (*b*) module, (*c*) pitch diameter (gear), (*d*) outside diameter (gear), (*e*) centre-distance, (*f*) addendum, (*g*) dedendum, (*h*) clearance, (*i*) whole depth of teeth, (*j*) tooth thickness along pitch circle.

2. Gear has 48 teeth, pinion has 15 teeth, 1-in. circular pitch (p).
Find (*a*) diametral pitch, (*b*) to (*j*) as in Question 1.

3. Gear has 30 teeth of 1½ in.p, addendum = $0{\cdot}3183p$, dedendum = $0{\cdot}3683p$.
Find (*a*) addendum, (*b*) dedendum, (*c*) pitch diameter, (*d*) outside diameter, (*e*) clearance, (*f*) working depth, (*g*) whole depth of teeth.

ANSWERS FOR YOUR GUIDANCE

1. (*a*) 1·0472 in., (*b*) $0{\cdot}\dot{3}$, (*c*) 16 in., (*d*) $16{\cdot}\dot{6}$ in., (*e*) 10·5 in., (*f*) $0{\cdot}\dot{3}$ in., (*g*) 0·386 in., (*h*) 0·0524 in., (*i*) 0·719 in., (*j*) 0·5236 in.

2. (*a*) 3·1416, (*b*) 0·3183, (*c*) 15·280 in., (*d*) 15·915 in., (*e*) 10·027 in., (*f*) 0·3183 in., (*g*) 0·3683 in., (*h*) 0·05 in., (*i*) 0·6866 in., (*j*) 0·5 in.

3. (*a*) 0·4775 in., (*b*) 0·5525 in., (*c*) 14·325 in., (*d*) 15·280 in., (*e*) 0·0750 in., (*f*) 0·9549 in., (*g*) 1·0299 in.

USEFUL SUMMARY OF TOOTH PROPORTIONS

	Cast Gears (Approx. Sizes)	14½° Full-depth Machine-cut Gears (B. & S.)	B.S.I. 20° Standard Racks
Addendum	0·3p	0·3183$p = m$	0·3183$p = m$
Dedendum	0·4p	0·3683p	0·3979p to 0·4583p
Whole Depth	0·7p	0·6866p	0·7162p to 0·7766p
Tooth Thickness	0·48p	0·5p	0·5p
Space Width	0·52p	0·5p	0·5p

Notes. (1) Teeth on spur gears may be machine cut, extruded, drawn, machine moulded, cast from patterns, or made separately and secured to the rim as in mortise gears. The majority are machine cut, (i.e. the teeth are cut in the rim of the gear).

(2) *B. & S.* is short for *Brown & Sharpe*. *B.S.I.* is short for *British Standards Institution*. *B.S.* is short for *British Standard*.

UNDERCUTTING. This generally arises from mistakes in design rather than production. It is a defect commonly found in involute pinions. We have already stressed that correct involute contact occurs along the *line of action*, i.e. the tangent to the base circles. It is tangential to the base circles at the *interference points*. From diagrams given in this book it will be evident that the maximum size of the addendum of either of a pair of mating gears is obtained when the addendum circle passes through the interference point which, as previously

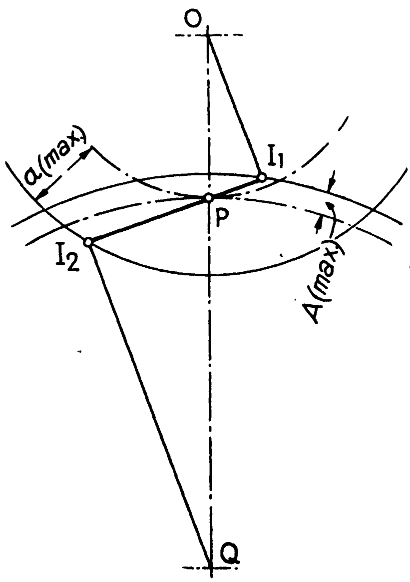


FIG. 18. THEORETICAL MAXIMUM ADDENDA

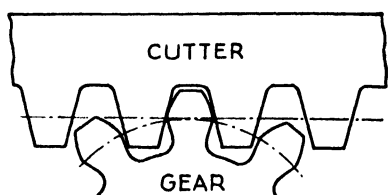


FIG. 19. UNDERCUT TEETH

stated, is on the base circle. This gives the greatest useful length of addendum if the whole of the tooth profile from tip to base circle could be made a true involute curve. Thus Fig. 18 shows the theoretical maximum addenda (a for pinion, A for gear). However, during the tooth-cutting operation undercutting may occur, due to the corners of the cutter biting into the true involute curve on the flank of the tooth. This "biting into," or "cutting away," of parts of the involute profile is called "interference" and it results in "undercutting."

Fig. 19 shows part of a small pinion generated by means of a rack cutter of $14\frac{1}{2}°$ pressure angle, full-depth teeth. The teeth are badly undercut for part of the involute profile has been mutilated.

Fig. 20 shows the difference that results from displacement of the rack cutter, which is set in the extreme position to generate involute teeth without undercutting. The line *PI* is the *line of contact*, i.e. the line along which contact takes place between the involute profile of the pinion teeth and the faces of the rack

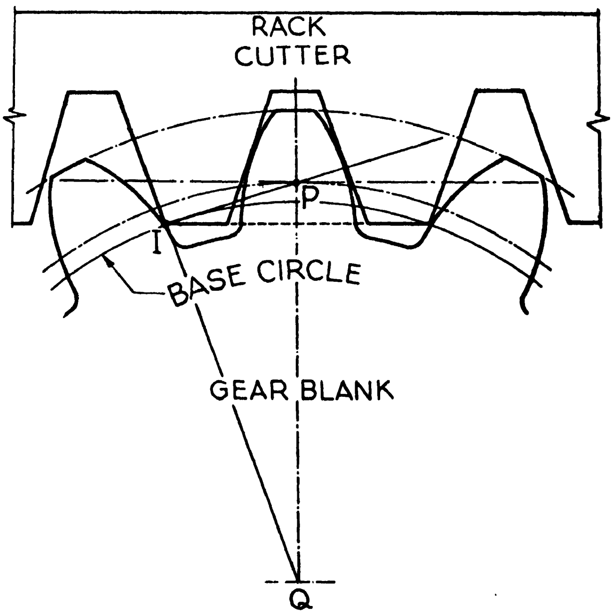


FIG. 20. CUTTER SET SO AS TO AVOID INTERFERENCE

teeth. If the corner of the rack cutter does not extend beyond *I*, undercutting will not occur.

The minimum number of teeth below which undercutting occurs in the rack-planing or hobbing process can be found by calculation.

Thus in the case of $14\frac{1}{2}°$ full depth involute teeth undercutting will occur when the number of teeth is *less than* 32. If the number of teeth is *less than* 22, the undercutting may be excessive. In the case of 20° full-depth involute teeth, undercutting will occur when the number of teeth is *less than* 18. If the number of teeth is *less than* 14, the undercutting may be excessive.

CORRECTION FOR UNDERCUTTING. If the reader examines the standard basic racks described in Chapter III he will find that the addendum is given in terms of pitch. Thus, on the $14\frac{1}{2}°$ full depth involute rack the addendum is given as $0{\cdot}3183\,p$ = module. In actual practice, however, it is not usual to work strictly to these proportions and the designer may resort to addendum modification as one means of avoiding undercutting of the teeth in pinions.

In the **B.S. involute system** the length of addendum for any gear (spur, helical, or internal) can be determined by means of a formula given in B.S. No. 436. Study of ensuing paragraphs will demonstrate that the form of a tooth depends upon (1) the shape of the basic rack, (2) the correction factor.

In order to obtain full involute action, to avoid undercutting when a pinion of a small number of teeth is in contact with a 20° basic rack, and to obtain better zone and strength factors, *the addendum varies with the pitch and the number of teeth in the mating gears.* The recommended values for the addendum are given by—

$$a = \frac{p_n}{\pi}(1 + k_p) = \frac{1}{P_n}(1 + k_p)$$

$$A = \frac{p_n}{\pi}(1 + k_w) = \frac{1}{P_n}(1 + k_w)$$

where k_p and k_w, the **correction factors** for pinion and wheel respectively, are determined thus—

(*a*) If $(t + T) \sec^3 \sigma$ *be not less than* 60 (i.e. is 60 or over), then

$$k_p = 0{\cdot}4\left(1 - \frac{t}{T}\right)$$

or $0{\cdot}02(30 - t \sec^3 \sigma)$, whichever is the greater, and $k_w = \quad k_p$

(*b*) If $(t + T) \sec^3 \sigma$ be less than 60, then the centre-distance shall be extended by an amount $\frac{\Delta p_n}{\pi}$

$$k_p = 0{\cdot}02(30 - t \sec^3 \sigma); \; k_w = 0{\cdot}02(30 - T \sec^3 \sigma).$$

The value of Δ (Extension of centre-distance factor) can be obtained by reference to Chart 13 in B.S. No. 436—1940.

(*c*) *For Internal Gears*—

$$k_p = 0{\cdot}4; \; k_w = -k_p, \text{ irrespective of numbers of teeth.}$$

Additional Note. To maintain the tip thickness if the number of teeth in the pinion is such that $t \sec^3 \sigma$ is less than 17, the outside diameter shall be reduced by an amount $\frac{p_n}{\pi} \times 0{\cdot}04\,(17 - t \sec^3 \sigma)$, the pitch and root diameters remaining unaltered. Approximate values of $\sec^3 \sigma$ are given in the following table—

	Spur	Helical		
Spiral angle σ	0°	22·5°	30°	45°
$\sec^3 \sigma$	1·000	1·268	1·540	2·83

(The foregoing notes are authoritative, being reproduced by permission from B.S. No. 436—1940.)

Explanatory Notes on Symbols and Terms Used in Preceding Paragraphs on Addendum Modification. The symbol p_n stands for *normal pitch.* (See the definition of *normal pitch* on page 81, helical gears.) Pitch (as a linear measurement) can be measured in two ways on a helical gear, as shown in Fig. 21. The normal pitch (p_n) and the *normal tooth thickness* are both measured on a helix lying on the pitch surface normal to the teeth. The circular pitch (p) is measured "at the side of a gear," i.e. round the pitch circle which, we have seen, lies in a plane normal to the gear axis. If σ represents the *helix angle* or *spiral angle* of a helical gear the relationship between p and p_n is expressed by: $p = p_n \sec \sigma$. The circular pitch (p) is sometimes called *transverse circular pitch*, to distinguish it from normal circular pitch, in which case it is represented by the symbol p_t. Thus, $p_t = p_n \sec \sigma$. Obviously the spiral angle on a spur gear is 0°. In regard to the normal diametral pitch (P_n) note that $P_n = P \sec \sigma = \dfrac{\pi}{p_n}$.

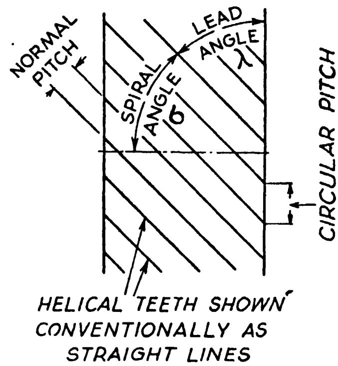


FIG. 21. PITCHES AND ANGLES OF HELICAL TEETH

Thus the expression for *tooth sum*, viz. $(t + T) \sec^3 \sigma$, can be replaced by $(t + T)$ for spur gears. Exactly the same applies to any other mathematical expression or formula containing the term $\sec^3 \sigma$, which occurs when applying such formulae to spur gears, straight-toothed internal gears, and straight-toothed pinions and racks. Thus the correction coefficient of a spur pinion (k_p), in a case where the tooth sum is greater than 60, is limited to a minimum value of $k_p = 0{\cdot}02\,(30 - t)$. Applying this formula to a ten-toothed pinion used in such a drive, we find it has a minimum correction factor (k_p) of 0·4. For twenty teeth, $k_p = 0{\cdot}2$. Similarly, if the tooth sum is less than 60, the correction factor of the spur pinion (k_p) can be expressed as $k_p = 0{\cdot}02\,(30 - t)$; the combined correction factors are made equal to $k_p + k_w = 0{\cdot}02[60 - (T + t)]$.

Instructive Examples from B.S. No. 436—1940.

(1) Pair of spur gears 26/73 teeth, 0·5 in. pitch

Then $(t + T) \sec^3 \sigma = (26 + 73) \times 1{\cdot}00 = 99$

This is greater than 60, therefore the pinion correction factor

$$k_p = 0{\cdot}4\left(1 - \frac{t}{T}\right), \text{ or } 0{\cdot}02(30 - t \sec^3 \sigma),$$

whichever is the greater.

$$0{\cdot}4\left(1 - \frac{t}{T}\right) = 0{\cdot}4\left(1 - \frac{26}{73}\right) = 0{\cdot}4 \times 0{\cdot}646 = 0{\cdot}258$$

$$0{\cdot}02(30 - t \sec^3 \sigma) = 0{\cdot}02(30 - 26) = 0{\cdot}02 \times 4 = 0{\cdot}08$$

Hence $\qquad k_p = 0{\cdot}258$

$$\text{Pinion addendum} = \frac{p_n}{\pi}(1 + k_p) = \frac{0{\cdot}5}{\pi} \times 1{\cdot}258 = 0{\cdot}200 \text{ in.}$$

$$\text{Wheel addendum} = \frac{p_n}{\pi}(1 + k_w) = \frac{0{\cdot}5}{\pi} \times 0{\cdot}742 = 0{\cdot}118 \text{ in.}$$

(2) Pair of helical gears, 11/22 teeth, 22·5° spiral angle, 4 diametral pitch.

Then $(t + T) \sec^3 \sigma = (11 + 22) \times 1{\cdot}268 = 41{\cdot}8$.

This is less than 60, therefore the pinion correction factor

$$k_p = 0{\cdot}02(30 - t \sec^3 \sigma) = 0{\cdot}02(30 - 11 \times 1{\cdot}268) = 0{\cdot}321,$$

and wheel correction factor

$$k_w = 0{\cdot}02(30 - 22 \times 1{\cdot}268) = 0{\cdot}042.$$

Hence $k_p + k_w = 0{\cdot}321 + 0{\cdot}042 = 0{\cdot}363$.

From Chart 13 in B.S. No. 436—1940, extension of centre-distance factor (Δ) = 0·343.

$$\text{And extension of centre-distance} = \frac{\Delta p_n}{\pi} = \frac{0{\cdot}343}{4} = 0{\cdot}086 \text{ in.}$$

Therefore centre-distance $= \dfrac{(11 + 22) \times \sec \sigma}{2 \times 4} + 0{\cdot}086$

$= 4{\cdot}465 + 0{\cdot}086 = 4{\cdot}551$ in.

And the wheel addendum $= \dfrac{p_n}{\pi}(1 + k_w) = \dfrac{1{\cdot}042}{4} = 0{\cdot}260$ in.

The pinion addendum would have been $\dfrac{p_n}{\pi}(1 + k_p) = \dfrac{1{\cdot}321}{4} = 0{\cdot}330$ in.

But $t \sec^3 \sigma = 11 \times 1{\cdot}268 = 13{\cdot}948$, which is less than 17.

Therefore the pinion outside diameter should be reduced by—

$$\frac{p_n}{\pi} \times 0{\cdot}04\,(17 - t \sec^3 \sigma) = \frac{0{\cdot}04(17 - 13{\cdot}948)}{4} = 0{\cdot}030 \text{ in.}$$

Therefore the pinion addendum should actually be made—

$$0{\cdot}330 - \frac{0{\cdot}030}{2} = 0{\cdot}315 \text{ in.}$$

In general the methods of avoiding or "correcting" for undercutting of teeth in pinions are (1) increasing the pressure angle, (2) increasing the outside diameter, (3) increasing pressure angle and increasing outside diameter, (4) correction by radial displacement of basic rack cutter as shown in Fig. 20. If teeth of B.S. form are used the *correction factor method* previously described is most readily applied.

An increase in the pressure angle brings the interference point nearer to the centre of the gear. If 21 teeth were being cut, they would ordinarily be undercut if the rack cutter had a 14½° angle, but would be well-shaped if it had a 20° angle.

If outside diameter is increased, it does not alter the velocity ratio nor the procedure in generation by hobbing or the rack cutter process. The following formula enables calculation of the outside diameter of a pinion with few teeth to avoid undercutting.

$$\text{Enlarged outside diameter } (J) = (\cos^2 \psi \times d) + (2 \times w)$$

(ψ = pressure angle; d = pitch diameter; w = working depth.)

If the centre-distance is to remain unchanged and the increase in pinion diameter is, say, 0·3 in., then the corresponding gear diameter must be reduced by the same amount (in this case 0·3 in.). When the pinion diameter is increased, the effect is to push the teeth outwards (relative to the pitch circle) from the centre of the pinion. This, of course, results in an increase in the circular tooth thickness (measured on the pitch circle). Conversely, the circular tooth thickness of the teeth on the mating gear, measured on its pitch circle, is reduced if its outside diameter is reduced. In both gear and pinion the teeth are cut to the standard total depth.

An increased addendum results in a more pointed shape of tooth. There is a well-known empirical rule that the tip width should be at least one-twentieth of circular pitch. Obviously an excessively pointed tooth has low tip strength. If standard clearance and tooth depth are to be maintained, it is obvious that when the addendum of one gear is increased, the addendum of its mating gear must be reduced correspondingly.

Apropos change of centre-distance, if this changes from C_1 to C_2 and the pressure angle changes from ψ_1 to ψ_2, the following relationship holds good—

$$C_1 \cos \psi_1 = C_2 \cos \psi_2$$

Tip Easing or Tip Relief. When loaded gear teeth make engagement they undergo deflection, which tends to cause a slight "bending backwards" of driving teeth and "bending forwards" of driven teeth, so that adjacent teeth about to commence engagement are slightly misplaced and thus do not tend to engage smoothly and silently. Therefore on high-speed and heavily-loaded gears, such as those used on modern machine-tools and in automobile and aero work, some tip relieving of the tooth form is often considered necessary.

In Fig. 22 is shown the relieved or thinned tip of the tooth of a pinion, part of the profile of the face being modified. A well-chosen amount of tip relief improves tooth action, and gives smoother and therefore quieter running under load. The B.S. Basic racks shown in Fig. 32 have specified tip relief. The actual amount of tip relief for any pair of gears depends upon the load. As the load changes, so the extent of tooth deflection changes also. The engineer's aim is to provide an amount of relief which should suit the gears when working under their normal rated loading, with even bedding on the teeth of the driver stretching up to the tips of its teeth. This is considered to improve lubrication.

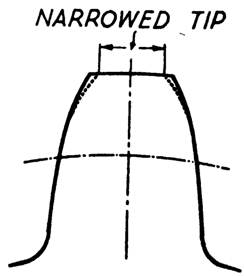


FIG. 22. TIP RELIEF

Gear Noises. The extensive use of non-metallic gears has played a major role in overcoming noise in gear assemblies. Noise arising from spur gear drives is sometimes known as "edge-squeal," i.e. when the noise is presumed to arise through gear teeth making contact only at their extreme ends, the contact possibly changing from one tooth edge to the other extreme end of the tooth during a complete revolution of the gear. This state of affairs usually arises through misalignment and deflection under load.

Many special gear-finishing processes have been evolved to control and eliminate "noise," in many instances with marked improvement.

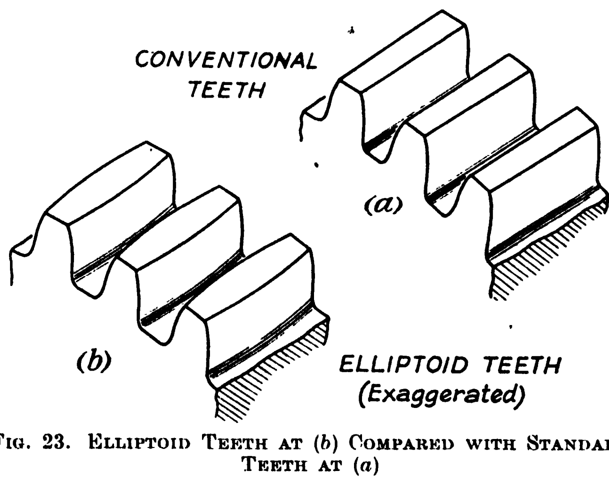


FIG. 23. ELLIPTOID TEETH AT (b) COMPARED WITH STANDARD TEETH AT (a)

However, no gear-finishing operation can rectify errors arising through misalignment and deflection, therefore modifications to standard tooth proportions and tooth form have been experimented with. For instance, there has been the introduction of what is called the "**Elliptoid**" tooth, better known as a "**crowned tooth.**"

To illustrate the Elliptoid principle, a drawing of a very exaggeratedly "crowned" tooth is shown in Fig. 23, from which it will be seen that the tooth has edge-relief. In practice, the latter seldom exceeds 0·0003 to 0·0004 in. on spur or helical gearing. The centre portion of the tooth is thickest with a gradual "thinning-out" towards each edge of the tooth to compensate for small errors of alignment and deflection, also for "tip-interference."

The Elliptoid process allows location of "crowning" at any pre-determined position along the teeth. The result is a self-aligning action of mating gears and elimination of "edge-squeal."

Fairly common noises are "hammering," due to excessive backlash of gears running under a varying load, intermittent "squealing" due to pitch errors of

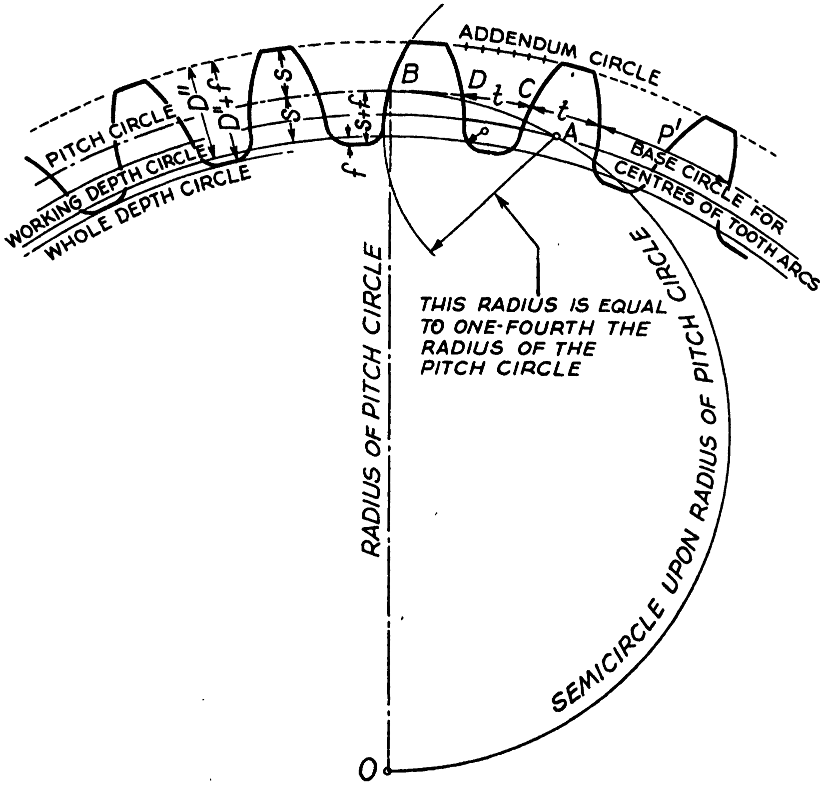


FIG. 24. APPROXIMATE METHOD OF LAYING OUT TEETH: GEARS WITH 30 TEETH OR MORE

adjacent teeth and eccentricity, "humming," and vibrational noises due to insufficient rigidity of assembly.

DRAWING INVOLUTE TEETH. Methods suitable for setting out teeth and marking them on patterns and blocks on wheel-moulding machines.

(1) Brown & Sharpe Approximate Method of Laying Out Teeth for a $14\frac{1}{2}°$ Gear with 30 Teeth or More. Fig. 24. In the B. & S. book, *Practical Treatise on Gearing*, the following method is given. (Its reproduction in this book is by express permission from the Brown & Sharpe Mfg. Co.) Whilst the tooth outlines obtained in this way are

not exactly true to form they are usually acceptable when it is necessary to show teeth on drawings, or in the pattern shop. The tracing paper method of drawing involute teeth is shown in *Intermediate Engineering Drawing*, by A. C. Parkinson (Pitman).

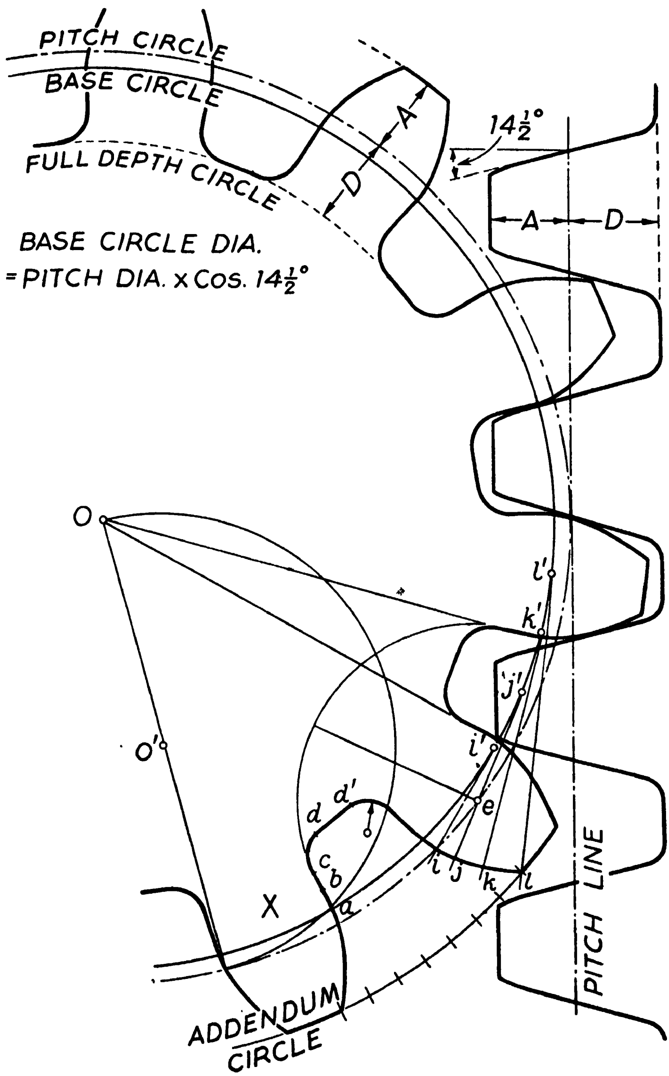


FIG. 25. APPROXIMATE METHOD OF LAYING OUT TEETH: GEARS WITH FEWER THAN 30 TEETH

Fig. 24. The method is employed for gears having 30 teeth or more. A single arc is used for tooth outlines, its radius being one-fourth the *radius* of the pitch circle. The radius of the fillet is one-seventh of the widest part of the tooth space as measured on the addendum circle.

Commence by drawing the pitch circle and mark it off into divisions each equal to half the circular pitch. From *B*, any one of these points draws a line *BO*, a radius of the pitch circle. On it describe a semicircle. Next draw the addendum-, working depth-, and whole depth-circles.

With *B* as centre, and radius equal to one-fourth the radius of the pitch circle. describe an arc to cut the semicircle in *A*. With centre *A*, and the same radius, describe an arc to pass through *B*. The first tooth outline has now been obtained.

With centre *O* describe an arc to pass through *A*. It will be parallel to the addendum—and other circles previously obtained. All other arcs, such as that passing through point *C*, have their centres on the same circle as that on which point *A* is situated.

The fillets or radii joining tooth outlines to the whole depth circle are described with a radius equal to one-seventh of the widest space between tooth corners, as measured on the addendum circle.

(2) **B. & S. Approximate Method of Laying Out Teeth. Gear with less than 30 teeth, say 12. Fig. 25.** In the B. & S. book, *Practical*

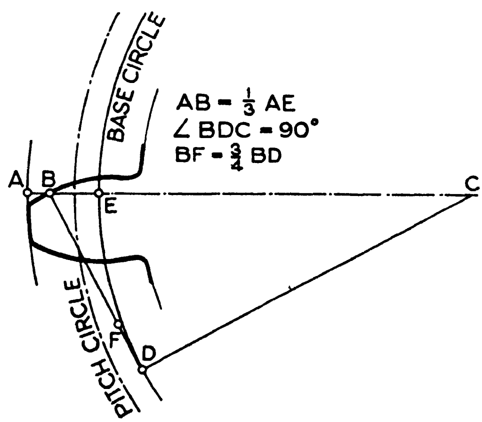


FIG. 26. PROFESSOR UNWIN'S METHOD

Treatise on Gearing, interesting instructive exercises are given showing methods of drawing teeth of pinions to avoid interference with those of mating racks. Examination of Fig. 25 will show

1. Pitch line of rack is tangential to pitch circle of pinion.
2. Curves of pinion teeth outside the base circle are drawn approximately involute by the method shown in Fig. 8 in this book. Points i^1, j^1, k^1, l^1 are successive instantaneous centres. The point i^1 is one-quarter of the radius of the pitch circle from *i*, distances i^1j^1, j^1k^1, k^1l^1, etc., being about one-third to one-quarter of circular pitch apart. If the addendum circle is beyond point *l*, it will be necessary to extend the approximate involute curve by marking off another centre m^1 on the base circle and drawing a tangent from it. Of course, curves of tooth outlines pass through points on the pitch circle separated by distances of $\frac{1}{4}$ circular pitch.
3. The outlines of the teeth below the base circle are, in part, radial lines, i.e. lines converging towards the centre of the circle. They are radial for a distance *ab*—

$$ab = \frac{3{\cdot}5}{\text{No. of teeth} \times \text{circular pitch}}$$

We next have to obtain the portion *bc* on tooth *X*.

The semicircle of centre O^1 is drawn as in the previous example. Point *e* lies on the pitch circle in the centre of the tooth next to *X*. With centre *e*, and radius *eb*, describe arc *bc*. From *c* to *d* a fillet arc is represented. Its radius is one-seventh of the width of space at the addendum circle.

Note that the faces of the rack teeth are "eased" or "relieved" so as to avoid *fouling* or *interference* with the teeth of the pinion. This is often necessary when racks or gears mate with pinions having few teeth. If the flanks in any gear will clear the addenda of a rack, they will clear the addenda of any other gear except an internal gear.

(3) **Professor Unwin's Method. Fig. 26.** Describe the pitch, addendum and dedendum circles. Obtain the base circle for the selected angle of obliquity, using the method shown in Fig. 10. Make $AB = \frac{1}{3} AE$ and draw BD tangential to the base circle. Make $BF = \frac{3}{4} BD$ F is the centre for an arc passing through B.

CHAPTER III

RACKS

A RACK has been defined as a portion of a spur gear of infinite diameter. An involute spur gear with a great many teeth has a large base circle; the greater the number of teeth the more nearly straight is a short part of the base circle. If we imagine a spur gear with an infinite number of teeth we picture a base circle of infinite diameter: in other words, the base circle will be a straight line. *The involute of a circle of infinite diameter is a straight line.*

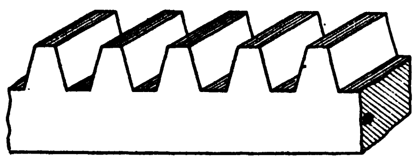


FIG. 27. A RACK

A rack and its mating pinion is shown in Fig. 28. The teeth of the rack are straight sided and are formed relative to a "pitch plane" instead of to a "pitch cylinder" as in the case of a spur gear. Most rack and pinion arrangements are employed to convert rotary into linear motion, but in some mechanisms the rack is employed as the driver.

In the involute system of gears the rack has a working profile formed of straight lines inclined to the vertical at the pressure angle ψ, as shown in Fig. 28. All involute gears having the same pressure angle, pitch and tooth proportions can be used interchangeably; all would work with the *common basic rack*. The involute rack, then, is the foundation of the standard system of involute gears, indeed, it is the basis of various "generating" methods of tooth cutting. For instance, in the *rack generating process*, a rack-shaped cutter having straight sides is employed. One such cutter serves to cut any gear having the same pitch and pressure angle, whatever the number of teeth. All gears cut in this manner will work correctly together provided that no "interference" occurred during the process of generation.

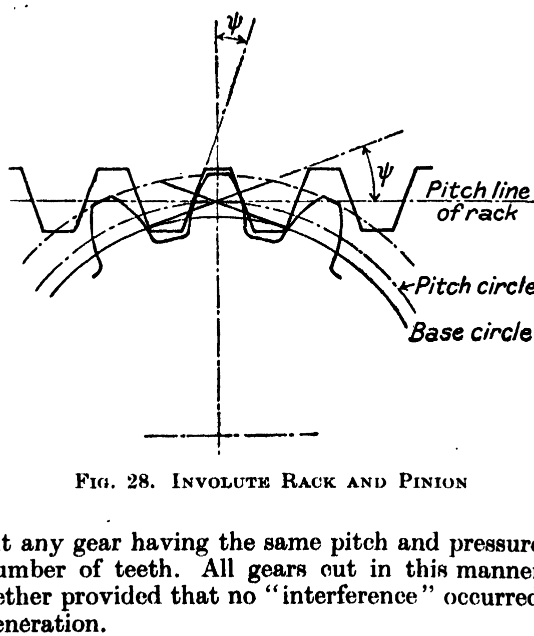


FIG. 28. INVOLUTE RACK AND PINION

We now give details of modern basic racks, leaving the British

Standard Racks, the most important in modern British practice, until the end.

RACK PROPORTIONS

(1) **$14\frac{1}{2}°$ Full-depth Involute System. A.S.A. Standard.** (Fig. 29 (*a*).) As far as British practice is concerned this system is gradually being superseded. In its original form it was closely associated with the use

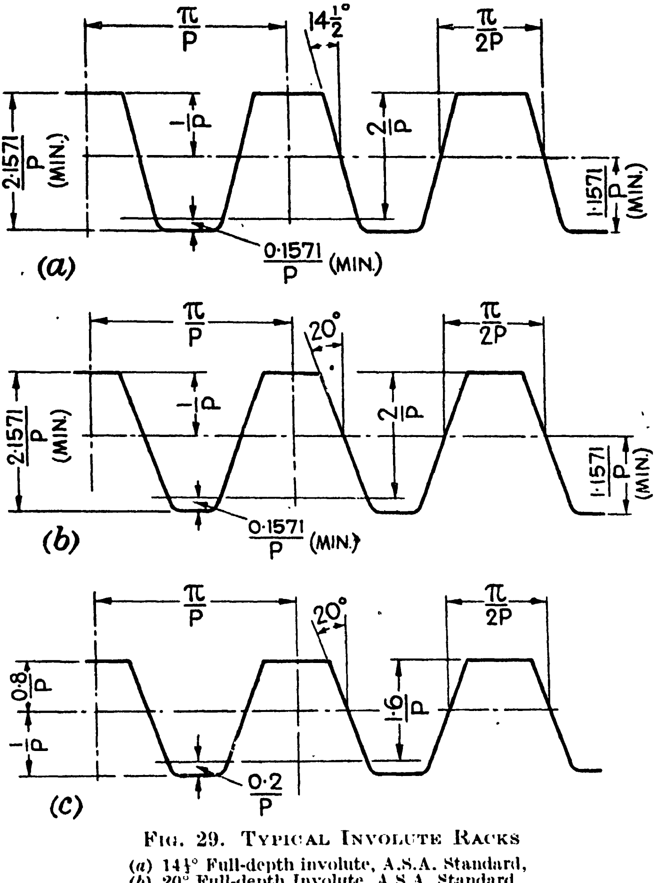


Fig. 29. Typical Involute Racks

(*a*) $14\frac{1}{2}°$ Full-depth involute, A.S.A. Standard,
(*b*) 20° Full-depth Involute, A.S.A. Standard,
(*c*) 20° Stub Tooth Involute, A.S.A. Standard.

of Brown & Sharpe rotary milling cutters. The principal proportions are as follows—

Addendum (A) $= \frac{p}{\pi} = 0{\cdot}3183p = \frac{1}{P} = 1{\cdot}0m$

Minimum dedendum (B) $= 0{\cdot}3683p = \frac{1{\cdot}1571}{P} = 1{\cdot}1571m$

Working depth = twice addendum $= 0{\cdot}6366p = \frac{2}{P} = 2m$

Minimum total depth $= 0{\cdot}6866p = \frac{2{\cdot}1571}{P} = 2{\cdot}1571m$

Minimum clearance $= 0{\cdot}05p = \dfrac{0{\cdot}1571}{P} = 0{\cdot}1571m$

Pitch line thickness $= 0{\cdot}5p = \dfrac{1{\cdot}5708}{P} = 1{\cdot}5708m$

Radius of fillet at root $= 1\frac{1}{3} \times$ clearance

For modern gears, produced by generating processes, this combination of proportions is not considered ideal. The working depth (twice the addendum) may be satisfactory but the pressure angle and clearance are too small; in regard to pressure angle because of the liability of fully-generated teeth on pinions to undercutting, and in regard to clearance because of the insufficient fillet radius. Note that the clearance is described in the drawing (Fig. 29) as "minimum" in order to allow for varying amounts of cutter clearance, depending on the method of gear-cutting employed.

(2) **20° Full-depth Involute System. A.S.A. Standard** (Fig. 29 (*b*).) Same proportions as to addendum, dedendum, tooth thickness, whole depth, etc., as in previous example, but the straight profiles of the teeth have greater slope in view of the increased pressure angle. In the A.S.A. standard 20° full depth involute rack the fillet radius is $1\frac{1}{2} \times$ clearance; in the $14\frac{1}{2}°$ rack the recommended fillet radius is $1\frac{1}{3} \times$ clearance.

(3) **20° Stub Tooth Involute Rack. A.S.A. Standard.** (Fig. 29 (*c*).) Many stub tooth systems have been introduced, all being characterized by large pressure angles and relatively small tooth depths. In the example given the pressure angle is 20°. Other proportions are as follows—

Addendum $= 0{\cdot}2546p = \dfrac{0{\cdot}8}{P} = 0{\cdot}8m$

Minimum dedendum $= 0{\cdot}3183p = \dfrac{1}{P} = 1{\cdot}0m$

Working depth $= 0{\cdot}5092p - \dfrac{1{\cdot}6}{P}$

Basic tooth thickness $= 0{\cdot}5p = \dfrac{1{\cdot}5708}{P} = 1{\cdot}5708m$

Minimum clearance $= 0{\cdot}0637p = \dfrac{0{\cdot}2}{P} = 0{\cdot}2m$

Minimum radius of fillet at root $= 1\frac{1}{2} \times$ clearance

The proportions given above are those approved by the A.G.M.A. (American Gear Manufacturers' Association).

(4) **Fellows' 20° Stub Tooth System.** This involves the use of two diametral pitches respectively applying to circular and radial dimensions. Thus, a pitch described as $\frac{7}{9}$ pitch has a pitch and tooth thickness corresponding to 7 diametral pitch, and an addendum, dedendum, etc., corresponding to 9 diametral pitch. Inasmuch as the dedendum is always 25 per cent greater than the addendum, the clearance is always one-quarter of the addendum. The standard combinations are: $\frac{4}{5}$, $\frac{5}{7}$, $\frac{6}{8}$, $\frac{7}{9}$, $\frac{8}{10}$, $\frac{9}{11}$, $\frac{10}{12}$, $\frac{12}{14}$.

(5) **Nuttall Stub Tooth System.** In this system, employed fairly widely in the U.S.A., the tooth proportions are given in terms of

circular pitch (p). The pressure angle is 20°, addendum is 0·25 p, dedendum is 0·3 p.

(6) **14½° Composite System. A.G.M.A. Standard.** (Fig. 30.) The proportions are the same as those given in Fig. 29 (a) for the 14½° full depth involute rack, but the tooth shape or profile is slightly different

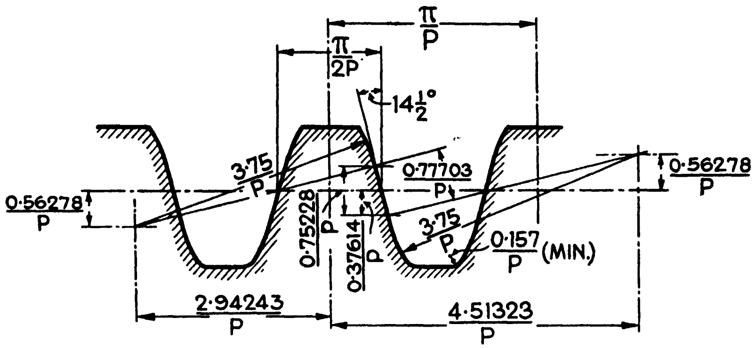


FIG. 30. 14½° COMPOSITE SYSTEM RACK

—the straight profile of the involute rack being modified by the introduction of approximately cycloidal curves above and below the pitch line. The curved portions are circular arcs of radii 3·75 ÷ P and are very similar to cycloidal curves. Gears made to this system are usually form-milled, but they can also be hobbed, provided that the form of the relieving tool used in producing the hob has exactly the same form as the basic rack space. A standard series of eight formed milling cutters in each pitch covers the range from 12 teeth to a rack.

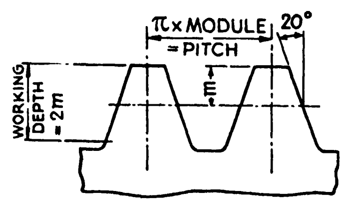


FIG. 31. GERMAN STANDARD RACK

(7) **German Standard Rack for Spur and Bevel Gears (DIN—867),** (Fig. 31). This is based on the involute system so that the sides of the teeth are straight. However, the shape of the root clearance depends on the method of cutting and special requirements. The clearance varies from 0·1 × module to 0·3 × module.

Addendum = Module = 0·3183p
Dedendum = 1·157m = 0·3683p (Formula 1)
or, = 1·167m = 0·3714p (Formula 2)
Working depth = 2 × module = 0·6366p
Total depth = 2·157m = 0·6866p (Formula 1)
or = 2·167m = 0·6898p (Formula 2)
Tooth thickness = 1·5708m = 0·5p

Formula 1 is used when clearance is 0·157m. Formula 2 is used when clearance is one-sixth module.

(8) **British Standard Basic Racks.** (Fig. 32.) The three forms of B.S. basic rack illustrated in B.S. No. 436—1940 are shown in Fig. 32. These B.S. racks are involute in form, except that a slight *easing* of the point is recommended. This easing is also known as *tip relief*, and

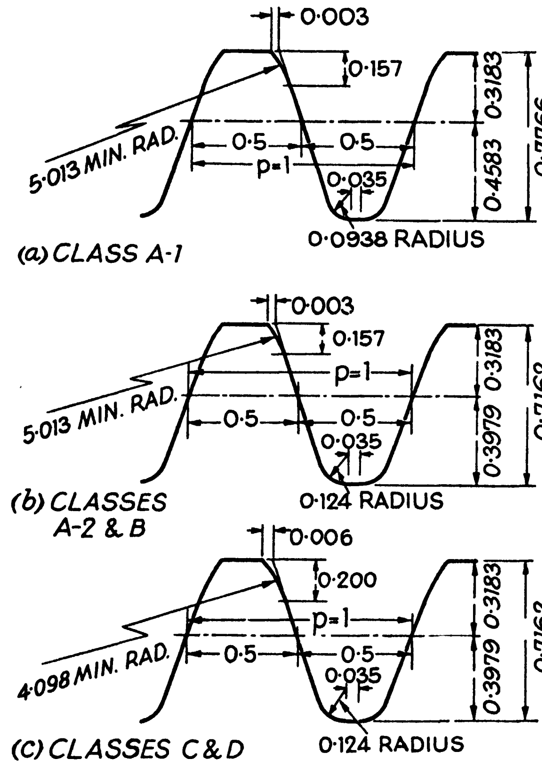


FIG. 32. BRITISH STANDARD BASIC RACKS

it is recommended in B.S. No. 436 that its amount should not exceed the following values on the basic rack—

Classes A1, A2, and B: Precision Ground, Precision Cut and High-class Cut Gears.

Easing (ε) = $0{\cdot}003p$ extending $0{\cdot}157p$ in depth.

Classes C and D: Commercial Cut and Large Internal Gears.

Easing (ε) = $0{\cdot}006p$ extending $0{\cdot}20p$ in depth.

Classes of Gears Covered in B.S. No. 436—1940

Class A1. Precision Ground Gears.

Class A2. Precision Cut Gears for peripheral speeds above 2000 ft. per minute.

Class B. High-class Cut Gears for peripheral speeds above 750 and below 3000 ft. per minute.

Class C. Commercial Cut Gears for peripheral speeds below 1200 ft. per minute.

Class D. Large Internal Gears.

Why the B.S.I. Recommend a 20° Pressure Angle. The form of tooth for machine-cut or ground circular cylindrical gears, straight spur and helical, external or internal to connect parallel shafts, is shown in Fig. 32. It has a pressure angle of 20°, a working depth of twice the module and a substantially semicircular clearance curve at the bottom of the tooth-space. The B.S.I. observations as to the reason for this recommendation, clearly set out in B.S. No. 436—1940, are reprinted with permission. They are as follows—

Experience has shown that the effect of a semicircular clearance curve is to increase the resistance of the teeth to repeated loadings, because the more gradual change of

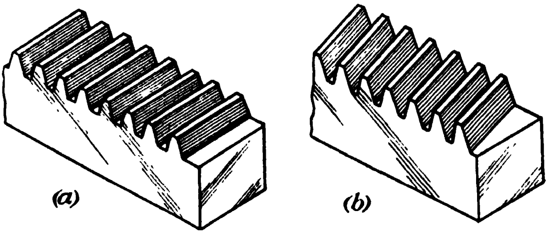


FIG. 33. COMPARISON OF RACKS FOR (*a*) STRAIGHT-TOOTHED SPUR GEARS, (*b*) HELICAL GEARS

section reduces the degree of stress concentration. It is fully realized that stub tooth gearing and gearing with a pressure angle of 14½° have been used with success for many years and will, for certain purposes, continue to be used.

The standard tooth form, having a 20° pressure angle, represents a well-balanced compromise between strength, resistance to wear, and quietness of running. It is comparable with the stub tooth of 20° pressure angle in respect of strength, and superior to it in wear resistance and quietness of running. It is superior to the full depth tooth of 14½° pressure angle in respect of strength and relative freedom from risk of undercutting. Although the arc of contact is a little shorter as compared with the 14½° pressure angle, the relative radius of curvature of the tooth faces is greater, which more than compensates for the reduction in the length of the arc of contact.

Bottom Clearance Space. The B.S.I. recommend that the bottom clearance space should have a smooth continuous surface and that it should be as nearly semicircular as the tooth form and system of cutting will permit. This recommendation is not meant to preclude the provision of slight "flats" at the bottoms of the semicircular clearance spaces, such "flats" being shown in Fig. 32 (*a*), (*b*), (*c*). These flats are for convenience in cutter manufacture and provide a definite diameter for measurement.

Addendum. In B.S. No. 436 (which is concerned with both spur and helical gears) *addendum modification* is dealt with as explained in the notes on *correction for undercutting* on page 23.

Basic Racks for Gears for Clockwork Mechanisms. These are shown in Fig. 48.

Helical Racks. Notes are given in Chapter VII on the form and proportions of helical racks, i.e. racks used in contact with helical gears. Fig. 33 (*b*) shows a pictorial view of a typical helical rack in contrast with a view of a typical rack for a straight-toothed spur gear shown at (*a*).

Other Simple Racks. Racks employed in lightly-loaded assemblies such as indicating mechanisms, assume simple forms as shown in the examples at (*a*) to (*d*) in Fig. 34, those at (*a*) and (*b*) being typical "press work" examples. Types (*c*) and (*d*) are used where lightness is of paramount importance, where the face-width of the mating pinion

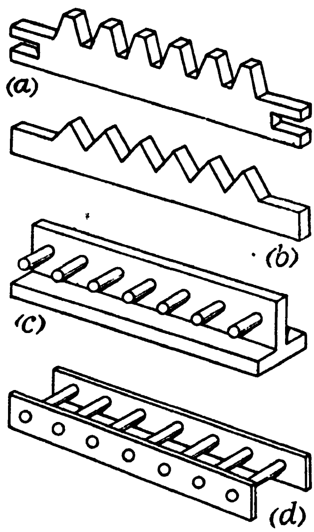


FIG. 34. SIMPLE RACK FORMS

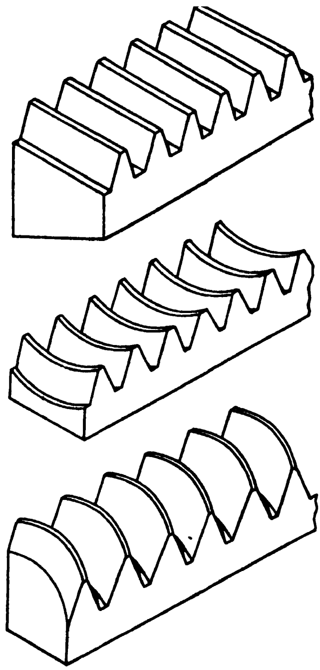


FIG. 35. SPECIAL RACK FORMS

is relatively wide so as to enable it to mesh with two or more gear wheels at the same time, i.e. as in "Marlborough" gearing, and where provision for endwise movement of the pinion during its rotation is necessary.

For special requirements the design of rack teeth sometimes deviates even further from customary standard designs. Some examples of special rack forms are shown in Fig. 35.

CHAPTER IV

INTERNAL SPUR GEARS

If teeth are cut on the inside curved surface of a ring we have an internal gear. Tooth contact takes place beyond the line of centres, i.e. the line joining the centres of the internal gear and its mating pinion. An internal gear and pinion rotate in the same direction, whereas the corresponding external gears rotate in opposite directions. Both straight- and helical-tooth internal gears are used for speed reduction purposes in view of their compact design and efficient tooth action.

The chief uses of internal gears are (1) internal gear drives or simple pairs, (2) internal differential drives, (3) internal planetary drives, (4) internal clutches.

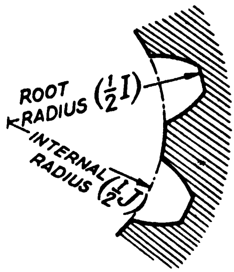


Fig. 36. Important Dimensions—Internal Gear

The same general principles as apply to the design of teeth for external spur gear drives apply also to teeth for internal gears. Limitations which apply to spur gear drives apply also to internal gear drives and in addition there are others peculiar to internal gears. Thus, in both cases interferences have to be avoided. Those applicable to internal gear drives are listed by Earle Buckingham as (1) *involute interference*, which is avoided by making the whole working profile of involute form, (2) *tip interference*, which is avoided by making the diameter of the spur pinion a sufficient amount smaller than that of the internal gear, (3) *fillet interference*, which arises when the tips of the teeth of one gear interfere with the fillets at the roots of the teeth of the other. This is avoided by proper proportioning of the teeth.

A practical difficulty inherent in many internal gear drives, and one that may be pronounced when the difference between the numbers of teeth in gear and pinion is small, is that it may be difficult in assembling to mate the gears together, except in an axial direction.

In further reference to *involute interference* it may be mentioned that in order to avoid it in internal gears of smaller diameters the internal radius must be increased over the conventional size. To avoid interference through contact between the tip of the internal gear tooth and the fillet of the pinion tooth, extra clearance must be provided at the tip of the gear tooth. To enable the generation of full involute profiles on the teeth of the internal gear it is recommended by Earle Buckingham that the cutter used should have not less than 16 teeth. Full data on the design and production of internal gears will be found in *Manual of Gear Design*, Vol. 2, by Earle Buckingham (Machinery Publishing Co.). For formulae covering minimum tooth number differences, extended centre-distances, settings to avoid *trimming*, i.e. removal of the tips of the teeth of the internal gear during generation, by a pinion cutter, etc., the reader is referred to *Gears*, by Dr. H. Merritt (Pitman).

Form of Tooth. The commonest form is the 20° full-depth involute, suitably corrected, but the 20° stub form is also used.

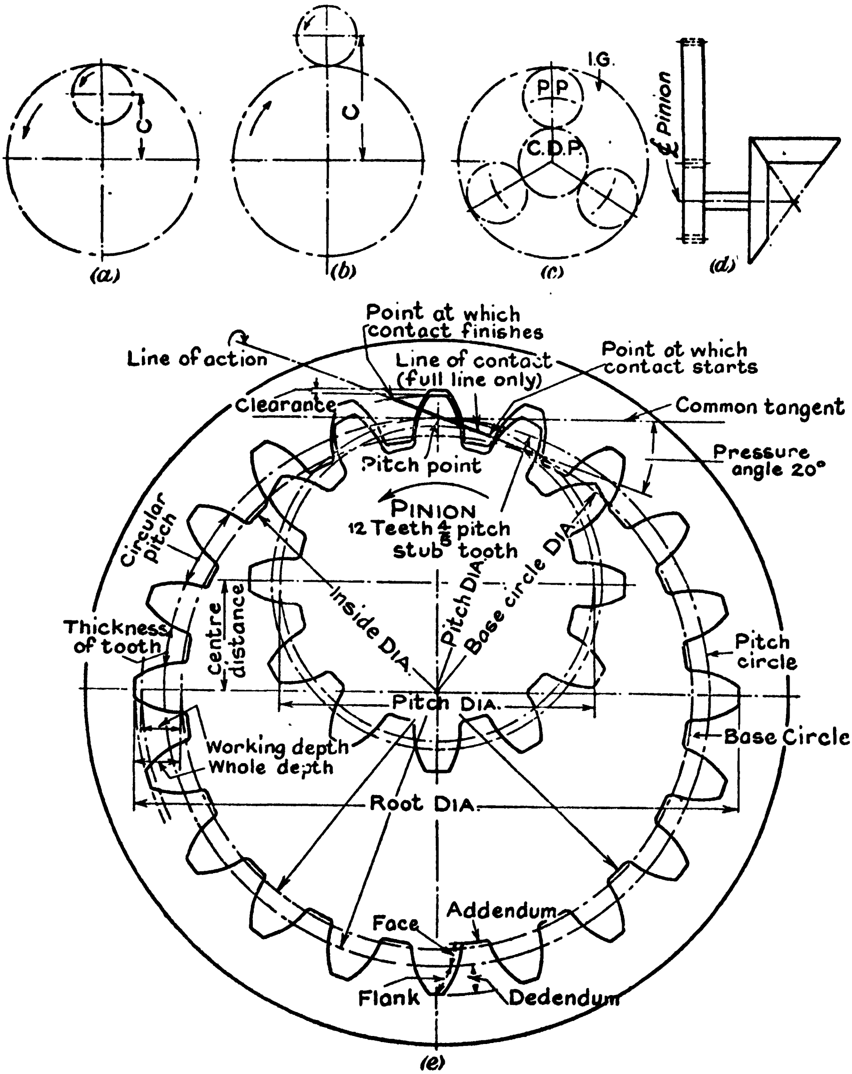


FIG. 37. INTERNAL GEAR AND PINION

The five classes of gears covered by B.S. No. 436, the basic racks of which are shown in Fig. 32, cover internal gears as well as spur and helical gears. In all five classes the working depth is twice the module. For formulae enabling the calculation of addenda and correction factors, see B.S. No. 436. It should be remembered that the addendum of an internal gear is measured radially inwards, for the gear resembles a spur gear turned "outside in," i.e. so that its addendum and dedendum occupy reverse positions.

Tooth Proportions: Corrected and Uncorrected

Wheel	*Pinion*
If D = Pitch diameter	If d = Pitch diameter
,, I = Root diameter	,, i = Root diameter
,, J = Internal diameter	,, j = External diameter
,, A = Addendum	,, a = Addendum
,, B = Dedendum	,, b = Dedendum
$J = D - 2A$	$j = d + 2a$
$I = D + 2B$	$i = d - 2b$

For uncorrected teeth the B.S. proportions for Classes $A2$, B, C, and D are—

$$A = a = 1{\cdot}0/P = 1{\cdot}0m = 0{\cdot}3183p$$
$$B = b = 1{\cdot}25/P = 1{\cdot}25m = 0{\cdot}3979p$$

Liberal correction of internal gears is always a good plan.

If the teeth are corrected according to the recommendations in B.S. No. 436, the correction factors are $k_p = 0{\cdot}4$, $k_w = -k_p$, irrespective of the numbers of teeth.

Wheel	*Pinion*
$A = (1 + k_w)/P$	$a = (1 + k_p)/P$
$B = (1{\cdot}25 - k_w)/P$	$b = (1{\cdot}25 - k_p)/P$

(In the foregoing formulae a minus sign for k_w, if applicable, must be allowed for. A negative correction coefficient on an internal gear means that the diameters of the addendum and dedendum circles are *increased*.)

EXAMPLE: Internal gear has 48 teeth, pinion has 18. $P = 6$.
In the B.S. system the addendum for a pinion meshing with an internal gear is $1{\cdot}4/P$, whilst the addendum of the internal gear is $0{\cdot}6/P$. The full depth of the tooth is $2{\cdot}25/P$.

Addendum of pinion (a) $= \dfrac{1{\cdot}4}{P} = \dfrac{1{\cdot}4}{6} = 0{\cdot}233$ in.

Dedendum of pinion (b) $= \dfrac{1{\cdot}25 - 0{\cdot}4}{P} = \dfrac{0{\cdot}85}{6} = 0{\cdot}142$ in.

Sum $= 0{\cdot}375$ in.

Whole depth of pinion tooth $= \dfrac{2{\cdot}25}{P} = \dfrac{2{\cdot}25}{6} = 0{\cdot}375$ in.

Addendum of gear (A) $= \dfrac{1 - 0{\cdot}4}{P} = \dfrac{0{\cdot}6}{6} = 0{\cdot}1$ in.

Dedendum of gear (B) $= \dfrac{1{\cdot}25 + 0{\cdot}4}{P} = \dfrac{1{\cdot}65}{6} = 0{\cdot}275$ in.

Sum $= 0{\cdot}375$ in.

Whole depth of gear tooth $= \dfrac{2{\cdot}25}{P} = 0{\cdot}375$ in.

Outside dia. of pinion = Pitch diameter (d) + 2 (addendum)
$= 3 + 2(0{\cdot}233) = 3{\cdot}466$ in.

Inside dia. of internal gear (J) = Pitch diameter (D) − 2 (addendum)
= 8 − 2(0·1) = 7·8 in.

$$\text{Centre distance} = \frac{48 - 18}{2 \times 6} = \frac{30}{12} = 2{\cdot}5 \text{ in.}$$

Caliper Settings or "Constant Chord" Settings for These Teeth

See table of "Constant Chord Caliper Settings" in B.S. No. 436.

Pinion ($k_p = 0{\cdot}4$)	*Internal Gear* ($k_w = -0{\cdot}4$)
Height (h) = 1·10079/P	Height (h) = 0·39437/P
= 0·183 in. approx.	= 0·0657 in. approx.
Thickness (g) = 1·64417/P	Thickness (g) = 1·12993/P
= 0·274 in. approx.	= 0·188 in. approx.

CHAPTER V

THE CYCLOIDAL SYSTEM

The Cycloidal System. Three important curves are used in this system, viz. (1) cycloid, (2) epicycloid, (3) hypocycloid.

The **cycloid** is the locus of a point on the circumference of a circle when the circle rolls along a fixed straight line.

The **epicycloid** is the locus of a point on the circumference of a circle when it rolls on the *outside* of a larger (*fixed* or *directing*) circle.

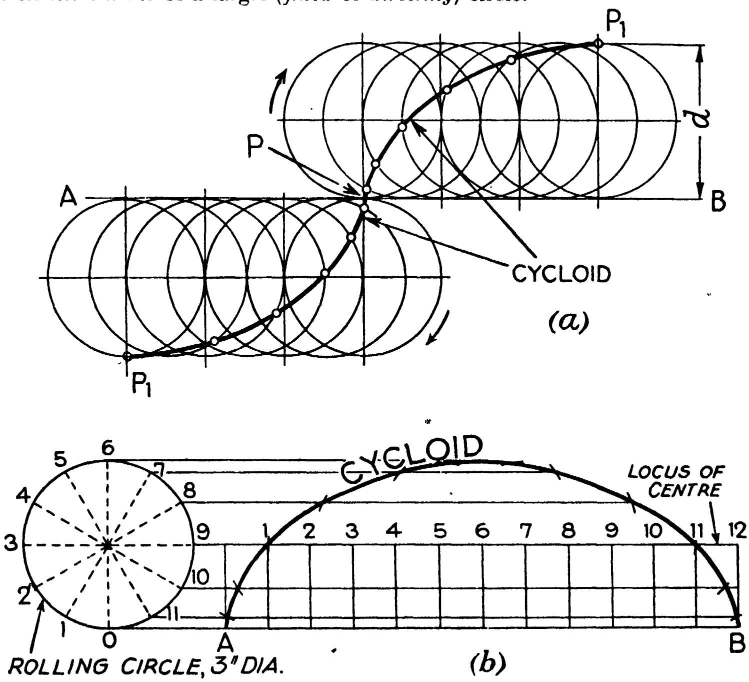


FIG. 38. CYCLOIDAL CURVES

The **hypocycloid** is the locus of a point on the circumference of a circle when it rolls on the *inside* of a larger (*fixed* or *directing*) circle.

Fig. 38. Cycloid. In Fig. 38 (*a*) the circle has a diameter d. It rolls along a fixed line AB. At the top of the diagram we have imagined that a point on the circle coincides with a point P on AB. If the circle is given a clockwise rotation it will roll along AB, the point P meanwhile tracing out the cycloidal curve shown. A cycloid is evidently the locus of a point on the tyre of a wheel if it rolls without slip along a flat road. At the bottom of the same diagram the circle is imagined as rolling on the underside of the straight line. The curves PP_1 are halves of a complete cycloid.

In Fig. 38 (*b*) we show a **geometrical construction.** Divide the circle into, say, twelve equal parts. This is easily done by drawing lines through the centre, using the 30°–60° set-square for the purpose. Mark off AB equal to the circumference of the circle. Thus if the circle has a diameter of 3 in., its circumference is $\pi \times 3 = 9{\cdot}43$ in. Divide AB into twelve equal parts also. Imagine the circle lifted up and placed with point O on A. Then, if the circle rolls along AB, the locus of its centre is the numbered horizontal line shown. Project horizontal lines parallel to AB from points 0, 6; 5, 7; 4, 8; 3, 9; 2, 10; 1, 11. With centre 1, and radius equal to that of the rolling circle, describe an arc cutting

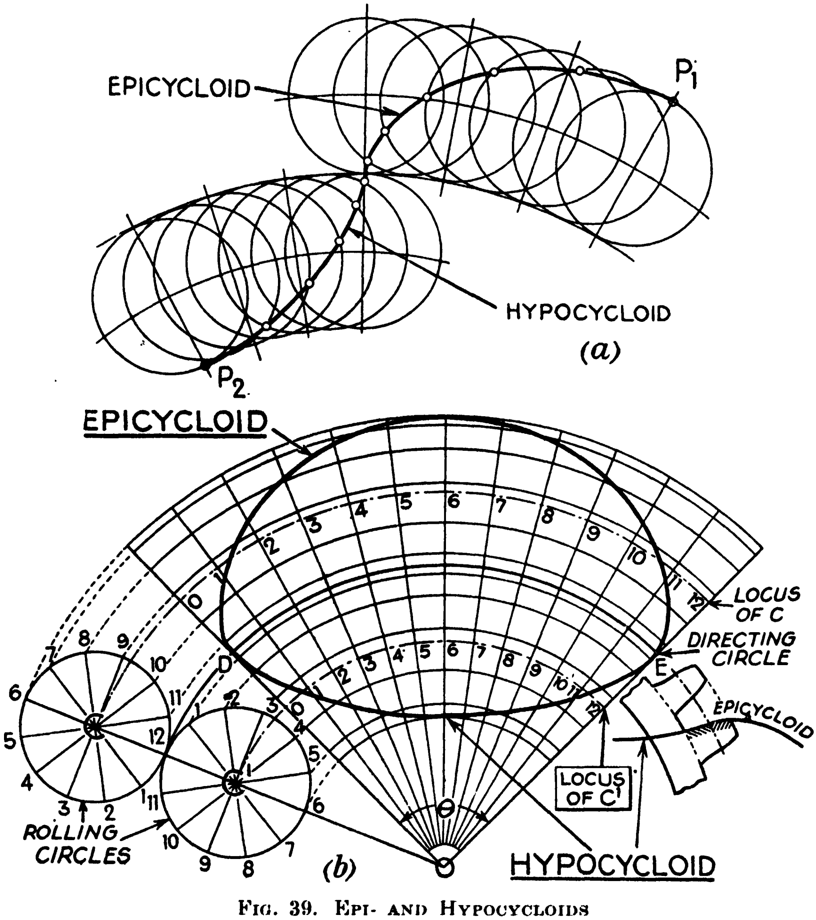


FIG. 39. EPI- AND HYPOCYCLOIDS

the projector from point No. 1 on the circumference. With centre 2, and the same radius, cut the projector from point No. 2 on the circumference. Proceed similarly from left to right, describing arcs from points Nos. 3 to 12 in order. Sketch through the points obtained, endeavouring to obtain a symmetrical and graceful curve. One of the most recent applications of cycloids has been in the form of the surfaces of modern rotary pumps.

Fig. 39. Epi- and Hypocycloids. At (*a*) in Fig. 39 are shown epi- and hypocycloidal curves traced out by a point on the circumference of a *rolling* circle when it rolls respectively on (1) the outside, (2) the inside, of a *fixed* or *directing circle.*

In (*b*) we show a **geometrical construction.** Suppose the directing circle has a diameter of 10 in., whilst the rolling circle has a diameter of $2\frac{1}{2}$ in. If the rolling circles be imagined as lifted up and placed with points No. 12 on *D* (on the directing circle), then it is evident that when these circles have rotated once completely, they will have traversed an arc *DE* equal in length to $\pi \times 2\frac{1}{2}$ in. Instead of measuring off this distance round the directing circle, we may obtain the length of the arc *DE* by measuring off the angle θ which subtends it. This angle may be formed by using the formula—

$$\text{Angle } \theta = \frac{360 \times \text{Rolling circle diameter}}{\text{Directing circle diameter}}$$

$$= \frac{360 \times 2\frac{1}{2}}{10} = 90^\circ$$

The rolling circle can be divided into some convenient number of equal parts (in the illustration we have chosen 12), and the angle *DOE*, or θ, divided into

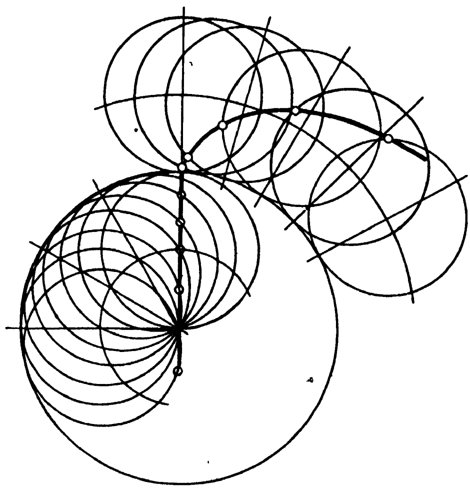


FIG. 40. SPECIAL CASE—HYPOCYCLOID A STRAIGHT LINE

the same number of equal parts also. The loci (or paths) of the centres *C* and C^1 are arcs concentric with *DE*, i.e. described from the centre *O*. Concentric "projectors" from the points 1–11 on the circles may then be described and their points of intersection with the radial lines, drawn from 0, numbered as shown. With centre 1 (on the centre-line locus) cut the projector from point No. 1 on the circumference of the rolling circle. With centre 2 cut the projector from point No. 2 on the circumference—in each case the radius being equal to the radius of the rolling circle. Proceed similarly, using all remaining points Nos. 3 to 12 in due order. Fair curves can then be sketched through the plotted points of intersection. Great care is necessary in sketching those parts of both curves which just spring away from the directing circle.

Fig. 40. The Hypocycloid May Be a Straight Line. When the diameter of the rolling circle is exactly one-half the diameter of the directing circle, the hypocycloid is a radial line.

CYCLOIDAL OR DOUBLE-CURVE TEETH. In cycloidal teeth the tooth outline changes at the pitch line or pitch circle. The pitch line of a rack and the pitch circle of a gear represent the *directing circle* mentioned in preceding geometrical exercises. It will thus be seen that the cycloid is really a special case of both the epi- and hypocycloid in which the directing circle is assumed to be of infinite diameter, so that a small portion of it could be considered as a straight line.

Although there are exceptions it is usual to find that the cycloid is used for the faces and flanks of rack teeth; the epi- and hypocycloids being used respectively for the faces and flanks of gear teeth. In all gears that part of the tooth outline above the pitch circle is convex, whereas in some gears in this system the outlines of the teeth below the pitch circle are convex, in some radial, in some concave. Convex faces and concave flanks are, however, the most familiar. In interchangeable sets of fair-sized cycloidal gears, for assembly into trains, *one* gear in each set, or of each pitch, has radial flanks. It is usually the smallest pinion—having 12 or 15 teeth. If it has, say, 15 teeth, we should call 15 teeth the *base* of this particular set. As the flanks of its teeth are radial it is evident that the diameter of the rolling circle used in tracing out the hypocycloid (in this case a straight line) must be half the diameter of the pitch circle of the pinion or gear in question. The *faces* of all other mating gears will be epicycloids traced out by a rolling circle of the same diameter as the one used for the *flanks* of the base gear. Mating parts of tooth profiles *must* be produced by equal rolling circles.

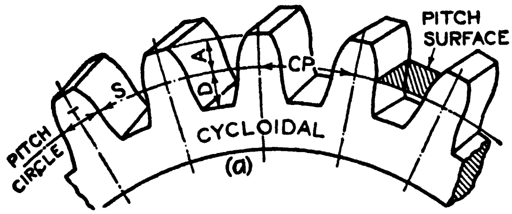


FIG. 41. CYCLOIDAL TEETH

Contact Between Cycloidal Teeth. (See Fig. 42.) The pitch circles of two gears are shown as APB and CPD. The rolling circle X, rolling *inside* the directing circle APB describes part of a hypocycloid, indicated as EP_1G. Similarly, if it rolls on the outside of circle CPD it traces out part of the epicycloid FP_1H. The shaded part of EG represents a *flank* of a tooth on APB, whilst the shaded part of FH represents a *face* of a tooth on CPD, both having been described by the rolling circle X. The face and flank can be imagined in engagement at P_1, the point of contact. This point P_1 must be on the rolling circle X when that circle touches both pitch circles, because at that instant the rolling circle could describe either the curve EP_1G or the curve FP_1H. The line P_1P lies in the direction of the common normal to these tangential curves at their point of contact P_1. Therefore, the gears will mesh correctly if the faces of the teeth are epicycloids and the mating flanks are hypocycloids described by the same rolling circle.

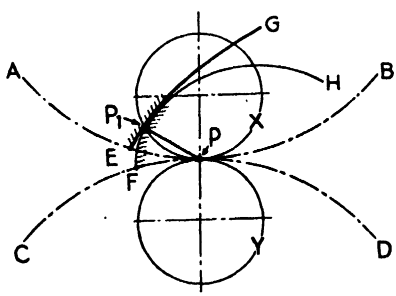


FIG. 42. CONTACT BETWEEN CYCLOIDAL TEETH

If the curves touch at P_1 they will also touch each other at every other position of P_1 as the circles rotate.

The Path of Contact. (See Fig. 43.) The point of contact must lie on circle X or circle Y, either on arc NP of circle X or PM_1 of circle Y. Points N are located at the intersection of the tip or addendum circle of the lower gear with rolling circle X, no tooth contact being possible beyond the tip of a tooth. In a similar way, points M lie at the intersection of the tip circle of the upper gear with circle Y. The path of contact is in this case a **curve of contact** made up of arcs of the rolling circles. The **line of pressure** is the line joining P to P_1 (wherever the latter is situated at the instant considered). In Fig. 43 **the pressure angle** is P_1PT, but, of course, the pressure angle changes as P_1 changes

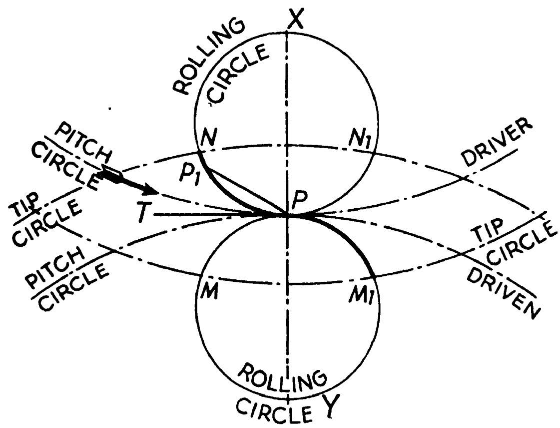


FIG. 43. THE PATH OF CONTACT

its position. When the point of contact falls on the pitch point P the magnitude of the pressure angle is zero, and pressure will therefore act in the direction of the common tangent to the pitch circles. The maximum angle of pressure is reached when a pinion gears with a rack and about $21\frac{1}{2}°$ is reached.

More Notes on the Path of Contact. Referring again to Fig. 43, suppose the upper gear to be the driver, its direction of motion being as indicated by the arrow. At the point N, which is on the flank of a tooth of the driver, a point will come in contact with a point on the face of a tooth on the lower or driven gear. As the motion continues, the flank of the tooth on the driver will slide on the face of the tooth on the driven, until the point of contact has moved from N to P along the arc NP. The face of the tooth of the driver will then slide on the flank of the tooth of the driven until the point of contact has moved from P to M_1 along the arc PM_1. The arc NP is called the **arc of approach,** whilst the arc PM_1 is called the **arc of recess.** Had the driver been rotating in the opposite direction, the complete curve of contact would have been the arc N_1PM.

Rolling and Sliding. In Fig. 44 a cycloidal pinion is shown mating with a gear. On two mating tooth outlines are shown portions of such outlines or profiles as engage with each other. Thus, the blackened-in portion 4 on the pinion slides over the blackened portion 4 on the wheel. Similarly the blackened portion 10 on the wheel slides over the blackened portion 10 on the pinion. Bear in mind that the path of contact (AB) consists of parts of the rolling circles. Once this path of contact has been drawn it is easy to find points on the mating profiles which make contact with one another, as will now be shown.

Describe an arc from the centre of the pinion to meet the arc of contact in any point *C* and the pinion tooth profile in *D*. Describe an arc from the centre of the wheel to pass through *C* and to meet the wheel tooth profile in *F*. The point *F* on the wheel tooth will make contact with point *D* on the pinion tooth.

It will be clear that a good deal of sliding action takes place below the pitch circle of the pinion, because the blackened-in lengths 9, 10, 11, and 12 on the wheel are longer than portions similarly numbered on the pinion. This causes

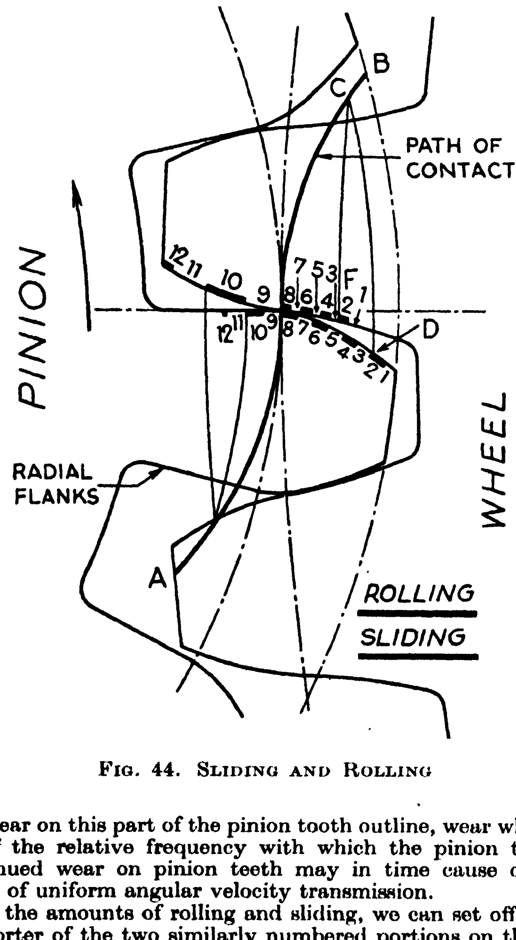


FIG. 44. SLIDING AND ROLLING

considerable wear on this part of the pinion tooth outline, wear which is increased on account of the relative frequency with which the pinion teeth come into action. Continued wear on pinion teeth may in time cause change of form, noise, and loss of uniform angular velocity transmission.

To compare the amounts of rolling and sliding, we can set off on one line the sum of the shorter of the two similarly numbered portions on the tooth curves. This line can be marked "rolling," as at the foot of Fig. 44. The sum of these shorter lengths, then, being represented by one line, gives a measure of the *rolling motion*. The difference in the lengths of any two similarly numbered lengths gives a measure of the *sliding motion*. The sum of these differences can be represented by one line, which can be marked "sliding." In the particular example considered, the two black lines at the foot of Fig. 44 are approximately equal.

An examination of the diagram shows that sliding takes place over the wheel profile below the wheel pitch circle, and changes into sliding over the pinion profile below the pinion pitch circle. Thus at the pitch point there is no sliding, but there is pure rolling.

THE DIAMETER OF THE ROLLING CIRCLE. No hard-and-fast rule can be given for the determination of the diameter of the rolling circle, although several important considerations arise to guide its selection. For cycloidal teeth (of the same pitch) to gear correctly, it is necessary that the faces of one be epicycloids and the flanks of the other, or mating tooth, be hypocycloids, described by the same size rolling circle. Now, if the diameter of the rolling circle be half that of the directing circle, *the resulting hypocycloid is a straight line* (a diameter of the directing circle). The flanks of the teeth outlines so developed are radial straight lines, i.e. they point from the pitch circle to the centre of the wheel. But if the diameter of the rolling circle be greater than half that of the directing circle, the flanks of the teeth would be curved inwards, so as to weaken and undercut the roots. When a small train of wheels is being considered, the diameter of the rolling circle is often taken as half that of the pitch circle of the smallest wheel in the train. Another rule is to make it half the diameter of a 12- or 15-tooth pinion of the same pitch: yet another is to take its diameter as $2\frac{1}{2}p$; another to take it as $6p \div \pi$. It is apparent that the determination of the diameter of the rolling circle is decided according to the exigencies of the situation, and is, at best, a compromise.

SUMMARIZING: CYCLOIDAL TEETH. If two wheels are in gear the faces of the teeth in one are epicycloids, and the flanks of the teeth of the other are hypocycloids, both curves being obtained by the *same rolling circle.*

It is evidently not necessary that the flanks of the teeth on two wheels which gear together be described by the same rolling circle, but the rolling circle which describes the flanks on one tooth must be used to describe the faces of the mating tooth.

The hypocycloid becomes a straight line passing through the centre of the pitch circle when the diameter of the rolling circle equals the radius of the directing circle. In such an event the flanks of the wheel teeth are radial, i.e. normal to the pitch circle.

Proportions of Cast Cycloidal Teeth. Proportions commonly employed are, addendum $= 0{\cdot}3p$, dedendum $= 0{\cdot}4p$, width of space $0{\cdot}52p$, width of tooth $= 0{\cdot}48p$.

INVOLUTE AND CYCLOIDAL SYSTEMS COMPARED. The cycloidal system, once widely used, has largely been supplanted by the involute system. The principal **advantages of the involute system** are as follows—

1. Economy in machining, fewer cutters being required for each pitch.
2. Both face and flank form one continuous curve on a wheel tooth, or one straight line on a rack tooth, and not two as in cycloidal gear teeth. This facilitates machining.
3. Involute gears are the *only* gears that can run at varying centre-distances and yet transmit uniform angular velocity. (This slight variation of gear centres necessarily modifies the pressure angle and increases the backlash—the main effect of *increasing* the pressure angle being to *increase* the load on the bearings,

as this varies with the secant of the pressure angle.) The property of driving satisfactorily at varying centre-distance makes involute gearing useful for driving rolls and other rotating pieces, the centre-distances between which vary from time to time.

4. Involute teeth are stronger for the same pitch, being stronger at the roots.

Disadvantages of involute gears are as follows—

1. There is a pressure on the bearings tending to force them apart. However, this also arises with cycloidal gears, except when actual contact occurs at the pitch point.
2. The contacting surfaces are both convex, except in internal gearing.

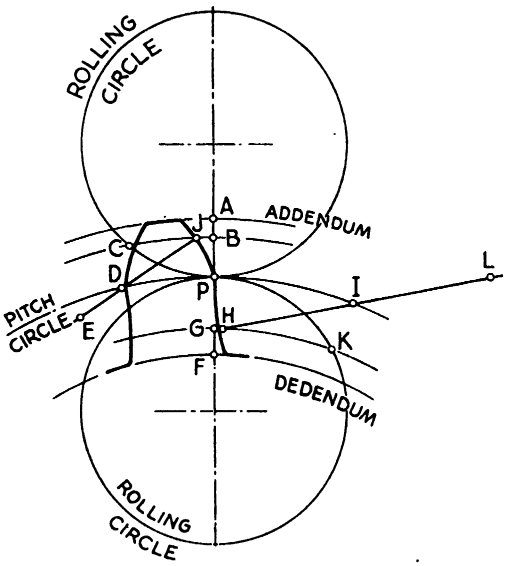


FIG. 45. PROFESSOR UNWIN'S METHOD

The supersession of the cycloidal form of tooth by the involute form indicates the superiority of the latter for most purposes, so that few **advantages of the cycloidal form** can be given.

However, it should be mentioned in favour of cycloidal teeth, that with contact at the pitch point there is no resultant pressure tending to thrust the bearings apart. Secondly, the form of the teeth is considered a good form for supporting loads, because the surfaces in contact are "one convex fitting into one concave."

Other Reminders

(1) The path of contact for cycloidal teeth is a curved line; for involute teeth it is a straight line.

(2) The pressure angle in all gears, except involute, constantly changes.

Approximate Drawing Methods

(1) **Professor Unwin's Approximate Method.** (Fig. 45.) Describe pitch, addendum, dedendum, and rolling circles. Make $PB = \frac{3}{4}PA$. Describe arc BC concentric with pitch circle. Make arc PD = arc PC. With centre D, and radius equal to chord PC, mark point J in arc BC. J is a point in the true epicycloid.

