WORKED EXAMPLES IN ENGINEERING SCIENCE

By

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PREFACE

THIS book is an attempt to bring together in a convenient form a number of worked examples in engineering science, in order to supplement the reading of students preparing for the Ordinary National Certificate in Mechanical Engineering. The author also hopes that those students who are working for the Joint Section A examination in Applied Mechanics of the Associate membership of the Institution of Mechanical or Electrical Engineers, and the many other examinations where the syllabuses are similar, will find the book useful.

The author has chosen typical examples in the syllabus, rather than a large number of similar examples, as he has found from experience that students have little opportunity of working out examples under expert guidance, since the majority of the teaching time in National Certificate work is taken up with theory and experimental work.

All the problems are either from past examination papers set at the sessional examinations or are of equivalent standard. The questions cover the first and second years of the National Certificate as well as the third year or Ordinary National Certificate examination.

As some colleges have Heat in the National Certificate course, some questions on this subject have been added, as well as a few on elementary hydraulics.

By repeated checking of the solutions, it is hoped that a high standard of accuracy has been achieved. In this connection the author desires to acknowledge the help he has received from D. Benton, Esq., M.A., and W. G. Evans, Esq., B.Sc., in correcting the manuscript and proofs. He also desires to thank all those who have assisted him with information.

H. J. C.

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DEFINITION OF TERMS

To assist the reader, some of the terms used in the examples are defined below. The symbols and abbreviations used comply as far as possible with B.S. 560.

1. *Mass.* The name given to inertia when expressed as a measurable quantity, or the quantity of matter in a body.
2. *Inertia.* The property of resisting a change of motion.
3. *Force.* That which tends to change the motion of bodies either in magnitude or direction.
4. *Moment of a Force.* The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.
5. *Weight* of a body is the force with which the earth attracts it.
6. *Bow's Notation.* A method of lettering a space diagram by assigning two letters placed one on each side of the line of action of a force.
7. *Space Diagram.* A diagram showing correctly the inclinations of the lines of action of the various known forces to one another and to some scale their relative amounts.
8. *Stress.* The equal and opposite action and reaction which take place between two bodies or two parts of the same body transmitting a force, divided by the area in contact perpendicular to the force.
9. *Strain.* The alteration of shape or dimensions resulting from stress.
10. *Centre of Gravity.* That point of a body about which all its parts can be balanced or which, being supported, the whole body will remain at rest, though acted upon by gravity.
11. *Friction.* The resistance which a body meets with from the surface on which it moves.
12. *Angle of Friction.* The angle which a plane makes with the horizontal when a body lying on it is just ready to slide down it.

13. *Velocity.* The rate of change of position of a body.
14. *Mean Velocity*, or average velocity of a body, is the number of units of length covered, divided by the number of units of time taken.
15. *Acceleration* of a moving body is its rate of change of velocity.
16. *Vector.* A straight line having a definite length and direction, but not a definite position in space.
17. *Work.* If a force acts upon a body and causes motion, it is said to do work.
18. *Power.* The rate of doing work, or work done per unit of time.
19. *Energy.* If a body is capable of doing work, it is said to possess energy.
20. *Potential Energy* is the energy a body has by virtue of its position.
21. *Kinetic Energy* is the energy a body has in virtue of its motion.
22. *Moment of Inertia* of a body about an axis is the sum of the products of the mass of each particle of matter in the body multiplied by the square of its distance from that axis of rotation.
23. *Second Moment of Area* of a body is the sum of the products of the area of each particle of matter in the body, multiplied by the square of its distance from any given axis.
24. *Centre of Gyration.* That point in a rotating body at which the whole mass might be concentrated (theoretically) without altering the resistance of the inertia of the body to angular acceleration.
25. *Radius of Gyration* of a body about a given axis is the radius at which, if an equal mass were concentrated, it would have the same moment of inertia.
26. *Momentum.* The quantity of motion in a moving body, and it is proportional to mass × velocity.
27. *Angular Momentum.* If a particle of mass m lb. is revolving in a circle of r ft. radius and has a linear velocity of v ft./sec. at any instant in the direction of the tangent,

the product of the linear momentum of the particle mv and the perpendicular distance r from the centre is called the Angular Momentum of the particle.

28. *Young's Modulus* is the modulus of elasticity for pure tension with no other stress acting, and is equal to the tensile stress per unit of linear strain. It is denoted by the letter E.

29. *Modulus of Rigidity*, or Shearing Modulus, is the modulus expressing the relation between the intensity of shear stress and the amount of shear strain. It is denoted by the letter C or N.

30. *Angular Velocity* of a rigid body is its time rate of angular displacement, i.e. rate of change of angular position.

31. *Angular Acceleration*. The rate of increase of angular velocity.

32. *Torque*. That which tends to produce torsion.

33. *Gravity* (g). The acceleration given to a body of mass 1 lb. when falling freely under its own weight (1 lb.).

34. *Bending Moment* at a section is the sum of the product of all the forces on one side of that section and the distances of these forces from the neutral axis of that section.

35. *Shearing Force* at a section is the sum of all the vertical forces acting on one side of the section.

36. *Density* is the mass per unit volume of a substance.

37. *Specific Gravity* is the ratio of the weight of any volume of that substance to the weight of an equal volume of water at 4° C.

38. *Calorific Value of a Fuel*. The quantity of heat given out during the complete combustion of unit mass or unit volume of the fuel in oxygen when the products of combustion are cooled down to atmospheric temperature. This is called the Higher Calorific Value. The Lower Calorific Value is the quantity of heat given out as above, but the products of combustion are not cooled and the steam is not condensed. Hence the value is less than the H.C.V. by the latent heat of steam formed.

39. *Latent Heat of Steam* is the amount of heat required to convert 1 lb. of water into steam without change of temperature.

40. *Dryness Fraction.* The mass of pure saturated steam contained in unit mass of wet steam.

41. *Units of Heat.* Quantities of heat are measured by the heat required to raise the temperature of unit mass of water through a given range. *The Centigrade heat Unit* (C.H.U.) is the one-hundredth part of the amount of heat required to raise 1 lb. of water from 0° C. to 100° C. *The British Thermal Unit* is the one hundredth and eightieth part of the amount of heat required to raise 1 lb. of water from 32° F. to 212° F.

42. *Coefficient of Linear Expansion* for a material is the average increase in any dimension, per degree rise in temperature, per unit length.

43. *Dry, Dry Saturated, Saturated Steam* is steam that is generated and absorbs the maximum latent heat so that there is no water in suspension.

44. *Superheated Steam* is dry saturated steam that is heated at constant pressure in a separate vessel—that is, not in contact with its water of generation; its volume increases and the steam becomes superheated, absorbing a quantity of heat equal to $W \times S_p \times$ degree of superheat.

45. *Thermal Efficiency of a Boiler* is the ratio of the heat supplied to produce steam per lb. of fuel to the calorific value of the fuel.

46. *Indicated Horse Power* (*I.H.P.*) is the work done in the cylinder in ft.-lb. per minute divided by 33,000, or mean effective pressure in lb./sq. in. multiplied by area of piston in sq. in. multiplied by length of stroke in ft. multiplied by the number of effective strokes per min. divided by 33,000.

47. *Brake Horse Power* is the work done by engine in ft.-lb. per min. at the brake divided by 33,000.

48. *One Horse Power* is 33,000 ft.-lb. per min.

49. *Thermal Efficiency* of an engine is the heat converted to mechanical work, divided by the heat supplied.

50. *Molecular Weight* of a compound is the sum of the atomic weights of its constituents.

51. *Intensity of Pressure* of a fluid is the pressure per unit area.

52. *Static Head* at any point is the height of the free surface of the liquid above the point.

53. *Centre of Pressure.* The point at which the resultant pressure acts.

54. *Centre of Buoyancy* is the centre of gravity of the volume of liquid displaced by the body.

55. *Metacentre.* An imaginary point about which a floating body is assumed to swing for infinitely small displacements.

56. *Bernoulli's Theorem.* For any mass of water in which there is a continuous connection between all the particles, the total energy of each particle is the same.

CHAPTER I

1.

A spring is elastically stretched 3 in. by a load of 20 lb. How much will it be stretched by a load of 5 lb.?

Since the extension is proportional to the load,

$$\frac{e}{W}=\frac{e_1}{W_1} \quad \therefore \frac{3}{20}=\frac{e_1}{5}\,;\; e_1=\frac{15}{20}=\underline{0{\cdot}75 \text{ in.}}$$

2.

A wire 0·05 in. diameter carries a load of 40 lb. Find the average intensity of stress in lb. per sq. in.

$$\text{Area of cross-section of wire}=0{\cdot}7854\times(0{\cdot}05)^2$$
$$=0{\cdot}00196 \text{ sq. in.}$$

$$\text{Stress intensity}=\frac{40 \text{ lb.}}{0{\cdot}00196 \text{ sq. in.}}=\underline{20{,}410 \text{ lb. per sq. in.}}$$

3.

What diameter of round steel rod is required to carry a tensile load of 10 tons if the intensity of stress is not to exceed 6 tons per sq. in.?

Let d=diameter of rod in in.

$$\text{Stress intensity}=\frac{\text{Load}}{\text{Area}}$$

$$\therefore 6=\frac{10}{0{\cdot}7854\times d^2}\,;\; d=\sqrt{\frac{10}{6\times 0{\cdot}7854}}$$
$$=\underline{1{\cdot}45 \text{ in.}}$$

4.

A round tie-bar of mild steel 1·5 in. diameter carries a tensile load of 7 tons. Find the intensity of tensile stress in the bar.

$$\text{Cross-sectional area of bar}=\frac{\pi d^2}{4}=\frac{\pi\times(1{\cdot}5)^2}{4}=1{\cdot}765 \text{ in.}^2$$

$$\text{Stress intensity}=\frac{7}{1{\cdot}765}=\underline{3{\cdot}96 \text{ ton per sq. in.}}$$

5.

Two men pull on a rope, one end of which is fastened to a fixed iron ring. If they each pull with a force of 100 lb., what is the tension in the rope, and what force will the ring exert on the rope?

Tension in rope = sum of two forces = 100 + 100

= 200 lb.

Since action and reaction are equal and opposite, the ring exerts a force of 200 lb.

6.

What thickness of cast-iron pipe 12 in. internal diameter will be required to stand an internal pressure of 60 lb. per sq. in. if the hoop stress in the pipe is not to exceed 1000 lb. per sq. in.?

Bursting force = resisting force

$$p \times d \times l = 2f \times t \times l$$

where p = internal pressure in lb./in.²
d = internal diameter of the C.I. pipe
t = thickness of pipe in in.
f = hoop stress in lb./in.²
l = length of pipe

$$t = \frac{pd}{2f} = \frac{60 \times 12}{2 \times 1000} = \frac{720}{2000} = 0{\cdot}36 \text{ in.}$$

CHAPTER II

7.

A mild steel tie-bar 15 ft. long has a diameter of 1·5 in., and carries a load of 10 tons. Find how much it will stretch if E=13,000 ton per sq. in.

$$\text{Cross-sectional area} = 0{\cdot}7854 \times (1{\cdot}5)^2 = 1{\cdot}767 \text{ sq. in.}$$

$$\text{Stress} = \frac{\text{total load}}{\text{area}} = \frac{10}{1{\cdot}767} = 5{\cdot}66 \text{ ton/in.}^2$$

$$E = \frac{\text{Stress intensity}}{\text{Strain}}$$

$$13{,}000 \text{ ton/in.}^2 = \frac{5{\cdot}66 \text{ ton/in.}^2}{\text{stretch in in. per in.}}$$

$$\text{Stretch in in. per in.} = \frac{5{\cdot}66}{13{,}000} = 0{\cdot}000435$$

$$\text{Total stretch on 15 ft. (180 in.)} = 180 \times 0{\cdot}000435$$

$$= \underline{0{\cdot}0783 \text{ in.}}$$

8.

A bar of mild steel 1 in. in diameter is subjected to a pull of 15 tons. What is the total stretch on an 8-in. length of the bar? And what is the fractional strain?

$$E = 13{,}000 \text{ ton/in.}^2$$

$$\text{Cross sectional area} = 0{\cdot}7854 \times (1)^2 = 0{\cdot}7854 \text{ in.}^2$$

$$\text{Stress} = \frac{\text{total load}}{\text{area}} = \frac{15}{0{\cdot}7854} = 19{\cdot}1 \text{ ton/in.}^2$$

$$\text{Strain in in. per in.} = \frac{\text{stress intensity}}{E}$$

$$= \frac{19{\cdot}1 \text{ ton/in.}^2}{13{,}000 \text{ ton/in.}^2} = \underline{0{\cdot}00147}$$

$$\text{Total stretch on 8 in.} = 8 \times 0{\cdot}00147 = \underline{0{\cdot}01176 \text{ in.}}$$

9.

A load of 250 lb., when applied at the free end of a vertical wire 20 ft. long, produces an extension of 0·045 in. What is the diameter of the wire, if E=12,000 tons/sq. in?

Let d=diameter of wire in inches

Area of cross-section $=0{\cdot}7854\times d^2$ sq. in.

$$\text{Intensity of stress}=\frac{\text{total load}}{\text{area}}=\frac{250\text{ lb.}}{0{\cdot}7854d^2\text{ sq. in.}}$$

Total stretch $=0{\cdot}045$ in. in 20 ft.

$$\text{Strain}=\frac{0{\cdot}045}{20\times 12}=0{\cdot}0001875$$

$$\text{Strain}=\frac{\text{stress intensity}}{E}$$

$$0{\cdot}0001875=\frac{\dfrac{250}{0{\cdot}7854d^2}}{12{,}000\times 2240}$$

$$d^2=\frac{250}{0{\cdot}0001875\times 0{\cdot}7854\times 12{,}000\times 2240}$$

$$d=\underline{0{\cdot}2484\text{ in.}}$$

10.

A hole 2 in. square is punched out of a metal plate 0·125 in. thick. If the ultimate shear stress intensity of the metal is 45,000 lb. per sq. in., what load is required? The punching tool is attached to a ram 3 in. diameter. What is the compressive stress intensity in the ram?

$$\text{Ultimate strength in shear}=\frac{\text{load}}{\text{perimeter of hole}\times\text{thickness of plate}}$$

$$45{,}000=\frac{\text{load}}{8\times 0{\cdot}125}$$

Load $=45{,}000$ lb.

$$\text{Stress intensity in ram}=\frac{\text{load}}{\text{area}}=\frac{45{,}000\text{ lb.}}{0{\cdot}7854\times(3)^2\text{ sq. in.}}$$

Stress intensity $=\underline{6366\text{ lb. per sq. in.}}$

11.

A hollow cylindrical cast-iron column is 12 in. external diameter and 9 in. internal diameter and 20 ft. long. If a load of 100 tons is placed on top of the column, find how much it will shorten.

$$E = 8000 \text{ tons per sq. in.}$$

$$\text{Area of cross-section} = \frac{\pi}{4}(d_1^2 - d_2^2)$$

$$= 0{\cdot}7854\,(12^2 - 9^2) = 49{\cdot}48 \text{ in.}^2$$

$$\text{Intensity of stress} = \frac{\text{total load}}{\text{area}} = \frac{100}{49{\cdot}48} = 2{\cdot}02 \text{ ton/sq. in.}$$

$$\text{Proportional strain} = \frac{\text{stress intensity}}{E} = \frac{2{\cdot}02 \text{ ton per sq. in.}}{8000 \text{ ton per sq. in.}}$$

$$= 0{\cdot}000252$$

$$\text{Total shortening on 20 ft.} = 20 \times 12 \times 0{\cdot}000252$$

$$= \underline{0{\cdot}0606 \text{ in.}}$$

12.

A strain gauge is fixed to a tie-bar of a rectangular cross-section 3 in. by 0·5 in. The gauge indicates a stretch of 0·0045 in. on a length of 8 in., when the tie-bar is under load. Find the intensity of stress and the total load carried by the tie-bar.

$$E = 13{,}000 \text{ tons per sq. in.}$$

$$\text{Strain} = \frac{0{\cdot}0045 \text{ in.}}{8 \text{ in.}}$$

$$= 0{\cdot}0005625$$

$$\text{Intensity of stress} = 0{\cdot}0005625 \times 13{,}000$$

$$= 7{\cdot}31 \text{ ton/in.}^2$$

$$\text{Total load carried} = 7{\cdot}31 \times 3 \times 0{\cdot}5 = \underline{11 \text{ tons.}}$$

13.

A steel overhead wire 1 in. diameter has an ultimate tensile strength of 80 tons per sq. in. If the factor of safety is 5, calculate the allowable pull on the wire and find the corresponding elongation on a 100-ft. span.

$$E = 13{,}500 \text{ tons per sq. in.}$$

$$\text{Area of cross-section} = \frac{\pi d^2}{4} = 0{\cdot}7854 \times 1^2$$

$$= 0{\cdot}7854 \text{ sq. in.}$$

$$\text{Working stress} = \frac{80}{5} = 16 \text{ tons per sq. in.}$$

$$\text{Allowable pull} = 16 \times 0{\cdot}7854 = 12{\cdot}57 \text{ tons}$$

$$\text{Stretch} = \frac{\text{stress intensity}}{E} = \frac{16{\cdot}0}{13{,}500} = 0{\cdot}00119$$

$$\text{Total elongation} = 100 \times 12 \times 0{\cdot}00119$$

$$= \underline{1{\cdot}428 \text{ in.}}$$

14.

A steam pipe 100 ft. long at a temperature of 15° C. has steam at 200° C. passed through it. What will its length be if it is free to expand? What stress will be induced in the material if expansion is prevented?

$$E = 6500 \text{ ton/sq. in.}$$
$$a = 0{\cdot}000012 \text{ per degree C.}$$

Change in temperature $= 200 - 15 = 185°$ C.

Increase in length $= Tal$

$$= 185 \times 0{\cdot}000012 \times 100$$
$$= \underline{0{\cdot}222 \text{ ft.}}$$

Stress induced in material $= TaE = 185 \times 0{\cdot}000012 \times 6500$

$$= \underline{14{\cdot}43 \text{ tons per sq. in.}}$$

15.

The diameter of the cylinder of an I.C. engine is 2·4 in. and the maximum compression pressure in the cylinder is 250 lb. per sq. in. The cylinder head is held by four bolts whose core diameter is 0·5 in. and length 2 in. What is the maximum tensile stress in each bolt, and the elongation of each bolt?

$$E = 30 \times 10^6 \text{ lb. per sq. in.}$$

Maximum force exerted on each bolt

$$= \frac{\pi d^2}{4} \times \frac{250}{4} = 0{\cdot}7854 \times 2{\cdot}4^2 \times \frac{250}{4}$$
$$= 282{\cdot}7 \text{ lb.}$$

Maximum stress in each bolt $= f_t = \dfrac{\text{max. force}}{\text{area}}$

$$= \frac{282{\cdot}7}{0{\cdot}7854 \times (0{\cdot}5)^2} = \underline{1440 \text{ lb./in.}^2}$$

Elongation of each bolt $= \dfrac{f_t \times l}{E} = \dfrac{1440 \times 2}{30 \times 10^6}$

$$= \underline{0{\cdot}0000953 \text{ in.}}$$

16.

A wire 0·025 in. in diameter is required to stretch 0·39 in. under a load of 15 lb. Find the length of the wire, if $E = 13{,}000$ tons/in.^{2}

Let l=length of wire in inches

$$\text{Cross-sectional area of wire}=\frac{\pi d^2}{4}=\frac{\pi\times(0\cdot025)^2}{4}\text{ sq. in.}$$

$$\text{Stress intensity}=\frac{\text{Load}}{\text{Area}}=\frac{15\times4}{\pi\times(0\cdot025)^2}$$

$$\text{Modulus of elasticity } E=\frac{\text{Stress}}{\text{Strain}}=\frac{\text{Stress}}{\delta l/l}$$

$$\therefore\ 13{,}000\times2240=\frac{\dfrac{15\times4}{\pi\times(0\cdot025)^2}}{\dfrac{0\cdot39}{l}}=\frac{15\times4\times l}{\pi\times(0\cdot025)^2\times0\cdot39}$$

$$l=\frac{13{,}000\times2240\times\pi\times(0\cdot025)^2\times0\cdot39}{15\times4}=\underline{371\cdot5\text{ in.}}$$

17.

A brass and a steel wire, each 12 ft. in length and of diameter 0·07 and 0·09 in. respectively, hang vertically from two points in the same horizontal plane, and 6 in. apart. The lower ends of the two wires are connected to a light rod, and this rod carries a weight of 120 lb. placed midway between the wires. Calculate the angle at which the rod will be inclined to the horizontal, if E for brass$=9\times10^6$ lb./in.^{2} and for steel $E=30\times10^6$ lb./in.^{2}.

$$\text{Cross-sectional area of brass wire}=\frac{\pi d^2}{4}=\frac{\pi\times(0\cdot07)^2}{4}\text{ sq. in.}$$

$$\text{Cross-sectional area of steel wire}=\frac{\pi d_1^2}{4}=\frac{\pi\times(0\cdot09)^2}{4}\text{ sq. in.}$$

Since the load is midway between the wires,

$$\text{Load per wire}=\frac{120}{2}=60\text{ lb.}$$

$$E=\frac{\text{Stress}}{\text{Strain}}=\frac{\text{Stress}}{\delta l/l}$$

where δl=increase in length in in.

$$\text{For brass wire } 9\times10^6=\frac{60}{\dfrac{\pi\times(0\cdot07)^2}{4}}\Big/\delta l/144=\frac{60\times4}{\pi\times(0\cdot07)^2}\times\frac{144}{\delta l}$$

$$\therefore\ \delta l=\frac{60\times4}{\pi\times(0{\cdot}07)^2}\times\frac{144}{9\times10^6}=0{\cdot}249 \text{ in.}$$

For steel $$\delta l=\frac{60\times4}{\pi\times(0{\cdot}09)^2}\times\frac{144}{30\times10^6}=0{\cdot}0453 \text{ in.}$$

Difference in lengths $=0{\cdot}249-0{\cdot}0453=0{\cdot}2037$ in.

If θ = Angle of rod to horizontal,

$$\sin\theta=\frac{0{\cdot}2037}{6}=0{\cdot}0339$$

$$\therefore\ \theta=\underline{56'}.$$

18.

A thin liner is to be shrunk on to a cylindrical shaft 9 in. in diameter. What should be the internal diameter of the liner before heating, so that it may grip the shaft with a tension of 5 ton/sq. in.? Assume $E=30\times10^6$ lb./in.^{2} and neglect the compressive strain in the shaft.

Let d = original internal diameter in inches

x = extension of diameter in inches

then $$x+d=9$$

$$E=\frac{\text{Stress}}{\text{Strain}}$$

$$\therefore\ \frac{x}{d}=\frac{5\times2240}{30\times10^6}=0{\cdot}000373$$

$$\therefore\ 1{\cdot}000373d=9$$

$$d=\underline{8{\cdot}996 \text{ in.}}$$

19.

Steel rails are welded together and are unstressed at a temperature of 60° F. They are prevented from buckling and cannot expand or contract. Find the stresses at 120° F., taking steel as expanding 0·0012 of its length for a temperature change of 180° F. $E=30\times10^6$ lb./in.^{2}. If the elastic limit is 40,000 lb./in.^{2}, at what temperature would it be reached?

Elongation per unit length at

$$120^\circ\text{ F.}=\frac{(120-60)\times0{\cdot}0012}{180}$$

$$=0{\cdot}0004$$

$$E=\frac{\text{Stress}}{\text{Strain}}$$

$\therefore$ Stress $= E \times$ Strain

$= 30 \times 10^6 \times 0{\cdot}0004 = 12{,}000$ lb./sq. in.

If stress reaches 40,000 lb./sq. in.,

then $40{,}000 = 30 \times 10^6 \times \text{strain}$

$$\text{Strain} = \frac{40{,}000}{30 \times 10^6} = \frac{\text{Rise in temperature} \times 0{\cdot}0012}{180}$$

$$\therefore \text{Rise in temperature} = \frac{40{,}000 \times 180}{30 \times 10^6 \times 0{\cdot}0012} = 200^\circ \text{ F.}$$

Actual temperature $= 200 + 60 =$ 260° F.

20.

A surveyor's tape nominally 100 ft. long is 0·5 in. wide and 0·04 in. thick. The length is correct when used at a temperature of 60° F. and under a pull of 15 lb. By how much will it be in error when used at a temperature of 100° F. and under a pull of 10 lb.?

$$E = 30 \times 10^6 \text{ lb./in.}^2$$

Coefficient of linear expansion

$a = 0{\cdot}000006$ per °F.

Let l = length with no tension at 60° F.

x = extension under load of 15 lb.

$\therefore l + x = 100 \times 12$ in.

$$\frac{x}{l} = \text{Strain} = \frac{\text{Stress}}{E} = \frac{15}{0{\cdot}5 \times 0{\cdot}04 \times 30 \times 10^6} = \frac{1}{40{,}000}$$

$\therefore 40{,}000x = l$

$\therefore 40{,}000x + x = 1200$

$x = 0{\cdot}03$ in.

$l = 1199{\cdot}97$ in.

If x for load of 15 lb. $= 0{\cdot}03$ in.

$\therefore x_1$ for load of 10 lb. $= 0{\cdot}02$ in.

$\therefore$ Length under pull of 10 lb. is $1199{\cdot}97 + 0{\cdot}02 = 1199{\cdot}99$

Hence for an increase of temperature of 40° F.,

$xt = 40 \times 0{\cdot}000006 \times 1199{\cdot}99 = 0{\cdot}288$ in.

Length of tape $= 1199{\cdot}99 + 0{\cdot}288 = 1200{\cdot}28$ in.

$\therefore$ Error = 0·28 in. too large.

21.

Two vertical wires of equal length are rigidly fixed at their upper ends at a distance 3 ft. apart. Their bottom ends are connected by a horizontal bar. One of the wires is made of steel and is 0·1 in. diameter; the other is made of brass and is 0·2 in. diameter. Where on the horizontal bar must a weight be placed so that the bar remains horizontal after the extension of the wires?

E for steel $= 30 \times 10^6$ lb./in.^{2}

E for brass $= 15 \times 10^6$ lb./in.^{2}

Let x = distance of load from steel wire in ft. (see Fig. 1)

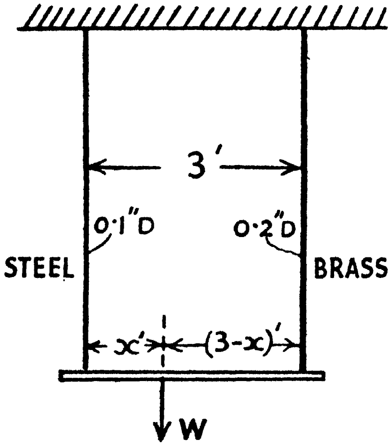


FIG. 1.

Let W = load in lb.

Then load carried by brass wire $= \dfrac{Wx}{3}$ lb.

load carried by steel wire $= \dfrac{W(3-x)}{3}$ lb.

Strain in each wire must be equal if bar is to remain horizontal.

$$\therefore \frac{Wx \times 4}{3\pi(0\cdot2)^2 \times 15 \times 10^6} = \frac{W(3-x)\times 4}{3\pi(0\cdot1)^2 \times 30 \times 10^6}$$

$$\frac{x}{(0\cdot2)^2} = \frac{(3-x)}{(0\cdot1)^2 \times 2}$$

$$0\cdot02x = 0\cdot04(3-x)$$

$$x = 2 \text{ ft.}$$

Weight must be placed at a distance of 2 ft. from the steel wire.

CHAPTER III

Bow's Notation and Vectors

Before working out any problems on the applications of the triangular and polygon force diagrams, it is desirable to state the way in which the forces are named by letters. The sides of

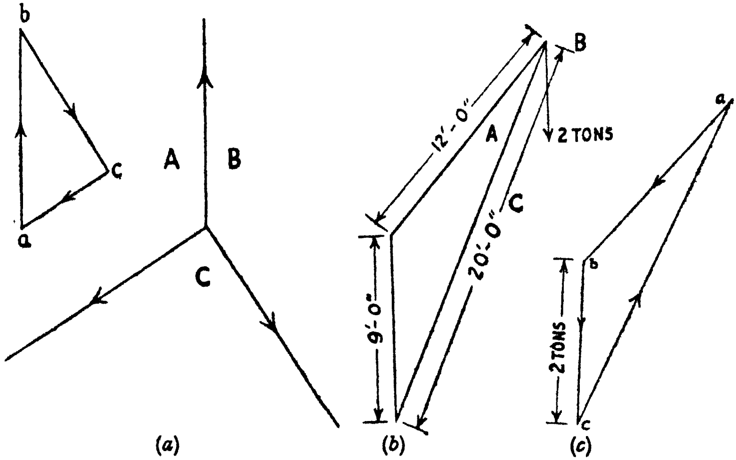


FIG. 2.

a triangle or polygon which represent the forces in magnitude and direction are called *vectors*. The small letters of the alphabet are used for the *force* or *vector* diagram. The capital letters are used for the *space* diagram. Fig. 2 (*a*) illustrates the system of lettering and is known as Bow's Notation.

22.

A jib crane has a jib 20 ft. long and a tie-rod 12 ft. long attached to a post 9 ft. vertically above the foot of the jib. Find the forces in the jib and tie rod when a load of 2 tons is lifted by the crane.

Fig 2 (*b*) sets out the problem in the space diagram, and Fig. 2 (*c*) gives the solution in the vector diagram. First draw the space diagram to scale, in this case 5 ft.=1 in. Then set off to scale 1 ton=1 in. *bc* parallel to *BC*. The directions of *ca* and *ab* are known and are drawn parallel to the jib *CA* and tie-rod *AB*, and the vector triangle *abc* is complete. The lengths of *ba* and *ca* are now measured to scale. The direction of the forces is also found from the vector diagram.

Force in jib=4·44 ton thrust
Force in tie-rod=2·66 ton pull

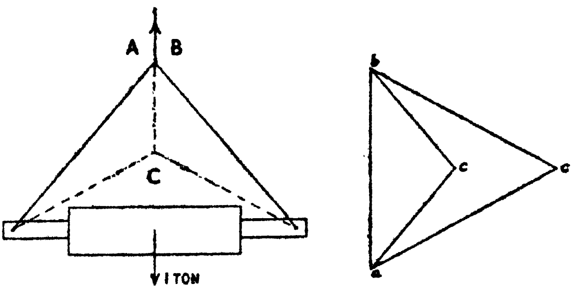
Since the triangular vector diagram is geometrically similar to the space diagram, the forces can be easily calculated without drawing the diagrams to scale. Thus $ca=2$ tons$\times 20/9=4\frac{4}{9}$ tons.

$$ab=2\text{ tons}\times\frac{12}{9}=\underline{2\tfrac{2}{3}\text{ tons.}}$$

23.

The rotor of a generator weighing 1 ton is slung from a crane. If the crane hook is vertically above the centre of gravity of the rotor, and the sling is placed symmetrically, what is the tension in the two sling ropes, and what would be the effect of shortening the sling?

Draw the space diagram Fig. 3 (*a*) and assign the letters



(*a*) (*b*)

FIG. 3.

A, *B* and *C* to the spaces round the junction of the forces at the hook, and draw the vector diagram for the three forces which are in equilibrium at the hook. Set off *ab* upwards to represent the force of 1 ton to scale, 3 in.=1 ton, and draw *ac* parallel to *AC* to meet *bc* drawn parallel to *BC* in *c*. The length of *bc* represents the pull of the rope *BC* on the hook. It scales 0·63 ton and the pull on the rope CA by symmetry must be the same (Fig. 3 (*b*)).

Redrawing the figure with a shorter sling—it will be noticed that the inclination to the horizontal is less and c lies further from ab, and the tension is increased, in this case to 1 ton.

24.

Determine the force acting in each member of the given structure, a roof truss which is simply supported at the ends of its span.

It carries a load of 6000 lb. at the apex.

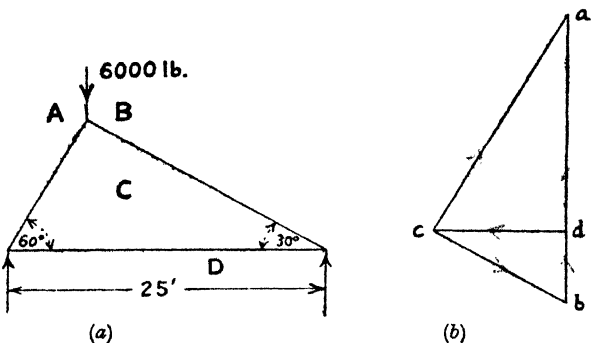


FIG. 4.

Letter the spaces A, B, C, D as shown and drawn in Fig. 4 (*a*). Then draw the vector ab downwards to represent to scale 1 in. = 2000 lb. the only known force at the apex joint (Fig. 4 (*b*)). Complete the vector triangle; since these three forces AB, BC and CA are in equilibrium bc scales 3000 lb. representing the thrust in rafter BC pushing from b to c at the apex, and ca scales 3200 lb., this is the thrust in rafter CA at the apex joint. Now consider joint at right-hand support. The joint is in equilibrium due to the action of three forces, the thrust in the rafter from a to c, the reaction of the support and the force exerted by the tie-rod. We can complete the vector triangle by using the vector ca already drawn and drawing cd parallel to CD. Then dac represents the vector triangle for the joint in that sequence of letters. We have now determined the remaining force in the frame *cd* which is a tie, and equal to 2600 lb., the reaction *da*=4500 lb. and *db*=1500 lb. This could have been solved by using the principle of moments.

25.

A large lamp weighing 100 lb. is suspended from the ceiling by a chain. It is pulled aside by a horizontal cord until the chain

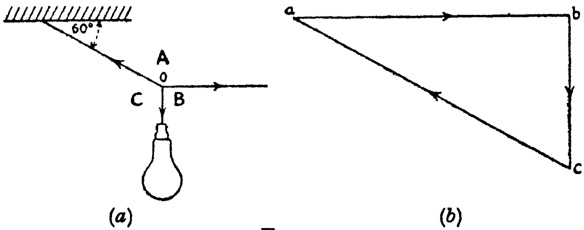


FIG. 5.

makes an angle of 60° with the ceiling. Find the tensions in the cord and the chain.

Fig. 5 (*a*) represents the space diagram. Letter the spaces as shown and draw the vector diagram to scale (Fig. 5 (*b*)), in this case 1 in.=50 lb. Since the forces acting at the point *O* are in equilibrium, the sides of the vector triangle represent the tensions in the cord and chain to scale. Pull in chain *ca*=200 lb., pull in cord *ab*=173·2 lb.

26.

A ladder 20 ft. long rests at an angle of 60° to the horizontal, its upper end resting against a smooth vertical wall. The lower end is on rough ground. If the ladder weighs 100 lb. and its c.g. is 10 ft. from the lower end, what will be the reactions of the wall and the ground on the ladder?

This is an example of Concurrency in the equilibrium of three forces.

Fig. 6 (*a*) represents the space diagram. The weight of the ladder, 100 lb., acts vertically downwards through the centre of gravity *G*.

The wall, being "smooth," means that the reaction *F* exerted by the wall is perpendicular to the wall. The "smooth" wall

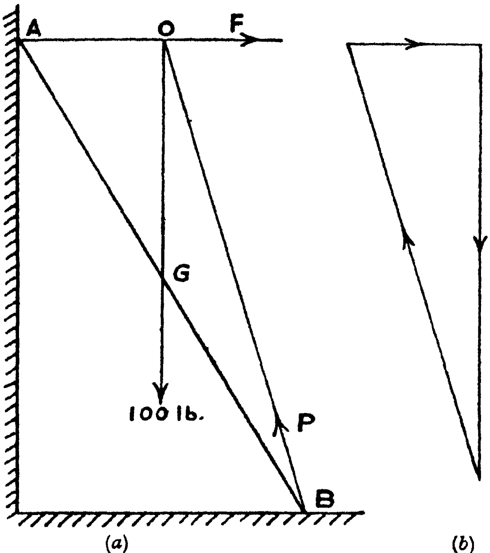


FIG. 6.

offers no upward support. Therefore the reaction F at A will meet the vertical line through G at O.

By the Concurrency theorem the reaction P of the ground at B must pass through O. Hence by drawing the vector diagram to scale 25 lb. = 1 in., the force F and P can be found (Fig. 6 (*b*)).

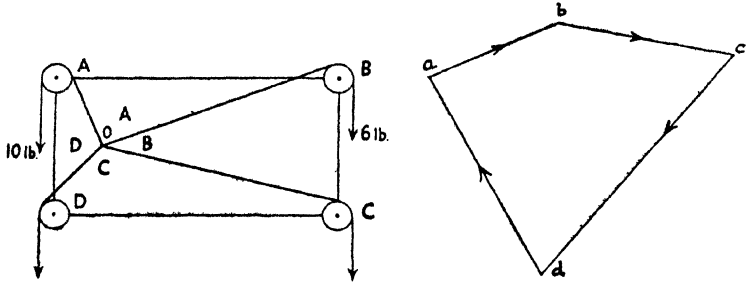
From diagram $\underline{F = 28 \text{ lb.}}$

From diagram $\underline{P = 104 \text{ lb.}}$

27.

A rectangular board $ABCD$ has the following dimensions: AB is 6 ft., AD is 3 ft. The board is fixed in a vertical plane with AB horizontal. Four pulleys are fixed at the corners and over these pass four strings knotted together at O. O is 1·5 ft. from DC and 1 ft. from AD. If the string OA carries 10 lb., and string OB 6 lb., find the weight carried by OC and OD, when O is in equilibrium.

Fig. 7 (*a*) illustrates the position of the strings drawn to scale. Using Bow's Notation, letter the space diagram as shown. Since



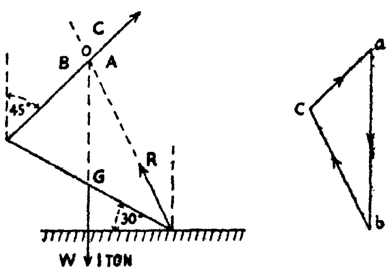
(*a*) (*b*)

FIG. 7.

O is in equilibrium, the four forces can be represented by the sides of a polygon taken in order. To find the two unknown forces *OC* and *OD*, take any point *d* and draw vector *da* parallel to *DA* to represent by its length, 10 lb. (scale 1 in. to 4 lb.) (Fig. 7 (*b*)). Then draw *ab* parallel to *AB* and representing 6 lb. on the same scale. Then draw from *b* a vector parallel to *BC*, and from *d* draw a vector parallel to *CD*. Since the forces are in equilibrium these two forces must meet at *c*. Scale off *dc* and *bc*. These vectors represent the forces in strings *OD* and *OC*.

Force in *OD*=12·8 lb.

Force in *OC*=7·6 lb.



(*a*) (*b*)

FIG. 8.

28.

A uniform rectangular steel plate 20 ft.×6 ft. rests on the ground. The plate weighs 1 ton and is held at an angle of 60° to the vertical by the chain of a crane, the chain making an angle of 45° with the vertical. The chain is attached to the middle of the upper short side of the plate. Find the tension in the chain and the magnitude and direction of the reaction of the ground on the plate.

Fig. 8 (*a*) represents the space diagram for the steel plate to scale. Since the plate is in equilibrium due to the action of the three forces *T* the tension in the chain, *W* the weight of the plate acting through its c.g., *G* and *R* the reaction of the ground,

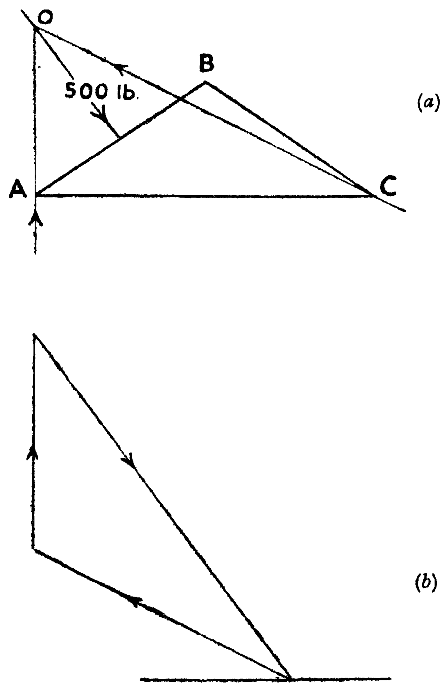


FIG. 9.

they must all meet in the point O. Letter the space diagram and draw the vector diagram Fig. 8 (b) to scale 1 ton=2 in.

From the diagram read off the tension in the chain, and the magnitude and direction of the Reaction R.

T=0·45 ton

R=0·75 ton upward at 65° to surface of ground.

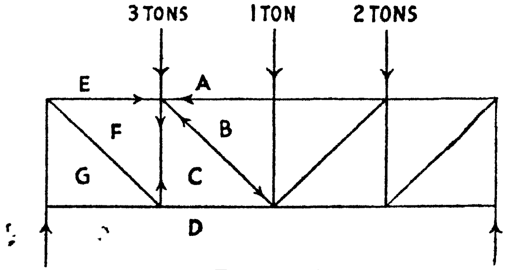


FIG. 10 (a).

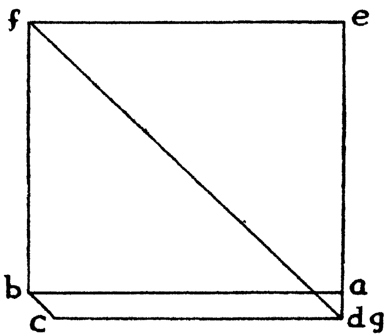


FIG. 10 (b).

29.

The structure ABC is supported at two points A and C. AB is 11 ft. long, BC is 11 ft. long and AC is 18 ft. long. The reaction at A is vertical and the reaction at C is oblique. A force of 500 lb. acts at the centre of AB and at right angles to it. Find the magnitude of the reactions at A and C and the inclination of the reaction at C to the vertical.

Fig. 9 (*a*) represents the structure. Since the forces are in equilibrium, the reaction at *A* and the force of 500 lb. meet at *O*. Therefore the reaction at *C* must also meet at this point. Draw the triangle of forces, choosing a suitable scale. This is shown in Fig. 9 (*b*).

The reactions at *A* and *C* can now be scaled off and are equal to reaction at *A* 322·5 lb., reaction at *C* 257·5 lb. Inclination of reaction at *C* to vertical=54°.

30.

A girder is loaded as shown in Fig. 10 (*a*). Find the values of the forces in the members *AB* and *CD*. State whether the forces are compressive or tensile.

Re-draw the space diagram to scale and letter according to figure. In order to find the forces in the members *AB* and *CD* it will be necessary to draw the force diagram as shown in Fig. 10 (*b*). Scale off from the diagram the force in *AB* which equals 3·5 tons and the member is in compression. Scale off the force in member *CD*, and it is found to be equal to 3·2 tons and is in tension.

31.

The framework shown in Fig. 11 is supported at its ends in the manner indicated and loaded as shown. The depth of the framework is 10 ft. and the loads are spaced at distances of 5 ft. apart. Find the stress in each member.

Letter the diagram as in Fig. 11 (*a*). Then draw the load line *da* making *dc*=5 ton, *cb*=10 ton and *ba*=5 ton to some suitable scale. Since the Warren girder is uniformly loaded, the reactions at the ends will be equal; hence bisect *da* at *e*. Now draw the triangle of forces *edl* for point *EDL*. Then complete the stress diagram in the usual manner (Fig. 11 (*b*)).

The stress in the member can now be scaled off.

DL, *CK*, *BG*, *AF* are all in tension and=5 tons each

LK, *GF* are both in tension and=5 tons each

KH, *HG* are both in tension and=5·5 tons each

EH is in compression and=7·5 tons

EL, *EF* are in compression and=11·5 tons each.

32.

The framework shown in Fig. 12 (*a*) on the opposite pag consists of three equilateral triangles, the length of each side bein

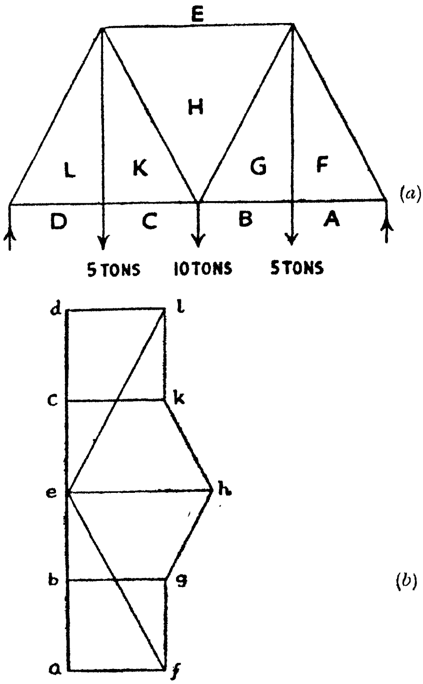


FIG. 11.

10 ft. It is supported in the manner indicated and loaded as shown. Find the stress in each member.

Letter the diagram as in Fig. 12 (*a*). Then draw the load line *ab* parallel to *AB*, using a suitable scale.

Since the girder is uniformly loaded, the reactions at the ends will be equal. Hence bisect *ab* at *c* (Fig. 12 (*b*)). Commence at reaction *R*. Draw *ad* parallel to *AD* and from *c* draw *cd* parallel

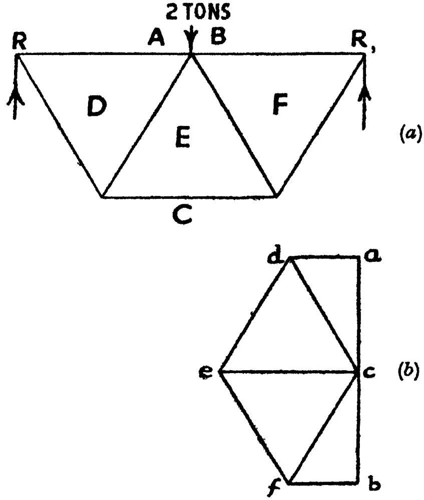


FIG. 12.

to *CD*. Then work round each joint in order and complete the stress diagram. Scale off the stresses in each member.

Member	*Stress* (*tons*)
AD	0·6 compressive
BF	0·6 ,,
DE	1·18 ,,
EF	1·18 ,,
DC	1·18 tension
EC	1·15 ,,
FC	1·15 ,,

CHAPTER IV

33.

A horizontal steel girder of uniform section 20 ft. long rests on two supports, one at the end of the beam, and the second 5 ft. from the other end of the beam. (The weight of the beam is 600 lb.) Find the upward reactions of the support.

Let R_A and R_B be the reactions at the supports A and B, as shown in Fig. 13.

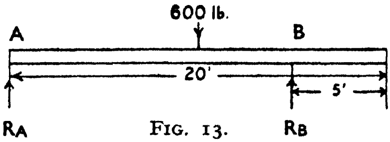


FIG. 13.

Taking moments about A,

$$\text{Clockwise moment} = 600 \times 10 = 6000 \text{ lb.-ft.}$$

$$\text{Anti-clockwise moment} = R_B \times 15 = 15R_B \text{ lb.-ft.}$$

Since beam is in equilibrium, clockwise moment balances anti-clockwise moment.

$$\therefore 6000 = 15R_B$$

$$400 \text{ lb.} = R_B$$

$$\text{Total upward force} = R_A + R_B = 600$$

$$\therefore R_A = 600 - 400 = \underline{200 \text{ lb.}}$$

This result can also be obtained by taking moments about B.

34.

A uniform steel girder 30 ft. long weighs 4500 lb. It is supported at one end and at a point 10 ft. from the other end. It carries a weight of 3000 lb. 10 ft. from the supported end and a load of 2000 lb. at the other end. Find the reactions of the supports.

Fig. 14 represents the loads on the beam.

Taking moments about A (Fig. 14),

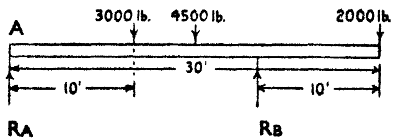


FIG. 14.

Clockwise moment $=(3000\times 10)+(4500\times 15)+(2000\times 30)$
$=157{,}500$ lb.-ft.

Anti-clockwise moment $=R_B\times 20$

Equating moments, $20R_B=157{,}500$

$\underline{R_B=7875 \text{ lb.}}$

Equating upward forces to downward forces,

$R_A+R_B=3000+4500+2000=9500$ lb.

$R_A=9500-7875=\underline{1625 \text{ lb.}}$

35.

A bell crank lever shown in Fig. 15 has the following dimensions: *AB* is 3 in., *BC* is 2 in., and angle *ABC* is 73°. The force

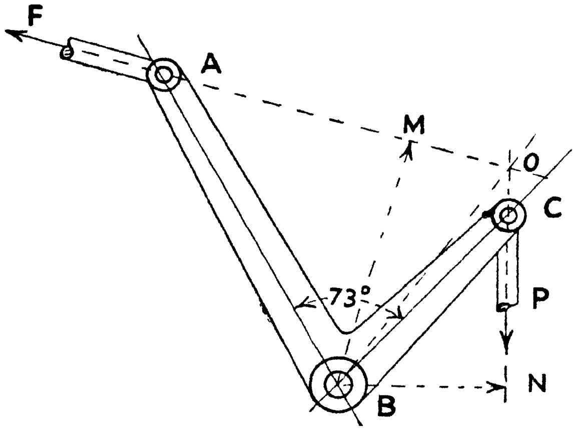


FIG. 15.

at *F* is 12 lb., and the perpendicular distance *MB* is 2·2 in. Find the pull acting at *P* if the perpendicular distance *BN* is 1·35 in.

Taking moments about *B*,

Clockwise moment $=P\times BN=P\times 1{\cdot}35$ lb.-in.

Anti-clockwise moment $=F\times BM=12\times 2{\cdot}2=26{\cdot}4$ lb.-in.

Equating moments, $P\times 1{\cdot}35=26{\cdot}4$

$$\therefore\ P=\frac{26{\cdot}4}{1{\cdot}35}=19{\cdot}55 \text{ lb.}$$

$\underline{\text{Pull acting at } P=19{\cdot}55 \text{ lb.}}$

This question can also be solved by producing the line of action of the forces F and P to meet at O (Fig. 15)). Join O and B and draw the triangle of forces.

36.

A wall crane is represented diagrammatically in Fig. 16 (*a*), and carries a load as shown. Find the tension in the tie-rod AB.

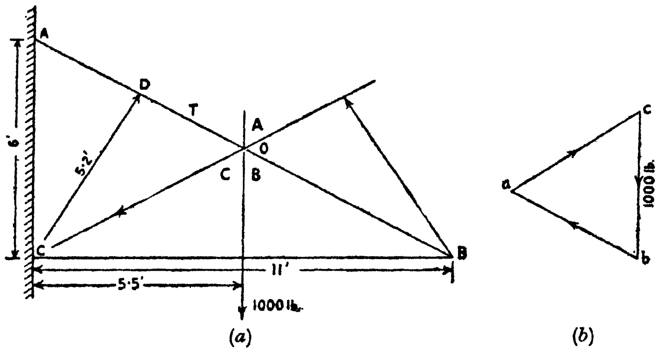


FIG. 16.

The forces acting on the member BC are: the weight of 1000 lb., the unknown tension in AB and whatever force exerted upon it through the pivot at the end C. This force is also unknown. If we take moments about an axis perpendicular to the diagram and through C, there will be no moment of the force about this axis. Draw the diagram to scale and find the perpendicular distance CD. In this case scale, 2 ft. $=$ 1 in., it is 5·2 ft.

Taking moments about C,

$$\text{Clockwise moment} = 1000 \times 5{\cdot}5 = 5500 \text{ lb.-ft.}$$

$$\text{Anti-clockwise moment} = T \times 5{\cdot}2 = 5{\cdot}2\ T \text{ lb.-ft.}$$

Since there is no rotation about the axis, we have

$$5{\cdot}2T = 5500$$

$$\underline{T = 1058 \text{ lb.}}$$

An alternative method is by drawing the triangle of forces for the three forces which meet at O. This is done in Fig. 16 (*b*).

37.

Find the c.g. of the T-shaped piece of thin, uniform sheet metal shown in Fig. 17. The sheet weighs 4 lb. per sq. in.

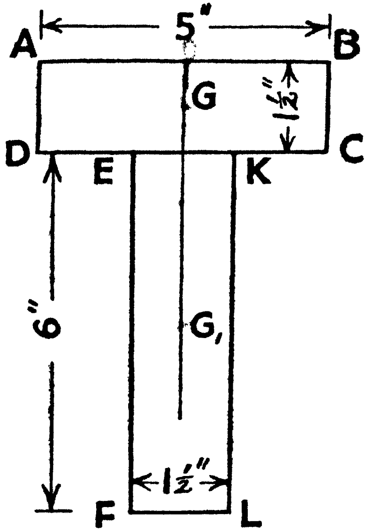


FIG. 17.

Area of cross-piece $ABCD = 5 \times 1{\cdot}5 = 7{\cdot}5$ sq. in.
$Weight = 7{\cdot}5 \times 4 = 30$ lb.
Area of stem $EFLK = 6 \times 1{\cdot}5 = 9$ sq. in.
$Weight = 9 \times 4 = 36$ lb.

c.g. of $ABCD$ is at G in centre line 0·75 in. from AB
c.g. of $EFLK$ is at G_1 in centre line and $1{\cdot}5 + 3$ in. $= 4{\cdot}5$ in. from AB

Taking moments about axis AB. Since the moments of the whole piece must equal the sum of the moments of its parts, let x = distance of c.g. from AB in inches.

We have $$x \times (30+36) = 0{\cdot}75 \times 30 + 4{\cdot}5 \times 36$$

$$x = \frac{184{\cdot}5}{66} = 2{\cdot}795 \text{ in.}$$

Hence the c.g. is in a line GG, at a distance of 2·795 in. from AB.

38.

A thin, uniform plate is in the shape of a square 3 in. side with an equilateral triangle described outwardly on one side

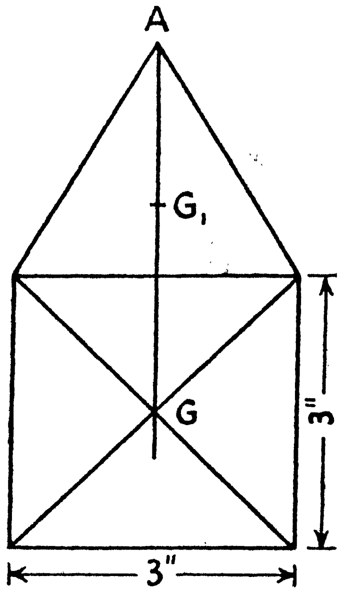


FIG. 18.

(Fig. 18). Find the distance of the c.g. of the plate from the apex of the triangle.

Let W=weight per sq. in. of plate

Weight of square part of plate$=3\times3\times W=9W$ lb.

Weight of triangular part of plate

$$=\tfrac{1}{2}\times3\times2\cdot598\times W=3\cdot897W \text{ lb.}$$

c.g. of square is at G in centre line of square 1·5 in. from edge; and

$$2\cdot598+1\cdot5=4\cdot098 \text{ in. from } A$$

c.g. of triangle is at G_1 in centre line of triangle and 1·732 in. from apex A and in line with G.

Taking moments about A and letting X=distance of c.g. of whole plate from A, we have:

$$X \times (9W + 3{\cdot}897W) = 1{\cdot}732 \times 3{\cdot}897W + 4{\cdot}098 \times 9W$$

$$X = 3{\cdot}383 \text{ in.}$$

Distance of c.g. from apex $A = 3{\cdot}383$ in.

39.

A connecting rod of an engine weighs 600 lb. In order to find its c.g., it is placed on two knife-edge supports, near its ends

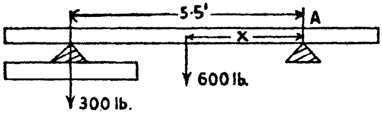


FIG. 19.

and 5·5 ft. apart. The pressure on one support is found to be 300 lb. by a weighing machine. What is the distance of the c.g. of the rod from the other support?

Let X = distance in feet (Fig. 19)

Taking moments about a horizontal axis perpendicular to the rod at the support A, the moment of the weight of the rod about A must balance the moment of the supporting force about A.

$$X \times 600 = 5{\cdot}5 \times 300$$

$$X = \frac{5{\cdot}5 \times 300}{600} = \underline{2{\cdot}75 \text{ ft.}}$$

Note that this is a simple method for finding the c.g. of large pieces of machinery.

40.