Join *JD*, and on *JD* produced, find, by trial, point *E* from which to describe arc *PJ*. *PJ* produced to the addendum gives the face of the tooth. Make $PG = \frac{2}{3}PF$. Describe arc *GK* concentric with pitch circle. Make arc *PI* = arc *PK*. With *I* as centre, and radius equal to chord *PK*, mark point *H* in arc *GK*. *H* is a point on the true hypocycloid. Join *HI*, and on *HI* produced, find, by trial, point *L* from which to describe arc *HP*. *PH* produced to the dedendum circle forms the flank of the tooth.

(2) **Brown & Sharpe Approximate Methods.** Approximate "circle constructions" yielding teeth for racks, gears, and pinions are described at length in the B & S publication, *Practical Treatise on Gearing*. These ingenious methods

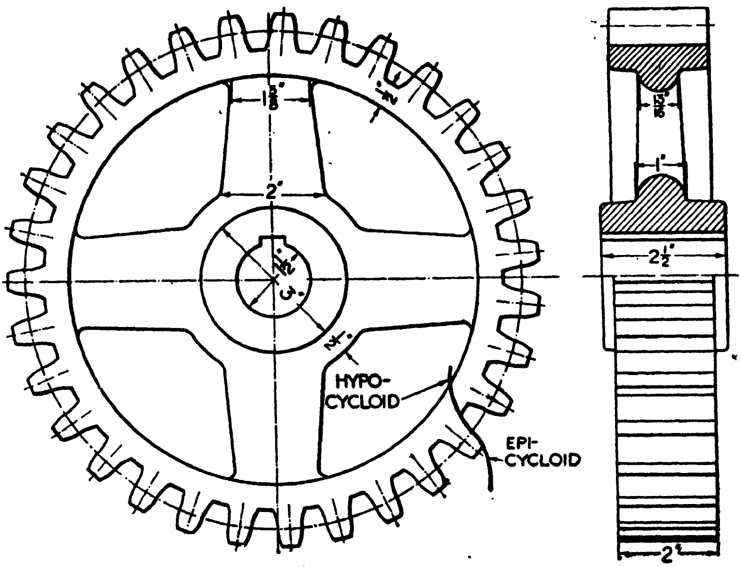


Fig. 46. Spur Gear with Cycloidal Teeth

comprise close approximations, and are suitable for laying out teeth of good form for patterns or for showing teeth on drawings.

(3) **Tracing Paper Method.** As a simple application of this method the following exercise has been devised. It is reproduced from *A First Year Engineering Drawing* by A. C. Parkinson. Re-examination of Fig. 39 will show a directing circle of 10 in. dia. and a rolling circle of $2\frac{1}{2}$ in. dia. Using these dimensions, the student should draw the two curves. Suppose we now think of a spur wheel having 31 teeth of 1 in. circular pitch (p). Then its pitch circle diameter is $31/\pi$, or 9·9 in. For the purpose of this introductory exercise, we can pardonably take this as 10 in. In Fig. 46 two views of this gear are given. The pitch circle being a bare 10 in. dia., 31 equal divisions can be stepped off around it. Each of these divisions can then be subdivided into two parts, in the ratio of 0·48 to 0·52, respectively for *tooth* and *space*. Thus in Fig. 47, AB = 0·48 in., BC = 0·52 in., proportions usually employed for cast cycloidal teeth. The next step is to take a piece of tracing paper and apply it to the scale drawing of Fig. 39, so as to take a clear tracing of (1) the pitch circle, (2) the epicycloid, as shown at (*b*). The tracing paper is then placed on the spur wheel drawing with the point *D* immediately over point *A* and with the tracing of the pitch circle in coincidence with the pitch circle on the spur wheel drawing. It is then possible to sketch over the tracing of the epicycloid curve with pressure sufficient to leave an impression on the drawing paper, which can later be inked in or

intensified in pencil, possibly with the assistance of a French curve. The tracing paper can then be shifted round so that *D* falls on *C*, *E*, etc., and the procedure repeated. In this way the left-hand faces are obtained. The right-hand faces are obtained by reversing the tracing paper and tracing the epicycloid through points *B*, *D*, *F*, etc. The flanks of the teeth are obtained in a similar manner by means of a tracing of the pitch circle and part of the hypocycloid, as shown at (*c*) in Fig. 47.

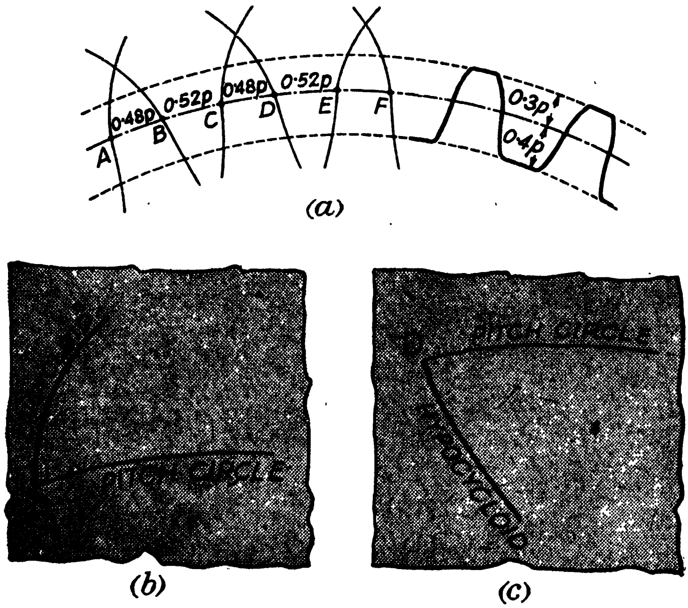


FIG. 47. TRACING PAPER METHOD

CHAPTER VI

GEARS FOR CLOCKWORK MECHANISMS

THE relevant B.S. Specification is No. 978—1941 (amended 1942). It was drawn up as a first step in standardizing gears used in clockwork mechanisms in order particularly to enable a reduction in the number of hobs required in manufacturing such gears. The term *clockwork mechanism* is defined as including gears used in clocks, watches, meters, and instruments. One manufacturer informed the Committee that he was being asked to make hobs of 800 different types—mainly because no universally recognized standard form of tooth existed. Much benefit will undoubtedly accrue to the tool-making industry and ultimately to the precision watch- and clock-making industry if further standardization can be agreed upon.

Involute, Cycloidal and Circular Arc Tooth Forms. In some clockwork mechanisms it is of the utmost importance to obtain the closest possible approximation to uniform velocity ratio. In such cases, the Committee state, *the involute tooth form is preferable* because it can more easily be produced to a high standard of accuracy than any other form in common use. Furthermore, only one hob is required for each pitch to cover all numbers of teeth. Its use is therefore recommended in all cases *except* (1) where the pinion has fewer than thirteen teeth, (2) in spring-driven escapement mechanisms for watches and clocks, particularly if the pinion has less than thirteen teeth and is the *driven* member of a pair.

For the purpose mentioned at the end of the preceding paragraph, a **cycloidal tooth form (or an approximation to it)** has heretofore been preferred, but tests made at the N.P.L. on an alternative form—based on a rack tooth profile composed of two circular arcs—showed no appreciable difference in efficiency as compared with the cycloidal form. Further, this **circular arc form** has the advantage that a single hob can be used to produce gears of any one pitch, irrespective of their numbers of teeth; whereas the cycloidal form requires at least eight hobs per pitch. The circular arc form is therefore officially recommended in B.S. No. 978 for spring-driven escapement mechanisms involving pinions having less than thirteen teeth. The arcs of approach and recess with this form are very similar to those with the cycloidal form.

Final B.S. Recommendation. The tooth form of gears for clockwork mechanisms shall wherever practicable be of **involute form,** except that for gears comprising a spring-driven escapement mechanism for clocks and watches, where the pinion is the driven member and it has fewer than thirteen teeth, the tooth form shall be based on the **double circular arc rack tooth form** (generally called the *circular arc form*).

3—(T.392)

B.S. NOTATION FOR THESE GEARS

A =	Wheel addendum	i =	Pinion root diameter
a =	Pinion addendum	J =	Wheel outside diameter
B =	Wheel dedendum	j =	Pinion outside diameter
b =	Pinion dedendum	m =	Module
C =	Centre distance	P =	Diametral pitch
c =	Bottom clearance	P_n =	Normal diametral pitch
D =	Wheel pitch diameter	p =	Circular pitch
d =	Pinion pitch diameter	p_n =	Normal pitch
F =	Face width	T =	Number of teeth in wheel
I =	Wheel root diameter	t =	Number of teeth in pinion

BASIC RACKS. The basic racks are shown in Fig. 48. The shape and proportions of teeth of **helical gears** on a section at right angles to

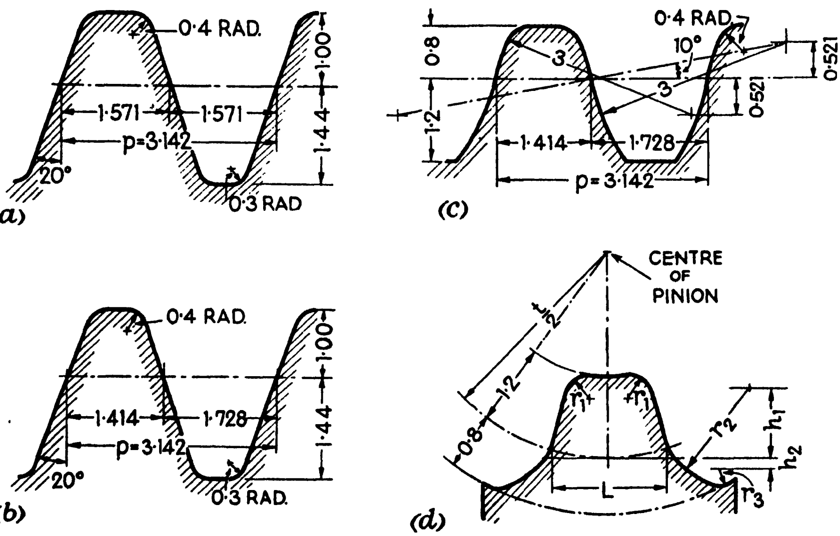


FIG. 48. RACKS AND HOB FOR GEARS FOR CLOCKWORK MECHANISMS

(*a*) Basic rack for pinions and wheels of involute form for minimum backlash.
(*b*) Basic rack for pinions and wheels of involute form with backlash allowance.
(*c*) Basic rack for pinions and wheels, teeth of circular arc form.
(*d*) Section of milling cutter for pinions of six to twelve teeth to mesh with gears generated from a circular arc rack. The tabulated sizes on page 56 refer to this diagram.
All dimensions are in terms of the module (m).

the helix shall correspond to the basic rack tooth forms shown in Fig. 48. It should be remembered that the **normal pitch** (p_n) of a helical gear is the distance between similar faces of successive teeth measured along a helix lying on the pitch cylinder normal to the teeth. The **normal diametral pitch** (P_n) is the diametral pitch of the normal basic rack.

$$P_n = P \sec \sigma = \pi / p_n$$

Definitions of *helix angle* (σ) and of other terms used in connection with helical gears will be found in Chapter VII

Range of Modules. B.S. No. 978 recommends that the sizes of the gear wheels should be expressed in terms of the module in mm. to conform to one of the following standard sizes—

0·08	0·22	0·5292 (48*P*)
0·085	0·24	0·55
0·09	0·2540 (100*P*)	0·60
0·095	0·26	0·6350 (40*P*)
0·1	0·28	0·65
0·11	0·2822 (90*P*)	0·70
0·12	0·2988 (85*P*)	0·7055 (36*P*)
0·13	0·30	0·7470 (34*P*)
0·14	0·3387 (75*P*)	0·75
0·15	0·35	0·7938 (32*P*)
0·16	0·3969 (64*P*)	0·80
0·17	0·40	0·8467 (30*P*)
0·18	0·4233 (60*P*)	0·90
0·19	0·45	1·0
0·20	0·4703 (54*P*)	
0·2116 (120*P*)	0·50	

When the use of intermediate sizes is unavoidable, they should be midway between the two standard sizes—excluding the diametral pitch sizes shown as *P* in the foregoing list.

B.S. Tolerances on Gears for Clockwork Mechanisms

(1) *Tolerance on Diameter of Gears.* (Module expressed in mm.)

Range of Module	Tolerance on Diameter	
	Wheel	Pinion
0 up to 0·12	+ 0 − 0·050	+ 0·050 − 0
Above 0·12 up to and including 1	+ 0 − 0·075	+ 0·075 − 0

(2) *Tolerance on Pitch of Teeth of Gears.* (Expressed in mm.)

$$\text{Tolerance} = \pm [0{\cdot}01 + 0{\cdot}003m\,(1 + Z)]$$

where Z = number of pitches over which measurement is made.

(3) *Tolerance on Thickness of Teeth of Gears.* (Expressed in mm.)

$$\text{Tolerance} = \begin{array}{l} +\,0 \\ -\,(0{\cdot}05m + 0{\cdot}005) \end{array}$$

(4) *Tolerance on Tooth Profile of Gears.* (Expressed in mm.)

The profile of a gear tooth shall not deviate from the theoretical one which it intersects at the pitch circle by more than:

$$\begin{array}{l} +\,0 \\ -\,(0{\cdot}005m + 0{\cdot}01) \text{ millimetre.} \end{array}$$

(5) *Tolerance on Centre-distance of Assembled Gears.* (Expressed in mm.)

The tolerance shall be to Grade A or Grade B as specified by the purchaser:

$$\text{Grade } A \pm \left(\frac{m}{50} + 0{\cdot}004\right), \text{ Grade B} \pm \left(\frac{m}{50} + 0{\cdot}006\right)$$

Hobs and Cutters

Tolerance on the pitch of teeth of hobs: $\pm[0{\cdot}007 + 0{\cdot}002m(1 + Z)]$ millimetres, where Z = number of pitches over which measurement is made.

Tolerance on thickness of teeth of hobs: $\begin{cases} +\ (0{\cdot}03m + 0{\cdot}003) \\ -\ 0 \end{cases}$

Tolerance on tooth profile of hobs: The profile of a hob or cutter shall not deviate from the theoretical one which it intersects at the pitch line by more than:

$$\begin{cases} +\ (0{\cdot}004m + 0{\cdot}007) \text{ millimetre.} \\ -\ 0 \end{cases}$$

The positive sign indicates an excess of tooth material. Note that the B.S.I. define *profile error* as *the maximum deviation of any point on the tooth surface from the designed profile passing through the point of intersection of the actual curve and the pitch line.*

Dimensions of Cutters. (See Fig. 48 (*d*).)

t	6	7	8	9	10	11	12
$\frac{t}{2}$	3	3·5	4	4·5	5	5·5	6
r_1	0·370	0·430	0·460	0·460	0·460	0·460	0·460
r_2	1·575	1·615	1·653	1·690	1·730	1·767	1·805
r_3	0·295	0·319	0·342	0·365	0·365	0·365	0·365
h_1	1·015	0·930	0·870	0·820	0·775	0·740	0·706
h_2	0·135	0·165	0·190	0·190	0·214	0·238	0·262
L	1·704	1·710	1·714	1·717	1·719	1·7205	1·722

The foregoing table shows dimensions of milling cutters for pinions of six to twelve teeth to mesh with gears generated from a circular arc rack. All dimensions are expressed as multiples of the module. See Fig. 48 (*d*), which, as well as this table, is reproduced with permission from B.S. No. 978—1941 (amended 1942).

A U.S.A. Standard. Professor Earle Buckingham, in Vol. 2 of his *Manual of Gear Design*, describes a 20° basic rack system of involute

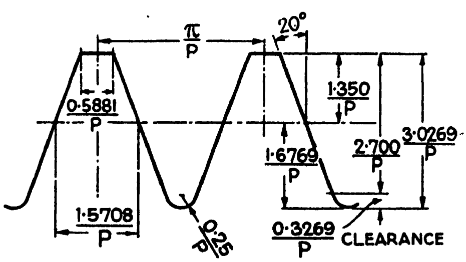


Fig. 49. A Basic Rack for Fine Pitch Gears (Earle Buckingham)

gears of fine pitch for instruments and clock mechanisms. The proportions of the basic rack are shown in Fig. 49.

The teeth of fine pitch gears are deep and the clearance is liberal. If the number of teeth in a pinion is less than 30, he recommends that the diameters be enlarged to avoid undercutting, resulting in an increase in the centre-distance when the pinion is meshed with its mating gear. In the *Manual of Gear Design* will be found detailed tables giving the proportions of 1 D.P. gears, the increase in centre-distance for non-standard gears, and the contact ratio. For other diametral pitches, the tabulated dimensions are divided by the D.P. used. If the pinion has less than 30 teeth, Buckingham recommends that the backlash be obtained by cutting the teeth of the mating gear deeper. Where the pinion has more than 30 teeth, the recommendation is to cut the teeth deeper in both mating gears.

Swiss firms, Mikron, for instance, supply hobs for clockwork mechanisms, and in this connection it may be noted that DIN standards give an addendum of $1{\cdot}0m$, with dedenda $1{\cdot}1m$, $1{\cdot}2m$, or $1{\cdot}3m$, for gears for fine instruments, for clockwork mechanisms, etc., having involute tooth forms. The pressure angle of a Mikron hob is normally $14\frac{1}{2}°$ or $20°$, but $17\frac{1}{2}°$ is used occasionally. Readers will note that in B.S. No. 978—1941 (Gears for Clockwork Mechanisms) a dedendum of $1{\cdot}44m$ is recommended. During the 1939–45 war small ordinary relieved hobs were produced in England and America, with pitches as fine as 200 D.P. and hobs ground on form of 180 D.P. It is interesting to note that the A.G.M.A. include diametral pitches of 30 to 200 in their current lists; also that for gears of 30 D.P. and finer a new series of war-time recommendations were made, allowing additional clearances at the bottoms of tooth spaces, partly for "dirt clearance" and partly to overcome the dulling of the tips on the generating tool. The A.G.M.A. formulae given below utilize a variable whole depth factor, which provides a greater percentage of clearance as the pitch becomes finer.

$$\text{Working depth} = \frac{2{\cdot}000''}{\text{D.P.}}$$

$$\text{Whole depth} = \frac{2{\cdot}200''}{\text{D.P.}} + 0{\cdot}002''$$

$$\text{Clearance} = \frac{0{\cdot}200''}{\text{D.P.}} + 0{\cdot}002''$$

GENERAL NOTES ON GEAR TEETH USED IN HOROLOGICAL WORK. For very many engineering purposes standard full-depth involute teeth are employed, the addendum being made equal to the module. On such gears the overall diameter is equal to the pitch diameter plus twice the module. To many watch- and clock-makers such gears look as though they have been "topped" and not "rounded up" afterwards. In watch and clock work, where the ratio of the number of teeth in the wheel to the number of teeth in the pinion is large, it is usually advisable to retain the whole of the addendum right up to where the two opposed profile curves meet in a point. The *driven pinions* of watches usually have teeth (or "leaves") with semicircular roundings above the pitch circle, such roundings being convenient to reproduce on small watch pinions. Quite commonly the width of the pinion leaf at the pitch circle equals one-third of the circular pitch, and addendum equals one-sixth of the circular pitch. In such cases the overall diameter equals pitch diameter plus one-third of circular pitch.

If the pinion has to drive a wheel the semicircular rounding of its leaves is not suitable. If the cycloidal system is used the "faces" of the leaves will be epicycloidal.

The teeth on many wheels and pinions used in both clocks and watches have "flanks" which are *radial*, i.e. they lie on lines passing through the centre of the gear and meeting (in a sharp corner) a "root circle" concentric with the pitch circle.

These "square-rooted" teeth are weak; "filleted" or "radiused" roots are stronger. The modern tendency is to provide curved or filleted corners to the roots throughout these trains. After all, the points of the addenda of neither wheel nor pinion can ever get into the sharp re-entrant angles of straight-flanked tooth spaces and, apart from considerations of strength, it is evident that sharp corners harbour accumulations of dirt. In regard to the contacting parts of mating teeth, the fundamental condition for the transmission of uniform velocity ratio at any instant applies, no matter how small or how large the job: *the common normal to the curves at their point of contact must pass through the pitch point.*

Large numbers of watch and clock gears and pinions in use throughout the world have teeth of "cycloidal" form, i.e. hypocycloidal and epicycloidal curves are used, both being traced out by equal rolling circles. In Chapter V we emphasize that when the diameter of the rolling circle is one-half that of the

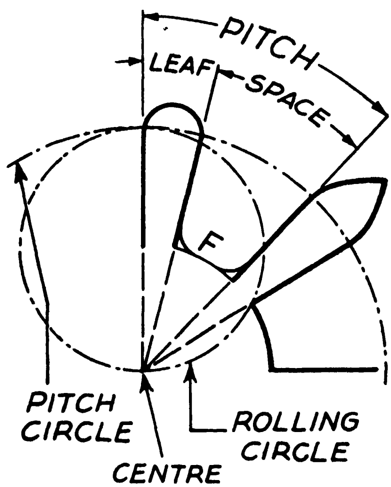


FIG. 50. TEETH OR LEAVES OF PINION—TYPICAL FORMS

directing circle (inside which it rolls), the resulting hypocycloid is a straight line (a radial line). In watch and clock work we almost always find that the flanks of the teeth of pinions are radial. Thus the rolling circle used to trace out the epicycloid, used for the faces of the teeth in the mating gear, must have a diameter equal to one-half of the pitch diameter of the pinion.

Suppose we have a pinion with 10 leaves driven by a wheel with 75 teeth. The rolling circle must be half the size of the pitch circle of the pinion. Thus if the diameter of the rolling circle equals 5 units, and the diameter of the pitch circle of the wheel equals 75 units, the ratio is $\frac{1}{15}$. Using the rule given in Chapter V, in relation to Fig. 39, we see that

$$\text{Angle } \theta = \frac{360 \times \text{Rolling circle diameter}}{\text{Directing circle diameter}}$$

$$= \frac{360 \times 1}{15} = 24^\circ$$

We could take inch-units and follow the construction fully explained in relation to Fig. 39. In that case, radius *OD* would be $7\frac{1}{2}$ in., and an arc of radius 8 in. would be the locus of the centre of the rolling circle. The angle *DOE* would be 24°. The epicycloid would be drawn as explained in Chapter V, but the hypocycloid would be a radial straight line, e.g. *DO* or *EO*. Of course, only a comparatively small part of the epicycloid is used. In the wheel of 75 teeth the pitch is nearly 5° out of the total angle of 24°. If tooth space equals tooth width, this reduces to about $2\frac{1}{2}$°. Therefore if we set off an angle of $2\frac{1}{2}$°, inwards from

DO and *EO*, the parts of the curves within these angles represent the opposed faces of a tooth.

Fig. 50. Teeth or Leaves of Pinion. Typical Forms. Rolling circle diameter equals half the pitch circle diameter, thus flanks are radial straight lines. The leaf occupies one-third of circular pitch measured at the pitch circle. One leaf is shown with semicircular rounding above the pitch circle, the centre for the semicircle being on the centre of the chord of the tooth-arc. The pointed tooth is suitable for a pinion which has to drive a wheel twice its size. The curved

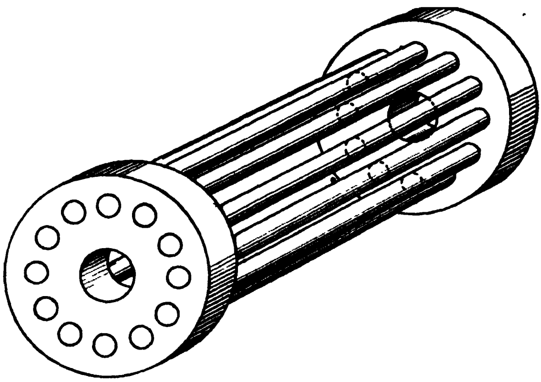


FIG. 51. LANTERN PINION

faces of the pointed teeth are epicycloids; the flanks of the mating wheel would be radial straight lines. The fillet curves shown at *F* illustrate previous remarks on filletted roots.

Lantern Pinions. (Fig. 51.) These are sometimes found in the going parts of turret clocks. Instead of having leaves integral with the material forming the body of the pinion, they have cylindrical pins mounted between two metal discs. Whilst not strong enough for striking parts, they may be quite suitable for the going train. The pitch circle is on the centre of the pins, the diameter of which should be rather less than half the circular pitch. The mating gear has teeth with epicycloidal faces traced out by a rolling circle, which proves suitable if its diameter equals half the diameter of the pitch circle of the lantern pinion minus the diameter of one pin.

CHAPTER VII

INTRODUCTION TO GEAR-CUTTING PRINCIPLES AND PRACTICE

GEAR cutting is a very extensive subject and therefore cannot be covered in detail in a single chapter. However, it is advantageous for draughtsmen, inspectors, etc., to have an outline knowledge of the processes commonly employed in cutting the teeth on the various classes of gears. Every inspector concerned with the examination of gears should endeavour to link up his work with practical observation of actual gear cutting, of as many different types of gear cutting machines as possible, and of the cutters employed. Different methods are applied to different classes of work, each method having its own relative advantages and defects.

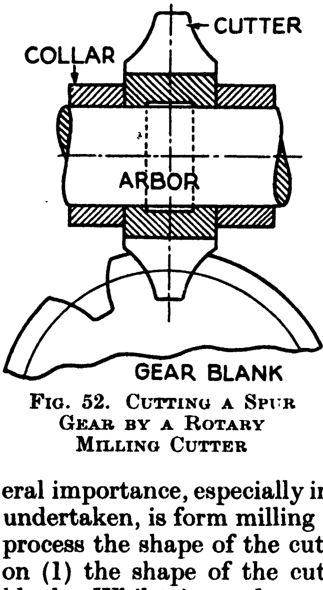


FIG. 52. CUTTING A SPUR GEAR BY A ROTARY MILLING CUTTER

The various processes employed in gear cutting can be divided into two classes, viz. **forming** and **generating.**

The **forming process** is a **copying process** in which we use an existing profile on a formed disc-shaped milling cutter, a formed end-milling cutter, a planing tool, or a form-dressed grinding wheel. Alternatively it may be the shape of a template or former used, as in the form-planing process, as part of the mechanism which guides a reciprocating single-point tool round each individual tooth profile. So far as spur gearing is concerned, however, the only forming process of any general importance, especially in smaller shops, where very large work is not undertaken, is form milling by means of rotary milling cutters. In this process the shape of the cut groove, or tooth-space, depends primarily on (1) the shape of the cutter, (2) its setting in relation to the gear blank. Whilst in modern well-equipped works one of the generating processes is nearly always used, especially in mass-production, form-milling has by no means been superseded entirely, especially in jobbing shops where gears are produced singly or in small numbers. In many works there is no modern special-purpose gear-cutting equipment, but equipment with universal milling machines, or with horizontal milling machines having dividing heads, enables the employment of the form milling cutter process which is very straightforward although comparatively slow. For some very large gears the only practicable means of cutting the teeth is by end-milling cutters.

SPUR GEARS

FORMING BY MEANS OF ROTARY DISC-CUTTERS. Fig. 52 is a simple line drawing of a rotary milling cutter in the process of cutting teeth in a pinion blank.

Suppose it were necessary by this means to cut a range of spur gears having the same pitch but different numbers of teeth. If teeth of strictly accurate shape were required it would be necessary to employ a different cutter for every different number of teeth. The shape of the teeth and tooth-spaces of, say, 10*P* are not the same on a pinion of 20 teeth as they are on a wheel of 130 teeth. On a small gear the curvature of the involute tooth profile is immediately obvious, on a large gear it is less, on a rack it is non-existent. In practice considerations of cost preclude the purchase of cutters for every possible number of teeth of the same pitch, so a compromise is made by the use of a set of eight cutters for involute teeth of the same diametral pitch. Actually, of course, the shape of any one of these cutters is correct only for one particular number of teeth, but it can be used for other numbers as shown in the following table.

No. of B & S Cutter	To Cut Gears from	No. of B & S Cutter	To Cut Gears from
1	135 teeth to a rack	5	21 to 25 teeth
2	55 to 134 teeth	6	17 to 20 teeth
3	35 to 54 teeth	7	14 to 16 teeth
4	26 to 34 teeth	8	12 and 13 teeth

From the table we see that a No. 5 cutter can be used to cut gears of a certain diametral pitch having 21 to 25 teeth, but actually the tooth profiles would be correct only for the 21-tooth gear, i.e. the lowest in the series. When the No. 5 cutter is used to cut a 25-tooth gear it mills away too much metal from the tips of the teeth. This is not entirely disadvantageous for, as we have seen, *easing* of the tips tends to facilitate smooth running and to reduce noise. However, if the tooth faces are too sharply curved the effective length of tooth face is reduced with consequent reduction of the length of the arc of engagement. These adverse effects, however, are of very limited extent and in the great majority of cases are of little moment. If greater accuracy is desired cutters are obtainable in half-numbers, as shown in the following table.

No. of B & S Cutter	To Cut Gears from	No. of B & S Cutter	To Cut Gears from
1½	80 to 134 teeth	5½	19 to 20 teeth
2½	42 to 54 teeth	6½	15 to 16 teeth
3½	30 to 34 teeth	7½	13 teeth
4½	23 to 25 teeth	—	—

The cutters are sold in three pitches, viz. circular, diametral and module, also as roughing or stocking cutters, and as finishing cutters. In all cases the teeth are form-relieved.

Accuracy of the Tooth Profile. The shape of the tooth profile, and consequently also of the tooth-space, depends primarily upon (1) the shape of the cutter,

(2) the setting of the cutter in relation to the blank, (3) the setting and rigidity of the machine. If the cutter is set off-centre, it is obvious that misshapen teeth will be produced. See Fig. 53, which shows an obviously exaggerated example, the result of setting the cutter off-centre to a considerable extent.

If the work-table of the machine is set square with the cutter arbor, but the cutter is slightly off-centre in relation to the blank, it can be detected by taking a cut through a surplus or dummy blank; afterwards, having turned the blank end-for-end, another cut can be taken through the tooth-space originally cut. If the setting is correct, no metal will be removed during the second pass; if the setting is incorrect, the cutter will, by milling away metal, reveal the inaccuracy. Of course, the machine table must be set square with the cutter arbor or the teeth will be "out of parallel" with the axis of the blank.

Forming by End-milling Cutters. This is the method resorted to for machining teeth of spur and helical gears of large pitch. (See Fig. 77.) End milling cutters are cheap by comparison with form cutters of disc or rotary type.

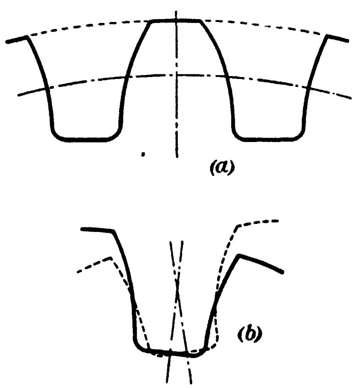


FIG. 53. TYPICAL EFFECTS OF SETTING THE CUTTER OFF-CENTRE

GENERATING METHODS

Sunderland Rack-planing Process. This is probably the easiest method to understand and is certainly the best to start with in an elementary text-book, for it has such an obviously direct connection with the basic rack.

We have seen that in the involute system the rack is straight-sided. Hence a cutter of rack-like form is easily and cheaply made, even when ground all over. The gear blank is mounted on a mandrel which is geared to a slide carrying the cutter *C* (Fig. 54). The cutter is practically identical in form with the rack that would mesh correctly with the gear being cut. The cutting face of the rack is perpendicular to the axis of the gear being cut. If the blank is made of some soft material it is evident that if the rack moves uniformly, the blank meanwhile rotating at correct relative speed, teeth of correct involute form will be moulded or generated on the blank.

In practice, of course, the gear blank consists of some hard metallic material, so that the rack must not merely "mould" but cut. It must therefore be made of hardened tool steel and in addition to the straight line motion indicated in Fig. 54 must be given a "to-and-fro," "planing," or "rectilineal reciprocating" motion parallel to the axis of the blank. We have, therefore, (1) cutting strokes of the cutter across the face of the blank, (2) a rotary motion of the blank, (3) a straight line motion of the cutter tangential to the pitch surface of the blank. Motions (2) and (3) are geared to act in unison and together comprise the *generating action*, comparable to the rolling together of a rack and pinion, the rack being represented by the cutter.

Rack cutters of short length are employed so that when the last tooth of the cutter has grazed the outside of the blank (Position 4 in

Fig. 54) the generating motion ceases, the rack cutter is withdrawn from the work and moved back to Position 1, a distance of one or more complete pitches. During this interruption of cutting—or "indexing" as it is called—the blank remains stationary. The cutting or reciprocating motion of the cutter is at a fairly high speed, whereas the generating motions are relatively slow. On small work the teeth may

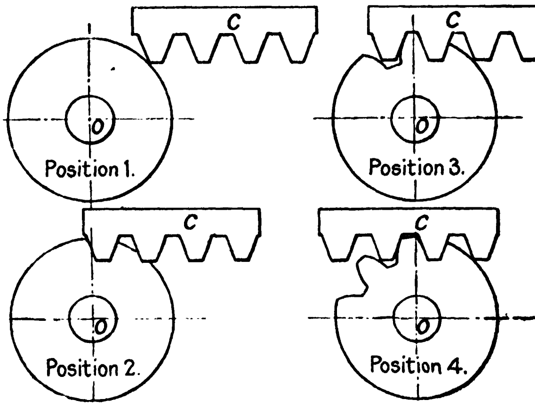


FIG. 54. CYCLE OF OPERATIONS IN GENERATION BY RACK CUTTER

be finish-cut during one rotation of the blank, but when teeth of the larger pitches are being generated, two or more rotations are necessary.

All involute gears of the same pitch and pressure angle are interchangeable with one another and with the same basic rack. Thus, one rack cutter enables us to cut any gear of a given pitch whatever its number of teeth. This generating system was first introduced by Sunderland. Messrs. J. Parkinson & Son, Shipley, Yorks, make a range of "Sunderland" gear-planing machines for spur and helical gears as well as auxiliary gear-cutting and gear-measuring equipment.

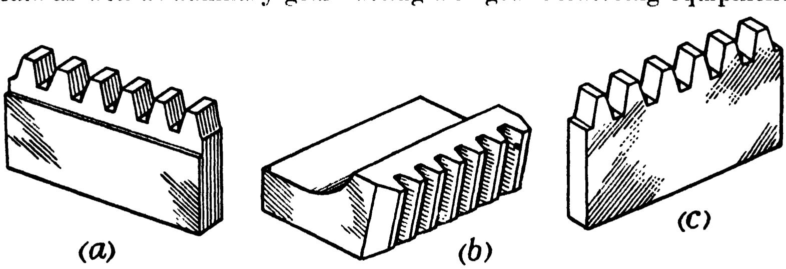


FIG. 55. RACK CUTTERS (*a*) AND (*b*). BACKING PLATE AT (*c*)

All gears which mesh correctly with the same rack have the same base pitch.

Fig. 55 (*a*) shows a *regular* cutter, and at (*b*) an *upset cutter* as sometimes used to cut the smaller portions of compound gears. So as to support the teeth and prevent breakage a *backing plate* is required. (See Fig. 55 (*c*).)

Fig. 56 (*a*) shows views of a rack cutter without top rake. It could generate a basic rack and a series of gears of pressure angle ψ_n. The front cutting face *CC* is perpendicular to the cutting motion of the cutter.

Fig. 56 (*b*) shows views of a rack cutter with top rake for free cutting. If such a cutter were being checked by optical projection the profile

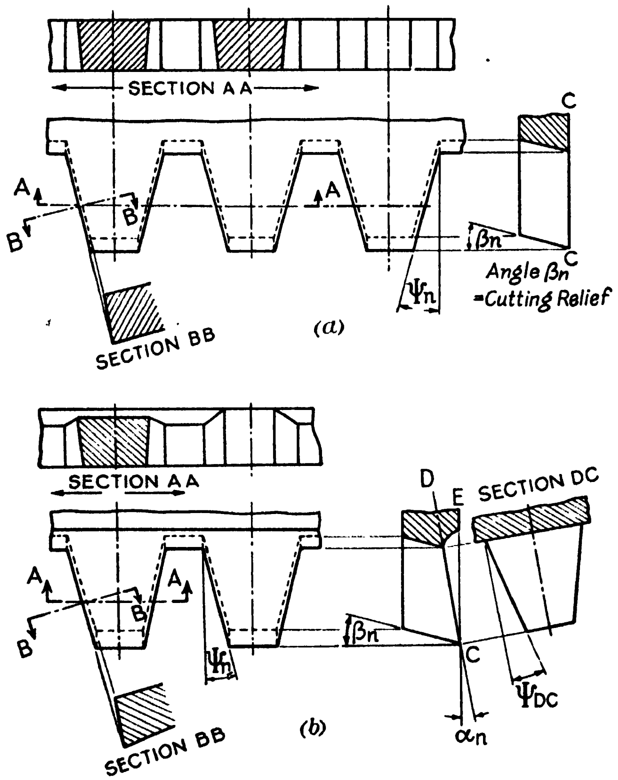


FIG. 56. RACK CUTTERS WITH AND WITHOUT TOP RAKE

in the plane *DC* would be checked, the inclination ψ_{DC} being calculated from,

$$\tan \psi_{DC} = \tan \psi_n \times \cos \alpha_n$$

The Maag Rack-planing Process. This generating process is basically similar in principle to the Sunderland process, although different mechanically. The blank is caused to roll along the pitch line of a reciprocating cutter of rack-like shape just as if the blank were a finished gear meshing with the rack represented by the cutter.

The gear blank is mounted either upon a horizontal work-table or on a vertical arbor projecting from it, whilst the tool moves up and down so as to cut across its face, rather like a tool in a slotting machine. On its down-stroke the tool cuts the teeth in the blank; before moving upwards on its return stroke, the tool is withdrawn out of contact with the blank.

In the *generating motion* of the blank we have two co-ordinated motions, viz. (1) *Rotation* resulting from the rotation of the work-table of the machine, (2) *Motion of the work-table in a straight line* parallel to the pitch line of the rack cutter. Thus the blank moves "round and along," but only during the upward movement of the cutter.

Indexing is necessary as in the Sunderland process, the blank being "brought back" after a generating motion of two or three pitches. During indexing the cutter remains stationary in its top-most position and the rotation of the work-table ceases, i.e. there is an interim suspension of both cutting and generating motions.

The *cutters* are ground with flat faces for cutting aluminium, bronze, cast iron, etc.; but for steels and other hard materials the upper faces of a cutter are advantageously hollow-ground over the whole face for pitches below module 10, or hollow ground along each cutting edge for larger pitches, as shown in Fig. 57.

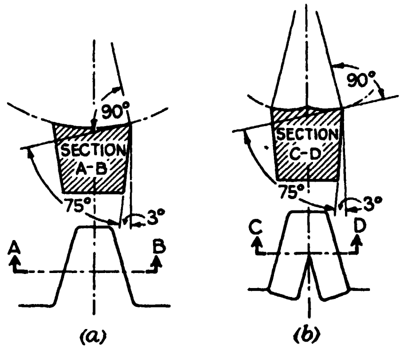


FIG. 57. HOLLOW-GROUND FACES OF MAAG RACK CUTTERS

The Gear Shaper or Pinion-type Cutter Process. This rapid and popular generating process was first introduced by the Fellows Gear Shaper Co. (Springfield, U.S.A.), who employed it for cutting straight-toothed gears and later for cutting helical gears. Whilst Fellows gear shapers are largely employed all over the world, substantially the same generating methods are incorporated in other machines such as the Sykes and Drummond "Maxicut" machines, in which pinion-shaped cutters are used.

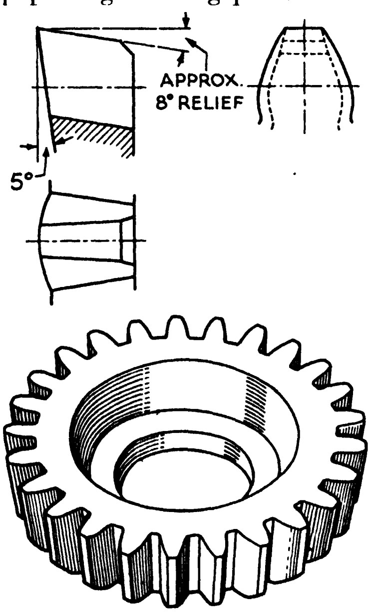


FIG. 58. PINION-TYPE CUTTER FOR GEAR SHAPER

The pinion cutter (see Fig. 58) has teeth of involute form and consequently we may look upon it as derived from an involute rack of the same pitch and pressure angle. Thus, all gears produced by a particular cutter mesh correctly with a common basic rack and therefore with one another. Only one cutter is necessary in order to generate teeth on any number of gears of the same pitch, although their tooth-numbers may all be dissimilar, whereas a rotary milling cutter is correct only for the particular number of teeth for which its profile is designed.

The Cutting Process. The cutter passes rapidly up and down across the face of the blank, making up to 600 strokes per minute (S.P.M.)

on high-speed machines. For instance, the number of S.P.M. on the *Maxicut Gear Shaper*, made by Messrs. Drummond Bros., Guildford, Surrey, varies from 400 to 600 according to material and length of stroke. These machines will cut either on the *pull* or *push* strokes and can be arranged to finish the gear in one, two, or three complete turns —two being the most common. The choice of either the *pull* or *push* method is made according to the type of job, the *push* method probably being most commonly used on Maxicut machines, whereas the *pull* method is recommended for use wherever possible by the makers of the Fellows machines. On jobs like cluster gears there is insufficient

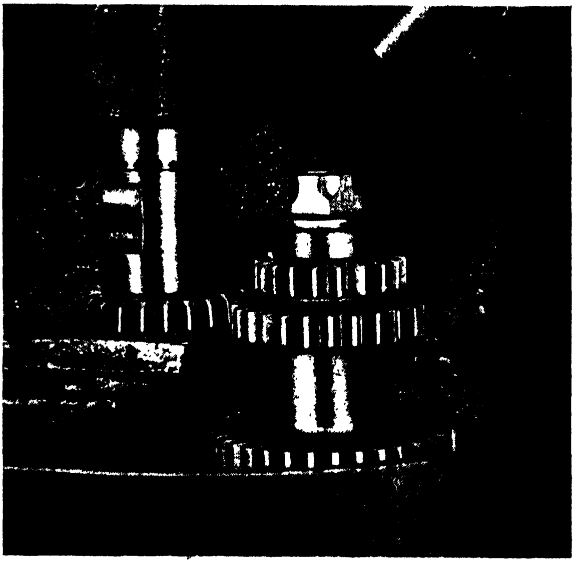


FIG. 59. GEAR SHAPING BY MEANS OF PINION-TYPE CUTTER IN THE D.B.S. WORKS

clearance to allow for the cutter to work except face downwards. On internal gears the *push* cut must be used. The work spindle or mandrel is withdrawn from the cutter on the return stroke by a *relieving mechanism*, subsequently being returned to its location for the cutting stroke. When cutting on the *push* cut the work relieving mechanism must be set to operate on the opposite stroke to that employed for the *pull* cut. When down-cutting, the relief takes place on the up-stroke of the cutter, whilst the up-cutting relief takes place on the down stroke. The cutter is first gradually fed inwards until the correct depth has been reached. Then the generating movements are commenced so that the cutter and the gear in course of generation revolve together like two mating gears in mesh at rates inversely proportional to their respective numbers of teeth. This process, like the hobbing process, is continuous, i.e. it is not interrupted by reversing or indexing movements such as occur in the rack planing method.

Direction of Rotation. When cutting external spur gears, the cutter spindle and the work spindle are rotated in opposite directions. When cutting internal spur gears the spindles rotate in the same direction.

Fig. 60 shows methods of mounting cutters on a *Maxicut* machine.

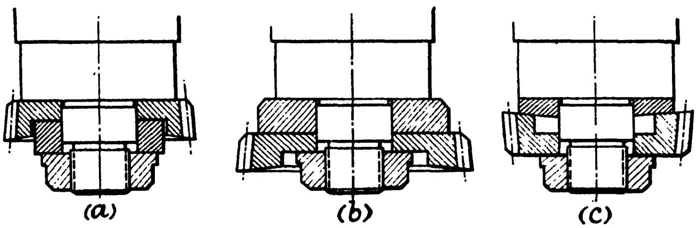


FIG. 60. METHODS OF MOUNTING STANDARD CUTTERS ON MAXICUT MACHINES

At (*a*) and (*b*) the setting is for *push* or down-cutting; at (*c*) for *pull* or up-cutting. Other types of cutters, known as *shank cutters* and *hub cutters* are supplied for jobs like internal gears of small pitch diameter, particulars of which are obtainable from the makers.

Fig. 61 shows the cutting of grouped blanks, plate clutch discs, on

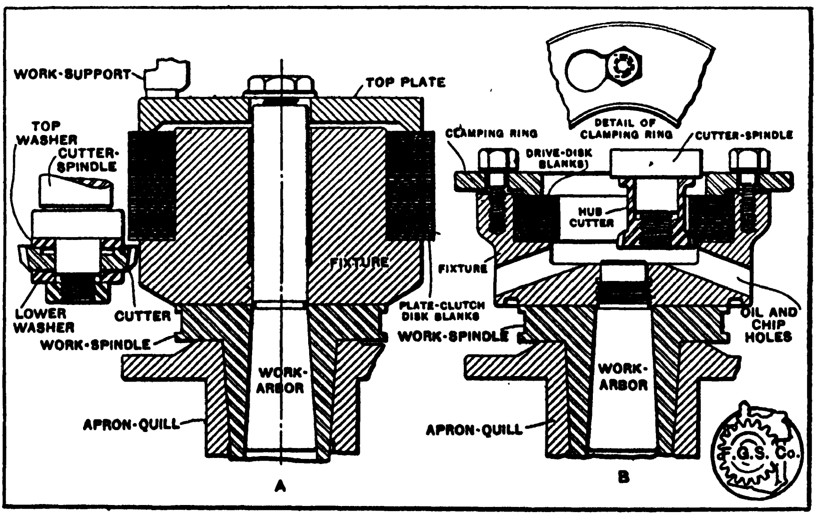


FIG. 61. METHODS OF HOLDING PLATE CLUTCH DISCS ON FELLOWS MACHINE

Fellows machines. In each case a special fixture is used. At *A* the *pull* stroke is used, at *B* the *push* stroke is used for the hub-cutter.

The Hobbing Process. We have previously explained that in the rack-cutter process of gear tooth generation the gear blank is rolled with the equivalent of a rack, and that in the pinion-cutter process the blank is rolled with the equivalent of a pinion. In the hobbing process the teeth on the blank are generated during the rolling of the blank with the equivalent of a spiral gear,[1] the cutter being a hob, i.e. a worm-like cutter which may be likened to a special form of spiral gear provided with relieved cutting edges. By means of a hob it is possible to generate teeth on spur and spiral gears which would mesh correctly with any other gears of the same pitch produced, for instance, by means of a rack cutter used to generate the teeth in the spiral gear of which the hob is the prototype.

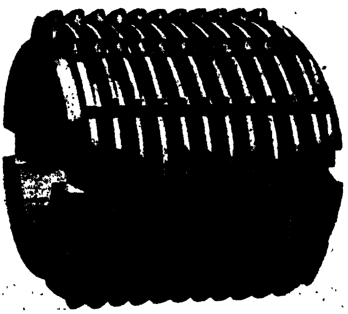


FIG. 62. SINGLE-START HOB FOR GENERATING SPUR AND HELICAL GEARS

Fig. 62 shows a typical single-start hob, its general likeness to a worm being readily apparent: It has a number of flutes or gashes milled across it at right angles to its thread helix. Between the gashes the teeth are so relieved or "backed off" that if their cutting faces are radially ground during sharpening, the outside diameter of the hob is reduced but the normal sections of the teeth remain unchanged. The normal pitch of the teeth of the hob is equal to the pitch of the teeth on the gear being cut. The speed of rotation of a hob is selected to give a suitable cutting speed for the material being cut.

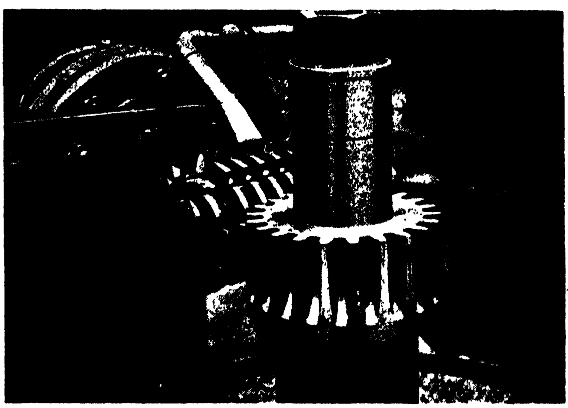


FIG. 63. HOBBING A SPUR GEAR

[1] A *worm* may be considered as a special form of spiral gear, i.e. a spiral gear having a single tooth and a very large spiral angle (nearly 90°). Single-start worms are readily threaded in a lathe, or can be thread-milled or thread-ground.

Fig. 63 shows the teeth on a spur gear being cut by means of a single-start hob. The blank is mounted on a vertical spindle, whereas the hob is mounted on a spindle so inclined that the threads on the hob are set parallel to the axis of the gear blank in that part of the rim where cutting occurs. Remember that the pitch of a spur gear hob is measured normal to its thread helix. The B.S. symbol for the lead angle of a worm is λ.

On hobs with small lead angle (e.g. single-start hobs) the normal section of the tooth-space is the same as that of the tooth on the basic rack corresponding to the gear being cut. As on a worm or helical gear, the axial pitch of the hob is larger than the normal pitch, but the difference is slight on a hob having a small lead angle. The normal pitch of the hob must be equal to the normal pitch of the gear being cut.

The spindles on which a single-start hob and gear blank are mounted are connected by a train of gears[1] so that the hob rotates once for every "pitch" that the gear blank rotates. This assumes that the hob is single-start, i.e. has one helical thread. When fed into the work, and across the face of the blank, the hob generates teeth in the blank. Therefore in the co-ordinated rotation of the hob and blank we may imagine the meshing of the generated gear with a spiral gear (a single-start worm) represented by the hob.

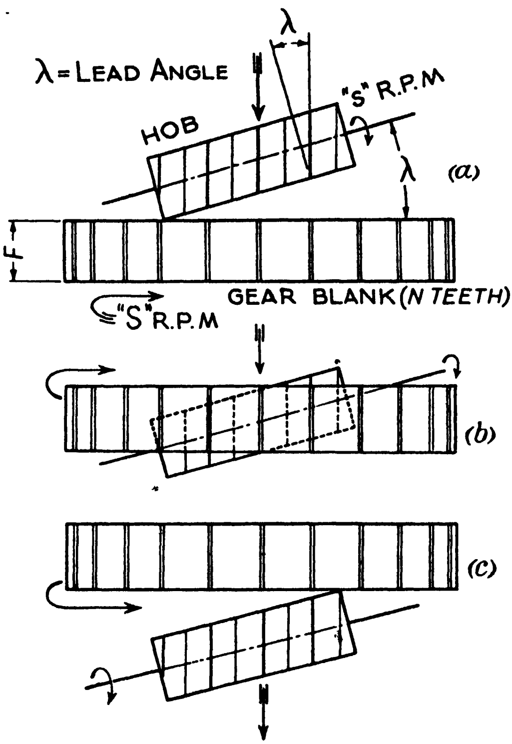


Fig. 64. The Hob in Relation to the Spur Gear Blank

Thus, in hobbing a spur gear we have (1) rotation of hob, (2) rotation of blank, (3) feed or traverse of hob parallel to axis of blank. This is illustrated in Fig. 64 which shows three positions of the hob in relation to the blank. Teeth on the length F of the rim of the blank are cut during the traverse of the hob from position (a) to position (c), during which time the blank has rotated in proportion.

[1] The *gear ratio* is as follows: If N = No. of teeth in required gear, n = No. of starts in hob, s = r.p.m. of hob, S = r.p.m. of gear blank, then $S \times N = s \times n$; $S/s = n/N$.

Advantages of this process. Hobbing is a smooth continuous process at regular speed, without reciprocating motion. This conduces to accuracy. No time is wasted in indexing. The hob is a multi-toothed cutter requiring less frequent grinding than cutters having only one cutting edge or face.

Climb-hobbing, or "climb-cutting," developed in the production of automobile splined shafts, has been successfully applied to gear cutting.

Climb-hobbing differs from the conventional method of hobbing, inasmuch as in climb hobbing the cutting action starts at the surface of the part and finishes at the root of the tooth, whereas in the usual method of hobbing the cutting action starts at the root and finishes at the surface of the part.

Climb-hobbing results in a better surface finish with little or no burr, accompanied by a lower power consumption, in addition to increased hob-life amounting to about 25 per cent more production per hob.

D.B.S. Hobs for Spur and Helical Gears. These high-speed steel hobs are generated to cut gears with 20° pressure angle and are supplied from stock in many sizes, from 1*P* with an outside diameter of 10¾ in., to 50*P* with an outside diameter of 1⅛ in. A standard hob in this series is right-hand single thread and is designed to generate gears that will mesh with a British Standard Basic Rack for Class *A*—2 and *B* Precision and High-class Cut Gears (see Fig. 32 in this book). Hobs from 1*P* to 6*P* have helical flutes; hobs from 7*P* to 50*P* have straight flutes.

Identification Marks. All hobs are marked with data useful in setting and sharpening, including pitch and pressure angle of gears cut, cutting depth, hand, number of threads, worm angle, lead of gashes.

D.B.S. MANUFACTURING TOLERANCES.
HOBS FOR SPUR AND HELICAL GEARS

(Data supplied by the makers)

Pitch (*P*) (diametral)	Spacing between adjacent gashes (inches)	Accumulated spacing between any two gashes (inches)	Pressure Angle	Axial spacing between successive teeth (inches)	Axial spacing between any two teeth (inches)	Parallelism of end faces (inches)	Bore Diameter (inches)
1	0·0021	0·0037	+ 0 – 0° 3′	0·0012	0·0021	0·0004	+ 0·0002 – 0·000
1¼	0·0017	0·0030	+ 0 – 0° 3′	0·0010	0·0017	0·0004	+ 0·0002 – 0·000
1½, 1¾	0·0015	0·0027	+ 0 – 0° 3′	0·0008	0·0014	0·0003	+ 0·0002 – 0·000
2, 2¼, 2½	0·0010	0·0019	+ 0 – 0° 3′	0·0006	0·0011	0·0003	+ 0·0002 – 0·000
2¾, 3, 3½	0·0009	0·0017	+ 0 – 0° 3′	0·0005	0·0009	0·0002	+ 0·0002 – 0·000
4, 5, 6	0·0008	0·0014	+ 0 – 0° 3′	0·0004	0·0007	0·0002	+ 0·0002 – 0·000
7, 8, 9, 10	0·0007	0·0012	+ 0 – 0° 3′	0·0003	0·0006	0·0001	+ 0·0002 – 0·000
12, 14, 16, 18 20, 22, 24	0·0005	0·0009	+ 0 – 0° 3′	0·0003	0·0005	0·0001	+ 0·0002 – 0·000
26 to 50 inclusive	0·0004	0·0007	+ 0 – 0° 3′	0·0003	0·0003	0·0001	+ 0·0002 – 0·000

D.B.S. Sharpening Hints. It is essential to maintain hobs in a satisfactory condition. If the cutting edges are allowed to become blunt, the tooth flanks are soon affected by wear, more time is required for grinding, and the tool life is reduced. Furthermore, a sharp hob will cut much faster, use less power, and produce more accurate work, with a superior finish to that obtainable from a hob not in first-class condition.

A correctly sharpened hob tooth *A* (Fig. 65 (*a*)) has neither of the common errors shown at *B* and *C*. When a tooth is ground with a hook, as at *B*, it will cut a gear with a reduced pressure angle. When ground ahead of centre, as at *C*, the gear tooth will have an increased pressure angle. Either of these errors will produce an incorrect form, and should be carefully avoided.

Care must be taken to maintain correct flute spacing. If some of the hob teeth are ground farther back than others, as at *D*, causing a reduction in tooth

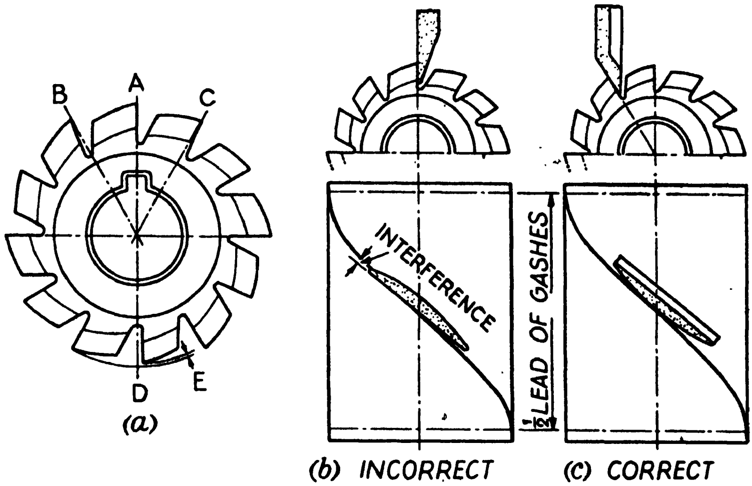


FIG 65. REGRINDING A HOB
Illustrating the notes: *D.B.S.* Sharpening hints

height *E*, the remaining teeth will do all the work. This results in more rapid wear and irregularity of cutting action to the detriment of the finish.

For sharpening, a saucer-shaped abrasive wheel should be used; the normal recommendation is a 60 K wheel at a peripheral speed of 6000 ft. per minute. No coolant is required if only light cuts are taken.

It is necessary to grind by means of the convex side of the abrasive wheel, as the flat side would not produce a surface containing straight radial lines when sharpening hobs with spiral gashes, owing to the interference as shown in diagram in Fig. 65 (*b*) and (*c*).

Further General Notes on Hob Fluting or Gashing. In the smaller sizes the flutes or gashes may be milled with their cutting faces lying in a plane containing the axis, but in the larger sizes they are almost always milled and ground normal to the pitch diameter helix angle, i.e. at right angles to the hob thread. This gives improved cutting action by more evenly distributing the *cut* and eliminating the hammering and resulting *chatter* associated with "straight" flutes, or gashes.

Lead of Gashes. The tangent of the pitch diameter helix angle equals—

$$\frac{\text{Pitch diameter} \times 3{\cdot}1416}{\text{Hob lead}},$$

using the term helix angle as shown in Fig. 70 (*a*), and the lead of the hob gashing is found thus—

$$\frac{(\text{Pitch diameter} \times 3{\cdot}1416)^2}{\text{Hob lead}}$$

Typical errors of flute grinding, associated particularly with hob resharpening, are shown in Fig. 66, which also illustrates the effect of such errors upon the depth of tooth produced. Effects on pressure angle are mentioned in a preceding paragraph.

At *A* (in Fig. 66) is shown the flute of a hob correctly ground (radially). Suppose the depth of cut produced in the gear blank to equal 1 in., as indicated by the rule divisions.

At *B* is shown incorrect fluting, which would produce a *shallow* tooth and a "rubbing" rather than a "cutting" action.

At *C* is shown a *hooked* tooth, which would cut *deeply*.

At *D* is shown the effect of run-out of fluting, which produces uneven distribution of cutting action.

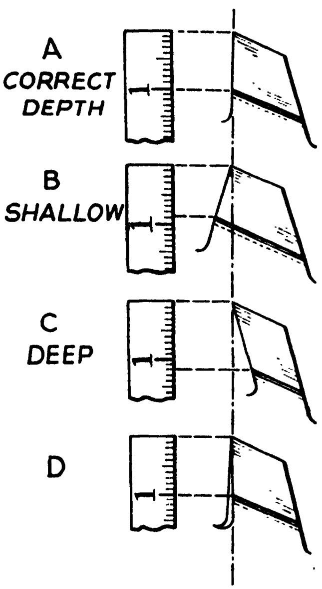


FIG. 66. HOB FLUTING

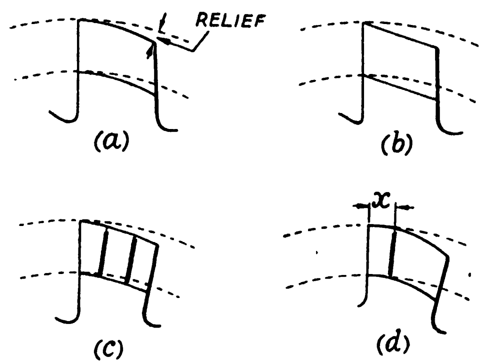


FIG. 67. HOB TOOTH RELIEF

Of the many kinds of form-relief in use, the principal types are shown in Fig. 67. At (*a*) is shown what is probably the commonest, viz. *Archimedean spiral relief*; at (*b*) is shown *straight cut relief*; and at (*c*) a *progressive relief*, which may be a double or a treble relief. Special note should be made of the relief shown at (*d*).

If the hob is to be employed for cutting rack, spur, or spiral gears, then resharpening of a hob with relief (*a*), (*b*), or (*c*) would have no ill-effect upon the product. If, however, the hob were used for generating a worm gear wheel, resharpening, which, of course, reduces the diameter of the hob, would cut a form of tooth of a smaller corresponding worm diameter, and the wormwheel would only match-up with an *undersize* worm.

To overcome this difficulty, an allowance for resharpening is made (shown by x in (*d*) of Fig. 67), which would allow the hob to be reground in the flutes until such time as the concentric section, x, is removed.

INTERNAL GEARS

Production. A larger range of gear-cutting methods is available for spur gears than for internal gears. Of course, casting is, for certain purposes, a suitable process, for others it may be practicable to cut the teeth by shaping or slotting[1] with a formed tool, or by milling with a formed cutter. For the more accurate medium-sized internal gears now made in such large quantities the method employed is a generating one, viz. shaping by means of a pinion-shaped cutter (Fellows type). Great care, however, must be expended in the selection of the cutter because if it is too large the tips of teeth in the internal gear will be trimmed as the cutter is fed to depth: if it is too small, imperfect tooth forms result. When form milling is resorted to a special cutter is

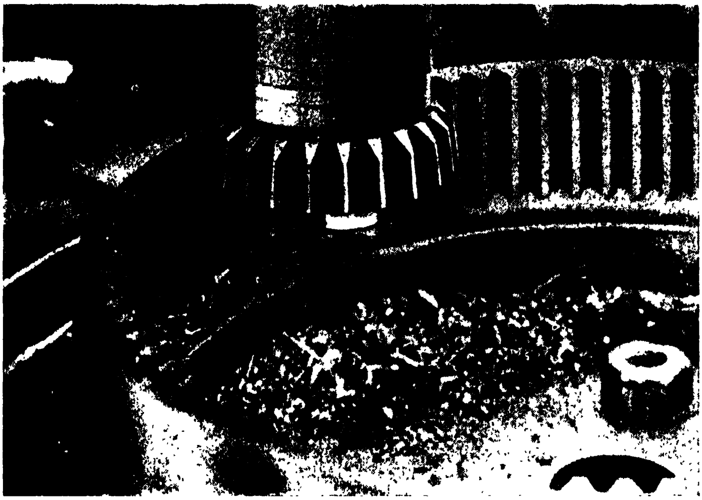


FIG. 68. CUTTING AN INTERNAL GEAR IN THE D.B.S. WORKS

generally employed because the tooth spaces of an internal gear are not the same as those of a corresponding external gear. For hints on cutter selection see *Machinery's Handbook*.

ADDITIONAL GENERAL NOTES ON GEAR CUTTING

LIMITS. Economical manufacture requires a basis of precision by determination of the *limits* for the more important items, e.g. pitch, profile, concentricity of pitch circle, etc. Tighter limits than are necessary for a certain purpose are as uneconomical as too little precision. Admissible limits for gears depend upon stress or load, peripheral speed, type of duty to be performed, conditions of running, etc.

Limits for Holes. The A.G.M.A. table of plus and minus limits for holes in gears is given in *Machinery's Handbook*.

In this system holes in gears are divided into three groups or classes. Class 1 applies to precision gears, such as are used in aircraft, printing machinery, etc. Class 2 applies to gears for automobiles, machine-tools, etc. Class 3 applies to pumps, hoisting machinery, general jobbing gears, etc.

The tables include hole diameters from ¼ in. to 12 in. The tolerance of Class 2 is about double that of Class 1, and that of Class 3 about double that of Class 2.

[1] The shaping or slotting tool must be produced with a form or profile developed from the basic rack form by generation.

Centre-distance, Pitch diameter, Outside diameter. In the same *Handbook* will be found a table headed "Manufacturing Limits for Gearing," applicable to spur gearing used under ordinary conditions. In this table, opposite a list of commonly-used diametral pitches, are given limits for centre-distance, pitch diameter, and outside diameter.

B.S. Limits. The various B.S. gear specifications should be consulted. Thus in B.S. No. 436 (helical and straight spur), B.S. No. 545 (bevel), and B.S. No. 721 (worm gears) will be found tables giving formulae enabling the calculation of tolerances for (1) pitch and profile, (2) tooth thickness, for gears of various classes of accuracy, (3) centre-distance (helical and straight spur).

THE ACCURACY OF GENERATED GEARS. In machines working on the generating principle the blank is steadily rotated by the dividing motion. Simultaneously the cutter receives a motion, either rotation or traverse. Unless both these motions are accurate, errors in tooth spacing and tooth form are inevitable. The accuracy of the pitch of the teeth in a gear is primarily dependent on the accuracy of the index wheel in the gear-cutting machine. In many such machines the dividing wheel is made of larger diameter than the biggest work it is likely to be used to index. Thus pitch errors in the wheel are reduced in the work-piece in ratio to the respective diameters of the driving wheel and the work. In the Fellows type of gear generator a continuous rotation of the cutter is imparted to both the cutter and the work by means of two separate dividing wormwheels connected by gears. Accuracy of tooth spacing and tooth profile necessarily depends to a considerable extent on the accuracy of these wormwheels. In hobbing machines the accuracy of the products depends on various separate elements, e.g. the index wheel and worm, the hob, the intermediate gears, the rigidity of the machine, etc.

Apart, however, from the design and suitability of the machine and the tool, accuracy of cut gears depends also on the shape of the gear, the machinability of the material, the coolant employed, the dimensions and workmanship of the bore and rim of the blank, and the suitability of the fixing devices.

Without accurately turned blanks, no gear-cutting department can produce accurate smooth-running work. The following quotations show what two well-known firms have to say on this subject—

"Inaccurate blanks are the prime source of poor gears. This applies to all gear cutting on any machines. It is not sufficient to have the bore true with the outside faces of the blank. The faces of the boss or portion which is clamped must be true with the bore also. It is obvious that if a boss face is out of truth, the error will be multiplied at the pitch circle of the gear, when the blank is clamped on the mandrel." (*Drummond Bros., Ltd., Guildford.*)

"To ensure quiet smooth-running gears, all blanks sent to us to be cut should be properly sized and turned, and bored true to centre with hubs and sides of rims faced. We ask that particular attention be given to bevel gears to see that they are machined to the correct outside diameter and face angle. We find that from 10 to 20 per cent of the blanks sent to us are machined incorrectly. . . ."

(*The Horsburgh & Scott Company, Cleveland, Ohio, U.S.A.*)

PROS AND CONS OF CUTTING NUMBERS OF BLANKS SIMULTANEOUSLY

(1) **In Form Milling.** Much time may be saved by setting up a milling machine to cut a number of blanks simultaneously. The form

milling cutter must travel a certain distance before burying itself to correct cutting depth in the work so that there is considerable excess travel necessary in order to finish the width of face. The amount of this excess travel is governed by the diameter of the cutter and the depth to which it has to penetrate the job. Therefore, on a milling machine, it is generally economical to group numbers of blanks together so as to save on the idle time required to bury the cutter in the blank.

(2) **In Gear Shaping.** On a gear shaper using a pinion cutter, however, these considerations do not wholly apply, and it can be shown

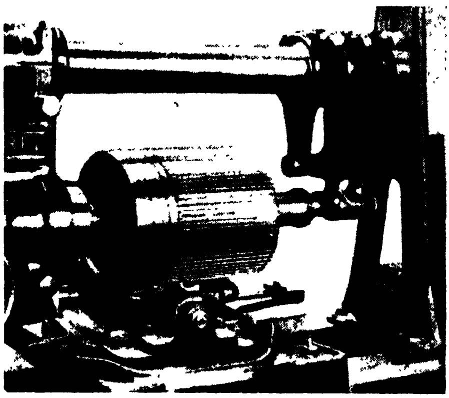


FIG. 69. HOBBING A NUMBER OF BLANKS SIMULTANEOUSLY
(By courtesy of Brown & Sharpe Co.)

that there is little or no advantage in grouping the blanks. The gear shaper uses a planing cutter set to travel a distance only slightly greater than the width of face of the blank. Therefore there is no advantage in a long travel of the cutter. All gear-cutting firms experience trouble due to inaccurate blanks, often through rims being out of truth with the holes. It is therefore obvious that by reducing the number of blanks to be machined at a time the effect of these errors is reduced. The Fellows Co. state that under ordinary conditions the best results are obtained by limiting the travel of the cutter to between 2 and 3 in. Messrs. Drummond Bros. recommend high-speed cutting of one blank at a time, so that cutter stroke is small and cutter speed high.

CHAPTER VIII

HELICAL GEARS

HELICAL gears, like spur gears, are used to connect parallel shafts lying in the same plane. On a spur gear the tooth spirals[1] run straight across the pitch surface parallel to the axis of the gear; on a helical gear the tooth spirals run "helically" across the pitch surface.

In spur gearing the lines of contact between and across the profiles of mating teeth are parallel to the axes of the gears and of the teeth. A tooth makes engagement along its whole length and passes out of engagement in the same way, the lines of contact moving from top to bottom of the teeth, or vice versa. These drives, even with precision-finished teeth, can never be entirely smooth and, owing to deflection of the mounting, deflection of the teeth, uneven distribution of wear across the teeth, etc., there is progressive lack of uniformity in the drive. When, however, helical teeth are used, more than one pair of teeth are in engagement at any instant, the lines of contact between any two such teeth being short diagonal or sloping lines extending across the width of the teeth, their angle of slope being equal to the base spiral angle[2] of the gear. The number of pairs of teeth in contact at any instant depends upon the spiral angle. Whereas in spur gears the lines of contact move from top to bottom of the teeth, those of helical gears move axially along the teeth. There are thus differences in the tooth engagement of spur gears and helical gears, the consequences of which include the superior load-carrying capacity and the quieter running associated with helical teeth, even at high speeds and after considerable wear has taken place.

What is a Helix? In Fig. 70 is shown a cylinder *along and around* which a point has moved with *helical motion* to trace out the curve *ABCD*. This curve is known as a helix. It is evident that a helix is the "screw-like" path traced out by a point on a cylindrical surface when the point moves with co-ordinated *forward-circular motion*. The helix is *not* a plane curve.

HELICAL RACKS. In connection with involute spur gears, we saw that the teeth of the mating rack run "straight across the rack," i.e. they are parallel to the axis of the pinion meshing with the rack. When the teeth of the gear are helical it is evident that those on the mating rack must be inclined to the axis of the gear. Fig. 71 shows at (*a*) the plan of a rack for a straight-toothed spur gear; at (*b*) is shown a helical rack. On the latter note that the teeth are inclined to its front face at an acute angle known as the *spiral angle* or *helix angle* of the mating helical gear, i.e. the spiral angle at the pitch circle of engagement. If the helical pinion rotates, the rack moves at right angles to the axis of the pinion.

[1] **Tooth Spirals.** By the term tooth spiral is meant the line in which the profile of a tooth intersects the pitch surface of the gear. The tooth spirals of spur gears are straight lines; those of helical gears are helical lines.

[2] **Spiral Angle.** See the B.S.I. definition of *helix angle* on page 82. In connection with helical gears, the terms spiral angle (σ) and helix angle (σ) mean exactly the same thing, i.e. the complement of the lead angle (λ). Thus $\sigma + \lambda = 90°$. Whilst in B.S. No. 436 (Helical Gears) the angle is described as *helix angle* in Fig. 1 (*b*), the term *spiral angle* is used on page 19. In B.S. No. 721 (Worm Gears) the angle is described as *spiral angle* in Fig. 1 (*a*).

Pitches Employed. Refer to Figs. 71 and 72.

The pitch of the teeth of a helical rack in a plane normal to the axes of the teeth is called the **normal pitch** (p_n). The pitch which corresponds to circular pitch on a spur gear rack is conveniently called **transverse circular pitch** or **transverse pitch** (p_t) on a helical rack.

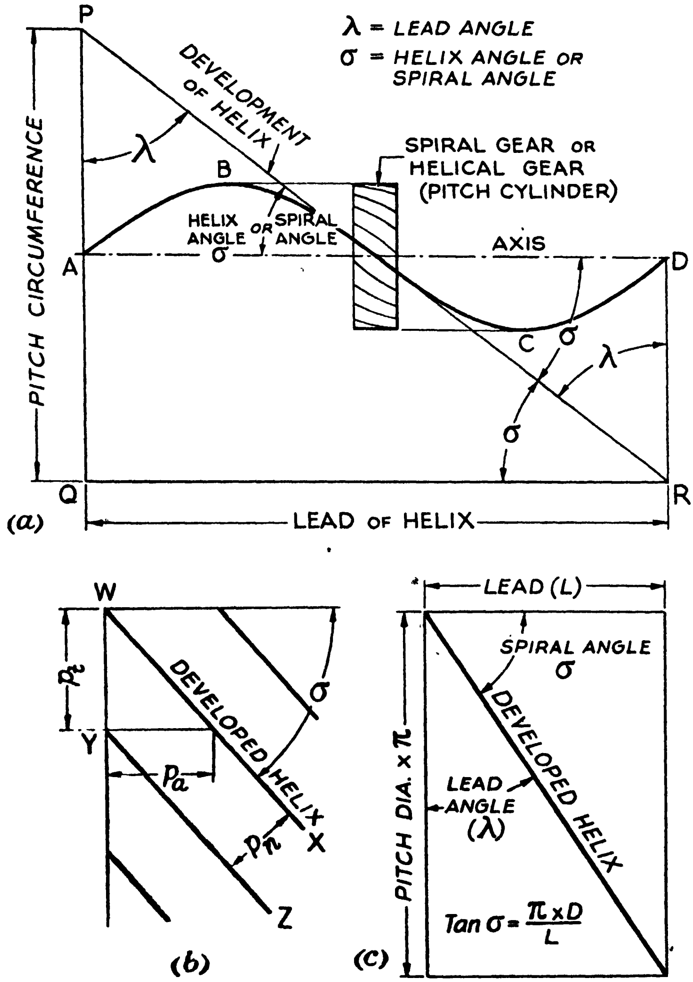


FIG. 70. TERMS EMPLOYED IN CONNECTION WITH HELICAL GEARS

Note that $p_t = p_n \sec \sigma$; $p_n = p_t \cos \sigma$.

These relationships apply both to helical gears and helical racks.

Developed Helices. Fig. 70 (*a*) shows the development of the helix lying in the pitch surface of a helical gear. In the shops the terms *helix* and *spiral* are used rather indiscriminately, hence the fairly general employment of both the terms *spiral angle* and *helix angle* to denote the angle between the tangent to the tooth helix at any point on the pitch surface and the plane containing the axis of the gear and passing through that point. In the example shown the development of the helix is the hypotenuse of the right-angled triangle *PQR*, the dimensions of which will be clear from the diagram.

Suppose that Fig. 70 (*b*) represents a part of the pitch surface of a helical gear developed, or spread out flat. Helical lines on the gear

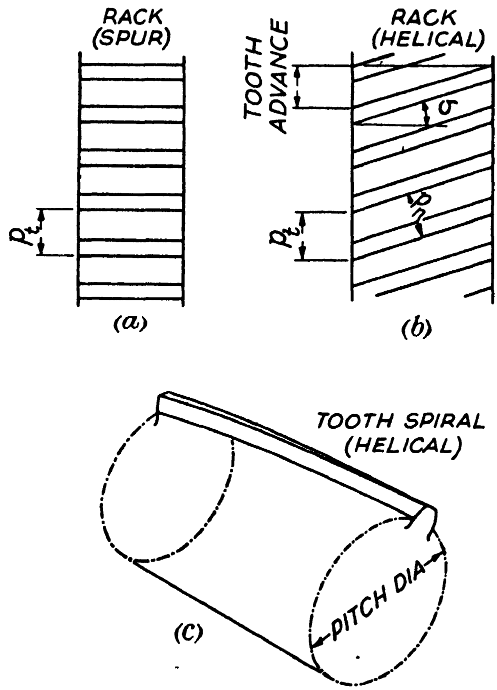


FIG. 71. (*a*) RACK FOR SPUR GEAR, (*b*) RACK FOR HELICAL GEAR, (*c*) TOOTH SPIRAL ON HELICAL PINION

become straight lines in the development. In the diagram *WX* and *YZ* represent short lengths of developed helical teeth. The dimension p_a represents the **axial pitch,** p_t the **transverse pitch,** and p_n the **normal pitch.**

The three pitches are related as follows—

$p_t = p_n \sec \sigma$;	$p_n = p_t \cos \sigma$;	$p_a = p_t \cot \sigma$
$p_t = p_a \tan \sigma$;	$p_n = p_a \sin \sigma$;	$p_a = p_n \operatorname{cosec} \sigma$
$P_n = P_t \sec \sigma$;	$P_t = P_n \cos \sigma$;	$P_a = P_n \sin \sigma$
$P_n = P_a \operatorname{cosec} \sigma$;	$P_t = P_a \cot \sigma$;	$P_a = P_t \tan \sigma$

Obviously the pitch of the cutter must equal the dimension p_n on the gear.

Spiral Angle and Lead. In Fig. 70 (*c*) is shown the development of the pitch surface of a helical gear (not to scale), the tooth spiral being a straight line in the development. Note the following relationships:

Spiral angle (σ) + lead angle (λ) = 90°; Tan $\sigma = \pi D/L$

$L = \pi D \cot \sigma = \pi D \tan \lambda$; Tan $\lambda = L/\pi D$

(where D = diameter of helix, L = lead, σ = spiral angle, λ = lead angle.)

Of course we could draw the helices for other diameters (e.g. tip diameter and root diameter) on the same gear, but all would have the same lead. The magnitude of the spiral angle depends on diameter as well as lead, but no confusion need arise because the expression *spiral*

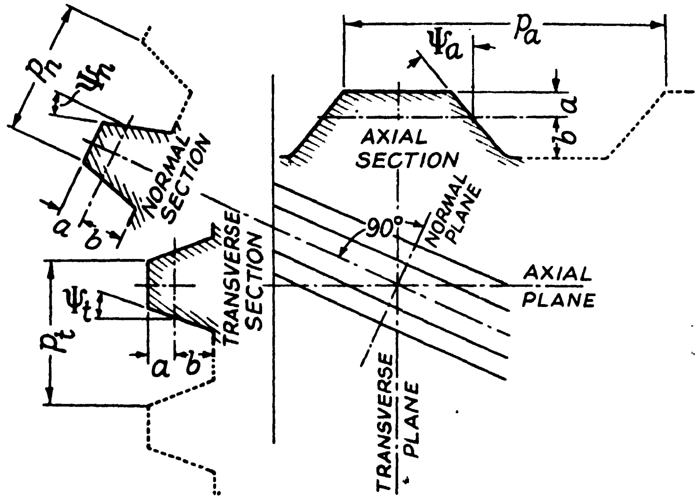


FIG. 72. PRESSURE ANGLES AT DIFFERENT SECTIONS OF A RACK

angle is normally taken to mean the spiral angle at the pitch diameter. A common value for the spiral angle is 30°.

When helical gears are cut by means of rotary cutters on milling machines it is the general practice to set the table to the spiral angle of the teeth at the pitch surface, or to the angle at some point on the tooth just below the pitch surface.

Lead and Diameter. Consider two mating helical gears having the same pitch diameters, the same number of teeth, and the same spiral angles. Evidently the lead of the spiral (or *helix* as it is more accurately called) will be the same in both gears. If, however, one gear has more teeth than the other, the lead of spiral in the larger gear should be longer in the same ratio.

If one gear has 40 teeth and the other 20, the lead of spiral in the larger gear should be twice that in the smaller, but the spiral angles will be the same. Of course, the diameter of the pitch circle of the 40-toothed gear will be twice that of the 20-toothed pinion.

Suppose a helical gear has a pitch diameter of 5 in. and a lead of 48 in. Then the tangent of the spiral angle $= \frac{\pi \times 5}{48} = 0{\cdot}3272$. From tables of tangents we find that the spiral angle = 18° 7′.

Pressure Angles at Different Sections of the Rack. (Fig. 72.) Three sections are shown, viz. normal, transverse, axial. In each case the tooth depths are the same but the pressure angles are different. Note that the normal pressure angle is less than the transverse pressure angle, just as normal pitch is less than corresponding transverse pitch.

Note that $\tan \psi_t = \tan \psi_n \times \sec \sigma$. Also $\tan \psi_t = \tan \psi_a \tan \sigma$
Note that $\tan \psi_n = \tan \psi_t \times \cos \sigma$. Also $\tan \psi_n = \tan \psi_a \sin \sigma$

Hand. If, when a tracing point travels along a helix in a clockwise direction, looking along the axis, it moves away from the observer, the helix is "right" hand; if it moves towards the observer the helix is "left" hand. The helix in Fig. 70 is right-hand.

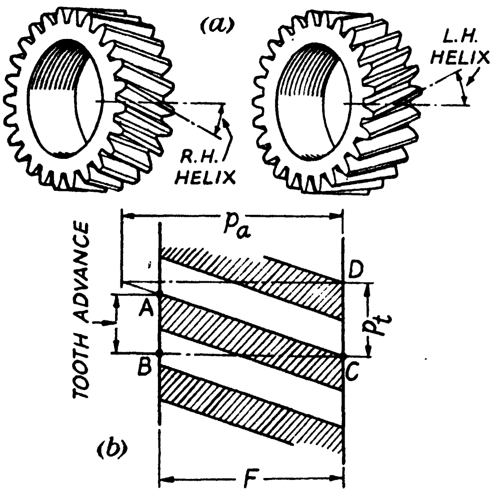


FIG. 73. (*a*) THE HAND OF HELICAL GEARS, (*b*) ILLUSTRATING OVERLAP RATIO

The tooth spirals of right-hand helical gears are right-hand helical curves; those of left-hand helical gears are left-hand. See Fig. 73 (*a*).

If two helical gears are to mesh correctly the spirals of engaging teeth must have the same spiral angle and be of opposite hand, unless one of them is an internal gear.

Overlap Ratio. (Fig. 73 (*b*).) AC is a tooth spiral having a *tooth advance* AB in a face-width BC. The overlap ratio is AB/p_t, or BC/p_a. There should be at least *one pitch overlap* and in order that this condition be complied with,

$$\text{Minimum face-width} = \frac{\pi}{P_n \times \sin \sigma}$$

Small spiral angles give small overlap ratios.

The Virtual Spur Gear. When cutting helical teeth, it is evident that a cutter must be chosen to suit the shape of the required tooth-space in a section normal to the tooth helix. The shape of teeth on such a section is very different from the shape of the teeth on a transverse section, i.e. from the shape of the teeth on a spur gear having the same pitch and number of teeth. The extent of the difference will depend on the spiral angle. It must be remembered that if a normal section be taken across a tooth of a helical involute gear the profiles are *not* of true involute form. However, whatever the shape of the teeth on a helical gear it is possible to find a spur gear having some larger number of teeth which will have *approximately the same shape.* The latter is called the **virtual or equivalent spur gear** corresponding to the assumed spiral gear. If the helical teeth are cut by means of a rotary milling cutter then the same cutter is used as would be suitable for cutting the virtual spur gear.

If D_v is the pitch diameter of the virtual spur gear, D the pitch diameter of the helical gear, T_v the number of teeth in the virtual spur gear, and T the number of teeth in the helical gear,

$$D_v = D \sec^2 \sigma;\ T_v = T \sec^3 \sigma = T/\cos^3 \sigma$$

Fig. 74 shows a section of a pitch cylinder normal to the helix at P. The resulting section is an ellipse, the minor axis (P^1Q) of which must equal the pitch diameter (D) of the cylinder, and the centre of which is lettered O^1. The

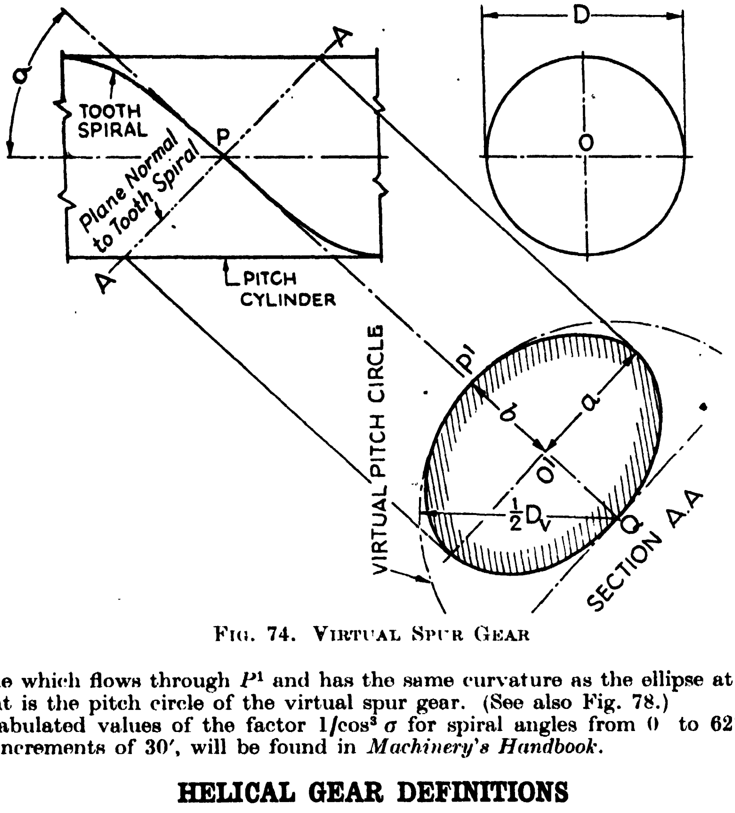


FIG. 74. VIRTUAL SPUR GEAR

circle which flows through P^1 and has the same curvature as the ellipse at that point is the pitch circle of the virtual spur gear. (See also Fig. 78.)

Tabulated values of the factor $1/\cos^3 \sigma$ for spiral angles from 0 to 62° 30′, by increments of 30′, will be found in *Machinery's Handbook*.

HELICAL GEAR DEFINITIONS

(*All the "Spur Gear Definitions" given on pages* 18–19 *apply equally to spur and helical gears. The following additional definitions are reprinted by permission from B.S. No.* 436—1940.)

The **normal pitch** (p_n) of a helical gear is the distance between similar faces of successive teeth measured along a helix lying on the pitch cylinder normal to the teeth.

The **normal diametral pitch** (P_n) is the diametral pitch of the normal basic rack. Note: $P_n = P \sec \sigma = \dfrac{\pi}{p_n}$.

The **axial pitch** (p_a) of a helical gear is the distance measured parallel to the axis between similar faces of successive teeth.

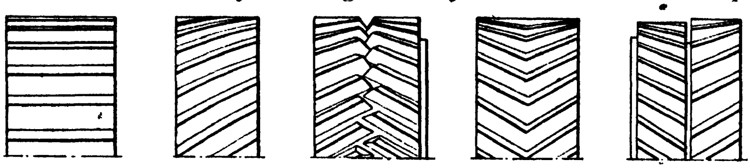
The **lead** (L) of a helical gear is the distance measured parallel to the axis by which each tooth advances per revolution, and is equal to the product of the axial pitch and the number of teeth in the gear.

The **helix angle** (σ) of a helical gear is the angle between a tangent to the tooth helix at any point on the pitch cylinder and the plane containing the axis of the gear, and passing through that point.

The **normal tooth thickness** of a helical gear is the distance between opposite faces of the same tooth measured along the helix lying on the pitch cylinder normal to the teeth.

Tooth Forms Employed. Variously proportioned tooth forms are employed. For instance many helical gears have teeth of the standard 20° full-depth involute form (see the rack proportions in Chapter III) in which addendum equals the module. Further simplification follows if the pitch diameters and spiral angle are so chosen that the pitch is a standard diametral pitch on both circular ("axial") and normal sections.

If the 20° full-depth involute system is used the whole depth of the teeth can be obtained by dividing 2·157 by the normal diametral pitch



(a) (b) (c) (d) (e)

FIG. 75. SPUR AND HELICAL TEETH

(*a*) Spur. (*b*) Single Helical. (*c*) Staggered Double Helical. (*d*) Continuous Double Helical. (*e*) Dissimilar Double Helical

(P_n) of the gear, for it must be remembered that the teeth of helical gears are proportioned from normal pitches and not circular pitches. Similarly, in this system tooth thickness at the pitch line equals 1·571 divided by P_n.

B.S. Tooth Proportions. In B.S. No. 436—1940 it is recommended that the shape and proportions of teeth of helical gears in a section at right angles to the helix (or, more accurately, on a helix normal to the helix of the tooth spiral) shall correspond to the basic rack tooth forms for spur gears. (See Fig. 32.) Normal pitch, however, must be substituted for circular pitch. (Note that $p_n = p \cos \sigma$.)

Where uncorrected tooth proportions are in accord with the standard basic rack on the normal section, pitch being expressed on the transverse section,

$$\text{Addendum} = 1{\cdot}0 \cos \sigma/P = 1{\cdot}0m \cos \sigma = 1{\cdot}0p \cos \sigma/\pi$$

$$\text{Dedendum} = 1{\cdot}25 \cos \sigma/P = 1{\cdot}25m \cos \sigma = 1{\cdot}25p \cos \sigma/\pi$$

Formulae relating to the effects of correction at standard centres, and to gears at extended centres, will be found in *Gears* by Dr. H. Merritt (Pitman).

A.G.M.A. Proportions: Helical Teeth

$$\text{Addendum} = \frac{0{\cdot}8}{P}; \qquad \text{Dedendum} = \frac{1{\cdot}0}{P}$$

$$\text{Whole depth} = \frac{1{\cdot}8}{P}; \qquad \text{Working depth} = \frac{1{\cdot}6}{P}$$

$$\text{Pressure angle} = 20°; \qquad \text{Spiral angle} = 23°$$

USEFUL FORMULAE: HELICAL GEARS

Pitch Diameter (D)

$$= \frac{\text{No. of teeth} \times \text{Secant of spiral angle}}{\text{Diametral pitch of cutter}} = T \sec \sigma / P_n$$

$$= \frac{\text{Transverse circular pitch} \times \text{No. of teeth}}{\pi} = Tp_t/\pi$$

$$= Tp_n \sec \sigma/\pi = Tm_n \sec \sigma = Tm_t = T/P_t$$

Centre Distance (C)

$$= \tfrac{1}{2}(T + t) \sec \sigma / P_n = \tfrac{1}{2} m_n (T + t) \sec \sigma$$

$$= \tfrac{1}{2} p_n (T + t) \sec \sigma / \pi = \tfrac{1}{2}(D + d)$$

Secant of Spiral Angle or Helix Angle (Sec σ)

$$= \frac{\text{Pitch diameter} \times \text{Normal diametral pitch of cutter}}{\text{No. of teeth}}$$

$$= \frac{\text{Transverse circular pitch}}{\text{Circular pitch of cutter}}$$

$$= \frac{\text{Pitch diameter}}{\text{No. of teeth} \times \text{Module of cutter}}$$

Cosine of Spiral Angle

$$= \frac{\text{No. of teeth}}{\text{Normal diametral pitch} \times \text{Pitch diameter}} = \frac{T}{P_n \times D}$$

Outside Diameter = Pitch diameter + 2 (addendum)

$$= \text{Pitch diameter} + \frac{2}{\text{Diametral pitch of cutter}} = D + 2/P_n$$

Transverse Diametral Pitch (P_t)

$$= \frac{\text{No. of teeth}}{\text{Pitch diameter}}$$

$$= \frac{\text{Normal diametral pitch of cutter}}{\text{Secant of spiral angle}} = P_n \cos \sigma$$

Transverse Module (m_t)

$$= \frac{\text{Pitch diameter}}{\text{No. of teeth}}$$

$$= \text{Module of cutter} \times \text{Secant of spiral angle} = m_n \sec \sigma$$

Transverse Circular Pitch (p_t)

$$= \text{Pitch of cutter} \times \text{Secant of spiral angle} = p_n \sec \sigma$$

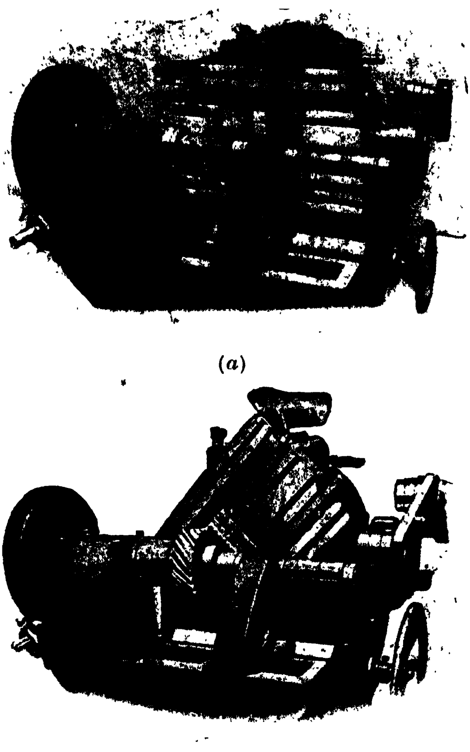
$$= \frac{\pi \times \text{Pitch diameter}}{\text{No. of teeth}} = \pi D/T$$

Number of Teeth Marked on Cutter $= \dfrac{T}{\text{Cos}^3 \sigma}$

Lead $= \pi D \cdot \cot \sigma = \pi D \tan \lambda = \pi D / \tan \sigma$

(See also Spiral Gear Formulae on page 125.)

DOUBLE HELICAL OR HERRING-BONE GEARS. When single helical gears are in engagement an axial thrust is set up, the tendency of which is to cause the mating gears to slide out of engagement along their axes. This may be reduced by using a small spiral angle or by the use of thrust bearings such as ball bearings of the radial type, but an alternative method of overcoming this drawback is to use double-helical or herring-bone teeth. A double-helical gear is really the equivalent of two single-helical gears except that their respective tooth spirals are of opposite hands. Thus the end thrust on one portion of the gear is "balanced out" by the equal and opposite end thrust on the other portion. Double helical gears are extensively used for heavy power transmission at high speeds, on account of their good wearing qualities and quiet smoothness of action. As the nature of the tooth action eliminates shock, the pitch required for any given set of conditions can be finer than if a spur gear drive were used.



(a)

(b)

FIG. 76. GEAR CUTTING ON A SUNDERLAND GEAR PLANER

(a) Cutting a Spur Gear, (b) Cutting a Spiral or Helical Gear

Fig. 76 (*a*) shows a straight-toothed pinion being cut on a Sunderland Rack Planing Machine in which a rack-shaped cutter is used to generate the tooth form.

Fig. 76 (*b*) shows a helical pinion being cut on a similar machine.

HELICAL GEAR-CUTTING METHODS SUMMARIZED

The teeth on helical gears, like those on spur gears, can be produced by both forming and generating processes. In the ensuing notes the following generating processes are described: Sunderland rack planing process, pinion-type cutter process, hobbing process.

FORMING PROCESS

(*a*) *Using End Milling Cutters* which are relatively inexpensive. Form milling of this kind is generally resorted to for cutting continuous vee teeth in large gears of coarse pitch. In this process the cutter rotates about its long axis whilst the gear blank is rotated. When the cutter reaches the middle of the face the direction of rotation of the blank is changed so that the remaining half of the tooth is cut with

FIG. 77. D.B.S. END MILLING MACHINES FOR CUTTING ROLLING MILL PINIONS

opposite hand. (See Fig. 77, showing double-helical pinions of 8½ in. pitch, in the D.B.S. works.)

(*b*) *Using Rotary Milling Cutters.* On smaller single-helical gears this process is sometimes employed, for it is usually possible to cut spirals up to 45° on a universal milling machine. The cutter selected must suit the tooth shape required on a plane normal to the tooth

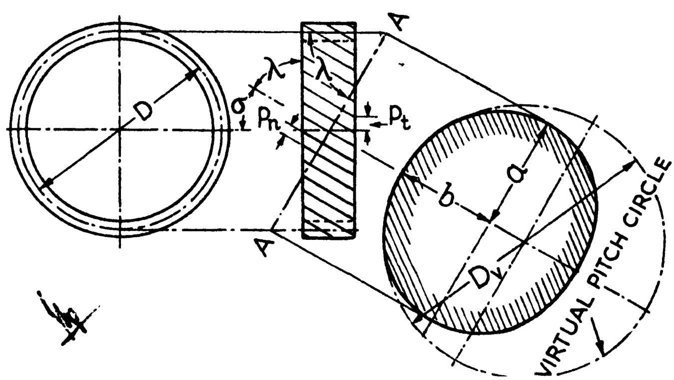


FIG. 78. THE VIRTUAL SPUR GEAR OF A TYPICAL HELICAL GEAR

helix, in other words the plane of the cutter must coincide with the plane of what we may call the "tooth-space helix." The shape of the tooth on such a section depends mainly upon the spiral angle of the gear. This is referred to on page 80 under the heading "Virtual Spur Gears."

(*c*) *Using Form Shaping Tools.* The Michigan Shear-Speed gear-cutting machine copes with both spur and helical gears with a maximum outside diameter of 7 in. in the largest capacity model.

All teeth are cut simultaneously by radially-fed form tools, giving a balanced cutting pressure around the gear, so maintaining concentricity. In action the cutter head is locked in position over the work, which reciprocates vertically—on each upward stroke the blades being advanced inwards an equal amount. On the return stroke they are withdrawn to provide clearance.

High rates of production with a good finish are claimed for both roughing and semi-finishing.

In Fig. 78 a line diagram of a helical gear is shown. *AA* represents the trace of a plane perpendicular to the helix at the pitch surface. The ellipse represents "Section *AA*," in other words a section of the

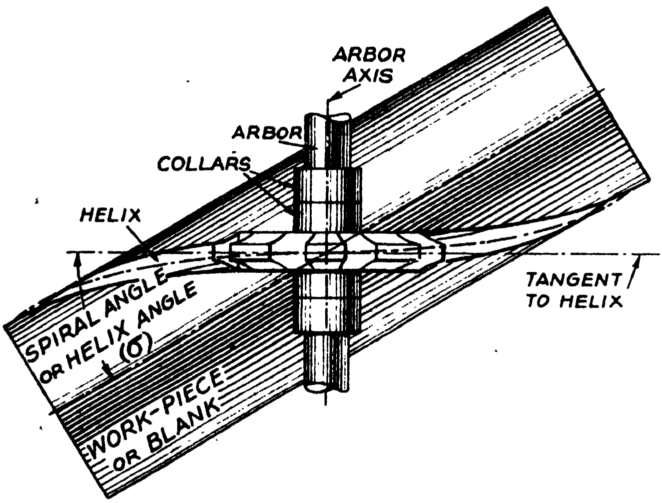


Fig. 79. Milling a Helical Groove in a Cylinder

cylindrical pitch surface. The spiral angle (or helix angle) is σ; the lead angle is λ; the normal pitch is p_n: the transverse pitch is p_t; the diameter of the virtual pitch circle is D_v, its radius is R_v, which equals the radius of curvature of the ellipse at the end of its minor axis.

Then
$$a = \frac{D}{2 \sin \lambda}; \; b = \frac{D}{2}$$

$$R_v = \frac{a^2}{b} = \frac{D}{2 \sin^2 \lambda}$$

$$T_v = \frac{\pi D_v}{p_n} = \frac{T}{\sin^3 \lambda} = \frac{T}{\cos^3 \sigma}$$

For detailed practical notes on the set-up of the machine and the choice of cutter the reader is referred to the *Practical Treatise on Gearing* published by the Brown & Sharpe Co. Inc. Tables of cutter sizes are given in *Machinery's Handbook.* The process of milling a helical groove in a cylinder is illustrated in Fig. 79 and while this is not comparable in all respects with form milling of helical teeth, it clearly shows what is meant by the coincidence of (1) the central plane of the cutter, and (2) the tangent to the helix at the pitch surface.

GENERATING PROCESSES

Sunderland Rack-planing Process. This can be used with equal facility for generating single-helical teeth or double helical continuous vee-teeth. See Fig. 80, which shows part of a typical machine. As when cutting spur gears by this process a rack-shaped cutter is used. For generating double-helical teeth, two such cutters are employed. They reciprocate alternately in directions parallel to the tooth spirals of the right- and left-hand parts of the gear being cut. At the end of

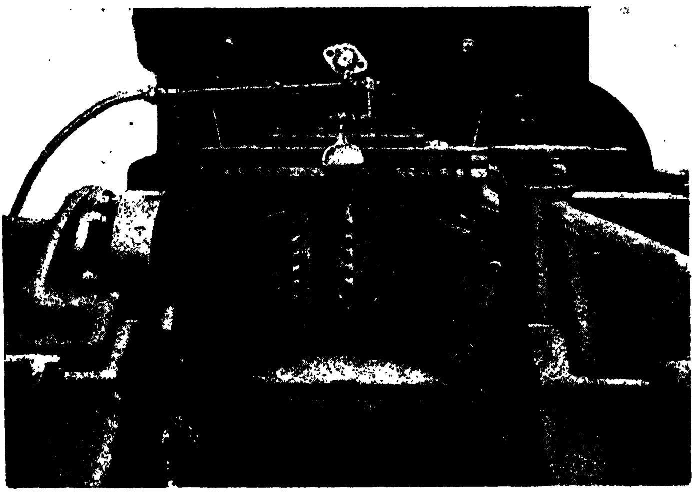


FIG. 80. "SUNDERLAND" DOUBLE HELICAL MACHINE CUTTER HEAD

the cutting stroke the front face of each cutter comes to a stop slightly beyond the middle of the face of the rotating blank. In this way the cutters sweep out the surfaces of a double-helical rack with which the generated teeth on the gear being cut are conjugate. One pair of rack cutters will cut gears having any number of teeth of the same pitch. Double-helical gears generated by this process have spiral angles fixed by the inclination of the cutter guides, which is generally 30°.

The Rack Cutters. These have teeth inclined to the front face which lies in a plane perpendicular to the axis of the gear blank. The cutters should be sharpened together and kept "paired" because, excepting that they are opposite handed, they are duplicates. Two opposite-handed cutters and two corresponding backing plates are required per pitch, the backing plates being used to support the cutters and lengthen their working lives in spite of the thinning that results from repeated sharpenings. Fig. 81 shows a pair of cutters separately and matched or paired.

Sunderland cutters for double helical gears are supplied by the makers for 20° pressure angle and for either $22\frac{1}{2}°$ or 30° spiral angles.

Grinding the Cutters. The cutters are ground all over in pairs. The makers recommend that the teeth be lipped along the edge *D* shown in Fig. 82, i.e. the edge of the obtuse-angle portion of the tooth, also that the edge *E* of the acute-angle portion be chamfered to remove the fragile corner.

Pinion-type Cutter Process. Cutters of the Fellows' type used for generating the teeth on spur gears have straight teeth. If employed for cutting helical gears, the teeth on pinion cutters are of helical form corresponding in lead but of opposite hand to the teeth being cut. In operation the cutters are given a helical reciprocating motion of correct lead, this helical motion being controlled by helical guides readily fitted to the machine. It should be noted that this helical motion is independent of, and is additional to, the slow and regular *generating rotation* of the cutter.

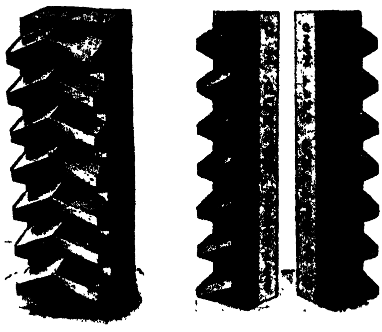


FIG. 81. HELICAL RACK CUTTERS—SEPARATELY AND PAIRED

In the *Sykes process* for cutting double-helical continuous vee teeth by means of pinion-type cutters having teeth of helical form, two cutters are reciprocated towards and from the centre of the gear face, at the same time being given a helical motion.

Hobbing Process. Whilst large numbers of helical gears (single, double, and triple) are generated by hobbing, it is worth noting at the outset that continuous vees cannot be produced by this method. A

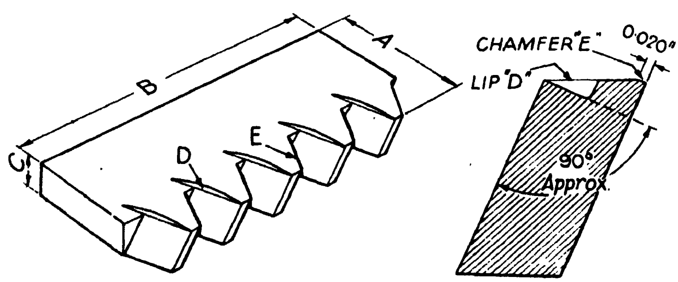


FIG. 82. LIPPING AND CHAMFERING OF HELICAL RACK CUTTERS

gap has to be left between the inner ends of the teeth of opposite hand. The hobbing process, skilfully applied, is characterized by a high degree of accuracy. It is employed to generate teeth on gears of large diameter in cases where machines for handling large gears by the other processes are not available. For large gears required on high-speed work, of the type used on turbine reduction units, the hobbing process is considered pre-eminent. The motions of work and cutting being purely rotational at regular speed, no jars or shocks ensue. The hob is a multi-toothed cutting tool the axis of which, when set for cutting, is inclined at an angle which brings its thread parallel to the tooth spiral being generated. In other words, the tangent to the hob tooth helix must coincide with the tangent to the helix being cut. For

FIG. 83. HOBBING A HELICAL OR SPIRAL GEAR

cutting either right- or left-hand teeth with the smaller spiral angles a hob of opposite hand can be used, but where the spiral angle is large it is best to use a hob of the same hand as that of the tooth being hobbed, so that the direction of the cut is against that of the blank. The hob must have the same normal pitch as the helical gear being cut.

Inclination of Hob Spindle. Single helical or spiral gears are hobbed in a manner similar to that previously explained in connection with spur gears but, as shown in Fig. 84, the axis of the hob is inclined to the face of the gear blank at an angle of $90° - (\lambda_g + \lambda_w)$ when both the hob and the gear being cut have the same hand. Note that λ_g = the lead angle of the helical or spiral gear, and λ_w = the lead angle of the hob. When the helix angle of the helical or spiral gear is less than, say, 20°, the hob should be of opposite hand to the gear, in which case the hob axis is inclined at an angle of $90° - (\lambda_g - \lambda_w)$ to the face of the gear.

Thus, the rule for determining the inclination of the hob spindle is: *Add or subtract the lead angle of the worm from the lead angle of the gear, according to whether gear and hob are of the same hand. The complement of the result, i.e. its difference from 90°, gives the angle between the axis of the hob and the face of the gear blank.*

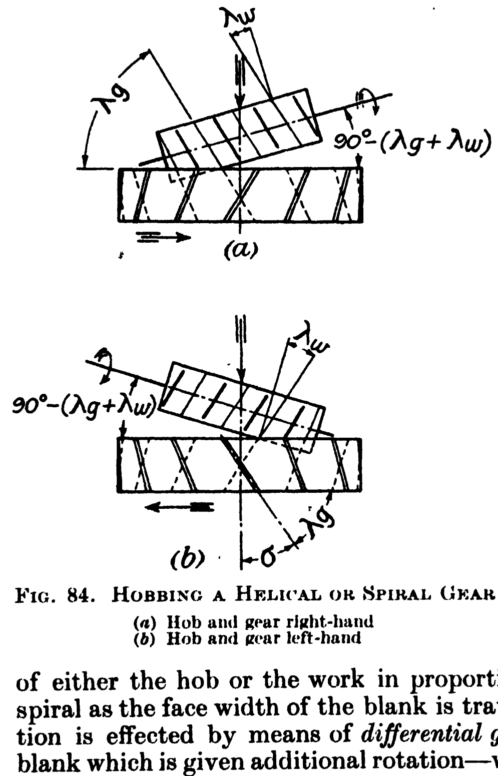


FIG. 84. HOBBING A HELICAL OR SPIRAL GEAR
(*a*) Hob and gear right-hand
(*b*) Hob and gear left-hand

A set of *index change gears* is used to connect the hob- and gear blank-spindles so that for every pitch advanced by the hob threads the blank rotates one pitch, exactly as in the hobbing of spur gears.

In Fig. 84 we have indicated by means of arrows (1) the generating revolutions of the gear blank, (2) the cutting revolutions of the hob, (3) the feed or traverse of the hob across the gear blank. In addition, it is necessary to arrange for *further* rotation of either the hob or the work in proportion to the slope of the tooth spiral as the face width of the blank is traversed. This additional rotation is effected by means of *differential gears*. Usually it is the gear blank which is given additional rotation—which may be called a "creeping motion," additional to that given to it in respect of the number of teeth required, necessitated by the need to cause the hob to "roll" with its teeth traversing the helical tooth spaces of the gear being cut. Helical gears are sometimes hobbed without recourse to the differential gears, but we do not propose to discuss other methods in this book.

Hobbing Double Helical Gears. Double helical gears are hobbed by the same methods as single helical gears, but as continuous vee-teeth cannot be produced by hobbing—owing to the "run-out" of the hob—a gap must be left between the two portions having opposed spiral angles. The width of the gap is approximately equal to the pitch. The teeth in the two separate halves of a double helical gear should be so disposed on the face that either they would meet if produced across the gap, or a tooth on one half would meet the centre of a tooth space in the other, thus giving either "continuous-V," or "staggered teeth." If the latter design is preferred the inner ends of the teeth are boldly chamfered. The adoption of the staggered tooth design enables the width of the gap to be reduced.

Large double-helical gears, such as are used for turbine reduction units, are cut on *double-headed hobbing machines* with astonishing accuracy. (See Fig. 85, reproduced by courtesy of David Brown & Sons, Ltd.) Each head carries a hob, one starting at the top on one side of

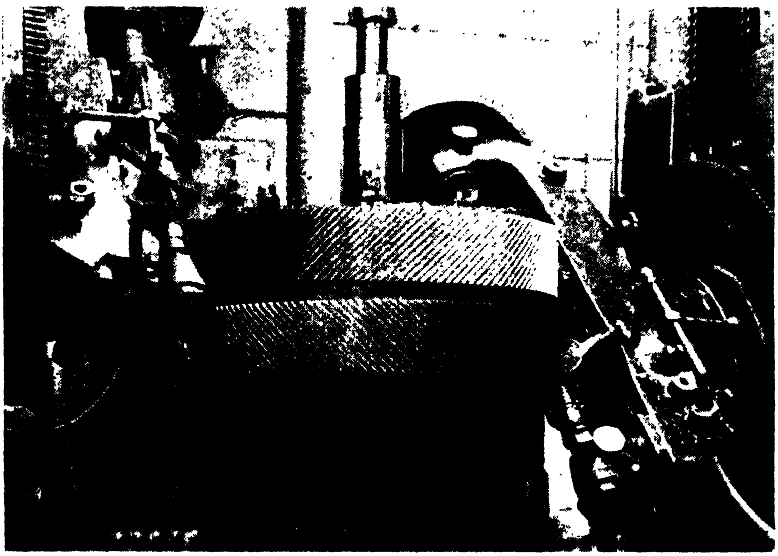


FIG. 85. D.B.S. DOUBLE-HEADED HOBBING MACHINE

the blank, the other at the bottom on the other side. Whilst the two halves are roughed out simultaneously, the finish-cutting is usually done by one hob only.

Profile Grinding: Single Helical Gears. If a profile-ground helical gear has teeth of true involute form they are capable of being generated by a rack having straight-sided teeth. If in the grinding operation this rack is replaced by an abrasive wheel regularly trued by a diamond across the face in a straight line, it will be clear that the teeth thus produced will be of correct form. For helical teeth a grinding operation based on generation is most widely used, grinding by a formed wheel process being troublesome.

The fact that teeth are subsequently to be ground does not obviate the necessity for keeping hardening distortions within the smallest limits, since to obtain the correct thickness and structure in the surface of a case-hardened gear the amount of material to be removed by grinding must be small and uniform. The most accurate control of the heat treatment process is thus an essential preliminary to any tooth grinding operation.

CHAPTER IX

BEVEL GEARS

BEVEL gears are used to connect shafts in the same plane whose axes would intersect if produced. In many instances the shafts are at right-angles, but they may be at an acute angle (less than 90°) or at an obtuse angle (greater than 90°). The conventional symbol for shaft angle is Σ.

Pitch Cones. The tooth action is designed to give the same relative motion to the driver and driven shafts as if they were connected by slipless conical friction wheels, or pitch cones. Examples of pitch cones are shown at (*a*) in Fig. 87. Note that the apexes (or *apices*) of these cones are coincident (i.e. the cones have a common apex) and that the pitch surfaces of the cones represent the pitch surfaces of the corresponding bevel gears. Above and below them teeth are formed. The diameter of the base of a pitch cone is taken as the *pitch diameter* of the corresponding bevel gear. Actually in bevel drives, whether friction or toothed, the whole conical surfaces are not employed. Thus, as shown at (*a*) in Fig. 87, the "friction cones" are really parts of right cones, the parts being known as *frusta* (singular: *frustum*). A frustum of a right cone is a solid bounded by top and bottom faces (plane, circular and parallel) and the curved lateral surface between such faces.

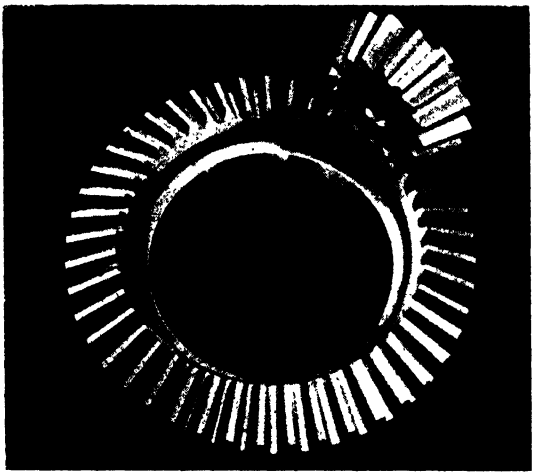


FIG. 86. STRAIGHT-TOOTHED BEVEL GEARS
(*By courtesy of Gleason Co.*)

Back Cones. At (*b*) in Fig. 87, it will be seen that, in addition to the pitch cones shown at (*a*) there are *back cones*, or *normal cones*, having the same bases as the pitch cones. The line OP is called the **common generator** of the pitch cones. If it be imagined as rotating about the fixed axis OQ_1, and at a constant angle to it, it will "sweep out" a solid cone. OP is the generator of a pitch cone. Similarly, OP could rotate about the fixed axis OQ_2, the angle between OQ_2 and OP remaining unchanged, in which case the larger pitch cone would be "generated" by OP. Thus, OP is the generator of both pitch cones, hence it is called the *common generator*.

Now examine the back cones. The line Q_1P is the generator of the back cone of the pinion, whilst PQ_2 is the generator of the back cone of the gear. Note that Q_1P and PQ_2 are each at right angles to OP. The outlines of the tooth profiles are set out on the surfaces of the

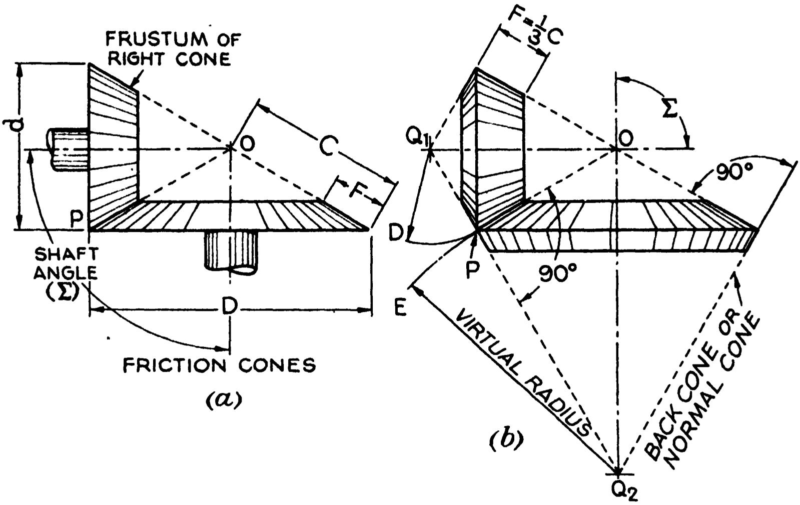


FIG. 87. PITCH CONES AND BACK CONES OF BEVEL GEARS

back cones when drawing the teeth on bevel gears. This will be understood from Fig. 88 where $Q_1P_1Q_2$ is vertical and the surfaces of the back cones are shown partly "developed," or "spread out flat." The tooth profiles seen in a view at right-angles to $Q_1P_1Q_2$ are approximately correct in form. They are disposed about **virtual pitch circles,** arcs of which are shown as PE and PD in Fig. 87 (b). It will thus be seen that the profiles of the teeth of bevel gears are different from those of spur gears having the same pitch and ratio, inasmuch as, although these teeth are actually *set out* on pitch circles corresponding to d and D in Fig. 87 (a), their profiles are those of spur gears of radii Q_1P and Q_2P respectively, the latter radii being known as the **virtual pitch radii** of the gears. If their virtual diameters be represented by d_v and D_v,

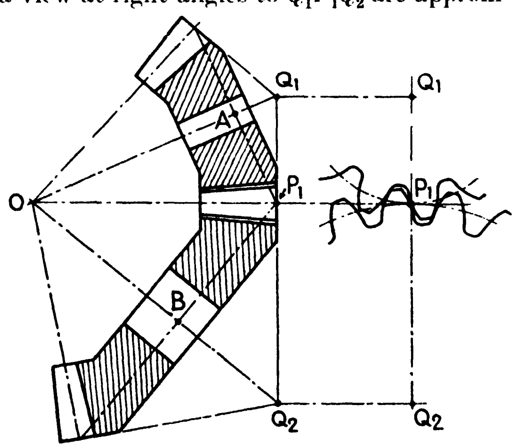


FIG. 88. THE VIRTUAL PITCH CIRCLES

d_v = Pitch diameter of pinion × Secant of its pitch angle,

D_v = Pitch diameter of gear × Secant of its pitch angle.

Pitch Cones and Corresponding Gears. (Fig. 89.) Here are shown some typical outline bevel arrangements, together with the corresponding pitch cones, which, neglecting slip, would drive each other in the same velocity ratio as the actual bevels. At (*a*) is shown a wheel and pinion right-angled drive; at (*b*) and (*c*) the **shaft angles** are respectively acute and obtuse.

In theory, the shaft angles may vary between 0° and 180°; in practice, they normally vary between 30° and 150°, 90° being the angle most commonly used. Bevel gears of the same size, which are used to connect shafts with their axes at 90°, are known as **mitres**. Shafts driven by mitre bevel gears rotate at equal speeds.

Why Frusta and not Whole Cones are Used. Any section of a right cone taken perpendicular to its axis is a circle. All circles are *similar*, i.e. have the same shape, the diameters of such circular sections of a cone being directly proportional to their distances from the apex. So, too, the shapes of bevel gear teeth on sectional planes at right-angles

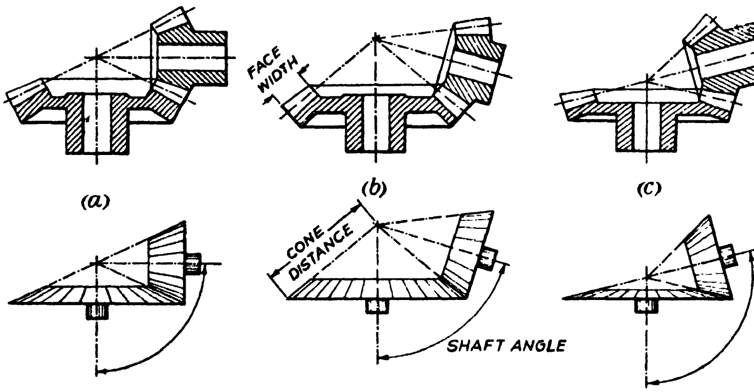


FIG. 89. PITCH CONES AND CORRESPONDING GEARS

to the axis of the gear are *similar*, but of course their dimensions decrease in proportion to the distance from the apex, where they have no dimensions whatever. The strength of any cross-section of a tooth is proportional to its distance from the apex, and near the apex the strength is so slight as to be negligible. For a given pitch diameter the strength does not increase uniformly with increase in face width (F), the useful limit of which is usually considered to be one-third of the cone distance (C). It is difficult to maintain uniform loading along straight teeth. Shaft deflection disturbs it.

Face Width. *It is recommended that the face width should be limited to one-third the cone distance, and that the pitch should be not less than one-third of the face width. (B.S. No. 545.)*

It may be added that some engineers prefer to make the face width 1½ to 2½ times the circular pitch. On spiral bevels, face width is often made one quarter of the cone distance.

Virtual Numbers of Teeth. If t = number of teeth in pinion, T = number of teeth in gear, t_v = virtual number of teeth in pinion, T_v = virtual number of teeth in gear,

$$t_v \doteq t \times \text{Secant of pitch angle of pinion}$$
$$T_v = T \times \text{Secant of pitch angle of gear.}$$

Thus, we have the formulae—

$$d_v = d \sec \theta_p \qquad D_v = D \sec \theta_w$$
$$t_v = t \sec \theta_p \qquad T_v = T \sec \theta_w$$

B.S. Definition. The virtual number of teeth is the number of teeth in a complete gear having a pitch radius equal to that on the developed back cone, and is equal to $T \cdot \sec \theta$. (B.S. No. 545–1934.)

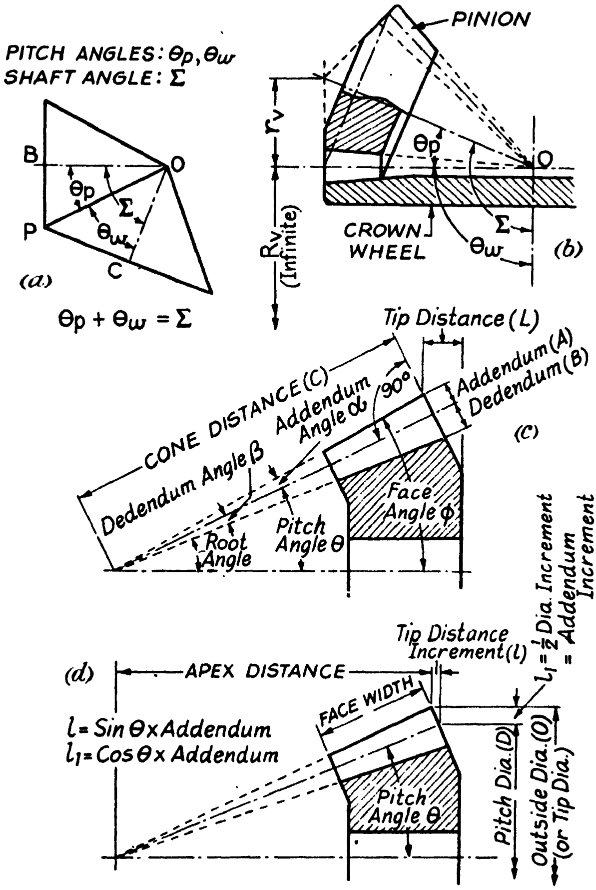


FIG. 90. IMPORTANT DIMENSIONS

BEVEL GEAR ANGLES. (Fig. 90.) The following are B.S. definitions:

The *pitch angle* (θ) is the angle between the axis and the cone generating line.

The *back cone angle* is the angle between the back cone generating line and the axis of the gear, and is equal to 90° minus the pitch angle.

The *addendum angle* (α) is the angle between the pitch cone surface and the tip of the tooth.

The *dedendum angle* (β) is the angle between the pitch cone surface and the root of the tooth.

The *face angle* (ϕ) is the angle between the tip of the tooth and the axis of the gear, and is equal to the pitch angle plus the addendum angle.

The *shaft angle* (Σ) is the angle between the intersecting axes within which the pitch point lies, and is equal to the sum of the pitch angles of the wheel and pinion.

Σ = Shaft angle | θ = Pitch angle | ϕ = Face angle
α = Addendum angle | β = Dedendum angle | t = Teeth in pinion
T = Teeth in gear | $T : t$ = Tooth ratio (R)

PITCH CONE ANGLES

(1) **Shaft Angle (Σ) less than 90°.**

$$\textit{Pinion}, \text{ Tan } \theta_p = \frac{\sin \Sigma}{\cos \Sigma + \frac{T}{t}}$$

$$\textit{Gear}, \text{ Tan } \theta_w = \frac{\sin \Sigma}{\cos \Sigma + \frac{t}{T}}$$

Note that $\theta_w = \Sigma - \theta_p$

(2) **Shaft Angle (Σ) greater than 90°.** Same formulae can be used as in (1), but the following will form a check—

$$\textit{Pinion}, \text{ Tan } \theta_p = \frac{\cos (\Sigma - 90°)}{\frac{T}{t} - \sin (\Sigma - 90°)}$$

(3) **Shaft Angle (Σ) equal to 90°.**

$$\textit{Pinion}, \text{ Tan } \theta_p = 1 \div \frac{T}{t} = \frac{t}{T}.$$

$$\textit{Wheel}, \text{ Tan } \theta_w = \frac{T}{t}$$

(4) **Crown Wheel and Pinion.** (Fig. 90 (*b*).)

In the case shown, Σ is greater than 90°, so that

$$\theta_w = 90°; \; \theta_p = \Sigma - 90°.$$

ADDENDUM AND DEDENDUM ANGLES

$$\text{Tan } \alpha_p = \frac{\text{Addendum}}{\text{Cone distance}} = \frac{a}{C}; \qquad \text{Tan } \beta_p = \frac{\text{Dedendum}}{\text{Cone distance}} = \frac{b}{C}$$

$$\text{Tan } \alpha_w = \frac{\text{Addendum}}{\text{Cone distance}} = \frac{A}{C}; \qquad \text{Tan } \beta_w = \frac{\text{Dedendum}}{\text{Cone distance}} = \frac{B}{C}$$

Face Angle (ϕ) = Pitch angle + Addendum angle = $\theta + \alpha$

Root Angle = Pitch angle − Dedendum angle = $\theta - \beta$

Angle Between Back Cone Generator and Gear Axis
= 90° − Pitch angle

Increment Dimensions. (See Fig. 90 (*c*) and (*d*).)

l_1 = $\frac{1}{2}$-diameter increment = Cos θ × Addendum.

l = tip distance increment = Sin θ × Addendum.

B.S. TOOTH PROPORTIONS. STRAIGHT BEVEL GEARS. The B.S. form of tooth is shown in Fig. 91. It represents the "B.S. Basic Rack Tooth Shape with 20° Pressure Angle." It is generated by a straight-sided tool having an angle of 20°, except that a slight **easing** of the tip is permissible. The amount of such easing, as given in B.S. No. 545, may not exceed the following values measured on the basic rack—

Class B—High-class Cut Gears: 0·004 × circular pitch extending 0·125 × circular pitch in depth.

Class C—Commercial Cut Gears: 0·008 × circular pitch extending 0·125 × circular pitch in depth.

The shape and proportions of **B.S. teeth for helical and spiraloid bevel gears** on a section at right-angles to the tooth spiral are the same as those shown in Fig. 91, normal pitch being substituted for circular pitch.

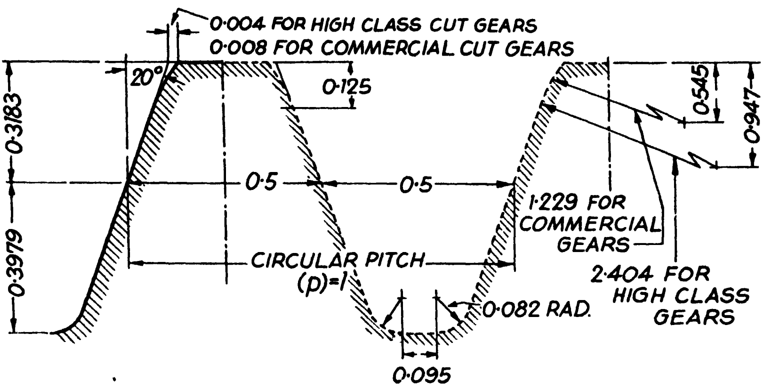


FIG. 91. B.S. BASIC RACK TOOTH SHAPE, 20° PRESSURE ANGLE

Addendum and Working Depth. In the B.S. system, **addenda** of bevel gears are calculated by the formulae used for spur and helical teeth except that the virtual numbers of teeth (T_v and t_v) are used instead of the actual numbers. **The working depth** equals the sum of the addenda of wheel and pinion and this is always $2p/\pi = 0{\cdot}6366p$ (whatever the values of T and t). In every case the dedendum $= 0{\cdot}7162p -$ addendum. Dimensions like pitch, addendum and dedendum apply to the large or outer ends of the teeth always.

Notes on **virtual diameters** are given on page 93. Now revise Fig. 88. Two back cones are shown, their generators being Q_1P_1 and Q_2P_1 respectively. In the drawing, the teeth are spaced out round virtual pitch circles of radii Q_1P_1 and Q_2P_1. Note that whilst the teeth are actually spaced out round circles of AP_1 and BP_1 respectively, the tooth profiles are those of spur gears of radii Q_1P_1 and Q_2P_1 respectively.

Pressure Angles. Both 14½° and 20° pressure angles are used for bevel gears. The B.S.I. recommend a 20° pressure angle full depth tooth, the radius at the root of the tooth being approximately semi-circular at the small end of the space, "as nearly semicircular in form as the tooth-space and system of cutting will allow."

When a pressure angle of $14\frac{1}{2}°$ is used, undercutting occurs when the virtual number of teeth is less than 32, whereas 17 is the minimum number when 20° is used. When it is necessary to use a smaller number of teeth, "addendum correction" can be applied. Actually this means

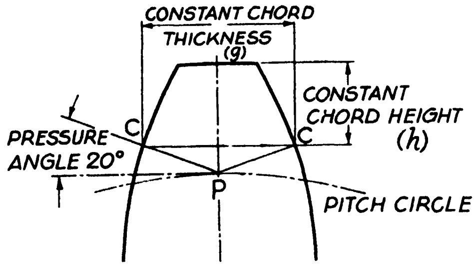


FIG. 92. CORRECTED ADDENDA AND CONSTANT CHORD CALIPER SETTINGS

modifying both addendum and dedendum[1] while maintaining standard working depth. As stated by the B.S.I. in B.S. No. 545, "*in order to avoid undercutting of the pinion and/or to give conjugate tooth action the addendum of the pinion shall be increased and that of the mating gear decreased by the same amount.*"

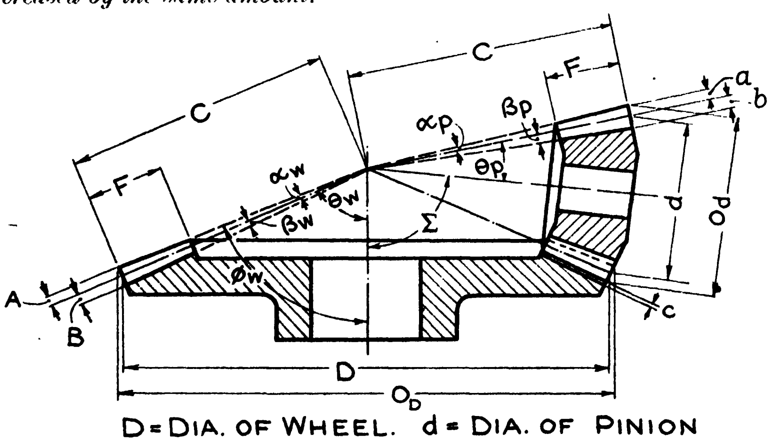


FIG. 93. BEVEL GEAR AND PINION. B.S.I. TERMINOLOGY

Σ = Shaft angle	θ = Pitch angle	t = No. of teeth (pinion)
C = Cone distance	α = Addendum angle	p = Circular pitch
D = Pitch dia. (gear)	β = Dedendum angle	P = Diametral pitch
d = Pitch dia. (pinion)	T = No. of teeth (gear)	m = Module
A = Addendum	B = Dedendum	

Face-width and Pitch. In B.S. No. 545 it is recommended that face-width be limited to one-third of the cone radius and that pitch be not less than one-third of the face width.

[1] The **correction** is the amount of modification to the addenda and dedenda of the wheel and pinion to avoid undercutting and/or to improve tooth action.
The **correction coefficient** is the amount of correction which would be applied to a tooth of 1 D.P. (B.S. No. 545).

BRITISH STANDARD NOTATION FOR BEVEL GEARS

For Straight and Spiral Bevel Gears (see B.S. No. 545 and Fig. 93).

Name	Symbol
Pitch diameter	D
Cone distance	C
Addendum	A
Dedendum	B
Outside diameter	O
Face width	F
Pitch angle	θ
Addendum angle	α
Dedendum angle	β
Face angle	ϕ
Shaft angle	Σ

For Shafts at Right-angles.

$$A = m\left[0{\cdot}6 + 0{\cdot}4\left(\frac{t}{T}\right)^2\right];\quad a = m\left[1{\cdot}4 - 0{\cdot}4\left(\frac{t}{T}\right)^2\right]$$

$$C = \tfrac{1}{2}\sqrt{(D^2 + d^2)}, \text{ or } \tfrac{1}{2}m\sqrt{(T^2 + t^2)},$$

$$= \frac{p}{2\pi}\sqrt{(T^2 + t^2)}, \text{ or } \frac{\sqrt{(T^2 + t^2)}}{2P}$$

For Shafts at Any Angle.

$$C = \frac{\text{Pitch diameter}}{2 \sin \theta}$$

$$a = \frac{1}{P_n}\left[1{\cdot}4 - 0{\cdot}4\,\frac{t \sec \theta_p}{T \sec \theta_w}\right]$$

$$A = \frac{1}{P_n}\left[0{\cdot}6 + 0{\cdot}4\,\frac{t \sec \theta_p}{T \sec \theta_w}\right]$$

$$b = \frac{1}{P_n}\left[0{\cdot}85 + 0{\cdot}4\,\frac{t \sec \theta_p}{T \sec \theta_w}\right]$$

$$B = \frac{1}{P_n}\left[1{\cdot}65 - 0{\cdot}4\,\frac{t \sec \theta_p}{T \sec \theta_w}\right]$$

$$\tan \alpha = \frac{\text{Addendum}}{\text{Cone distance}};\quad \tan \beta = \frac{\text{Dedendum}}{\text{Cone distance}}$$

$$\tan \theta \text{ (pinion)} = \frac{\sin \Sigma}{\cos \Sigma + \dfrac{T}{t}};\quad \tan \theta \text{ (gear)} = \frac{\sin \Sigma}{\cos \Sigma + \dfrac{t}{T}}$$

Face angle $\phi = \alpha + \theta$

Root angle or cutting angle $= \theta - \beta$

Outside (tip) dia. $=$ Pitch dia. $+ 2 \times$ Addendum $\times \cos \theta$.

Root dia. $=$ Pitch dia. $- 2 \times$ Dedendum $\times \cos \theta$.

Pitch dias. $d = t \times m;\quad D = T \times m$

$$= \frac{t \times p}{\pi};\quad = \frac{T \times p}{\pi}$$

$$= \frac{t}{P};\quad = \frac{T}{P}$$

GLEASON WORKS SYSTEM OF BEVEL GEARS. This is very fully described in *Machinery's Handbook*. In this system the gear addendum is decreased and the pinion addendum is increased as the ratios of the teeth in gear and pinion become greater. The lowest pressure angle is employed without sacrificing strength by permitting extensive undercutting. For straight-toothed bevel gears made under this system, there are three pressure angles: 14½°, 17½°, 20°, applicable where there are more than ten teeth in the pinion. For spiral bevel gears the usual angle is 14½°. The ratios to which the various angles apply are given in *Machinery's Handbook*, together with tables of addenda, etc.

When Ordering Bevel Gears. It is best to supply a drawing on which the following essential dimensions must be shown. The larger firms specializing in gear manufacture are ready to advise as to material, pressure angle, pitch, etc., especially where the gears are required for

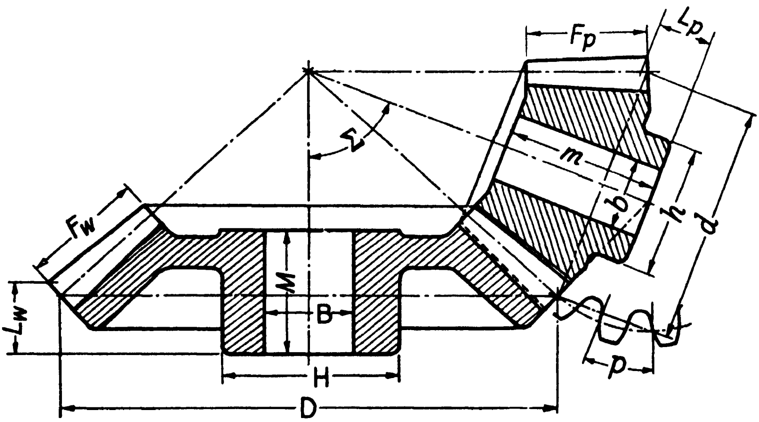


FIG. 94. ESSENTIAL DIMENSIONS FOR ORDERING BEVEL GEARS

high speeds or heavy duties. (See Fig. 94, where essential dimensions are given for a pair of cheap cast-iron gears.)

GEAR		PINION	
Number of teeth		Number of teeth	
Pitch		Pitch	
Face	$= F_w$	Face	$= F_p$
Bore	$= B$	Bore	$= b$
Pitch diameter . . .	$= D$	Pitch diameter . . .	$= d$
Tip distance	$= L_w$	Tip distance	$= L_p$
Length through hub . .	$= M$	Length through hub . .	$= m$
Diameter of hub . . .	$= H$	Diameter of hub . . .	$= h$
Keyway		Keyway	
Material		Material	

When ordering either Gear or Pinion, always give number of teeth of mate. Do not fail to give shaft angle (Σ).

When machined blanks are sent to a gear-cutting firm, the inspection department should see that they are properly sized and turned, bored true to centre with hubs, and with sides of rim faced. One well-known firm of gear-cutters, after experiencing much trouble due to defective blanks, issued the following appeal to customers: "*We would ask that particular attention be given to bevel gear blanks, to see that they are machined to the correct outside diameter and face angle.*"

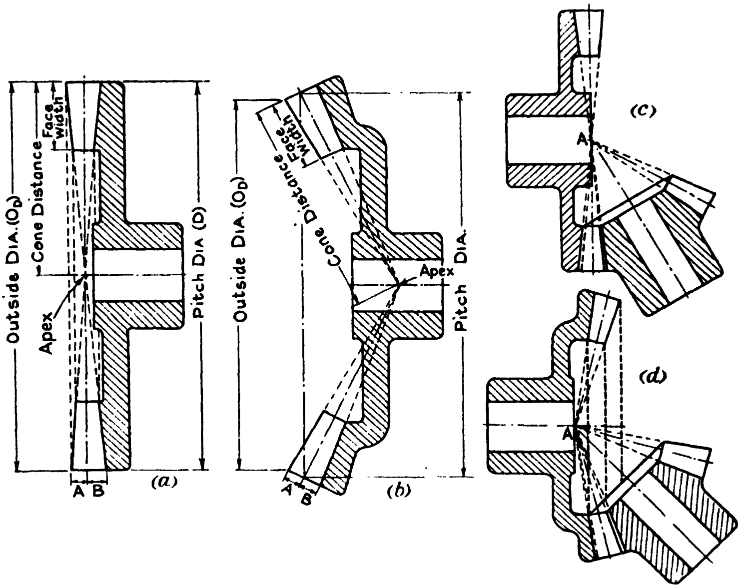


FIG. 95. CROWN GEARS AND INTERNAL BEVEL GEARS
(a) Crown Gear. (b) Internal bevel gear. Teeth easily cast but not readily cut. (c) Crown gear and pinion. (d) Internal bevel gear and pinion.

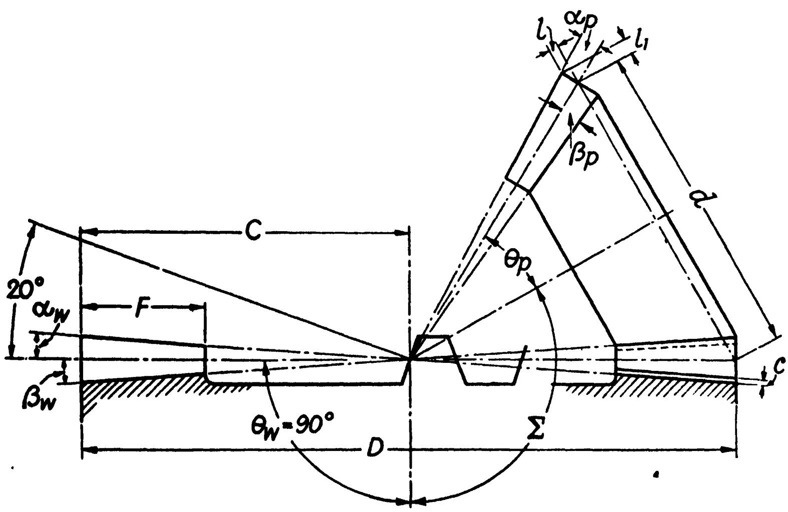


FIG. 96. BASIC CROWN GEAR

C = Cone distance F = Face width D = Pitch diameter
α_w = Addendum angle β_w = Dedendum angle θ_w = Pitch angle
Σ = Shaft angle

CROWN GEARS OR CROWN WHEELS. The pitch surface of a crown wheel has been described as a cone without vertical height. In other words, it is a plane or flat surface. The pitch angle of a crown wheel is 90° and its virtual pitch radius is infinite. When a crown wheel and pinion are used the shaft angle Σ must be greater than 90°. If the involute system is employed the form of the teeth of the crown wheel corresponds to that of a rack having straight-sided teeth, a matter referred to again in connection with the generation of teeth on bevel gears. The B.S.I. dimensioning of a crown wheel and pinion pair is shown in Fig. 96.

Octoid Form. When generating teeth on bevel gears, the basis of the process is a crown gear with straight-sided teeth. The cutting tool represents the side of such a tooth and is thus straight-sided, the angle of its sloping side being equal to the pressure angle of the gear. However, the teeth of a true involute crown gear are very slightly curved, so that theoretically a tool having a slightly curved edge should be used. In practice, straight-sided tools are generally used, the resulting tooth forms being known as *octoid forms*.

BEVEL GEAR AND PINION EXAMPLES

1. B & S Teeth. Wheel has 45 teeth (T), pinion has 27 teeth (t), 3 diametral pitch (P). Face width 3 in. Shaft angle (Σ) = 90°. These are machine-cut gears having 14½° full-depth "B & S" teeth with addendum = 0·3183p and dedendum = 0·3683p. Dedendum is thus smaller than on the B.S. tooth for bevel gears.

To Find	Rule and Working
Circular pitch (p)	$p = \frac{\pi}{P} = \frac{\pi}{3} = 1{\cdot}0472$ in.
Pitch diameter (D) (gear)	(a) $\frac{T \times p}{\pi} = \frac{45 \times 1{\cdot}0472}{\pi} = 15$ in.
	(b) $\frac{T}{P} = \frac{45}{3} = 15$ in.
Pitch diameter (d) (pinion)	(a) $\frac{t \times p}{\pi} = \frac{27 \times 1{\cdot}0472}{\pi} = 9$ in.
	(b) $\frac{t}{P} = \frac{27}{3} = 9$ in.
Pitch angle (θ_w) (gear)	$\frac{T}{t} = \tan \theta_w$
	$\frac{45}{27} = 1{\cdot}667 = \tan 59°\ 2'$ $\quad \theta_w = 59°\ 2'$
Pitch angle (θ_p) (pinion)	$\Sigma = 90°$. Thus $\theta_p = 90° - \theta_w = 30°\ 58'$
Outside diameter of gear (O_D)	$O_D = \cos \theta_w \times 2$ (addendum) $+ D$ $= 0{\cdot}51454 \times 2\ (0{\cdot}3333) + 15$ $= 15{\cdot}343$ in.
Outside diameter of pinion (O_d)	$O_d = \cos \theta_p \times 2$ (addendum) $+ d$ $= 0{\cdot}85747 \times 2\ (0{\cdot}3333) + 9$ $= 9{\cdot}572$ in.
Cone distance (C)	$\frac{\text{Pitch dia. } (D)}{2 . \sin. \text{ pitch angle } (\theta_w)} = \frac{15}{2 . \sin . 59°\ 2'}$ $\frac{15}{1{\cdot}7149} = 8{\cdot}747$ in.

To Find	Rule and Working
Addendum angle of gear (α_w)	(a) $\tan \alpha_w = \frac{\text{Addendum}}{\text{Cone distance}} = \frac{0{\cdot}3333}{8{\cdot}747}$
	$= 0{\cdot}0381$
	$\therefore \alpha_w = 2^\circ\, 11'$
	(b) $\tan \alpha_w = \frac{2\,.\,\sin\,.\,\text{pitch angle}}{\text{No. of teeth}}$
	$= \frac{1{\cdot}7149}{45} = 0{\cdot}0381$
	$\therefore \alpha_w = 2^\circ\, 11'$
Dedendum angle of gear (β_w)	(a) $\tan \beta_w = \frac{\text{Dedendum of tooth}}{\text{Cone distance}} = \frac{0{\cdot}3857}{8{\cdot}747}$
	$= 0{\cdot}04410$
	$\therefore \beta_w = 2^\circ\, 31'$
	(b) $\tan \beta_w = \frac{2{\cdot}3141 \times \sin\,.\,\text{pitch angle}}{\text{No. of teeth}}$
	$= \frac{2{\cdot}3141 \times 0{\cdot}8575}{45} = 0{\cdot}04410$
	$\therefore \beta_w = 2^\circ\, 31'$
Face angle of gear (ϕ_w)	$\phi_w = \theta_w + \alpha_w$
	$= 59^\circ\, 2' + 2^\circ\, 11' = 61^\circ\, 13'$
Cutting angle (or Root angle) of gear	Cutting Angle $= \theta_w - \beta_w$
	$= 59^\circ\, 2' - 2^\circ\, 31' = 56^\circ\, 31'$
Face angle of pinion (ϕ_p)	$\phi_p = \theta_p + \alpha_p$
	$= 30^\circ\, 58' + 2^\circ\, 11' = 33^\circ\, 9'$
Cutting angle (or Root angle) of pinion	Cutting Angle $= \theta_p - \beta_p$
	$= 30^\circ\, 58' - 2^\circ\, 31' = 28^\circ\, 27'$
Tip distance increment (gear) (l)	$l = \sin \theta_w \times$ Addendum
	$= 0{\cdot}286$ in.

2. B.S. Teeth. Same gear and pinion drive with B.S. proportions. Circular pitch, pitch diameters and pitch angles are the same as in Exercise 1.

$$\text{Addendum } (A) \text{ of gear} = \frac{1}{P}\left[0{\cdot}6 + 0{\cdot}4\left(\frac{t}{T}\right)^2\right]$$

$$= \frac{1}{3}\left[0{\cdot}6 + 0{\cdot}4\left(\frac{27}{45}\right)^2\right]$$

$$= 0{\cdot}248 \text{ in.}$$

$$\text{Addendum } (a) \text{ of pinion} = \frac{1}{P}\left[1{\cdot}4 - 0{\cdot}4\left(\frac{t}{T}\right)^2\right]$$

$$= 0{\cdot}4187 \text{ in.}$$

$$\text{Dedendum } (B) \text{ of gear} = \frac{1}{P}\left[1{\cdot}65 - 0{\cdot}4\left(\frac{t}{T}\right)^2\right]$$

$$= 0{\cdot}502 \text{ in.}$$

$$\text{Dedendum } (b) \text{ of pinion} = \frac{1}{P}\left[0{\cdot}85 + 0{\cdot}4\left(\frac{t}{T}\right)^2\right]$$

$$= 0{\cdot}3313 \text{ in.}$$

$$\tan \alpha_w = \frac{0{\cdot}248}{8{\cdot}747} = 0{\cdot}02835$$

$$\therefore \alpha_w = 1^\circ\, 37'$$

$$\tan \alpha_p = \frac{0{\cdot}4187}{8{\cdot}747} = 0{\cdot}04787$$

$$\therefore \alpha_p = 2^\circ\, 44'.$$

$$\text{Tan } \beta_w = \frac{0.502}{8.747} = 0.05739$$

$$\therefore \beta_w = 3^\circ 17'$$

$$\text{Tan } \beta_p = \frac{0.3313}{8.747} = 0.03788$$

$$\therefore \beta_p = 2^\circ 10'$$

Face angle of gear (ϕ_w) $= 59^\circ 2' + 1^\circ 37' = 60^\circ 39'$
Face angle of pinion (ϕ_p) $= 30^\circ 58' + 2^\circ 44' = 33^\circ 42'$
Cutting angle of gear $= 59^\circ 2' - 3^\circ 17' = 55^\circ 45'$
Cutting angle of pinion $= 30^\circ 58' - 2^\circ 10' = 28^\circ 48'$.

3. Gears to connect shafts at right angles. Speed ratio is 3 : 2. Teeth are $10P$. Pitch diameter of pinion is 2·6 in. Check the following calculated results. Teeth are $14\frac{1}{2}^\circ$ Brown & Sharpe type.

Dia. of gear = 3·9 in. $T = 39$, $t = 26$. Addendum = 0·1 in., dedendum = 0·1157 in. Pitch angles = $33^\circ 41'$ (pinion), $56^\circ 19'$ (gear) Addendum angle = $2^\circ 27'$ Dedendum angle = $2^\circ 49'$ Face angles = $36^\circ 8'$ (pinion), $58^\circ 46'$ (gear). Root angles = $30^\circ 52'$ (pinion), $53^\circ 30'$ (wheel). Outside diameters = 2·766 in. (pinion), 4·011 in. (gear).

4. Find the cone distances in the following cases: (*a*) bevel gears, 17 and 69 teeth, $4P$; (*b*) bevel gears, 25 and 76 teeth, $2P$.
Shafts are at right angles.

(*Answers: (a) 8·88 in.; (b) 20 in.*)

BEVEL GEARING TERMS AND DEFINITIONS

The **pitch circles** of a pair of bevel gears are those circles which form the bases of the two pitch cones. The pitch circle may also be considered as the circle in which the pitch cone of a gear intersects its back cone.

The **pitch diameter** of a bevel gear is the diameter of its pitch circle.

The **cone distance** (C) of a pair of bevel gears is the length of the generating line of the two cones from the pitch circles to the apex.

The **pitch point** of a pair of bevel gears is the point of tangency of the pitch circles.

The **back cones** of a pair of bevel gears are the complementary cones generated by lines at right angles to the pitch cone generator at the pitch circles and intersecting the axes.

The **pressure angle** is the acute angle between the common normal to the tooth curves at the pitch point and the common tangent to the two pitch circles passing through the pitch point.

The **circular pitch** (p) of a bevel gear is the length of arc of the pitch circle between similar faces on successive teeth, measured on the back cone.

The **diametral pitch** (P) of a bevel gear is the number of teeth divided by the pitch diameter in inches.

The **module** (m) of a bevel gear is the pitch diameter divided by the number of teeth. It is the reciprocal of the diametral pitch.

(*Note:* When no units are stated, the module is understood to be in millimetres; the unit implied should, however, be specified. For example: 10 mm. module; 0·3937 in. module.)

The **normal pitch** of a helical bevel gear is the circular pitch multiplied by the cosine of the spiral angle at the pitch circle.

The **normal tooth thickness** of a helical bevel gear is the circular tooth thickness multiplied by the cosine of the spiral angle at the pitch circle.

The **addendum** of a bevel gear is the height from the pitch circle to the tip of the tooth measured along the back cone generator.

The **dedendum** of a bevel gear is the depth of the tooth space below the pitch circle measured along the back cone generator.

The **working depth** of a bevel gear is the depth in the tooth space to which the tooth of the mating gear extends along the back cone generator and is equal to the sum of the addenda of the two gears.

The **outside diameter** (*O*) of a bevel gear is the diameter at the tip of the teeth.

The **root diameter** of a bevel gear is the diameter at the bottom of the tooth spaces at the large end of the tooth.

The **constant chord** is the chord between two points on the gear tooth profile which make contact with the basic rack.

The **spiral angle** of a helical or spiral bevel gear at any point is the angle between the cone generator passing through that point, and the line formed by the intersection of the tooth surface and the pitch cone surface.

The **tip distance increment** (l) of a bevel gear is the distance, measured parallel to the axis, from the tip of the tooth at the large end to the plane containing the pitch circle. (See dimension l in Fig. 90 (*c*).)

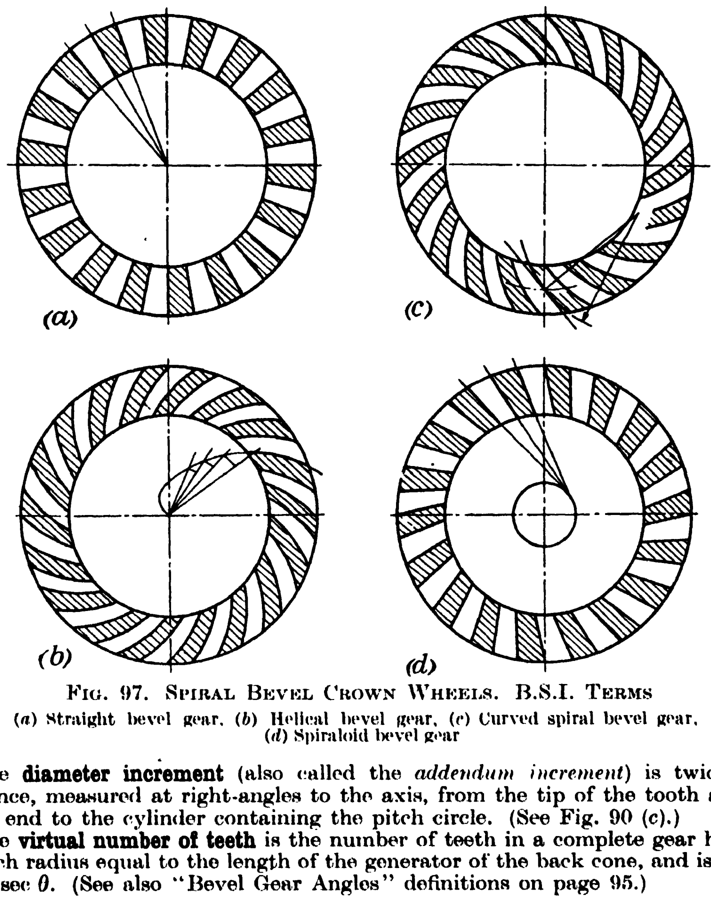


FIG. 97. SPIRAL BEVEL CROWN WHEELS. B.S.I. TERMS

(*a*) Straight bevel gear, (*b*) Helical bevel gear, (*c*) Curved spiral bevel gear, (*d*) Spiraloid bevel gear

The **diameter increment** (also called the *addendum increment*) is twice the distance, measured at right-angles to the axis, from the tip of the tooth at the large end to the cylinder containing the pitch circle. (See Fig. 90 (*c*).)

The **virtual number of teeth** is the number of teeth in a complete gear having a pitch radius equal to the length of the generator of the back cone, and is equal to $T \sec \theta$. (See also "Bevel Gear Angles" definitions on page 95.)

SPIRAL BEVEL GEARS. Spiral bevel gears have become very much more widely used in recent years. In a very general way it may be said that they stand in the same relation to straight-toothed bevels as do helical gears to straight-toothed spur gears. The principal object of specifying "inclined" teeth is to obtain the smoother and quieter drive which results when teeth engage gradually from end to end, and to increase the number of teeth in engagement at any given moment. Overlapping of tooth action reduces noise at high speeds. A further advantage sometimes mentioned is that the position of the pinion can be adjusted to a greater extent than is possible with straight bevel gears without serious detriment to the efficiency of the drive.

In the early days of the automobile industry much patient work was done in order to reduce gear noises, especially in gear-boxes and back axles. Straight-toothed bevel drives have never been quiet at high speeds and, as engine speeds rose and the bevel pinions were made smaller and the mating gears larger, it became still more difficult to get the "crown wheel" flat and true, particularly after hardening—hence the introduction of spiral bevels for these final drives. Nowadays spiral bevel gears are largely used in automobile back axles,[1] on good-class machine-tools, etc., where an efficient, smooth, and quiet drive is required.

Spiral bevels are similar in general form to straight bevels, except that the teeth have some form of "spiral," "twist," or "bend" in the

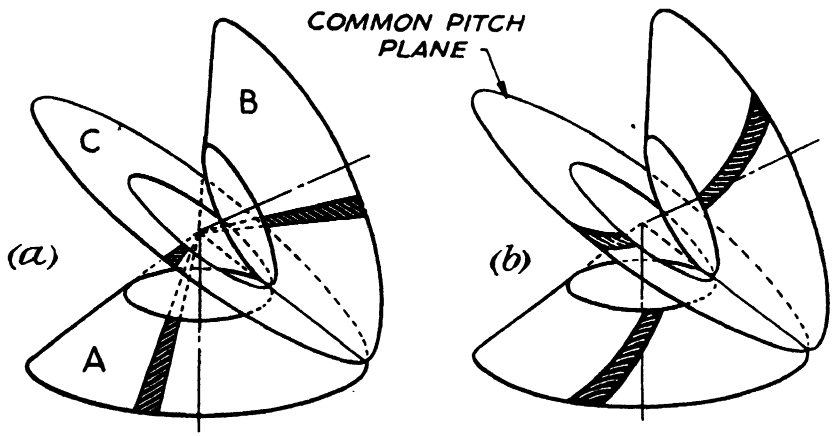


FIG. 98. COMMON PITCH PLANES OF BEVEL GEARS

longitudinal direction. A very large number of forms of "spiral" could be used—but in practice it becomes necessary to use a limited number readily produced by standard machines. The British standard names for three different forms of spiral teeth are given in B.S. No. 545. (See Fig. 97.) Most spiral bevel gears have teeth like those shown at (*c*) for they are readily cut by the Gleason process.

(1) Direction of Tooth Inclination, (2) End Thrust. The teeth of the gear are inclined, or curved, in the opposite direction to those of the pinion, one being right hand and the other left. This inclination causes an end-thrust on the pinion, such thrust acting towards, or away from, the apex according to the direction of inclination, hand of spiral, and gear proportions. The pitch angle causes the pinion to thrust out from its apex. Therefore when it happens that these forces act in opposite directions, the load on the thrust bearings is reduced, and is equal to their difference. When, however, the direction of rotation is reversed, the thrust load on the bearing is increased and is equal to the sum of the forces mentioned above.

Conjugate Tooth Spirals. In Fig. 98 (*a*) are shown conjugate tooth spirals for straight-toothed bevels. The common pitch plane or disc

[1] In heavy automobiles the rear-axle drive is generally a worm and worm gear (very suitable for the high reduction ratios required in such drives).

shown at (*a*) has radial lines indicating tooth spirals. Evidently such lines on the common pitch plane would, when transferred to the pitch cones, represent tooth spirals for straight-toothed bevels. Supposing that the lines representing tooth spirals on the common pitch plane are curves, then the conjugate tooth spirals on the pitch cones will also be curves, such as are used for the teeth on **spiral bevel gears.**

As previously stated, different kinds of curves could be used for the purpose, but in practice their choice is narrowed down to those that can be machine-generated in the simplest and most economical manner. The common pitch plane is the pitch surface of the imaginary common crown wheel. If bevel gear teeth are conjugate to each other, they are also conjugate to the straight-sided teeth of the common crown wheel, this being the principle on which the generating process is founded, no matter whether the teeth be straight or "spiral."

Tooth Spirals and Spiral Angles. The tooth spirals of the straight-toothed crown wheel shown in Fig. 97 (*a*) cut the pitch surface in straight lines, all of which pass through the centre of the plane. In Fig. 97 (*d*), however, the straight lines are all tangential to a circle. In Fig. 97 (*b*) and (*c*) the lines are curved, those at (*c*) being circular arcs such as occur on spiral bevels easily cut on the Gleason system.

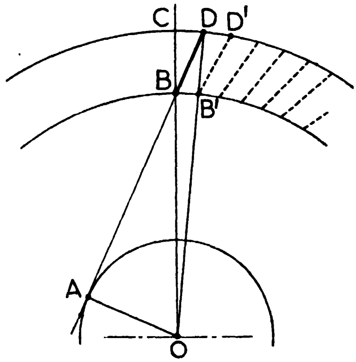


Fig. 99. Part of Spiraloid Crown Wheel

In a preliminary discussion it may be easier to think of the teeth on the imaginary common crown wheel rather than those on the bevel gear conjugate to it. Thus, Fig. 97 (*d*) shows what the B.S.I. describe as a spiraloid bevel gear. A view of part of the crown wheel is shown in Fig. 99.

At B the tooth has a certain spiral angle unequal to the spiral angle at other points along BD. At D the spiral angle is ODA. In the diagram (Fig. 99) we have an obtuse-angled triangle OBD, in which the exterior angle at B equals the sum of the interior and opposite angles at D and O. Thus $O\hat{B}A = O\hat{D}A + D\hat{O}B$. It is therefore evident that the difference between the spiral angles at B and D is the angle BOD. Now, suppose the next tooth is B^1D^1. The angles BOD and B^1OD^1 will be equal and each will be equal to $360/n$, where n = number of teeth.

The spiral angle σ at any point along BD can be calculated from $\sin \sigma = OA/r$, where r = distance of the point from the centre of the gear. The point to note is that the spiral angle is not the same at different points along a spiral tooth.

Overlap Ratio. This is defined by Merritt as the ratio of the angle α subtended by a tooth spiral to the angle β subtended by one pitch, these angles being at the centre of the gear.

Three B.S. Definitions

The **spiral angle** of a helical or spiral bevel gear *at any point* is the angle between the cone generator passing through that point, and the line formed by the intersection of the tooth surface and the pitch cone surface.

The **normal pitch** of a helical bevel gear is the circular pitch multiplied by the cosine of the spiral angle at the pitch circle.

The **normal tooth thickness** of a helical bevel gear is the circular tooth thickness multiplied by the cosine of the spiral angle at the pitch circle.

In the case of a crown wheel both the spiral angle and the pressure angle of a tooth is the same at all points *in its depth* which are situated at a certain circular distance from the centre of the wheel, whereas for any other spiral bevel gear, other than a crown wheel, both the spiral angles and the pressure angles are *not* the same at different depths.

Mean Spiral Angle. Fig. 100 shows a spiraloid tooth projected on a plane parallel to *OP*, a pitch cone generator. If we look at the tooth in a direction normal to *OP* we see that there is an angle between the cone generator and the

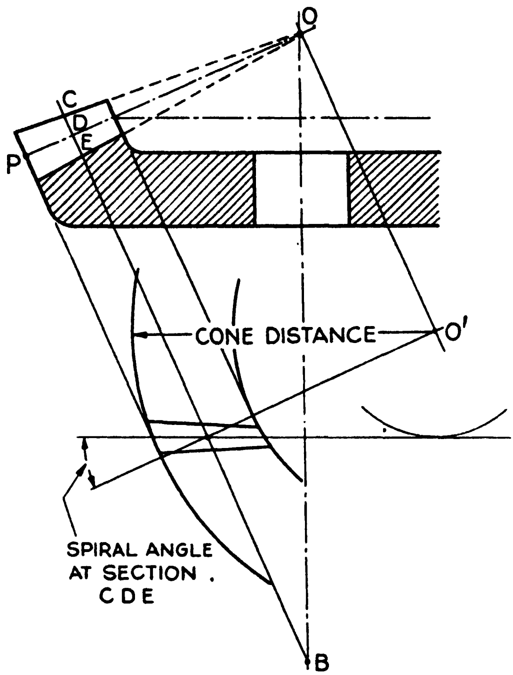


FIG. 100. ILLUSTRATING TERM "SPIRAL ANGLE"

centre-line of a tooth (lying in the pitch surface). When these lines intersect at the *mid-point of face-width*, the angle between them is called the **nominal spiral angle**, which in practice is generally 30° to 40°. The nominal spiral angle can be taken in practice as equal to the **mean spiral angle,** a quantity used in calculations of thrusts and loads.

Tooth Correction[1]**: Spiral Bevel Teeth.** It is recommended by the B.S.I. that spiral bevel gears having curved teeth, a normal pressure angle of $14\frac{1}{2}$°, and a spiral angle of not less than 30°, may be corrected according to the following formula for correction coefficient—

$$k_p = 0{\cdot}4\left[1 - \left(\frac{t}{T}\right)^2\right]$$

where t and T are the actual numbers of teeth on the pinion and wheel respectively.