A uniform steel disc, 2 ft. diameter, is used as a flywheel. The c.g. is found to be 0·2 in. from the centre. What diameter of hole must be drilled in the disc, 6 in. from the centre, to bring the c.g. to the centre?

Referring to Fig. 20,

Let d = diameter of hole in inches

$$\text{Area} = \frac{\pi d^2}{4} \text{ sq. in.}$$

Let D = diameter of disc

$$\text{Area of steel disc} = \frac{\pi D^2}{4} = \frac{\pi (24)^2}{4} \text{ sq. in.}$$

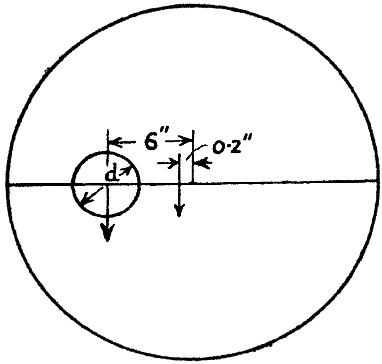


FIG. 20.

Taking moments about centre:

Moment of original disc about centre—moment of hole about centre=moment of disc with hole about centre.

But moment of disc with hole about centre=0

$$\therefore \quad \frac{\pi(24)^2}{4} \times 0{\cdot}2 - \left(\frac{\pi d^2}{4} \times 6\right) = 0$$

$$d^2 = \frac{24 \times 24}{6} \times 0{\cdot}2$$

$$\underline{\underline{d = 4{\cdot}38 \text{ in.}}}$$

41.

In Question 40 the weight of the disc is 40 lb. Find the weight of metal that must be removed to bring the c.g. back to the centre.

Let X=weight of metal to be removed in pounds

$$\frac{X}{40} = \frac{0{\cdot}2}{6}$$

$$X = \frac{40 \times 0{\cdot}2}{6} = \underline{\underline{1\tfrac{1}{3} \text{ lb.}}}$$

42.

A cone and hemisphere made of the same material are arranged so that the cone sits on the hemisphere. The diameter of the base of the cone is 18 in. What will be the height of the cone when the hemispherical surface rests on a horizontal plane in neutral equilibrium?

The equilibrium will be neutral when the c.g. is at O, the centre of the hemisphere (Fig. 21).

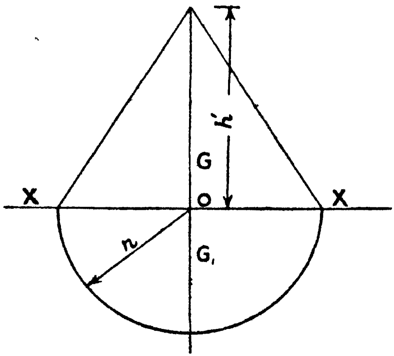


FIG. 21.

$$\text{Volume of cone} = \frac{\pi d^2}{4} \times \frac{h}{3} \text{ cu.ft.}$$

$$= \frac{\pi \times (1 \cdot 5)^2}{4} \times \frac{h}{3}$$

$$\text{c.g. of cone } G = \frac{h}{4} \text{ from } O.$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times (0 \cdot 75)^3$$

c.g. of hemisphere is $\frac{3}{8} \times 0 \cdot 75$ from O at G_1

$$\frac{OG}{OG_1} = \frac{\frac{h}{4}}{\frac{3}{8} \times 0 \cdot 75} = \frac{\text{Weight of hemisphere}}{\text{Weight of cone}} = \frac{\frac{2}{3}\pi \times (0 \cdot 75)^3}{\frac{\pi \times (1 \cdot 5)^2}{4} \times \frac{h}{3}}$$

$$\frac{h}{1 \cdot 125} = \frac{1 \cdot 4925}{h}$$

$$h = \sqrt{1 \cdot 4925 \times 1 \cdot 125} = \underline{1 \cdot 297 \text{ ft.}}$$

If h is less than 1·297 ft. the equilibrium will be stable; if greater, unstable.

43.

A beam 25 ft. long is supported horizontally by two supports, one at the end A and the other 20 ft. from A. The beam carries

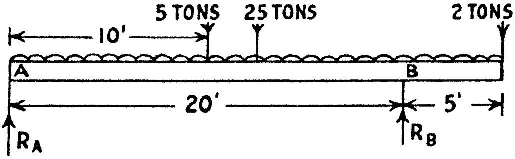


FIG. 22.

a load of 1 ton per foot uniformly spread over its whole length, with a load of 5 tons midway between the supports and 2 tons at the overhung end. Find the reactions at the supports.

Fig. 22 represents the loading of the beam.

Taking moments about A, we have, equating clockwise to anti-clockwise moments.

$$10\times5+12{\cdot}5\times25+25\times2=R_B\times20$$
$$50+312{\cdot}5+50=R_B\times20$$
$$\frac{412{\cdot}5}{20}=R_B$$
$$R_B=\underline{20{\cdot}625 \text{ tons}}$$
$$\therefore R_A=32-20{\cdot}625=\underline{11{\cdot}375 \text{ tons}}$$

This could also be found by taking moments about R_B.

44.

A horizontal beam 25 ft. long is supported at its ends A and B. Vertically loads of 1, 1·5 and 2 tons are applied at 5, 10 and 15 ft. respectively from A. Find the reactions at the supports.

Fig. 23 represents the loading of the beam.

Taking moments about A and equating clockwise to anti-clockwise moments,

$$5\times1+10\times1{\cdot}5+15\times2=R_B\times25$$
$$5+15+30=R_B\times25$$
$$R_B=\frac{50}{25}=\underline{2 \text{ tons}}$$

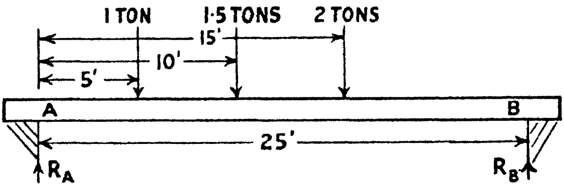


FIG. 23.

$$R_A + R_B = 1 + 1{\cdot}5 + 2$$
$$\therefore R_A = 4{\cdot}5 - 2 = \underline{2{\cdot}5 \text{ tons.}}$$

This result could also be found by taking moments about R_B.

45.

A 30-ft. ladder is carried by two men *A* and *B*. If *A* is bearing five-eighths of the weight, find the position of the centre of gravity

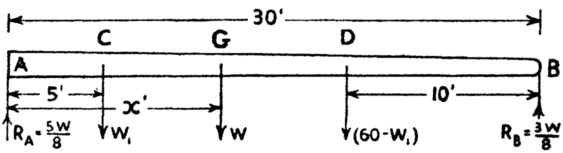


FIG. 24.

of the ladder and the weights of the parts *AG* and *BG* of the ladder. Assume the ladder weighs 60 lb. and the centre of gravity of part *AG* is 5 ft. from *A* and the centre of gravity of part *BG* is 10 ft. from *B*.

Fig. 24 illustrates the ladder diagrammatically.

Let the centre of gravity of the ladder be at *G* distance *x* ft. from *A*.

Taking moments about *A*,

$$x \times W = 30 \times \frac{3W}{8}$$

$$\therefore x = \frac{90}{8} = \underline{11{\cdot}25 \text{ ft.}}$$

To find the weights of AG and BG,

Let W_1 lb. = weight of part AG

then Weight of part $BG = (60 - W_1)$ lb.

R_A now equals $\frac{5 \times 60}{8}$ lb. and $R_B = \frac{3 \times 60}{8}$ lb.

Taking moments about A,

$$5 \times W_1 + (60 - W_1)20 = \frac{3 \times 60}{8} \times 30$$

$$5W_1 + 1200 - 20W_1 = \frac{5400}{8} = 675$$

$$1200 - 675 = 15W_1$$

$$W_1 = \frac{525}{15} = 35 \text{ lb.}$$

$\therefore$ Weight of part $BG = 60 - 35 = \underline{25 \text{ lb.}}$

CHAPTER V

46.

A horizontal rod, 5 ft. long, is hinged at A while the other end rests on a smooth roller at the same level. Forces of 10, 12 and 15 lb. act on the rod, their lines of action being in the same

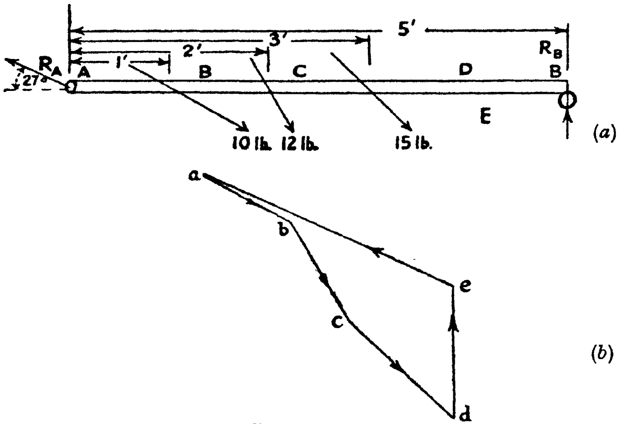


FIG. 25.

vertical plane and intersecting it at distances of 1, 2 and 3 ft. respectively from A, and making angles of 30°, 60° and 45° respectively with AB. Find the magnitude of the reactions at A and B. See Fig. 25 (a).

Taking moments about A,

$$\text{Clockwise moments} = 10 \times 1 \sin 30° + 12 \times 2 \sin 60° + 15 \times 3 \times \sin 45°$$
$$= 5 + 23{\cdot}184 + 41{\cdot}815$$
$$= 69{\cdot}99 \text{ lb./ft.}$$

Anti-clockwise moments

$$= 5 \times R_B$$
$$5R_B = 69{\cdot}99 \text{ lb./ft.}$$
$$R_B = 13{\cdot}99 \text{ lb.}$$

R_A can now be found by drawing the vector polygon (Fig. 25 (b)); the closing side will represent R_A. From diagram,

$$R_A = 27{\cdot}97 \text{ lb.}$$

acting at A, making an angle of 27° with the horizontal.

By resolving the forces horizontally and vertically we have:
Resolving horizontally:

$$H_A - 10 \cos 30 - 12 \cos 60 - 15 \cos 45 = 0$$
$$H_A = 10 \times 0{\cdot}866 + 12 \times 0{\cdot}5 + 15 \times 0{\cdot}7071 = 25{\cdot}26 \text{ lb.}$$

Resolving vertically:

$$V_A - 10 \sin 30° - 12 \sin 60° - 15 \sin 45° + 13{\cdot}99 = 0$$
$$V_A = 10 \times 0{\cdot}5 + 12 \times 0{\cdot}866 + 15 \times 0{\cdot}7071 - 13{\cdot}99 = 12 \text{ lb.}$$

By compounding the two rectangular forces we have:

$$R_A = \sqrt{(25{\cdot}26)^2 + (12)^2}$$
$$R_A = \underline{27{\cdot}97 \text{ lb.}}$$

47.

A ladder 20 ft. long rests against a smooth vertical wall, its lower end on rough ground and 8 ft. from the foot of the wall.

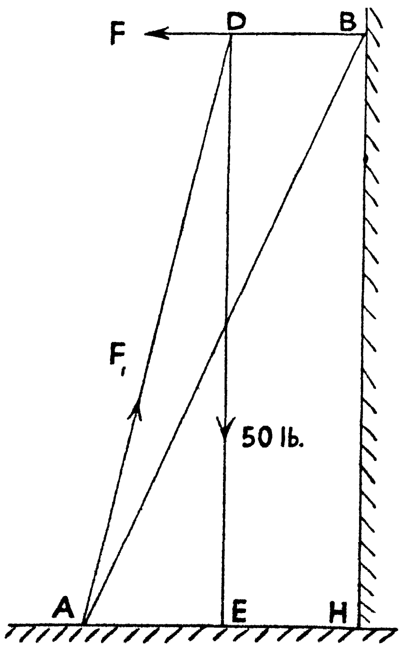


FIG. 26.

If the ladder is of uniform section and weighs 50 lb., find the magnitude and direction of the forces exerted by the wall and ground on the ladder.

There are four methods of solving this question:

(1) By moments.
(2) By drawing the vector diagram.
(3) By applying the converse of the triangle of forces.
(4) By resolving the forces F and F_1 and 50 lb. horizontally and vertically.

The solution given here is No. 3, but the student is advised to solve it by the other three methods.

Method 3

Fig. 26 represents the ladder and the forces acting on it which meet at D.

The sides of the triangle ADE are proportional to the forces.

Hence $$F/w = \frac{EA}{DE}, \quad \frac{F_1}{w} = \frac{AD}{DE}$$

$$AE = \tfrac{1}{2}AH = 4$$

$$DE = BH = \sqrt{20^2 - 8^2} = \sqrt{336}$$

$$AD = \sqrt{4^2 + 336}$$

$$F = \frac{50 \times 4}{\sqrt{336}} = \underline{10 \cdot 9 \text{ lb.}}$$

$$F_1 = \frac{50\sqrt{352}}{\sqrt{336}} = \underline{51 \cdot 17 \text{ lb.}}$$

48.

The braced support carries a load of 2000 lb. at a distance of 10 ft. from the wall. Find the stresses in each member.

Draw vector diagram for joint P to scale 1000 lb. = 1 in. This diagram is shown in Fig. 27 (*b*).

Then draw vector diagram for joint Q to same scale. This is shown in Fig. 27 (*c*).

Combining these two diagrams as in Fig. 27 (*d*), we have stresses in members from the diagram as follows:

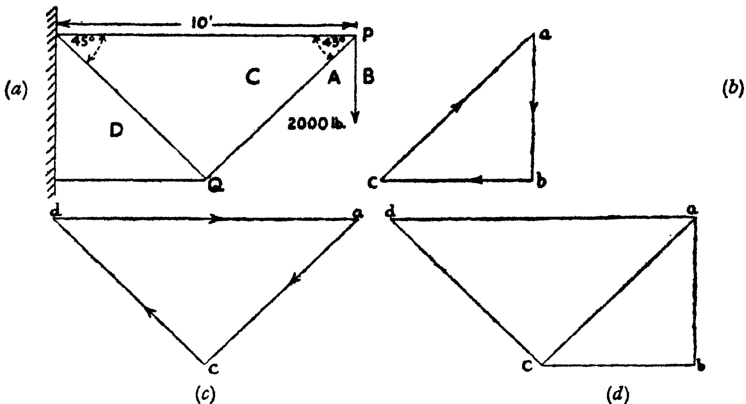


FIG. 27.

Member AC=2800 lb. compressive
Member CD=2800 lb. tensile
Member BC=2000 lb. tensile
Member DA=4000 lb. compressive

49.

An iron gate 4 ft. wide and 5 ft. high is attached to its post as in Fig. 28 (*a*). The top hinge is a plain bearing, and the weight of the gate is taken by a socket at ground level. The gate weighs 150 lb., and its c.g. is at its geometric centre. Find the magnitude,

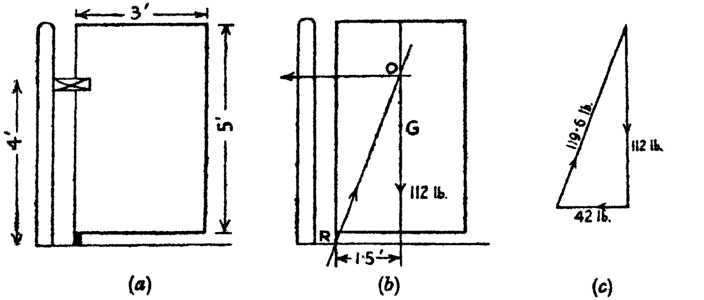


FIG. 28.

direction and sense of the force acting at the ground socket of the gate.

Fig. 28 (*b*) represents the space diagram to scale.

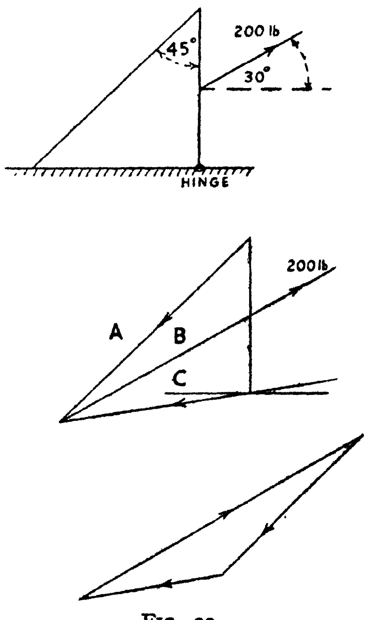
Since the top hinge is a plain bearing, it is assumed that the reaction is normal to the hinge and therefore horizontal. This force and the weight 112 lb. intersect at *O*, and since the forces are in equilibrium the reaction of the lower socket *R* must pass through this point. Choosing a scale of 1 in.=50 lb., draw the Force Diagram as in Fig. 28 (*c*). The magnitude direction and sense of the force acting at the ground can be read off.

Magnitude=119·6 lb.

Direction=upward at angle of 70° to the horizontal.

50.

A vertical post is 20 ft. high and hinged at the bottom. A force of 200 lb. is applied to the post 10 ft. from the ground and at an



(*a*)

(*b*)

(*c*)

FIG. 29.

angle of 30° to the horizontal. The post is stayed by a rope which makes an angle of 45° with the post. Find the tensile force on the rope.

Fig. 29 (*a*) represents the arrangement. Since the post is hinged at the bottom, the vertical component deflecting the rope will be nil.

The horizontal component of the 200 lb. force equals

$$200 \cos 30° = 173 \cdot 2 \text{ lb.}$$

Taking moments about the hinge,

Horizontal force P acting at top $\times 20 = 173 \cdot 2 \times 10$

Hence $$P = 86 \cdot 6 \text{ lb.}$$

Therefore $$\text{Force in rope} = \frac{86 \cdot 6}{\sin 45} = \underline{122 \cdot 5 \text{ lb.}}$$

This can also be solved by drawing the force diagram.

It will be seen that the three forces acting on the post must meet at the point O. Fig. 29 (*b*). Fig. 29 (*c*) represents the force diagram. Scaling this off, we obtain the force in the rope = 123 lb.

CHAPTER VI

51.

A block weighing 1 cwt. rests on a horizontal floor. If it requires a horizontal force of 40 lb. to keep the block moving uniformly along the floor, what is the coefficient of friction?

Consider the forces keeping the block moving uniformly on the floor (Fig. 30). This will best be understood by first assuming the

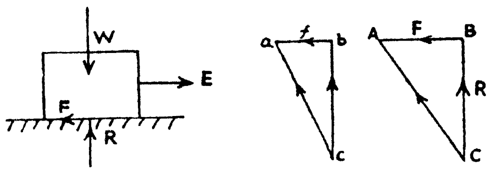


FIG. 30.

block to be at rest. Then the forces keeping it at rest are: Weight W acting vertically downwards, and the vertical upward pressure R of the floor on the block, which must equal W. This, like the weight, acts through the centre of gravity of the block. Now apply a horizontal force E in the direction shown, and assume it is not large enough to produce motion. The force exerted on the block by the floor is now the resultant of R, and the friction f equal and opposite to E. This is found by adding the forces as in vector triangle *abc*, the resultant being parallel and proportional to *ca*.

As E increases, the resultant pressure will move to the right, and the contra clockwise moment of W and R balance the clockwise moment of E and F. When the limit of friction is reached by increasing E, f increasing to F, the inclination of the resultant to the vertical is the angle of friction, ϕ.

$$\text{Tan } \phi = \frac{AB}{BC} = \frac{F}{R} = \frac{F}{W} = \mu .$$

So we have the very important relation—
Tangent of angle of friction = coefficient of friction.

$$\text{The coefficient of friction } \mu = \frac{F}{W} = \frac{E}{W} = \frac{40}{112} = \underline{0{\cdot}35.}$$

52.

The slide valve of a steam engine measures 10 in. by 16 in., and the steam pressure at the back of the valve is 120 lb./in.^{2}. If the coefficient of friction between the two steel faces is 0·15, find the horizontal force required to move the valve.

Area of valve $=10\times16=160$ in.^{2}

Total pressure $=160\times120=19{,}200$ lb.

$$\mu=\frac{E}{W}$$

$$\therefore E=\mu W=0{\cdot}15\times19{,}200$$

$$=\underline{2880\text{ lb.}}$$

53.

What is the greatest weight a man weighing 200 lb. can pull along a horizontal floor, by a horizontal rope, if the coefficient of friction between the floor and his boot soles is 0·50 and between the weight and the floor 0·30?

Let F = maximum force man can exert

then $\mu W=F=0{\cdot}5\times200=100$ lb.

If this force is applied horizontally to a weight W_1 lb.

$$\mu=\frac{F}{W_1}$$

$$\therefore W_1=\frac{F}{\mu}=\frac{100}{0{\cdot}30}=\underline{333{\cdot}3\text{ lb.}}$$

54.

A planing machine, the table of which weighs 5 cwt., makes 6 forward and 6 backward strokes per min. If the length of the stroke is 6 feet, and the coefficient of friction between the sliding surfaces is 0·05, calculate the work done in foot-pounds per min. in moving the table.

Force required to move table $F=\mu W=0{\cdot}05\times5\times112=28$ lb.

Distance moved per min. by table $=12\times6$ ft. $=72$ ft.

Work done per min. $=28\times72=\underline{2016\text{ ft.-lb.}}$

55.

A lathe spindle 2·5 in. diameter revolves at 100 r.p.m. The load on the spindle is 2·5 cwt. If the coefficient of friction between the bearing and the spindle is 0·025, find the foot-pounds of work per min. wasted in friction.

Total frictional force $=2{\cdot}5\times112\times0{\cdot}025=7$ lb.

Circumference of spindle $=\frac{\pi\times2{\cdot}5}{12}$ ft.

Distance moved per min. $=\frac{\pi\times2{\cdot}5}{12}\times100=65{\cdot}4$ ft.

Work lost per min. $=7\times65{\cdot}4=\underline{457{\cdot}8\text{ ft.-lb.}}$

56.

A weight of 100 lb. rests on an inclined plane whose slope is 30°. If the coefficient of friction between the weight and the plane is 0·40, find the force which, acting parallel to the plane, will just make the weight move up.

E must equal F before the weight will move up the plane.

We have $E=W\sin30°+\mu W\cos30°$

$$\therefore E=100\times{\cdot}5+0{\cdot}4\times100\times0{\cdot}8660=\underline{84{\cdot}6\text{ lb.}}$$

This can also be solved by drawing the triangle of forces, and the reader is advised to do so.

57.

A weight of 100 lb. rests on an inclined plane whose slope is 30°. If the coefficient of friction between the weight and the plane is 0·40, find the force which, acting at an angle of 20° to the plane, will just make the weight move up.

Let Fig. 31 represent the forces acting on the weight which

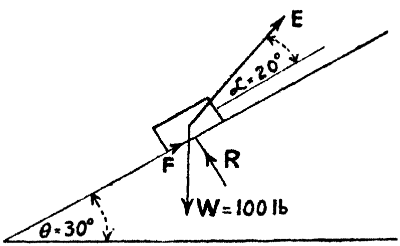


FIG. 31.

rests on the plane, and let E be the force acting at angle α (in our case 20°) with the plane that must pull the weight up the plane.

Resolving the forces,

We have

$$E \cos \alpha = F + W \sin \theta$$
$$E \sin \alpha = W \cos \theta - R$$
$$F = \mu R$$
$$F = E \cos \alpha - W \sin \theta = \mu R \quad . \quad . \quad . \quad . \quad (1)$$
$$R = -E \sin \alpha + W \cos \theta \quad . \quad . \quad . \quad . \quad (2)$$

Multiplying Equation (2) by μ and subtracting from Equation (1), we have:

$$\mu R = \mathrm{E} \cos \alpha - W \sin \theta$$
$$\mu R = -E\mu \sin \alpha + \mu W \cos \theta$$
$$O = E \cos \alpha + E\mu \sin \alpha - W \sin \theta - \mu W \cos \theta$$

$$\therefore W (\sin \theta + \mu \cos \theta) = E (\cos \alpha + \mu \sin \alpha)$$
$$E = W \frac{(\sin \theta + \mu \cos \theta)}{(\cos \alpha + \mu \sin \alpha)}$$

Substituting in this formula,

We have

$$E = 100 \frac{(\sin 30^\circ + 0{\cdot}4 \cos 30^\circ)}{(\cos 20^\circ + 0{\cdot}4 \sin 20^\circ)}$$
$$E = 100 \times 0{\cdot}786 = \underline{78{\cdot}6 \text{ lb.}}$$

58

A body which weighs 18 lb. is placed on an inclined plane. If the coefficient of friction between the plane and the body is 0·16, at what angle of inclination of the plane will the body slide down the plane at a uniform speed? What horizontal force would be required to move the body at a uniform speed, if the plane were horizontal?

Coefficient of friction $\mu = \tan \theta$

$$\therefore \tan^{-1} \theta = 0{\cdot}16$$
$$\theta = 9^\circ\ 6'$$

Horizontal force required to maintain steady motion

$$= 0{\cdot}16 \times 18 = \underline{2{\cdot}88 \text{ lb.}}$$

59.

A smooth inclined plane rises 1 ft. in 9 ft. of its length. What effort is required to draw a body weighing 500 lb. up the plane, when the effort is: (1) parallel to the plane, (2) horizontal to the plane, (3) at an angle of 30° to the plane?

(1) Applying the principle of work, the effort E_p lb. travels 9 ft. along the slope.

$\therefore$ Work done by effort $= E_p$ lb. $\times 9$ ft.

Work done on load $= 500$ lb. $\times 1$ ft. $= 500$ ft.-lb.

$$E_p \times 9 \text{ ft.} = 500 \text{ ft.-lb.}$$

$$\therefore E_p = \underline{55 \cdot 55 \text{ lb.}}$$

(2) Let x = length of horizontal base in feet.

Then
$$9^2 = x^2 + 1^2$$

$$\therefore x = \sqrt{9^2 - 1^2} = 8 \cdot 94 \text{ ft.}$$

The effort travels 8·94 ft. whilst the load is lifted 1 ft.

By the principle of work,

$$E_h \text{ lb.} \times 8 \cdot 94 \text{ ft.} = 500 \text{ ft.-lb.}$$

$$\therefore E_h = 55 \cdot 92 \text{ lb.}$$

Note. It makes very little difference whether we take the force parallel to the plane or horizontal to it.

(3) Component of E_a parallel to the plane

$$= E_a \cos 30°$$

$$= 0 \cdot 866 \, E_a$$

Work done by effort $= 0 \cdot 866 \, E_a \times 9$

Work done on load $= 500$ lb. $\times 1$ ft.

$$\therefore 0 \cdot 866 \, E_a \times 9 = 500 \text{ ft.-lb.}$$

$$E_a = \underline{64 \cdot 15 \text{ lb.}}$$

60.

A train weighing 400 tons is pulled up a gradient of 1 in 200. If the tractive resistance on the level is 12 lb. per ton, what tractive effort and horse-power will be required to draw the train at 25 miles per hour up the gradient?

Effort required to overcome tractive resistance

$$= 12 \times 400 = 4800 \text{ lb.}$$

Effort required to pull train up gradient assuming no friction

$$= 400 \times 2240 \times \frac{1}{200} = 4480 \text{ lb.}$$

Total tractive effort $= 4800 + 4480 = \underline{9280 \text{ lb.}}$

$$\text{Distance travelled per second} = \frac{25 \times 1760 \times 3}{3600} = 36 \cdot 67 \text{ ft.}$$

Work done per sec. $= 9280$ lb. $\times 36 \cdot 67$ ft.

$$= 340{,}198 \text{ ft.-lb.}$$

$$\text{h.p. required} = \frac{340{,}198}{550} = \underline{618 \text{ h.p.}}$$

61.

In example 60 what is the horse-power required to draw the train down the gradient?

When train is being drawn down gradient, gravitation will assist in overcoming the tractive resistance of 4800 lb. opposing motion. Resolving the weight of the train along the gradient, we have component of weight along gradient

$$=400\times 2240\times\frac{1}{200}=4480 \text{ lb.}$$

$$\text{Force required to draw train}=4800-4480=\underline{320 \text{ lb.}}$$

$$\text{Work done per sec.}=320 \text{ lb.}\times 36{\cdot}67 \text{ ft.}$$

$$=\underline{11{,}734 \text{ ft.-lb.}}$$

$$\text{h.p. required}=\frac{11{,}734}{550}=\underline{21{\cdot}3 \text{ h.p.}}$$

CHAPTER VII

62.