[1] See notes on *Tooth Correction* and *Correction Coefficient* applied to straight-toothed bevel gears on page 98.

EXAMPLE. To find the correction for a pair of spiral bevel gears, 30° spiral angle, $14\frac{1}{2}°$ normal pressure angle, 20 × 60 teeth, 5 normal diametral pitch.

$$\text{Correction coefficient} = 0{\cdot}4\left\{1 - \left(\frac{20}{60}\right)^2\right\} = 0{\cdot}355$$

$$\text{Amount of correction} = 0{\cdot}355 \times \frac{1}{5} = 0{\cdot}071 \text{ in.},$$

i.e. addendum of pinion is increased, and that of wheel decreased by 0·071 in.

Zerol Spiral Bevel Gear.[1] On Zerol gears the teeth are curved, but the spiral angle at mid-face is zero. Zerol gearing may be said to correspond to double helical gearing used to connect parallel shafts,

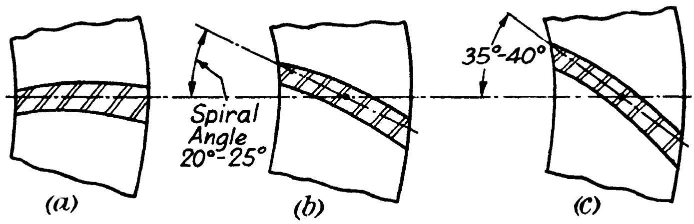


FIG. 101. ZEROL AND SPIRAL TEETH SUMMARY

(*a*) Zerol teeth combine low axial thrust of straight bevel gears with localized tooth contact of curved spiral teeth

(*b*) Spiral bevels are preferred for industrial drives where provision can be made for moderate axial thrusts

(*c*) Spiral bevels for quietest operation at high speeds. Bearings must be designed for increased axial thrusts

inasmuch as it has the smooth and quiet running which characterizes all spiral bevels and also limits end thrust to the amount which arises in corresponding straight bevels.

Like straight bevel gears, Zerol gears have the advantage of no inward axial thrust under any conditions. In fact, with zero-degree spiral angle, the thrust loads are the same as for equivalent straight bevel gears. Thus Zerol gears can be substituted for straight bevel gears without changing the thrust bearings.

Another advantage is that Zerol gears yield the localized tooth contact of spiral bevels. Gear drives with localized tooth contact give smoothness, quietness, and strength even under heavy loads. "Localized" tooth bearing does not extend over an entire tooth surface. This is one of the important advantages of curved teeth. Since no gear mounting or housing is absolutely rigid, displacements of the gears are bound to occur under load. A localized tooth bearing, however, allows for slight displacements, as shown in Fig. 102. Curved tooth spiral bevels, hypoids, and Zerols cut in Gleason machines can be mounted in a testing machine, their teeth painted lightly with a marking compound, and the gears run to show the tooth contact. Localized tooth contact, more or less in the middle of the tooth, eliminates load concentrations on the tooth ends. The usual operating position for the teeth of a Gleason-cut gear and pinion is shown at (*a*), while the position after displacement is shown at (*b*). In the displaced position the load is still not concentrated at the ends of the teeth.

Tooth Proportions. Like all other bevel gears, Zerol teeth can be designed with long and short addenda for smooth running and with large fillet radii for maximum strength.

Manufacture. Gleason machines used for generating spiral bevel and hypoid gears can also be used for generating Zerol bevel gears. The Gleason process employs cutters, which are mounted round the periphery of a central holder

[1] Trade-marked name—Gleason Co.

roughly of cylindrical form. On heavily-loaded precision drives, the curved teeth of Zerol gears are often ground. (See Fig. 103.) The same Gleason machines used for grinding spiral bevel and hypoid gears can be used to grind Zerol gears. Burnishing of Zerol pinions can be carried out on the same machine as is used

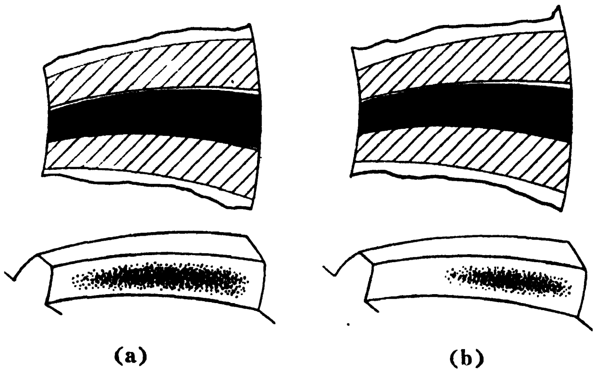


FIG. 102. LOCALIZED TOOTH CONTACT
Effects of displacement on lengthwise position of tooth contact

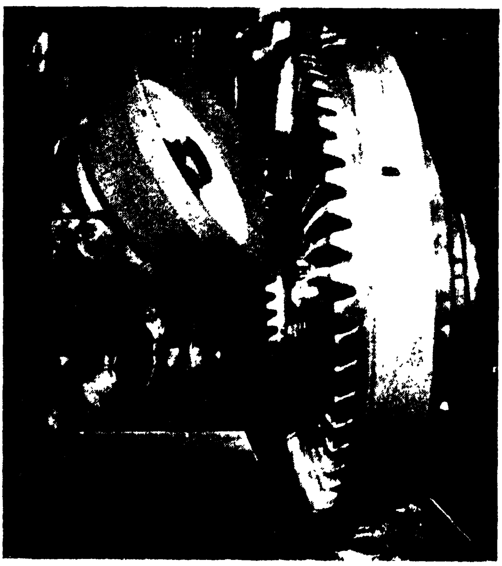


FIG. 103. GRINDING A HARDENED ZEROL GEAR ON THE GLEASON NO 14. GEAR GRINDER. THE MACHINE IS ALSO USED FOR SPIRAL BEVEL AND HYPOID GEARS

for burnishing spiral bevel pinions. Exactly the same applies to lapping and testing.

Double Helical Bevels, with teeth of herring-bone form, had a period of popularity, the idea being to overcome end thrust. They are not often used, being expensive to manufacture, indeed, in view of the success of single-hélical or spiral bevels, there seems no great future for this design. The teeth are end-milled—a process always slow and, therefore, expensive.

Hypoid Bevel Gears. (Fig. 104.) The illustration shows that hypoid bevel gears are similar in general appearance to spiral bevel gears, the difference being that the pinion shaft is offset relative to the gear axis.

Hypoid bevels are used in many automobile rear axle designs, as they enable the propeller shaft to be carried nearer the ground, thus assisting in lowering the body and floor level without employing an entirely different type of drive, such as worm gearing. The direction of offset determines the hand of the spiral. In rear axles a pinion below centre will have a left-hand spiral; if above, a right-hand spiral. Whilst the load-carrying capacity of hypoid gears is about equal to that of corresponding spiral bevel gears, there is more sliding between the teeth, so that efficiency is slightly lower.

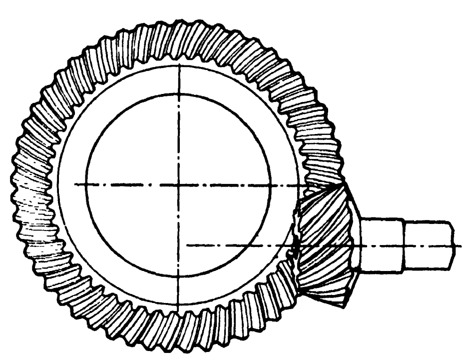


FIG. 104. HYPOID GEARS

Tooth Action. As on spiral bevels, the teeth on hypoid gears are oblique and curved. They engage gradually from one end to the other, having continuous contact at the pitch line. There are always at least two pairs of teeth in contact to share the load, which is thus transmitted smoothly and quietly.

Spiral Angles. Owing to the offset, the spiral angle of a hypoid pinion must be different from that of the mating gear. In the usual arrangement, the pinion spiral angle is larger. In order to keep the pinion spiral angle from being too high, with resultant high axial thrust, the gear spiral angle is generally made somewhat smaller than on a corresponding spiral bevel gear. The Gleason Co. recommend the following spiral angles for hypoid pinions: 6 to 13 teeth, 50°; 14 to 15 teeth, 45°; 16 teeth and over, 40°. It is reiterated that the gear spiral angle must be different from the pinion spiral angle. It depends upon the ratio, offset, and general dimensions of the gears.

Pressure Angles. Usual pressure angles for hypoid gears are 17½° on the concave side of the pinion teeth and convex side of the gear teeth, and 25° on the opposite sides of the teeth.

Pinion Diameter and Amount of Offset. As a result of the offset and the increased spiral angle, a hypoid pinion is larger in diameter than a corresponding spiral bevel pinion. Teeth can therefore be thicker, while the pinion shank can be proportioned more boldly. The Gleason recommendations are that the maximum pinion offset be limited to ⅛ the gear diameter on ratios 3 to 1 and higher, and to ⅕ the gear diameter on 1 to 1 ratios. Between these ratios the maximum offset should be taken proportionally to the values given. In automobile passenger-car drives, the maximum offset of ⅛ the gear diameter is usually approached in order to obtain a low floor level.

Manufacture. Gleason spiral cutters of the same design as those used for cutting spiral bevels are used for generating hypoid gears up to 30 in. diameter. For gears above this diameter, it is usual to employ the planing generator method in which a single tool is used. Teeth on hypoid gears produced by the latter process appear to be straight, but in actuality are slightly curved. The Gleason Spiral Bevel and Hypoid Gear Grinder enables teeth on both types of gear to be **ground** very rapidly, a flaring cup wheel being used. The wheel is carried from end to end of the teeth, both sides of the tooth space being ground simultaneously. The lengthwise sliding of hypoid teeth has been mentioned previously. It is a characteristic that facilitates **lapping.** Further notes on manufacture are given on page 119.

GEAR-CUTTING PROCESSES SUMMARIZED

GENERATION OF STRAIGHT BEVEL GEARS. It has already been stated that a crown wheel, i.e. a bevel gear with a flat pitch surface, stands in the same relation to a bevel gear as does a "basic

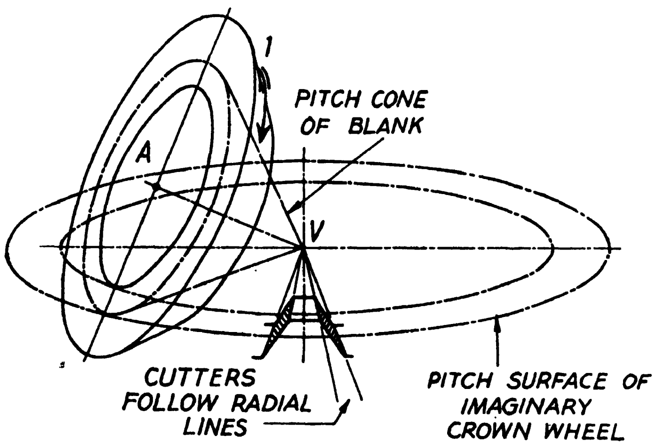


Fig. 105. Rolling a Blank on the Pitch Surface of a Crown Wheel

rack" to a spur or helical gear. It will be remembered that the tooth surfaces of a crown wheel are generally made flat throughout, so that the tooth profiles are straight at both the large end and the small.[1] These considerations, added to the fact that straight-sided cutters are readily made, enable the crown wheel to form the theoretical basis of a generating process employing straight-sided cutters for straight-toothed bevel gears. These cutter blades are given the required shape and motion to enable them to sweep out the profiles of the teeth of the basic crown wheel—such teeth being conjugate to those produced on the bevel gear blank. The process can therefore be described as rolling the blank over the surface of an imaginary crown wheel of which the reciprocating cutter represents one tooth. One tooth is generated during this rolling action, after which the work is indexed and the process repeated. In this way the teeth of the blank are generated correctly, so as to enable

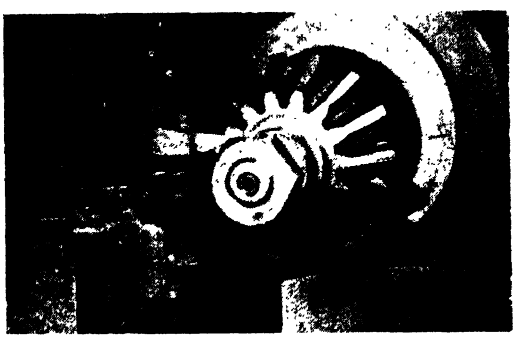


Fig. 106. Generating Teeth on a Straight-toothed Bevel Pinion in the D.B.S. Works

[1] See notes on "octoid" form on page 102.

them to mesh correctly with those on any other pinion which has been cut by rolling it over the same imaginary crown wheel.

Perhaps the simple underlying theory will be clear from a study of Fig. 105, where a blank is shown with its pitch surface tangential to that of a crown wheel. This diagram is a simplified version of a line drawing given in Dr. Merritt's *Gears* (Pitman), which clearly, indeed ingeniously, illustrates the basic principles of generation of teeth in both straight and spiral bevel gears. The blank is caused to rotate on its axis *VA*, as well as to move about the axis of the crown wheel. In other words, the conical pitch surface of the blank rolls over the flat pitch surface of the crown wheel. In doing so, it is subjected to the cutting action previously described. In practice, the machines are arranged to give both the blank and the cutter-head slow rotational motion. Two facing profiles are generated at once, and when the cutters have completely finished these profiles, indexing ensues, and the process is repeated until all the teeth have been generated.

Note that the *cutter head* carries two *cutter boxes*. It is free to rotate, whilst the straight-edged tools pursue their reciprocating motion along straight lines converging to the apex of the blank, which, as previously stated, coincides with the axis of the crown wheel.

Generating the teeth on a mating bevel gear is done similarly after the machine has been re-set to suit its pitch angle and number of teeth. In this way both mating bevel gears have teeth so cut as to be conjugate to a common imaginary counterpart crown wheel, and therefore capable of gearing together satisfactorily.

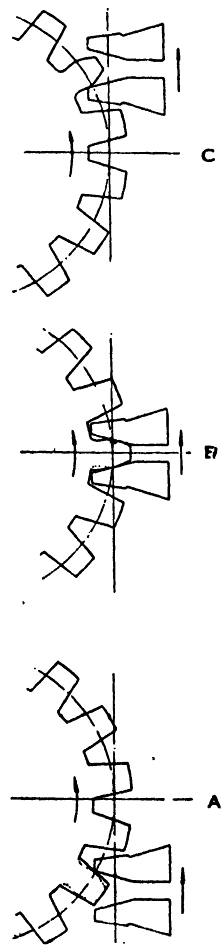


Fig. 107

The first machine to be designed on these principles was the Bilgram machine, in which the tool is mounted on a reciprocating ram similar to that on a shaping machine. The rolling motion of the blank is obtained by securing a cone to the work spindle, the cone being caused to roll by means of steel bands as the head carrying the work spindle is swivelled round a vertical axis. Automatic gear planers of the Reinecker-Bilgram type are largely used for generating teeth on straight and spiral bevels. They are described in detail in *Workshop Practice*, by F. Johnstone Taylor (Virtue & Co.), and in *Engineering Materials, Machine Tools and Processes*, by Steed (Longmans).

Machines using the principles described in previous paragraphs, but employing two reciprocating cutters, finishing both sides of a tooth at once, are made by the **Gleason Co.** Generation is accomplished through a rolling motion between the reciprocating tools and the gear blank. The latter rotates on a fixed axis, whilst the reciprocating cutters are carried in a cradle, representing the imaginary crown wheel, which rotates in geared relation to the blank to give the necessary relative rolling action.

The views in Fig. 107 (reproduced by courtesy of the Gleason Co.) show three positions of the tool and work: *A* at the beginning of the generating roll, *B* at the middle, and C at the end. After a tooth is finished, the work is automatically withdrawn and indexed for the next tooth. Simultaneously, the tools return to their original positions in readiness for the next cycle. The Gleason Co. make a range of machines for generating straight bevel gears from ¼ to 35½ in. diameter. All operate automatically from the time the starting button is pushed until they stop after the last tooth has been cut. To cut teeth from a solid blank, two cuts are generally required—a roughing cut made without generation, and a finishing cut made with generation.

Fig. 108 shows the tools and tool slides on a **12-in. Gleason Straight Bevel Gear Generator.** The tools are crank-driven and operate alternately on opposite sides of a tooth. The tool head is mounted on the cradle, which oscillates on

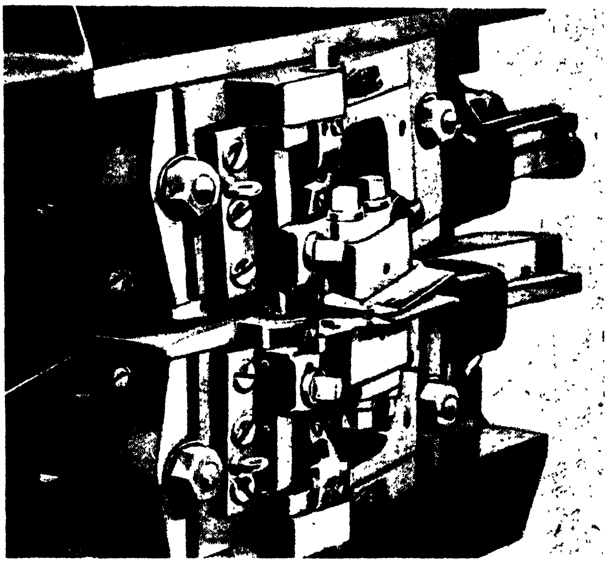


FIG. 108. TOOLS AND TOOL SLIDES. 12 IN. STRAIGHT BEVEL GEAR GENERATOR (GLEASON)

circular ways during the generating motion. Cutting tools are simple in design and made of high-speed steel. The indexing mechanism is of the worm and worm gear type, operated through a differential and stop plate. All numbers of teeth from 10 to 100, and most of the numbers up to 200, can be cut with the regular machine equipment. When roughing gears on this machine, the generating motion is not used and the cradle is locked in position.

The same company have introduced a faster method of cutting small straight bevels with pitches of 30*P* and finer. They call it the "Duplete" method. It enables the teeth to be cut in one operation by using Duplete cutters, which have two cutting edges arranged in tandem, the first edge for roughing and the second for finishing.

Undercutting in Bevel Gears. This may occur with bevel gears just as with spur gears, and is corrected or avoided by the same methods. One has to regard the tooth profiles of the bevel gears as corresponding to those of the virtual spur gears.

FORM PLANING OF STRAIGHT-TOOTHED BEVEL GEARS

From the illustrations of bevel gears previously given it will be evident that the straight lines joining any points on the outer edges of the teeth to the apex lie wholly in the tooth profiles. On this fact rests the principle of the form-planing process.

A slide is arranged so that one end of it may swivel about two axes, (1) horizontal and (2) vertical, both of which pass through the apex of the gear, whilst at the other a roller is carried. The latter rolls over the former or templet, the profile of which is similar in shape to that of the gear tooth being cut.

The tool, normally a point tool, is held in a tool box, which has reciprocating motion along the slide. Whatever the inclination of the slide, the line of action

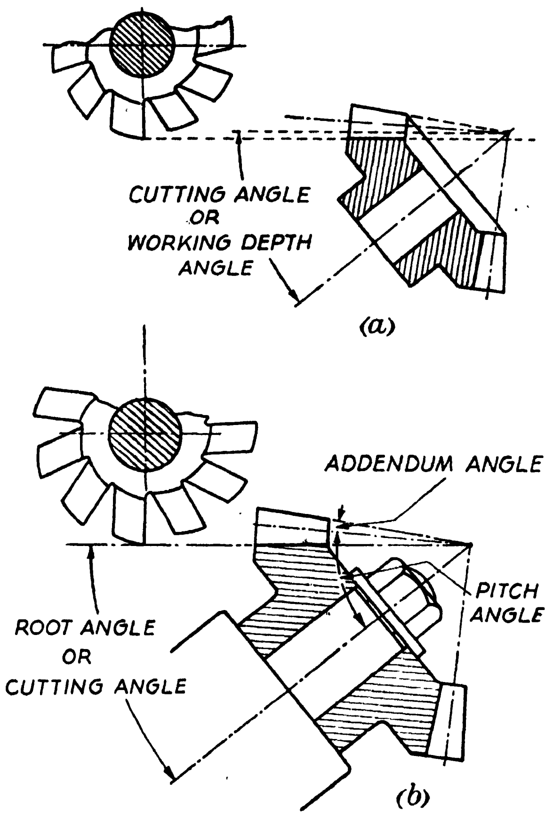


FIG. 109. METHODS OF SETTING THE BLANK IN RELATION TO THE CUTTER

of the tool always converges towards the apex of the gear being cut. Its cutting point, therefore, sweeps out lines lying on the required tooth form.

Form planing is the method nowadays usually resorted to for bevel gears of comparatively large pitch.

FORM MILLING OF STRAIGHT-TOOTHED BEVEL GEARS BY SINGLE DISC CUTTER. Just as spur gear teeth can be cut by a rotary disc cutter on a milling machine equipped with an indexing arrangement, so, too, straight bevel gear teeth can be cut similarly. However, it is neither an accurate nor a rapid method, indeed, it should be regarded as a makeshift to be resorted to when it is impracticable to cut the teeth by a generating process—even then it can only be considered as justified for gears that are to run at low speeds, to carry light loads and have small pitch, i.e. about 4 or 5 *DP* or finer. The only exception is a crown wheel, the teeth of which can be accurately milled by a straight-sided rotary cutter.

The relative positions in which the disc cutter and the gear blank are commonly set are shown in Fig. 109 (*a*), where the cutting angle[1] is shown the same as the working depth angle. This results in more suitable rounding of the insides of the teeth and lessens the amount to be filed off. When blanks are cut in this way the surface of the bottom of the tooth space in one gear is parallel to the outermost surface of the teeth in the mating gear, and thus does *not* lie radially with respect

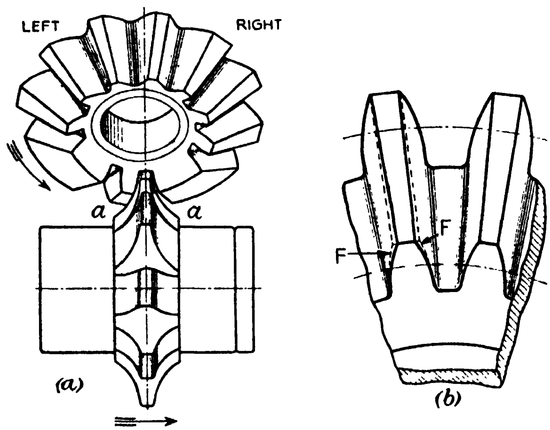


FIG. 110. (*a*) CUTTER SET OUT OF CENTRE, (*b*) DOTTED LINES (F) INDICATE TEETH AS LEFT BY CUTTER. FULL LINES SHOW TOOTH OUTLINE AFTER FILING

to the cone apex. The gear blank is mounted in a universal milling machine on an arbor inserted in the spindle of the spiral head or universal index centre, which is set to the cutting angle desired.

Another method of setting the gear blank is shown at Fig. 109 (*b*). Here the cutting angle equals the pitch angle minus the dedendum angle.

Bevel gear teeth taper regularly from the large to the small end, i.e. from "heel" to "toe." Thus circular pitch changes from end to end, so that the cutter, if of correct form for the large ends of the teeth, cannot be right for the small ends. If the face width is greater than one-third of the cone distance, the variaton in form is too serious unless corrected by filing.

When cutting the teeth, one set of flanks is operated on at a time by one side of the cutter. Two cuts through each tooth-space are necessary to secure the approximately correct form. Frequently the teeth have to be rounded over at the small ends by filing (see *F* in Fig. 110 (*b*).) The longer the teeth, the more filing is required. Note that the filing is done above the pitch circle. Of course, the cutter must be thin enough to pass through the small ends of the spaces, so that the large end has to be cut to correct width by adjusting either the cutter or the blank sideways, then rotating the blank and cutting twice around. Thus, in Fig. 110 (*a*), a cutter and gear blank are shown set so as to have a space

[1] **Cutting Angle.** This term, when used in connection with bevel gears cut by a rotary cutter, means the angle made by the path of the cutter with the axis of the gear.

widened at the large end *aa*, and the last chip to be cut off by the left side of the cutter, the cutter having been moved to the right and the blank rotated as shown by the arrow. In a universal milling machine the same result would be obtained by moving the blank to the right and rotating it in the direction of the arrow.

Selection of Cutter. When cutting a pair of mitre gears it is, of course, practicable to use one cutter for both gears, but when cutting a wheel and pinion it is obvious that one cutter would not be suitable for both. One method of cutter selection for any gear is as follows: measure or calculate the slant height of its developed back cone and so find the virtual diameter (D_v) of the bevel gear. D_v multiplied by diametral pitch (P) gives the number of teeth for which to select a cutter[1] from the table of involute cutters for bevel gears. This method is based on the idea of shaping the teeth as nearly right as conveniently possible at the large end and then filing the small end above the pitch circle. Of course, the cutter must be thin enough to pass through the thin end of the space. As previously mentioned, the process is makeshift in basis and any rules given in regard to cutter selection and operation must necessarily be empirical.

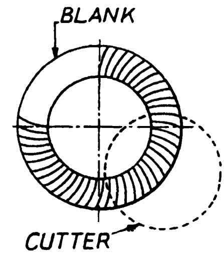


FIG. 111. CUTTER AND BLANK ON SPIRAL BEVEL GEAR

Two Brown & Sharpe Rules

(1) Multiply the back cone radius by 2, and multiply this product by the diametral pitch of the gear. By referring to a table of involute cutters for bevel gears, the number of the cutter can be determined by taking this final result and finding which cutter in the table corresponds to it.

(2) Divide the number of teeth in the bevel gear by the cosine of the pitch angle. The result will be the number of teeth for which the cutter is selected.

BEVEL GEARS WITH TEETH OF PARALLEL DEPTH. Very occasionally, and to a decreasing extent, teeth having the same depth throughout their length are cut by means of rotary cutters. The number of the cutter is chosen corresponding to the number of teeth in a circle equal to the average of the virtual diameters of the mating bevels. For a detailed description of the cutting operation, see the Brown & Sharpe *Practical Treatise on Gearing*.

Roughing Out. Various processes and machines are employed for roughing out or gashing the blanks, a process often resorted to in mass-production, as it relieves much of the work required from the costly finishing machines. Roughing out facilitates production and saves considerable time by taking the heavy first cuts. It is done by means of parting tools, planer tools, rotary milling cutters, hobs, Gleason-type cutters, etc., according to the size of the work and the equipment available.

Generation of Gleason Spiral Bevel Gears. These gears have tooth spirals which are approximately circular arcs, the teeth being produced very rapidly in the best-known Gleason process by means of a large-diameter cutter consisting of a cylindrical block or disc carrying a number of inserted cutter blades. Simultaneously with a slow rotation of the gear blank the cutter block rotates about its own axis, whilst the latter itself slowly rotates about another axis passing through the

[1] Special rotary cutters are obtainable for bevel gears. In the Brown & Sharpe series they are made in sets of eight per pitch.

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centre of the pitch surface of the imaginary crown wheel. The crown wheel has teeth of circular arc form and the cutting process is a generating one. Thus, the curved teeth of the imaginary crown wheel are represented by the rotating cutter blades, of which there may be twenty or thirty in a block, so arranged that half of them generate one side of a tooth whilst the other half generate a side of an adjacent tooth in a smoothly continuous manner. After each generating process is completed in regard to the opposed profiles of adjacent teeth the

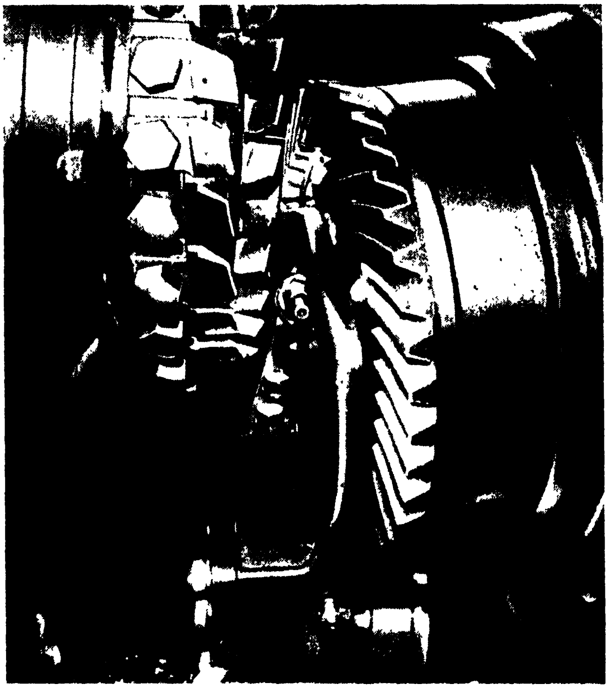


FIG. 112. CUTTER, GEAR AND CHAMFERING ARM ON GLEASON NO. 22 ROUGHING MACHINE

cutter block is automatically withdrawn from the blank, which is then given an extra rotation corresponding to one pitch, in readiness for the next series of generating motions.

Roughing Out. The *Gleason No. 22 Roughing Machine* for spiral bevel, Zerol, and hypoid gears employs no generating motion. The tooth slots are cut by a simple depth feed motion of the cutter into the work. After each slot has been cut the cutter is withdrawn from the work to permit indexing for the next cut.

Operation is very speedy. As on many other modern machine tools, the time required for removing the roughed-out gear from the machine and replacing it by a fresh blank is reduced to a minimum. De-chucking automatically ejects a gear which is then lifted from the arbor and replaced by a fresh blank. Clamping is done hydraulically (600 lb.).

The Chamfering Attachment is for chamfering the acute angles of both ends of the gear teeth simultaneously with the roughing operation. Each tooth is chamfered just after it has been rough cut and immediately before the index operates. The amount of chamfer put on the teeth is of sufficient width to leave a chamfer on the tooth after it has been finish cut.

The Burring Attachment has a burring tool which rides on the back angle of the gear. As the gear "indexes," the burring tool neatly trims off the burr, which is usually left after cutting.

Finish-cutting. The *Gleason No. 22 Single-cycle Finishing Machine* for spiral bevel, Zerol, and hypoid gears accepts gears previously roughed out on the roughing machine described in previous paragraphs and finishes them off at great speed. During the actual cutting only the cutter is in motion. Successive blades in the cutter are set radially beyond the preceding blades so that each blade takes a light cut. There is a gap between the last blade and the first blade which allows the blank to be indexed at the time the gap is over the face width of the gear. A single revolution of the cutter finishes a tooth-space, and when the number of revolutions equals the number of teeth to be cut, the gear is finished.

Very accurate tooth cutting and spacing is obtained because of the light cuts and even distribution of the load taken by each blade, the rigidity of the machine, and the hardened and ground dividing plate, which is mounted directly on the work spindle.

Gears cut by this method are known by the company as *Formate* gears, a name derived from the fact that *such gears are formed without generation*, the pinion being mated to the gear in a correct manner by generation. The machine cannot be used for gears having pitch angles less than 71° 34′ (3 to 1 ratio), because of special requirements for generating the pinion.

Gleason Planing Generator for Spiral Bevel and Hypoid Teeth. Besides the generating machines employing circular cutter blocks for cutting curved teeth on bevel gear blanks the Gleason Co. make a *planing generator* (No. 40) which employs a tool moving in a straight slide. The required curves on the tooth profiles are obtained by the usual method of generation in which a rolling motion takes place between the straight-sided tool and the teeth being generated. The tool represents a tooth of the crown gear with which the blank is rolled.

The crank-driven tool reciprocates in a straight slide, while the blank rotates on its axis at a uniform rate. The cradle which carries the tool slide is given a slight rocking motion, which, combined with the rotation of the blank and the movement of the tool in a straight line, produces the desired curve of the tooth across the face of the gear. The single tool cuts in a different tooth space on each stroke, so that during one revolution of the blank the tool makes as many strokes as there are teeth in the gear being cut. The set-up for finish-cutting is the same as that for rough-cutting, but different tools are used and each side of the tooth is finished separately.

Production of Teeth on Hypoid Gears. A hypoid *gear* is manufactured by methods similar to those used for spiral bevel gears; a hypoid *pinion*, however, must be generated in an offset position corresponding to the position in which it meshes with the mating gear, i.e. it must be cut on a generator in which the work spindle can be placed above or below centre.

Hypoid *gears* up to 30 in. diameter are cut with Gleason spiral cutters. Those beyond the capacity of the Gleason spiral cutters, and up to 100 in. diameter, can be cut on Gleason planing generators in which a single tool is used.

It has already been mentioned that spiral bevels cut on Gleason machines may be either generated or "formate," and the same applies to hypoid gears. If generated, they are produced in the usual manner by means of a relative rolling motion between the cutter and the gear. By this method the finishing time for a hypoid gear with 41 teeth of $4\frac{1}{2}P$, in steel, is approximately 30 seconds per tooth. Roughing is done without roll in a roughing machine at the rate of 5 seconds per tooth.

The gear of a "formate" pair has straight tooth profiles, being cut without generation. The pinion is mated to the gear by the generating method.

The *single-cycle method* of finishing "formate" gears is the fastest mass-production finishing process. One revolution of the cutter finishes one tooth

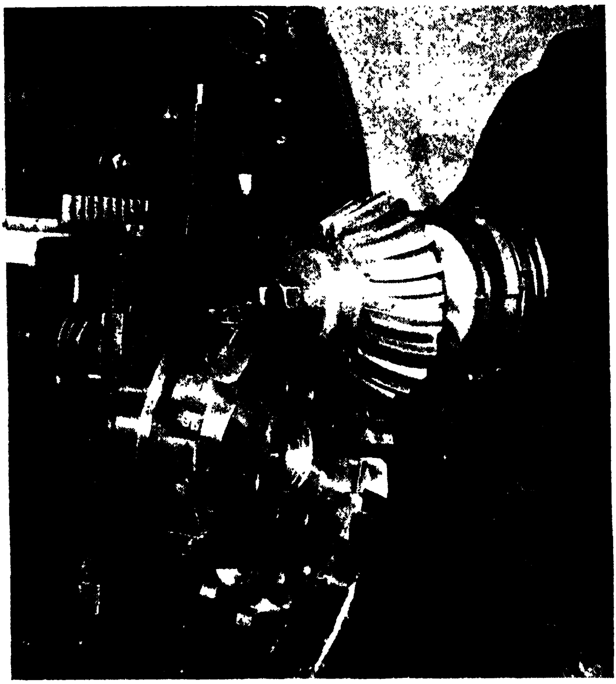


FIG. 113. FINISH CUTTING A ZEROL PINION ON A GLEASON NO. 16 HYPOID GENERATOR

space. By this method, finishing time for the 41-tooth gear, previously mentioned, is 5 seconds per tooth. Roughing also takes 5 seconds per tooth. Gears of this type, whilst a recent development, are used in enormous quantities in the U.S.A. for automobile rear axle drives.

Production of Teeth on Zerol Gears. Zerol gears from $\frac{3}{16}$ to 100 in. diameter are manufactured on the same Gleason machines as are used for spiral bevel and hypoid gears. Thus, they can be produced on the Gleason No. 40 Gear Planing Generator, on the No. 16 Hypoid Generator (using circular cutters), on the No. 22 Roughing Machine, and the No. 22 Single-Cycle Finishing Machine (for "formate" gears). Fig. 113 shows the cutter on a No. 16 Hypoid Generator.

CHAPTER X

SPIRAL GEARING

HELICAL gears used to connect skew shafts are called spiral gears. Either of a pair of spiral gears, considered by itself, is geometrically the same as a helical gear of large or small spiral angle—but, as we have seen, the term helical gear is reserved for gears having helical teeth but used to connect parallel and co-planar shafts. In the case of a pair of mating helical gears the tooth spirals are of opposite hand; whereas on spiral gears the tooth spirals are very commonly of the same hand.

FIG. 114. PAIR OF SPIRAL GEARS CUT ON A SUNDERLAND GEAR PLANER

The axes of the pitch cylinders of spiral gears are not parallel to each other, although generally they lie in parallel planes, furthermore these pitch cylinders make only point contact with each other. Spiral gears are usually employed for connecting non-intersecting shafts at 90°, and for light duty, because they have only theoretical point contact at any one instant, instead of the line contact which we have seen is associated with helical gears. The teeth are generally produced by hobbing, planing, or shaping, but form milling is resorted to occasionally. It may be added that if the gears work at right-angles to each other it does not make a great deal of difference if there is a slight error in the spiral angle of one or both of them. As the shaft angle decreases, however, the adverse effects of such inaccuracies become more noticeable, until in the case of helical gears connecting parallel shafts it is essential that the spiral angles be exactly the same if smooth working is to result.

Materials. For a lightly-loaded high-speed drive, a steel-bronze combination is commonly used; but for heavier duty, case-hardened steel gears are used. Soft mild steel gears rapidly abrade, for there is longitudinal sliding motion between the teeth of spiral gears.

Shaft and Spiral Angles. The spiral angle of any helical or spiral gear is the angle between the tangent to the helix at the pitch surface and the axis of the gear. The angle may be described as right-hand or left-hand. Conventionally, right-hand spirals are classed as positive and left-hand spirals negative. The algebraic addition of spiral angles (i.e. using algebraic addition methods in regard to the + and − signs) gives a sum known as the *shaft angle* (Σ).

Thus, in Fig. 115 we have shafts AA_1 and BB_1 disposed as shown in respect of a common perpendicular which passes through O. The angles AOB_1 and AOB together make 180° and are therefore *supplementary*. Either of these angles may be considered as the shaft angle, the one actually selected being dependent upon the directions of rotation. If we think of a pair of *spur gears* we have connected shafts which are parallel and which rotate in opposite directions. If we think of a pair of *helical gears* we also have connected shafts which are parallel and which rotate in opposite directions—the teeth on the gears having equal spiral angles, such angles being of opposite "hand" or sign. Thus, the algebraic sum of these spiral angles is zero (the magnitude of the shaft angle in this case). In the case of spiral gears the following method is explained at length in Pitman's *Engineering Educator*—

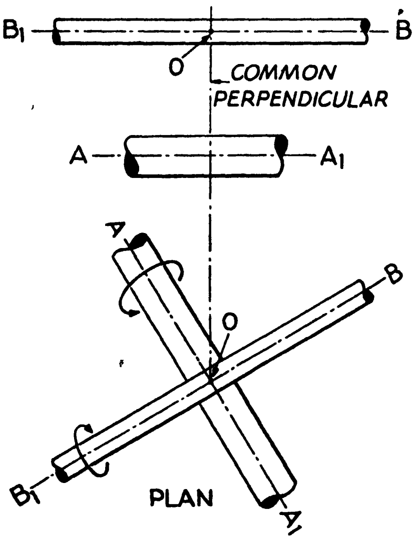


FIG. 115. SHAFT ANGLES

Find the angle through which the more distant shaft must be rotated about the common perpendicular in order to bring the shafts parallel whilst rotating in opposite directions.

In Fig. 115 the more distant shaft is AA_1, and it must be rotated about the common perpendicular through the angle AOB_1. This is an anti-clockwise rotation conventionally considered negative. We may, therefore, have two left-hand spiral angles, their algebraic sum being equal to the shaft angle. Alternatively, the larger spiral angle will be left-hand and the algebraic sum (or numerical difference) will be equal to the shaft angle.

Dr. Merritt, in his classic book *Gears* (Pitman), summarizes the rules as follows—

1. Looking along a shaft towards the common perpendicular, clockwise rotation is positive and counter-clockwise negative.

2. The shaft angle is the angle between the parts of the respective shafts, which have opposite directions of rotation when viewed along the common perpendicular.

3. The shaft angle is positive or negative, according to whether the angle traced out from the negative to the positive shaft (looking along the common perpendicular with the positive shaft nearer) is clockwise or anti-clockwise.

EXAMPLE. Fig. 116 shows the plan of two skew shafts having a distance of 8 in. between their axes.

If we look along the common perpendicular O, we see that the more distant shaft *BB* must be rotated in a clockwise direction in order to bring the shafts parallel whilst rotating in opposite directions. The shaft angle (Σ) is $+ 75°$.

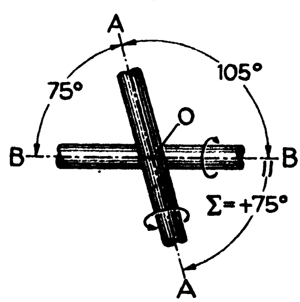


FIG. 116. ILLUSTRATING SHAFT ANGLE EXAMPLE

Further Useful Notes on Angles and Hands

When Shaft Angle is 45°. Tooth spirals are the same hand if the spiral angle of each gear is less than 45°, the angle of one gear being equal to 45° minus the angle of the other gear.

If the spiral angle of one of the gears exceeds 45°, the tooth spirals are of opposite hand, the angle of one gear being equal to 45° plus the angle of the other gear.

On a spiral gear having a spiral angle of 45°, the circumference is the same as the lead, for tan 45° = 1.

Shafts at Any Angle. When the spiral angle on both gears is less than the shaft angle, it will be found that the addition of the spiral angles of the two

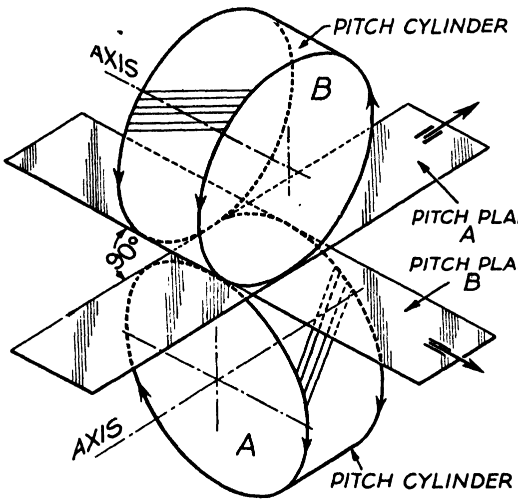


Fig. 117. Pitch Surfaces and Pitch Planes of Mating Spiral Gears

gears gives a sum that equals the shaft angle. In this case the hand of the spiral on each gear is the same.

When the spiral angle of one of the gears exceeds the shaft angle, it will be found that the difference between the spiral angles of the two gears equals the shaft angle. In this case, the hands of the spirals are opposite.

Shafts at Right Angles. Most spiral gear drives are for shafts at right angles and with pitch cylinders tangential. The pitch planes coincide near the pitch point and travel in directions at right-angles to each other. Fig. 117 shows that in these cases each pitch plane

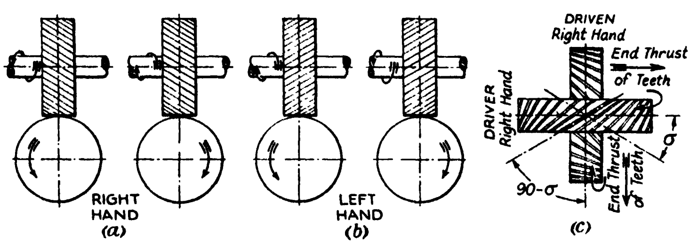


Fig. 118. Directions of Rotation—Spiral Gears

moves in the direction of rotation of its mating gear. In Fig. 117 arrow heads indicate the directions of movement of the planes A and B. These directions are obviously dependent on the hands of the tooth spirals. In Fig. 117 both gears have left-hand spirals. Generally the spiral (or *helix*) angles of both of a pair of mating spiral gears are the same hand (and also of the same hand as the shaft angle), and the shaft angle (Σ) equals the sum of the spiral angles. When the shaft angle is small the teeth may be of opposite hand. If the spiral angles are of the same hand the gear with the larger spiral angle should be the driver. Typical calculations and design data are given in *Manual of Gear Design* (Vol. 3), by Earle Buckingham (Machinery Publishing Co.), and in *Machinery's Handbook*.

If the spiral angles are respectively σ_p and σ_w then $\Sigma = \sigma_p + \sigma_w$. When σ_p and σ_w are nearly the same in magnitude the efficiency is highest. A very high spiral angle for the driven gear should be avoided.

Equivalent Diameter. (D_e and d_e.) This is the diameter of an ordinary straight-toothed spur gear which has the same number of

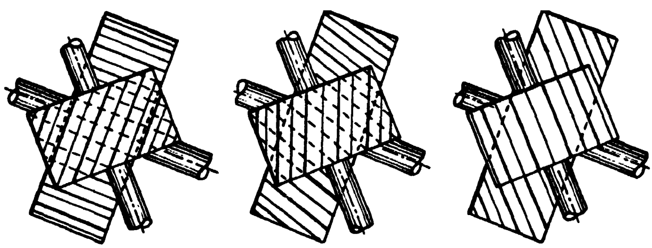


FIG. 119. SPIRAL ANGLES ALTERED, BUT SHAFT ANGLES EQUAL

teeth of the same normal pitch as the spiral gear considered. The following formulae apply—

$$d_e = \frac{t}{P_n}; \quad D_e = \frac{T}{P_n}$$

The Normal Pitch of the Teeth. In spiral gearing the pitch is the same for both gears of a pair only if measured normal to the tooth spirals. The cutting faces of the cutters used in both the hobbing and rack-planing processes are set at right-angles to the direction of the tooth spirals at the pitch surface. Hence, in order to use standard cutters it is necessary to design spiral gears with reference to normal pitch. Fig. 119 emphasizes that it is possible to alter the spiral angles on both gears and yet keep the shaft angle unaltered. However, as the spiral angles alter so also the centre-distance alters.

The circular pitch of a spiral or helical gear is always greater than the normal pitch, their relationship being as follows—

$$\frac{\text{Circular pitch } (p)}{\text{Normal pitch } (p_n)} = \frac{1}{\cos \sigma}; \qquad p_n = p_t \cos \sigma, \quad p_t = p_n \sec \sigma$$

If, therefore, there is an increase of spiral angle for a given equivalent diameter, the effect is an increase in the pitch diameter in proportion to $1/\cos \sigma$. Thus—

$$\textbf{Pitch diameter} = \frac{\text{Equivalent pitch diameter}}{\cos \sigma}$$

Thus
$$d = \frac{d_e}{\cos \sigma_p} \qquad D = \frac{D_e}{\cos \sigma_w}$$
$$= \frac{t}{P_n \cos \sigma_p} \qquad = \frac{T}{P_n \cos \sigma_w}$$
$$= t \sec \sigma_p \div P_n \qquad = T \sec \sigma_w \div P_n$$

Centre Distance. (C) is half the sum of the pitch diameters.

Thus
$$C = \tfrac{1}{2}(d + D) = \frac{d_e}{2 \cos \sigma_p} + \frac{D_e}{2 \cos \sigma_w}$$

(when pitch planes coincide)
$$= \frac{d_e}{2}\left(\frac{1}{\cos \sigma_p} + \frac{R}{\cos \sigma_w}\right)$$
$$= \frac{1}{2P_n}(t \sec \sigma_p + T \sec \sigma_w)$$

If σ_w and σ_p are equal the formula simplifies to—
$$C = \frac{T + t}{2(P_n \times \cos \sigma)}$$

In case other normal pitches are given, note that—
$$1/2P_n = m_n/2 = p_n/2\pi.$$

Tooth Proportions and Blank Diameters. Addendum and dedendum are based on the normal pitch in the same proportions as spur gears. Root and outside diameters should be obtained from—

Overall diameter = Pitch diameter + 2 (addendum)

Root diameter = Pitch diameter – 2 (dedendum)

Addendum = $1/P_n$; Whole depth of tooth = $2{\cdot}157/P_n$;

Normal tooth thickness at pitch line = $1{\cdot}571/P_n$

Alternative Method. It should be mentioned that in some cases correction is calculated as for the virtual spur gears, using the "plus and minus formulae" already given in Chapters II and IX. In the case of spiral gears we have—

$$A = m\left[0{\cdot}6 + 0{\cdot}4\,\frac{t \sec^3 \sigma_p}{T \sec^3 \sigma_w}\right]; \quad a = m\left[1{\cdot}4 - 0{\cdot}4\,\frac{t \sec^3 \sigma_p}{T \sec^3 \sigma_w}\right]$$
$$B = m\left[1{\cdot}65 - 0{\cdot}4\,\frac{t \sec^3 \sigma_p}{T \sec^3 \sigma_w}\right]; \quad b = m\left[0{\cdot}85 + 0{\cdot}4\,\frac{t \sec^3 \sigma_p}{T \sec^3 \sigma_w}\right]$$

In applying the above formulae the pinion is taken as the one with the smaller number of virtual teeth. These four formulae are rarely employed and have not been used in the following examples.

OTHER SIMPLE RULES

$$\text{Pitch Diameter} = \frac{\text{No. of teeth } (T)}{P_n \cos \sigma} = T \sec \sigma / P_n$$
$$= \frac{\text{Normal circular pitch} \times \text{No. of teeth}}{\pi \cos \sigma} = p_n T \sec \sigma / \pi$$
$$= \frac{\text{Transverse circular pitch} \times \text{No. of teeth}}{\pi} = p_t T / \pi$$

Outside Diameter = Pitch diameter + 2 (addendum)
= Pitch diameter + $2/P_n$

(See similar applicable rules in Chapter VIII—Helical Gears.)

Simple Example. Pair of spiral gears, each has 36 teeth. Spiral angle = 45°. Normal diametral pitch = 4.

Equivalent diameter (d_e), i.e. "normal pitch diameter"

$$= \frac{t}{P_n} = \frac{36}{4} = 9 \text{ in.}$$

Pitch diameter (d) $= \frac{d_e}{\cos\sigma} = \frac{9}{0{\cdot}70711} = 12{\cdot}728$ in.

Outside diameter = Pitch diameter + 2 (addendum)

$= 12{\cdot}728 + 2\,(0{\cdot}25) = 13{\cdot}228$ in.

Lead of spiral $= \pi d \times \cot\sigma$

$= \pi \times 12{\cdot}728 \times 1 = 39{\cdot}986$ in.

Centre-distance (C). Rule (a): $\frac{d_w + d_p}{2} = 12{\cdot}728$ in.

or, Rule (b): $= \frac{T + t}{2(P_n \times \cos\sigma)} = \frac{72}{8 \times 0{\cdot}70711} = 12{\cdot}728$ in.

(*Note:* Rule (b) applies only to 45° gears.)

No. of teeth = Pitch dia. $\times P_n \times \cos\sigma$

$= 12{\cdot}728 \times 4 \times 0{\cdot}70711 = 36$

No. of teeth for which to select cutter (N_1) $= \frac{t}{(\cos\sigma)^3}$

$= \frac{36}{(\cos 45)^3} = 102$ approx.

Cosine of spiral angle $= \frac{T}{P_n \times \text{pitch dia.}} = \frac{36}{4 \times 12{\cdot}728} = 0{\cdot}70711$

Note. To check calculations for Pitch Diameter (d) and Centre Distance (C) use the following—

$$(t \times \tan\sigma_p) + T = 2 \times C \times P_n \times \sin\sigma_p$$

USING A WORM IN A SPIRAL GEAR DRIVE. Sometimes a worm is used in conjunction with a spiral gear to connect shafts at 90°—especially when an axial movement of the spiral gear is required whilst it is rotating. A parallel worm is simply a spiral gear with a very large spiral angle. Also, as its threads have true helical form, it is a screw. If a worm is the driver of a pair of spiral gears for shafts at right-angles, the speeds of the shafts being equal, then the circumference of the pitch circle of the driven gear must have the same length as the lead of the worm.

Suppose the worm is 2 in. pitch diameter, 3 in. lead, and ⅜ in. pitch. It will evidently be 8-start. Pitch diameter of driven gear $= \frac{3}{\pi} = 0.9548$ in. The number of teeth in the driven gear = 8. A common pressure angle for these worms is 14½°; and if this applies in the present case, the teeth of the driven gear, if of usual "standard" proportions, will be undercut badly—so that extensive addendum modification will be necessary.

The design of such a pair involves the use of many-termed formulae given, with worked examples, in Vol. 3 of *Manual of Gear Design*, by Earle Buckingham.

PRODUCTION METHODS SUMMARIZED

GENERATION OF SPIRAL GEARS Like spur and helical gears, spiral gears can be produced by both forming and generating processes. Of the latter, the two most commonly used for spiral gears are the hobbing and rack-shaping processes. When **hobbing spiral gears**

it is, of course, necessary for the hob to have the same normal pitch as that of the required spiral gear. The hob spindle must be adjusted so that the direction of cutting is parallel to the teeth of the gear, i.e. it is adjusted to an angle which equals the spiral angle of the gear, plus or minus the lead angle of the hob threads (according to whether the gear and the hob are of the same or opposite hands). In other respects the hobbing process is the same as has previously been described in connection with helical gears. When using the **rack cutter process,** already adequately described in previous pages, it is

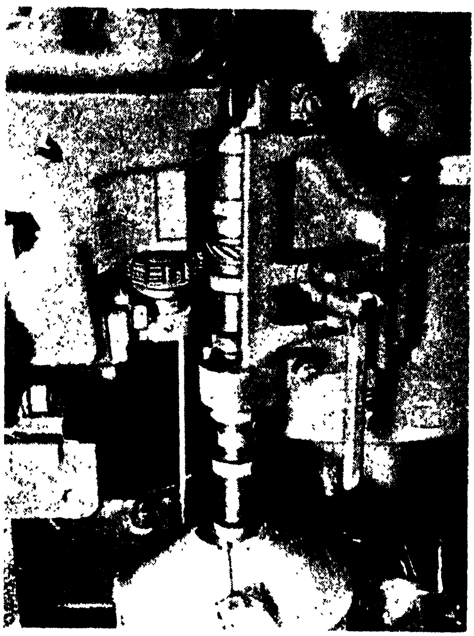


FIG. 120. HOBBING ON A LEES-BRADNER HOBBING MACHINE. THE JOB IS A SPIRAL GEAR IN CENTRE OF AUTOMOBILE CAMSHAFT

necessary to select a cutter of the same normal pitch as that of the required spiral gear, so inclined as to reciprocate at the desired spiral angle. The rolling mechanism must be set up for the circular pitch, as fully explained in the *Sunderland Operators' Handbook*. The **pinion cutter process,** as previously described in connection with helical gears, is largely used for spiral gears of small diameter and low spiral angle. The direction of reciprocation of the pinion cutter is parallel to the axis of the blank, hence its teeth must be inclined at a spiral angle equal to that of the blank. Thus, the cutter reciprocates with a "spiral" motion, the latter having a fixed lead controlled by helical guides as previously explained.

FORM MILLING OF SPIRAL GEARS

Pitch of Form-milling Cutter for Spiral Teeth. The thickness of the rotary cutter at the pitch line should be one-half the normal circular pitch. The cutter in a diametral pitch series must therefore be selected in relation to the normal diametral pitch of the gear. As we have seen, $P_n = P/\cos\sigma$, i.e. normal P = real $P/\cos\sigma$. In the Brown & Sharpe system there are usually available eight different groups of cutters per pitch, each shaped differently. Therefore, when P_n has been found, the number of cutter for that particular pitch must be determined as follows.

In cutting *spur gears* by form milling the proper cutter to select depends upon (1) the pitch of the teeth, (2) the number of teeth in the gear. However, in *spiral milling* the cutter is not selected with reference to the actual number of teeth in the gear, but to the number of teeth in the virtual spur gear, i.e. the virtual number of teeth (T_v).

The following rule is employed—

$$\text{No. of teeth for which to select cutter} = \frac{\text{Actual no. of teeth in gear}}{(\cos\sigma)^3}$$

The formula is independent of the pressure angle.

The result gives the number of teeth for which to select the cutter from the table given on page 61.

EXAMPLE. Spiral gear has 40 teeth. Spiral angle is 45°.

$$\text{Use formula } T_v = \frac{T}{(\cos\sigma)^3} = \frac{40}{(0{\cdot}7071)^3} = 113 \text{ (approx.)}$$

This calls for a No. 2 cutter for spur gears having any number of teeth between 55 and 134.

The table of the machine is swivelled round to the spiral angle of the teeth to be cut, the work mounted in a dividing head geared up to the lead of the teeth or threads, and a tooth space milled.

See also *Cutting Helical Teeth on a Milling Machine* on page 86, and *The Virtual Spur Gear* on page 80. In *Machinery's Handbook* is given a *Table for Selecting Cutter for Milling Spiral Gears.* It gives two columns, viz. *Angle of Spiral*, and K. The factor K is equal to $1/\cos^3\sigma$. The use of this table saves a good deal of time.

CHAPTER XI

WORM GEARS

WORM gears, like spiral gears, are used to connect "skew" shafts, i.e. shafts at any angle and not in the same plane. The shafts are non-intersecting, are usually a fair distance apart, and in practice are usually at right-angles. Thus, B.S. No. 721 is expressly limited to worm gears for shafts at 90°. Fig. 121 represents a typical shaft

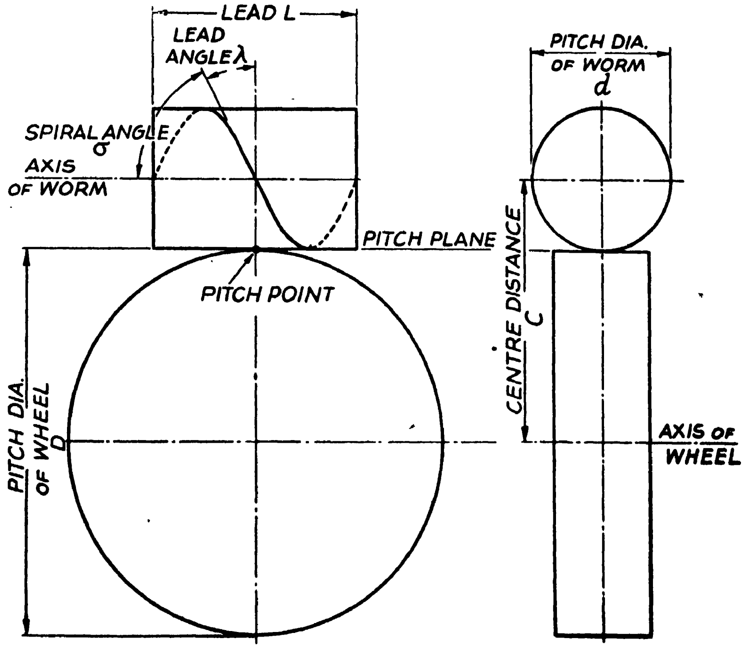


FIG. 121. BASIC DIMENSIONS OF WORM GEARS

arrangement for a worm and wormwheel drive. The worm is a screw and usually it is the driving member. When it rotates its threads press against the teeth in the rim of the wheel, causing it to rotate.

The relative positions occupied by the shafts would enable a pair of spiral gears to be used for the drive. But the tooth contact on spiral gearing is theoretically a point, whereas on worm gearing there is line contact. Worm gears are stronger than spiral gears, which are noted for their relatively small load-carrying capacity.

Worm gearing was formerly looked upon mainly as a means of obtaining a compact high speed reduction. For instance, old editions of Dr. Unwin's classic, *Machine Design*, refer to worm gearing as disadvantageous on account of heavy friction and wear. It is now much more successfully used not only when a large reduction in velocity is required, but also as a means of transmitting considerable power with high efficiency. However, there are, and always will be, many applications in engineering where what is called "high efficiency" and "high performance" is unnecessary, and where emphasis is mainly on a speed-reduction drive that is self-sustaining, free from backlash for a long period, and satisfactory for intermittent service.

The Pitch Surfaces of a Worm and Wormwheel. When dealing with mating spur and helical gears we imagined the existence of cylindrical pitch surfaces, each tangential to a common rack or pitch plane. In the case of bevel gears we saw that the pitch surfaces were cones tangential to a common disc. To employ a similar analogy to suit a worm and wormwheel we imagine the wormwheel to have a pitch cylinder which rolls with a pitch plane—the latter being associated with the worm. Worm threads being helical, the worm corresponds

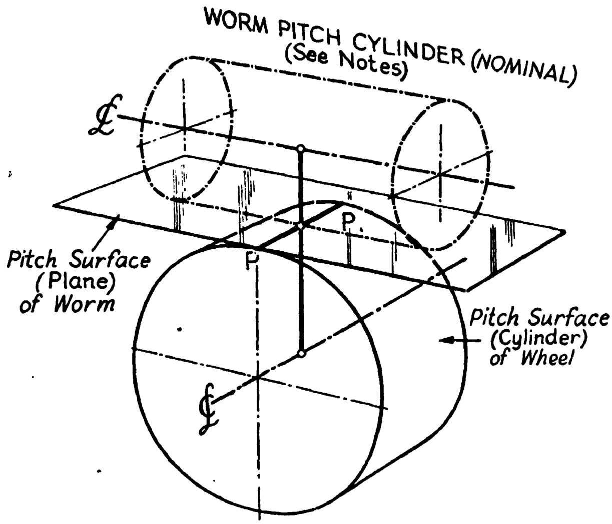


FIG. 122. PITCH SURFACES OF WORM AND WORMWHEEL
The wormwheel has a pitch cylinder which rolls with a pitch plane representing the pitch surface of the worm

to a rack. A normal-to-helix section of a worm for a right-angled drive shows rack-like teeth which, due to rotation of the worm, appear to move parallel to its axis. This "translation" is either to the left or right according to (1) the hand of the worm, (2) the direction of its rotation. Thus, in Fig. 122 is represented the pitch surfaces, cylindrical and plane, of a wheel and worm respectively. The position of the plane has no relation to the dimensions of the worm, i.e. to the *nominal pitch diameter* from which detail dimensions are calculated. Whilst the "worm cylinder" shown in the diagram may be called the "worm pitch cylinder," its surface is not actually a pitch surface. See the definition of "pitch cylinder of a worm" and the ensuing text on page 132. Fig. 122 shows the line of contact *PP* between the pitch surfaces. It may be called the *pitch surface generator* of the wheel pitch cylinder. At any point of contact the common normal to the pitch surfaces must pass through the pitch surface generator.

It will simplify matters at this stage to give commonly accepted definitions of terms used in connection with worm gearing. Some of these definitions apply only to worms, like the British Standard worm, having threads which are involute in section at right-angles to the axis.

THE WORM

A worm is essentially a screw thread. Hence it has various diameters, angles, pitch, and lead. If it has a high lead angle, we readily see its resemblance to a single helical involute gear with one or more teeth.

Distinction between Pitch and Lead. On a single-start thread (or worm), lead equals pitch. On a 2-start thread, lead equals twice pitch. On a 3-start thread, lead equals three times pitch. Pitch is always the distance between corresponding points on adjacent threads measured parallel to the axis, and when the term is used without qualification, *axial pitch* or *linear pitch* is assumed.

The **lead** (L) is the distance, measured parallel to the axis, by which each thread advances per revolution. Lead = Axial pitch × No. of starts.

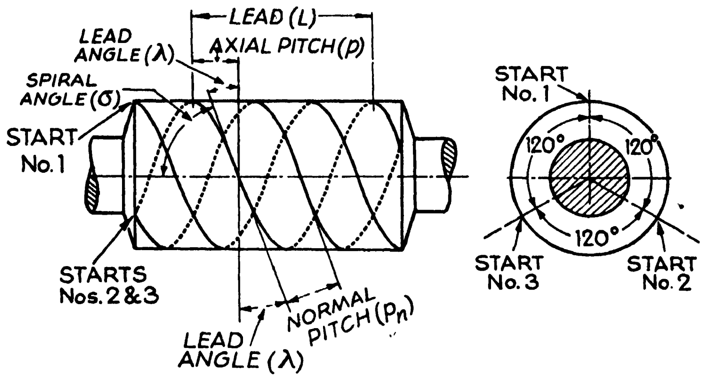


FIG. 123. THREE-START WORM

The **axial pitch** (p) is the distance, measured parallel to the axis, between similar faces of successive threads.

The **normal pitch** (p_n) is the length of arc between similar faces of successive threads measured along a helix lying on the pitch cylinder normal to the thread helix, and is equal to the normal pitch of the generating rack.

The **module** (m) is the axial pitch of the worm divided by π.

The **lead angle of a worm** (λ) is the angle between a tangent to the thread helix on the pitch cylinder and a plane perpendicular to the axis of the worm. High-efficiency worms have high lead angles.

The **spiral angle of a worm** (σ) is the complement of the lead angle.

The **base lead angle of a worm** (λ_0) is the angle between a straight line lying in the surface of the thread tangential to the base cylinder, and a plane at right angles to the axis. This is equal to the lead angle on the base cylinder.

The **axial pressure angle of a worm** (ψ_a) is the acute angle between the axis of the worm and a normal to the profile on an axial section at a point on the pitch cylinder.

The **normal pressure angle of a worm** (ψ_n) is the acute angle between a normal to the thread surface at any point on the pitch cylinder and the tangent plane to the pitch cylinder at that point. The B.S. normal pressure angle is 20°.

The **axial thickness of the worm thread** (g_a) is the distance between opposite faces of the same thread measured on the pitch cylinder in a direction parallel to the axis.

The **normal thickness of the worm thread** (g_n) is the length of arc between opposite faces of the same thread along a helix which lies in the pitch cylinder and is normal to the thread helix.

The **pitch cylinder of a worm** is that cylinder co-axial with the worm which touches the pitch cylinder of the wheel.

Note. It may be well to repeat that the pitch cylinder of any worm is not of fixed diameter, but depends upon the centre distance at which the worm runs with a given wheel. It only becomes a fixed dimension when the centre distance becomes fixed. The real pitch diameter of the worm is not necessarily equal to the *nominal pitch diameter* used in calculating detail dimensions.

The manner in which the tooth action of worm gearing appears to correspond with that of a rack and pinion has been mentioned previously. Fig. 124 shows a section of a worm, the threads of which, being helical, have apparent motion ("translation") parallel to the worm axis. Hence the straight line PP corresponds to the pitch line of a rack. It is tangential to the pitch circle at P, at which point we imagine the pitch circles to make contact. Note that wherever we take a section of the worm, the pitch circle of the wheel has the same

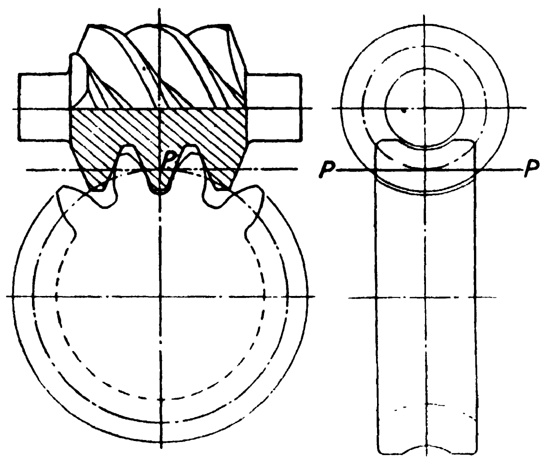


FIG. 124

diameter and all the points P collectively form the line PP, i.e. the pitch line of the gear. The circle which is tangential to PP is therefore the pitch circle of the worm in this arrangement. If the worm is separated from the wheel shown in the diagram, it cannot truly be said to have any particular pitch surface or pitch diameter.

$$\text{Pitch dia. of wormwheel} = \frac{\text{No. of teeth} \times \text{Axial pitch of worm teeth}}{\pi}$$

$$D = \frac{T \times p}{\pi} = Tm$$

Nominal pitch dia. of worm $(d) = 2C - D$, where C = centre distance, T = number of teeth in gear, D = nominal pitch diameter of gear. Measured round the pitch circle of the gear the *circular pitch* of the teeth (p) equals the axial pitch (p) of the worm teeth.

The **pitch diameter** (d) is the diameter of its pitch cylinder.

The **base diameter** (d_0) is the diameter of its base cylinder.

The **base cylinder** is the cylinder from which the involute thread form is developed.

The **number of starts** (t) is the number of thread sections in a plane at right angles to the axis.

Hand. Misunderstandings about the meaning of the terms *left-hand* and *right-hand* applied to threads, gears, tools, etc., are frequent. There is perhaps fairly general agreement about the "hand" of a thread, but in 1941 the B.S.I. considered it opportune to issue an Addendum to B.S. No. 721—1937 as follows—

"Right"- or "Left"-hand Helix. If when a point is moved along a helix in a clockwise direction looking along the axis, it moves away from the observer, the helix is "right-hand"; if it moves towards the observer, the helix is "left-hand."

A right-hand worm or wormwheel is one in which the general direction of the thread or teeth follows a right-hand helix; a left-hand worm or wormwheel is one in which the general direction of the thread or teeth follows a left-hand helix.

A worm and wormwheel meshing together with their axes at right-angles must have threads or teeth of the same hand.

THE WORMWHEEL

The **pitch cylinder of a wormwheel** is that cylinder on which the pitch is the same as the axial pitch of the worm.

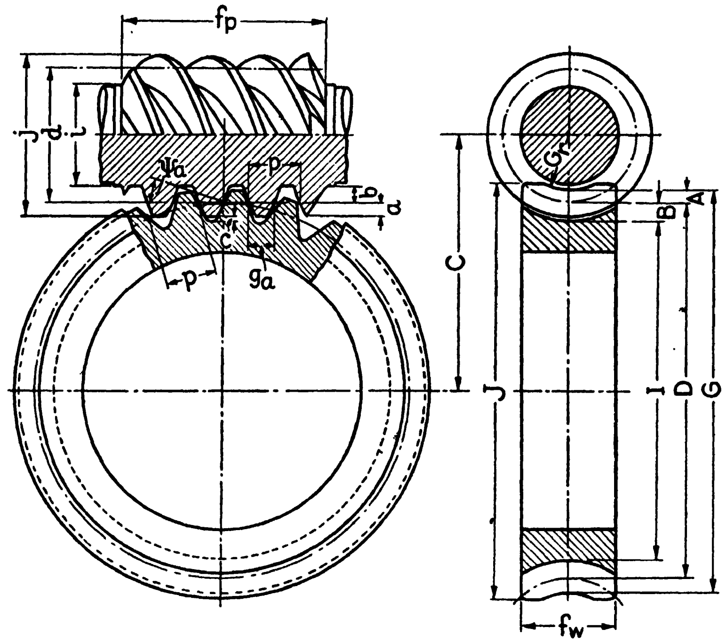


FIG. 125. NOTATION FOR MODERN B.S. WORM GEARING. REPRODUCED FROM B.S. NO. 721

The **pitch diameter of a wormwheel** (D) is the diameter of its pitch cylinder.

The **throat diameter** (G) is the diameter at the tips of the teeth in the central plane.

The **outside or overall diameter** (J) is the maximum diameter of the wheel.

The **root diameter** (I) is the diameter at the bottom of the wheel teeth in the central plane.

The **gorge radius** (G_r) is the radius of the section of the curved throat of the wormwheel in a plane containing the axis.

The **circular pitch** (p) of the wheel is the length of circular arc of the pitch cylinder between similar faces of successive teeth and is equal to the axial pitch of the worm.

The **tooth thickness** is the length of arc of the pitch cylinder between opposite faces of the same tooth measured in the central plane.

The **overall face-width of the wheel** (f_w) is the width of the wheel rim in the direction of its axis.

The **effective face-width of the wheel** (f_{ew}) is that part of the face-width which for purposes of calculation is regarded as carrying the load. It is equal to the width of that portion of the wheel pitch cylinder which is intersected by the cylinder containing the tips of the worm threads or to the width of the wheel face, whichever is the less.

THE WORM AND WORMWHEEL

The **pitch point** of a worm and wheel is the point of tangency of the pitch circles or cylinders.

The **centre distance** (C) is the length of the common perpendicular to the axes of the worm and wheel.

The **shaft angle** (Σ) is the angle between the axes of the shafts when viewed along the common perpendicular.

The **clearance** (c) is the shortest distance between the tip of the tooth or thread and the bottom of its mating space.

The **ratio** (R) **of worm and wheel** is the ratio of the number of teeth of the wheel to the number of starts of the worm.

The **backlash** is the total play between the surfaces of the worm threads and the wormwheel teeth measured normal to the surfaces.

THE TWO MAIN CLASSES OF CYLINDRICAL OR PARALLEL WORMS. In a general way it is possible to divide these worms into two main groups by considering the shape of their threads on certain sections—

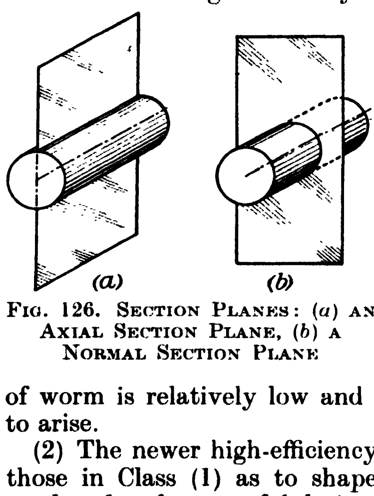


FIG. 126. SECTION PLANES: (*a*) AN AXIAL SECTION PLANE, (*b*) A NORMAL SECTION PLANE

(1) The older design, still widely used, mainly single-start, having straight-sided rack-like threads *in an axial section*, usually having a low lead angle and a low pressure angle (14½° or 15°) also. Typical example: Fig. 127.

A low lead angle entails considerable frictional loss and, in general, low efficiency. Furthermore, the load-carrying capacity of this type of worm is relatively low and undercutting of the wheel teeth tends to arise.

(2) The newer high-efficiency worms, generally quite different from those in Class (1) as to shape in axial section. Various accurately produced and successful designs are available but we shall confine our attention to the *British Standard type*, covered by B.S. No. 721, these worms having thread surfaces which are involute helicoids. In a section *normal to the axis* (see Fig. 126) the threads of such a worm have involute profiles. On a B.S. worm, then, the screw surface is an **involute helicoid,** or a screw surface, the *transverse section* (perpendicular to the worm axis) of which is an involute of a circle concentric with the axis. The normal pressure angle of this type of worm is 20° and it usually has a high lead angle. On a plane containing the axis of the worm the shape of the thread profile is a curve to which no particular geometrical term can be applied. If the lead angle of the worm is small the curve is so flat as to be nearly straight.

14½° INVOLUTE WORM GEARING. This was the standard system originally sponsored by the Brown & Sharpe Mfg. Co. and adopted very largely. It has rendered yeoman service all over the world. Other manufacturers introduced similar systems which, like the B & S system, were intended mainly for large ratios of reduction and, as a consequence, generally envisaged the use of single-start worms having a low pressure angle and, of course, a low lead angle. Such gearing, when properly mounted and lubricated, gives quiet and steady transmission. It is usually self-locking, but theoretically inefficient as generally designed and used.

A typical **14½° single-start (right-hand involute) worm** is shown in

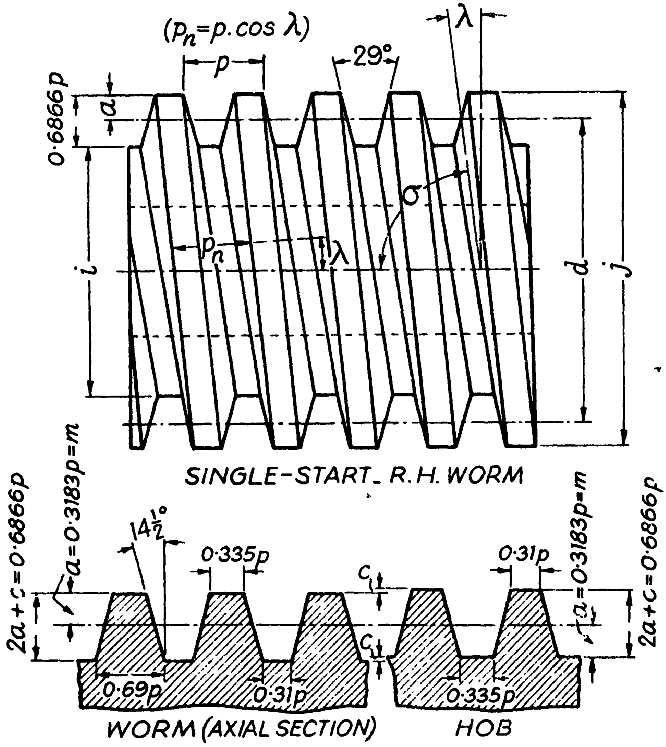


FIG. 127. SINGLE-START INVOLUTE WORM, WITH CORRESPONDING HOB PROPORTIONS

Fig. 127, together with views showing axial sections of the worm and also of the hob for milling the teeth on the rim of the mating gear.

The form of the worm thread, seen on an axial section is straight-sided, having the same general shape and dimensions as the full-depth 14½° involute rack tooth of the same pitch. (Refer back to Fig. 29 (*a*).) Unlike the rack, however, most worm threads of this type are not filleted at the root corners. (See, however, B.S. No. 721, where, for B.S. high-efficiency worms, a root fillet is recommended, its radius to be not less than the clearance (*c*), also the thickness of the thread is to be one-half the axial pitch, measured half-way down the tooth.)

The analogy between worm gears and a rack and pinion, previously mentioned, no doubt influenced the widespread use in the past of a worm thread profile shaped and proportioned exactly like the section of an ordinary straight-sided involute rack. The central section of the wheel teeth, cut to mesh with this type of worm, yields profiles like those of an involute spur gear having the same pressure angle, circular pitch, and number of teeth. Possibly that is why these worm gears became generally known as *involute worm gears*.

In the following example it will be seen that the nominal pitch diameter of the worm is selected arbitrarily, and the threads are proportioned like the full-depth teeth of involute spur gears. When the lead angle exceeds 20°, proportions are usually calculated on normal pitch. Note that $p_n = p_a \cos \lambda$.

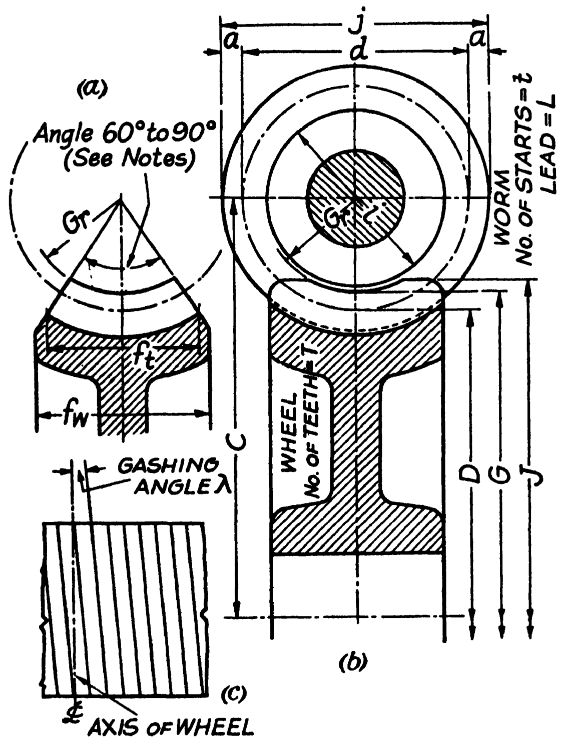


FIG. 128. WORM AND WHEEL DIMENSIONS

Formulae Applicable to 14½° Involute Worm Gearing. (Fig. 127.)

Notation. The notation employed in these formulae is that recommended by the B.S.I., but the rules given on p. 137 are in many instances *not* applicable to the design of modern high-efficiency worm gearing. The latter is well covered in B.S. 721.

Note 1. **Worms with Large Lead Angle.** Many engineers change the formulae given in the following table when dealing with worms of this type, having large lead angles (15° and over is suggested in *Machinery's Handbook*) as follows—

(1) Axial pitch (p) is replaced by normal pitch (p_n), for such threads are measured normal to the helix.

Then $p_n = p \times \cos \lambda$. (Note that axial pitch is often represented by p_a.)

(2) Addendum (a) = $0{\cdot}3183p_n$.

(3) Whole depth of worm tooth = $0{\cdot}6866p_n$.

(4) Width of thread tool at end = $0{\cdot}31p_n$.

WORM GEARS

To Find	Rule
Circular pitch (p) of Wheel	$p = \frac{\text{Throat dia. of wheel} \times \pi}{\text{No. of teeth} + 2} = \frac{G \times \pi}{T + 2}$ (*Note:* Circular pitch of wheel equals axial pitch of worm)
Diametral pitch (P) of Wheel	$P = \frac{\pi}{\text{Circular pitch (wheel)}}$, or $= \frac{\pi}{\text{Axial pitch (worm)}}$ $= \frac{\pi}{p}$
Axial pitch of Worm (p)	$p = \frac{\text{Lead of worm}}{\text{No. of starts}} = \frac{L}{t}$
Addendum of Worm Tooth (a)	$a = \text{Axial pitch} \times 0{\cdot}3183 = 0{\cdot}3183p$
Pitch dia. of Worm (d)	$d = \text{Outside dia. of worm} - \text{Twice addendum}$ $= j - 2a$, or, $d = \text{Twice centre distance} - \text{Pitch dia. of wheel}$ $= 2C - D$
Bottom or root dia. of Worm (i)	Outside dia. of worm − Twice whole depth of tooth $i = j - 2(0{\cdot}6866p)$
Width of Worm thread tool at end	$\text{Width} = 0{\cdot}31p$
Outside or overall dia. of Worm (j)	Pitch dia. of worm + Twice addendum $j = d + 2a$
Pitch dia. of Wheel (D)	$D = \frac{\text{No. of teeth in wheel} \times \text{Circular pitch}}{\pi} = \frac{T \times p}{\pi}$ $= \text{No. of teeth in wheel} \times \text{Addendum} = T \times a$
Throat dia. of Wheel (G)	$G = $ Twice addendum of worm tooth + Pitch dia. of wheel $= 2a + D$
Centre-distance between Worm and Wheel (C)	$C = \frac{\text{Pitch dia. of wheel} + \text{Pitch dia. of worm}}{2}$ $= \frac{D + d}{2}$
Radius of Wheel throat, or Gorge radius (G_r)	$G_r = \frac{1}{2}(\text{Outside dia. of worm}) - 2 \times \text{addendum}$ $= \frac{j}{2} - 2a$
Outside or Overall dia. of Wheel (J) (See note below)	Various rules are employed— (1) $J = \text{Pitch dia.} + 3\,(\text{addendum})$ (2) $J = \text{Throat dia.} + 0{\cdot}4775p$, for single- and double-threaded worms $= \text{Throat dia.} + 0{\cdot}3183p$, for triple- and quadruple-threaded worms
Lead angle of Worm (λ)	$\cot \lambda = \frac{\pi \times \text{pitch dia.}}{\text{lead}} = \frac{\pi d}{L}$ or $\tan \lambda = \frac{\text{lead}}{\pi \times \text{pitch dia.}} = \frac{L}{\pi d}$

Note 2. **Rims of Wheels.** For this type of gear, with low-pressure angle, it is seldom necessary to calculate the overall diameter of the wheel if an accurate drawing is made. A **bevelled or chamfered rim** is shown in Fig. 128 (*a*). It is not the style used for worm gearing for heavy power transmission, inasmuch as it reduces the tooth contact and necessitates additional machining. The "square rim" is employed for modern high-efficiency gearing (see Fig. 128 (*b*)). An average angle for the bevelled type of rim is 75°, but angles from 60° to 90° are used. The use of a larger angle results in the gear being carried round a larger part of the circumference of the worm. Little advantage would appear to accrue from this, especially when steep pitches are used. For the type of gear under consideration, the width of the wheel at the root of the tooth may be taken as $f_t = \frac{3}{5}j$ to $\frac{2}{3}j$. The radial thickness of the rim should be at least as great as the full depth of the tooth. Rim proportions for high efficiency gears, in terms of module, are given in B.S. No. 721.

B.S.I. Recommendation. Wormwheel Rim Section. The B.S.I., in B.S. No. 721, recognize that worm gears will continue to be made with hobs based on a straight-sided axial section, and in regard to wheels cut by such hobs, recommend as follows (see Fig. 132 (*d*)): A line drawn from the centre of the worm to meet the intersection of the outside diameter of the worm and the pitch cylinder of the wormwheel should make an angle θ with the line of centres not less than the value given by

$$\tan \theta = (\pi j \tan \psi_a)/L$$

or, $0{\cdot}5[j \cos \theta + Tm]$ must not be less than C

Exercise. Use the foregoing tabulated formulae to calculate the leading dimensions of a worm and wheel, and make a conventional drawing for the arrangement on the lines of Fig. 128. The worm is single-threaded, 1 in. axial pitch and has a pitch diameter of 4 in. Wheel has 48 teeth.

Answer Hints. $p = 1$ in.; $P = 3{\cdot}1416$; $D = 15{\cdot}279$ in.; $J = 15{\cdot}915$ in.; $j = 4{\cdot}6366$ in.; $i = 3{\cdot}2634$; $G_r = 1{\cdot}6817$ in.; $\lambda = 4° 33'$; $C = 9{\cdot}6395$ in.; f_p (length of worm) $= 6{\cdot}23$ in.

UNDERCUTTING OF TEETH WITH LOW-PRESSURE ANGLES: HOW AVOIDED

The usual way to calculate the throat diameter of a wormwheel is to add twice the wheel tooth addendum to the pitch diameter, just as in calculating the outside diameter of a spur gear. However, if this rule is followed for wheels having 30 teeth or less it is found that the flanks of the teeth are undercut by the hob. Not only are such teeth mechanically weak but also they give the worm a poor effective bearing surface below the pitch line. This *undercutting* we have described elsewhere as a consequence of the *interference* of rack teeth with gear teeth. When using a straight-sided worm thread the corresponding wheel tooth profile, on an axial section, is involute, so that we have an evident rack and spur gear analogy. With a pressure angle of 14½° undercutting begins at 31 teeth and affects nearly the whole flank of a pinion having 12 teeth. In the case of a 20° pressure angle, undercutting begins at 17 teeth.

One way of avoiding this, recommended by the Brown & Sharpe Co., is to increase the throat diameter, calculating it thus: throat diameter = (pitch dia. of wheel × 0·937) + (4 × addendum of wheel). The overall or outside diameter is usually obtained from a reasonably accurate drawing.

If it is necessary to maintain the same centre-distance, the outside diameter of the worm must be reduced by the same amount as the throat diameter of the gear is increased.

Increasing Centre-distance. The B.S.I. refer to this problem in B.S. No. 721 under the heading: *Gears made with hobs based on a straight-sided axial section—Addendum of Worm Threads*, as follows—

Below a certain minimum number of teeth in the wheel, interference at the roots of the teeth will occur. This may be avoided by increasing the centre-distance at which the wheel is cut and meshed with the worm. The minimum centre-distance shall be—

$$C = 0{\cdot}5\,[\,j + D\cos^2\psi_a]$$

Increasing centre-distance to remedy undercutting is called "correction." Other remedies suitable in some cases, are to increase the pressure angle or to adjust the addenda and dedenda, as in spur gearing.

Centre-distance of cutting or generation is not to be confused with centre-distance of running or mounting. The latter may be altered slightly for spur and helical drives, but worm gears should be run only at the designed centre-distance.

Diameter of Worm. As stated previously, the pitch diameter of a worm is quite an arbitrary dimension. A worm can be cut to any pitch diameter suitable to enable it to work at the centre distance at which it is to engage with a given wheel. When the centre-distance is fixed, the pitch diameter of the worm can be fixed. Given the pitch and the number of teeth in the wormwheel we can calculate its pitch diameter, but the diameter of the mating worm may be of any size to suit a given centre-distance. If diameters of worms were made to a general rule based on pitch, fewer hobs would be required to finish the teeth on the wheels.

This is exemplified in a note issued by an American gear-cutting firm, to its prospective customers, as follows: "To facilitate the designing of worm gear drives and to eliminate the additional cost of hobs, we will be pleased to furnish, upon receipt of necessary data, dimensions of our standard hobs to suit customers' requirements."

Naturally, many empirical rules have been devised in connection with involute worms. For instance, the Brown & Sharpe Co. state: *It is unusual to have the diameter of the worm much less than four times the linear pitch, but the worm can be of any larger diameter, five or ten times the linear pitch if desired.*

Length of Worm. The aim is usually to make the worm just long enough to engage the teeth of the wormwheel in contact with it at one time, i.e. for "complete action."

Brown & Sharpe Rule

f_p (minimum) $= 2\sqrt{D''(G - D'')}$

Another American Rule

f_p (minimum) $= \sqrt{G^2 - (G - 4a)^2}$

A.G.M.A. Rule for length of worm (shell-type, i.e. bored) for wheels having 30 to 40 teeth is: Length of worm = axial pitch × [4·5 + (No. of teeth ÷ 50)]

The B.S. Rule (B.S. No. 721) is

$f_p = \sqrt{(J^2 - D^2)}$

Machinery's Rule

f_p (minimum) $= \sqrt{8D - a}$

f_p = Length of worm
D'' = Working depth of tooth
J = Overall dia. of wheel
D = Pitch dia. of wheel
G = Throat dia. of wheel
a = Addendum

Direction of Rotation. Fig. 129 gives the direction of rotation of worm-wheels gearing with worms of right- and left-hand threads respectively.

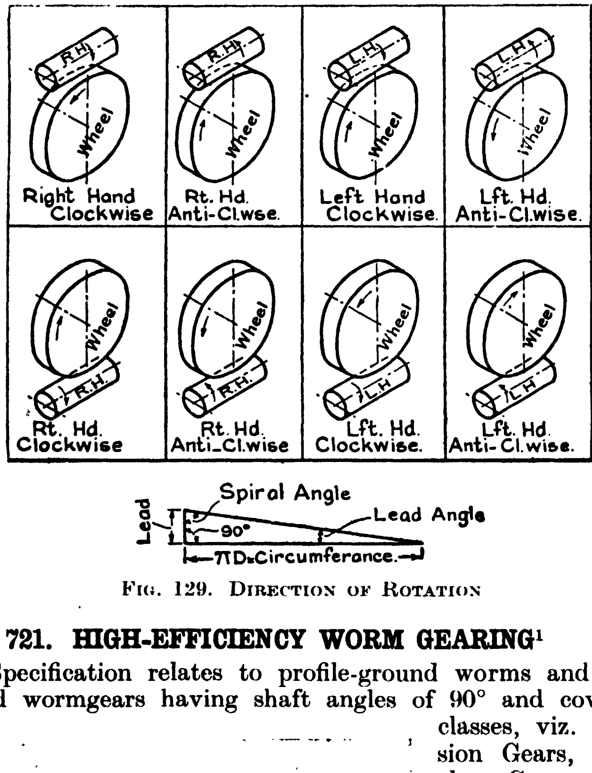


FIG. 129. DIRECTION OF ROTATION

B.S. No. 721. HIGH-EFFICIENCY WORM GEARING[1]

This Specification relates to profile-ground worms and correctly generated wormgears having shaft angles of 90° and covers three classes, viz. (1) Precision Gears, (2) High-class Gears, (3) Commercial Gears.

FIG. 130. MODERN WORM GEARS
(By courtesy of David Brown & Sons, Ltd., Huddersfield.)

Basic Thread Form. Such as can be generated from a basic rack surface which is straight-sided in normal section, so as to simplify cutting, profile grinding, measurement and geometrical analysis. The worm thread to be involute in section in a plane perpendicular to the axis, thus to be an involute helicoid. The fillet radius of the worm threads to be not less than the clearance. It will be

[1] A list of British Standards is obtainable from the British Standards Institution, Publications Department, 28 Victoria Street, London, S.W.1.

evident that this type of worm could accurately be described as a *single helical pinion* or a *spiral pinion.*

Addendum of Worm (a) = Module (m).

Working depth = Twice the normal module = $2m \cdot \cos \lambda$.

Clearance (c) = One-tenth of working depth = $0{\cdot}2m \cos \lambda$.

Normal Pressure Angle (ψ_n) = 20°.

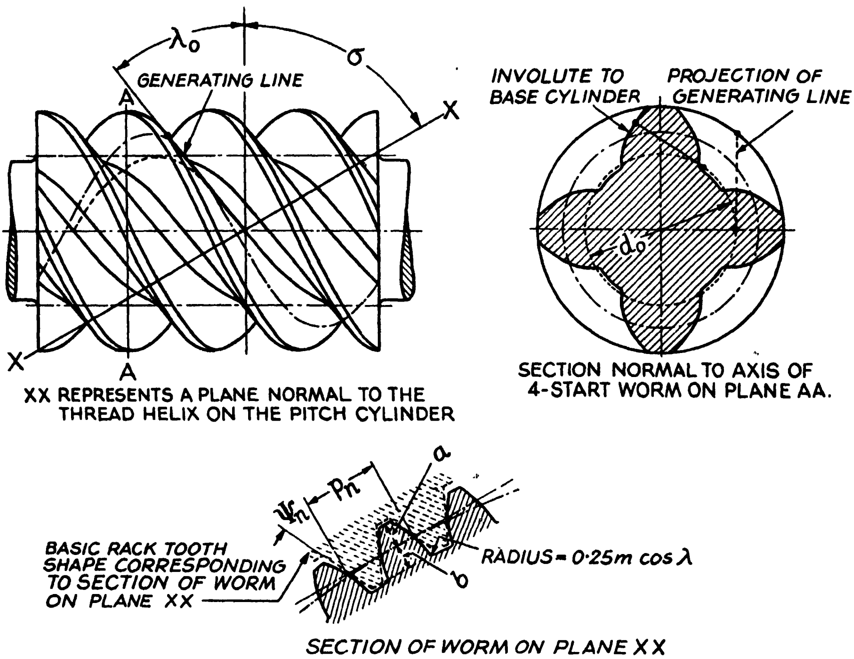


FIG. 131. BASIC DIMENSIONS, WORM THREAD AND GENERATING RACK, BRITISH STANDARD

Thickness of Worm Thread. The normal chordal thickness of the worm threads to be such that *the axial thickness of the worm threads at one-half the total depth is equal to one-half of the axial pitch.* (Not, as formerly, at one-half the working depth.)

Designation of Pitch. Pitch to be designated by the module in inches.

$$\text{Module} = m = \text{axial pitch}/\pi = 0{\cdot}3183 \times \text{axial pitch}.$$

Section of Rim of Wheel. Preferably to have straight parallel sides with corner radii of $0{\cdot}5m$. The maximum effective face-width determining load-carrying capacity on a basis of wear, to be

$$f_{ew} = 2m\sqrt{(q+1)}$$

If the overall face width is less than this, the value of the effective face width used in the calculations to be reduced accordingly.

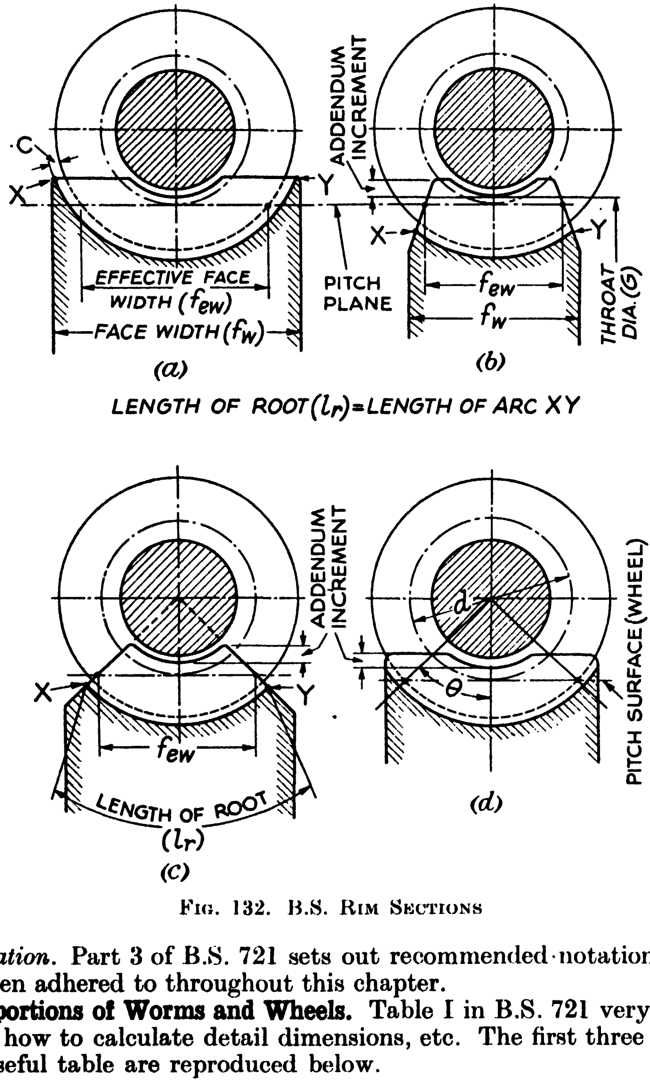


FIG. 132. B.S. RIM SECTIONS

Notation. Part 3 of B.S. 721 sets out recommended notation which has been adhered to throughout this chapter.

Proportions of Worms and Wheels. Table I in B.S. 721 very clearly shows how to calculate detail dimensions, etc. The first three lines of this useful table are reproduced below.

Required	Given or Assumed	Formula
m . . .	C, T, q	$m = 2C/(T + q)$
C . . .	T, q, m	$C = 0{\cdot}5m\,(T + q)$
λ . . .	t, q	$\tan \lambda = t/q$

(t, T, q must be selected according to the conditions of the drive.)

To Find	Formula	Working
Pitch dia. wheel (D)	$D=T\times p/\pi$ or, $D=T\times m$	$D=60\times\frac{\pi}{4}\times\frac{1}{4}\times\frac{1}{\pi}$ $=3{\cdot}75$ in.
Module (m)	$m=\frac{D}{T}$	$m=\frac{3{\cdot}75}{60}=0{\cdot}0625$ in. units
Diameter quotient (q)	$q=\frac{d}{m}$	$q=\frac{0{\cdot}86}{0{\cdot}0625}=13{\cdot}76$
Lead angle (λ)	$\text{Tan }\lambda=\frac{\text{Starts in worm}}{q}$	$\text{Tan }\lambda=\frac{4}{13{\cdot}76}=0{\cdot}29070$ $\therefore\ \lambda=16^\circ\ 13'$
Addendum (worm) (a)	$a=m$	$a=0{\cdot}0625$ in.
Dedendum (worm) (b)	$b=m(2{\cdot}2\cos\lambda-1)$	$b=0{\cdot}0625(2{\cdot}2\times0{\cdot}96021-1)$ $=0{\cdot}0695$ in.
Addendum (wheel) (A)	$A=m(2\cos\lambda-1)$	$A=0{\cdot}0625(2\times0{\cdot}96021-1)$ $=0{\cdot}0575$ in.
Dedendum (wheel) (B)	$B=m(1+0{\cdot}2\cos\lambda)$	$B=0{\cdot}0625(1+0{\cdot}2\times0{\cdot}96021)$ $=0{\cdot}0745$ in.
Overall dia. (wheel) (J)	$J=D+2A+m$	$J=3{\cdot}75+0{\cdot}1151+0{\cdot}0625$ $=3{\cdot}9276$ in.
Throat dia. (wheel)(G)	$G=D+2A$	$G=3{\cdot}75+0{\cdot}1151=3{\cdot}8651$ in.
Root dia. wheel (I)	$I=D-2B$	$I=3{\cdot}75-0{\cdot}1490=3{\cdot}6010$ in.
Outside dia. (worm) (j)	$j=d+2a$	$j=0{\cdot}86+0{\cdot}125=0{\cdot}985$ in.
Root (dia.) (worm) (i)	$i=d-2b$	$i=0{\cdot}86-0{\cdot}1391=0{\cdot}7209$ in.
Normal pitch (worm) p_n	$p_n=p\cos\lambda$	$p_n=\frac{0{\cdot}7854}{4}\times0{\cdot}96021$ $=0{\cdot}1885$ in.
Normal pressure angle (worm) (ψ_n)	Standard $=20^\circ$	
Centre-distance (C)	$C=0{\cdot}5m\ (T+q)$	$C=0{\cdot}03125\ (60+13{\cdot}76)$ $=2{\cdot}305$ in.
	Note. This formula gives the basic centre-distance, but in B.S. No. 721 it is laid down that if desired for any reason (e.g. the use of existing tools), the *actual centre-distance* may, subject to limitations recommended in the Specification, differ from the *basic centre-distance* by an amount not exceeding $\pm0{\cdot}25m$, i.e. in above example by $+0{\cdot}0156$.	
Nominal pitch dia. (worm)	The *nominal pitch diameter* of the worm (used for calculation of detail dimensions) is given in B.S. No. 721 by $d=qm$, where q is the *diameter quotient* and is preferably an integer. The *real pitch diameter* is given by $d=2C-Tm$.	
Nominal pitch dia. (wheel)	The *nominal pitch diameter* of the wheel (used for the calculation of detail dimensions) is given by $D=2C-qm$. The *real pitch diameter* is given by $D=Tm$.	

(Relating to example on page 144)

The Symbol " q ": Worm Diameter Quotient. The diameter quotient (q) is a ratio, preferably an integer, expressed in the table on page 143 as $q = d/m$. Thus we have $d = mq$. It is therefore evident that q is the ratio of the nominal pitch diameter of the worm to the module. From this it follows that the pitch diameter of the worm is equal to that of a spur gear having q teeth of the same module. This simplifies the relationship between pitch, pitch diameters, and centre-distance. Thus in B.S. No. 721, Table I, we have $C \doteqdot 0{\cdot}5m\ (T + q)$.

Maximum Permissible Pitch Errors and Tolerances. These are tabulated in respect of pitch, tooth thickness, and backlash as applicable to different classes of gears.

Strength and Horse-power Ratings. Part 6 gives adequate guidance to designers.

Temperature Rise, Efficiency, Lubrication, and Materials. See Part 7.

Examples of Calculations. Fully-worked examples are given in the Appendix to B.S. No. 721, showing how the various clauses, tables, and charts are applied to actual problems.

Typical Simple Example. Detail Dimensions Obtained by Methods Tabulated in B.S. No. 721

Worm. Four start. Lead, 0·7854 in. Pitch diameter, 0·86 in. **Wheel,** 60 teeth. See calculations on page 143.

Worm Gear Zone of Contact. Worm threads and wormwheel teeth make curved line contact. By somewhat arduous analytical geometry, or by a straightforward graphical method clearly explained in *Gears*, by Dr. H. E. Merritt (Pitman), it is possible to plot the positions and lengths of the **lines of contact.** They move over what is called the **zone of contact,** which may be said to comprise the successive positions of all the lines of contact over the whole period of engagement. As the gears rotate so the lines of contact move upon the zone of contact, about which the designer needs information when estimating load-carrying capacity and fixing maximum useful face-widths for worm and wheel. Formulae facilitating all such calculations will be found in B.S. No. 721.

Materials and Lubrication. The sliding velocity between the threads of the worm and the teeth of the wheel being relatively high it is incumbent upon designers to select materials with care and to arrange for adequate and suitable lubrication. Heavy and continuous friction always leads to rise of temperature, loss of power, wearing away of contacting surfaces, and noise. Properly designed worm gearing, accurately cut, mounted, and lubricated, yields a quiet mechanical drive.

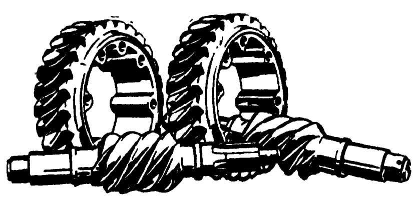


FIG. 133. D.B.S. HIGH-EFFICIENCY WORM GEARS

In B.S. No. 721 the table of materials for worms includes five case-hardened steels and two normalized high-carbon steels. For wormwheels (or their rims), three phosphor bronzes are recommended (respectively sand-cast, chill-cast, and centrifugally-cast), and one cast iron. High-speed worm gears are usually jet lubricated, others splash lubricated; one object, apart from the maintenance of an oil film on contacting surfaces, being to carry away the heat generated. On high-speed work the choice of a lubricant depends on so many factors that it is impracticable to discuss it satisfactorily in brief compass.

Readers are therefore referred to *Gears*, by Dr. H. E. Merritt (Pitman), where gear tooth lubrication is discussed in detail. Most engineers agree that for high-efficiency drives having the worm of steel, case-hardened, ground, and polished, and the wheel of bronze, the most suitable lubricant is castor oil.

Efficiency. B.S. No. 721 gives the following well-known formulae for efficiency (excluding bearing and oil-churning losses) for gears made in accordance with that specification

$$E \text{ (worm driving)} = \frac{\tan \lambda}{(\tan \lambda + \phi)}$$

$$E \text{ (wormwheel driving)} = \frac{\tan (\lambda - \phi)}{\tan \lambda}.$$

where $\tan \phi$ = coefficient of friction (μ) between worm and wormwheel corresponding to the rubbing speed (ascertained from Chart 3 in B.S. No. 721).

Note: In the derivation of these formulae, contact at the pitch point was assumed. Power losses at bearings are reduced by the use of efficient anti-friction bearings; oil-churning losses can be estimated fairly closely by specialists in this work. It will be evident that for high efficiency d should be small and λ large. Efficiency increases as λ increases, up to a certain point; but for angles of inclination larger than 25° to 30°, the efficiency increases very little, especially if the coefficient of friction (μ) is low. Experiments show that maximum efficiency is reached when λ = 45° (approx.). Actual tests of high-efficiency worm gears with ball bearings and adequate lubrication show efficiencies of 95 to 97 per cent and higher.

Irreversibility. When λ exceeds 9° the drive may be reversible. Relative inefficiency is therefore a characteristic of irreversible gears. The worm cannot be rotated by the wheel if the lead angle (λ) is equal to, or less than, the angle of friction (ϕ). In this connection the following extract from B.S. No. 721 No. has interesting relevance—

"Since the angle of friction changes rapidly with the rubbing speed, and the static angle of friction may be reduced by external vibration, it is usually impracticable to design irreversible worm gears with any security. If the effect of irreversibility is desired, it is therefore recommended that some form of brake be employed."

Merritt's definition is as follows: *Worm gears are theoretically irreversible when the lead angle is equal to or less than the static angle of friction.*

Rubbing Speed (v_s). This is the relative velocity of the worm threads and the wheel teeth at the pitch point. In B.S. No. 721 its value in feet per minute is given by—

$$v_s = 0{\cdot}262 dn \sec \lambda = 0{\cdot}262 nm \sqrt{(t^2 + q^2)}$$

where n = revolutions per minute of worm, t = number of starts in worm, d = worm pitch diameter, etc., the same notation being employed as elsewhere in this chapter.

American Practice: High-efficiency Worms. Multi-start worms are necessary for high efficiency, and larger thread angles than 29° are necessary on straight-sided sections to avoid excessive undercutting during hobbing. The A.G.M.A. recommended practice is to use a thread angle of 40° for worms having 3- or 4-starts. For worms with large lead angles some manufacturers use a thread angle of 60°. Where the lead angle exceeds 15° to 20° it is customary to reduce the depth of the thread by using normal pitch instead of axial pitch in the calculations. Thus, depth $= 0{\cdot}6866p_n$ instead of $0{\cdot}6866p$. It has been mentioned previously that $p_n = p \times \cos \lambda$.

In the A.G.M.A. recommended practice, the whole depth of thread for single- and double-start worms is $0{\cdot}6866p$, but for triple- and quadruple-start worms is $0{\cdot}623p$.

The Brown & Sharpe Co. prefer to cut worms in a thread-milling machine, using a milling cutter having straight sides, the included angle being as follows—

Lead Angle of Worm	12° or less	12° to 20°	20° to 25°	25° or more
Included Angle of Cutter	29°	40°	45°	50°

The 40° cutter, too, is used for worms working with wheels having 24 teeth or less. The Brown & Sharpe Co. use p_n instead of p in calculations for depth of teeth of worms having a lead angle of 12° or more. (See the B & S publication, *Practical Treatise on Gearing.*)

Some U.S.A. manufacturers of triple- and quadruple-threaded worms make $A = a = 0{\cdot}286p$, and $B = b = 0{\cdot}332p$.

Globoidal or Hollow-faced Worms. The worms dealt with up to the present have all been of the parallel type, having pitch surfaces of cylindrical form. Less frequently one comes across "globoidal," "hour-glass," or "hollow-faced" worms. Both the worm and the gear are concave on their respective axial sections. The design has frequently been introduced in a variety of forms but appears to make little headway towards general acceptance. The Hindley worm is of this type, its pitch line being a circular arc corresponding to the pitch circle of the gear.

PRODUCTION METHODS SUMMARIZED

(1) **WORMS.** A parallel worm may be considered either as a screw or as a special form of spiral gear having one tooth and a very large spiral angle. For production purposes its similarity to a screw is the most apt conception; for a worm thread, like any other screw thread, can readily be **cut in a lathe.** Whilst worms (single- or double-start) having large spiral angles are easily threaded in a lathe, those used for heavy power transmission, being multi-start and having smaller spiral angles, are nowadays milled in large quantities in special worm thread-milling machines, rotary cutters being employed. After heat treatment, these worms are **profile-ground** in thread-grinding machines, an operation which removes the effects of distortion arising through the heat treatment process and leaves the thread brightly polished. The mirror-like finish on modern high-efficiency worms is obtained by **lapping** after grinding. Thread grinding is readily applied to B.S. worms of involute helicoid form because, if it is set correctly, a flat-sided abrasive wheel can be used. The trimming of such wheels by means of a diamond is a simple process.

In Fig. 127, side by side, are shown dimensioned sections of an "involute worm" and the corresponding hob. The hob is *not* an exact facsimile of the worm. Thus,

(1) Outside diameter of hob

= (Outside diameter of worm) + (0·1 × Linear pitch of worm)

(2) Root diameter of hob

= (Outer diameter of worm) − (1·2732 × Linear pitch of worm).

The straight-sided form tool used for cutting the threads in the worm would be set with its cutting face in an axial plane and would have its outer end $0{\cdot}31p$ wide, whereas the tool used for threading the hob would be $0{\cdot}335p$ wide, if the included angle is 29° in each case. Hobs of this type supplied by the Brown & Sharpe Co. have an additional diameter increase of 0·005 in. to 0·010 in. for small sizes to allow for wear. Hobs must be long enough for the largest worm-wheels they are likely to have to cut. No radii or fillets are shown in the diagram, but many hobs of this kind have their outer corners radiused and inner corners filleted, the radii being as large as the clearance allows, viz. about $p/20$.

FIG. 134. D.B.S. WORM, WHEEL, AND HOB

Worms can also be produced by a **hobbing process.** Hobs, however, are expensive tools so that, unless a suitable hob is in stock, the hobbing process is customarily employed only when a large number of worms of the same kind are required. Milled worms are sometimes finished in a lathe by using a form tool. So also are cast worms, in the cleaning of which cemented carbide tools are useful.

WORM MILLING. Worms of low lead angle, approximately straight-sided in axial section, are, as stated above, conveniently cut in a lathe, but worms with higher lead angles are thread-milled in large numbers. During the milling operation the blank is moved axially, or "traversed," at the same time being rotated against a revolving milling cutter. Involute worms with low lead angles are milled by means of *disc cutters* having straight sides inclined to each other at an angle of 29°. The teeth on some of these cutters alternate on either side, one tooth having its cutting edges immediately opposite to each other for purposes of inspection.

In connection with single-start worms with small lead angles cut in a lathe the *pitch* most commonly referred to is the *axial pitch* (p), sometimes called *linear pitch* or, in most screw thread work, *pitch.* The thickness of the worm thread in the larger sizes is conveniently *measured* in a plane normal to its sides by means of a gear tooth vernier caliper.

The normal thickness at any depth is equal to the axial thickness (at that depth) multiplied by the cosine of the lead angle (at the same depth). Particularly bear in mind that the lead angle changes from the tip to the root of the thread. A careful definition of *lead angle* is given on page 131. To calculate the lead angle (λ), we use the formula, $\tan \lambda = \text{lead}/\pi d$. Thus $\lambda = \tan^{-1}(L/\pi d)$, where d is the nominal pitch diameter.

The following table shows how lead angle decreases as diameter increases, if pitch remains the same; it shows the lead angles for a single-start screw thread having 2 threads per inch.

Diameter	$1\frac{5}{8}$ in.	2 in.	$2\frac{3}{4}$ in.	$3\frac{1}{2}$ in.	4 in.
Lead Angle	$6^\circ 59'$	$5^\circ 26'$	$3^\circ 45'$	$2^\circ 52'$	$2^\circ 29'$

In the course of years large numbers of expensive hobs for involute wormwheels have been accumulated, so that worm gearing of this type continues to be made. Its gradual displacement, however, is being effected in view of the call for gears with smaller ratios of reduction and improved performance.

Involute worms of high lead angle are milled by cutters having outlines different from the shape of the groove on the axial section of the worm. During the milling operation the cutter is inclined at an angle equal to the lead angle of the worm thread, in other words the plane of the cutter is tangential to the helix being cut. If the thread is to be straight-sided in axial section the cutter profile is slightly convex so as to avoid "interference." Worm threads are comparatively deep in relation to pitch. Lead angle at various points on the thread profile differs from a maximum near the root to a minimum near the crest. Thus, the profile of the cutter must be determined by calculation or by a graphical method from the desired axial section of the worm thread. The heights of the two profiles remain the same but the widths are different. Thus, if a be the width of the required profile, in an axial plane, at a height h, λ being the lead angle at that height; then the width b of the cutter profile at the same height is $b = a \cos \lambda$.

Multiple start worms have every separate thread groove milled as a separate operation. After one thread has been cut the work is indexed and another groove commenced. Sometimes ganged cutters are used to rough out all the grooves at one pass of the work.

Worm milling is sometimes undertaken as a roughing operation, the worm threads subsequently being finished in a screw-cutting lathe or before or after heat-treatment, on a **worm thread grinder.**

Calculating Change Wheels for Cutting Worm Threads in a Screw-cutting Lathe
Methods of calculating change wheels for both straightforward and odd pitches are explained in *Screw Thread Cutting and Measurement* by A. C. Parkinson It is often necessary to resort to the "repeated division" method.

GRINDING WORM THREADS AND HOBS. Only in very recent years has the profile grinding of worm threads passed from a tool-room to a mass-production stage. Machine-tool development has advanced so rapidly that, with few exceptions, it is becoming more economical to produce worm threads "from the solid" by form-grinding than by any other method.

One general advantage of the grinding process applied to hardened material is, of course, the removal of heat-treatment distortion. Material in unhardened condition is thread ground also, and it is a fact that production times of thread grinding of material in its "soft" state compare very favourably with those of more conventional processes of thread production. As to accuracy, it is now an accepted fact that accuracy associated with modern form-grinding meets the most exacting demands of the trade.

The range of sizes accommodated by modern thread-grinding machines is sufficient to cater for all types of work required in general engineering practice. A good example is the **Jones & Lamson, 12 × 45, Automatic Universal Thread Grinding Machine,** which can be used to grind a worm thread or hob up to 12 in. in diameter, having length between centres of over 45 in. Conversely, the same machine may be used on work as small as $\frac{1}{64}$ in. diameter and a mere fraction of an inch in length, in both instances the result being a tool-room product at a mass-production rate of output. This type of machine, and others on the market, has enabled the profile grinding process to reach a stage of first-rate importance.

Important Factors. In planning any gear-grinding operation, the two main factors to be considered are: (*a*) accuracy required, and (*b*) economical production. In regard to (*a*) it is axiomatic in all production work that the tighter the limits, the greater the production costs. In thread grinding, therefore, the designer should fix his limits on form and pitch, and his requirements as to surface finish as widely and liberally as possible, consistent with the successful working of the finished job.

"Economical production" does not simply imply completing the work in the least possible time. To illustrate this point, consider the following example which the writers have encountered in recent experience. Production of a gross of worms, to be produced at minimum cost, started off at a time schedule of twenty per hour, the product being wholly satisfactory. The output was then increased by 50 per cent, i.e. to thirty worms per hour, the work parts still being produced within prescribed dimensions and with good surface finish. At first sight this would appear to have achieved more economical production, but investigation proved the reverse to be the truth. Compiling of time statistics over an extended period indicated conclusively that a rate of twenty worms per hour was more economical than thirty per hour, the faster production being countered by more rapid deformation of the grinding wheel and premature dulling of the diamonds used for wheel-dressing. Thus maintenance and replacement costs more than offset the seemingly advantageous production step-up, and thus the governing factors of accuracy coupled with economy are dependent on the production of worms acceptable as to pitch, profile, and surface finish, as well as on the maintenance of diamonds and the abrasive wheel.

Pitch. Accuracy of pitch in the product largely depends upon the relative accuracy of the machine-tool equipment. The variable elements within the influence and control of the operator are, firstly, the correct calculation, selection, and assembly of change-gears. On pre-cut work, i.e. work previously roughed out, and perhaps, subsequently heat treated, correct choice of grinding wheel is of paramount importance also, otherwise any form and/or pitch errors of the roughing-out operation may be duplicated in the finished product. For detailed information on change-gear calculations, the reader is referred to *Screw Thread Cutting and Measurement* by A. C. Parkinson (Pitman), and *Thread Grinding and Measurement*, by A. C. Parkinson and W. H. Dawney (Pitman).

Errors of Form and Surface Finish. Errors of form are generally attributable to one or more errors in regard to the wheel, cams, stylus, and diamonds; alternatively, to incorrect speeds and feeds or haphazard mounting of the work. A common trouble is "burning" or "blueing" of the work. This is sometimes due to excessive frictional heat caused by very heavy depth-of-cut, alternatively to glazing of the wheel resulting from the use of too hard a wheel and too high a wheel speed. Overheating is aggravated by insufficient supply of coolant or cutting oil, and, of course, the latter may be unsuitable as to viscosity and freedom from impurities. If the profile desired has a controlled radius, as, for example, "easing" or "tip relief," then irregular and flatted curves will result from worn diamonds and breaking down of the wheel form.

The Wheel and the Diamond. Dulled and fractured diamonds are responsible for premature wheel-wear. If the diamonds are not sharp edged, they tend to rub and crush the wheel rather than to cut it crisply when trimming it to shape. If the formed edges of the diamonds have become blunted, it is impracticable to dress the wheel to a sharp profile or to any small radii.

A deformed stylus will cause erratic tracking on the master cam, resulting in the reproduction of such errors by the diamond on the abrasive wheel.

The Coolant. A good flow of coolant which actually does contact both the wheel and the work is essential to reduce heat generation and blueing of the work surface. Air-currents set up by rotation of the wheel tend to blow aside the flow of cutting-oil so that the "grits and grains" of the wheel are lubricated after it grinds or cuts the work. Very few, if any, sparks should be visible in the grinding operation.

Cutting-oils fall within the heading of organic compounds, mineral oils, or animal oils, and soluble oil coolants. Recommendations of the manufacturers should be studied in cases of difficulty.

"Chatter." "Chatter" and "hammering" marks are attributable, in most instances, to the wheel being out of balance. Many devices are marketed for wheel balancing, and may be classified under two headings, viz. Static Balancing Equipment and Dynamic Balancing Equipment.

Other causes of "chatter" are bad "centres," loose mounting, backlash in transmission, and vibration of the machine.

Diamonds. In ensuing paragraphs we give useful hints on the use of diamonds. Care must be exercised in setting and using them, not purely because of their first cost, but also in view of the difficulty of obtaining suitable replacements.

(1) Before even introducing a diamond to the abrasive wheel, it should be ascertained that the latter is well balanced, otherwise attendant fracture of the diamond is a foregone conclusion.

(2) Ascertain that the diamond is a good fit in its holder, is not loose in its seating, and lies in a position to cut against the direction of wheel rotation.

(3) See that a plentiful supply of coolant flows over the diamond and not only over its metal holder. If, by chance, a diamond contacts the wheel without coolant, do not turn on the coolant and so "quench" and therefore crack the diamond; wait until the diamond and holder cool down.

(4) Take light cuts, and use as heavy a diamond as is consistent with desired results.

Selection of Abrasive Wheel. Use of an incorrect wheel can be the cause of so much trouble, that we feel the following notes justified. Detailed and extensive information on abrasive wheels may be obtained from publications issued by the grinding wheel manufacturers. The following notes apply specially to gear-grinding.

Firstly, let us consider the action of the abrasive wheel, which, to illustrate its principles, we may liken to a rotary disc milling cutter. The "teeth" of the wheel are represented by the "grits" of abrasive, and so we have a different cutting action in using a wheel with, say, 200 grits (teeth) per inch of diameter than when using a wheel with 50 grits (teeth) per inch of diameter. The grits or teeth are periodically sharpened by "dressing" with a diamond. The fewer the number of teeth, within reason, the greater is the stock we can remove per revolution of the wheel. Similarly, the fewer the teeth, the more liable they are to breakage at slow speeds. If, for instance, a wheel having 50 grits per inch of diameter travels at 500 revolutions per minute, we get the effect of a wheel with a greater number of teeth by increasing the r.p.m. of the wheel.

If the wheel speed is increased, each grit is brought more often into contact with the work, so that many light cuts may be taken in preference to few but heavier cuts.

The more cuts and the less the amount of stock removed, the fewer become the feed marks, and hence the better the resulting finish.

In gear grinding, the wheels generally used are selected from two groups, namely, vitrified wheels and resinoid wheels.

The vitrified wheel is strongest and, owing to its resiliency, does not "flex" in use. The resinoid wheel, however, may be safely run at higher wheel speeds and may take heavier cuts, but is more liable to flex in use. Thus it should not be employed in finish-grinding of pre-cut work, as the wheel tends to flex, and so follow any errors in pitch and (to a lesser extent) errors of form.

Tables of wheels recommended for gear grinding are given on pages 254 and 255.

(2) **WORMWHEELS.** The teeth on a wormwheel may be (1) *formed* or (2) *generated by means of hobs*, the forming process being of little practical interest nowadays except in small shops. Most wormwheel teeth are generated by **hobs,** especially in the case of gears produced in quantities for heavy power transmission. The parallel part of a hob must be of substantially the same form (as to tooth shape) as the worm, and one hob can be used only for one design of worm. The usual differences between the hob and the worm are (1) the hob is slightly larger in outside diameter, so as to cut the necessary clearance at the bottom of the wormwheel tooth space, (2) the hob is usually tapered at one end, (3) the hob is gashed and its teeth are relieved.

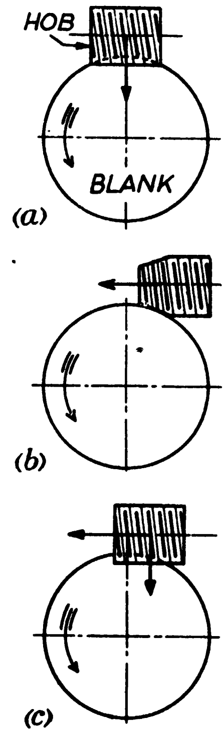


FIG. 135. METHODS OF FEEDING THE HOB

The hob and the wheel blank are mounted on arbors set square with each other, and with their axes lying in one plane, so that the hob corresponds to the worm with which the wheel would mesh. Gearing connects the hob and blank spindles in a manner such that they rotate at speeds in the same ratio as that applicable to the finished wheel and its mating worm, i.e. in proportion to the number of starts in the hob and the number of teeth in the wheel. Every cutting edge on the hob has helical motion. Generation results when the co-ordinated rotations of the hob and wheel blank are added to the helical motion of the numerous cutting edges of the hob.

Feed of the Hob. (1) *The In-feed or Radial Feed Method.* See Fig. 135 (*a*).) The hob is fed in the direction of the radius of the wheel: this method being used for gears having a lead angle of about 6° to 8°.

(2) *The Tangential or Axial Feed Method.* (See Fig. 135 (*b*).) The hob is chamfered or tapered at its leading end and is fed in a direction parallel to its axis, the centre-distance between the axes of the hob and the wheel-blank being equal to the theoretical centre-distance from start to finish of the cutting. An extra rolling motion of the hob is necessary and this is given to it through *differential gears*; the rolling motion being *extra* to the rotary motions of the hob and blank (similar to the extra rolling motion required when hobbing spiral gears).

(3) *Combined In-feed and Tangential Feed Method.* (See Fig. 135 (*c*).) In this method, limited to gears of low lead angle, the roughing-out

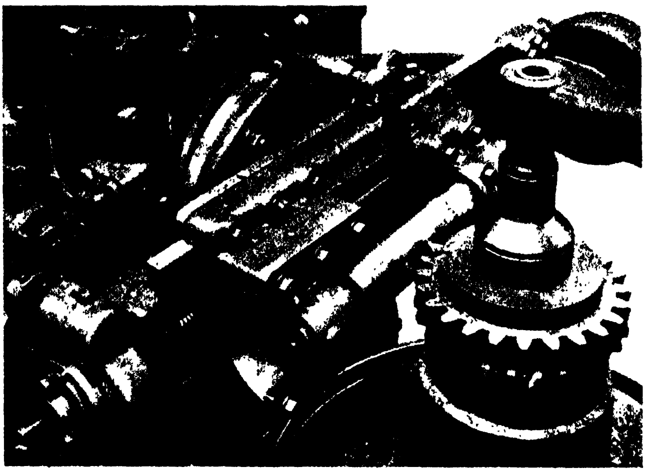


FIG. 136. HOBBING A WORMWHEEL, USING CROSS-FEED HEAD AND FLYCUTTERS ON A PFAUTER GEAR HOBBING MACHINE

is done by the in-feed method followed by finishing by the tangential feed method.

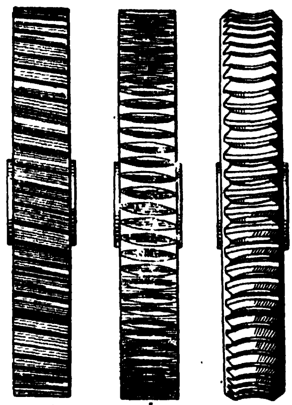
Production in Quantity. Cutting may be done in two operations, viz. roughing and finishing. In the finishing operation the hob used has many cutting edges. This reduces the spacing of the flats on the wheel teeth which are an accompaniment of any hobbing process. The D.B.S. serrated hob, for instance, has closely-spaced cutting edges.

Use of a Fly-cutter. When a small number of wormwheels is required it may not pay to make or buy a hob, in which case an inexpensive fly-cutter or fly-hob can be used. A hob is multi-toothed, whereas a fly-cutter has only one tooth—hence the process is slow. A few notes relating to the process of "fly-cutting" which, customarily, does not involve the use of the cross-feed head and where the wormwheel is not driven, will enable a better understanding of the principles involved in hobbing as well as indicate the resulting erratic zone of contact of a worm and a fly-cut wormwheel. (See Fig. 136.)

Firstly, it is readily seen that a hob incorporates a series of cutting edges, each of which is presented to the wormwheel face at different angles of approach. For instance, as the lead angle at the outside diameter of a hob differs from the lead angle at the root diameter, it follows that the tip of the hob tooth is actually cutting away the wormwheel blank at a different angle from that resulting from the cutting action of the root of the hob tooth.

Let us assume that the tip diameter lead angle is 10° and the root diameter lead angle is 25°, then the tip of the hob tooth represents a conventional cutting tool set at an angle of 10° to the wormwheel axis; the root of the hob representing a tool set at an angle of 25°. The hob tooth flanks now act as an infinite number of tools set at angles gradually varying from 10° to 25°.

When using a fly-cutter, however, we have a choice of setting the cutter to approach the work at any one lead angle, usually accepted as the mean lead angle. Thus if the radius of the travel of the fly-cutter is developed graphically



(a) (b) (c)

FIG. 137. ALTERNATIVE RIMS (*See Notes*)

we find that we have one straight line, whereas if we develop the travel of any one tooth of the hob we have an infinite series of straight lines within a boundary of 10° to 25°.

In use, therefore, the fly-cutter will cut a groove in the rim of the wormwheel, the profile of the groove corresponding to that of the cutter; the groove itself lying in a single path corresponding in its direction to the helical setting of the wormwheel.

Its emphasized effect would be to produce a groove which is practically straight, whereas a hobbed groove is not straight, but is of spiral form.

If, then, we rest a worm in the fly-cut wormwheel, the tooth of the worm will only make point-contact, for its thread is helical, whereas its "mating" or 'meshing" groove is straight.

Universal Milling Machine Method. In connection with 14½° involute worms with low lead angles proportioned according to the original Brown & Sharpe system, or of similar type, it may be well to give an outline description of the process of cutting the wormwheel teeth in universal milling machines. Figs. 138, and 139, illustrating descriptive notes of their own methods, were kindly supplied by the Brown & Sharpe Manufacturing Company. Two methods may be employed, viz. (1) *using a formed rotary cutter only*, (2) *using a formed rotary cutter for roughing, followed by finishing-cutting with a hob.* A single-start worm of low pressure angle could be paired, more or less satisfactorily, for slow intermittent motion, with wheels having teeth of the form shown in Fig. 137. At (*a*) are shown straight teeth cut diagonally across the face in a straight line inclined at the lead angle of the worm. This might be suitable for a lightly loaded feed mechanism—but note that only point contact takes place between the worm and the wheel, the latter being shaped rather like a helical gear with a small spiral angle. Another crudely cut rim is shown at (*b*), whilst at (*c*) is shown a typically hobbed rim.

(1) *Using Rotary Cutter Only.* This must be regarded as a makeshift method by comparison with a full generating[1] process, but it continues

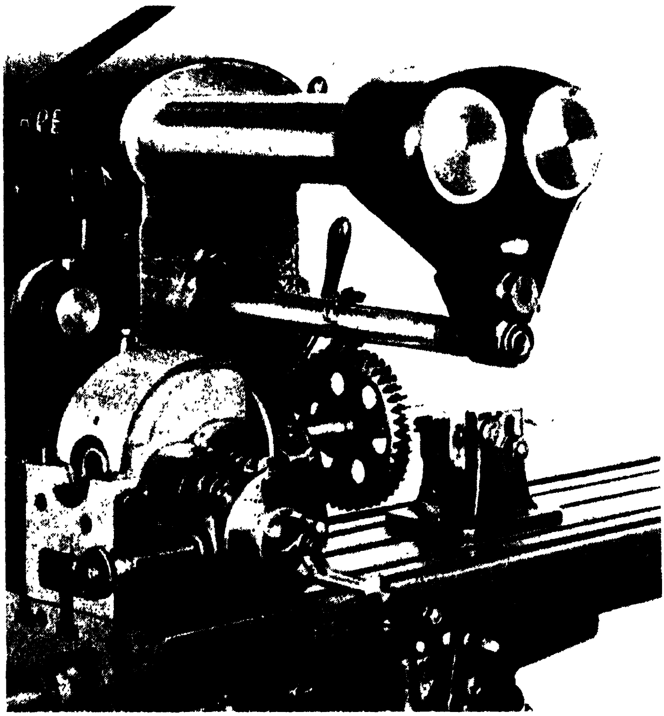


FIG. 138. GASHING THE TEETH ON A WORMWHEEL

to be used in smaller shops for low lead angle worm gears, and for emergency jobs, and therefore cannot be ignored. It would be more suitable for the type of rim shown at (*a*) in Fig. 137 rather than for those shown at (*b*) and (*c*).

(2) *Using Rotary Cutter for Gashing and Hob for Finishing.* (See Figs. 138 and 139.) Fig. 138 shows the teeth in the wormwheel being gashed. The blank and cutter are mounted on arbors, the blank is dogged to the dividing head spindle and the swivel table swung round to the required lead angle. The vertical feed is used and the teeth

[1] The B.S.I. define "correct generation" as "generation by means of a hob or tool which in its form or movement corresponds to a worm thread."

are indexed as in cutting a spur gear. Most of the stock is removed in gashing, only enough being left to allow the hob to take a light finishing cut.

Fig. 139 shows the same wheel being hobbed. The set-up, as

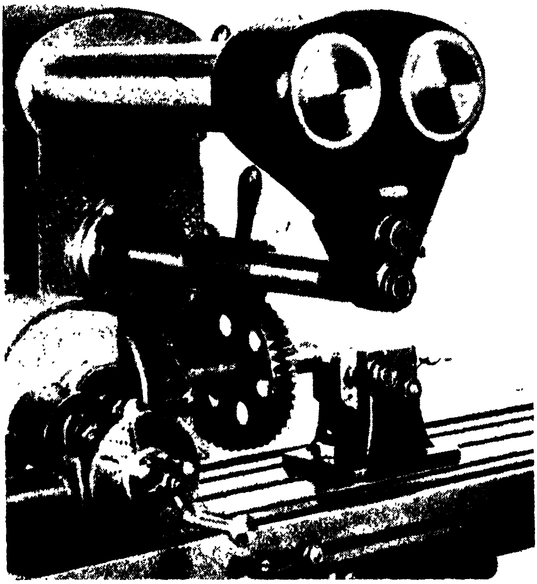


FIG. 139. HOBBING THE TEETH ON A WORMWHEEL

described by the Brown & Sharpe Company, is practically the same as in the gashing operation except that the dog on the arbor is removed

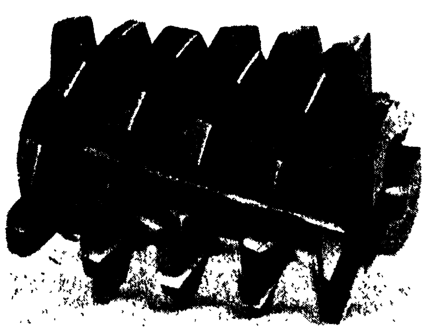


FIG. 140. TYPICAL BROWN & SHARPE HOB FOR A WORMWHEEL

and the swivel table is set at zero. The wormwheel arbor revolves freely between centres, being rotated by the hob. The hob is rotated and fed more deeply into the rim of the blank until the teeth are finished. Hobbing to correct depth can be controlled by using a steel rule at the back of the knee to measure a distance equal to the centre-distance of the worm and wheel from a line marked "Centre," on the vertical slide to the top of the knee. This line on the vertical slide indicates the position of the top of the knee when the index centres are at the same height as the centre of the machine spindle. The milling machine method is not recommended for hobbing wormwheels with large lead angles. If wheels are to be produced by this method in quantities, the Brown & Sharpe Company recommend the use of a

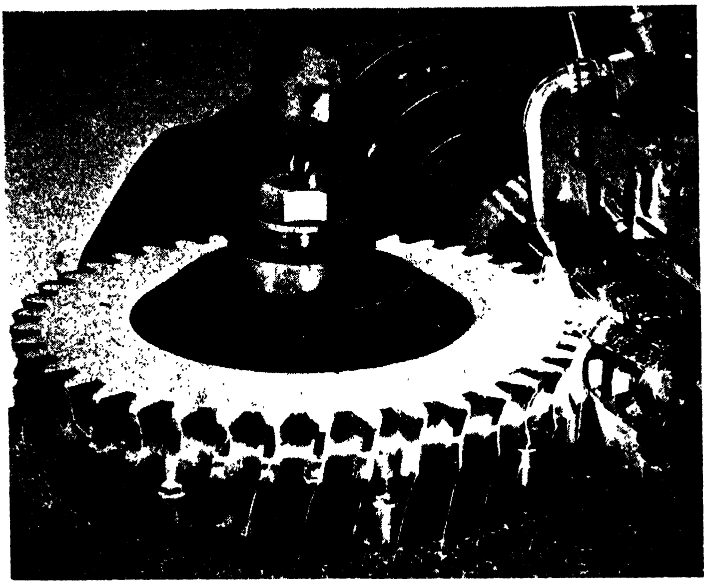


FIG. 141. GENERATION OF WORMWHEEL ON D.B.S. GENERATOR
The icing-sugar effect on the rim is due to the way the cutting lubricant has spread over the top face of the blank

machine in which the work spindle is driven by gearing, so that the hob may cut the teeth from the solid without preliminary gashing.

A typical Brown & Sharpe wormwheel hob is shown in Fig. 140. In Fig. 141 is shown the generation of a wormwheel on a D.B.S. Generator, the block having been loaned by Messrs. David Brown & Sons (Huddersfield), Ltd.

A Reminder. When Measuring the Pitch of Hobs—

(1) **For Spur Gears:** Measure normal to the thread at the pitch surface.

(2) **For Wormwheels:** Measure parallel to axis at the pitch surface.

Normal pitch (p_n) = Axial pitch (p) $\times$ Cos lead angle (λ).

CHAPTER XII

OTHER GEAR PRODUCTION METHODS

NON-METALLIC GEARS. These are used primarily and most effectively where quietness of operation at high speed is essential.

Originally **rawhide** was the commonest non-metallic material employed, and it still has considerable use.

It is cut into circular blanks, which are cemented together under great pressure between brass or gun-metal shrouds or side plates. Rivets (or bolts in the larger sizes) bind the whole together, and the teeth are subsequently cut clean through the side plates and the rawhide blanks. For larger wheels, cast-iron side plates are used and the pinion is bushed—the bush being attached to one of the plates. Fig. 142 (*a*) shows a rawhide pinion in engagement with a

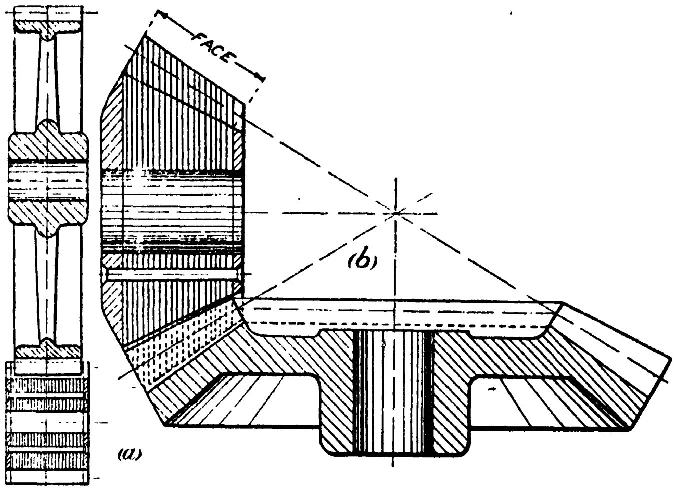


FIG. 142. RAWHIDE PINIONS

cast-iron gear. The shrouds in this example extend to the tips of the pinion—the face width of the rawhide being wider than that of the mating gear, thus obviating any tendency of the latter to make contact with the brass shrouds or side plates. Pinions of this type are often used for electric motor speed-reduction drives. They reduce noise and absorb shocks. Rawhide bevels are built up on similar lines. For lubricating rawhide gears, blacklead or french chalk may be applied sparingly until the teeth take on a fine surface.

A **Prescott paper pinion,** in section, is shown in Fig. 143. An advantage of all non-metallic gears in common use is that lubrication is unnecessary. They are used dry in electrical switchgear and drives where the application of a lubricant would lead to electrical breakdown.

Phenolic Laminated Materials. Nowadays a whole range of proprietary materials for non-metallic gearing is on the market under such well-known trade names as Fabroil, Trefoil, Phenolite, Textolite, etc., in the main consisting of layers of canvas fabric impregnated with bakelite and compressed tightly together under hydraulic pressure. With the application of heat the material forms a dense and rigid mass. Such gears are usually considered to have the power transmitting capacity of cast iron, whilst their resiliency enables them to withstand shocks, impact and abrasion, more successfully than cast iron.

Speeds. Non-metallic gears of phenolic laminated materials are mainly applied to high speed jobs, i.e. for pitch line velocities of about 600 ft. per minute upwards to 6000.

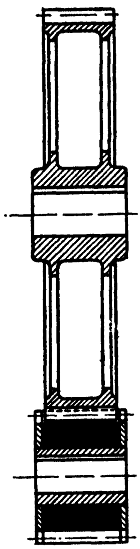


FIG. 143 PRESCOTT PAPER PINION

Tooth Form. The 20° stub form appears to be favoured widely. As such gears are usually made as pinions for speed reduction drives, the driven gear being metallic, it is sometimes possible to have the non-metallic pinion tooth of all-addendum form, the tooth of the driven gear being cut to standard form.

Phenolic laminated materials have a very much lower modulus of elasticity than applies to metals (about one-thirtieth that of steel), so that the effects of small errors of tooth shape and spacing are absorbed in operation, with little adverse effect on the strength of the gears.

Power Transmitting Capacity. The makers of the various non-metallic gear materials publish useful guidance on allowable stresses, pitch line velocities, etc., as well as on the machining of the materials. It is best to consult such references when designing non-metallic drives. Where the safe working stress is known, it is possible to use formulae devised by the A.G.M.A., or to determine horse-power capacity by using the ordinary Lewis formula for spur gears. It is best, however, to consider these gears in a class by themselves.

When phenolic laminated material is used for the driver, the load-carrying capacity is limited by the cutting action of the edges of the teeth of the metallic mating gear, owing to the relatively large amount of deformation.

Safe Working Stress for a Given Velocity. According to the recommended practice of the American Gear Manufacturers' Association (A.G.M.A.), which is fairly generally followed,

$$f = f_1 \left(\frac{150}{200 + v} + 0{\cdot}25 \right)$$

where f = allowable working stress for a given velocity, and f_1 = allowable static stress (usually taken as 6000 lb. per sq. inch for non-metallic material).

The well-known **Lewis formula**, which connects stress, dimensions, and tangent line load can be applied to these gears. The Lewis formula was introduced by Wilfred Lewis and described by him in a paper read before the Philadelphia Engineers' Club in 1892. Both in its original and modified forms it has been used extensively for determining the strength of gearing. For a detailed description of its application, readers may refer to books on machine design, or to *Machinery's Handbook*.

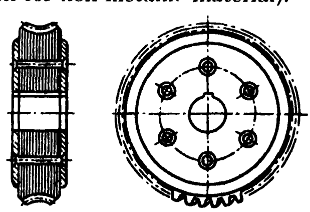


FIG. 144. A NON-METALLIC WORMWHEEL

If applying the B.S. formulae for strength and power capacity, the reader may note that in B.S. No. 436 the value of the *bending stress factor* (S_b) for *fabric* is given as 4500 lb. per sq. in., and the value of the *surface stress factor* (S_c) as 560.

Further Points. Non-metallic pinions should be mated with wheels having machined teeth. Cast gears have teeth which are relatively irregular in form and spacing. This produces strain on the teeth of a mating gear. Furthermore, the rough surfaces of cast teeth wear out the pinion fairly quickly. Some authorities consider that the best results accrue when the mating gear is of hardened steel (over 400 Brinell) or, as a second choice, machined cast iron. The use of brass, bronze, or soft steel (under 400 Brinell) for the mating member of a phenolic laminated gear drive leads to excessive abrasive wear.

Preferred Pitches. The A.G.M.A. have sponsored a table of preferred pitches for both rawhide and phenolic laminated types of materials. These pitches have a reasonable relation either to the horse-power, or speed, or to the applied torque. The table is given in *Machinery's Handbook*.

Different Materials for Gear and Pinion. In B.S. No. 436 it is pointed out that fracture of a gear tooth is almost invariably the result of fatigue, that "pitting" is the commonest form of surface failure, and that "pitting" is a compressive fatigue failure of the contacting

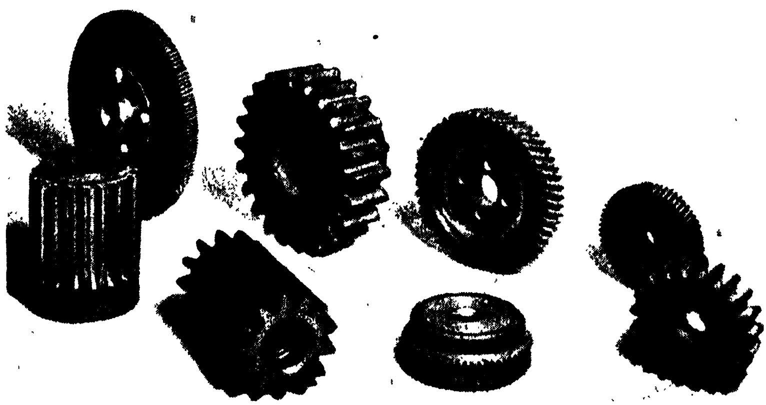


FIG. 145. VARIOUS TYPES OF NOISELESS GEARS
(*By courtesy of Microfil, Zurich*)

surface. It continues as follows: "In the determination, therefore, of both allowable bending and allowable surface loads, the probable number of repetitions of stress must be taken into account, and a speed factor based on revolutions per minute contributes to this end, whereas one based on pitch line velocity fails to do so. A means of differentiating between the severity of load conditions on the pinion and on the wheel is thus obtained, and this leads to the employment of a stronger material for the pinion than for the wheel.

"Each time gear teeth pass through contact a minute amount of material is worn off the working surfaces, and it is evident that if the materials are similar the wear on the pinion teeth will be greater than on the wheel teeth in proportion to the speed ratio."

EXTRUDED AND SOLID-DRAWN PINIONS. First of all—what is meant by "extrusion"? The following paragraphs, descriptive of the process, have been compiled in collaboration with The Delta Metal Company, Ltd., Greenwich, S.E.10, the firm which developed the "Dick Process" for producing bars, rods, and sections of many shapes in malleable metals and alloys.

The Extrusion Process. The word extrusion means *to expel*, or *to force out*, and the operation may be briefly described as follows—

The metal to be extruded is heated to a suitable temperature, and while in a more or less plastic condition is submitted to very heavy compression in specially designed hydraulic machinery. The pressure forces the metal through a die of the required cross-section, from which it "extrudes" in the form in which the die has been cut.

The extrusion press consists of a front and back cross-head held together by two or more massive columns, and is usually placed in a horizontal position. In the centre of the rear cross-head is situated the cylinder of a large hydraulic ram and, to the front, the receptacle or container into which the heated metal block or billet is placed. On the opposite side of the container to the ram is placed the die, held in position by a slide or wedge. To the front of the large hydraulic ram, one of smaller diameter is attached, the working length of which is of such dimensions that it will enter and pass right through the container. Cylinders and rams are also provided for effecting the withdrawal of the ram at the end of the extrusion operation and for putting the die in position, etc. The container is usually heated to a temperature of 200° to 300° C., though this will depend on the nature of the metal being extruded. Power is provided from a hydraulic accumulator or pump.

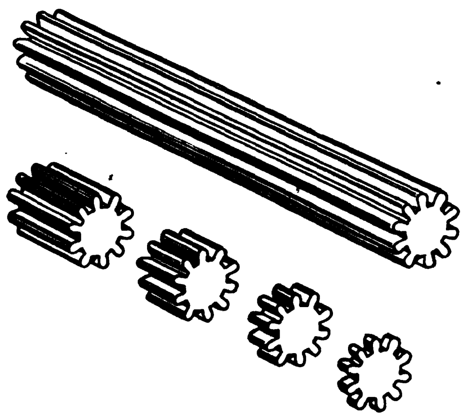


FIG. 146. SKETCH OF RIGBY'S PINION WIRE

After the metal block has been heated to the correct temperature, in an oven specially built for the purpose, it is cleaned and put into the container of the extrusion press. Pressure is then supplied to the ram and the metal is extruded out in the desired shape through the die.

Finishing by Drawing. The extruded bar is then allowed to cool, when it is passed through drawing dies if a special degree of accuracy is required. It is then cut up into the desired lengths, straightened by hydraulic or mechanical power, cleaned, weighed, and packed in cases ready to be dispatched to the user.

An advantage of the extrusion process is that the working faces of the gears become compressed or work-hardened, resulting in more dense hard-wearing working surfaces.

Three of the best-known British firms, manufacturing pinion rods and wires, have supplied information from which the following notes are brief extracts.

The Delta Metal Company, Ltd., Greenwich, S.E.10, can supply extruded pinion-section bars in brass, bronze, and other non-ferrous alloys from ¼ in. diameter to 3 to 4 in. diameter. They maintain that extruded pinions, especially in the larger sizes, unless subsequently machined, are only suitable for comparatively rough applications, e.g. impellers in gear pumps, etc. This company produces large quantities of fluted rods, ratchet sections, serrated rods suitable for milled head screws, knobs, grips, etc.; also toothed sections of special shapes used in instruments and machinery of many types.

Messrs. Birmabright, Ltd., of Birmingham, supply light alloy extruded racks and pinions, especially suitable for slow-moving parts, such as switchgears.

Broaching. Internal or ring gears are produced by a rather similar method. The blanks, which may be parted off from the bar, or may be hot-stampings or of "cast" form, are drilled, reamed, and finally broached to produce the gear-form.

Pressing. Many types of small gears are mass-produced, from the strip, by means of power presses.

GEAR GRINDING. References to grinding processes have been made in various earlier chapters (see the index). Grinding removes distortion from hardened and case-hardened gear teeth and ensures uniformity. It removes machining marks from the tooth surfaces and blends the profile and the fillet into the bottom land in one continuous curve. Ground teeth operate more smoothly, more silently, more durably.

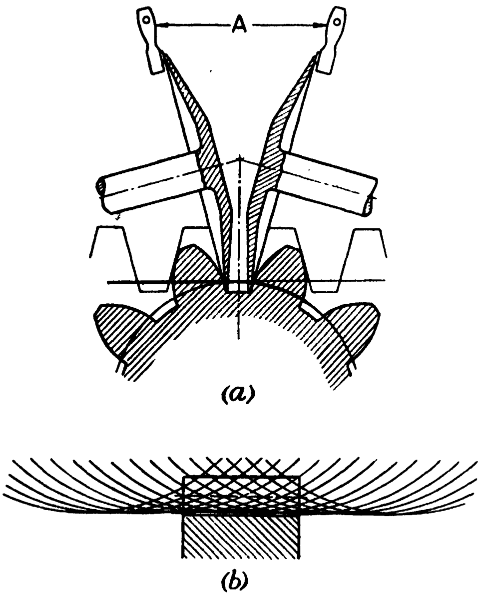


FIG. 149. MAAG GRINDING PROCESS

Grinding processes may be grouped as **forming** or **generating.** Among the well-known gear-grinding machines are the Maag, Churchill, Lees-Bradner, Orcutt, Pratt and Whitney, etc. Wheels recommended by grinding wheel manufacturers are tabulated at the end of this book. The table should serve as a guide in the selection of suitable wheels for gear grinding on practically all makes of grinding machines.

In the *formed wheel process* (mainly applied to spur gears) an abrasive wheel, usually trued by cam-controlled diamonds to the required shape of the tooth-space, is used in a special machine in a manner similar to an ordinary rotary form-milling cutter, i.e. the wheel is traversed to and fro parallel to the axis of the workpiece. The machine provides for automatic truing of the shape of the wheel as it wears, and for simultaneously lowering it to compensate for the reduction in wheel diameter which results from the truing operation. The process lends itself to the production of teeth with profiles modified from true involute.

The *generating process* of profile grinding is carried out on a number of well-known machines. In these a concave or saucer-shaped abrasive wheel sweeps out the plane surface of the basic rack, past which the gear to be ground rolls in much the same way as when being generated by means of a rack cutter. The Maag process is illustrated in Fig. 149 (*a*). The abrasive wheels in this case are comparatively small in diameter and are so shaped that only their outer edges (or narrow bands adjacent to their edges) touch the sides of the teeth.

When rotating, these edges form a circle, and the two planes containing these circles may be considered as the flanks of a corresponding rack tooth on which the gear blank is rolling during the generating process. Two opposed involute profiles are produced simultaneously. As every grinding circle touches the tooth flank, and as axial feed is displacing the circles, the tooth profiles show characteristic criss-cross marks. (See Fig. 149 (*b*).)

So that the wheels may sweep out the full plane surface of the basic rack, the work is fed longitudinally past the wheels, in addition to being given a rolling motion controlled by steel tapes fastened to a cylindrical pitch block, i.e. a disc having the same diameter as that of the pitch circle of the gear being ground. In other machines the rotary movement of the work spindle is produced by a master gear and rack. In Fig. 149 (*a*) the parts marked *A* are feeler levers, which are part of the arrangement for the automatic compensation of grinding wheel wear, an especially necessary feature of machine design in cases

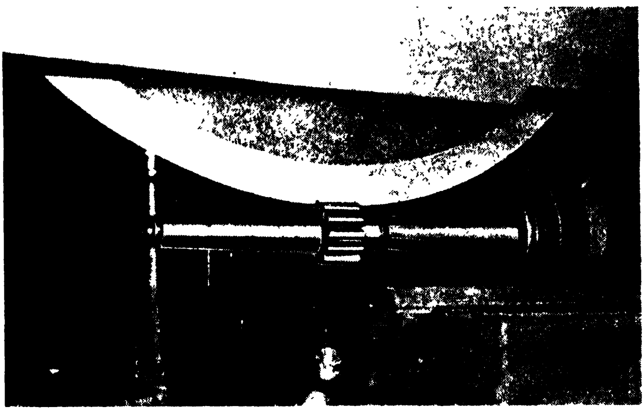


FIG. 150. PINION GRINDING ON THE LEES BRADNER GEAR GRINDER, EMPLOYING A 30 IN. DIAMETER GRINDING WHEEL, GRINDING ON ITS FLAT SIDE FACE. A MASTER GEAR AND MASTER RACK CONTROL THE ROLL OF THE WORK

where small grinding wheels are employed. This ensures that the face of the wheel is kept coincident with the side of the imaginary rack tooth. In the Maag machines the generating motion is given entirely to the gear blank, which is also traversed to and fro axially.

Whatever the grinding process, absolute accuracy is impossible of attainment owing to the wear of the wheel-dressing mechanism, of the diamond, and of such machine parts as the rack and pinion, formers, etc. Thus, as in other machining operations, a tolerance is necessary, not only on dimensions, but on tooth form also.

An "Approximate" Method of Wheel Dressing. The following method is adopted for dressing the wheel to enable it to reproduce the gear tooth-space profile. This method is sometimes used when dressing wheels on grinding machines not designed for gear grinding.

The diamond setting dimensions are obtained by calculation or, more simply, from an enlarged scale drawing or from Grant's Odontograph. (See *Machinery's Handbook*, etc.)

The root radius (*A* of Fig. 151) is dressed on the grinding wheel by the usual form of diamond tool and adjustable tool holder, the readings of the graduated dials on the table traverse being noted so that the centre of the diamond holder may be used as a datum point. The table, with diamond adjusted to sweep out radius B, is traversed a horizontal distance Y and the perpendicular distance X, the flank radius being formed on the wheel by rotating the diamond tool. The process is repeated when dressing the opposite face of the wheel. Note that in

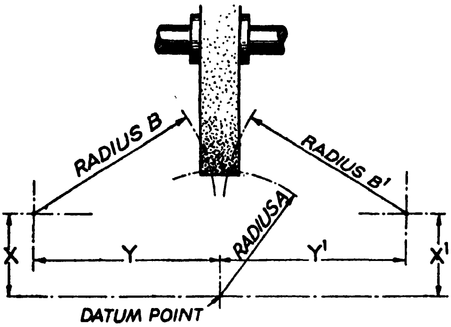


FIG. 151. WHEEL DRESSING

Fig. 151, $X = X^1$, $Y = Y^1$, and $B = B^1$. The radial motion of the diamond may be repeated by adjusting the radius and also the position of the datum point, so that a series of arcs are formed on the grinding wheel to approximate closely to the involute required. Fig. 152 shows the sequence of the wheel-

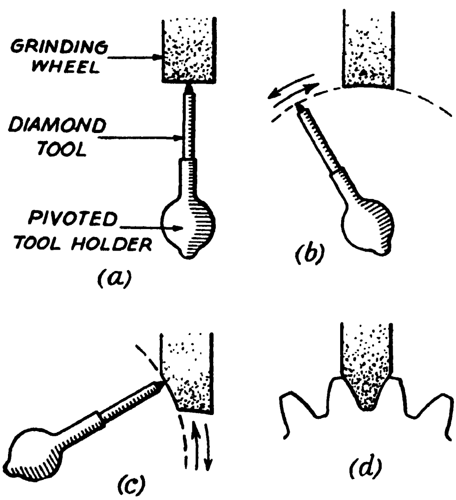


FIG. 152. WHEEL DRESSING—SEQUENCE OF OPERATIONS

dressing operations. The dressing of the wheel-face indicated at (*c*), applied to both faces of the wheel, results in the wheel-form shown at (*d*).

Wheel Dressing by Crush Grinding. Where gears are to be mass-produced by finish grinding, the wheel-form is sometimes obtained by "crush grinding." In this method the abrasive wheel is dressed to the required form of tooth-space by an accurately made "crushing roller" of specially prepared tool steel.

The crushing roller is illustrated in Fig. 153. As shown at (*a*), the bevelled side of the roller is used to dress away the square corners of the abrasive wheel, the process being repeated on the other side of the wheel as at (*b*). The resultant roughed-out form is finish-dressed, as shown at (*c*) by the centre portion of the crushing roller, the form of which is identical to the profile desired.

The crushing roller is made to crush away the abrasive wheel, the latter being force-fed against the crusher while running at a very low speed. In some arrangements the crusher is made, and geared, to drive the grinding wheel. Suitable relief of roller teeth (*e*) allows regrinding along the flutes to remove the effects of wear and tear.

A "Copying" Method of Wheel Dressing. This system of forming the abrasive wheel employs a templet which is made to an enlarged scale, say 50 to 1, and which represents the involute form of the tooth space. A diamond tool is attached to one end of a pantograph instrument which is set to the same ratio as that between the templet and

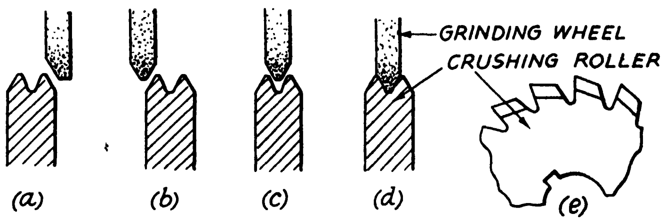


FIG. 153. WHEEL DRESSING BY CRUSH GRINDING

the form required (say 50 to 1). At the other end of the pantograph is fitted a spherical stylus. The stylus is brought into contact with the templet profile and the diamond into contact with the grinding wheel. Traversing the stylus along the templet results in the diamond duplicating the form on the wheel, but, of course, fifty times smaller than the templet form.

LAPPING OF GEAR TEETH. Lapping, like grinding, is an abrasive process applied to the accurate finishing of gears both as to size and profile. Both are finishing processes differing in the speed of the abrasive surface and in the methods employed for controlling the cut, i.e. the means of guiding the cutting points relative to the work. Lapping and burnishing are processes similar in some respects, but differing in that lapping removes surplus metal, whereas burnishing forces the metal to the correct shape.

In most lapping operations the finished shape of the work is a copy of the lap in reverse. Usually the lap is a piece of metal soft enough to enable particles of abrasive to be embedded in it, with slight protrusion beyond its surface. When the work and the lap are rubbed together, with a lubricant, small chips are cut from the work. In production-lapping bonded laps of very fine abrasive are often used.

Profile grinding is relatively expensive and in cases where a large quantity of small case-hardened steel gears are required it is sometimes cheaper to correct errors and distortions introduced during hardening by lapping rather than grinding. This presupposes that heat treatment is skilfully done, in which case resulting distortions are small.

However, lapping is seldom a cheap operation; it takes rather a long time, and may necessitate the provision of two or more master lapping gears which are expensive in themselves.

Various makes of lapping machines are on the market. The process which we shall describe entails running the gear in contact with a lapping gear (or gears) having a different motion and relative setting from that of the gear with which the lapped gear will ultimately be paired. The lapping gear is often of cast iron and has the same normal pitch and pressure angle as the gear being lapped—but has a slightly different spiral angle. Thus, when mated together the lap and the gear being lapped have axes out of parallel, entailing sliding at all points in the zone of contact. An abrasive paste with which the lapping gear is coated laps away the high points on the tooth of the gear, whilst the form of the softer lapping gear is very little affected. In addition to its rotary motion the lapping gear is given a rapid axial reciprocation parallel to its centre-line, so that it covers the face-width of the gear. Alternatively, the gear to be lapped is given a "stroking" or backwards-and-forwards movement as well as being rotated in contact with the lapping wheel or wheels. Cast-iron lapping wheels remain serviceable for a long time, some being capable of lapping about 10,000 gears.

Spiral bevel gears are frequently lapped, and references to this process will be found in Chapter X. The spiral bevels employed for automobile final drives are almost always made of case-hardened steel, and errors due to distortion are generally removed by lapping. No separate lap is employed, but the wheel and pinion to be used as a pair are run together, the pinion driving, under a light brake load.

In the **Gleason Combination Testing and Lapping Machine** a mixture of abrasive and oil is pumped to the meshing point. A complete cycle laps both sides of the teeth over the entire surface. The best results are obtained if, during the time of lapping, the position of the gear is changed continuously and automatically to effect a combined horizontal and vertical movement of the gear relative to the pinion. The gear spindle is journaled in an eccentric sleeve, which is oscillated by a cam driven from a separate motor. Provision is made for a backlash cam to keep the backlash constant during lapping. A high polish is produced on the tooth surface.

Worm threads are given a high mirror-like polish after the grinding operation by running the worm with a lap in the form of a wooden wormwheel, its teeth being dressed with fine lapping paste. The worm rotates the wheel and, so that all its threads may be lapped, it is given an axial as well as a rotary motion.

Lapping Compounds. Whilst many are marketed, they generally consist of emery, carborundum, or diamond dust mixed with a bonding agent of sperm oil, lard oil, etc. An old method of grading the compound is to make a thorough mix of the abrasive powder and oil in a vessel which has, say, five bungs or taps equally spaced. In a few hours the substance settles, large particles sinking to the bottom. The top bung is then removed and the compound above that level drained off into a container. The process is repeated at regular intervals. The first quantity obtained is labelled "Fine Grade No. 5," the last is labelled "Coarse Grade No. 1," intermediate grades being labelled Nos. 2, 3, 4. These various "grades" can themselves be allowed to settle, after which they can be poured again, and so on, until the desired fineness of grade is obtained. As a rule, the coarsest grade is used first and the finest grade last, after which the lapped gears are very thoroughly washed in benzine before use.

Running-in by Lapping. The running-in of gears by adding lapping compounds should be avoided if at all possible, and certainly should never be applied to gear set-ups which have to run at high speeds. No matter what detailed precautions are taken to clean the gears after the lapping process, there must remain a film of abrasive compound infused or impregnated into the working parts of the tooth profiles. This soon causes excessive wear of mating teeth when they operate together at high speed under load. The process may be warranted in the case of slow-running gear trains.

GEAR-SHAVING. Gear-shaving is a profile-finishing operation employed in the finish-sizing of gears, such as hob-cut gears, which are intended to fulfil working conditions of precision in a "soft" or unhardened state. The amount of stock removed is usually limited to about 0·003 in. depth. In the hobbing or other generating process a shaving allowance of a few thousandths must be left on the profiles of the teeth, so that the generating cutters must be modified accordingly.

Surface smoothness is an essential pre-requisite to the avoidance of gear noises and the shaving process removes the feed-wave marks left by the previous gear-cutting operation. Thus, a very smooth tooth surface is obtained and, in addition, there is the accompanying removal of some of the compression strains due to gear cutting.

The many gear finishing machines on the market may be classified into two separate types, viz. the "Circular Cutter," and the "Rack Cutter."

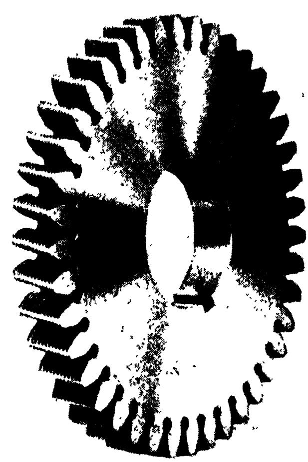


FIG. 154. "RED RING" GASHED HELICAL SHAVING CUTTER

The Circular Cutter Process. In this process the gear to be shaved is run in contact with a shaving cutter shaped like a spiral or helical gear, and having the same normal pitch and pressure angle as the gear being processed. This shaving cutter is of tool steel suitably hardened and profile-ground, and may be likened to a precision master gear. For shaving spur gears, the cutter represents a helical gear having a helix angle of about 10° to 15°. For shaving helical gears, the cutter is in the form of a similar helical gear, but of opposite hand.

The circular cutter method has been considerably developed during recent years, with the result that machine-tool manufacturers have produced equipment capable of gear-finishing in remarkably fast times and to a very high degree of accuracy, e.g. with the "Red Ring" gear-shaving machine, a 20-tooth gear of 9·252 diametral pitch, 20° pressure angle, and 35° helix angle is completely processed in about 30 seconds floor-to-floor time.

Fig. 154 shows a typical "Red Ring" shaving cutter, which takes the form of a helical gear with a series of gashes or serrations extending from tip to root of each tooth. The gashes are at right angles to the long axis of the gear and form virtual cutting tools. The "cutting rake" is obtained by presenting the cutter to the gear at a predetermined angle depending on the helix angle of the cutter and the work.

Suitable clearance is provided at the root of each tooth to enable chip-removal and an unrestricted flow of coolant.

As the work gear is driven by the cutter or "shaving gear," the accuracy of the work is governed by the relative accuracy of the cutter and is uninfluenced by errors in the machine transmission. The chips removed by the cutter are in the form of minute hair-like shavings. The cutter is resharpened by accurate profile grinding of the teeth.