If a motor car travels at a uniform speed of 30 miles per hour, how many feet will it travel in 30 seconds, and how long will it take to cover 50 miles?

Distance travelled per hour $=30\times5280$ ft.

$$\text{Distance travelled per sec.}=\frac{30\times5280}{60\times60}=44 \text{ ft.}$$

Let s = space travelled in feet per second
v = speed in feet per second
t = time in seconds

Then $s=vt$ and $t=\frac{s}{v}$

∴ Distance travelled in 30 sec. $s=vt=44\times30=$ 1320 ft.

$$\text{Time to travel 50 miles } t=\frac{s}{v}=\frac{50}{30}=1\cdot66 \text{ hour}$$

$= 1$ hour 40 min.

63.

The following table gives the distance a body is from its starting position at the given times. Calculate the average velocity in each 5 seconds. Plot a speed-time curve and from it find the speed after 35 secs.

Time t (secs.)	0	5	10	15	20	25	30	35	40	45
Distance s feet	0	100	400	750	1125	1475	1800	2100	2350	2500

$$\text{Average velocity in 1st 5 sec.}=\frac{0+100}{5}=20 \text{ ft. per sec.}$$

(This can be taken as the velocity after 2·5 sec.)

$$\text{Average velocity in 2nd 5 sec.}=\frac{400-100}{5}=60 \text{ ft. per sec.}$$

(Take this as velocity at 7·5 sec.)

Average velocity in 3rd 5 sec. $= \frac{750-400}{5} = 70$ ft. per sec. at 12·5 sec.

Average velocity in 4th 5 sec. $= \frac{1125-750}{5} = 75$ ft. per sec. at 17·5 sec.

Average velocity in 5th 5 sec. $= \frac{1475-1125}{5} = 70$ ft. per sec. at 22·5 sec.

Average velocity in 6th 5 sec. $= \frac{1800-1475}{5} = 65$ ft. per sec. at 27·5 sec.

Average velocity in 7th 5 sec. $= \frac{2100-1800}{5} = 60$ ft. per sec. at 32·5 sec.

Average velocity in 8th 5 sec. $= \frac{2350-2100}{5} = 50$ ft. per sec. at 37·5 sec.

Average velocity in 9th 5 sec. $= \frac{2500-2350}{5} = 30$ ft. per sec. at 42·5 sec.

Now plot their average velocities and times and the curve shown in Fig. 32 is obtained from the curve scale of the velocity at 35 sec., which is approximately 55 ft. per sec.

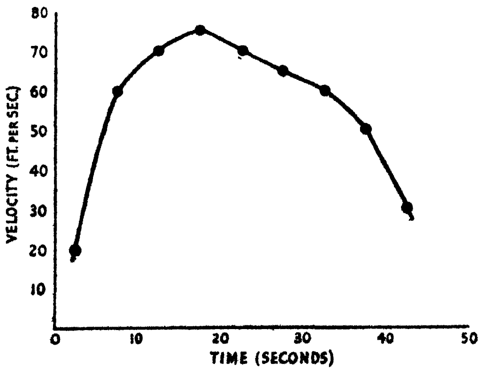


FIG. 32.

64.

A car starting from rest has its speed uniformly increased until at the end of one minute it is moving at the rate of 30 miles an hour. What distance has it covered in this time?

$$30 \text{ miles per hour} = \frac{30 \times 5280}{60 \times 60} = 44 \text{ ft. per sec.}$$

$$\text{Average speed} = \frac{0+44}{2} = 22 \text{ ft. per sec.}$$

$$\text{Distance covered } s = vt = 22 \times 1 \times 60 = \underline{1320 \text{ ft.}}$$

65.

A fly-wheel of 6 ft. diameter is keyed to a shaft which makes 300 r.p.m. Calculate the angular and linear speed of the rim of the fly-wheel.

$$300 \text{ r.p.m.} = \frac{300}{60} = 5 \text{ r.p.s.}$$

Radians turned through per revolution $= 2\pi$

Angular speed of fly-wheel $= 2\pi \times 5 = 10\pi = \underline{31{\cdot}41 \text{ radians per sec.}}$

Circumference of fly-wheel $= \pi d = \pi \times 6 = 18{\cdot}84$ ft.

Linear speed of rim $= 18{\cdot}84 \times 5 = \underline{94{\cdot}2 \text{ ft. per sec.}}$

66.

A body has an initial velocity of 50 ft. per sec., and moves with an acceleration of 5 ft. per sec. per sec. Find the distance passed over in 5 sec.

Let s = space passed over
u = initial velocity
a = acceleration
t = time in seconds.

$$s = ut + \tfrac{1}{2}at^2 = 50 \times 5 + \tfrac{1}{2}5 \times 5^2 = \underline{312{\cdot}5 \text{ ft.}}$$

67.

A body has an initial velocity of 50 ft. per sec., and moves with a retardation of 6 ft. per sec. per sec. Find when it will be 200 ft. from the starting point.

$$s = ut - \tfrac{1}{2}at^2$$

$$200 = 50t - \tfrac{1}{2} \times 6t^2$$

Solving this quadratic equation, we find

$$t = 10 \text{ sec.}$$

68.

Find the velocity with which a jet of water must be projected vertically upwards in order to reach a height of 100 ft.

If v = the velocity required

then $v^2 = 2gs$

where g = acceleration due to gravity

$$v^2 = 2 \times 32{\cdot}2 \times 100$$

$$\therefore v = \sqrt{6440} = \underline{80 \text{ ft. per sec.}}$$

69.

A bullet is fired vertically upwards with an initial velocity of 2500 ft. per sec. Calculate the height to which it will rise, and the time taken to reach the ground again. Neglect air resistance.

$$v^2 = 2gs$$

$$\therefore s = v^2/2g$$

$$s = \frac{2500 \times 2500}{2 \times 32{\cdot}2} = \underline{97{,}049 \text{ ft.}}$$

Since the time taken for the bullet to rise will be the same as that for it to fall, we have

$$s = \tfrac{1}{2}gt^2$$

$$t^2 = \frac{2s}{g} = \frac{2 \times 97049}{32{\cdot}2}$$

$$t = \sqrt{\frac{2 \times 97049}{32{\cdot}2}} = 77{\cdot}6 \text{ sec.}$$

$$\therefore \text{Total time} = 2 \times 77{\cdot}6 = \underline{155{\cdot}2 \text{ sec.}}$$

70.

The speed of a car increases uniformly for the first 4 minutes after starting, and during this time travels 1 mile. Calculate the speed of the car at the end of 4 minutes, and the space covered in the first 2 minutes.

$$s = \tfrac{1}{2}at^2$$

$$5280 = \tfrac{1}{2}a \times (4 \times 60)^2$$

$$\therefore a = \tfrac{11}{60} \text{ ft. per sec. per sec.}$$

Speed at the end of 4 minutes or velocity gained in covering 1 mile:

$$v^2 = 2as$$
$$\therefore\ v^2 = 2 \times \tfrac{11}{60} \times 5280$$
$$v = \sqrt{1936} = \underline{44 \text{ ft. per sec., or } 30 \text{ miles per hour}}$$

Space covered in first 2 minutes:

$$s = \tfrac{1}{2}at^2 = \tfrac{1}{2}\tfrac{11}{60} \times (2 \times 60)^2$$
$$= \underline{1320 \text{ ft.} = \tfrac{1}{4} \text{ mile.}}$$

71.

A body moves with uniform acceleration and covers 20 ft. in the fifth second, and 30 ft. in the tenth second after starting. What is its initial velocity, and acceleration?

If we assume that the initial velocity is u, and that the body moves with uniform acceleration a, space covered equals (average velocity) × (time). The space covered in the nth second is equal to the average velocity during the nth second; but since the acceleration is constant, the average velocity is the actual velocity at the middle of the nth second or

$$u + (n - \tfrac{1}{2})a$$

Thus the spaces covered in the first, fourth, . . . nth seconds are:

$$u + \tfrac{1}{2}a,\ u + (4 - \tfrac{1}{2})a,\ u + (n - \tfrac{1}{2})a$$

From the above we see we have two simultaneous equations involving u and a:

$$20 = u + (5 - \tfrac{1}{2})a$$
$$30 = u + (10 - \tfrac{1}{2})a$$

From these equations we have:

$$\underline{u = 11 \text{ ft. per sec.}}$$
$$\underline{a = 2 \text{ ft. per sec. per sec.}}$$

72.

A car accelerating uniformly from rest passes in succession three lamp-posts spaced 100 yards apart. The time between the first and second lamp-post and the second and third lamp-post is 10 seconds and 6 seconds respectively. Find (1) the acceleration of the car, (2) the distance of the first lamp-post from the starting point.

Let u = velocity of car at first lamp-post
v = velocity of car at second lamp-post
v_1 = velocity of car at third lamp-post

Space passed over $(s) = \left(\frac{u+v}{2}\right)t$

$$\therefore\ 300 = \left(\frac{u+v}{2}\right) \times 10$$

$$300 = \left(\frac{v+v_1}{2}\right) \times 6$$

$$600 = \left(\frac{u+v_1}{2}\right) \times 16$$

Solving these three equations, we have:

$$300 = 5u + 5v \quad \ldots \quad (1)$$
$$300 = 3v + 3v_1 \quad \ldots \quad (2)$$
$$600 = 8u + 8v_1 \quad \ldots \quad (3)$$

From Equation (2),

$$v_1 = (100 - v)$$

Substituting this value in Equation (3) and adding it to Equation (1), we have:

$$-25 = u - v$$
$$60 = u + v$$
$$35 = 2u$$
$$u = 17 \cdot 5 \text{ ft. per sec.}$$
$$v = 42 \cdot 5 \text{ ft. per sec.}$$

Substituting the value of $v = 42 \cdot 5$ ft. per sec. in Equation (2), we have:

$$v_1 = 57 \cdot 5 \text{ ft. per sec.}$$

Again, $v^2 = u^2 + 2fs$

$$42 \cdot 5^2 = 17 \cdot 5^2 + 2f \times 300$$

$$f = \frac{42 \cdot 5^2 - 17 \cdot 5^2}{2 \times 300} = \underline{2 \cdot 5 \text{ ft. per sec. per sec. acceleration of car}}$$

To find distance between first lamp-post and start,

$$v^2 = u^2 + 2fs$$

In this case, $u = 0$

$$\therefore\ 17 \cdot 5^2 = 2 \times 2 \cdot 5 \times s$$

$$s = \frac{17 \cdot 5^2}{2 \times 2 \cdot 5} = \underline{61 \cdot 25 \text{ ft.}}$$

73.

A body is projected upwards with an initial velocity of 320 ft./sec. To what height will it rise? How long will it take to reach a point 600 ft. high on its downward path? $g=32$ ft./sec.^{2}.

To find the height to which it will rise,

$$v^2-u^2=2gs$$

$$u=320 \text{ ft./sec.}$$

$$v=0$$

$$2g=-64 \text{ ft./sec.}^2 \text{ (a retardation)}$$

$$\therefore s=\frac{320^2}{64}=1600 \text{ ft.}$$

$$\text{Time } t \text{ sec. to reach highest point}=\sqrt{\frac{1600}{16}}=10 \text{ sec.}$$

$$\text{Time } t_1 \text{ sec. to fall 1000 ft.}=\sqrt{\frac{1000}{16}}=7\cdot9 \text{ sec.}$$

$\therefore$ Time taken to reach a point 600 ft. high on the downward path

$$=t+t_1=10+7\cdot9=\underline{17\cdot9 \text{ sec.}}$$

74.

A rope passes over a frictionless pulley. On one end of the rope is a weight of 125 lb., and on the other end a weight of 185 lb. The 185-lb. weight is held steady, then let go. What vertical distance will it fall in two seconds?

Assuming a frictionless pulley and also that it has no mass,

Accelerating force$=185-125=60$ lb.

Mass accelerated$=185+125=310$ lb.

$$\text{Acceleration}=\frac{\text{accelerating force}\times g}{\text{mass accelerated}}=\frac{60\times g}{310}=6\cdot24 \text{ ft./sec.}^2$$

$$s=\tfrac{1}{2}at^2=\tfrac{1}{2}\times6\cdot24\times2^2=\underline{12\cdot48 \text{ ft.}}$$

75.

A bullet is fired with a barrel velocity of 1650 ft./sec. from an aeroplane travelling at 350 m.p.h.

(*a*) If the bullet is fired in the same direction as the plane (i.e. forwards), how long will it take to travel 440 yds.?

(*b*) If the bullet is fired in the opposite direction to the plane (i.e. backwards), how far will it travel in 1·75 seconds?

$$\text{Velocity of plane forward} = \frac{350 \times 5280}{3600} = 513 \cdot 3 \text{ ft./sec.}$$

Velocity of bullet forward relative to plane

$$= 1650 + 513 \cdot 3 = 2163 \cdot 3 \text{ ft./sec.}$$

$$\text{Time taken to travel 440 yds.} = \frac{440 \times 3}{2163 \cdot 3} = \underline{0 \cdot 61 \text{ sec.}}$$

Velocity of bullet backwards relative to the earth

$$= 1650 - 513 \cdot 3 = 1136 \cdot 6 \text{ ft./sec.}$$

$$\text{Distance travelled in } 1 \cdot 75 \text{ sec.} = 1136 \cdot 6 \times 1 \cdot 75 = \underline{1990 \text{ ft.}}$$

76.

A shell weighing 20 lb. is fired from a gun weighing 2 tons with a velocity of 1300 ft./sec.

(*a*) Find the velocity of recoil of the gun.

(*b*) If the gun moves a distance of 1 ft., what is the average force resisting recoil?

Let V_1=velocity of shell and W_1 its weight
V_2=velocity of recoil and W_2 weight of gun

Then

$$W_1 V_1 = W_2 V_2$$

$$20 \times 1300 = 4480 \times V_2$$

$$V_2 = \frac{20 \times 1300}{4480} = 5 \cdot 8 \text{ ft./sec.}$$

$$\text{Kinetic energy of recoiling gun} = \frac{W_2 V_2^2}{2g} = \frac{4480 \times (5 \cdot 8)^2}{64 \cdot 4}$$

$$= 2340 \text{ ft.-lb.}$$

If gun is arrested in 1 ft.,

Average force acting=<u>2340 lb.</u>

77.

A motor car weighs 1·5 tons and is running on a level road at 60 m.p.h. Find its kinetic energy in foot-pounds. How far will the car run without driving force or brakes if the resistance to motion is 40 lb./per ton? 60 m.p.h.=88 ft./sec.

$$\text{Kinetic energy} = \frac{\frac{1}{2} W V^2}{g} = \frac{1 \cdot 5 \times 2240 \text{ (lb. force)} \times 88^2 \text{ (ft./sec.)}^2}{2 \times 32 \cdot 2 \text{ ft./sec.}^2}$$

$$= 404{,}035 \text{ ft.-lb.}$$

To destroy this energy,

Work done in resistance $=40$ lb. $\times 1{\cdot}5 \times$ distance run

$$404{,}035 \text{ ft.-lb.} = 40 \text{ lb.} \times 1{\cdot}5 \times \text{distance}$$

$$\frac{404{,}035}{40 \times 1{\cdot}5} = \text{distance} = 6733{\cdot}9 \text{ ft.}$$

Alternative method:

$$\frac{f}{g} = \frac{P}{W} = \frac{40 \times 1{\cdot}5}{3360}$$

$$\therefore f = \frac{40 \times 1{\cdot}5 \times 32{\cdot}2 \text{ ft./sec.}^2}{3360} = 0{\cdot}575$$

$$= 0{\cdot}575 \text{ ft./sec./sec.}$$

Time in sec. to reduce speed by 88 ft./sec.

$$= \frac{88 \text{ ft./sec.}}{0{\cdot}575 \text{ ft./sec.}^2} = 153{\cdot}0 \text{ sec.}$$

Distance travelled in this time at average speed of

$$44 \text{ ft./sec.} = 44 \times 153{\cdot}0 = \underline{6732 \text{ ft.}}$$

CHAPTER VIII

78.

A lifting-crab having a velocity ratio of 15 lifts a load of 210 lb. with an efficiency of 60 per cent. What effort is required, and what will be the mechanical advantage?

Useful work per foot of lift=210 ft.-lb.

Work done by effort P per foot of lift=$P\times 15$ ft.-lb.

Since the efficiency is 60 per cent.,

$$0{\cdot}60P\times 15=210$$

$$P=\frac{210}{15\times 0{\cdot}60}=23{\cdot}3 \text{ lb.}$$

$$\text{Mechanical advantage}=\frac{\text{load}}{\text{effort}}=\frac{210}{23{\cdot}3}=9{\cdot}0$$

$$\text{Velocity ratio}\times\text{efficiency}=\text{mechanical advantage}$$

$$15\times 0{\cdot}6=\underline{9}.$$

79.

In a lifting machine, the efficiency is 35 per cent., and the velocity ratio 24. What effort is required to raise a load of 2000 lb.?

$$\text{Mechanical efficiency}=\frac{\text{useful work}}{\text{work done by effort}}=\frac{W}{P\times V}$$

$$\therefore \frac{2000 \text{ lb.}}{P \text{ lb.}\times V \text{ ft.}}=0{\cdot}35=\frac{2000}{P\times 24}$$

V=velocity ratio, P=effort in lb., and W=load lb.

$$\text{Hence Effort } P=\frac{2000}{0{\cdot}35\times 24}=\underline{238 \text{ lb.}}$$

80.

An experiment on a lifting machine gave the following results:

Load W lb.	5	10	15	20	25	30
Effort P lb.	0·50	0·72	0·96	1·20	1·45	1·7

The velocity ratio was found to be 28·8. Plot Effort against Load and find the "law of the machine" and calculate the efficiency of the machine when the load is 26·0 lb.

First plot to some scale (Fig. 33) the Load W lb. against the

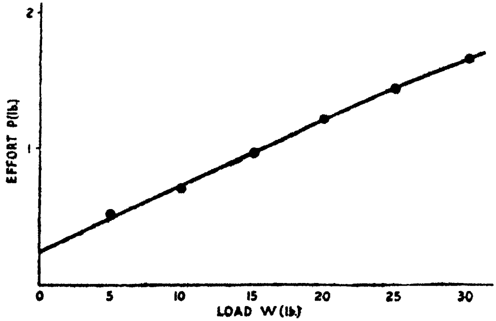


FIG. 33.

Effort P lb. (in this case 5 lb.=1 in. for W and 1 lb.=2 in. for P). The graph is a straight line, but does not pass through the zero point O. This indicates that some effort 0·25 lb. is required to move the machine without any useful load.

The law of the graph is $P=mW+C$.

From the graph $C=0 \cdot 25$ lb., and by taking two points A and B on the graph and measuring the slope we obtain m. From graph $m=0 \cdot 049$.

The law of the machine is $\underline{P=0 \cdot 049W+0 \cdot 25.}$

From graph find Effort $P=1 \cdot 5$ lb. for load 26·0 lb.

$$\text{Then Efficiency}=\frac{\text{useful work}}{\text{work done by effort}}=\frac{W}{P\times V}=\frac{26 \cdot 0}{1 \cdot 5\times 28 \cdot 8}$$
$$=0 \cdot 60=\underline{60 \text{ per cent.}}$$

81.

In an experiment with a lifting machine, the following results were obtained:

Load W lb.	50	100	150	200	250
Effort P lb.	6·5	9·9	13·2	16·6	20·0

The velocity ratio was found to be 30.

Determine the efficiency for each load and plot a curve on a load base to show the efficiency of the machine at all loads from 0 to 250 lb.

Work out the efficiency$=\dfrac{W}{P\times V}$ for each load and thus obtain Table I.

Table I

Load W lb.	50	100	150	200	250
Efficiency	0·256	0·336	0·379	0·402	0·416

Plot graph of Load W against Efficiency to some scale, in this case 1 in.=50 lb. for W and 1 in.=10 per cent. Fig. 34 shows this graph.

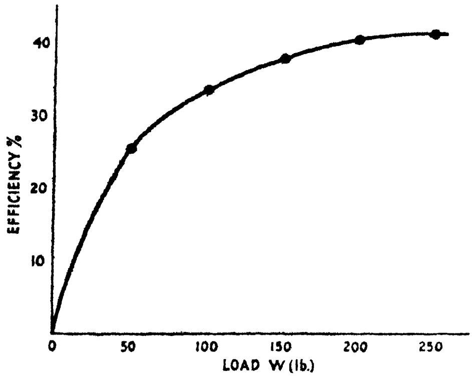


FIG. 34.

82.

A simple crab has a pinion with 20 teeth on the shaft to which the handle is fitted. This gears with a wheel having 200 teeth on the drum shaft. An effort of 10 lb. acts perpendicular to the handle at a radius of 15 in. from the axis of the shaft. This lifts a load of 200 lb. suspended by a rope round the drum, which has an effective radius of 6 in. Find the efficiency of the crab at this load. Fig. 35 shows the machine in diagrammatic form.

For one revolution of the drum,

The handle shaft makes $200/20=10$ revolutions
The effort moves $10\times2\pi\times15=300\pi$ in.
Work done by effort $=300\pi\times10=3000\pi$ in.-lb.
The load moves $2\pi\times6=12\pi$ in.
Work done $=12\pi\times200=2400\pi$ in.-lb.

$$\text{Efficiency}=\frac{\text{useful work}}{\text{work done by effort}}=\frac{2400\pi}{3000\pi}=0{\cdot}80$$

or 80 per cent.

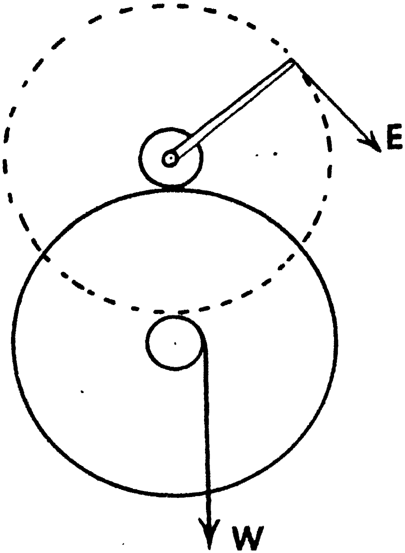


FIG. 35.

83.

Two gallons of water are raised from a well in a can weighing 5 lb. by means of a wheel and differential axle. If the wheel is 2 ft. diameter and the diameters of axle 10 and 6 in. respectively, and a force of 4 lb. is required, what is the efficiency of the machine?

$$\text{Velocity ratio } V = \frac{\text{distance } E \text{ moves}}{\text{distance } W \text{ moves}} = \frac{\pi d_2}{\frac{\pi(d_1 - d)}{2}}$$

$$= \frac{2d_2}{d_1 - d} = \frac{2 \times 24}{10 - 6} = 12$$

2 gallons of water weigh 20 lb.

$$W = 20 + 5 = 25 \text{ lb.}$$

$$\text{Efficiency} = \frac{\text{useful work}}{\text{work done by effort}} = \frac{W}{P \times V} = \frac{25}{4 \times 12}$$

$$= 0{\cdot}52 = \underline{52 \text{ per cent.}}$$

84.

A screw-jack lifts a load of 1000 lb. The lead of the screw is $\frac{3}{8}$ in. Find the effort required at the end of a 12-in. arm. If the efficiency of the screw-jack is 30 per cent., what is the velocity ratio and the mechanical advantage of the screw-jack?

Let P=the effort

Work done by effort per revolution$=P\times\pi d$

$=P\times\pi\times24$ in.-lb.

Work done in raising W per revolution$=1000\times\frac{3}{8}$ in.-lb.

$$\text{Efficiency}=\frac{\text{useful work per revolution}}{\text{work done by effort per revolution}}=\frac{1000\times\frac{3}{8}}{P\times24\pi}$$

$$0\cdot30=\frac{375}{24\pi P}$$

$$P=\frac{375}{0\cdot30\times24\pi}=\underline{16\cdot6\text{ lb.}}$$

$$\text{Velocity ratio } V=\frac{\text{circumference of effort circle}}{\text{lead of screw}}$$

$$=\frac{\pi\times24}{\frac{3}{8}}=201$$

Mechanical advantage

$$=\frac{\text{load}}{\text{effort}}=\frac{1000}{16\cdot6}=\underline{60\cdot3}$$

Or mechanical advantage

$$=\text{velocity ratio}\times\text{efficiency}$$

$$=201\times0\cdot30=\underline{60\cdot3}.$$

85.

In a Weston Differential Pulley block the sheaves are 12 and 13-in. diameter respectively. Find the force required to lift 1 ton if the efficiency is 40 per cent. What length of chain must be pulled over to make the weight rise 1 ft.?

Referring to Fig. 36,

Let R=radius of large sheaf

r=radius of small sheaf

For one revolution effort moves a distance$=2\pi R$

Chain shortens by a length $2\pi(R-r)$.

Load rises a distance of $\pi(R-r)$.

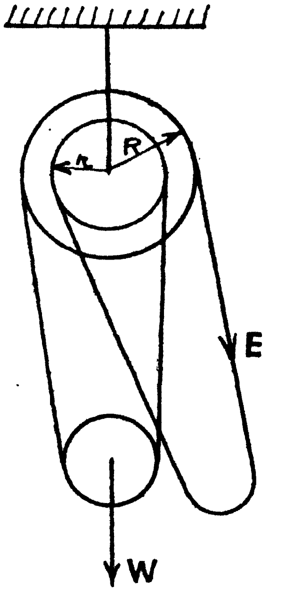


FIG. 36.

$$\text{Velocity ratio} = \frac{2R}{R-r} = \frac{2\times 13}{13-12} = 26$$

$$\frac{\text{Load } (W)}{\text{Effort } (E)} = \frac{2R}{R-r} \times \eta$$

$$\frac{2240}{E} = \frac{2\times 6\cdot 5}{6\cdot 5 - 6} \times 0\cdot 40$$

$$\text{Effort} = \underline{215\cdot 4 \text{ lb.}}$$

$$\text{Length of chain pulled out} = VR \times \text{distance } W \text{ moves}$$

$$= 26 \times 1 = \underline{26 \text{ ft.}}$$

CHAPTER IX

86.

A motor lorry pulls a trailer with a steady pull of 100 lb. at 20 miles per hour. How much work is expended upon the trailer in 1 hour?

$$\text{Distance travelled in 1 hour} = 20 \times 5{,}280 \text{ ft.}$$
$$= 105{,}600 \text{ ft.}$$

$$\text{Work done in 1 hour} = 100 \text{ lb.} \times 105{,}600 \text{ ft.} = \underline{10{,}560{,}000 \text{ ft.-lb.}}$$

87.

If a trailer being pulled by a motor car at 30 miles per hour absorbs 100,000 ft.-lb. of work per minute, what will be the pull on the trailer?

$$\text{Distance moved per hour} = 30 \times 5{,}280 \text{ ft.}$$
$$= 158{,}400 \text{ ft.}$$
$$\text{Distance moved per minute} = \frac{158{,}400}{60} = 2640 \text{ ft.}$$
$$\text{Work done per minute} = \text{pull in lb.} \times \text{distance ft.}$$
$$100{,}000 = \text{pull} \times 2640$$
$$\text{pull in lb.} = \frac{100{,}000}{2640} = \underline{37 \cdot 9 \text{ lb.}}$$

88.

An electric motor does 100,000 ft.-lb. of work per minute at a speed of 1000 revolutions per minute. What is the torque on the shaft?

$$\text{Angle turned through per minute} = 1000 \times 2\pi$$
$$= 2000\ \pi \text{ radians}$$
$$\text{Torque (lb.-ft.)} \times 2000\pi = 100{,}000 \text{ ft.-lb.}$$
$$\text{Torque} = \frac{100{,}000}{2000\pi} = \underline{15 \cdot 9 \text{ lb.-ft.}}$$

89.

A chain 100 ft. long weighs 6 lb. per foot, and hangs vertically. Find the work done in winding it up on a drum at the top, and the work done in winding up the first 50 ft.

This question can be solved by two methods:

(1) By plotting the force against the distance and finding the area under the graph.

(2) Mathematically.

Method 1. Plot to some scale the Force (lb.) against distance in ft. Fig. 37 shows the diagram of work. If the scales chosen

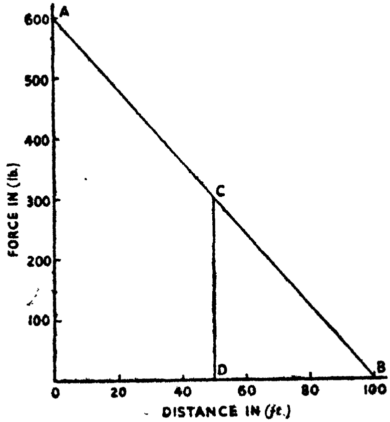


FIG. 37.

are 100 lb.=1 in. and 20 ft.=1 in., then 1 sq. in. represents $100 \times 20 = 2000$ ft./lb.

The total area of the diagram equals

$$\tfrac{1}{2}OB \times OA = \tfrac{1}{2} \times 6 \times 5 = 15 \text{ sq. in.}$$

Work done=15 sq. in.×2000 ft.-lb. per sq. in.=30,000 ft.-lb.

From the diagram we see that the *average* force is half *OA*. Therefore the work done is

$$\tfrac{1}{2}600 \times 100 = 30{,}000 \text{ ft.-lb.}$$

The work done in the first 50 ft. of lift is represented by the area *OACD*. It is not necessary to calculate the area, as it is evident the *average* force during the 50-ft. lift is=450

Hence work done=$450 \times 50 = \underline{22{,}500 \text{ ft.-lb.}}$

Method 2. When a force varies uniformly, and the minimum force is zero, the work done is $\frac{1}{2}$ maximum force $\times$ total distance.

$$\text{Work done} = \tfrac{1}{2}600 \times 100 = 30{,}000 \text{ ft.-lb.}$$

Work done in first 50-ft. lift is

$$\textit{average} \text{ force} \times \text{distance} = \frac{600+300}{2} \times 50 = 450 \times 50 = \underline{22{,}500 \text{ ft.-lb.}}$$

90.

A load is pulled by a tractive force F lb., which varies with the distance X ft. moved as given in table. Find the work done.

F (lb.)	800	780	740	680	660	680	740
X (ft.)	0	20	40	60	80	100	120

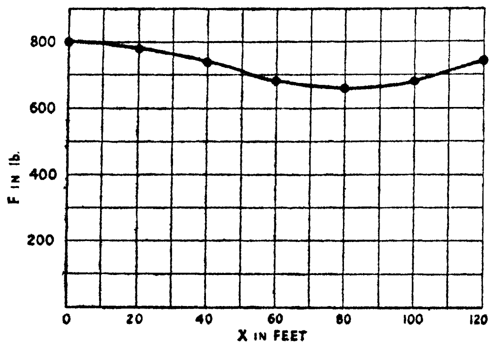


FIG. 38.

Plot the graph as in Fig. 38, using scales 1 in. = 20 ft., 1 in. = 200 lb. Then the area under the curve represents the work done. By the scale used, 1 sq. in. = $20 \times 200 = 4000$ ft.-lb.

The area can be found by (1) counting the squares, (2) mid-ordinate rule, (3) by planimeter.

(1) Number of squares = 21·5

$$\text{Work done} = 21{\cdot}5 \times 4000 = 86{,}000 \text{ ft.-lb.}$$

(2) Divide the base into 9 and get the average height of the diagram of work, and multiply by the length of the base line.

Average height $\times$ length of base = work done

$$722 \text{ lb.} \times 120 \text{ ft.} = 86{,}640 \text{ ft.-lb.}$$

(3) Area by planimeter = 21·55 sq. in.

$$\text{Work done} = \text{area} \times 4000 = 86{,}200 \text{ ft.-lb.}$$

91.

A spring is extended 6 in. from its unstretched state. If the spring requires a force of 10 lb. per inch of stretch, find the work done. What work is required stretching the last 2 in.?

Maximum force required $=6\times10=60$ lb.

Average force during stretch $=\dfrac{60}{2}=30$ lb.

Work done $=30\times6=$ 180 in.-lb.

Force at 6-in. stretch $=60$ lb.
Force at 4-in. stretch $=40$ lb.

Average force for last 2 in. $=\dfrac{60+40}{2}=50$ lb.

Work done in stretching last 2 in. $=50\times2=$ 100 in.-lb.

92.

A barge is towed along a canal by a horse at 2 miles per hour. The rope is inclined at an angle of 30° to the direction of motion of the barge. If the pull on the rope is 70 lb., calculate (1) the work done per hour, and (2) the h.p. required.

Work done $=$ force $\times$ distance moved $\times\cos30^\circ$

$=70\times2\times5280\times0{\cdot}8660$

$=$ 640,147 ft.-lb. per hour

The h.p. required $=\dfrac{640{,}147}{33{,}000\times60}=$ 0·323 h.p.

93.

The pull on the lower side of a belt is 160 lb., and on the upper side 20 lb. If the pulley is 2 ft. diameter and rotates at 400 r.p.m., calculate the power transmitted and the torque on the pulley.

Circumference in ft. $=2\pi$

Distance moved by rim of pulley per min.

$=400\times2\pi=800\pi$ ft.

Work done per min. $=$ effective force $\times$ distance

$=(160-20)\times800\pi$

$=140\times800\pi=$ 351,680 ft.-lb.

Torque $=140$ lb. $\times 1$ ft. $=140$ lb.-ft.

Angle turned through per min. $=400\times 2\pi=800\pi$ radians

$$\text{h.p.}=\frac{140\times 800\pi}{33{,}000}=\underline{10\cdot 66.}$$

94.

An I.C. engine has one cylinder 9·5 in. diameter, and the piston stroke is 19 in. If the mean effective pressure is 105 lb. per sq. in. and the number of explosions is 80 per min., find the I.H.P.

Area of piston $=0\cdot 7854d^2=0\cdot 7854\times 9\cdot 5^2$

Work done per cycle = force on piston × stroke (ft.)

$$=105\times 0\cdot 7854\times 9\cdot 5^2\times\frac{19}{12}$$

$$=11{,}790 \text{ ft.-lb.}$$

$$\underline{\text{I.H.P.}}=\frac{11{,}790\times 80}{33{,}000}=\underline{28\cdot 6.}$$

95.

Power is transmitted from an engine shaft by a belt passing over a pulley 30 in. diameter. The speed of the shaft is 120 r.p.m., and the tension on the tight and slack sides of the belt are 1350 lb. and 700 lb. respectively. Calculate the h.p. transmitted

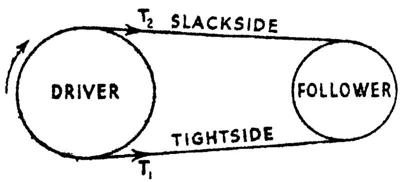


FIG. 39.

by the shaft, and the maximum stress in the belt. The belt is 5 in. × $\frac{3}{8}$ in. cross-section.

Referring to Fig. 39, we have:

Difference of tensions $=(T_1-T_2)$ lb. $=1350-700$
$=650$ lb.

Circumference of driver $=\pi d=\pi\times 2\cdot 5$ ft.

Speed of belt = circumference of driver × revolutions of driver per minute

$$= \pi \times 2{\cdot}5 \times 120$$

$$\text{h.p.} = \frac{650 \times \pi \times 2{\cdot}5 \times 120}{33{,}000} = \underline{18{\cdot}55}$$

$$T_1 = 1350 \text{ lb. in tight side}$$

$$\therefore \underline{\text{Maximum tension}} = \frac{1350}{\text{Area}} = \frac{1350}{5 \times \frac{3}{8}}$$

$$= \underline{720 \text{ lb. per sq. in.}}$$

96.

Two spur wheels in gear are transmitting 10 h.p. The driver has 60 teeth, and the pitch is 1 in. If the driver makes 150 r.p.m., what will be the tangèntial pressure between the teeth?

Power transmitted = pressure between teeth (lb.) × speed of pitch circle (ft. per min.)

$$\text{h.p.} = \frac{P \times \pi d N}{33{,}000}$$

P = tangential pressure

N = r.p.m.

$$P = \frac{33{,}000 \times \text{h.p.}}{\pi d \times N} \text{ lb.}$$

Circumference of pitch circle of driver

$$= 60 \times \frac{1}{12} = \frac{60}{12} = 5 \text{ ft.}$$

$$\therefore P = \frac{33{,}000 \times 10}{5 \times 150} = \underline{440 \text{ lb.}}$$

97.

A pinion gearing with a rack has 30 teeth of ½-in. pitch, and rotates at 80 r.p.m. The rack is fixed to the table of a planing machine. If the pressure between the teeth of the rack and pinion is 400 lb., at what speed will the table be driven? And what is the horse-power transmitted?

Circumference of pitch of pinion $=30\times\frac{1}{2}=15$ in.

$$=\frac{15}{12}\text{ ft.}$$

Speed of pitch circle = speed of rack $=\frac{15}{12}\times 80=100$ ft. per min.

Speed of table = 100 ft. per min.

$$\text{h.p. transmitted}=\frac{\text{pressure between teeth}\times\text{speed}}{33{,}000}$$

$$=\frac{400\times 100}{33{,}000}=\frac{40}{33}=1{\cdot}2\text{ h.p.}$$

CHAPTER X

98.

A flywheel weighing 2 tons has a radius of gyration of 3 ft. If there is a driving torque of 600 lb.-ft. and a resisting torque of 250 lb.-ft., find the time required to increase its speed from 20 to 80 revolutions per minute.

The effective accelerating torque $=600-250=350$ lb.-ft.

Torque or turning moment equals mass $\left(\frac{W}{g}\right)\times$(rate of change of velocity) $(a)\times$(radius of gyration)^2 $(k)^2$.

$$\therefore \text{Torque}=\frac{W}{g}k^2\times \text{rate of change of angular velocity } a$$

$$=\frac{W}{g}k^2\times a$$

where $\qquad a=$angular acceleration

$$\text{Torque}=\frac{Wk^2a}{g}$$

$$350=\frac{2\times 2240}{32\cdot 2}\times 3^2\times a$$

Angular acceleration a

$$=\frac{350\times 32\cdot 2}{2\times 2240\times 3^2}=0\cdot 28 \text{ radians per sec.}^2$$

Speed increase $=80-20=60$ revolutions per minute

$$=\frac{2\pi}{60}\times 60=2\pi \text{ radians per sec.}$$

$$\text{Time required}=\frac{2\pi}{0\cdot 28}=\underline{22\cdot 4 \text{ sec.}}$$

99.

A flywheel weighing 10 tons has a radius of gyration of 4·5 ft. and is revolving at 100 r.p.m. The coupling connecting it to the engine breaks, and the wheel is then subjected to a constant frictional torque of 450 lb.-ft. Calculate the number of revolutions the wheel will make before coming to rest, and the amount of work it will do against friction.

Resisting torque=450 lb.-ft.

$$\text{Resisting torque}=\frac{W}{g}k^2\times\text{angular retardation}$$

$$\therefore 450=\frac{10\times2240}{32\cdot2}\times(4\cdot5)^2\times\text{angular retardation}$$

Angular retardation=0·0336 radian per sec.²

$$100 \text{ r.p.m.}=\frac{100\times2\pi}{60}=\frac{10\pi}{3} \text{ radians per sec.}$$

$$\text{Time for wheel to come to rest}=\frac{\frac{10\pi}{3}}{0\cdot0336}=311\cdot3 \text{ sec.}$$

$$=5 \text{ min. } 11\cdot3 \text{ sec.}$$

$$\text{Average angular speed}=\frac{\text{maximum}}{2}=\frac{100}{2}=50 \text{ r.p.m.}$$

$$\text{Revolutions in coming to rest}=\frac{50\times311\cdot3}{60}=259\cdot4$$

Torque × angle in radians = work done per revolution

$$\therefore \text{Total work done}=450\times2\pi\times259\cdot4$$

$$=\underline{733{,}300 \text{ ft.-lb.}}$$

100.

In a fly-press for punching holes in iron plates, the two balls are placed at a radius of 30 in. from the axis of the screw, and the weight of each ball is 10 lb. If the pitch of the screw is 0·5 in. and the speed of the balls 240 r.p.m., calculate the energy of rotation, and the energy of translation, of the balls and the speed of revolution of the balls after punching a plate 0·25 in. thick if the average resistance is 5 tons.

$$\text{Kinetic energy of wheel}=\frac{\omega^2}{2g}\times I \text{ ft.-lb.}$$

Where ω=the angular velocity, in radians per sec.
I=the moment of inertia of the wheel, lb. and ft. units

$$\text{K.E. of rotation}=\frac{\left(\frac{240\times2\pi}{60}\right)^2}{64\cdot4}\times2\times10\times\left(\frac{30}{12}\right)^2=1225 \text{ ft.-lb.}$$

Vertical downward velocity of balls$=\dfrac{240\times 0\cdot 5}{66\times 12}=\frac{1}{6}$ ft./sec.

Kinetic energy of translation$=\dfrac{W}{2g}V^2$

$$=\frac{2\times 30}{64\cdot 4}\times\left(\frac{1}{6}\right)^2=\underline{0\cdot 026\text{ ft.-lb.}}$$

Work done in shearing plate$=\dfrac{5\times 2240\times 0\cdot 25}{12}=233\cdot 3$ ft.-lb.

Neglecting the K.E. of translation, which is small, and letting ω radians per sec.$=$velocity of the balls after shearing,

$$\text{K.E.}=\frac{\omega^2 I}{2g}$$

$$\therefore\ 1225-233\cdot 3=\frac{\omega^2\times 2\times 10\times\left(\frac{30}{12}\right)^2}{64\cdot 4}$$

$$\therefore\ \omega=22\cdot 6\text{ radians per sec.}$$

$$\text{Final velocity}=\underline{216\text{ r.p.m.}}$$

101.

A flywheel weighing 5 tons has a radius of gyration of 3·5 ft. and a speed of 200 r.p.m. Find its kinetic energy. If 100,000 ft.-lb. of energy are taken from it, to what speed will it fall?

$$\text{K.E. of wheel}=\frac{I\omega^2}{2g}=\frac{5\times 2240}{64\cdot 4}\times(3\cdot 5)^2\times\left(\frac{2\pi\times 200}{60}\right)^2$$

$$=934{,}000\text{ ft.-lb.}$$

Let N=speed in revolutions per minute after energy has been removed.

Since K.E. of wheel is proportional to the square of the speed, we have:

$$\frac{N^2}{200^2}=\frac{934{,}000-100{,}000}{934{,}000}=\frac{834}{934}$$

$$N=\sqrt{\frac{200^2\times 834}{934}}=\underline{189\text{ r.p.m.}}$$

102.

A weight of 40 lb. is attached to a cord which is wrapped round the 2-in. diameter spindle of a flywheel. The descending weight makes the wheel rotate. If the weight descends 6 ft. in 10 sec., and the friction of the bearing is equivalent to a force of 3 lb. at the circumference of the spindle, find the moment of inertia of the flywheel. If the flywheel weighs 212 lb., what is its radius of gyration?

Potential energy (P.E.) = kinetic energy of weight + kinetic energy of flywheel

$$\therefore 40 \times 6 = \frac{40V^2}{2g} + \frac{\omega^2 I}{2g} + 3 \times 6 \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Average velocity } (v) = \frac{6}{10} = 0{\cdot}6 \text{ ft./sec.}$$

$$\therefore \text{Maximum velocity } (V) = 2 \times 0{\cdot}6 = 1{\cdot}2 \text{ ft./sec.}$$

If r = radius of spindle in feet

V = speed in feet per sec.

ω = angular velocity radians per sec.

$$\omega = \frac{V}{r} = \frac{\frac{1{\cdot}2}{1}}{\frac{1}{12}} = 14{\cdot}4 \text{ radians/sec.}$$

Substituting in Equation (1), we have:

$$(40-3)6 = \frac{40 \times (1{\cdot}2)^2}{64{\cdot}4} + \frac{(14{\cdot}4)^2 I}{2 \times 32{\cdot}2}$$

$$222 = 0{\cdot}89 + 3{\cdot}22I$$

$$\frac{(222 - 0{\cdot}89)}{3{\cdot}22} = I = \underline{68{\cdot}6 \text{ lb.-ft.}^2}$$

$$I = \frac{W}{g}k^2$$

$$\therefore 68{\cdot}6 = \frac{212}{32{\cdot}2} \times k^2$$

$$k = \sqrt{\frac{68{\cdot}6 \times 32{\cdot}2}{212}}$$

$$= \underline{3{\cdot}2 \text{ ft.}}$$

103.

A flywheel and axle weighing 250 lb. is rotated by a falling weight of 50 lb. attached to a string wrapped round the 2·5-in.

diameter axle. The weight falls 5 ft. in 8 sec. Find the tangential frictional resisting force at the circumference of the axle assuming it to be constant. The radius of gyration of the wheel and axle is 7·75 in.

Let V=the velocity of the 50 lb. weight after falling 5 ft.
ω=the angular velocity of the flywheel at same instant
T=the frictional torque

$$V=\frac{5}{8}\times 2=1{\cdot}25 \text{ ft./sec.}$$

$$\omega=\frac{1{\cdot}25\times 12}{1{\cdot}25}=12{\cdot}0 \text{ radians/sec.}$$

I=moment of inertia of wheel and axle

$$=Mk^2=250\times\left(\frac{7{\cdot}75}{12}\right)^2=104{\cdot}2 \text{ lb.-ft.}^2$$

Number of revolutions made by wheel

$$=\frac{5\times 12}{\pi\times 2{\cdot}5}=7{\cdot}64$$

Energy lost by falling weight$=50\times 5=250$ ft.-lb.

$$\text{K.E. of weight}=\frac{W}{2g}V^2=\frac{50}{64{\cdot}4}\times(1{\cdot}25)^2=1{\cdot}21 \text{ ft.-lb.}$$

$$\text{K.E. of wheel and axle}=\frac{\omega^2 I}{2g}=\frac{12^2\times 104{\cdot}2}{2\times 32{\cdot}2}=233{\cdot}0 \text{ ft.-lb.}$$

$$\text{Energy lost in friction}=T\times 7{\cdot}64\times 2\pi=48{\cdot}0T$$

$$\therefore\ 250=1{\cdot}21+233{\cdot}0+48{\cdot}0T$$

$$T=\frac{15{\cdot}79}{48}=0{\cdot}329 \text{ lb.-ft.}=\underline{3{\cdot}95 \text{ lb.-in.}}$$

104.

The total mass of a railway truck is 5 tons, of which 1 ton is the mass of the wheels and axles. The wheels are 3 ft. 6 in. diameter, and the radius of gyration of each pair of wheels and axle is 12 in. The truck starts from rest on an incline of 1 in 100. Find the velocity and acceleration of the truck after travelling a distance of 500 yd. down the incline.

The vertical height dropped through by the truck is 5 yd.=15 ft.

Potential energy lost by truck (P.E.) equals kinetic energy (K.E.) gained by body+kinetic energy (K.E.) gained by wheels.

$$\therefore 5\times 15=\frac{5}{64\cdot 4}V^2+\frac{1}{64\cdot 4}\omega^2$$

But
$$\omega=\frac{V}{r}$$

$$\therefore 75=\frac{5}{64\cdot 4}V^2+\frac{1}{64\cdot 4}\times\frac{V^2}{(1\frac{3}{4})^2}$$

$$75=\frac{5}{64\cdot 4}V^2+\frac{16}{64\cdot 4\times 49}V^2$$

$$75=V^2\left(\frac{5}{64\cdot 4}+\frac{16}{64\cdot 4\times 49}\right)$$

$$V^2=906\cdot 7$$

$$\underline{V=30\cdot 1 \text{ ft./sec.}}$$

Acceleration of truck $V^2=u^2+2fs$

In this case, $u=0$, $s=1500$ ft.

$$f=\frac{V^2}{2s}=\frac{906\cdot 7}{2\times 1500}$$

$$=\underline{0\cdot 30 \text{ ft./sec.}^2}$$

105.

The rotor of a hydraulic turbine weighs 25 tons and has a radius of gyration of 5 ft. When running at 200 r.p.m., the rotor is suddenly relieved of part of its load so that its speed rises to 205 r.p.m. in 1 sec. Find the accelerating torque exerted, assuming this uniform throughout the change of speed. Calculate the change in the kinetic energy of the rotor.

Increase of rotor speed in 1 sec.

$$=205-200=5 \text{ r.p.m.}$$

$$\therefore \text{Angular acceleration } (\alpha)=\frac{5\times 2\pi}{60}\text{ radians/sec.}^2=\frac{\pi}{6}\text{ radians/sec.}^2$$

$$I \text{ for rotor}=\frac{W}{g}k^2=\frac{25\times 2240}{32\cdot 2}\times 5^2=43{,}500 \text{ lb./ft.}^2\text{/sec.}^2$$

$$\text{Accelerating torque}=I\times\alpha=43{,}500\times\pi/6$$

$$=\underline{22{,}750 \text{ lb.-ft.}}$$

$$=\underline{10\cdot 15 \text{ ton-ft.}}$$

Change of kinetic energy (K.E.)

$$=\tfrac{1}{2}I(\omega_2^2-\omega_1^2)$$

$$\omega_2=\frac{2\pi\times 205}{60}=21{\cdot}44 \text{ radians/sec.}$$

$$\omega_1=\frac{2\pi\times 200}{60}=20{\cdot}92 \text{ radians/sec.}$$

∴ Change of kinetic energy (K.E.)

$$=\tfrac{1}{2}\frac{143{,}500}{2240}(21{\cdot}44^2-20{\cdot}92^2)$$

$$=\underline{214 \text{ ft.-ton.}}$$

106.

In a plate-shearing machine the shears derive their energy from a fly-wheel. In shearing a particular plate 500,000 ft.-lb. of energy is required, and this reduces the speed of the fly-wheel from 300 to 200 r.p.m. Calculate the moment of inertia of the fly-wheel about its axis, and assuming the mass of the fly-wheel to be effective at 8 ft. diameter, estimate the mass of the wheel.

Let ω=angular velocity, radians/sec. at 300 r.p.m.
ω_1=angular velocity, radians/sec. at 200 r.p.m.

$$\omega=\frac{300\times 2\pi}{60}=10\pi \text{ radians/sec.}$$

$$\omega_1=\frac{200\times 2\pi}{60}=6\frac{2}{3}\pi \text{ radians/sec.}$$

$$\text{Loss of K.E. of fly-wheel}=\frac{I}{2g}(\omega^2-\omega_1^2)$$

$$\therefore 500{,}000 \text{ ft.-lb.}=\frac{I}{64{\cdot}4}\left[(10\pi)^2-\left(6\frac{2}{3}\pi\right)^2\right]$$

$$I=\frac{500{,}000\times 64{\cdot}4}{\pi^2\left(10^2-\left(6\frac{2}{3}\right)^2\right)}$$

$$=58{,}750 \text{ lb.-ft.}^2$$

$$I=Mk^2$$

k in this case $=4$ ft.

$$\therefore 58{,}750=M\times 4^2$$

$$M=\underline{3{,}672 \text{ lb. or } 1{\cdot}64 \text{ ton.}}$$

107.

An engine in starting exerts on the crank-shaft for 1 minute a constant turning moment of 1200 lb.-ft., and there is a uniform resisting moment of 600 lb.-ft. The fly-wheel has a radius of gyration of 4 ft., and weighs 2500 lb. Assuming that the only inertia is that of the fly-wheel, what speed will the engine attain after 1 minute from starting?

Let ω=angular velocity required in radians/sec.

$$\frac{\omega}{2}=\text{average angular velocity}$$

$$\text{Angle turned through in 1 minute}=60\times\frac{\omega}{2}=30\omega \text{ radians}$$

$$\text{Work done on fly-wheel in 1 minute}=(1200-600)30\omega$$
$$=18{,}000\omega \text{ ft.-lb.}$$

$$\text{K.E. of wheel}=\frac{W}{2g}(4\omega)^2=\frac{2500}{64\cdot4}\times(4\omega)^2$$

$$\therefore\ 18{,}000\omega=\frac{2{,}500\times16\omega^2}{64\cdot4}$$

$$\frac{18{,}000\times64\cdot4}{2500\times16}=\omega=29 \text{ radians/sec.}$$

$$=\frac{29\times60}{2\pi}=\underline{277 \text{ r.p.m.}}$$

108.

The crank arms and crank pin of a crank-shaft are equivalent to a mass of 650 lb. at 1 ft. radius. The shaft is supported in two

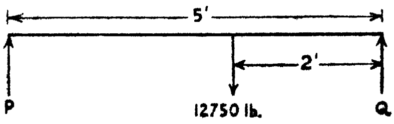


FIG. 40.

bearings 5 ft. from centre to centre, and the centre plane of the crank is 2 ft. from the centre of one of the bearings. Find the dynamical load on each bearing when the shaft is running at 240 r.p.m.

$$\text{Centrifugal force} = \text{mass} \times \text{acceleration}$$
$$= \frac{W}{g} \times \frac{V^2}{r} \text{ or } \frac{W}{g}\omega^2 r \text{ lb.}$$
$$\omega = \frac{240 \times 2\pi}{60} = 8\pi \text{ radians/sec.}$$
$$\therefore \text{Centrifugal force} = \frac{650}{32 \cdot 2} \times (8\pi)^2 \times 1 = 12{,}750 \text{ lb.}$$

Let this force at any position of the crank be balanced by the forces P and Q at the bearings (see Fig. 40).

Then, taking moments about P, we have:

$$3 \times 12{,}750 = Q \times 5$$
$$\therefore Q = 7{,}650 \text{ lb.}$$

and $$P = 5{,}100 \text{ lb.}$$

109.

A metal cylinder, with its axis vertical, weighs 2 lb. and falls vertically through 6 ft. on to a similar cylinder weighing 6 lb., which is supported on a spring. Find the speed with which the two bodies move downward before the effect of the spring retards the motion. How much of the energy is converted into heat in the impact?

Here potential energy is converted into kinetic energy.

So if v=velocity of 2 lb. cylinder at impact, we have:

$$\text{K.E.} = \text{P.E.}$$
$$\frac{2 \times v^2}{2 \times 32 \cdot 2} = 2 \times 6 \text{ ft.-lb.}$$
$$\therefore v^2 = \frac{2 \times 6 \times 64 \cdot 4}{2}$$

or $$v = 19 \cdot 6 \text{ ft./sec.}$$

$$\text{Momentum before impact} = \text{momentum after impact}$$
$$\therefore 2 \times 19 \cdot 6 = (2 + 6)v_1$$

Where v_1=velocity of the two cylinders immediately after impact.

$$\therefore v_1 = \frac{2 \times 19 \cdot 6}{8} = \underline{4 \cdot 9 \text{ ft./sec.}}$$

$$\text{K.E. immediately after impact} = \frac{8}{2 \times 32 \cdot 2} \times 4 \cdot 9^2 = 2 \cdot 9 \text{ ft.-lb.}$$

$$\text{Loss of K.E. at impact} = 12 \text{ ft.-lb.} - 2 \cdot 9 \text{ ft.-lb.} = \underline{9 \cdot 3 \text{ ft.-lb.}}$$

This is converted into heat.

CHAPTER XI

110.

A motor car weighing 25 cwt. is moving at 30 miles per hour. What is its momentum in engineers' units?

$$\text{Mass}=\frac{\text{Weight}}{g}$$

$$\text{Mass of car}=\frac{25\times 112}{32{\cdot}2}$$

$$\text{Velocity}=\frac{30\times 5280}{60\times 60}=44 \text{ ft. per sec.}$$

$$\text{Momentum}=\frac{\text{Weight in lb.}}{g}\times\text{velocity in ft. per sec.}$$

$$=\frac{25\times 112}{32{\cdot}2}\times 44=\underline{3826 \text{ units.}}$$

111.

A truck weighing 10 cwt. is moving at 10 miles per hour. After 30 sec. it is moving at 20 miles per hour. What is the average force acting upon it during this time?

$$\text{Change in velocity}=20-10=10 \text{ miles per hour}$$

$$=\frac{10\times 5280}{60\times 60}=14{\cdot}66 \text{ ft. per sec.}$$

$$\text{Change in momentum}=\frac{10\times 112}{32{\cdot}2}\times 14{\cdot}66=34{\cdot}8 \text{ units}$$

$$\text{Average force}=\frac{34{\cdot}8}{30}=\underline{1{\cdot}16 \text{ lb.}}$$

112.