The Rack Cutter Process. This process entails the use of a shaving tool shaped like a rack, of suitable form and pitch to mesh with the work gear. The teeth of the rack cutter are gashed like those on the circular type of cutter, so as to produce cutting edges. The rack is geared to reciprocate and so drive the work with a rolling action along the rack teeth. Pressure (usually hydraulic) is applied to retain the work in force-fed contact with the rack, the latter lying at a slight angle to the work gear axis. A straight-toothed rack is used for shaving helical gears, a helical rack being used for spur gears.

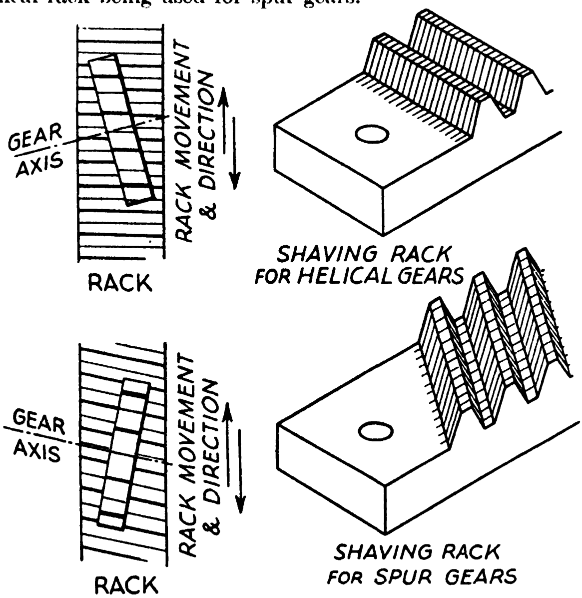


FIG. 155. SHAVING RACKS

The rolling action of the work gear, coupled with the pressure applied to force the gear teeth against the teeth of the rack, sets up a cutting or shaving action, the gashes representing conventional cutting tools.

The teeth on the rack are of the same design as those on the basic rack, with the same form and pitch as the work gear.

In Fig. 155 are shown typical gashed shaving racks for helical and spur gears.

Additional Notes. The shaving of shoulder or cluster gears may present difficulty, dependent on the amount of clearance possible between the shaving cutter, when tilted, and the adjacent shoulder on the cluster gear. For this type of job less difficulty is encountered when using the circular cutter than when using the rack type of cutter.

For shaving internal gears, the circular type cutter is used exclusively, for it is obviously impracticable to shave internal or ring gears with a rack-like cutter. Fig. 156 shows the shaving of an internal gear.

Errors of form and pitch, which are present in the rack type cutter, are readily transferred to the work gear, for each tooth of the latter always meshes with the same tooth space of the rack. The use of a "hunting tooth" in the circular cutter will average the cutter errors, if any, around the whole circumference of the work gear. Modifications to the tooth proportions of the gear teeth of work and cutter are sometimes necessary to avoid "tip" or other interference.

BURNISHING. A wide range of gear work is nowadays finished with a burnishing operation. In general, the process is applied only to gears which are not hardened. Burnishing is a straightforward operation wherein the work gear is driven, under pressure, between two or more hardened burnishing gears, made of special hammered steel. These impart a squeezing action and so press the work-gear tooth profiles to the correct form, with a very smooth surface finish.

Many burnishing machines are marketed with completely motorized and automatic features including timed rotation and reversing mechanisms which may be set to give any desired amount of burnishing and to release the pressure automatically. It is essential that the work gears be cut as nearly correctly as possible prior to burnishing. The burnishing master gears are manufactured to a high order of accuracy and, in use, correct slight errors of pitch, form and concentricity.

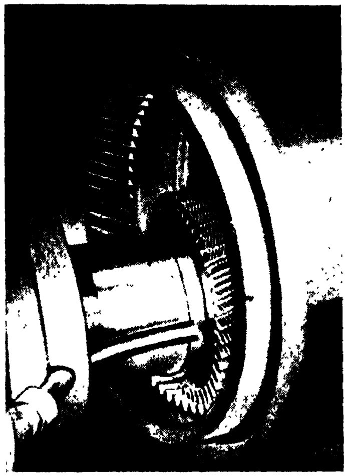


FIG. 156. "RED RING" METHOD OF SHAVING INTERNAL GEARS

Metal Plating of Gears. Although metal plating of gears is sometimes done in gear production, the process is usually applied to the reclamation of gears which have become badly worn or abused in use. Several firms specialize in plating processes involving the addition of a coating of chromium or other hard-wearing metal to gear teeth, the purpose being to reclaim worn gears or to provide a very durable wear-resisting surface to teeth, thus ensuring long working life under strenuous conditions. The gears are generally profile ground after the plating process. When this method is adopted the gears are cut about 0·04 in. under the desired size, a deposit of chrome 0·06 in. thick being applied before finishing the gears to size by grinding.

Powder Metallurgy. Progress in powder-metallurgy developments indicate favourable conditions for its application to gear production. The process entails use of powdered metal which is pressed into a mould, the latter being a very accurate replica of the contour of the gear, the whole being baked at a very high temperature.

The process results in the production of an accurate, hard-wearing yet non-abrasive gear, bulk-produced if need be, at little cost compared to the more conventional methods.

Glass Gears. Remarkable results achieved by the glass-working industry lead to confident expectations of substituting glass for metal in the production of certain gears, particularly so in the range of small-size gear wheels. The high order of accuracy associated with present-day complex moulding suggests that more extensive use of glass gears, in whole or in part, may confidently be anticipated. The use of glass for gauges, surface plates, and other items of inspection equipment has been increasing during recent years.

PART II

CHAPTER XIII

INTRODUCTION TO GEAR MEASUREMENT AND TESTING

THE gauging and measurement of gears is nowadays a matter so extensive and complicated that it cannot possibly be covered exhaustively in a book of small compass. However, an attempt is made in ensuing pages to give a balanced description of leading principles underlying many current methods of gear measurement and inspection which, linked up with worked examples from actual practice, should enable a beginner to lay foundations for subsequent study of more specialist volumes. Higher degrees of accuracy are required to an increasing extent in practically all branches of mechanical engineering and this demand has led to widespread developments of inspection technique and methods generally. In connection with gears many elaborate methods have been introduced to check and measure the highly finished tooth surfaces produced by generation, grinding, lapping, etc. The accuracy of gears, both as to shape and size, has considerable effect on smoothness of operation, freedom from noise, and length of working life.

In ensuing pages we describe a large range of tests and methods of measurement, some of which may appear over-refined for the average run of work. Naturally the means must justify the ends, and methods suitable for a pair of slow-moving cast-iron gears would be inappropriate to a pair of hardened and ground gears capable of transmitting a considerable load at high speed. All gears, therefore, are not tested by the whole range of methods described in these pages.

For many purposes it is sufficient to judge a pair of gears by the noise they make—and a **noise test** is really more exacting than appears at first sight. The difference in relative dimensions (size or location), in tooth form, or in finish, between a noisy pair and a quiet pair is sometimes very small indeed. If a pair of gears work together at the designed speed and under load, with little noise, they are considered satisfactory for many purposes. If the drive is noisy, then examination of individual items is undertaken. Complete inspection of all elements is rarely necessary, although it would very likely be put in hand when comparing the results obtained by different gear-cutting processes.

For a gear drive to be quiet and vibrationless, it is necessary that (1) the distance between adjacent tooth profiles of both gear and pinion be the same; (2) that the tooth profiles are those which correspond exactly with correct-size base circles; (3) that engaging teeth run concentrically without binding.

Running Tests. It is often emphasized in technical books and journals that the best method of inspection of a pair, or a train, of mating gears is one that tests them under the actual operating conditions.

In the final resort, the gears are suitable or otherwise in relation to the way in which they run in assembly and under load. In subsequent paragraphs we refer to gear testing machines in which pairs of gears can be mounted. If such a machine is built to run a pair of gears in their correct relative positions and under load, the result may be considered an *omnibus test* of all the conditions required for smooth and quiet running, e.g. uniform tooth spacing, concentricity of teeth cylinders with bore or shank, correct tooth shapes in relation to definite base circles, etc. Before running the gears into mesh in these testing machines, the teeth of one of them is smeared with some colouring matter, e.g. printer's ink, red lead, copper sulphate, etc. After a few revolutions under a lightly applied load, information as to tooth contact is given by the way in which the colouring matter is distributed on (or removed from) the teeth of the mating gear teeth.

Measurements made on gears may, in a very general way, be placed into one or other of two broad classifications, viz. (1) Readily-made workshop measurements which usually show the combined effect of various factors (they are sometimes called *combination tests*), (2) Direct measurements focused upon *one element at a time*. Thus, some tests are concentrated on one particular tooth element without any reference to its position relative to the axis of rotation of the complete gear.

Once again we start by referring to a straight-toothed spur gear. Errors which may be present include (1) errors in tooth profile, (2) spacing errors,[1] (3) errors in the tooth spiral, (4) errors of concentricity. A little thought will demonstrate that it is generally difficult to measure the errors in one group independently of those in another. Let us next glance at some causes of error.

Some Possible Causes of Error. Firstly, it is useful to picture the production of the gear blank. The usual method is to turn the outside of the rim, face one side of the rim, and bore the hole in one setting. Thus the probability is that the bore and the outside of the rim are concentric. Teeth, however, are cut later in a different machine. The setting of the turned blank in the gear-cutting machine may not be accurate, in which case the teeth will have different depths. If the blank is tilted, the teeth will not be parallel with the bore. Indeed, whatever is the particular inaccuracy in the set-up, the result will be that actual tooth spirals will differ from the designed tooth spirals. When mounting the blank in a gear-cutting machine, **errors of setting** may occur in connection with the work arbor, the tapered socket of the work spindle nose, the tapered shank of the arbor, etc.

Granted that a gear blank is accurately set in the generating machine, the **accuracy of pitch** in the workpiece primarily depends upon the accuracy of the index-wheel or the final drive of the generating machine. Slight pitch errors sometimes comprise the most potent causes of noisy running. The change gears of the machine may also produce certain cyclic errors. The process employed in using the machine has some bearing on this matter. Thus if teeth are form-milled, the blanks are usually indexed one tooth at a time, in which event the index wheel is relieved of load while the actual cutting is being done. In machines working on the generating principle, the index wheel is usually of large diameter—larger, in fact, than any work customarily accepted by the machine. Spacing errors in the index wheel are therefore reduced in proportion to the ratio existing between the diameters of the index wheel and the blank being cut.

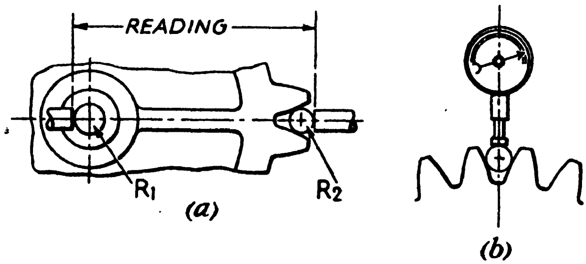
If the blank is accurately set, and the gearing and slides of the generating machine are accurate, errors may arise in spacing if the machine as a whole lacks rigidity or the blank becomes overheated, with consequent unequal expansion.

Errors in the setting and in the form of cutters manifestly produce corresponding errors in the teeth produced. If errors are due to faulty setting, they reveal themselves in different forms, depending on the particular process employed.

[1] By *tooth spacing* is meant the distance between either the driving or trailing profiles of two adjacent teeth.

SOME SIMPLE DIRECT WORKSHOP TESTS. (Fig. 157 (*a*).) In this **concentricity test** two accurately-made cylinders or rollers of any convenient size are used in conjunction with a micrometer. One of the cylinders is placed in the hole at the centre of the gear, or in an adaptor fitting the hole, and the other in a tooth space. The cylinders are shown as R_1 and R_2 in Fig. 157 (*a*). A micrometer measurement is taken across the rollers and is compared with the measurements obtained when R_2 is placed in successive tooth-spaces right round the gear.

Another and somewhat similar method of examining the gear for concentricity errors would be to mount it between centres, or on an arbor fixed to an angle plate, place the roller in a tooth-space, and take a dial indicator reading on the plug. If the plug is placed in each



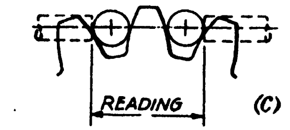


FIG. 157. SIMPLE TESTS WITH ROLLERS OR CYLINDERS

tooth-space in turn further readings reveal any concentricity errors. (See Fig. 157 (*b*).) If the gear is assumed as perfect in respect of other errors, these tests would reveal errors of concentricity.

A simple direct **test on uniformity of spacing** is shown in Fig. 157 (*c*), which shows two cylinders, equal in diameter and of any convenient size, placed in adjacent tooth-spaces. A micrometer measurement is taken across their projecting ends. If this process is continued, using each tooth-space in turn, differences in micrometer readings will reveal "adjacent pitch errors."

This test, in which a series of chordal measurements are taken, could accurately be described as a *spacing test*, provided that the assumption is made that no other errors are present, e.g. errors of concentricity, of profile, of tooth spiral, etc.

A rapid means of **checking chordal spacing** is shown in Fig. 158, where the **Clement Gear and Serration Comparator** is illustrated by courtesy of the makers, Messrs. J. E. Baty & Co. Fundamentally a serrated shaft is the same as a spur gear, differing only in tooth form. Different designs of Clement Comparators are supplied for checking and comparing (1) pitch diameter of external work, (2) chordal spacing of external work, (3) pitch diameter of internal work.

Apart from normal spur gears and components with continuous evenly-spaced serrations round their peripheries, there exist cases where it is essential to leave one serration uncut in order to ensure that the mating component can only be assembled in a predetermined position. An example is the airscrew shaft of an aero engine.

The production of a serrated shaft with one serration left uncut by a generation process is impracticable, so that the single indexing method must be used. Using an optical dividing head and a modern jig-boring machine, an indexing plate can be manufactured to fine limits. However, whilst the results may be satisfactory when the fixture is in a new condition, the accuracy of the spacing may be questionable when the bushes and indexing plunger have become worn. This is a typical example where the Clement Comparator, shown in Fig. 158, enables the rapid testing of chordal spacing.

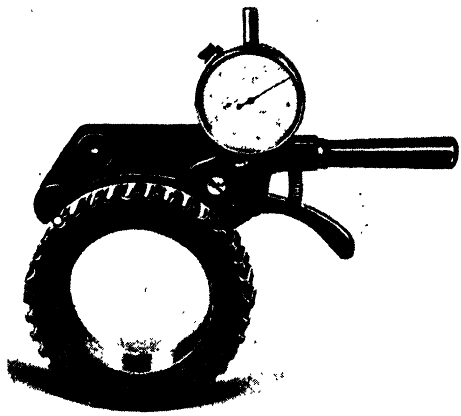


FIG. 158. CLEMENT EXTERNAL COMPARATOR FOR CHECKING CHORDAL SPACING OF SERRATIONS OR GEAR TEETH

The Parkson Gear Tester. (Fig. 159.) A typical simple "**combination test,**" suitable alike for workshop and inspection room, consists of meshing a gear with a "standard," "template," or "master" gear of known accuracy in a fixture which permits a variation in the centre-distance between the shafts. A general idea of the arrangement will be gathered from Fig. 159, which shows a typical gear tester made by Messrs. J. Parkinson & Son, Shipley, Yorks.

The gears are mounted on parallel arbors, each of which is fixed to guide ways on the bed. The adjustable carriage can be clamped to the bed, whilst the floating carriage—shown at the right of the illustration—moves easily on balls against spring pressure. A scale is attached to the adjustable carriage and a vernier to the other, so that the distance between the centres of the gears can be measured to 0·001 in. On some machines a dial indicator is attached to the end of the floating carriage. Thus, movements of the carriage, caused by irregularities in the gears as they are rolled together, are shown by the indicator. On more elaborate machines, of the type shown, a recorder is fixed in place of an indicator. When the recorder is fitted, a permanent record of the movements of the floating carriage is traced by a needle on a circular waxed chart, which makes one revolution for each revolution of the gear mounted on the floating carriage.

FIG. 159. PARKSON GEAR TESTER

Assuming a pair of gears to be mounted, the adjustable carriage is moved to bring the teeth into contact, in which position it is clamped to the bed. The adjusting screw at the end of the bed is then regulated to give the desired pressure to the floating carriage, the spring plunger holding the teeth in close mesh. The centre-distance can then be read on the vernier. If the gears are rotated, it enables the freedom and smoothness of the rolling action to be judged.

If the floating carriage is not yet fixed by the regulating screws, but is only acted upon by the gear teeth on one side and the spring plunger on the other side, it will be found that as the gears are rotated, slight movements are imparted to the floating carriage. The extent of these movements determines the value of the gears, and they are shown on the dial indicator mounted at the end of the bed. The indicator has an adjusting screw for making correct contact with the floating carriage. Indicators are furnished to read 0·001 in. or 0·01 mm.

The indicator shows immediately the effect of a thick tooth in one of the wheels, how bad tooth forms produce a "jumpy, jerky" action, and will not

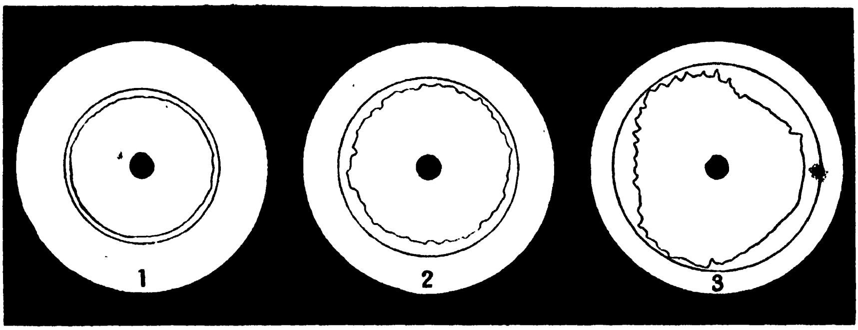


FIG. 160. TYPICAL RECORDER CHARTS (PARKSON GEAR TESTER)

only show when a wheel blank has not been mounted concentrically for cutting, but shows the extent of the error.

The regulating screw which acts opposite the plunger may now be brought into action sufficiently to show a movement of 1, 2, or more thousandths of an inch on the dial indicator, until the scale and vernier show the correct reading for centre-distance. This will enable the gears to be rotated in working position. When thus tried there should be no movement of the dial. If there is a dial movement, the teeth require cutting deeper, or the addendum reducing. If the dial movement is at one part of the rotation only, either there are some bad teeth or the teeth are not concentric with the bore.

The recording device may be fitted to all styles and sizes. It traces a line on a circular chart, corresponding with the movements of the floating carriage, and thus records in permanent and graphic form the character of the gears. (See the reproductions of actual charts in Fig. 160.)

The inner lines, which are more or less irregular, are the records, and show the irregularities in the gears, whilst the circles just beyond the records are drawn at the same mounting to show the amount of variation from the true circle which a perfect gear would give. The actual errors are magnified about fifty times, so that No. 1 shows an excellent gear, No. 2 a moderate one, and No. 3 a bad one. No. 3 was made when testing a 15-tooth pinion with a 45-tooth wheel; the pinion is shown to be eccentric, and as it made three revolutions to one revolution of the chart, the error of eccentricity is shown by the three high points.

Parkson Gear Testers are obtainable for spur and helical gears, spiral gears, worm gears and bevel gears.

The Monarch Gear-testing Machine, manufactured by the Monarch Tool Co., Ltd., Kirkheaton, Huddersfield, is adaptable for testing spurs, helicals, bevels, spirals, and worm gears, by the *rolling test method.*

For instance, spur gears are quickly tested for actual backlash at the prescribed centre-distance, or, alternatively, close in mesh, without backlash at a reduced centre-distance. The commonest test is probably for concentricity or for individual errors existing in one or more teeth in regard to pitch or profile. The machine is equipped with a chart mounted on a disc. Generally, if only one of the gears has concentricity errors, there is one steady movement of the indicator pointer during the whole of a revolution; whilst if concentricity errors exist in both gears, the errors of the pinion would overlap those of the gear in proportion to the ratio of the tooth numbers. In addition, any individual errors existing in any tooth in pitch or profile will cause additional movement of the pointer. The errors, if plotted out, would give a curve of more-or-less sine form. This result is the sum of the errors of both gears; but if a carefully ground and accurate *master gear* of correct pitch and form is available, the errors of each individual gear can be determined.

The Maag Centre Distance and Concentricity Testing Instrument is designed on very similar lines; but the recorder, instead of being a flat disc, is a cylindrical drum.

The principal function of the instrument is to measure the play between the pinion and wheel profiles when the gears are located at their correct centre-distance, in order to determine whether there is possibility of the teeth binding —owing, for instance, to increase in temperature when the unit is in operation. Also available are Maag Pitch-measuring Instruments and Maag Profile-testing Instruments. When two gears are set in the Centre-distance and Concentricity-testing Instrument, and rotated without backlash, the indicator on the apparatus will show to what extent both sets of teeth are concentric with their axes, and whether the engagement of the teeth is free from shock. At the same time, by carefully listening while the gears are rotated some indication can be obtained as to how they will actually run in service. To determine the play between the centres of a pair of gears it is only necessary to note the difference between the correct centre-distance and the centre-distance at which they mesh without backlash.

The Gleason Universal Gear-testing Machine can be adapted to test spur, spiral, double helical, bevel, and worm gears.

It is a bigger machine altogether, rather like a milling machine at first glance, designed to test a pair of gears under power, indeed under conditions which approximate as closely as possible to those under which they are expected to operate. It tests them under load applied through a hand-brake, the rigidity of the machine enabling reliable tests to be made for size, spacing, noise, run-out, and tooth bearing.

PITCH ERROR MEASUREMENT. In a works where gears are produced regularly it pays to install pitch-measuring machines which enable the gear to be rotated exactly through a definite angle corresponding to a movement of circular pitch at the pitch circle. A dial reading can then be taken on each tooth flank in turn. Errors in pitch are indicated by differences in the dial readings.

Another method, illustrated in Fig. 161 (reproduced by permission from Pitman's *Workshop Practice*) consists of measuring the spacing of each pair of teeth in turn, thus determining the "adjacent pitch errors." In order to obtain the "accumulated error" these are added, usually graphically. The "David Brown" instrument shown in Fig. 161 incorporates two measuring points, each operating a dial indicator. One of these is brought to zero, the variation being read off the other.

MAAG PITCH-MEASURING INSTRUMENTS. These are designed to measure from one driving profile to an adjacent driving profile, and do so with ease and exactitude.

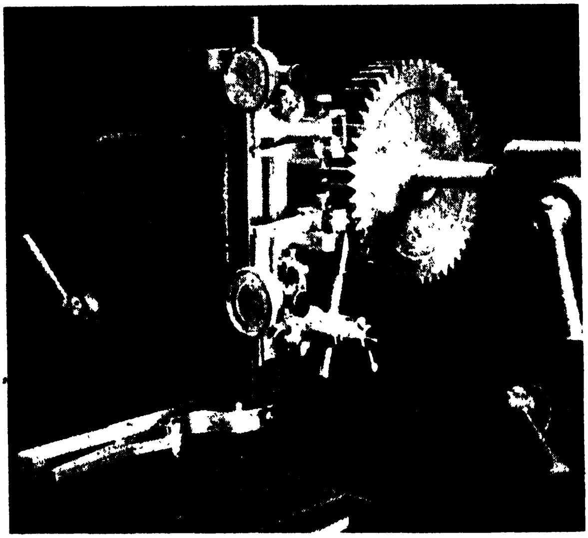


FIG. 161. INSTRUMENT FOR MEASURING PITCH ERROR

There are several ways of taking such measurements, but in the Maag pitch-measuring instruments the pitch of gears is compared either (1) in the direction of the line of action, i.e. at right angles to

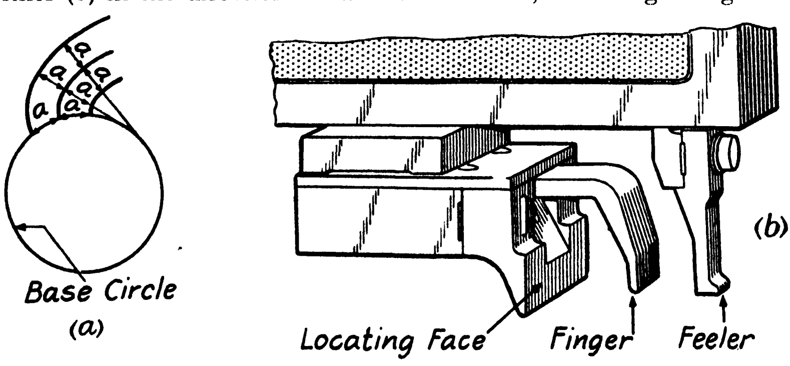


FIG. 162. (*a*) PARALLEL INVOLUTES, (*b*) CONTACTS OF MAAG TME PITCH-MEASURING MACHINE

the tangent to the tooth profile, or (2) on the pitch circle or a concentric circle.

Pitch-measuring Instrument Based on Method (1). Fig. 162 (*a*) emphasizes a matter previously touched upon, viz. that with parallel involutes the distances *a*, between adjacent involute tooth profiles, when measured along lines tangential to the base circle, are equal. The normal distance *a* between such parallel involutes is equal to the distance *a* measured as an arc of the base circle, in other words to *base pitch.* As we have seen previously—

Base pitch = Circular pitch × Cos of pressure angle.

The base pitch is an important dimension. All gears which mesh correctly with the same rack have the same base pitch. In order to transmit uniform motion the ratio between the base circle diameters must be exactly the same as between the numbers of teeth in the gears. Hence, if the base diameters are given, the *angular velocity ratio* will be equal to the ratio of the base diameters. These conditions are only fulfilled and these results obtained when all portions of the tooth profile are generated from the same base circle. A glance at Fig. 162 will make it clear that any modification to the tooth curve *must* affect the pitch when it is measured along the line of action. As this method gives comparative readings for the pitch measurements along the base circle and also provides a ready means of testing the correctness of the profile, it enables a good indication to be obtained as to whether the transmission will be smooth and uniform. Instead of calculating the base circle diameter by means of the readings and the number of teeth it can be obtained by comparing the average value of the measurements *a* taken on pinion and wheel teeth.

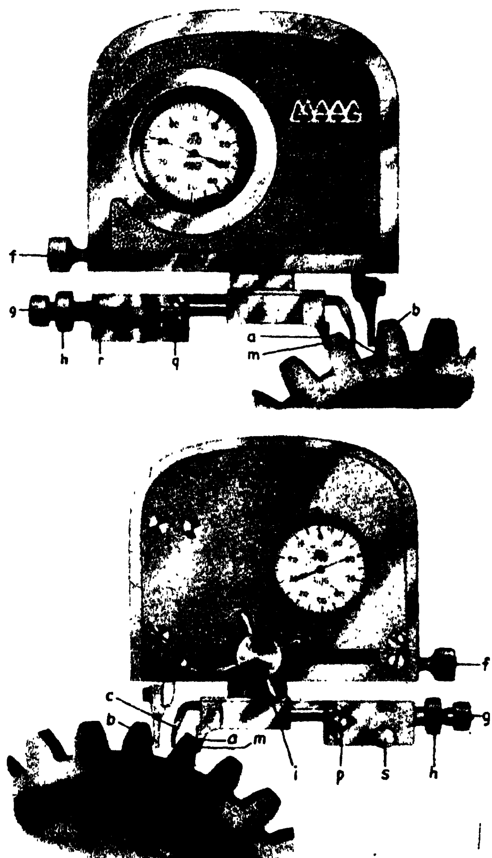


FIG. 163. MAAG PITCH-MEASURING INSTRUMENT

The instrument, type TME, *for measuring in the direction of the line of action* is shown in Figs. 163 and 164. The dial indicators on the front and back are graduated to 0·001 mm. or, alternatively, to 0·0001 in. To enable the instrument to be set for various pitches, a locating face *a* is carried on a slide; while a finger *c*, resting on the opposite profile, assists in supporting the instrument, maintaining at the same time a certain measuring position. Feeler *b* is provided in various sizes to cover all pitches (modules 2 to 20, diametral pitches 12·7 to 1·27) within the range of the instrument.

Setting. Finger *c* is moved by screw *g* until a portion of locating face *a*, above its mark *m*, lies tangential on the tooth profile. Locking screw *i* and adjusting screw *f* are moved until the distance from locating face *a* to feeler *b* is approximately equal to the base pitch. Locking screw *i* is then tightened.

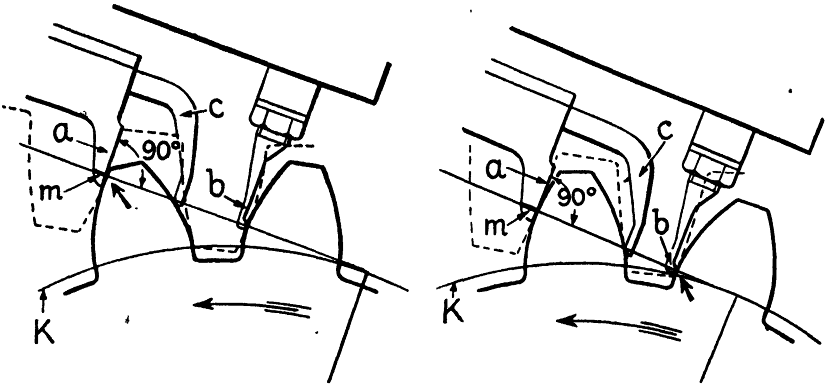


FIG. 164. MEASURING TOOTH THICKNESS

Measuring. By swinging the instrument lightly round the tooth lying between *a* and *c*, the base pitch is given by the *maximum reading* obtained on the dial indicator. The locating face *a* should be set so that a minimum amount of about 50 divisions (0·05 mm.) is recorded on the indicator pointer. All pairs of tooth profiles passing through engagement may similarly be measured.

Of importance is the comparison of the reading obtained for the base pitch at the points where the teeth come into engagement and leave engagement. The screw *g* can be turned until finger *c* is sufficiently open to locate mark *m* at the tip of a tooth. (See Fig. 164.)

Screw *p* is then tightened. The pitch reading will be one of the limit values. To obtain the other value, screw *h* is loosened until, by pressing on lever *r*, finger *c* opens sufficiently to allow feeler *b* to sink to the profile root (see Fig. 164). Readings are then taken. When the instrument is lifted clear of the teeth, finger *c* will automatically revert to its original position for locating at the tip of the tooth. By tightening screw *s*, the finger *c* is disengaged and the adjusting bush fixed. Thus the two extreme positions for measuring the base pitches from the tip of the tooth to the bottom are positively determined.

For rapidly testing large quantities of small gears, such as those used in automobile manufacture, the instrument just described can be mounted on a fixture described as a measuring table.

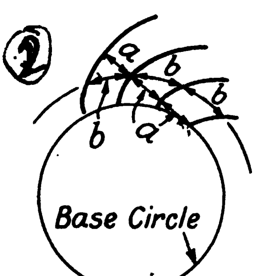


FIG. 165. BASE PITCH AND CIRCUMFERENTIAL PITCH

Maag Pitch-measuring Instrument Based on Method 2. In the case of gears made with a cutter having *more than one tooth* the accuracy of the *base pitch* is chiefly dependent on the cutter and is not influenced by errors in the generating motion of the machine. If the base pitch were compared for all the teeth of a gear at a similar depth on the profile, no differences would be observed even though the cutter itself were incorrect. The pitch of such gears should therefore be measured *circumferentially*.

This can be done by means of the Maag Pitch Measuring Instrument, Type TMA, by means of which both circumferential and base pitch can be measured.

On helical gears normal pitch measurements can be taken with both

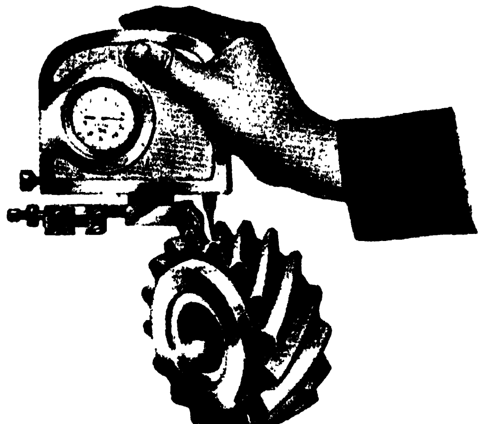


FIG. 166. MEASURING THE NORMAL PITCH

the TME and TMA types of Maag Pitch Measuring Instruments. (See Fig. 166.)

PERMISSIBLE ERRORS AND TOLERANCES (B.S. No. 436—1940)

Indexing or Division and Tooth Shape. The permissible errors of indexing or division on the pitch circle and shape shall not exceed the figures given by the formulae in the table on p. 180, reprinted by permission from B.S. No. 436—1940, for the particular form of error and class of gear. (All linear dimensions are expressed in inches.)

TESTING FOR PROFILE ERROR. Methods of ascertaining and measuring profile errors can be classified into two general groups, viz. (1) mechanical, (2) optical. The job is to measure the total departure of the working part of the actual profile from the designed profile—which comprises the theoretical involute together with any specified modifications such as tip relief. Of the "direct" mechanical methods of checking there are two main classes, viz. (1) measuring chordal thicknesses at various radii, (2) gauging by a moving point, to which an indicator is attached, the point moving along the designed involute profile.

The first of these mechanical methods, requiring the use of the gear tooth vernier caliper shown in Fig. 185, is a typical "combination test," inasmuch as it shows the combined effect of the departures of two involute profiles from the designed profile. These opposing profiles, "leading" and "trailing," ought not to be assumed as identical in

PITCH AND PROFILE TOLERANCES

(In thousandths of an inch. See notes on page 179.)

Class of Gear	Adjacent Pitch Error	Accumulated Error	Profile Error (See Note 2)
A1. Precision Ground	$0{\cdot}1\delta + 0{\cdot}5$	$0{\cdot}1L_a + 1{\cdot}0$	$+ 0 - \frac{1{\cdot}8}{P}$
A2. Precision Cut			$+ 0 - \frac{3}{P}$
B. High-class Cut	$0{\cdot}2\delta + 1{\cdot}0$	$0{\cdot}2L_a + 2{\cdot}0$	$+ 0 - \frac{3}{P}$
C. Commercial	$0{\cdot}3\delta + 1{\cdot}5$	$0{\cdot}3L_a + 3{\cdot}0$	$+ 0 - \frac{6}{P}$
D. Large Internal	$0{\cdot}6\delta + 3{\cdot}0$	$0{\cdot}3L_a + 3{\cdot}0$	$+ 0 - \frac{6}{P}$

δ = Tolerance factor $= \left[\frac{(T + 60)}{10P} + 1\right] = \left[p\frac{(T + 60)}{10\pi} + 1\right]$

P = Diametral pitch, reciprocal inches.

p = Circular pitch, inches.

L_a = Length of arc of pitch circle on which accumulated error exists, in inches.

Note:

(1) The allowable profile error shall not permit an excess of material either above or below the pitch point.

(2) In view of the difficulties of measurement, the tolerances on profile error shall only apply when expressly agreed between purchaser and manufacturer.

form and position relative to a radial centre-line. Furthermore, nothing is revealed in this test as to how the tooth form varies between the measuring points selected. Other defects of the method are that it requires two settings of the vernier gauge for every measurement taken, depends on individual "feel" and cannot be relied upon to less than 0·001 to 0·0005 in. The calculation of chordal thickness is referred to on page 203. The use of balls or cylinders is similar in principle and has the advantage that readings can be taken by means of a vernier or a micrometer with or without a dial indicator. It should be remembered that whilst an actual involute may be correct as an involute, it may not be the involute of the correct base circle.

The second of the mechanical methods is more dependable, rapid, and accurate to 0·0001 in., and a number of firms have developed apparatus incorporating it. The underlying principle is that the gear is given a rolling motion past a gauging point in such a way that the point describes an involute curve relative to the gear. A disc is attached to the gear, the diameter of the disc being equal to its base diameter. Both are then rolled along a flat surface or "slide." The gauging point is mounted in the plane of the latter surface and a dial gauge shows its movement. If the needle of the clock shows no movement the profile is correct. A typical instrument of this form is shown in Fig. 167 (reproduced by permission from Pitman's *Workshop Practice*).

Optical projection methods are described in Chapter XVI.

The Sykes machine is one of several specially designed contour measuring machines on the market. Its principle is covered by the notes in the preceding paragraph. The gear is mounted on a disc having a diameter exactly equal to that of its base circle, and a stylus is brought to rest against the flank of a tooth. The stylus is connected to a dial indicator giving readings in ten-thousandths. When the base circle disc and the gear are rolled along the slide, any departure of the tooth profile from true involute is indicated by the needle of the clock as a measurement taken normal to the profile of the tooth.

The Maag machine [1] is designed on basically similar principles, but in place of the clock indicator there is a system of links which controls the movement of a

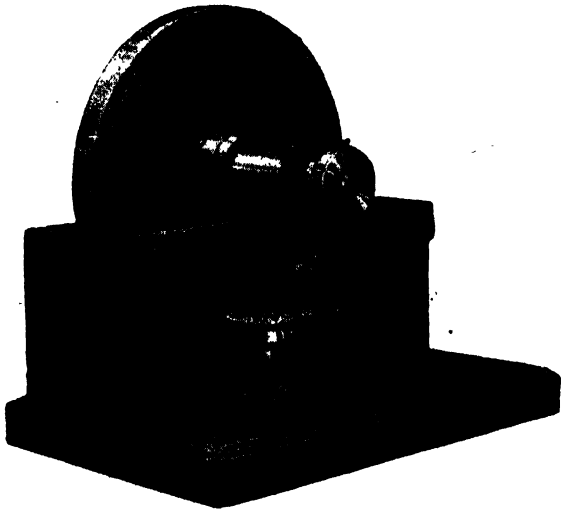


FIG. 167

pen. The latter draws a graph showing how the actual profile differs from a true involute to a magnification of 350.

The Fellows machine has novel features. Unlike the Maag machine, it does not require the manufacture of an accurately-made base circle disc corresponding to every gear tested. It produces a graph showing departures from the true involute to a magnification of 400.

The Sykes Gear Tooth Comparator. (Figs. 168 and 169.) The comparator has a rigid frame carrying a pair of adjustable jaws operated by an adjusting screw with right- and left-hand threads so that both jaws move simultaneously. The jaws have inclined faces representing the sides of a rack tooth. In the illustration the jaws are shown in contact with a gauge block, the latter being one of a set supplied with the instrument. Each gauge block represents an involute rack tooth of a certain pitch and pressure angle. The plunger rests with one end on the gauge block and is connected at the other to the dial indicator.

[1] Sole agents in Great Britain: Messrs. Burton, Griffiths & Co., Sparkbrook, Birmingham.

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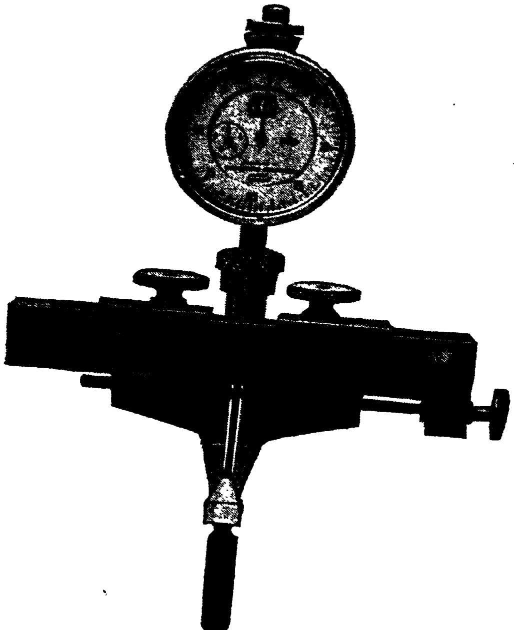

Fig. 168. Sykes Gear Tooth Comparator, with Gauge Block

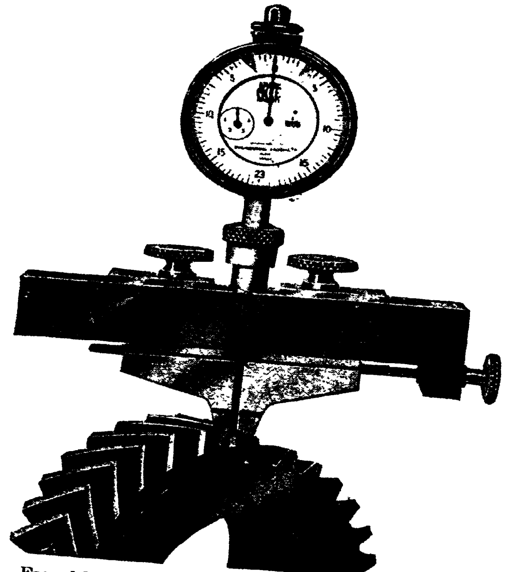

Fig. 169. Sykes Gear Tooth Comparator Checking a Double Helical Gear

In use the gauge block is placed between the jaws of the comparator, which are then locked in position, and the dial turned so that the indicator hand points to zero. The instrument is then placed on the tooth to be gauged. If the tooth is thin, it will enter farther between the jaws than the gauge block, the difference being shown as thousandths of an inch on the dial. If the tooth is thick, it will not enter as far as the gauge block, the difference showing on the dial as before.

As the inclined faces of the jaws are tangential to the profiles of the teeth being inspected, there is little wear, especially as the point of tangency varies with each gear that is measured. No calculations have to be made; no chordal

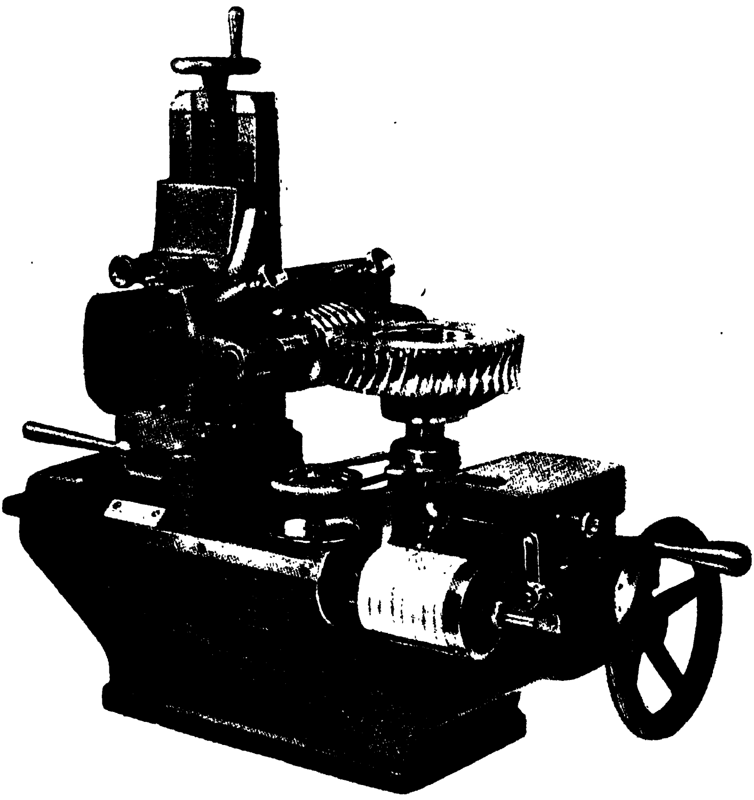

FIG. 170. MAAG CONCENTRICITY-TESTING APPARATUS FOR WORM GEARS

heights have to be figured. The instrument is portable—gears can just as readily be measured in the field or in the gear-cutting machine. The dial reading is 1/1000 in. and, if a gear with $14\frac{1}{2}°$ pressure angle is being measured, one division on the dial represents 1/1·933 in. or 0·000517 in. on the thickness of the tooth; or, for all practical purposes, 0·001 in. on the dial represents half of one-thousandth of an inch on tooth thickness. The comparators shown in Figs. 168 and 169 are described as *Model E*. Particulars of other Sykes' measuring instruments are obtainable from Messrs. W. E. Sykes, Ltd., Staines, Middlesex.

TESTING WORM GEARS. Worm gears made in large quantities for interchangeable use must, of course, be machined to close limits. Whilst it is possible to check the worm threads for accuracy of pitch and thread form, it is not usual to apply direct checks on the tooth forms of the wormwheel. The same applies to the teeth of bevel gears. The usual method of testing a wormwheel is to run it in contact with a worm so as to discover the position of the contact area. Testing for backlash and tooth form is best done by mounting each wheel with a master worm and each worm with a master wheel. Various "worm gear testers" are available. For instance, the **Parkson Machine** provides for the wheel to be mounted on a vertical arbor whilst the worm is mounted on a horizontal shaft in bushed bearings, or between centres, with means for vertical adjustment. The same Tester can be used for spiral gears with axes at 90°, providing that the diameter of the smaller gear does not exceed the maximum diameter specified by the makers for worms.

On worm gear testing machines there are adjustments for the centre-distance and for the lateral position of the wheel with respect to the worm. Backlash is measured by the amount of rotation obtained at the rim of the wheel when the worm is fixed. Tooth bearing is checked by applying Prussian blue, red lead, or some suitable colouring material to the threads of the worm, which is then rotated under light load in contact with the wheel. Markings on the wheel teeth indicate where contact occurs. The marked part of a wormwheel tooth should lie near to the edge at which the worm thread slides out of contact.

The **Maag Concentricity Testing Apparatus,** complete with recorder, is shown in Fig. 170. The **Gleason Universal Gear Testing Machine** is readily adapted for worm gear work.

CHAPTER XIV

USING CYLINDERS OR BALLS FOR GEAR MEASUREMENT

MEASURING OVER CYLINDERS, ROLLERS, OR BALLS[1]

THE pin or ball method of checking gear tooth thickness consists of measuring across the outsides of two cylindrical rollers, pins, or needles, or alternatively of balls, placed in opposite tooth-spaces of gears with an even number of teeth, or nearly opposite spaces of gears with an odd number. The idea is shown in Fig. 171.

Cylinder methods of measurement are commonly used. Readings depend upon the shapes and relative situations of opposed tooth profiles, hence these methods are typical *combination tests*, i.e. tests enabling the determination of

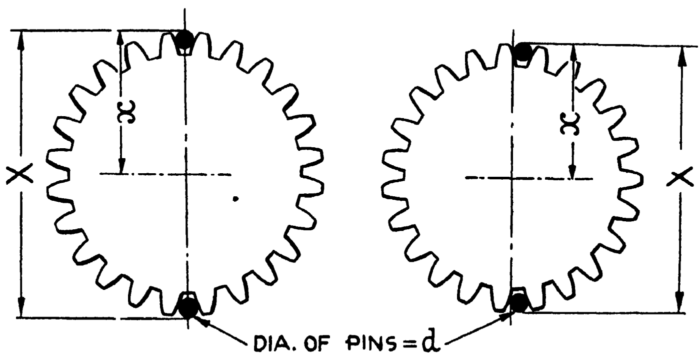

FIG. 171. MEASURING OVER ROLLERS

the extent of a number of possible errors. Such tests may be accepted as accurate pointers to widths of space arcs of specified radius, *provided that circular pitch is correct.* Again, measurements across cylinders in adjacent tooth-spaces may enable the measurement of pitch, or of space width, *provided that the tooth profiles are accurately formed.*

The use of balls for checking gears is not in general very satisfactory. Unless special equipment is available, the method cannot be considered any more reliable than the use of a vernier tooth caliper. One obvious difficulty is that of holding the balls truly in position in a normal-to-axis plane, especially with bevel gears. Another drawback to the use of balls is that it is often impossible to obtain balls of the diameter required.

Three Cases. Different opinions prevail in the trade as to the most suitable diameters of the measuring cylinders in relation to pitch. Three different methods of calculating these diameters are commonly employed. All three are described in ensuing notes. (See Fig. 172.)

[1] The cylindrical measuring pieces are variously known as *cylinders*, *rollers*, *pins*, *needles*, etc. In the paragraphs describing this method of measurement, we have, in general, employed the word cylinder. Here and there the terms *roller*, *pin*, *wire*, *needle* are used. All these terms are commonly used in reference books and in shop parlance. It should be noted also that throughout these paragraphs T stands for *arc width of tooth*, w for *arc width of tooth space*, d for *cylinder diameter*, and X for *measurement over cylinders.*

First Case. Axes of measuring cylinders lie on the cylindrical pitch surfaces of the gears being measured, i.e. the cylinders make contact with the tooth profiles on the line of action. The calculations for these measuring cylinders or rollers are relatively simple, but the cylinders lie below the crests of the teeth. Furthermore, it may happen that the contact between the cylinders and the tooth profiles is close to the base circle on gears liable to have undercut teeth.

Second Case. Measuring cylinders touch tooth profiles at the pitch surface, and *not* below it as in the *first case.* The calculations are more involved.

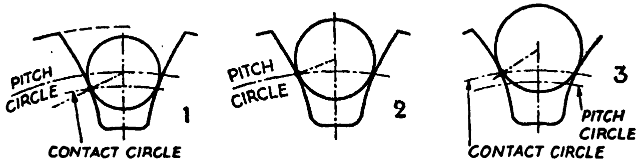

FIG. 172. MEASURING CYLINDERS—THREE CASES

Third Case. Measuring cylinders make contact with tooth profiles at *any* suitable radius.

(1) Simple Special Case. AXIS OF ROLLER COINCIDENT WITH PITCH SURFACE OF GEAR. (See Fig. 173.)

(*a*) **Ignoring Backlash.** Here the roller is of such a size that its long axis exactly coincides with the pitch surface of the gear. Thus, in Fig. 173 the point *P* lies on the pitch circle of the gear and at the centre of the roller. The line *PB* is a common normal to the involute profile, to the circle representing the roller, and to the rack tooth profile. The broken lines represent the imaginary mating rack, one of them passing through *A*, a point common to the rack, the spur gear and the roller profiles. The line *PT* represents a tangent to the pitch circle through the point *P*. It may also be regarded as the pitch line of the rack, in relation to the tooth of which it lies symmetrically. The distance between its points of intersection with the flanks of the rack teeth (i.e. half the circular pitch of the rack) equals the distance, measured round the pitch circle, between the flanks of adjacent gear teeth profiles (i.e. half the circular pitch of the gear). The pressure angle (ψ) is represented by the angles *CPB* and *BOP*.

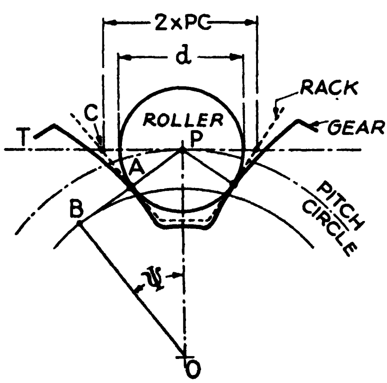

FIG. 173. ROLLER DIAMETER = ARC-WIDTH OF SPACE × COS ψ
(Diagram not to scale)

Then, Diameter of cylinder or roller (d) $= 2PC \times \cos\psi$

Radius of cylinder or roller (r) $= PC \times \cos\psi$.

Thus the *diameter* of the roller is *the arc of the tooth-space multiplied by the cosine of the pressure angle.* (Tables of cylinder sizes are given at the end of the book.)

A roller of this size will seat itself between two tooth profiles, with its axis on the pitch surface. On a gear having an even number of teeth the diameter of the pitch cylinder plus the diameter of one roller gives the measurement over two such rollers set in opposite tooth-spaces. The same cylinder can be used for the measurements of all gears of the same pitch. A further advantage is that, with teeth of correct thickness and form, the centre-distance between cylinders placed in adjacent tooth-spaces is equal to the chordal pitch.

Dimension X Over Rollers. Even Number of Teeth. Spur Gears. Suppose a gear having an even number of teeth is being measured, the rollers being equal, sized as in the foregoing example, and placed in opposite tooth spaces as shown in Fig. 171; the measurement (X) *over* rollers will be: pitch diameter of gear + diameter of one roller. Similarly, the distance *between* rollers will be: pitch diameter of gear − diameter of one roller. In the foregoing notes and formulae, backlash and effects of correction have been ignored.

Spur Gears. If R = radius of pitch circle of gear, r = radius of roller, t = number of teeth, p = circular pitch, T = arc width of tooth, X = measurement over rollers.

For Even Number of Teeth, $$X = 2R + 2r = \frac{tp}{\pi} + (p - T)\cos\psi$$

For Odd Number of Teeth, $$X = 2R\cos\left(\frac{90^\circ}{t}\right) + 2r = \frac{tp}{\pi}\cos\left(\frac{90^\circ}{t}\right) + (p - T)\cos\psi$$

EXAMPLE 1. An uncorrected spur gear has 26 teeth of $8P$, pressure angle 20°. Find the diameter of a roller and the measurement across two rollers in opposite tooth spaces. Ignore backlash.

$$p = \frac{\pi}{8} = 0{\cdot}3927 \text{ in. Arc of tooth space } (w) = 0{\cdot}19635 \text{ in.}$$

$$\textit{Roller dia.} = (p - T)\cos\psi = 0{\cdot}19635 \cos 20^\circ = 0{\cdot}1845 \text{ in.}$$

$$X = \frac{tp}{\pi} + 0{\cdot}1845 = \frac{26 \times 0{\cdot}3927}{\pi} + 0{\cdot}1845 = 3{\cdot}4345 \text{ in.}$$

(*Note:* The standard addendum of this gear = 1/8 = 0·125 in. Thus the roller lies within the crest cylinder of the gear by an amount 0·125 − 0·0922 = 0·0328 in.)

EXAMPLE 2. Suppose the gear in Example (1) had 25 teeth. Find the roller diameter and the dimension X. Rollers are placed in spaces as nearly as possible opposite. Ignore backlash.

$$\textit{Roller dia.} = (p - T)\cos\psi = 0{\cdot}1845 \text{ in.}$$

$$X = \frac{tp}{\pi}\left(\cos\frac{90^\circ}{t}\right) + \text{Roller diameter} = \frac{25}{8}\left(\cos\frac{90^\circ}{25}\right) + 0{\cdot}1845 = 3{\cdot}3033 \text{ in.}$$

EXAMPLE 3. An internal gear has 40 teeth of $10P$; pressure angle is 20°. Supposing the teeth to be cut without thinning for backlash and without addendum modification, find the theoretical diameter of the cylinders and the measurement between them.

$$\text{Pitch dia.} = \frac{\text{No. of teeth}}{P} = \frac{40}{10} = 4 \text{ in.} \quad (p = 0{\cdot}3142 \text{ in.})$$

Cylinder dia. (if centre of cylinder lies on pitch circle)
$$= \text{Arc of space width} \times \cos\psi = 0{\cdot}1571 \times 0{\cdot}9397 = 0{\cdot}1476 \text{ in.}$$

$$\begin{aligned}\text{Distance between cylinders} &= \text{Pitch dia.} - \text{Cylinder dia.}\\ &= 4 - 0{\cdot}1476\\ &= 3{\cdot}8524 \text{ in.}\end{aligned}$$

If the measurement between the cylinders is less than 3·8524 in., it is evident that the arc of space width is less than 0·1571 in.

Precaution. Dimension X should be checked at various places round a gear. If the measurement varies when the cylinders are placed in different pairs of opposite tooth spaces, there is an indication of faults such as lack of uniformity in tooth spacing, tooth thickness, tooth shape, or possibly of mal-alignment of teeth with the axis of the gear.

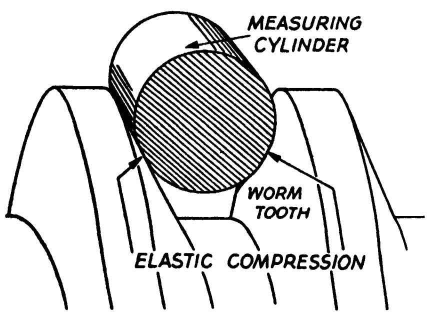

FIG. 174. EXAGGERATED EFFECT OF ELASTIC COMPRESSION OF MEASURING CYLINDER AND FLANKS OF WORM TEETH

Practical Measurement. In worked example (1) on page 187 it is shown that if the diameter of the cylinder is obtained from the rule $d = (p - T)\cos\psi$, and its axis lies in the pitch surface of the gear being measured, the cylinder itself lies within the crest cylinder of the gear. This introduces certain minor "over cylinder" measurement difficulties. The measuring tips of micrometers and verniers can be provided with special tips to drop into the tooth-spaces without fouling the crests of the teeth. Larger cylinders could be used, as shown in succeeding pages, but at the expense of simplicity of calculation. Measurements may be made over projecting ends of cylinders.

Correction for Elastic Compression. When using cylinders for gear measuring, some form of fiducial, or spring-loaded, indicator should be used if utmost accuracy is required. Measurements obtained by "feel" of hand are often misleading; different persons obtaining different readings. Pressure applied to a cylinder resting in a groove results in a wedging action or "elastic compression" (see Fig. 174). This is a matter discussed at length in *Notes on Screw Gauges* (H.M.S.O.), with special reference to the measurement of screw threads by means of cylinders and micrometers.

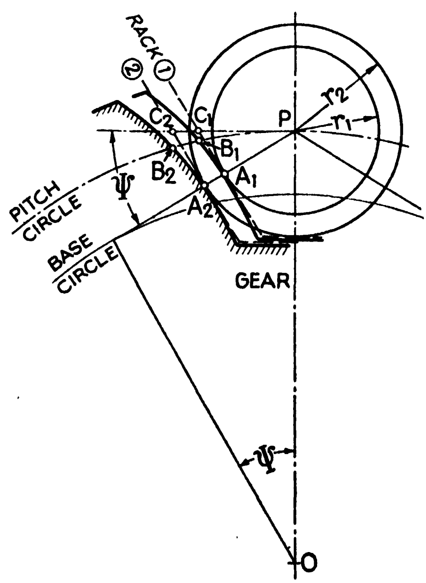

FIG. 175. ILLUSTRATING BACKLASH CALCULATION

(*b*) **Allowing for Backlash.** (See Fig. 175.) It has previously been stated that one method of obtaining the required amount of backlash is to cut tooth-spaces more deeply. If this be done, whilst still retaining the original pitch-circle diameter, it is clear that a roller (having a diameter obtained as explained in the preceding section (*a*)) will sink more deeply into the tooth-space with its centre displaced from the pitch circle. If we still desire to use a very simple formula for calculating the dimension X across the cylinders, the condition of the axis of the roller coinciding with the pitch surface of the gear *must* be fulfilled. Clearly this necessitates an increase in the diameter of the roller to meet the required condition. The writers consider that for most practical and everyday purposes in connection with gears of small pitch, the following formula gives satisfactory results, provided it is possible to ignore effects of any addendum modifications.

In Fig. 175 outline (1) represents a profile of a tooth having no backlash allowance. Outline (2) represents a profile of a tooth thinned to provide backlash. As stated previously, backlash is conventionally measured, at the pitch circle, normal to the tooth surfaces. Here, however, for our present purpose, and on this spur gear example, we take it as $2B_1B_2$, which is more than what is usually described as backlash in the shops.

It can be shown that arc $PB_2 = PC_2$; also that $r_2 = PC_2 \cos \psi$. (Note that $r_2 = PA_2$.)

Thus, $$2r_2 = 2PC_2 \cos \psi = 2PB_2 \cos \psi.$$

Thus the **diameter of the cylinder,** the axis of which will coincide with the pitch surface of the gear, is equal to *the arc of the actual tooth space multiplied by the cosine of the pressure angle.*

If p = pitch of cutter, t = no. of teeth in gear, d = cylinder dia., X = measurement over rollers *in opposite spaces,* T = actual arc width of tooth on generating pitch circle,

$$d = (p - T) \cos \psi; \qquad X = \frac{tp}{\pi} + d$$

Note. The arc width of the tooth space equals half the circular pitch plus half the backlash. The arc width of the tooth equals half the circular pitch minus half the backlash.

EXAMPLE. A spur gear of 5 in. P.C.D. has 30 teeth, with 20° pressure angle. If the backlash allowance per tooth is 0·003 in., what should be the diameter of a cylinder if its centre is to fall on the pitch circle, and what will be the measurement X over the cylinders?

Theoretical circular pitch, ignoring backlash $= \frac{\pi \times 5}{30} = 0{\cdot}5236$ in.

Tooth space arc width $= \frac{1}{2} \times 0{\cdot}5236 = 0{\cdot}2618$ in. (neglecting backlash allowance).

$$\text{Dia. of cylinder} = (0{\cdot}2618 + 0{\cdot}0015) \cos 20^\circ = 0{\cdot}2474 \text{ in.}$$

Measurement over cylinders $(X) = 5{\cdot}2474$ in.

Using These Cylinders for Helical Gears. If the addendum of wheel and pinion are equal—

Dia. of cylinder (d)

= Normal arc width of space × cos normal pressure angle

$= (p_n -$ normal tooth thickness$) \times \cos \psi_n$

$= \frac{1}{2}(p_n +$ backlash$) \times \cos \psi_n$

This cylinder will lie in a tooth space with its axis in the pitch surface.

Measurement (X) over cylinders, for EVEN number of teeth,
= Pitch diameter of gear + Diameter of one cylinder.

Measurement (X) over cylinders, for ODD number of teeth,

$$= \text{Pitch diameter of gear} \left(\cos\frac{90^\circ}{t}\right) + \text{Diameter of one cylinder}$$

(where t = number of teeth).

Note: On a helical or spiral gear, Circular pitch (p) = Normal circular pitch (p_n) × sec of spiral angle (σ). That is, $p = p_n \sec \sigma$. The normal pitch is the pitch of the cutter employed. *Circular pitch* (p) is sometimes called *transverse pitch* (p_t).

Two important formulae:

$$p_t = p_n \sec \sigma;\ p_n = p_t \cos \sigma.$$

Similarly, $\tan \psi_t = \tan \psi_n \sec \sigma;\ \tan \psi_n = \tan \psi_t \cos \sigma.$

General Formulae. For Both Spur and Helical Gears, with an EVEN number of teeth, and cylinders placed in opposite tooth spaces, the following is a general formula,

$$X = \frac{t \times p_n}{\pi} \sec \sigma + (p_n - T_n) \cos \psi_n \quad . \quad . \quad (1)$$

(where the suffix n indicates a normal measurement. On spur gears there is no spiral angle, whence $\sigma = 0$).

For **spur gears** with ODD number of teeth,

$$X = \frac{tp_n}{\pi} \cos\frac{90^\circ}{t} + (p_n - T_n) \cos \psi_n \quad . \quad . \quad (2)$$

For **helical gears** with ODD number of teeth, the following formula is approximately correct,

$$X = \frac{tp_n}{\pi} \sec \sigma \,.\, \cos\frac{90^\circ}{t} + (p_n - T_n) \cos \psi_n \quad . \quad (3)$$

Further Simplification. Suppose we have an ordinary uncorrected tooth with normal diametral pitch (P_n) = 1, normal pressure angle (ψ_n) = 20°, arc of tooth space (T_n) = $p_n/2$. If $P_n = 1$, $p_n = \pi$.

For an EVEN number of teeth general formula (1) reduces to,

$$X = t \,.\, \sec \sigma + 1{\cdot}476 \quad . \quad . \quad . \quad (4)$$

And for an ODD number of teeth general formula (2) reduces to,

$$X = t \,.\, \cos\left(\frac{90^\circ}{t}\right) + 1{\cdot}476 \quad . \quad . \quad . \quad . \quad (5)$$

The cylinder diameter is 1·476 in. in both cases. More accurate values are:

	Unit diametral pitch or module	Unit circular pitch
For 20° pressure angle, d =	1·476066	0·469846
For 14½° pressure angle, d =	1·520763	0·484074

To use these formulae on any particular job we have simply to substitute values of t and σ and divide by the normal diametral pitch or multiply by the given circular pitch or module.

Example 1. Helical gear, 24 teeth, $P_n = 6$, spiral angle = 30°, pressure angle (ψ_n) = 20°.

Working: $X = t \sec \sigma + 1{\cdot}476 = \frac{1}{6}(24 \times 1{\cdot}1547 + 1{\cdot}476)$
$= \underline{4{\cdot}865 \text{ in.}}$

Cylinder diameter $= 1{\cdot}476 \div 6 = \underline{0{\cdot}246 \text{ in.}}$

Let us check the simple formula employed in the foregoing example

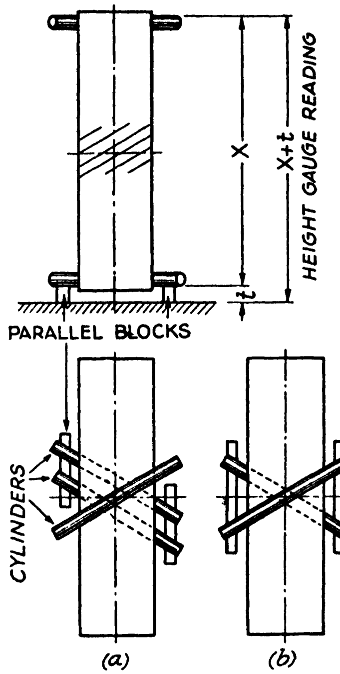

Fig. 176. Checking a Helical Gear by Means of Cylinders
(a) Odd-toothed gear—3 pins—Plan and Elevation.
(b) Even-toothed gear—2 pins—Plan only.

by substitution of the given values in the general formula (1) given previously,

$$X = \frac{tp_n}{\pi} \sec \sigma . + (p_n - T_n) \cos \psi_n .$$

$$t = 24;\ p_n = \frac{\pi}{P_n} = \frac{\pi}{6};\ (p_n - T_n) = \frac{\pi}{12};\ \psi_n = 20°;\ \cos \psi_n = 0{\cdot}9397$$

$$X = \frac{24}{\pi} . \frac{\pi}{6} \sec 30° + \frac{\pi}{12} \times 0{\cdot}9397$$
$$= \frac{4}{\cos 30°} + \pi \times 0{\cdot}0783$$
$$= 4{\cdot}619 + 0{\cdot}246$$
$$= \underline{4{\cdot}865 \text{ in.}}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$
$$\therefore \frac{4}{\cos 30} = \frac{4 \times 2}{\sqrt{3}}$$
$$= \frac{8\sqrt{3}}{3} = 4{\cdot}619$$

Diameter of cylinder $= \frac{\pi}{12} \cos 20° = \underline{0{\cdot}246 \text{ in.}}$ (to nearest thousandth)

EXAMPLE 2. Helical gear; $6P$; pressure angle (ψ_t) 20° in transverse section, i.e. in a plane perpendicular to its axis; spiral angle (σ) 25° at pitch surface. Find (1) its normal tooth thickness (T_n), (2) its normal pressure angle ψ_n, (3) diameter of cylinder having its axis in the pitch surface. Ignore backlash.

Working:

Circular pitch $(p_t) = \frac{\pi}{6}$; $\cos 25° = 0{\cdot}90631$; $\tan 20° = 0{\cdot}36397$;

Transverse tooth thickness $(T_t) = \frac{\pi}{12} = 0{\cdot}26179$

(1) Normal tooth thickness $(T_n) = T_t \times \cos\sigma$
$= 0{\cdot}26179 \times 0{\cdot}90631$
$= \underline{\underline{0{\cdot}2373 \text{ in.}}}$

(2) $\operatorname{Tan}\psi_n = \tan\psi_t \cos\sigma$
$= 0{\cdot}36397 \times 0{\cdot}90631 = 0{\cdot}3299$
$\therefore\ \psi_n = \underline{\underline{18° 16'}}$

(3) Cylinder dia. $(d) = T_n \times \cos\psi_n$
$= 0{\cdot}2373 \times \cos 18° 16'$
$= 0{\cdot}2373 \times 0{\cdot}94961 = \underline{\underline{0{\cdot}2253 \text{ in.}}}$

(2) Another Special Case. CYLINDER DIAMETER SUCH AS TO ENABLE IT TO TOUCH THE TEETH SIMULTANEOUSLY AT A

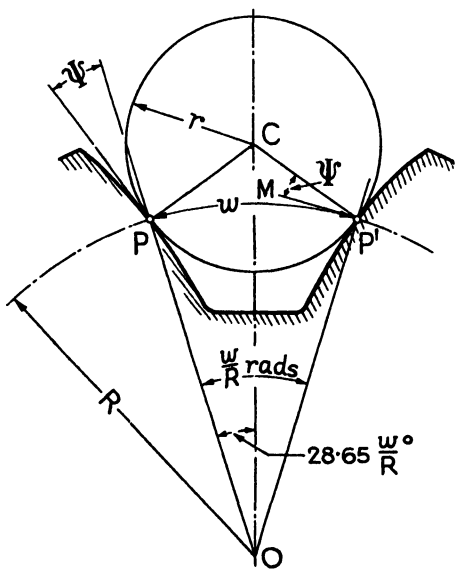

FIG. 177. ROLLER TOUCHING TOOTH PROFILE AT PITCH CIRCLE

GIVEN RADIUS R, THE GEAR HAVING A KNOWN SPACE WIDTH w. (See Fig. 177.)

Given: Pressure angle of spur gear tooth at radius R is ψ. Arc width at that radius is w.

Required (1) Radius (r) of cylinder that will just touch adjacent tooth surfaces simultaneously at radius R.

Working. Consider the point of contact P';

$P'M$ is a tangent to the pitch circle. Since the tangent to the tooth profile is inclined at an angle ψ to the radius OP' (property of involute teeth), angle $MP'C = \psi$.

The centre of the cylinder will lie along a line normal to the surface, i.e. at point C.

$$\angle POC = \tfrac{1}{2}\angle POP' = \tfrac{1}{2}\left(\frac{w}{R}\right) \text{radians} = 28{\cdot}65\,\frac{w}{R} \text{ degrees.}$$

$$\text{In } \triangle POC,\ \angle PCO = 90 - \left(\psi + 28{\cdot}65\,\frac{w}{R}\right)$$

Applying the sine rule—

$$PC = \frac{R \sin\left(28{\cdot}65\frac{w}{R}\right)}{\text{Sin}\left[90 - \left(\psi + 28{\cdot}65\,\frac{w}{R}\right)\right]}$$

$$\textbf{Diameter of cylinder} = \frac{2R \sin\left(28{\cdot}65\,\frac{w}{R}\right)}{\cos\left(\psi + 28{\cdot}65\,\frac{w}{R}\right)} \quad . \quad . \quad (1)$$

Required (2). Distance between centre of cylinder C and centre of gear O.

Working. By the sine rule, since $\angle OPC = (90 + \psi)$

$$OC = \frac{R \sin(90 + \psi)}{\text{Sin}\left[90 - \left(\psi + 28{\cdot}65\,\frac{w}{R}\right)\right]}$$

$$\textbf{Distance between centres} = \frac{R \cos\psi}{\text{Cos}\left(\psi + 28{\cdot}65\,\frac{w}{R}\right)} \quad . \quad (2)$$

Note. The above two formulae apply to all cases where a cylinder touches the teeth at a specified radius, the pressure angle at that radius being known. This will not generally be the normal pressure angle, unless the radius specified equals the pitch circle radius.

EXAMPLE. Pressure angle of spur gear at radius 1·625 in. = 20°. Arc width of space at radius 1·625 in. = 0·19635 in.

$$\text{Then } 28{\cdot}65\,\frac{w}{R} = 28{\cdot}65 \times \frac{0{\cdot}19635}{1{\cdot}625} = 3{\cdot}463^\circ = 3^\circ\ 27{\cdot}8'$$

$$\text{Dia. of cylinder} = \frac{2 \times 1{\cdot}625 \sin 3^\circ\ 27{\cdot}8'}{\cos 23^\circ\ 27{\cdot}8'}$$

$$= \frac{2 \times 1{\cdot}625 \times 0{\cdot}06041}{0{\cdot}9173} = 0{\cdot}214 \text{ in.}$$

Distance (OC) between centres of cylinder and gear

$$= \frac{1{\cdot}625 \cos 20^\circ}{\cos 23^\circ\ 27{\cdot}8'} = \frac{1{\cdot}625 \times 0{\cdot}93969}{0{\cdot}9173} = 1{\cdot}665 \text{ in.}$$

Distance (X) over these cylinders

(1) If the number of teeth in the gear is EVEN and equal cylinders are placed in opposite spaces,

$$X = \text{Dia. of one cylinder} + 2\ (\text{distance between axes of gear and cylinder}).$$

$$= 2r + 2 \times OC$$

$$= \frac{2R\left(\sin 28{\cdot}65 \frac{w}{R} + \cos \psi\right)}{\text{Cos}\left(\psi + 28{\cdot}65 \frac{w}{R}\right)}$$

(2) If the number of teeth (N) in the gear is ODD and equal cylinders are placed in spaces as nearly opposite as possible,

$$X = \left(2r + 2\,(OC) \cos \frac{90}{N}\right)$$

or,

$$X = \frac{2R\left(\cos \psi \cos\left(\frac{90}{N}\right) + \sin 28{\cdot}65 \frac{w}{R}\right)}{\text{Cos}\left(\psi + 28{\cdot}65 \frac{w}{R}\right)}$$

Thus, on the spur gear in the preceding example, for 26 teeth,

$X = 2r + 2\ (OC) = 0{\cdot}214 + 3{\cdot}330 = 3{\cdot}544$ in.

Similarly, for 25 teeth,

$X = 2r + 2\ (OC) \cos\left(\frac{90}{N}\right) = 0{\cdot}214 + 3{\cdot}330 \times 0{\cdot}998 = 3{\cdot}537$ in.

EXAMPLE. A spur gear has 35 teeth and a pitch circle diameter of 6 in. Cylinders are placed between teeth as nearly opposite as possible, touching the teeth at the pitch circle. What is the cylinder diameter and the measurement across nearly opposite cylinders? The pressure angle is 20° and backlash is neglected.

$$\text{Circumferential Pitch} = \frac{\pi D}{t} = \frac{3{\cdot}1416 \times 6}{35} = 0{\cdot}5386 \text{ in.}$$

Arc width (w) = 0·2693 in.

$$\text{Dia. of Cylinder} = \frac{6 \times \sin\left(\frac{28{\cdot}65 \times 0{\cdot}2693}{3}\right)}{\cos\,(20 + 2{\cdot}572)} = \frac{6 \times 0{\cdot}044867}{0{\cdot}92339} = 0{\cdot}292 \text{ in.}$$

$$\text{Measurement over Cylinders} = \frac{6\,(\cos 20^\circ \cos 2^\circ\, 34{\cdot}3' + \sin 2^\circ\, 34{\cdot}3')}{\cos\,(22^\circ\, 34{\cdot}3')}$$

$$= \frac{6 \times (0{\cdot}93969 \times 0{\cdot}99899 + 0{\cdot}044867)}{0{\cdot}92339}$$

$$= 6{\cdot}391 \text{ in.}$$

(3) **USING OTHER CYLINDERS OR ROLLERS.** It may not be convenient to make or obtain cylinders having their diameters equal to arc width of tooth-space multiplied by the cosine of the pressure angle, or to other special dimensions, so that it may be necessary to use cylinders already in stock. This involves more tedious calculations, as shown in the following examples.

(1) GIVEN: (1) Arc tooth thickness and pressure angle of spur gear at a given radius, (2) Diameters of equal rollers for use on a gear with an EVEN number of teeth.

REQUIRED: Measurement over cylinders. (See Fig. 178.)

Let T = Arc tooth thickness at radius R_1
R_1 = Given radius
N = No. of teeth in gear
ψ_1 = Pressure angle at R_1
ψ_2 = Pressure angle at R_2, i.e. at centre of roller
r = Radius of cylinder
R_2 = Radius to centre of cylinder
X = Measurement over cylinders

Then,
$$\text{Inv. } \psi_2 = \frac{T}{2R_1} + \text{inv. } \psi_1 + \frac{r}{R_1 \cos \psi_1} - \frac{\pi}{N}$$
$$X = \frac{2R_1 \cos \psi_1}{\text{Cos } \psi_2} + 2r$$

Diameter of Cylinder. A commonly-used rule for spur gears is to have cylinder or roller diameter = 1·68 ÷ diametral pitch, or a little larger.

EXAMPLE. In a given case, the pitch circle diameter is 5·000 in., the radius of the cylinder (r) is 0·141 in., and the number of teeth (N) is 30. Pressure angle is 20°. The arc tooth thickness at the pitch line is 0·262 in. We require the measurement over the cylinders (X).

$$\text{Inv. } \psi_2 = \frac{0{\cdot}262}{5{\cdot}000} + \text{inv. } 20^\circ + \frac{0{\cdot}141}{2{\cdot}5 \times 0{\cdot}9397} - \frac{\pi}{30}$$
$$= 0{\cdot}0524 + 0{\cdot}014904 + 0{\cdot}06002 - 0{\cdot}104720$$
$$= 0{\cdot}022604$$

The next step in the problem is to find the angle whose involute function is 0·022604.[1]

In six-figure tables of involute functions we find that inv. 22° = 0·020054. Using equation (3), on page 244, ψ_2 is given by

$$22^\circ + 3440\left[\frac{(0{\cdot}022604 - 0{\cdot}020054)}{\tan^2 22^\circ} - \frac{(0{\cdot}022604 - 0{\cdot}020054)^2 \sec^2 22^\circ}{\tan^5 22^\circ}\right]'$$
$$= 22^\circ + 3440\left[\frac{0{\cdot}00255}{(0{\cdot}4040)^2} - \frac{(0{\cdot}00255)^2\,(1{\cdot}0785)^2}{(0{\cdot}4040)^5}\right]'$$
$$= 22^\circ + 3440\,(0{\cdot}01562 - 0{\cdot}0007028)'$$
$$\therefore \psi_2 = 22^\circ\ 51{\cdot}3'$$

$$X = \frac{5{\cdot}000 \cos 20^\circ}{\cos 22^\circ\ 51{\cdot}3'} + 2 \times 0{\cdot}141$$
$$= \frac{5{\cdot}000 \times 0{\cdot}939693}{0{\cdot}92149} + 0{\cdot}282$$
$$= 5{\cdot}09877 + 0{\cdot}28200 = 5{\cdot}38077 \text{ in.}$$

Note: The diametral pitch is $\frac{30}{5} = 6$. Then $1{\cdot}68/P = 1{\cdot}68/6 = 0{\cdot}28$. A cylinder of 0·141 in. radius is therefore suitable.

(2) GIVEN: (1) Arc tooth thickness and pressure angle of spur gear at a given radius, (2) Diameter of equal cylinders for use on a gear with an ODD number of teeth.

[1] *Involute functions* are explained on page 242.

REQUIRED: Measurement over rollers. (See Fig. 179.)

Let T = Arc tooth thickness at radius R_1
R_1 = Given radius
N = No. of teeth in gear
ψ_1 = Pressure angle at R_1
ψ_2 = Pressure angle at R_2
r = Radius of cylinder
R_2 = Radius to centre of cylinder
X_1 = Measurement over cylinders

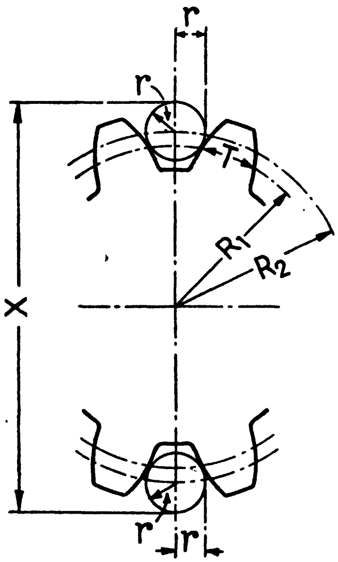

FIG. 178. MEASURING OVER CYLINDERS. GENERAL CASE —EVEN NUMBER OF TEETH

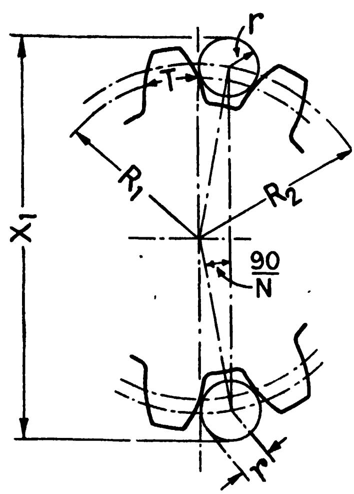

FIG. 179. MEASURING OVER ROLLERS. GENERAL CASE —ODD NUMBER OF TEETH

Then,

$$\text{Inv. } \psi_2 = \frac{T}{2R_1} + \text{inv. } \psi_1 + \frac{r}{R_1 \cos \psi_1} - \frac{\pi}{N}$$

$$R_2 = \frac{R_1 \cos \psi_1}{\cos \psi_2}$$

$$X_1 = 2\left(R_2 \cos \frac{90}{N} + r\right)$$

EXAMPLE. In a certain gear having 31 teeth the pitch circle diameter is 5·1666 in., where the arc tooth thickness is 0·2618 in. Evaluate X_1 if the radius of the cylinder is 0·1406 in. Take pressure angle as 20°.

$$\text{Inv. } \psi_2 = \frac{0{\cdot}2618}{5{\cdot}1666} + \text{inv. } 20^\circ + \frac{0{\cdot}1406}{2{\cdot}5833 \cos 20^\circ} - \frac{\pi}{31}$$
$$= 0.05067 + 0{\cdot}014904 + 0{\cdot}0579 - 0{\cdot}1013417$$
$$= 0{\cdot}022132$$

From tables, the involute function of 22° is 0·020054

$$\psi_2 = 22^\circ + 3440\left[\frac{0{\cdot}002078}{\tan^2 22^\circ} - \frac{(0{\cdot}002078)^2 \sec^2 22^\circ}{\tan^5 22^\circ}\right]'$$
$$= 22^\circ + 3440\,(0{\cdot}01273 - 0{\cdot}0004666)'$$
$$= 22^\circ\ 42{\cdot}2'$$

EXAMPLE. An internal gear having 30 teeth and a pitch circle diameter of 5·000 in. has an arc tooth thickness of 0·2618 in. What should be the measurement between cylinders of ¼ in. dia., if the pressure angle is 20°?

$$\text{Inv. } \psi_2 = \frac{\pi}{30} + \text{inv. } 20^\circ - \frac{0{\cdot}2618}{5{\cdot}0000} - \frac{0{\cdot}125}{2{\cdot}500 \times 0{\cdot}93969}$$
$$= 0{\cdot}104720 + 0{\cdot}014904 - 0{\cdot}05236 - 0{\cdot}053209$$
$$= 0{\cdot}014055$$

Tables give the inv. of 19° as 0·0127151

$$\psi_2 = 19^\circ + 3440\left[\frac{0{\cdot}001340}{(0{\cdot}3443)^2} - \frac{(0{\cdot}001340)^2\,(1{\cdot}0576)^2}{(0{\cdot}3443)^5}\right]'$$
$$= 19^\circ + 3440\,(0{\cdot}01130 - 0{\cdot}0004150)'$$
$$= 19^\circ\,37{\cdot}4' \ldots (\textit{Note: } \cos 19^\circ\,37{\cdot}4' = 0{\cdot}941921)$$
$$X_2 = \frac{5{\cdot}000 \times 0{\cdot}939693}{0{\cdot}941921} - 0{\cdot}25000$$
$$= 4{\cdot}98820 - 0{\cdot}25000 = 4{\cdot}73820 \text{ in.}$$

(4) GIVEN: (1) Arc tooth thickness and pressure angle of an internal gear at a given radius, (2) Diameter of equal cylinders for use on an internal gear with an ODD number of teeth.