A train travelling at 45 m.p.h. is pulled up gradually, the retardation being 3·5 ft./sec./sec. (*a*) How far will it travel before coming to rest? (*b*) If the retarding force is 70 tons, what is the weight of the train?

$$45 \text{ m.p.h.}=\frac{45\times 5280}{60\times 60}=66 \text{ ft. per sec.}$$

$$v^2=u^2-2fs$$

$$\therefore 0=66^2-2\times 3{\cdot}5\times s$$

$$s=\frac{66^2}{7}=\underline{622\tfrac{2}{7} \text{ ft.}}$$

Let W=weight of train in tons

$$\text{Mass of train} = \frac{W \times 2240}{32 \cdot 2} \text{ units}$$

$$P = mf$$

$$\therefore 70 \times 2240 = \frac{W \times 2240}{32 \cdot 2} \times 3 \cdot 5$$

$$W = \frac{70 \times 32 \cdot 2}{3 \cdot 5} = \underline{644 \text{ tons.}}$$

113.

A train weighing 400 tons has a frictional resistance of 20 lb. per ton. What average pull will be required if it is to attain a speed of 60 m.p.h. from rest in 4 min. on the level, and what will be the horse-power required at the end of this time?

60 m.p.h.=88 ft. per sec.

$$\text{Acceleration (average)} = \frac{88}{60 \times 4} = 0 \cdot 366 \text{ ft. per sec./sec.}$$

$$\text{Accelerating force} = \frac{400 \times 2240}{32 \cdot 2} \times 0 \cdot 366 = 10{,}180 \text{ lb.}$$

Force required to overcome frictional resistance

$$= 400 \times 20 = 8000 \text{ lb.}$$

$$\therefore \text{Total pull} = 8000 + 10{,}180 = 18{,}180 \text{ lb.}$$

$$\text{Horse-power} = \frac{18{,}180 \times 88}{550} = \underline{2909 \text{ h.p.}}$$

114.

A motor car weighing 30 cwt. has to attain a speed of 30 m.p.h. in 30 sec. from rest up an incline of 1 in 50, the frictional resistance, etc., being 1 lb. per cwt. What is the minimum horse-power required?

Force required to overcome friction=30×1=30 lb.

Force required to overcome gravity=$W \sin \alpha$

Where W=weight of car, and α=the angle made by the incline with the horizontal, where α is small, $\sin \alpha = \tan \alpha$, very nearly.

$$\therefore W \sin \alpha = W \tan \alpha = 30 \times 112 \times \frac{1}{50} = 67 \cdot 2 \text{ lb.}$$

Acceleration is 44 ft. per sec. in 30 sec.

$$= \frac{44}{30} \text{ ft. per sec. per sec.}$$

$$\text{Mass} \times \text{acceleration} = \text{accelerating force}$$

$$\frac{30 \times 112}{32 \cdot 2} \times \frac{44}{30} = \text{accelerating force} = 153 \text{ lb.}$$

$$\text{Total force required} = 67 \cdot 2 + 153 + 30 = 250 \cdot 2 \text{ lb.}$$

$$\text{Horse-power} = \frac{250 \cdot 2 \times 44}{550} = \underline{20 \text{ h.p.}}$$

115.

A lift has an upward acceleration of 4 ft. per sec./sec. A man weighing 170 lb. is in the lift. What force will he exert on the floor of the lift? If the lift is descending with the same acceleration, what force will the man exert on the floor of the lift?

Since action and reaction are equal and opposite, the man exerts a downward pressure equal to the upward pressure of the floor on the man. The upward pressure must exceed the man's weight by an amount equal to that required to give him an acceleration of 4 ft. per sec./sec.

$$\text{Upward pressure (lift ascending)} = 170 + \text{mass} \times \text{acceleration}$$

$$= 170 + \frac{170}{32 \cdot 2} \times 4 = \underline{191 \cdot 1 \text{ lb.}}$$

When the lift is descending the upward pressure will be less than the man's weight.

$$\therefore \text{Upward pressure (lift descending)} = 170 - \frac{170}{32 \cdot 2} \times 4 = \underline{148 \cdot 9 \text{ lb.}}$$

116.

A train has a mass of 400 tons. The frictional resistance is 15 lb. weight per ton. What steady pull must the engine exert, if the speed on the level is increased from 20 to 60 miles per hour, in 2 min.?

Let T=pull required in lb.

F=total frictional resistance lb.

P=resultant force producing acceleration in lb.

Then
$$P=T-F=\frac{mf}{g}$$
$$F=400\times 15=6000 \text{ lb.}$$
$$\text{Initial velocity}=20 \text{ m.p.h.}=\frac{88}{3} \text{ ft. per sec.}$$
$$\text{Final velocity}=60 \text{ m.p.h.}=88 \text{ ft. per sec.}$$
$$\text{Acceleration}=f=\frac{88-\frac{88}{3}}{120}=\frac{176}{360} \text{ ft. per sec./sec.}$$
$$P=T-F=\frac{mf}{g}$$
$$\therefore T-6000=\frac{400\times 2240}{32\cdot 2}\times\frac{176}{360}$$
$$T=6000+13{,}600=19{,}600 \text{ lb.}=\underline{8 \text{ tons.}}$$

117.

The mass of a train is 400 tons. The frictional resistance is 15 lb. weight per ton. Its speed at the top of an incline of 1 in 100 is 60 m.p.h., and the incline is 1 mile long. If the train runs down the incline with steam shut off, what will be its speed when it reaches the bottom of the incline?

Resolving the weight of the train W into two forces T and R, one parallel to the plane, the other perpendicular to it, we have:
$$T=W \sin \alpha=W \tan \alpha \text{ (where } \alpha \text{ is small)}$$
$$T=400\times 2240\times\frac{1}{100}=8960 \text{ lb.}$$
$$F=400\times 15=6000 \text{ lb.} \checkmark$$
$$P=T-F=8960-6000=2960 \text{ lb.}$$
$$P=\frac{mf}{g}$$
$$f=\frac{Pg}{m}=\frac{2960\times 32\cdot 2}{400\times 2240}=0\cdot 111 \text{ ft. per sec./sec.}$$
$$\text{Initial velocity } u=60 \text{ m.p.h.}=88 \text{ ft. per sec.}$$
$$v^2=u^2+2fs$$
$$\therefore v^2=88^2+2\times 0\cdot 111\times 5280$$
$$v=\sqrt{88^2+0\cdot 222\times 5280}=94\cdot 4 \text{ ft. per sec.}$$
$$=\underline{64\cdot 3 \text{ m.p.h.}}$$

118.

The head of a sledge-hammer weighs 14 lb., and is moving at 15 ft. per sec. It is brought to rest in $\frac{1}{300}$ sec. Find the average force of the blow.

$$\text{Change of momentum} = \frac{14}{32 \cdot 2} \times 15 = 6 \cdot 5 \text{ units}$$

Change of momentum per sec. = average force

$$\frac{14 \times 15}{32 \cdot 2} \div \frac{1}{300} = \text{average force} = \underline{1957 \text{ lb.}}$$

119.

A steam hammer weighs 5 tons, the piston of which has a diameter of 6 in. If the effective steam pressure is 100 lb. per sq. in., find (1) the acceleration with which the hammer comes down, (2) the velocity of the hammer after it has fallen 2 ft., and (3) the average force of the blow if the hammer is brought to rest in $\frac{1}{300}$ sec.

Total weight = weight of hammer + total steam pressure on piston

$$= 5 + \frac{0 \cdot 7854 \times 6 \times 6 \times 100}{2240} = 6 \cdot 26 \text{ tons}$$

$$\frac{\text{Force}}{\text{Mass}} = \text{acceleration}$$

$$\therefore f = \frac{6 \cdot 26 \times 2240}{\frac{5 \times 2240}{32 \cdot 2}} = \underline{40 \cdot 3 \text{ ft./sec.}^2}$$

$v^2 = u^2 + 2fs$; in this case, initial velocity $u = 0$

$$\therefore v = \sqrt{2 \times 40 \cdot 3 \times 2} = 2\sqrt{40 \cdot 3} = \underline{6 \cdot 3 \text{ ft. per sec.}}$$

Change of momentum = mass × change of velocity

$$= \frac{W}{g} \times V$$

$$= \frac{5 \times 2240}{32 \cdot 2} \times 6 \cdot 3 \text{ units}$$

Average force = change of momentum per sec. + steam load

$$= \frac{5 \times 2240}{32 \cdot 2} \times 6 \cdot 3 \div \frac{1}{300} \text{ lb.} + 1 \cdot 26$$

$$= \frac{5 \times 6 \cdot 3 \times 300}{32 \cdot 2} + 1 \cdot 26 = \underline{294 \cdot 8 \text{ tons.}}$$

120.

A block of iron 2 in. by 3 in. by 2 in. is fixed to the arm of a wheel, its centre of gravity being 2·5 ft. from the axis. If the wheel is rotating at 1000 r.p.m., what is the force tending to fracture the fastening? (1 cub. in. of iron weighs 0·26 lb.)

$$\text{Weight of block } (W)=2\times3\times2\times0{\cdot}26=3{\cdot}12 \text{ lb.}$$

$$\text{Centrifugal force}=\frac{WV^2}{gr}=\text{force on fastening}$$

$$V=2\pi\times2{\cdot}5\times\frac{1000}{60}=261{\cdot}7 \text{ ft./sec.}$$

$$\text{Force on fastening}=\frac{WV^2}{gr}=\frac{3{\cdot}12\times(261{\cdot}7)^2}{32{\cdot}2\times2{\cdot}5}=\underline{2654 \text{ lb.}}$$

121.

A crank 18 in. long is driven by a piston, and rotates at 180 r.p.m. What is the acceleration of the piston when it is 2 in. from the end of the stroke? (Assume motion to be simple harmonic.)

Fig. 41 illustrates the position of the piston X, 2 in. from the end of the stroke.

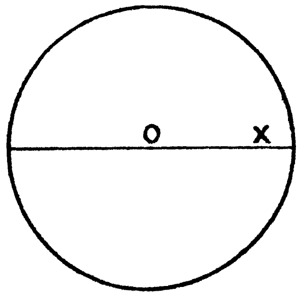


FIG. 41.

$$\text{Acceleration}=\frac{V^2}{r}\times\frac{OX}{r}$$

where r=radius of crank

$=1{\cdot}5$ ft.

and $V=2\pi\times\frac{18}{12}\times\frac{180}{60}=28{\cdot}26$ ft./sec.

$$\frac{OX}{r}=\frac{18-2}{18}$$

$$\therefore \text{Acceleration}=\frac{(28{\cdot}26)^2}{1{\cdot}5}\times\frac{16}{18}=\underline{473{\cdot}1 \text{ ft./sec.}^2}$$

CHAPTER XII

122.

A beam is 15 ft. long and is simply supported at each end. It carries loads of 2 tons and 4 tons at points 3 ft. and 8 ft. respectively from the left-hand support.

(*a*) Draw to scale a shear force diagram. State the value and position of the max. shear force.

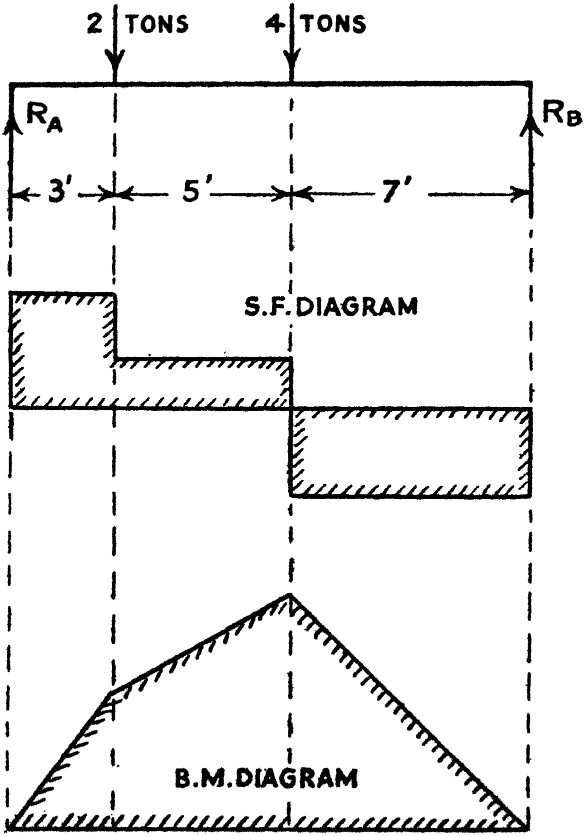


FIG. 42.

(*b*) Draw to scale the bending moment diagram. State the value and position of the max. bending moment.

Fig. 42 represents to scale—1 ton=$\frac{1}{4}$ in. the shearing force diagram for the beam.

The max. S.F. equals 3·467 tons at the left-hand support.

The bending diagram is drawn to a scale of 10 ton.-ft.=1 in., and the max. B.M. is found to equal 17·71 ton.-ft. at the 4-ton load.

123.

A cantilever 10 ft. long is loaded as follows: 5 tons 3 ft. from the wall, 2 tons 5 ft. from the wall, and 1 ton at the free end. Calculate the bending moment and shearing force at each load

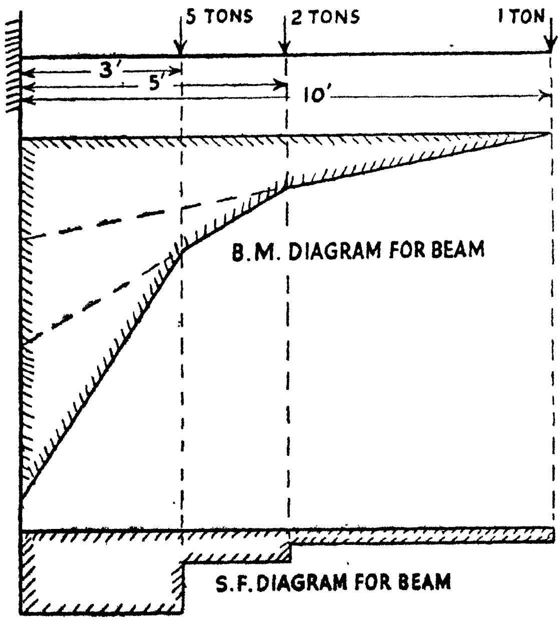


FIG. 43.

and at the wall, and draw the bending moment and shearing force diagrams.

The loading of the beam is shown in Fig. 43.

B.M. at wall $=3\times5+5\times2+10\times1=35$ ton-ft.
B.M. at 3 ft. $=2\times2+7\times1=11$ ton-ft.
B.M. at 5 ft. $=5\times1=5$ ton-ft.
S.F. at wall $=5+2+1=8$ ton
S.F. at 3 ft. $=2+1=3$ ton
S.F. at 5 ft. $=1$ ton

The bending moment and shearing force diagrams can now be drawn.

124.

A rectangular wooden beam 12 ft. long has a breadth of 6 in. and a depth of 12 in. It carries a load of 30 cwt. at the middle of the span, and is simply supported at the ends. Calculate the greatest stress in the beam.

Since the beam is simply supported and centrally loaded, the reaction at each end will be half the total load 15 cwt.

$$\text{The greatest B.M.}=\frac{15}{20}\times\frac{12}{2}\times12=54 \text{ tons-in.}$$

$$\text{Bending moment}=f\times\frac{bd^2}{6}$$

Where f=maximum intensity of stress, b=breadth and d=depth.

$$\therefore f=\frac{BM}{\frac{bd^2}{6}}=\frac{54}{\frac{6\times12^2}{6}}=\underline{0{\cdot}38 \text{ tons/in.}^2}$$

125.

A rolled steel I section girder 20 ft. long has the following dimensions: overall depth, 20 in.; overall width, 8 in.; thickness

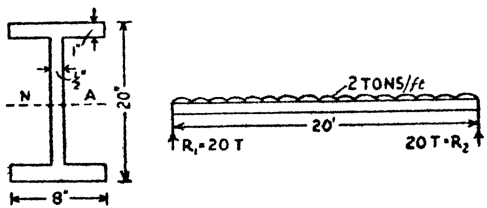


FIG. 44.

of web, 0·5 in.; thickness of flanges, 1 in. It is supported at each end and carries a uniformly distributed load of 2 tons per foot run. Determine the longitudinal stress.

Fig. 44 represents the section and loading of the beam.

$$\frac{M}{I}=\frac{f}{y}$$

where M=the bending moment, lb.-in.

I=moment of inertia of section, about the neutral axis in inch units

f=stress in lb./sq. in. due to bending at a distance y from the neutral axis

$$I_{xx}=\frac{BD^3-bd^3}{12}=\frac{8\times20^3-7\cdot5\times18^3}{12}=1686 \text{ in.}^4 \text{ units}$$

$$M=\frac{WL}{8}=\frac{20\times2\times20}{8}=100 \text{ ton-ft.}$$

$$=1200 \text{ ton-in.}$$

$$\therefore f=\frac{M}{I}\times y=\frac{1200\times10}{1686}=7\cdot11 \text{ ton/sq. in.}$$

126.

A simply supported beam spans 20 ft. It carries a uniformly distributed load of 100 lb. per foot for the first 8 ft. from the

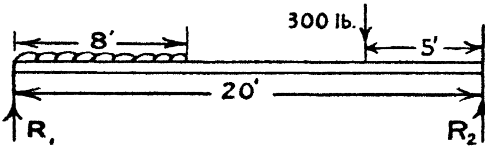


FIG. 45.

left-hand support, and a concentrated load of 300 lb. 15 ft. from the left-hand support. Determine the value and position of the maximum bending moment. If a rectangular section beam 9 in. deep and 4 in. wide is used, what is the maximum stress in it?

Fig. 45 represents the loading.

To find the reactions R_1 and R_2, take moments about support R_1; then

$$20R_2=300\times 15+800\times 4$$
$$R_2=385 \text{ lb.} \quad R_1=715 \text{ lb.}$$

By drawing the B.M. diagram we find

Max. B.M. = 2556 lb.-ft. 7·15 ft. from left-hand support

$$\frac{M}{I}=\frac{f}{y}$$
$$I=\frac{BD^3}{12}=\frac{4\times 9^3}{12}=243 \text{ in.}^4 \text{ units}$$
$$f \text{ max.}=\frac{M}{I}\times y=\frac{2556\times 12}{243}\times\frac{9}{2}=555 \text{ lb./sq. in.}$$

127.

A girder spans 20 ft. and is freely supported at each end. It consists of a 14-in. by 6-in. B.S.B. with a plate 10 in. by 1·5 in. attached to each flange. I_{xx} for the B.S.B. is 533·34 in.^{4} units. The weight per foot of the compound girder is 150·5 lb. What is the maximum uniformly distributed load in lb./ft.

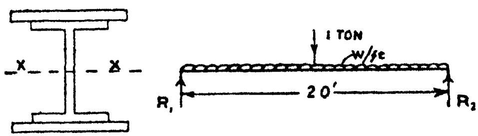


FIG. 46.

which the girder can carry in addition to its own weight, and a concentrated load of 1 ton at the centre of the span? Maximum allowable stress intensity equals 5 ton/sq. in.

Fig. 46 shows the section of the beam and the method of loading.

Total moment of inertia about axis xx

$$=I_{xx}=533\cdot 34+2\left(\frac{10\times(1\cdot 5)^3}{12}+10\times 1\cdot 5\times(7\cdot 75)^2\right)$$
$$=2340\cdot 84 \text{ in.}^4 \text{ units}$$

Let the total distributed load per foot run $=w$ lb.

To find the reactions R_1 and R_2 we have:

$$R_1=R_2=\frac{2240}{2}+\frac{20w}{2}=(1120+10w) \text{ lb.}$$

$M_{max.}$ at centre of span $=(1120+10w)10-10w\times 5$ lb.-ft.

$=11,200+50w$ lb.-ft.

$$\frac{M}{I}=\frac{f}{y}$$

$$\therefore M=\frac{f}{y}\times I=\frac{5\times 2240\times 2340\cdot 84}{8\cdot 5}$$

$$=\underline{3,083,000 \text{ lb.-in.}}$$

$$\therefore (11,200+50w)12=3,083,000$$

$$w=\frac{3,083,000-12\times 11,200}{12\times 50}=4913 \text{ lb.-ft.}$$

$\therefore$ Distributed load in addition to weight of beam

$$=4913-150\cdot 5=\underline{4763 \text{ lb.-ft.}}$$

128.

A floor has to carry a load of 3 cwt. per square foot. The floor joists are 12 in. deep and 4·5 in. wide and have a span of 14 ft. Determine the distance apart, from centre to centre, at which these joists must be spaced if the maximum stress is not to exceed 1000 lb. per sq. in.

Let d=distance in feet between centres of joist

Area of floor supported per joist $=14\times d$ sq. ft.

Total distributed load per joist $=3\times 112\times 14\times d$ lb.

$=336d$ lb. per ft.

$$\text{Maximum B.M.}=\frac{wl^2}{8}=\frac{336d\times 14^2}{8}=42\times 14^2 d \text{ lb.-ft.}$$

$$I \text{ for joist}=\frac{BD^3}{12}=\frac{4\cdot 5\times 12^3}{12}=4\cdot 5\times 144 \text{ in.}^4 \text{ units}$$

$$f=\frac{M}{I}\times y$$

$$y=\frac{12}{2}=6 \text{ in.}$$

$$\therefore 1000=\frac{(42\times 14^2\times d)\times 12}{4\cdot 5\times 144}\times 6$$

$$d=\frac{144,000\times 4\cdot 5}{42\times 14^2\times 6\times 12}=\underline{1\cdot 093 \text{ ft.}}$$

129.

The compression flange of a cast-iron girder is 4 in. wide and 1·5 in. deep. The tension flange is 12 in. wide by 2 in. deep, and the web 10 in. by 1·5 in. Find (*a*) the distance of the centroid

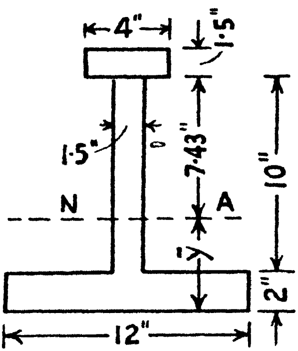


FIG. 47.

from the tension edge, (*b*) the moment of inertia about the neutral axis, and (*c*) the load per ft. run which may be carried over a 10-ft. span by this beam simply supported at its ends without the skin tension exceeding 1 ton per sq. in. What is then the maximum intensity of compressive stress?

Fig. 47 represents the section of the C.I. girder.

Total area of section

$$=4\times1\cdot5+1\cdot5\times10+2\times12$$
$$=45 \text{ sq. in.}$$

Taking moments about the base, we have:

$$45\times\bar{Y}=(12\times2)\times1+(10\times1\cdot5)\times7+(4\times1\cdot5)\times12\cdot75$$

$$\bar{Y}=\frac{205\cdot5}{45}=\underline{4\cdot57 \text{ in.}}$$

To find the moment of inertia about N.A., we have:

$$I_{\text{N.A.}}=\frac{12\times4\cdot57^3}{3}-\frac{10\cdot5\times2\cdot57^3}{3}+\frac{4\times8\cdot93^3}{3}-\frac{2\cdot5\times7\cdot43^3}{3}$$
$$=381\cdot76-59\cdot4+950-342$$
$$=\underline{930\cdot36 \text{ in.}^4 \text{ units}}$$

Maximum bending moment for uniformly distributed load

$$=\frac{wl^2}{8}=\frac{w\times(10\times12)^2}{8}$$

$$\frac{M}{I}=\frac{f}{y}$$

$$\therefore\ \frac{w\times(10\times12)^2}{8\times930{\cdot}36}=\frac{1}{4{\cdot}57}$$

$$w=\frac{8\times930{\cdot}36}{4{\cdot}57\times(10\times12)^2}$$

$$w=0{\cdot}113 \text{ ton/in. length}=\underline{1{\cdot}355 \text{ ton/ft. length}}$$

Since stress is proportional to distance from N.A.,
Maximum compressive stress

$$=\frac{1}{4{\cdot}57}\times8{\cdot}93=\underline{1{\cdot}94 \text{ ton/sq. in.}}$$

130.

A rolled steel joist 16 in. deep, with flanges 6 in. wide and 1 in. thick, the web being 0·75 in. thick, is used to support a

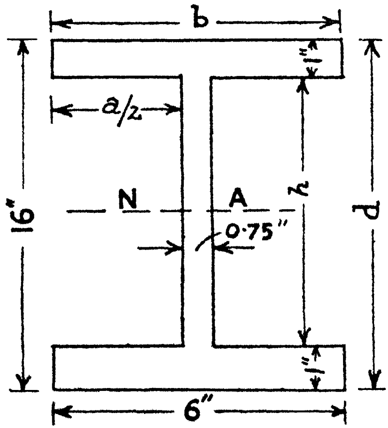


FIG. 48.

uniformly distributed load of 2 ton per ft. run. If the span is 12 ft. 6 in., what is the maximum direct stress in the lower flange?

Fig. 48 illustrates a section of the beam.

Moment of inertia about N.A. $=I=\frac{bd^3}{12}-\frac{ah^3}{12}$

$$I=\frac{6\times16^3}{12}-\frac{5\cdot25\times14^3}{12}=848\text{ in.}^4\text{ units}$$

$$\text{Maximum B.M.}=\frac{wl^2}{8}$$

where w = load per ft. run

$$\text{B.M.}=\frac{2\times(150)^2}{12\times8}$$

$$\frac{M}{I}=\frac{f}{y}$$

$$\therefore f=\frac{My}{I}=\frac{2\times(150)^2\times8}{12\times8\times848}=\underline{4\cdot42\text{ ton/in.}^2}$$

131.

A beam is freely supported in the horizontal position and spans 20 ft. It carries a total load of 5 tons. Calculate the maximum bending moment and shearing force on the beam: (1) when the load is concentrated at the middle of the span, (2) when the load is uniformly distributed over the span, and (3) when the intensity of loading increases gradually from zero at one end of the span.

$$(1)\ \text{B.M.}_{max.}=\frac{WL}{4}=\frac{5\times20}{4}=\underline{25\text{ ton-ft.}}$$

$$S_{max.}=\frac{W}{2}=\frac{5}{2}=2\cdot5\text{ tons}$$

$$(2)\ \text{B.M.}_{max.}=\frac{WL}{8}=\frac{5\times20}{8}=\underline{12\cdot5\text{ ton-ft.}}$$

$$S_{max.}=\frac{W}{2}=\frac{5}{2}=2\cdot5\text{ tons}$$

$$(3)\ \text{B.M.}_{max.}=\frac{2WL}{9\sqrt{3}}=0\cdot128WL=0\cdot128\times5\times20$$

$$=\underline{12\cdot8\text{ ton-ft.}}$$

$$S_{max.}=\frac{2W}{3}=\frac{2\times5}{3}=\underline{3\cdot33\text{ tons.}}$$

132.

A bar of mild steel is 10 ft. long and 0·75 in. diameter. Determine the instantaneous stress induced in the bar when a weight of 30 lb. falls freely from rest through 8 ft. and is then suddenly pulled up by a collar on the end of the bar. $E=30\times10^6$ lb. per sq. in.

Fig. 49 represents the arrangement.

Work done by load $=W(h+x)$

Where W = the weight 30 lb.
h = height through which it falls
x = the elongation of rod

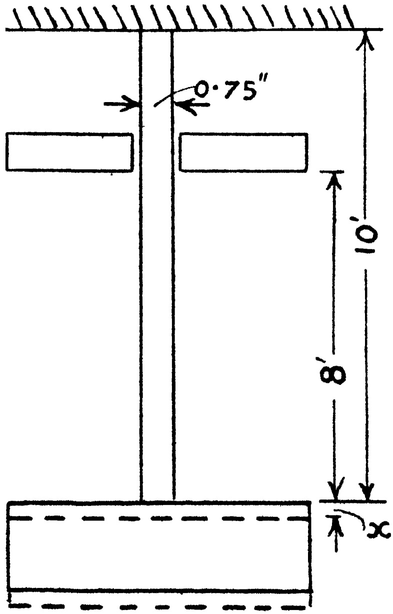


FIG. 49.

If f = maximum stress induced in rod
A = cross-sectional area of rod

Then $$W\left(h+\frac{fl}{E}\right)=\tfrac{1}{2}f^2\frac{Al}{E}$$

$$\therefore\ 30\left(12\times8+\frac{f\times120}{E}\right)=\frac{1}{2}\times\frac{f^2}{E}\times\frac{\pi}{4}\times\frac{9}{16}\times120$$

$$12\times8\times(30\times10^6)+f\times120=0{\cdot}88f^2$$

This is a quadratic equation in f which can be solved in the usual way.

$$f = \underline{57{,}275 \text{ lb./sq. in.} = 25{\cdot}5 \text{ ton/sq. in.}}$$

133.