REQUIRED: Measurement between cylinders. (See Fig. 181.)

Let, T = Arc tooth thickness at R_1
R_1 = Given radius
N = No. of teeth
ψ_1 = Pressure angle at R_1
ψ_2 = Pressure angle at R_2, i.e. at centre of cylinder
r = Radius of cylinders
R_2 = Radius to centre of cylinder
X_3 = Measurement between cylinders

Then,
$$\text{Inv. } \psi_2 = \frac{\pi}{N} + \text{inv. } \psi_1 - \frac{T}{2R_1} - \frac{r}{R_1 \cos \psi_1}$$
$$R_2 = \frac{R_1 \cos \psi_1}{\cos \psi_2}$$
$$X_3 = 2\left(R_2 \cos \frac{90}{N} - r\right)$$

EXAMPLE. An internal gear of pitch circle diameter 5·1666 in., having 31 teeth, has a tooth thickness of 0·2618 in. on the pitch circle. If the diameter of the cylinders is ¼ in. and the pressure angle is 20°, find the measurement between the cylinders.

$$\text{Inv. } \psi_2 = \frac{\pi}{31} + \text{inv. } 20^\circ - \frac{0{\cdot}2618}{5{\cdot}1666} - \frac{0{\cdot}125}{2{\cdot}5833 \times \cos 20^\circ}$$
$$= 0{\cdot}1013417 + 0{\cdot}014904 - 0{\cdot}0506716 - 0{\cdot}0514933$$
$$= 0{\cdot}0140808$$

From tables the involute function of 19° is 0·0127151

$$\psi_2 = 19^\circ + 3440\,(0{\cdot}01152 - 0{\cdot}0004309)'$$
$$= 19^\circ\,38{\cdot}1' \quad (\textit{Note: } \cos 19^\circ\,38{\cdot}1' = 0{\cdot}941852)$$
$$R_2 = \frac{2{\cdot}5833 \times 0{\cdot}939693}{0{\cdot}941852} = 2{\cdot}5774$$
$$\frac{90^\circ}{N} = \frac{90^\circ}{31} = 2^\circ\,54{\cdot}1' \ldots (\cos 2^\circ\,54{\cdot}1' = 0{\cdot}9987176)$$
$$X_3 = 2\,(2{\cdot}5774 \times 0{\cdot}9987176 - 0{\cdot}125)$$
$$= 4{\cdot}89839 \text{ in.}$$

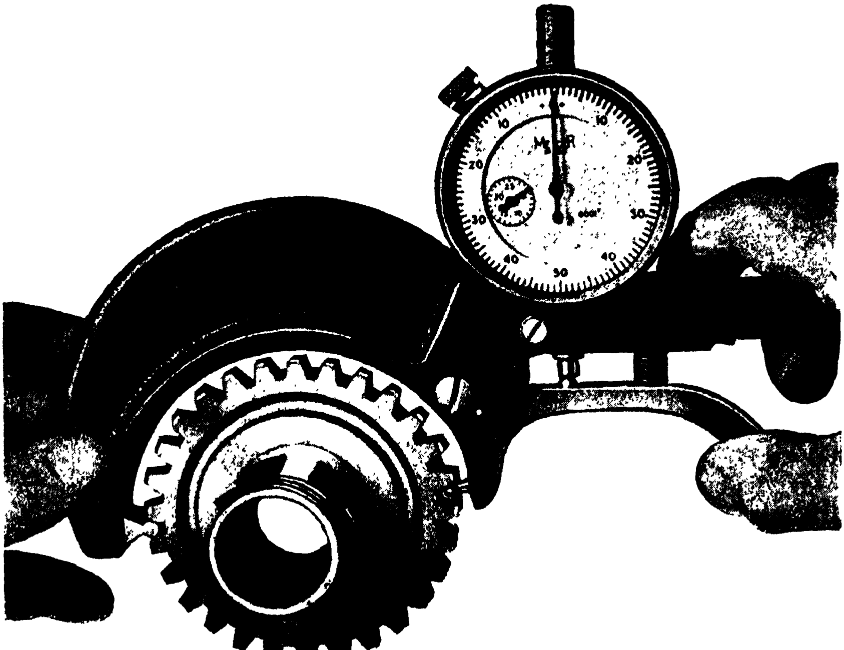

FIG. 182. THE CLEMENT GEAR COMPARATOR

The Clement Gear Comparator. This is shown in Fig. 182, kindly supplied by Messrs. J. E. Baty & Co., Ltd., the makers. Essentially it consists of a metal frame carrying a Mercer dial indicator graduated to 0·0001 in. On the outer end is a hardened and ground cylinder or roller which rests in a tooth-space, whilst at the other end a steel ball is held in one arm of a spring-loaded lever pivoted in the frame. Movement of the ball is transmitted to the dial gauge. This gauge is essentially a comparator using the principle of roller measurement to disclose variations in pitch diameter.

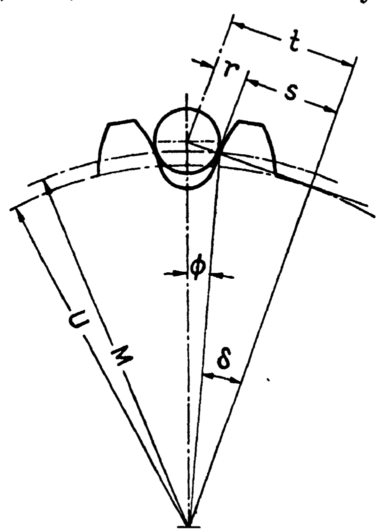

FIG. 183. DIMENSIONS OF ROLLER IN CLEMENT COMPARATOR

The Dimensions of Roller and Ball in the Clement Comparator. The makers inform us that these are dimensioned, as shown in Fig. 183, so that the centre of the roller (or ball) lies beyond the pitch circle, on the normal to the tangent at the point of contact, the normal actually being tangential to the base circle. The roller method of measuring gears for pitch diameter suffers from the disadvantage that "feel" in adjusting the micrometer is involved. In this type of comparator, in which the rollers are integral with the means of measurement (or comparison), "feel" does not enter into the matter.

In Fig. 183, notation used by J. E. Baty & Co., Ltd.,

r = Radius of roller
U = Radius of base circle = $M \cos \delta$
M = Radius of pitch circle
$\phi = 90 \div$ No. of teeth
δ = Pressure angle

$$r = t - s$$
$$t = U \tan (\delta + \phi)$$
$$s = M \sin \delta$$

CHAPTER XV

VERNIER AND MICROMETER METHODS OF GEAR TESTING

GEAR TOOTH VERNIER CALIPER. For checking the thickness of teeth at predetermined distances from the centre of the gear, or from the tips of the teeth, the **gear tooth-vernier caliper** is widely used. It enables us to take two measurements on a tooth simultaneously. Whilst this particular gear-measuring operation, which is a typical "combination test," might more conveniently have been discussed in earlier pages, it is dealt with here as it links up satisfactorily

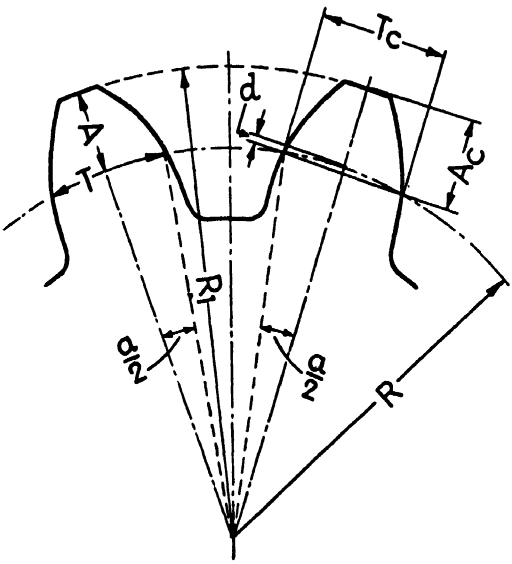

FIG. 184. CHORDAL TOOTH THICKNESS CORRECTION

with other related tests and calculations shortly to be described. The following remarks apply to spur gears unless the contrary is stated.

Disadvantages of this method: (1) The calipers require two settings for each pitch measured; (2) they require a different setting for any variation in the number of teeth for a given pitch; (3) wear is concentrated on the corners of the two jaws; (4) reads only to 0·001 in.

Chordal Thickness. What is called chordal thickness is clearly shown in Fig. 184. The *arc T* cannot be measured directly, whereas the *chord* T_c[1] can be measured directly by means of the gear tooth vernier caliper shown in Fig. 185. At one setting these calipers enable us to measure (1) *chordal thickness* (T_c), and (2) *height setting* A_c (also known as *corrected addendum* and *chordal addendum*).

The adjustable tongue or slide of the vernier caliper is set so that

[1] A chord of a circle is a straight line joining any two points in its circumference.

when it rests on the top of a tooth the lower ends or corners of the caliper jaws touch the sides of the tooth at the pitch circle (when using the caliper to test tooth thickness at the pitch circle).

On mating gears having equal tooth thickness, and without addendum modifications, the length of the arc T actually equals half the circular pitch minus half the backlash. More accurately it becomes necessary to subtract half the maximum and minimum allowance for backlash, thus obtaining the upper and lower limits for tooth thickness.

A useful tip is not to spend time on measuring tooth thickness until after having checked the outside diameter of the gear. If an error is discovered in the latter dimension, it must be allowed for when measuring tooth thickness.

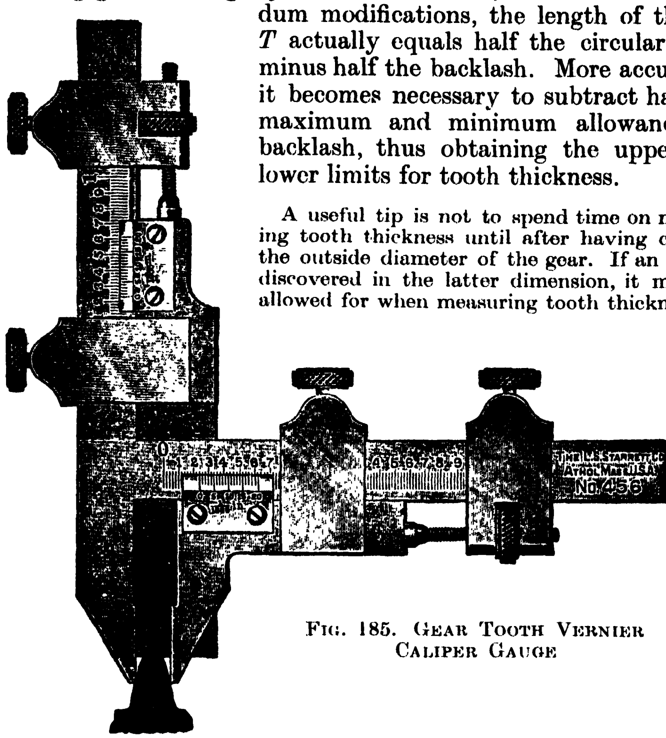

FIG. 185. GEAR TOOTH VERNIER CALIPER GAUGE

$14\frac{1}{2}°$ Involute Gear Example. (Outside Diameter Standard.)

Let A = True addendum
A_c = Corrected addendum, chordal addendum, or height setting
d = Addendum correction or height of arc
T = Circular tooth thickness
T_c = Chordal tooth thickness
R = Pitch radius of gear
R_1 = Tip radius of gear
a = Angle of tooth in degrees
t = Number of teeth in gear

$$\frac{a}{2} = \frac{90}{t} \text{ degrees} = \frac{\pi}{2t} \text{ radians}; \quad A_c = A + d; \quad T = \frac{CP}{2} = \text{Arc}\,\frac{a}{2} \times 2R$$

$$d = R\left(1 - \cos\frac{a}{2}\right); \qquad T_c = 2R \times \sin\frac{a}{2}$$

These formulae apply when the basic rack has its axis of symmetry, along which tooth thickness equals tooth-space, coincident with its pitch-line, the gear being "uncorrected" and backlash ignored.

The reader will find that on gears of small pitch the difference between T and T_c is very small. Indeed, it is customary when dealing with gears having equal tooth thickness to calculate T_c and then subtract half the backlash allowance from the result.

EXAMPLE. Find the chordal thickness and chordal addendum for a gear of 8 diametral pitch (outside diameter standard), 14½° involute full-depth form, 36 teeth. Pitch diameter = 4½ in.

Working (refer to Fig. 184):

$$\frac{a}{2} = \frac{90}{36} = 2{\cdot}5°;\ \text{Addendum} = 0{\cdot}125\ \text{in. (from table)}$$

$$T_c = 2R \times \sin\frac{a}{2} = 4{\cdot}5 \times \sin 2{\cdot}5° = 0{\cdot}1963\ \text{in.}$$

$$d = R\left(1 - \cos\frac{a}{2}\right) = 2{\cdot}25\,(1 - \cos 2{\cdot}5°) = 2{\cdot}25\,(1 - 0{\cdot}99905) = 0{\cdot}0021\ \text{in.}$$

$$A_c = A + d = 0{\cdot}125 + 0{\cdot}0021 = 0{\cdot}1271\ \text{in.}$$

These four-figure calculations emphasize one of the shortcomings of the tooth-vernier, for, whilst we have calculated dimensions to four decimal places, the smallest reading we obtain from the vernier is to the third decimal place.

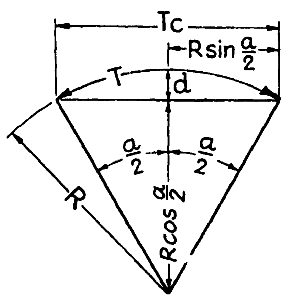

FIG. 186. NOTATION EMPLOYED IN CHORDAL MEASUREMENT FORMULAE

Using a Table. Many engineering handbooks contain tables of chordal thickness similar in form to that given on page 204. From this table it is possible to obtain chordal thickness and chordal addendum with sufficient accuracy for ordinary requirements. The table given on page 204 is taken from the Maxicut Handbook but has been extended to include pinions with 8 to 11 teeth. Bear in mind that the table is for gears of one D.P. **To use the table for 14½° gears,** read off the chordal tooth thickness (T_c) corresponding to the number of teeth in the gear, and divide by the diametral pitch.

EXAMPLE. Gear has 20 teeth, 6 diametral pitch.

$$T_c = \frac{1{\cdot}5686}{6} = 0{\cdot}2614\ \text{in.}$$

To find corrected or chordal addendum (A_c), divide the corrected addendum value given in the table by diametral pitch.

Thus, $$A_c = \frac{1{\cdot}0362}{6} = 0{\cdot}1727\ \text{in.}$$

To Use the Table for Stub Teeth

(*a*) *To Find Chordal Thickness.* Divide the value shown in the table by the first or numerator figure in the pitch fraction.

Example. Stub tooth gear of 7/9 pitch, 28 teeth.

$$T_c = \frac{1{\cdot}5698}{7} = 0{\cdot}2243 \text{ in.}$$

(*b*) *To Find Corrected Addendum.* Take the correction value from the table, divide by the numerator of the pitch fraction, and add the result to the normal addendum reading obtained from a table of the form given on page 246. Thus, using the foregoing gear as an example,

$$A_c = \frac{0{\cdot}0237}{7} + 0{\cdot}1111 = 0{\cdot}0034 + 0{\cdot}1111 = 0{\cdot}1145 \text{ in.}$$

No. of Teeth in Gear	Chordal Tooth Thickness (T_c)	Corrected Addendum (A_c)	Correction (d)
8	1·5607	1·0769	0·0769
9	1·5628	1·0684	0·0684
10	1·5643	1·0616	0·0616
11	1·5654	1·0559	0·0559
12–13	1·5663	1·0514	0·0514
14–16	1·5675	1·0440	0·0440
17–20	1·5686	1·0362	0·0362
21–25	1·5694	1·0294	0·0294
26–34	1·5698	1·0237	0·0237
35–54	1·5702	1·0176	0·0176
55–133	1·5706	1·0112	0·0112
134	1·5707	1·0047	0·0047

(In the table the dimensions given are for gears of 1 D.P. See accompanying explanatory notes. A fuller table is given on page 250.)

If the Outside Diameter is Large. The outside diameter of the pinion may be enlarged to avoid undercut or be large through error in turning it. In such a case the tooth will probably be situated on a pitch circle of larger diameter than "standard." Therefore the tooth thickness will be increased. In such a case we use the following rule to find T_c—

Find the difference between the standard addendum and the long addendum, and multiply difference by the tangent of the pressure angle; multiply this product by 2 and *add* result to one-half of circular pitch, thus obtaining arc thickness on pitch circle (T). Having obtained the arc thickness, it is necessary to calculate the corresponding chordal thickness (T_c), so that the tooth can be measured by the gear tooth vernier.

If the Outside Diameter is Small. If the outside diameter of a pinion is increased, it is necessary to decrease the diameter of the mating gear by the corresponding amount if the pair are to run at the standard centre-distance. If this be done, the arc tooth thickness of the *gear*, measured on the pitch circle, will be small. In such a case we can use the following rule to find T_c—

Find the difference between the standard addendum and the short addendum, and multiply difference by the tangent of the pressure angle; multiply this product by 2 and *subtract* the result from one-half the circular pitch, thus obtaining arc thickness (T) on pitch circle. The corresponding chordal thickness (T_c) can then be calculated.

Chordal Thickness (T_c)

1. *When Outside Diameter is Standard*

$$T_c = D \times \sin \frac{90}{t}$$

2. *When Outside Diameter is Non-Standard*

$$T_c = D \times \sin \frac{90 \times T}{\pi \times R}$$

D = Pitch diameter
R = Pitch radius
t = No. of teeth
T = Arc tooth thickness

Teeth of Internal Gears. Reference back to Fig. 184, which applies to spur gears, will show that *for spur gears* $A_c = A + d$, i.e. height setting = true addendum + height of arc. When using the tooth vernier for measuring teeth on internal gears it is customary to consider it sufficiently accurate to find the height setting by *subtracting* the height of arc from the true addendum, using the rule $A_c = A - d$. But inasmuch as the tips of the internal teeth are slightly concave this is not a strictly correct method, for we should add to the value of A_c

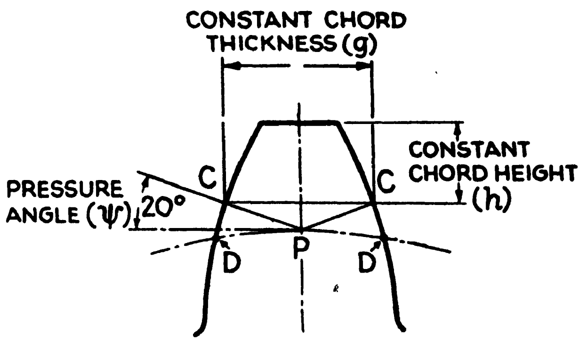

FIG. 187. CONSTANT CHORD DIMENSIONS AND SYMBOLS

so obtained the height of arc at the tips of the teeth. Considering the limitations of the tooth vernier and the smallness of the latter amount this addition is usually not considered important enough to bother about.

CONSTANT CHORD CALIPER MEASUREMENT METHOD. Certain drawbacks to the tooth vernier method of measurement have already been mentioned. Another is that the two settings have to be calculated and made afresh when measuring teeth on other gears of the same pitch but having a different number of teeth. This drawback is overcome by using the *constant chord method of measurement*, a method which enables us to employ a constant pair of settings for all mating gears, i.e. gears having the same pitch and pressure angle, irrespective of their numbers of teeth. All teeth similarly generated from the same rack have a "constant chord." In Fig. 187 the line CC represents the constant chord, the points C being the points of contact between the tooth shown and the teeth with which it mates, provided that the two normals CP pass through P.

$$\text{Constant chord } (g) = \frac{\pi}{2P} \cos^2 \psi = \frac{m \times \pi}{2} \cos^2 \psi$$

$$\text{Constant chord height } (h) = \frac{1}{P}\left(1 - \frac{\pi}{4}\cos\psi\sin\psi\right)$$
$$= m\left(1 - \frac{\pi}{4}\cos\psi\sin\psi\right)$$

B.S. Definitions—B.S. No. 436—1940

The *constant chord* is the shortest distance between the lines of contact of a gear tooth with the adjacent teeth of the basic rack when the latter is symmetrically placed in relation to the centre line of the gear tooth. The constant chord lies in a normal section of the gear tooth, and is shown in Fig. 187. The position and length of this chord is independent of the number of teeth, and depends only on the proportions of the normal section of the basic rack, the pitch, and the correction factor.

The *constant chord thickness* (g) (Fig. 187) is the length of the constant chord.

The *constant chord height* (h) (Fig. 187) is the length of the intercept of the centre line of the gear tooth between the tip of the tooth and the constant chord.

Caliper Settings. The constant chord caliper settings are given in the following table, reprinted by permission from B.S. No. 436—1940, for gears of unit normal diametral pitch, British Standard normal basic rack proportions, and correction factors from − 0·5 to + 0·5; settings for intermediate values of the correction factor may be directly interpolated. Note that the caliper height h increases uniformly by 0·088302, and the caliper thickness g by 0·064279 for every 0·1 increase in k.

CONSTANT CHORD CALIPER SETTINGS

20° Normal Pressure Angle — Full Depth

Correction Factor k	Caliper Settings Height h	Caliper Settings Thickness g	Correction Factor k	Caliper Settings Height h	Caliper Settings Thickness g
− 0·5	0·30607	1·06566	+ 0·5	1·18909	1·70845
− 0·4	0·39437	1·12993	+ 0·4	1·10079	1·64417
− 0·3	0·48267	1·19421	+ 0·3	1·01249	1·57989
− 0·2	0·57098	1·25849	+ 0·2	0·92418	1·51561
− 0·1	0·65928	1·32277	+ 0·1	0·83588	1·45133
0	0·74758	1·38705	0	0·74758	1·38705

The figures are for unit normal diametral pitch. The corresponding values for other pitches are obtained by dividing by the normal diametral pitch. In Table 5 (page 249) constant chords are given for uncorrected teeth of 14½° and 20° pressure angles.

Addendum of Internal Gear. See the notes on pages 24 and 41 on the B.S. method of calculating addenda, taking *correction factors* into account. Note that,

$$a = \frac{p_n}{\pi}\left(1 + k_p\right) = \frac{1}{P}\left(1 + k_p\right)$$
$$A = \frac{p_n}{\pi}\left(1 + k_w\right) = \frac{1}{P}\left(1 + k_w\right)$$

For internal gears $k_p = 0{\cdot}4,\ k_w = -\,k_p$

Pinion. **Addendum = 1·4/P; height setting 1·10079/P, tooth thickness 1·64417/P**

Wheel. **Addendum = 0·6/P; height setting 0·39437/P, tooth thickness 1·12993/P**

The tooth-vernier settings given are taken from the B.S. table—*Constant Chord Caliper Settings*, given on page 206, and thus do not depend upon the numbers of teeth in the gears.

Using the Tooth-Vernier: Practical Hints

In general, the degree of accuracy attained with the tooth-vernier is restricted to 0·002 in., or with practice 0·001 in., which is sufficiently accurate for general work.

The "anvils" have a very small area of contact and should periodically be inspected for wear-and-tear.

Measurement should be taken by "sight" rather than by "feel," a strong-light background being advantageous so that any tendency of the tooth-vernier to "rock" or change its position will be evident.

Note that unlike the customary vernier scale, the tooth-vernier has the 0·1 in. division graduated into "five" and not "four" subdivisions.

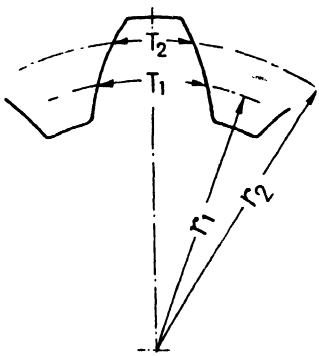

FIG. 188. SPUR GEAR TOOTH THICKNESSES AT DIFFERENT RADII

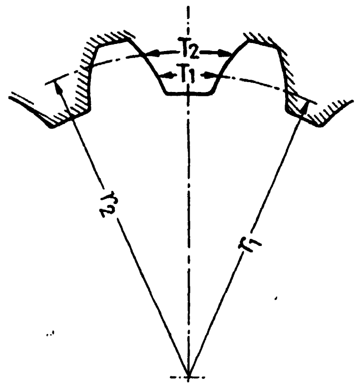

FIG. 189. INTERNAL GEAR TOOTH THICKNESSES AT DIFFERENT RADII

When holding the gauge in position on the gear-tooth, any pressure exerted to obtain the measurement should be exerted with the index finger of the left hand, placed on top of the vertical column leaving the right hand free for adjusting the horizontal column.

Calculating Base Radius. When considering involute teeth, it is necessary to refer to base radius (r_0). As mentioned previously, this can be calculated if we know the pressure angle (ψ) at some radius (r). Thus,

$$r_0 = r \cos \psi$$

EXAMPLE. Find the base radius of a gear having 30 teeth of 8 diametral pitch, 20° pressure angle.

Clearly this means that where the pitch is $8P$, the pressure angle is 20°. Therefore the diameter in question is

$$30/8 = 3{\cdot}75 \text{ in., whence } r = 1{\cdot}875 \text{ in.}$$

Then

$$r_0 = 1{\cdot}875 \cos 20° = 1{\cdot}875 \times 0{\cdot}9397$$
$$= 1{\cdot}7619 \text{ in.}$$

Arc Tooth Thicknesses at Different Radii. (See Fig. 188.) Suppose the arc tooth thickness and pressure angle of an involute spur gear are known, we can calculate its tooth thickness at any other radii thus, Let r_1 = Given radius — r_2 = Radius at which tooth thickness is required

ψ_1 = Pressure angle at r_1 — ψ_2 = Pressure angle at r_2

T_1 = Arc thickness at r_1 — T_2 = Arc thickness at r_2

Then,
$$\text{Cos } \psi_2 = \frac{r_1 \cos \psi_1}{r_2}$$

$$T_2 = 2r_2\left[\frac{T_1}{2r_1} + \text{inv. } \psi_1 - \text{inv. } \psi_2\right]$$

Note: The terms inv. ψ_1 and inv. ψ_2 respectively mean "the involute function of the angle ψ_1" and "the involute function of the angle ψ_2." The reader will find an explanation of involute functions in the Addendum (page 242).

Internal Gear. Arc Tooth Thicknesses at Different Radii. (See Fig. 189.)

Let r_1 = Given radius — r_2 = Radius at which tooth thickness is required

ψ_1 = Pressure angle at r_1 — ψ_2 = Pressure angle at r_2

T_1 = Arc thickness at r_1 — T_2 = Arc thickness at r_2

Then,
$$\text{Cos } \psi_2 = \frac{r_1 \cos \psi_1}{r_2}$$

$$T_2 = 2r_2\left[\frac{T_1}{2r_1} - \text{inv. } \psi_1 + \text{inv. } \psi_2\right]$$

MEASURING INVOLUTE TOOTH SPACING WITH A MICROMETER OR VERNIER

Opposed Involutes and Maximum Chords. The opposed involutes in Fig. 190 (*a*) intersect a tangent to the base circle in points A and B, giving the maximum chord AB. It can be shown that the lengths of the maximum chords produced by any other tangents have the same length. Thus, $AB' = A_1B_1$. These chords also equal the arc CC_1.

Then,
$$AB = A_1B_1 = CC_1 = g_o = 2r_oy_o$$

Note that the angle y_o is measured in radians.

Fig. 190 (*b*) shows how the chord increases by p_o (the base pitch) for every additional tooth included between the outermost opposed involutes. If there are n additional pitches,

$$\text{Maximum chord } (z) = 2r_oy_o + np_o$$

For 1 diametral pitch or 1 in. module.

Maximum chord $(z) = (t \cos \psi)\, y_o + \pi n \cos \psi$, where t = no. of teeth in pinion.

In Fig. 190 (*b*) M stands for micrometer and V for vernier.

Alternative Well-known Formula. An alternative formula for calculating the length of the maximum chord is given by Earle Buckingham. It requires reference to a table of involute functions. (See Fig. 191.)

Suppose it is required to find the length of the maximum chord, i.e. the vernier setting, when given the arc tooth thickness and pressure angle of a spur gear at a given radius.

Let r = Given radius
ψ = Pressure angle at radius r
Z = Length of maximum chord (required measurement)
T = Arc tooth thickness at radius r
t = No. of teeth in gear
n = No. of tooth spaces between measured profiles

$$Z = r \cos \psi \left(\frac{T}{r} + \frac{2\pi n}{t} + 2 \text{ inv. } \psi \right)$$

EXAMPLE. A spur gear of 5 in. P.C.D. has an arc tooth thickness of 0·2618 in. What should be the vernier measurement across 4 teeth if there are 30 teeth and the pressure angle is 20°?

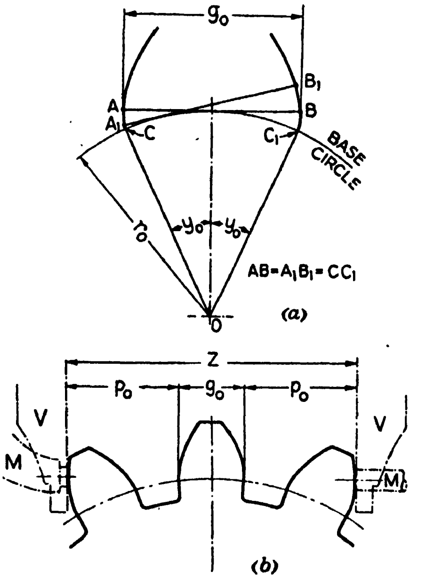

FIG. 190. DISTANCES ACROSS OPPOSED INVOLUTES

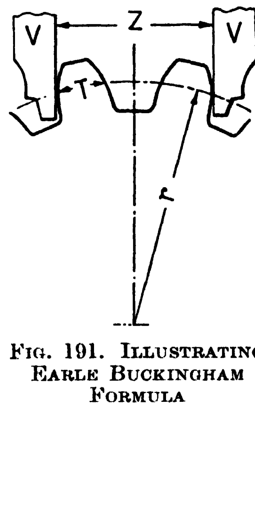

FIG. 191. ILLUSTRATING EARLE BUCKINGHAM FORMULA

If there are 4 teeth, the number of spaces between the profiles being examined is 3.

$$Z = 2{\cdot}500 \cos 20° \left\{ \frac{0{\cdot}2618}{2{\cdot}5} + \frac{2\pi 3}{30} + 2 \text{ inv. } 20° \right\}$$
$$= 2{\cdot}500 \times 0{\cdot}93969\ (0{\cdot}10472 + 0{\cdot}62832 + 0{\cdot}029808)$$
$$= 1{\cdot}7921 \text{ in.}$$

An Even Simpler Formula. In one of the most useful of the well-known *Mechanical World Monographs*,[1] viz. *Checking Spur Gear Teeth*, F. W. Shaw develops a simple formula and applies it to worked examples. The formula, using the same notation as in the preceding paragraph, is

$$Z_o = r_o\,(\alpha + 2 \text{ inv. } \psi + n\beta)$$

Note: If the maximum chord of one tooth is to be found, $n = 0$; if of two teeth, $n = 1$; if of three teeth, $n = 2$.

[1] Published by Emmott & Co., Ltd., King Street West, Manchester.

In Fig. 192, based on Fig. 2 in Shaw's booklet, note that z_o = (two arcs inv. ψ) + (two arcs β) + (arc α). If the base radius (r_o) be regarded as unity (1) it makes it convenient to simplify the formula to

$$z_o = \alpha + 2 \text{ inv. } \psi + n\beta.$$

Having obtained z_o, the vernier measurement for what Shaw calls the "basal" gear, i.e. the gear having a unit base radius, the vernier measurement for the actual gear can be found by multiplying the value so obtained by the base radius of the actual gear (r_o). Thus $Z_o = r_o \times z_o$.

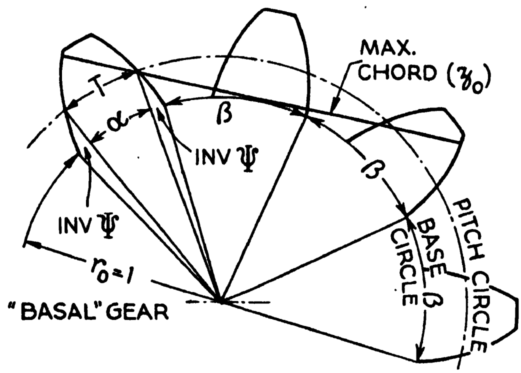

FIG. 192. ILLUSTRATING BASIS OF SIMPLE FORMULA

Tooth Thickness Angle (α) in radians is equal to the length of the corresponding arc T at the base circle of the basal gear. Thus $\alpha = T/R$, where T = arc tooth thickness and R = pitch radius.

The Base Pitch (β) of the "basal" gear can be found by the formulae:

$$\beta = p/R, \text{ or } \beta = 2\pi/t,$$

where p = circular pitch at radius R, t = number of teeth in gear, 2π = circumference of circle of unit radius.

The Number of Tooth-spaces (n). One method of finding n is to scribe its base circle on an actual gear and, placing the edge of a rule or straight-edge tangential to it, see whether it would be preferable to gauge across two, three, or more tooth spaces, so that the vernier points could "rock" over fairly long parts of the opposed tooth profiles. A table is published in Vol. 2, *Manual of Gear Design*, by Earle Buckingham, giving number of tooth-spaces (from 1 to 8) between teeth measured by vernier caliper, for five different pressure angles. For a pressure angle of 20°, the following numbers are taken—

No. of Tooth Spaces	From	To	No. of Tooth Spaces	From	To	No. of Tooth Spaces	From	To
1	12	18	4	37	45	7	64	72
2	19	27	5	46	54	8	73	81
3	28	36	6	55	63	—	—	

The Term 2 inv. ψ. The involute function is equal to the difference between the tangent of the angle and its circular measure. Thus 2 inv. ψ = 2 (tan ψ − arc ψ).

Suppose $\psi = 20°$. Reference to a table of involute functions gives inv. 20° = 0·0149044. In Shaw's booklet, *Checking Spur Gear Teeth*, will be found a table of involute functions (× 2) of angles from 10° to 30°. In this table the value of 2 inv. 20° is given as 0·02980876. For most purposes this could be taken as 0·0298. Similarly, the value of 2 inv. $14\frac{1}{2}°$ is given as 0·01108968.

EXAMPLE. Gear has pitch diameter of 5 in., 30 teeth, $P = 6$, $p = 0{\cdot}5236$ in., $\psi = 14\frac{1}{2}°$. Teeth are uncorrected. Assume measurement is taken across 3 teeth, so that $n = 2$.

First find values of quantities to be used when substituting in the formula for Z_o.

$$T = 0{\cdot}2618 \text{ in.}, \; 2 \text{ inv. } 14\tfrac{1}{2}° = 0{\cdot}01109$$

$$R_o = r \cos \psi = 2\tfrac{1}{2} \times 0{\cdot}96815 = 2{\cdot}4204$$

$$\alpha = T/R = 0{\cdot}2618 \div 2{\cdot}5 = 0{\cdot}1047$$

$$\beta = p/R = 0{\cdot}5236 \div 2{\cdot}5 = 0{\cdot}2094$$

$$= 2\pi/t = 6{\cdot}2832 \div 30 = 0{\cdot}2094$$

$$n\beta = 2 \times 0{\cdot}2094 = 0{\cdot}4188$$

Add

$$\alpha = 0{\cdot}1047$$

$$2 \text{ inv. } \psi = 0{\cdot}01109$$

$$n\beta = 0{\cdot}4188$$

Sum

$$= 0{\cdot}53459$$

$$Z_o = R_o \times 0{\cdot}5346 = 1{\cdot}2939 \text{ in.}$$

Thus the caliper measurements over three teeth, neglecting backlash, should be 1·2939 in. and this should allow the measuring points to rock over a reasonably long portion of opposed profiles, away from tips (which may be modified) and from lower parts of flanks (which may be undercut).

The reader may now find it interesting to check the result obtained from this formula, given by Shaw in his *Checking Spur Gear Teeth*, with the result obtained by substitution in the formula, given in a previous paragraph, taken from Earle Buckingham's *Manual of Gear Design*. Results are practically identical, provided that calculations are made in respect of the same pressure angle.

Making Allowance for Backlash. The B.S.I. define backlash as "the total free movement *at the pitch circle* of one gear in the direction of its circumference, when the other member of the pair is fixed, and the bearing clearances are eliminated." When using micrometers or verniers to take measurements between opposite-facing tooth profiles it is not usually thought essential to differentiate between backlash at the base circle and backlash at the pitch circle. The difference between them is usually very small indeed. If it is thought desirable to calculate base backlash the following formula could be employed: base backlash = circular backlash × cos. pressure angle at the pitch circle. Similarly, circular backlash = base backlash × sec ψ (both formulae applying to spur gears).

The chordal measurement Z having been calculated, the backlash allowance must be deducted from it.

Measuring Wormwheel Teeth. Wormwheels are generally manufactured to closer limits than are normally controlled effectively by the use of a tooth caliper. The following may, however, serve to check the work during the gear-cutting operation. (See Fig. 193.)

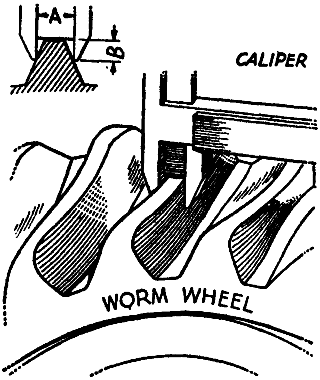

Fig. 193. Checking Worm-wheel with Tooth Caliper

$$A_c \text{ (chordal addendum)} = D\left(\sin \frac{90°}{T}\right)\cos \lambda$$

$$T_c \text{ (chordal thickness)} = \frac{G - D\left(\cos \frac{90°}{T}\right)}{2}$$

(where D = pitch diameter; λ = lead angle; G = throat diameter; T = no. of teeth).

Using a Flange Micrometer. The minimum addendum and chordal measurement that can be checked with standard-type tooth-verniers is about 0·04 in., and on the smaller range of gears it is impracticable to use the tooth-vernier. In practice extensive use is made of a Flange Micrometer for accurate chordal measurement. Figs. 194 and 195 show such a measurement being made by means of flange micrometers which may, of course, be used advantageously on the larger range of

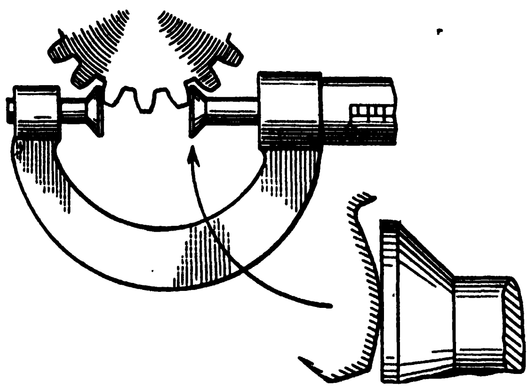

Fig. 194. Using a Flange Micrometer

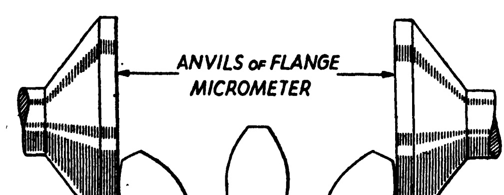

Fig. 195. The Anvils in Contact with Teeth

gears within the scope of the tooth-vernier but to a greater degree of accuracy. The rigidity and the ease with which a micrometer can be handled, together with the advantage of its more prominent graduations, enable a measurement to be taken to the fourth decimal part of an inch, i.e. 0·0001 in., compared with the 0·001 in. on tooth-vernier readings.

A flange micrometer is more readily applicable to this kind of measurement than the conventional micrometer. When using the latter, the operator frequently finds that the frame rides on adjacent teeth. On the other hand, the use of the flange micrometer enables anvil contact nearer the roots of the gear tooth.

Where the size of work permits, use may be made of slip blocks and anvils for accurately measuring chordal dimensions, as shown in Fig. 196. The flange micrometer and slip gauge method is readily used for measuring the thickness of every individual tooth where undercutting is present.

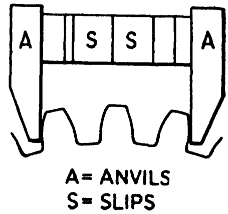

FIG. 196. USING SLIPS AND ANVILS

Further Rack-measurement Methods. The tooth-vernier may be used to check variation in thickness and pitch of rack teeth by taking measurements successively over one tooth, then two adjacent teeth, three teeth, and so on, and noting if increments of measurements obtained are constant. Variations in increments indicate errors of thickness and spacing. The method is shown in Fig. 197 (*a*) and (*b*).

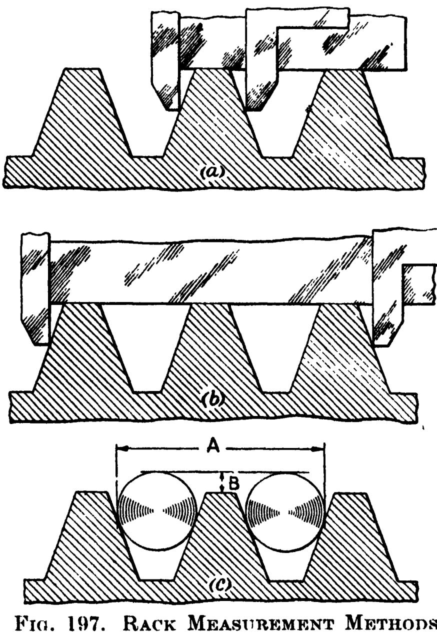

FIG. 197. RACK MEASUREMENT METHODS

On large-size work, cylinders may be used, being placed in adjacent tooth-spaces as shown in Fig. 197 (*c*), measurement *A* over projecting ends of cylinders, and measurement *B*, from top of cylinders to tip of rack tooth, being noted. The cylinders are then removed and placed in next adjacent tooth-spaces, further measurements *A* and *B* being taken. The process is repeated all along the rack and any varia-

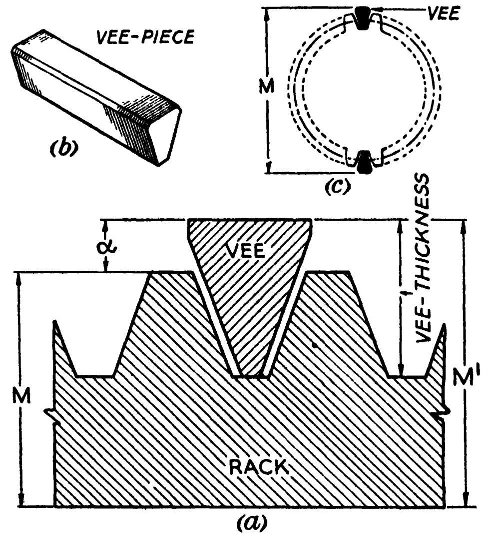

Fig. 198. Using Vee-pieces

tions in *A* and *B* measurements indicate variation in thickness and spacing of teeth.

Vee-pieces, or "prisms," made of hardened, ground, and lapped tool steel are used (as shown in Fig. 198) for checking depth of tooth.

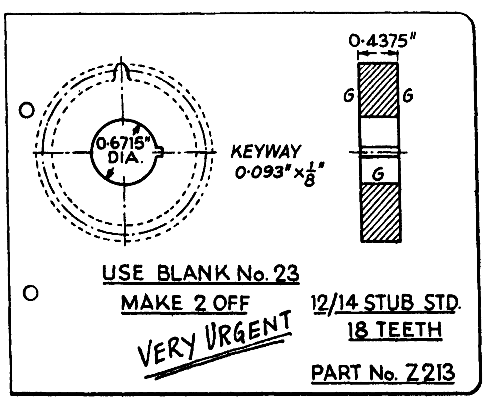

Fig. 199. Gear to be Examined

The "vees" are made to a definite size such as 0·1 in., 0·5 in., etc (depending on range of work sizes in use) to simplify calculation.

On long-running jobs the vees are made equal in thickness to tooth dept required, measurement being taken over vees and rack with a micrometer o with a comparator to check difference between dimension obtained on tips of teeth and dimension obtained over top of "vee" resting in tooth-spaces. The vees are also used for measuring root diameters of gear wheels. The difference between micrometer measurements M^1 and M gives the dimension α. Then, if $(M^1 - M)$ be subtracted from the vee-thickness, we have the depth of the rack tooth. In Fig. 198 (c) the measurement M minus twice thickness of vees equals the root diameter of the gear.

TYPICAL WORKSHOP EXAMPLE SIMPLY EXPLAINED. A rough drawing, Fig. 199, is received with the request for two gears. As shown, the only information given is: "12/14 D.P. STUB," but by reference to appropriate tables, as given at the end of the book, further dimensions necessary for measuring the gear are readily obtained. The numerator "12" indicates the ratio of the number of teeth relative to the pitch diameter, the latter being denoted by D in accordance with British Standard Notation, and being found by dividing the number of teeth (T) by the numerator, thus—

$$\text{Pitch diameter } (D) = \frac{T}{12} = \frac{18}{12} = 1{\cdot}5 \text{ in.}$$

The circular pitch, p, is obtained by dividing the circumference of the pitch circle by the number of teeth.

$$\therefore \text{ Circular pitch } (p) = \frac{\pi D}{T} = \frac{3{\cdot}1416 \times 1{\cdot}5}{18} = 0{\cdot}2618 \text{ in.}$$

The tooth thickness along the pitch circle equals one-half the circular pitch (backlash being ignored).

$$\therefore \text{ Tooth thickness} = \frac{p}{2} = \frac{0{\cdot}2618}{2} = 0{\cdot}1309 \text{ in.}$$

An alternative way of finding "tooth thickness" is as follows—

$$\text{Tooth thickness} = \frac{\pi D}{2T} = \frac{3{\cdot}1416 \times 1{\cdot}5}{2 \times 18} = 0{\cdot}1309 \text{ in.}$$

To find A, the wheel tooth addendum, we find the reciprocal of the denominator.

$$\therefore \text{ Addendum } (A) = \frac{1}{14} = 0{\cdot}0714 \text{ in.}$$

The dedendum B equals the addendum A plus clearance c, and for Stub[1] teeth the clearance c is found by dividing a constant, 0·25, by the denominator.

$$\therefore \text{ Clearance } (c) = \frac{0{\cdot}25}{14} = 0{\cdot}0178 \text{ in.}$$

The dedendum therefore equals addendum plus clearance.

$$\therefore \text{ Dedendum } (B) = A + c = 0{\cdot}0714 + 0{\cdot}0178 = 0{\cdot}0892 \text{ in.}$$

Of course, the depth of tooth is the sum of addendum and dedendum.

$$\therefore \text{ Depth of tooth} = A + B = 0{\cdot}0714 + 0{\cdot}0892 = 0{\cdot}1606 \text{ in.}$$

The "tip" diameter J is the pitch diameter plus twice the addendum.

$$\therefore \text{ Wheel tip diameter } (J) = D + 2A = 1{\cdot}5 + 2 \times 0{\cdot}0714 = 1{\cdot}6428 \text{ in.}$$

The "root" diameter I is the pitch diameter minus twice the dedendum.

$$\therefore \text{ Wheel root diameter } (I) = D - 2B = 1{\cdot}5 - 2 \times 0{\cdot}0892 = 1{\cdot}3216 \text{ in.}$$

[1] Unless otherwise stated, "Stub" refers to Fellows Stub.

Summary. From the foregoing calculations we have obtained the following—

Gear tooth addendum $(A) = 0{\cdot}0714$ in.
Gear tooth dedendum $(B) = 0{\cdot}0892$ in.
Gear pitch diameter $(D) = 1{\cdot}5000$ in.
Gear tip diameter $(J) = 1{\cdot}6428$ in.
Gear root diameter $(I) = 1{\cdot}3216$ in.
Circular pitch $(p) = 0{\cdot}2618$ in.

MEASURING THE GEAR

Cylinder Diameter. Using "standard equipment" only, we proceed to the measurement of this gear, giving attention firstly to the pitch diameter (D).

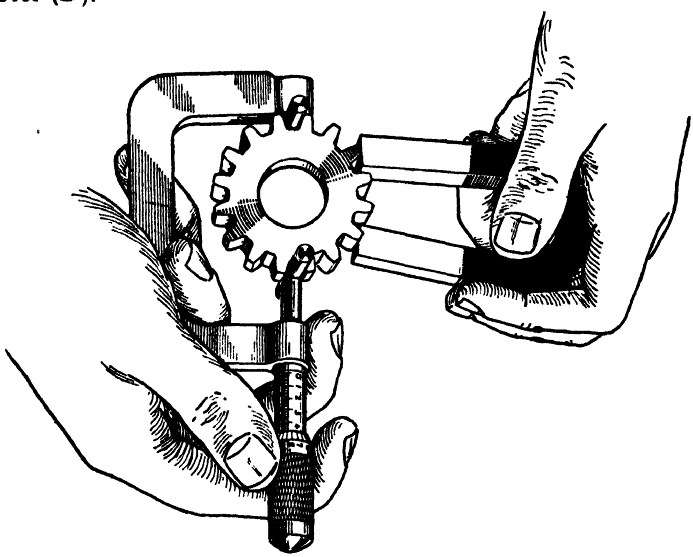

FIG. 200. USE OF MAGNET TO FACILITATE MEASUREMENT OVER CYLINDERS

We use the cylinder method, for this is easily applied and theoretically perfect. It is suggested that the simple formula be committed to memory. The diameter of cylinders is found by multiplying the cosine of the pressure angle ψ by half the circular pitch p.

$$\text{Diameter of cylinder} = \frac{p}{2}\cos\psi_n = \frac{0{\cdot}2618}{2} \times \cos 20^\circ$$

$$= \frac{0{\cdot}2618}{2} \times 0{\cdot}93969 = 0{\cdot}123 \text{ in.}$$

All dimensions required are worked out in this example, but the reader is reminded that such information is readily found in the tables supplied at the end of the book.

The dimension over the cylinders, when these are placed in opposite tooth-spaces, the pitch diameter being correct to size, is obtained as follows—

Measurement over cylinders $(X) = D + R$, where R is the cylinder diameter.

$$\therefore X = D + R = 1{\cdot}5 + 0{\cdot}123 = 1{\cdot}6230 \text{ in.}$$

The cylinder diameter is not "standard" and, therefore, may not readily be obtained. This often proves to be the case, and to overcome this difficulty the tables of gear measurements, given at the end of the book, referring to cylinders of standard size, have been specially compiled. The diameters may be those of standard steel rods, twist-drills, dowels, etc.

To facilitate measurement over the cylinders, the latter may be held in place with the aid of petroleum-jelly, plasticine, or by a rubber band placed around, and over, the gear and cylinders. A very convenient method is shown in Fig. 200, and entails the use of a simple type of magnet placed in contact with the gear wheel, magnetism holding the cylinders in position. The arrangement allows plenty of room for manipulating the micrometer.

Tooth Thickness. Using the tooth-vernier as shown in Fig. 185, we check the thickness of tooth at the pitch circle. Studying Fig. 184, we see that allowance has to be made for the differences between

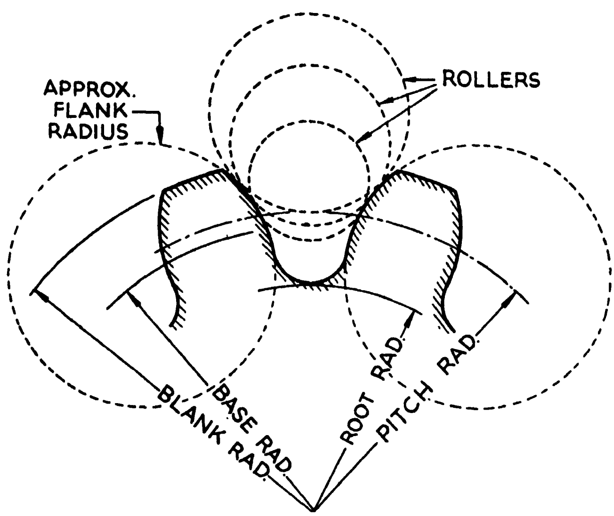

FIG. 201. CHECKING PRESSURE ANGLE

chordal addendum and chordal thickness, and the normal or standard addenda and arc thickness.

Referring to Fig. 184, we see that the "height of arc" d has to be added to the tooth normal addendum, and gives us the dimension A_c at which we set the vertical column of the tooth-vernier.

Tabulated values of d and chordal addendum are given on pages 204 and 249, but we continue by calculating them as follows—

$$\text{Chordal Addendum } (A_c) = A + d, \text{ where } d = R\left(1 - \cos\frac{a}{2}\right)$$
$$= 0{\cdot}0743 \text{ in.}$$

$$\text{Chordal Thickness } (T_c) = 2R \times \sin\frac{a}{2} = 0{\cdot}1308 \text{ in.}$$

The chordal thickness is measured by adjusting the horizontal column of the tooth-vernier to make good contact between the measuring anvils and the gear tooth at the correct depth.

Pressure Angle. Check of tooth profiles by optical projection is described in Chapter XVI. Any errors of pressure angle will be disclosed by checking the pitch diameter with different-sized rollers as shown in Fig. 201.

CHAPTER XVI

OPTICAL PROJECTION

OPTICAL projection or "shadowgraph" apparatus provides a rapid yet precise method of gear examination. The following notes comprise a brief outline of the process, typical workshop examples being provided.

Some of the uses of a projector may be summarized as follows—

The checking of formed components, profiles, and templates by projection against an enlarged master lay-out.

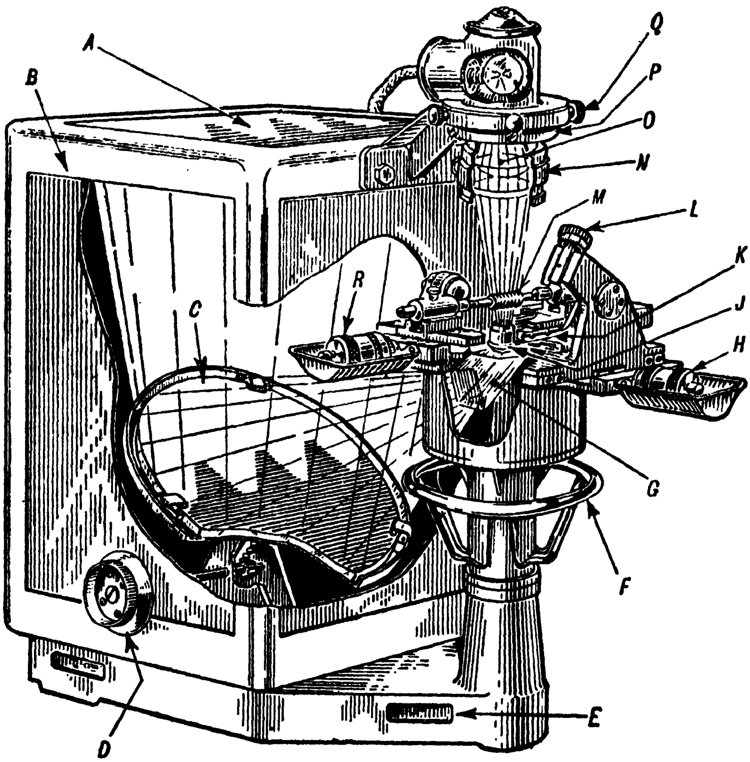

FIG. 202. VICKERS CONTOUR PROJECTOR

The inspection of small components which can only be checked with difficulty by any other method of gauging.

For checking screwed components, e.g. taps, male thread gauges, etc., and particularly when installed adjacent to thread- and form-grinding machines.

For checking elaborate form tools and form gauges to the finest limits of accuracy.

For checking gear hobs, worm hobs, and thread-milling hobs.

For quality control of mass-produced components (the instrument being installed near the machines).

For inspection of work which cannot be dealt with by direct projection, such as vernier scales and surface markings, small instrument assemblies, and similar work.

Fig. 202 shows a **Vickers Contour Projector** (courtesy of Messrs. Cooke, Troughton & Simms).

This projector is of very compact design, its principal uses being—

1. (a) The *measurement* of magnified silhouettes of templates and other plain objects, or (b) their *comparison* with translucent masters drawn to an enlarged scale.

2. (a) The *measurement* of screw threads, or (b) their *comparison* under magnification with master screw thread templates.

The projector is so designed that the object and image planes are close together. All controls may be manipulated whilst the image is under observation. Since projection is from beneath the screen, the observer does not tend to produce shadows across the projected image when taking measurements with a rule, protractor, or the like.

The illumination is sufficient to allow the use of the instrument in a well-lighted room, whilst, when necessary, a hood may be used to cut off stray light from the screen. The projector can produce an image whose size bears a fixed relationship to that of the object, i.e. exactly 10, 25, or 50 times.

The *dimensions of the object* may be determined, in this "inspection without gauges" method, in two ways—

1. By measuring the projected image and dividing the result by the appropriate magnification factor; or
2. By translating the projected image in relation to a fiducial line on the projection screen, and by noting on the micrometer screws of the measuring stage the value of the movements necessary to bring this about.

In the first method, measurement of the image may be made conveniently with a glass scale, or from a template made to the appropriate scale. Such templates may be produced photographically from a master component, drawn on tracing paper or formed out of sheet metal.

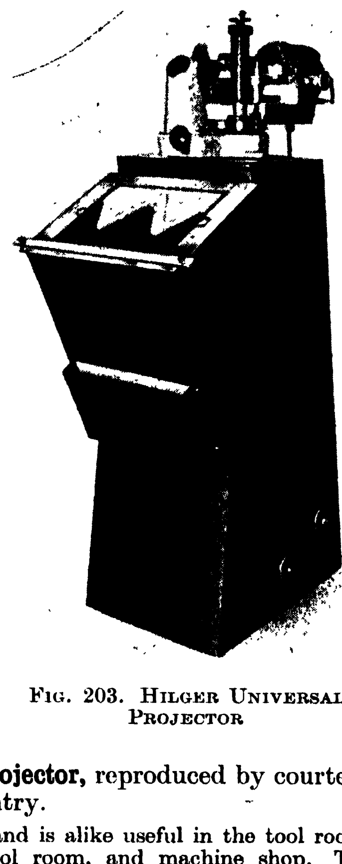

FIG. 203. HILGER UNIVERSAL PROJECTOR

Fig. 203 shows a **Hilger Universal Projector,** reproduced by courtesy of Messrs. Alfred Herbert, Ltd., Coventry.

This is one of various Hilger projectors and is alike useful in the tool room, detail inspection department, gauge control room, and machine shop. The model is compact and portable, and a point worth noting is that the image can be observed in daylight without screening. For work which necessitates measurement of dimensions to an accuracy of 0·0001 in., in addition to ordinary projection, this Universal Projector is very suitable. Co-ordinate measurements to 0·0001 in. and angular measurements to one minute can be made.

A simpler projector for use on production checking of formed components against enlarged lay-outs; for checking thread forms, gear tooth forms, etc., is called the **Hilger Inspection Enlarger.** It can be plugged in directly to the shop lighting circuit, and can be operated in daylight by unskilled labour.

Projection Apparatus is usually classified under the headings "Screen-type projectors" or "Toolmakers' Microscopes." The screen-type projector has the advantage of a large working field, some makes allowing for silhouettes up to about five feet in size to be examined as a whole. An important advantage of the toolmakers' microscope is that it enables an operator to view, through the eyepiece, the actual job and *not* its shadow.

Fig. 204 shows an **O.M.T. Toolmaker's Microscope,** the block having been provided by Optical Measuring Tools, Ltd., Slough. The work is held above the table, whilst a beam of light casts a shadow of the contour of the workpiece on a line templet within the microscope eyepiece. The worktable may be rotated, or moved transversely or longitudinally, for making angular or linear measurements. These microscopes are largely used for checking thread forms, hence oculars are supplied for thread form and general contour inspection. The first embodies a revolving templet carrying several ranges of thread forms and cross-lines of the full thread angles. This main templet lies below a fixed one, which carries an angular scale for measuring the error in each half angle of thread form inspected. The second ocular carries a simple cross-line surrounded by a full circular scale. This circular scale is observed by a separate integral microscope for reading direct to one minute of arc.

FIG. 204. O.M.T. TOOLMAKER'S MICROSCOPE

A projection screen may be fitted over the eyepiece of either ocular, and transparent drawings may be mounted on the screen to serve as line templets. A special scale is provided for preparing such drawings and making measurements on the screen.

Checking Small Gears. In the mechanical checking and measuring of the various gear elements there are many difficulties encountered when the work parts are rather small. If we consider, for instance, gear components of watch and clock assemblies we realize the difficulty of applying the general means of gear measurement, such as the roller, tooth-vernier, and similar mechanical methods. The greatest difficulty arises from the very small size of the gear-work which may be outside the capacity of the standard measuring equipment. By recourse to optical projection the size difficulty is eliminated by projecting the outline of the gear profile which is magnified, ten, twenty-five, or fifty times its full size and comparing the enlarged image of the profile of the job with a scale drawing of the profile.

Example. Small Pinion. As an example, let us assume it necessary to check the pinion shown in Fig. 205. We note that the total depth of tooth is only 0·0431 in. This factor alone rules out the use of ordinary measuring tools. The drawing states that the *form of tooth*

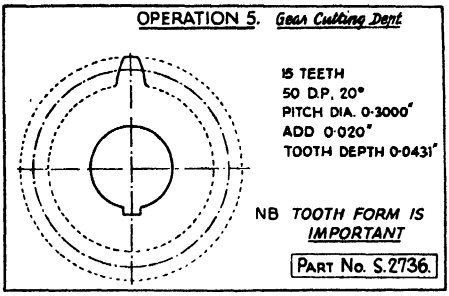

FIG. 205. SMALL PINION TO BE CHECKED

is important; thus attention must be focused on this individual element.

The gear particulars are given as: addendum = 0·020 in., whole depth = 0·0431 in., pressure angle 20°, etc. We choose a suitable magnification factor, in this case 50×. The given sizes are multiplied throughout by fifty, which now gives us an addendum of 1 in., etc.

Using the enlarged dimensions, we carefully draw the form of tooth on a sheet of translucent paper. The pinion is then scrupulously cleaned in solvent spirit, dried and dusted with a camel-hair brush, and placed on the "work-holding stage" of the shadowgraph. The result is shown in Fig. 206, where the

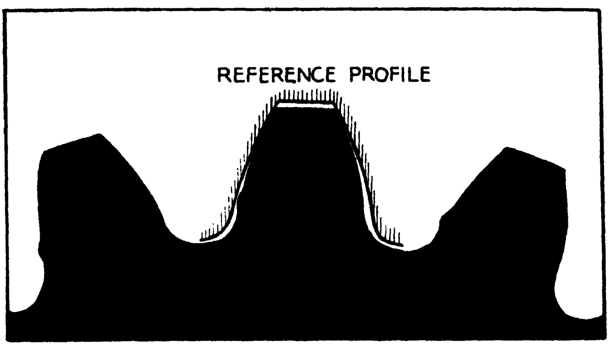

FIG. 206. CHECKING TOOTH PROFILE

projected silhouette is compared with the reference or "master" contour, already drawn and attached with adhesive tape to the projector screen. By comparing this enlarged profile with the silhouette of the actual gear tooth profile, we are able to determine the presence of errors in the tooth form, which are rather prominent in this instance.

It is a fact that if we rule out expensive "laboratory" methods, many small "jobs" of this nature can only be checked speedily and economically by "optical projection." Of course, the process is equally well applicable to a widely varying range of gear sizes and profiles.

Set-up for Checking Meshing. Another use for "projection" is in the visual examination of the meshing action of gears, e.g. of a rack-and-pinion, gear-and-pinion, etc.

A typical example is shown in Fig. 207. This illustrates an important check on gear wheels required to transmit heavy loads. In practice,

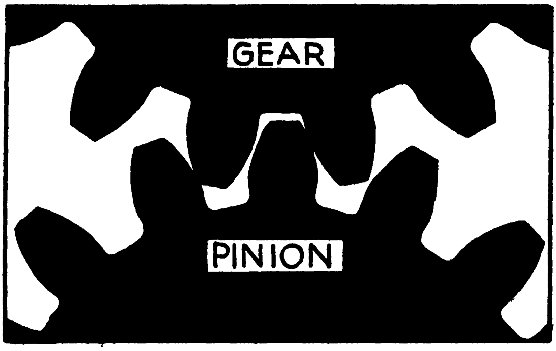

FIG. 207. CHECKING MESHING OF GEAR AND PINION

an erratic meshing zone of contact is often the cause of intermittent hammering of the gear teeth, resulting in excessive heat, noisy running, and ensuing rapid wear.

In the set-up of the gears being examined for "meshing," the gears are mounted on suitable spindles to allow the wheels to be rotated by hand. Examination of the "mesh" of each mating tooth and tooth-space is then carried out by observing the shadow of each tooth action in turn.

The number of teeth to be examined depends upon the number of teeth in each gear. A point often overlooked, however, is that it is usually necessary to examine the meshing which results from more than one complete revolution of the gears. One complete revolution of both gears is correct only if the number of teeth in one gear wheel is exactly the same as the number of teeth in the mating gear. If the number of teeth in one gear is odd and greater than the number of teeth in the mating gear, then one tooth will engage a different space until it has meshed with every tooth-space of the mating gear. Thus it may be necessary, dependent on the relative numbers of teeth in the two gears, to rotate each gear several times until every tooth in one gear has meshed with every space on its mating gear.

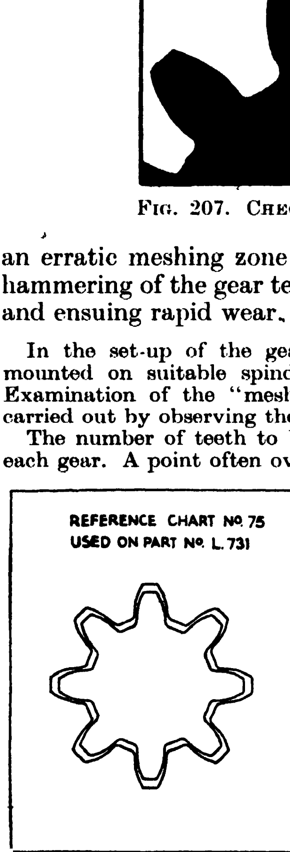

FIG. 208. TYPICAL REFERENCE CHART

Controlling Quality of Gear Production. On the repetition checking of small gears and pinions, it is customary to use **Tolerance Charts,** a typical chart being shown in Fig. 208, in which two complete gear profiles are drawn about one common axis. These profiles are, however, of different size; one (the smaller) represents the minimum allowable size of gear, whilst the larger represents the maximum gear size. In use, the silhouette of the work being checked must fall within the prescribed maximum and minimum outlines, or "tolerance lines," on the tolerance chart.

On high-class work, this method would be used for checking the "tip" and "root" diameters, the tooth form being checked as a separate item; but in a similar manner, i.e. maximum and minimum tooth thickness profiles would be drawn as a tolerance chart.

Checking the Pressure Angle. As already explained earlier in this book, the pressure angle plays an important part in the strength of the gear tooth, its load capacity, quiet running, and similar characteristics which combine to form the hall-mark of a satisfactory gear tooth. The use of profile charts enables accurate control in checking the pressure angle.

SELECTED EXAMPLES

To enable the reader more easily to understand the methods of measuring gears by means of projection apparatus we have selected

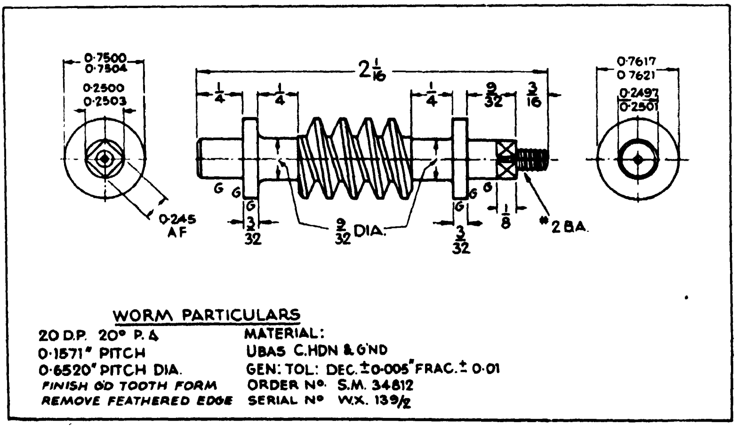

FIG. 209. SINGLE-START WORM

typical examples from the workshop and these, illustrated by simple diagrams, will, it is hoped, explain in a clear manner the complete process of gear measurement and examination by the "shadowgraph" method.

Example. **Single-start Worm.** For our first example, let us assume that we require to check the "worm" shown in Fig. 209. This book is devoted entirely to gear work[1] and it is therefore suggested that the reader examine only those dimensions given in Fig. 209 which appertain to tooth form and diameters, neglecting all other dimensions.

This worm example refers to an involute worm straight-sided in axial section, i.e. having a basic form on an axial section, and not normal to the axis, as in the case of involute helicoid worms. In the main the methods discussed apply to the optical measurement of any type of worm.

Fig. 209 gives the following information relative to the worm: 20 D.P., 20° pressure angle, 0·1571 in. pitch, 0·6520 in. pitch diameter. The use of diametral pitch on worms is *not* generally recommended.

To utilize and practise the formulae given in Chapter XI, the data given on

[1] Readers who desire a general knowledge of inspection are advised to refer to *Engineering Inspection* by A. C. Parkinson (Pitman).

the drawing have purposely been curtailed, so that the reader has to calculate for himself any other data he may require.

The worm is cleaned very carefully and mounted on the work-stage of the "projector." In this case the work is "centred" at each end, and so we hold the worm between centres. The following notes and diagrams are equally well applicable to checking by means of any standard type of projector.

Unless we tilt the worm, it is obvious that interference caused by curvature of the worm thread will produce a silhouette of a form greatly different from the

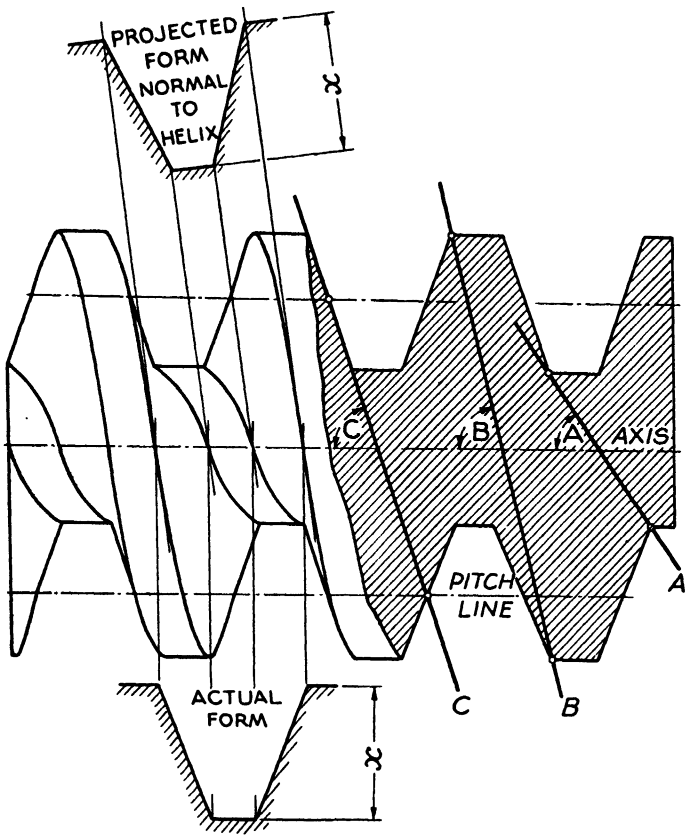

FIG. 210. THE DIFFERENT SPIRAL ANGLES OF A WORM

worm tooth. Thus we set the lead angle indicator of the projector to correspond with the lead angle of the worm.

Referring to Fig. 210, and also to Fig. 123, we see that there is a different spiral angle for each diameter. It is at its minimum at the core diameter, and at a maximum at the "tip" or outside diameter, indicated in the illustration by *A* and *B* respectively. We have to compromise between these two helices and, without exception on lead angles not exceeding about 12°, it is the practice to set the worm to the spiral angle at the "Pitch Circle Diameter," which we calculate as follows:—

$$\text{Tan of lead angle} = \frac{\text{Lead}}{\pi \times \text{Pitch diameter}}$$

The worm in question is "single start." Thus the pitch equals the lead. The pitch is given as 0·1571 in. and the pitch diameter equals 0·6520 in.

$$\therefore \quad \text{Tan lead angle} = \frac{0{\cdot}1571}{3{\cdot}1416 \times 0{\cdot}6520}$$
$$= 0{\cdot}0766$$
$$\therefore \quad \text{Lead angle} = 4^\circ\ 23'.$$

The lead angle indicator is now adjusted to tilt the worm to 4° 23'; the

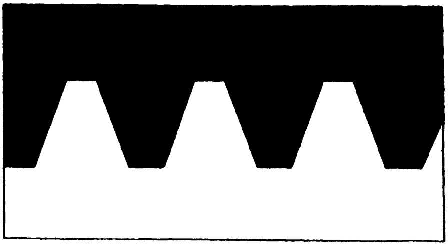

FIG. 211. SILHOUETTE OF TOOTH PROFILE

silhouette is brought into focus and the outline of the worm appears on the screen as shown in Fig. 211.

Many of the less modern "projectors" are not fitted with indicative lead arrangements, so that the setting has to be decided by trial-and-error methods. In such cases the following process is adopted: If we bring the image or "shadow" of the worm into focus, without tilting at any lead angle, the tooth form will appear as at (*a*) in Fig. 212. We note that while one flank appears sharp in outline, the other flank seems distorted and out of focus, due to interference and reflected light-rays.

If we gradually tilt the worm, the interference disappears and the reflected light rays become of equal intensity on each side of the tooth form, thus signifying that the worm is tilted to the mean-lead angle. Its image is then viewed on the screen as shown at (*b*) in Fig. 212. Bringing the "shadow" into proper focus results in a clear outline, as illustrated in (c) of Fig. 212.

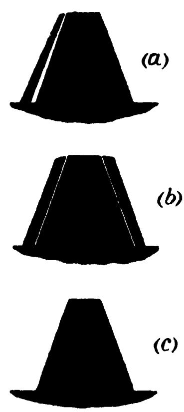

FIG. 212. STAGES IN SETTING CORRECT LEAD ANGLE

Diameter and Tooth-depth Measurements. Fig. 213 explains how we may measure the outside diameter, the root diameter, and the total depth of tooth.

The "hair-line," or reference line, is brought into optical contact with the top of the tooth by actuating the work-stage micrometer and noting the indicated measurement. At this operation in the selected example the micrometer reading was 0·9132 in., and the "shadow" appeared as *A* in Fig. 213. Revolving the micrometer drum until the reference line was in coincidence with root of tooth, as at *B*, the micrometer reading was 0·8047 in. Thus we have measured the total depth of tooth, which is clearly the difference between the two readings so obtained, i.e. 0·9132 minus 0·8047. The tooth depth in this example is evidently 0·1085 in.

At *C*, in Fig. 213, we show the next operation, the reference line coinciding with the opposite outline of the root diameter. The micrometer reading was 0·2695 in. The root diameter therefore is the micrometer value of the difference between *B* and *C*, that is, 0·8047 minus 0·2695. Root diameter is thus 0·5352 in.

To complete the measurement of outside diameter, we bring the reference line to the tips of the teeth, as shown at *D*, the micrometer reading being 0·1610 in.

Summary.

A–D equals outside diameter
B–C equals root diameter
A–B and C–D equals tooth depth
∴ Outside diameter = 0·7522 in.
Root diameter = 0·5352 in.
Tooth depth = 0·1085 in.

Pitch Measuring. See Fig. 214. To measure the pitch of the worm, we incline the reference line to coincide with the flank angle and note the reading of the longitudinal micrometer. In this instance A was 0·4731 in. As shown in Fig. 214 (B) we then move the work stage

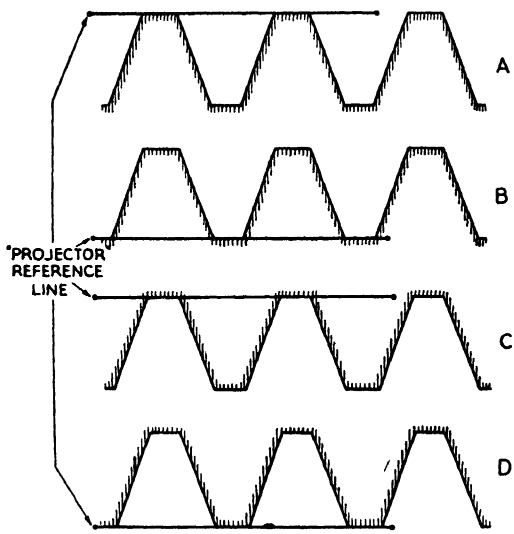

FIG. 213. DIAMETER MEASURING BY OPTICAL PROJECTION

until the next tooth flank matches up with the reference line, the micrometer reading being 0·3160 in.

The measured pitch of the worm is therefore 0·4731 − 0·3160 = 0·1571 in. Thus the pitch of the worm is correct. (See the dimension on Fig. 209.)

Measuring Pitch Diameter. The pitch diameter may be checked in a manner similar to the roller method of gear measurement described in Chapter XIV. A circle representing the roller is drawn to an enlarged scale, corresponding to the ratio of "shadow" magnification, and brought into optical contact with the projected tooth-space flanks. The micrometer reading is noted and the operation repeated by bringing the diametrically opposite tooth flanks into contact with the circular outline representing the roller. The difference in micrometer readings may be arranged to represent either the distance between the centres of the rollers, or the distance over the rollers as may be preferred. The calculations will, of course, be the same as described in Chapter XIV in connection with gear measurement by rollers.

In the tables on page 251 will be found a tabulated list of roller diameters suitable for all standard tooth proportions. The reader should consult this table before drawing the enlargement. When bringing the drawn circle into contact with the opposite tooth-space

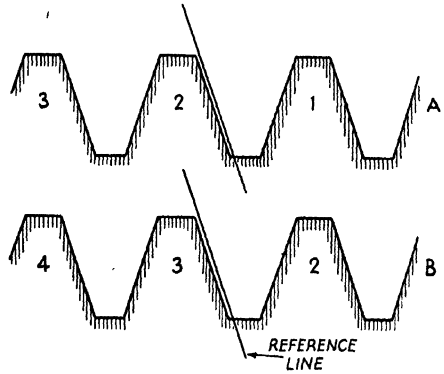

FIG. 214. PITCH MEASUREMENT BY "SHADOWGRAPH" METHOD

it is necessary to reverse the helix setting from right- to left-hand, or vice versa as the case may be.

We next explain a more direct method of measuring pitch diameter; a method which has the advantage of freedom from "roller calculations." The desired or "given" tooth thickness is spaced-off (to enlarged scale) and set parallel to worm axis on the projector screen. The tooth shadow is adjusted to bring the tooth-space dimension to bridge, without overlapping either way, the shadow of projected tooth. The micrometer reading is noted and the work holding stage traversed by actuating the micrometers to bring the opposite tooth shadow to line up with the marked-off tooth thickness on the screen. The difference in micrometer readings yields the magnitude of the pitch diameter directly.

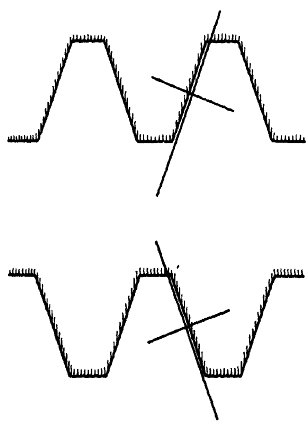

FIG. 215. MEASURING FLANK ANGLES

A further method is to draw the complete tooth profile and match it with the silhouette. By repeating the process previously discussed, i.e. taking micrometer readings of each line-up of opposite teeth, we can determine the pitch diameter by noting the difference between recorded micrometer readings.

Measuring Flank Angles. Fig. 215 illustrates the checking of flank angles by using the reference line which is incorporated in the vernier protractor-plate. The use of this accessory enables angular measurements to be taken within a minute of arc, values being read on the protractor scale.

For examining flank angle errors on the ordinary class of work, the use of carefully-drawn profile charts will be found sufficiently accurate, although care must be taken in the actual scale drawing of profiles.

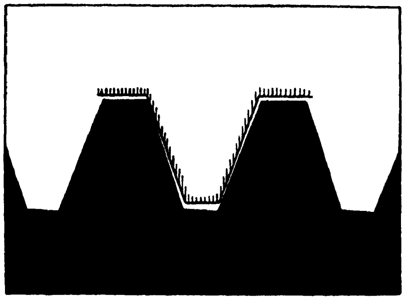

FIG. 216. "TILT" OF WORM TEETH

Fig. 210 shows the "true" and "projected" tooth-form. The angular difference is calculated as follows—

Let
A = Axial form angle
B = Projected form angle
C = Linear pitch
D = Normal pitch (projected)
E = Angular depth to sharp vee
F = Lead angle

In the previous calculations we found that F in this example equals 4° 23′.

$$E = \frac{\frac{1}{2}C}{\text{Tan } \frac{1}{2}A} = \frac{0{\cdot}07855}{\text{Tan } 20^\circ} = \frac{0{\cdot}07855}{0{\cdot}36397}$$

$$\therefore\ E = 0{\cdot}2158 \text{ in.}$$

$$\text{Tan } \tfrac{1}{2}B = \tfrac{1}{2}\frac{D}{E}; \text{ and } D = C \times \text{cosine } F.$$

Then,

$$\text{Tan } \tfrac{1}{2}B = \text{Tan } \tfrac{1}{2}A \cos F$$
$$= \text{Tan } 20^\circ \cos 4^\circ\ 23' = 0{\cdot}36397 \times 0{\cdot}997075$$
$$= 0{\cdot}362909$$
$$\therefore\ \tfrac{1}{2}B = 19^\circ\ 57'.$$

If the flank angles of the selected example are correct, then the included angle measured on the screen will be 39° 54′, compared with the true axial angle of 40°.