Determine the maximum tensile and compressive str intensities in a beam, the section of which is shown in Fig. 5

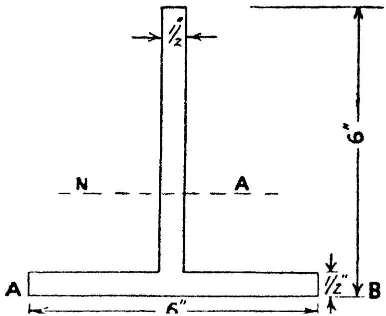


FIG. 50.

which is simply supported over a span of 12 ft., and carries a uniformly distributed load of 112 lb. per foot.

First find the neutral axis N.A. by taking moments about A.B. and let x=distance N.A. from A.B.

Then $$(6\times0{\cdot}5+5{\cdot}5\times0{\cdot}5)x=(6\times0{\cdot}5)0{\cdot}25+(5{\cdot}5\times0{\cdot}5)3{\cdot}25$$

$$\therefore\ 5{\cdot}75x=0{\cdot}75+8{\cdot}938$$

$$x=\frac{9{\cdot}688}{5{\cdot}75}=\underline{1{\cdot}7 \text{ in.}}$$

The moment of inertia $I_{N.A.}$

$$=\frac{6\times(\cdot5)^3}{12}+6\times\cdot5\times(1{\cdot}45)^2+\frac{\cdot5\times(5{\cdot}5)^3}{12}+\cdot5\times5{\cdot}5\times(1{\cdot}55)^2$$

$$=\cdot062+6{\cdot}3+13{\cdot}864+6{\cdot}607=\underline{26{\cdot}83 \text{ in.}^4 \text{ units}}$$

Maximum B.M.

$$=\frac{WL^2}{8}=\frac{(112\times12)\times12^2}{8}=\underline{24{,}190 \text{ lb.-in.}}$$

Let f_t=maximum tensile stress intensity
f_c=maximum compressive stress intensity

Then $f_t=\dfrac{My}{I}$

where $y=1\cdot7$ in.

$$f_t=\frac{24{,}190\times1\cdot7}{26\cdot83}=1532\text{ lb./sq. in.}=\underline{0\cdot68\text{ ton/sq. in. tension}}$$

$$f_c=\frac{My}{I}$$

where $y=4\cdot3$ in.

$$f_c=\frac{24{,}190\times4\cdot3}{26\cdot83}=3891\text{ lb./sq. in.}=\underline{1\cdot7\text{ ton/sq. in. compression.}}$$

134.

A rolled steel joist 8 in. deep has flanges 5 in. wide by 0·75 in. thick. Find the maximum span that may be used, if the maximum intensity of stress is not to exceed 6 ton/in.^{2} and the beam carries a uniformly distributed load of 1·5 ton per foot run.

Let l=length of span in feet, and w=load per foot run

Maximum bending moment will be at mid-span and

$$=\frac{wl^2}{8}\text{ ton-ft.}$$

$$=\frac{w12l^2}{8}\text{ ton-in.}$$

Moment of inertia I_x for beam$=\dfrac{BD^3-bd^3}{12}$

$$=\frac{5\times8^3-4\cdot25\times(6\cdot5)^3}{12}=116\text{ in.}^4$$

Modulus of resistance $(Z)=I_x\Big/\dfrac{D}{2}$

$$=\frac{116\times2}{8}=29$$

Bending moment=moment of resistance$=fZ$

where f=max. intensity of stress

$$\therefore\ \frac{1\cdot5\times12l^2}{8}=6\times29$$

$$l^2=\frac{6\times8\times29}{1\cdot5\times12}=77\cdot3$$

$$l=\underline{9\text{ ft. approx.}}$$

CHAPTER XIII

135.

A steel rod, 1 in. diameter, is subjected to an axial couple of 120 lb.-ft., and is found to twist through an angle of 40′ measured over a length of 10 in. Calculate the rigidity modulus.

$$\frac{T}{I_p}=\frac{C\theta}{l}$$

where T=twisting moment in lb.-in.
I_p=polar moment of inertia of section
C=rigidity modulus in lb./in.^{2}
θ=angle of twist in radians in a length of l in.

$$T=120\times 12 \text{ lb.-in.}$$

$$I_p=\frac{\pi}{2}r^4=\frac{\pi}{2}(0{\cdot}5)^4 \text{ in units}$$

$$\theta=\frac{40}{60}\times\frac{\pi}{180} \text{ radian}$$

$$l=10 \text{ in.}$$

$$\therefore\ C=\frac{T\times l}{I_p\times\theta}=\frac{120\times 12\times 10\times 2\times 60\times 180}{\pi\times(0{\cdot}5)^4\times 40\times\pi\times 2240}$$

$$=\underline{5895 \text{ ton/in.}^2}$$

136.

What is the maximum horse-power that can be safely transmitted by a hollow shaft, 10 in. external diameter, and 8 in. internal diameter, at 300 r.p.m., if the shear stress in the material is limited to 6000 lb./sq. in.?

Let R=external radius of shaft in inches
r=internal radius of shaft in inches
f_s=shear stress in lb./sq. in. at a radius R in.
I_p=polar moment of inertia of section in inch units
T=twisting moment in lb.-in.

Then
$$T=\frac{f_s I_p}{R}$$

$$I_p=\frac{\pi}{2}(R^4-r^4)=\frac{\pi}{32}(D^4-d^4)=\frac{\pi}{32}(10^4-8^4)$$

$$=\frac{\pi}{32}(10{,}000-4090)=580 \text{ in.}^4 \text{ units}$$

$$\therefore T=\frac{6000\times 580}{5\times 12}=58{,}000 \text{ lb.-ft.}$$

$$\text{Horse-power}=\frac{2\pi NT}{33{,}000}$$

$$\text{h.p.}=\frac{2\pi\times 300\times 58{,}000}{33{,}000}=\underline{3320 \text{ h.p.}}$$

137.

Find the maximum stress in a propeller shaft 16 in. external diameter and 8 in. internal diameter when subjected to a twisting moment of 1800 tons/in. Compare the strength of this shaft with a solid one of the same material and weight.

$$T=\frac{f_s I_p}{R}$$

$$\therefore f_s=\frac{TR}{I_p}$$

$$I_p=\frac{\pi}{32}(D^4-d^4)=\frac{\pi}{32}(16^4-8^4)$$

$$I_p=6030 \text{ in.}^4 \text{ units}$$

$$f_s=\frac{1800\times 8}{6030}=\underline{2{\cdot}38 \text{ ton/sq. in.}}$$

$$T_{\text{hollow}}=\frac{f_s}{R}\frac{\pi}{2}(R^4-r^4)$$

$$T_{\text{solid}}=f_s\frac{\pi}{2}R_1^3$$

Where R_1=radius of solid shaft in inches

Since f_s is the same for both shafts, we have:

$$\frac{T_{\text{hollow}}}{T_{\text{solid}}}=\frac{R^4-r^4}{R\times R_1^3}$$

And since the weights are the same

$$R_1^2=R^2-r^2$$

$$\frac{T_{\text{hollow}}}{T_{\text{solid}}}=\frac{R^2+r^2}{RR_1}$$

Let $$n=\frac{R}{r}$$

Then $$\frac{T_{\text{hollow}}}{T_{\text{solid}}}=\frac{R}{R_1}\left(1+\frac{1}{n^2}\right)=\frac{n^2+1}{n\sqrt{n^2-1}}$$

$$\therefore \frac{\text{Strength of hollow shaft}}{\text{Strength of solid shaft}}=\frac{n^2+1}{n\sqrt{n^2-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Substituting $n=2$ in (1), we have

$$\frac{\text{Strength of hollow shaft}}{\text{Strength of solid shaft}}=\frac{2^2+1}{2\sqrt{2^2-1}}=\frac{5}{2\sqrt{3}}=\underline{1\cdot44}.$$

138.

A shaft is required to transmit 100 h.p. at 75 r.p.m. What diameter must it have in order to limit the shear stress to 8000 lb. per sq. in. if the maximum torque exceeds the mean by 30 per cent.? The modulus of rigidity is 12,000,000 lb. per sq. in. Calculate the angular twist on a length of 5 ft.

$$\text{Horse-power to be transmitted}=\frac{T\times 2\pi N}{33{,}000}$$

$$\therefore T_{\text{max.}}=\frac{33{,}000\times\text{h.p.}\times 1\cdot30\times 12}{2\pi N}$$

$$T_{\text{max.}}=\frac{33{,}000\times 100\times 1\cdot30\times 12}{2\pi\times 75}=111{,}800 \text{ lb.-in.}$$

$$\text{Torque}=\frac{\pi D^3}{16}f_s$$

$$\therefore 111{,}800=\frac{\pi D^3\times 8000}{16}$$

$$D=\sqrt[3]{\frac{111{,}800\times 16}{8000\times\pi}}=\underline{4\cdot14 \text{ in.}}$$

The angle of twist: $$\frac{T}{I_p}=\frac{C\theta}{l}$$

$$\therefore \theta=\frac{Tl}{I_pC}$$

$$=\frac{111{,}800\times 5\times 12}{\frac{\pi\times(2\cdot07)^4}{2}12\times 10^6}$$

$$\therefore \theta=0\cdot01937 \text{ radian.}$$

139.

What horse-power may be transmitted by a solid shaft 3 in. diameter, when running at 120 r.p.m.? The shear stress must not exceed 8000 lb. per sq. in. If the modulus of rigidity is 13,000,000 lb. per sq. in., what will be the angle of twist over a length of 60 ft.?

$$\text{Torque}=f_s\frac{\pi r^3}{2}=\frac{8000\pi(1\cdot5)^3}{2}=42{,}410 \text{ lb.-in.}$$

$$\text{h.p.}=\frac{2\pi N\frac{T}{12}}{33{,}000}=\frac{2\pi\times120\times42{,}410}{33{,}000\times12}=80\cdot76$$

From the torsion equation,

$$\frac{T}{I_p}=\frac{f_s}{R}=\frac{C\theta}{l}$$

C=rigidity modulus in lb./sq. in.

θ=angle of twist in radians in a length of l in.

$$\frac{f_s}{R}=\frac{C\theta}{l}$$

$$\therefore\ \theta=\frac{f_s l}{R\times C}=\frac{8000\times60\times12}{1\cdot5\times13{,}000{,}000}$$

$$\theta=\underline{0\cdot2954 \text{ radian}=16\cdot92^\circ}.$$

140.

A close-coiled helical spring is to be made of coils each 3 in. mean diameter, from wire of 0·25 in. diameter, and of such a stiffness that it will elongate axially 0·5 in. for an axial pull of 15 lb. How many coils will the spring have if the modulus of rigidity of the material is 13,000,000 lb./sq. in.?

Fig. 51 illustrates a close-coiled helical spring.

Let R=mean radius of the coils
W=axial load
δ=axial deflection
θ=angle of twist of the wire
d=diameter of the wire
n=number of coils
C=modulus of rigidity

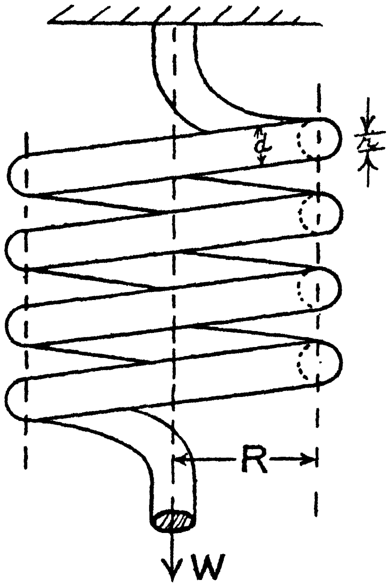


FIG. 51.

Work done by W = resilience of the spring

$$\therefore \frac{1}{2}W\delta = \frac{1}{2}T\theta$$

$$\delta = \frac{T}{W}\theta$$

But $$\theta = \frac{4WR^2n}{Cr^4}$$

$$\delta = \frac{4WR^3n}{Cr^4} = \frac{64WR^3n}{Cd^4}$$

$$\therefore n = \frac{Cd^4\delta}{64WR^3} = \frac{13\times10^6\times0{\cdot}5\times(0{\cdot}25)^4}{64\times15\times(1{\cdot}5)^3}$$

$$n = \underline{7{\cdot}8 \text{ coils.}}$$

[illegible] of the bolt circle of a shaft coupling is 12 in., and [illegible] 10 bolts each 1 in. diameter. What horse-power can be transmitted by the coupling at 100 r.p.m. if the shear stress for the bolts is not to exceed 3 tons per sq. in.?

$$\text{Area of each bolt} = \frac{\pi d^2}{4} = \frac{\pi \times 1^2}{4} = 0{\cdot}7854 \text{ sq. in.}$$

$$\text{Force on each bolt} = 3 \times 2240 \times 0{\cdot}7854 = 5277 \text{ lb.}$$

$$\text{Torque due to each bolt} = 5277 \times 12 = 63{,}324 \text{ lb.-in.}$$

$$\text{Total torque on coupling} = 63{,}324 \times 10 = 633{,}240 \text{ lb.-in.}$$

$$= 52{,}770 \text{ lb.-ft.}$$

$$\text{h.p. transmitted} = \frac{52{,}770 \times 2\pi \times 100}{33{,}000} = \underline{10{,}060 \text{ h.p.}}$$

142.

A closely-coiled spiral spring has 20 coils. The mean diameter of the coil is 3 in., and the diameter of the wire from which the spring is made is 0·25 in. Calculate the axial load that must be applied to the spring if it is to be elongated 4 in. The modulus of rigidity is 12,000,000 lb. per sq. in.

From Question 140 we have:

$$\delta = \frac{4WR^3n}{Cr^4}$$

$$\delta = 4 \text{ in.}$$

$$r = \frac{0{\cdot}25}{2} = 0{\cdot}125 \text{ in.}$$

$$R = \frac{3}{2} = 1{\cdot}5 \text{ in.}$$

$$\therefore W = \frac{Cr^4\delta}{4R^3n} = \frac{12{,}000{,}000 \times (0{\cdot}125)^4 \times 4}{4 \times (1{\cdot}5)^3 \times 20}$$

$$W = \underline{339{\cdot}3 \text{ lb.}}$$

CHAPTER XIV

143.

Turpentine boils at 159° C. and freezes at −10° C. What are these temperatures on the Fahrenheit scale?

To convert Centigrade readings into Fahrenheit readings, we have:

$$F. = \frac{9}{5}C. + 32$$

$$\therefore\ 159° \text{ C.} = 9/5 \times 159 + 32 = 318{\cdot}2° \text{ F.}$$
$$-10° \text{ C.} = 9/5 \times (-10) + 32 = 14° \text{ F.}$$

144.

The melting point of cast iron is 2192° F., and of tin 450° F. What are these temperatures on the Centigrade scale?

To convert Fahrenheit readings into Centigrade readings, we have:

$$C. = \frac{5}{9}(F. - 32)$$

$$\therefore\ 2192° \text{ F.} = 5/9(2192 - 32) = 1200° \text{ C.}$$
$$450° \text{ F.} = 5/9(450 - 32) = 232{\cdot}1° \text{ C.}$$

145.

A steam pipe is 100 ft. long at 60° F. What will be its length when full of steam at 400° F.? Take the coefficient of linear expansion of the pipe as 0·0000066.

The coefficient of linear expansion α

$$= \frac{\text{change in length}}{\text{original length} \times \text{change in temperature}}$$

or
$$\alpha = \frac{L_T - L_t}{L_t(T - t)}$$

where L_t = length at $t°$

L_T = length at higher temperature $T°$

$\therefore$ Increase in length $L_T - L_t$

$$= \alpha \times L_t(T - t)$$
$$= 0{\cdot}0000066 \times 100(400 - 60)$$
$$= 0{\cdot}0000066 \times 34{,}000$$
$$= 0{\cdot}2244 \text{ ft.}$$

$\therefore$ New length = <u>100·2244 ft.</u>

146.

The dimensions of a room are 20 ft. × 15 ft. × 12 ft. What is the weight of dry air in the room when its temperature is 20° C. and the pressure is 75 cm. of mercury? 1 cu. ft. of dry air at 0° C. and 76 cm. of mercury weighs 0·081 lb.

Volume of room = 20 × 15 × 12 = 3600 cu. ft.

By Gas Law $p_0 v_0 = RT_0$

where T = absolute temperature

$$\therefore \frac{p_0 v_0}{p_1 v_1} = \frac{T_0}{T_1}$$

where p_0 = pressure at 0° C.
v_0 = volume at 0° C.
p_1 = pressure at 20° C.
v_1 = volume at 20° C.

∴ To find the volume that the 3600 cu. ft. of air will occupy at 0° C. and 76 cm. of mercury

$$v_0 = v_1 \times \frac{p_1}{p_0} \times \frac{T_0}{T_1} = 3600 \times \frac{75}{76} \times \frac{273+0}{273+20}$$

$$= 3388 \text{ cu. ft.}$$

Weight of air = 3388 × 0·081 = <u>274·4 lb.</u>

147.

A block of metal weighing 30 lb. and at a temperature of 780° C. is to be cooled to a temperature of 60° C. by immersing it in cold water at a temperature of 10° C. What is the minimum weight of water required? Specific heat of the metal = 0·08.

Let W = weight of water in lb.

Heat lost by hot body = heat gained by cold body

$$\therefore 30 \times 0{\cdot}08(780° - 60°) = W \times (60° - 10°)$$

$$W = \frac{30 \times 0{\cdot}08 \times 720}{50} = \underline{34{\cdot}56 \text{ lb.}}$$

148.

A piece of metal weighing 57·5 gm. at a temperature of 100° C. is placed in a vessel containing 225 c.c. of water at 15° C. If the specific heat of the metal is 0·12, to what temperature will the water rise, assuming no heat lost to the vessel? Also, what is the water equivalent of the piece of metal?

Let t=temperature in ° C. to which the water will rise

Heat lost by hot body=heat gained by cold body

$$57 \cdot 5 \times 0 \cdot 12 \times (100-t) = 225(t-15)$$

$$690-6 \cdot 9t = 225t-3375$$

$$690+3375 = 225t+69t$$

$$17 \cdot 5^\circ \text{ C.} = t$$

Water equivalent=mass × sp. ht.=$57 \cdot 5 \times 0 \cdot 12$

$$= \underline{6 \cdot 9 \text{ gm.}}$$

149.

A ball of platinum weighing 100 gm. is placed in a furnace. When it has acquired the temperature of the furnace it is quickly transferred without loss of heat to a vessel containing water at 15° C. The temperature rises to 18° C. If the weight of water and water equivalent of the vessel is 500 gm., what is the temperature of the furnace? Specific heat of platinum=0·0365.

Let t=temperature of furnace in degrees Centigrade

Heat lost by hot body=heat gained by cold body

$$100 \times 0 \cdot 0365(t-18) = 500(18-15)$$

$$3 \cdot 65t - 65 \cdot 7 = 1500$$

$$t = \underline{392 \cdot 9^\circ \text{ C.}}$$

150.

The frictional force produced by a brake on the circumference of a 3-ft. diameter brake drum, rotating at 100 r.p.m., is 500 lb. What quantity of heat is produced per second?

Work done by brake per sec.

$$= \text{force} \times \text{circumference} \times \text{r.p.s.}$$

$$= 500 \times \pi \times 3 \times \frac{100}{60} = 7850 \text{ ft.-lb. per sec.}$$

Hence heat produced per sec.

$$= \frac{7850}{1400} = 5 \cdot 6 \text{ C.H.U.}$$

or

$$= \frac{7850}{778} = \underline{10 \text{ B.Th.U.}}$$

CHAPTER XIX

182.

If the mercury barometer reads 78 cm., what will be the pressure in ft. of water and lb. per sq. in.? Specific gravity of mercury=13·6. 1 cu. ft. of water weighs 62·4 lb.

$$78\times 13{\cdot}6=1060{\cdot}8 \text{ cm. of water}$$

$$\frac{1060{\cdot}8}{2{\cdot}54}=417{\cdot}6 \text{ in. of water}=\frac{417{\cdot}6}{12}=\underline{34{\cdot}8 \text{ ft. of water}}$$

$$wH=62{\cdot}4\times 34{\cdot}8=2171{\cdot}5 \text{ lb./sq. ft.}$$

$$\frac{2171{\cdot}5}{144}=\underline{15{\cdot}8 \text{ lb./sq. in.}}$$

183.

A U-tube is connected to two points in a horizontal water main in order to measure the difference in pressure at the two points. The difference in level of the mercury surfaces in the two limbs of the U-tube is found to be 10·5 in. What is this difference in pressure if the sp. gr. of mercury is 13·6?

Let the pressures at the two points be p_1 and p_2 respectively.

Then the difference in pressure in feet of water

$$=\frac{p_1-p_2}{62{\cdot}4}=h(s-1)$$

where h=difference in level of the liquid in the U-tube in feet and s=sp. gr. of the liquid.

$$\therefore \frac{p_1-p_2}{62{\cdot}4}=\frac{10{\cdot}5}{12}(13{\cdot}6-1)$$

$$p_1-p_2=\frac{62{\cdot}4\times 10{\cdot}5\times 12{\cdot}6}{12} \text{ lb./sq. ft.}$$

$$p_1-p_2=\frac{62{\cdot}4\times 10{\cdot}5\times 12{\cdot}6}{12\times 144}=\underline{4{\cdot}77 \text{ lb./sq. in.}}$$

184.

The ram of a hydraulic press is 6 in. diameter, and the diameter of the piston is 0·75 in. What load can the ram lift if the force on the piston is 30 lb.?

Let A=area of ram
a=area of piston
W=weight lifted by ram
F=force acting on piston

Since the intensity of pressure is the same throughout, we have:

$$\frac{W}{A}=\frac{F}{a}$$

$$\therefore\ W=\frac{FA}{a}=\frac{30\times\frac{\pi 6^2}{4}}{\frac{\pi(0\cdot75)^2}{4}}$$

$$\underline{W=1920 \text{ lb.}}$$

185.

The leverage of the handle of a hydraulic jack is 20 to 1. What must be the ratio of the diameter of the ram to the diameter of the piston which is worked by the handle, if a force of 30 lb. applied to the handle is to lift a load of 1 ton? Assume the efficiency of 80 per cent.

$$\text{Force on piston}=20\times30=600 \text{ lb.}$$

$$\text{Intensity of pressure in water } p=\frac{600}{\frac{\pi d^2}{4}} \text{ lb./sq. in.}$$

where d=diameter of piston in inches
Let D=diameter of ram in inches

$$\therefore\ 0\cdot8\times\frac{600}{\frac{\pi d^2}{4}}=\frac{2240}{\frac{\pi D^2}{4}}$$

$$\frac{D^2}{d^2}=\frac{2240}{0\cdot8\times600}$$

$$\therefore\ \frac{D}{d}=\sqrt{\frac{2240}{0\cdot8\times600}}=\underline{2\cdot15.}$$

186.

A pump ram has a stroke of 3 in. and a diameter of 1 in. The pump supplies water to a lift which has a ram of 6 in. diameter. The force driving the pump ram is 1600 lb. Neglecting all losses due to friction, etc., find (*a*) the weight lifted, (*b*) the work done in raising it 10 ft., and (*c*) the number of strokes made by the pump while raising the weight.

$$\text{Area of ram} = \frac{\pi d^2}{4} = \frac{\pi \times 1^2}{4} = 0{\cdot}7854 \text{ sq. in.}$$

$$\text{Area of the lift ram} = \frac{\pi d^2}{4} = \frac{3{\cdot}14 \times 6^2}{4} = 28{\cdot}27 \text{ sq. in.}$$

$$\text{Weight lifted } W = \frac{28{\cdot}27 \times 1600}{0{\cdot}7854} = \underline{57{,}578 \text{ lb.}}$$

$$\text{Work done} = 57{,}578 \times 10 = \underline{575{,}780 \text{ ft.-lb.}}$$

If N = number of strokes

$$N \times \frac{3}{12} \times 1600 = 575{,}780$$

$$N = \underline{1439{\cdot}45 \text{ strokes.}}$$

187.

A water tank is 4 ft. long, 2·5 ft. wide and 2·5 ft. deep, and is full of water. Find the total water pressure on (1) the bottom of the tank, (2) the side of the tank. Take water at 62·4 lb. per cu. ft.

Depth of c.g. of bottom below surface of water

$$= 2{\cdot}5 \text{ ft.}$$

$$\text{Pressure at depth of c.g.} = 2{\cdot}5 \times 62{\cdot}4 = 156 \text{ lb./sq. ft.}$$

(1) Total pressure = area × pressure $= 4 \times 2{\cdot}5 \times 156 = \underline{1560 \text{ lb.}}$

(2) Depth of c.g. of side below surface of water

$$= \frac{2{\cdot}5}{2} = 1{\cdot}25 \text{ ft.}$$

$$\text{Pressure at depth of c.g.} = 1{\cdot}25 \times 62{\cdot}4 = 78 \text{ lb./sq. ft.}$$

Total pressure = area × pressure $= 4 \times 2{\cdot}5 \times 78 = \underline{780 \text{ lb.}}$

188.

A water tank is 10 ft. broad in a direction perpendicular to the diagram (Fig. 53), which shows the elevation is 10 ft. deep. Find the total pressure on (1) the bottom of the tank, (2) the sloping side of the tank, and (3) the vertical end of the tank, when the tank is full of water.

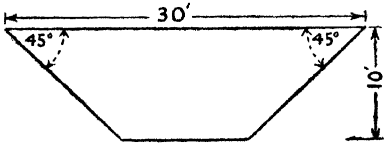


FIG. 53.

Total pressure on bottom of tank:

Depth of c.g. below surface of water

$=10$ ft.

Total pressure = area × pressure

$=10\times10\times10\times62\cdot4=\underline{62{,}400 \text{ lb.}}$

Total pressure on sloping side of tank:

Depth of c.g. below surface of water

$=\frac{10}{2}=5$ ft.

Length of sloping side $=10\sqrt{2}=14\cdot14$ ft.

Total pressure = area × pressure

$=14\cdot14\times10\times5\times62\cdot4=\underline{44{,}168 \text{ lb.}}$

Total pressure on vertical end of tank:

Rectangular area $=10\times10=100$ sq. ft.

Depth of c.g. below surface of water

$=\frac{10}{2}=5$ ft.

Total pressure = area × pressure $=100\times5\times62\cdot4$

$=\underline{31{,}200 \text{ lb.}}$

Area of two triangles $=2\times10\times10\times0\cdot5=100$ sq. ft.

Depth of c.g. below surface of water

$=\frac{10}{3}=3\cdot33$ ft.

Total pressure on these triangles = area × pressure

$=100\times3\cdot33\times62\cdot4$

$=20{,}779$ lb.

Total pressure $=20{,}779+31{,}200=\underline{51{,}979 \text{ lb.}}$

189.

A vertical flap valve in a dock wall closes the end of a 4-ft. diameter pipe. The pressure at the centre of the pipe is equal to a head of 10 ft. of water. Find (*a*) the whole pressure on the valve, and (*b*) the centre of pressure.

Total pressure on plate $= wA\bar{x}$

$$= 64 \times \frac{\pi d^2}{4} \times 10 = 64 \times \frac{\pi \times 4^2}{4} \times 10$$

$$= \underline{8038 \cdot 4 \text{ lb.}}$$

I_G = moment of inertia of figure about a horizontal axis through its centre of area

I_0 = moment of inertia of figure about the water line

$$I_0 = I_G + A\bar{x}^2$$

$$I_0 = \frac{\pi R^4}{4} + \pi R^2 \times 10^2 = 404\pi$$

If h = distance of centre of pressure from surface

$$h = \frac{I_0}{A\bar{x}} = \frac{404\pi}{\pi 2^2 \times 10} = 10 \cdot 1 \text{ ft.}$$

that is, <u>0·1 ft. below centre of valve.</u>

190.

A rectangular sluice gate 6 ft. wide by 4 ft. deep is inclined at an angle of 60° to the water surface, and its top edge is 5 ft.

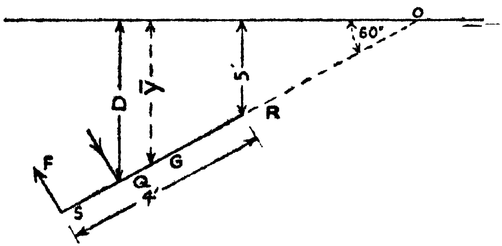


FIG. 54.

below the water level measured vertically. It is hinged along its top edge. What is the minimum force applied to its bottom edge required to open it?

Fig. 54 represents diagrammatically the sluice gate.

Let D=depth to centre of pressure
I_0=moment of inertia about water line
A=area of surface
$\bar{y}$=vertical depth to centre of area

Then
$$D=\frac{I_0 \sin^2\theta}{A\bar{y}}$$
$$\bar{y}=5+2\sin 60°=6{\cdot}732 \text{ ft.}$$
$$I_0=\frac{I_G+A\bar{y}^2}{\sin^2\theta}$$
$$=A\left(k_G^2+\frac{\bar{y}^2}{\sin^2\theta}\right)$$
$$D=A\left(k_G^2+\frac{\bar{y}^2}{\sin^2\theta}\right)\frac{\sin^2\theta}{A\bar{y}}=\frac{k_G^2\sin^2\theta}{\bar{y}}+\bar{y}$$
$$k_G^2=\frac{(SR)^2}{12}=\frac{16}{12}=1{\cdot}333 \text{ ft.}^2$$
$$D=\frac{1{\cdot}333\sin^2\theta}{6{\cdot}732}+6{\cdot}732=6{\cdot}881 \text{ ft.}$$

Let P=total force due to water on gate
F=minimum force required to lift gate

Then
$$P=wA\bar{y}=62{\cdot}4\times6\times4\times6{\cdot}732$$

Taking moments about hinge R,
$$F\times SR=P\times QR$$
$$SR=4 \text{ ft.}$$
$$QR=\frac{(D-5)}{\sin 60°}=2{\cdot}17 \text{ ft.}$$
$$4F=(62{\cdot}4\times6\times4\times6{\cdot}732)\times2{\cdot}17$$
$$F=5470 \text{ lb.}=\underline{2{\cdot}44 \text{ tons.}}$$

191.

A circuar conduit, 6 ft. diameter, which just runs full, is fitted with a sluice gate. It is required to balance this gate about a horizontal axis. Show that this axis should be 2 ft. 3 in. above the bottom of the conduit.

For the gate to balance it must be pivoted at the level of the centre of pressure.

Depth from top of conduit to centre of pressure

$$=\frac{k^2}{\bar{y}}$$

Where k=radius of gyration of gate about a horizontal axis through top of conduit

$\bar{y}$=depth of centre of area of gate below top of conduit

$$k^2=\frac{r^2}{4}+r^2$$

where r=radius of conduit

$$\bar{y}=r$$

Depth of c. of p.$=(\frac{1}{4}+1)r=1\frac{1}{4}\times 3=3{\cdot}75$ ft.

Height of pivot above bottom of conduit

$=(6-3{\cdot}75)$ ft.

$=$ <u>2·25 ft.</u>

Height of pivot above bottom of conduit

$=$ <u>2 ft. 3 in.</u>

CHAPTER XX

192.

A cruiser weighs 10,000 tons. When a load of 50 tons is shifted across the deck a distance of 30 ft., the angle of displacement of a plumb line is 2° 36′. Determine the metacentric height.

Let GM=distance from c. of g. of ship to metacentre
W=weight of ship
M=weight moved across deck
x=distance M is moved
θ=angle of displacement of plumb-line

Then $$GM=\frac{Mx}{W \tan\theta}=\frac{50\times 30}{10{,}000\times 0{\cdot}0454}=\underline{3{\cdot}3 \text{ ft.}}$$

193.

A buoy carries a beacon light and has its upper portion cylindrical 7 ft. diameter and 4 ft. deep. The lower portion,

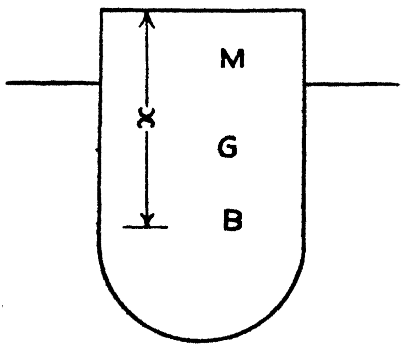


FIG. 55.

which is curved, displaces a volume of 14 cu. ft., and the centre of buoyancy of this portion is 4 ft. 3 in. below the top of the cylinder. The centre of gravity of the buoy is situated 3 ft. below the top of the cylinder, and the buoy weighs 2·6 tons. Find the metacentric height. Sea water weighs 64 lb. per cu. ft.

Let M, G and B in Fig. 55 represent respectively the metacentre, centre of gravity and centre of buoyancy of the buoy, and x=distance of B from top of cylinder.

$$\text{Volume of water displaced}=\frac{2{\cdot}6\times2240}{64}=91 \text{ cu. ft.}$$

$$\text{Volume of curved portion}=14 \text{ cu. ft.}$$

Volume of cylindrical portion of buoy immersed

$$=91-14=77 \text{ cu. ft.}$$

$$\text{Depth of cylindrical portion immersed}=\frac{77}{\pi\times(3{\cdot}5)^2}=2 \text{ ft.}$$

To find centre of buoyancy B of the whole buoy,

$$77\times3+14\times4{\cdot}25=91x$$

$$x=3{\cdot}2 \text{ ft.}$$

But $\quad$ Depth of $CG=3$ ft.

Therefore $\quad BG=3{\cdot}2-3=0{\cdot}2$ ft.

Also $\quad BM=\dfrac{I_0}{V}$

Where I_0=2nd moment of area of water line plane
V=total displacement

$$\therefore BM=\frac{\pi\times(3{\cdot}5)^4}{4\times91}=1{\cdot}3 \text{ ft.}$$

$$\therefore \text{Metacentric height } GM=BM-BG=\underline{1{\cdot}1 \text{ ft.}}$$

194.

Find the metacentric height of a rectangular vessel of 3,000 tons, of length 240 ft., beam 44 ft., and depth 20 ft. Sea water weighs 64 lb. per cu. ft.

$$\text{Underwater volume}=\frac{3000\times2240}{64}=105{,}000 \text{ cu. ft.}$$

$$\text{Draught}=\frac{3000\times2240}{64\times240\times44}=9{\cdot}96 \text{ ft.}$$

$$\text{Least moment of inertia}=I_0=\frac{240\times44^3}{12}=1{,}705{,}000 \text{ ft.}^4$$

$$BM=\frac{I_0}{V}=\frac{1{,}705{,}000}{105{,}000}=16{\cdot}2 \text{ ft.}$$

Height of centre of buoyancy B from base

$$=\frac{9\cdot96}{2}=4\cdot98 \text{ ft.}$$

$$\therefore \text{ Height of } M=4\cdot98+16\cdot2=21\cdot2 \text{ ft.}$$

$$\therefore GM=21\cdot2-10=\underline{11\cdot2 \text{ ft.}}$$

195.

A cylindrical buoy floats in the sea. It is 8 ft. in diameter and 5 ft. long, and weighs 4000 lb. The c.g. is 1·8 ft. from the bottom. If a load of 700 lb. is placed on the top of the buoy, find the height of its c.g. from the bottom so that the buoy remains in stable equilibrium. (1 cu. ft. of sea water weighs 64 lb.)

Let $\bar{x}$=required height of c.g. of weight

$$\text{Total weight of buoy}=4000+700=4700 \text{ lb.}$$

$$\text{Volume of water displaced}=V=\frac{4740}{64}=73\cdot44 \text{ cu. ft.}$$

$$\text{Depth of buoy below water line}=D=\frac{73\cdot44}{\frac{\pi}{4}8^2}=1\cdot46 \text{ ft.}$$

$$\text{Height of centre of buoyancy}=\frac{1\cdot46}{2}=0\cdot73 \text{ ft.}$$

$$\frac{I_0}{V}=\frac{\pi d^4}{64V}=\frac{\pi 8^4}{64\times73\cdot44}=2\cdot735 \text{ ft.}$$

$$\text{Height of } G_1 \text{ above bottom}=2\cdot735+0\cdot73=3\cdot465 \text{ ft.}$$

Taking moments, $\bar{x}\times700=4700\times2\cdot735-4000\times1\cdot8$

$$\bar{x}=\underline{8\cdot08 \text{ ft.}}$$

CHAPTER XXI

196.

The diameter of a pipe changes gradually from 6 in. at a point A, 20 ft. above datum, to 3 in. at B, 10 ft. above datum. The pressure at A is 15 lb./sq. in., and the velocity of flow 12 ft./sec. Neglecting losses between A and B, determine the pressure at B.

Let p_1=pressure at A
v_1=velocity at A
h_1=height above datum at A

Let p_2, v_2 and h_2 be the corresponding values at B.
Applying Bernoulli's theorem:

$$\frac{p_1}{w}+\frac{v_1^2}{2g}+h_1=\frac{p_2}{w}+\frac{v_2^2}{2g}+h_2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Since

$$a_1v_1=a_2v_2$$

$$\frac{\pi 6^2}{4}\times 12=\frac{\pi 3^2}{4}v_2$$

$$\therefore v_2=48 \text{ ft./sec.}$$

Substituting in Equation (1),

$$\frac{15\times 144}{62\cdot 4}+\frac{12^2}{64\cdot 4}+20=\frac{p_2\times 144}{62\cdot 4}+\frac{48^2}{64\cdot 4}+10$$

$$p_2=\underline{4\cdot 83 \text{ lb./sq. in.}}$$

197.

A venturi-meter having a diameter at the throat of 24 in. is inserted in a 6-ft. diameter water main. The pressure head at the throat gauge is 15 ft. of water and at the pipe gauge is 20 ft. of water. Find (*a*) the quantity of water flowing in cu. ft. per sec., and the velocity of the water through (*b*) the pipe, (*c*) the throat.

$$\text{Area of pipe}=\frac{\pi d_2}{4}=\frac{\pi\times 6^2}{4}=28\cdot 26 \text{ sq. ft.}$$

$$\text{Area of throat}=\frac{\pi d^2}{4}=\frac{\pi\times 2^2}{4}=3\cdot 14 \text{ sq. ft.}$$

Difference in pressure head=20−15=5 ft. of water

$$\text{Quantity } Q=\frac{a_1a_2}{\sqrt{a_1^2-a_2^2}}\sqrt{2g}\sqrt{H} \text{ cu. ft./sec.}$$

Where a_1=area main
a_2=area of throat
H=difference of pressure head in ft. of water
Q=quantity flowing in cu. ft./sec.

$$Q=\frac{28{\cdot}26\times3{\cdot}14}{\sqrt{28{\cdot}26^2-3{\cdot}14^2}}\sqrt{2\times32{\cdot}2}\times\sqrt{5}$$
$$=\underline{56{\cdot}7\text{ cu. ft./sec.}}$$

$$\text{Velocity in pipe}=\frac{Q}{a_1}=\frac{56{\cdot}7}{28{\cdot}26}=\underline{2\text{ ft./sec.}}$$

$$\text{Velocity in throat}=\frac{Q}{a_2}=\frac{56{\cdot}7}{3{\cdot}14}=\underline{18\text{ ft./sec.}}$$

198.

A venturi-meter having a main diameter of 8 in. and throat diameter of 2 in. has a coefficient of discharge of 0·96. If the difference in head for the two mercury limbs is 1·5 in., find (*a*) the discharge in gallons per hour, (*b*) the velocity at the throat, and (*c*) the velocity in the main.

$$\text{Area of main}=\frac{\pi d^2}{4}=\frac{\pi}{4}\times\left(\frac{8}{12}\right)^2=\frac{\pi}{9}\text{ sq. ft.}$$

$$\text{Head available}=\frac{1{\cdot}5}{12}\times12{\cdot}6\text{ ft. of water}=1{\cdot}575\text{ ft.}$$

$$Q=\frac{C_d\times A\sqrt{2gh}}{\sqrt{\left(\frac{D}{d}\right)^4-1}}$$

Where Q=quantity flowing
A=area of main
D=diameter of main
d=diameter of throat
C_d=coefficient of discharge
h=head

$$Q=\frac{0{\cdot}96\times\frac{\pi}{9}\sqrt{2\times32{\cdot}2\times1{\cdot}575}}{\sqrt{\left(\frac{8}{2}\right)^4-1}}=0{\cdot}21\text{ cu. ft./sec.}$$

$$0{\cdot}21\text{ cu. ft./sec.}=0{\cdot}21\times3600\times6{\cdot}25=\underline{4720\text{ gals./hour}}$$

$$\text{Velocity in throat} = 0{\cdot}21 \times \frac{144}{\pi} = \underline{9{\cdot}6 \text{ ft./sec.}}$$

$$\text{Velocity in main} = 0{\cdot}21 \times \frac{9}{\pi} = \underline{0{\cdot}6 \text{ ft./sec.}}$$

199.

The throat and full bore diameter of a venturi-meter are 0·75 in. and 2·25 in. respectively. Calculate the coefficient of discharge of the meter if the pressure at full bore section is 25 lb./sq. in. above that at the throat, when the meter is passing 68·45 gals./min. The meter is inclined to the horizontal, the throat section being 18 in. above the full bore section.

Let p_1=pressure at throat in lb./sq. in.
h_1=height above datum of throat in ft.
v_1=throat velocity in ft./sec.

p_2, h_2 and v_2 are the corresponding values for full bore section.
Applying Bernoulli's theorem:

$$\frac{p_1}{w} + h_1 + \frac{v_1^2}{2g} = \frac{p_2}{w} + h_2 + \frac{v_2^2}{2g} \quad \ldots\ldots \quad (1)$$

$$\text{Area of throat} \times v_1 = \text{area of full bore} \times v_2$$

$$\therefore \frac{v_1}{v_2} = \frac{\pi \times (2{\cdot}25)^2}{\pi \times (0{\cdot}75)^2} = 9$$

Substituting for v_1 in Equation (1),

$$\frac{80}{2g} v_2^2 = \frac{p_2 - p_1}{w} + h_2 - h_1$$

$$= \frac{25 \times 144}{62{\cdot}4} - 1{\cdot}5 = 56{\cdot}3$$

$$\therefore v_2 = \sqrt{\frac{56{\cdot}3 \times 2g}{80}} = 6{\cdot}7 \text{ ft./sec.}$$

$$\text{Flow} = C_d A_2 V_2 = C_d \times \frac{\pi (2{\cdot}25)^2 \times 6{\cdot}7}{4 \times 144}$$

$$= C_d \times 0{\cdot}185 \text{ cu. ft./sec.}$$

$$= C_d \times 69{\cdot}3 \text{ gals./min.}$$

Actual flow=68·45 gals./min.

$$\therefore C_d = \frac{68{\cdot}45}{69{\cdot}3} = \underline{0{\cdot}987.}$$

200.

A 42-in. pipe is deflected through 90°, the ends being anchored by tie rods at right angles to the pipe at the ends of the bend. If the pipe is delivering 63 cu. ft./sec., find the resultant force on the pipe and the load in each tie rod.

Fig. 56 represents the pipe.

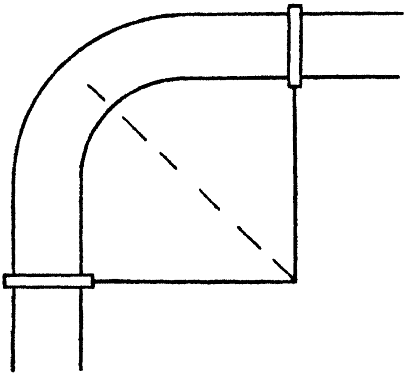


FIG. 56.

$$\text{Velocity of flow} = \frac{Q}{A} = \frac{63}{\frac{\pi}{4} \times \left(\frac{42}{12}\right)^2} = 6 \cdot 5 \text{ ft./sec.}$$

Force exerted on bend = rate of change of momentum of water

$$\text{Force exerted} = 2 \times 63 \times \frac{62 \cdot 4}{g} \times 6 \cdot 5 \sin 45°$$

$$= 1115 \text{ lb.}$$

$$\therefore \text{Force in each tie rod} = \frac{1115}{2} \times \sqrt{2} = \underline{790 \text{ lb.}}$$

201.

A jet of water has a velocity of 40 ft. per sec. If the diameter of the jet is 3 in., find the horse-power.

$$\text{Area of jet} = \frac{\pi d^2}{4} = \frac{\pi \times 3^2}{4} = 7 \cdot 07 \text{ sq. in.}$$

$$= 0 \cdot 049 \text{ sq. ft.}$$

$$\text{Horse-power} = \frac{wav^3}{2g \times 550} = \frac{62 \cdot 4 \times 0 \cdot 049 \times 40^3}{64 \cdot 4 \times 550}$$

$$= \underline{0 \cdot 553}.$$

CHAPTER XXII

202.

A jet 3 in. diameter issues from a nozzle connected to a water supply at a pressure of 400 lb./sq. in. Find (1) the velocity of the jet, (2) the flow in cu. ft./min., and (3) the horse-power available from the jet if 3 per cent. of the energy is wasted in friction.

Consider the quantities per lb. issuing:

$$\text{Volume}=\frac{1}{w}=\frac{1}{62{\cdot}4}\text{ cu. ft.}$$

$$\text{Work done}=\text{pressure lb./sq. ft.}\times\text{volume cu. ft./lb.}$$

$$=\frac{400\times 144}{62{\cdot}4}\text{ ft.-lb./lb.}=923{\cdot}1\text{ ft.-lb./lb.}$$

Deduct 3 per cent. loss $=27{\cdot}7$ ft.-lb./lb.

Kinetic energy of jet $=895{\cdot}4$ ft.-lb./lb.

$$895{\cdot}4=\frac{v^2}{2g}\quad\therefore\quad v^2=64{\cdot}4\times 895{\cdot}4$$

$$v=\underline{240{\cdot}1\text{ ft./sec.}}$$

$$\text{Flow in cu. ft./min.}=\frac{240{\cdot}1\times 60\times 0{\cdot}7854\times 9}{144\times 4}$$

$$=\underline{176{\cdot}5\text{ cu. ft./min.}}$$

Kinetic energy available per min. from jet

$$=176{\cdot}5\times 62{\cdot}4\times 895{\cdot}4$$

$$\therefore\ \text{Horse-power}=\frac{176{\cdot}5\times 62{\cdot}4\times 895{\cdot}4}{33{,}000}=\underline{299{\cdot}5}.$$

203.

What must be the diameter of a sharp-edged circular orifice in order for it to discharge 3000 gallons of water per hour under a constant head of 50 feet? Take the coefficient of discharge to be 0·62.

$$3000\text{ gals./hour}=3000\times\frac{1}{6{\cdot}24}\times\frac{1}{60^2}=0{\cdot}133\text{ cu. ft./sec.}$$

Let a = area of orifice in sq. ft.

$$Q=ka\sqrt{2gh}$$

where h=head in ft.
Q=volume of water in cu. ft. per sec.

$$0{\cdot}133=0{\cdot}62a\sqrt{2\times32{\cdot}2\times50}$$

$$a=\frac{0{\cdot}133}{0{\cdot}62\sqrt{3220}}=0{\cdot}00378 \text{ sq. ft.}$$

$$=0{\cdot}5445 \text{ sq. in.}$$

$$\text{Diameter}=\sqrt{\frac{0{\cdot}5445\times4}{\pi}}=\underline{0{\cdot}83 \text{ in.}}$$

204.

A circular orifice 1 sq. in. in area is made in the vertical side of a large tank. If the jet falls vertically through a distance of

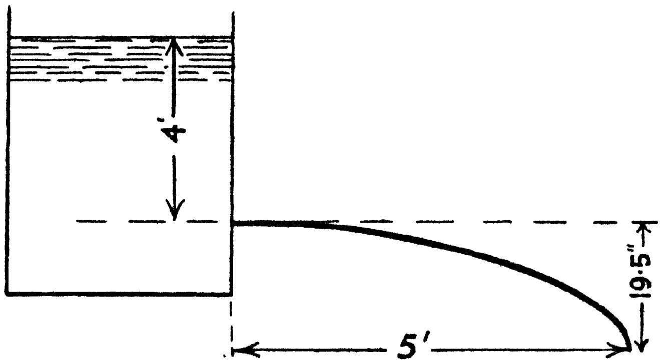


FIG. 57.

19·5 in. whilst moving horizontally through 5 ft. under a head of 4 ft., at the same time discharging 26 gals. per min., determine the three constants.

In Fig. 57,

Let x=horizontal co-ordinate from the orifice in ft.
y=vertical co-ordinate from the orifice in ft.
H=head above orifice in ft.
Cv=coefficient of velocity

$$C_v=\sqrt{\frac{x^2}{4yH}}=\sqrt{\frac{5^2}{4\times\frac{19{\cdot}5}{12}\times4}}=\sqrt{\frac{25}{26}}=\underline{0{\cdot}98}$$

Let C_d=coefficient of discharge
Q=quantity of water flowing in cu. ft./sec.
A=area of orifice in sq. ft.

$$C_d=\frac{Q}{A\sqrt{2gH}}=\frac{26\times\frac{1}{6\cdot25}\times\frac{1}{60}}{\frac{1}{144}\sqrt{2\times32\cdot2\times4}}=\underline{0\cdot623}$$

Let C_c=coefficient of contraction
Then actual velocity of jet × actual area of jet
=actual discharge

$$C_v\sqrt{2gH}\times C_cA=\text{actual discharge}$$

But $A\sqrt{2gH}$=theoretical discharge

$$\therefore C_d=C_v\times C_c$$

$$C_c=\frac{C_d}{C_v}=\frac{0\cdot623}{0\cdot98}=\underline{0\cdot635.}$$

205.

A circular orifice or area 1 sq. in. is made in the vertical side of a large tank. The tank is suspended from knife edges 5 ft. above the level of the orifice. When the head of water in the tank is 4 ft., the discharge is 261 lb./min., and a turning moment of 10·62 lb.-ft. has to be applied at the knife edges to keep the tank vertical. Determine (1) the coefficient of velocity, (2) the coefficient of contraction, and (3) the coefficient of discharge.

Let V=actual velocity of jet in ft./sec.

$$\text{Reaction of jet}=\frac{WV}{g}\text{ lb.}$$

where W=wt. issuing/sec.

$$\frac{WV}{g}=\frac{261V}{60\times g}$$

$$\text{Moment of jet about knife edge}=\frac{261V}{60g}\times5\text{ lb./ft.}$$

This must equal the applied moment:

$$\therefore 10\cdot62=\frac{261\times5V}{60\times32\cdot2}$$

$$V=\frac{60\times32\cdot2\times10\cdot62}{261\times5}=15\cdot7\text{ ft./sec.}$$

Theoretical velocity of flow $=\sqrt{2gH}$
where H = head in ft.

$$=\sqrt{2\times 32\cdot 2\times 4}=16\cdot 05 \text{ ft./sec.}$$

$$\text{Coefficient of velocity}=C_v=\frac{15\cdot 7}{16\cdot 05}=\underline{0\cdot 98}$$

$$\text{Area of jet at } \textit{vena contracta}=\frac{\text{vol. flowing/sec.}}{\text{actual velocity}}$$

$$=\frac{261}{60\times 62\cdot 4}\times\frac{144}{15\cdot 7}=0\cdot 638 \text{ sq. in.}$$

$$\text{Coefficient of contraction}=C_c=\frac{0\cdot 638}{1}=\underline{0\cdot 638}$$

$$\text{Coefficient of discharge}=C_d=C_c\times C_v=0\cdot 638\times 0\cdot 98$$
$$=\underline{0\cdot 625}$$

206.

An orifice 1·5 in. diameter and having a coefficient of discharge 0·6 is placed in the bottom of a vertically disposed cylindrical tank. Water flows into the tank at a uniform rate and is discharged through the orifice. If it takes 100 sec. for the head to rise from 2 ft. to 2 ft. 4 in., and 180 sec. for it to rise from 4 ft. to 4 ft. 4 in., find the rate of inflow.

Let A = area of tank in sq. ft.
a = area of orifice in sq. ft.
Q = rate of flow in cu. ft./sec.
C_d = coefficient of discharge

Fig. 58 represents the arrangement.

Then

$$(Q-C_d a\sqrt{2gh})dt=A\,dh$$

$$\frac{dh}{dt}=\frac{Q}{A}-C_d\frac{a}{A}\sqrt{2gh}$$

Substituting in this equation the values given:

$$(1)\ \frac{1}{3(100)}=\frac{Q}{A}-\frac{0\cdot 6}{A}\left(\frac{\pi}{4}\times\frac{2\cdot 25}{144}\right)\sqrt{2g}\sqrt{2\cdot 166}$$

$$(2)\ \frac{1}{3(180)}=\frac{Q}{A}-\frac{0\cdot 6}{A}\left(\frac{\pi}{4}\times\frac{2\cdot 25}{144}\right)\sqrt{2g}\sqrt{4\cdot 166}$$

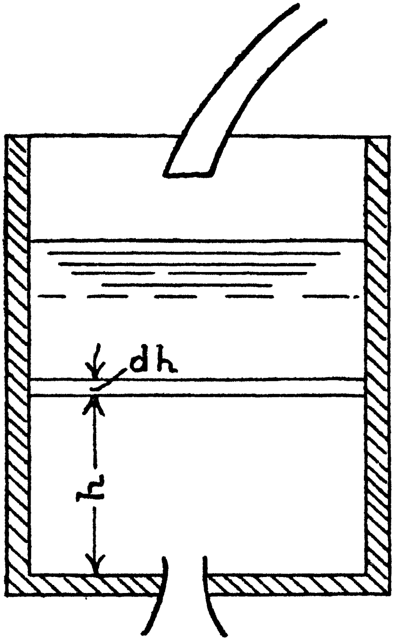


FIG. 58.

By subtraction,

$$A=\frac{6(540)}{8}\left(\frac{\pi}{4}\times\frac{2\cdot25}{144}\right)\sqrt{2g}(\sqrt{4\cdot166}-\sqrt{2\cdot166}$$

$$A=22\cdot62 \text{ sq. ft.}$$

From (1) we have:

$$Q=\frac{A}{300}+0\cdot6\left(\frac{\pi}{4}\times\frac{2\cdot25}{144}\right)\sqrt{2g}\sqrt{2\cdot166}$$

$$=0\cdot0754+0\cdot0864$$

$$=\underline{0\cdot162 \text{ cu. ft./sec.}}$$

207.

A pipe of cross-sectional area 1 sq. ft. suddenly enlarges to an area of 4 sq. ft. If the water pressure just prior to entering the enlargement is 100 lb./sq. ft., determine (*a*) the head lost due to shock, (*b*) the pressure in the enlargement, and (*c*) the work

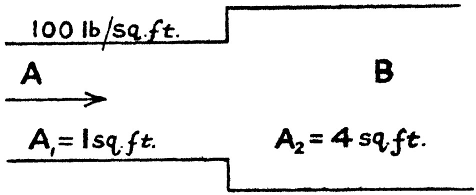


FIG. 59.

done in forcing the water through the enlargement, when the quantity of water flowing is 10 cu. ft./sec.

Fig. 59 illustrates the conditions in the pipe.

$$\text{Velocity at } A=\frac{Q}{A_1}=\frac{10}{1}=10 \text{ ft./sec.}$$

$$\text{Velocity at } B=\frac{Q}{A_2}=\frac{10}{4}=2{\cdot}5 \text{ ft./sec.}$$

$$\text{Loss of head due to shock}=\frac{(V_A-V_B)^2}{2g}=\frac{(10-2{\cdot}5)^2}{2g}=\underline{0{\cdot}875 \text{ ft.}}$$

To find the pressure in the enlargement, we have, applying Bernoulli's equation to points A and B:

$$\frac{p_A}{w}+\frac{V_A{}^2}{2g}=\frac{p_B}{w}+\frac{V_B{}^2}{2g}+0{\cdot}875$$

$$\frac{100}{62{\cdot}4}+\frac{10^2}{64{\cdot}4}=\frac{p_B}{62{\cdot}4}+\frac{(2{\cdot}5)^2}{64{\cdot}4}+0{\cdot}875$$

$$p_B=\underline{136{\cdot}6 \text{ lb./sq. ft.}}$$

Work done in forcing water through enlargement

$$=10\times 62{\cdot}4\times 0{\cdot}875=\underline{545 \text{ ft.-lb./sec.}}$$

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(Numbering refers to Problems)