In this example the angular difference totals six minutes of arc. However, if we had a large lead angle, as, for example, on a multi-start worm, then the angular difference might amount to a degree or more; hence its importance.

Tilted Tooth Form. Fig. 216 shows an error frequently found when measuring worm gears. It is seen that the tooth form is "tilted" about its axis and, although the included form-angle is correct, one flank angle is greater than the other. Such an error is not readily disclosed by mechanical methods of inspection but optical measuring indicates at a glance any "tilt" or "lean" of the tooth. In practice this error causes noise in the gear-assembly and distributes more load to one flank than the other. On "cut" worms the cause is generally traceable to the form-tool having been set out of square with the work axis.

Tooth Thickness Measurement. It is often found that the information supplied on gear drawings, blue-prints, etc., includes dimensions of the width of flat at the tip of the tooth, or of the flat at the root of the tooth-space. The provision of this information is essential to enable the determination of such essential cutter dimensions as tool-tip on "cut" worms or the apex width of wheel for "ground form" worms. The following remarks are equally applicable to measuring the cutter-form or the actual product, i.e. the worm itself.

Using hair-lines inclined at the desired flank angle and intersecting a datum or "centre" line (Fig. 217) the micrometer reading is noted when the left-hand flank coincides with the reference line as illustrated in *A* of Fig. 217. After moving the work-stage to bring the right-hand flank to "match-up," as in view *B*, the next reading of the micrometer can be compared with the micrometer reading *A*. The difference between the two readings will give the "width-of-flat."

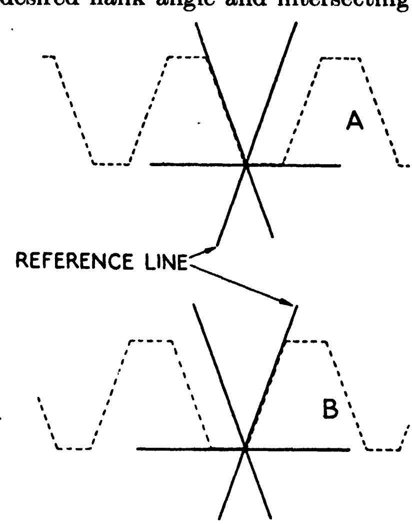

Fig. 217. Measuring "Width of Flat" of Worm Tooth or Groove

The reader will appreciate that this simple method enables the tooth thickness at any position of the tooth to be measured, e.g. the tooth thickness at any desired "addendum." The foregoing is a very simple operation. In practice the writers have found the master-plate (Fig. 218) very useful, inasmuch as it is not so readily liable to damage in use as is the usual paper master-plate. The plate referred to was made out of plate glass, the datum line and flank angles being marked out, while the plate was mounted on a sine-plate, by means of a height-gauge incorporating a diamond-tipped scriber (or cemented carbide tip).

Useful Hints on Measured Dimensions: Worms. (1) When measuring worm gears and hobs by optical projection always measure the gap and not the tooth.

(2) When supplying information about gear hobs and worm hobs, the following rules must be observed—

For Worm Hobs: supply the dimensions of the LINEAR or AXIAL FORM and PITCH (LINEAR).

For Gear Hobs: supply the dimensions of the NORMAL FORM and PITCH (NORMAL).

As shown in Fig. 210; the form projected on the shadowgraph screen is the normal form, therefore the form and pitch being viewed are respectively the NORMAL FORM and the NORMAL PITCH. Consequently, the flank angle being measured is the NORMAL ANGLE.

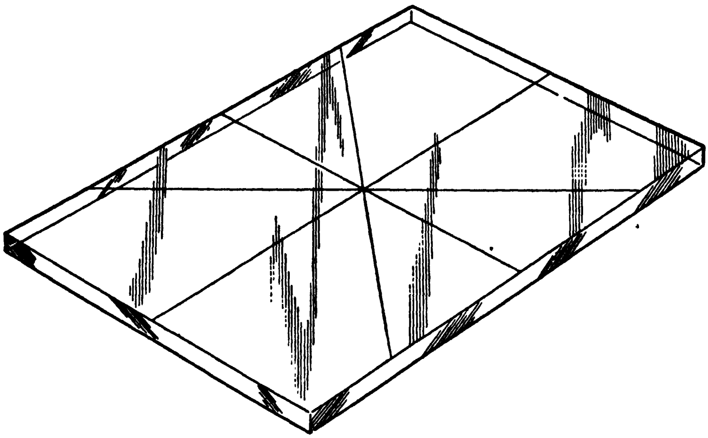

FIG. 218. A USEFUL PLATE

It is not the angle formed on an axial section, i.e. a section through the worm axis, but on a section normal to the pitch diameter helix. Errors may easily result if attention is not paid to this important, if not immediately obvious, difference.

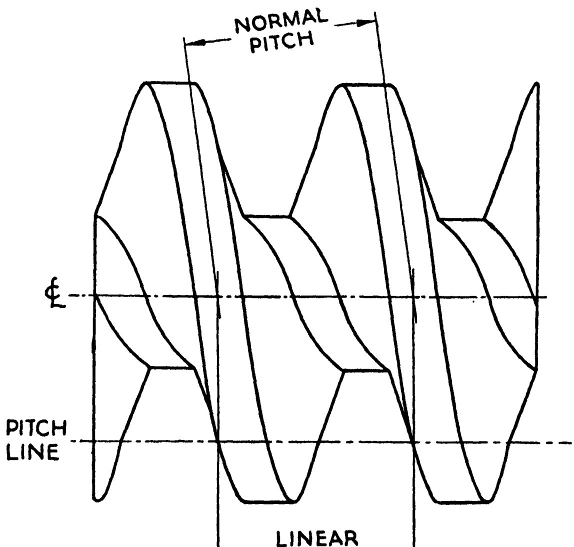

FIG. 219. NORMAL PITCH AND LINEAR PITCH

The difference between "normal" and "linear" pitch is clearly shown in Fig. 219. Normal as well as linear form is illustrated in

Fig. 210. Useful definitions and information on various pitches employed is given in Chapter XI.

EXAMINING OTHER FORMS OF GEARS: Whilst a worm was selected as a typical example to illustrate the use of projection apparatus the reader will appreciate that the checking of any other type of gear by this method is done similarly.

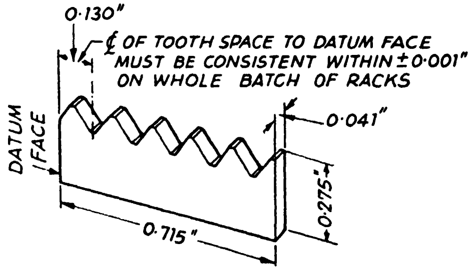

FIG. 220. RACK TO BE MEASURED

Racks. The measurement of racks is usually carried out by mechanical methods but on the smaller class of work optical checking is often more rapid and suitable. For repetition dimensional checking the optical method is ideal and, for accuracy, is equal to other gauging methods.

The rack shown in Fig. 220 is mass-produced and from the stipulated dimensions would be difficult to check mechanically and quickly, particularly in regard to the 0·130 in. dimension. Using a Tolerance

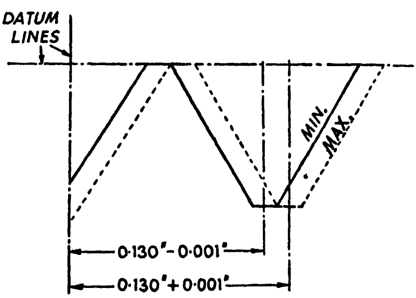

FIG. 221. TOLERANCE CHART FOR RACK
(*Not to scale*)

Chart (Fig. 221), drawn to enlarged scale (in this instance 50: 1), it is straightforward to place the rack on the work-holding stage, bring it into focus, and observe whether the silhouette falls within the tolerance zone.

The pitch, profile, pressure angle, tooth thickness, and depth are measured in a manner similar to that described in *optical measuring of a worm.*

Internal Gears, if too bulky to mount on the work-holding stage, are checked for tooth form, etc., by taking a "mould" or "wax-impression" of the teeth, this in turn being "projected" and examined. (See Fig. 222.)

Bevel Gears must be mounted so that light-rays are intercepted by the form of tooth exactly perpendicular to the pitch angle.

Helical and Spiral Gears. It is not practicable to project a spiral tooth form directly, owing to interference to the light-rays by the "curvature" of the spiral teeth. On repetition work it is customary to take a sample gear from a batch of work. A section, comprising two or more teeth, is then cut from the gear perpendicular to the spiral or helix angle. This section is then projected.

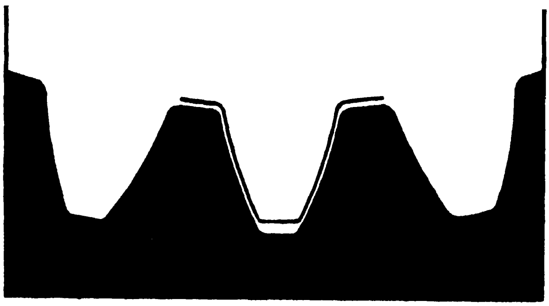

FIG. 222. PROJECTION OF INTERNAL GEAR

Geometrically, helicals and spirals are similar to a worm, and for the purpose of describing "shadowgraphing" no distinction need be made between "helical" and "spiral," as their difference has no bearing on their projection. It has been shown, in connection with the optical projection of a worm, that a marked difference exists between the axial form and the normal form. In view of the fact that the light-rays must follow the mean helical path of the tooth and tooth-space, it is apparent that, when correctly set up, the worm presents a projected section of infinite thickness. Similarly, in the projection of the profile of a helical or spiral tooth, the light-rays should follow the mean helix.

Of course, this is impracticable, for the beam of light-rays follows a straight and not a spiral path. The greater the face-width of the gear (i.e. the length of tooth), the longer becomes the spiral, resulting in an increase in the amount of twist, or controlled deflection of the light-rays, which would be necessary. This, of course, is impossible. A practical solution is to project a very thin section of *one* of the gear teeth (not more, because on a plane section of a helical or spiral gear no two teeth are alike).

If the section were produced by cutting at 90° to the longitudinal axis of the gear, thinned down, and projected, the silhouette would be that of a transverse profile with pressure angle (ψ_t). The usual method is to compare the shadow with an enlarged drawing—and in this case it would entail producing a drawing to show the transverse section of the teeth. Any errors revealed in the result would be errors in the transverse section and not the normal section. Calculations would have to be made to assess corresponding departures from accuracy in the normal form.

Obviously, then, it is advantageous to project the normal form and, to do this, as shown in Fig. 223, a section of the teeth is cut perpendicular to the mean helix angle. This can then be compared with an enlarged drawing of the normal profile, showing normal pitch and pressure angle (ψ_n). The thinned portion is usually about 0·03 in. thick.

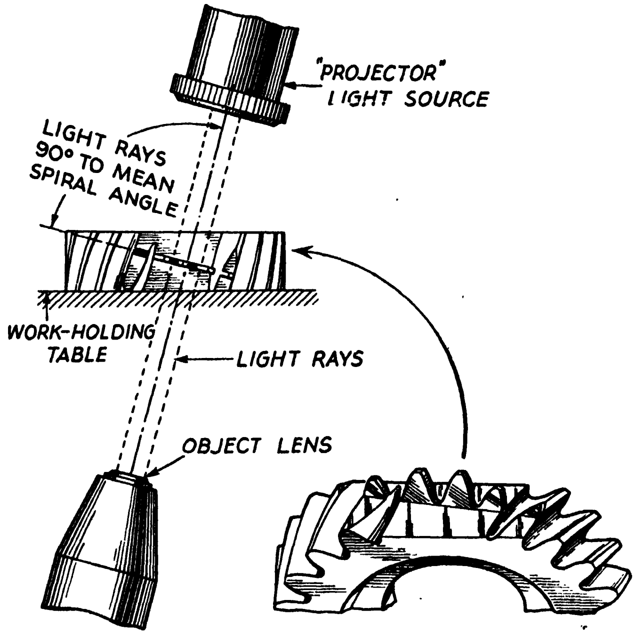

FIG. 223. PROJECTING SPIRAL GEAR

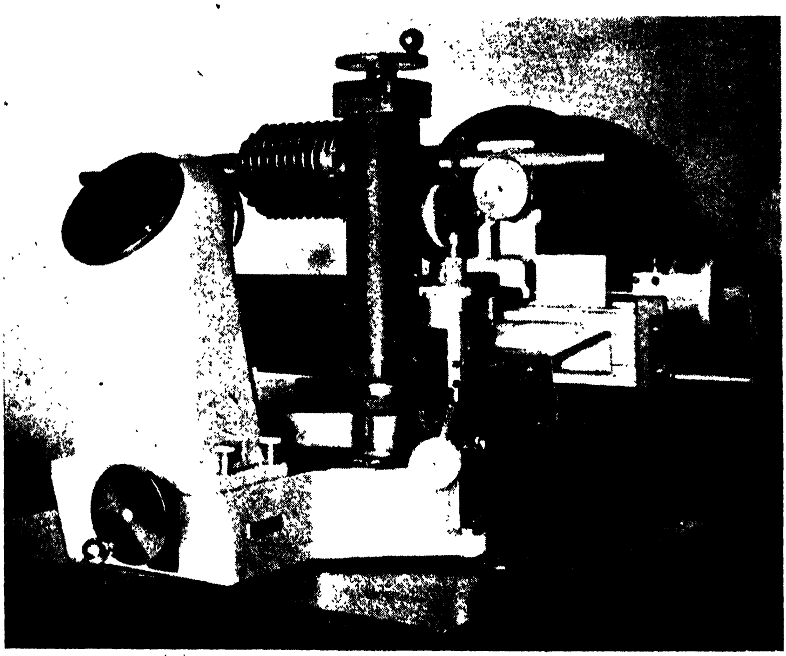

FIG. 224. CHECKING A GEAR HOB ON A HILGER PROJECTOR

The process mentioned entails destruction of a "sample" gear. Where this is impracticable, a "wax-impression" or "mould" may be used.

Gear Hobs. Optical projection methods can be employed to check hobs, the procedure being very similar to that described in connection with worms.

Fig. 224, reproduced by courtesy of Messrs. Alfred Herbert, Ltd., Coventry, shows a gear hob set between centres on a **Hilger Projector,** one of various useful types made by Messrs. Adam Hilger, Ltd., and supplied by Messrs. Alfred Herbert, Ltd., Coventry. The form of tooth is projected against an enlarged profile, the design of the apparatus enabling the depth of tooth to be measured to 0·0001 in., and both pitch and lead errors to be measured to the same degree of accuracy.

On this projector the horizontal travel of the work slide is 6 in., which is covered by 1 in. of adjustment on the micrometer and 5 in. by slip gauges. The vertical travel is 2 in., using 1 in. travel of micrometer and slip gauges up to 1 in.

Inspection of hobs by mechanical methods is explained in the chapter dealing with the measurement of gear-cutters.

CHAPTER XVII

USEFUL ADDITIONAL MEMORANDA

TANGENTS AND NORMALS TO ANY PLANE CURVES. References having been made to tangents and normals in various parts of the book, the following explanatory notes are provided to assist readers who have not previously come across these terms.

(See Fig. 225.) Let P and Q in Fig. 225 (*a*) be two points in any given curve. Suppose Q to move along the curve and to approach infinitely near to P. Then the chord PQ will turn round P, tending

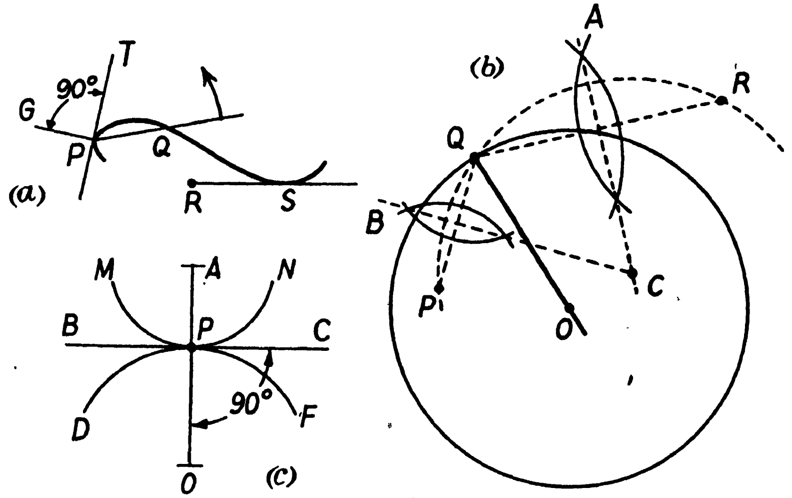

FIG. 225. TANGENTS AND NORMALS

towards a definite direction PT, becoming in the limit the *tangent* to the curve at P. The line PG, perpendicular to PT, is called the *normal* to the curve at P. This explains the well-known geometrical definitions:

The **tangent** *at a point in a curve is the straight line passing through the point and through a second point infinitely near the first. The* **normal** *to the curve at the point is the straight line passing through the point at right angles to the tangent.*

If it be desired to draw a tangent from a point R beyond the curve in Fig. 225 (*a*), this may be done satisfactorily for many purposes by applying a straight-edge and drawing the line RS to touch the curve as shown. (The word tangent comes from the Latin word *tango* = I touch.) This method still leaves the exact point of contact undetermined and, to find it more exactly, some additional construction would be necessary, such construction being based on some known property of the given curve.

Let P, Q, R in Fig. 225 (*b*) be any three points in a plane curve. Lines PQ and QR are chords. The perpendicular bisectors of the chords intersect at C. Then a circle with centre C passing through P will also pass through Q and R. Suppose now that the points P and R move along the curve and approach indefinitely near to Q. Then, in the limit, the two bisectors AC, BC coincide, each becoming a normal to the curve. They intersect at a definite point O, called the *centre of curvature*. The circle through Q, with centre O, is called the *circle of curvature* at Q, and OQ is the *radius of curvature*.

In Fig. 225 (*c*) BC is a tangent to a circular arc DF at the point P. The line APO is the normal through P. As the arcs MN and DF are tangential at P, the line BC is a **common tangent** and AO is a **common normal** to both arcs.

LINEAR VELOCITY AND ANGULAR VELOCITY. In connection with Fig. 1 we referred to surface speeds, or peripheral speeds of the discs or, more precisely, of points on the rims of the discs. As the rim surfaces touch each other and we have imagined that no slip occurs it is evident that points on both surfaces move through the same distance in a given time. For the present purpose we need not bother about the mathematician's differentiation between the respective meanings of *speed* and *velocity* and may define *linear velocity* as *the distance a point moves along a straight or curved line in a unit of time.*

When dealing with gears, we are mainly dealing with rotating objects, and so have to consider circular as well as linear movement. The line described by a point on the circumference of either of the rotating discs, shown in Fig. 1, is an arc of a circle. The length of the arc may be greater or less than the circumference of the cylinder; but if it is expressed in linear units, e.g. inches, feet, etc., described in a unit of time, then it is the linear velocity. Couple these terms in the mind: length of arc, unit of time, linear velocity.

ANGULAR VELOCITY. Referring again to Fig. 1, suppose the diameter of wheel W is twice that of wheel P, the *angle* through which W turns during any length of time is half the *angle* through which P turns in the same length of time. Provided that the wheels are accurately made and set, and no slip occurs, this relation holds good throughout any length of time (however short or long) and the cylinders are said to give *uniform velocity transmission.*

Supposing the smooth cylinders converted into accurately-made spur gears, wheel W having twice the number of teeth as pinion P, then P will make two revolutions while W is making one. If we restrict consideration to a whole number of revolutions of W, then the number of revolutions made by P will always be twice that made by W and apparently there will be *uniform velocity transmission.* When, however, we come to consider smaller angular movements of these gears the rotation of P is not necessarily twice that of W *at every instant.* Various causes may prevent *uniform* velocity transmission, e.g. the teeth may not be accurately spaced round the pitch circle or may be incorrectly formed. Under such conditions tooth action may be irregular, and a rotation of W through, say, half a degree may not produce a rotation of P through one degree. It will thus be seen that in order to secure uniform velocity transmission, the gears must be carefully mounted, the teeth must be cut to great accuracy, and must be designed to correct geometrical form.

The Radian. This is a scientific unit of angle measurement, being the angle subtended at the centre of a circle by an arc equal in length to the radius.

Thus, in Fig. 226, if the arc AB is equal in length to the radius r, then the angle θ is one radian.

2π radians correspond to 360°

$$\therefore \textbf{1 radian} = 360/2\pi = \textbf{57·3}° \text{ approximately.}$$

2π is the radian measure of a circle.

Angle in degrees	30	45	60	90	120	135	150	180
Angle in radians .	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

To convert degrees to radians, divide by 57·3.
To convert radians to degrees, multiply by 57·3.

Length of Circular Arc. The length of a circular arc is given by: arc $= r\theta$, r being the radius and θ the angle subtended at the centre in radians.

Angular Velocity. Suppose the point P in Fig. 226 moves once round the circumference of the circle of centre O. Then the line OP will trace out an angle of 2π radians. If P moves round the circle n times per second, OP will trace out $2\pi n$ radians. This is termed the angular velocity of the point P, and is denoted by the Greek letter ω.

The *linear* velocity of P = circumf. of circle × revs. p.s.

$$= 2\pi r n,$$

$$\therefore v = \omega r$$

As v is usually expressed in feet per second, r must be measured in feet also.

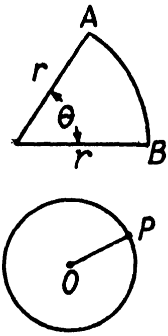

FIG. 226
ILLUSTRATING ANGULAR VELOCITY

EXAMPLE. A gear has a pitch diameter of 12 ft. It makes 85 revolutions per minute. Find its angular velocity in radians per second, and the linear velocity of a point on the pitch surface in feet per second.

$$\text{Angular velocity } (\omega) = 2\pi n$$

$$= \frac{2\pi \times 85}{60} = 8·9 \text{ rad. per sec.}$$

$$\text{Linear velocity} = \omega r = 8·9 \times 6 = 53·4 \text{ ft. per sec.}$$

(The angular velocity is independent of the diameter of the gear.)

VELOCITY RATIOS: SPUR GEARS. The prime mover is called the driving wheel or "driver." It meshes with a mating gear called the "driven," or "follower."

If N and N_1 be the speeds of driver and driven respectively, in revolutions per minute (written R.P.M. for short); if D and D_1 be their pitch diameters respectively; and if the driver has T teeth and the driven T_1 teeth, then the velocity ratio (V.R.) *of the driven gear to the driver gear*

$$= \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{N_1}{N} = \frac{D}{D_1} = \frac{T}{T_1}$$

Note that $ND = N_1D_1$; also $NT = N_1T_1$.

Thus, the velocity ratio of a pair of spur gears is the *inverse* of the ratio of their diameters or numbers of teeth. The V.R. of gear A to gear B

$$= \frac{\text{Speed of } A}{\text{Speed of } B} = \frac{\text{No. of teeth in } B}{\text{No. of teeth in } A} = \frac{\text{Diameter of } B}{\text{Diameter of } A}$$

B.S. Definition. The *ratio of a pair of gears* (R) is the ratio of the

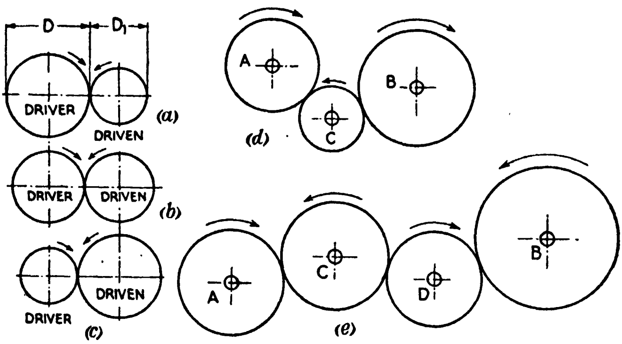

FIG. 227. VELOCITY RATIOS. SPUR GEAR DRIVES

number of teeth in the wheel to the number of teeth in the pinion $= \frac{T}{t}$. (B.S. No. 436—1940.)

A Useful General Rule

R.P.M. of driven gear

$$= \frac{\text{No. of teeth in driving gear}}{\text{No. of teeth in driven gear}} \times \text{R.P.M. of driver gear.}$$

Fig. 227 (*a*). "Large driving small." This is a "speed increase" drive. If the diameter of the driver is twice that of the driven, we have a V.R. of driven to driver of 2/1, or 2 to 1.

Fig. 227 (*b*). Two gears of equal size. V.R. is unity, or 1 to 1. Thus the gears and their respective shafts rotate at equal speeds.

Fig. 227 (*c*). "Small driving large." This is a "speed reduction" drive such as one sees on an electric motor drive to a machine tool. If $D = \frac{1}{2}D_1$, the V.R. of driven to driver is 1/2, or 1 to 2.

EXAMPLE 1. A gear having 120 teeth drives another having 75 teeth. If the speed of the driver is 150 R.P.M. find the speed of the follower.

$$\frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth in driven}}{\text{No. of teeth in driver}} \text{ i.e. } \frac{N}{N_1} = \frac{T_1}{T}$$

$$\therefore \frac{150}{\text{Speed of driven}} = \frac{75}{120}$$

$$\therefore \text{Speed of driven} = \frac{120 \times 150}{75} = 240 \text{ R.P.M.}$$

$$\text{The V.R. of driver to driven} = \frac{75}{120} = \frac{5}{8}$$

$$\text{the V.R. of driven to driver} = \frac{120}{75} = \frac{8}{5} = 1\tfrac{3}{5} \text{ to } 1$$

EXAMPLE 2. A gear A, 24 in. dia., drives a gear B, 6 in. dia. Find the V.R. of B to A, and the revolutions made by B while A makes 100 revolutions.

$$\text{V.R. of } B \text{ to } A = \frac{\text{Dia. of } A}{\text{Dia. of } B} = \frac{\text{Speed of } B}{\text{Speed of } A}$$

$$\therefore \text{ V.R.} = \frac{24}{6} = 4 = \frac{\text{Speed of } B}{\text{Speed of } A}$$

$\therefore$ No. of revolutions made by B while A makes 100 revolutions $= 4 \times 100 = 400$.

DIRECTION OF ROTATION. In all three examples in Fig. 227 (*a*) to (*c*) note that the connected shafts rotate in opposite directions. Now see Fig. 227 (*d*) where it is required to connect two parallel shafts so that they rotate in *the same direction*. This is done by interposing a third or "idler" wheel (C) between the driver (A) and the driven (B). The numerical value of the velocity ratio is independent of the number of intermediate gears in a *simple train*. Fig. 227 (*d*) and (*e*) shows two such simple trains, i.e. arrangements of more than two connected gears all of which lie in the same plane. In both cases A is the driver and B the driven, C being the idler in (*d*) and C and D being the idlers in (*e*). In both cases the V.R. of gear A to gear B

$$= \frac{\text{No. of teeth in } B}{\text{No. of teeth in } A}$$

COMPOUND TRAINS. A large V.R. cannot be obtained conveniently by using two spur gears unless one of them is very large—an impossibility usually if only on account of the large centre-distance that would be required. Therefore "compound trains" like that shown in Fig. 228 are arranged. In this case the driver is marked A. It drives gear B, which is thus the first driven gear. Keyed on the same spindle as gear B is gear C; hence these two gears have the same speed in revolutions per minute. Gear C drives gear D which is keyed to the same spindle as gear E. The latter drives gear F. Thus we have three drivers (A, C, E) and three driven (B, D, F). When A rotates it causes all the gears in the train to rotate.

The V.R. (or revolutions of the last shaft to one revolution of the first shaft) may be found thus—

$$\text{V.R.} = \frac{\text{Product of numbers of teeth in driver gears}}{\text{Product of numbers of teeth in driven gears}}$$

Alternatively the revolutions of the first shaft to one revolution of the last shaft may be found from

$$\text{V.R.} = \frac{\text{Product of numbers of teeth in driven gears}}{\text{Product of numbers of teeth in driver gears}}$$

EXAMPLE 1. Suppose that in a train of gears, similar to that shown in Fig. 228, the drivers have 60, 75, and 50 teeth; the followers have 100, 75, 90 teeth respectively.

$$\text{V.R.} = \frac{60 \times 75 \times 50}{100 \times 75 \times 90} = \frac{1}{3} \text{ (i.e. a "speed reduction")}$$

Suppose the first driver (A) to make 150 R.P.M., what will be the speed of the last driven gear (F)?

Speed of gear (F) $= \frac{1}{3} \times 150 = 50$ revolutions per minute.

EXAMPLE 2. Suppose the same compound train is employed but gear (F) is used as the first driver, gear (A) being the last driven. If the speed of (F) is 150 R.P.M., what is the speed of gear (A)?

$$\text{V.R.} = \frac{100 \times 75 \times 90}{60 \times 75 \times 50} = \frac{3}{1} \text{ (i.e. a "speed increase")}$$

Speed of last driven gear (A) $= 3 \times 150 = 450$ R.P.M.

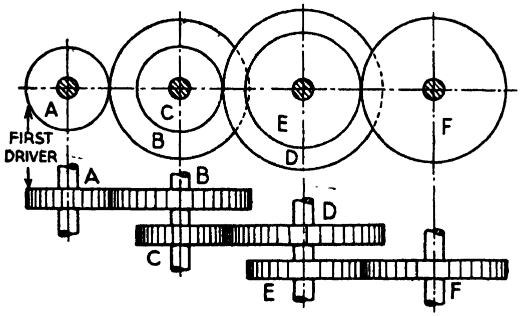

FIG. 228. A TRAIN OF SPUR GEARS

RACK AND PINION. When a pinion drives a rack, the linear speed of the pitch circle of the pinion, say in feet per minute, is equal to the linear speed of the pitch line of the rack in the same units.

EXAMPLE. Suppose a pinion gearing with a rack to drive the table of a planing machine has 22 teeth of $\frac{1}{2}$ in. pitch. When the pinion rotates at 110 R.P.M., what is the speed of the rack?

Circumference of pitch circle of pinion $= 22 \times \frac{1}{2} = 11$ in. $= \frac{11}{12}$ foot
Speed of rack $= \frac{11}{12} \times 110$ ft. per min. $= 100\frac{5}{6}$ ft. per min.

BEVEL GEARS. "The *ratio* of a pair of bevel gears is the ratio of their numbers of teeth ($T \div t$)." (B.S. No. 545.)

Thus the diameters of the pitch circles and the numbers of teeth of bevel gears are related to the required gear ratio in exactly the same way as in spur gearing.

WORM AND WORMWHEEL. "*The ratio* (R) *of a worm and wheel is the ratio of the number of teeth of the wheel to the number of starts of the worm.*" (B.S. No. 721.)

Imagine a single-start worm gearing with a wheel having 45 teeth. One revolution of the worm will cause the wheel to turn $\frac{1}{45}$ of a revolution. To cause the wheel to make one complete turn we shall have to rotate the worm 45 times.

Thus, $$\text{V.R.} = \frac{\text{Revolutions of worm}}{\text{Revolutions of wheel}} = \frac{45}{1}$$

If a worm is single-start and the wheel has n teeth we have the general formula, V.R. $= n$.

If the worm is two-start, one revolution of the worm will advance the wheel by two teeth. In this case, for a wheel with n teeth the V.R. $= \frac{1}{2}n$, or $n/2$.

If the worm has m starts and the wheel n teeth, the V.R. $= n/m$.

Worm speeds as high as 4000 R.P.M. are used in worm gear speed-reduction units connected directly to steam turbines. Double-reduction worm gear units can be used for reduction ratios up to 10,000 to 1. The ordinary low-pressure-angle "involute worm gear" is used for ratios up to about 70 to 1. Other types of gearing are not generally employed for ratios in excess of about 6 to 1 for a single pair of gears.

EPICYCLIC TRAINS. In these we have a train of wheels in which there is relative motion between two or more of the axes of the wheels comprising the train.

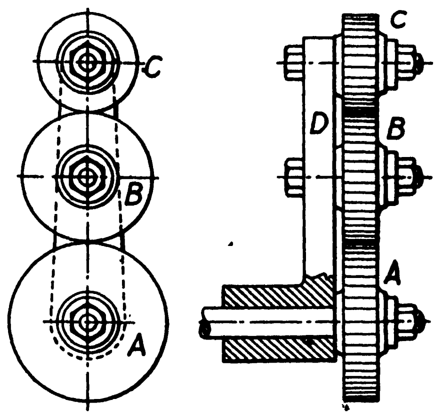

FIG. 229. A SIMPLE EPICYCLIC TRAIN

Thus in Fig. 229 we have an epicyclic train of three gears. *If the arm D is fixed*, the velocity ratio is easily calculated by using the rules previously given. In any case, the *value* (e) of such a train can be calculated from

$$e = \frac{\text{Speed of last wheel relative to the arm}}{\text{Speed of first wheel relative to the arm}}$$

The value of e is positive or negative according as the last wheel and the first wheel rotate in the same or in opposite directions when viewed from the same direction along the axis of either wheel. If f, l, a be the number of revolutions made in a given time by the first wheel, the last wheel and the arm respectively, relative to a fixed framework,

then $$\frac{l-a}{f-a} = e$$

EXAMPLE. Suppose in Fig. 229 the wheel A has 32 teeth and wheel C has 16 teeth.

First Case. If A is fixed: $f = 0$. The arm D makes one revolution clockwise. Thus $a = +1$ and $e = +2$.

$$\frac{l-a}{f-a} = \frac{l-1}{0-1} = +2, \quad \therefore l = -1$$

C therefore makes one revolution in space in an anti-clockwise direction.

Second Case. If C is fixed: $l = 0$. The arm D makes one revolution clockwise. Thus $a = +1$

$$\frac{l-a}{f-a} = \frac{0-1}{f-1} = +2 \qquad \therefore f = \tfrac{1}{2}.$$

A therefore makes half a revolution in a clockwise direction.

Epicyclic trains may incorporate gears of any kind and may be arranged in innumerable ways—simple and compound. Methods of calculating their velocity ratios are given at length in *Gears* by Dr. Merritt (Pitman) and in textbooks on Theory of Machines, such as *Theory of Machines* by Toft and Kersey (Pitman).

INVOLUTE FUNCTION. (Fig. 230.) *S* is shown as the origin of an involute of the base circle of radius r_b. From a point *P* on the involute is drawn the radial line *PTO*. Arc *SP* subtends an angle θ at *O*. This

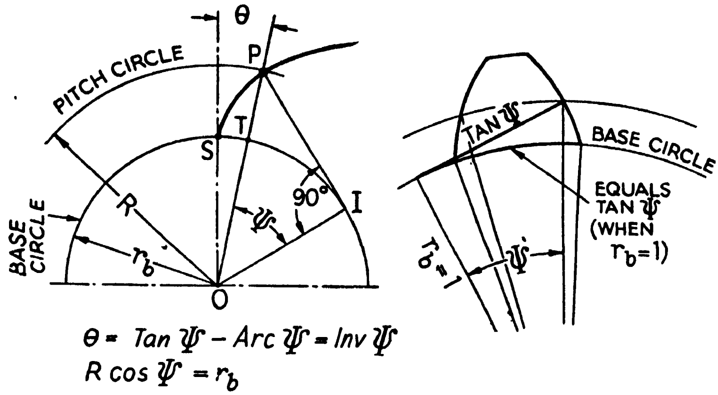

FIG. 230. INVOLUTE FUNCTION

vectoral angle, *in radians*, is the value of the involute function in respect to the arc *SP*.

$$\text{Angle } \theta, \text{ in radians}, = \frac{\text{arc}}{\text{radius}} = \frac{ST}{r_b}$$

It can be shown that

$$\text{Angle } \theta = \tan \psi - \text{arc } \psi = \text{Inv. } \psi$$

"Inv." is a shortened way of writing "involute function." From the formula we see that the involute function of an angle is equal to the difference between the tangent of the angle and its circular measure (in other words its value in radians).

Referring again to Fig. 230 it will be seen that the angle θ, subtended at the centre of the circle by the arc (*SP*) of an involute is the involute function of the angle (ψ) formed by the radius *PTO* through the point considered (i.e. point *P*) and the radius (*OI*) at which the tangent (*PI*) from the point meets the base circle.

To convert degrees to radians we multiply by $\pi/180$. To convert minutes to radians we divide by 3440. Thus the involute function of 12° is equal to,

$$\tan 12° - \frac{12 \times \pi}{180} = 0{\cdot}2125566 - 0{\cdot}2094395$$
$$= 0{\cdot}0031171$$

We write this, inv. 12° = 0·0031171 (to seven places).

Tables of involute functions are given in *Manual of Gear Design*, Vol. 1, by Professor Earle Buckingham.

For angles less than 6° it is simpler to use the formula

$$\text{inv. } \theta^\circ = \tfrac{1}{3}\,(0{\cdot}017453\,\theta)^3$$

Thus from table of involute functions, inv. 3° = 0·0000479. Using the foregoing rule the value is, $\tfrac{1}{3}\,(0{\cdot}017453 \times 3)^3 = 0{\cdot}0000478$.

It is economical of time to be able to refer to a complete table of involute functions showing values up to 45° in steps of one minute. Some tables, however, are arranged in steps of degrees or half-degrees, or in degrees and tenths of degrees.

Given the Angle in Degrees and Minutes. To Find the Involute Function. If it is required to use the table to find the involute function of an angle expressed in degrees and minutes, the following approximation may be adopted—

If the angle is $M^\circ N'$

$$\text{inv. } M^\circ N' = \text{inv. } M^\circ + \frac{N}{3440}\left\{\tan M^\circ\left(\frac{N}{2}\right)'\right\}^2 \qquad (1)$$

EXAMPLE. To find the involute function of 25° 24′ from a table. It is assumed that the tangent of 25° 12′ is known. Work to six decimal places.

$$\text{inv. } 25^\circ = 0{\cdot}029975;\ \tan 25^\circ\ 12' = 0{\cdot}470564$$

$$\therefore \text{inv. } 25^\circ\ 24' = 0{\cdot}029975 + \frac{24}{3440}(0{\cdot}470564)^2$$

$$= 0{\cdot}029975 + 0{\cdot}001545$$

$$= \underline{\underline{0{\cdot}031520}}$$

If the number of minutes is greater than 30 ($N > 30$) a satisfactory result is obtained if the involute function of the next highest complete degree is taken from tables and a corresponding correction substituted.

If $N > 30$,

$$\text{inv. } M^\circ N' = \text{inv. } (M+1)^\circ - \left(\frac{60-N}{3440}\right)\left\{\tan M^\circ\left(N + \frac{60-N}{2}\right)'\right\}^2 \qquad (2)$$

EXAMPLE. To find the involute function of 19° 46′.

$$\text{inv. } 20^\circ = 0{\cdot}014904$$

$$\tan 19^\circ\left(46 + \frac{60-46}{2}\right)' = \tan 19^\circ\ 53' = 0{\cdot}361666$$

$$\text{inv. } 19^\circ\ 46' = 0{\cdot}014904 - \frac{14}{3440} \times (0{\cdot}361666)^2$$

$$= 0{\cdot}014904 - 0{\cdot}000532$$

$$= 0{\cdot}014372$$

Given the Involute Function. To Find the Corresponding Angle. So far we have considered cases where the angle is given and the involute function determined therefrom. In many practical problems the involute function will be known and the corresponding angle will be required.

Let A be the given involute function. If you are given tables of involute functions in steps of whole degrees, find the tabulated value of an involute function nearest to A and just below it.

This value is B, say, and the corresponding angle is x degrees.

Then the required angle is given by,

$$x + 3440\left[\frac{A-B}{\tan^2 x} - \left(\frac{A-B}{\tan^2 x}\right)^2\left(\frac{1+\tan^2 x}{\tan x}\right)\right]$$

which may conveniently be simplified to

$$\text{Angle} = x + 3440\left[\frac{A-B}{\tan^2 x} - \frac{(A-B)^2\sec^2 x}{\tan^5 x}\right] \quad . \quad (3)$$

where the quantity to be added to x is already expressed in minutes (note that the angle x is in degrees).

EXAMPLE. The given involute function is 0·050000. Find the corresponding angle.

The nearest value to 0·050000 in the tables is 0·048164, the involute function of 29°.

Using formula (3) we must substitute $x = 29$, $A = 0{\cdot}050000$, $B = 0{\cdot}048164$.

$$\text{Required angle} = 29^\circ + 3440\left[\frac{0{\cdot}001836}{\tan^2 29^\circ} - \frac{(0{\cdot}001836)^2\sec^2 29^\circ}{\tan^5 29^\circ}\right]'$$

$$= 29^\circ + 3440\left[\frac{0{\cdot}001836}{(0{\cdot}5543)^2} - \frac{(0{\cdot}001836)^2\,(1{\cdot}1434)^2}{(0{\cdot}5543)^5}\right]'$$

$$= 29^\circ + 3440\,[0{\cdot}005977 - 0{\cdot}0000842]'$$

$$= 29^\circ + 3440 \times 0{\cdot}005893' = 29^\circ\,20'$$

The reader is advised to check the above calculation by finding the involute function of 29° 20′, using formula (1). The result should approximate closely to 0·05.

INVOLUTE FUNCTIONS: BRIEF TABLE

Angle ψ deg.	Involute Function $\tan\psi - \psi$	Angle ψ deg.	Involute Function $\tan\psi - \psi$	Angle ψ deg.	Involute Function $\tan\psi - \psi$
1	0·0000 018	11	0·0023 941	20	0·0149 044
2	0·0000 142	12	0·0031 171	21	0·0173 449
3	0·0000 479	13	0·0039 754	22	0·0200 538
4	0·0001 136	14	0·0049 819	23	0·0230 491
5	0·0002 222	14½	0·0055 448	24	0·0263 497
6	0·0003 845	15	0·0061 498	25	0·0299 753
7	0·0006 115	16	0·0074 927	26	0·0339 470
8	0·0009 145	17	0·0090 247	27	0·0382 865
9	0·0013 048	18	0·0107 604	28	0·0430 172
10	0·0017 941	19	0·0127 151	29	0·0481 636
				30	0·0537 514

1. STANDARD PITCHES

Circular Pitch (p) In.	Diametral Pitch (P)	Module (m) Mm.	Circular Pitch (p) In.	Diametral Pitch (P)	Module (m) Mm.	Circular Pitch (p) In.	Diametral Pitch (P)	Module (m) Mm.
1/8	25·133	1·011	**5/16**	10·053	2·526	0·6494	4·838	**5¼**
0·1257	**25**	1·016	0·3142	**10**	2·540	**2/3**	4·712	53·90
0·1309	**24**	1·058	**1/3**	9·425	2·699	0·6803	4·618	**5½**
0·1366	**23**	1·105	0·3401	9·237	**2¾**	**11/16**	4·570	5·559
0·1428	**22**	1·155	0·3491	**9**	2·822	0·7112	4·417	**5¾**
0·1496	**21**	1·209	0·3711	8·466	**3**	0·7421	4·238	**6**
0·1546	20·320	**1¼**	**3/8**	8·378	3·032	**3/4**	4·189	6·064
0·1571	**20**	1·270	0·3927	**8**	3·175	0·7730	4·064	**6¼**
0·1653	**19**	1·337	0·4020	7·815	**3¼**	0·7854	**4**	6·350
0·1745	**18**	1·411	0·4329	7·257	**3½**	0·8040	3·907	**6½**
0·1848	**17**	1·494	**7/16**	7·181	3·537	**13/16**	3·867	6·569
0·1855	16·933	**1½**	0·4488	**7**	3·628	0·8349	3·763	**6¾**
3/16	16·755	1·516	0·4638	6·774	**3¾**	0·8658	3·628	**7**
0·1964	**16**	1·587	0·4947	6·351	**4**	**7/8**	3·590	7·074
0·2094	**15**	1·693	**1/2**	6·283	4·042	0·8967	3·503	**7¼**
0·2164	14·514	**1¾**	0·5236	**6**	4·233	0·9277	3·864	**7½**
0·2244	**14**	1·814	0·5256	5·977	**4¼**	**15/16**	3·510	7·580
0·2417	**13**	1·954	0·5566	5·644	**4½**	0·9586	3·277	**7¾**
0·2474	12·699	**2**	**9/16**	5·585	4·548	0·9895	3·175	**8**
1/4	12·566	2·021	0·5875	5·347	**4¾**	**1**	3·1416	8·805
0·2618	**12**	2·117	0·6184	5·080	**5**			
0·2783	11·288	**2¼**	**5/8**	5·026	5·053			
0·2856	**11**	2·309	0·6283	**5**	5·080			
0·3092	10·160	**2½**						

Circular pitch (p) = $\pi \div$ Diametral pitch (P); $p = \pi \div P$; $P = \pi \div p$; Module (m) $= p \div \pi = 1 \div P$ (see Chapter II).

9—(T.392) 24 pp.

2. GEAR TOOTH PARTS

20° Stub Tooth Pitches

(Fellows' Gear Shaper Company Standards)

20° Stub Tooth English (*DP*)	20° Stub Tooth Module (*M*)	Dimensions in Inches: Circular Thickness (CTh)	Addendum (*A*)	Dedendum (*D*)	Whole Depth of Tooth (*WD*)	Double Depth of Tooth (*DD*)
3/4		0·5236	0·2500	0·3125	0·5625	1·1250
	8/6	0·4947	0·2362	0·2952	0·5314	1·0628
	7¾/6	0·4793	0·2362	0·2952	0·5314	1·0628
	7½/5¾	0·4638	0·2264	0·2830	0·5094	1·0188
	7¼/5¾	0·4484	0·2264	0·2830	0·5094	1·0188
	7/5¼	0·4329	0·2067	0·2584	0·4651	0·9302
	6¾/5¼	0·4174	0·2037	0·2584	0·4651	0·9302
	6½/5	0·4019	0·1969	0·2461	0·4430	0·8860
4/5		0·3927	0·2000	0·2500	0·4500	0·9000
	6¼/4¾	0·3865	0·1870	0·2338	0·4208	0·8416
	6/4½	0·3710	0·1772	0·2215	0·3987	0·7974
	5¾/4½	0·3556	0·1772	0·2215	0·3987	0·7974
	5½/4	0·3401	0·1575	0·1969	0·3544	0·7088
	5¼/4	0·3247	0·1575	0·1969	0·3544	0·7088
5/7		0·3142	0·1429	0·1786	0·3215	0·6430
	5/3¾	0·3092	0·1476	0·1845	0·3321	0·6642
	4¾/3½	0·2938	0·1378	0·1722	0·3100	0·6200
	4½/3¼	0·2783	0·1279	0·1599	0·2878	0·5756
	4¼/3¼	0·2628	0·1279	0·1599	0·2878	0·5756
6/8		0·2618	0·1250	0·1563	0·2813	0·5626
	4/3	0·2473	0·1181	0·1476	0·2657	0·5314
	3¾/2¾	0·2319	0·1082	0·1352	0·2434	0·4868
7/9		0·2244	0·1111	0·1389	0·2500	0·5000
	3½/2½	0·2164	0·0984	0·1230	0·2214	0·4428
	3¼/2½	0·2010	0·0984	0·1230	0·2214	0·4428
8/10		0·1964	0·1000	0·1250	0·2250	0·4500
	3/2¼	0·1855	0·0885	0·1106	0·1991	0·3982
9/11		0·1745	0·0909	0·1136	0·2045	0·4090
	2¾/2	0·1700	0·0787	0·0984	0·1771	0·3542
10/12		0·1571	0·0833	0·1042	0·1875	0·3750
	2½/2	0·1546	0·0787	0·0984	0·1771	0·3542
11/14		0·1428	0·0714	0·0893	0·1607	0·3214
	2¼/1¾	0·1391	0·0689	0·0861	0·1550	0·3100
12/14		0·1309	0·0714	0·0893	0·1607	0·3214
	2/1¾	0·1236	0·0689	0·0861	0·1550	0·3100
13/16		0·1208	0·0625	0·0781	0·1406	0·2812
14/18		0·1122	0·0556	0·0706	0·1262	0·2524
	1¾/1½	0·1082	0·0591	0·0741	0·1332	0·2664
16/21		0·0982	0·0476	0·0626	0·1102	0·2204
	1½/1¼	0·0927	0·0492	0·0642	0·1134	0·2268
18/24		0·0873	0·0417	0·0567	0·0984	0·1968
20/26		0·0785	0·0385	0·0535	0·0920	0·1840
	1¼/1	0·0773	0·0394	0·0544	0·0938	0·1876
22/29		0·0714	0·0345	0·0495	0·0840	0·1680
24/32		0·0654	0·0313	0·0395	0·0708	0·1416
	1/¾	0·0618	0·0295	0·0374	0·0669	0·1338
26/35		0·0604	0·0286	0·0363	0·0649	0·1298
28/37		0·0561	0·0270	0·0344	0·0614	0·1228
30/40		0·0524	0·0250	0·0320	0·0570	0·1140
32/42		0·0491	0·0238	0·0306	0·0544	0·1088
	¾/½	0·0464	0·0197	0·0256	0·0453	0·0906
34/45		0·0462	0·0222	0·0287	0·0509	0·1018
36/48		0·0436	0·0208	0·0270	0·0478	0·0956
38/50		0·0413	0·0200	0·0260	0·0460	0·0920
40/54		0·0393	0·0185	0·0242	0·0427	0·0854

Pitches finer than 30 are in agreement with the A.G.M.A. Fine Pitch Standard.

(*Table supplied by Fellows Co.*)

3. LEAD ANGLES (λ) OF DIAMETRAL PITCH: WORMS AND HOBS

D.P.	Pitch Diameter $\frac{1}{4}$	$\frac{9}{32}$	$\frac{5}{16}$	$\frac{11}{32}$	$\frac{3}{8}$	$\frac{13}{32}$	$\frac{7}{16}$	$\frac{15}{32}$	$\frac{1}{2}$	$\frac{17}{32}$	$\frac{9}{16}$	$\frac{19}{32}$	$\frac{5}{8}$	$\frac{21}{32}$	$\frac{11}{16}$	$\frac{23}{32}$	$\frac{3}{4}$	$\frac{25}{32}$	$\frac{13}{16}$	$\frac{27}{32}$	$\frac{7}{8}$	$\frac{29}{32}$	$\frac{15}{16}$	$\frac{31}{32}$	1″
10	21° 48′	19° 34′	17° 43′	16° 15′	14° 56′	13° 50′	12° 58′	12° 2′	11° 18′	10° 39′	10° 5′	9° 34′	9° 3′	8° 40′	8° 16′	7° 57′	7° 36′	7° 17′	7° 1′	6° 45′	6° 34′	6° 17′	6° 5′	5° 51′	5° 42′
11	19° 59′	17° 56′	16° 13′	14° 50′	13° 38′	12° 37′	11° 31′	10° 59′	10° 18′	9° 43′	9° 10′	8° 42′	8° 16′	7° 53′	7° 33′	7° 12′	6° 54′	6° 38′	6° 20′	6° 8′	5° 58′	5° 44′	5° 32′	5° 22′	5° 12′
12	18° 26′	16° 31′	14° 55′	13° 39′	12° 32′	11° 36′	10° 51′	10° 5′	9° 28′	8° 52′	8° 25′	7° 59′	7° 36′	7° 14′	6° 55′	6° 37′	6° 20′	6° 5′	5° 51′	5° 39′	5° 29′	5° 15′	5° 3′	4° 52′	4° 46′
13	17° 6′	15° 17′	13° 50′	12° 31′	11° 36′	10° 43′	10° 2′	9° 19′	8° 45′	8° 14′	7° 47′	7° 23′	7° 1′	6° 41′	6° 20′	6° 6′	5° 51′	5° 37′	5° 24′	5° 12′	5° 3′	4° 51′	4° 41′	4° 30′	4° 18′
14	15° 56′	14° 15′	12° 53′	11° 45′	10° 47′	9° 58′	9° 20′	8° 40′	8° 8′	7° 40′	7° 14′	6° 52′	6° 31′	6° 12′	5° 57′	5° 40′	5° 26′	5° 13′	5° 1′	4° 46′	4° 42′	4° 31′	4° 21′	4° 14′	4° 5′
15	14° 56′	13° 20′	12° 3′	10° 59′	10° 4′	9° 19′	8° 43′	8° 6′	7° 36′	7° 10′	6° 46′	6° 24′	6° 5′	5° 48′	5° 33′	5° 19′	5° 2′	4° 52′	4° 42′	4° 31′	4° 23′	4° 13′	4° 2′	3° 54′	3° 49′
16	14° 2′	12° 30′	11° 19′	10° 19′	9° 27′	8° 45′	8° 11′	7° 36′	7° 7′	6° 43′	6° 20′	6° 1′	5° 42′	5° 27′	5° 12′	4° 58′	4° 46′	4° 34′	4° 29′	4° 14′	4° 7′	3° 57′	3° 49′	3° 41′	3° 35′
18	12° 31′	11° 11′	10° 5′	9° 11′	8° 25′	7° 47′	7° 17′	6° 45′	6° 20′	5° 56′	5° 38′	5° 21′	5° 4′	4° 49′	4° 37′	4° 27′	4° 14′	4°	3° 54′	3° 44′	3° 36	3° 30′	3° 24	3° 17′	3° 10′
20	11° 18′	10° 5′	9° 7′	8° 17′	7° 36′	7° 1′	6° 34′	6° 5′	5° 42′	5° 23′	5° 2′	4° 49′	4° 35′	4° 21′	4° 9′	4°	3° 49′	3° 40′	3° 31′	3° 24′	3° 17′	3° 9′	3° 3′	2° 55′	2° 52′
22	10° 18′	9° 11′	8° 16′	7° 33′	6° 55′	6° 23′	5° 58′	5° 32′	5° 12′	4° 53′	4° 37′	4° 23′	4° 9′	3° 58′	3° 47′	3° 37′	3° 28′	3° 20′	3° 12′	3° 5′	3°	2° 56′	2° 47′	2° 41′	2° 36′
24	9° 28′	8° 26′	7° 36′	6° 55′	6° 20′	5° 51′	5° 28′	5° 5′	4° 46′	4° 28′	4° 16′	4	3° 49′	3° 38′	3° 28′	3° 19′	3° 9′	3° 4′	2° 56′	2° 49′	2° 43′	2° 38′	2° 32′	2° 28′	2° 23′
26	8° 45′	7° 47′	7° 1′	6° 20′	5° 51′	5° 24′	5° 3′	4° 41′	4° 24′	4° 9′	3° 54′	3° 42′	3° 31′	3° 21′	3° 12′	3° 4′	2° 58′	2° 49′	2° 43′	2° 37′	2° 30′	2° 26′	2° 21′	2° 16′	2° 11′
28	8° 8′	7° 12′	6° 31′	5° 56′	5° 26′	5° 1′	4° 45′	4° 21′	4° 3′	3° 51′	3° 38′	3° 27′	3° 16′	3° 6′	2° 58′	2° 51′	2° 43′	2° 37′	2° 31′	2° 23′	2° 20′	2° 16′	2° 11′	2° 7′	2° 3′
30	7° 36′	6° 45′	6° 5′	5° 33′	5° 2′	4° 42′	4° 23′	4° 2′	3° 49′	3° 30′	3° 23′	3° 14′	3° 3′	2° 54′	2° 47′	2° 39′	2° 32′	2° 26′	2° 21′	2° 15′	2° 11′	2° 6′	2° 2′	1° 57′	1° 54′
32	7° 7′	6° 18′	5° 42′	5° 12′	4° 46′	4° 24′	4° 7′	3° 49′	3° 35′	3° 22′	3° 12′	3° 1′	2° 52′	2° 43′	2° 36′	2° 29′	2° 23′	2° 16′	2° 12′	2° 7′	2° 3′	1° 58′	1° 54′	1° 51′	1° 48′
40	5° 42′	5° 5′	4° 35′	4° 20′	3° 49′	3° 31′	3° 17′	3° 3′	2° 52′	2° 42′	2° 32′	2° 24′	2° 17′	2° 14′	2° 5′	2°	1° 54′	1° 50′	1° 45′	1° 42′	1° 39′	1° 35′	1° 32′	1° 28′	1° 25′

EXAMPLE. Find Lead Angle of Worm 15 D.P. $\frac{9}{16}$ in. pitch diameter; 15 D.P. is found in extreme left-hand column: $\frac{9}{16}$ in. pitch diameter is found under $\frac{9}{16}$ in. top row. *Answer*: 6° 46′.

Based on Linear Pitch and Pitch Diameter

Lead in Inches	Pitch Diameter in Inches: 1/4	9/32	5/16	11/32	3/8	13/32	7/16	15/32	1/2	17/32	9/16	19/32	5/8	21/32	11/16	23/32	3/4	25/32	13/16	27/32	7/8	29/32	15/16	31/32	1″
1″												28° 12′	26° 59′	25° 52′	24° 50′	23° 53′	23°	22° 10′	21° 44′	20° 27′	20°	19° 21′	18° 54′	18° 10′	17° 39′
15/16												26° 41′	25° 31′	24° 27′	23° 29′	22° 33′	21° 43′	20° 55′	20° 30′	19° 16′	18° 50′	18° 14′	17° 39′	17° 8′	16° 37′
7/8									29° 7′	27° 40′	26° 20′	25° 8′	24° 1′	22° 54′	22° 3′	21° 11′	20° 23′	19° 38′	19° 13′	18° 4′	17° 39′	17° 5′	16° 32′	16° 3′	15° 34′
13/16									27° 21′	25° 58′	24° 42′	23° 32′	22° 29′	21° 33′	20° 35′	19° 46′	19° 2′	18° 17′	17° 56′	16° 51′	16° 28′	15° 56′	15° 25′	14° 57′	14° 30′
3/4									25° 31′	24° 12′	23°	21° 54′	20° 54	19° 59′	19° 8′	18° 23′	17° 39′	17°	16° 38′	15° 37′	15° 16′	14° 46′	14° 17′	13° 51′	13° 26′
11/16					30° 16′	28° 19′	26° 37′	25° 2′	23° 38′	22° 23′	21° 15′	20° 14′	19° 17′	18° 26′	17° 38′	16° 56′	16° 17′	15° 39′	15° 19′	14° 22′	14° 2′	13° 35′	13° 7′	12° 44′	12° 21′
5/8					27° 57′	26° 6′	24° 27′	23°	21° 42′	20° 32′	19° 28′	18° 32′	17° 39′	16° 52′	16° 7′	15° 28′	14° 59′	14° 17′	14°	13° 7′	12° 48′	12° 23′	11° 59′	11° 36′	11° 15′
9/16					25° 31′	23° 47′	22° 14′	20° 54′	19° 42′	18° 38′	17° 39′	16° 47′	15° 58′	15° 15′	14° 36′	13° 59′	13° 27′	12° 55′	12° 38′	11° 51′	11° 34′	11° 11′	10° 48′	10° 29′	10° 9′
1/2	32° 29′	29° 30′	27°	24° 51′	23°	21° 23′	19° 59′	18° 45′	17° 39′	16° 40′	15° 48′	15° 1′	14° 17′	13° 38′	13° 2′	12° 29′	11° 58′	11° 33′	11° 16′	10° 34′	10° 19′	9° 58′	9° 38′	9° 20′	9° 2′
7/16	29° 1′	26° 20′	24° 1′	22° 3′	20° 23′	18° 55′	17° 40′	16° 35′	15° 39′	14° 41′	13° 54′	13° 12′	12° 34′	11° 58′	11° 26′	10° 58′	10° 32′	10° 7′	9° 53′	9° 16′	9° 2′	8° 43′	8° 27′	8° 11′	7° 55′
3/8	25° 31′	23°	20° 54′	19° 9′	17° 39′	16° 22′	15° 16′	14° 17′	13° 24′	12° 40′	11° 59′	11° 23′	11° 18′	10° 18′	9° 51′	9° 26′	9° 3′	8° 42′	8° 30′	7° 57′	7° 46′	7° 30′	7° 15′	7° 1′	6° 48′
5/16	21° 42′	19° 28′	17° 39′	16° 8′	14° 52′	13° 45′	12° 50′	11° 59′	11° 13′	10° 36′	10° 2′	9° 31′	9° 2′	8° 37′	8° 13′	7° 53′	7° 34′	7° 16′	7° 6′	6° 39′	6° 29′	6° 16′	6° 3′	5° 52′	5° 41′
1/4	17° 39′	15° 48′	14° 18′	13° 2′	11° 59′	11° 5′	10° 19′	9° 38′	9° 2′	8° 31′	8° 3′	7° 38′	7° 15′	6° 57′	6° 36′	6° 19′	6° 3′	5° 49′	5° 41′	5° 19′	5° 12′	5° 1′	4° 51′	4° 42′	4° 33′
3/16	13° 26′	11° 59′	10° 48′	9° 51′	9° 2′	8° 21′	7° 46′	7° 15′	6° 48′	6° 25′	6° 3′	5° 44′	5° 27′	5° 12′	4° 57′	4° 45′	4° 33′	4° 22′	4° 16′	4°	3° 54′	3° 46′	3° 38′	3° 31′	3° 25′
1/8	9° 2′	8° 3′	7° 46′	6° 36′	6° 4′	5° 28′	5° 12′	4° 51′	4° 33′	4° 17′	4° 3′	3° 50′	3° 38′	3° 28′	3° 18′	3° 10′	3° 3′	2° 55′	2° 51′	2° 40′	2° 35′	2° 31′	2° 26′	2° 21′	2° 17′

EXAMPLE. Find Lead Angle for 1/4 in. Lead, 3/16 in. Pitch Diameter. 1/4 in. Lead is found in extreme left-hand column; 3/16 in. pitch diameter is found

5. CONSTANT CHORDS—CALIPER SETTINGS

(*h* = **Constant Chord Height,** *g* = **Constant Chord Thickness.** See Fig. 87.)

D.P.	20° Pressure Angle		14½° Pressure Angle		D.P.	20° Pressure Angle		14½° Pressure Angle	
	h	*g*	*h*	*g*		*h*	*g*	*h*	*g*
1	0·7476	1·3871	0·8096	1·4723	**16**	0·0467	0·0867	0·0506	0·0920
1¼	0·5981	1·1097	0·6476	1·1780	**17**	0·0439	0·0816	0·0476	0·0866
1½	0·4984	0·9247	0·5398	0·9816	**18**	0·0415	0·0770	0·0450	0·0818
1¾	0·4270	0·7926	0·4628	0·8412	**19**	0·0394	0·0730	0·0426	0·0775
2	0·3738	0·6935	0·4048	0·7361	**20**	0·0374	0·0694	0·0405	0·0736
2¼	0·3321	0·5764	0·3600	0·6544	**22**	0·0339	0·0631	0·0368	0·0669
2½	0·2990	0·5548	0·3238	0·5890	**24**	0·0311	0·0578	0·0338	0·0614
2¾	0·2715	0·5043	0·2944	0·5352	**25**	0·0299	0·0555	0·0324	0·0589
3	0·2492	0·4624	0·2699	0·4908	**26**	0·0288	0·5033	0·0311	0·0566
3½	0·2135	0·3963	0·2314	0·4206	**28**	0·0267	0·0496	0·0289	0·0526
4	0·1869	0·3468	0·2024	0·3681	**30**	0·0249	0·0462	0·0267	0·0491
5	0·1495	0·2774	0·1619	0·2945	**32**	0·0234	0·0433	0·0253	0·0460
6	0·1246	0·2312	0·1349	0·2454	**34**	0·0220	0·0408	0·0238	0·0432
7	0·1068	0·1982	0·1157	0·2103	**36**	0·0208	0·0385	0·0225	0·0409
8	0·0934	0·1734	0·1012	0·1840	**38**	0·0197	0·0365	0·0213	0·0398
9	0·0830	0·1441	0·0900	0·1636	**40**	0·0187	0·0347	0·0202	0·0368
10	0·0748	0·1387	0·0810	0·1472	**42**	0·0178	0·0330	0·0193	0·0357
11	0·0679	0·1261	0·0736	0·1338	**44**	0·0169	0·0315	0·0184	0·0335
12	0·0623	0·1156	0·0675	0·1227	**46**	0·0162	0·0301	0·0176	0·0319
13	0·0575	0·1067	0·0623	0·1132	**48**	0·0156	0·0289	0·0169	0·0307
14	0·0534	0·0991	0·0578	0·1052	**50**	0·0149	0·0277	0·0162	0·0295
15	0·0498	0·0925	0·0540	0·0982					

6. CHORDAL THICKNESSES AND CHORDAL ADDENDA

The following table will be found useful for checking gears when there is doubt about the accuracy of blank diameter, about concentricity, etc. (See notes on Gear Tooth Vernier Caliper.) The table refers to one diametral pitch—spur gears and bevel gears. To use it for any other diametral pitch, divide the tabulated values by the required diametral pitch. When using the table for helical gears, divide by the normal diametral pitch. Note that the table relates to standard full-depth teeth having addendum equal to module, pressure angle 14½°.

No. of Teeth	Divide by Diametral Pitch to obtain		No. of Teeth	Divide by Diametral Pitch to obtain		No. of Teeth	Divide by Diametral Pitch to obtain	
	Chordal Thickness	Chordal Addend.		Chordal Thickness	Chordal Addend.		Chordal Thickness	Chordal Addend.
8	1·5607	1·0769	53	1·5706	1·0117	97	1·5707	1·0064
9	1·5628	1·0684	54	1·5706	1·0114	98	1·5707	1·0063
10	1·5643	1·0616	55	1·5706	1·0112	99	1·5707	1·0062
11	1·5654	1·0559	56	1·5706	1·0110	100	1·5707	1·0062
12	1·5663	1·0514	57	1·5706	1·0108	101	1·5707	1·0061
13	1·5669	1·0474	58	1·5706	1·0106	102	1·5707	1·0060
14	1·5675	1·0440	59	1·5706	1·0105	103	1·5707	1·0059
15	1·5679	1·0411	60	1·5706	1·0103	104	1·5707	1·0059
16	1·5683	1·0385	61	1·5706	1·0101	105	1·5707	1·0059
17	1·5686	1·0362	62	1·5706	1·0099	106	1·5707	1·0058
18	1·5688	1·0342	63	1·5706	1·0098	107	1·5707	1·0057
19	1·5690	1·0324	64	1·5706	1·0096	108	1·5707	1·0057
20	1·5692	1·0308	65	1·5706	1·0095	109	1·5707	1·0056
21	1·5694	1·0294	66	1·5706	1·0093	110	1·5707	1·0056
22	1·5695	1·0280	67	1·5706	1·0092	111	1·5707	1·0056
23	1·5696	1·0268	68	1·5707	1·0091	112	1·5707	1·0055
24	1·5697	1·0257	69	1·5707	1·0089	113	1·5707	1·0055
25	1·5698	1·0247	70	1·5707	1·0088	114	1·5707	1·0054
26	1·5699	1·0237	71	1·5707	1·0087	115	1·5707	1·0054
27	1·5699	1·0228	72	1·5707	1·0085	116	1·5707	1·0053
28	1·5700	1·0219	73	1·5707	1·0084	117	1·5707	1·0053
29	1·5700	1·0212	74	1·5707	1·0083	118	1·5707	1·0052
30	1·5701	1·0206	75	1·5707	1·0082	119	1·5707	1·0052
31	1·5701	1·0199	76	1·5707	1·0081	120	1·5707	1·0051
32	1·5702	1·0193	77	1·5707	1·0080	121	1·5707	1·0051
33	1·5702	1·0187	78	1·5707	1·0079	122	1·5707	1·0051
34	1·5702	1·0181	79	1·5707	1·0078	123	1·5708	1·0050
35	1·5702	1·0176	80	1·5707	1·0077	124	1·5708	1·0050
36	1·5703	1·0171	81	1·5707	1·0076	125	1·5708	1·0050
37	1·5703	1·0167	82	1·5707	1·0075	126	1·5708	1·0049
38	1·5704	1·0162	83	1·5707	1·0074	127	1·5708	1·0049
39	1·5704	1·0158	84	1·5707	1·0073	128	1·5708	1·0048
40	1·5704	1·0154	85	1·5707	1·0072	129	1·5708	1·0048
41	1·5704	1·0150	86	1·5707	1·0072	130	1·5708	1·0047
42	1·5704	1·0147	87	1·5707	1·0070	131	1·5708	1·0047
43	1·5704	1·0144	88	1·5707	1·0069	132	1·5708	1·0047
44	1·5705	1·0140	89	1·5707	1·0069	133	1·5708	1·0047
45	1·5705	1·0137	90	1·5707	1·0069	134	1·5708	1·0046
46	1·5705	1·0134	91	1·5707	1·0068	135	1·5708	1·0046
47	1·5705	1·0131	92	1·5707	1·0067	136	1·5708	1·0045
48	1·5705	1·0128	93	1·5707	1·0066	137	1·5708	1·0045
49	1·5705	1·0126	94	1·5707	1·0066	138	1·5708	1·0045
50	1·5705	1·0123	95	1·5707	1·0065	139	1·5708	1·0044
51	1·5706	1·0121	96	1·5707	1·0064	140	1·5708	1·0044
52	1·5706	1·0119						

7A. DIAMETERS OF CYLINDERS FOR CIRCULAR PITCH GEARS

Pressure Angles $\psi = 14\frac{1}{2}°$ and $20°$

Circ. Pitch (In.)	Dia. of Cylinder		Circ. Pitch (In.)	Dia. of Cylinder		Circ. Pitch (In.)	Dia. of Cylinder	
	$14\frac{1}{2}°$	$20°$		$14\frac{1}{2}°$	$20°$		$14\frac{1}{2}°$	$20°$
4	1·9363	1·8794	$1\frac{1}{4}$	0·6051	0·5873	$\frac{1}{2}$	0·2420	0·2349
$3\frac{1}{2}$	1·6943	1·6445	1	0·4841	0·4698	$\frac{7}{16}$	0·2125	0·2055
3	1·4522	1·4095	$\frac{15}{16}$	0·4538	0·4405	$\frac{3}{8}$	0·1815	0·1762
$2\frac{3}{4}$	1·3312	1·2921	$\frac{7}{8}$	0·4236	0·4111	$\frac{5}{16}$	0·1518	0·1468
$2\frac{1}{2}$	1·2102	1·1746	$\frac{13}{16}$	0·3939	0·3817	$\frac{1}{4}$	0·1210	0·1175
$2\frac{1}{4}$	1·0892	1·0572	$\frac{3}{4}$	0·3631	0·3524	$\frac{3}{16}$	0·0911	0·0881
2	0·9681	0·9397	$\frac{11}{16}$	0·3339	0·3229	$\frac{1}{8}$	0·0605	0·0587
$1\frac{3}{4}$	0·8471	0·8222	$\frac{5}{8}$	0·3025	0·2936	$\frac{1}{16}$	0·0303	0·0294
$1\frac{1}{2}$	0·7261	0·7048	$\frac{9}{16}$	0·2732	0·2643			

Cylinder diameter = Arc width of space × Cos ψ. This is the most easily applied system. The axis of the cylinder lies in the pitch surface of the gear. This simplifies calculation for the distance from the centre of the gear to the centre of the cylinder equals the pitch radius of the gear. Exactly the same applies to the next table for diametral pitch gears.

7B. DIAMETERS OF CYLINDERS FOR DIAMETRAL PITCH GEARS

D.P.	Dia. of Cylinder		D.P.	Dia. of Cylinder	
	$14\frac{1}{2}°$	$20°$		$14\frac{1}{2}°$	$20°$
$\frac{1}{2}$	3·0414	2·9521	18	0·0845	0·0820
1	1·5207	1·4760	19	0·0800	0·0768
$1\frac{1}{4}$	1·2166	1·1808	20	0·0760	0·0738
$1\frac{1}{2}$	1·0138	0·9840	21	0·0724	0·0703
$1\frac{3}{4}$	0·8688	0·8436	22	0·0691	0·0670
2	0·7603	0·7380	23	0·0661	0·0642
$2\frac{1}{4}$	0·6756	0·6560	24	0·0634	0·0615
$2\frac{1}{2}$	0·6082	0·5904	25	0·0608	0·0590
$2\frac{3}{4}$	0·5528	0·5364	26	0·0585	0·0568
3	0·5069	0·4920	27	0·0563	0·0546
4	0·3802	0·3690	28	0·0543	0·0527
5	0·3041	0·2952	29	0·0524	0·0509
6	0·2534	0·2460	30	0·0507	0·0492
7	0·2172	0·2109	32	0·0475	0·0461
8	0·1901	0·1845	34	0·0447	0·0434
9	0·1689	0·1640	36	0·0423	0·0410
10	0·1521	0·1476	38	0·0400	0·0384
11	0·1382	0·1341	40	0·0380	0·0369
12	0·1267	0·1230	50	0·0304	0·0295
13	0·1169	0·1135	60	0·0253	0·0246
14	0·1086	0·1054	70	0·0217	0·0211
15	0·1014	0·0984	80	0·0190	0·0185
16	0·0950	0·0922	90	0·0169	0·0164
17	0·0895	0·0868	100	0·0152	0·0148

8. TABLE FOR REFERENCE WHEN MEASURING PITCH DIAMETERS OF GEARS BY USE OF CYLINDERS

Refer to Fig. 171. N_t = No. of teeth in gear; d = dia. of cylinder; x = dimension from outside of cylinder to centre of gear; X = Dimension over cylinders.

For 20° Pressure Angle

N_t	d	x	X	N_t	d	x	X	N_t	d	x	X
15	1·74456	8·71350	17·34074	44	1·69808	23·15369	46·30738	73	1·68537	37·64058	75·26645
16	1·73942	9·20739	18·41478	45	1·69378	23·65061	47·26762	74	1·68521	38·14006	76·28011
17	1·73691	9·70290	19·32984	46	1·68766	24·19654	48·29308	75	1·68452	38·63937	77·26340
18	1·73103	10·19705	20·39409	47	1·67975	24·64383	49·26395	76	1·68275	39·13809	78·27617
19	1·72675	10·65111	21·23568	48	1·68989	25·14831	50·29661	77	1·68105	39·63674	79·25810
20	1·72439	11·18894	22·37787	49	1·68921	25·64724	51·26760	78	1·68100	40·13653	80·27306
21	1·72079	11·68517	23·30977	50	1·69143	26·19750	52·29499	79	1·67676	40·63397	81·25209
22	1·71924	12·18229	24·36458	51	1·69038	26·64741	53·26928	80	1·67998	41·13543	82·27086
23	1·71863	12·68047	25·29556	52	1·69333	27·19668	54·29337	81	1·69062	41·64034	83·26430
24	1·71479	13·17703	26·35405	53	1·69248	27·64554	55·26963	82	1·68760	42·13864	84·27728
25	1·71407	13·67496	27·29870	54	1·68964	28·19601	56·29202	83	1·68203	42·63460	85·25201
26	1·71182	14·16295	28·32580	55	1·69223	28·64665	57·27109	84	1·68327	43·13581	86·27161
27	1·70994	14·67177	29·29656	56	1·69256	29·19606	58·29212	85	1·68509	43·63639	87·25567
28	1·70936	15·16920	30·33840	57	1·69365	29·64596	59·26891	86	1.67246	44·12965	88·25930
29	1·70538	15·66626	31·28894	58	1·68984	30·14351	60·28702	87	1·68060	44·63355	89·24958
30	1·70576	16·16508	32·33016	59	1·68061	30·64010	61·26223	88	1·67808	45·13203	90·26406
31	1·70239	16·66307	33·28508	60	1·68778	31·14434	62·28868	89	1·67723	45·63157	91·24501
32	1·70598	17·16400	34·32800	61	1·68464	31·64956	63·28066	90	1·68201	46·13348	92·26695
33	1·70628	17·66346	35·28990	62	1·68530	32·14889	64·29778	91	1·68608	46·63518	93·25226
34	1·70357	18·16122	36·32243	63	1·69267	32·65165	65·29089	92	1·67504	47·12948	94·25896
35	1·69723	18·65755	37·27950	64	1·68220	33·13925	66·27850	93	1·68024	47·63174	95·25472
36	1·69971	19·15744	38·31587	65	1·68495	33·63974	67·25977	94	1·66884	48·12579	96·25158
37	1·70027	19·65748	39·28012	66	1·69016	34·14238	68·28476	95	1·68651	48·63446	97·25339
38	1·69705	20·15449	40·30899	67	1·67615	34·63482	69·24627	96	1·67543	49·12357	98·25713
39	1·70393	20·65733	41·28294	68	1·67729	35·13487	70·26973	97	1·68278	49·63206	99·24456
40	1·69666	21·15416	42·30832	69	1·67936	35·63560	71·25063	98	1·66854	50·12447	100·24894
41	1·70008	21·65410	43·27479	70	1·68278	36·14034	72·28068	99	1·67951	50·62984	101·23979
42	1·69584	22·15264	44·30528	71	1·68354	36·64016	73·27602	100	1·68160	51·13075	102·26150
43	1·69396	22·65064	45·27054	72	1·68478	37·14210	74·28420				

This table is based on Unit Diametral Pitch. For use with any other D.P., divide given values by No. of D.P. required, e.g. 12 D.P., 41 teeth, d = 0·14167, x = 1·8045, X = 3·6062.

9. TABLE FOR REFERENCE WHEN MEASURING RACKS BY MEANS OF CYLINDERS. PRESSURE ANGLE = 14½°

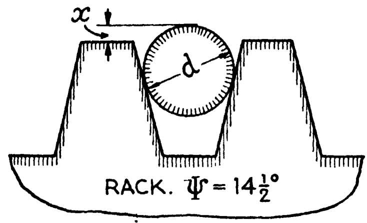

Fig. 231

(Dimensions in inch units)

D.P.	d	x	D.P.	d	x	D.P.	d	x
½	3·2450	0·0286	14	0·1159	0·0010	31	0·0523	0·0005
1	1·6225	0·0143	15	0·1082	0·0010	32	0·0507	0·0005
1½	1·0817	0·0095	16	0·1014	0·0009	33	0·0492	0·0004
2	0·8113	0·0072	17	0·0954	0·0008	34	0·0477	0·0004
2½	0·6490	0·0057	18	0·0902	0·0008	36	0·0451	0·0004
3	0·5408	0·0048	19	0·0854	0·0008	37	0·0438	0.0004
4	0·4056	0·0036	20	0·0811	0·0007	38	0·0427	0·0004
5	0·3245	0·0029	22	0·0738	0·0007	39	0·0416	0·0004
6	0·2704	0·0024	23	0·0705	0·0006	40	0·0406	0·0004
7	0·2318	0·0020	24	0·0676	0·0006	44	0·0369	0·0003
8	0·2028	0·0018	25	0·0649	0·0006	50	0·0325	0·0003
9	0·1803	0·0016	26	0·0624	0·0006	60	0·0270	0·0002
10	0·1623	0·0014	27	0·0601	0·0005	70	0·0232	0·0002
11	0·1475	0·0013	28	0·0580	0·0005	80	0·0203	0·0002
12	0·1352	0·0012	29	0·0560	0·0005	90	0·0180	0·0001
13	0·1248	0·0011	30	0·0541	0·0005	100	0·0162	0·0001

10. DIAMETERS OF SEWING NEEDLES FOR GEAR MEASUREMENT

Packet No.	11	10	9	8	7	6	5	4	3	2	1	1′0	2′0
Size (In.)	0·015	0·017	0·020	0·023	0·026	0·029	0·033	0·036	0·038	0·042	0·047	0·049	0·055

When using cylinders or wires in the measurement of small gears, ordinary household needles can be used. The table gives the packet designating number and diameter. If it is necessary to use cylinders of larger diameter, it may be possible to use the shanks of standard twist drills. (See the table on page 254.)

9A—(T.392)

11. TABLE OF STANDARD TWIST DRILLS

No.	Size	No.	Size	No.	Size	No.	Size
1	0·2280	**16**	0·1770	**31**	0·1200	**46**	0·0810
2	0·2210	**17**	0·1730	**32**	0·1160	**47**	0·0785
3	0·2130	**18**	0·1695	**33**	0·1130	**48**	0·0760
4	0·2090	**19**	0·1660	**34**	0·1110	**49**	0·0730
5	0·2055	**20**	0·1610	**35**	0·1100	**50**	0·0700
6	0·2040	**21**	0·1590	**36**	0·1065	**51**	0·0670
7	0·2010	**22**	0·1570	**37**	0·1040	**52**	0·0635
8	0·1990	**23**	0·1540	**38**	0·1015	**53**	0·0595
9	0·1960	**24**	0·1520	**39**	0·0995	**54**	0·0550
10	0·1935	**25**	0·1495	**40**	0·0980	**55**	0·0520
11	0·1910	**26**	0·1470	**41**	0·0960	**56**	0·0465
12	0·1890	**27**	0·1440	**42**	0·0935	**57**	0·0430
13	0·1850	**28**	0·1405	**43**	0·0890	**58**	0·0420
14	0·1820	**29**	0·1360	**44**	0·0860	**59**	0·0410
15	0·1800	**30**	0·1285	**45**	0·0820	**60**	0·0400

(On the left is given the designating number of the drill, followed by its diameter in inch units.)

12. RECOMMENDED WHEELS FOR GRINDING WORMS AND HOBS

Maker's Name	Grain and Grade	Pitch of Worm/Hob
Norton	38–50L—5BE	Very Coarse
,,	38–120K—8	Coarse
,,	38–120J	,,
,,	38–220K—8	Medium
,,	38–320L	Fine Pitch
,,	38–500L	Very Fine Pitch
Carborundum	A120—K600	Coarse Pitch
,,	A150—L—PH8	,, ,,
,,	A46—M—600	Very Coarse Pitch
,,	A54—N—600	,, ,, ,,
,,	A60—M—600	,, ,, ,,
,,	A180—K—800	Fine Pitch

GEAR GRINDING

Maker's Name	Grain and Grade	Purpose
Carborundum	E50—KA 10R	Form Precision and Generative Precision Grinding
,,	E50—K 12R	
,,	A54—M—600	
,,	A60—N—PL6	
Norton	38-60K—5B	Form Precision Grinding
,,	38-60J—5B	Generative Precision Grinding
,,	38-46L	Internal Grinding
	Fluting of Hobs	Size of Hob
Norton	3850-K5BE	Average
,,	3846-18BE	Large

GRINDING GEARS (HARDENED)

NORTON WHEELS

Type of Wheel	Type of Machine
38-46—K—5BE 38-60—K—5BE	Churchill
38-60—L—5BE	Maag
38-60—K—5BE	Pratt & Whitney
38-60—I—6BE 38-60—L—5BE	Lees Bradner
38-60—K—5BE 38-46—K—5BE	Orcutt

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